

Updates on Tripartite Entanglement

$$e^+e^- \rightarrow \tau^+\tau^-\gamma$$

Tomography for a bipartite system

$$\langle \cos\theta_n^{(\tau^-\pi^-)} \rangle$$

$$\langle \cos\theta_r^{(\tau^-\pi^-)} \rangle$$

$$\langle \cos\theta_k^{(\tau^-\pi^-)} \rangle$$

$$\langle \cos\theta_n^{(\tau^+\pi^+)} \rangle$$

$$\langle \cos\theta_r^{(\tau^+\pi^+)} \rangle$$

$$\langle \cos\theta_k^{(\tau^+\pi^+)} \rangle$$

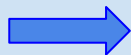
Possible
because the τ
decays weakly
(self-analyzing)

How can we do photon polarimetry? **With conversion pairs!**

$$\gamma \rightarrow e^+e^-$$

arxiv.org/abs/2607.08816v1

arxiv.org/pdf/hep-ph/0005069



$$B_n = 2 \langle \cos(2\phi) \rangle$$

where

$$B_r = 2 \langle \sin(2\phi) \rangle$$

$$\phi = \arctan\left(\frac{\vec{p}_{e^-} \cdot \hat{r}_\gamma}{\vec{p}_{e^-} \cdot \hat{n}_\gamma}\right)$$

Basis used:

$$\hat{k}_\gamma = \frac{\vec{p}_\gamma}{|\vec{p}_\gamma|} \quad \hat{r}_\gamma = \frac{\hat{p} - \cos\Theta_\gamma \hat{k}_\gamma}{\sin\Theta_\gamma} \quad \hat{n}_\gamma = \frac{\hat{p} \times \hat{k}_\gamma}{\sin\Theta_\gamma} \quad \cos\Theta_\gamma = \hat{p} \cdot \hat{k}_\gamma \quad \hat{p} = (0, 0, 1)$$

What happens with B_k ?

B_k measures the circular polarization $\longrightarrow B_k^\gamma = \frac{|A_R|^2 - |A_L|^2}{|A_R|^2 + |A_L|^2}$

Photon helicity is invisible in e^+e^- kinematics (QED parity conservation)

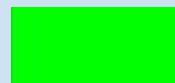
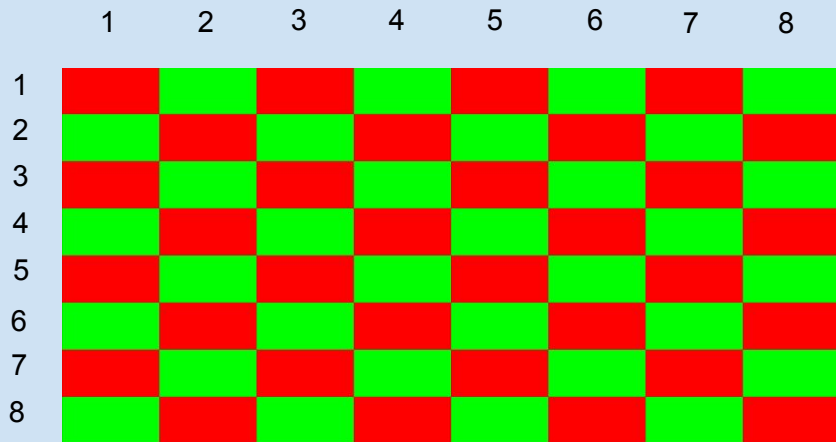
We cannot do a complete tripartite entanglement analysis at colliders!!



Which terms of the spin density matrix can we access?

Spin density matrix we can measure with kinematics

$$\rho = \frac{1}{8} \left[\mathbb{I} \otimes \mathbb{I} \otimes \mathbb{I} + \sum_i B_i^{(1)} \sigma_i^{(1)} \otimes \mathbb{I} \otimes \mathbb{I} + \sum_j B_j^{(2)} \mathbb{I} \otimes \sigma_j^{(2)} \otimes \mathbb{I} + \sum_\ell B_\ell^{(3)} \mathbb{I} \otimes \mathbb{I} \otimes \sigma_\ell^{(3)} \right. \\ \left. + \sum_{i,j} C_{ij}^{(12)} \sigma_i^{(1)} \otimes \sigma_j^{(2)} \otimes \mathbb{I} + \sum_{i,\ell} C_{i\ell}^{(13)} \sigma_i^{(1)} \otimes \mathbb{I} \otimes \sigma_\ell^{(3)} + \sum_{j,\ell} C_{j\ell}^{(23)} \mathbb{I} \otimes \sigma_j^{(2)} \otimes \sigma_\ell^{(3)} \right. \\ \left. + \sum_{i,j,\ell} C_{ij\ell}^{(123)} \sigma_i^{(1)} \otimes \sigma_j^{(2)} \otimes \sigma_\ell^{(3)} \right].$$



32/64 accessible components



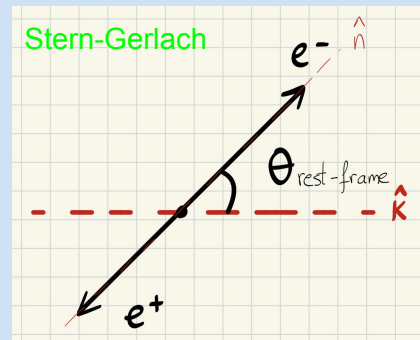
32/64 inaccessible components

What can we do now?

There are three open questions I want to discuss:

- How much does the circular component contribute to the spin density matrix?
Could it be neglected, allowing us to do the tomography?
- Can we build a quantum witness using the available coefficients, following the idea of D_n , that can tell us if there is any tripartite entanglement?
- MadGraph gives us access to the electron's helicities of the conversion pair.
Can we build a truth MC observable with this information?

I am searching for the observable $B_k \propto \langle h \cos \theta_{\gamma-rest}^{(e^- \gamma)} \rangle$



Request for theoretical values

In order to check that I am doing the calculation of the coefficients correctly, it would be nice to have the theoretical values of:

$$B_i^{(\tau^+, \tau^-, \gamma)} \quad C_{ij}^{(\tau^+, \tau^-, \gamma)} \quad C_{ijk}^{(\tau^+, \tau^-, \gamma)}$$

In this basis (same helicity basis $\{n, r, k\}$ for τ^- , τ^+ and γ):

$$\hat{k}_\gamma = \frac{\vec{p}_\gamma}{|\vec{p}_\gamma|} \quad \hat{r}_\gamma = \frac{\hat{p} - \cos \Theta_\gamma \hat{k}_\gamma}{\sin \Theta_\gamma} \quad \hat{n}_\gamma = \frac{\hat{p} \times \hat{k}_\gamma}{\sin \Theta_\gamma} \quad \cos \Theta_\gamma = \hat{p} \cdot \hat{k}_\gamma \quad \hat{p} = (0, 0, 1)$$