# PhD33:
# Resolution analysis ATF May26

Laura Karina Pedraza, Nuria Fuster, Daniel Esperante, Benito Gimeno, Marça Boronat

PhD Journal, 20/07/2026

laura.pedraza@ific.uv.es

# PhD33: Resolution analysis ATF May26

PhD Journal, 20/07/26

INSTITUT DE FÍSICA CORPUSCULAR

I. Two SVD methods:
   A) Pseudo-inverse for position estimation
   B) SVD analysis for beam coherent motion removal
   C) SVD + Pseudo-Inverse

II. Estimation of the resolution error
   A) Artificial noise on de-noised matrix
   B) Pseudo-experiments on pseudo-inverse method

# I. Two SVD methods

## A) Pseudo-inverse for position estimation

Taking a long set of data in a unique beam position:

$k$ : refers to the BPM under test

$P = 1000$ is the number of pulses

$M = 19$ is the number of stripline BPMs used

Resolution measurements: Model Independent Analysis $\quad \mathbf{d_k} = \mathbf{D}\cancel{\mathbf{k}} \cdot \mathbf{v}$

cBPM measured values

$$\begin{pmatrix} d_{k0} \\ d_{k1} \\ \vdots \\ d_{kP} \end{pmatrix} = \begin{pmatrix} d_{00} & d_{10} & \dots & d_{i\neq k,0} & \dots & d_{M0} \\ d_{01} & d_{11} & \dots & d_{i\neq k,1} & \dots & d_{M1} \\ \vdots & \vdots & & \vdots & & \vdots \\ d_{0P} & d_{1P} & \dots & d_{i\neq k,P} & \dots & d_{MP} \end{pmatrix} \cdot \begin{pmatrix} v_0 \\ v_1 \\ \vdots \\ v_M \end{pmatrix}$$

correlation coefficients

stripline BPM measured values

To find the correlation coefficients: $\mathbf{v} = \mathbf{D}_{\cancel{k}}{}^{-1}\mathbf{d}_k.$ where the inverse of $D_k$ is calculated using the SVD method: $SVD(\mathbf{D}_k) = \mathbf{USV^T},$

$$D\cancel{k}^{-1} = V^{TT} S^{-1} U^T$$

$$= V \begin{pmatrix} 1/\sigma_1 & & \\ & \ddots & \\ & & 1/\sigma_M \end{pmatrix} U^T$$

The residuals between measured position and estimated position are calculated as $\mathbf{R_k} = \mathbf{d_k} - \mathbf{D}_{\cancel{k}} \cdot \mathbf{v}.$

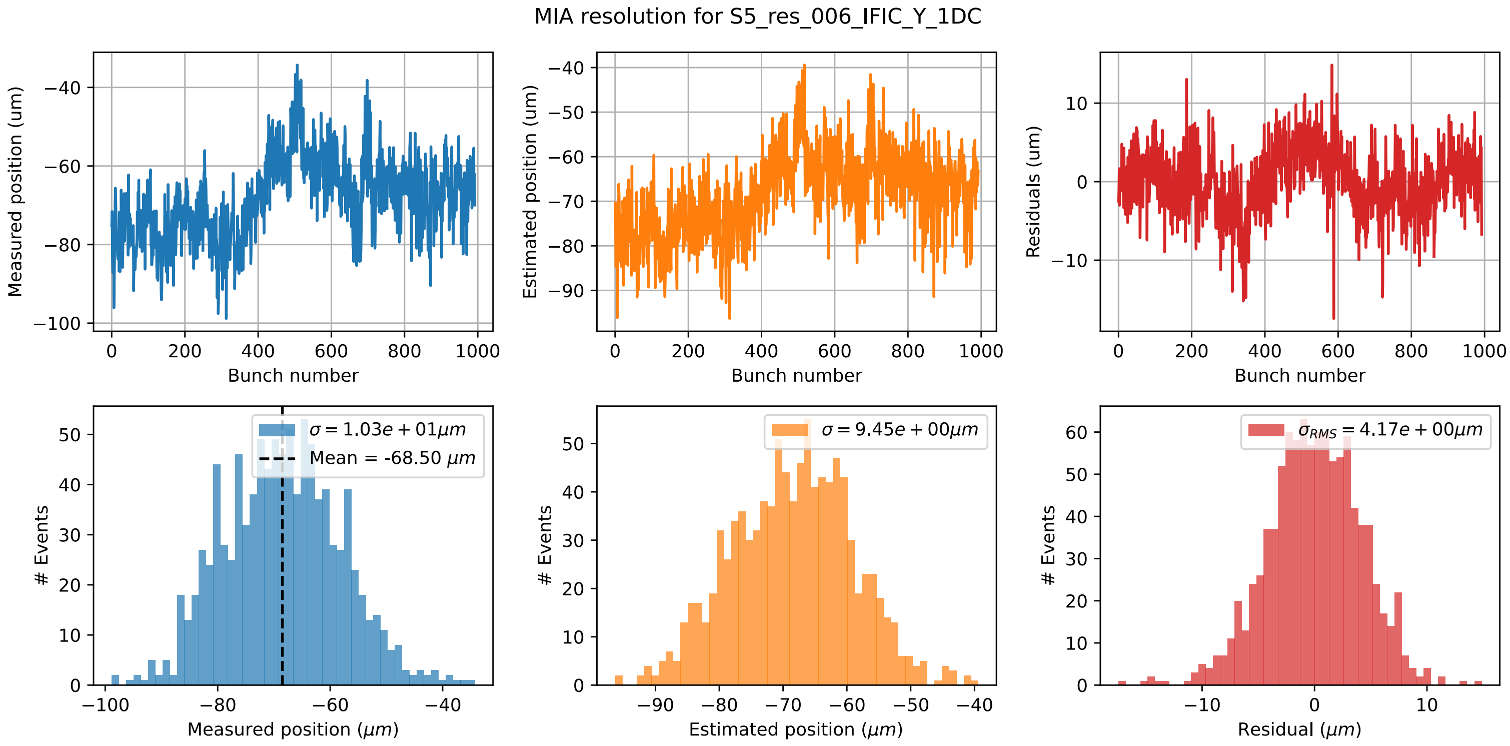
And the resolution is estimated do be the rms of the residuals:

$$\sigma_k = \sqrt{\frac{\Sigma_i^P \; R_{ki}^2}{P}}$$

**BPMs taken:** up-stream: ML1L .... ML15L, ML1P, ML2P, ML3P ; down-stream: ML4P

# I. Two SVD methods

## A) Pseudo-inverse for position estimation

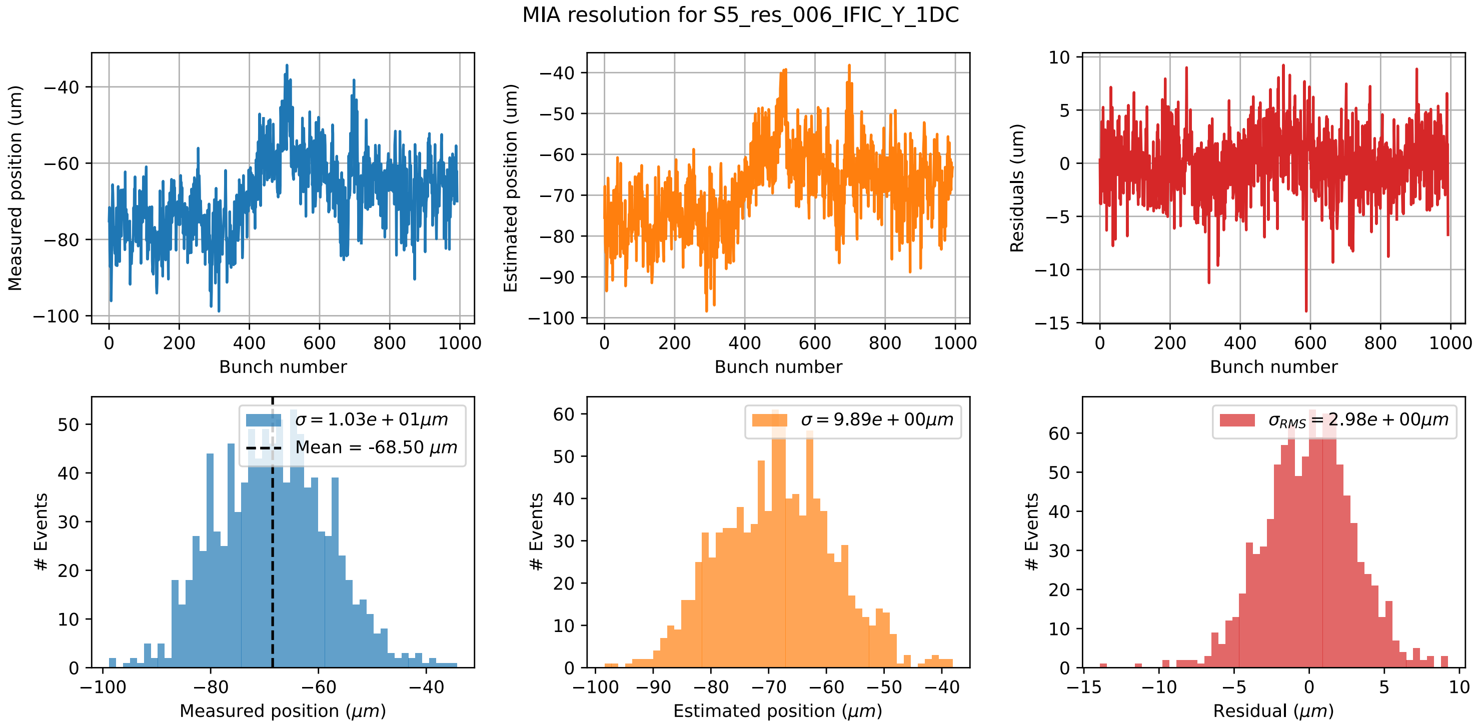


# I. Two SVD methods

With **only striplines** for position estimation: 4.26 μm

## A) Pseudo-inverse for position estimation

With **Saclay** cBPM for position estimation:



# I. Two SVD methods

## B) SVD analysis for beam coherent motion removal

Taking a long set of data in a unique beam position:

$k$ : refers to the BPM under test
$P = 1000$ is the number of pulses
$M = 19$ is the number of stripline BPMs used

$$D_k = \begin{pmatrix} d_{00} & d_{10} & \dots & d_{k0} & \dots & d_{M0} \\ d_{01} & d_{11} & \dots & d_{k1} & \dots & d_{M1} \\ \vdots & \vdots & & \vdots & & \vdots \\ d_{0P} & d_{1P} & \dots & d_{kP} & \dots & d_{MP} \end{pmatrix} = U\Sigma V^T = \left[\ \mathbf{u_1} \ \ |\dots|\ \ \mathbf{u_M}\ \right] \begin{bmatrix} \sigma_1 & & \\ & \ddots & \\ & & \sigma_M \end{bmatrix} \left[\ \mathbf{v_1} \ \ |\dots|\ \ \mathbf{v_P}\ \right]^T$$

$u_i$ **= beam spatial vectors**
columns of $U$ describe a behaviour pattern along space

every vector $u_i$ describes the pattern of motion of mode $i$ along the $M$ BPMs

$\sigma_i$ **= singular values**
represent the strength of the different modes of the beam motion

$\sigma_i$ are arranged from higher to lower so the dominant modes are found first

\# modes $\leq$ min$(M, P)$

$v_i$ **= beam temporal vectors**
columns of $V$ describe a behaviour pattern along time

every vector $v_i$ describes the pattern of motion of mode $i$ along the $P$ pulses

# I. Two SVD methods

## B) SVD analysis for beam coherent motion removal

Taking a long set of data in a unique beam position:

$k$ : refers to the BPM under test

$P = 1000$ is the number of pulses

$M = 19$ is the number of stripline BPMs used

Reconstruction of $D_k$ as:

$$\tilde{D}_k = U\tilde{\Sigma}V^T = \left[\ \mathbf{u_1}\ \ |\ldots|\ \ \mathbf{u_M}\ \right] \begin{bmatrix} \tilde{\sigma}_1 & & \\ & \ddots & \\ & & \tilde{\sigma}_M \end{bmatrix} \left[\ \mathbf{v_1}\ \ |\ldots|\ \ \mathbf{v_P}\ \right]^T$$

A threshold singular value $\sigma_t$ is defined such as $\sigma_1 \geq \sigma_t \geq \sigma_M$ where singular values above $\sigma_t$ are attributed to coherent beam motion and singular values under $\sigma_t$ represent the noise

**Beam coherent motion**

setting all singular values $\tilde{\sigma} < \sigma_t$ to zero

Allows the reconstruction of $\tilde{D}_k$ keeping only the information of the beam jitter and betatron oscillations observed along all BPMs

$$\tilde{D}_{k,dominant} = U \begin{bmatrix} \tilde{\sigma}_1 & & & & \\ & \ddots & & & \\ & & \tilde{\sigma}_t & & \\ & & & \ddots & \\ & & & & 0 \end{bmatrix} V^T$$

**Noise of the devices**

setting all singular values $\tilde{\sigma} \geq \sigma_t$ to zero

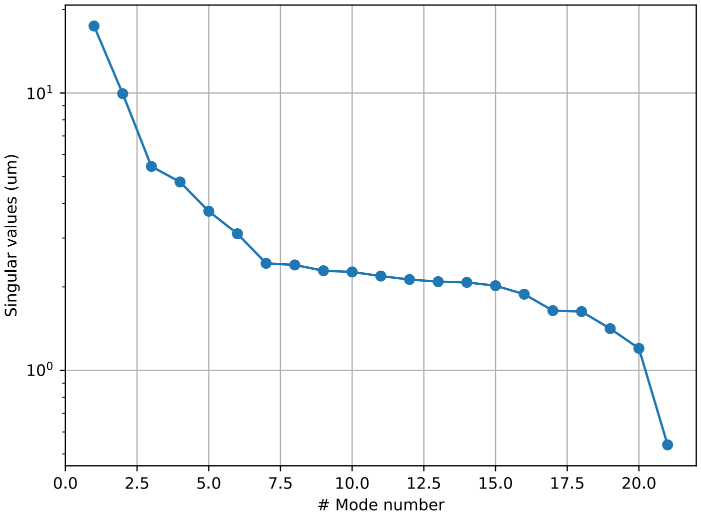
Allows the reconstruction of $\tilde{D}_k$ keeping only the information of the measurements noise, individual for each BPMs

$$\tilde{D}_{k,noise} = U \begin{bmatrix} 0 & & & & \\ & \ddots & & & \\ & & 0 & & \\ & & & \ddots & \\ & & & & \tilde{\sigma}_M \end{bmatrix} V^T$$

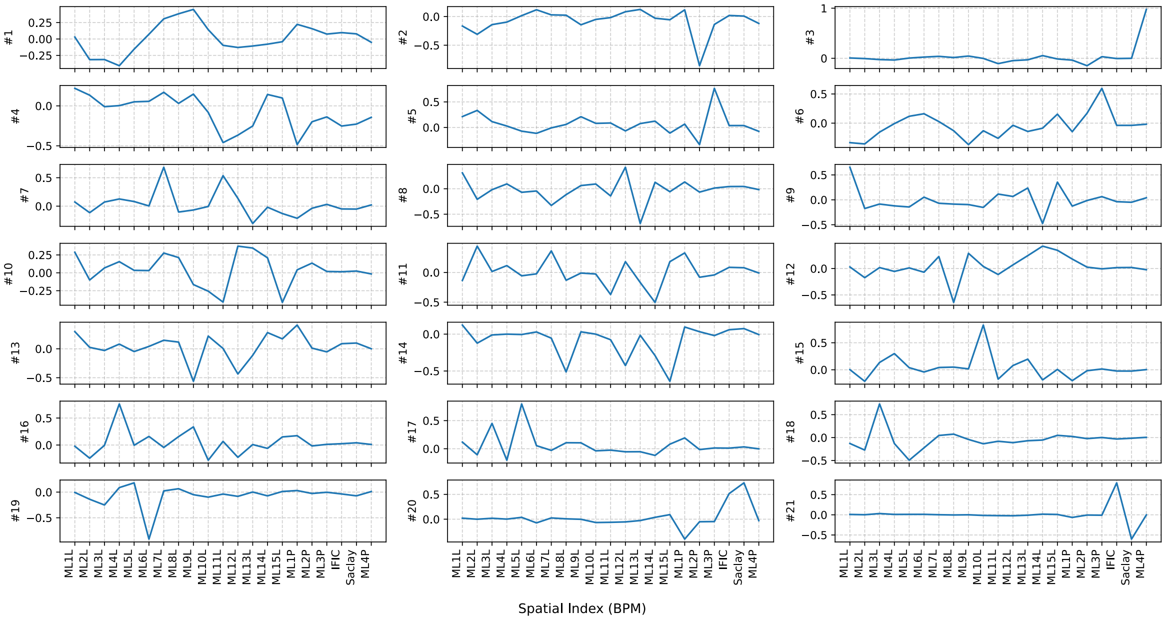
# I. Two SVD methods

## B) SVD analysis for beam coherent motion removal

**Singular values**

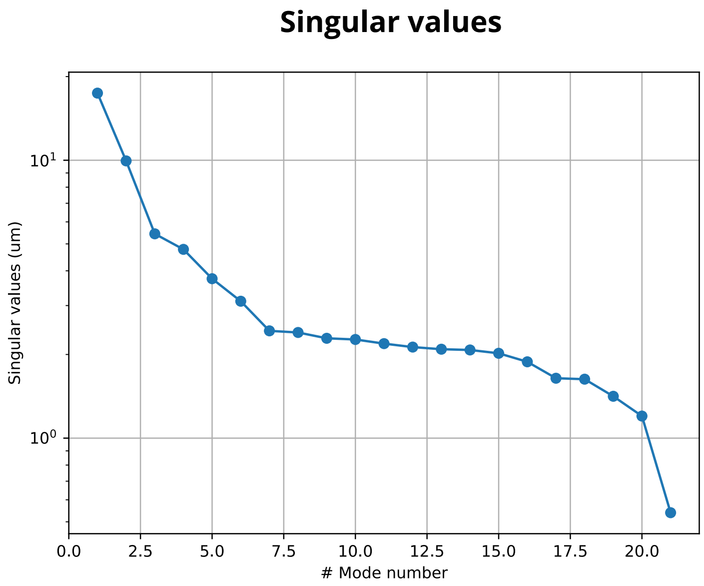


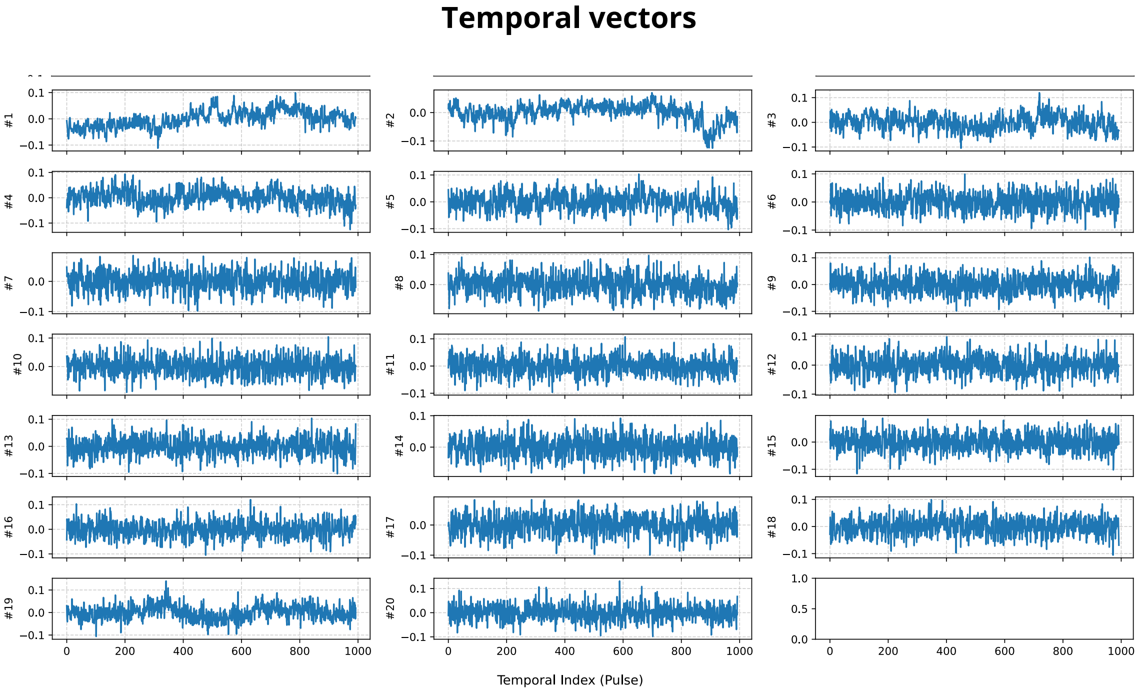

**Spatial vectors**



# I. Two SVD methods

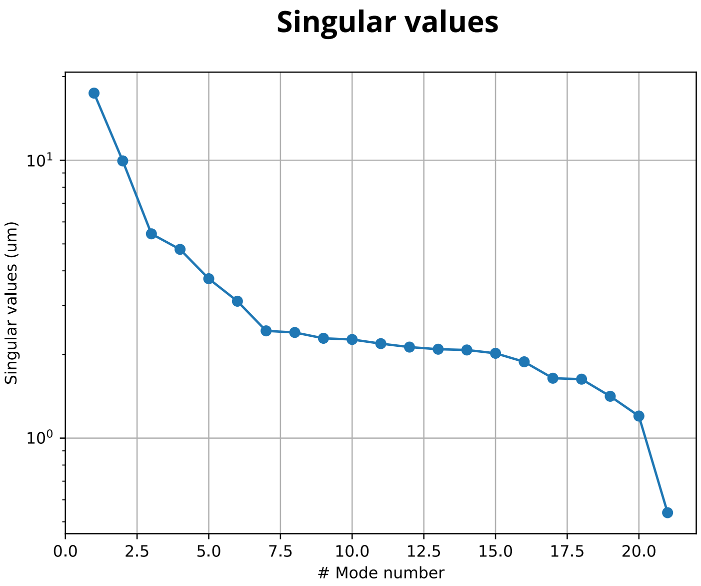
## B) SVD analysis for beam coherent motion removal

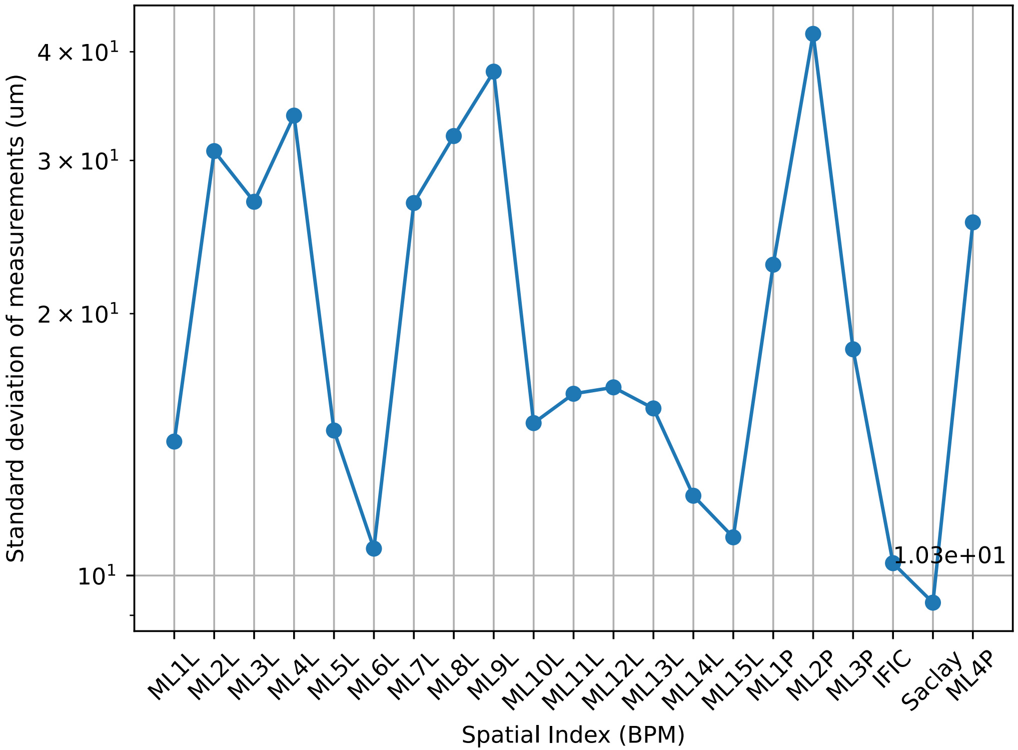


# I. Two SVD methods

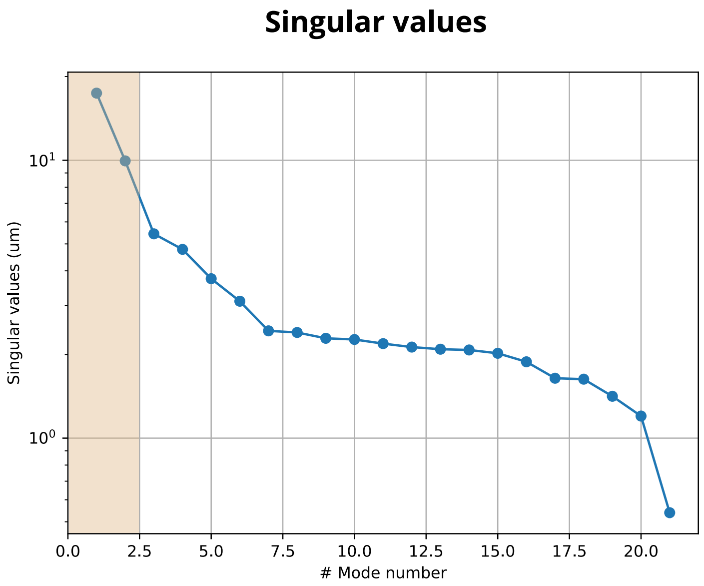
## B) SVD analysis for beam coherent motion removal

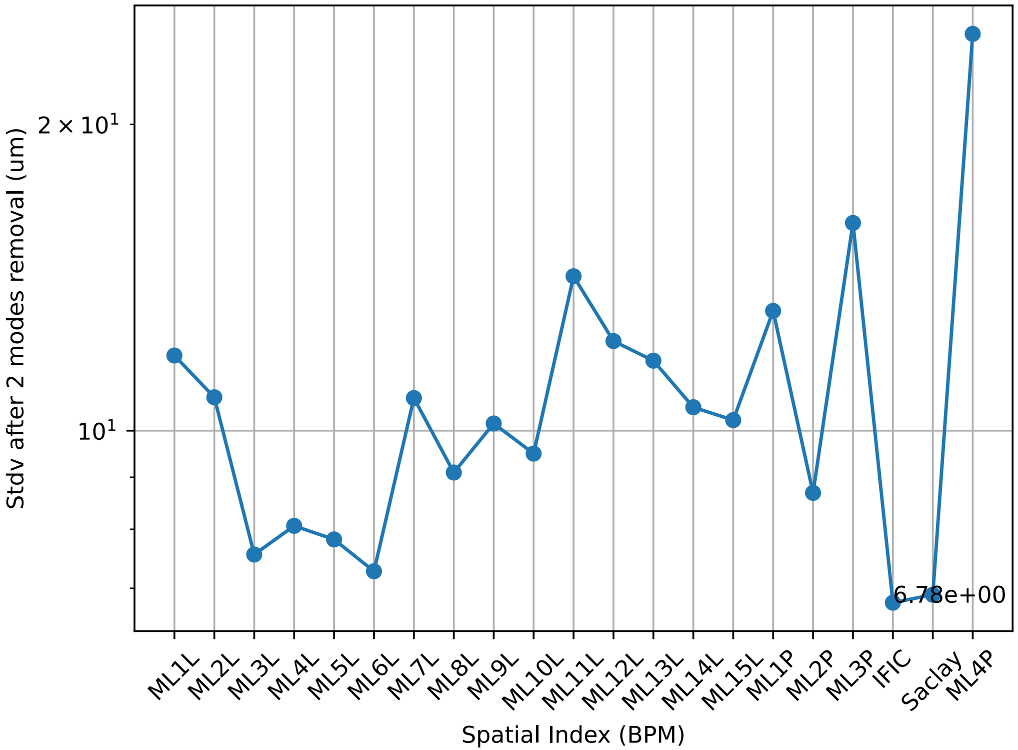


# I. Two SVD methods

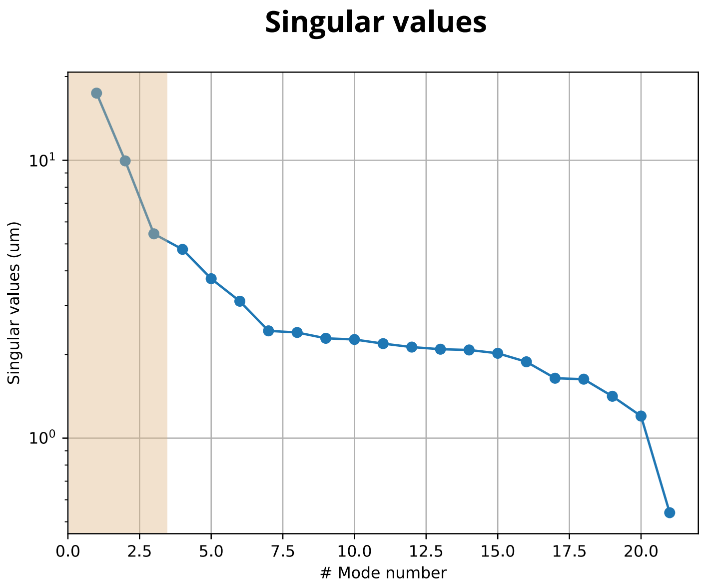
## B) SVD analysis for beam coherent motion removal

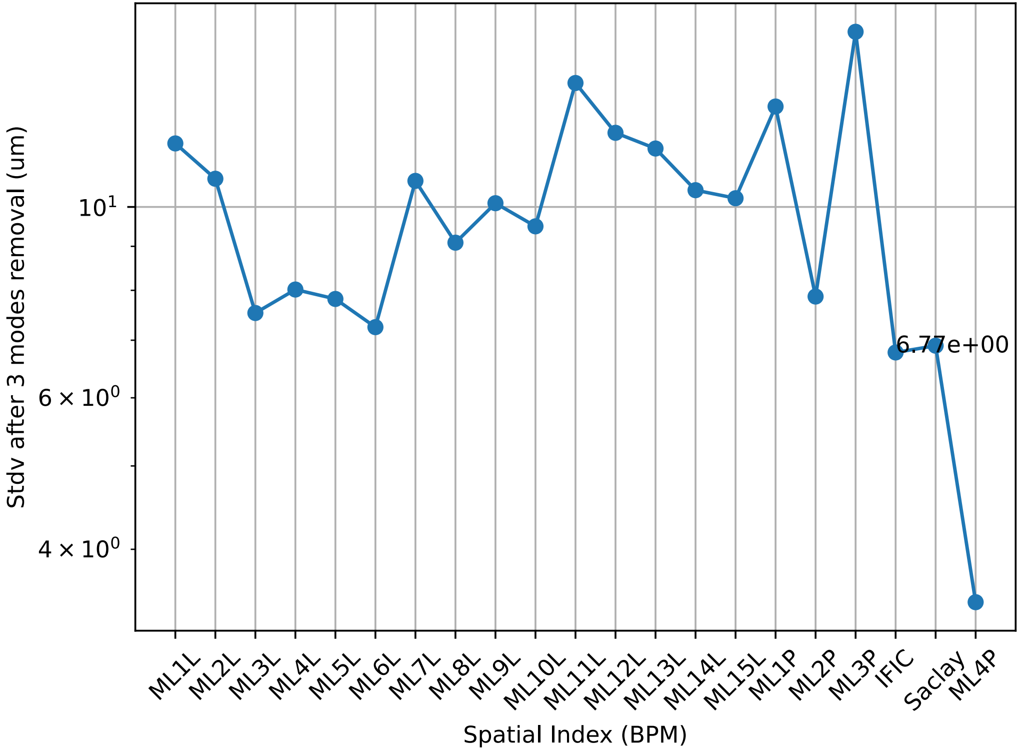


# I. Two SVD methods

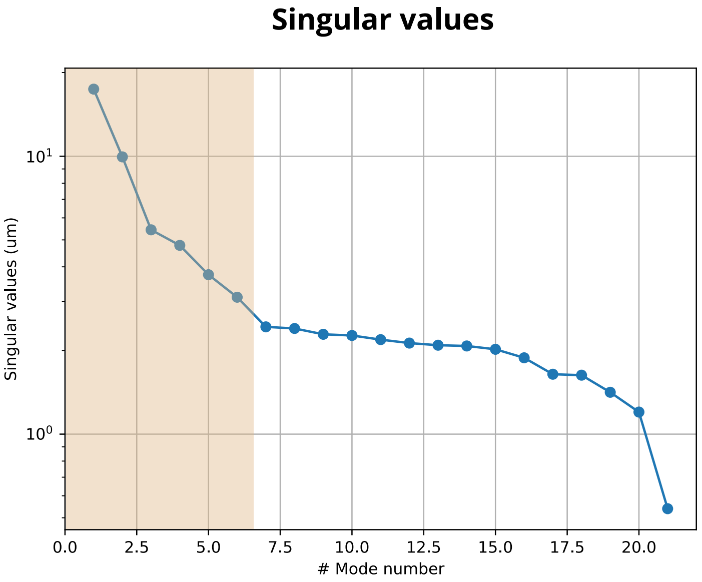
## B) SVD analysis for beam coherent motion removal

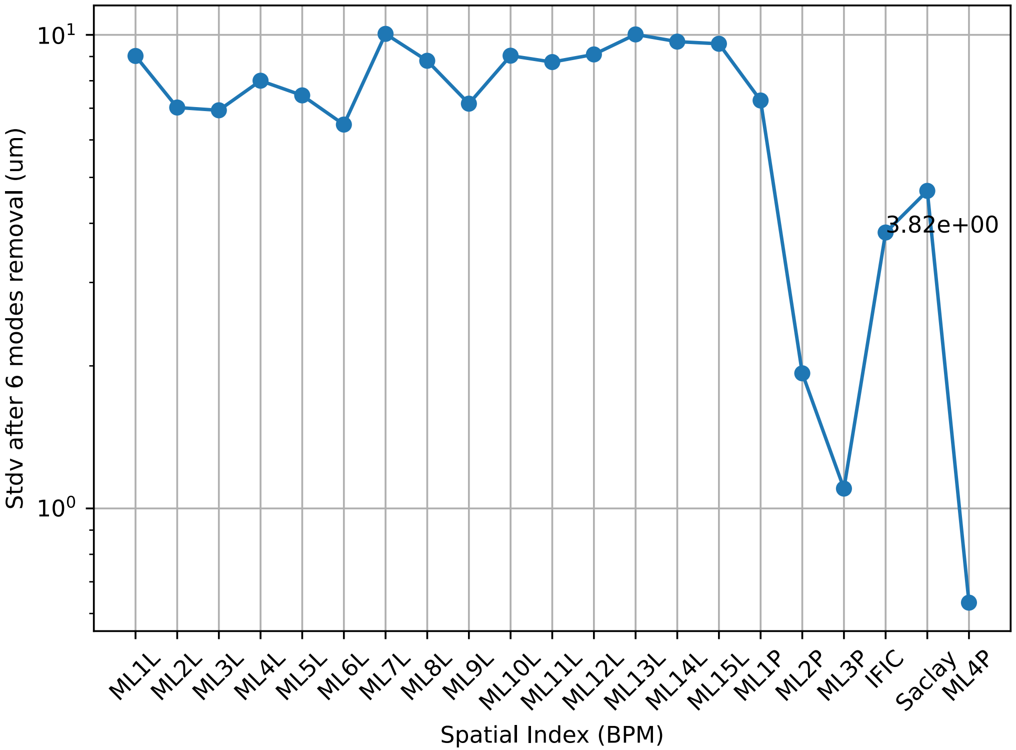


# I. Two SVD methods

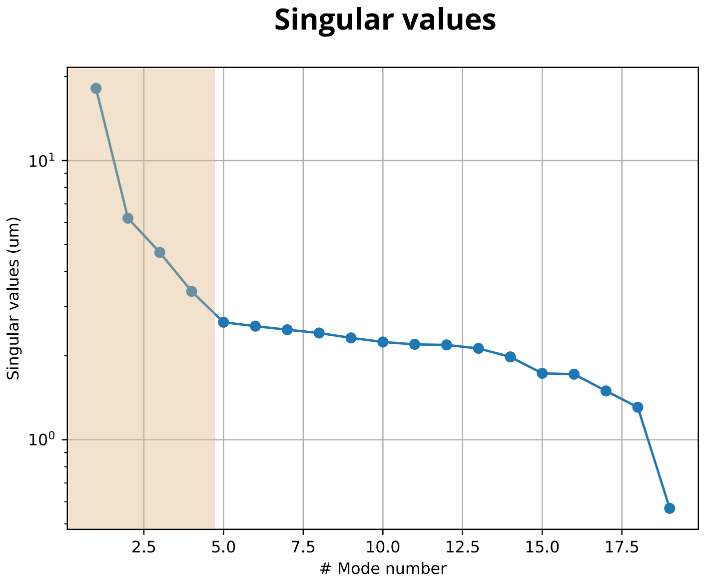
## B) SVD analysis for beam coherent motion removal

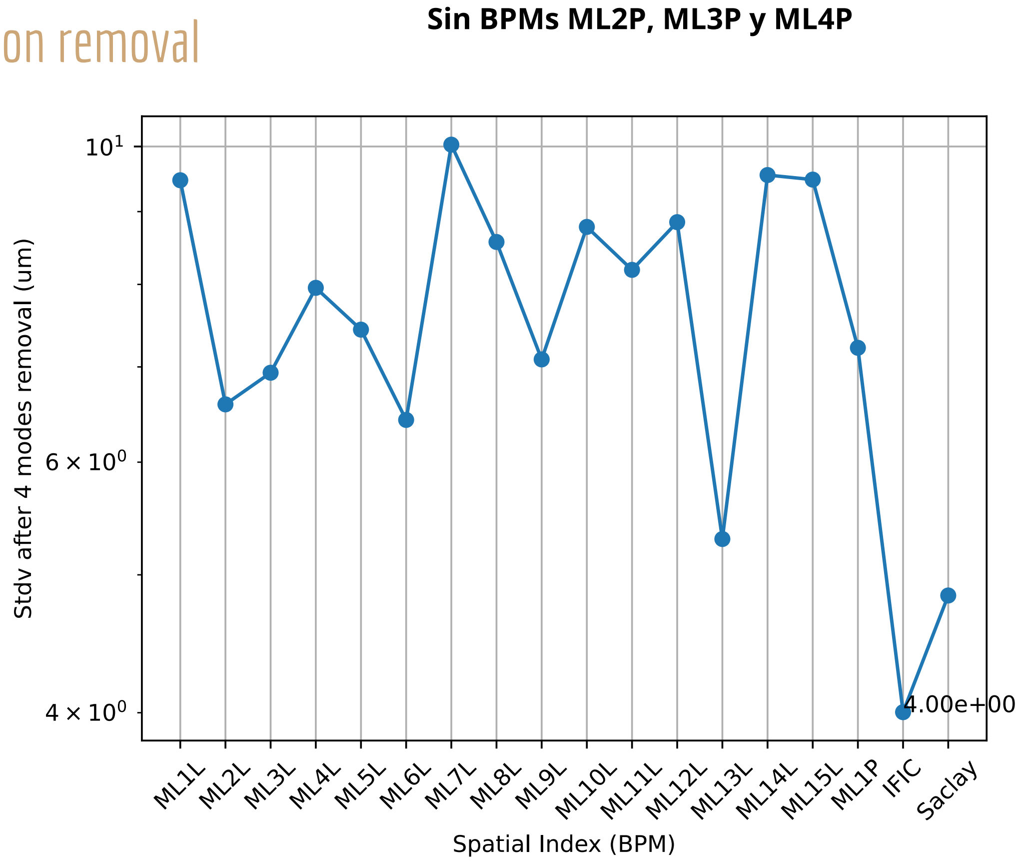


# I. Two SVD methods

## B) SVD analysis for beam coherent motion removal

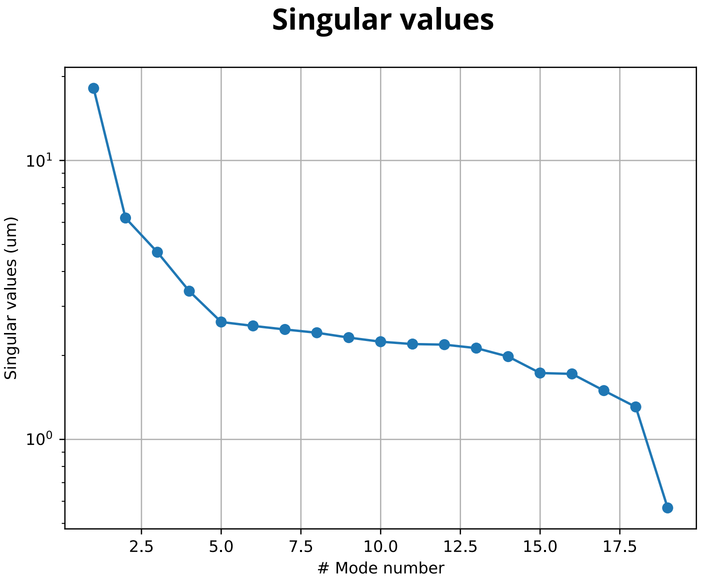


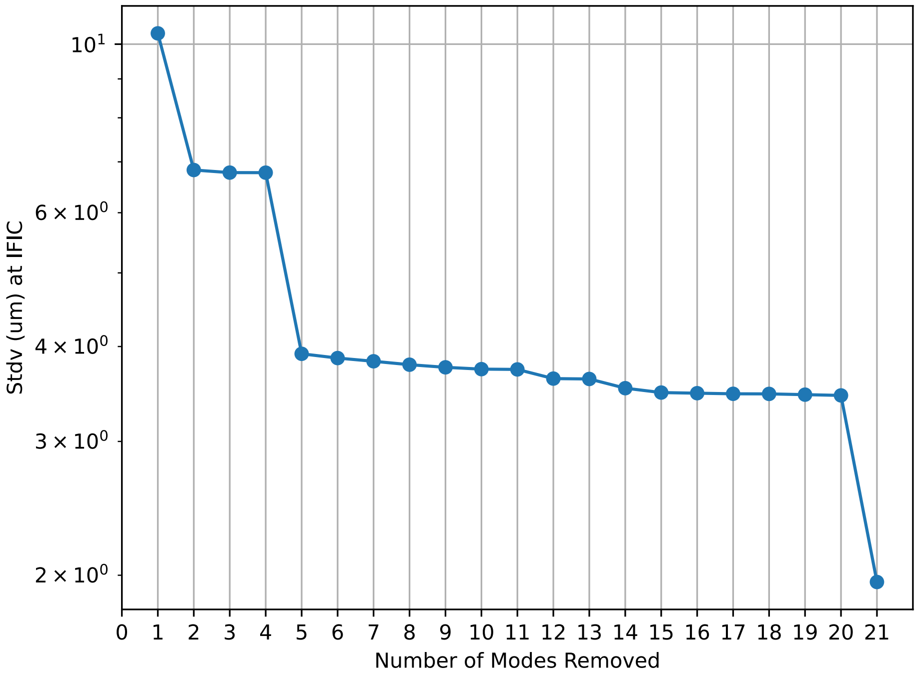



# I. Two SVD methods

## B) SVD analysis for beam coherent motion removal

**Resolution on IFIC BPM as a function of the modes removed:**



# I. Two SVD methods

## C) SVD analysis + Pseudo-Inverse

Taking a long set of data in a unique beam position:

$k$ : refers to the BPM under test

$P = 1000$ is the number of pulses

$M = 19$ is the number of stripline BPMs used

$$\mathbf{d_k} = \mathbf{D}\cancel{\mathbf{k}} \cdot \mathbf{v}$$

cBPM measured values →

$$\begin{pmatrix} d_{k0} \\ d_{k1} \\ \vdots \\ d_{kP} \end{pmatrix} = \begin{pmatrix} d_{00} & d_{10} & \dots & d_{i\neq k,0} & \dots & d_{M0} \\ d_{01} & d_{11} & \dots & d_{i\neq k,1} & \dots & d_{M1} \\ \vdots & \vdots & & \vdots & & \vdots \\ d_{0P} & d_{1P} & \dots & d_{i\neq k,P} & \dots & d_{MP} \end{pmatrix} \cdot \begin{pmatrix} v_0 \\ v_1 \\ \vdots \\ v_M \end{pmatrix}$$

← correlation coefficients

↑ stripline BPM measured values

To find the correlation coefficients: $\tilde{\mathbf{v}} = \tilde{\mathbf{D}}\cancel{\mathbf{k}}^{-1}\mathbf{d_k}$ where the inverse of $D_k$ is calculated using the SVD method: $SVD(\mathbf{D}_k) = \mathbf{USV^T}$,
The non dominant modes are removed when calculating the pseudo-inverse.

$$\tilde{D}\cancel{k}_{dominant} = U\tilde{\Sigma}V^T = \left[\ \mathbf{u_1}\ \ |\dots|\ \ \mathbf{u_M}\ \right] \begin{bmatrix} \tilde{\sigma}_1 & & & \\ & \ddots & & \\ & & \tilde{\sigma}_t & \\ & & & \ddots \\ & & & & 0 \end{bmatrix} \left[\ \mathbf{v_1}\ \ |\dots|\ \ \mathbf{v_P}\ \right]^T \Rightarrow \tilde{D}\cancel{k}^{-1}_{dominant} = V \begin{bmatrix} 1/\tilde{\sigma}_1 & & & \\ & \ddots & & \\ & & 1/\tilde{\sigma}_t & \\ & & & \ddots \\ & & & & 0 \end{bmatrix} U^T$$

The residuals between measured position and estimated position are calculated as $\mathbf{R_k} = \mathbf{d_k} - \tilde{\mathbf{D}}\cancel{\mathbf{k}} \cdot \tilde{\mathbf{v}}$

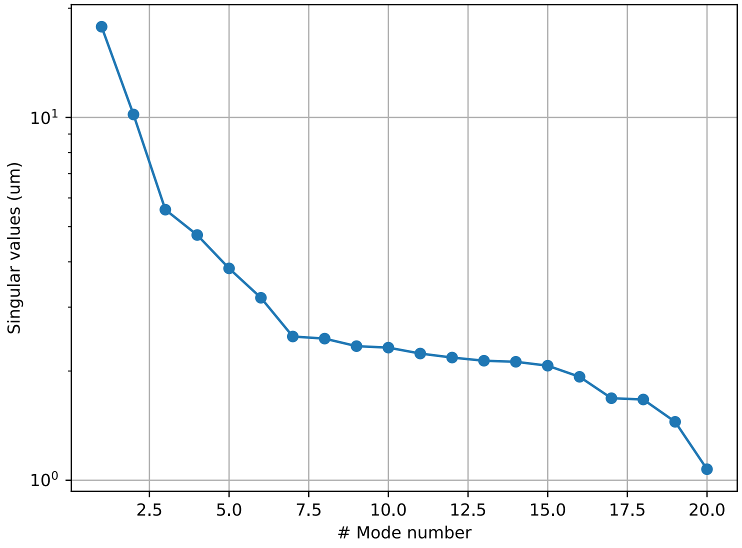
And the resolution is estimated do be the rms of the residuals:

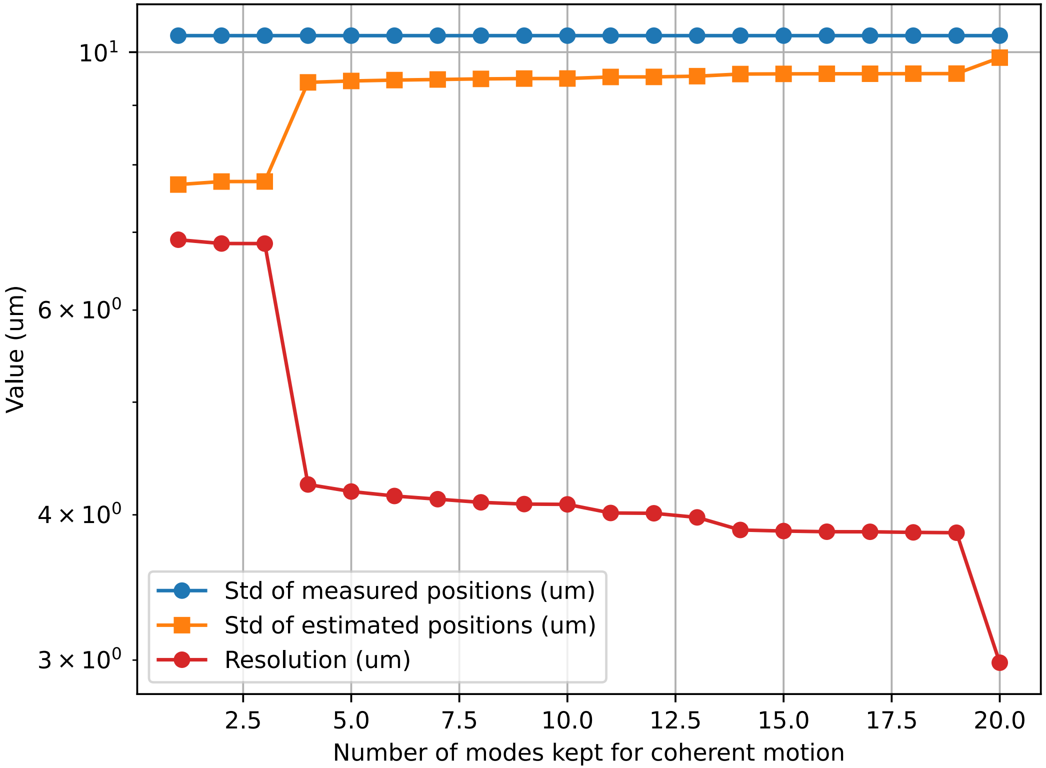
$$\sigma_k = \sqrt{\frac{\sum_i^P \ R_{ki}^2}{P}}$$

# I. Two SVD methods

## C) SVD analysis + Pseudo-Inverse

Taking a long set of data in a unique beam position:



PhD33:
Resolution analysis
ATF May26

PhD Journal, 20/07/26

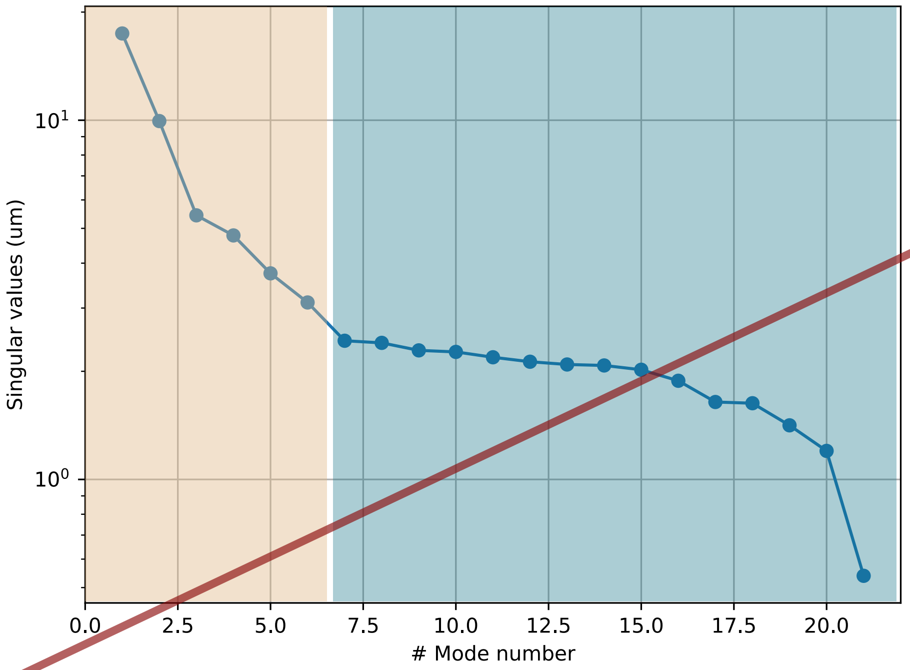
INSTITUT DE FÍSICA CORPUSCULAR

I. Two SVD methods:
   A) Pseudo-inverse for position estimation
   B) SVD analysis for beam coherent motion removal
   C) SVD + Pseudo-Inverse

II. Estimation of the resolution error
   A) Artificial noise on de-noised matrix
   B) Pseudo-experiments on pseudo-inverse method

# II. Estimation of the resolution error

## A) Adding artificial noise to a de-noised matrix




**idea 2:**

Generate an $D_{coherent}$ matrix keeping only the dominant modes, to which the noise is added as:

$D_{sim} = D_{coherent} + E$

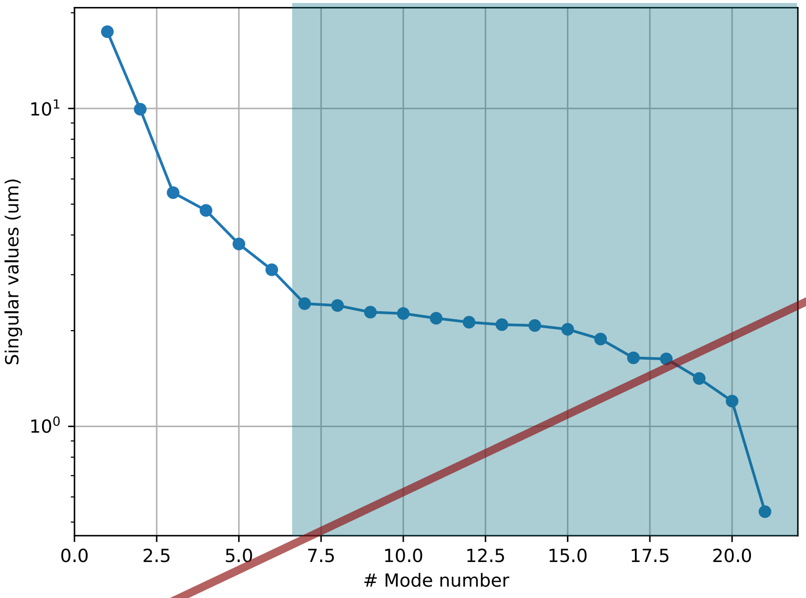
$E$ is Gaussian noise added independently to each channel: $\sigma_{strip,i}$ for striplines.

Perform SVD on $D_{sim}$ removing the dominant modes to obtain $D_{sim,noise}$ and compute $\sigma_{sim,cBPM}$

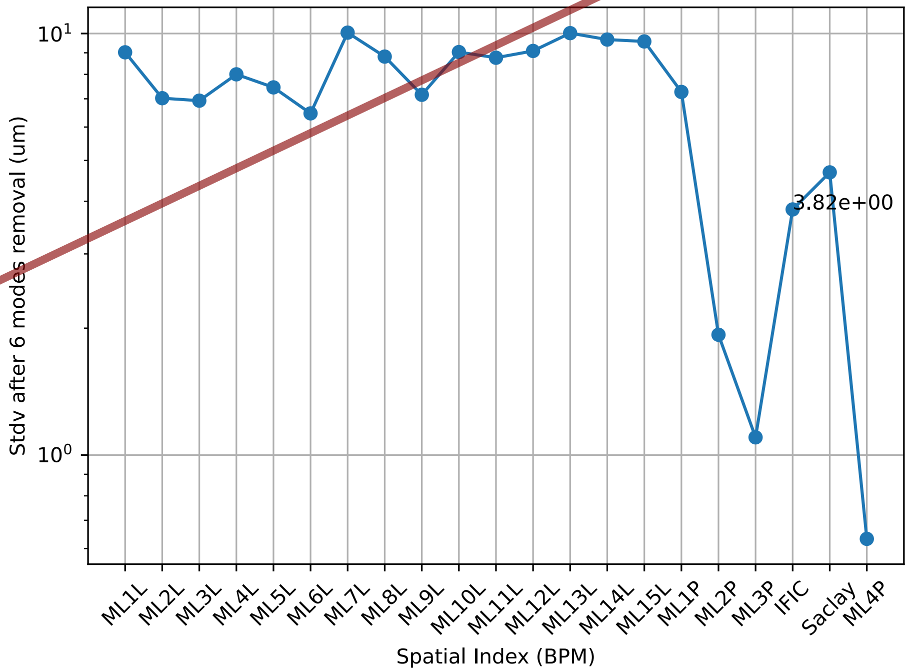
The systematic error provoked by the stripline limitations is $\Delta\sigma_{cBPM} = \sigma_{sim,cBPM} - \sigma_{cBPM}$

# II. Estimation of the resolution error

## A) Adding artificial noise to a de-noised matrix

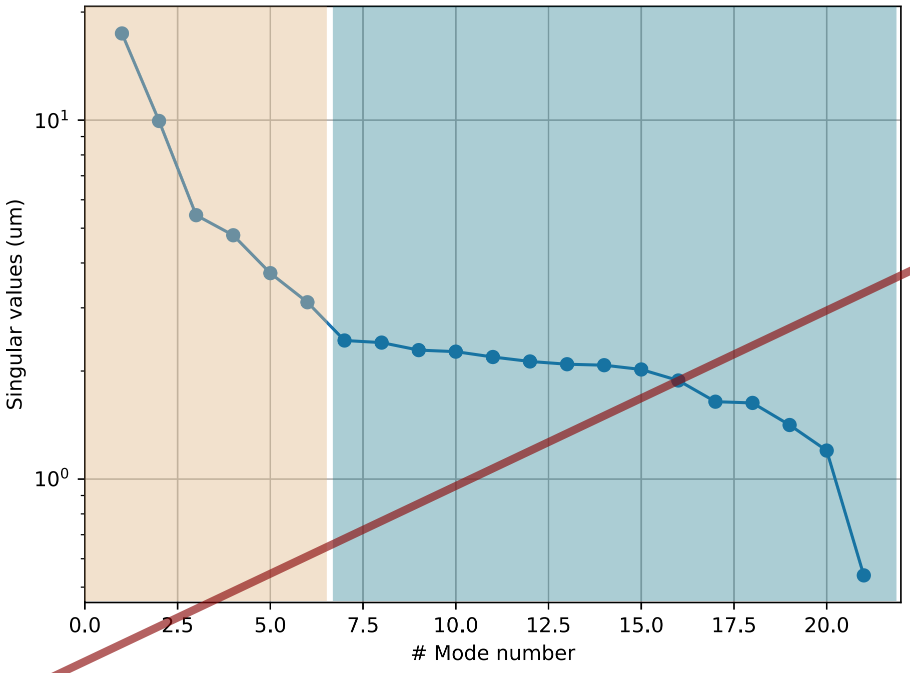



Resolutions from each BPM $\sigma_{strip,i}$ are extracted from this:



# II. Estimation of the resolution error

## A) Adding artificial noise to a de-noised matrix




Generate an $D_{coherent}$ matrix keeping only the dominant modes, to which the noise is added as:

$$D_{sim} = D_{coherent} + E$$

$E$ is Gaussian noise added independently to each channel: $\sigma_{strip,i}$ for striplines.

Perform SVD on $D_{sim}$ removing the dominant modes to obtain $D_{sim,noise}$ and compute $\sigma_{sim,cBPM}$

The systematic error provoked by the stripline limitations is $\Delta\sigma_{cBPM} = \sigma_{sim,cBPM} - \sigma_{cBPM}$

Measured resolution after removing 6 modes: 3.82e+00 um
Simulated resolution after removing 6 modes: 5.76e+00 um
The difference in resolution between the measured and simulated data is: **1.94e+00 um**

# II. Estimation of the resolution error

## A) Adding artificial noise to a de-noised matrix

Generate an $D_{coherent}$ matrix keeping only the dominant modes, to which the noise is added as:

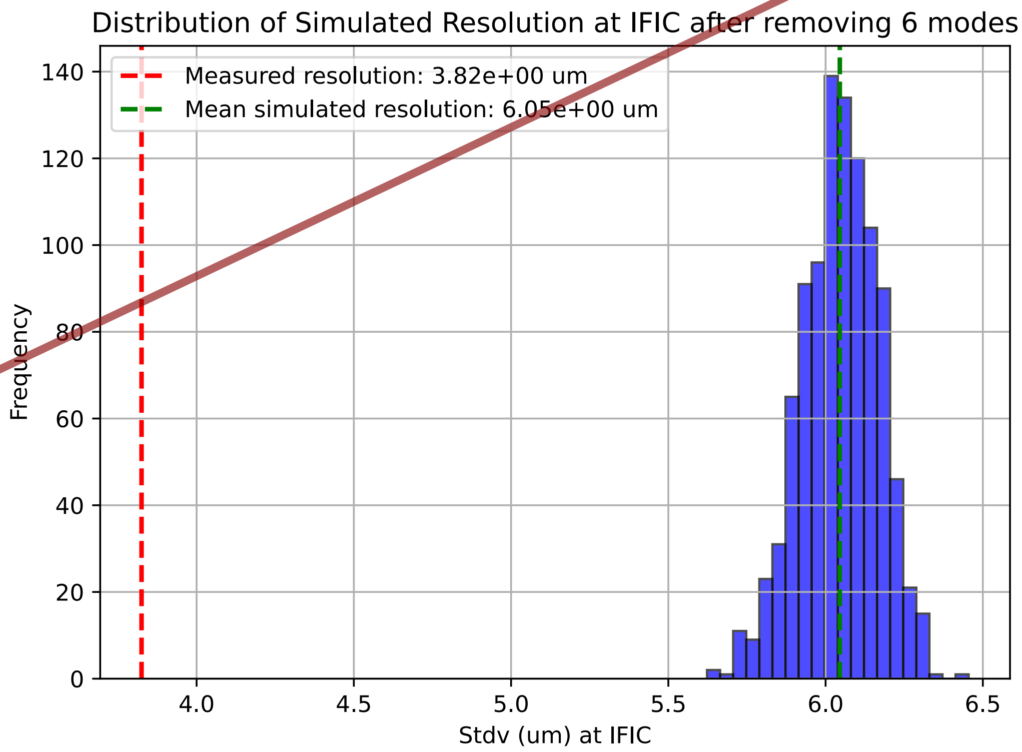
$$D_{sim} = \boxed{D_{coherent}} + E$$

$E$ is Gaussian noise added independently to each channel: $\sigma_{strip,i}$ for striplines.

Perform SVD on $D_{sim}$ removing the dominant modes to obtain $\boxed{D_{sim,noise}}$ and compute $\sigma_{sim,cBPM}$

The systematic error provoked by the stripline limitations is $\Delta\sigma_{cBPM} = \sigma_{sim,cBPM} - \sigma_{cBPM}$

**The difference in resolution between the measured and mean simulated resolution is: 2.22e+00 um**



# II. Estimation of the resolution error

## B) Pseudo-experiments on pseudo-inverse

Pseudoexperimentos:

1. Seleccionas aleatoriamente X pulsos (500? si pueden ser más mejor)
2. Calculas los factores de correlación $v_i$.
   - Al coger 500 pulsos ya estamos teniendo en cuenta el posible error en las $v_i$ (cuantos más pulsos mejor)
3. Cogemos un pulso de los que queda
   - Y metemos ruido aleatorio a las medidas dadas por los otros BPMs en un rango de sigma= 10 um (se puede estudiar el efecto de este valor en el error asociado al sistema de medida) y estimamos la posición en nuestro BPM para unos 1000 o 10000 casos
4. Repetimos el paso 3 para más pulsos y promediamos $\boldsymbol{\sigma}_{ref}$

Cómo evaluamos el error introducido por $v_i$ y $n_i$?

$$R_k = (x_k^{beam} - \sum_{i\neq k}^{N} v_i x_i^{beam}) + (n_k - \sum_{i\neq k}^{N} v_i n_i)$$

Generamos ruido en los BPMs $d_{00} = d_{00} + R0[0,1]*\boldsymbol{\sigma}_0$

$$d_{estim} = d_{00}\, v_0 + d_{10}\, v_1 + \cdots d_{N0}\, v_N$$

$d_{10} = d_{10} + R1[0,1]*\boldsymbol{\sigma}_1$

$\sigma_{ref,1} = std(d_{estim})$ or $std(R_k)$ ?

Este método debería tener en cuenta el:

- ruido de los BPMs,
- incertidumbre de los coeficientes de correlación,
- estadística

Con esto estimamos el error del sistema de medida $\boldsymbol{\sigma}_{ref}$ , después podríamos hacer que:

$$\sigma_{BPM-IFIC} = \sqrt{\sigma^2_{medida,BPM-IFIC} - \sigma^2_{ref}}$$

Si $\boldsymbol{\sigma}_{ref}$ > $\boldsymbol{\sigma}_{ref}$ … malo

# II. Estimation of the resolution error

## B) Pseudo-experiments on pseudo-inverse

$k$ : refers to the BPM under test
$P = 1000$ is the number of pulses
$M = 19$ is the number of stripline BPMs used
$L = 10000$ is the number of simulations

**INFLUENCE OF BPMs RESOLUTION:**

$$\mathbf{d_k} = \mathbf{D}\cancel{\mathbf{K}} \cdot \mathbf{v}$$

cBPM measured values → 

$$\begin{pmatrix} d_{k0} \\ d_{k1} \\ \vdots \\ d_{kP} \end{pmatrix} = \begin{pmatrix} d_{00} & d_{10} & \dots & d_{i\neq k,0} & \dots & d_{M0} \\ d_{01} & d_{11} & \dots & d_{i\neq k,1} & \dots & d_{M1} \\ \vdots & \vdots & & \vdots & & \vdots \\ d_{0P} & d_{1P} & \dots & d_{i\neq k,P} & \dots & d_{MP} \end{pmatrix} \cdot \begin{pmatrix} v_0 \\ v_1 \\ \vdots \\ v_M \end{pmatrix}$$

← correlation coefficients

↑ stripline BPM measured values

To find the correlation coefficients: $\mathbf{v} = \mathbf{D}\cancel{\mathbf{K}}^{-1}\mathbf{d_k}$ → done **for all bunches**

**For a given bunch i and a simulation l:** We simulate a random noise with $\sigma_{strip,i}$ in $\mathbf{E}_{l,i}$ which is added to $\mathbf{D}\cancel{\mathbf{K}}_i$

The simulated estimation position is: $\mathbf{D}\cancel{\mathbf{K}}_{sim,l,i} \cdot \mathbf{v_i}$ where $\mathbf{D}\cancel{\mathbf{K}}_{sim,l,i} = \mathbf{D}\cancel{\mathbf{K}}_\mathbf{i} + \mathbf{E}_{l,i}$

The simulated residuals between measured position and estimated position are calculated as: $\mathbf{R}_{\mathbf{k}_{sim,l,i}} = \mathbf{d}_{\mathbf{k}_i} - \mathbf{D}\cancel{\mathbf{K}}_{sim,l,i} \cdot \mathbf{v}_i$

We observe the standard deviation of the estimated position and the residuals for a given bunch $i$ across all $L$ simulations:

$\sigma_{estim,i}(\mathbf{D}\cancel{\mathbf{K}}_{sim,l,i} \cdot \mathbf{v}_i)$ $\sigma_{residuals,i}(\mathbf{R}_{\mathbf{k}_{sim,l,i}})$ $\Rightarrow$ Performing simulations over several bunches: $\bar{\sigma}_{estim}$ $\bar{\sigma}_{residuals}$

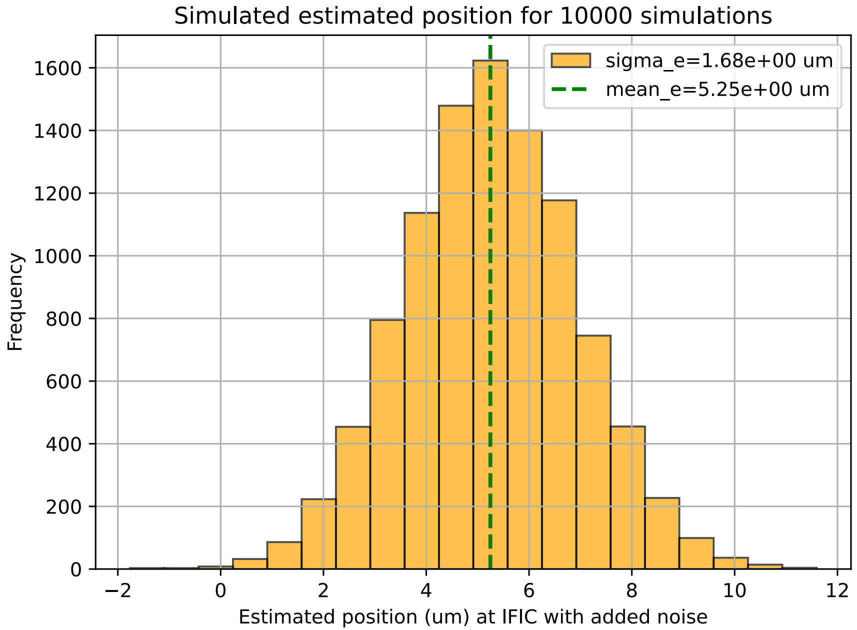
# II. Estimation of the resolution error

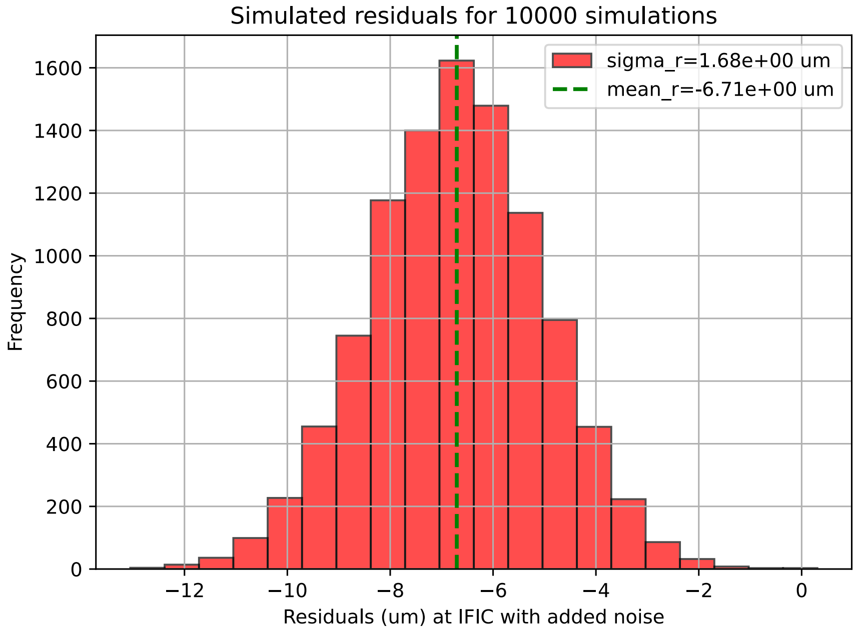
## B) Pseudo-experiments on pseudo-inverse

$k$ : refers to the BPM under test
$P = 1000$ is the number of pulses
$M = 19$ is the number of stripline BPMs used
$L = 10000$ is the number of simulations

**INFLUENCE OF BPMs RESOLUTION:**

Pseudo-experiment considering $\sigma_{strip,i}$ obtained with SVD and **only the last bunch**



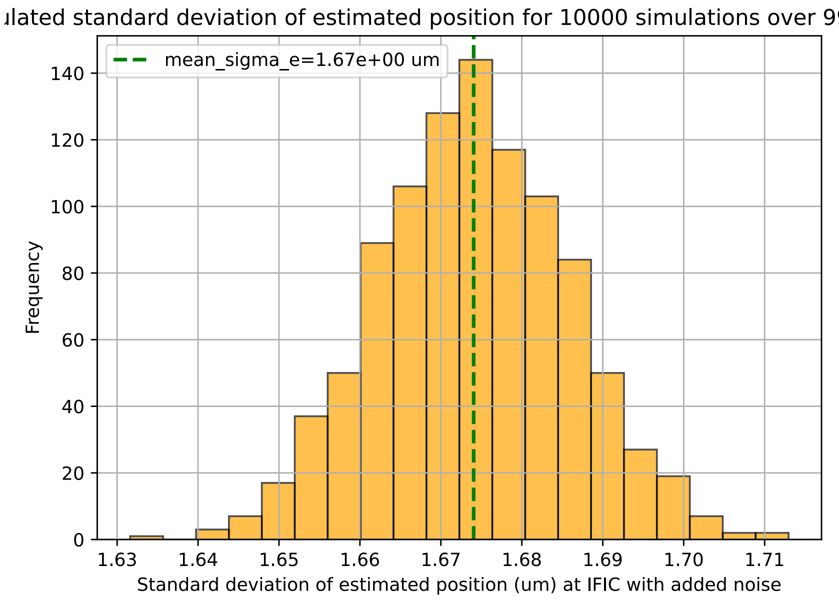
# II. Estimation of the resolution error

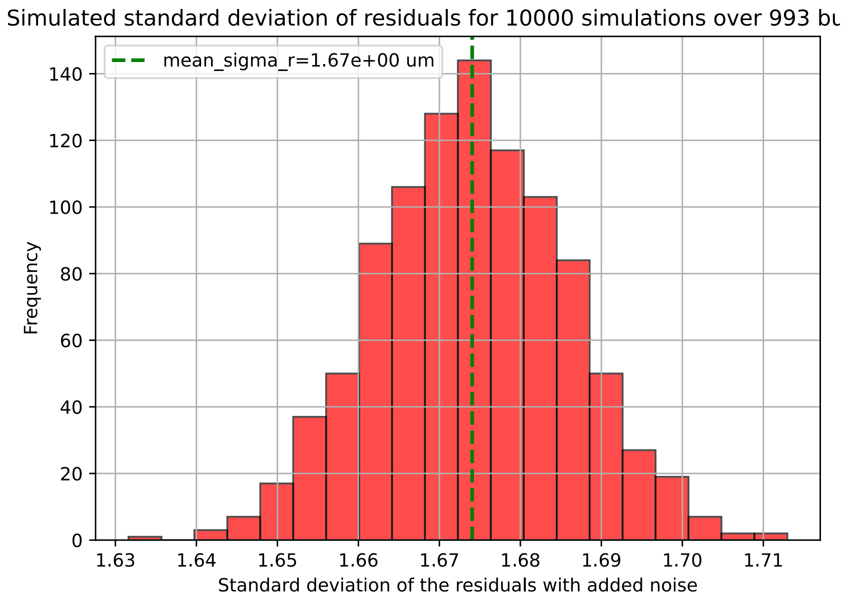
## B) Pseudo-experiments on pseudo-inverse

$k$ : refers to the BPM under test
$P = 1000$ is the number of pulses
$M = 19$ is the number of stripline BPMs used
$L = 10000$ is the number of simulations

**INFLUENCE OF BPMs RESOLUTION:**

Pseudo-experiments considering $\sigma_{strip,i}$ obtained with SVD and **observing the stdv results over all bunches**



# II. Estimation of the resolution error

## B) Pseudo-experiments on pseudo-inverse

$k$ : refers to the BPM under test
$P = 1000$ is the number of pulses
$M = 19$ is the number of stripline BPMs used
$L = 10000$ is the number of simulations

**INFLUENCE OF CORRELATION COEFFICIENTS ERROR:**

$$\mathbf{d_k} = \mathbf{D}\cancel{\mathbf{K}} \cdot \mathbf{v}$$

cBPM measured values → 

$$\begin{pmatrix} d_{k0} \\ d_{k1} \\ \vdots \\ d_{kP} \end{pmatrix} = \begin{pmatrix} d_{00} & d_{10} & \dots & d_{i\neq k,0} & \dots & d_{M0} \\ d_{01} & d_{11} & \dots & d_{i\neq k,1} & \dots & d_{M1} \\ \vdots & \vdots & & \vdots & & \vdots \\ d_{0P} & d_{1P} & \dots & d_{i\neq k,P} & \dots & d_{MP} \end{pmatrix} \cdot \begin{pmatrix} v_0 \\ v_1 \\ \vdots \\ v_M \end{pmatrix}$$

← correlation coefficients

↑ stripline BPM measured values

To find the correlation coefficients: $\mathbf{v} = \mathbf{D}\cancel{\mathbf{K}}^{-1}\mathbf{d_k}$ → done **for all bunches**

We assume bunches are not correlated between each other.
**Bootstraping:** procedure for estimating the distribution of an estimator by resampling (often with replacement) one's data.

**We perform $n_{bootstrap} = 1000$ bootstrap simulations in order to quantify the statistical uncertainty of $v_i$ due to finite sample size and random noise/jitter across the measured pulses:**

- ➡ how robust $\mathbf{v}$ is? For another set of $P$ pulses under the exact same conditions, $v_i$ would likely fall within $\pm\sigma(v_i)$
- ➡ how much $v_i$ fluctuates due to random electronic noise in the BPMs, beam position jitter, or pulse-to-pulse intensity fluctuations.

# II. Estimation of the resolution error

## B) Pseudo-experiments on pseudo-inverse

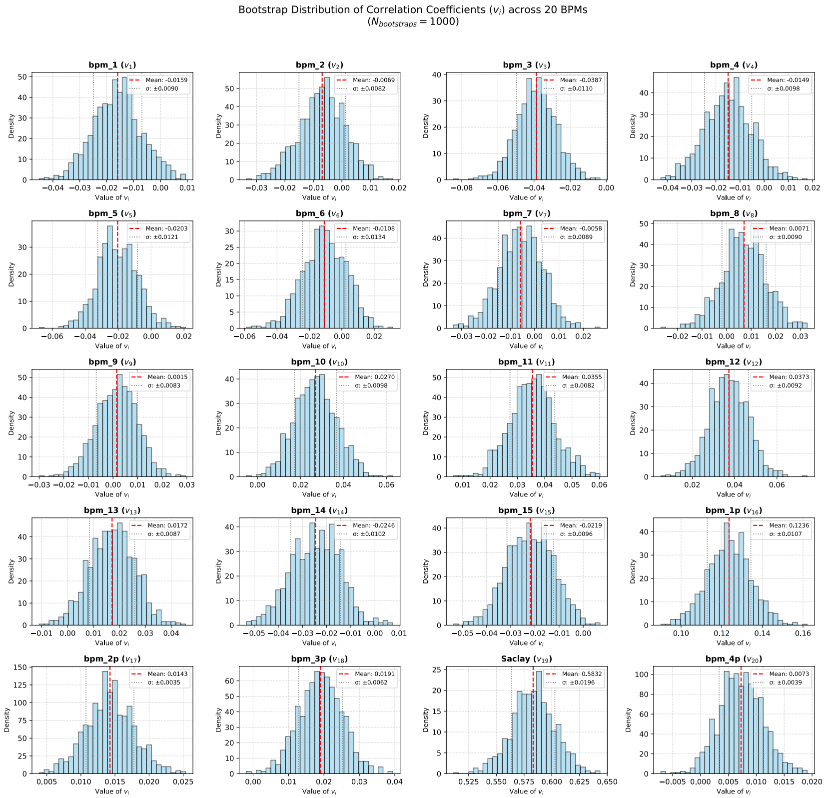
$k$ : refers to the BPM under test
$P = 1000$ is the number of pulses
$M = 19$ is the number of stripline BPMs used
$n_{bootstraps} = 1000$ is the number of simulations

**INFLUENCE OF CORRELATION COEFFICIENTS ERROR:**

**BPM Reliability / Contribution:** If a specific $v_i$ has a very large $\sigma(v_i)$relative to its mean value, that particular BPM's contribution to predicting $d_k$ is noisy or unstable.

When performing bootstraping, it is possible to evaluate:

$$SNR_i = |\mu(v_i)/\sigma(v_i)|$$




To know if $v_i$ is reliable:

$$|\mu(v_i)/\sigma(v_i)| < 1$$

SNR of v_i(bpm_1): 1.80, reliable?: Yes
SNR of v_i(bpm_2): 0.81, reliable?: No
SNR of v_i(bpm_3): 3.30, reliable?: Yes
SNR of v_i(bpm_4): 1.60, reliable?: Yes
SNR of v_i(bpm_5): 1.59, reliable?: Yes
SNR of v_i(bpm_6): 0.69, reliable?: No
SNR of v_i(bpm_7): 0.66, reliable?: No
SNR of v_i(bpm_8): 0.84, reliable?: No
SNR of v_i(bpm_9): 0.19, reliable?: No
SNR of v_i(bpm_10): 2.67, reliable?: Yes
SNR of v_i(bpm_11): 4.36, reliable?: Yes
SNR of v_i(bpm_12): 3.98, reliable?: Yes
SNR of v_i(bpm_13): 1.95, reliable?: Yes
SNR of v_i(bpm_14): 2.52, reliable?: Yes
SNR of v_i(bpm_15): 2.20, reliable?: Yes
SNR of v_i(bpm_1p): 12.00, reliable?: Yes
SNR of v_i(bpm_2p): 4.00, reliable?: Yes
SNR of v_i(bpm_3p): 3.00, reliable?: Yes
SNR of v_i(Saclay): 28.05, reliable?: Yes
SNR of v_i(bpm_4p): 1.86, reliable?: Yes

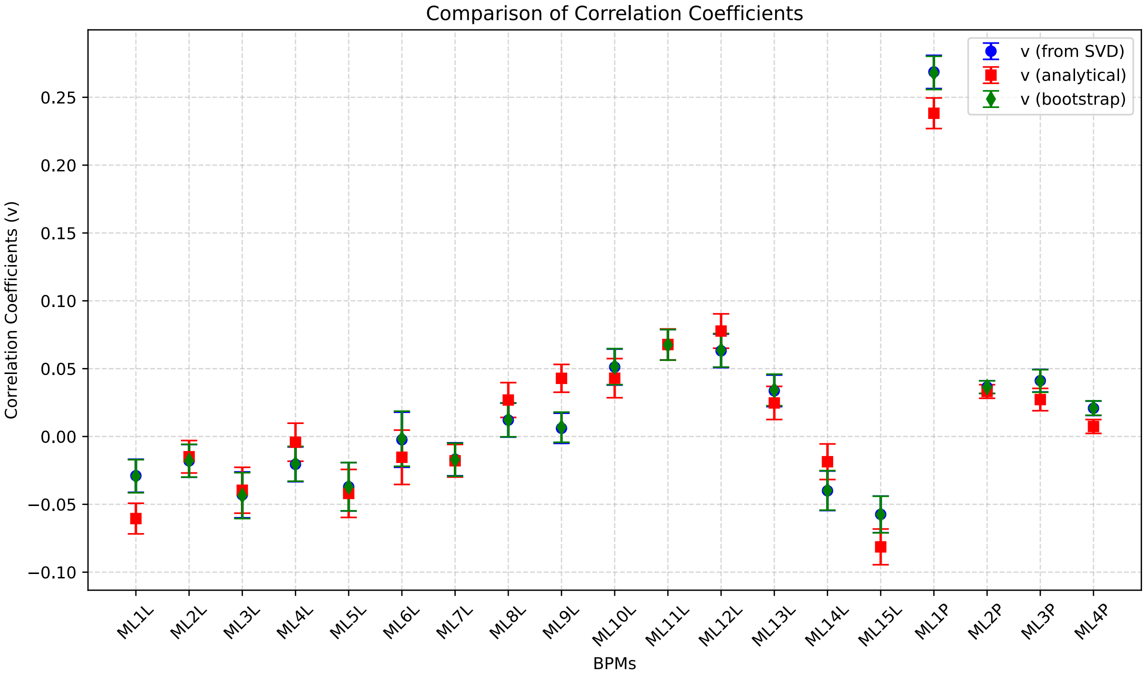
# II. Estimation of the resolution error

## B) Pseudo-experiments on pseudo-inverse

$k$ : refers to the BPM under test
$P = 1000$ is the number of pulses
$M = 19$ is the number of stripline BPMs used
$n_{bootstraps} = 1000$ is the number of simulations

**INFLUENCE OF CORRELATION COEFFICIENTS ERROR:**

Comparison with theoretical formula:



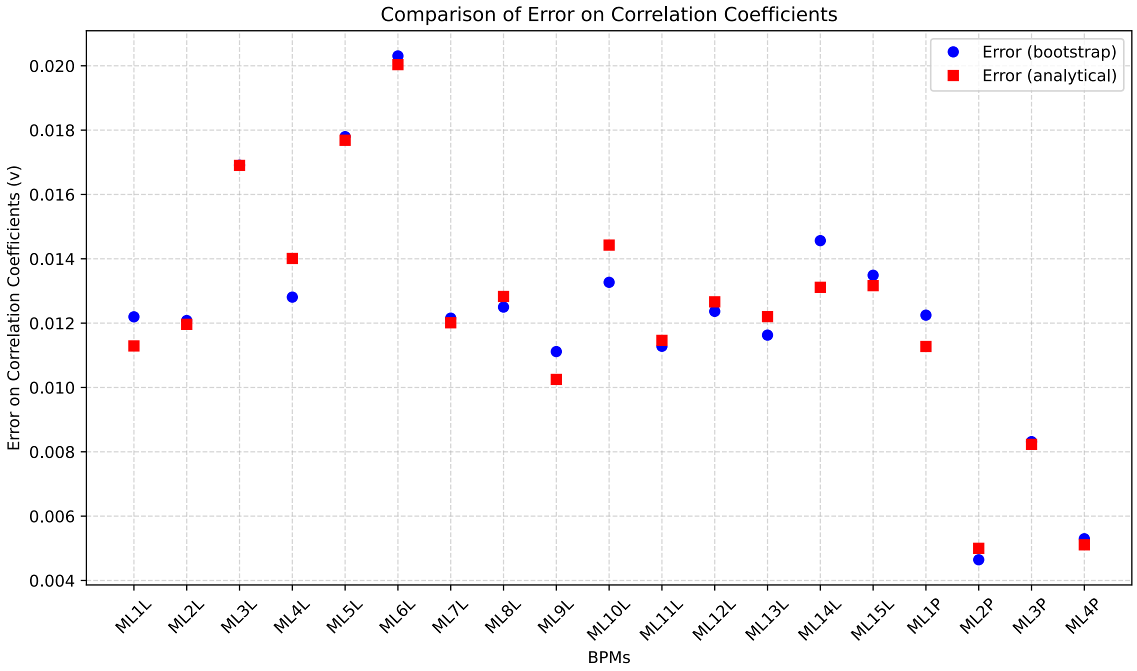
# II. Estimation of the resolution error

## B) Pseudo-experiments on pseudo-inverse

$k$ : refers to the BPM under test
$P = 1000$ is the number of pulses
$M = 19$ is the number of stripline BPMs used
$n_{bootstraps} = 1000$ is the number of simulations

**INFLUENCE OF CORRELATION COEFFICIENTS ERROR:**

Comparison with theoretical formula:



# II. Estimation of the resolution error

## B) Pseudo-experiments on pseudo-inverse

$k$ : refers to the BPM under test
$P = 1000$ is the number of pulses
$M = 19$ is the number of stripline BPMs used
$L = 10000$ is the number of simulations

**INFLUENCE OF CORRELATION COEFFICIENTS ERROR:**

$$\mathbf{d_k} = \mathbf{D}\not\mathbf{K} \cdot \mathbf{v}$$

cBPM measured values $\longrightarrow$

$$\begin{pmatrix} d_{k0} \\ d_{k1} \\ \vdots \\ d_{kP} \end{pmatrix} = \begin{pmatrix} d_{00} & d_{10} & \dots & d_{i\neq k,0} & \dots & d_{M0} \\ d_{01} & d_{11} & \dots & d_{i\neq k,1} & \dots & d_{M1} \\ \vdots & \vdots & & \vdots & & \vdots \\ d_{0P} & d_{1P} & \dots & d_{i\neq k,P} & \dots & d_{MP} \end{pmatrix} \cdot \begin{pmatrix} v_0 \\ v_1 \\ \vdots \\ v_M \end{pmatrix}$$

$\longleftarrow$ correlation coefficients

$\uparrow$ stripline BPM measured values

To find the correlation coefficients: $\mathbf{v} = \mathbf{D}\not\mathbf{K}^{-1}\mathbf{d_k}$ $\rightarrow$ done **for all bunches**

**For a given bunch i and a simulation l:** We simulate the uncertainty on the correlation coefficients $\sigma_{v_i}$ in $\mathbf{E}_{l,i}$ which is added to $\mathbf{v}_i$

The simulated estimation position is: $\mathbf{D}\not\mathbf{K}_i \cdot \mathbf{v}_{sim,l,i}$ where $\mathbf{v}_{sim,l,i} = \mathbf{v}_i + \mathbf{E}_{l,i}$

The simulated residuals between measured position and estimated position are calculated as: $\mathbf{R}_{\mathbf{k}_{sim,l,i}} = \mathbf{d}_{\mathbf{k}_i} - \mathbf{D}\not\mathbf{K}_i \cdot \mathbf{v}_{sim,l,i}$

We observe the standard deviation of the estimated position and the residuals for a given bunch $i$ across all $L$ simulations:

$\sigma_{estim,i}(\mathbf{D}\not\mathbf{K}_i \cdot \mathbf{v}_{sim,l,i})$ $\quad$ $\sigma_{residuals,i}(\mathbf{R}_{\mathbf{k}_{sim,l,i}})$ $\Rightarrow$ Performing simulations over several bunches: $\bar{\sigma}_{estim}$ $\quad$ $\bar{\sigma}_{residuals}$

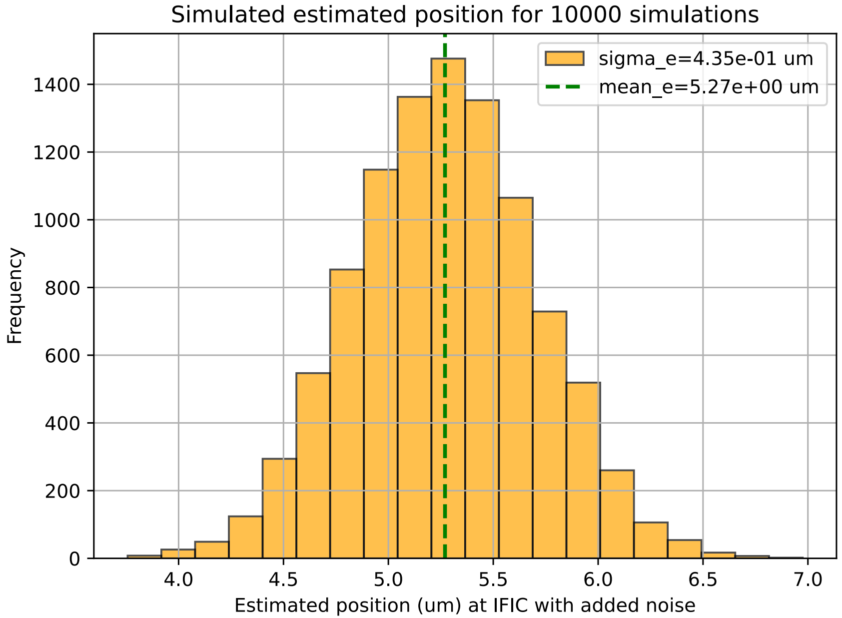
# II. Estimation of the resolution error

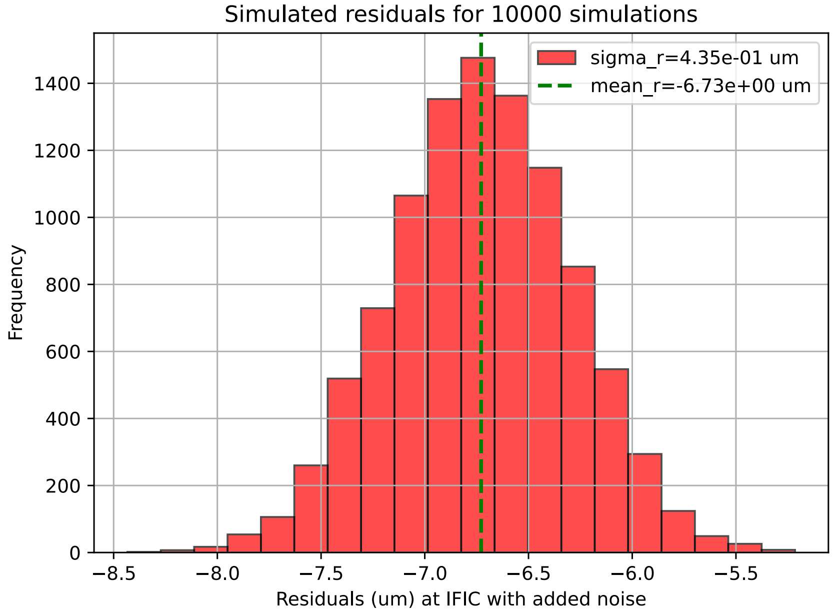
## B) Pseudo-experiments on pseudo-inverse

$k$ : refers to the BPM under test
$P = 1000$ is the number of pulses
$M = 19$ is the number of stripline BPMs used
$L = 10000$ is the number of simulations

**INFLUENCE OF CORRELATION COEFFICIENTS ERROR:**

Considering $\sigma_{v,i}$ obtained with **Bootstraping for only the last bunch**



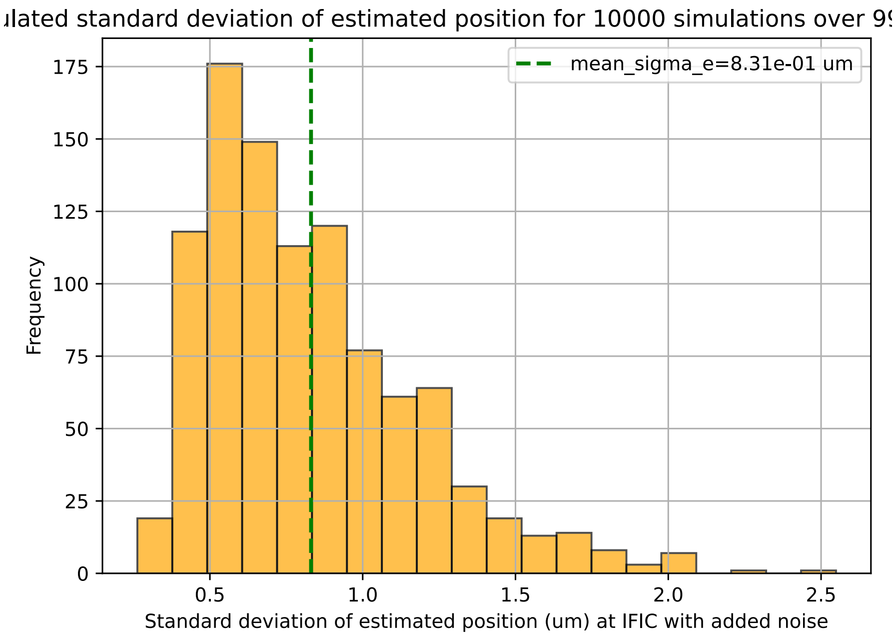
# II. Estimation of the resolution error

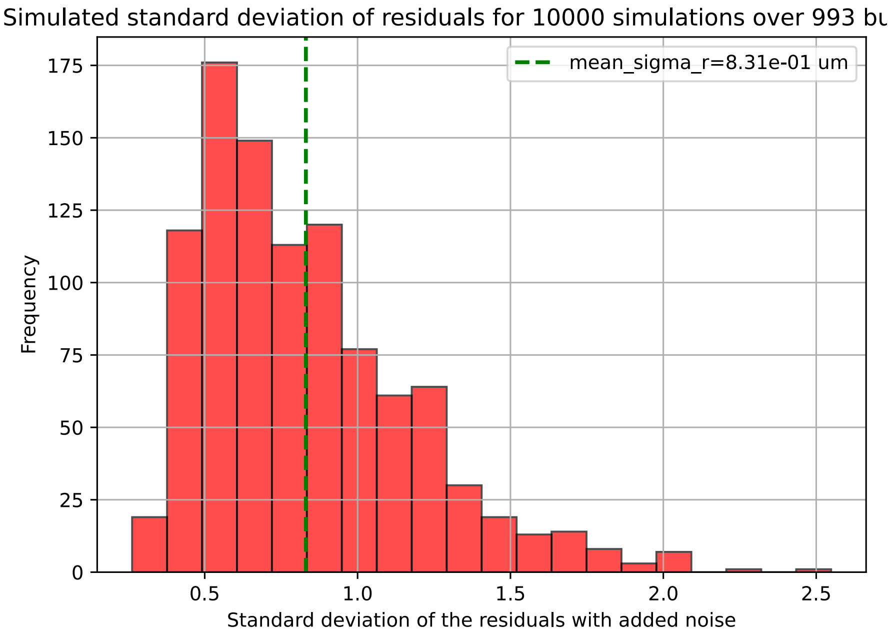
## B) Pseudo-experiments on pseudo-inverse

$k$ : refers to the BPM under test
$P = 1000$ is the number of pulses
$M = 19$ is the number of stripline BPMs used
$L = 10000$ is the number of simulations

**INFLUENCE OF CORRELATION COEFFICIENTS ERROR:**

Considering $\sigma_{v,i}$ obtained with **Bootstraping for ALL BUNCHES**



# II. Estimation of the resolution error

## B) Pseudo-experiments on pseudo-inverse

$k$ : refers to the BPM under test
$P = 1000$ is the number of pulses
$M = 19$ is the number of stripline BPMs used
$L = 10000$ is the number of simulations

**INFLUENCE OF BPMs RESOLUTION AND CORRELATION COEFFICIENTS ERROR:**

$$\mathbf{d}_k = \mathbf{D}_{\not k} \cdot \mathbf{v}$$

cBPM measured values →

$$\begin{pmatrix} d_{k0} \\ d_{k1} \\ \vdots \\ d_{kP} \end{pmatrix} = \begin{pmatrix} d_{00} & d_{10} & \dots & d_{i\neq k,0} & \dots & d_{M0} \\ d_{01} & d_{11} & \dots & d_{i\neq k,1} & \dots & d_{M1} \\ \vdots & \vdots & & \vdots & & \vdots \\ d_{0P} & d_{1P} & \dots & d_{i\neq k,P} & \dots & d_{MP} \end{pmatrix} \cdot \begin{pmatrix} v_0 \\ v_1 \\ \vdots \\ v_M \end{pmatrix}$$

← correlation coefficients

stripline BPM measured values

To find the correlation coefficients: $\mathbf{v} = \mathbf{D}_{\not k}^{-1} \cdot \mathbf{d}_k$ → done **for all bunches**

**For a given bunch i and a simulation l:** We simulate the noise with $\sigma_{strip,i}$ and $\sigma_{v_i}$ in $\mathbf{E}_{l,i}$ which is added to $\mathbf{D}_{\not k_i}$ and $\mathbf{v}_i$:

The simulated estimation position is: $\mathbf{D}_{\not k_{sim,l,i}} \cdot \mathbf{v}_{sim,l,i}$ where $\mathbf{D}_{\not k_{sim,l,i}} = \mathbf{D}_{\not k_i} + \mathbf{E}_{D,l,i}$ and $\mathbf{v}_{sim,l,i} = \mathbf{v}_i + \mathbf{E}_{v,l,i}$

The simulated residuals between measured position and estimated position are calculated as: $\mathbf{R}_{\mathbf{k}_{sim,l,i}} = \mathbf{d}_{\mathbf{k}_i} - \mathbf{D}_{\not k_{sim,l,i}} \cdot \mathbf{v}_{sim,l,i}$

We observe the standard deviation of the estimated position and the residuals for a given bunch $i$ across all $L$ simulations:

$\sigma_{estim,i}(\mathbf{D}\not{\mathbf{k}}_{sim,l,i} \cdot \mathbf{v}_{sim,l,i})$ $\sigma_{residuals,i}(\mathbf{R}_{\mathbf{k}_{sim,l,i}})$ ⇒ Performing simulations over several bunches: $\bar{\sigma}_{estim}$ $\bar{\sigma}_{residuals}$

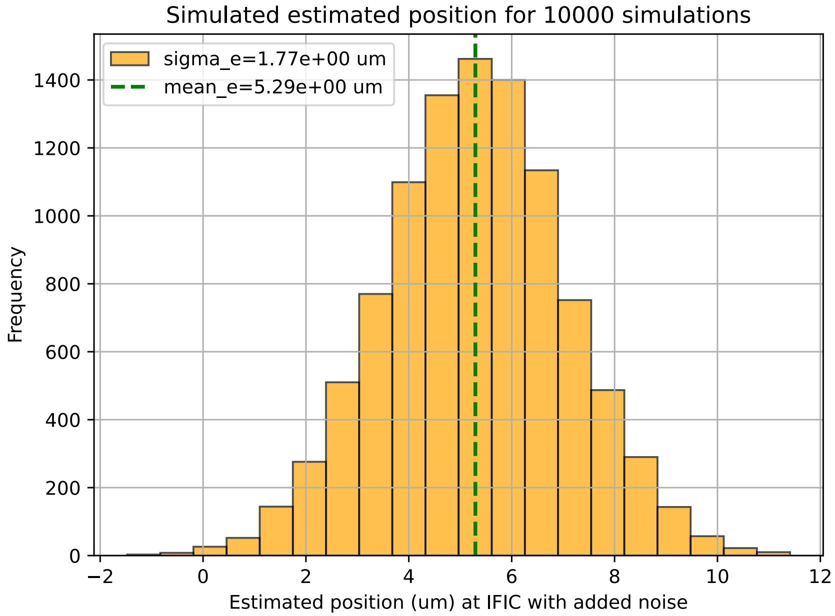
# II. Estimation of the resolution error

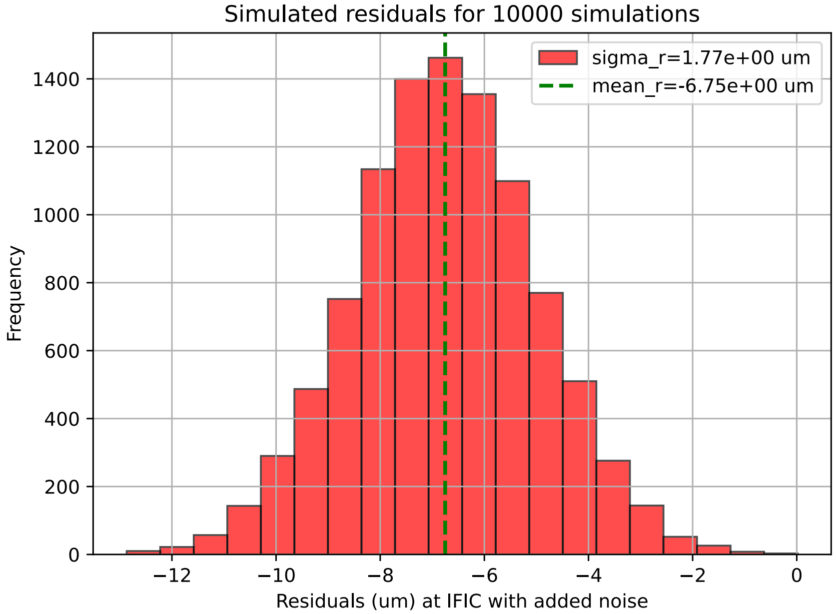
## B) Pseudo-experiments on pseudo-inverse

$k$ : refers to the BPM under test
$P = 1000$ is the number of pulses
$M = 19$ is the number of stripline BPMs used
$L = 10000$ is the number of simulations

**INFLUENCE OF BPMs RESOLUTION AND CORRELATION COEFFICIENTS ERROR:**

Pseudo-experiment considering $\sigma_{strip,i}$ and $\sigma(v_i)$ **only the last bunch**



# II. Estimation of the resolution error

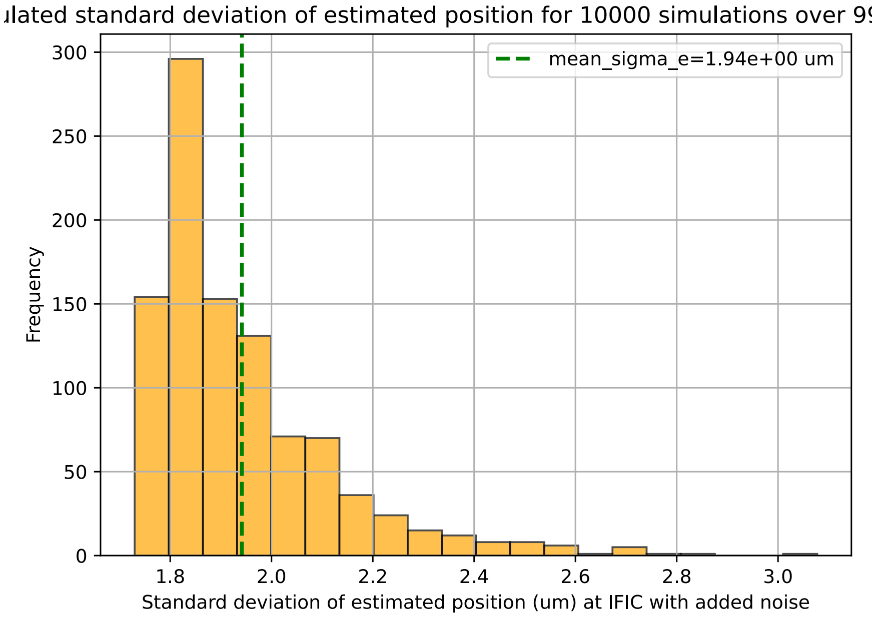
## B) Pseudo-experiments on pseudo-inverse

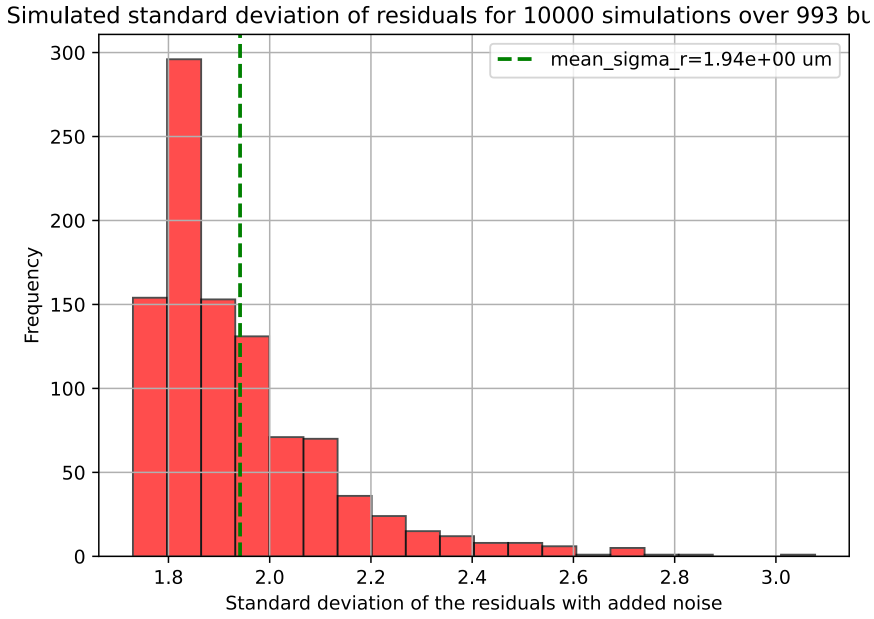
$k$ : refers to the BPM under test
$P = 1000$ is the number of pulses
$M = 19$ is the number of stripline BPMs used
$L = 10000$ is the number of simulations

**INFLUENCE OF BPMs RESOLUTION AND CORRELATION COEFFICIENTS ERROR:**

Pseudo-experiment considering $\sigma_{strip,i}$ and $\sigma(v_i)$ **observing the stdv results over all bunches**

Analytically:
$\sqrt{\sigma^2_{res,BPMs} + \sigma^2_{v_i}} = 1.86$ µm



# Thank you for your attention

We gratefully acknowledge the ATF staff for their assistance during the installation and BPM measurements. Special thanks to Toshiyuki Okugi, Alex Aryshev, and Konstantin Popov for their support. We also thank Toshihiro Matsumoto and Hiroshi Kaji for providing the necessary equipment for the measurements.

This work has the support of "Plan de Recuperación, Transformación y Resiliencia (PRTR) 2022 (ASFAE/2022/013)", funded by Conselleria d'Innovació, Universitats, Ciència i Societat Digital from Generalitat Valenciana, and NextGenerationEU from European Union.
This work was partially supported by the European Union's Horizon Europe Marie Sklodowska-Curie Staff Exchanges programme under grant agreement no. 101086276.

laura.pedraza@ific.uv.es

Financiado por la Unión Europea NextGenerationEU
Gobierno de España – Ministerio de Ciencia, Innovación y Universidades
Plan de Recuperación, Transformación y Resiliencia
Generalitat Valenciana – Conselleria de Educación, Universidades y Empleo
GVA NEXT – Fondos Next Generation en la Comunitat Valenciana
EAJADE
Excelencia Severo Ochoa