

Radiative corrections to pion β decay and $\tau \rightarrow \pi\pi\nu$:
an effective field theory approach

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IFIC seminar
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*Based on V.Cirigliano, M. Hoferichter, N. Valori:
arXiv 2602.11253 [hep-ph]*

Outline

Introduction

Cabibbo anomalies

Muon $g-2$ puzzle

Radiative corrections

The ChPT framework

A tower of EFT

Applications

Pion β decay

HVP of muon $g-2$

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Is the SM the ultimate theory ?

Most likely no! Some open questions cannot be addressed within the SM!

How to deal with the search of New Physics ?

Neutrino masses

Baryon asymmetry

Dark matter

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How to deal with the search of New Physics ?

Direct searches

- *Production of new particles at colliders, e.g. VL fermions, SUSY particles,...*
- *Typically conducted at high energy facilities, e.g. LEP, LHC,...*

Indirect searches

- *Look for processes extremely suppressed in the SM: e.g. EDMs, LFV,...*
- *Look for discrepancies between exp. measurements and SM theory predictions.*

Cabibbo anomalies

CKM matrix rules flavor physics in the quark sector $V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$

Unitarity condition: $\sum_k V_{ik} V_{jk}^* = \delta_{ij}$

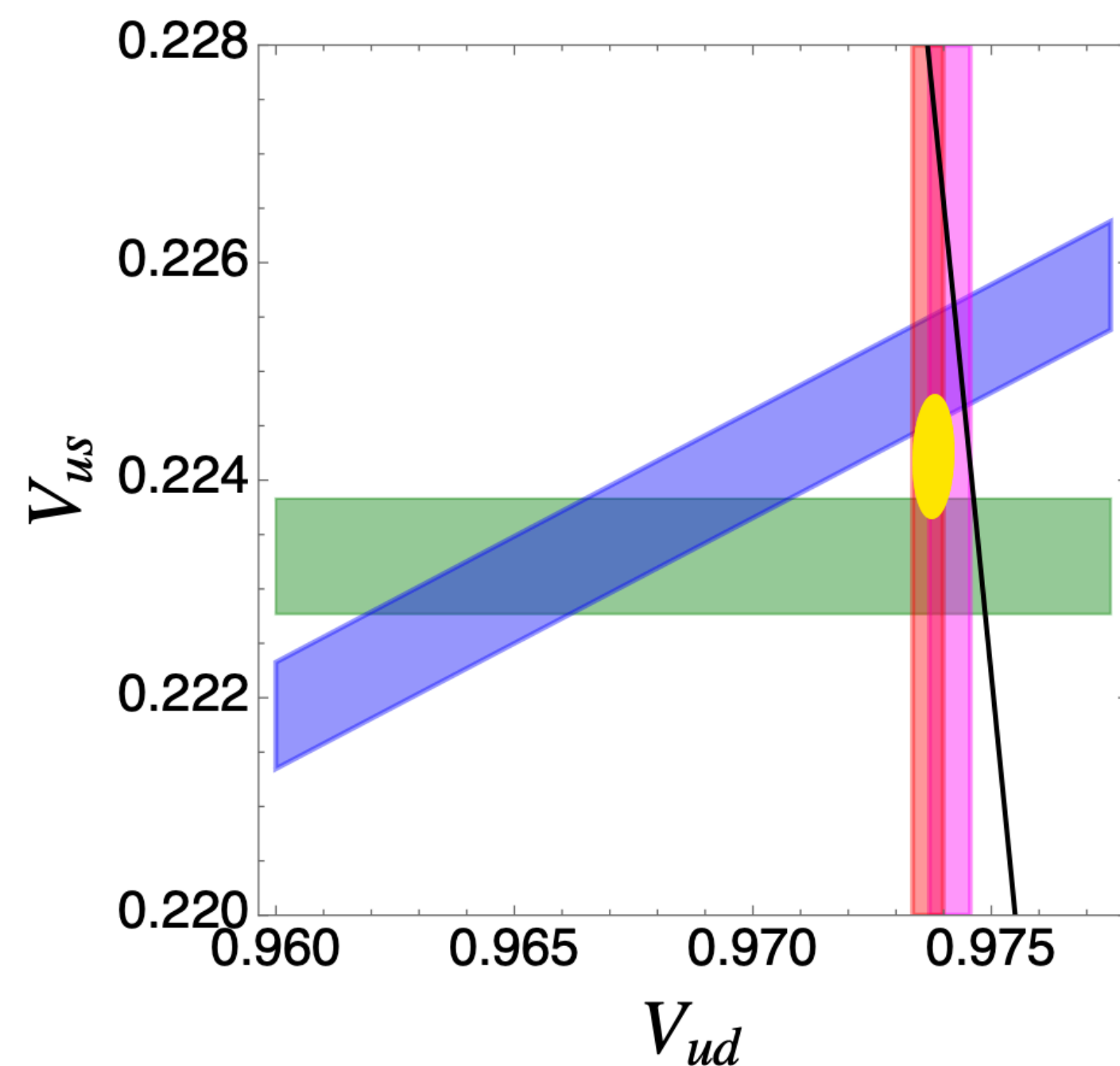
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Any deviation from unitary hints at possible dynamics beyond the SM!

[Cirigliano et al., *Phys.Lett.B* 838 (2023)]



Present day tensions in the first row of CKM challenge the SM picture

Cabibbo Anomalies

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 \neq 1 \sim 3\sigma$$

$\delta V_{ud} \sim 10^{-4}$
 $\delta V_{us} \sim 10^{-4}$
 $\sim 10^{-5}$ Irrelevant at this given accuracy!

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Select a process involving $u \leftrightarrow d$ transition: Semileptonic processes are free from QCD rad. correction

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Particle Data Group, PTEP 2022, 083C01 (2022).

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Process	Advantages	Limitations	V_{ud}
$0^+ \rightarrow 0^+ (e \nu_e)$	Many isotopes to average	Nuclear uncertainty	0.97367 (11) _{exp} (13) _{RC} (27) _{NS} [32]

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$\pi^\pm \rightarrow \pi^0 e^\pm \nu_e$	Theoretically clean	Experimentally extremely challenging	$0.97386 (281)_{\text{Br}} (9)_{\tau_\pi} (14)_{RC} (28)_{I_{\pi\ell}} [283]$ *Before CHV 2026

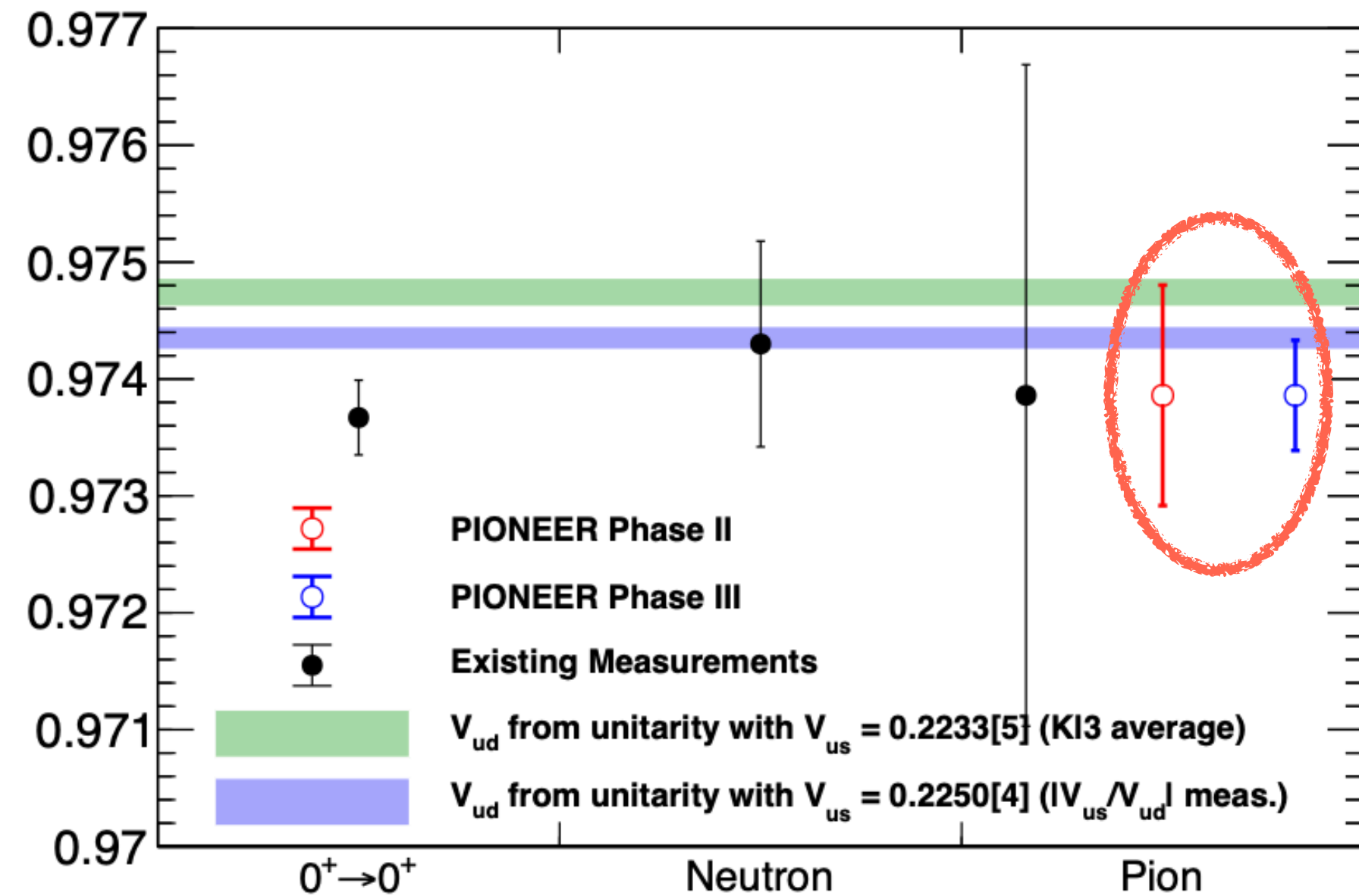
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Why is timely to deal with radiative corrections in Pion Beta decays?

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[PIONEER collab., 2504.06375]

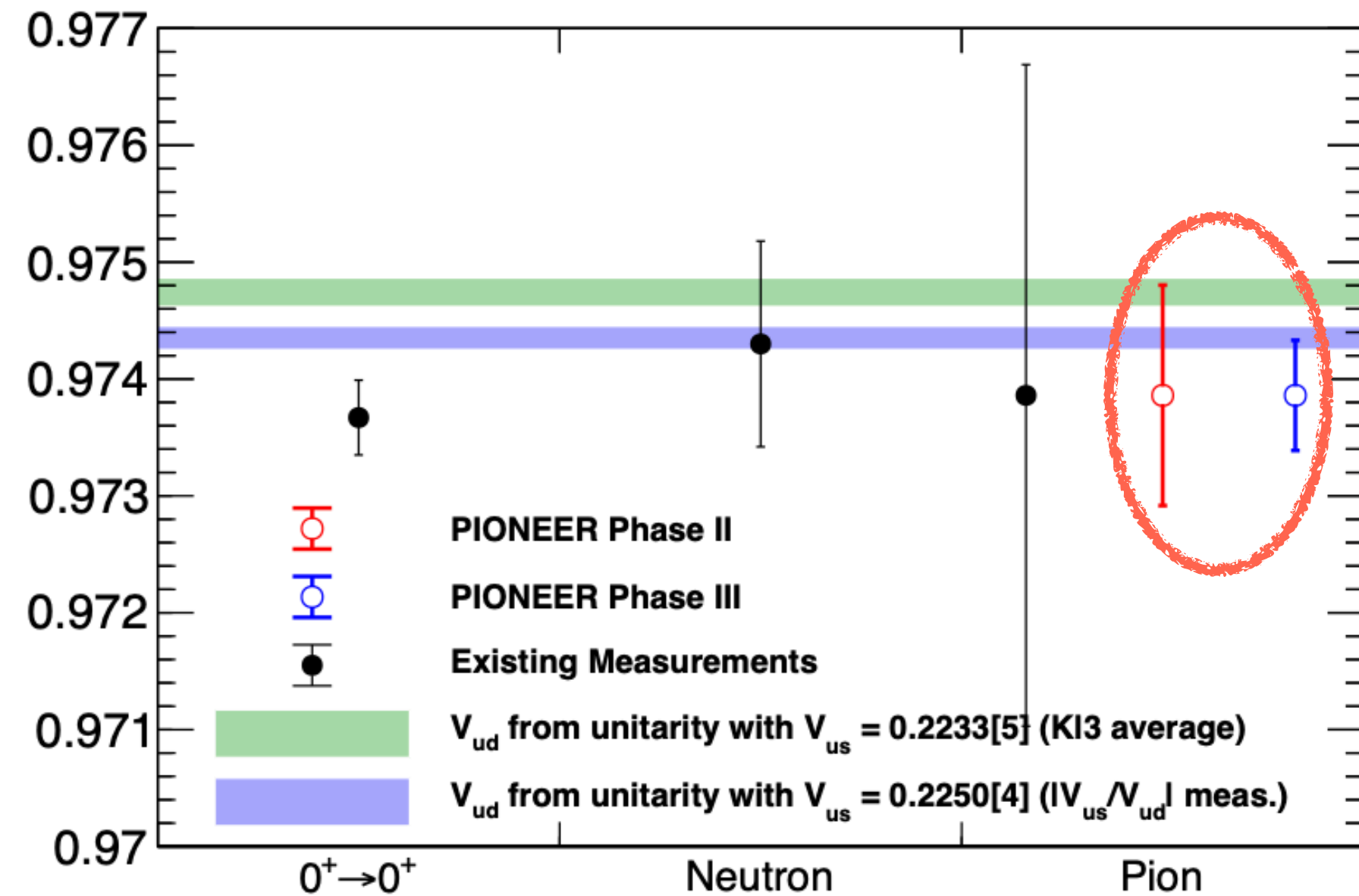


- *PIONEER exp. at PSI aims at reducing exp. unc. by one OoF*
- *V_{ud} extraction based on pion beta decay competitive with alternative approaches*
- *Reducing the th. uncertainty is crucial to keep with exp. progresses*

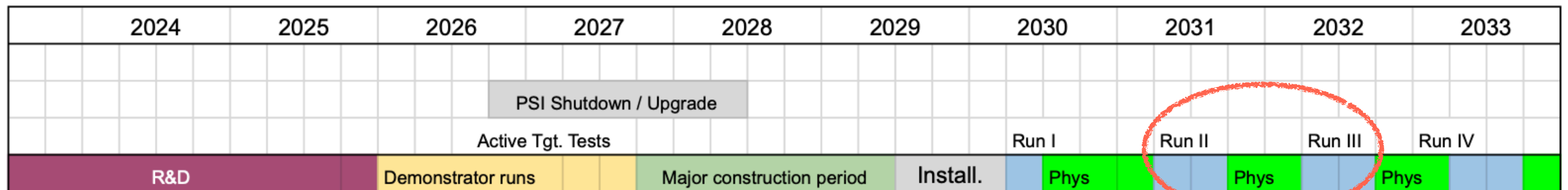
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Muon g-2

$$H = -g \frac{Q_f}{2m_f} \vec{B} \cdot \vec{S}$$

- *Proportionality constant in an interaction between particles with spin and an external magnetic field*
- *Relativistic QM predicts $g=2$: exact relation!*
- *Quantum correction from SM loops leads to a deviation from $g=2$: $a_\mu = \frac{g-2}{2}$*

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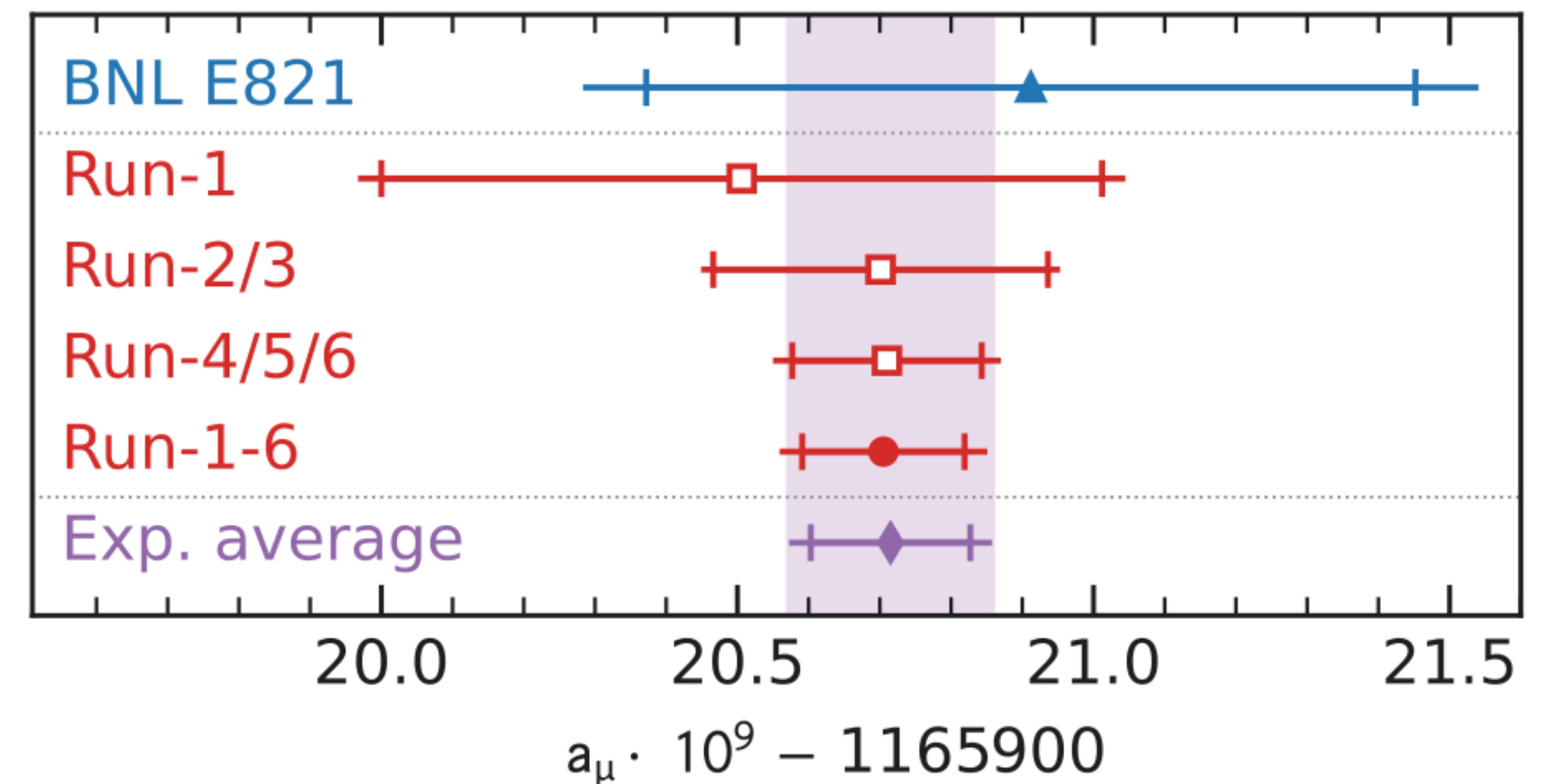
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Experimentally measured with a precision of 127 ppb:

$$a_\mu(\text{exp}) = 116\,592\,071,5 (14,5) \times 10^{-11}$$

Great consistency between BNL and Fermilab results!

[Muon g-2 collab., 2025]



Muon $g-2$

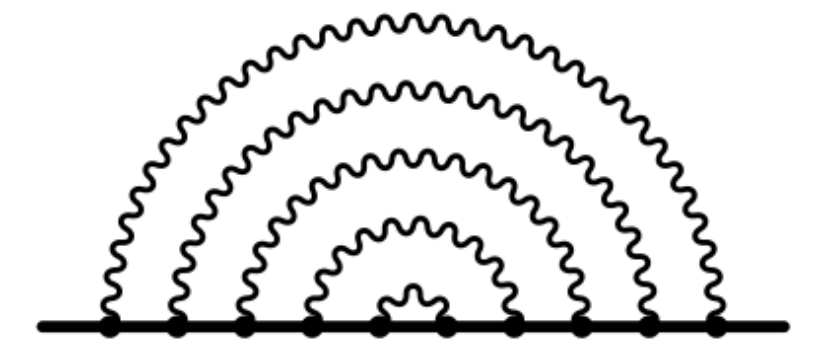
[WP., 2025]

SM theory prediction exhibits an uncertainty a factor of 4 larger:

$$a_{\mu}(\text{th.}) = 116\,592\,033\,(62) \times 10^{-11}$$

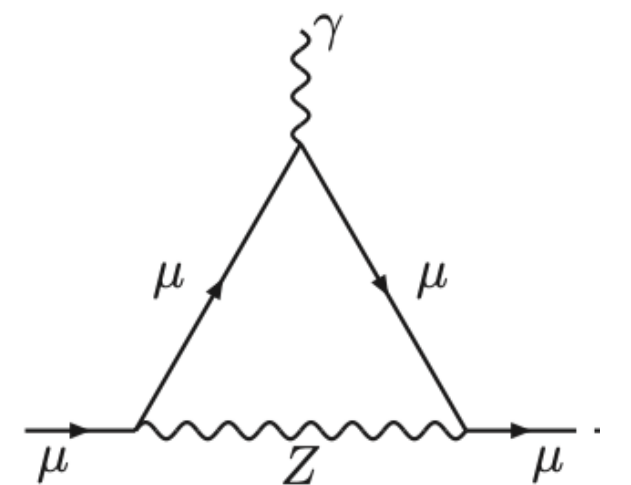
• QED:

- 99,99% of the whole contribution: $a_{\mu}^{QED} = 116\,584\,718,8\,(2) \times 10^{-11}$
- Know at 5 loops accuracy: negligible uncertainty



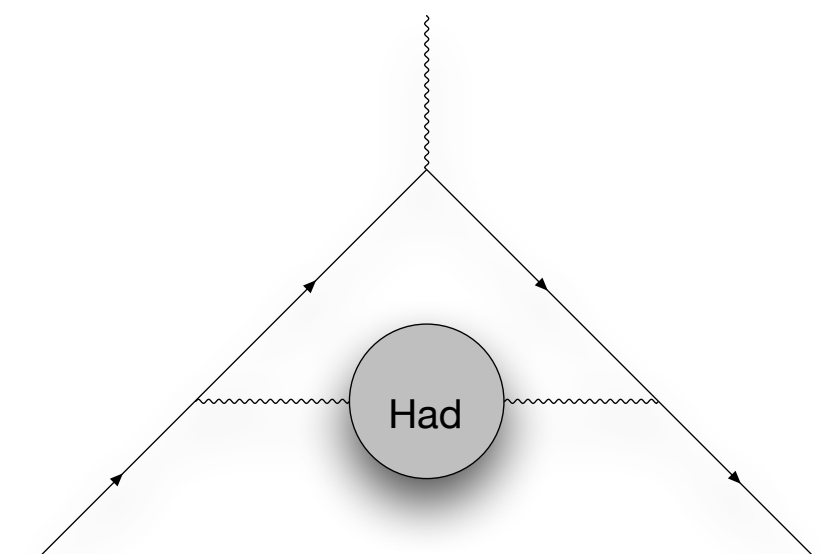
• EW:

- Known at 2 loops accuracy: negligible uncertainty: $a_{\mu}^{EW} = 154.4\,(4) \times 10^{-11}$
- Same order of magnitude of $\delta\Delta a_{\mu}$: benchmark value for sensitivity to BSM physics



• Hadronic:

- $a_{\mu}^{Had} \sim 7000\,(62) \times 10^{-11}$ accounts for the whole theory uncertainty



Hadronic Vacuum Polarization

$$\text{Had.} = i(q^2 g_{\mu\nu} - q_\mu q_\nu) \Pi(q^2)$$

- ▶ Data driven approach: $e^+e^- \rightarrow hadrons(\gamma)$
 - ▶ Dispersion relation based on unitarity and analyticity relates $\Pi(q^2)$ and $\sigma(e^+e^- \rightarrow had.)$

$$a_{\mu}^{HVP} = \frac{m_{\mu}^2}{12\pi^3} \int_{s_{thr}}^{\infty} ds \frac{K(s)}{s} \sigma(e^+e^- \rightarrow had.(\gamma))$$

- $K(s) \rightarrow 1/s$: $\pi^+\pi^-$ channel is $\sim 70\%$ of the whole contribution and the error budget:

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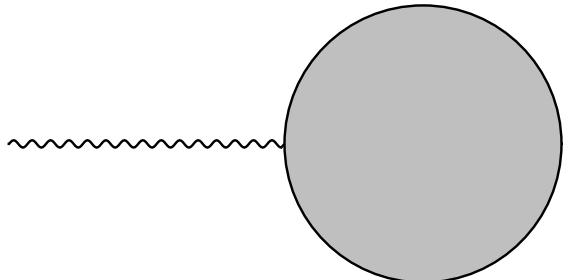
- ▶ Lattice QCD

- *First principle determination of the two point function. No need of data as external input.*

- ▶ Nowadays well established method to evaluate the HVP to Muon $g-2$.

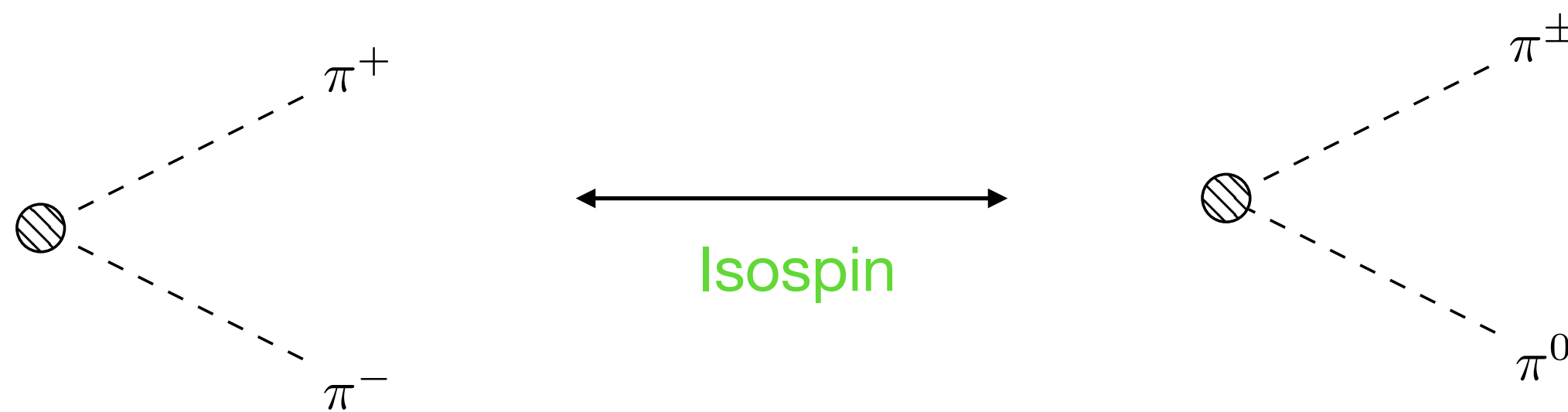
Muon g-2

Hadronic Vacuum Polarization



$$= i(q^2 g_{\mu\nu} - q_\mu q_\nu) \Pi(q^2)$$

- Data driven approach: τ -based evaluation



$$\sigma(e^+e^- \rightarrow \pi^- \pi^+) \propto \frac{d\Gamma[\tau \rightarrow \pi\pi\nu_\tau(\gamma)]}{ds}$$

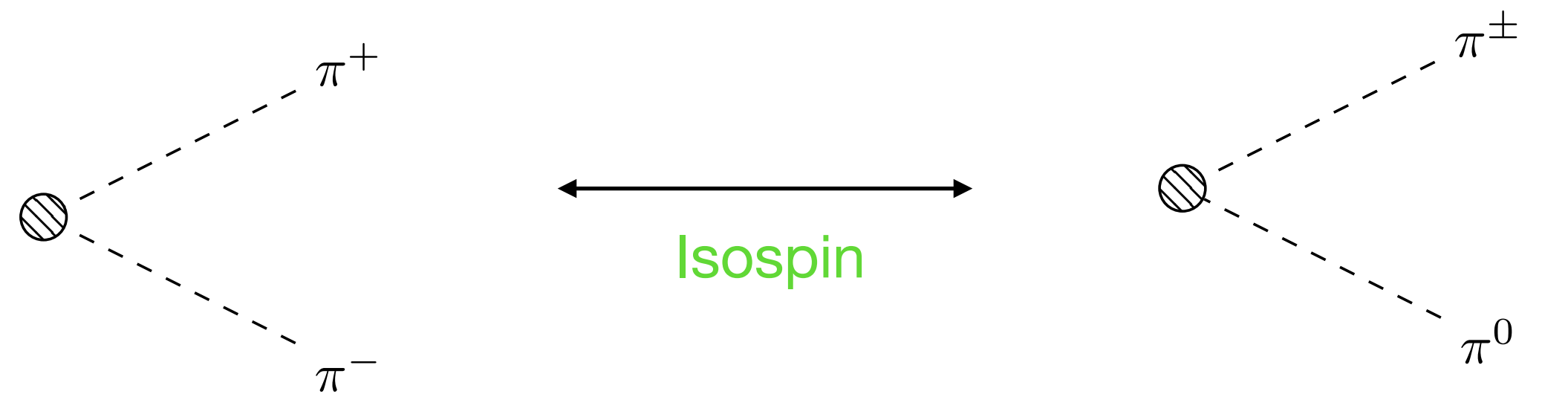
- If isospin is exact:
$$a_\mu^{HVP,LO}[\pi\pi, \tau] = \frac{1}{4\pi^3} \int_{s_{thr}}^{\infty} ds K(s) \left[\frac{K_\sigma(s)}{K_\Gamma(s)} \frac{d\Gamma_{\pi\pi(\gamma)}}{ds} \right]$$

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Hadronic Vacuum Polarization

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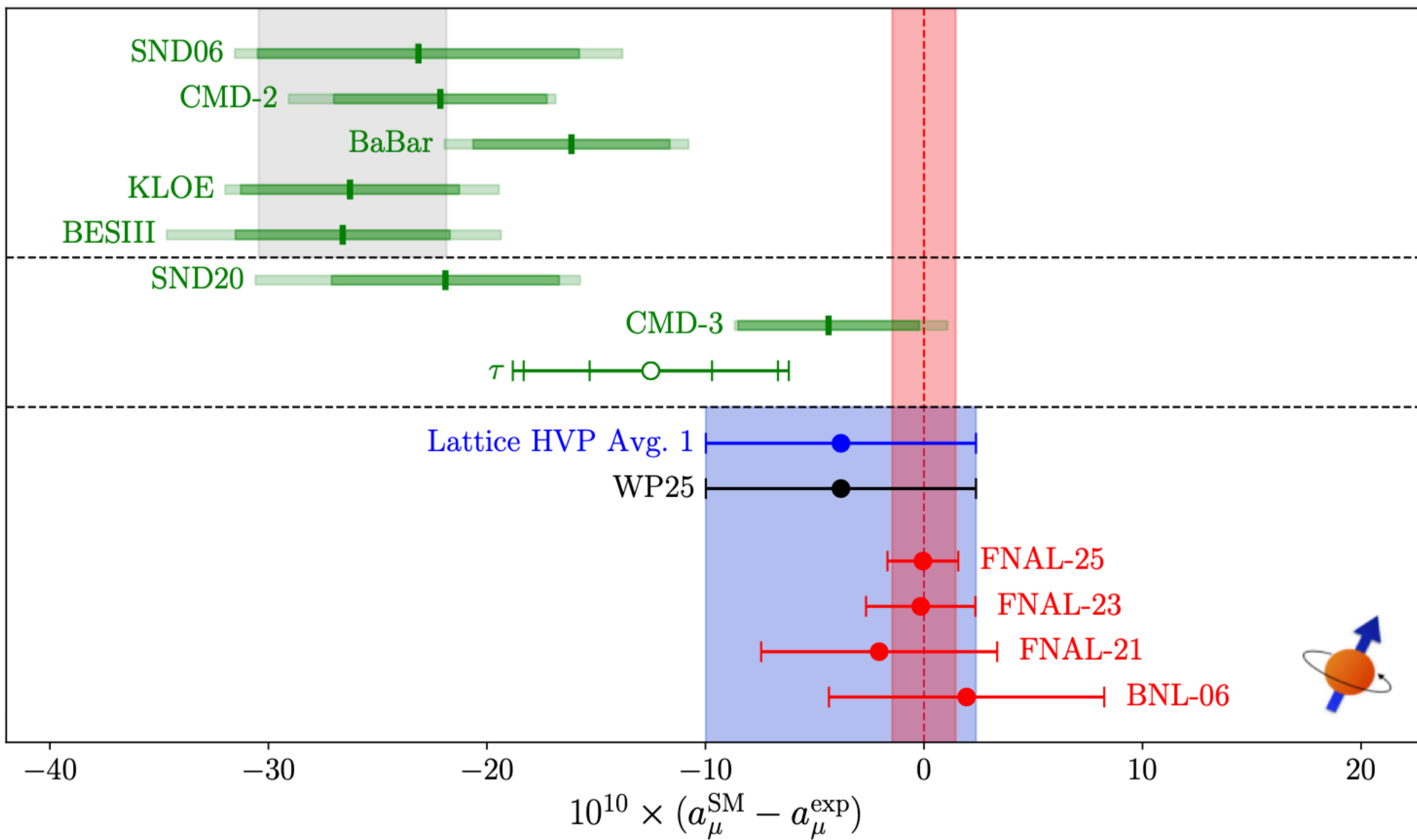
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► $m_u \neq m_d$, $O(\alpha)$ effects break isospin symmetry: **a correction factor is needed!**

$$a_\mu^{HVP,LO}[\pi\pi, \tau] = \frac{1}{4\pi^3} \int_{s_{thr}}^{\infty} ds K(s) \left[\frac{K_\sigma(s)}{K_\Gamma(s)} \frac{d\Gamma_{\pi\pi(\gamma)}}{ds} \right] \times R_{IB}(s)$$

Muon g-2

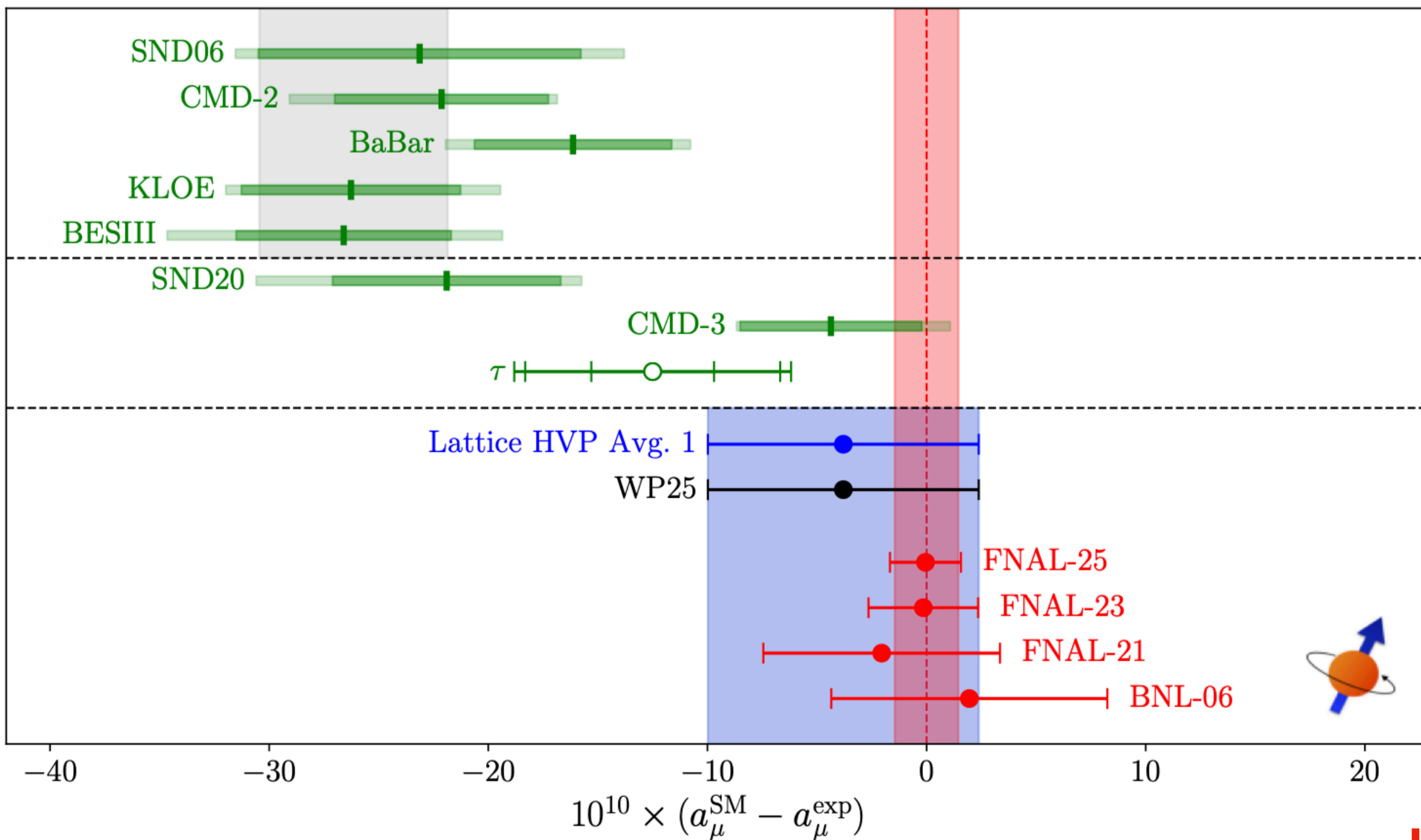
White Paper 25



- *Tensions in $\sigma(e^+e^- \rightarrow \pi^+\pi^-)$ data renders HVP prediction unreliable at present.*
- *Lattice QCD results show a great agreement between theory prediction and experimental measurement*
- *τ data offer an ind. channel to shed light on the Muon g-2 puzzle*

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However

$$R_{IB}(s) = \frac{\text{FSR}(s)}{S_{EW} G_{EM}(s)} \frac{\beta_{\pi^+\pi^-}(s)}{\beta_{\pi^\pm\pi^0}(s)} \left| \frac{F_\pi^V(s)}{f_+(s)} \right|^2$$

- *IB corr. not under control: τ data based HVP is model dependent.*
- *Reducing the model dependence in R_{IB} is crucial!*

Radiative corrections

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Comparison between experiments and theoretical predictions require the evaluation of radiative corrections

Low energy processes typically involve large separation in the relevant energy scale



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Top-down EFT approach

PRO

- *Effective field theory approach allows to factorize short-distance (Wilson coefficients) and long distance (matrix elements) components of an amplitude. $A_{full} = C(\mu) \times A_{EFT}(\mu)$*
- *RGI Pt enables the resummation of large logarithms ($LL \rightarrow (\alpha \log \frac{\mu_f}{\mu_i})^n$; $NLL \rightarrow \alpha^{n+1} (\log \frac{\mu_f}{\mu_i})^n$) in the scale dependent Wilson coefficient $C(\mu)$*

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CONS

- *EFT may introduce scheme dependences: scale and scheme dependence cancel if matching and running are treated consistently*

Chiral Perturbation Theory

ChPT is the natural framework for dealing with light mesons

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- *Light mesons (π, K) (pseudo) Goldstone modes encoded in an exponential representation: e.g. $U = e^{i \frac{\phi}{F_\pi}}$*
- *Lagrangian invariant under global $SU(3)_L \times SU(3)_R$ symmetry + symmetry breaking terms*
- *Effective perturbative expansion in powers of meson momenta p/Λ_χ*

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$$\mathcal{L}_2 = \frac{F_0^2}{4} \text{Tr}[D_\mu U (D^\mu U)^\dagger] + \frac{F_0^2}{4} \text{Tr}(\chi U^\dagger + U \chi^\dagger)$$

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Loop diagrams at $O(p^2)$ generate divergent $O(p^4)$ terms: ren. by LEC at order $O(p^4)$

[Gasser, Leutwyler, *Annals Phys.* 158 (1984) 142]

$$\mathcal{L}_4 = L_1 \left\{ \text{Tr}[D_\mu U (D^\mu U)^\dagger] \right\}^2 + L_2 \text{Tr} [D_\mu U (D_\nu U)^\dagger] \text{Tr} [D^\mu U (D^\nu U)^\dagger] + \dots \quad 12 \text{ operators total}$$

Renormalized LEC expressed in terms of the underlined QCD parameters!

Chiral Perturbation Theory

Adding EM effects must follow the same organizing principle!

[R. Urech, hep-ph/9405341, M. Knecht et al., hep-ph/9909284, B. Moussallam hep-ph/9701400, S. Descote Geno et al., hep-ph/0505077]

- *Electroweak rad. correction break the chiral symmetry: $Q_u \neq Q_d$ and V_{CKM}*
- *Spurion technique to control the symmetry breaking effects, e.g. mass terms*

LEFT

$$\mathcal{L}_{QCD+Fermi} = -eA_\lambda (\bar{\psi}_L Q_L \gamma^\lambda \psi_L + \bar{\psi}_R Q_R \gamma^\lambda \psi_R) + G_F \left(\bar{l}_L \gamma_\lambda \nu_L \bar{\psi}_L Q_W \gamma^\lambda \psi_L + \bar{\nu}_L \gamma_\lambda l_L \bar{\psi}_L Q_W^\dagger \gamma^\lambda \psi_L \right)$$

$$Q_L \rightarrow L Q_L L^\dagger$$

$$Q_R \rightarrow R Q_R R^\dagger$$

$$Q_W \rightarrow L Q_W L^\dagger$$

ChPT at $O(p^2)$

- *Inclusion of leptons and photons to the chiral Lagrangian through covariant derivative*
- *Assign to the weak and electric charge an $O(p)$ to respect chiral counting*

$$D_\mu U(x) = \partial_\mu U(x) + i(r_\mu - eQ_R A_\mu) U(x) - iU(x) \left(l_\mu - eQ_L A_\mu + G_F \left(Q_W \bar{l}_L \gamma_\lambda \nu_L + Q_W^\dagger \bar{\nu}_L \gamma_\lambda l_L \right) \right)$$

Chiral Perturbation Theory

Electromagnetic Chiral LECs renormalize Chiral loops and encode short distance physics

[R. Urech, hep-ph/9405341, M. Knecht et al., hep-ph 9909284, B. Moussallam hep-ph/9701400, S. Descote Geno et al., hep-ph/0505077]

- Build an $O(p^4)$ operators basis with ChPT degrees of freedom + spurions respecting the chiral symmetry
- $O(\alpha)$ effects encoded in Chiral operators with two spurions

$$\begin{aligned}
 \mathcal{L}_\gamma = & \frac{1}{2}K_1F_0^2 \langle D_\mu U D^\mu U^\dagger \rangle \langle Q_L Q_L + Q_R Q_R \rangle + K_2F_0^2 \langle D_\mu U D^\mu U^\dagger \rangle \langle Q_L U^\dagger Q_R U \rangle \\
 & + K_3F_0^2 \left(\langle D_\mu U Q_L U^\dagger \rangle^2 + \langle D_\mu U^\dagger Q_R U \rangle^2 \right) + K_4F_0^2 \langle D_\mu U Q_L U^\dagger \rangle \langle D_\mu U^\dagger Q_R U \rangle \\
 & + K_5F_0^2 \langle D_\mu U^\dagger D^\mu U Q_L Q_L + D_\mu U D^\mu U^\dagger Q_R Q_R \rangle \\
 & + K_6F_0^2 \langle D_\mu U^\dagger D^\mu U Q_L U^\dagger Q_R U + D_\mu U D^\mu U^\dagger Q_R U Q_L U^\dagger \rangle \\
 & + \frac{1}{2}K_7F_0^2 \langle \chi^\dagger U + U^\dagger \chi \rangle \langle Q_L Q_L + Q_R Q_R \rangle + K_8F_0^2 \langle \chi^\dagger U + U^\dagger \chi \rangle \langle Q_L U^\dagger Q_R U \rangle \\
 & + K_9F_0^2 \langle (\chi^\dagger U + U^\dagger \chi) Q_L Q_L + (\chi U^\dagger + U \chi^\dagger) Q_R Q_R \rangle \\
 & + K_{10}F_0^2 \langle (\chi^\dagger U + U^\dagger \chi) Q_L U^\dagger Q_R U + (\chi U^\dagger + U \chi^\dagger) Q_R U Q_L U^\dagger \rangle \\
 & + K_{11}F_0^2 \langle (\chi^\dagger U - U^\dagger \chi) Q_L U^\dagger Q_R U + (\chi U^\dagger - U \chi^\dagger) Q_R U Q_L U^\dagger \rangle \\
 & + K_{12}F_0^2 \langle U D^\mu U^\dagger [D_\mu Q_R, Q_R] + U^\dagger D^\mu U [D_\mu Q_L, Q_L] \rangle \\
 & + K_{13}F_0^2 \langle D_\mu Q_R U D^\mu Q_L U^\dagger \rangle. \\
 \mathcal{L}_{\text{leptons}} = & e^2 \sum_l \left\{ F_0^2 G_F \left[X_1 \bar{l}_L \gamma_\mu \nu_L \langle u^\mu \{ \mathcal{Q}_R, \mathcal{Q}_W \} \rangle + \hat{X}_1 \bar{l}_L \gamma_\mu \nu_L \langle u^\mu \{ \mathcal{Q}_L, \mathcal{Q}_W \} \rangle \right. \right. \\
 & + X_2 \bar{l}_L \gamma_\mu \nu_L \langle u^\mu [\mathcal{Q}_R, \mathcal{Q}_W] \rangle + \hat{X}_2 \bar{l}_L \gamma_\mu \nu_L \langle u^\mu [\mathcal{Q}_L, \mathcal{Q}_W] \rangle \\
 & + X_3 M_l \bar{l}_R \nu_L \langle \mathcal{Q}_R \mathcal{Q}_W \rangle \\
 & \left. + iX_4 \bar{l}_L \gamma_\mu \nu_L \langle \mathcal{Q}_L^\mu \mathcal{Q}_W \rangle + iX_5 \bar{l}_L \gamma_\mu \nu_L \langle \mathcal{Q}_R^\mu \mathcal{Q}_W \rangle + h.c. \right] \\
 & + \hat{X}_6 \bar{l} (i \not{\partial} + e \not{F}) l + \hat{X}_7 M_l \bar{l} l \left. \right\}.
 \end{aligned}$$

Renormalized LEC expressed in terms of the underlined SM parameters!

Chiral Perturbation Theory

Electroweak radiative corrections in ChPT are parametrized via chiral LECs

- *Chiral LECs encode short-distance as well as non perturbative dynamics*
- *Treatment of radiative corrections within ChPT historically limited due to unknown QCD dynamics*
- *NDA or ad hoc models to parametrize chiral LEC: Model dependent results!*

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V. Cirigliano, M. Hoferichter, NV 2026

- *Progresses on lattice nowadays allow for a consistent parametrization of the electromagnetic LECs*
- *Perturbative methods + lattice input required for a consistent treatment of radiative corrections beyond leading logarithms within a ChPT framework*

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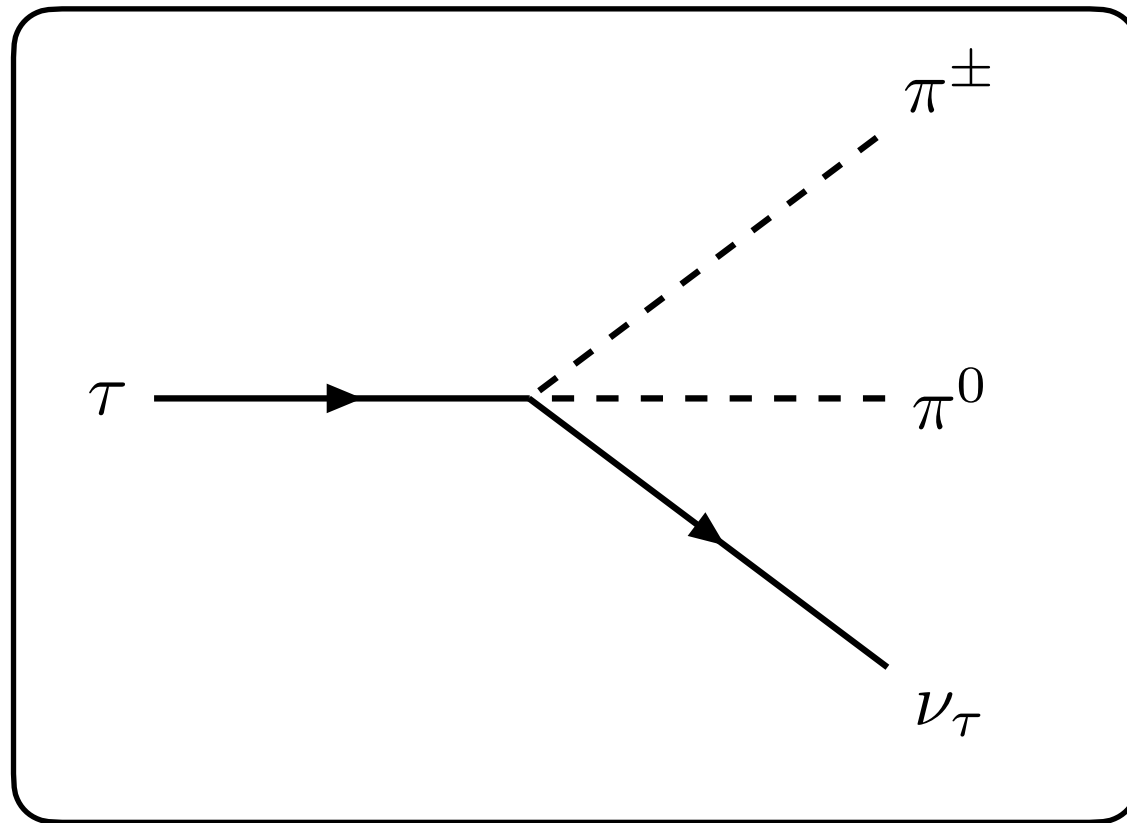
Application to Pion β decay and $\tau \rightarrow \pi \pi \nu$

Why together?

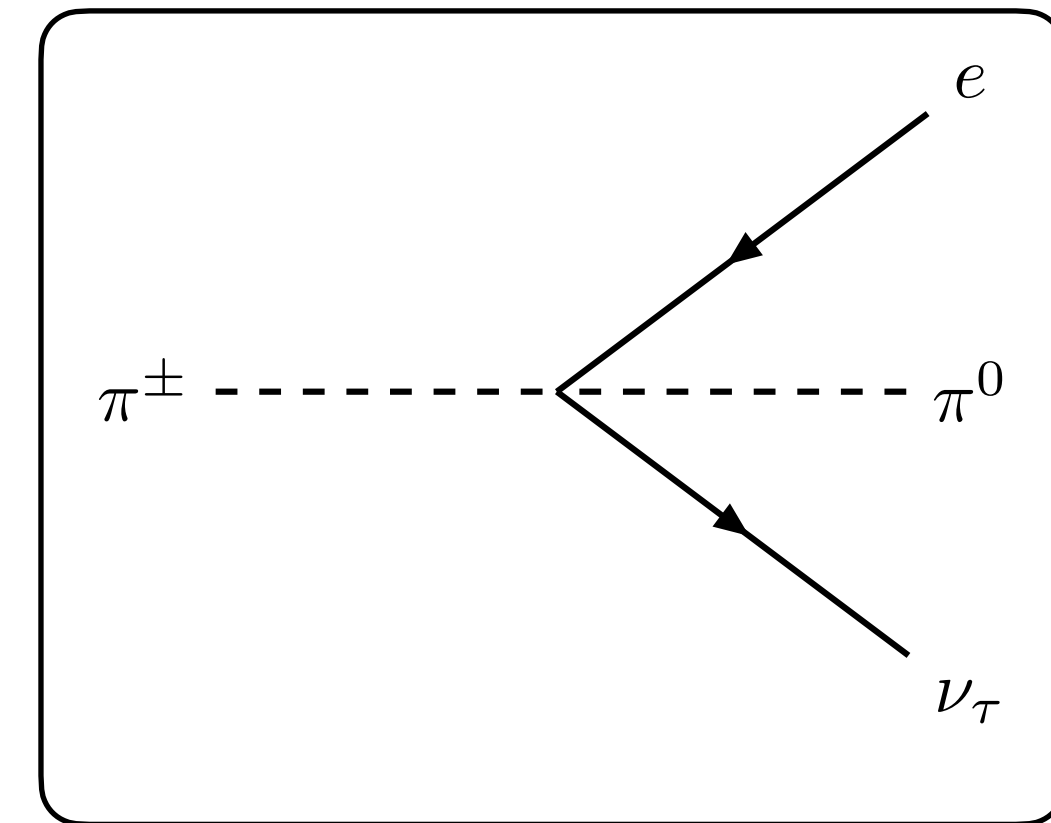
A common framework for both processes

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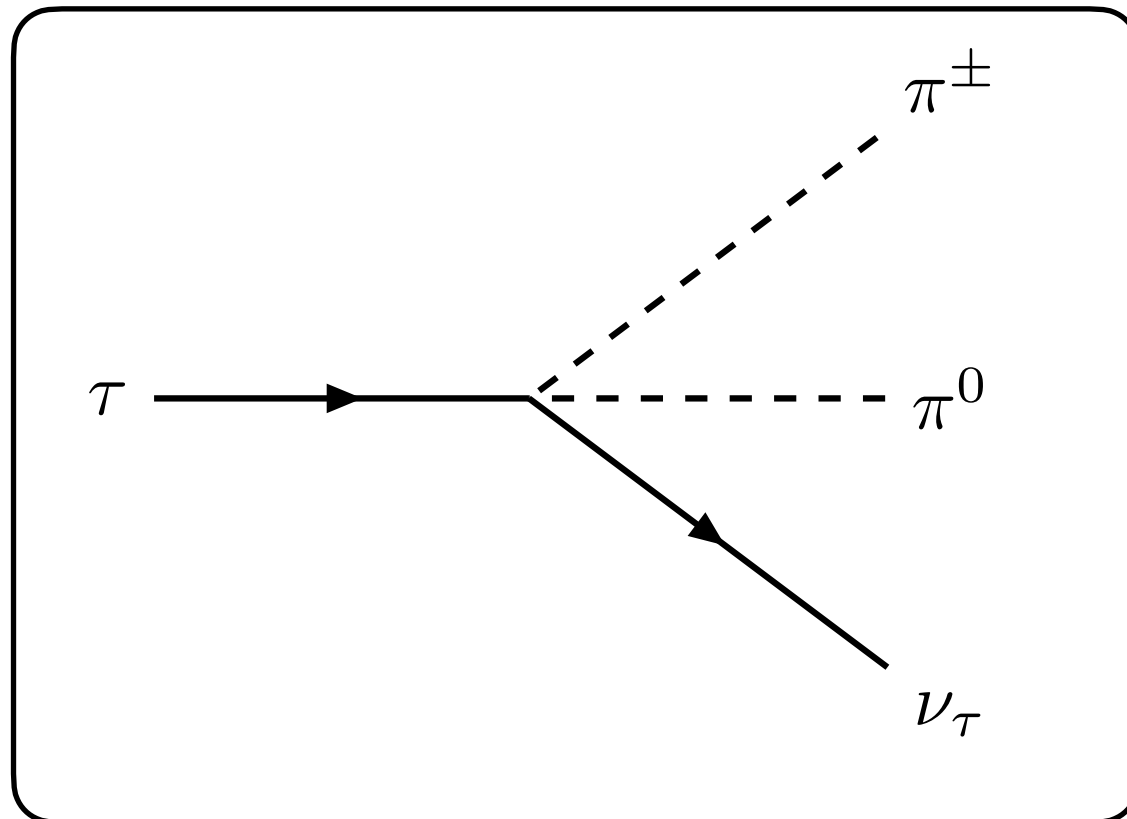



LFU + crossing symmetry

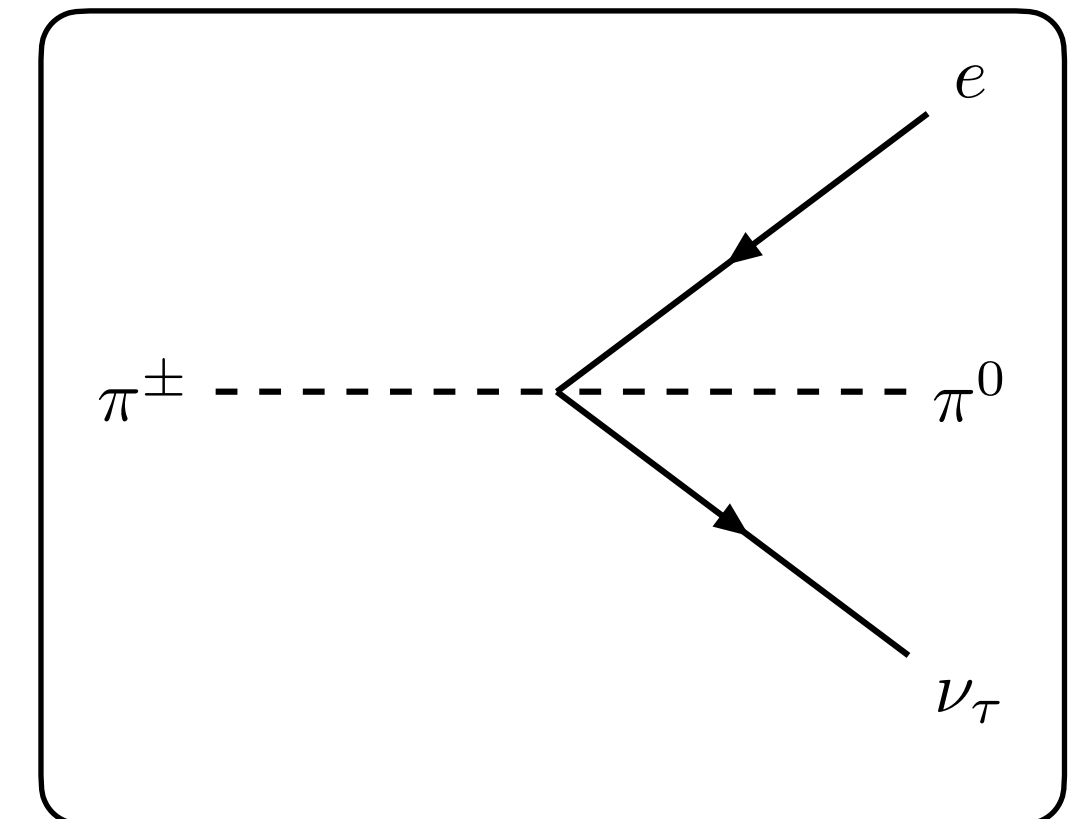


Why together?

A common framework for both processes



↔
LFU + crossing symmetry



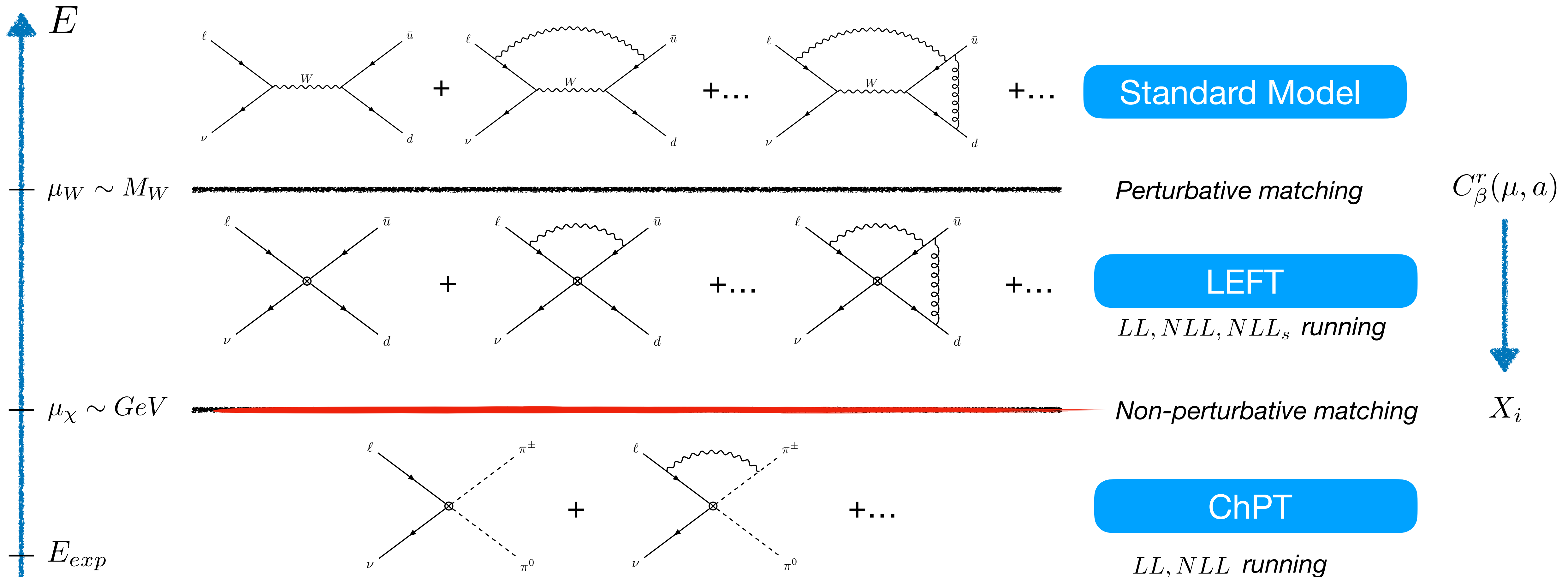
$$\mathcal{L}_{eff} = -\sqrt{2} G_F V_{ud} g_V^\pi(\mu_\chi) (\bar{\ell} \gamma_\mu P_L \nu_\ell) (\partial_\mu \pi^- \pi^0 - \pi^- \partial_\mu \pi^0) + \text{h.c.}$$

• Short distance radiative correction encoded in $g_V^\pi(\mu_\chi)$ factorized as:

$$g_V^\pi(\mu_\chi) = C_\beta^r(\mu, a) \left[1 + 4\pi\alpha \left(2K_{12} - 2\hat{X}_2 - \frac{\hat{X}_6}{2} - \frac{2}{3}(X_1 + \hat{X}_1) \right) \right]$$

A tower of EFTs

Typical multiscale processes involve a series of EFT



LEFT

Above the ChPT scale, the RGE flow is the same as neutron decays: universality

[V. Cirigliano et al. Phys. Rev. D. 108 053033 (2023)] [M.Gorbahn et al., hep-ph/2510.27648]

SM-LEFT matching beyond leading log

$$\mathcal{L}_{LEFT} = -2\sqrt{2}G_F \bar{e}_L \gamma_\rho \mu_L \bar{\nu}_{\mu L} \gamma^\rho \nu_{eL} - 2\sqrt{2}G_F V_{ud} C_\beta^{(f)}(\mu, a) \bar{e}_L \gamma_\rho \nu_{eL} \bar{u}_L \gamma^\rho d_L$$

- Short distance rad. corrections factorized as G_F (muon decays) and $C_\beta^f = 1 + O(\alpha) + O(\alpha\alpha_s)$ (S_{EW} at LL)
- Finite pieces depend on the γ_5 scheme (we use NDR) and evanescence scheme convention

$$\gamma^\alpha \gamma^\rho \gamma^\beta P_L \otimes \gamma_\alpha \gamma_\rho \gamma_\beta P_L = 4(4 - a\epsilon) \gamma^\rho P_L \otimes \gamma_\rho P_L + E(a)$$

Running in LEFT

$$\mu \frac{dC_\beta^{(f)}}{d\mu} = \gamma(\alpha, \alpha_s) C_\beta^{(f)} \quad \gamma(\alpha, \alpha_s) = \gamma_0 \frac{\alpha(\mu)}{4\pi} + \gamma_1^{(f)} \frac{\alpha^2(\mu)}{(4\pi)^2} + \tilde{\gamma}_0 \frac{\alpha\alpha_s(\mu)}{(4\pi)^2} + \tilde{\gamma}_1^{(f)} \frac{\alpha\alpha_s^2(\mu)}{(4\pi)^3}$$



LEFT-ChPT matching

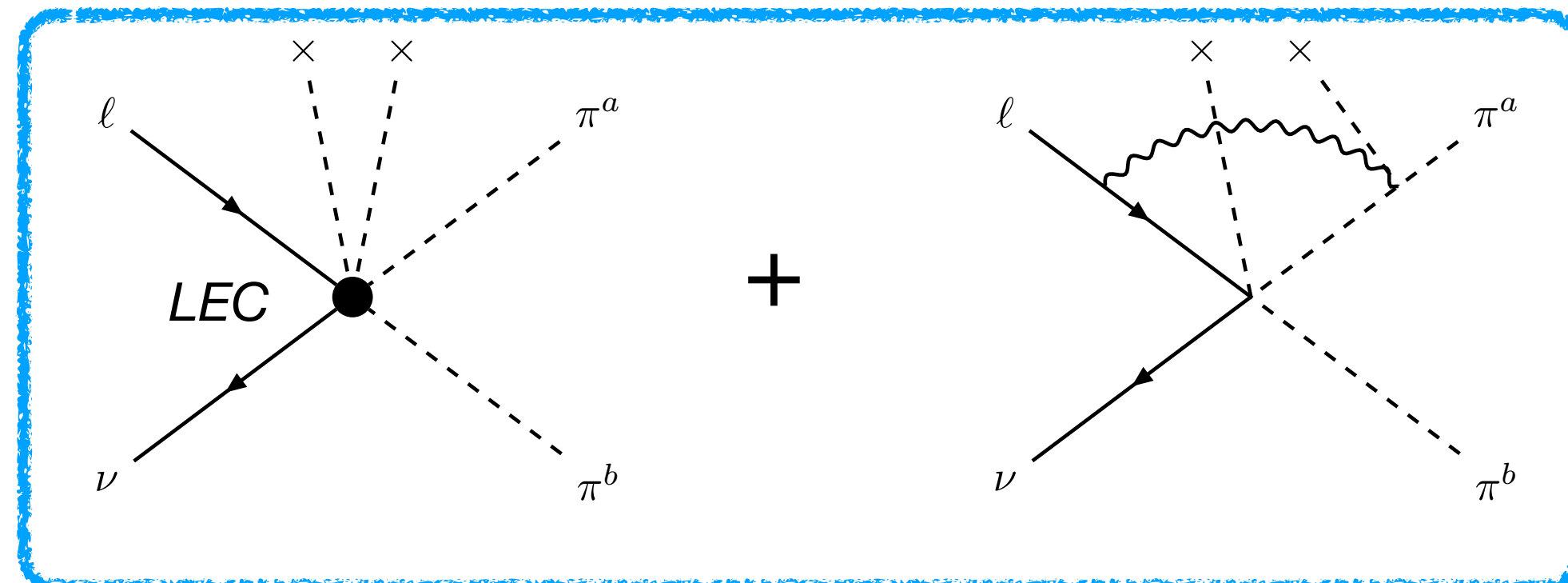
Electroweak rad. correction explicitly break $SU(3)_L \times SU(3)_R$: matching using the spurion method

- ▶ Promote electric charge and V_{CKM} to spurions in ChPT and LEFT
- ▶ Chiral LECs can be individually extracted from one-loop matching at the level of green functions, obtained by differentiating the generating functional with respect to the electric and weak spurions

$$\text{e.g. } \int d^4x \langle l(p) \bar{\nu}(q) \pi^d(s) \left| \frac{\delta^2 W(Q_L, Q_R, Q_W)}{\delta Q_{R(L)}^c \delta Q_W^d} \right| \pi^a(r) \rangle$$

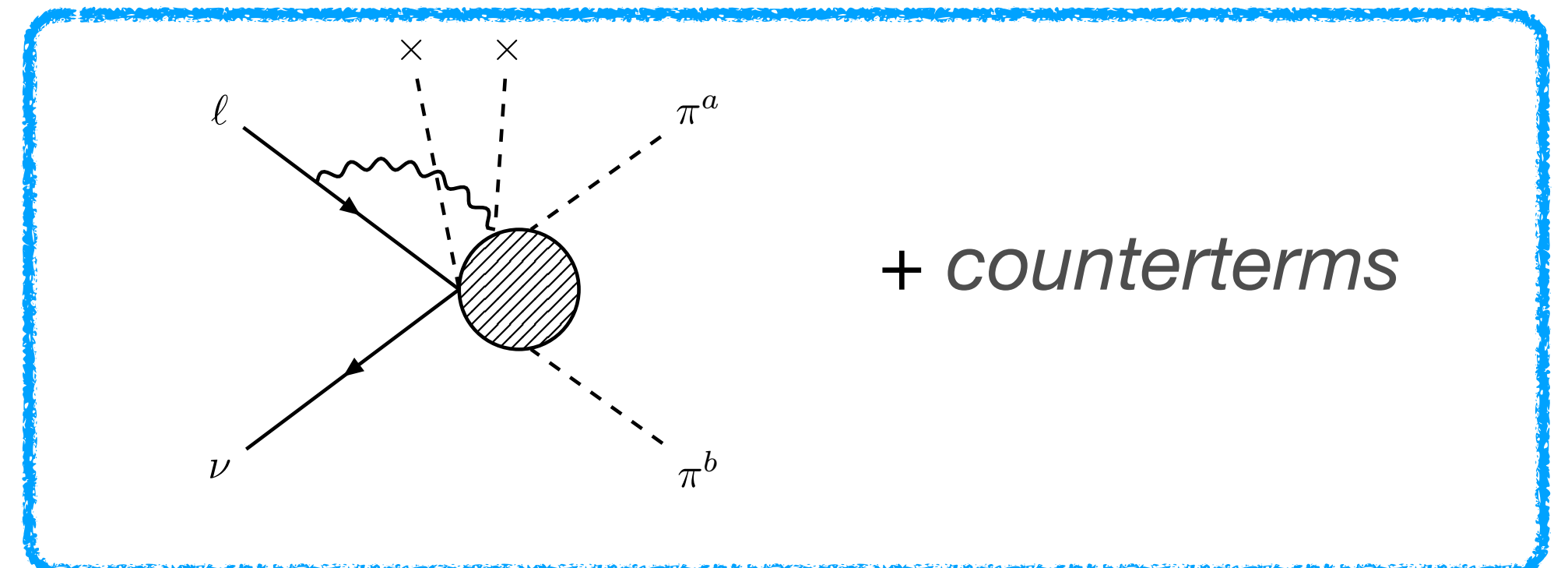
- ▶ Equating $O(\alpha)$ terms in ChPT and LEFT

ChPT



=

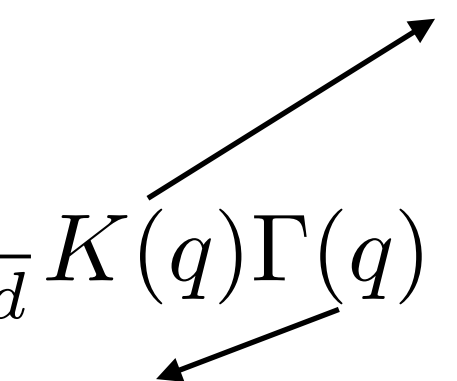
LEFT



LEFT-ChPT matching

LEFT-ChPT matching

Kernel involving lepton/photon propagators

$$LEC^{(r)}(\mu, \mu_\chi, a) = -f_{ChPT}^{(loop)}(\mu_\chi) + g_{LEFT}^{CT} + \int \frac{d^d q}{(2\pi)^d} K(q) \Gamma(q)$$


Correlation functions of QCD currents in the meson system

$$\int d^4 x e^{iqx} \langle \pi^a(p) | T \{ J_\mu^{em}(x) J_\nu^{W,A}(0) \} | \pi^b(p) \rangle$$

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- OPE result in dim. reg. introduce scheme dependence that will cancel the one in $C_\beta^f(\mu, a) = 1 + O(\alpha)$

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$$\bar{\Gamma}(q) = \Gamma(q) - \Gamma(q)|_{UV}$$

- $\int \frac{d^4 q}{(2\pi)^4} K(q) \bar{\Gamma}(q)$ is UV finite: It can be evaluated in $d=4$ by using Lattice data

Short-distance contribution

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“Inner” corrections at $O(\alpha, \alpha\alpha_s)$

$$g_V^\pi(\mu_\chi) \equiv \bar{C}_\beta^{(3)}(\mu) \left[1 + \square_\pi^{V(3)}(\mu_0) + \frac{\alpha^{(3)}(\mu)}{4\pi} \left(\frac{17}{6} - \frac{3}{2} \log \frac{\mu_\chi^2}{\mu_0^2} + 2 \log \frac{\mu^2}{\mu_0^2} \right) + \frac{\alpha \alpha_s^{(3)}(\mu)}{(4\pi)^2} \left(-\frac{130}{81} - 2 \log \frac{\mu^2}{\mu_0^2} \right) \right].$$

Factorization into
scheme independent WC

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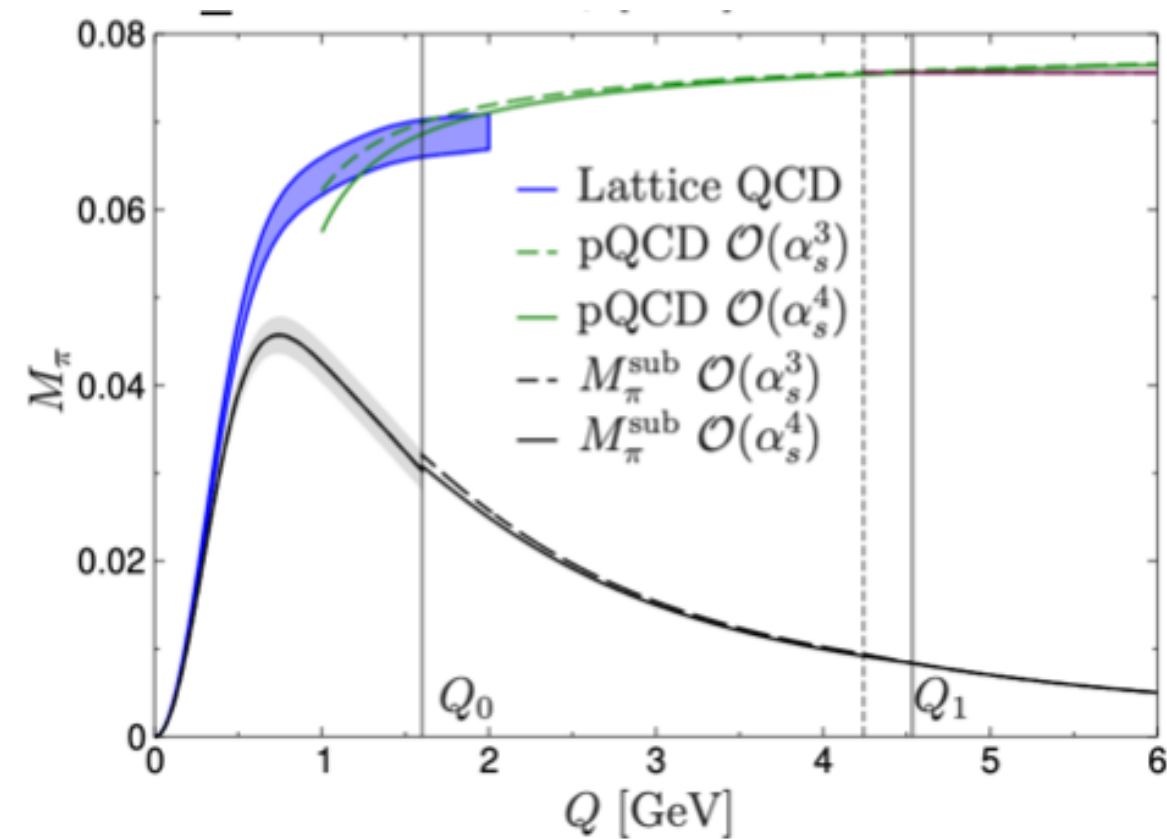
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$$\bar{\square}_\pi^V(\mu_0) = \frac{3\alpha}{2\pi} \int_0^\infty \frac{dQ^2}{Q^2} \left[M_\pi(Q^2) - \frac{1}{12} \frac{Q^2}{Q^2 + \mu_0^2} \left(1 - \frac{\alpha_s(\mu_0)}{\pi} \right) \right].$$



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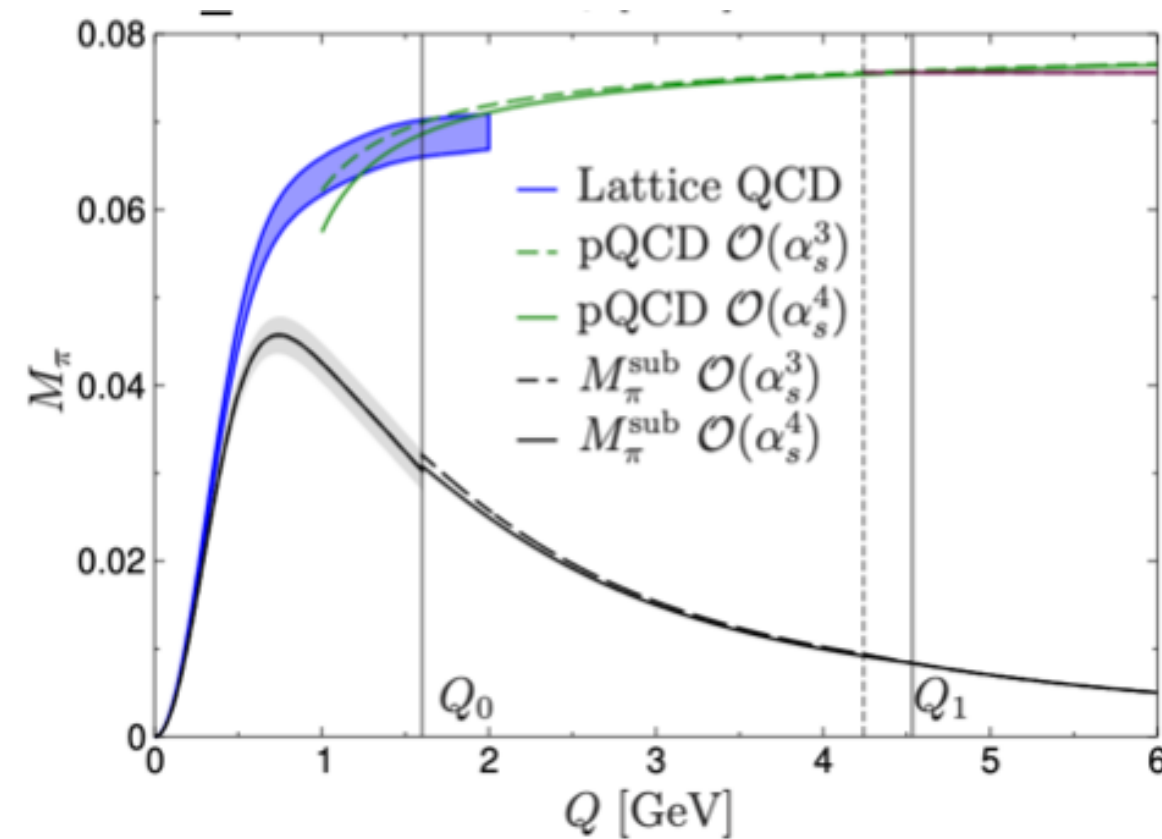
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Universal to all β -decay: no dependence on the spin or mass
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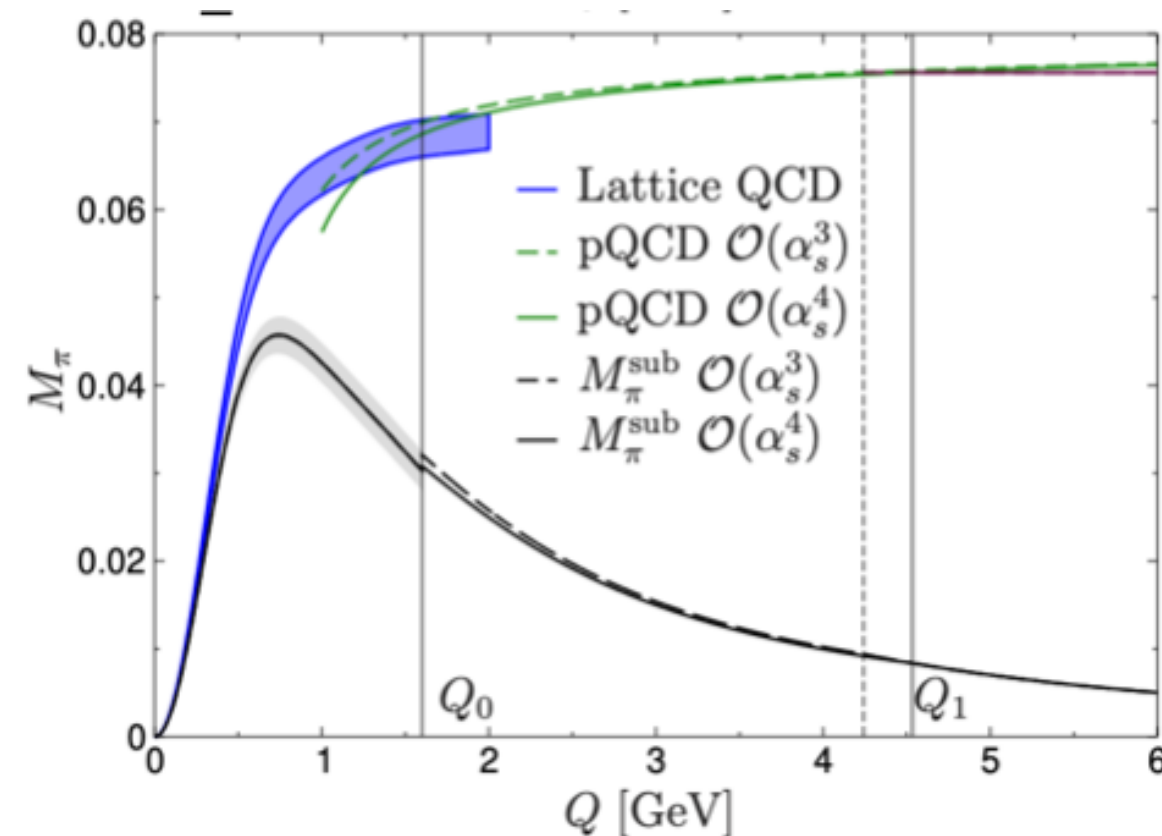
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Adapted from neutron decay

[M.Gorbahn et al., hep-ph/2510.27648]

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Application

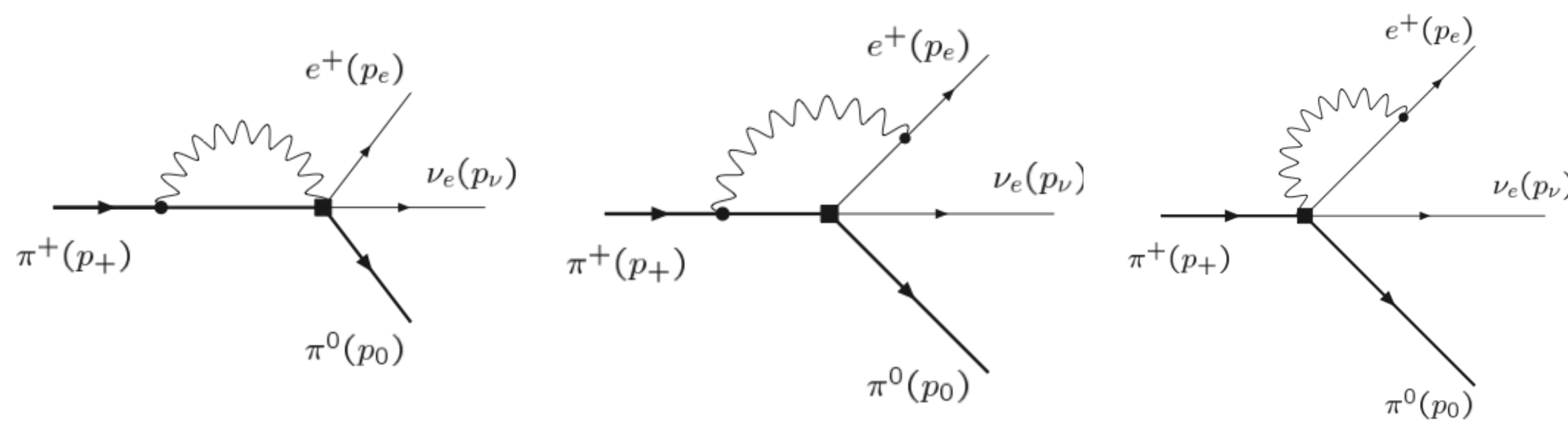
Pion β Decay

$$\Gamma[\pi^+ \rightarrow \pi^0 e^+ \nu_e(\gamma)] = \frac{G_F^2 |V_{ud}|^2 M_\pi^5 |f_+^\pi(0)|^2}{64\pi^3} (1 + \Delta_{\text{RC}}^{\pi\ell}) I_{\pi\ell}.$$

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- Long-distance radiative correction evaluated in ChPT : Cirigliano et al., hep-ph/0209226



$$X_i \sim \frac{1}{16\pi^2} + 100\% \text{ uncertainty}$$

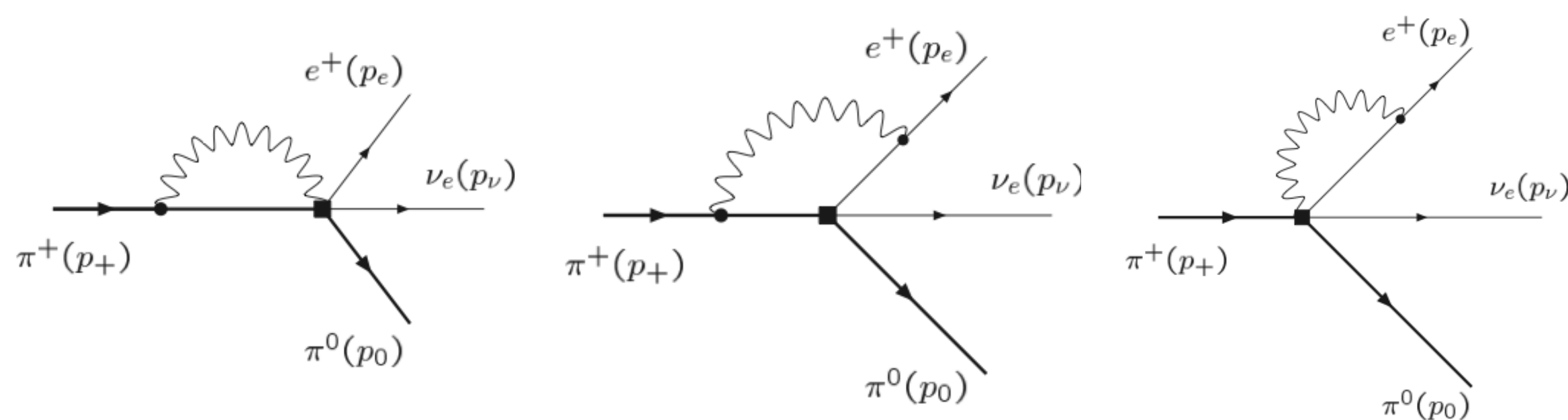
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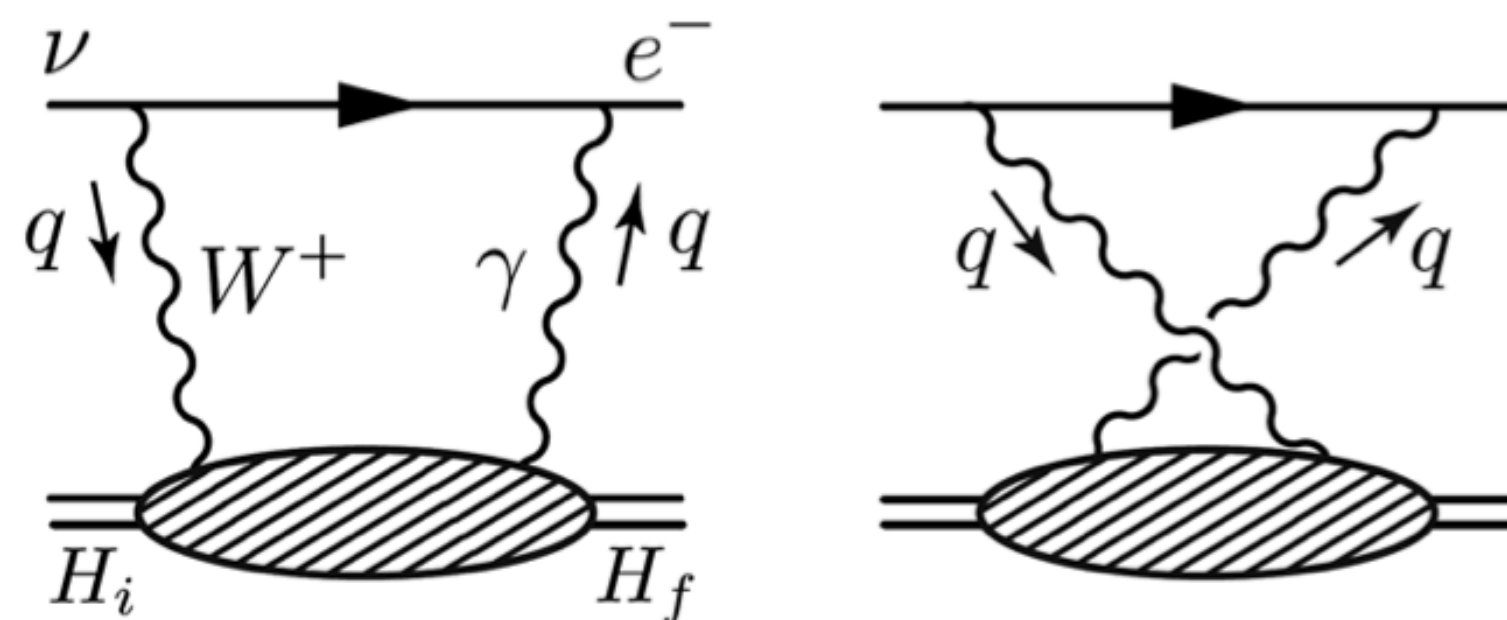


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- Radiative correction evaluated á la Sirlin + lattice input: A.Sirlin, Rev. Mod. Phys. 50 573 (1978]
X. Feng et al., hep-lat/2003.09798



Fixed order approach with SM dof

$$\Delta_{RC}^{\pi\ell} = 0.0332(2)$$

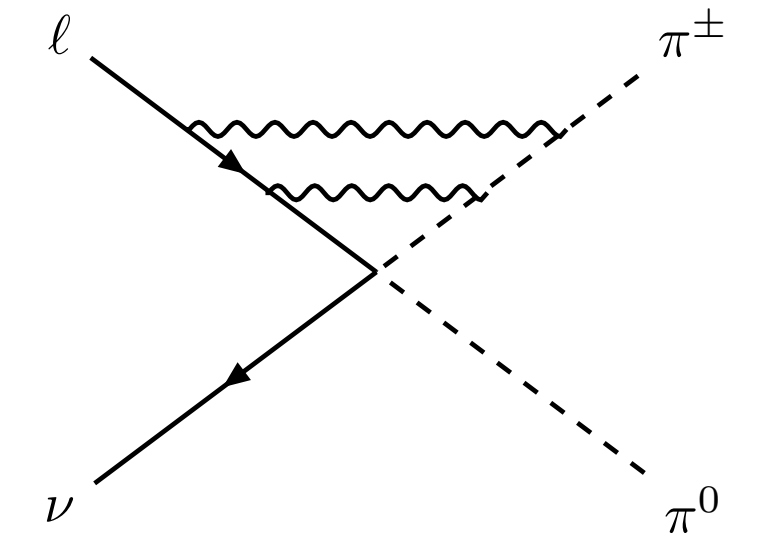
$$V_{ud}^{\pi, \text{LQCD}} = 0.97386(281)_{Br(9)\tau_\pi} (14)_{\Delta_{RC}^{\pi\ell}} (28)_{I_{\pi\ell}}$$

Pion β Decay

V. Cirigliano, M. Hoferichter, NV 2026

- ▶ Radiative correction in ChPT + lattice input for short-distance matching
- ▶ Large log resummation beyond leading log accuracy from M_W to end point energy
- ▶ Typical energy scale of pion β decay: $\text{MeV} \ll \Lambda_\chi$

Two-loop anomalous dimension



[V. Cirigliano et al. Phys. Rev. D. 108 053033 (2023)]

$\Delta_{RC}^{\pi\ell} = 0.03403(11)$: Large shift of 2.6σ in the central value compared to X. Feng et al., hep-lat/2003.09798:

Unaccounted effects from large log resummation in ChPT at LL $\sim \frac{\alpha^2}{\pi^2} \log^2 \frac{M_\rho}{m_e}$

$V_{ud}^{\pi, CHV} = 0.97386(281)_{Br}(9)_{\tau\pi} (5)_{\Delta_{RC}^{\pi\ell}} (28)_{I_{\pi\ell}}$: Rad correction uncertainty slashed by a factor of 3!

PIONEER final goal: $(281)_{Br} \rightarrow \sim (30)_{Br}$: theory uncertainties negligible beyond the PIONEER reach!

HVP contribution to Muon g-2

$$R_{IB}(s) = \frac{\text{FSR}(s)}{S_{EW} G_{EM}(s)} \frac{\beta_{\pi^+\pi^-}(s)}{\beta_{\pi^\pm\pi^0}(s)} \left| \frac{F_\pi^V(s)}{f_+(s)} \right|^2$$

- Radiative corrections to τ decays break isospin $\rightarrow$ must be subtracted
- Rad correction typically separately encoded in S_{EW} (short-distance) and G_{EM} (long-distance)

$$\Delta a_\mu^{HVP,LO}[\pi\pi, \tau] = \frac{1}{4\pi^3} \int_{s_{thr}}^{\infty} ds K(s) \left[\frac{K_\sigma(s)}{K_\Gamma(s)} \frac{d\Gamma_{\pi\pi(\gamma)}}{ds} \right] \times (r_{IB}(s) - 1)$$

$$r_{IB} = \left\{ \frac{[\beta_{\pi\pi}]^3}{[\beta_{\pi\pi^0}]^3} \times \frac{1}{S_{EW}} \times \frac{1}{G_{EM}(s)} \right\}$$


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However

- Short and long-distance correction separately scale and scheme dependent beyond leading log approx

$$S_{EW} \rightarrow C_\beta^r(\mu, a)$$

- Phase space beyond the ChPT energy regime: ChPT works only in the low energy region!

HVP contribution to Muon $g-2$

$$\frac{d\Gamma[\tau^\pm \rightarrow \pi^\pm \pi^0 \nu_\tau(\gamma)]}{ds} = K_\Gamma(s) [\beta_{\pi\pi^0}(s)]^3 |f_+(s)|^2 S_{\text{EW}}^{\pi\pi} G_{\text{EM}}(s).$$

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- ChPT-like strategies for rad.corr. need to be corrected within specific models: [V. Cirigliano et al. JHEP, (2002), Flores-Baez et al, hep-ph/0608084, Miranda, Roig, hep-ph/2007.11019]
- Novel approach based on dispersive parametrization of the FF allows for a model independent quantification of G_{EM} [G. Colangelo et al., Phys. Rev. Lett, 136 (2026) 10]: retain the ChPT description in the low energy region to enforce RGE effects

$$\Delta a_\mu^{\text{HVP,LO}}[\pi\pi, \tau] = -24.8(0.1)_{\text{exp}}(0.5)_{\text{th}}(1.3)_{\text{SD}} \times 10^{-10}$$

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V. Cirigliano, M. Hoferichter, NV 2026

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Negligible unc. From the short-distance matching: significant step forward in the τ based determination of HVP

Conclusion

General result

- *First full EFT analysis of semileptonic processes involving pions beyond leading log in ChPT*
- *Explicit model independent expression of the Chiral LECs up to order $O(\alpha, \alpha\alpha_s)$*

V_{ud} extraction

- *Reduction of theory uncertainty from radiative corrections by a factor of 3*
- *Theory uncertainty negligible for future determination of V_{ud} based on Pion beta decay*

Muon g-2

- *Elimination of scheme dependence ambiguity in $G_{EM} \times S_{EW}$: negligible uncertainty from the short-distance matching*
- *Improved robustness for HVP contribution to Muon g-2 based on τ data*

Back up

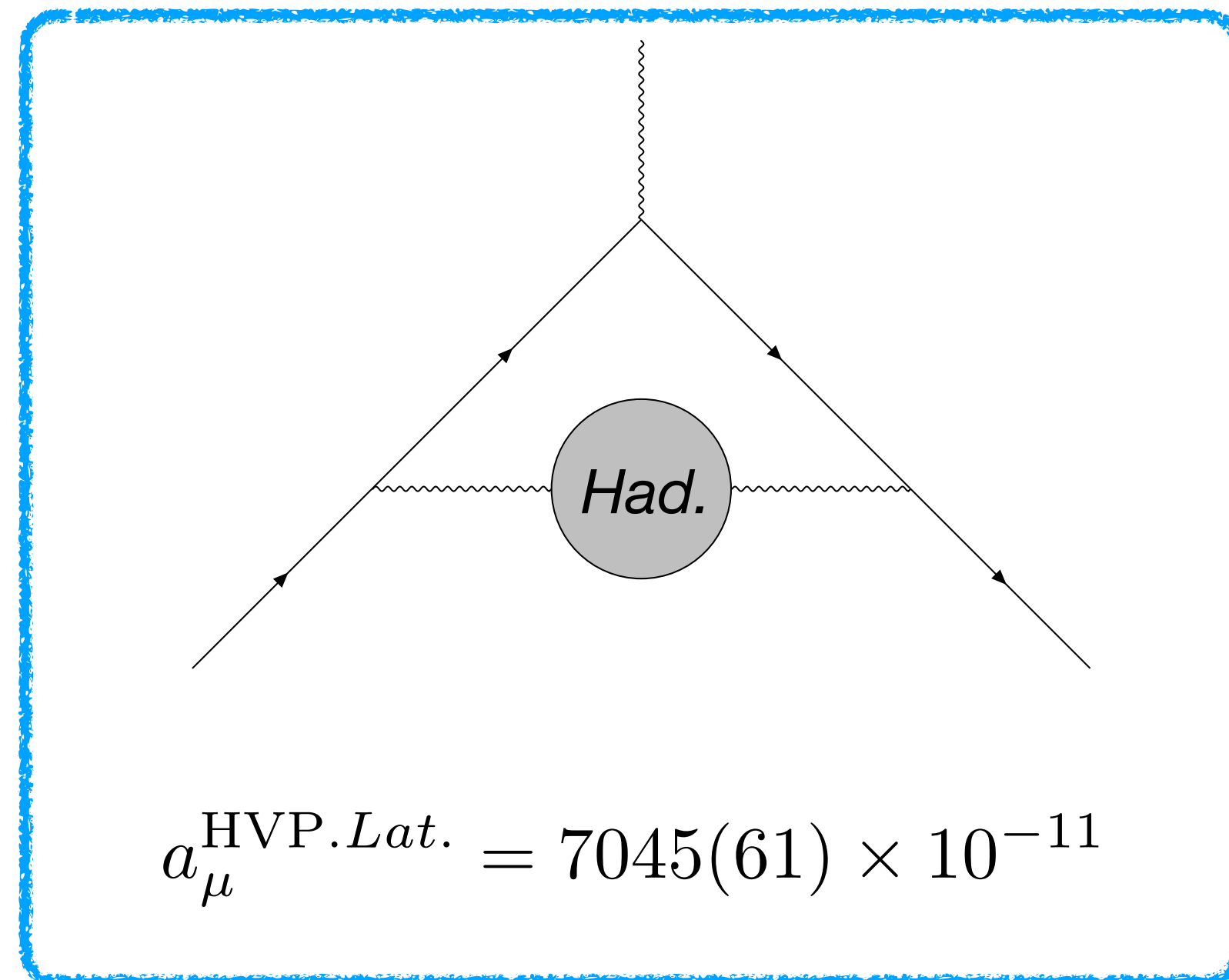
Muon g-2

Hadronic contribution to Muon g-2 [WP., 2025]

- *Perturbation theory fails for hadronic contribution in the low energy region*
- *Alternative methods must be employed to assess the hadronic contribution to a_μ^{Had}*

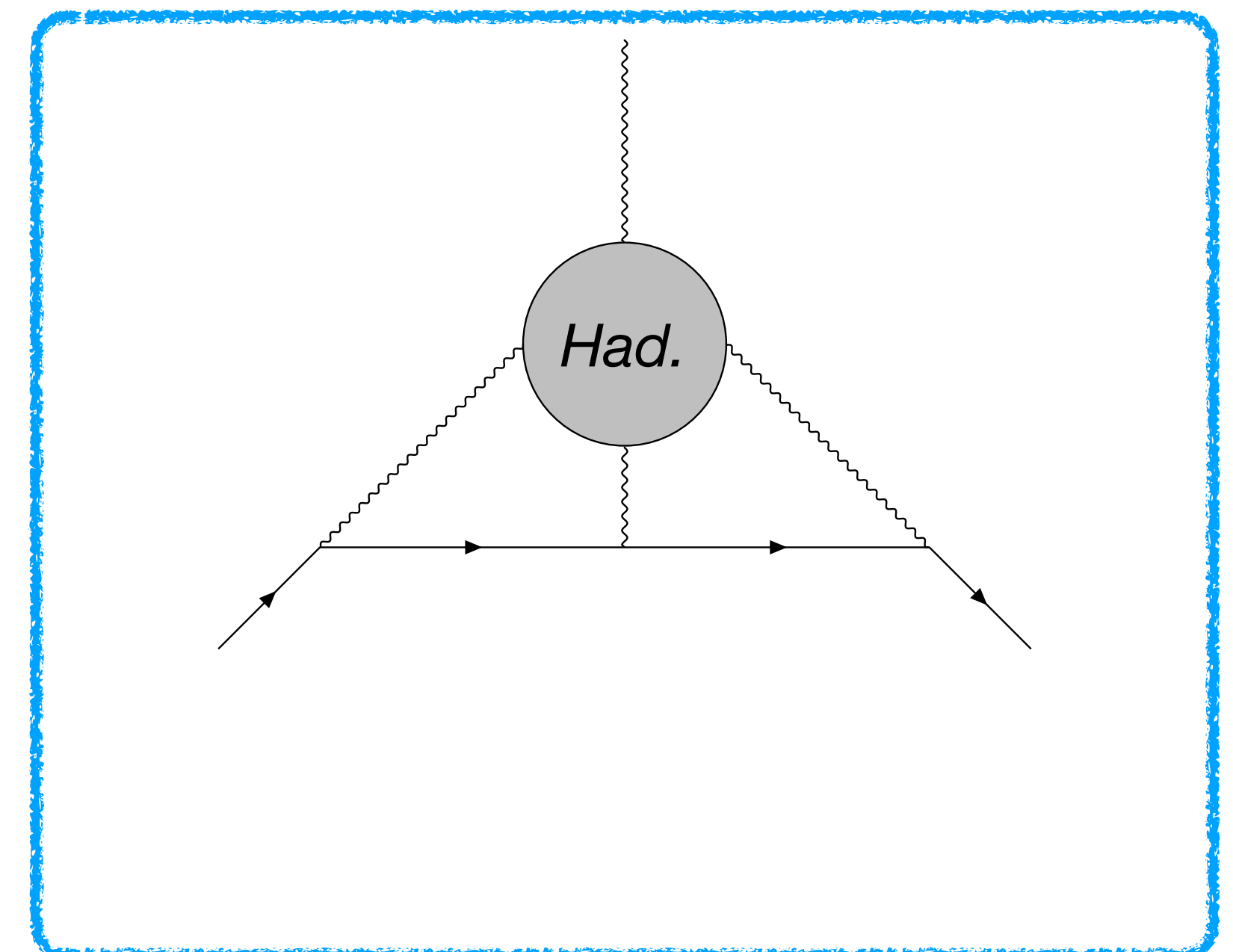
Two classes of diagrams

HVP



*Dominates the th. uncertainty

HLbL



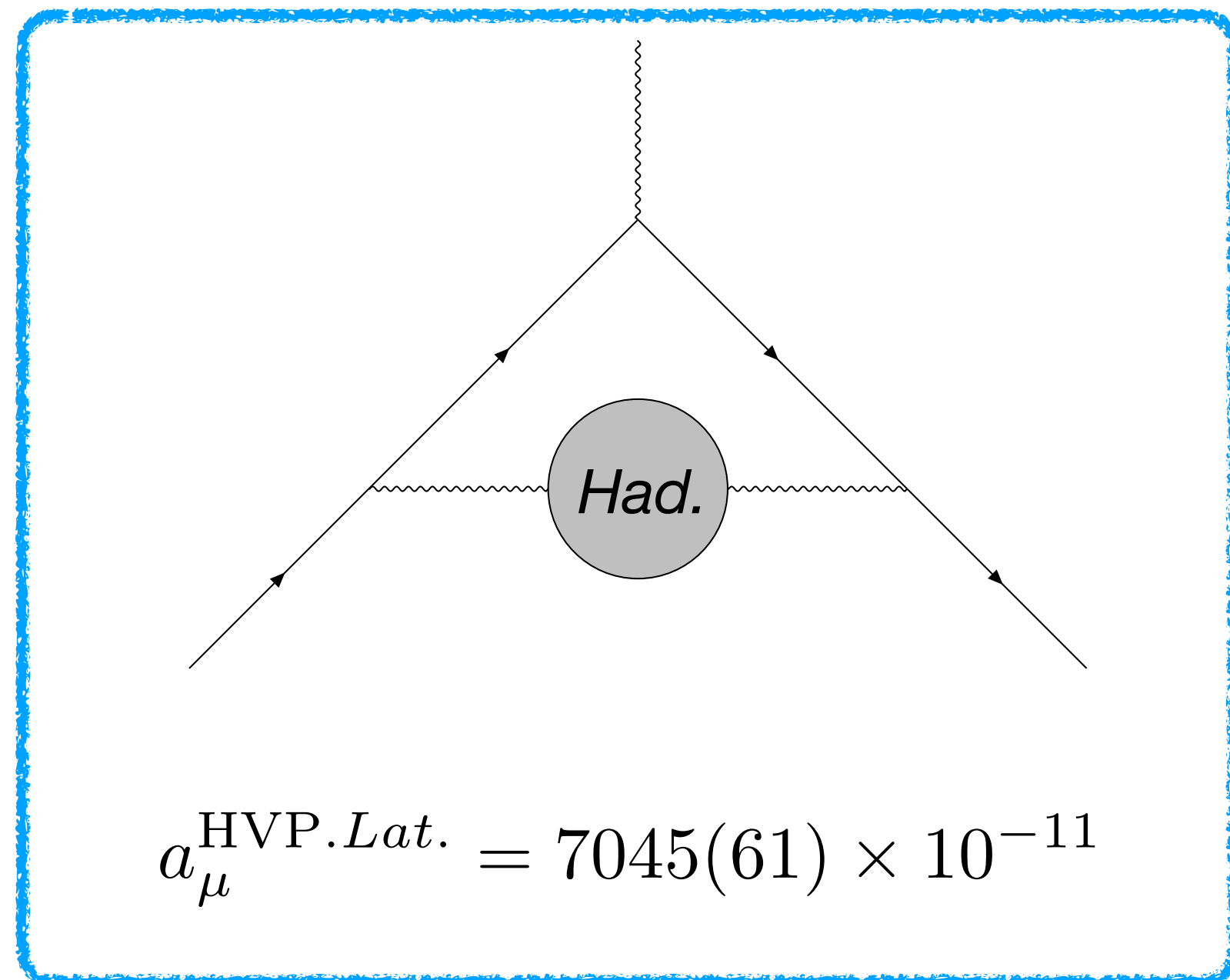
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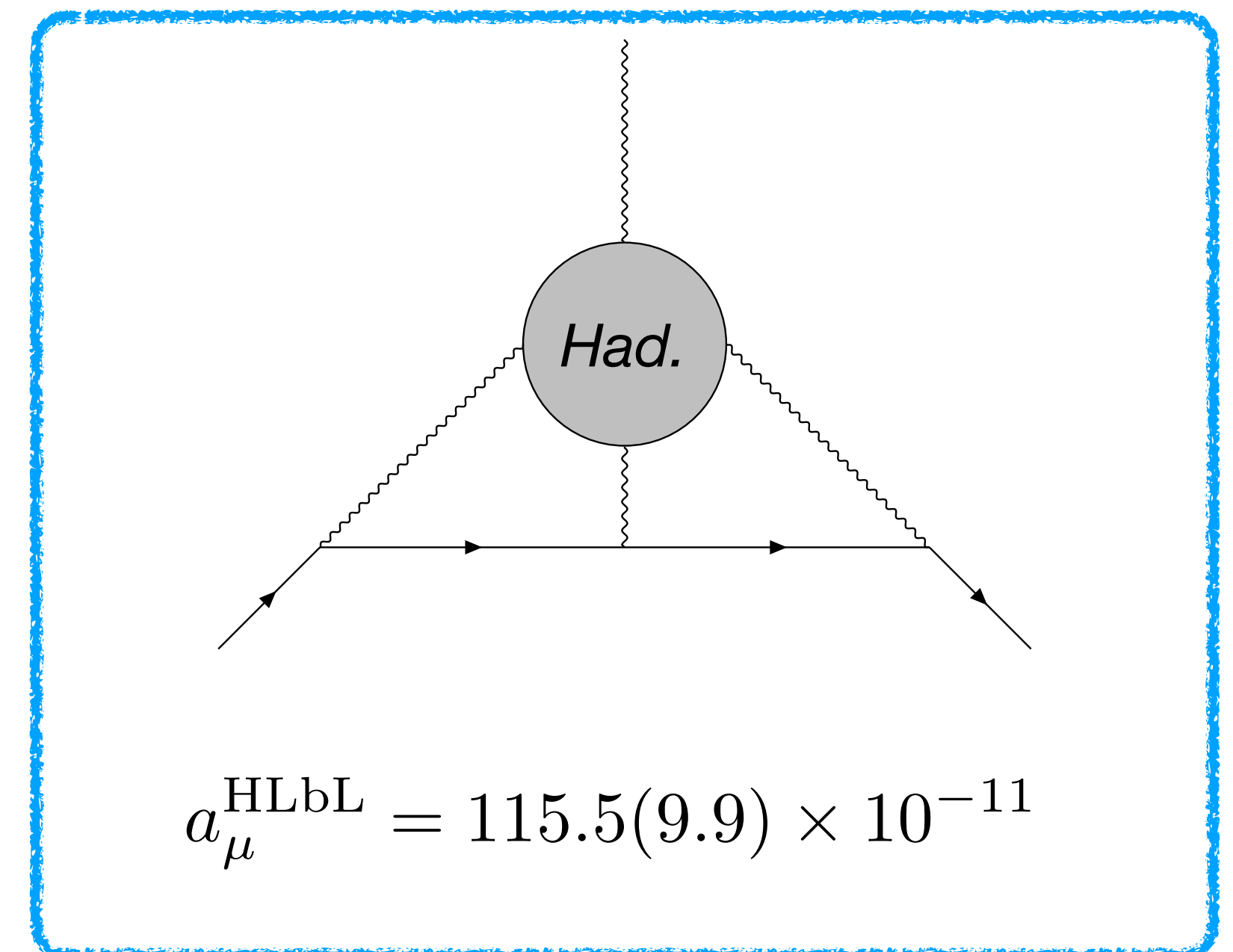
Two classes of diagrams

HVP



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LEFT-ChPT matching

Electroweak rad. correction explicitly break $SU(3)_L \times SU(3)_R$: matching using the spurion method

Spurion method

(a) Identify LEFT Lagrangian terms that break $SU(3)_L \times SU(3)_R$

(b) Promote parameters that break $SU(3)_L \times SU(3)_R$ to dynamical fields (spurions)

(c) Impose spurion transformations such that the chiral symmetry is restored.

(d) Build an operator basis in ChPT with spurions and light dof which is invariant under $SU(3)_L \times SU(3)_R$

e.g.

Mass term

(a) quark mass terms explicitly break $SU(3)_L \times SU(3)_R$

$$\mathcal{L}_M = -\bar{q}_R M q_L - \bar{q}_L M^\dagger q_R, \quad M = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix}$$

(b + c) $M \mapsto R M L^\dagger$ chiral symmetry is restored

(d) At leading order in ChPT:

$$\mathcal{L}_{\text{s.b.}} = \frac{F_0^2 B_0}{2} \text{Tr}(M U^\dagger + U M^\dagger) \quad U = e^{i \frac{t^a \phi^a(x)}{F_0}}$$

LO expansion and M to physical value:

$$\text{Tr}[\phi^2 M] = 2(m_u + m_d)\pi^+\pi^- + (m_u + m_d)\pi^0\pi^0 + \dots$$

LEFT–ChPT matching

Electroweak rad. correction explicitly break $SU(3)_L \times SU(3)_R$: matching using the spurion method

Spurion method

[S. Descote Genon et al., hep-ph/0505077]

[V. Cirigliano et al. Phys. Rev. D. 108 053033 (2023)]

(a) *Identify LEFT Lagrangian terms that break $SU(3)_L \times SU(3)_R$*

$$\mathcal{L}_{QCD+Fermi} = -eA_\lambda (\bar{\psi}_L Q_L \gamma^\lambda \psi_L + \bar{\psi}_R Q_R \gamma^\lambda \psi_R) + G_F \left(\bar{l}_L \gamma_\lambda \nu_L \bar{\psi}_L Q_W \gamma^\lambda \psi_L + \bar{\nu}_L \gamma_\lambda l_L \bar{\psi}_L Q_W^\dagger \gamma^\lambda \psi_L \right)$$

$$Q_{L,R} = \text{diag}(2/3, -1/3, -1/3) \qquad Q_W = \frac{-4}{\sqrt{2}} \begin{pmatrix} 0 & V_{ud} & V_{us} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

(b) *Promote parameters that break $SU(3)_L \times SU(3)_R$ to dynamical fields (spurions)*

$$Q_i(x) = \frac{\tau^a}{2} Q_i^a(x) \text{ with } \tau^a \text{ Gell-Mann matrices}$$

(c) *Impose spurion transformations such that the chiral symmetry is restored*

$$Q_L \rightarrow L Q_L L^\dagger$$

$$Q_R \rightarrow R Q_R R^\dagger$$

$$Q_W \rightarrow L Q_W L^\dagger$$

LEFT-ChPT matching

Electroweak rad. correction explicitly break $SU(3)_L \times SU(3)_R$: matching using the spurion method

Spurion method

(d) *Build an operator basis in ChPT with spurions and light dof which is invariant under $SU(3)_L \times SU(3)_R$*

I) *Write down the ChPT Lagrangian at $O(p^2)$ with electroweak currents: it encodes IR physics*

II) *Write down an operator basis $O(e^2 p^2)$ whose Wilson coefficients (LECs) encode short-distance physics*

$$O(p^2)$$

I) *Low energy ChPT lagrangian $O(p^2)$ with the introduction of external vector sources:*

$$\mathcal{L}^{(2)} = \frac{F_0^2}{4} \text{Tr}[D^\mu U^\dagger D_\mu U]$$

$$D_\mu U(x) = \partial_\mu U(x) + i(r_\mu - eQ_R A_\mu)U(x) - iU(x)\left(l_\mu - eQ_L A_\mu + G_F\left(Q_W \bar{l}_L \gamma_\lambda \nu_L + Q_W^\dagger \bar{\nu}_L \gamma_\lambda l_L\right)\right)$$

LEFT–ChPT matching

Electroweak rad. correction explicitly break $SU(3)_L \times SU(3)_R$: matching using the spurion method

Spurion method

(d) Build an operator basis in ChPT with spurions and light dof which is invariant under $SU(3)_L \times SU(3)_R$
 $O(e^2 p^2)$

II) Relevant operators basis $O(e^2 p^2)$ [R. Urech, hep-ph/9405341, M. Knecht et al., hep-ph/9909284, S. Descote Geno et al., hep-ph/0505077]

$$\begin{aligned} \mathcal{L}_{\text{leptons}} = e^2 \sum_l \Big\{ & F_0^2 G_F \left[X_1 \bar{l}_L \gamma_\mu \nu_L \langle u^\mu \{ \mathcal{Q}_R, \mathcal{Q}_W \} \rangle + \hat{X}_1 \bar{l}_L \gamma_\mu \nu_L \langle u^\mu \{ \mathcal{Q}_L, \mathcal{Q}_W \} \rangle \right. \\ & + X_2 \bar{l}_L \gamma_\mu \nu_L \langle u^\mu [\mathcal{Q}_R, \mathcal{Q}_W] \rangle + \hat{X}_2 \bar{l}_L \gamma_\mu \nu_L \langle u^\mu [\mathcal{Q}_L, \mathcal{Q}_W] \rangle \\ & + X_3 M_l \bar{l}_R \nu_L \langle \mathcal{Q}_R \mathcal{Q}_W \rangle \\ & + i X_4 \bar{l}_L \gamma_\mu \nu_L \langle \mathcal{Q}_L^\mu \mathcal{Q}_W \rangle + i X_5 \bar{l}_L \gamma_\mu \nu_L \langle \mathcal{Q}_R^\mu \mathcal{Q}_W \rangle + h.c. \Big] \\ & \left. + \hat{X}_6 \bar{l} (i \not{\partial} + e \not{F}) l + \hat{X}_7 M_l \bar{l} l \right\} . \end{aligned} \quad \mathcal{L}_{\text{Urech}}^{(12)} = -ie^2 F_0^2 K_{12} \langle u_\mu ([\mathcal{Q}_L^\mu, \mathcal{Q}_L] - [\mathcal{Q}_R^\mu, \mathcal{Q}_R]) \rangle$$

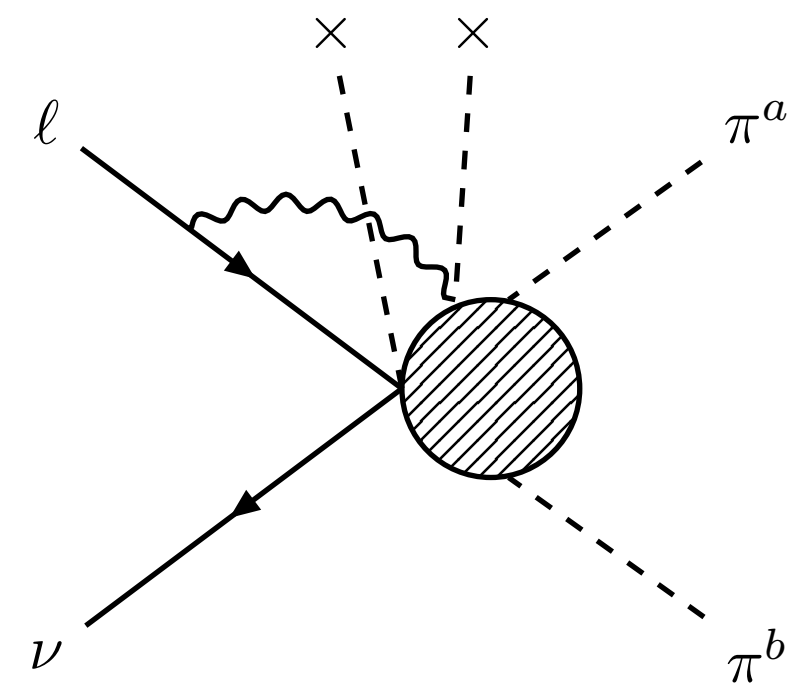
$$u_\mu = i u^\dagger D_\mu U u^\dagger \quad U = u^2$$

$$\mathcal{Q}_R = u^\dagger Q_R u, \quad \mathcal{Q}_L = u Q_L u^\dagger, \quad \mathcal{Q}_W = u Q_W u^\dagger$$

$$\mathcal{Q}_R^\mu = u^\dagger D^\mu Q_R u, \quad \mathcal{Q}_L^\mu = u D^\mu Q_L u^\dagger$$

OPE

Lattice QCD data to evaluate the non perturbative input: box diagrams



$$J_\mu \sim \bar{\psi} \gamma_\mu (\gamma_5) \psi$$

$$\int \frac{d^d q}{(2\pi)^d} K(q) \Gamma(q) \sim \int \frac{d^d q}{(2\pi)^d} \frac{\bar{u} \gamma_\mu \gamma_\alpha \gamma_\beta P_L v}{q^4} \times \int d^4 x e^{iqx} \langle \pi^a(p) | J_\mu(x) J_\nu(0) | \pi^b(p) \rangle q^\alpha \quad \text{OPE} \quad q \rightarrow \infty$$

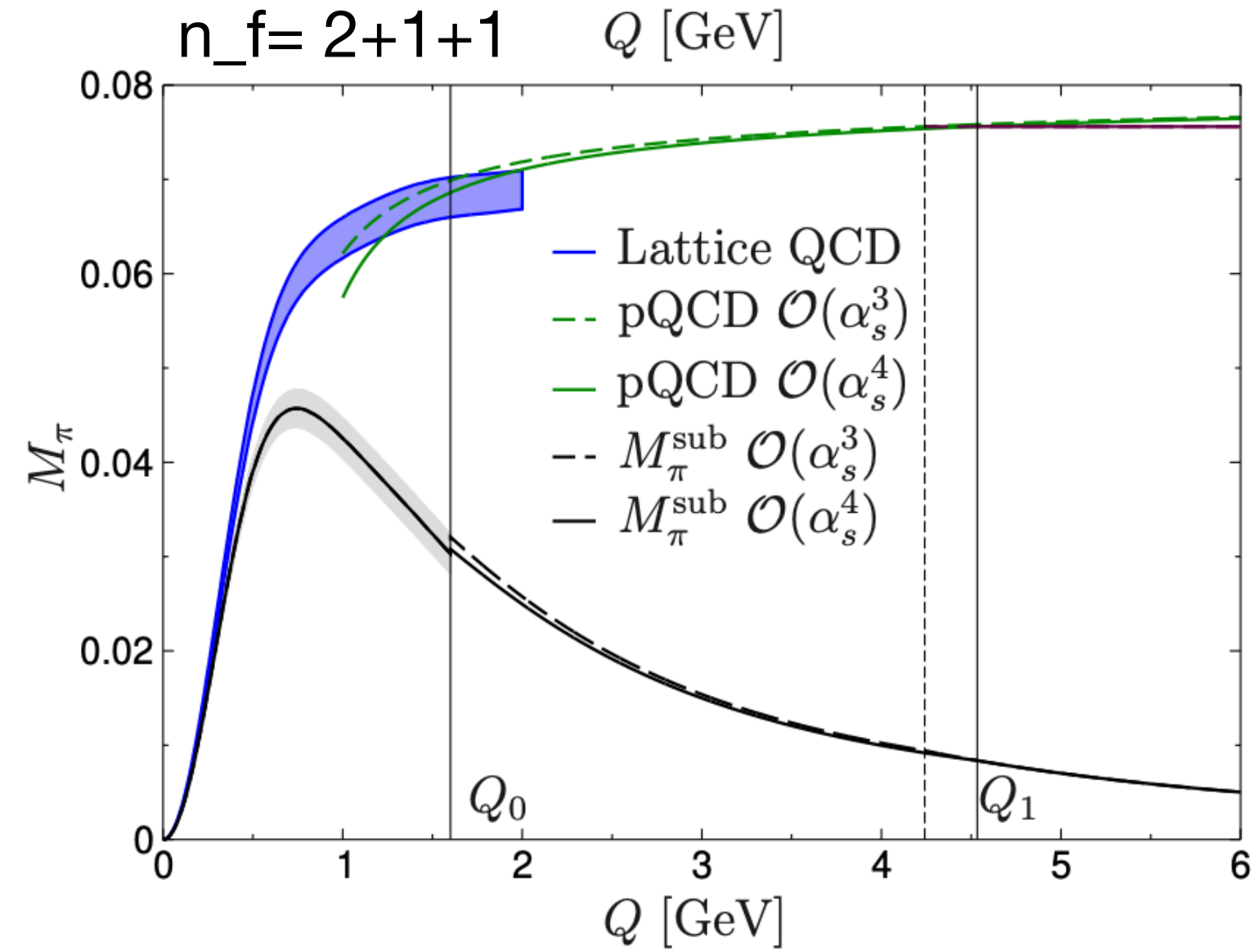
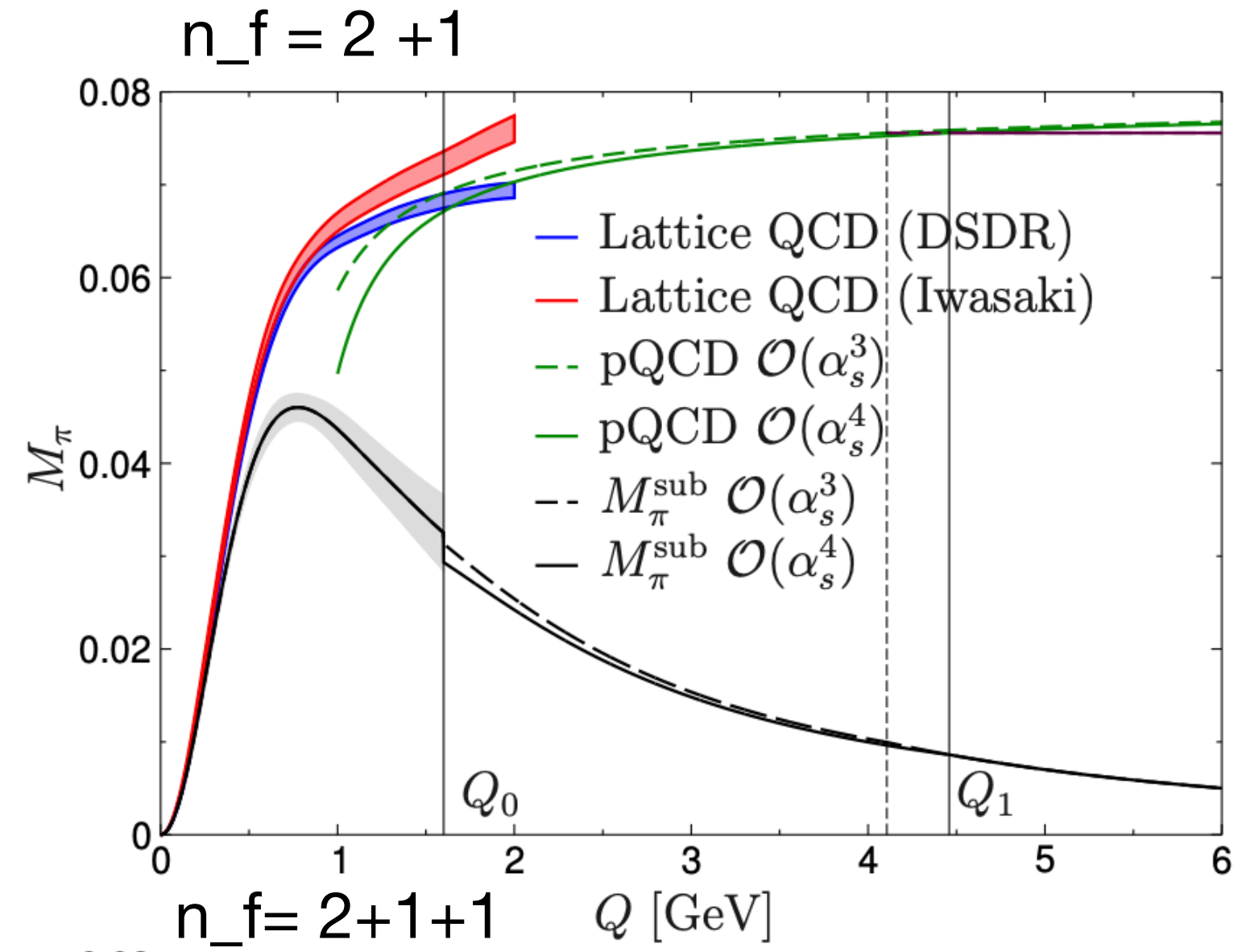
$$\sim \int \frac{d^d q}{(2\pi)^d} \frac{\bar{u} \gamma_\mu \gamma_\alpha \gamma_\beta P_L v}{q^4} \times \langle \pi^a(p) | \bar{\psi}(0) \gamma_\beta \gamma_\alpha \gamma_\mu P_L \psi(0) | \pi^b(p) \rangle$$

$$\sim \bar{u} \gamma_\mu P_L v \times (p^\mu) \times 4(4 - a\epsilon) \times \frac{1}{\epsilon}$$

Divergent and scheme dependent pieces

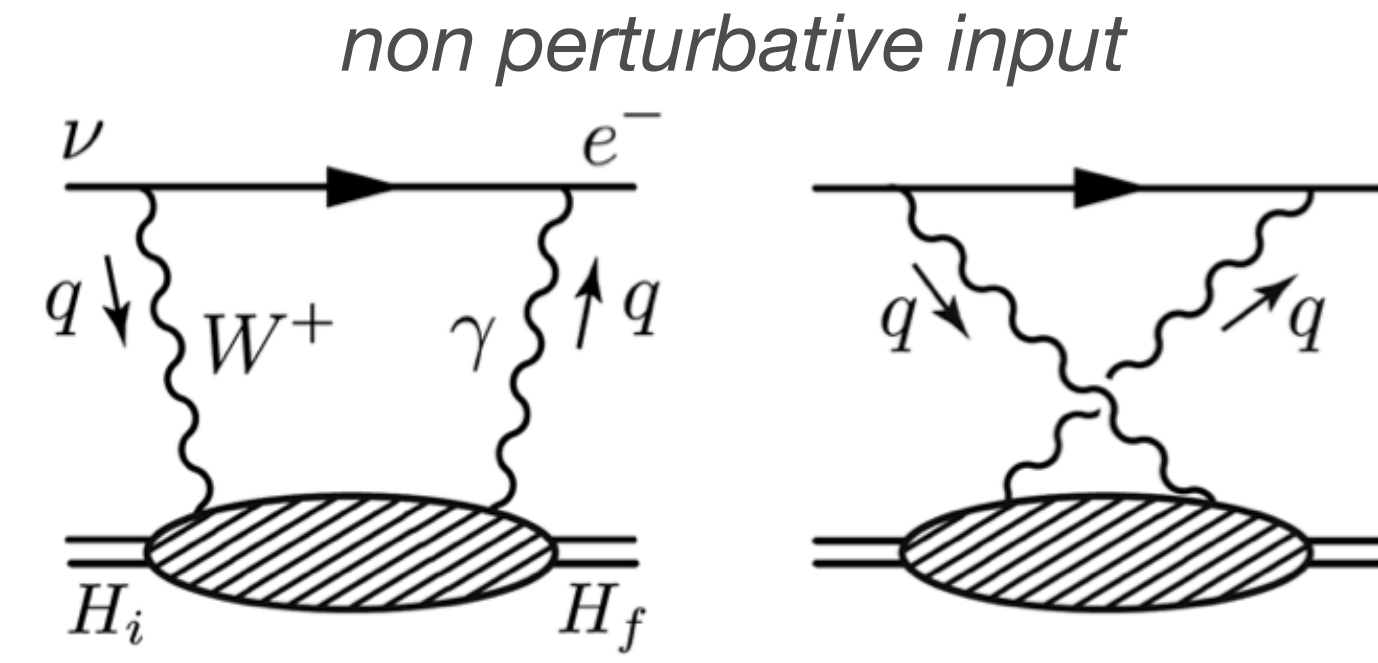
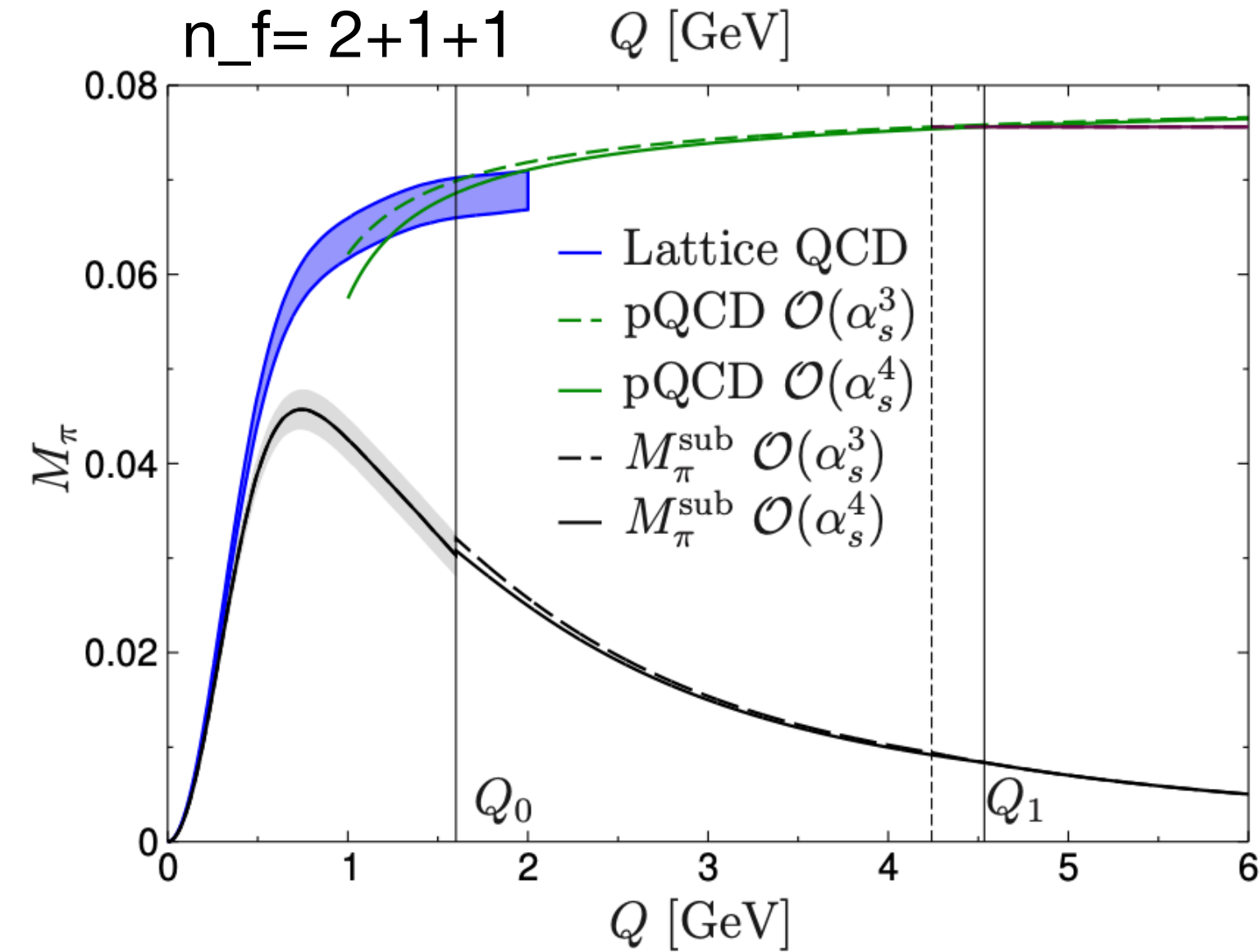
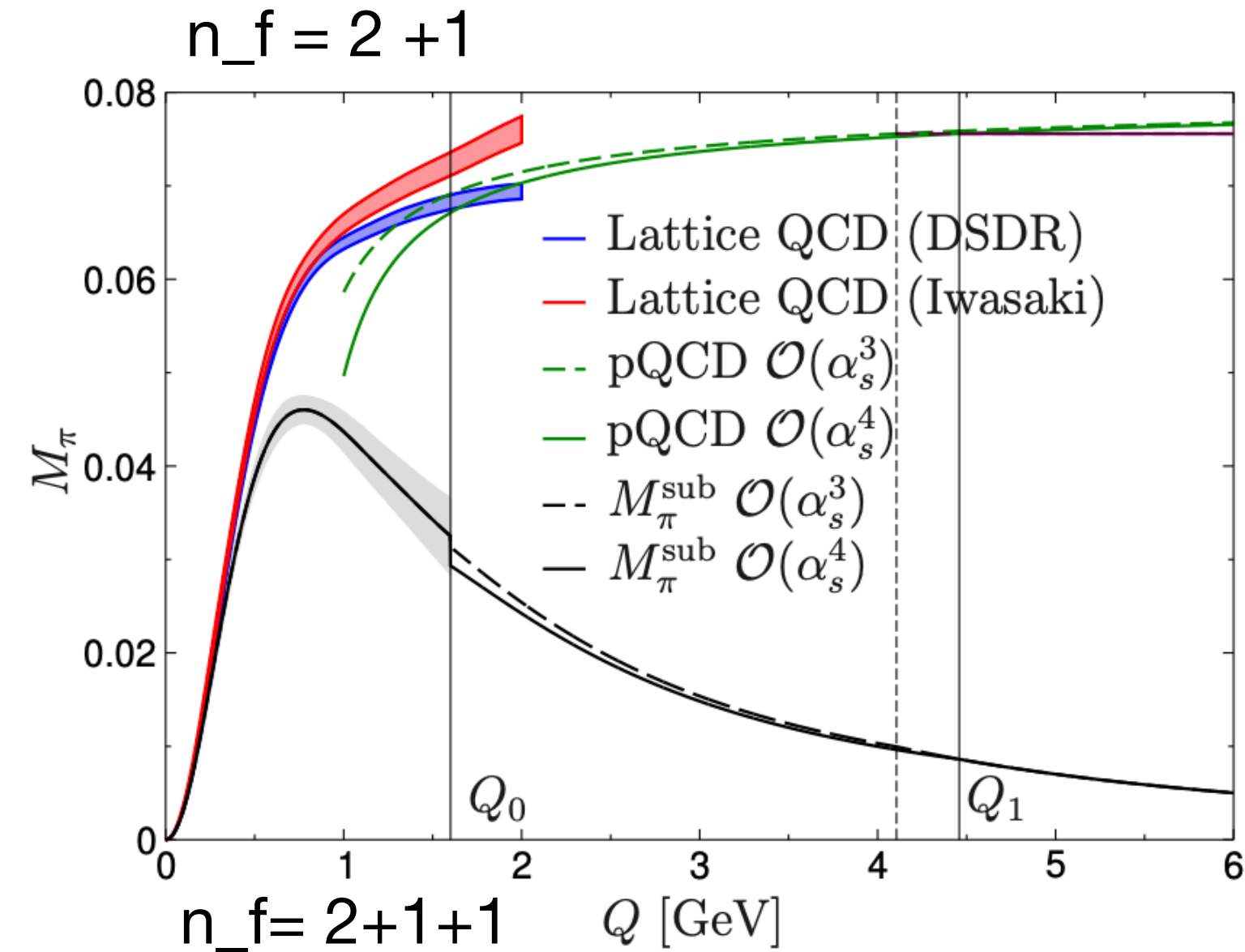
$$\gamma^\alpha \gamma^\rho \gamma^\beta P_L \otimes \gamma_\alpha \gamma_\rho \gamma_\beta P_L = 4(4 - a\epsilon) \gamma^\rho P_L \otimes \gamma_\rho P_L + E(a)$$

Lattice input



$$\bar{\square}_\pi^V(\mu_0) = \frac{3\alpha}{2\pi} \int_0^\infty \frac{dQ^2}{Q^2} \left[M_\pi(Q^2) - \frac{1}{12} \frac{Q^2}{Q^2 + \mu_0^2} \left(1 - \frac{\alpha_s(\mu_0)}{\pi} \right) \right].$$

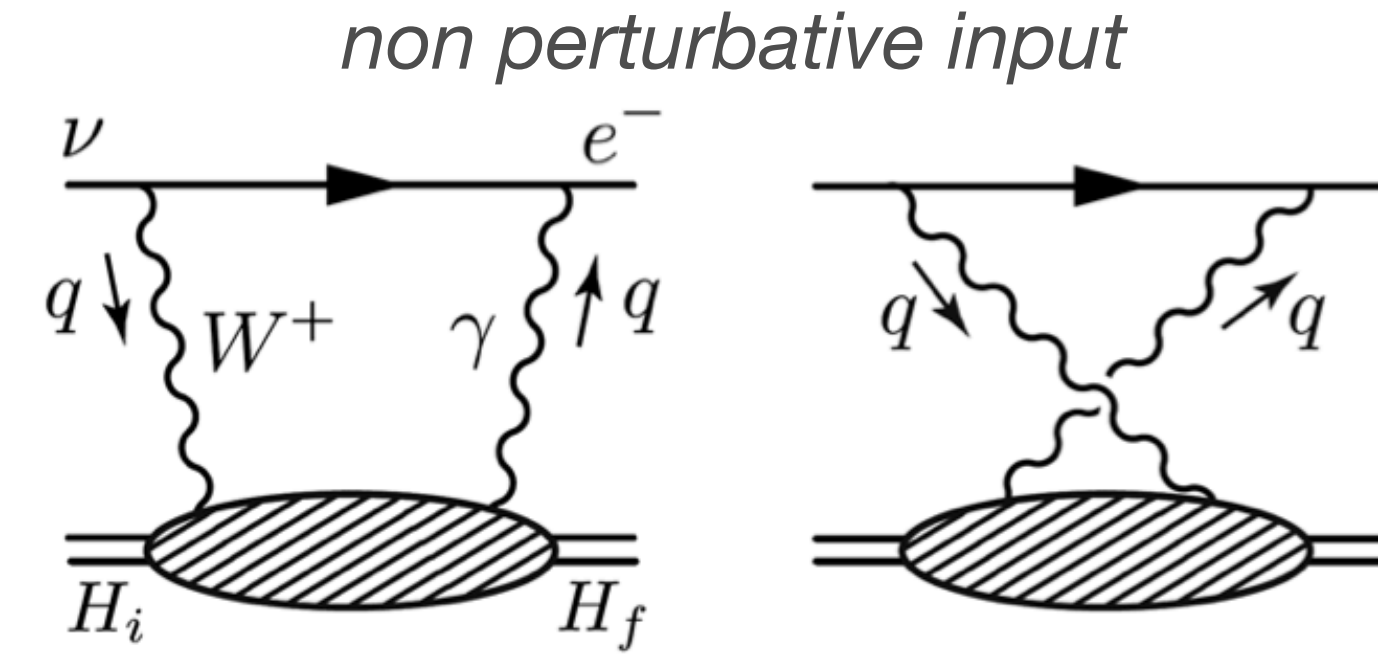
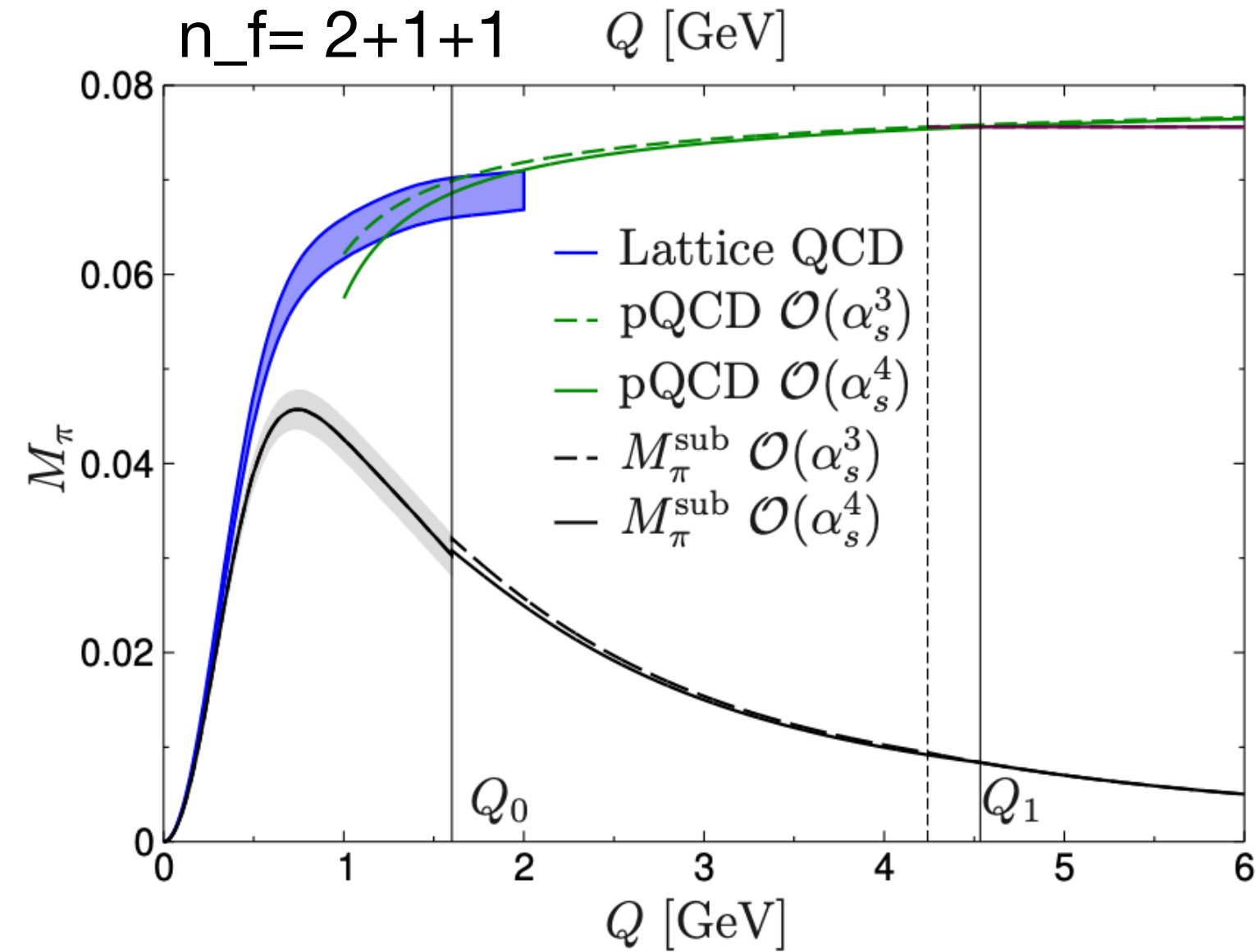
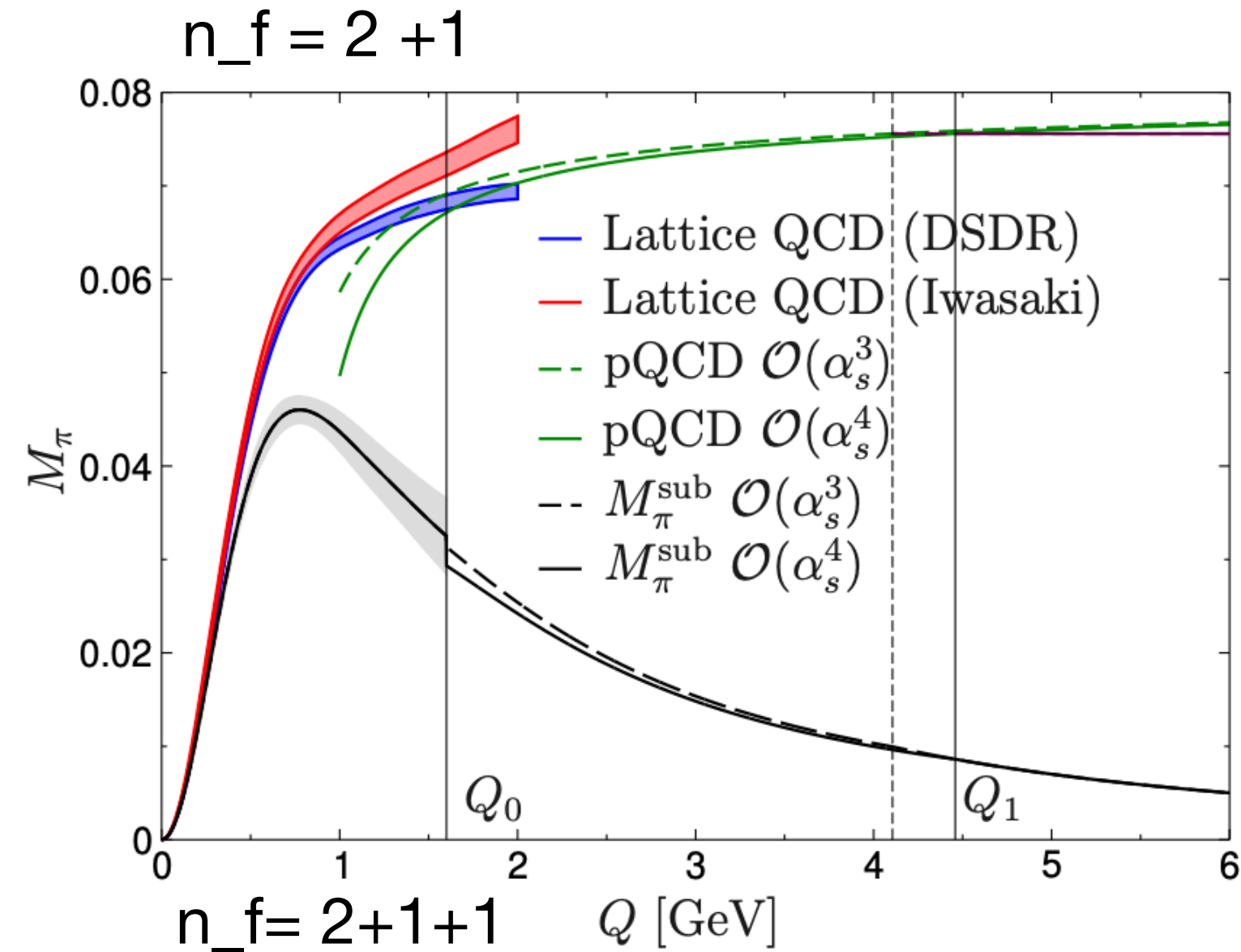
Lattice input



[\[X.Feng et al., Phys. Rev. Lett. 124 192002 \(2020\) \]](#)
[\[J.S. Yoo et al., Phys. Rev. D 108 034508 \(2023\) \]](#)

$$\bar{\square}_\pi^V(\mu_0) = \frac{3\alpha}{2\pi} \int_0^\infty \frac{dQ^2}{Q^2} \left[M_\pi(Q^2) - \frac{1}{12} \frac{Q^2}{Q^2 + \mu_0^2} \left(1 - \frac{\alpha_s(\mu_0)}{\pi} \right) \right].$$

Lattice input



[X.Feng et al., Phys. Rev. Lett. 124 192002 (2020)]
 [J.S. Yoo et al., Phys. Rev. D 108 034508 (2023)]

Only the axial-vector contribution to the box amplitude sensitive to the hadronic scales

[A.Sirlin., Rev. Mod. Phys. 50, 573 (1978)]

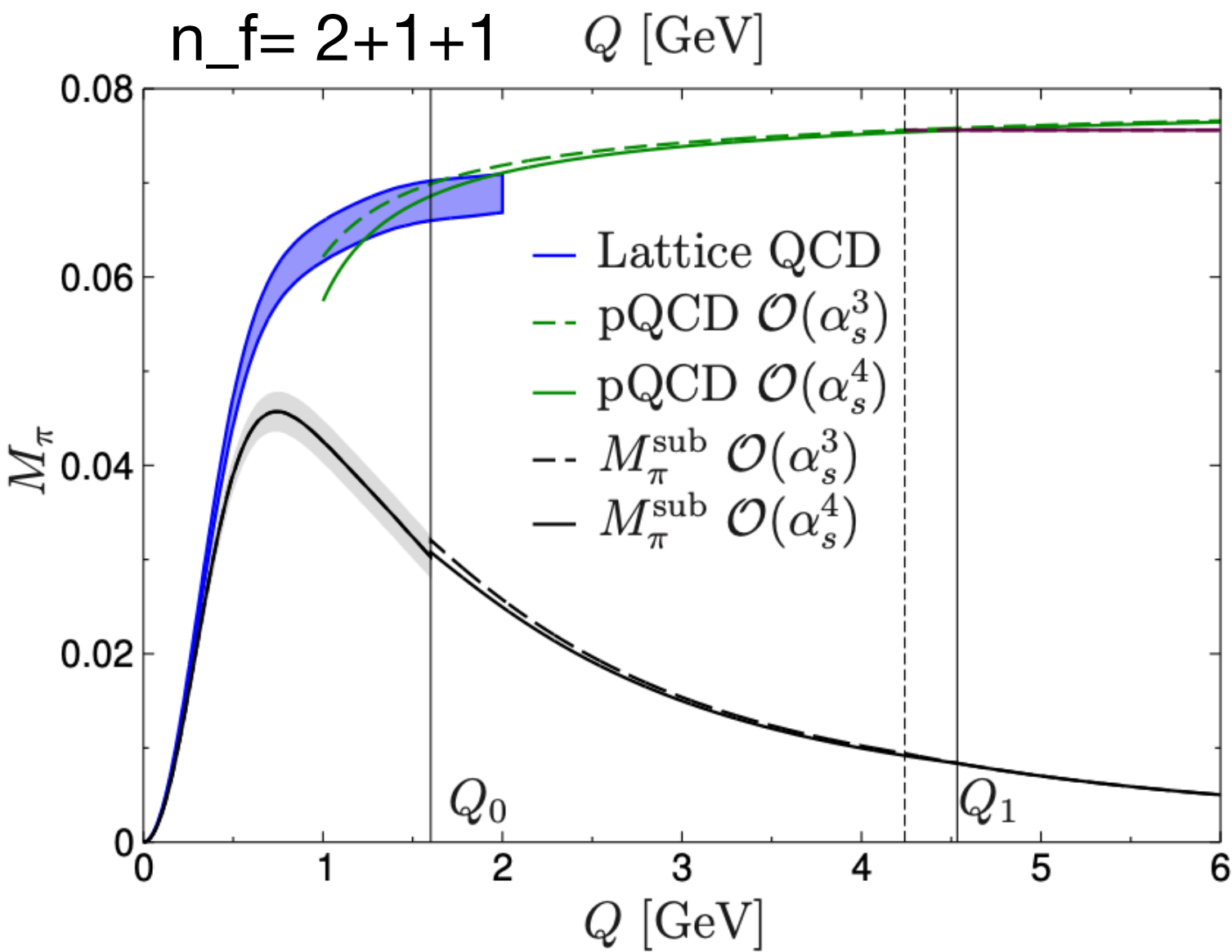
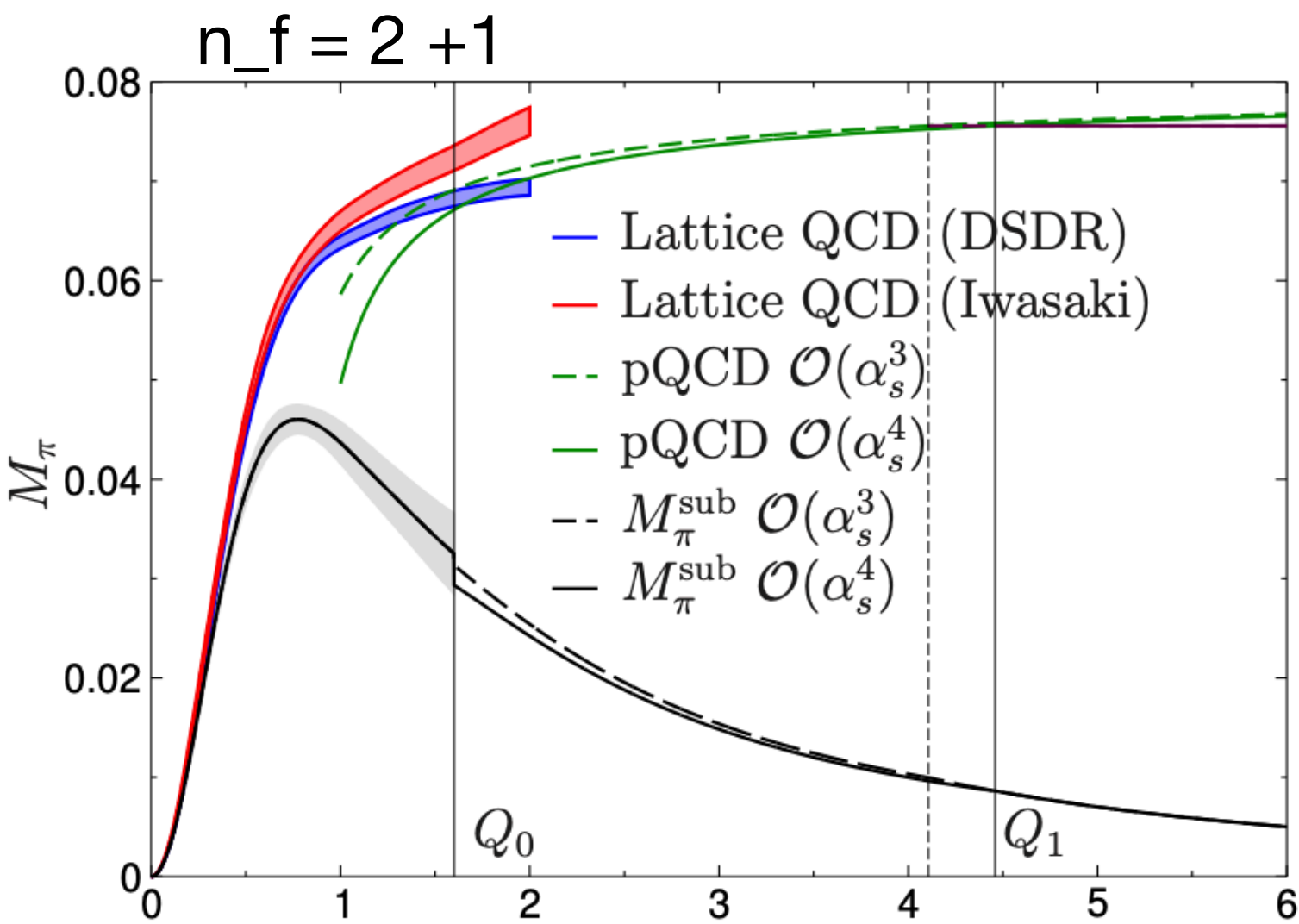
$$T_{\mu\nu}^{VA} = \frac{1}{2} \int d^4x e^{iqx} \langle \pi^0(p) | T \{ J_\mu^{em}(x) J_\nu^{W,A}(0) \} | \pi^-(p) \rangle = i \epsilon_{\mu\nu\alpha\beta} q^\alpha p^\beta \frac{T_3}{m_\pi \nu}$$

$$\bar{\Box}_\pi^V(\mu_0) = -ie^2 \int \frac{d^4q}{(2\pi)^4} \frac{\nu^2 + Q^2}{Q^4} \times \left[\frac{T_3^\pi(\nu, Q^2)}{2M_\pi \nu} - \frac{2}{3} \frac{1}{Q^2 + \mu_0^2} \left(1 - \frac{\alpha_s(\mu_0)}{\pi} \right) \right].$$

$$\bar{\Box}_\pi^V(\mu_0) = \frac{3\alpha}{2\pi} \int_0^\infty \frac{dQ^2}{Q^2} \left[M_\pi(Q^2) - \frac{1}{12} \frac{Q^2}{Q^2 + \mu_0^2} \left(1 - \frac{\alpha_s(\mu_0)}{\pi} \right) \right].$$

Box diagram

$$\bar{\Box}_\pi^V(\mu_0) = \frac{3\alpha}{2\pi} \int_0^\infty \frac{dQ^2}{Q^2} \left[M_\pi(Q^2) - \frac{1}{12} \frac{Q^2}{Q^2 + \mu_0^2} \left(1 - \frac{\alpha_s(\mu_0)}{\pi} \right) \right].$$



$$M_\pi(Q^2)$$

$$Q_0$$

$$M_\pi(Q^2) = \frac{C_d(Q^2)}{12}, \quad C_d(Q^2) = 1 - \frac{\alpha_s}{\pi} + \sum_{n \geq 2} c_n \left(\frac{\alpha_s}{\pi} \right)^n,$$

$$\mathcal{O}(\alpha_s)$$

$$Q_1$$

$$Q_0$$

$$Q_1$$

$$Q$$

$$Q_0$$

LEFT-ChPT matching

LECs in $\overline{MS} + \overline{MS}_\chi$

Matching conditions are arbitrary: $\pi^a \rightarrow l\nu$ is simpler!

$$\hat{X}_6 = \frac{1}{16\pi^2} \left(1 - \log \frac{\mu_\chi^2}{\mu^2} \right) : \text{Lepton wave function renormalization}$$

$$\hat{X}_2 = i \frac{3}{8} \int \frac{d^d k}{(2\pi)^d} \left[\Xi(k^2) - \frac{1}{k^2(k^2 - \mu_0^2)} \right] + \frac{1}{16\pi^2} \left(-\frac{5}{4} \log \frac{\mu^2}{\mu_0^2} - \frac{7}{8} \log \frac{\mu_0^2}{\mu_\chi^2} - \frac{19}{16} \right),$$

Contributions sensitive to the structure of hadrons cancel in the amplitude: $\hat{K}_{12} - \hat{X}_2$.

$$K_{12} = i \frac{3}{8} \int \frac{d^d k}{(2\pi)^d} \left[\Xi(k^2) - \frac{1}{k^2(k^2 - \mu_0^2)} \right] + \frac{1}{16\pi^2} \left(-\frac{1}{4} \log \frac{\mu^2}{\mu_0^2} + \frac{1}{8} \log \frac{\mu_0^2}{\mu_\chi^2} - \frac{3}{16} \right).$$

[S.E. Abers et al., Phys. Rev. 167 1461 (1968),
A.Sirlin, Rev. Mod. Phys. 50 573 (1978)]

Only $X_1 + \hat{X}_1$ sensitive to the hadronic structure and dependent on the evanescent scheme

$$\hat{X}_1 + X_1 = i \frac{3}{4} \int \frac{d^d k}{(2\pi)^d} \left[\frac{\Gamma_{VV}(k, r)}{k^2} - \frac{1}{k^2(k^2 - \mu_0^2)} \right] + \frac{1}{16\pi^2} \left[-\frac{3}{4} \log \frac{\mu^2}{\mu_0^2} + \frac{3}{2} B(a) + \frac{33}{8} + \frac{\alpha_s}{4\pi} \left(3 \log \frac{\mu^2}{\mu_0^2} + \frac{3}{2} B_s(a) - 3 \right) \right]$$



Non-perturbative input

$$\int d^4 x e^{ik \cdot x} \langle 0 | T \{ V_\mu^b(x) V_\nu^c(0) \} | \pi^a(r) \rangle = i F_0 \epsilon_{\mu\nu\rho\gamma} k^\rho r^\gamma d^{abc} \Gamma_{VV}(k, r)$$



Adapted from neutron decay

[M.Gorbahn et al., hep-ph/2510.27648]

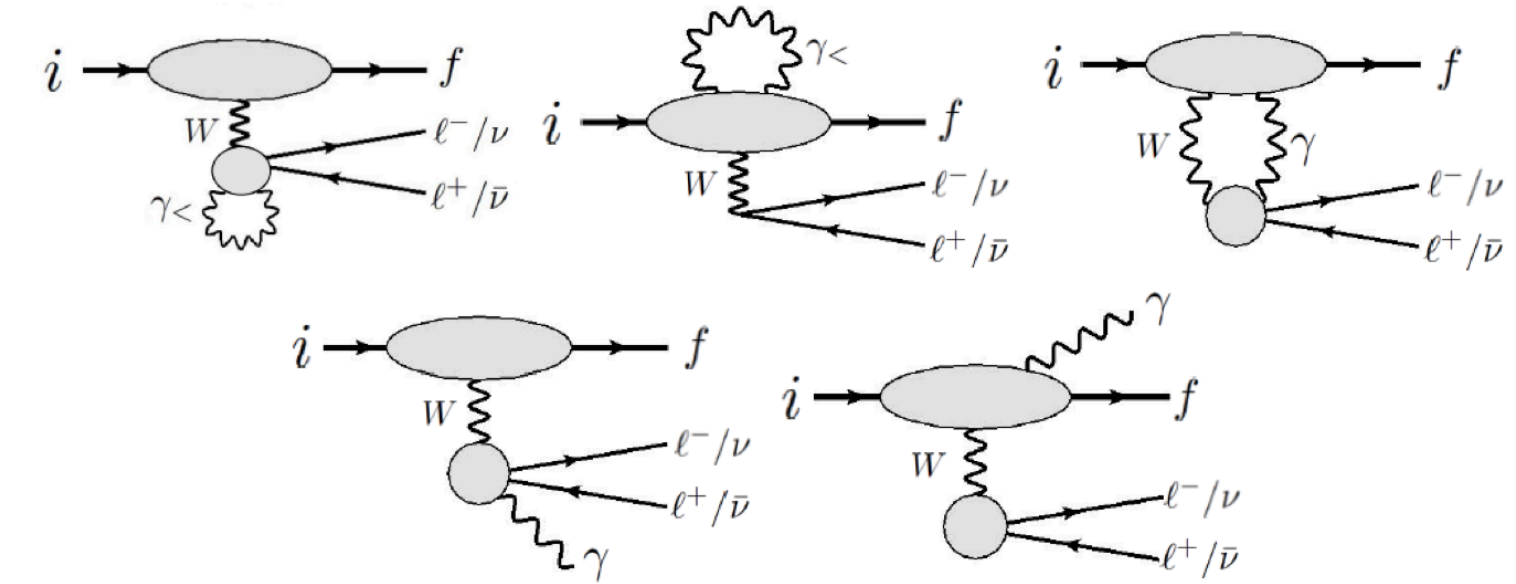
Rad correction in beta decays

$$\Gamma[\pi^+ \rightarrow \pi^0 e^+ \nu_e(\gamma)] = \frac{G_F^2 |V_{ud}|^2 M_\pi^5 |f_+^\pi(0)|^2}{64\pi^3} (1 + \Delta_{\text{RC}}^{\pi\ell}) I_{\pi\ell},$$

Previous result: Using the so-called Sirlin's representation: [X.Feng et al., Phys. Rev. Lett. 124 192002 (2020)]

$$\delta = \frac{\alpha_e}{2\pi} \left[\bar{g} + 3 \ln \frac{m_Z}{m_p} + \ln \frac{m_Z}{m_W} + \tilde{a}_g \right] + \delta_{\text{HO}}^{\text{QED}} + 2\Box_{\gamma W}^{VA} \quad \Delta_{\text{RC}} = 0.0332(3)$$

LL resummation in the short-distance part



Our result:

• Large scale separation $M_{\pi^+} - M_{\pi^0} \sim \text{MeV}$ large log resummation is important

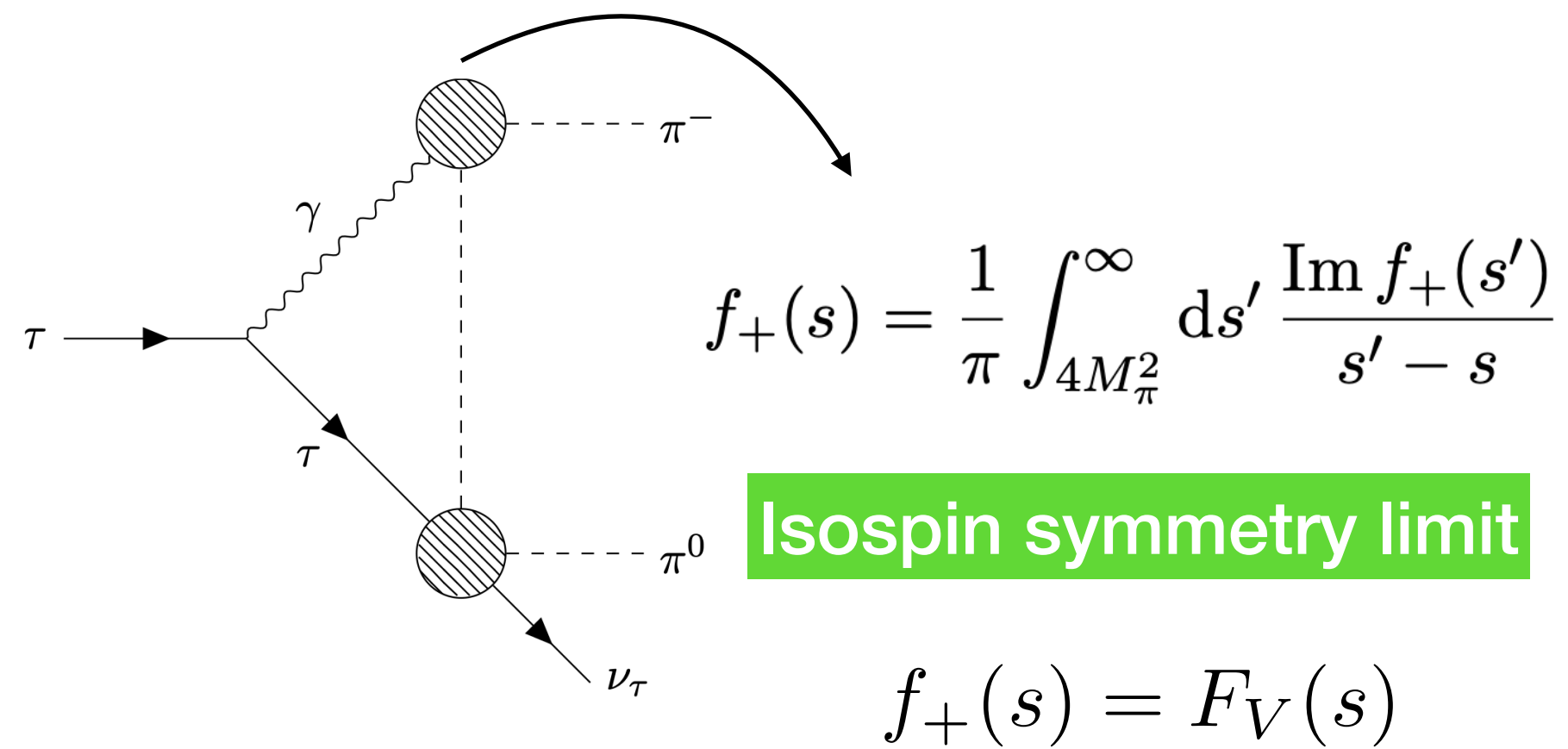
$$\Delta_{\text{RC}}^{\pi\ell} = 0.03403(4)_{\Box_\pi}(5)_\mu(5)_{\mu_\chi}(8)_{\text{ChPT}[11]}_{\text{tot}}.$$

Central value 2.6 sigma away from previous determination ———> Leading log resummation in the low energy region is important!

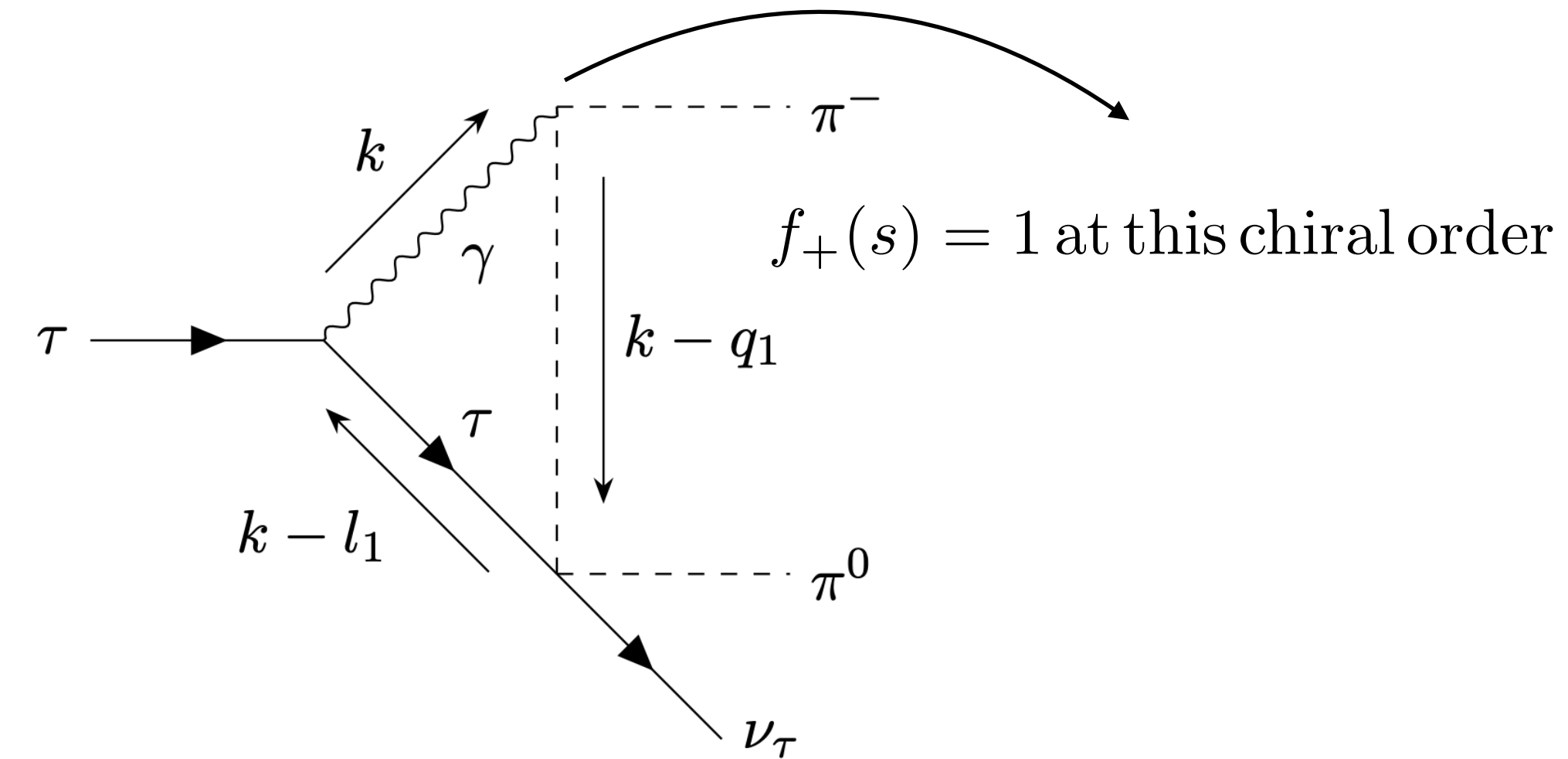
Dispersive approach vs ChPT

$$\mathcal{M} = G_F V_{ud}^* f_+(s) \bar{u}(l_2, \nu_\tau) \gamma^\mu (1 - \gamma_5) u(l_1, \tau) \left[(q_2 - q_1)_\mu (1 + \tilde{f}_+(s, t)) + (q_1 + q_2)_\mu \tilde{f}_-(s, t) \right] \rightarrow 0$$

Dispersive approach



ChPT



$$\tilde{f}_+^{\text{match}}(s, t) = \tilde{f}_+^{\text{disp}}(s, t) - \tilde{f}_+^{\text{disp}}(0, 0) + \tilde{f}_+^{\text{ChPT}}(0, 0)$$

- *Correct chiral logs*
- *Chiral LECs*

Muon g-2 status

White paper 25

$$R_{IB}(s) = \frac{\text{FSR}(s)}{S_{EW} G_{EM}(s)} \frac{\beta_{\pi^+\pi^-}(s)}{\beta_{\pi^\pm\pi^0}(s)} \left| \frac{F_\pi^V(s)}{f_+(s)} \right|^2$$

	Refs. [166, 194]	Ref. [209]	Refs. [237, 247]	Our estimate
Phase space	-7.88	-7.52	-	-7.7(2)
S_{EW}	-12.21(15)	-12.16(15)	-	-12.2(1.3)
G_{EM}	-1.92(90)	$(-1.67)^{+0.60}_{-1.39}$	-	-2.0(1.4)
FSR	4.67(47)	4.62(46)	4.42(4)	4.5(3)
ρ - ω mixing	4.0(4)	2.87(8)	3.79(19)	3.9(3)
$\frac{F_\pi^V}{f_+}$ (w/o ρ - ω)	ΔM_ρ	$0.20^{+27}_{-19}(9)$	$1.95^{+1.56}_{-1.55}$	-
	$\Delta\Gamma_\rho(\Delta M_\pi)$	4.09(0)(7)	3.37	-
	$\Delta\Gamma_\rho(\pi\pi\gamma)$	-5.91(59)(48)	-6.66(73)	-
	$\Delta\Gamma_\rho(g_{\rho\pi\pi})$	-	-	-
	Total	-1.62(65)(63)	$(-1.34)^{+1.72}_{-1.71}$	-1.5(4.7)
Sum	-14.9(1.9)	$(-15.20)^{+2.26}_{-2.63}$	-	-15.0(5.1)

Uncertainty dominated by the ratio of the two form factors. Still model dependent.