Experimental Prospects for Decoherence of Fermion Qubit Pairs from Radiation Emission at Colliders

***FlipPhysics* – Jardín Botánico de la Universidad de València – May 26th-29th, 2026**

Speaker: Guillermo Garcia-Mir

In collaboration with Jose Manuel Camacho, Marcel Vos, María Moreno Llácer (IFIC, CSIC-UV),

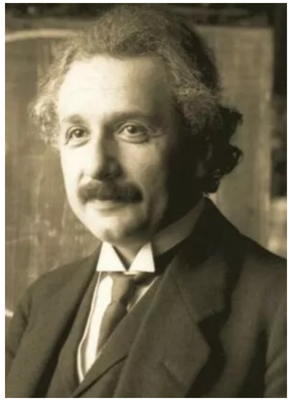
Many thanks to Rafael Aoude (Hamburg-DESY), Fabio Maltoni (Bologna/Louvain/CERN), Leonardo Satrioni, Valentin Durupt (Louvain)

Based on the paper arXiv:2604.16268. Soon to appear in XXX (2026)

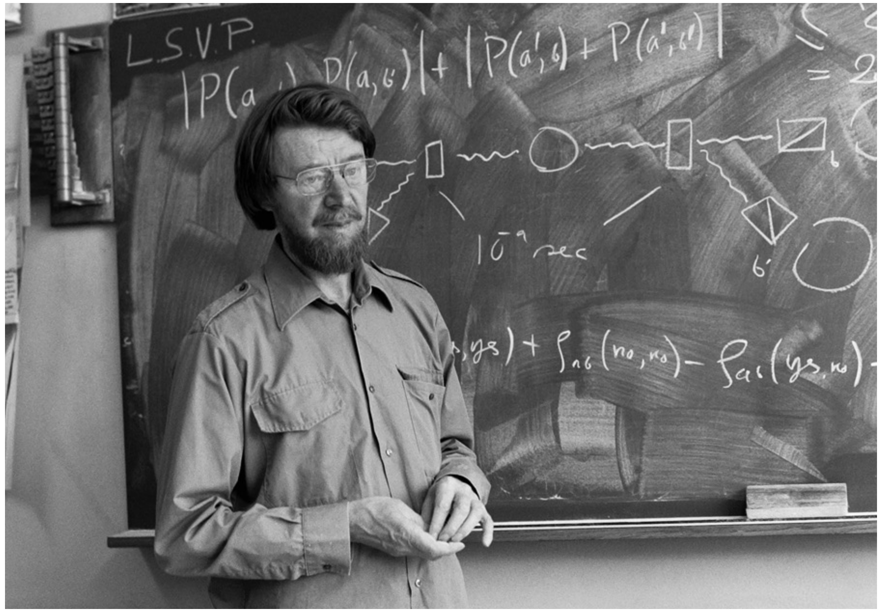
The role of quantum entanglement

Is quantum mechanics complete with a probabilistic interpretation?





Niels Bohr – Werner Heisenberg – Albert Einstein



Jonh S. Bell at CERN, 1982

References: H. Kragh (2002). Quantum generations;
W. H. Zurek (1025). Decoherence and Quantum Darwinism

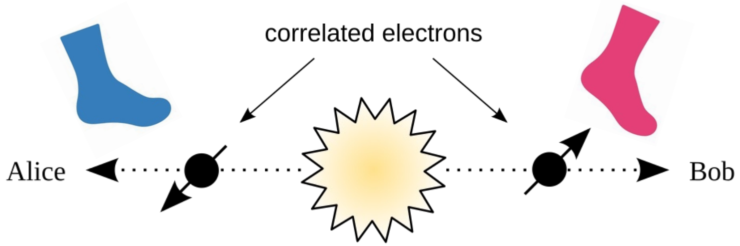
The role of quantum entanglement

Is quantum mechanics complete with a probabilistic interpretation?

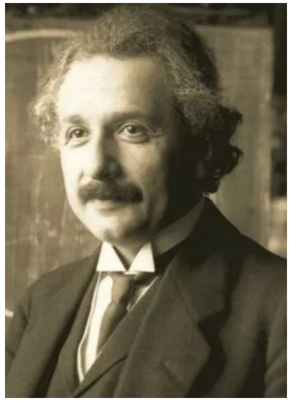
THE EPR THOUGHT EXPERIMENT:

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

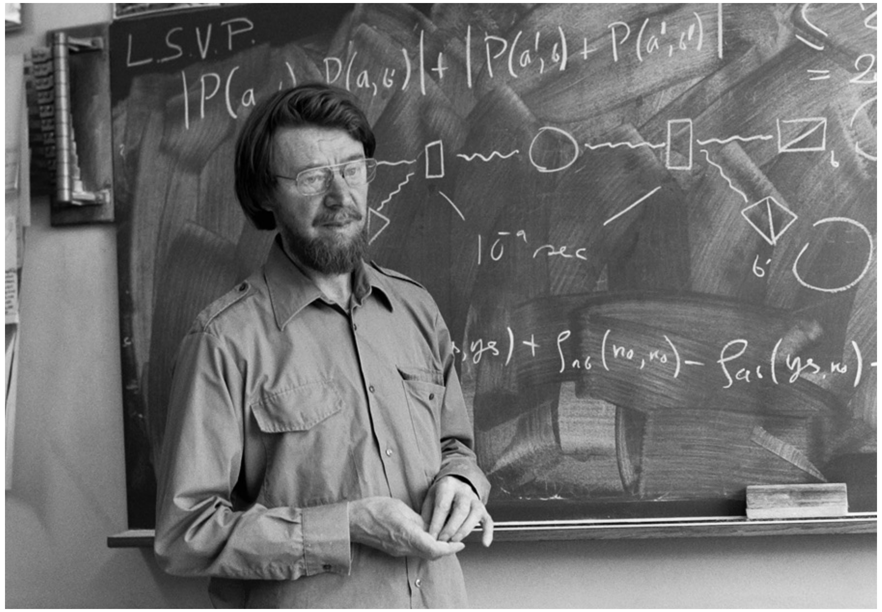
Singlet (S = 1) → Maximally entangled state ⇒ Alice and Bob observations are correlated







Niels Bohr – Werner Heisenberg – Albert Einstein



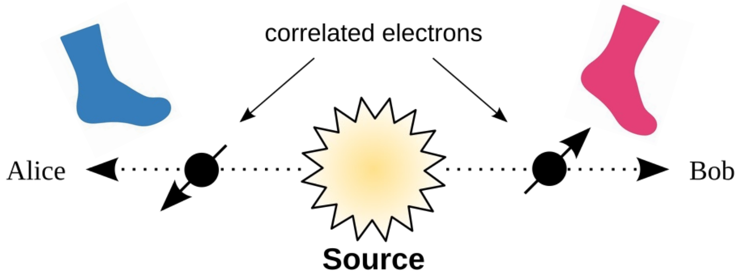
Jonh S. Bell at CERN, 1982

References: H. Kragh (2002). Quantum generations;
W. H. Zurek (1025). Decoherence and Quantum Darwinism

The role of quantum entanglement

Is quantum mechanics complete with a probabilistic interpretation?

THE EPR THOUGHT EXPERIMENT:

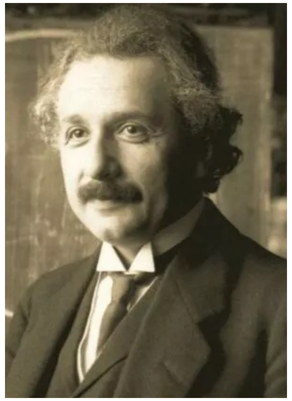


$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

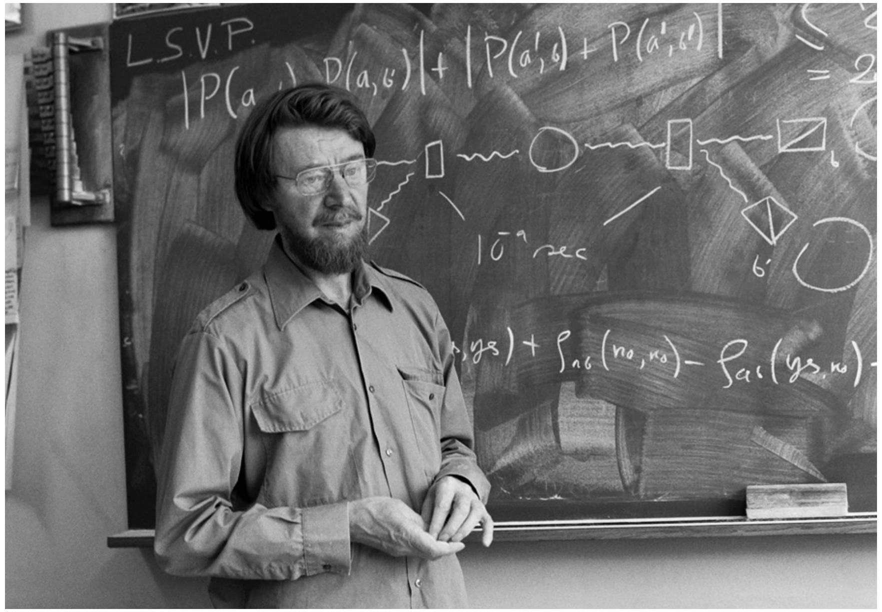
Singlet (S = 1) → Maximally entangled state ⇒ Alice and Bob observations are correlated

Hidden variable theory that predetermines the observation from source?





Niels Bohr – Werner Heisenberg – Albert Einstein



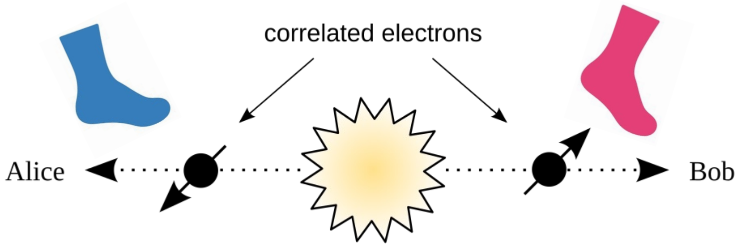
Jonh S. Bell at CERN, 1982

References: H. Kragh (2002). Quantum generations;
W. H. Zurek (1025). Decoherence and Quantum Darwinism

The role of quantum entanglement

Is quantum mechanics complete with a probabilistic interpretation?

THE EPR THOUGHT EXPERIMENT:



$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

Singlet (S = 1) → Maximally entangled state ⇨ Alice and Bob observations are correlated

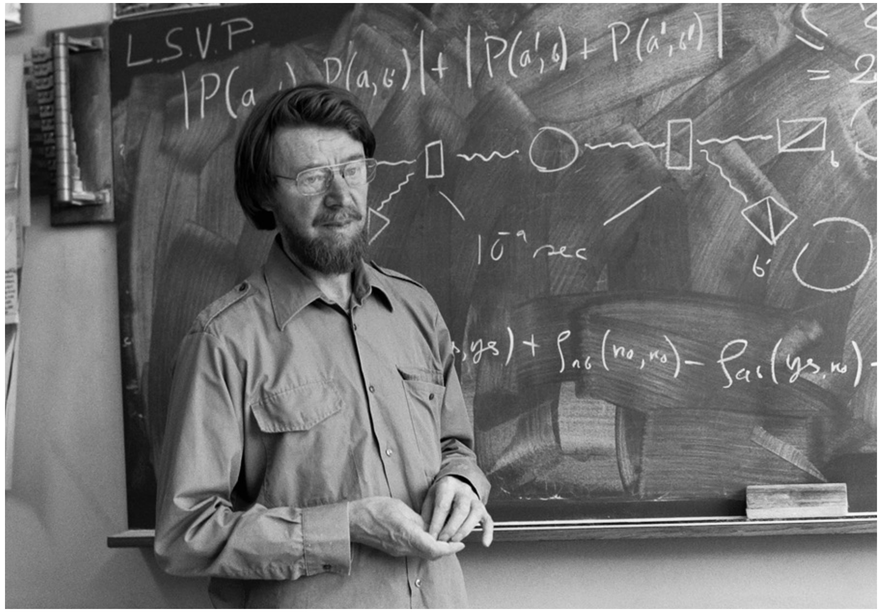
Hidden variable ... hat predetermines the ob ... from source? **NO!**

BELL THEOREM: $2 \leq |E(A_1, B_1) + E(A_1, B_2) + E(A_2, B_1) - E(A_2, B_2)| \leq 2\sqrt{2}$

Classical maximum — Quantum maximum

Aspect, Clauser, Zeilinger (2022 Nobel Prize)

Niels Bohr – Werner Heisenberg – Albert Einstein



Jonh S. Bell at CERN, 1982

References: H. Kragh (2002). Quantum generations;
W. H. Zurek (1025). Decoherence and Quantum Darwinism

Quantum decoherence. Motivations

How do the classical rules emerge from a fundamentally quantum world?

✦ First **observation of quantum spin entanglement** in 2024

→ $t\bar{t}$ pairs by ATLAS and CMS at LHC.

Article

Observation of quantum entanglement with top quarks at the ATLAS detector

ATLAS Collaboration. Nature 633, 542 (2024)

Measurement of the top quark polarization and $t\bar{t}$ spin correlations using dilepton final states in proton-proton collisions at $\sqrt{s} = 13$ TeV

CMS Collaboration. PRD 110, 112016 (2024)

Quantum decoherence. Motivations

How do the classical rules emerge from a fundamentally quantum world?

Article

Observation of quantum entanglement with top quarks at the ATLAS detector

ATLAS Collaboration. Nature 633, 542 (2024)

Measurement of the top quark polarization and $t\bar{t}$ spin correlations using dilepton final states in proton-proton collisions at $\sqrt{s} = 13$ TeV

CMS Collaboration. PRD 110, 112016 (2024)

- First **observation of quantum spin entanglement** in 2024

  → $t\bar{t}$ pairs by ATLAS and CMS at LHC.

- Decoherence —loss of *quantumness*— is at the heart of this topic → measurement problem and collapse of wave function.

Quantum decoherence. Motivations

How do the classical rules emerge from a fundamentally quantum world?

Article

Observation of quantum entanglement with top quarks at the ATLAS detector

ATLAS Collaboration. Nature 633, 542 (2024)

Measurement of the top quark polarization and $t\bar{t}$ spin correlations using dilepton final states in proton-proton collisions at $\sqrt{s} = 13$ TeV

CMS Collaboration. PRD 110, 112016 (2024)

- First **observation of quantum spin entanglement** in 2024

  → $t\bar{t}$ pairs by ATLAS and CMS at LHC.

- Decoherence —loss of *quantumness*— is at the heart of this topic → measurement problem and collapse of wave function.

- **OUR STUDY**: Minimal protocol for prospects of the **effects of energetic final-state radiation (FSR)** in the **entanglement of heavy fermion pairs:**
  - Modelling the source of decoherence → gluon and photon radiation (FSR) from $f\bar{f}$ → understood as interaction with the environment (Aoude, Barr, Maltoni and Satrioni. https://doi.org/10.48550/arXiv.2504.07030).
  - Two process studied: ⇒ **$t\bar{t}$** production at **LCF and LHC.** ⇒ **$\tau^+\tau^-$** production at **Belle II** and **Z-pole FCC-ee**.
  - We assess the experimental feasibility naively ⇒ statistical significance for observation of decoherence.

Quantum state tomography

How do we "*observe*" quantum mechanical systems in High Energy Physics?

1 ⟶ General two-qubit (spin ½) density matrix ρ:

$$\rho = \frac{1}{4}\left[\mathbb{1}_4 + \sigma^i \otimes \mathbb{1}_2 \cdot B_i^1 + \mathbb{1}_2 \otimes \sigma^j \cdot B_j^2 + \sigma^i \otimes \sigma^j C_{ij}\right]$$

SPIN POLARIZATION VECTOR $B_i^a = \langle S_i \rangle^a$

SPIN CORRELATION MATRIX $C_{ij} = \langle S_i S_j \rangle$

References: W. Bernreuther et. al. (2015)

Quantum state tomography

How do we "*observe*" quantum mechanical systems in High Energy Physics?

1 ⟶ General two-qubit (spin ½) density matrix ρ:

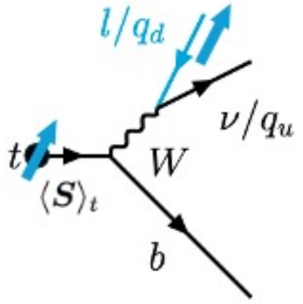
$$\rho = \frac{1}{4}\left[\mathbb{1}_4 + \sigma^i \otimes \mathbb{1}_2 \cdot B_i^1 + \mathbb{1}_2 \otimes \sigma^j \cdot B_j^2 + \sigma^i \otimes \sigma^j C_{ij}\right]$$

SPIN POLARIZATION VECTOR $B_i^a = \langle S_i \rangle^a$

SPIN CORRELATION MATRIX $C_{ij} = \langle S_i S_j \rangle$

2 ⟶ Angular distribution for the differential decay width of the top quark:

Expected value of the decay products 3-momenta → **correlated with the spin direction** of their parent top-quark.



$$\frac{1}{\Gamma_T}\frac{d\Gamma}{d\cos\chi_i} = (1 + \alpha_i \cos\chi_i)/2$$

α : spin analysing power

In $t\bar{t}$ → charged lepton (e/μ)
or down-type (d/s) quarks
In $\tau^+\tau^-$ → $\pi^\pm$.

} α = 1

References: W. Bernreuther et. al. (2015)

Quantum state tomography

How do we "*observe*" quantum mechanical systems in High Energy Physics?

1 ⟶ General two-qubit (spin ½) density matrix ρ:

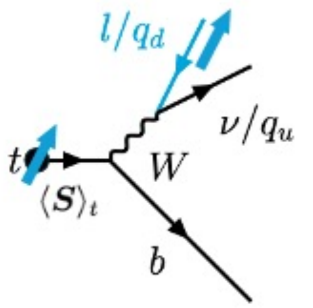
$$\rho = \frac{1}{4}\left[\mathbb{1}_4 + \sigma^i \otimes \mathbb{1}_2 \cdot B_i^1 + \mathbb{1}_2 \otimes \sigma^j \cdot B_j^2 + \sigma^i \otimes \sigma^j C_{ij}\right]$$

SPIN POLARIZATION VECTOR $B_i^a = \langle S_i \rangle^a$

SPIN CORRELATION MATRIX $C_{ij} = \langle S_i S_j \rangle$

2 ⟶ Angular distribution for the differential decay width of the top quark:

Expected value of the decay products 3-momenta → **correlated with the spin direction** of their parent top-quark.



$$\frac{1}{\Gamma_T}\frac{d\Gamma}{d\cos\chi_i} = (1 + \alpha_i \cos\chi_i)/2$$

α : spin analysing power

In $t\bar{t}$ → charged lepton (e/μ)
or down-type (d/s) quarks
In $\tau^+\tau^-$ → $\pi^{\pm}$.
} α = 1

3 ⟶ Normalized differential cross-section distribution:

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega_+ d\Omega_-} = \frac{1}{(4\pi)^2}\left(1 + \mathbf{B}_1' \cdot \hat{\ell}_+ + \mathbf{B}_2' \cdot \hat{\ell}_- - \hat{\ell}_+ \cdot C' \cdot \hat{\ell}_-\right)$$

$$B_i^a = \langle S_i \rangle^a = 3\,\frac{1}{\alpha}\,\langle \cos\theta_i^a \rangle \quad C_{ij} = \langle S_i S_j \rangle = -9\,\frac{1}{\alpha_1 \alpha_2}\,\langle \cos\theta_i^a \cos\theta_j^b \rangle$$

References: W. Bernreuther et. al. (2015)

Quantum state tomography

How do we "*observe*" quantum mechanical systems in High Energy Physics?

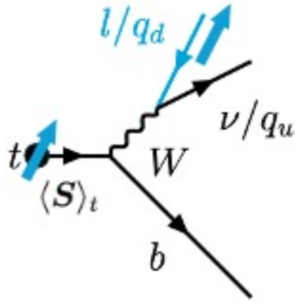
1 ⟶ General two-qubit (spin ½) density matrix ρ:

$$\rho = \frac{1}{4}\left[\mathbb{1}_4 + \sigma^i \otimes \mathbb{1}_2 \cdot B_i^1 + \mathbb{1}_2 \otimes \sigma^j \cdot B_j^2 + \sigma^i \otimes \sigma^j C_{ij}\right]$$

SPIN POLARIZATION VECTOR $B_i^a = \langle S_i \rangle^a$ **SPIN CORRELATION MATRIX** $C_{ij} = \langle S_i S_j \rangle$

2 ⟶ Angular distribution for the differential decay width of the top quark:

Expected value of the decay products 3-momenta ⟶ **correlated with the spin direction** of their parent top-quark.



$$\frac{1}{\Gamma_T}\frac{d\Gamma}{d\cos\chi_i} = (1 + \alpha_i \cos\chi_i)/2$$

α : spin analysing power

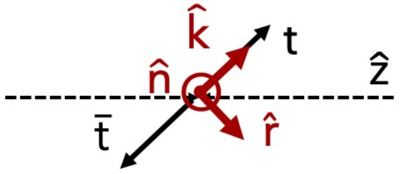
In $t\bar{t}$ ⟶ charged lepton (e/μ) or down-type (d/s) quarks
In $\tau^+\tau^-$ ⟶ $\pi^\pm$.
} $\alpha = 1$

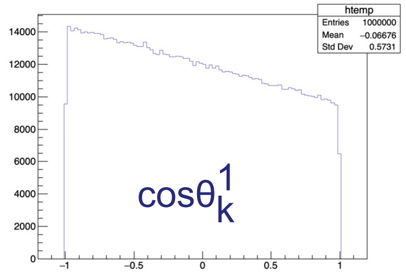
3 ⟶ Normalized differential cross-section distribution:

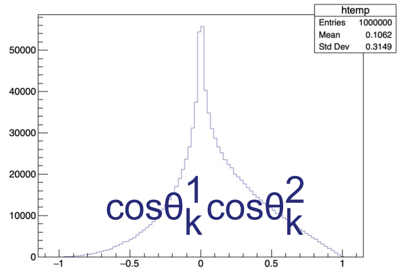
$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega_+ d\Omega_-} = \frac{1}{(4\pi)^2}\left(1 + \mathbf{B}_1' \cdot \hat{\ell}_+ + \mathbf{B}_2' \cdot \hat{\ell}_- - \hat{\ell}_+ \cdot C' \cdot \hat{\ell}_-\right)$$

$$B_i^a = \langle S_i \rangle^a = 3\,\frac{1}{\alpha}\,\langle \cos\theta_i^a \rangle \qquad C_{ij} = \langle S_i S_j \rangle = -9\,\frac{1}{\alpha_1\alpha_2}\,\langle \cos\theta_i^a \cos\theta_j^b \rangle$$

4 ⟶ The axis system chosen is the **helicity base:**

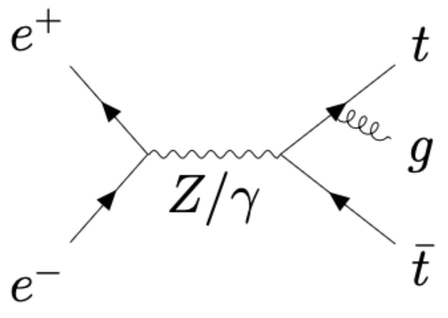






References: W. Bernreuther et. al. (2015)

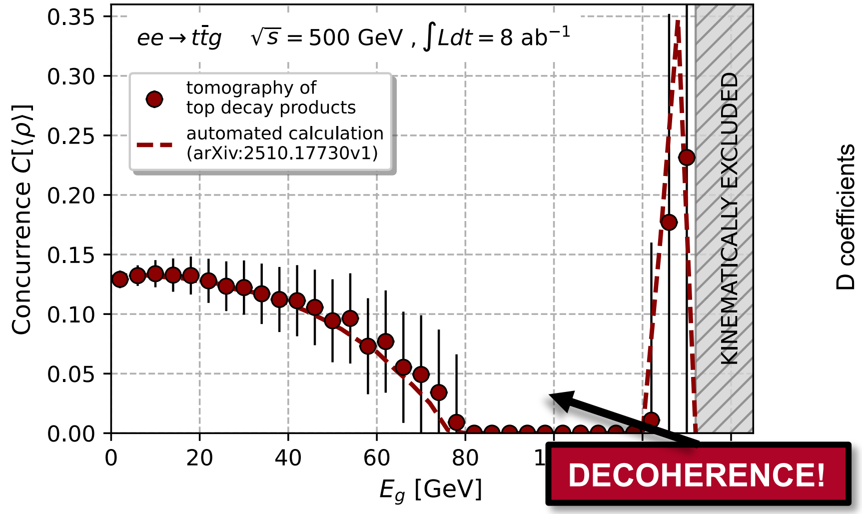
Decoherence prospects for $e^+e^- \to t\bar{t}$ from gluon FSR



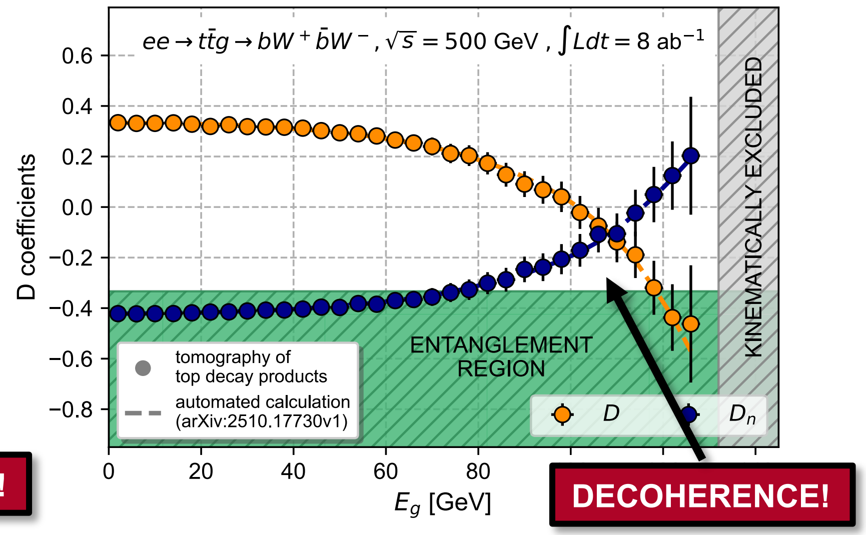
Simulation: MADGRAPH 5
[10.1007/jhep07(2014)079]

Decay mode: $\mu^+\mu^-$

CONCURRENCE



D MARKERS



Concurrence (C[ρ]):

Provides with a quantitative measurement of the degree of entanglement. Non-linear:

$$C\left[\langle\rho\rangle\right] = \max\left(0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\right)$$

λ_i increasing eigenvalues of

$$R = \sqrt{\sqrt{\rho}\sigma_2 \otimes \sigma_2\rho^*\sigma_2 \otimes \sigma_2\sqrt{\rho}}$$

D and D_n markers:

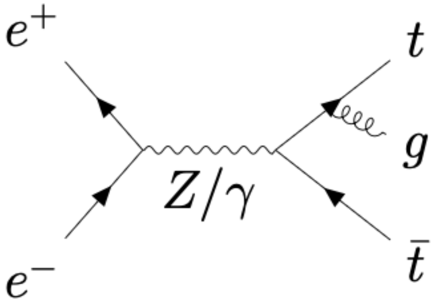
Provide with sufficient enough criteria for entanglement via linear expressions:

$$D = -\tfrac{1}{3}\left(C_{nn} + C_{rr} + C_{kk}\right)$$

$$D_n = -\tfrac{1}{3}\left(C_{nn} - C_{rr} - C_{kk}\right)$$

$D < -1/3$ or $D_n < -1/3 \to$ ENTANGLEMENT

Decoherence prospects for $e^+e^- \rightarrow t\bar{t}$ from gluon FSR

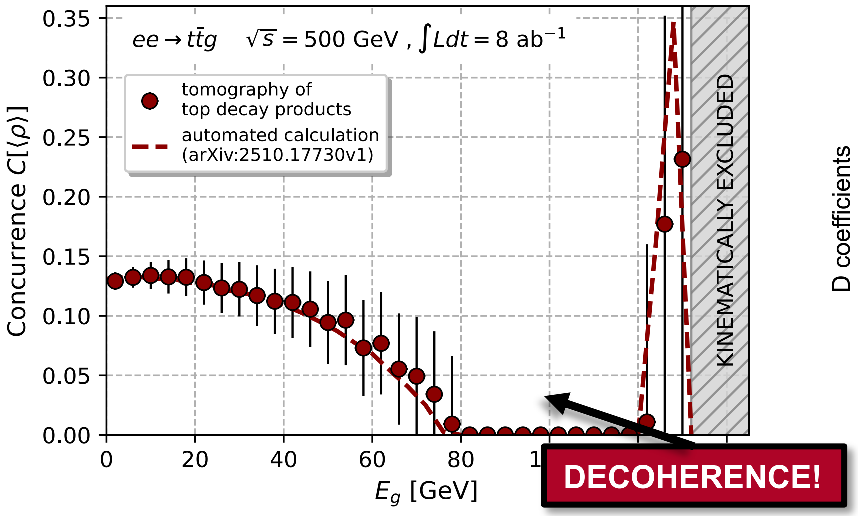


Simulation: MADGRAPH 5
[10.1007/jhep07(2014)079]

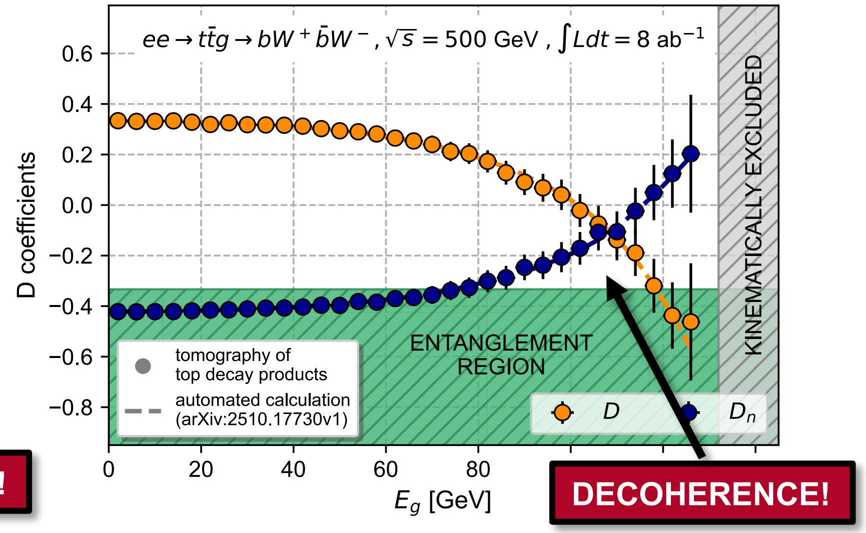
Decay mode: $\mu^+\mu^-$

Spin-spin transition! ⟶ Modification from the **triplet** (D_n < -1/3) at E_g < 80 GeV, to a **singlet** (D < -1/3), at the kinematic limit.

CONCURRENCE



D MARKERS



Concurrence (C[ρ]):

Provides with a quantitative measurement of the degree of entanglement. Non-linear:

$$C\left[\langle\rho\rangle\right] = \max\left(0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\right)$$

λ_i increasing eigenvalues of

$$R = \sqrt{\sqrt{\rho}\sigma_2 \otimes \sigma_2 \rho^* \sigma_2 \otimes \sigma_2 \sqrt{\rho}}$$

D and D_n markers:

Provide with sufficient enough criteria for entanglement via linear expressions:

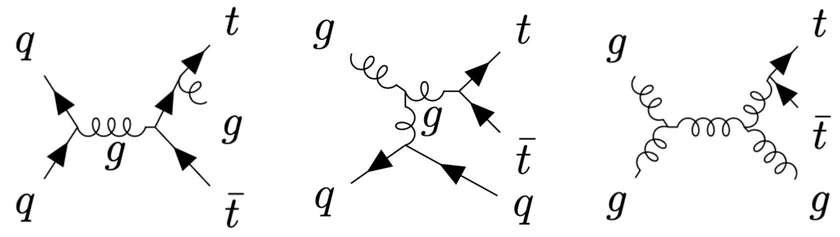
$$D = -\tfrac{1}{3}\left(C_{nn} + C_{rr} + C_{kk}\right)$$

$$D_n = -\tfrac{1}{3}\left(C_{nn} - C_{rr} - C_{kk}\right)$$

$D < -1/3$ or $D_n < -1/3 \rightarrow$ ENTANGLEMENT

Decoherence prospects for $pp \to t\bar{t}$ from gluon FSR

This processes may involve the radiation of quark and gluon jets:



etc.

Simulation: MADGRAPH 5
[10.1007/jhep07(2014)079]

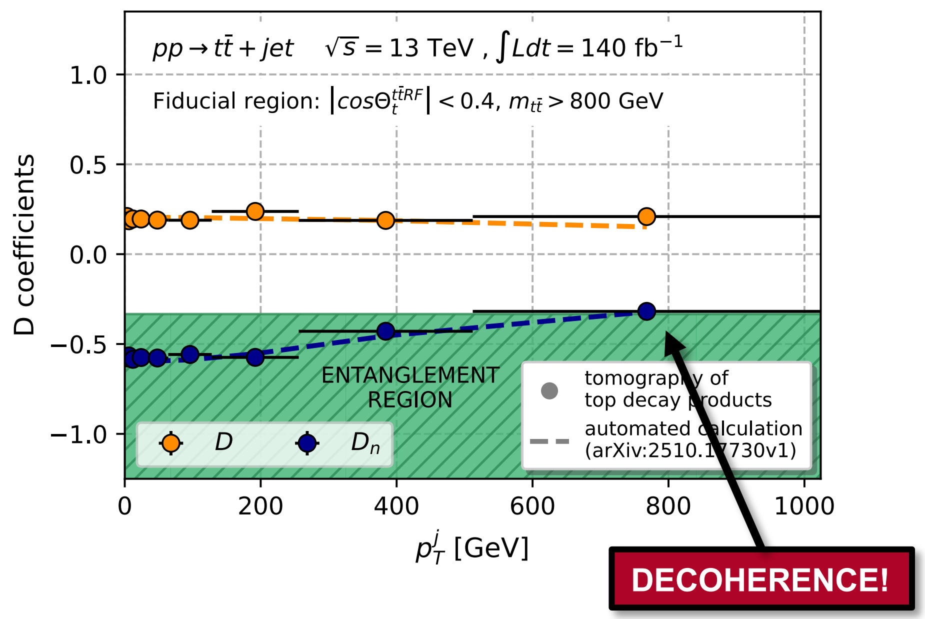
Decay mode: $\mu^+\mu^-$

$$\left|\cos\Theta_t^{t\bar{t}RF}\right| < 0.4, \quad m_{t\bar{t}} > 800\ \text{GeV}$$

Restrict to the **fiducial region:**

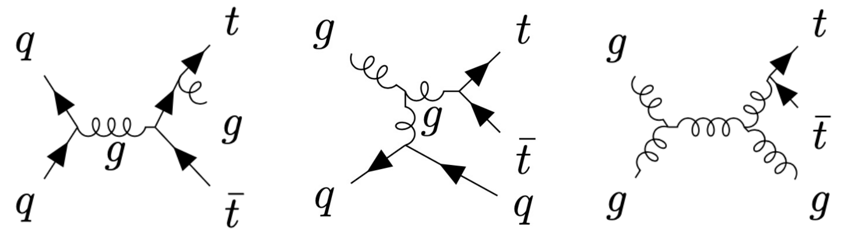
- Isolate qq → tt̄+jet topologies
- reduce contributions from ISR jets.

D MARKERS



Decoherence prospects for $pp \to t\bar{t}$ from gluon FSR

This processes may involve the radiation of quark and gluon jets:



etc.

Simulation: MADGRAPH 5 *[10.1007/jhep07(2014)079]*

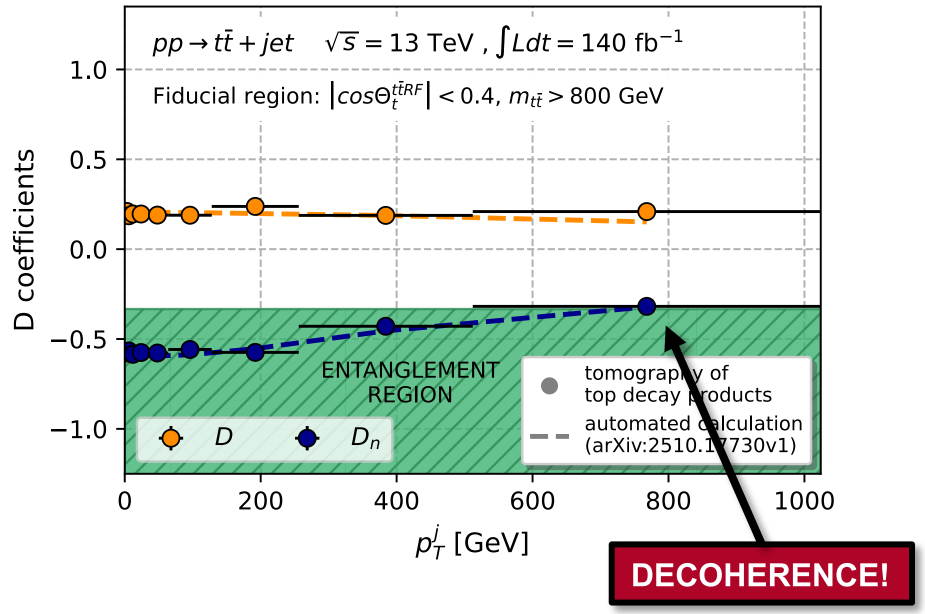
Decay mode: $\mu^+\mu^-$

$$\left|\cos\Theta_t^{t\bar{t}RF}\right| < 0.4, \quad m_{t\bar{t}} > 800 \text{ GeV}$$

Restrict to the **fiducial region:**

- Isolate qq → tt̄+jet topologies
- reduce contributions from ISR jets.

D MARKERS

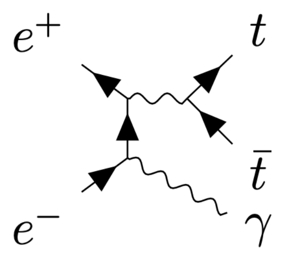


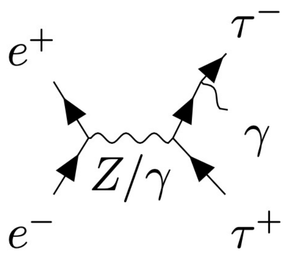
Decoherence present above 250 GeV.

Total separability found above 500 GeV.

Decoherence prospects for $e^+e^- \rightarrow \tau^+\tau^-$ *from gamma FSR*

This processes may involve the FSR and ISR:





We perform a tomography of hadronic **one-prong decays** (Fabbrichesi et. al.):

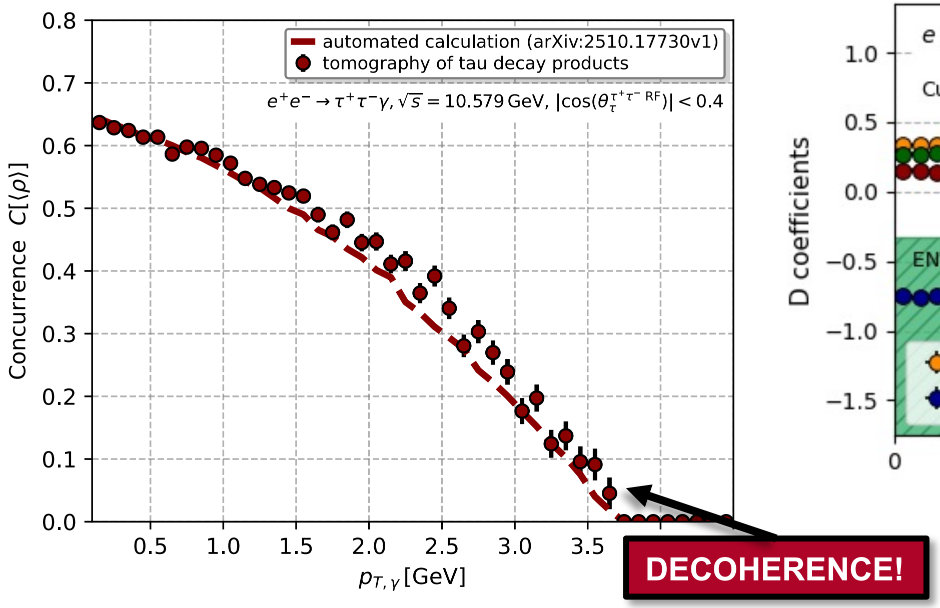
$\tau^\pm \rightarrow \pi^\pm \nu$

spin analysing power α = 1

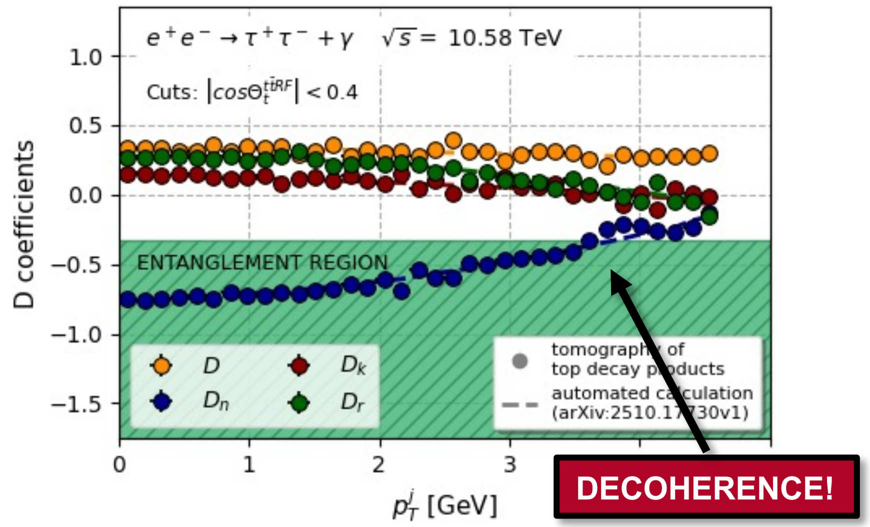
$$\left|cos(\Theta_\tau^{\tau^+\tau^- RF})\right| < 0.4$$

Restrict to central events to reduce contributions from ISR photons.

CONCURRENCE



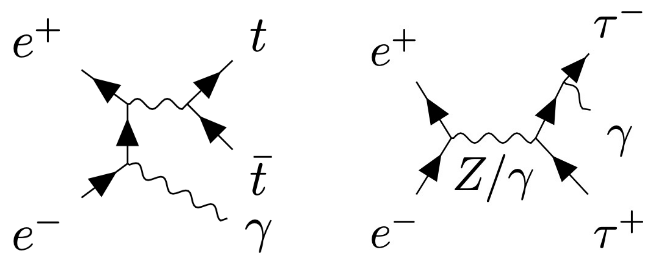
D MARKERS



Example for Belle II experiment ($\sqrt{s}$ = 10.58 GeV).

Decoherence prospects for $e^+e^- \rightarrow \tau^+\tau^-$ *from gamma FSR*

This processes may involve the FSR and ISR:



We perform a tomography of hadronic **one-prong decays** (Fabbrichesi et. al.):

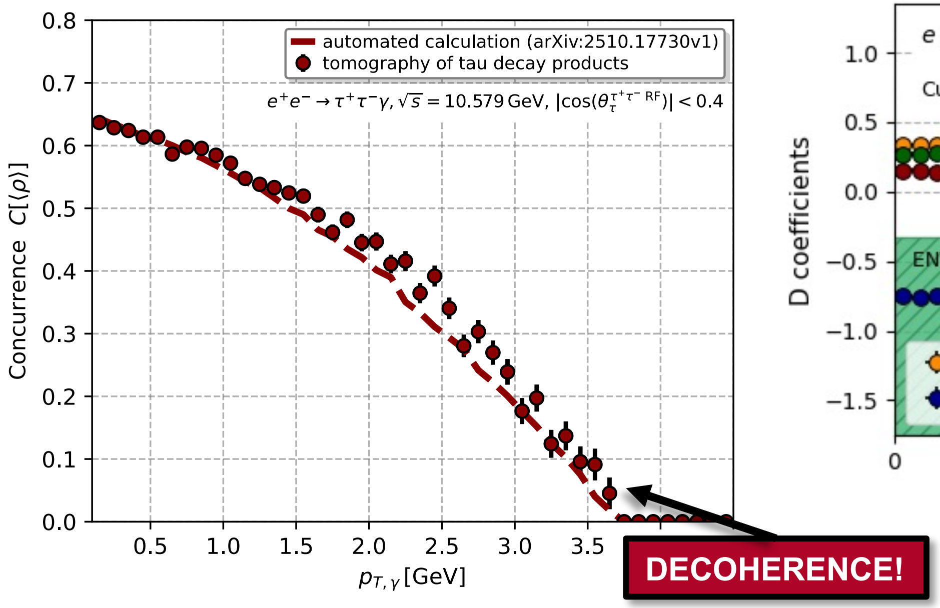
$\tau^\pm \rightarrow \pi^\pm \nu$

spin analysing power α = 1

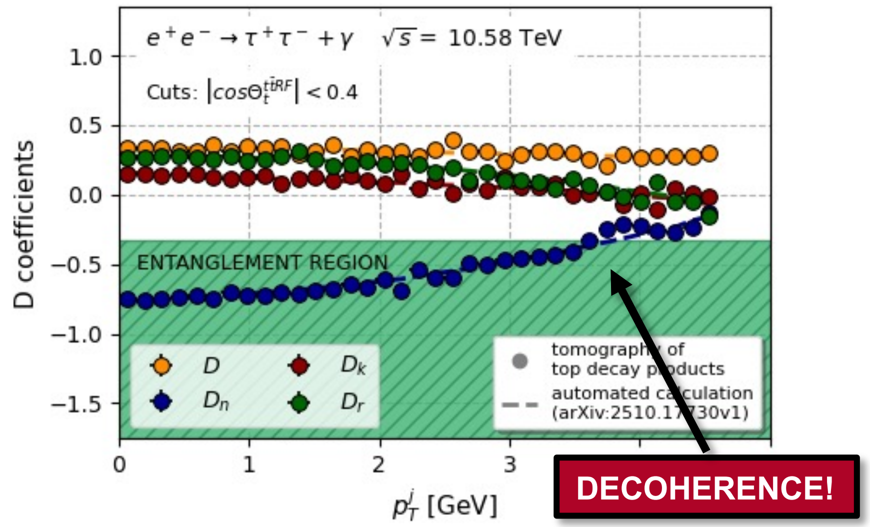
$$\left|cos(\Theta_{\tau}^{\tau^+\tau^- RF})\right| < 0.4$$

Restrict to central events to reduce contributions from ISR photons.

CONCURRENCE



D MARKERS



Example for Belle II experiment ($\sqrt{s}$ = 10.58 GeV).

Decoherence marked by the D_n marker

Total separability found above 500 GeV.

Statistical significances for the observation of decoherence

A simple feasibility study separates the data set in two regions

- Inclusive region → establish entanglement of the fermion pair (Cinc > 0)
- Exclusive region with FSR → establish decoherence (Cexc < Cinc)

Statistical significance

$$S = \frac{R - 1}{\sigma_R}$$

- R → Decoherence quotient
- 1 → Null hypothesis.
- σ_R → Statistical uncertainty

Statistical significances for the observation of decoherence

A simple feasibility study separates the data set in two regions

- Inclusive region → establish entanglement of the fermion pair (Cinc > 0)
- Exclusive region with FSR → establish decoherence (Cexc < Cinc)

Statistical significance

$$S = \frac{R - 1}{\sigma_R}$$

- R → Decoherence quotient
- 1 → Null hypothesis.
- σ_R → Statistical uncertainty

$e^+e^- \rightarrow t\bar{t}g$ SIGNIFICANCE $\int \mathcal{L}dt = 8\ \mathrm{ab}^{-1}$

$\sqrt{s}$ [TeV]	$C[\rho]_{t\bar{t}}$	$C[\rho]_{t\bar{t}g}$	$R = C_{t\bar{t}g}/C_{t\bar{t}}$	$\mathcal{S} = \frac{R-1}{\sigma_R}$
0.5	0.133	0.041 (p_T^g>30GeV)	0.310 ± 0.2 (dilep.) / 0.008 (all ch.)	**3.0σ** (dilep.) / **8.2 σ** (all ch.)
1.0	0.249	0.081 (p_T^g>120GeV)	0.33 ± 0.14 (dilep.) / 0.05 (all ch.)	**4.6σ** (dilep.) / **13 σ** (all ch.)

$pp \rightarrow t\bar{t}$+jet SIGNIFICANCE

$\sqrt{s}$ [TeV]	$\int \mathcal{L}dt$ [ab^{-1}]	$C[\rho]_{t\bar{t}}$	$C[\rho]_{t\bar{t}+jet}$	$R = C_{t\bar{t}+jet}/C_{t\bar{t}}$	$\mathcal{S} = \frac{R-1}{\sigma_R}$
13	0.14	0.310	0.118 (p_T^j>250GeV)	0.38 ± 0.16 (dilep.) / 0.07 (all ch.)	**4.0σ** (dilep.) / **11 σ** (all ch.)
14	3.00	0.431	0.096 (p_T^j>250GeV)	0.22 ± 0.06 (dilep.) / 0.02 (all ch.)	**12 σ** (dilep.) / **34 σ** (all ch.)

Significances above **5.0σ** if considering **leptonic and hadronic** final states.

Statistical significances for the observation of decoherence

A simple feasibility study separates the data set in two regions

- Inclusive region → establish entanglement of the fermion pair (Cinc > 0)
- Exclusive region with FSR → establish decoherence (Cexc < Cinc)

Statistical significance

$$S = \frac{R - 1}{\sigma_R}$$

Decoherence quotient

Null hypothesis.

Statistical uncertainty

$e^+e^- \to t\bar{t}g$ SIGNIFICANCE

$\int \mathcal{L}dt = 8\ \mathrm{ab}^{-1}$

$\sqrt{s}$ [TeV]	$C[\rho]_{t\bar{t}}$	$C[\rho]_{t\bar{t}g}$	$R = C_{t\bar{t}g}/C_{t\bar{t}}$	$\mathcal{S} = \frac{R-1}{\sigma_R}$
0.5	0.133	0.041 (p_T^g>30GeV)	0.310 ± 0.2 (dilep.) / 0.008 (all ch.)	**3.0σ** (dilep.) / **8.2 σ** (all ch.)
1.0	0.249	0.081 (p_T^g>120GeV)	0.33 ± 0.14 (dilep.) / 0.05 (all ch.)	**4.6σ** (dilep.) / **13 σ** (all ch.)

$pp \to t\bar{t}$+jet SIGNIFICANCE

$\sqrt{s}$ [TeV]	$\int \mathcal{L}dt$ [ab^{-1}]	$C[\rho]_{t\bar{t}}$	$C[\rho]_{t\bar{t}+jet}$	$R = C_{t\bar{t}+jet}/C_{t\bar{t}}$	$\mathcal{S} = \frac{R-1}{\sigma_R}$
13	0.14	0.310	0.118 (p_T^j>250GeV)	0.38 ± 0.16 (dilep.) / 0.07 (all ch.)	**4.0σ** (dilep.) / **11 σ** (all ch.)
14	3.00	0.431	0.096 (p_T^j>250GeV)	0.22 ± 0.06 (dilep.) / 0.02 (all ch.)	**12 σ** (dilep.) / **34 σ** (all ch.)

Significances above **5.0σ** if considering **leptonic and hadronic** final states.

$e^+e^- \to \tau^+\tau^-\gamma$ SIGNIFICANCE

$\sqrt{s}$ [GeV]	$C[\rho]_{\tau^+\tau^-}$	$C[\rho]_{\tau^+\tau^-\gamma}$	$R = C_{t\bar{t}+jet}/C_{t\bar{t}}$	$\mathcal{S} = \frac{R-1}{\sigma_R}$
10.58 Belle II [9]	0.727	0.235 p_T^γ>2GeV	0.323 ± 0.02 (0.5 ab^{-1}) / 0.002 (50.0 ab^{-1})	28σ (0.5 ab^{-1}) / ≫ 5σ (50.0 ab^{-1})
91.20 Z-Pole [10]	0.858	0.102 p_T^γ>30GeV	0.1190 ± 0.05 (0.1 ab^{-1}) / 0.0009 (300 ab^{-1})	16σ (0.1 ab^{-1}) / ≫ 5σ (300 ab^{-1})

$\tau^+\tau^-\gamma$ production provides the **cleanest measurement of FSR** decoherence, in a single decay channel.

Conclusions

- Fermion pairs produced at colliders show a **significant loss of entanglement** when considering **gluon and photon FSR**.

- Measurements of entanglement in the FSR samples **deviate more than 5.0σ** from the inclusive reference measurement.

- A minimal protocol, only considering the statistical significances, yields the greatest statistics for this observation at FCC-ee (in $\tau^+\tau^-$ production), in comparison with LHC, LCF and Belle II.

- The **target** data sets of Belle II, the HL-LHC phase, and the FCC-ee collider can perform **precision differential measurements**, with even greater statistical significance.

References

[1] Afik and Muñoz de la Nova. Eur. Phys. J. Plus 136, 907 (2021)

[2] ATLAS Collaboration. Nature 633, 542 (2024)

[3] CMS Collaboration. PRD 110, 112016 (2024)

[4] Aoude, Barr, Maltoni and Satrioni (arXiv.2504.07030)

[5] Bernreuther, Heislera and Sib. JHEP12(2015)026

[6] Alwall et al. arXiv:1405.0301

[7] Durupt, Maltoni and Mattelaera. arXiv:2510.17730v1

[8] Maltoni, Severi, Tentori and Vryonidou, JHEP 09 (2024) 001, [2404.08049].

[9] Ehatäht, Fabbrichesi , Marzola, and Veelken. Phys. Rev. D 109, 032005

[10] FCC collaboration. Eur. Phys. J. C 85, 1468 (2025).

[11] *H. Kragh (2002). Quantum generations*

[12] *W. H. Zurek (1025). Decoherence and Quantum Darwinism*

Based on the paper arXiv:2604.16268.
Soon to appear in XXX (2026)

Contact information:

guillermo.garcia-mir@uv.es

jose.m.camacho@uv.es

marcel.vos@ific.uv.es

maria.moreno@ific.uv.es

The author's work is funded by regional government Generalitat Valenciana (Spain) through the predoctoral grant **CIACIF/2024/194.** This work is also supported by Spanish State Research Agency (AEI) grants PID2024-158190NB-C21 and PID2024-156321NB-I00.