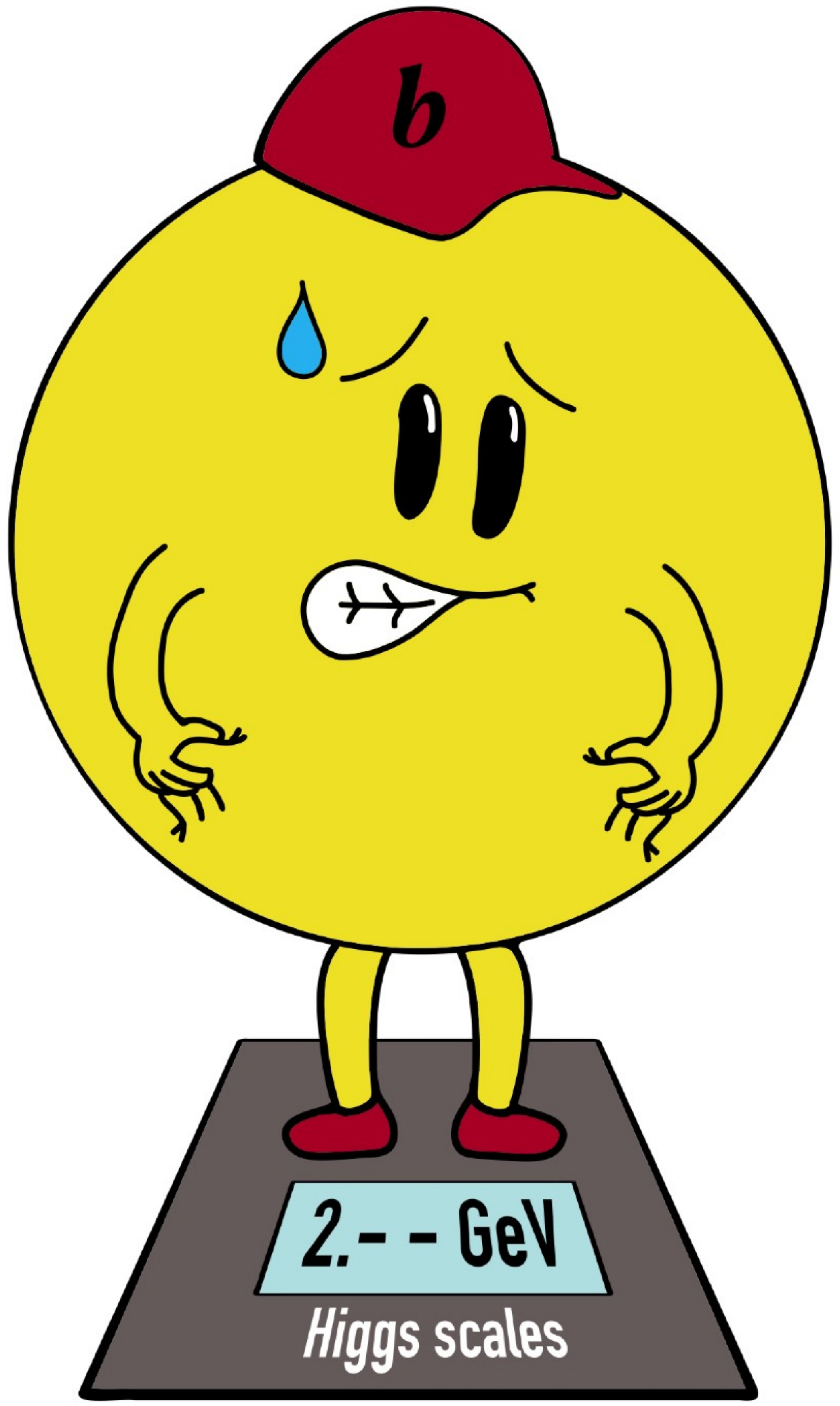
The bottom quark mass at high scale



Juan Ramírez Alfaro[1]

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Valencia, 26 May 2026

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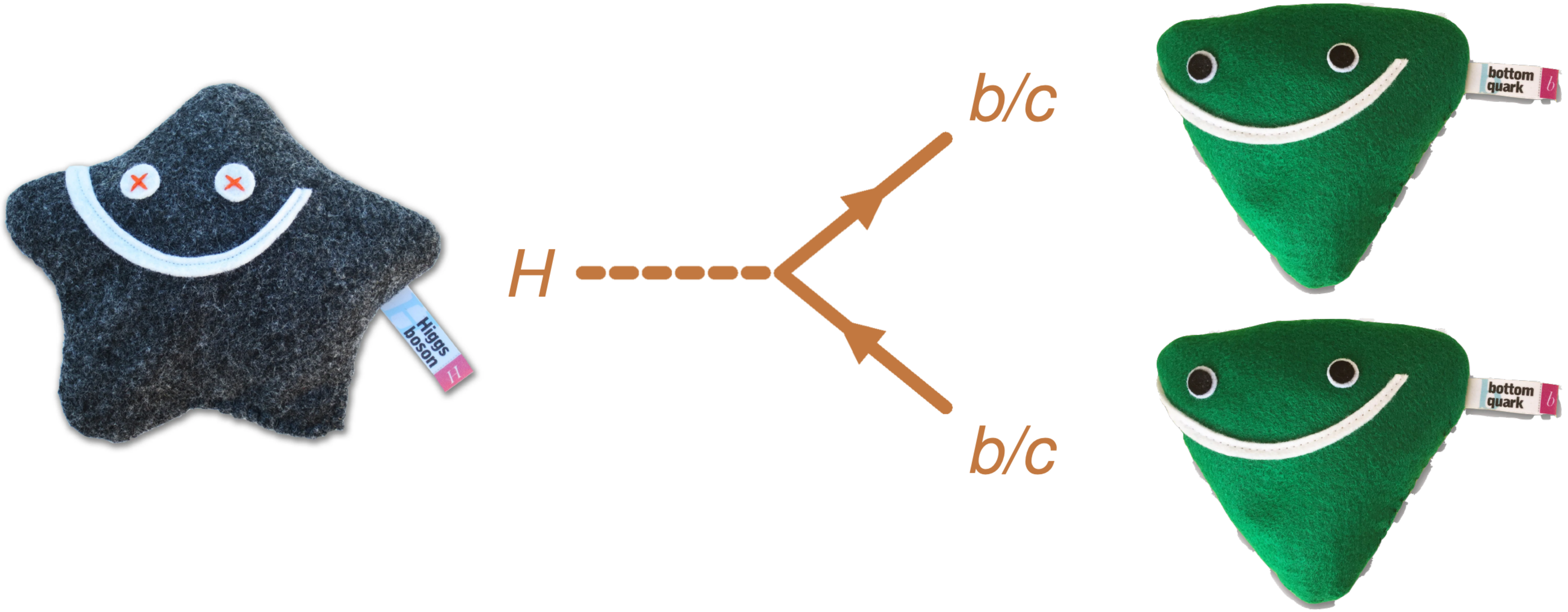
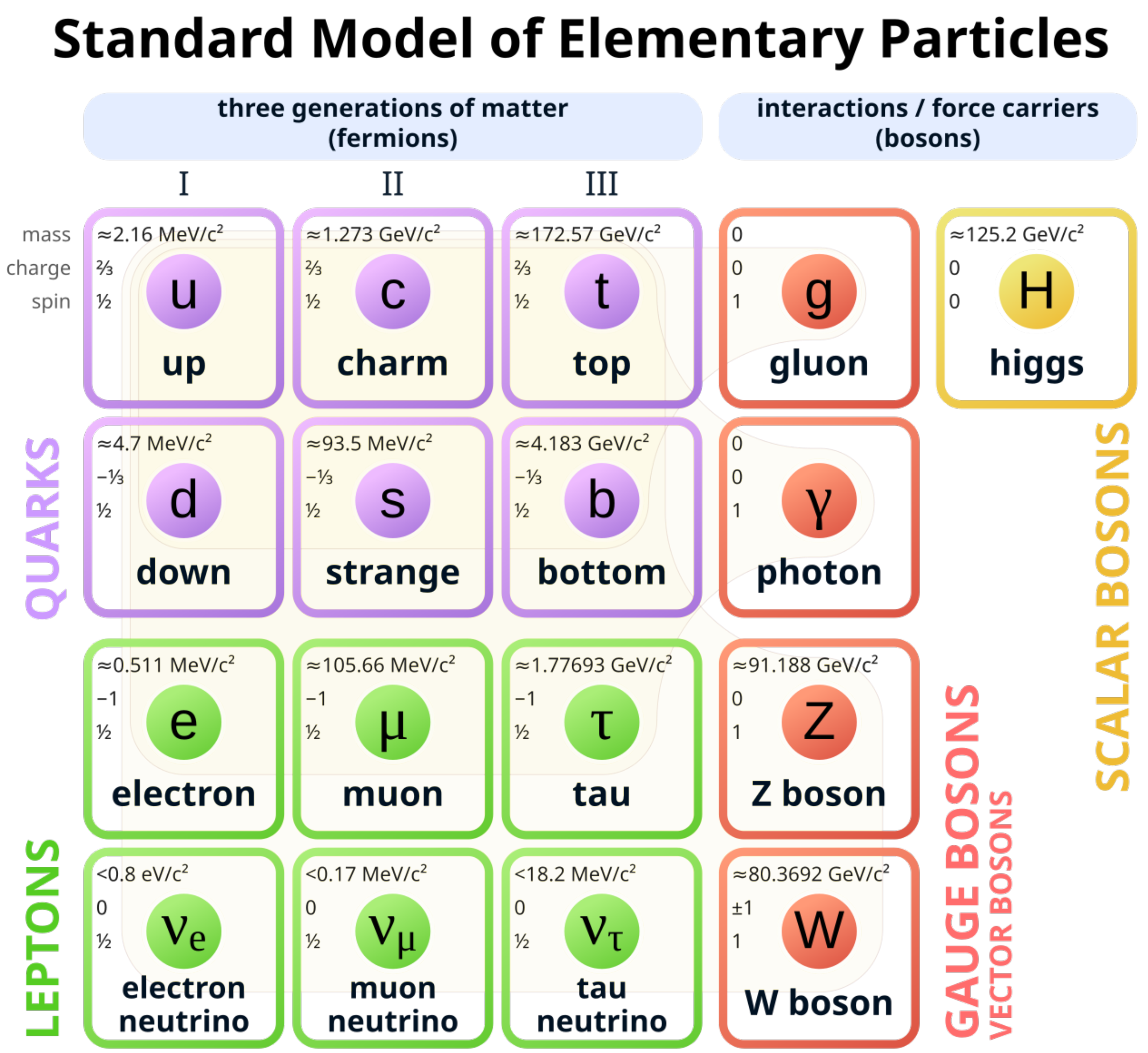
Introduction & Motivation



The SM and running parameters

- **Standard Model** (SM): best description of particle physics up to date.



The SM and running parameters

$$\not{D} = (\partial_\mu - gA_\mu)\,\gamma^\mu$$

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- In SM theory, some SM quantities (masses, couplings) are treated as **free parameters**, which in QFT undergo **dim. renormalization**(*); two key properties:

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i\bar{\psi}\not{D}\psi + \bar{\psi}_i y_{ij} \psi_j \phi + h.c. + |D_\mu \phi|^2 - V(\phi)$$

$$\psi_i\, y_{ii}\, \psi_i\, \phi \propto m_\psi\, \psi_i \psi_i \phi$$

(*): For this work we consider the *modified minimal subtraction* or $\overline{MS}$ renormalization scheme. Thus, we always assume $m_b \equiv m_b^{\overline{MS}}$

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  - This reference can be **evolved** to find the value at any other μ i.e. the parameter evolves/"runs" with the scale: **running constants**.

measured experimentally

$m_f(\mu_0) \equiv$ reference value

$m_f(\mu)$, dependent on μ and the reference value

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- QCD parameters' running (strong coupling $\alpha_s(\mu)$ and quark masses $m_q(\mu)$) is precisely predicted by theory: **Renormalization Group Equations** (RGE).

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$m_q(\mu_0) \equiv$ reference value

$$\frac{\partial m_q(\mu)}{\partial \log(\mu^2)} = \gamma_m[\alpha_s(\mu)]\, m_q(\mu)$$

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Performing several measurements at different energy scales allows us to test the renormalization scale dependence of these parameters experimentally!

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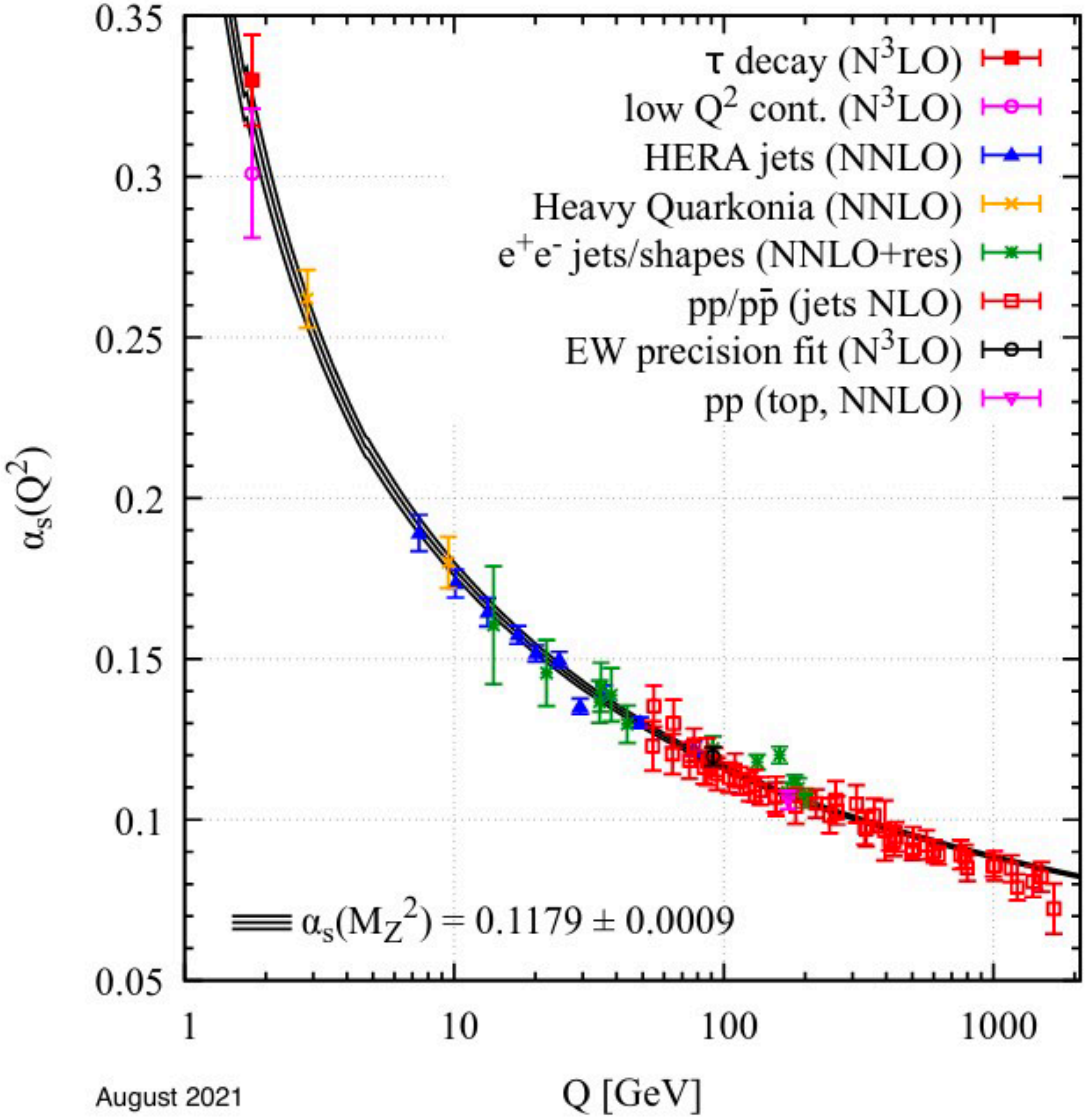
$m_b(m_H)$ and running

- α_s experimentally shows scale evolution…



P. Zyla *et al.* (Particle Data Group), PTEP 2020, 083C01 (2020)

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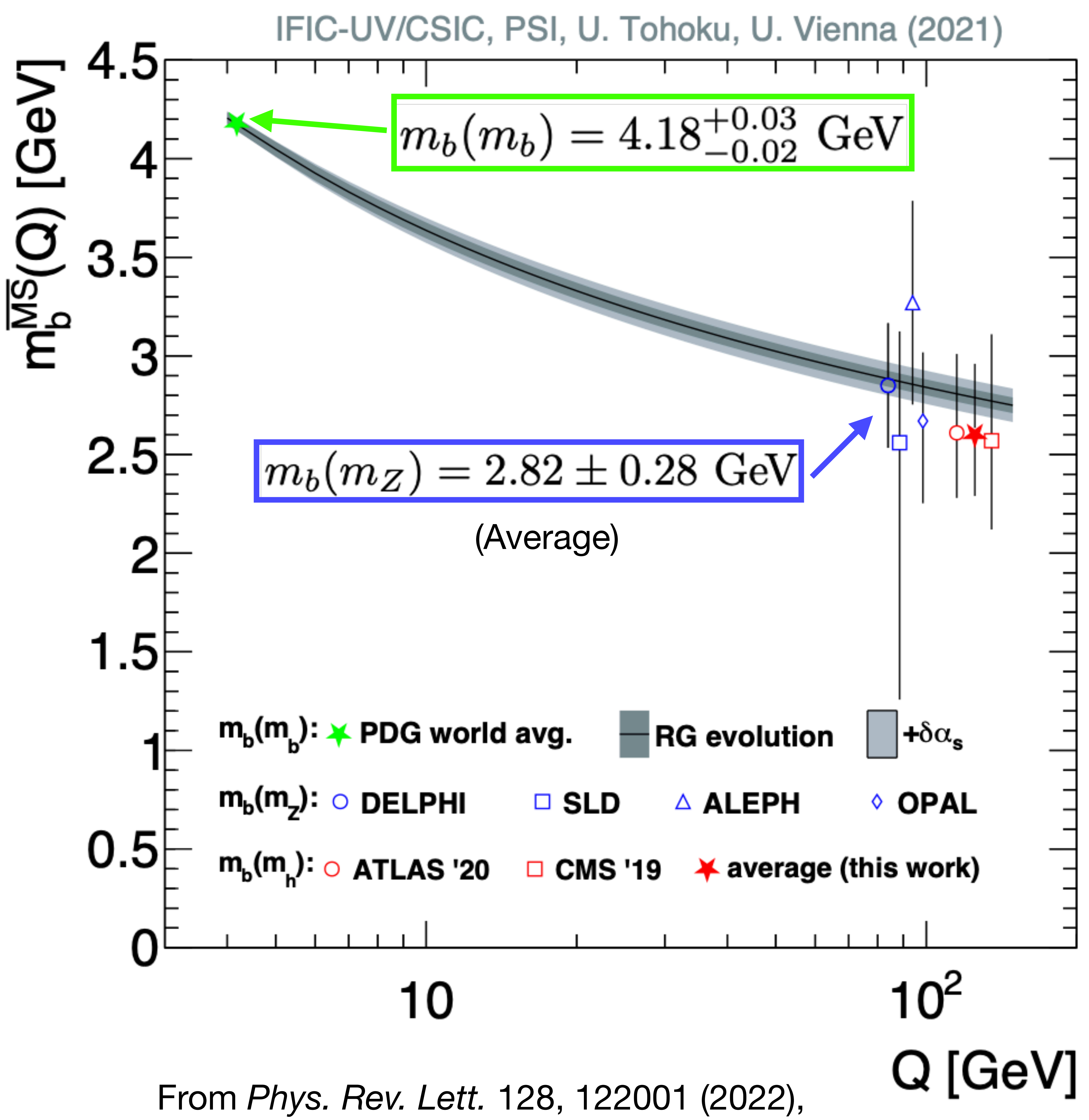

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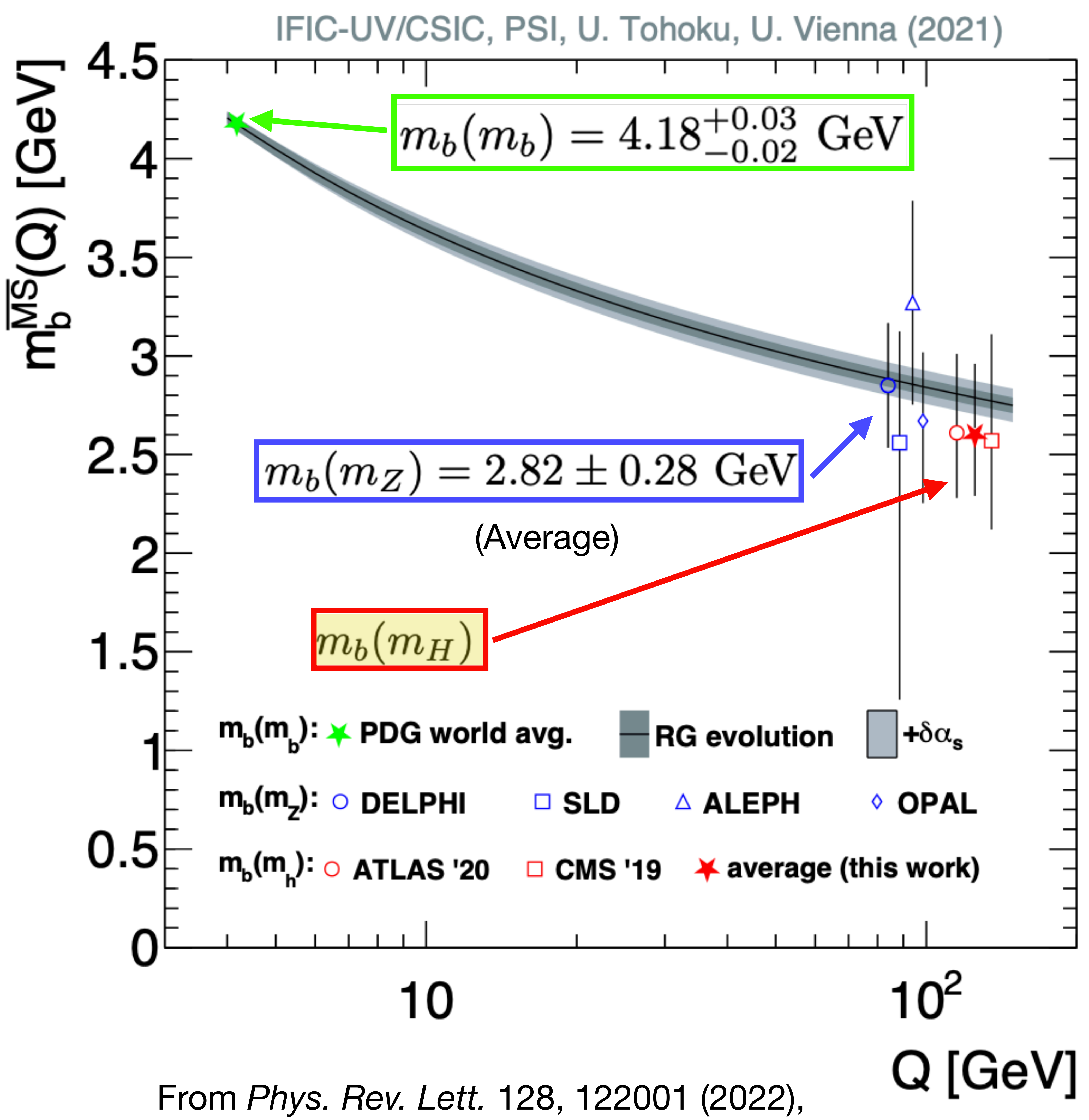

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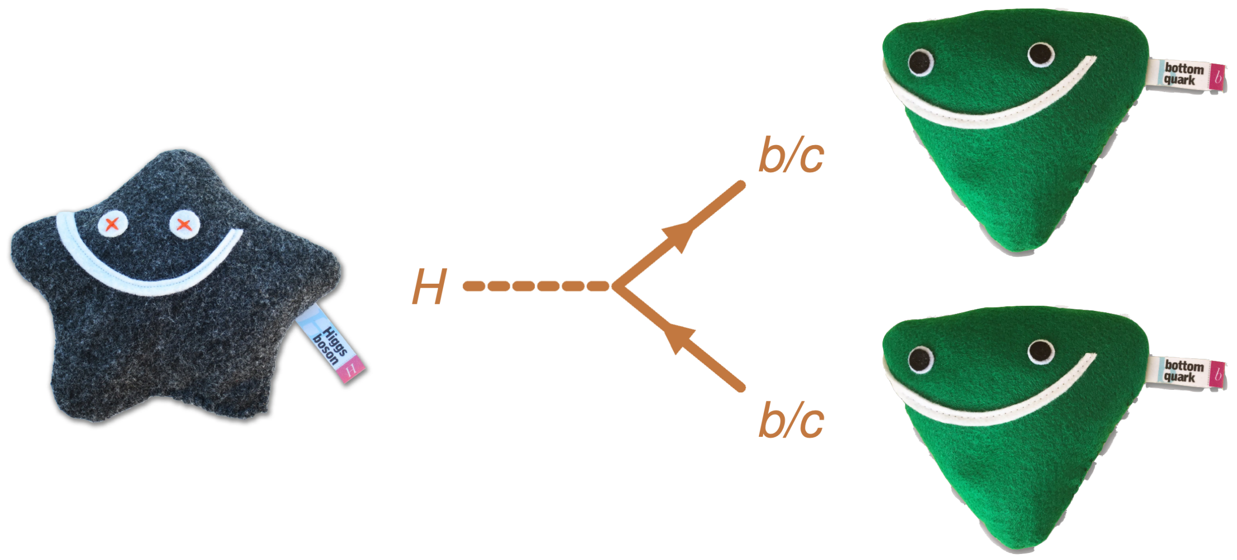

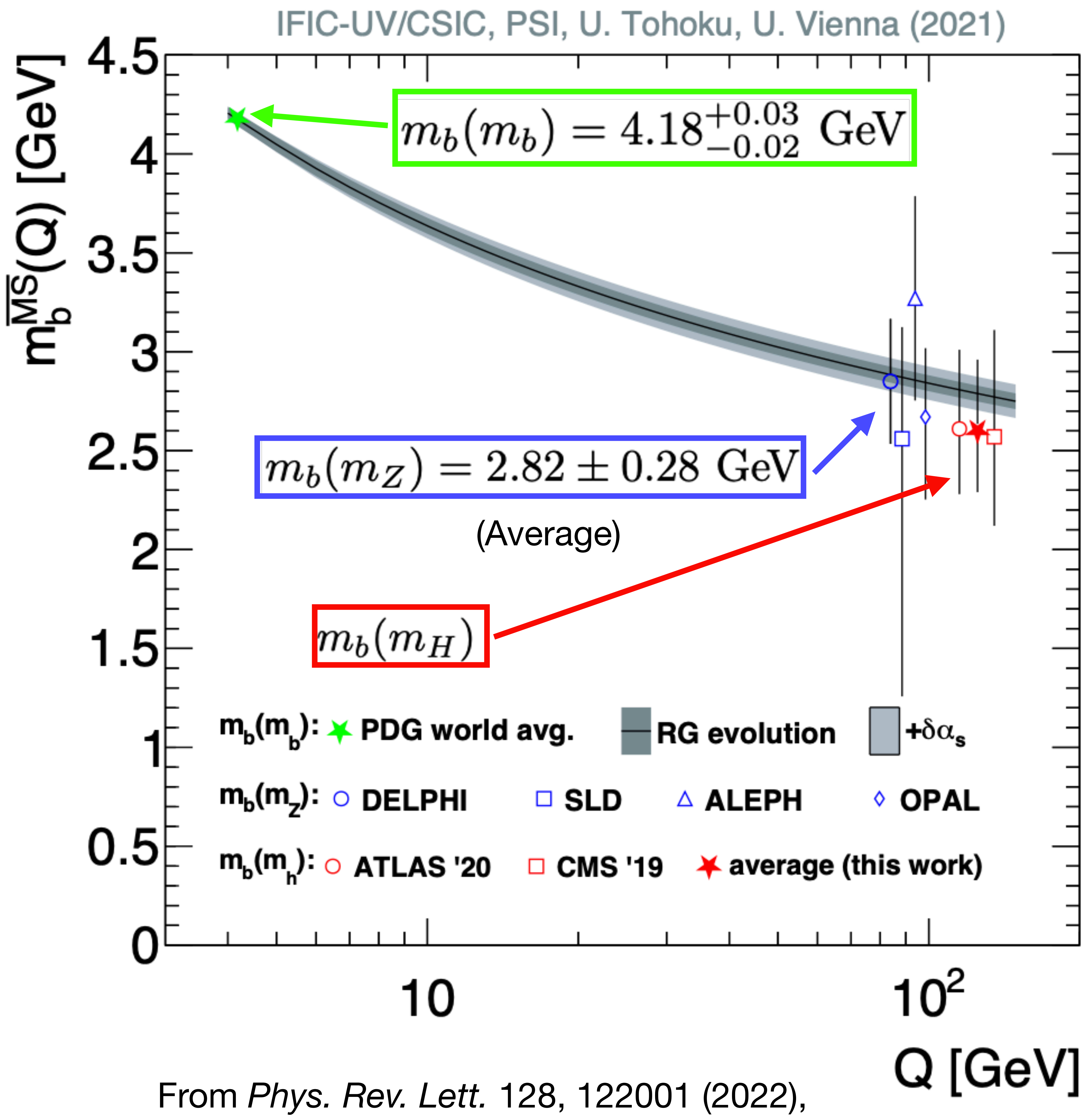

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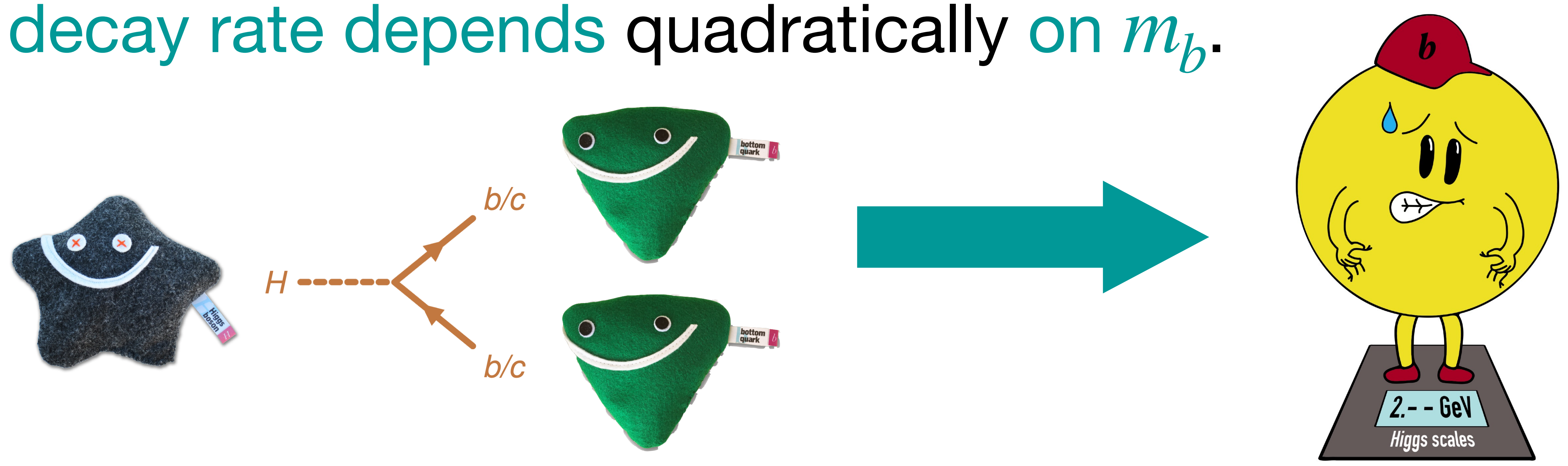

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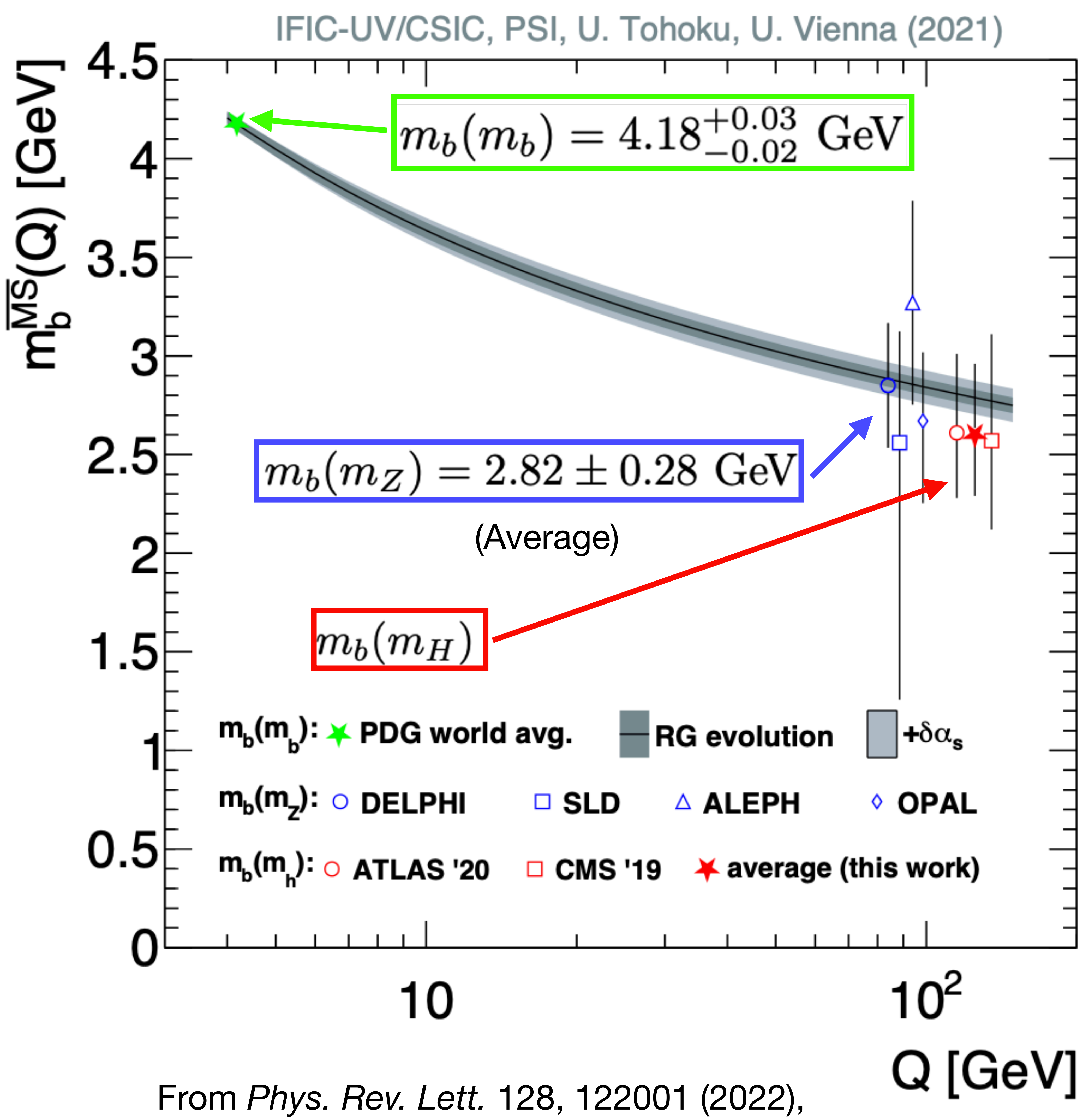

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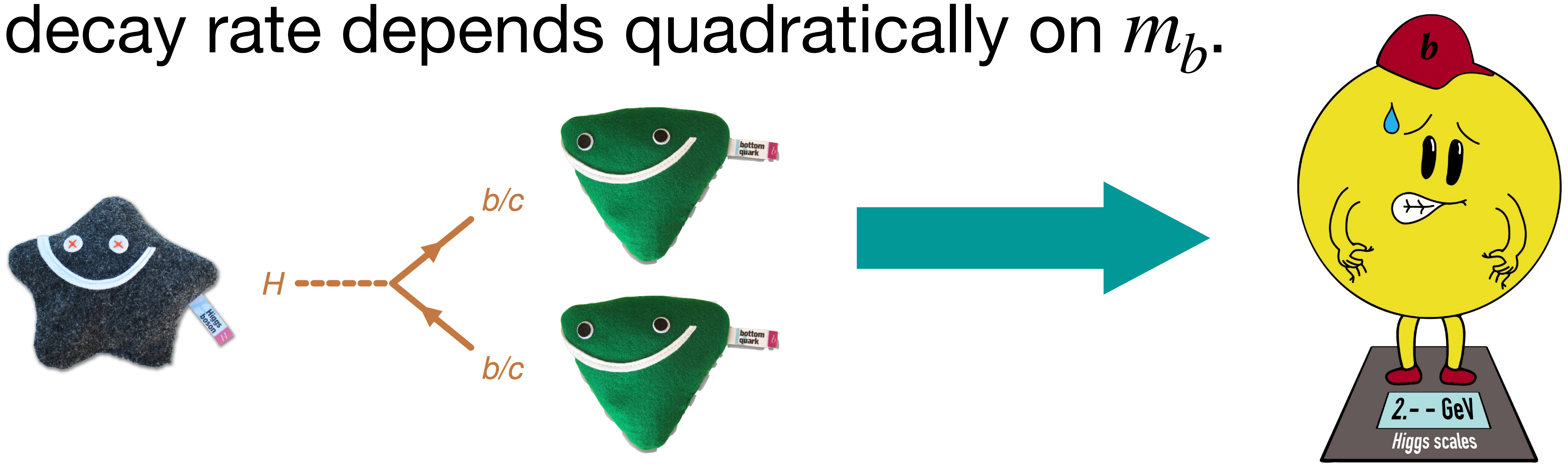

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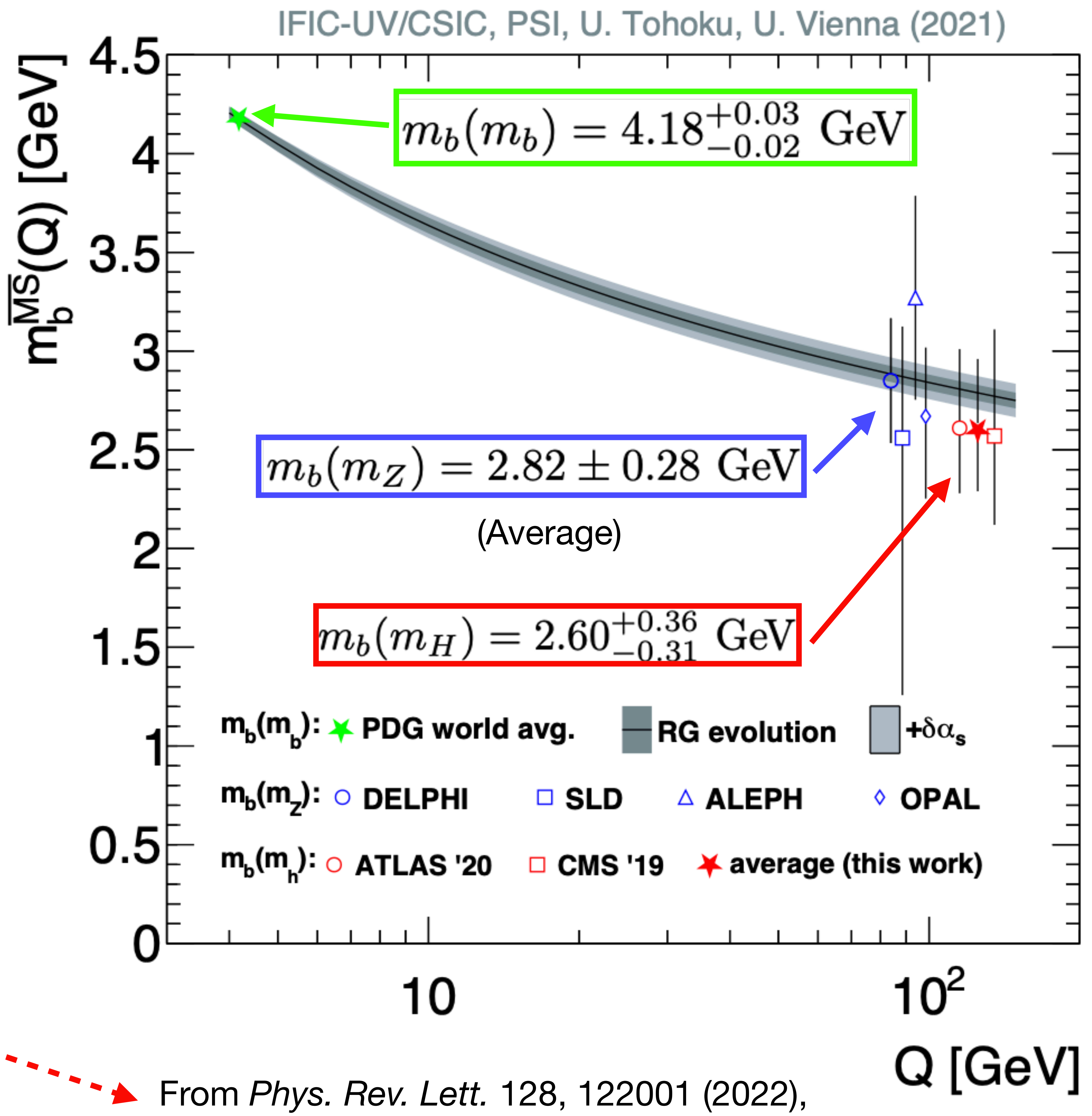
- **First result** uses ATLAS & CMS data, PRL 2022.
- **Test of running hypothesis**: compatible with SM within 1σ, **differs from no-running by nearly 7σ**.



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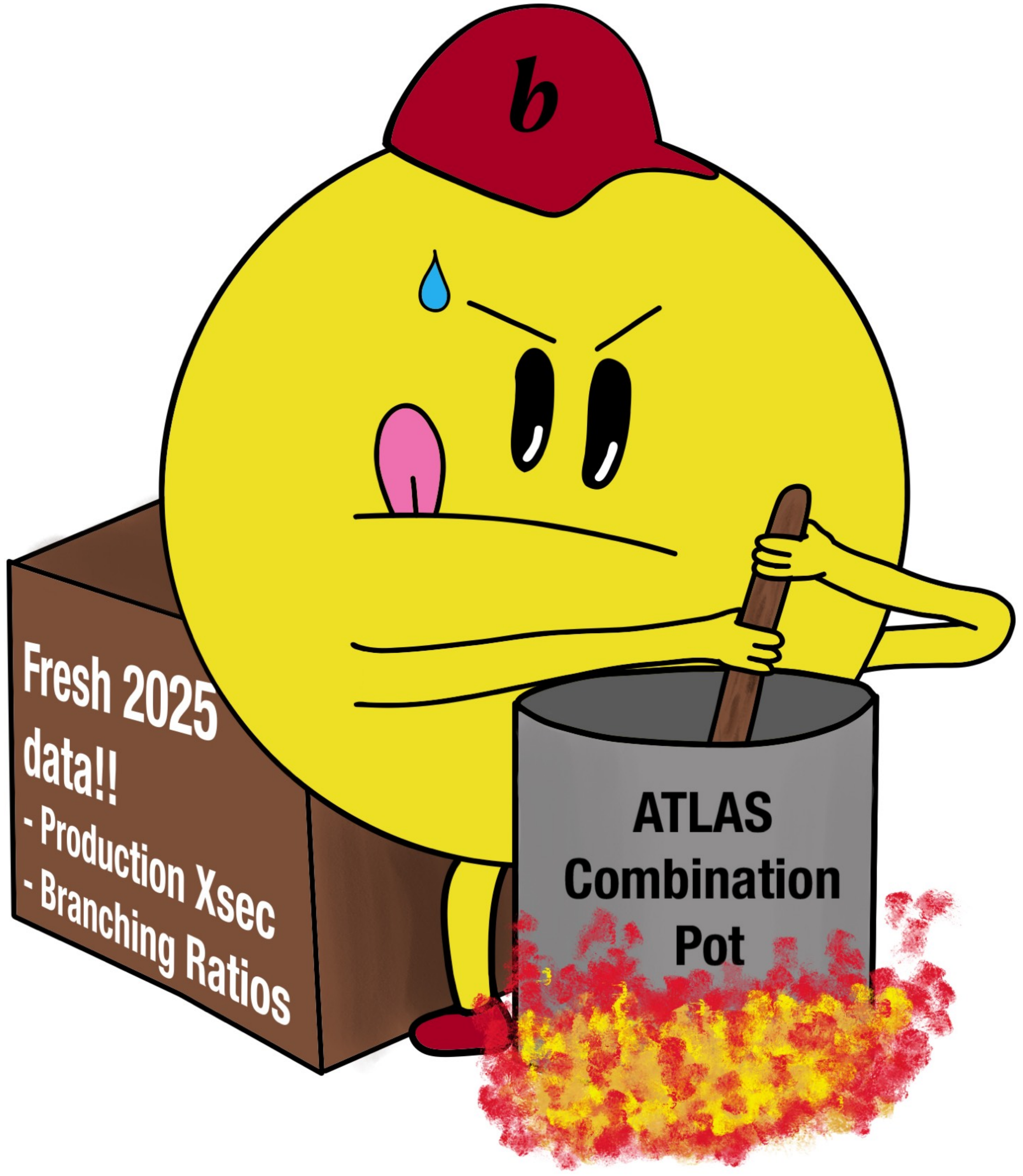
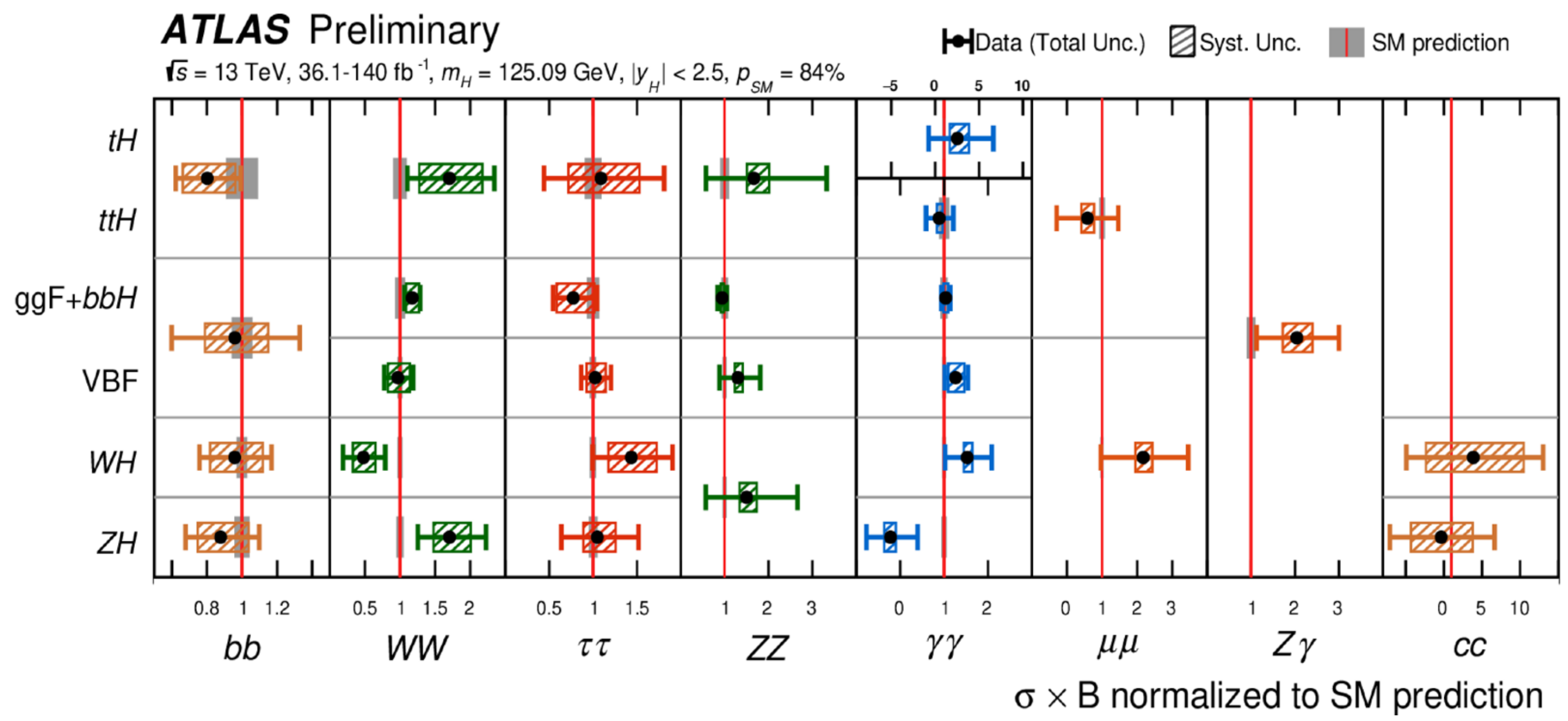
CSIC
Consejo Superior de Investigaciones Científicas
Universitat de València
IFIC Institut de Física Corpuscular
Excelencia Severo Ochoa
ATLAS Experiment

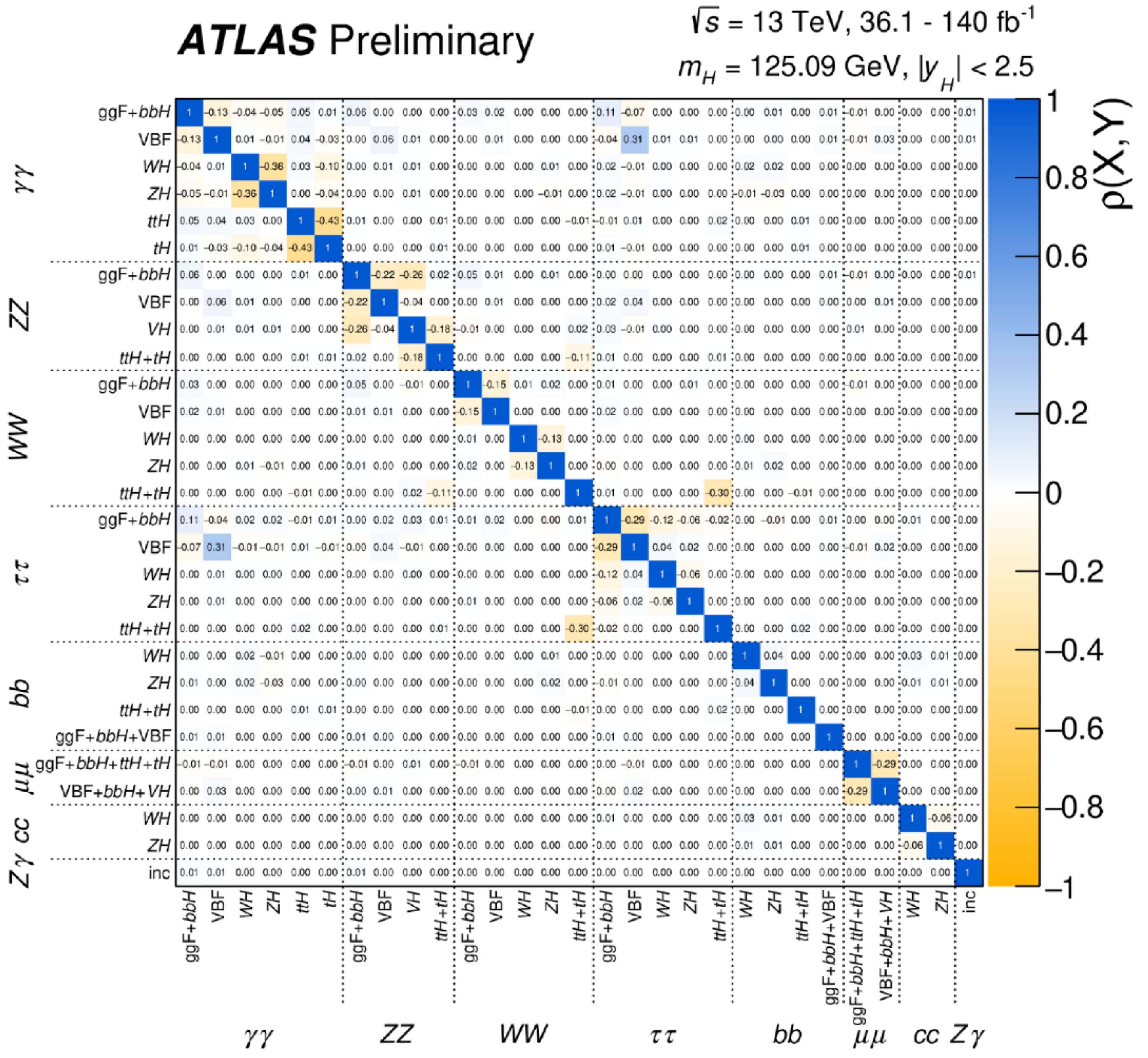
$m_b(m_H)$ measurement at LHC with updated ATLAS data



Inputs & procedure

- **ATLAS-CONF-2025-006**: **measurements** of Higgs <u>production cross-sections</u> σ_i and <u>decay rates</u> B_f, $\sigma_i \times B_f$, as well as the **full correlation matrix**.





From *https://atlas.web.cern.ch/Atlas/GROUPS/PHYSICS/CONFNOTES/ATLAS-CONF-2025-006/*

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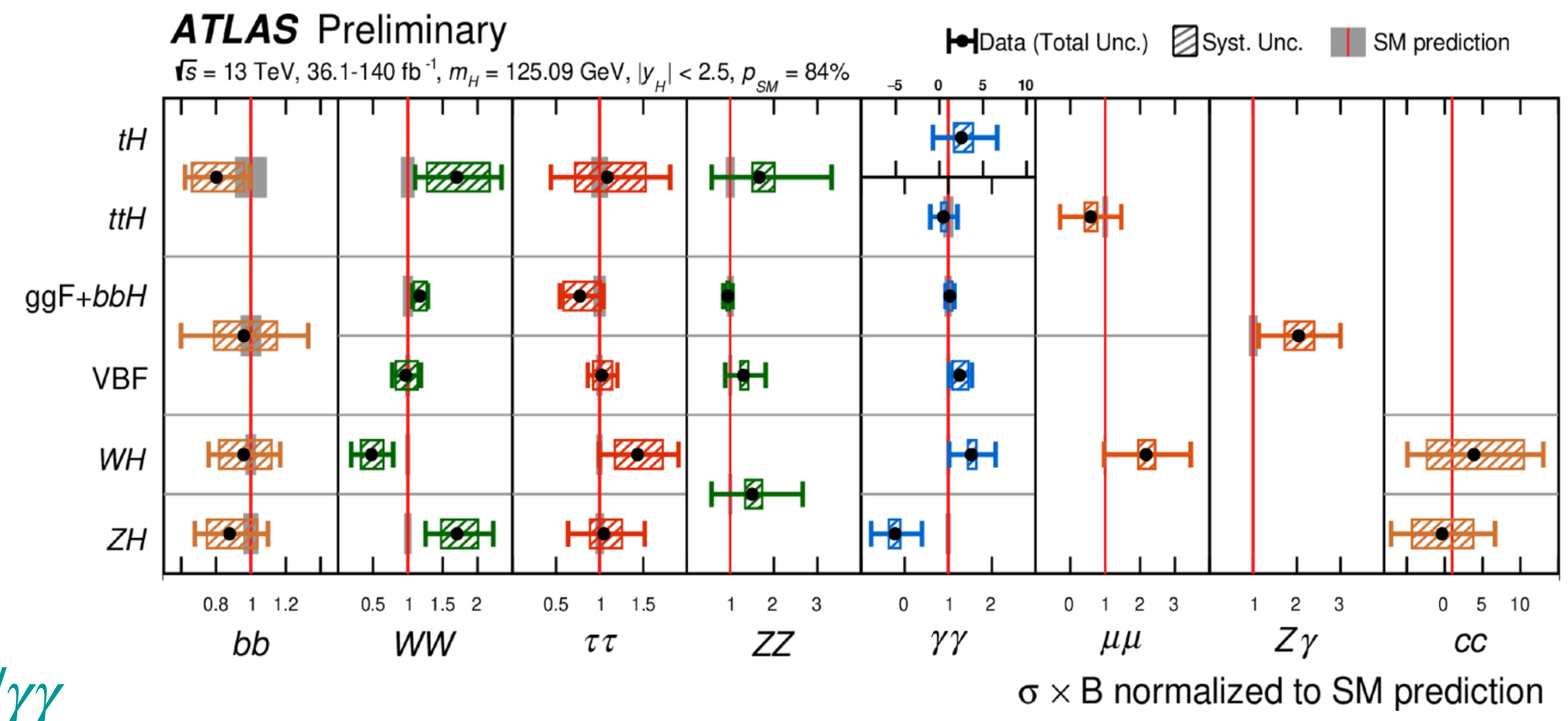

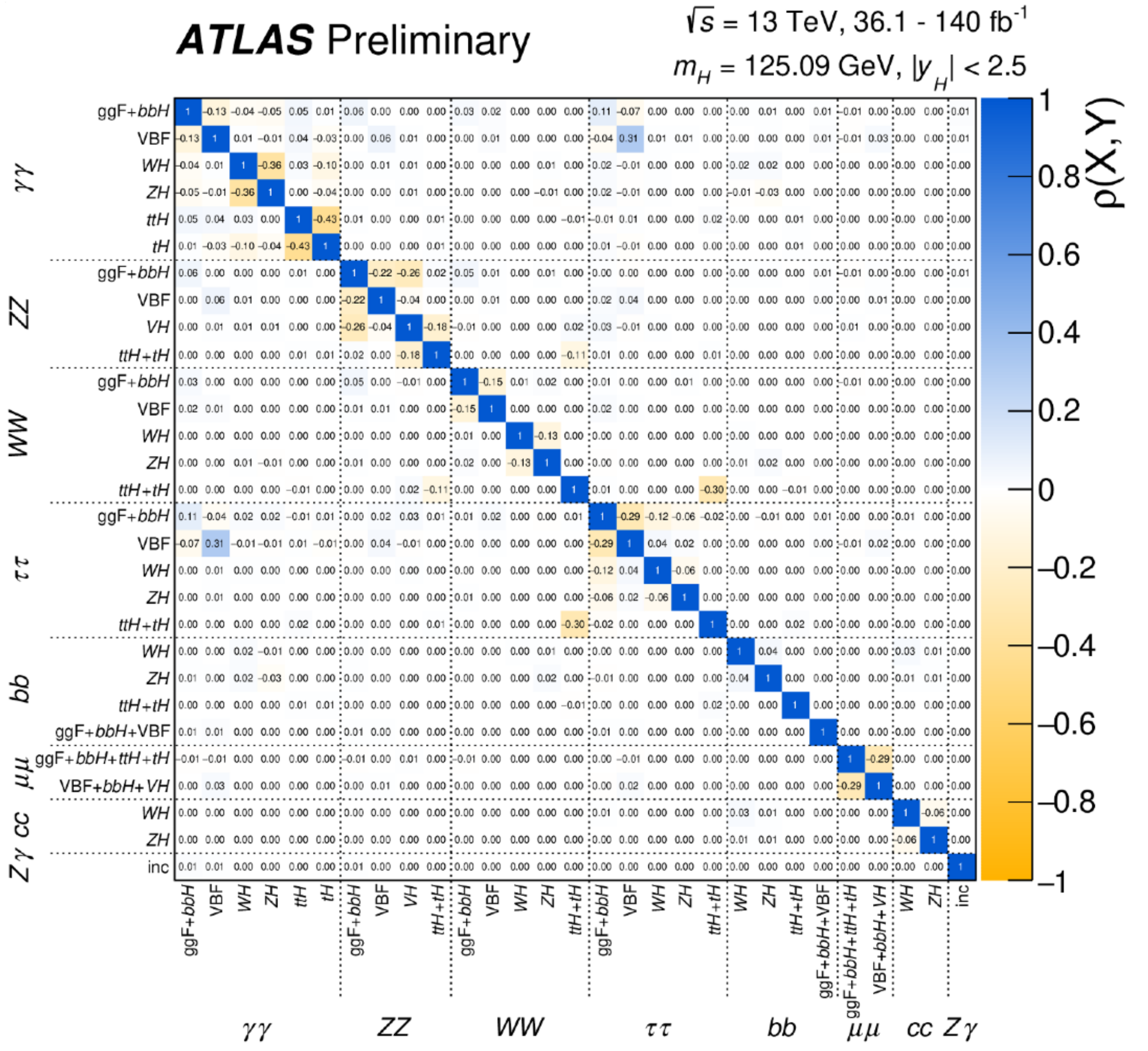

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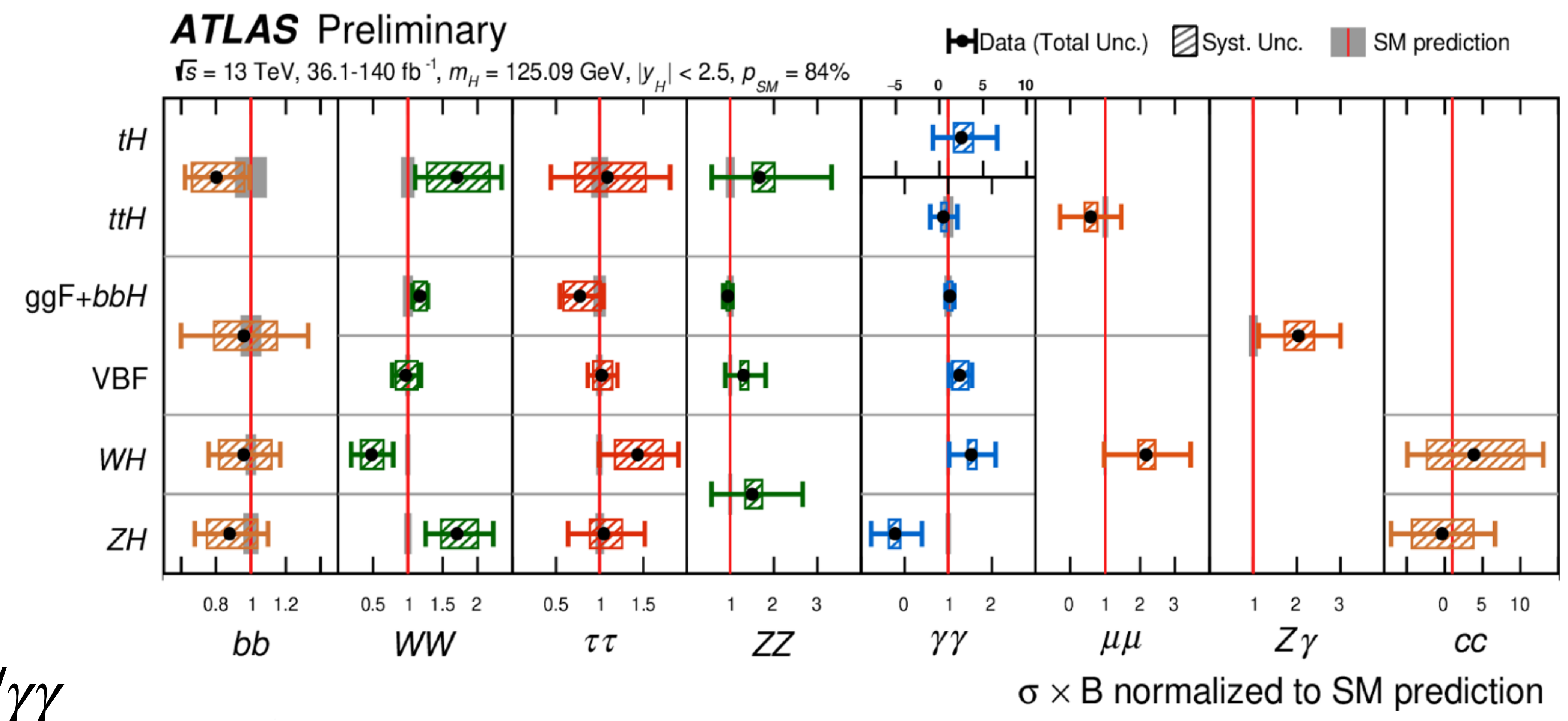

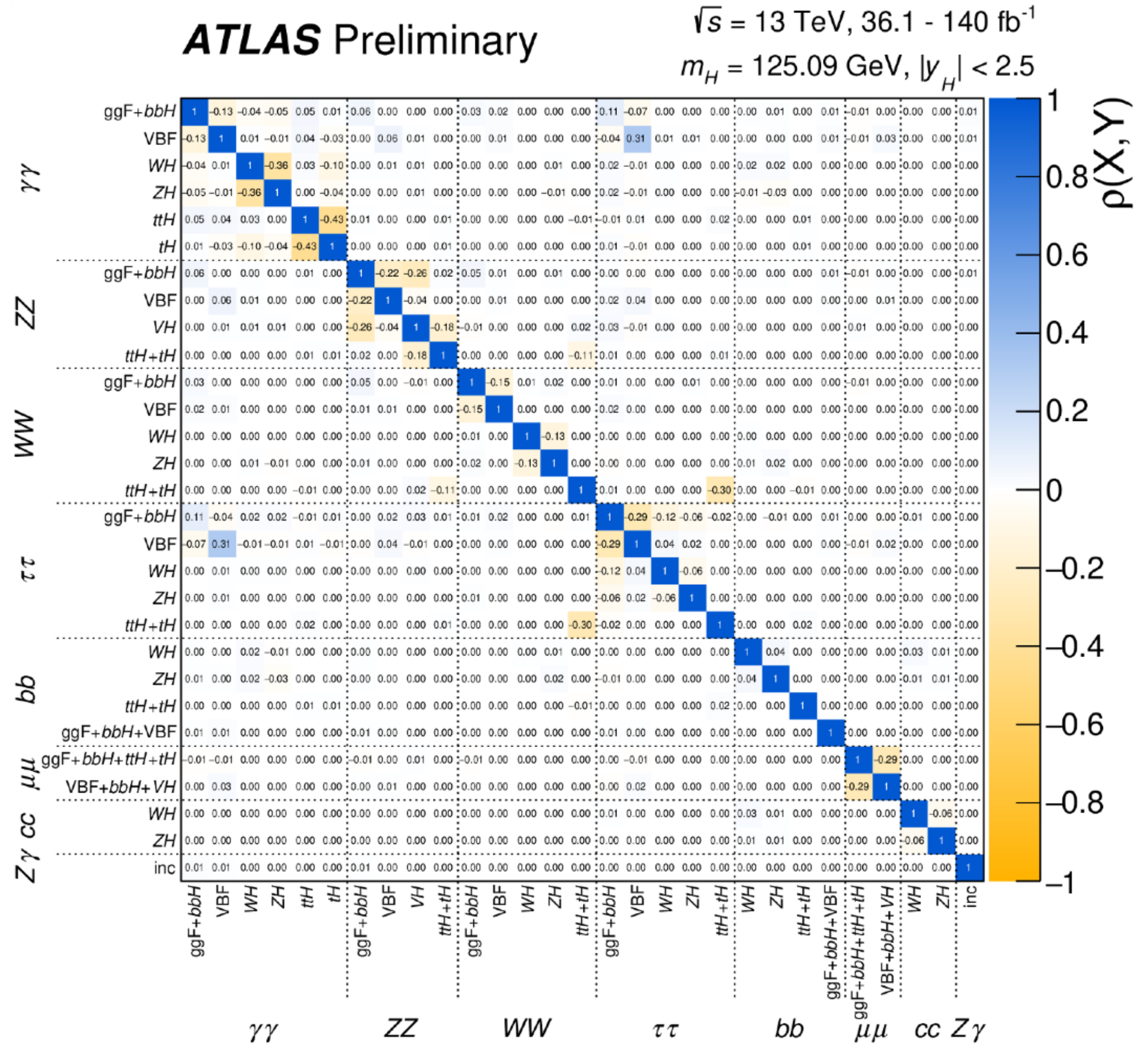

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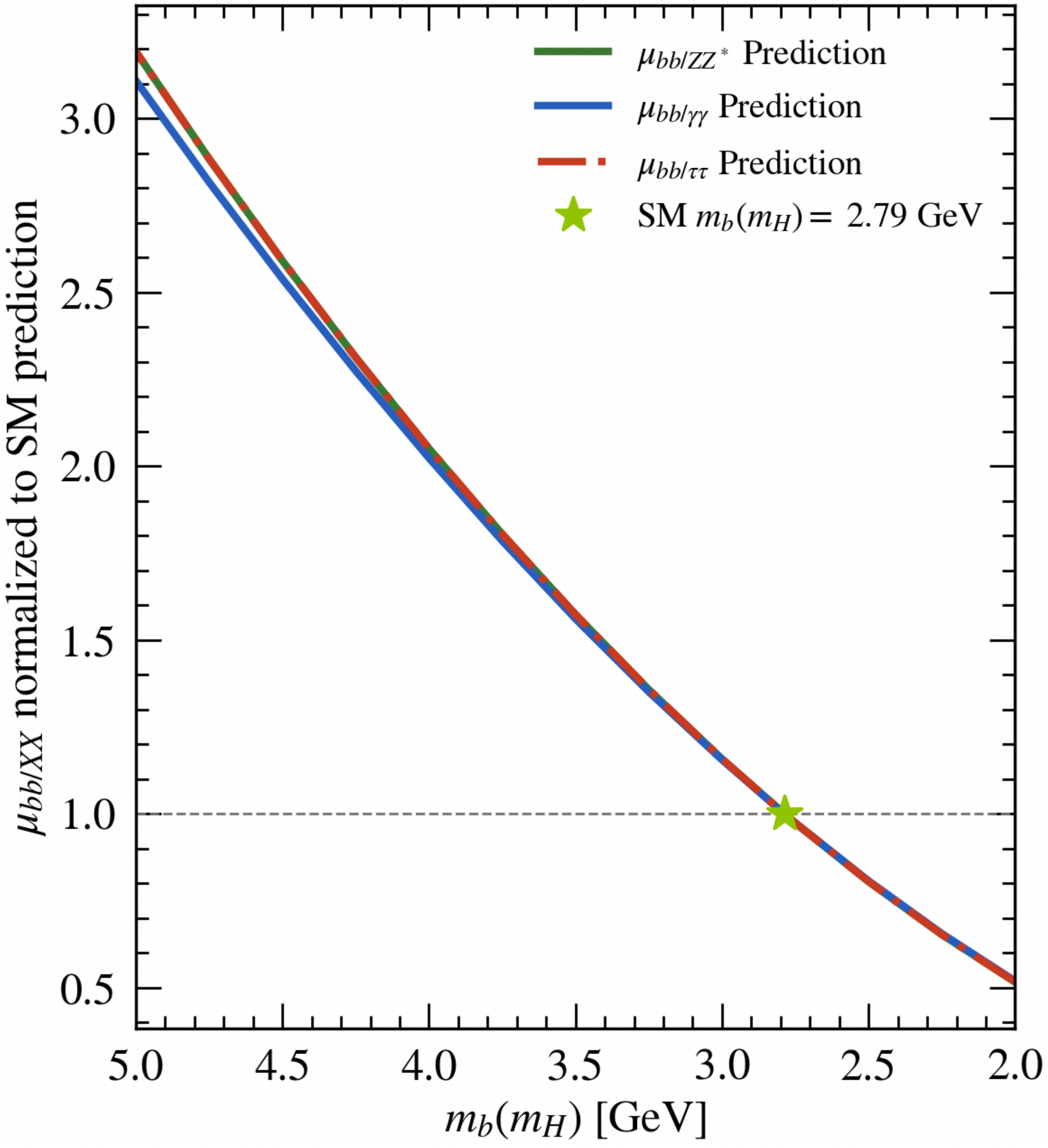

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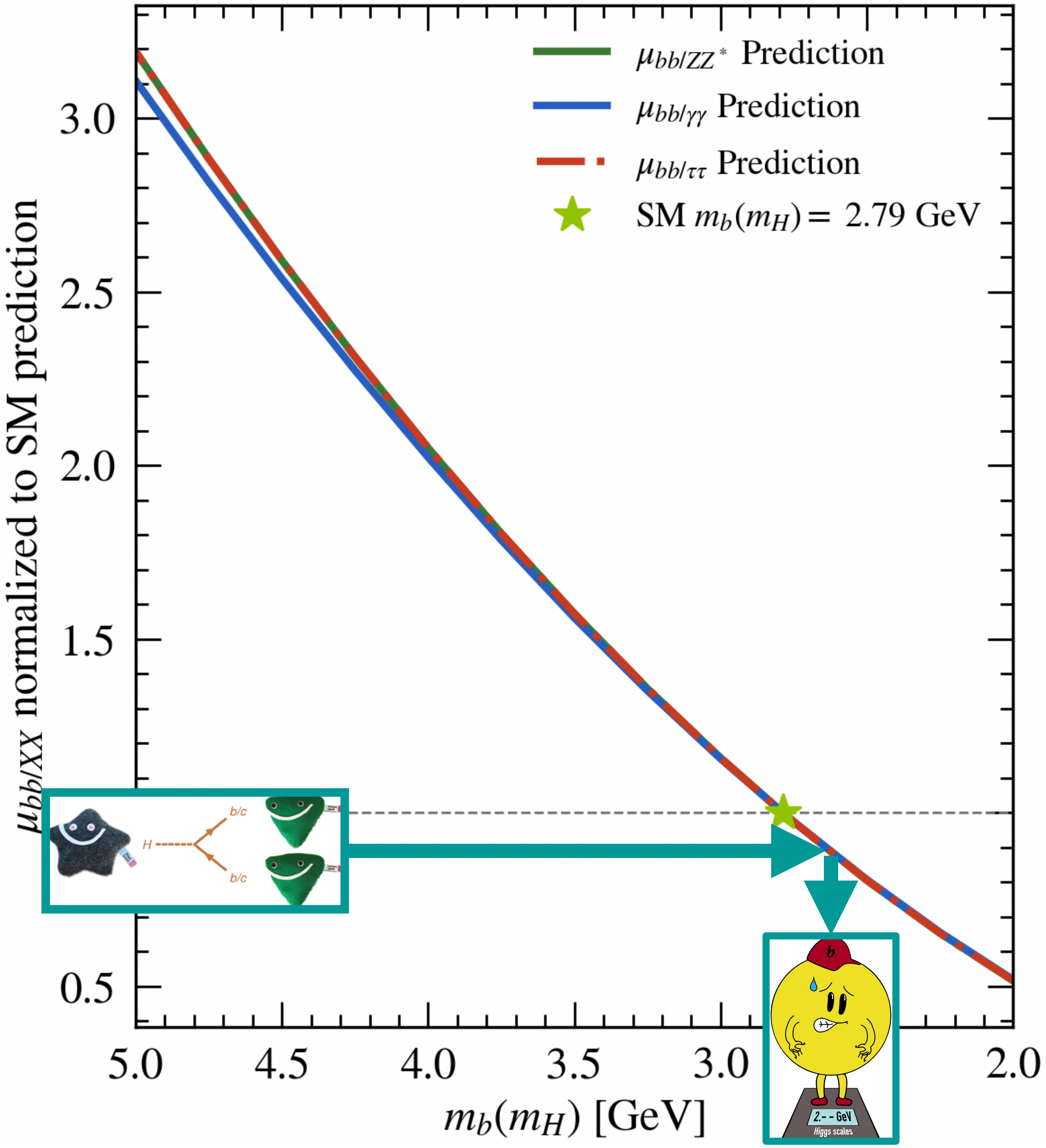
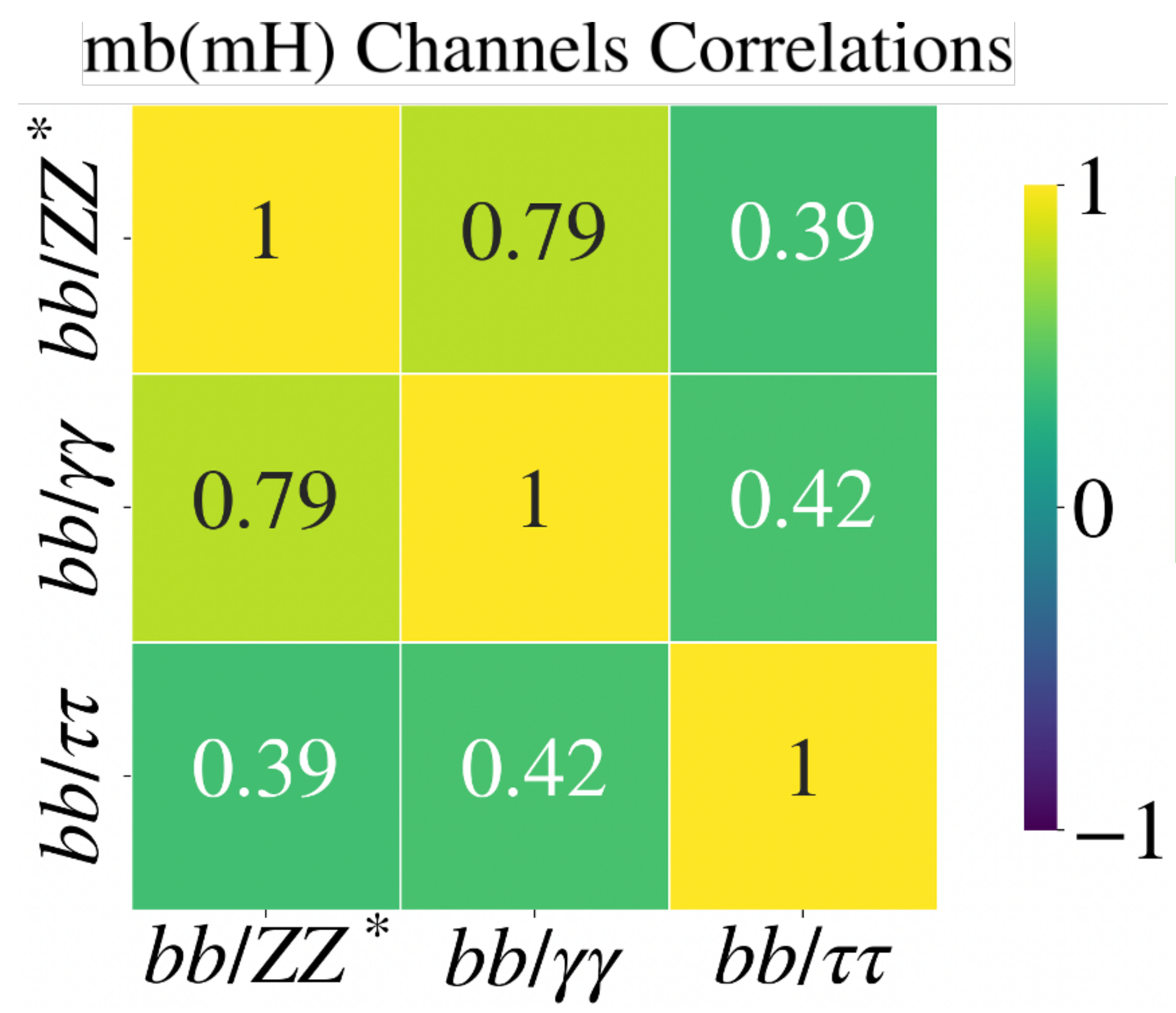


Results

Ratio	μ_{bb}/μ_{XX}	$m_b(m_H)$ [GeV]	
bb/ZZ^*	$0.69^{+0.25}_{-0.20}$	$2.31^{+0.41}_{-0.32}$	15.8%
$bb/\gamma\gamma$	$0.71^{+0.18}_{-0.16}$	$2.34^{+0.30}_{-0.26}$	**12.0 %**
$bb/\tau\tau$	$0.78^{+0.20}_{-0.17}$	$2.45^{+0.31}_{-0.26}$	**11.6 %**



Compatible results! <u>Combine</u>

Matrix obtained with Convino (J. Kieseler, *Eur. Phys. J. C* 77, 792 (2017), arXiv:1706.01681 [physics.data-an])

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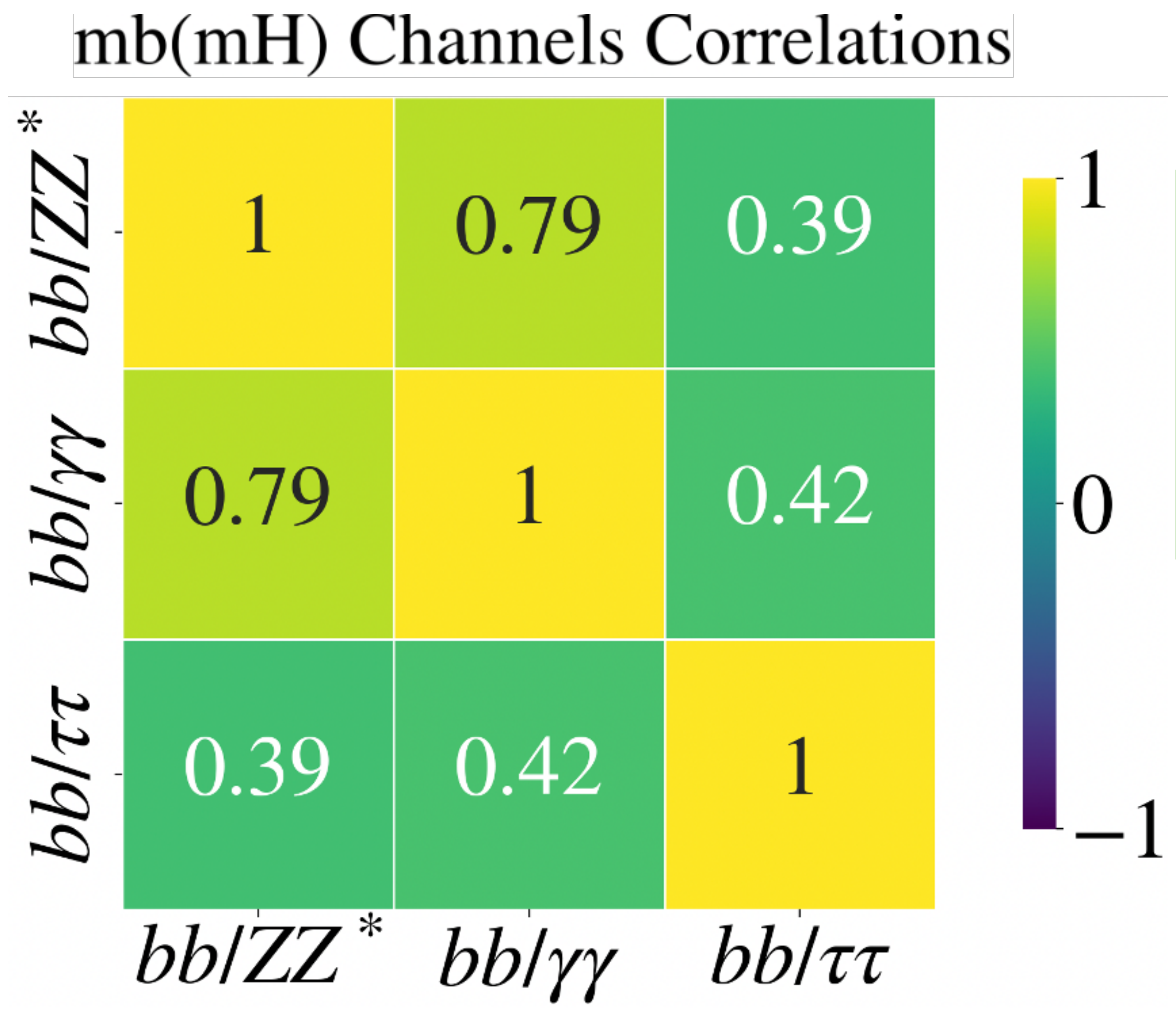
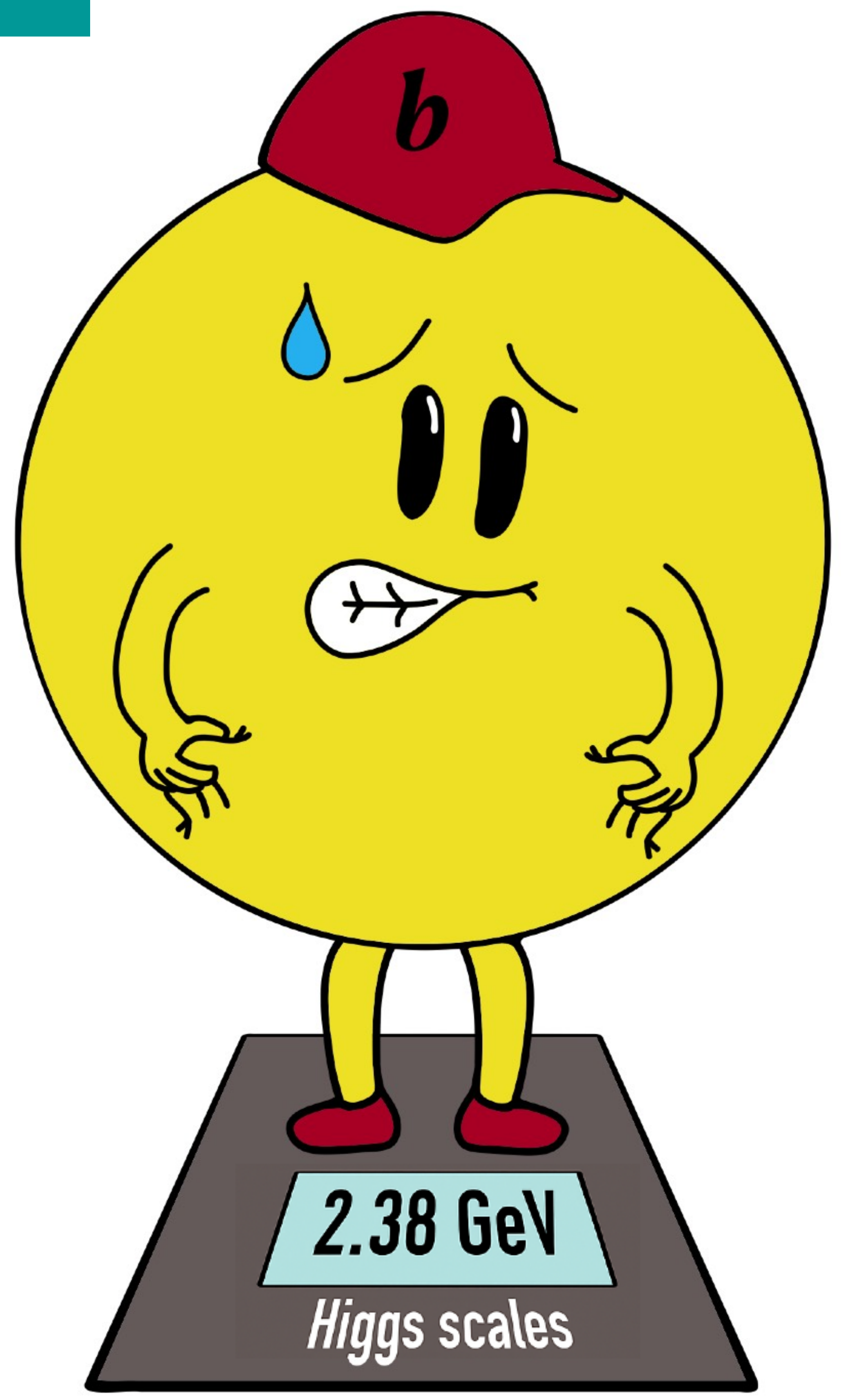


Compatible results!
<u>Combine</u>

$$m_b(m_H) = 2.38^{+0.24}_{-0.21}\ \mathrm{GeV}$$

- **Best channels** are $bb/\gamma\gamma$ & $bb/\tau\tau$, dominating comb.
- **9.5%** rel. unc. of the combined result
- **Combining** the channels brings **unc.** to **225 MeV**.
- Theo. uncertainties **negligible**! —> Precision will **improve** with **more data.**



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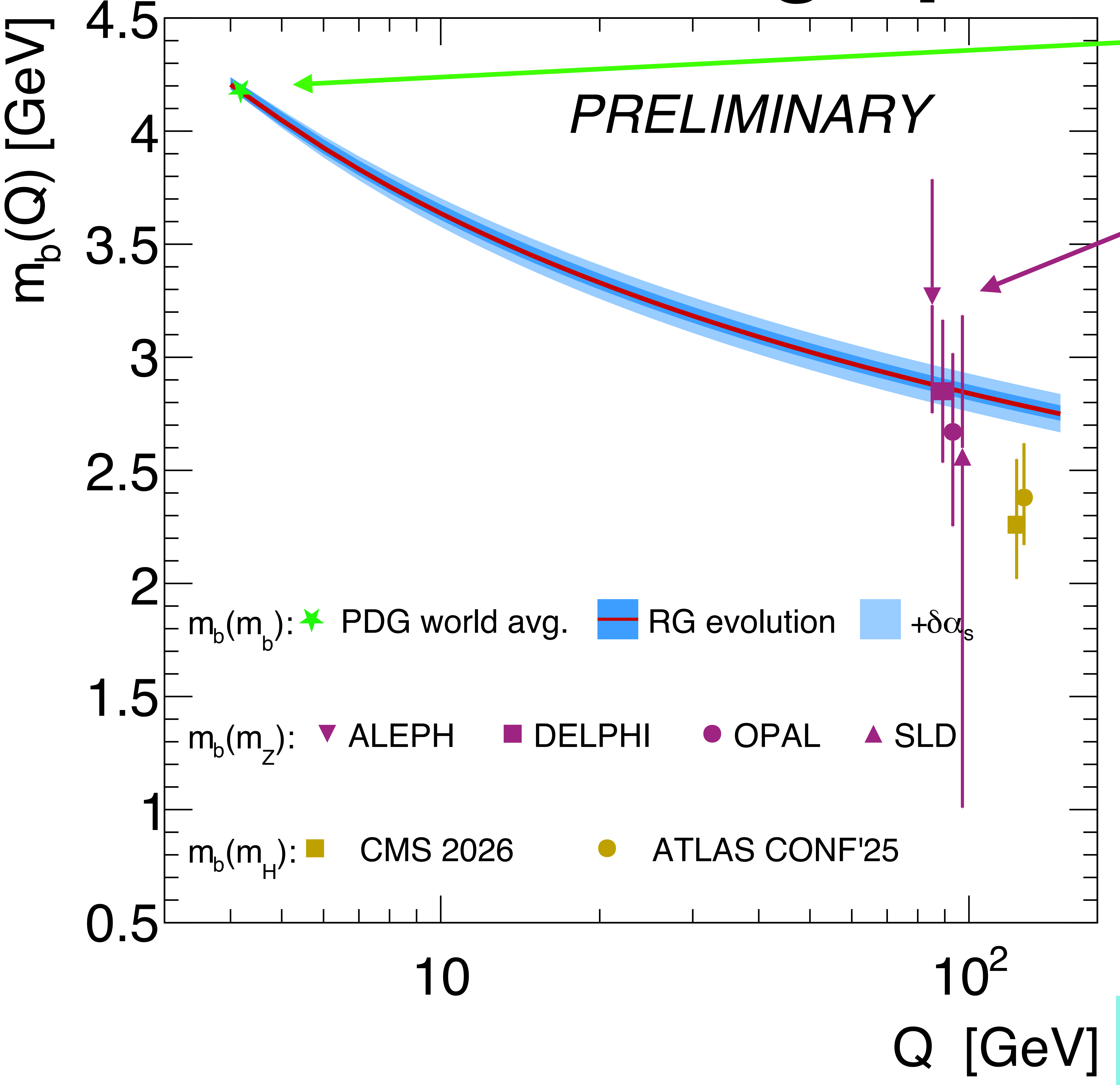

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$m_b(m_Z) = 2.82 \pm 0.28$ GeV

(Average)

- **Update** of the **running plot**!

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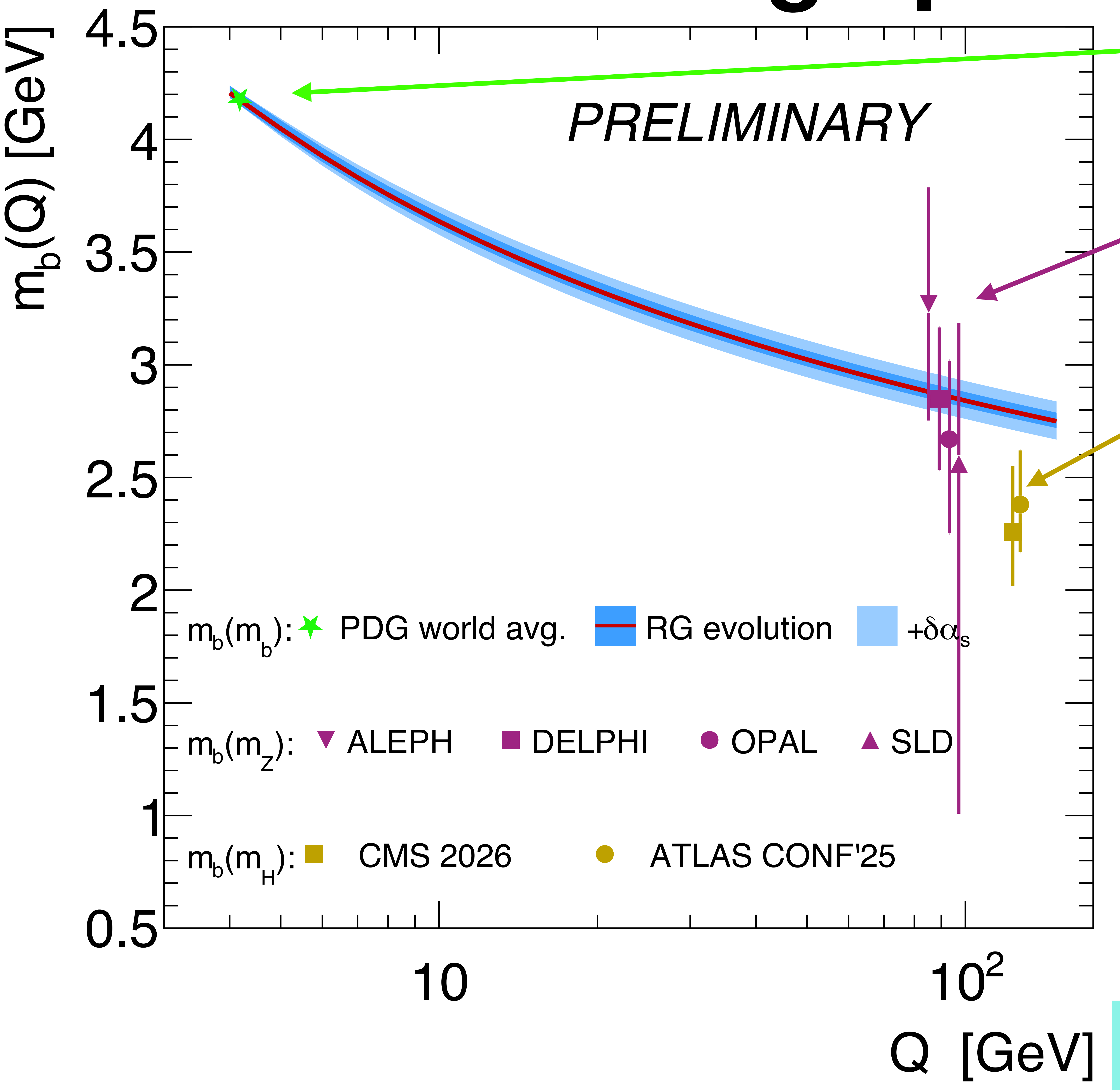

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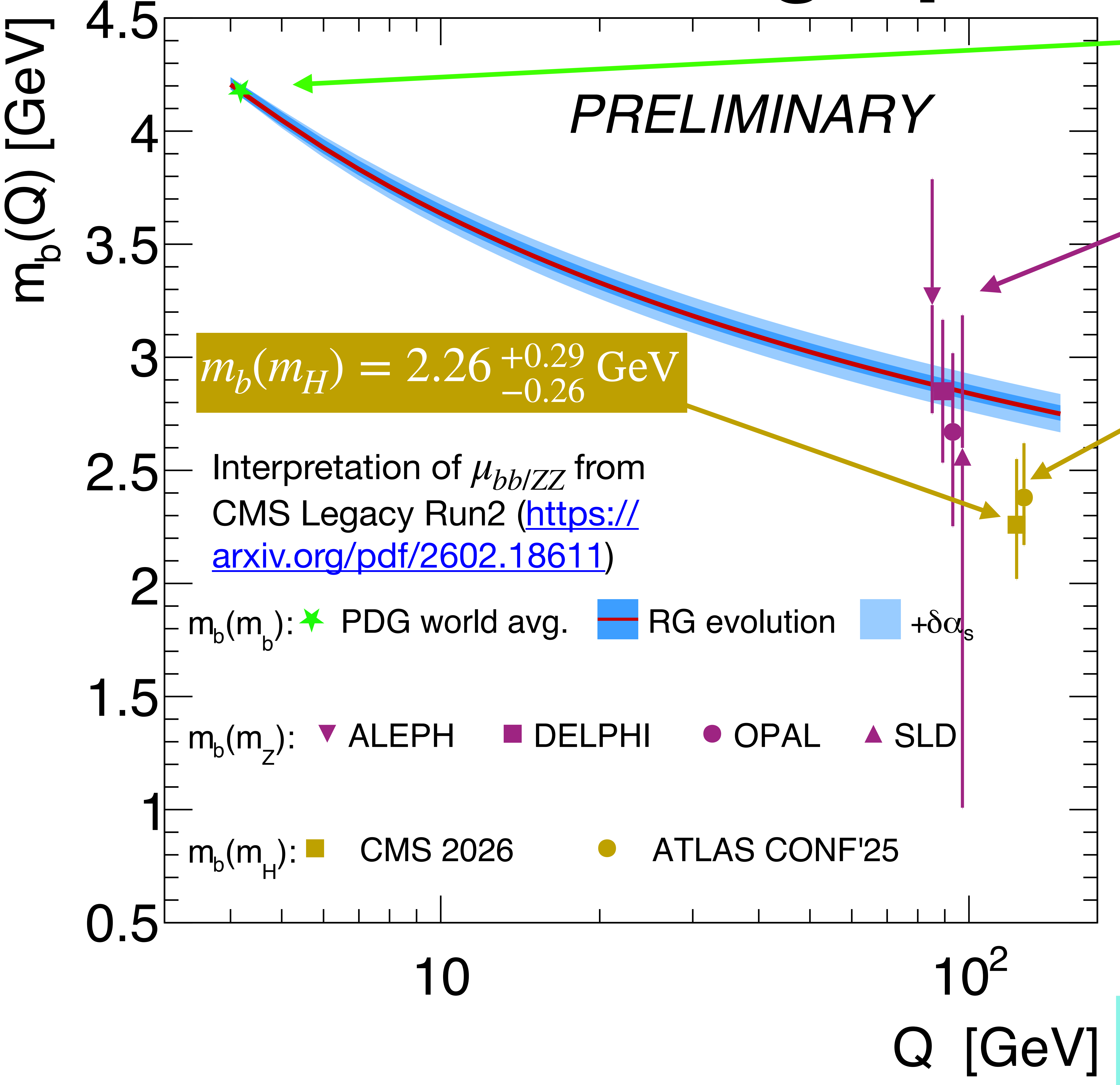

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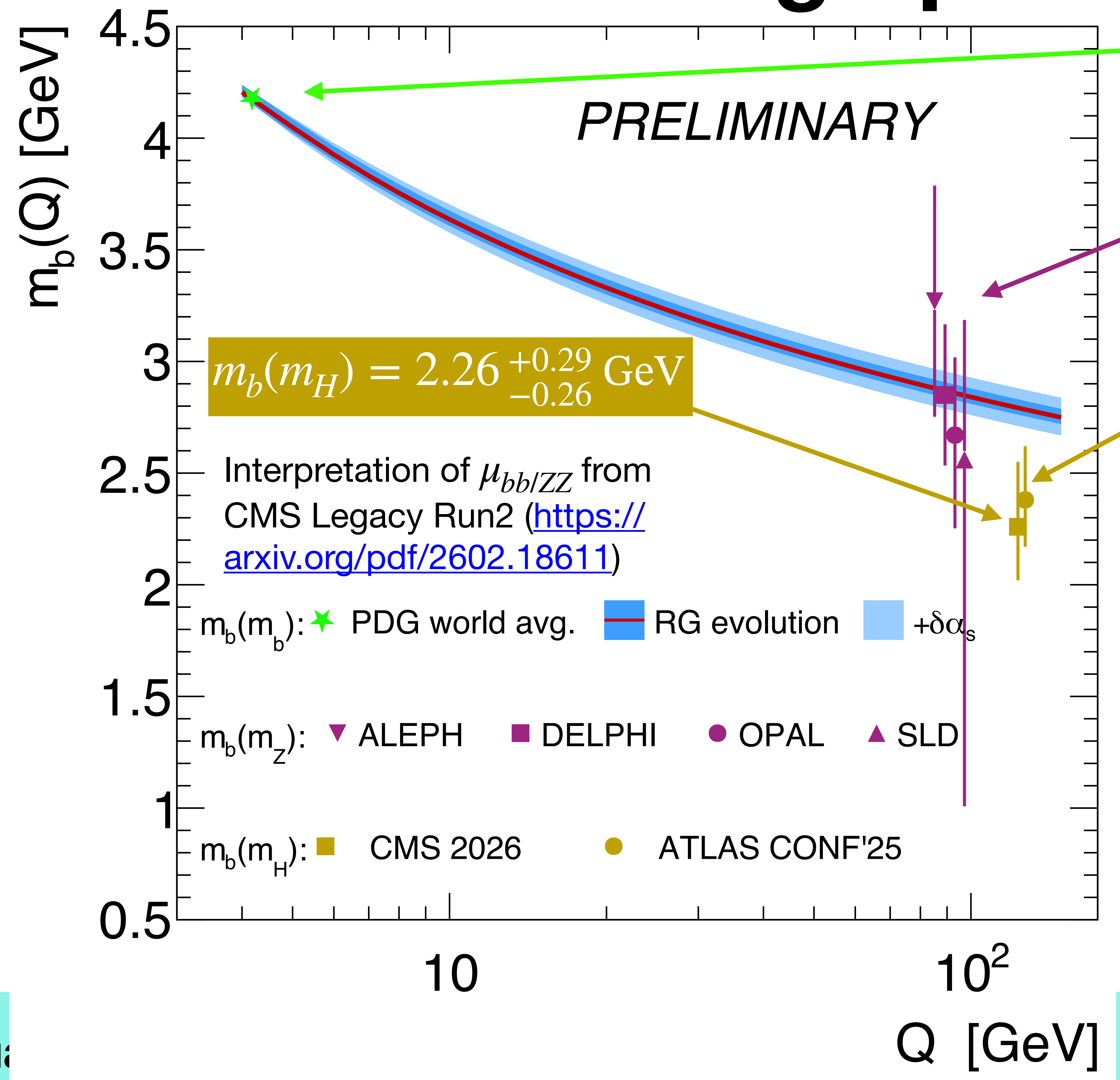

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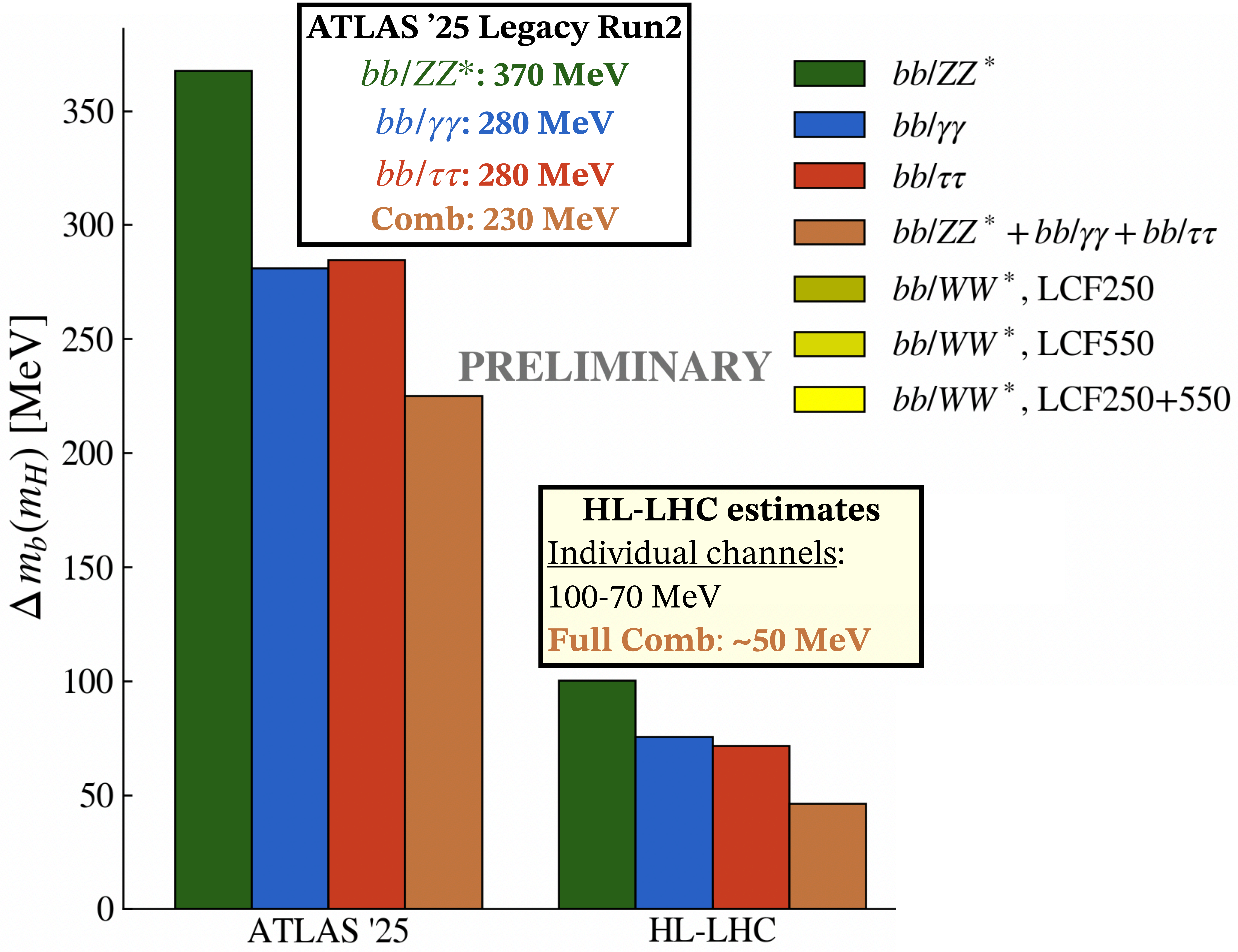
- **Update** of the **running plot**!
- **Using this work's** result of $m_b(m_H)$ **rules out** the **no-running** scenario by **>8σ** while remaining **compatible with SM** within 1.4σ.

Projected uncertainties for Future Facilities

- **HL-LHC estimates** (preliminarily) obtained by scaling the ATLAS ’25 exp. uncs. with luminosity (*Number of collisions per second*).



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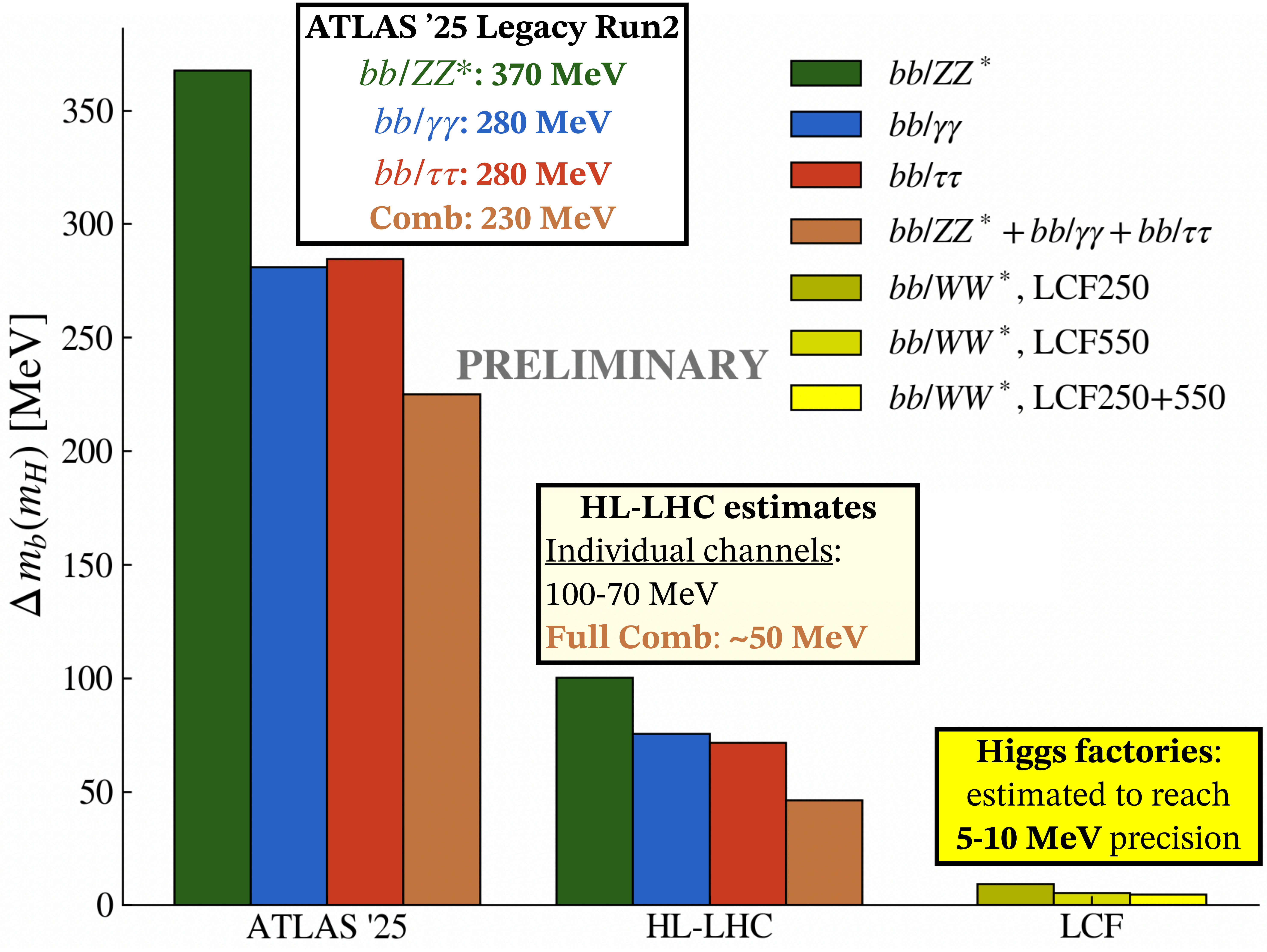

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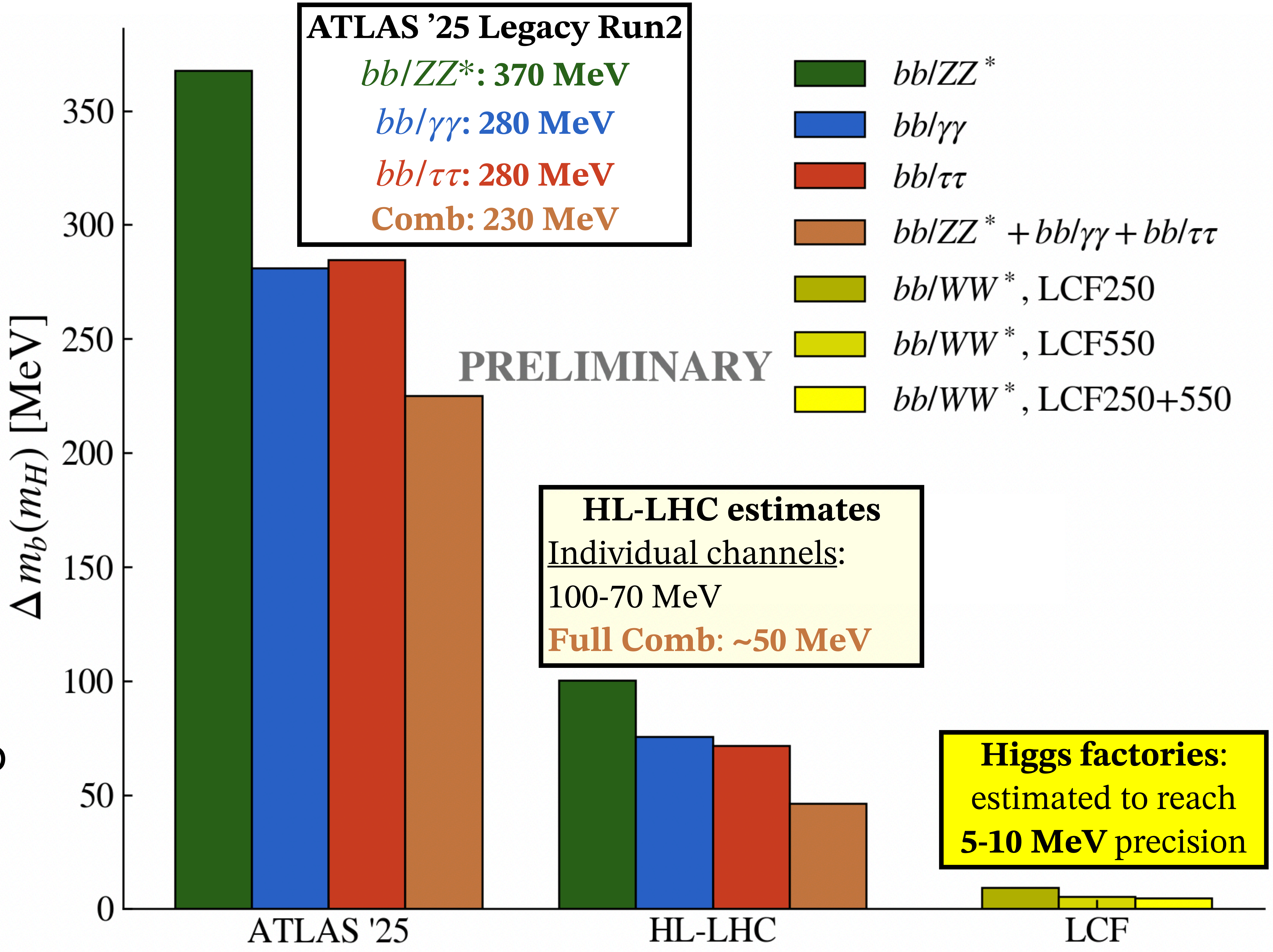
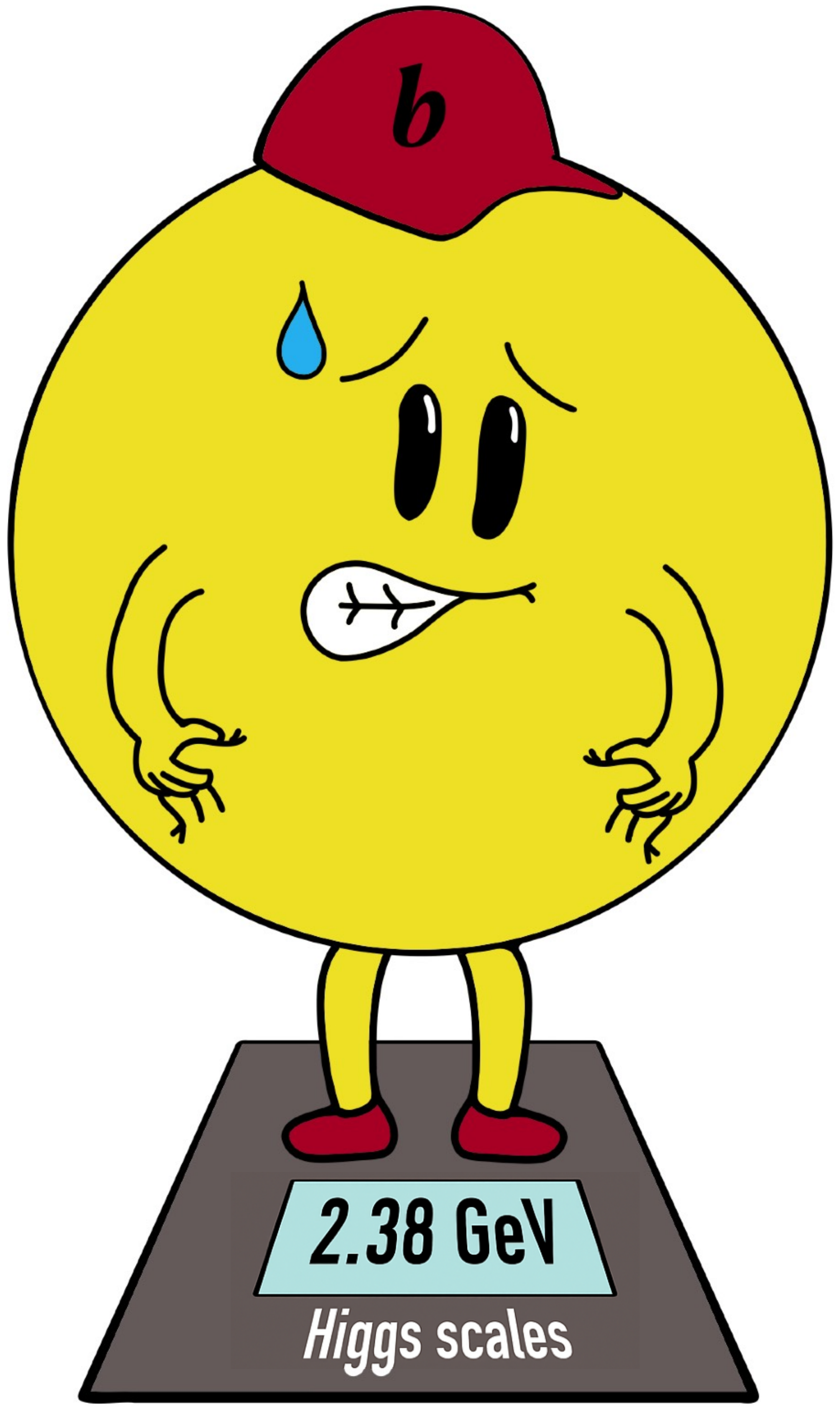
- At Higgs factories, $m_b(m_H)$ could **compete** with $m_b(m_b)$ (expected to go **<10 MeV**) for being the **most precise measurement of m_b!!**



Conclusion & Outlook



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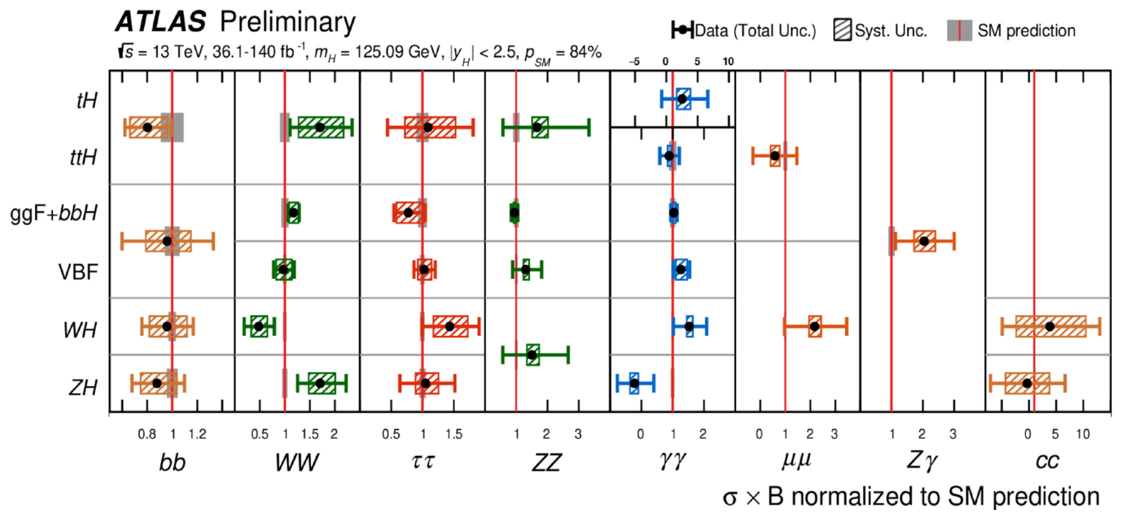
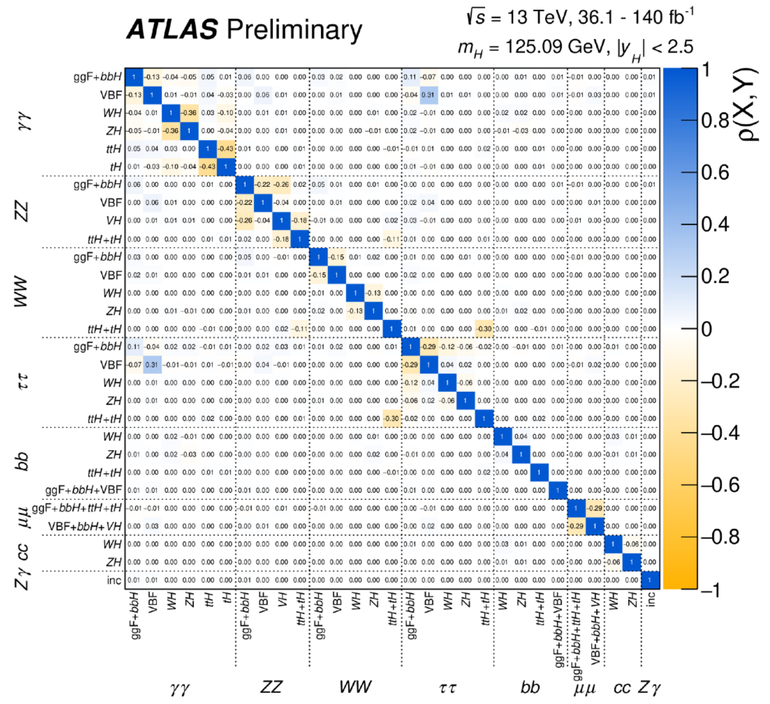
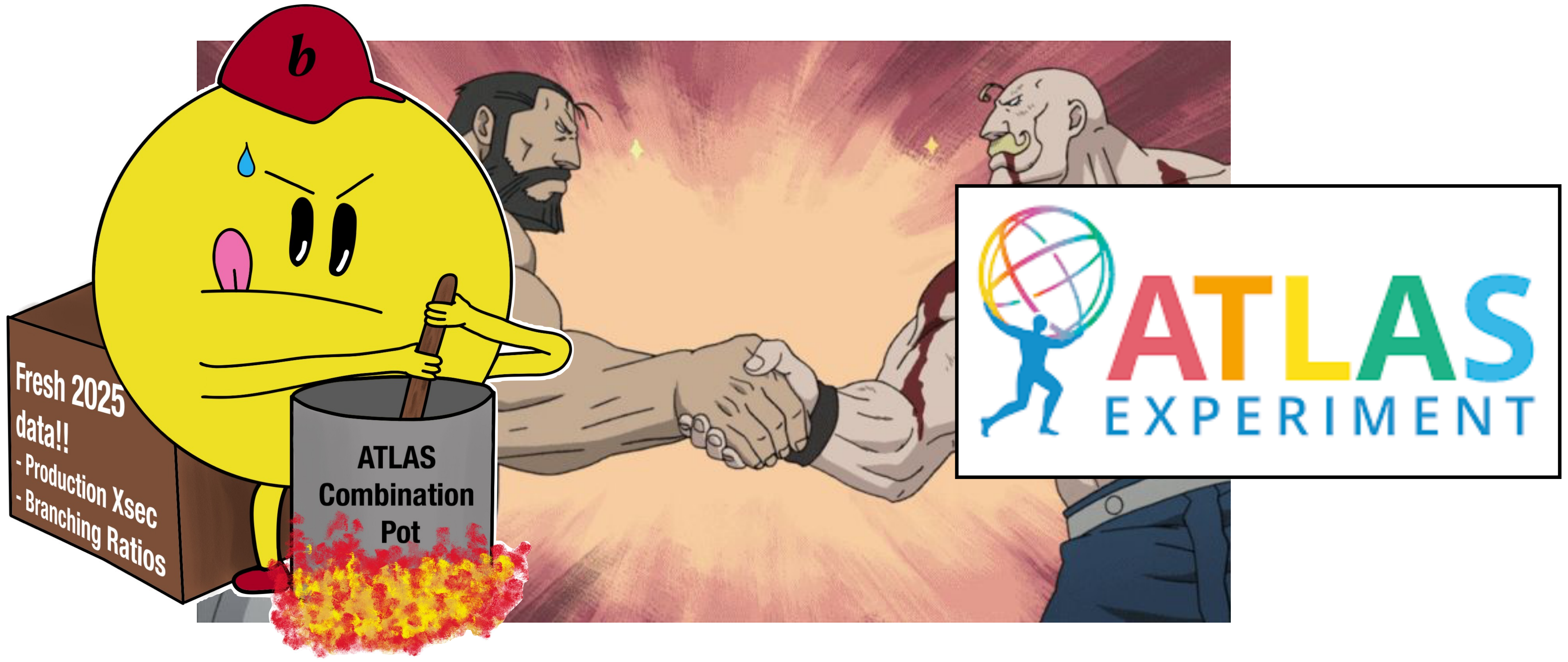
- Interpretation of public ATLAS results, but done **outside the Collaboration**…





Conclusion & Outlook

- The **bottom quark mass at high scale** can be **measured** to **good precision with Higgs data**, limited by current experimental uncertainties: **excellent prospects for improvement** at HL-LHC and Higgs factory.
- **Demonstrates** the **rejection of the no-running scenario** for quark masses, even assuming conservative uncertainties.
- Interpretation of public ATLAS results, but done **outside the Collaboration**… **HOWEVER** $m_b(m_H)$ is now a part of Higgs Combination effort, and a new result using the full glory & resources from ATLAS is on the works. Can't show anything today, but **stay tuned for news!**



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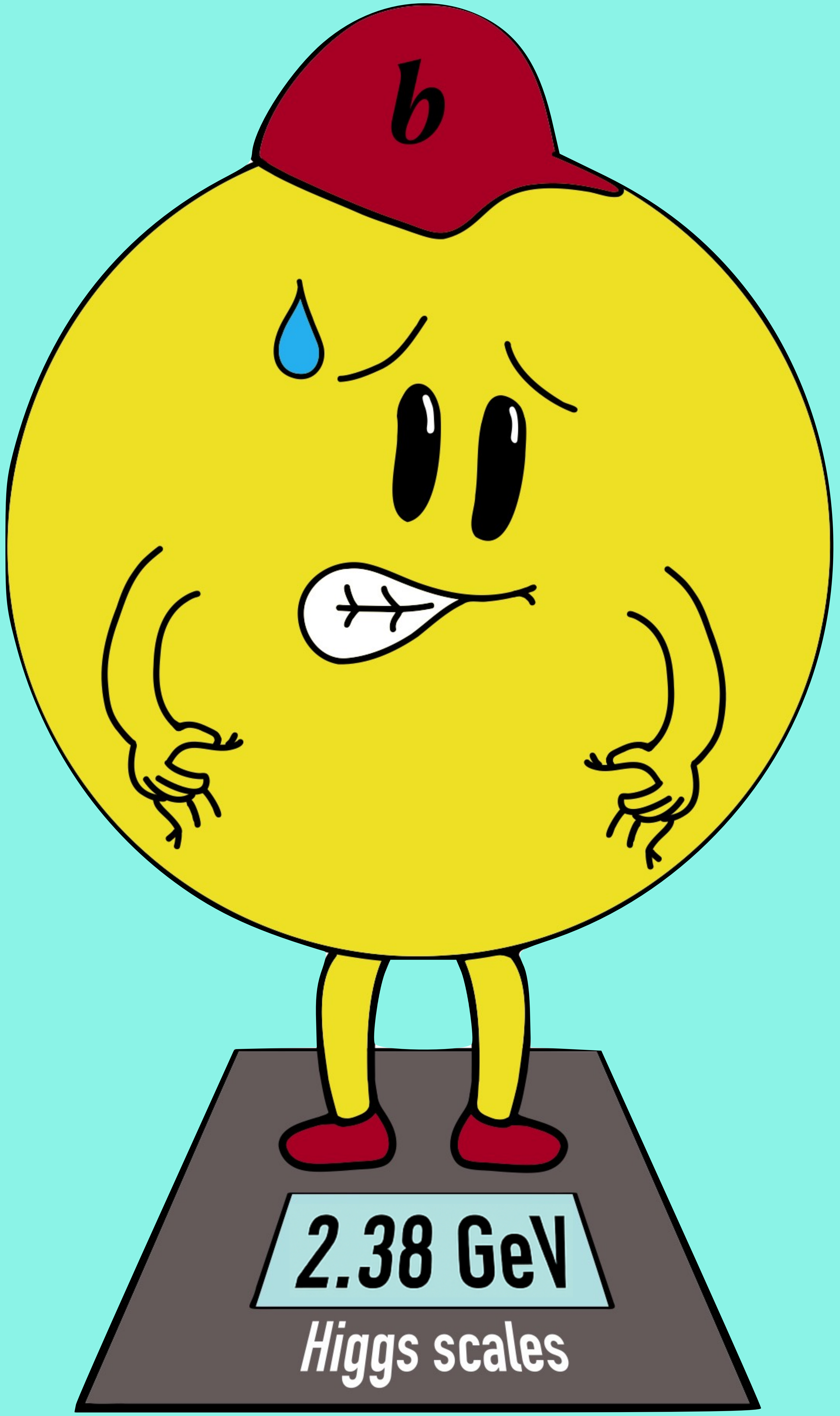
- **Future pheno study** to **evaluate feasibility** of simultaneous extraction of $m_b(m_H)$ and y_b. Ultimate goal of dropping SM-Yukawa assumption (**NOTE**: work independent of the analysis proposed in this talk).

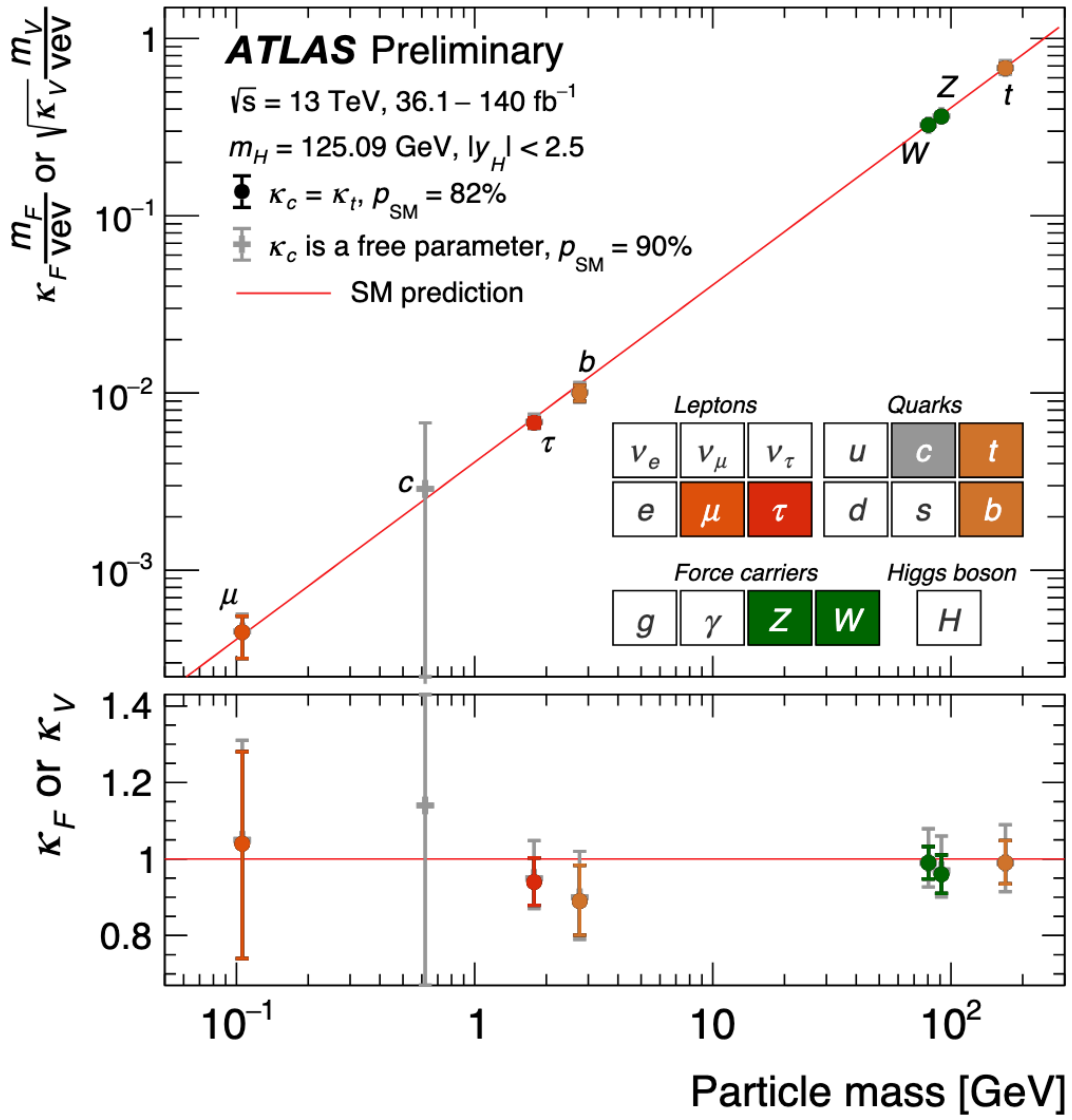
Thank you for your attention!



Backup

Backup: Why do we assume $(y_b)_{SM}$?

- So far, there has been **no evidence of the opposite**!
- Higgs coupling **measurements** up to date **align with the Standard Model** prediction: the coupling of the Higgs to SM-particles depends linearly on the particle's mass.
- If there are any new physics hiding around, they have a subleading effect.



From *https://atlas.web.cern.ch/Atlas/GROUPS/PHYSICS/CONFNOTES/ATLAS-CONF-2025-006/*

Backup: Check that m_b from $H \to b\bar{b}$ is indeed $m_b(m_H)$

- The point can be illustrated by considering the **convergence of the perturbative series.** When taking $\mu = m_H$ as the renormalization scale for both m_b and α_s, the leading QCD correction series for $H \to b\bar{b}$ when expanding in m_b^2/m_H^2 takes the form:

$$1 + \delta_{\mathrm{QCD}} = 1 + 0.2030 + 0.0374 + 0.0019 - 0.0014$$

- However, when taking $\mu = m_b$ the convergence is much poorer and presents large perturbative uncertainties due to logarithmic uncertainties $\ln(m_H/m_b)$, which are otherwise resumed to all orders in $\mu = m_H$:

$$1 + \delta_{\mathrm{QCD}} = 1 - 0.5665 + 0.0586 + 0.1475 - 0.1274$$

- Renormalization scales can also be chosen independently for m_b and α_s. However, the **best results** for the convergence are obtained when **both scales** are in the **order of m_H.**

Backup: A brief history of $m_b(m_H)$

- **First measurement** (the PRL one) relied on ATLAS & CMS analyses that **directly provided the ratio** B_{bb}/B_{ZZ*}; $m_b(m_H)$ was computed for **this ratio only**.
- When ATLAS published the "**Higgs 10th anniversary**" (*Nature 607 (2022) 52-59 & 60-68*), with partial Run2 data, the measurement of the ratio was no longer made public. However, the **full set of** $\sigma_i \times B_f$ **measurements**, together with the corresponding **covariance matrix**, were provided. From this new input, an update on $m_b(m_H)$ was made (unpublished work by M. Vos, B. Moser & V. Dao), following the methodology described in this talk, considering other ratios rather than $bb/ZZ*$ only.
- **This work** represents an update on the Nature paper result, using the same methodology for extracting the masses but with Legacy Run2 data, with the new addition of **combining the masses** from the different ratios.

Backup: A brief history of $m_b(m_H)$

Experiment	Input	Ratio	mb(mH) [GeV]
ATLAS + CMS, partial Run2	Direct ratio	bb/ZZ	2.60 +0.36 -0.31
ATLAS, partial Run2	Full σ x B covariance matrix	bb/ZZ	2.55 +0.29 -0.32
		bb/$\gamma\gamma$	2.52 +0.25 -0.28
ATLAS, Legacy Run2	Full σ x B covariance matrix	bb/ZZ	2.31 +0.41 -0.32
		bb/$\gamma\gamma$	2.34 +0.30 -0.26
		bb/$\tau\tau$	2.45 +0.31 -0.26
		bb/ZZ & bb/$\gamma\gamma$ & bb/$\tau\tau$	2.38 +0.24 -0.21

Backup: Combination of production channels for Branching function's ratios

- **Logic behind the previous combination**: the absolute measurements (not SM-normalized) can be added directly because they are measured as Number of events, so $N_{A+B} = N_A + N_B$. For the signal strengths (measurements normalized to SM), this turns into a "weighted average" with the SM production predictions as weights:

$$\mu_{BR_X}^{\sigma_{A+B}} = \frac{(\sigma_A^{SM} \times BR_X^{SM})\,\mu_{BR_X}^{\sigma_A} + (\sigma_B^{SM} \times BR_X^{SM})\,\mu_{BR_X}^{\sigma_B}}{\sigma_A^{SM} \times BR_X^{SM} + \sigma_B^{SM} \times BR_X^{SM}} = \frac{\sigma_A^{SM}\,\mu_{BR_X}^{\sigma_A} + \sigma_B^{SM}\,\mu_{BR_X}^{\sigma_B}}{\sigma_A^{SM} + \sigma_B^{SM}}$$

- Production predictions are taken from the Yellow Report, for $\sqrt{s} = 13$ TeV & $m_H = 125.09$ GeV:

Production Mode	Cross-Section [fb]
ggF	48 510
VBF	4 006
WH	1 370.00
ZH	882.10
ttH	506.5
tH	74.26
bbH	486.30

Backup: Correlation treatment

- Starting from the **ATLAS full Covariance matrix** and the **reported uncertainties** as input, we apply the **standard formalism of uncertainty propagation** in the case of multiple, multi-variable functions, **all the way to the final** $m_b(m_H)$:
  - Have a set of N variables $x_i\,,\;\; i = 1, 2\ldots, N$, with uncertainties $\{\sigma_i\}$ that are correlated according to the correlations, $\rho_{ij} = \sigma_{ij}/(\sigma_i\,\sigma_j)$ where σ_{ij} is the covariance b/w the variables x_i and x_j, and it holds that $\sigma_{ii} = \sigma_i^2$.
  - Then, if we define a series of M functions $f_k(x_i)\,,\;\; k = 1, 2\ldots, M$ that depend on the variables x_i, we can compute their covariances (and thus their correlations and uncertainties by using identical definitions as the ones for x_i) as:

$$\{\mathcal{V}[x]\}_{ij} = \sigma_{ij}(x)$$

Covariance matrix of the variables

$$\{\mathcal{V}[f]\}_{ij} = \sigma_{ij}(f)$$

Covariance matrix of the functions

$$\mathcal{V}[f] = \mathcal{S}^T\,\mathcal{V}[x]\,\mathcal{S}$$

$$\{\mathcal{S}\}_{ij} = \frac{\partial f_j}{\partial x_i}$$

Columns (j): functions

Rows (i): variables

Matrix of derivatives

Backup: Correlated weighted average

- For the combination of correlated measurements (either the ratios per prod. chan. or the final $m_b(m_H)$), we perform a **correlated weighted average**, following the analytical formulae:
  - For asymmetric uncertainties, we would have 2 covariance matrices, C^+ & C^-, from which we define a symmetric covariance matrix (and from that the correlation matrix) C.
  - With these 3 matrices, and the set of measurements $\{x_i\}$ to be combined, the **weighted average** x_{avg} and its **uncertainty** (both up and down, $\sigma_{avg,\pm}$) are defined as

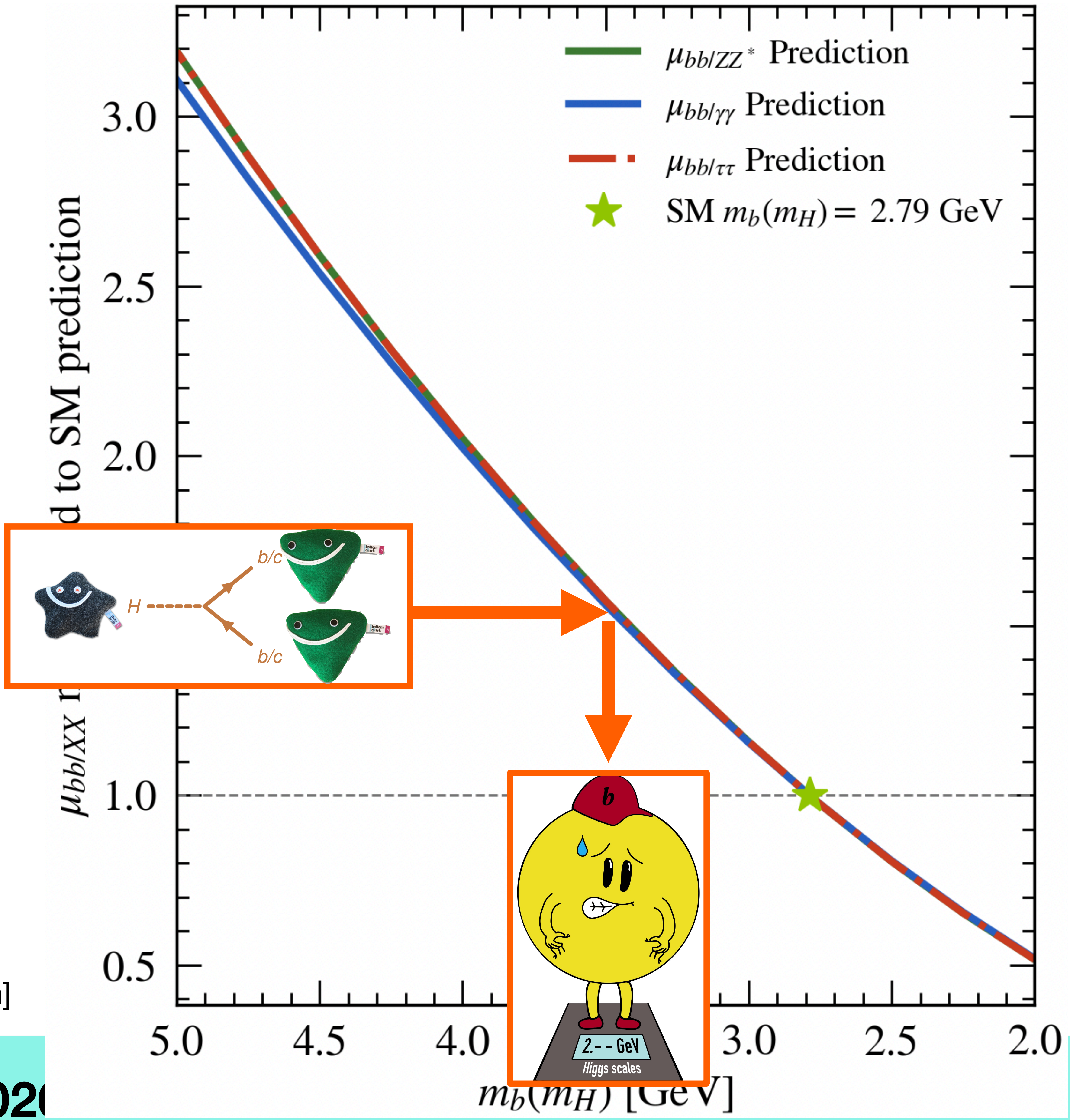
$$C_{ij} = \frac{C^+_{ij} + C^-_{ij}}{2}$$

$$x_{avg} = \frac{1}{\sum_{p=1}^{N} \sum_{q=1}^{N} (C^{-1})_{pq}} \left\{ \sum_{i=1}^{N} x_i \left[\sum_{j=1}^{N} (C^{-1})_{ij} \right] \right\}$$

$$\sigma^2_{avg,\pm} = \frac{\sum_{i=1}^{N} \sum_{j=1}^{N} \left\{ \left[\sum_{s=1}^{N} (C^{-1})_{si} \right] \left[\sum_{r=1}^{N} (C^{-1})_{rj} \right] (C^{\pm})_{ij} \right\}}{\left(\sum_{p=1}^{N} \sum_{q=1}^{N} (C^{-1})_{pq} \right)^2}$$

Backup: Extraction of $m_b(m_H)$ from ratios of B_f s

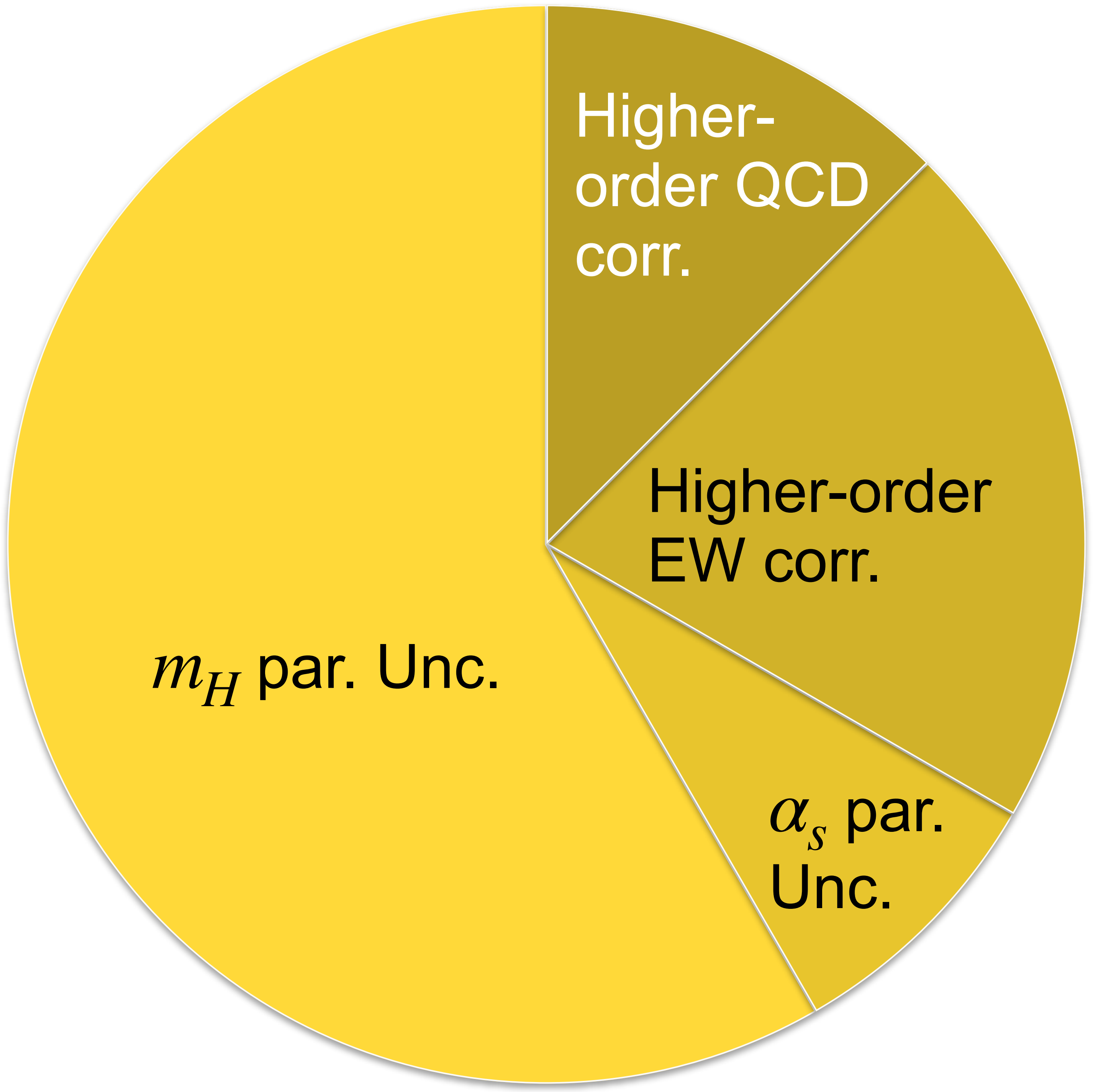
- Each measured $\mu_{bb/XX}$ is fed to a **parametrization** of its **prediction** (obtained from **HDecay**) as a **polynomial of** $m_b(m_H)$, $\mu_{bb/XX} = f(m_b(m_H))$, **giving a value for** $m_b(m_H)$**!**
- The masses **for each ratio** are **extracted independently** of each other. If they are found to be **compatible**, they are combined using a correlated weighted average to provide a **single, final value for** $m_b(m_H)$.



HDecay: *Comput. Phys. Commun.* 238, 214 (2019), arXiv:1801.09506 [hep-ph]

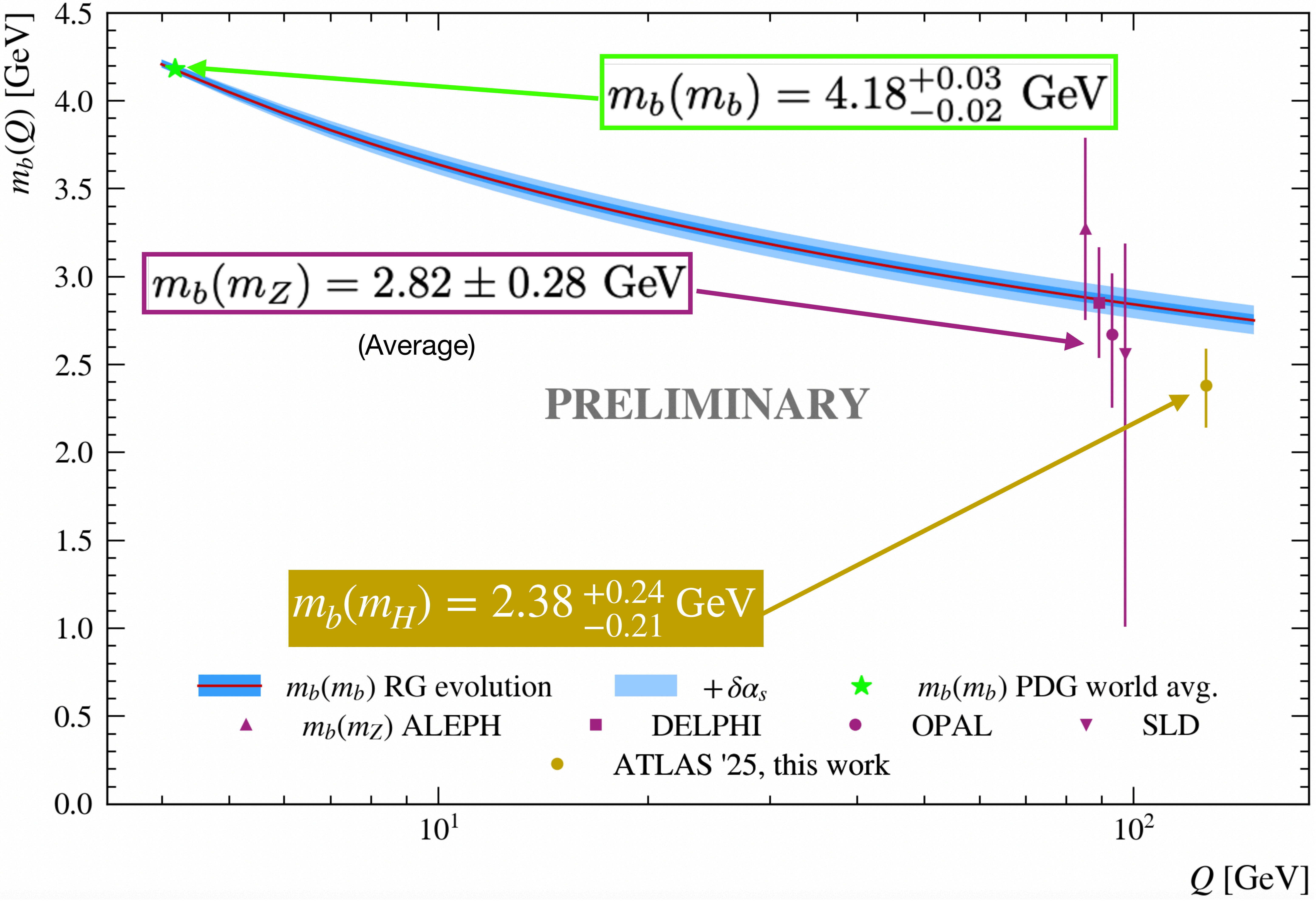
Backup: Theoretical Uncertainties

- **Absolute theo. uncertainty** estimated to be **~30 MeV**.
- **Theoretical uncertainties** include (values given as relative uncertainties):
  - Higher-Order QCD corrections (Scale variations) and EW corrections (estimated from Higgs Yellow Report): **0.3% - 0.5%**
  - Parametric uncertainty from α_s: $\Delta\alpha_s(m_Z) = 0.0009 \rightarrow$ **0.2%**
  - Parametric uncertainty from m_H: $\Delta m_H = 0.11$ GeV $\rightarrow$ **1.4%**
- Taking a **conservative** approach, **uncertainties** are **added up linearly**. These uncertainties were estimated for bb/ZZ^* originally in PRL work, but here they are considered for the full combination (following the conservative line).
- **Well below the ~200 MeV exp. uncertainties.**



Relative contribution to total Theo. Unc.

Backup: testing the running



- Running test done by fitting the following parametrization:

$$m(\mu; x, m_b(m_b)) = x\left[m_b^{\mathrm{RGE}}(\mu, m_b(m_b)) - m_b(m_b)\right] + m_b(m_b)$$

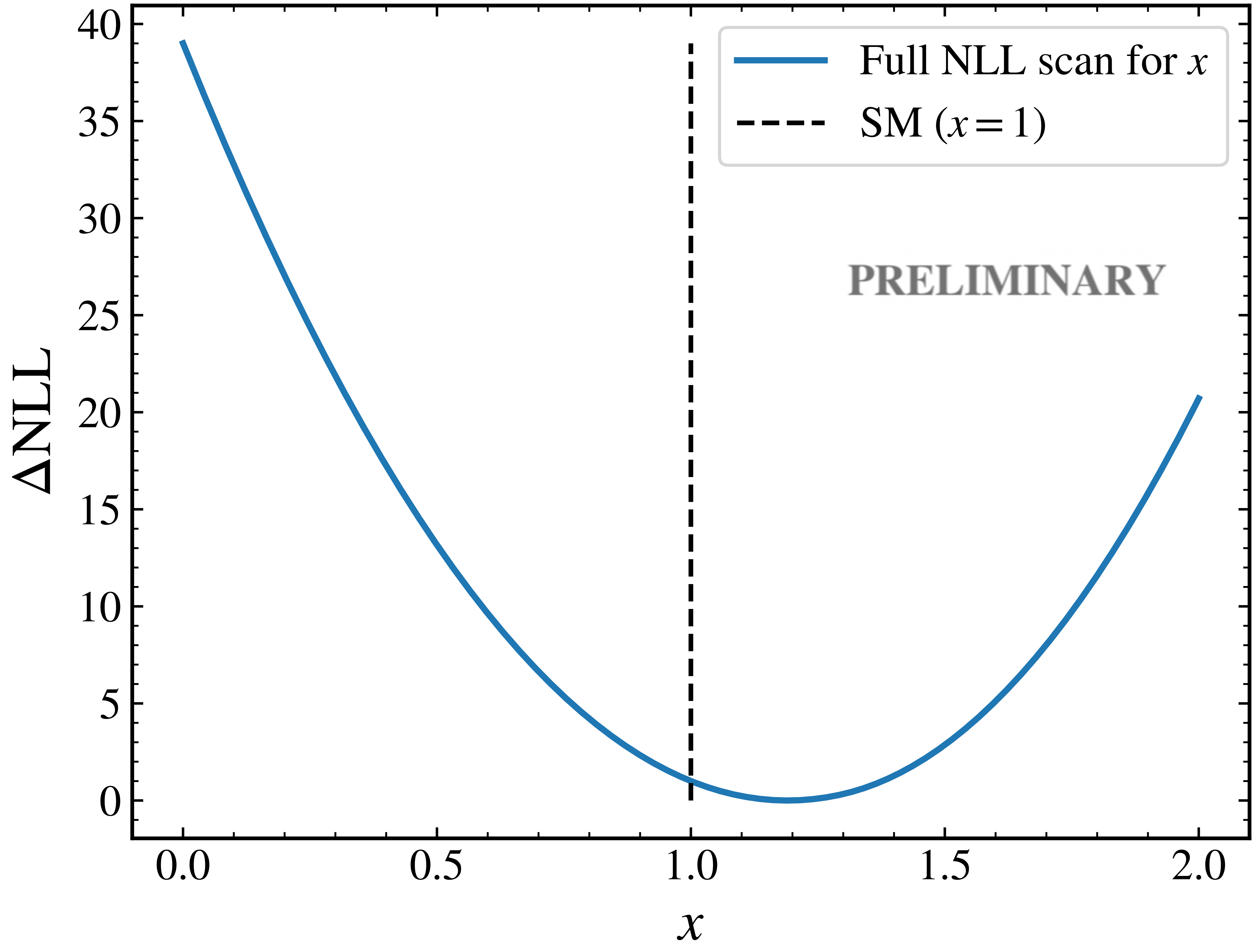
- $x = 0$ is no running; $x = 1$ is SM.
- **Using this work's** result of $m_b(m_H)$ gives

$$x = 1.19 \pm 0.13\,(\mathrm{exp})\,^{+0.06}_{-0.05}\,(\alpha_s)$$

- This **rules out** the **no-running** scenario by **>8σ** while remaining **compatible with SM** within 1.4σ.
- From fit, $m_b(m_b) = 4.18 \pm 0.02$ GeV. Compatible with PDG value!

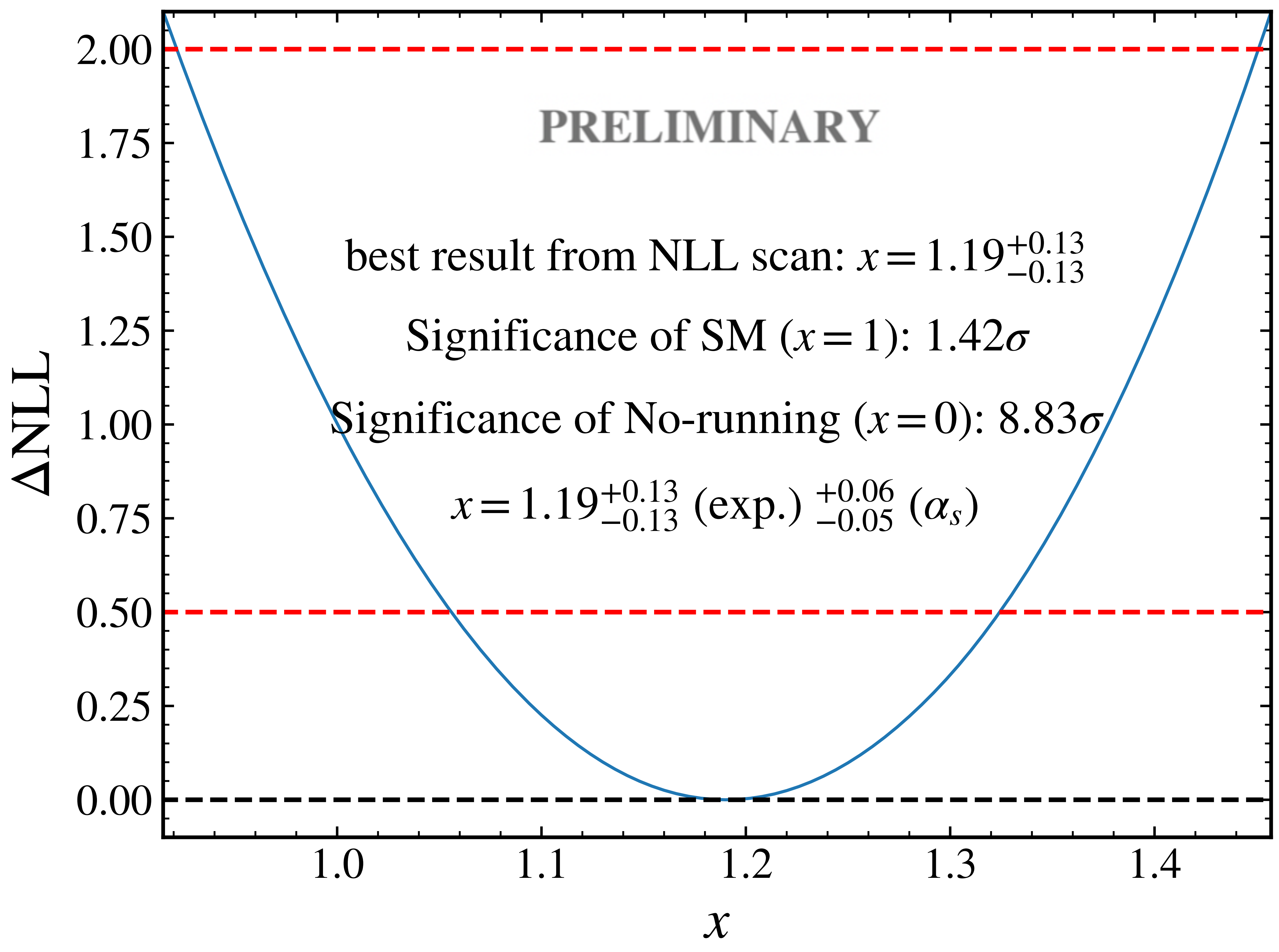
Backup: testing the running

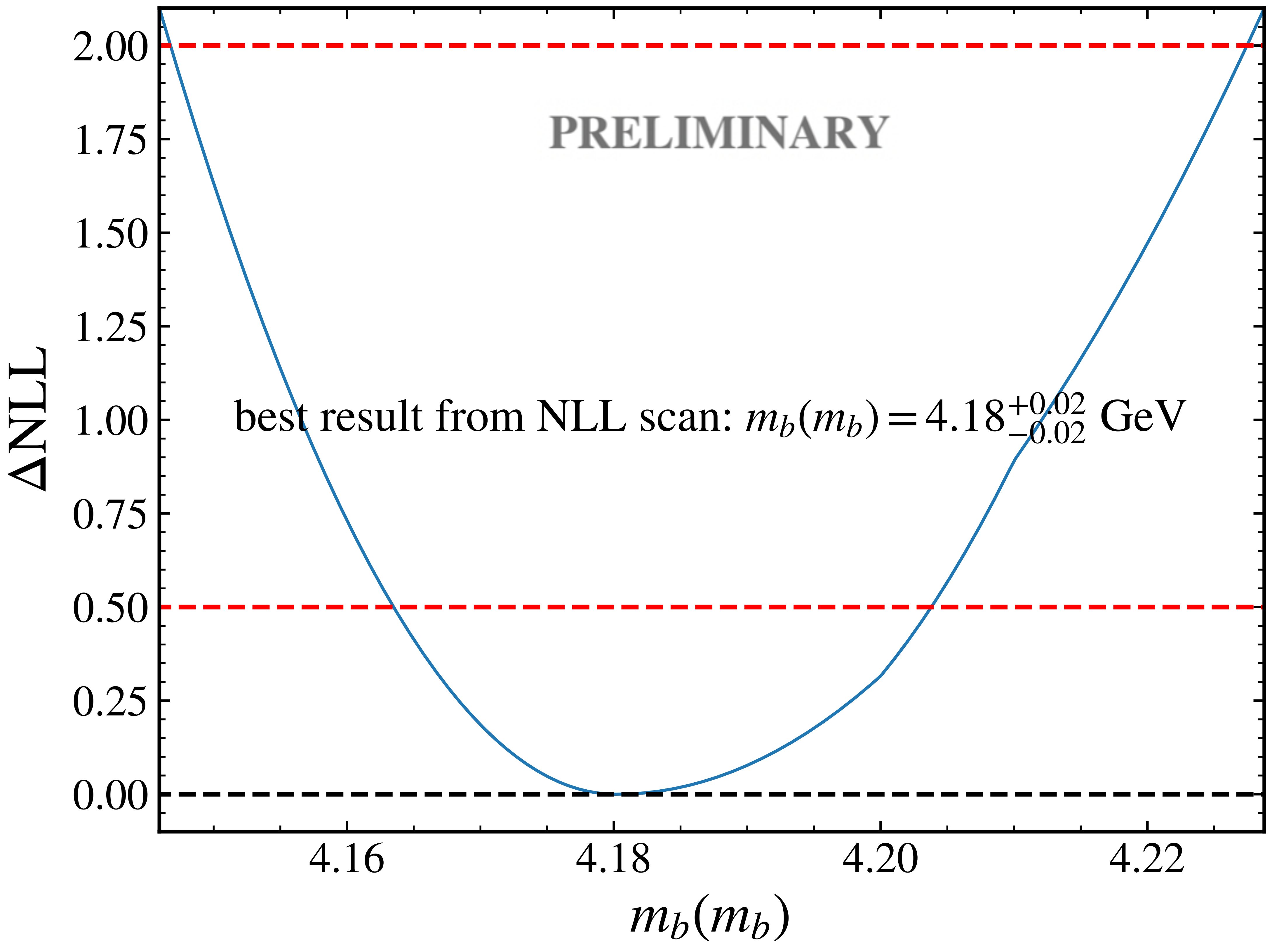
- Parameter scan of running parametrization's x. See how x=0 has a terrible NLL value



Backup: testing the running

- Parameter scans of running parametrization's x and $m_b(m_b)$



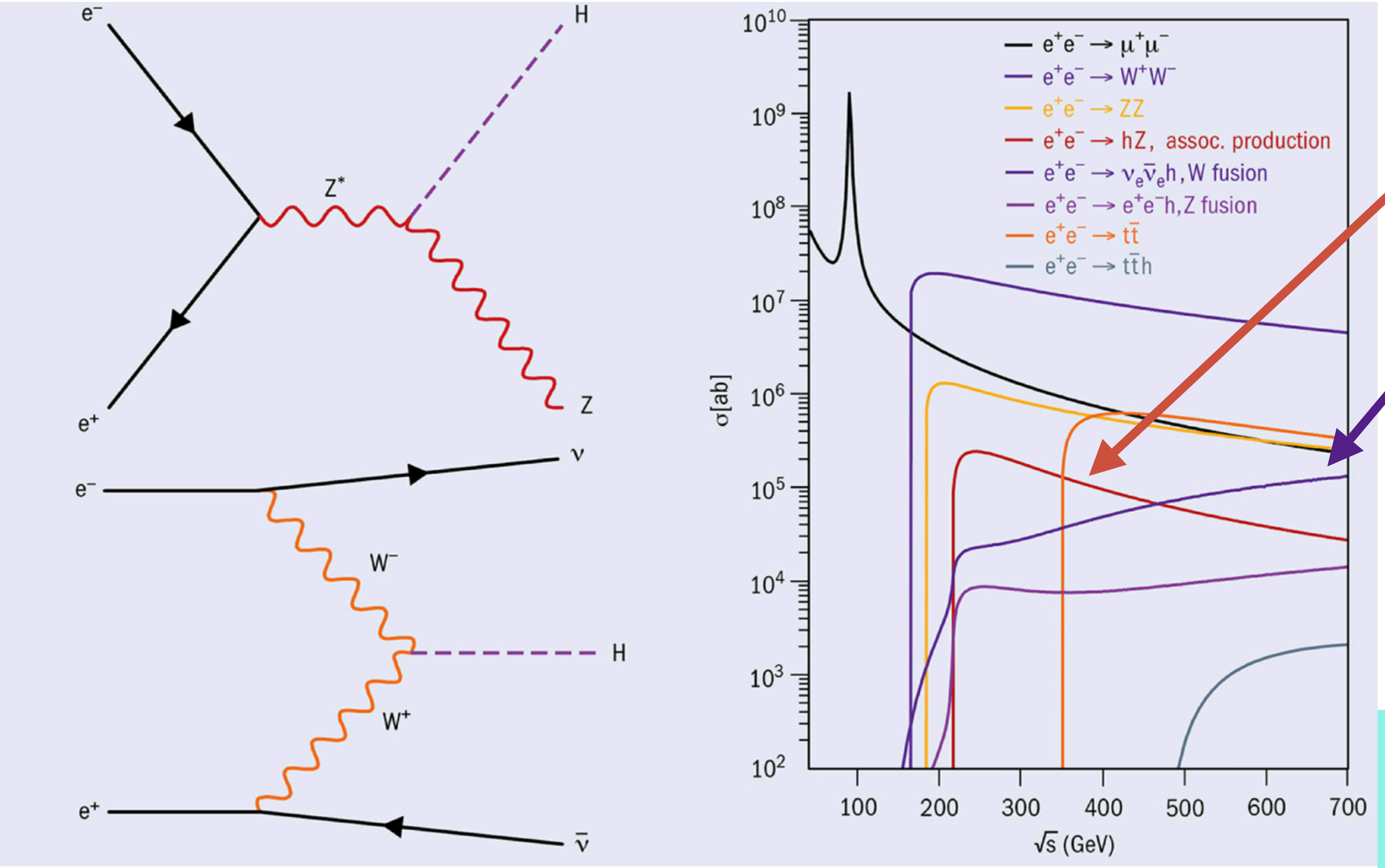


Backup: e^+e^- colliders as Higgs factories

From arXiv:2503.19983v3 [hep-ex]:

An e$^+$e$^-$ collider is the ideal machine for precision measurements. Because electron and positrons are elementary particles, their reactions are simple and display the structure of the underlying interactions directly. Backgrounds are dominated by electroweak processes, and these are also simple and — more importantly — precisely understood at the part-per-mil level. The low event rates relative to proton collisions allow the construction of low-material-budget, high-precision detectors and of trigger-less data taking. All of these features minimize systematic uncertainties. This makes it possible to measure small deviations from the SM with high confidence and credibility.

Credit: R. K. Ellis; R. K. Ellis / Durham. Taken from https://cerncourier.com/a/targeting-a-higgs-factory/



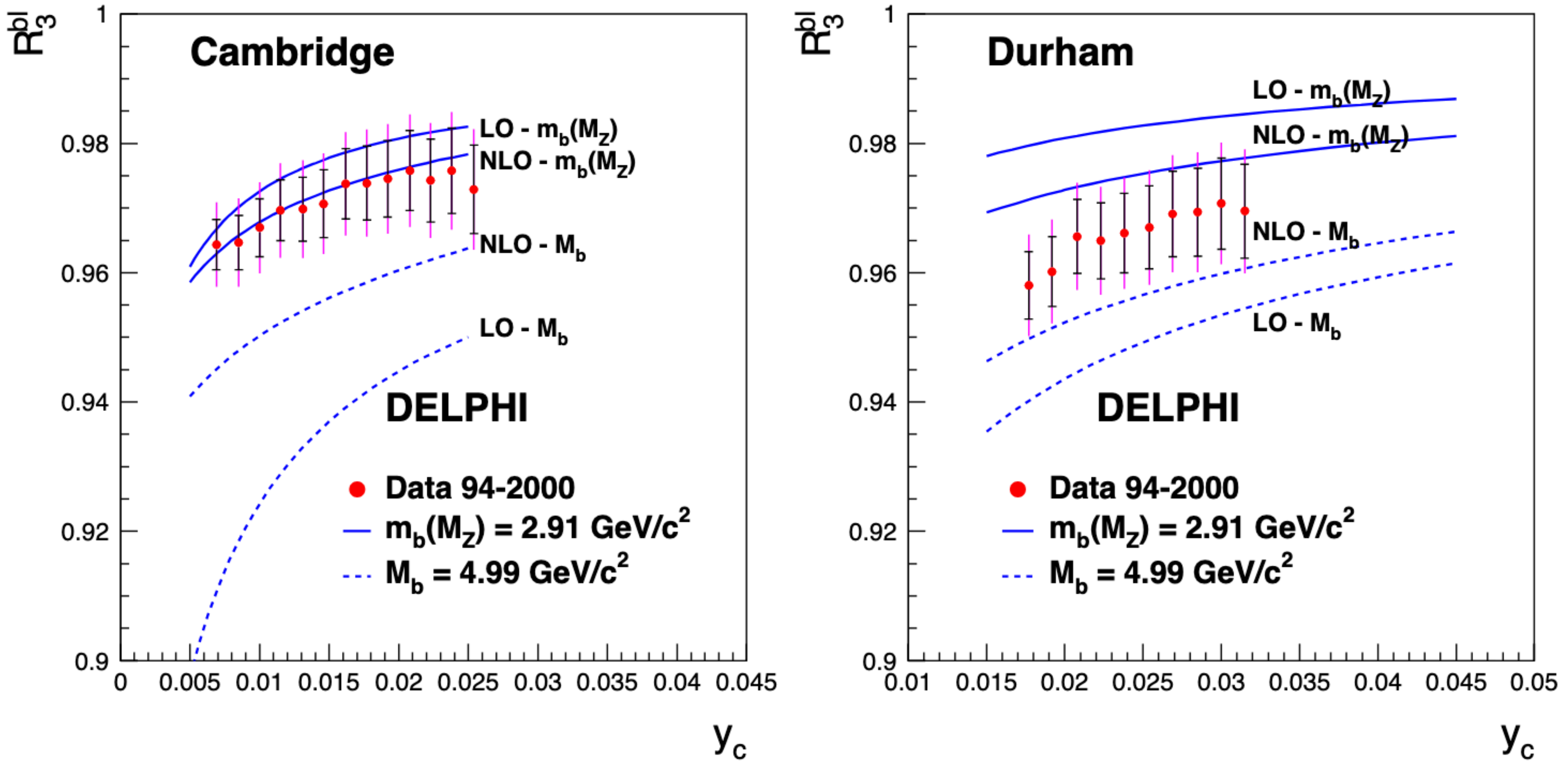
HZ associated production; dominant at ~[200, 450] GeV

WW fusion ($\nu_e\bar{\nu}_e H$); dominant above 450 GeV

- **LCF250 & FCCee240** are mostly dominated by HZ prod.
- **FCCee365** still dominated by HZ, but WW fusion is now relevant.
- **LCF550** explores the region where WW fusion is dominant.

Backup: SM-independent $m_b(m_H)$ measurement

- **Elephant in the room**: we are assuming SM for the Branching ratios. How could we avoid this?
- **Idea**: reconstruct R_{3b} in $H \to b\bar{b}\,(g)$ like what was done @ LEP for $Z \to b\bar{b}\,(g)$ (e.g. 2005 DELPHI result): ratio of three-jet and total decay width of $H \to b\bar{b}$. Extra jet (from gluon emission) is sensitive to mass effects!
  - Normalization depends on b-quark Yukawa, y_b.
  - Shape depends on $m_b(m_H)$.
- **In touch with theorists** to get higher order predictions (Germán Rodrigo, Pier Paolo Giardino). **NEW**: P.P. just uploaded his **NLO computation** of m_b & y_b independently (but no R_{3b}) to arXiv, https://arxiv.org/abs/2601.09599.



Simultaneous measurement of y_b and m_b:

- **Conceptually** under control & understood ✅
- **Feasibility** @LHC with enough precision: **NOT SO CLEAR ‼️**
- **Pheno study to be done** to evaluate it.

Backup: alternative to simultaneous $m_b(m_H)$ and y_b measurement

- If we can't make the simultaneous measurement of $m_b(m_H)$ and y_b, we would try to test some **scenarios to quantify the effect of new physics** in the measurement

1. **Best-case scenarios** 😇: New physics that only affect production, to show that the measurement isn't influenced by this.
2. **Not-so-great scenarios** ☹️: The branching ratios get affected. This way, we would expect different results for $m_b(m_H)$ on each different ratio bb/XX.
3. **Worst-case scenario (for us)** 💀: The only thing that changes is y_b, and nothing else. In this case, the only clue for this that we can expect is $m_b(m_H)$ failing to align with $m_b(m_b)$ and $m_b(m_Z)$ in the RG evolution.