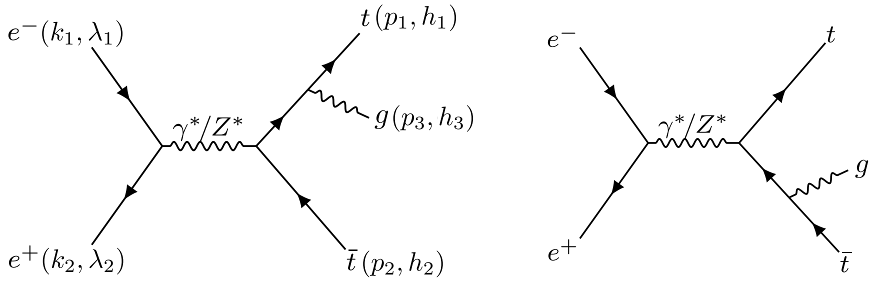
$e^-e^+ \to t\bar{t}g$ entanglement

We consider the $e^-e^+ \to t\bar{t}g$ process. The momentum and helicity assignment as well as the tree-level diagrams are shown below.



1 Kimematics

We work in the centre-of-mass frame, which is the same as the lab frame. The total momenta are constrained as

$$(k_1 + k_2)^\mu \;=\; (p_1 + p_2 + p_3)^\mu \;=\; (\sqrt{s}, \vec{0}\,), \tag{1}$$

where $\sqrt{s}$ is the centre-of-mass energy. We fix the initial momenta as

$$k_1^\mu \;=\; \tfrac{\sqrt{s}}{2}(1, 0, 0, 1), \qquad k_2^\mu \;=\; \tfrac{\sqrt{s}}{2}(1, 0, 0, -1). \tag{2}$$

The gluon and the $t\bar{t}$ subsystem are back-to-back. They are parametrised as

$$p_3^\mu \;=\; (E_g, \vec{p}_g), \qquad (p_1 + p_2)^\mu \;=\; (E_{t\bar{t}}, -\vec{p}_g), \qquad \vec{p}_g \;=\; E_g(\sin\theta_g, 0, \cos\theta_g), \tag{3}$$

where $E_{t\bar{t}} = \sqrt{m_{t\bar{t}}^2 + E_g^2}$ and $m_{t\bar{t}}$ are the energy and invariant mass of the $t\bar{t}$ subsystem.

At the rest frame of the $t\bar{t}$, the top and antitop momenta are parametrised as

$$p_1' = (\tfrac{m_{t\bar{t}}}{2}, \vec{q}), \qquad p_2' = (\tfrac{m_{t\bar{t}}}{2}, -\vec{q}), \qquad \vec{q} = q(\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta), \tag{4}$$

where $q = \sqrt{\frac{m_{t\bar{t}}^2}{4} - m_t^2}$ is the momentum magnitude. The lab frame momenta p_1 and p_2 are obtained by boosting p_1' to p_2' back to the lab frame with the boost vector $\vec{b}$

$$p_1 \;=\; \Lambda_{\vec{b}}(p_1'), \qquad p_2 \;=\; \Lambda_{\vec{b}}(p_2'), \qquad \vec{b} \;=\; -\frac{\vec{p}_g}{E_{t\bar{t}}}. \tag{5}$$

In this way, all final state momenta p_1, p_2 and p_3 are determined by the five kinematical parameters

$$\sqrt{s}, \quad E_g, \quad \theta_g, \quad \theta, \quad \phi\,. \tag{6}$$

The energy conservation $\sqrt{s} = E_g + E_{t\bar{t}}$ relates the gluon energy and the $t\bar{t}$ invariant mass as

$$m_{t\bar{t}}^2 \;=\; s - 2\sqrt{s}E_g\,. \tag{7}$$

The gluon energy ranges

$$0 \;\le\; E_g \;\le\; \frac{s - 4m_t^2}{2\sqrt{s}}\,. \tag{8}$$

2 Helicity amplitudes

$$\mathcal{M}^{\lambda_1,\lambda_2}_{h_1,h_2,h_3} = \sum_{I}^{\text{diagrams}} \mathcal{M}^{(I)}[\bar{u}(k_1,\lambda_1), v(k_2,\lambda_2), u(p_1,h_1), \bar{v}(p_2,h_2), \epsilon(p_3,h_3)], \tag{9}$$

where $\mathcal{M}^{(I)}$ is the contribution from the diagram I, which is a function of four Dirac spinors corresponding to e^-, e^+, t and $\bar{t}$ and the polarisation vector ϵ^μ of the gluon. The fermion and anti-fermion spinors $u(p,\lambda)$ and $v(p,\lambda)$ with the helicity $\lambda = \pm$ and the momentum $p^\mu = (E, \vec{p})$ with $E = \sqrt{m^2 + |\vec{p}|^2}$ and $\vec{p} = |\vec{p}|(\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$ are given by [1, 2, 3].

$$u(p,\lambda) = \begin{pmatrix} \omega_{-\lambda}(p)\chi_\lambda(p) \\ \omega_\lambda(p)\chi_\lambda(p) \end{pmatrix}, \quad v(p,\lambda) = \begin{pmatrix} -\lambda\omega_\lambda(p)\chi_{-\lambda}(p) \\ \lambda\omega_\lambda(p)\chi_{-\lambda}(p) \end{pmatrix}, \quad \omega_\lambda(p) = \sqrt{E + \lambda|\vec{p}|},$$
$$\chi_+ = \begin{pmatrix} e^{-i\phi/2}\cos\frac{\theta}{2} \\ e^{i\phi/2}\sin\frac{\theta}{2} \end{pmatrix}, \qquad \chi_- = \begin{pmatrix} -e^{-i\phi/2}\sin\frac{\theta}{2} \\ e^{i\phi/2}\cos\frac{\theta}{2} \end{pmatrix}. \tag{10}$$

When $\theta = 0$ and π, the phase ambiguity in $\chi_\pm$ is fixed by setting $\phi = 0$. For the gluon momentum $p_3^\mu = E_g(1, \sin\theta_g, 0, \cos\theta_g)$, the polarisation vector is given by

$$\epsilon^\mu(p_3, \pm) = \pm\frac{1}{\sqrt{2}}\left(0, -\cos\theta_g, \pm i, \sin\theta_g\right), \tag{11}$$

We explicitly evaluate the helicity amplitudes (9) with these spinors and polarisation vectors. Since the electron mass is neglected, the amplitudes vanish unless $\lambda_1 = -\lambda_2$

$$\mathcal{M}^{++}_{h_1,h_2,h_3} = \mathcal{M}^{--}_{h_1,h_2,h_3} = 0. \tag{12}$$

2.1 Spin Quantum States

For an initial polarisation (λ_1, λ_2), given the final state momenta, the unnormalised final state in the spin Hilbert space, $\mathcal{H} = \mathbb{C}^2 \otimes \mathbb{C}^2 \otimes \mathbb{C}^2$, is given by

$$|\widetilde{\psi}_{\lambda_1,\lambda_2}\rangle = \sum_{h_1,h_2,h_3} \mathcal{M}^{\lambda_1,\lambda_2}_{h_1,h_2,h_3}|h_1,h_2,h_3\rangle. \tag{13}$$

The unnormalised 8×8 density matrix is given by

$$R^{\lambda_1,\lambda_2}_{(h_1,h_2,h_3),(h_1',h_2',h_3')} = \mathcal{M}^{\lambda_1,\lambda_2}_{h_1,h_2,h_3}[\mathcal{M}^{\lambda_1,\lambda_2}_{h_1',h_2',h_3'}]^*. \tag{14}$$

For unpolarised beams, the corresponding R matrix is

$$R = \frac{1}{4}\left[R^{+-} + R^{-+}\right]. \tag{15}$$

The normalised density matrix is immediately obtained as $\rho = R/\mathrm{Tr}[R]$.

The diagonal elements of the R-matrix corresponds to the differential cross-section

$$\frac{d\sigma_{h_1,h_2,h_3}}{d\Phi} = R_{(h_1,h_2,h_3),(h_1,h_2,h_3)}. \tag{16}$$

In an experimental setup, we must average the density matrix ρ over the events in some selected region Σ, which leads to

$$\rho_\Sigma = \frac{R_\Sigma}{\mathrm{Tr}[R_\Sigma]}, \qquad R_\Sigma = \int_\Sigma d\sigma\,\rho = \int_\Sigma d\Phi\,R, \tag{17}$$

where the full phase-space integration is

$$\int_{\text{full}} d\Phi = \frac{1}{256\,\pi^4}\int_0^{E_g^{\max}} dE_g\,E_g\sqrt{\lambda(E_g)}\int_{-1}^{1} d\cos\theta_g\int_{-1}^{1} d\cos\theta\int_0^{2\pi} d\phi \tag{18}$$

with $\lambda(E_g) = 1 - \frac{4m_t^2}{s - 2\sqrt{s}\,E_g}$.

3 Entanglement

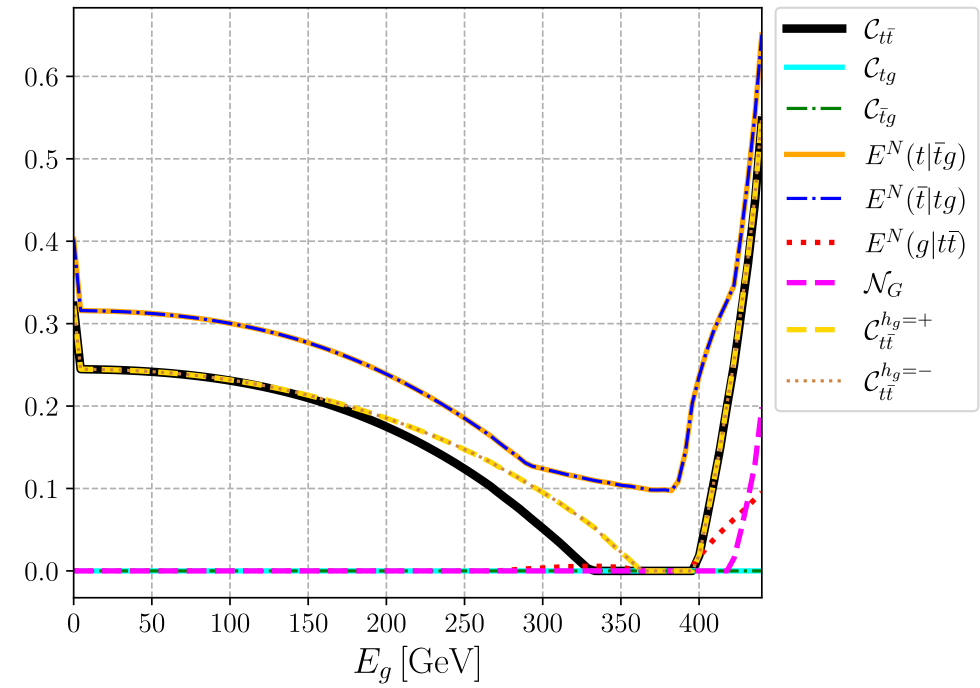


Figure 1: The concurrence, $\mathcal{C}_{ij}$, the logarighmic negativity, $E^N(i|jk)$ and the Genuine Multipartite Negativity (GMN), $\mathcal{N}_G$, are plotted. GMN is defined in [4, 5].

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