‘t Hooft anomalies and the chiral phase transition

Shi Chen, Aleksey Cherman, & RDP, 2603.09977

For three massless flavors at nonzero temperature

The linear sigma model is *wrong*

Predicts 1st, instead of 2nd, order chiral transition.

Lattice (today) finds 2nd order chiral transition.

And, linear model does not satisfy “ ‘t Hooft anomalies”.

The chiral phase transition in QCD is “beyond Landau-Ginzburg”

At least for three flavors, in the chiral limit…

Brookhaven National Laboratory

Ferromagnets & spin waves

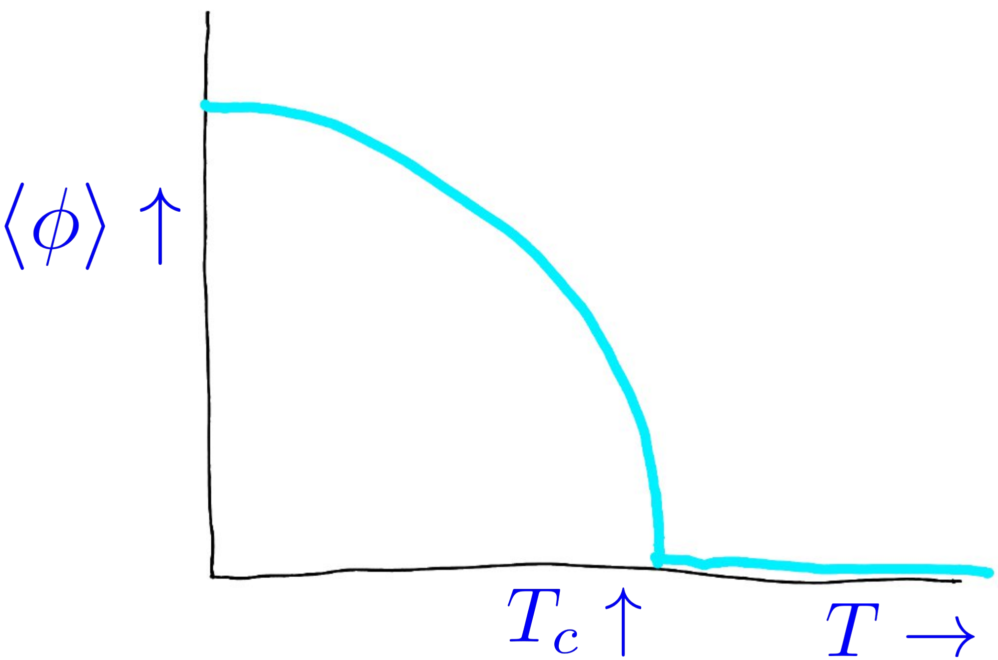
Ferromagnet: spins which like to line up with one another.

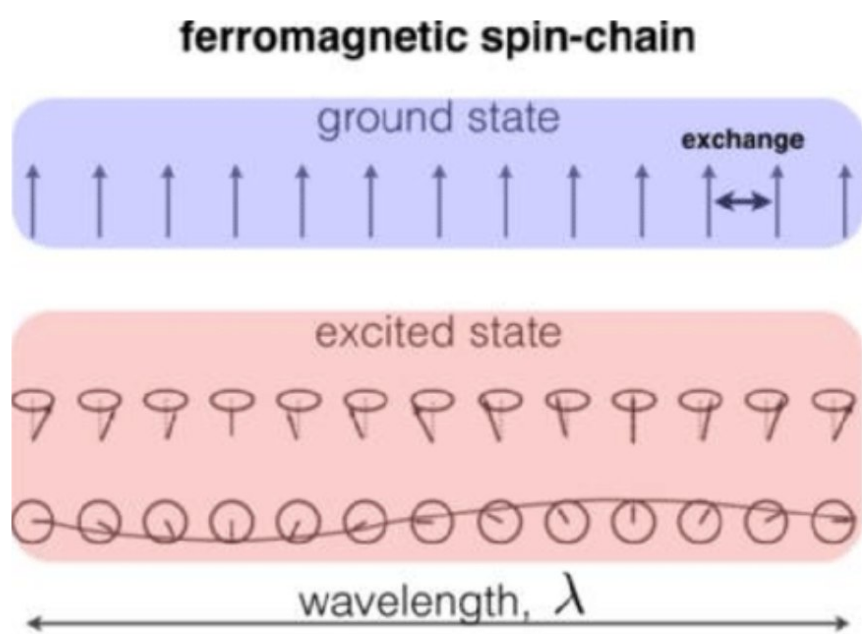
Low temperature: magnetization. High temperature: thermal fluc.'s disorder

So the magnetization goes ↓ as T goes ↑. Below for 2nd order trans. @ T_c.

If symmetry exact, *massless* excitations: Nambu- Goldstone bosons.

If symmetry almost exact, *light* excitations. Pions light because of approx. chiral sym.





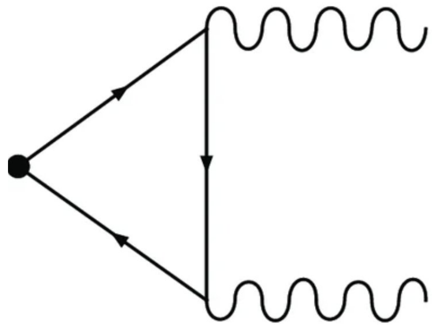
Chiral symmetry in QCD

For N_f flavors of massless quarks, the QCD Lagrangian is invariant under

$$q_L \rightarrow \mathrm{e}^{i\theta_V + i\theta_A}\, U_L\, q_L \; ; \; q_R \rightarrow \mathrm{e}^{i\theta_V - i\theta_A}\, U_R\, q_R \; ; \; U^\dagger_{L,R} U_{L,R} = \mathbf{1}_{N_f}$$

Global chiral symmetry, $G_{cl} = U(1)_V \times U(1)_A \times SU(N_f)_L \times SU(N_f)_R$.

Valid classically. $U(1)_A$ violated by *axial anomaly*, $U(1)_A \rightarrow Z(N_f)$:



$$\partial_\mu J^5_\mu \sim \mathrm{tr}\, G_{\mu\nu} \widetilde{G}^{\mu\nu} \; ; \; J^5_\mu = \overline{q} \gamma_\mu \gamma_5 q$$

$$Q_{\mathrm{top}} \sim \int d^4x\, \mathrm{tr}\, G_{\mu\nu} \widetilde{G}^{\mu\nu} = \mathrm{integer}$$

The isosinglet channel is special, and is *not* a symmetry of the quantum theory.

For three flavors, octet of π's, K's, & η ; singlet is η'.

When chiral symmetry is spontaneously broken, *light* octet, *heavy* singlet.

The linear sigma model

Natural order parameter for chiral symmetry:

$$\Phi = \overline{q}_L \, q_R \to \mathrm{e}^{i\theta_A} \, U_L^\dagger \, \Phi \, U_R$$

In vacuum, <Φ> ≠ 0, symmetry then $U(1)_B$ x $SU(N_f)_V$

$$q_{L,R} \to \mathrm{e}^{i\theta_V} \, U_V \, q_{L,R}$$

Construct effective Lagrangian from all terms invariant under the *un*broken symmetry:

$$\mathcal{L} = \mathrm{tr}|\partial_\mu \Phi|^2 + m^2 \mathrm{tr}\Phi^\dagger \Phi + \lambda_1 (\mathrm{tr}\Phi^\dagger \Phi)^2 + \lambda_2 \mathrm{tr}(\Phi^\dagger \Phi)^2$$

Can't be everything! Terms ~ $\Phi^\dagger \Phi$ are *invariant* under $U(1)_A$!

Anomalous terms

Because of the axial anomaly, *must* include terms ~ det Φ.

Under G_{cl} rotation, $\det\Phi \to e^{iN_f\theta_A}\det\Phi$

If $\theta_A = 2\pi/N_f$, det Φ invariant. Origin of $Z(N_f)$ symmetry.

Terms with det Φ *must* be included in the effective Lagrangian:

$$\mathcal{L}_A = c_1 \det\Phi + c_2\,(\mathrm{tr}\,\Phi^\dagger\Phi)\det\Phi + c_3\,(\det\Phi)^2 + \mathrm{c.c.}$$

These terms are related to "*instantons*", which carry topological charge, $Q_{\mathrm{top}} \sim \mathrm{tr}\, G_{\mu\nu}\widetilde{G}_{\mu\nu} \neq 0$

The coefficients of det Φ and tr $\Phi^\dagger\Phi$ det Φ are due to $Q_{top} = 1$;

That ~ $(\det\Phi)^2$ to $Q_{top} = 2$. RDP & Rennecke, 1910.14052

In vacuum, c_1 (or c_2 or c_3) *must be* large to make the η' heavy.

…and the chiral phase transition

The anomaly term depends *crucially* upon the number of flavors:

$$\det \Phi \sim \Phi^{N_f}$$

Two flavors: a mass term,

splits the η from the σ, and the a_0's from the π's

$$m_\eta^2 \sim m_\sigma^2 + c_1$$

Three flavors: again, splits the η' from the σ

$$m_{\eta'}^2 \sim m_\sigma^2 + c_1 \langle \Phi \rangle$$

Chiral phase transition: if second order, $\langle \Phi \rangle \to 0$

Two flavors: $\sigma + \pi = O(4)$ go critical, a_0's and η stay massive.

Three flavors: $\det \Phi$ is cubic. *Must* be first order if $c_1 \neq 0$ *Unavoidable.*

RDP & Wilczek, '84.

Phase transitions, 2 & 3 flavors

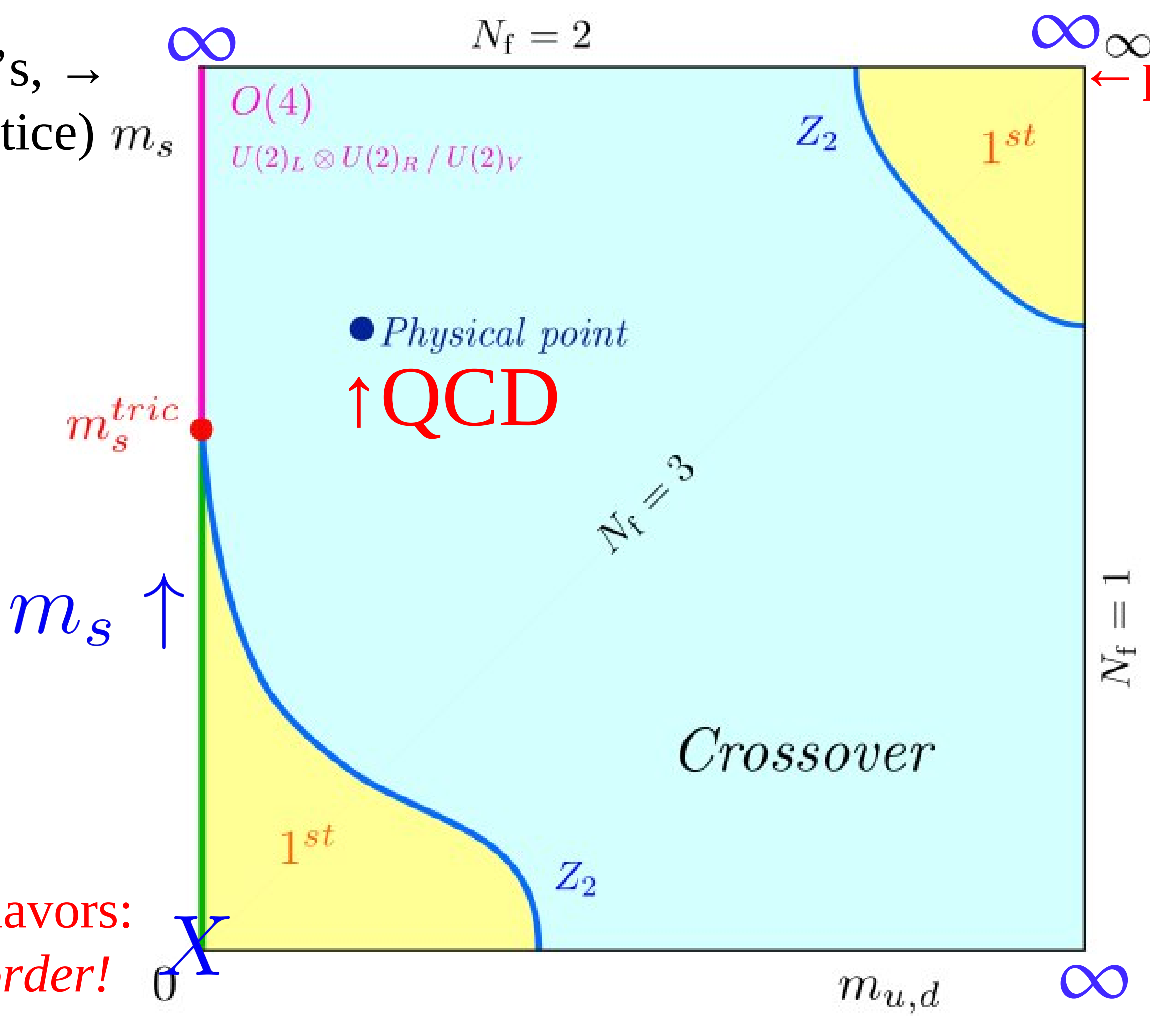
"Columbia" phase diagram. *Pure gauge: cubic invariant from Z(3), 1st order*.

2 massless flavors: 2nd order. *QCD = crossover*

3 massless flavors: expected 1st order

X

Two m=0 fl.'s, →
2nd order (lattice)



← pure gauge, *1st order* .

$m_u = m_d \rightarrow$

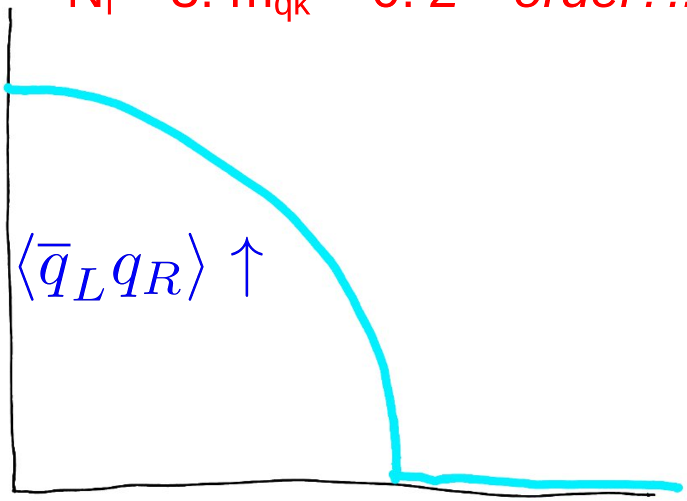
Three m=0 flavors:
must be 1st order!

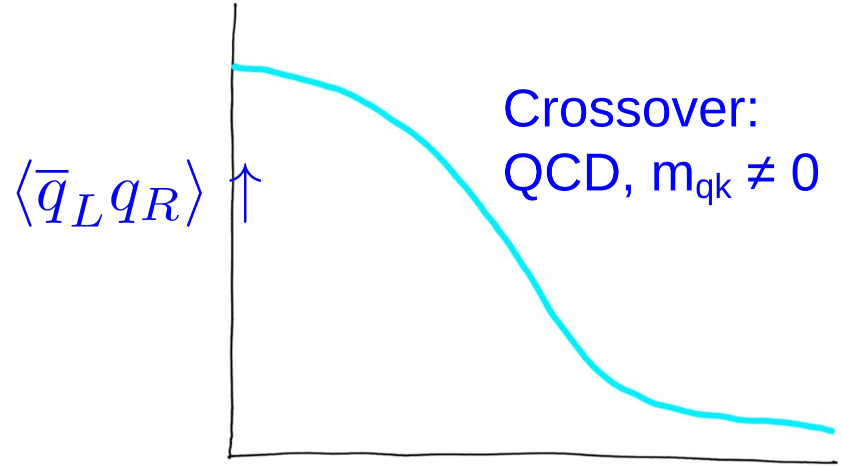
Phase transitions for the chiral transition: *2nd order for 3 flavo*

N_f = 2: m_{qk} = 0, 2nd order

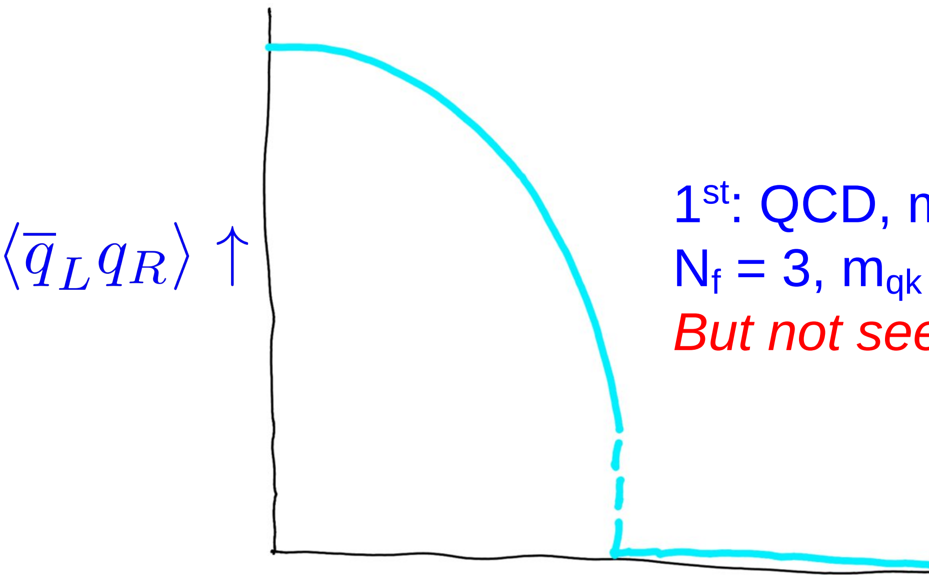
N_f = 3: m_{qk} = 0: *2nd order?!!!!*

$\mathrm{x-axis}: T \text{ or } \mu$





Crossover:
QCD, $m_{qk} \neq 0$



1st: QCD, $m_{qk} \neq 0$ possible

N_f = 3, m_{qk} = 0? *Expected.*

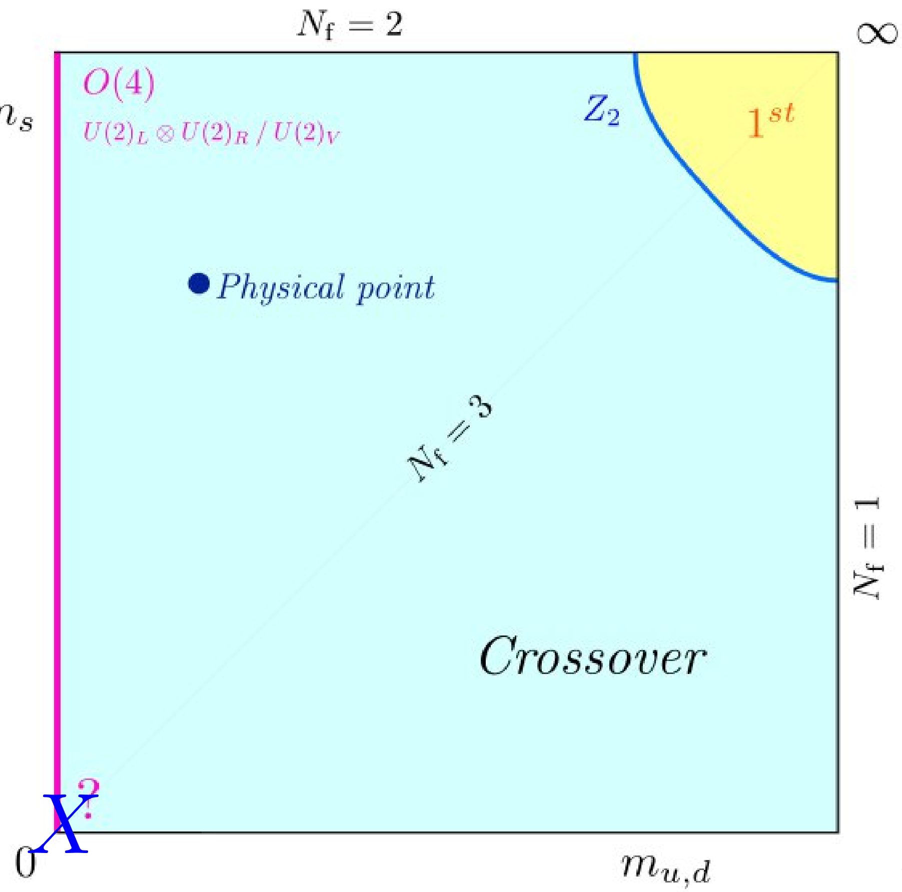
But not seen on the lattice! (yet)

Lattice: *no* sign of 1st order for 3 massless flavors!

"Goethe" phase diagram. Cuteri, Philipsen, Sciarra, 2112.11107:
Chiral transition appears to be 2nd order for 3 m=0 flavors! *As if $c_1 = 0$ at T_χ !*

Restore $U(1)_A$ at T_χ ? RDP & Rennecke, 2401.06130
Phase diagrams with c_1, c_2, c_3: Giacosa, Kovacs², RDP & Rennecke, 2410.08185

$m_s \uparrow$



Three m=0 flavors:
2nd order!

$m_u = m_d \rightarrow$

‘t Hooft anomalies

First, *completely* ignore $U(1)_A$, always broken (vanishes as $T \to \infty$….: $c_1 \sim 1/T^7$, etc)

Global symmetry group is

$$\mathcal{G}_{\rm qu} = \frac{U(1)_V \times SU(N_f)_L \times SU(N_f)_R}{Z(3) \times Z(N_f)}$$

After chiral symmetry breaking,

$$\mathcal{H} = \frac{U(1)_V \times SU(N_f)_V}{Z(N_f)}$$

Boundary conditions in imaginary time:
in *pure* gauge theory, gauge transf.’s
Ω can have Z(3) “twist”:

$$\Omega(1/T) = {\rm e}^{2\pi i/3}\,\Omega(0)$$

Gluons are invariant:

$$A_\mu(1/T) \to {\rm e}^{-2\pi i/3} A_\mu(0)\,{\rm e}^{+2\pi i/3} = A_\mu(0)$$

~ Polyakov loop, order parameter for deconfinement. *“1-form” order parameter*.

Quarks (in fund. rep.) are *not*:

$$q(1/T) = {\rm e}^{2\pi i/3} q(0)$$

Roberge-Weiss transition

Have $U(1)_B$, introduce chemical potential μ.

Boundary conditions now $$q(1/T) = \mathrm{e}^{\mu/T} q(0)$$

Thus for Z(3) twisted b.c.,

$$q(1/T) = \mathrm{e}^{2\pi i/3 - \mu/T} q(0) = q(0) \; ; \; \textit{if} \;\; \mu = \frac{2\pi\, T}{3} i$$

Roberge & Weiss '88. Thus the partition function is invariant under this shift:

$$\mathcal{Z}\left(\mu + \frac{2\pi}{3} i\right) = \mathcal{Z}(\mu)$$

An *imaginary* shift in the chemical potential! Who cares?

‘t Hooft anomalies

‘t Hooft ’79….Cordova, Freed, Lam, Seiberg, 1905.09315 ; 1905.13361
Yonekura, 1901.08188 ; Nishimura & Tanizaki, 1903.04014;
Kobayashi, Yokokura & Yonekura, 2305.01217

Add background *gauge* fields for the *global* symmetries: A_L^μ, A_R^μ: *not* dynamical!

t Hooft anomaly: $U(1)_V \times SU(N_f)_{L,R} \times SU(N_f)_{L,R}$. Constant (imag.) μ like A_0 for $U(1)_V$

$$\mathcal{Z}(\mu + 2\pi i/3, A_L, A_R) = \mathcal{Z}(\mu, A_L, A_R)\ \mathcal{J}(A_L)\ \mathcal{J}^{-1}(A_R)$$

$$\mathcal{J}(A) = \exp\left(\frac{i}{4\pi}\int d^3x\ \epsilon^{\alpha\beta\delta}\mathrm{tr}\left(A_\alpha\partial_\beta A_\delta - \frac{2}{3}iA_\alpha A_\beta A_\delta\right)\right)$$

↑ Chern-Simons term for the background gauge fields.

Computed perturbatively in the UV. Must hold for *any* effective theory in the IR;
‘t Hooft: introduce “spectator fermions”

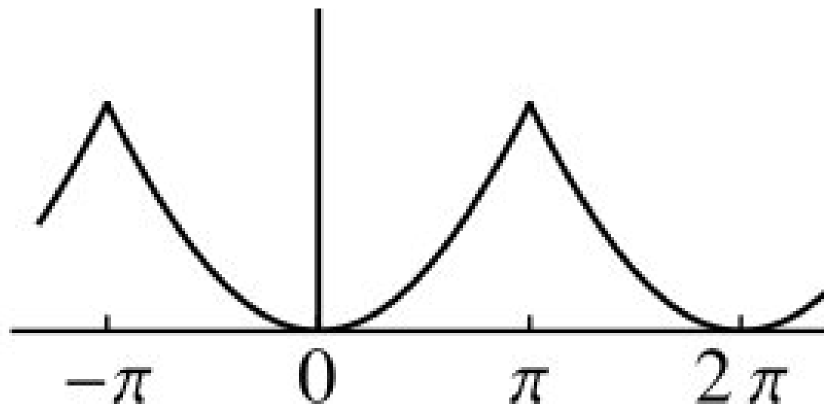
Strong constraint on effective theories! “Anomaly in space of coupling constants”

High T: 1st order transition

At high T, restore chiral symmetry, can compute in perturbation theory.

Imaginary chemical potential = constant, real A_0 field. *Easy* to compute @ 1-loop order!

$\mathcal{V}_{1-loop} \uparrow$



$\frac{3\,\mu_B^{imag}}{T} \rightarrow$

Roberge & Weiss '88: 1st order transition at $3\,\mu^{imag}_B/T = \pi$ T

Quark density imaginary, flips sign.

One way to satisfy 't Hooft anomaly is by 1st order transition.

$\mu^{imag}_B = 0$ *not* analytically connected to $3\, i\, \mu^{imag}_B/T = 2\,\pi$

RW transition exists at high T, presumably ends at some nonzero T. When?

‘t Hooft anomaly at *low* T

At low T, chiral symmetry is broken, $\Phi = \Omega\, \phi_0 \ ; \ \Omega^\dagger \Omega = \mathbf{1}$

Can rewrite the effective Lagrangian with the Goldstone bosons, the Ω’s

$$\mathcal{S}_{\rm GB} = \int d^3x \left(\frac{1}{g^2} {\rm tr}\, |\partial_\mu \Omega|^2 + \mu\, \frac{\epsilon^{ijk}}{24\pi^2}\, {\rm tr}(B_i B_j B_k) \right) \ ; \ B_i = \Omega^\dagger \partial_i \Omega$$

The second term is baryon number, related to $\pi_3(SU(N_f)_V)$.

If $3\, i\, \mu^{imag}_B/T$ shifts by 2π, $\exp(-S_{GB})$ shifts by $\exp(2\pi i)=1$.

Gauging S_{GB}, this generates the correct anomaly (Witten ‘84)

Anomalies arise from fermion loops, *and* bosons with topological terms

In linear sigma model:

$$Q = \int d^3x\ \epsilon^{ijk} {\rm tr}(\widetilde{B}_i \widetilde{B}_j \widetilde{B}_k) \ ; \ \widetilde{B}_i = \Phi^\dagger \partial_i \Phi$$

But not topological! *Q n*ot baryon number, just operator with high mass dimension

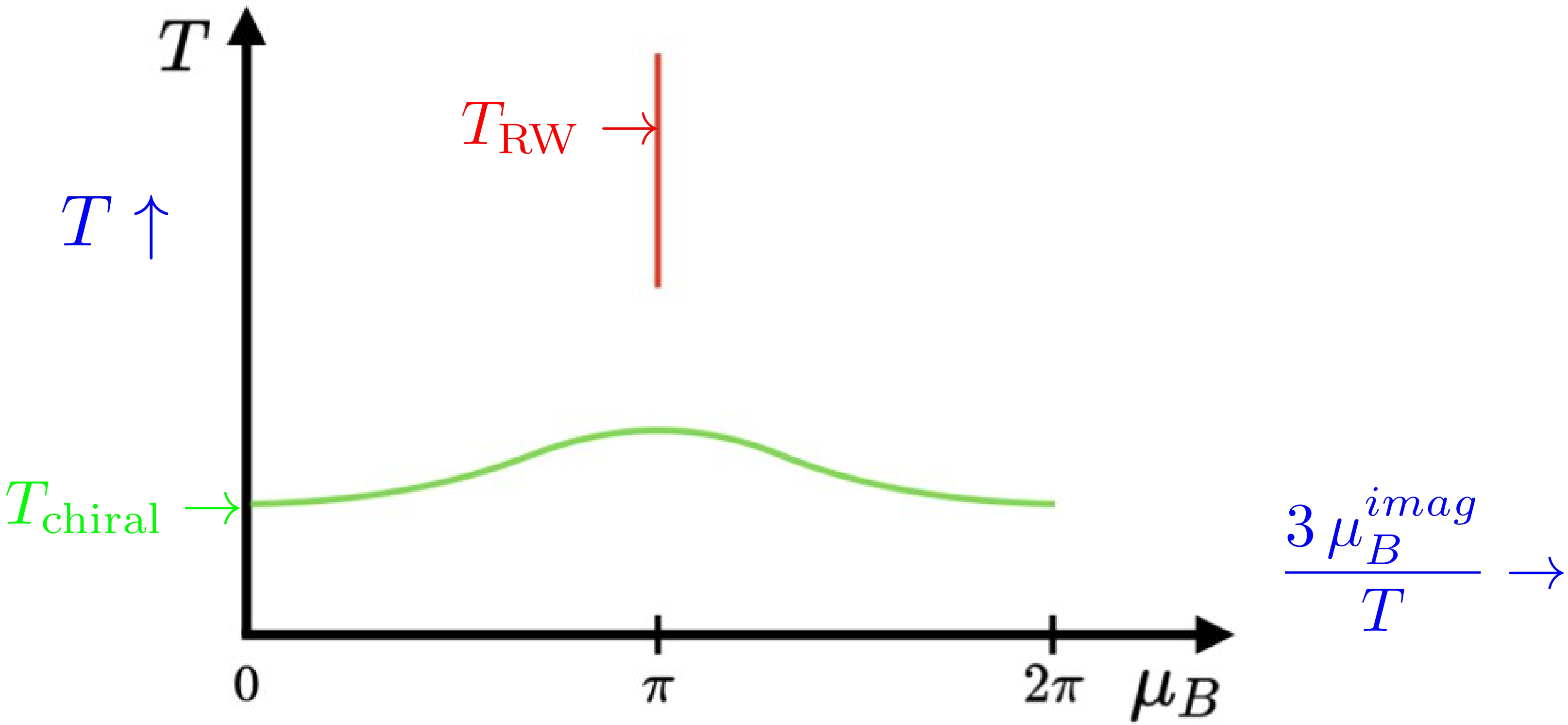
Excluded: $T_{RW} > T_{ch}$.

RW transition: 1st order at $3\ \mu^{imag}_B / T = \pi$ for high T, assume same at lower T.

How does the RW line end? *Not* above T_{chiral}!

Linear model does not satisfy 't Hooft anomaly. True for any # flavors.

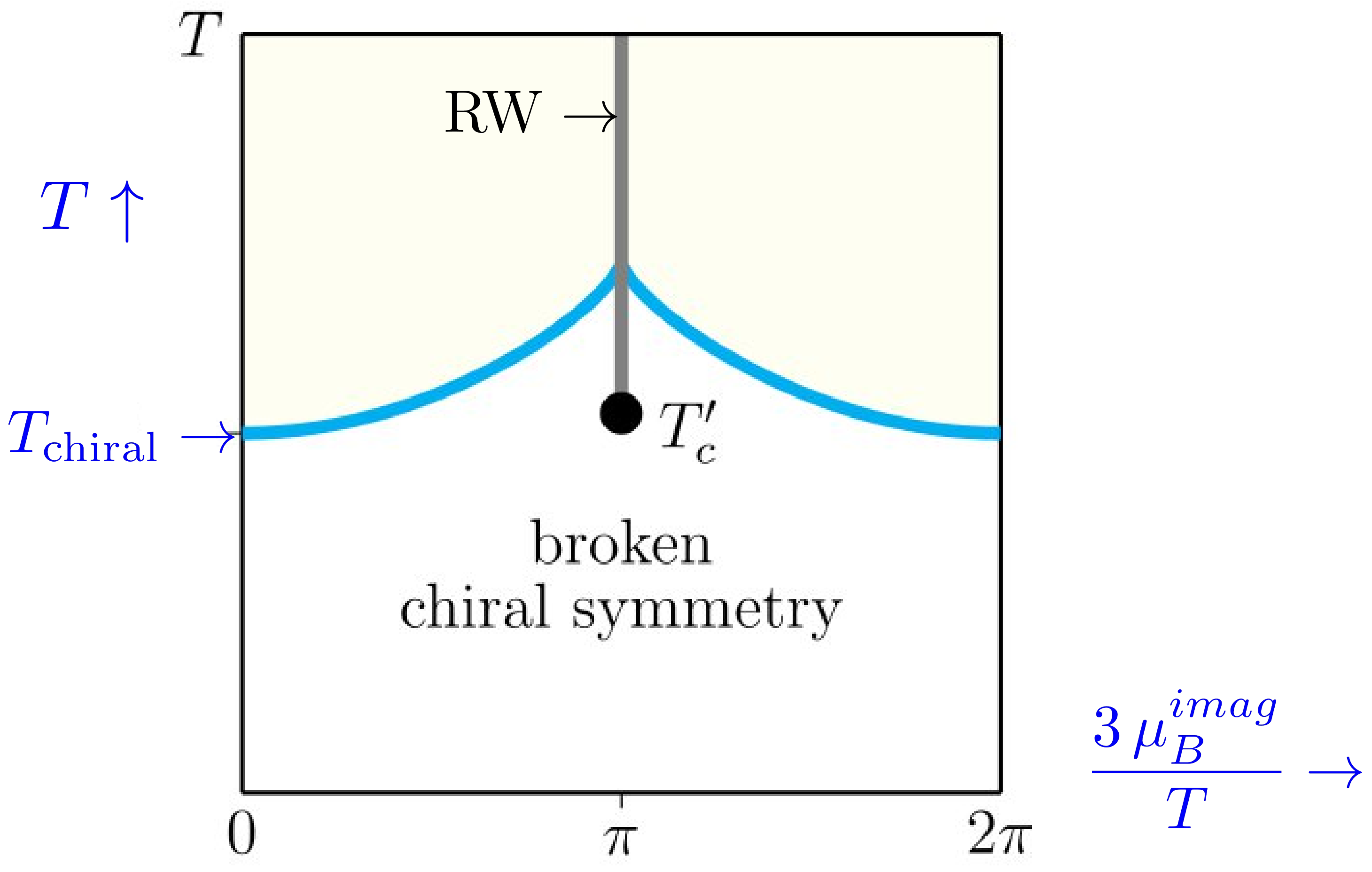
Kobayashi, Yokokura & Yonekura, 2305.01217 . Three (other) phase diagrams possible



Landau: $T_{RW} < T_{ch}$.

The Roberge-Weiss transition can end *below* the chiral, at standard critical endpoint.

Landau-Ginzburg works: in sym. phase, linear model+quarks for 1st order RW transition.

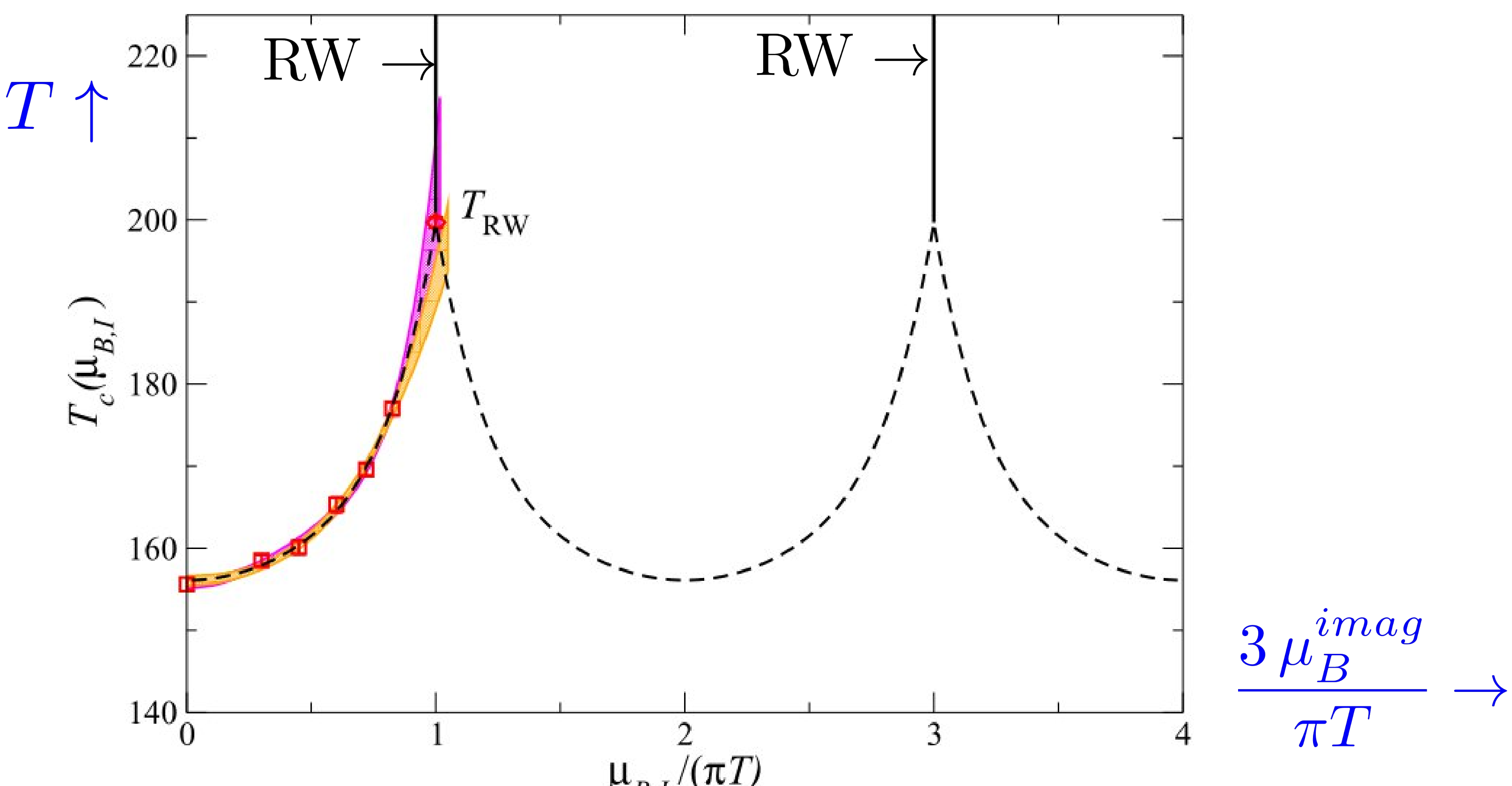


Lattice, 2+1 flavors: $T_{RW} > T_{ch}$.

Easy for lattice to compute at imaginary chemical potential, as real A_0.

Lattice: Bonati, D'Elia, Mariti, Mesiti, Negro 1602.01426, Cuteri+…2205.12707

QCD, 2+1 flavors: *crossover* at μ=0, remains crossover until $T_{RW} > T_{ch}$.



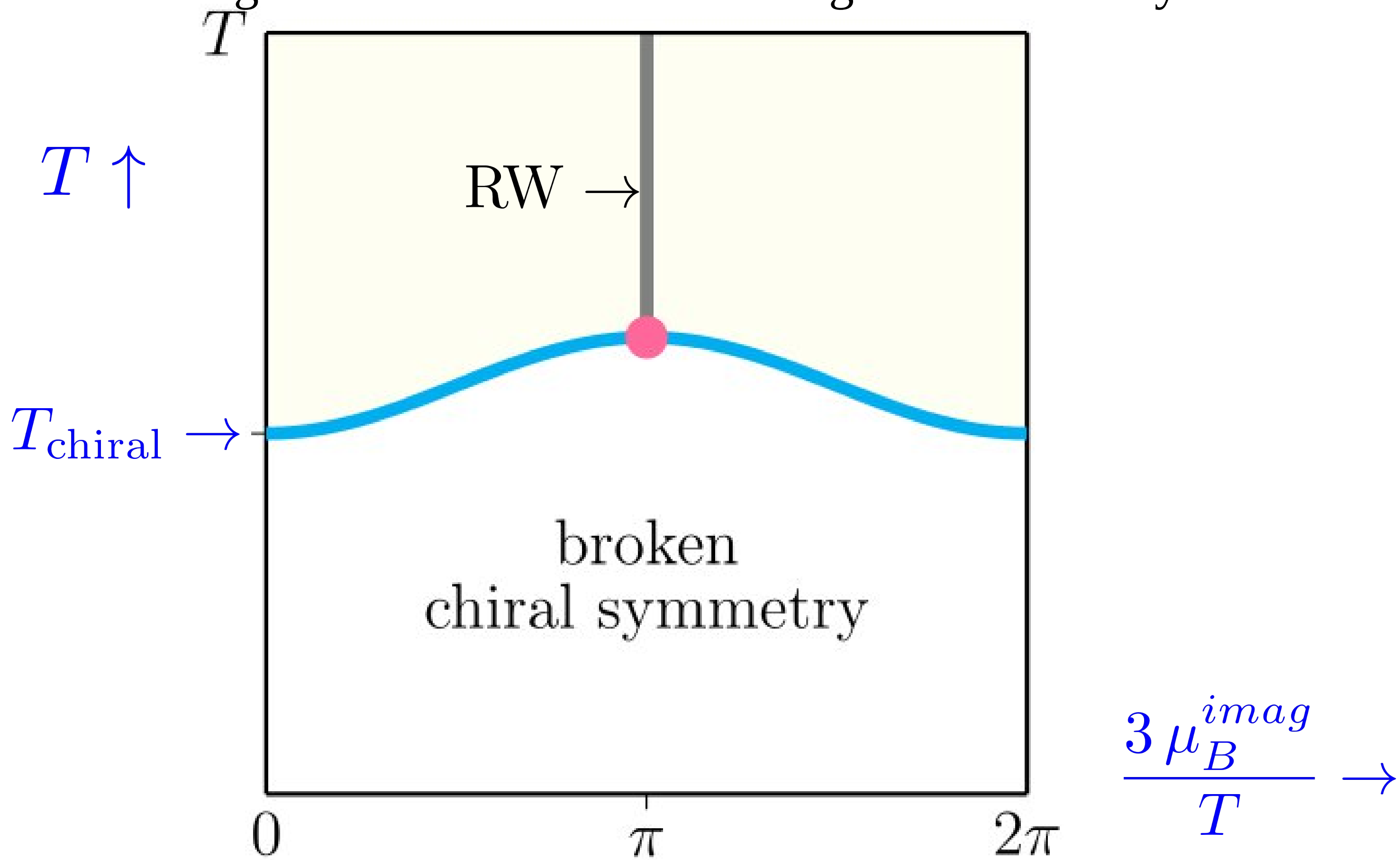
DQCP: $T_{RW} = T_{ch}$.

Back to chiral limit. RW transition can end on the critical line for the chiral transition.

1st order RW line ends at Z(2) critical endpoint. *Not* universality class of chiral trans.

"*Deconfined quantum critical point*"

"Beyond Landau-Ginzburg": L-G can*not* describe change in universality class.

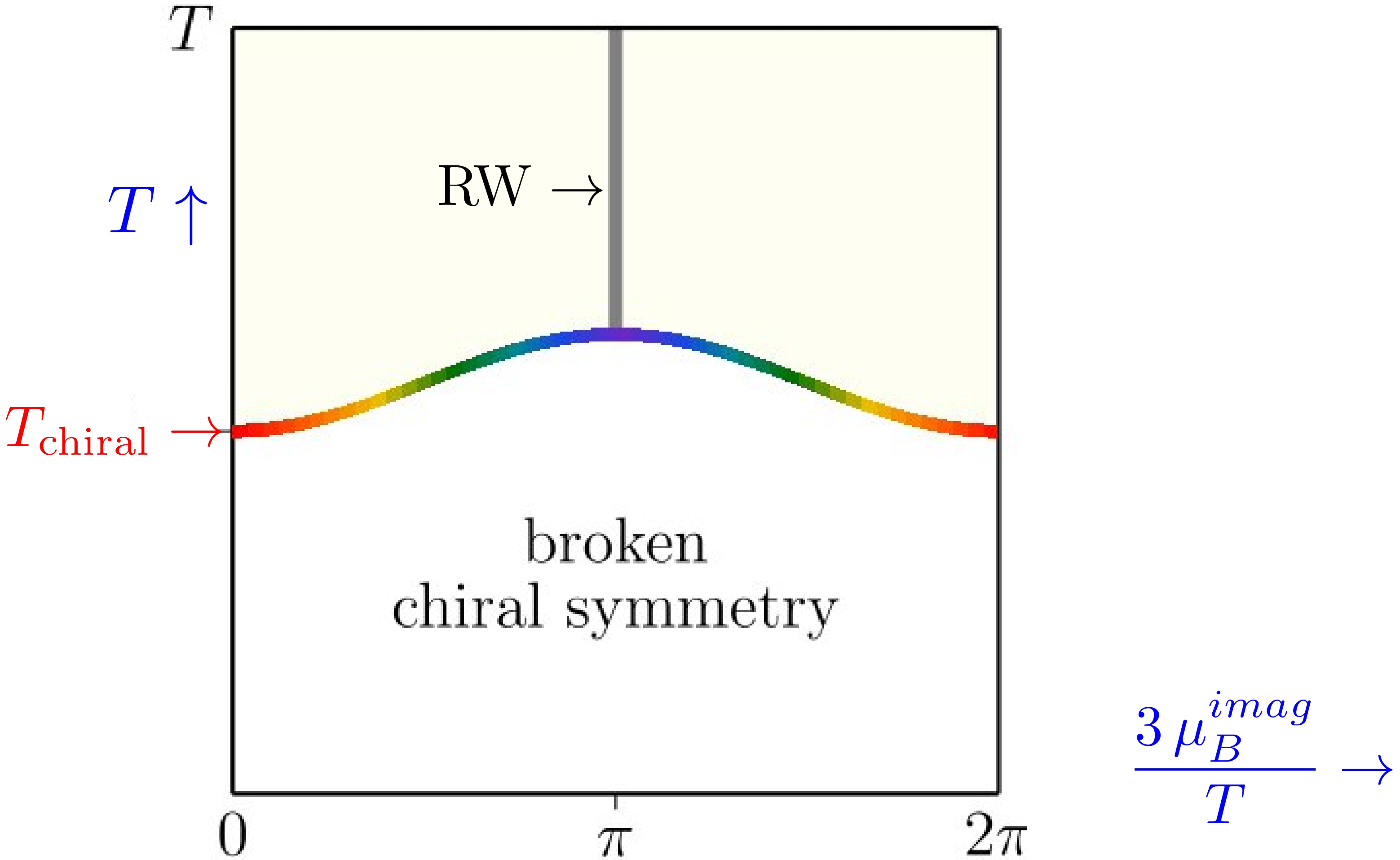


Continuous manifold: $T_{RW} = T_{ch}$.

Roberge-Weiss ends on critical line, but universality class changes *continuously* with μ_B

Requires that the baryon density is *exactly* marginal for *all* μ_B.

Beyond Landau-Ginzburg. *No* known example without SUSY. Unusual, possible.



Conclusions

For two flavors, chiral transition is 2^{nd} order, ok with linear model.

For three flavors, in the chiral limit trans. of 2^{nd} order, *not* 1^{st} order, as in linear model.

Linear model fails to describe 't Hooft anomaly; ok if 1^{st} order RW intervenes.

Extension to nonzero chemical potential?

Effective theory for three flavors? What about $\mu_B \neq 0$?

Instead of linear σ model, going *down* from 4 – ε to 3 dimensions,

work with a non-linear σ model, going *up* from 2 + ε dimensions to 3.

Really? A *non*-linear model?

Certainly must add quarks to any effective model to get the RW transition right.

Quarkyonic models, which always have quarks, appear inescapable.

How to cool a neutron star *very* fast

L. Brodie & RDP, *BP*, 2501.02055 , PRL 135 .

L. Brodie, R. Negreiros, J. Steinheimer, V. Dexheimer, & RDP, *BNSDP*, 2603.067

Punchline: in parity doubled models, for *limited* region of density, in *heavy* stars,
Urca with parity *doubled* nucleons opens up, *neutrino emission increases by 1*




Liam Brodie Rodrigo Negreiros Jan Steinheimer Veronica Dexheimer

Wash U → Kent St CaseWestern

“Cat Tech”

FIAS&GSI Kent &

Brookhaven National Laboratory

Neutron Stars (NS)

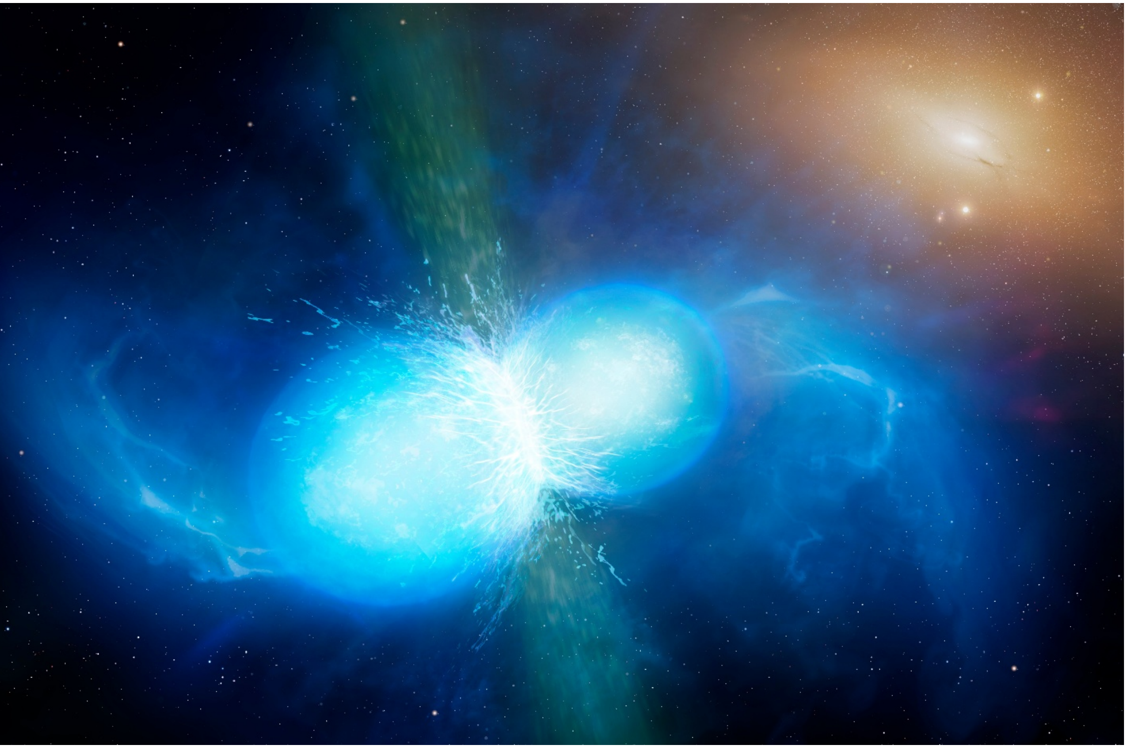
Supergiant stars, 10-25 M_o (M_o = solar mass) collapse through supernova
into black holes or neutron stars
If remnant too heavy, forms a black hole
Else a neutron star, M: 1.4 – 2.2 M_o . Radii 10-14 km.
Dense: weight of matchbox on NS = $(0.8\ km)^3$ on earth!

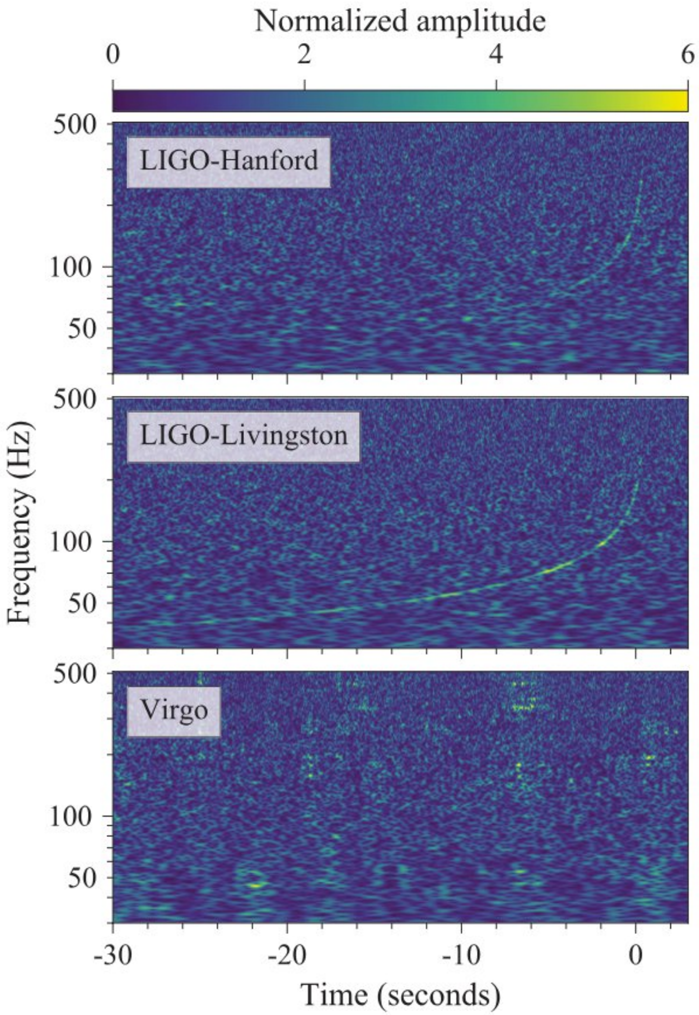
https://science.nasa.gov/asset/webb/crab-nebula-webb-and-hubble-

GW 170817

Merger of two neutron stars. Correlated with EM observations, Dawn of “multi-messenger” astronomy. Only *one* NS merger to date.

Forms kilonova, lasts weeks-months, EM radiation from radioactive decay of r-process nuclei





When is the next merger of neutron stars?

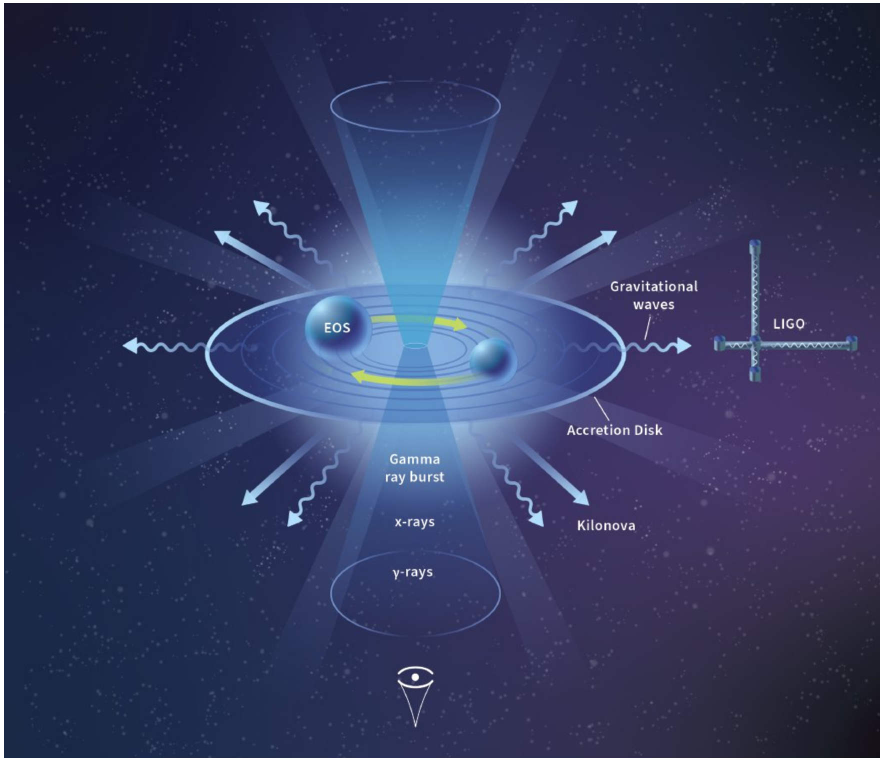
Golden Age in astrophysics

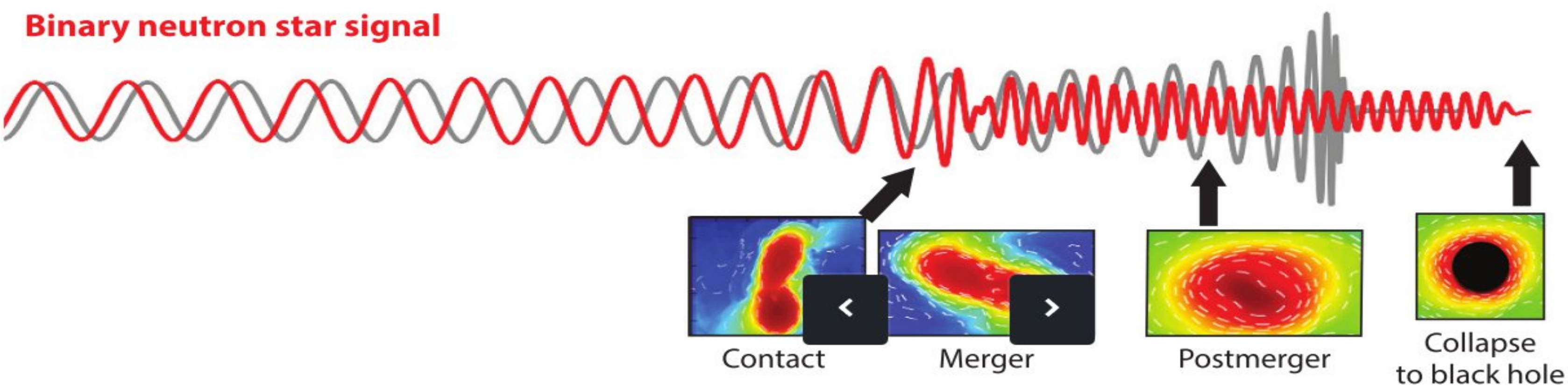
Can fix EoS from *post* ring down

Upgrades to LIGO and:

Cosmic Explorer & Einstein (2030's):
hundreds of NS mergers per *year*!

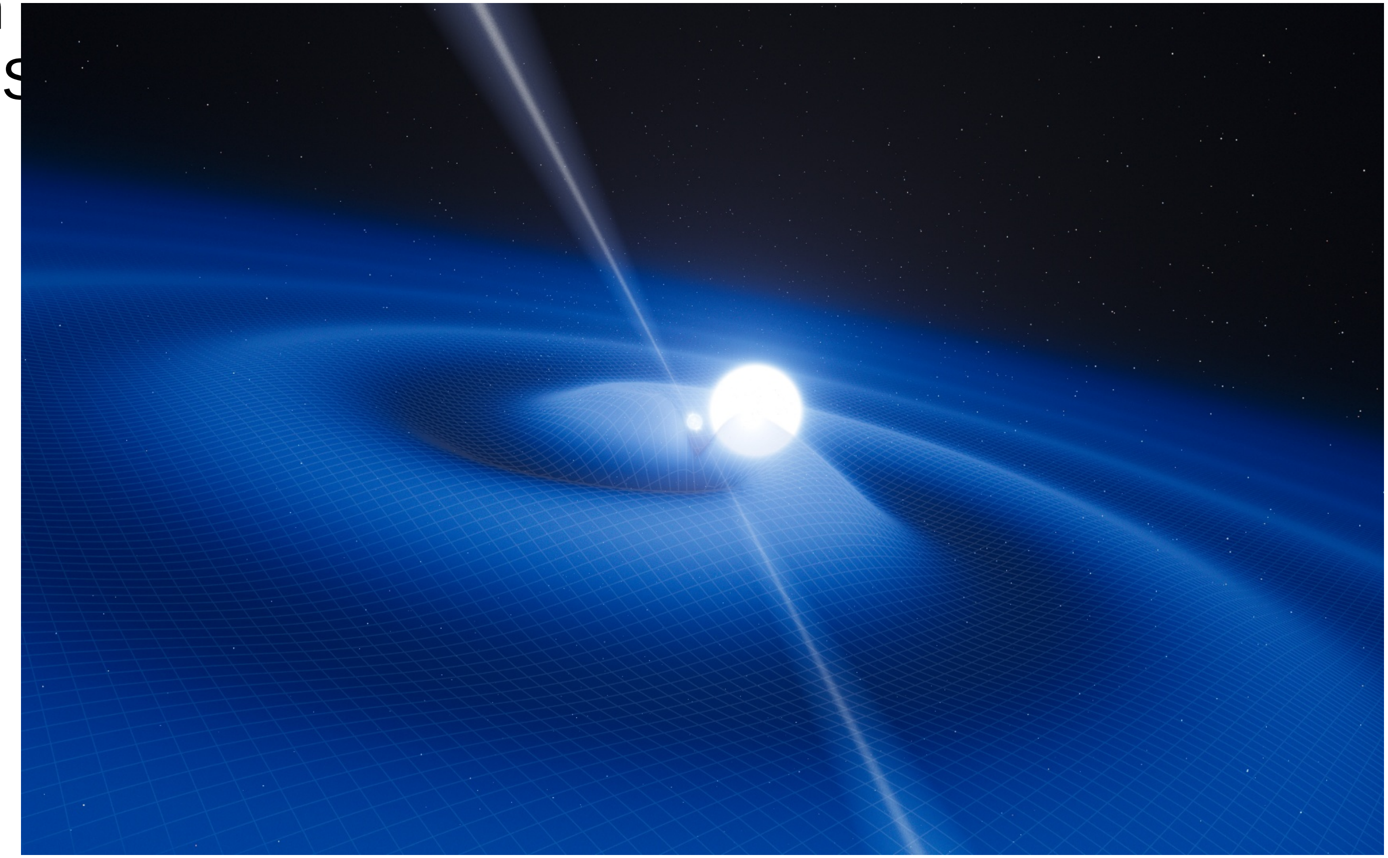
Radius from X-ray satellites, eg NICER





Heavy NS's!

For many years, most NS's had M ~ 1.44 M_o.
2006: Pulsating Source of Radiation PSR J1614-2230, M = 1.97 ± 0.04 M_o.
2013: PSR J0348-0432, M = 2.01 ± 0.04 M_o., below: + white dwarf companion
Heaviest PS



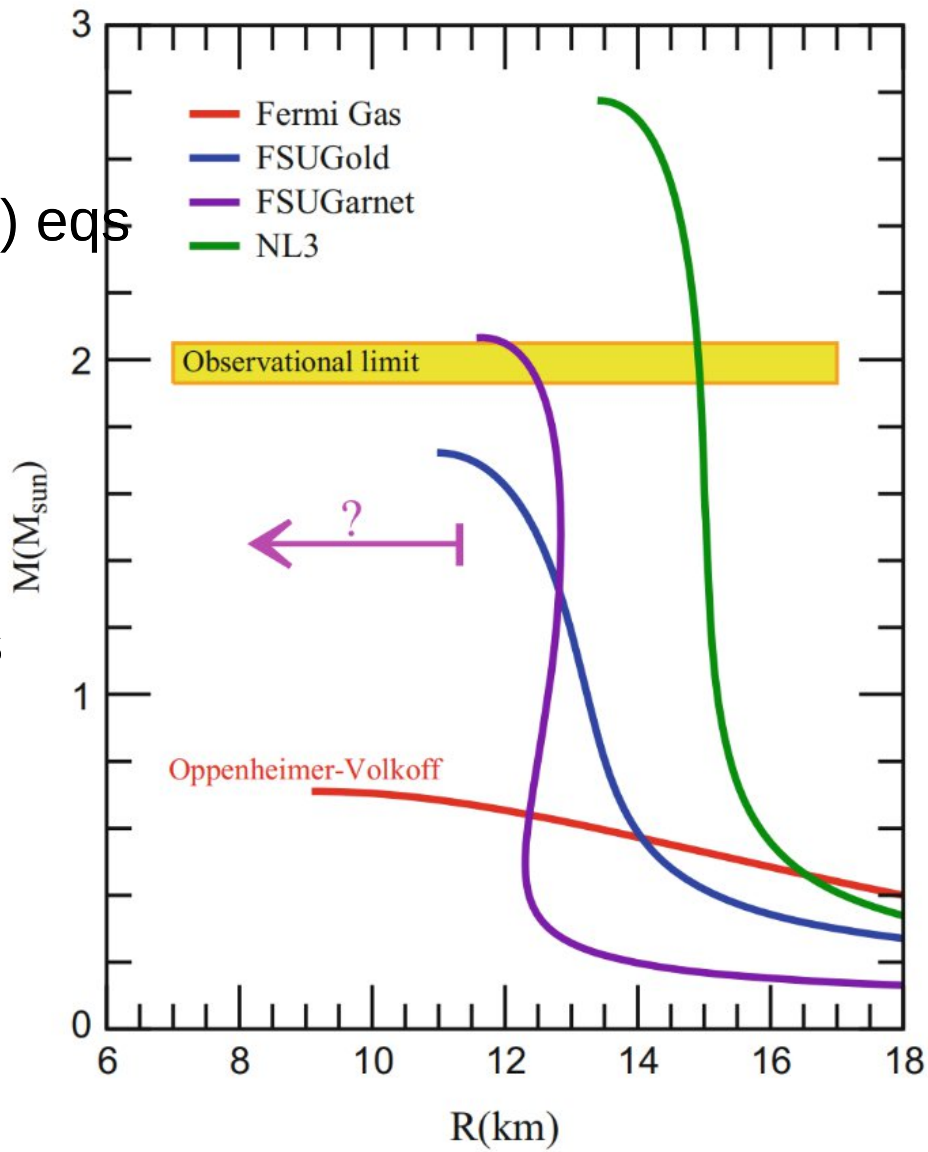
Masses, radii of NS's

Need the Equation of State, EoS!

Given p(e), solve the Tolman-Oppenheimer-Volkov (TOV) eqs

Z. Ji, J. Chen, G. Wu, https://arxiv.org/abs/2505.05241

Will eventually have mass vs radius to few %

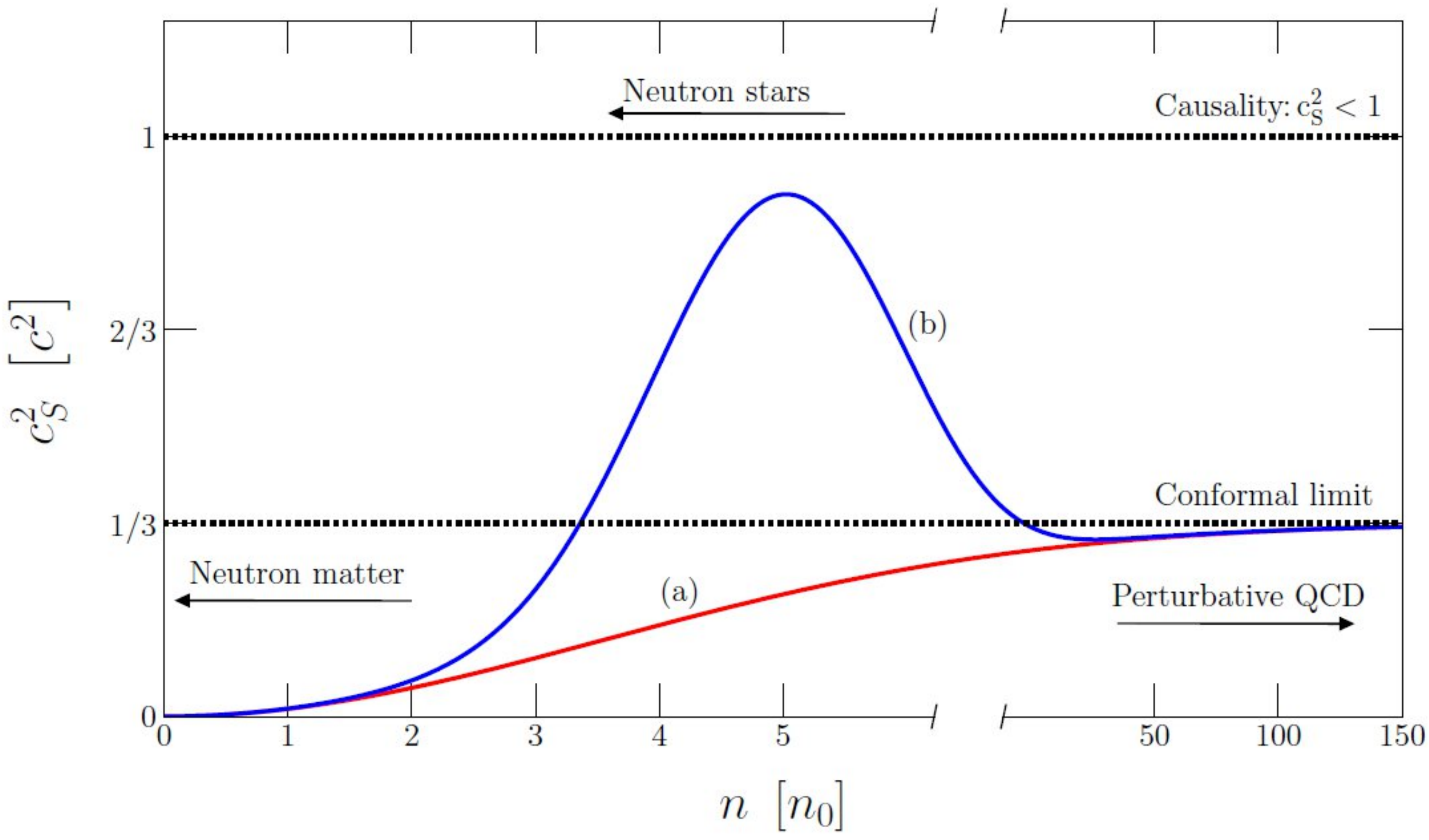


“Spike” in the speed of sound

Heavy NS’s imply the nuclear EoS is “stiff” in some region above ρ_0

All fits require that speed of sound^2 > 1/3 in some regime: a “spike”

RDP, https://arxiv.org/abs/2101.05813 : quantum pion liquid



One phase transition?

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EXPONENTIAL HADRONIC SPECTRUM AND QUARK LIBERATION

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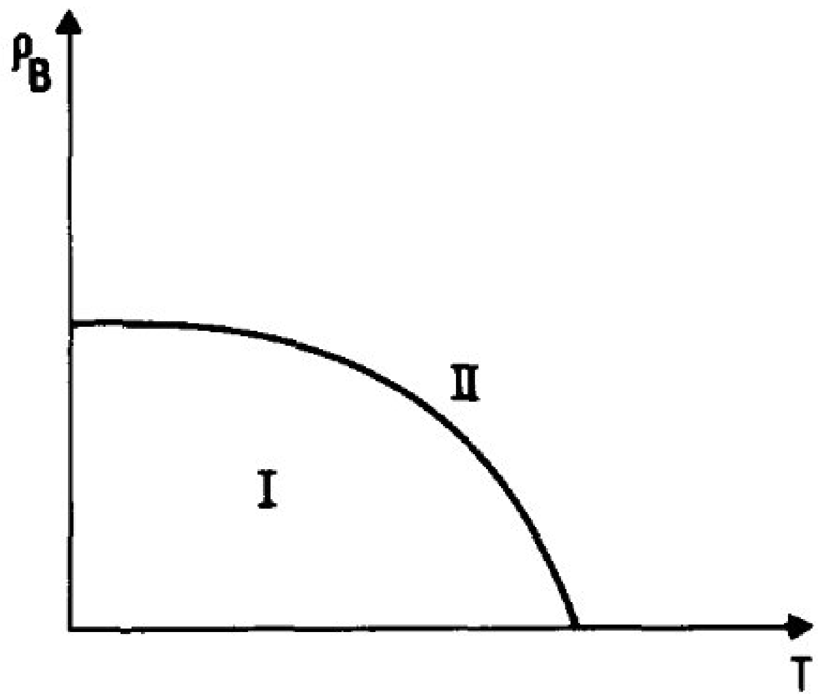
$$(T_d, \mu_d) = (T_\chi, \mu_\chi)$$



Wrong at T = 0, where $T_d \approx 2\ T_\chi$.

Reasonable first guest.

Historically, *dominant* for decades

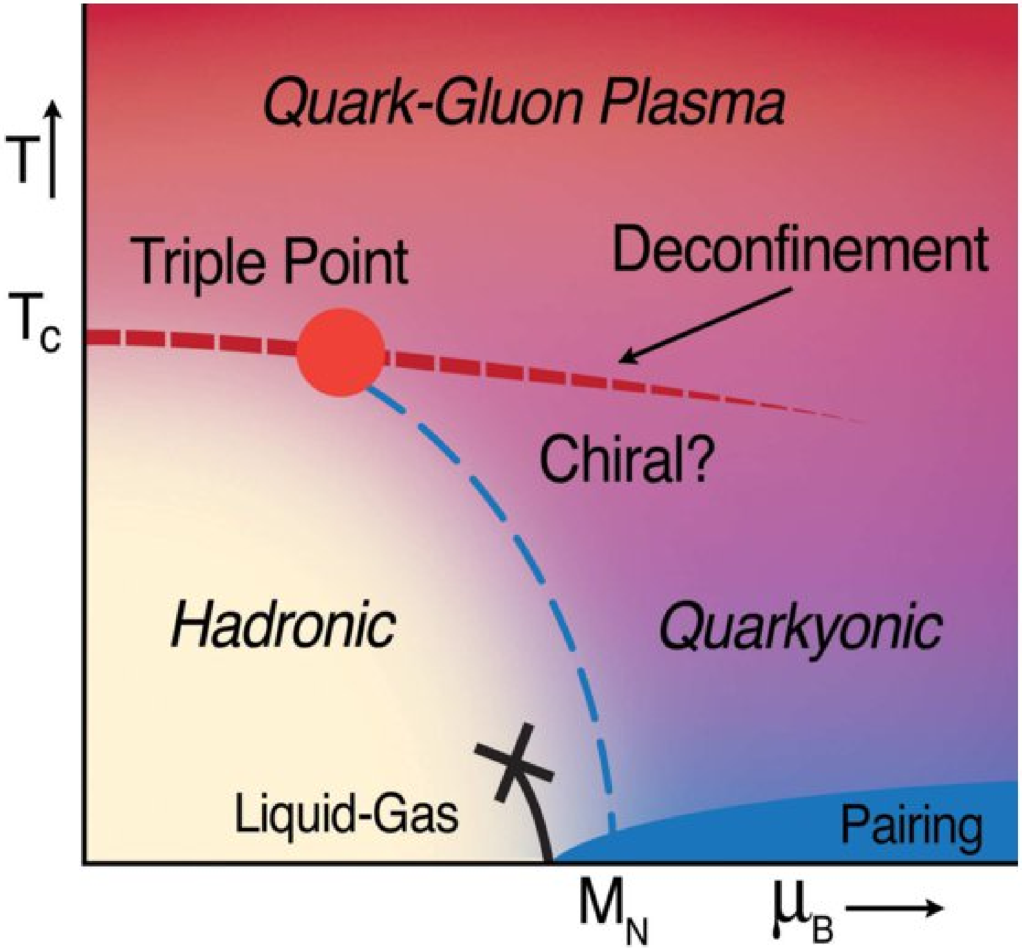
Still is, in some corners.

"Quarkyonic" phase diagram

When # colors >> # flavors, gluons (obviously) dominate, *"quarkyonic"*
L. McLerran & RDP, 0706.2191

At "moderate" μ, chiral symmetry restoration happens *before* deconfinement
Need model of chirally symmetric baryons.

Andronic…McLerran, RDP +…
0911.4806



Chirally symmetric, *massive* baryons

Massless quarks have chiral symmetry. For *massive* baryons? Detar and Kunihiro '89:

Usual nucleons, N(939), $J^P = ½^+$, *and* their parity partners, N*(1535), $J^P = ½^-$

$$\mathcal{L}_N = \overline{\psi}_1 \, \partial\!\!\!/ \, \psi_1 = \overline{\psi}_{1,L} \, \partial\!\!\!/ \, \psi_{1,L} + \overline{\psi}_{1,R} \, \partial\!\!\!/ \, \psi_{1,R}$$

$$\psi_{L,R} = \frac{1 \pm \gamma_5}{2} \, \psi$$

$$\mathcal{L}_{N^*} = \overline{\psi}_2 \, \partial\!\!\!/ \, \psi_2 = \overline{\psi}_{2,L} \, \partial\!\!\!/ \, \psi_{2,L} + \overline{\psi}_{2,R} \, \partial\!\!\!/ \, \psi_{2,R}$$

Duh. Big deal. But because N*'s parity differs, so do chiral transformations:

$$\psi_{1;L,R} \rightarrow U_{L,R} \, \psi_{1;L,R} \; ; \; \psi_{2;L,R} \rightarrow U_{R,L} \psi_{2;L,R}$$

Then can form chirally *symmetric* mass term for the N's *and* N*'s:

$$\mathcal{L}_m = m_0 (\overline{\psi}_{1,L} \gamma_5 \, \psi_{2,R} + \overline{\psi}_{1,R} \gamma_5 \, \psi_{2,L} + L \leftrightarrow R) = m_0 (\overline{\psi}_1 \gamma_5 \, \psi_2 - \overline{\psi}_2 \gamma_5 \, \psi_1)$$

Parity Doubled baryons

Couple these nucleons to $SU(2)_L \times SU(2)_R = O(4)$ invariant fields φ = (σ, π), along with vector mesons, ω_μ and ρ_μ :

$$\mathcal{L}_{\mathrm{PD'd}} = \overline{\psi}_1 (\partial\!\!\!/ - 2\, g_1\, (\sigma + i\gamma_5 \vec{\pi}\cdot\vec{\tau}) - g_\omega \omega\!\!\!/ - g_\rho \vec{\rho\!\!\!/}\cdot\vec{\tau})\psi_1$$

$$+\overline{\psi}_2\, (\partial\!\!\!/ - 2\, g_2\, (\sigma - i\gamma_5 \vec{\pi}\cdot\vec{\tau}) - g_\omega \omega\!\!\!/ - g_\rho \vec{\rho\!\!\!/}\cdot\vec{\tau})\, \psi_2$$

$$+m_0 \left(\overline{\psi}_2 \gamma_5 \psi_1 - \overline{\psi}_1 \gamma_5 \psi_2\right)$$

In a chirally symmetric phase, masses degenerate, = m_0.

When chiral symmetry is broken, $\langle\sigma\rangle = \sigma_0 \neq 0$, the masses of the N and N* are split

$$m_\pm = \pm(g_1 - g_2)\sigma_0 + \sqrt{(g_1 + g_2)^2 \sigma_0^2 + m_0^2}$$

"Know" g_1; *not* g_2 or m_0. In vacuum, fit masses to N(939) & N*(1535)

Parity Doubled Urca

After a few days, temperatures for neutron stars are *really* low, ~ 1 MeV.

Usual Urca: $n \to p + e + \overline{\nu}_e$ Can be hard to do, $k^F{}_p + k^F{}_e \geq k^F{}_n$, k^F = Fermi mom.

Urca emissivity: $$Q^{\rm Urca} \sim G_F^2 \, (1 + 3 \, g_A^2) \, T^6$$

But often Urca can't go.

Modified Urca, $n + N \to p + N + e + \overline{\nu}_e$ $$Q^{\rm mUrca} \sim G_F^2 \, g_A^2 \, T^8$$

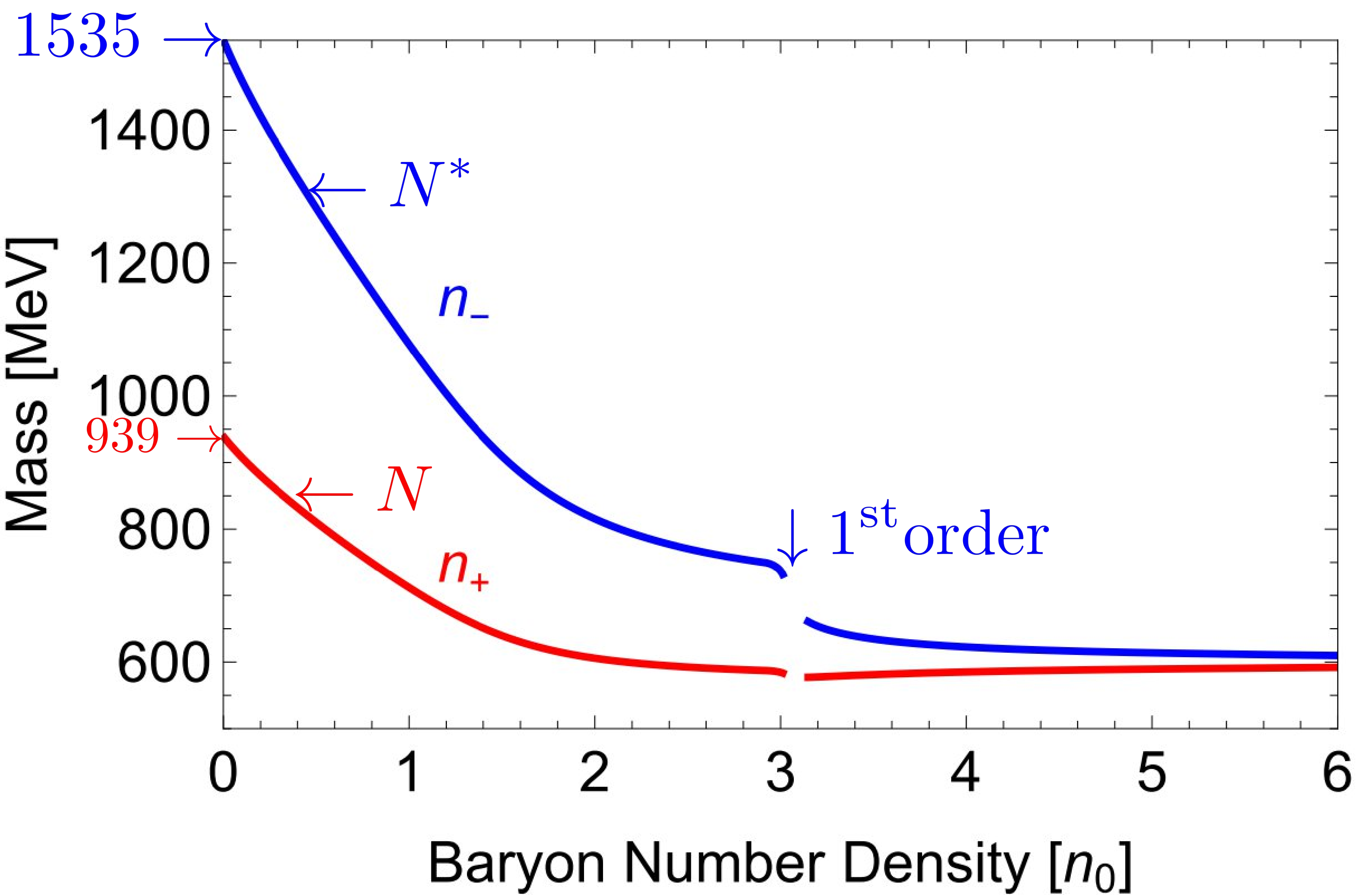
Parity doubled nucleons: PD'd Urca, $n* \to p + e + \overline{\nu}_e$

opens up in the *broken* phase! $$Q^{\rm PD'd\,Urca} \sim G_F^2 \, (1 + 3 \, \widetilde{g}_A^2) \, T^6$$

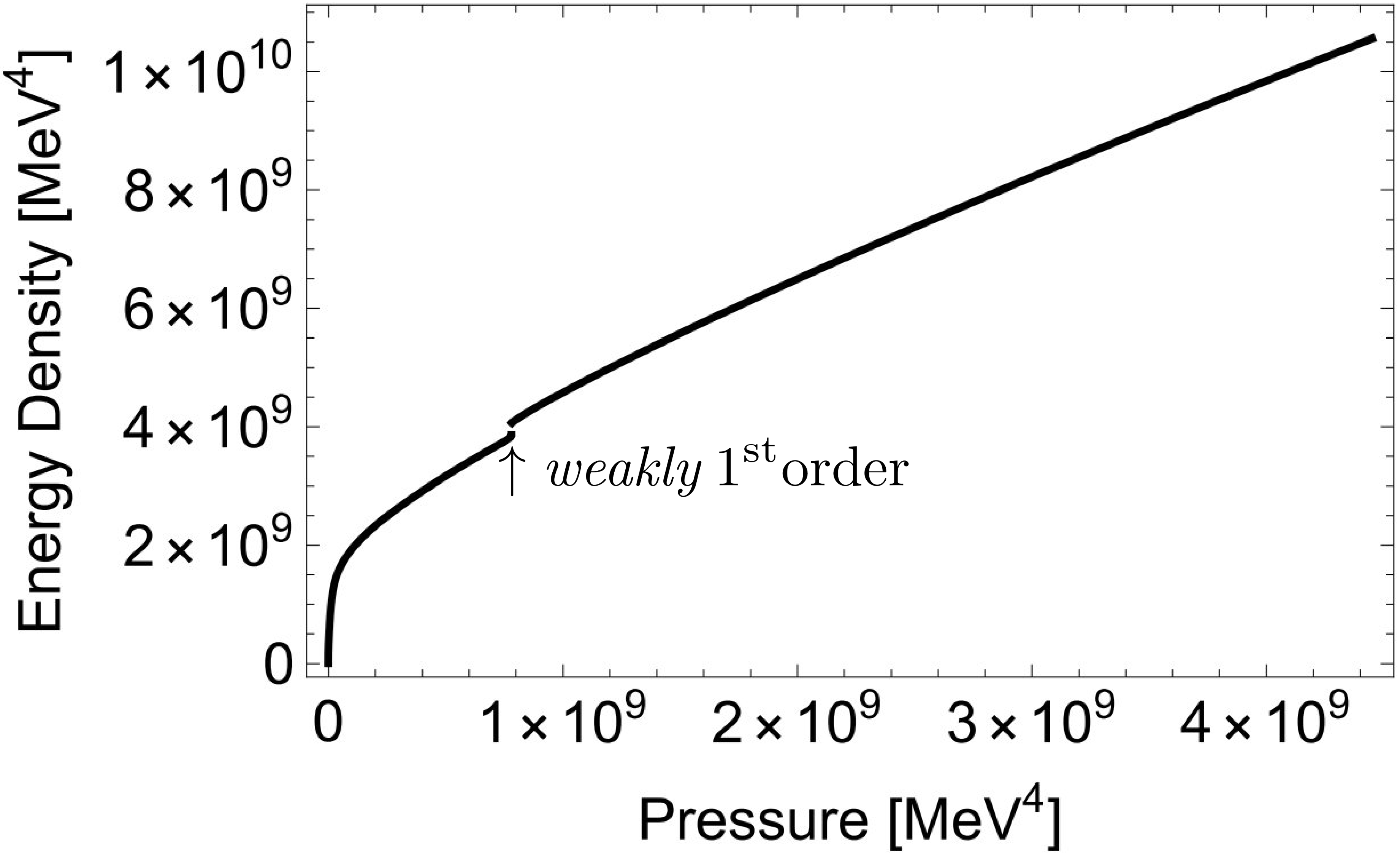
Find only in a *narrow* region of density for *heavy* neutron stars.

Sensitive to the value of g_A for PD'd nucleons $\widetilde{g}_A$;
Kummer, Leupold & von Smekal, 2512.03894

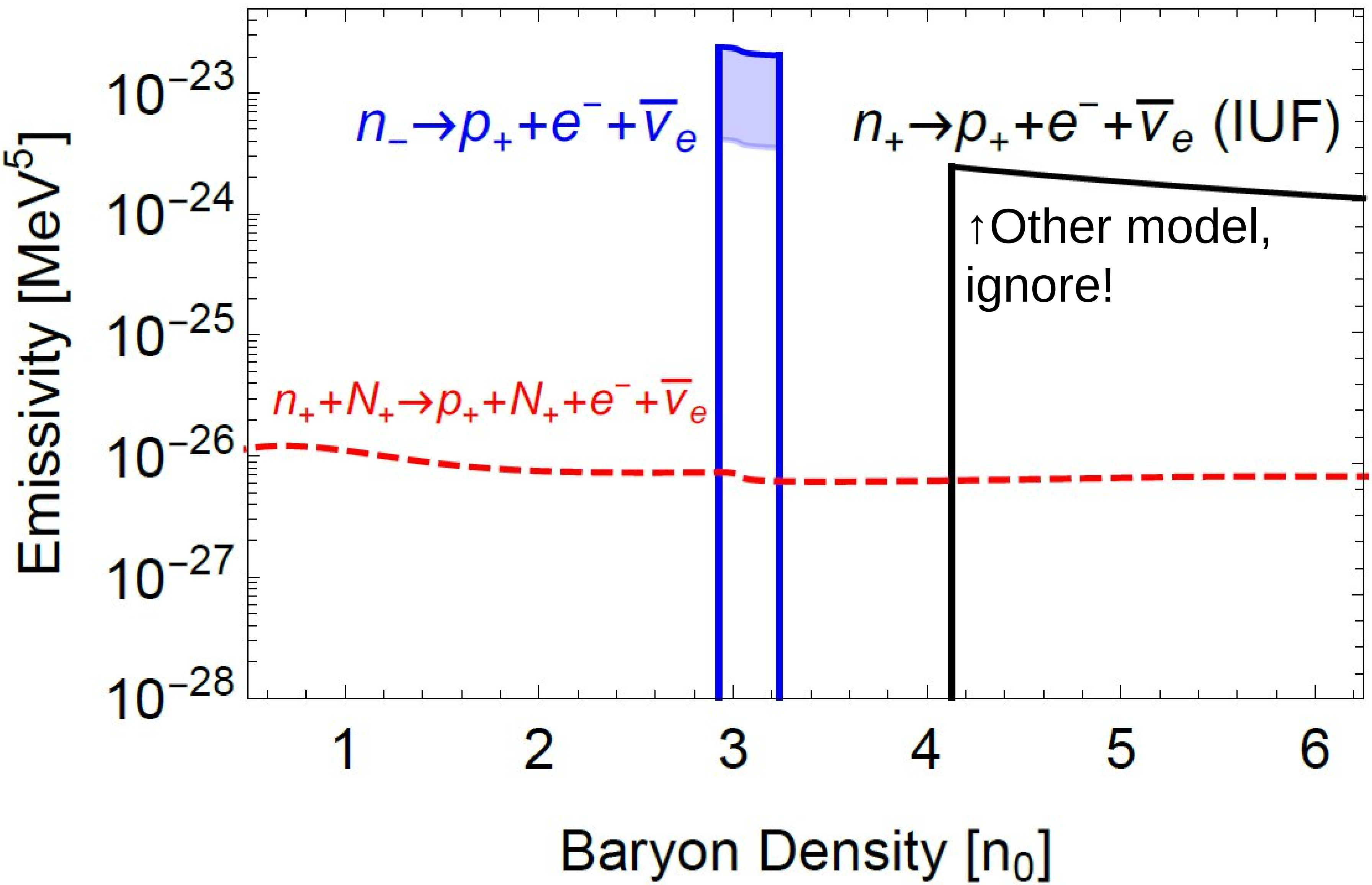
N* and N masses vs density



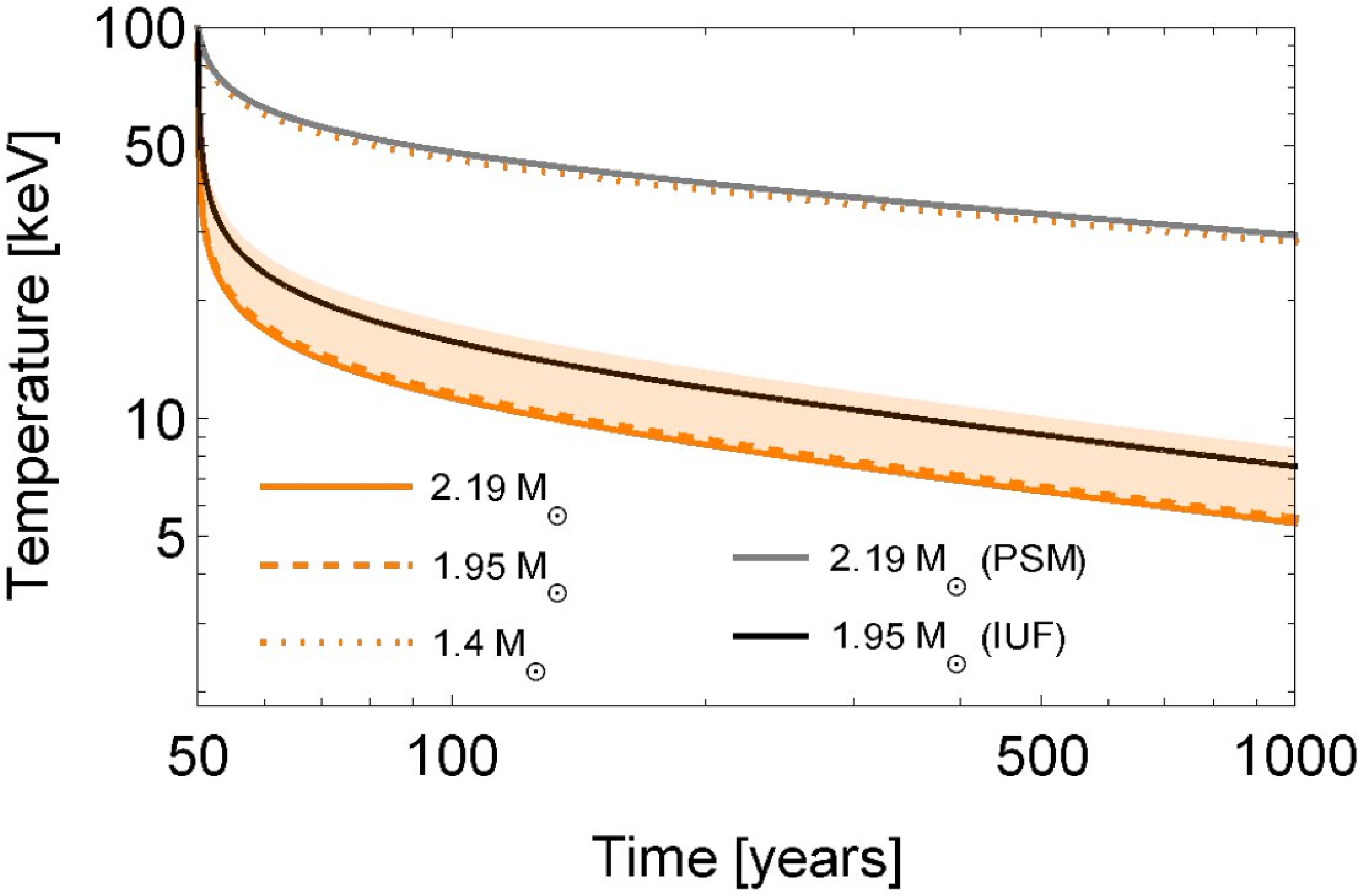
Energy vs pressure: *weakly* 1st order



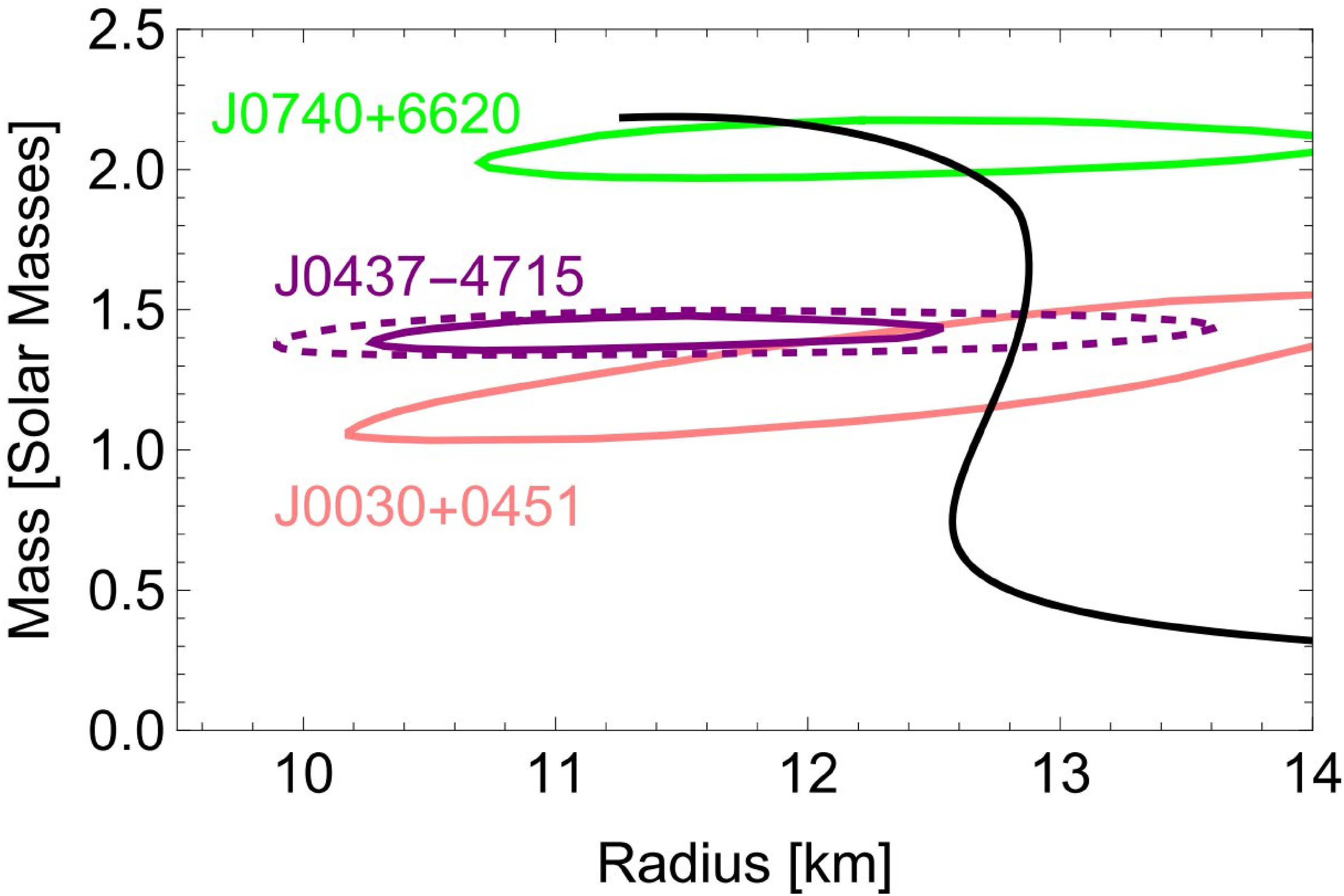
Emissivity for PD'd Urca vs mUrca: "Goldilocks regime"



Temperature vs time: heavy stars cool *much* faster, PD'd Ur



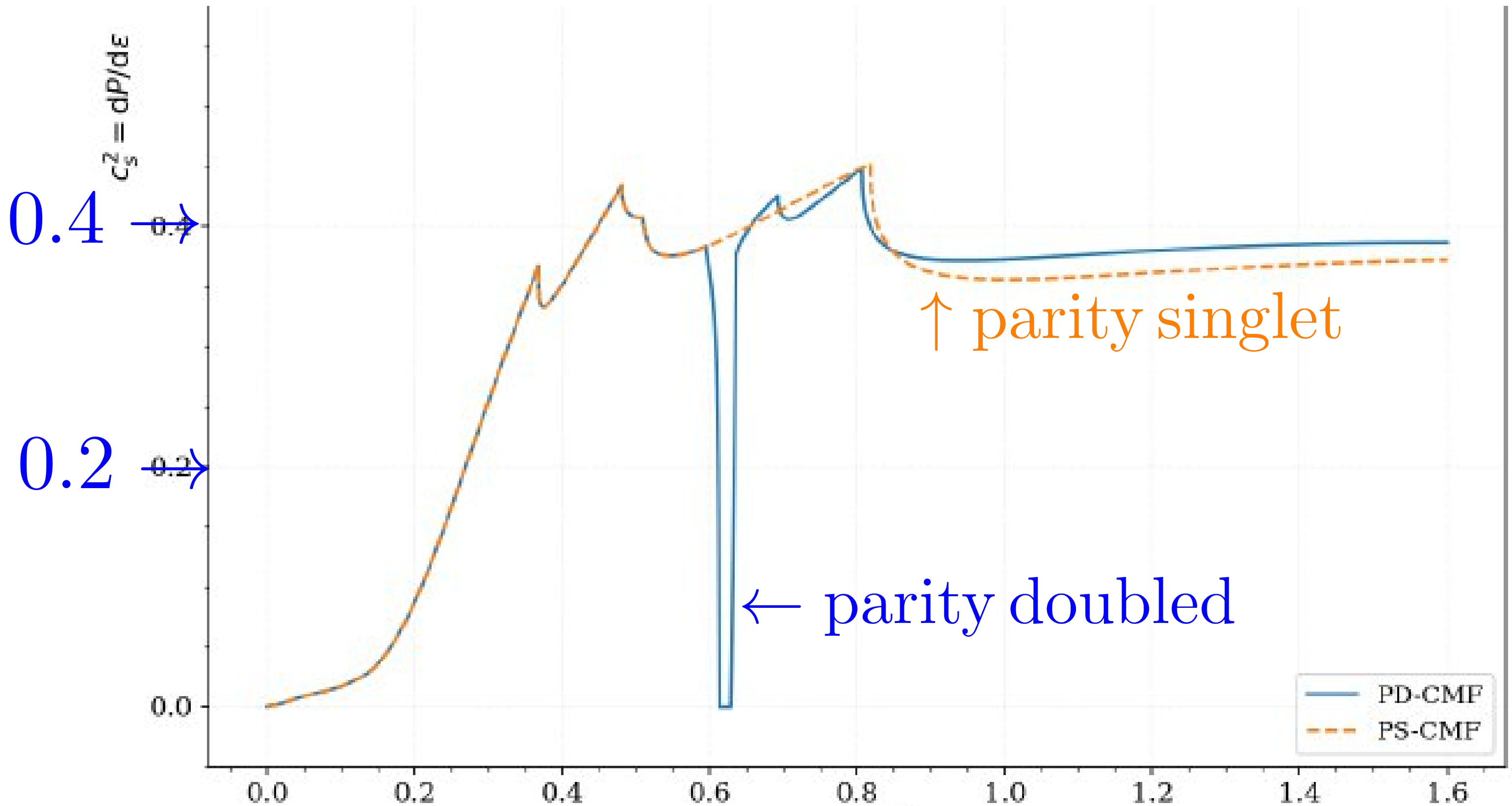
BP: Mass vs radius



Brodie, Negreiros + … (BNSDP): speed of sound

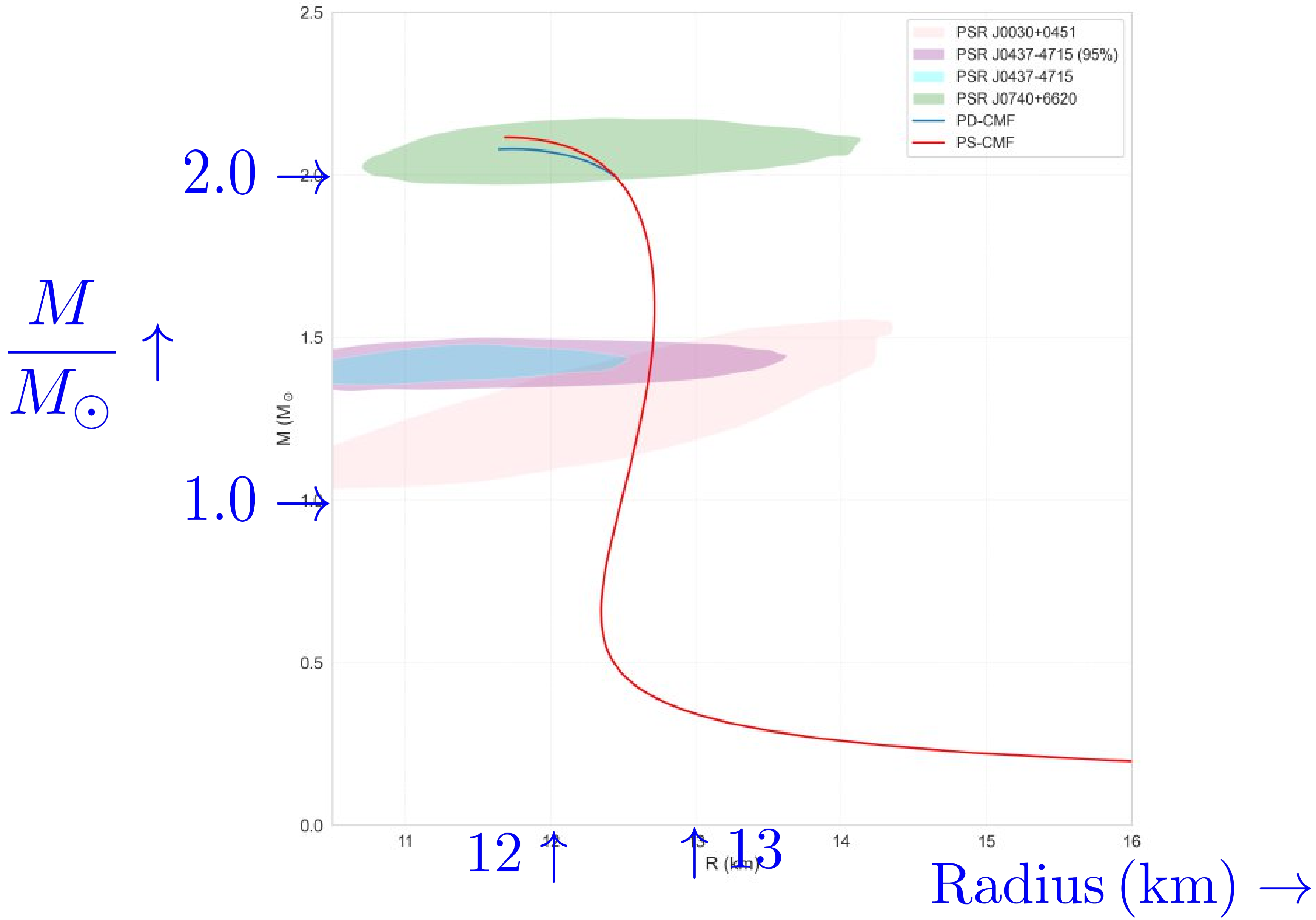
Include full decuplet of strange baryons
Thermal evolution with better thermal conductivity
Include possibility of quark core
Parity doubled (PD) has first order transition where N*'s form Fermi sea
Broad "spike" in the speed of sound. $c_s^2 \rightarrow 1/3$ as $n_B \rightarrow \infty$

$$c_s^2 = \frac{\partial p}{\partial \epsilon}$$

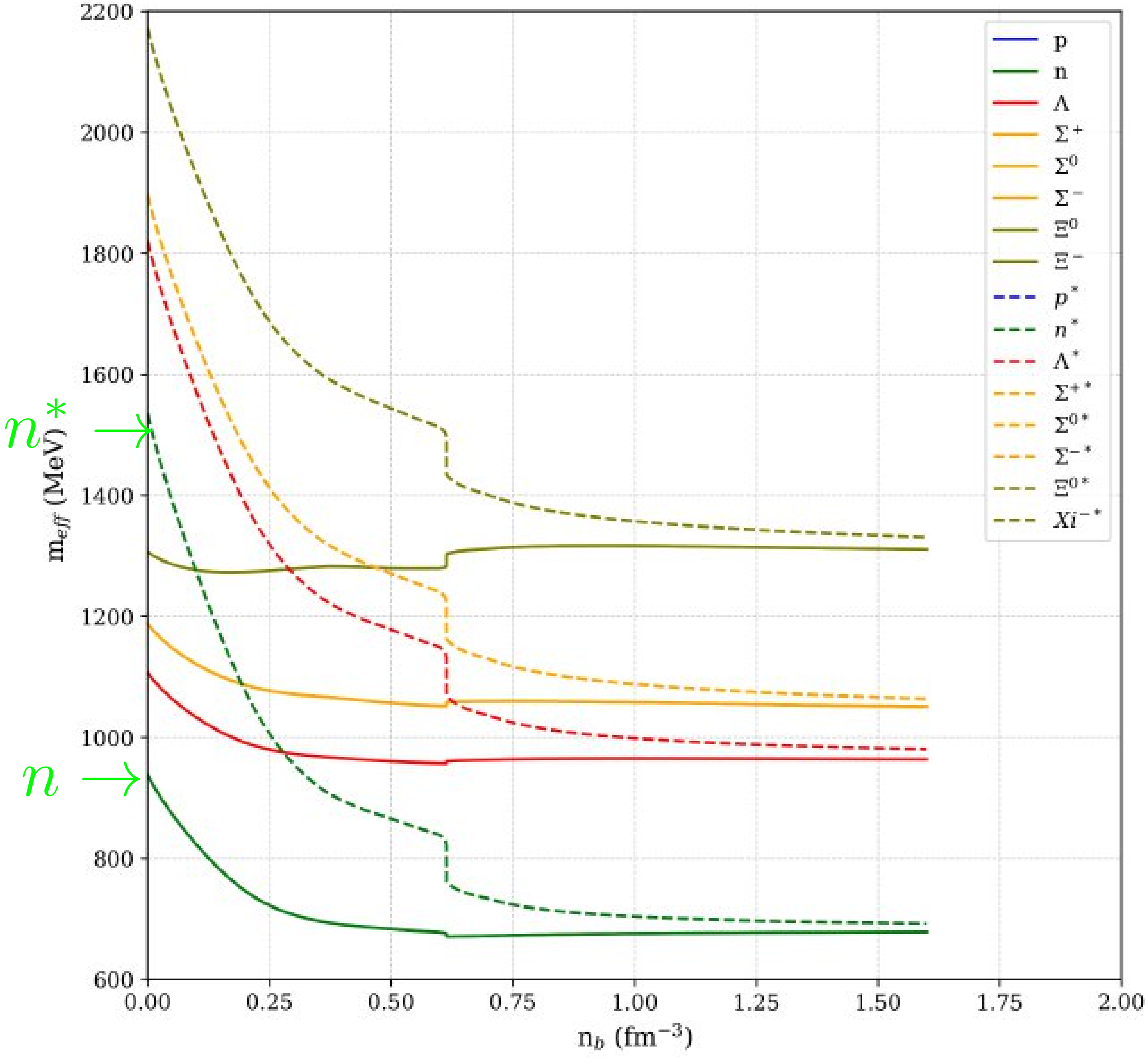


$n_B^{sat} = 0.16 \uparrow$ $\quad n_B(\text{fm})^{-3} \rightarrow$

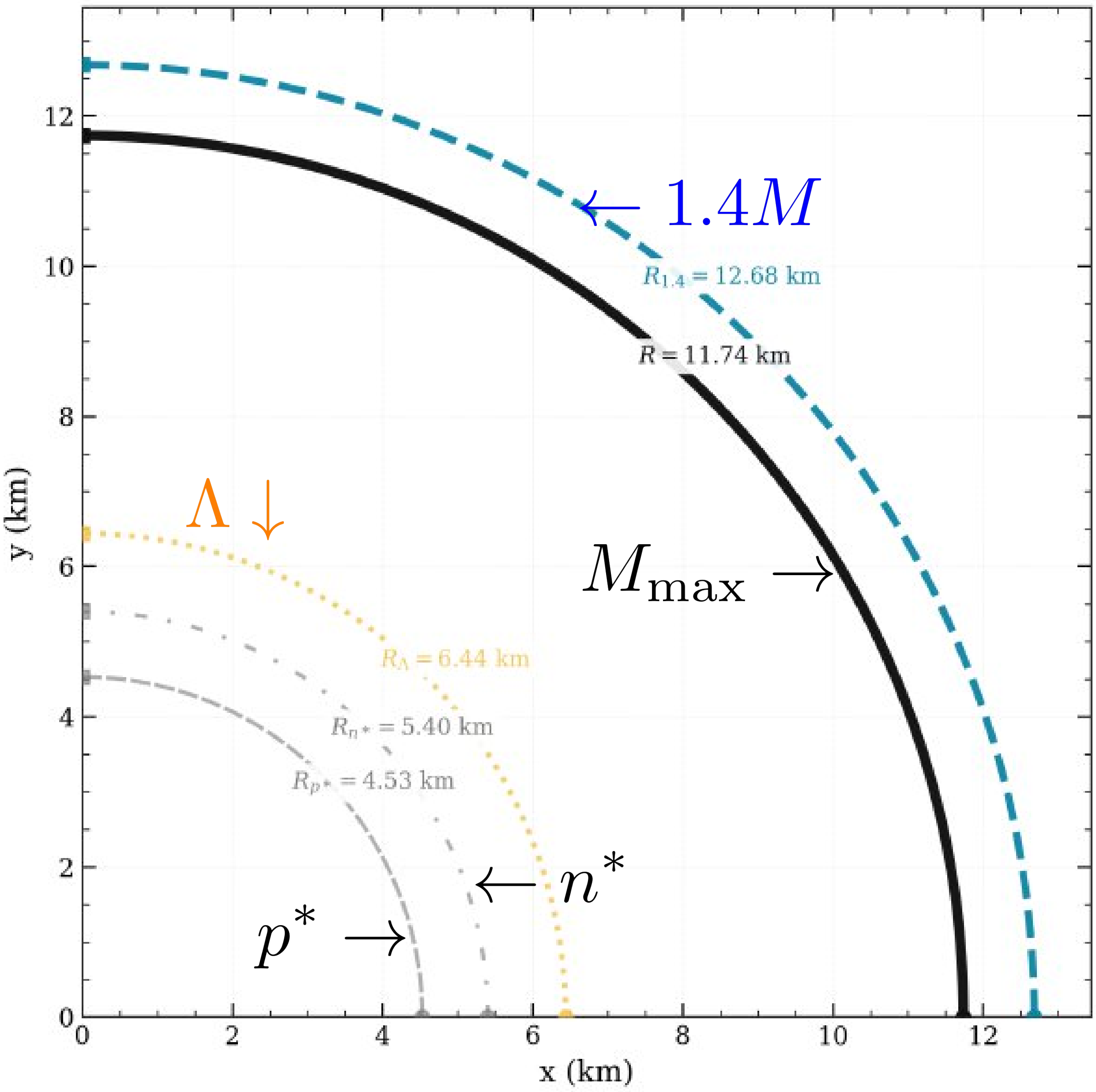
BNSDP: mass vs radius



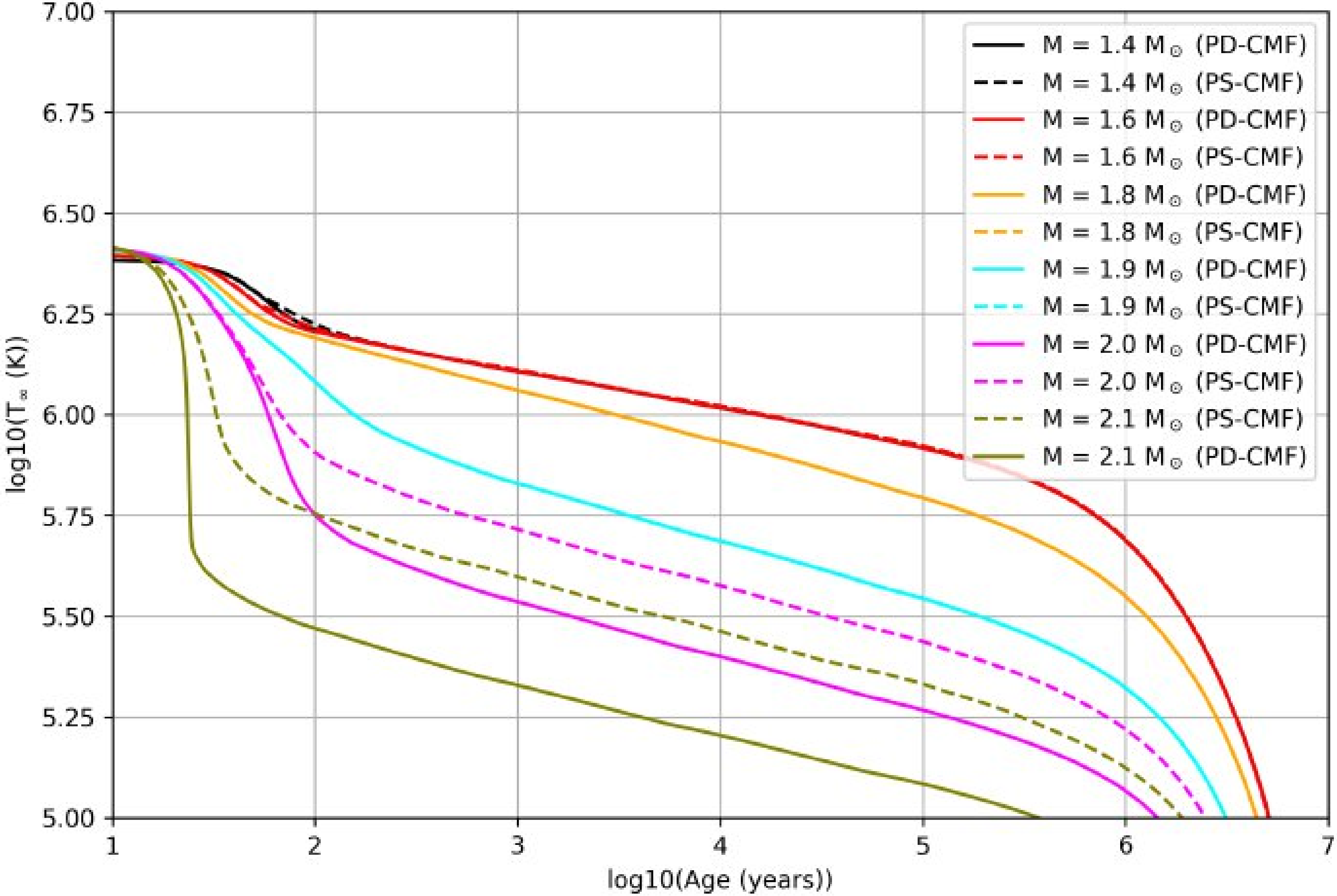
BNSDP: masses vs density



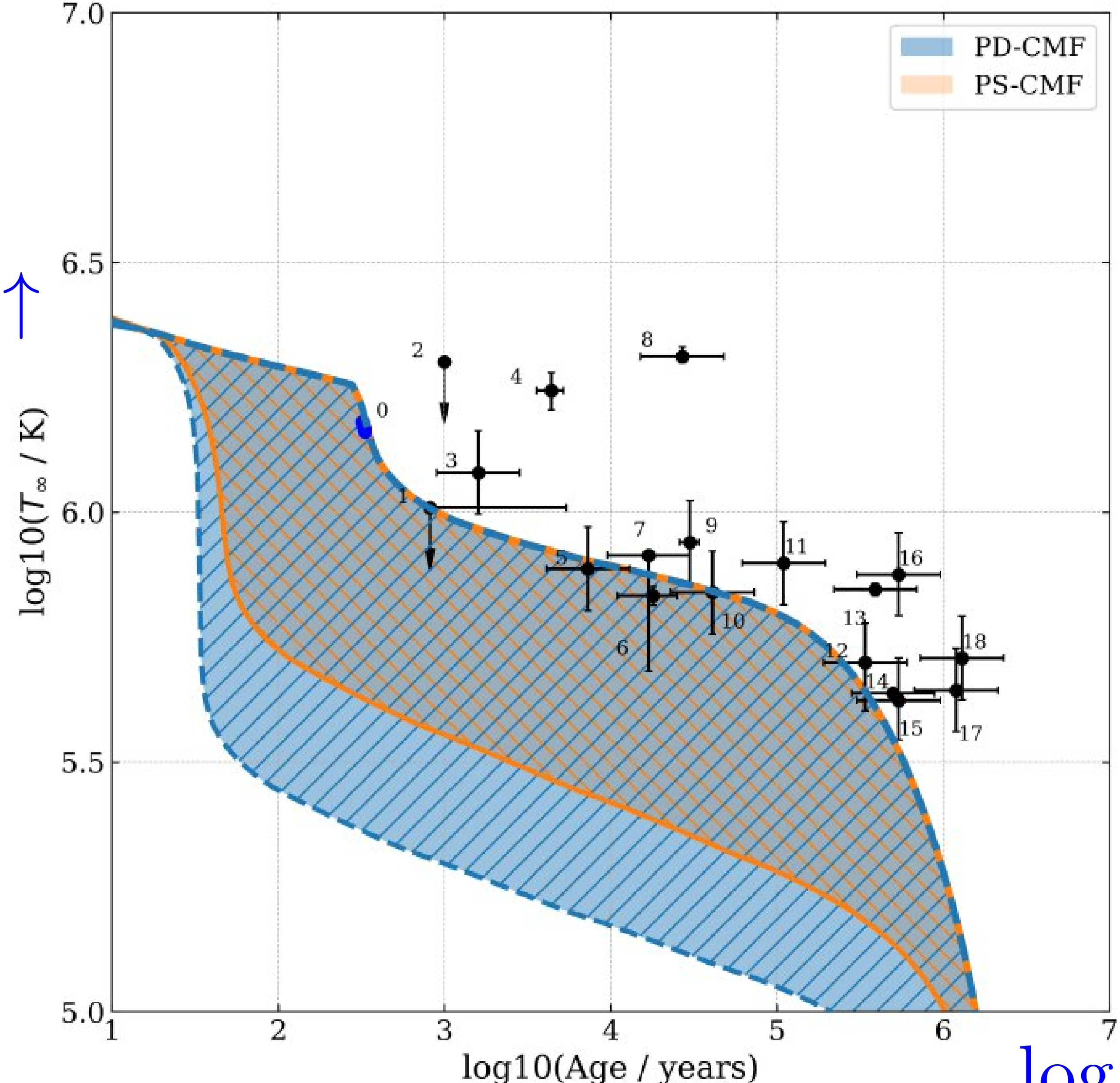
BNSDP: radii of different species



BNSDP: parity singlet (PS) vs doublet (PD) cooling



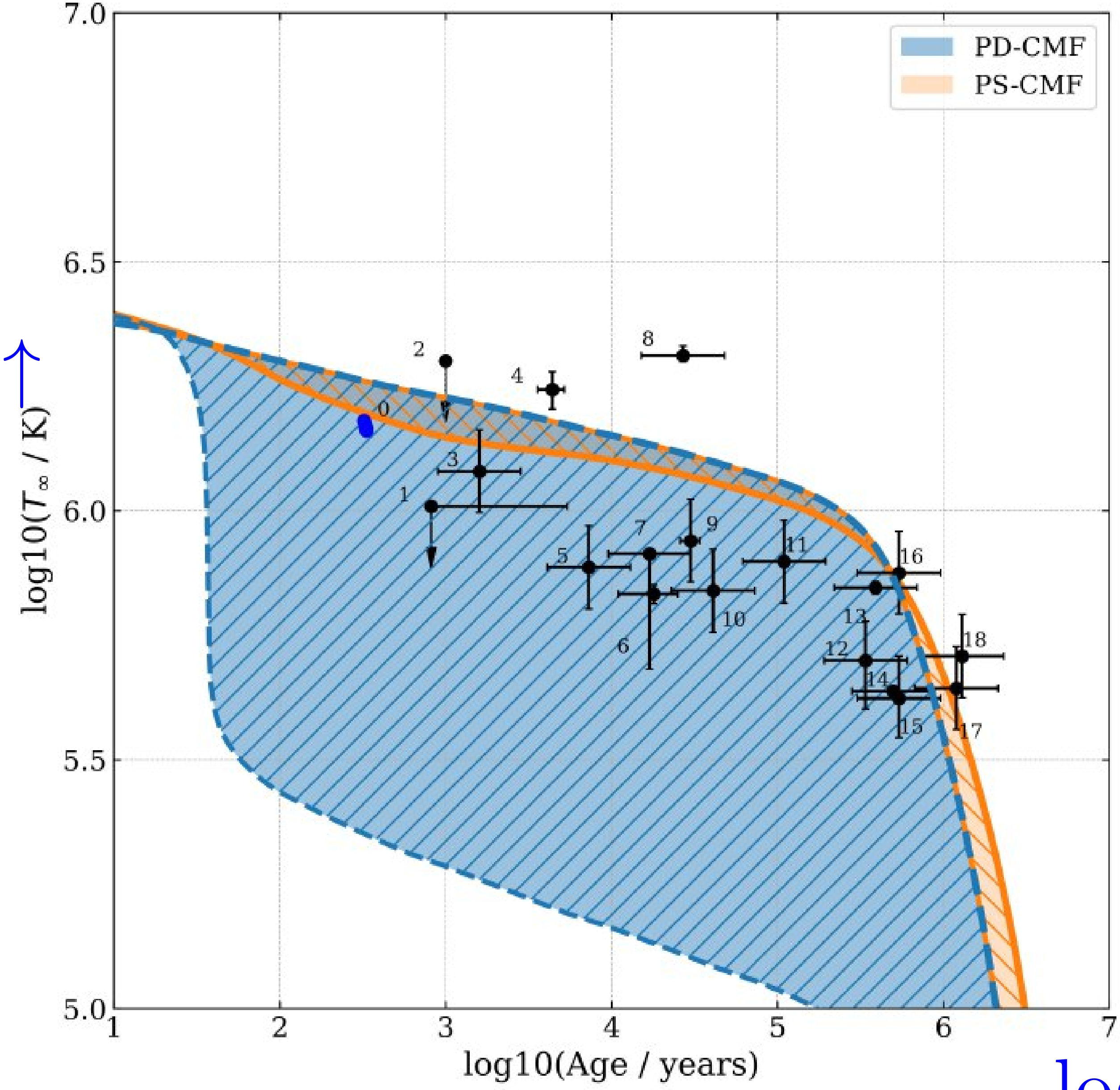
BNSDP: Temperature vs Age, PS & PD



$\log(T) \uparrow$

$\log(\text{age}) \rightarrow$

BNSDP: Temperature vs Age, PS & PD, *with* quark cooling



$\log(T) \uparrow$

$\log(\text{age}) \rightarrow$