

Finite-Range Effects and Coulomb Interactions in Femtosopic Correlation Functions

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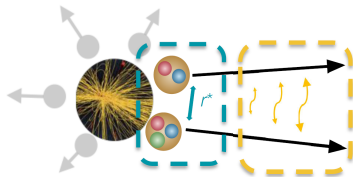


- ❶ Femtoscopic correlation function
- ❷ Amplitude and wave function
- ❸ Coulomb interaction in momentum space
- ❹ Finite-Range effects

The correlation function can be expressed in the Koonin-Pratt formalism:

$$C(\vec{k}) = \mathcal{N} \frac{N_{SE}(\vec{k})}{N_{ME}(\vec{k})} = \int d^3\vec{r} S(\vec{r}) |\psi(\vec{k}, \vec{r})|^2$$

- $S(\vec{r})$: probability distribution of the relative separation of the particles.
- $\psi(\vec{k}, \vec{r})$: relative wave function, containing the final-state interaction



We consider throughout this work a Gaussian source profile:

$$S(r) = \frac{1}{(4\pi R^2)^{3/2}} \exp\left(-\frac{r^2}{4R^2}\right)$$

The correlation function can be expressed in the Koonin-Pratt formalism:

$$C(\vec{k}) = \int d^3\vec{r} S(\vec{r}) |\psi(\vec{k}, \vec{r})|^2$$

For an S -wave interaction,

$$\psi(\vec{k}, \vec{r}) = \begin{cases} e^{i\vec{k}\vec{r}} - j_0(kr) & \text{Non-interacting WF (or Coulomb) w/o S-wave} \\ \phi_C(\vec{k}, \vec{r}) - \phi_C^{\ell=0}(k, r) & \text{Non-interacting WF (or Coulomb) w/o S-wave} \end{cases} + \begin{cases} \psi_S^{\ell=0}(k, r) & \text{Interacting S-wave WF (No Coulomb)} \\ \psi_{SC}^{\ell=0}(k, r) & \text{Interacting S-wave WF (Coulomb)} \end{cases}$$

The correlation function can be written as

$$C(k) = \begin{cases} 1 + 4\pi \int_0^\infty r^2 dr S(r) [|\psi_S^{\ell=0}(k, r)|^2 - j_0(kr)^2] & \text{(No Coulomb)} \\ C_C(k) + 4\pi \int_0^\infty r^2 dr S(r) [|\psi_{SC}^{\ell=0}(k, r)|^2 - |\phi_C^{\ell=0}(k, r)|^2] & \text{(Coulomb)} \end{cases}$$

We need the strong (in the presence of Coulomb) interacting wave function

Why $\bar{K}\Omega$?

- **High strangeness:** $S = -4$
- Scarcely explored meson–baryon interaction
- Single-channel system
- **Repulsive** interaction and clean access to **Coulomb** effects

Two charge sectors: $\bar{K}^0\Omega^-$ ($Q = -1$) , $K^-\Omega^-$ ($Q = -2$)

Chiral interaction [S. Sarkar, E. Oset, and M. J. Vicente Vacas, Nucl. Phys. A 750 294 (2005)]

The lowest-order chiral Lagrangian for Goldstone bosons and decuplet baryons is

$$\mathcal{L} = -i\bar{T}_{abc}^{\mu}\gamma^{\nu}\mathcal{D}_{\nu}T_{\mu}^{abc}.$$

In the non-relativistic limit, the S -wave interaction reduces to the contact term

$$\mathcal{V}_{\bar{K}\Omega}(p', p) = \frac{3}{4f^2} \left(2m_K + \frac{p^2 + p'^2}{2m_K} \right)$$

Unitarity is achieved by solving the off-shell Lippmann-Schwinger equation

$$T(p', p) = V(p', p) + \frac{\mu}{\pi^2} \int_0^\infty dq \frac{q^2 V(p', q) T(q, p)}{k^2 - q^2 + i\epsilon}$$

with $V(p', p) = V_S(p', p) + V_C(p', p)$

Strong interaction (UV regulator):

$$V_S(p', p) = \frac{\mathcal{V}_{\bar{K}\Omega}(p', p)}{2m_K} \exp\left[-\frac{p^2 - k^2}{\Lambda^2}\right] \exp\left[-\frac{p'^2 - k^2}{\Lambda^2}\right] \quad \Lambda = 800 \text{ MeV}$$

$$\mathcal{V}_{\bar{K}\Omega}(p', p) = \frac{3}{4f^2} \left(2m_K + \frac{p^2 + p'^2}{2m_K} \right)$$

The exact S -wave wave function can be obtained from the half-off-shell T -matrix

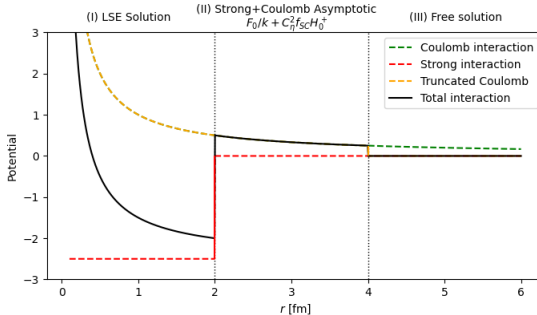
$$[\psi^{\ell=0}(k, r)]^* = j_0(kr) + \frac{\mu}{\pi^2} \int_0^\infty dq \frac{q^2 j_0(qr) T(k, q)}{k^2 - q^2 + i\epsilon}$$

$T(k, q)$: half-off-shell amplitude k on shell, q off shell

How can we implement the **Coulomb** interaction?

The Coulomb interaction is long-ranged and cannot be included directly in the momentum-space Lippmann–Schwinger equation, $V(p', p) = V_S(p', p) + V_C(p', p)$

Vincent-Phatak method: $V_C(r) = \frac{\alpha}{r} \theta(R - r)$ M. Vincent and S. C. Phatak, Phys. Rev. C 10, 391 (1974)



Problem: truncated Coulomb is oscillatory in momentum space

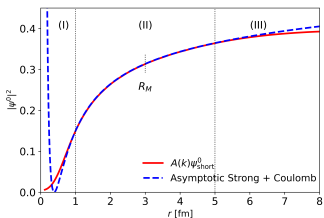
We follow a modified Vincent–Phatak construction with a Woods–Saxon regulator:

$$V_C(r) = \frac{Z_1 Z_2 \alpha}{r} g_{\text{WS}}(r) \quad g_{\text{WS}}(r) = \frac{1 + e^{-R_{\text{WS}}/b}}{1 + e^{(r-R_{\text{WS}})/b}}$$

Full exact wave function:

$$[\psi^{\ell=0}(k, r)]^* = \begin{cases} A(k) [\psi_{\text{short}}^{\ell=0}(k, r)]^*, & r \leq R_M, \\ e^{i\sigma_0} \left[\frac{F_0(\eta, kr)}{kr} + f_{SC}(k) C_\eta^2 \frac{H_0^+(\eta, kr)}{r} \right], & r > R_M \end{cases}$$

F_0 : regular Coulomb wave $H_0^+ = G_0 + iF_0$: outgoing Coulomb wave $f_{SC}(k)$: Coulomb-modified strong amplitude



- $\psi_{\text{short}}(k, r)$: solution of the LSE with the regularized strong+Coulomb potential
- Normalization $A(k)$ and amplitude $f_{SC}(k)$ fixed by matching the wave function and its derivative at $r = R_M$

With this method:

- Full wave function, at short and long distances
- On-shell strong scattering amplitude modified by Coulomb

Full exact wave function:

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Solution:

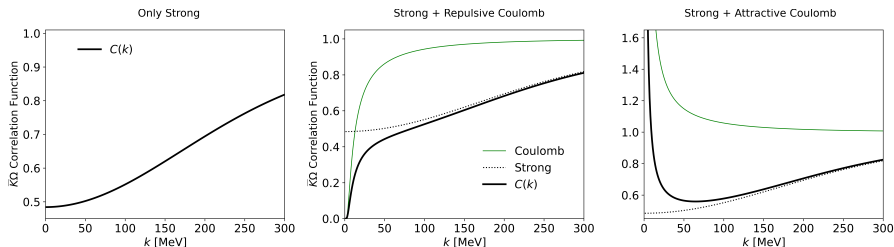
$$A(k) = \frac{e^{i\sigma_0}}{[H_0^+(\eta, kr), u_{\text{short}}^0(k, r)]_{R_M}^*},$$

$$f_{\text{SC}}^{-1}(k) = -C_\eta^2 k \left(i + \frac{[G_0(\eta, kr), u_{\text{short}}^0(k, r)]_{R_M}^*}{[F_0(\eta, kr), u_{\text{short}}^0(k, r)]_{R_M}^*} \right),$$

where

$$[\mathcal{F}(r), \mathcal{G}(r)]_{R_M} = \left(\mathcal{F}(r) \frac{d\mathcal{G}(r)}{dr} - \mathcal{G}(r) \frac{d\mathcal{F}(r)}{dr} \right)_{r=R_M},$$

- Source size: $R = 0.5$ fm
- Strong interaction only ($Z_1 Z_2 = 0$) and Strong+Coulomb interactions ($Z_1 Z_2 = \pm 1$)



- The modified Vincent-Phatak implementation of Coulomb reproduces the expected behavior
- Repulsive Coulomb: $C(k) \xrightarrow[k \rightarrow 0]{} 0$
- Attractive Coulomb: $C(k) \xrightarrow[k \rightarrow 0]{} \infty$
- At $k \rightarrow 0$, Coulomb dominates
- At $k \rightarrow \infty$, Strong dominates

Recall the correlation function:

$$C(k) = \int d^3r S(r) |\psi(k, r)|^2 = \begin{cases} 1 + 4\pi \int_0^\infty r^2 dr S(r) [|\psi^{\ell=0}(k, r)|^2 - j_0(kr)^2] & \text{(No Coulomb)} \\ C_C(k) + 4\pi \int_0^\infty r^2 dr S(r) [|\psi^{\ell=0}(k, r)|^2 - |\phi_C^{\ell=0}(k, r)|^2] & \text{(Coulomb)} \end{cases}$$

The wave function requires the half-off-shell scattering amplitude:

$$[\psi^{\ell=0}(k, r)]^* = j_0(kr) + \frac{\mu}{\pi^2} \int_0^\infty dq \frac{q^2 j_0(qr) T(k, q)}{k^2 - q^2 + i\epsilon} \quad \text{q off-shell}$$

Generally we have no access to the off-shell amplitude: it is only constrained on-shell

$$T(k, k) \longleftrightarrow f(k), \delta(k), \dots$$

Two different off-shell extensions can reproduce the same on-shell physics while modifying the short-distance wave function, and therefore the femtoscopic correlation function.

What can we calculate from the on-shell amplitude?

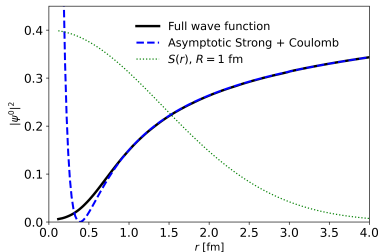
On-shell physics determine the asymptotic strong+Coulomb wave function:

$$[\psi_{\text{asy}}^{\ell=0}(k, r)]^* = \begin{cases} \frac{\sin(kr)}{kr} + f(k) \frac{e^{ikr}}{r} & \text{(No Coulomb)} \\ e^{i\sigma_0} \left[\frac{F_0(\eta, kr)}{kr} + f_{SC}(k) C_\eta^2 \frac{H_0^+(\eta, kr)}{r} \right] & \text{(Coulomb)} \end{cases}$$

Using the asymptotic wave function in the Koonin–Pratt formula defines the Lednický–Lyuboshitz (LL) approximation:

$$C_{LL}(k) = \begin{cases} 1 + 4\pi \int_0^\infty r^2 dr S(r) \left[|\psi_{\text{asy}}^{\ell=0}(k, r)|^2 - j_0(kr)^2 \right] & \text{(No Coulomb)} \\ C_C(k) + 4\pi \int_0^\infty r^2 dr S(r) \left[|\psi_{\text{asy}}^{\ell=0}(k, r)|^2 - |\phi_C^{\ell=0}(k, r)|^2 \right] & \text{(Coulomb)} \end{cases}$$

- The exact wave function is not equal to its asymptotic form in the region probed by the source.
- The short-distance behavior of the wave function is dependent on the off-shell model adopted.



FRP (Finite-range potential) correction:

$$C(k) = C_{LL}(k) + \delta C_{FRP}(k)$$

$$\delta C_{FRP}(k) = \int d^3r S(r) \left[|\psi^{\ell=0}(k, r)|^2 - |\psi_{asy}^{\ell=0}(k, r)|^2 \right].$$

For a:

- **Local** interaction, $V = V(r)$ (closely related to off-shellness)
- **Extensive source**: $S(r) \simeq S(0)$ in the interaction range

$$\delta C_{FRP}(k) \simeq -4\pi S(0) C_\eta^2 |f_{SC}(k)|^2 \frac{d}{dk^2} \left[\text{Re } f_{SC}^{-1} + 2\eta k h^\lambda(\eta) \right]$$

Phenomenological Parameter β

For a:

- **Local** interaction, $V = V(r)$ (closely related to off-shellness)
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We introduce a **phenomenological parameter β** to absorb off-shell and source size effects:

$$\delta C_{\text{FRP}}(k) \simeq -4\pi \beta S(0) C_\eta^2 |f_{\text{SC}}(k)|^2 \frac{d}{dk^2} \left[\text{Re } f_{\text{SC}}^{-1} + 2\eta k h^\lambda(\eta) \right]$$

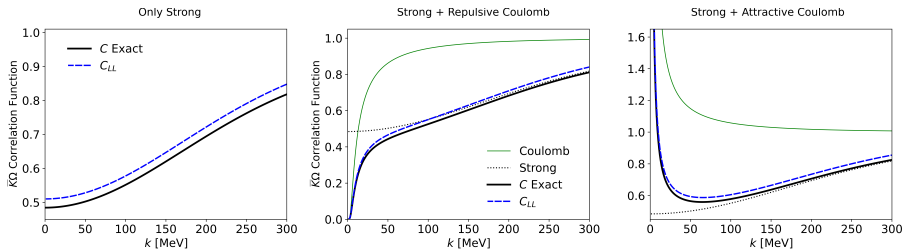
- $\beta \simeq 1$
- $\beta \neq 1$: accounts effectively for source-size and off-shell effects.

Therefore,

$$C(k) \simeq C_c(k) + \int_0^\infty r^2 dr S(r) \left[|\psi_{\text{asy}}^{\ell=0}(k, r)|^2 - |\phi_c^{\ell=0}(k, r)|^2 \right] - 4\pi \beta S(0) C_\eta^2 |f_{\text{SC}}(k)|^2 \frac{d}{dk^2} \left[\text{Re } f_{\text{SC}}^{-1} + 2\eta k h^\lambda(\eta) \right]$$

Dependent on: **On-shell** physics, **Coulomb** parameters, and an additional parameter β

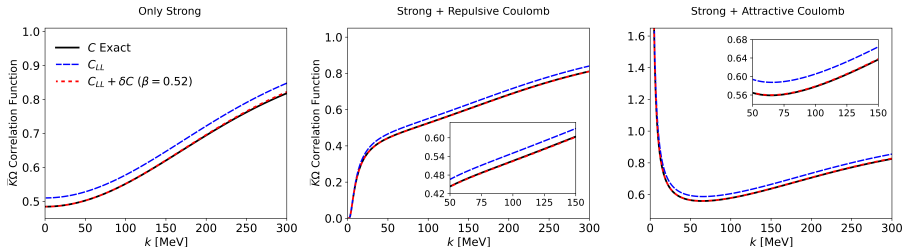
- Source size: $R = 0.5$ fm to enhance short-distance effects



- $C(k) \leftrightarrow$ exact wave function from the momentum-space LSE.
- $C_{LL}(k) \leftrightarrow$ asymptotic wave function using the extracted $f_{SC}(k)$.

The difference $C(k) - C_{LL}(k)$ is due to the short-distance region where $\psi_{\text{exact}} \neq \psi_{\text{asy}}$.

- The FRP correction accounts for the difference between the exact and asymptotic CFs



$$C(k) = C_{LL}(k) + \delta C(k),$$

$$\delta C_{FRP} \simeq -4\pi\beta S(0)C_{\eta}^2|f_{SC}|^2 \frac{d}{dk^2} \left[\text{Re } f_{SC}^{-1} + 2\eta kh^{\lambda} \right].$$

- β effectively absorbs the finite source and short-distance interaction effects.
- For the present interaction and source: $\beta = 0.52$

In a realistic analysis, the on-shell scattering amplitude f_{SC} plus the phenomenological parameter β provide an effective description of the full correlation function

- **Coulomb** effects can be incorporated in momentum space via the WS–VP method
- The correlation function is sensitive to the wave function at the **short distances** covered by the source.
- The short-distance wave function depends on the **half-off-shell** interaction, which is not fixed by on-shell scattering observables.
- The usual asymptotic (LL) approximation contains only the **on-shell** scattering amplitude and can therefore miss finite-range effects.
- The FRP correction, parametrized by β , provides an **effective way to account for the missing short-distance information**.