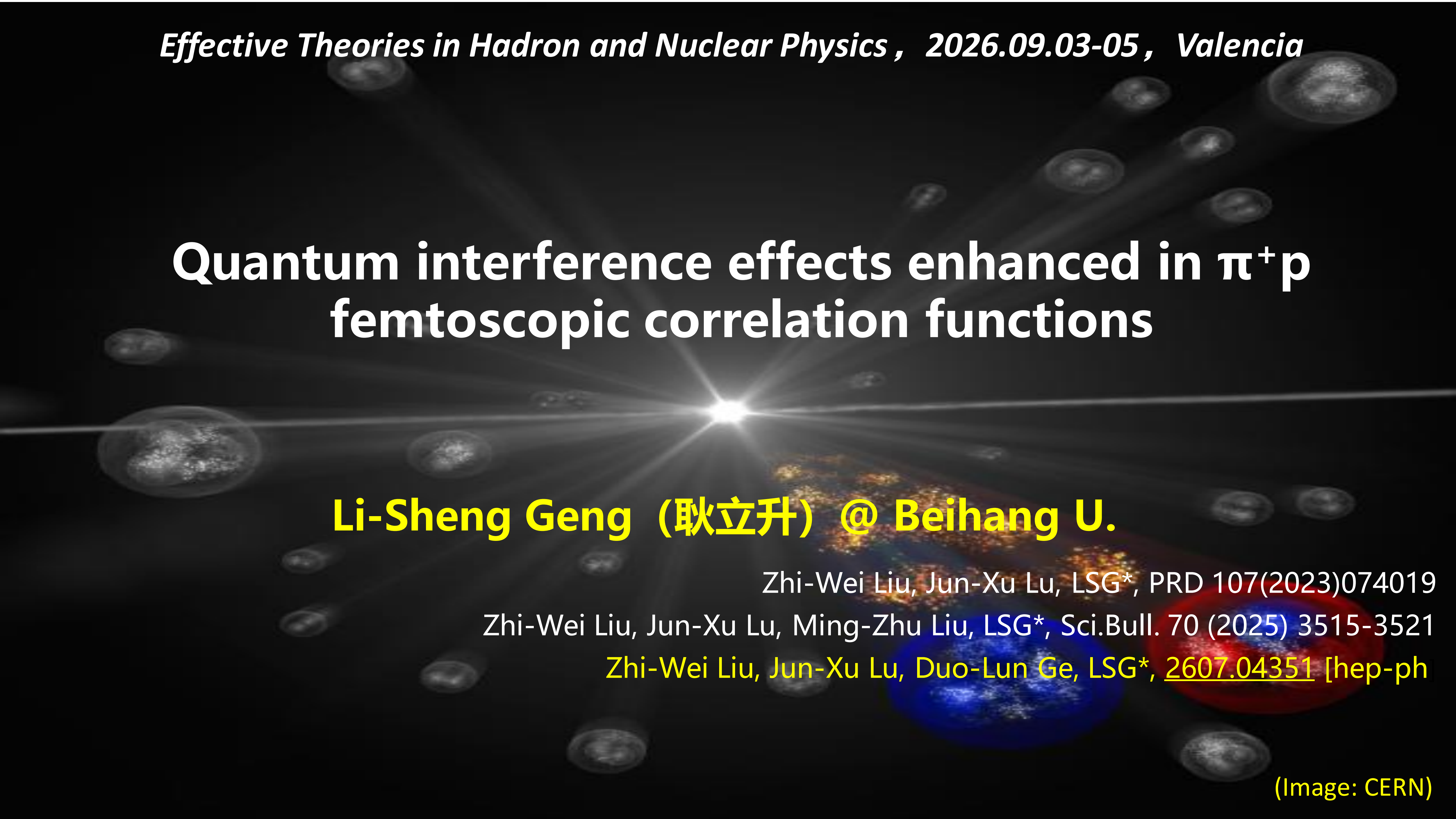
Effective Theories in Hadron and Nuclear Physics， 2026.09.03-05， Valencia



Quantum interference effects enhanced in π^+p femtoscopic correlation functions

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Zhi-Wei Liu, Jun-Xu Lu, LSG*, PRD 107(2023)074019

Zhi-Wei Liu, Jun-Xu Lu, Ming-Zhu Liu, LSG*, Sci.Bull. 70 (2025) 3515-3521

Zhi-Wei Liu, Jun-Xu Lu, Duo-Lun Ge, LSG*, 2607.04351 [hep-ph

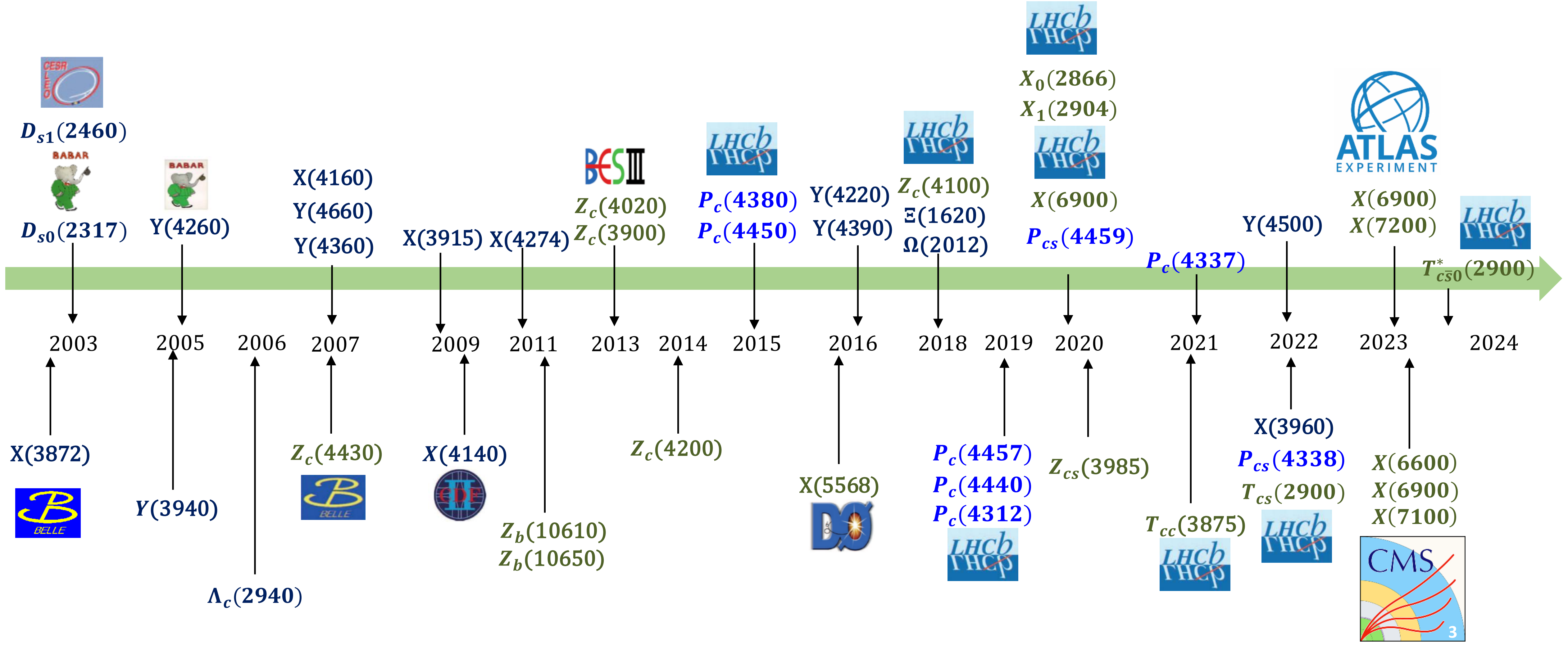
(Image: CERN)

Contents

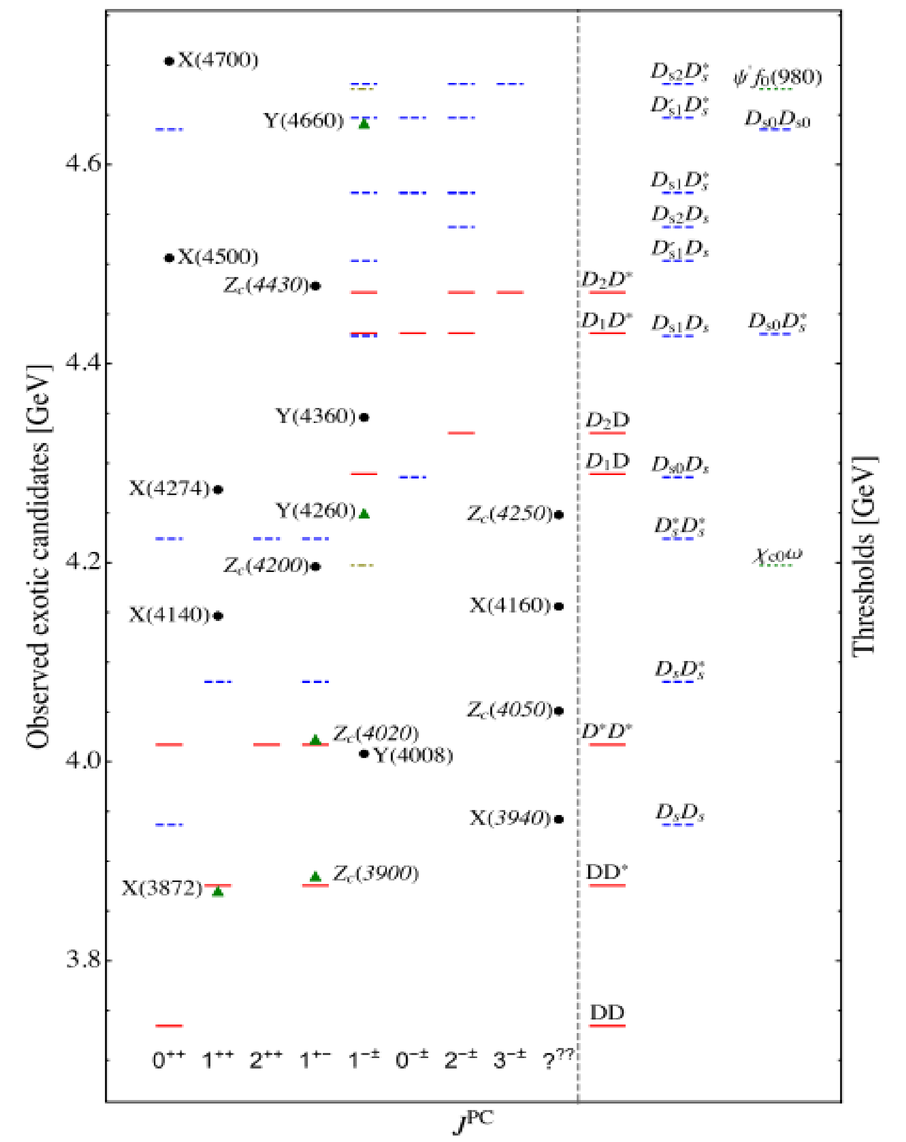
- ☞ Brief introduction: exotic hadrons and femtoscopy
- ☞ Femtoscopic correlation functions (CFs)—general features
- ☞ One recent application: $\pi^+ p$ CF
- ☞ Summary and outlook

Many more so-called exotic hadrons discovered since then

Exotic mesons and baryons Tetraquarks Pentaquarks



Many (if not all) of them close to thresholds—molecules



Feng-Kun Guo, Christoph Hanhart, Ulf-G. Meißner, Qian Wang,Qiang Zhao, Bing-Song Zou. Rev.Mod.Phys. 90 (2018) 015004

Richard F. Lebed, Ryan E. Mitchell,
Eric S. Swanson,
Prog.Part.Nucl.Phys. 93 (2017) 143

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PTEP 2016 (2016) 062C01

Hua-Xing Chen, Wei Chen, Xiang Liu, Shi-Lin Zhu,
Phys. Rept.639 (2016) 1

……

How to verify the molecular picture?

How to verify the molecular picture

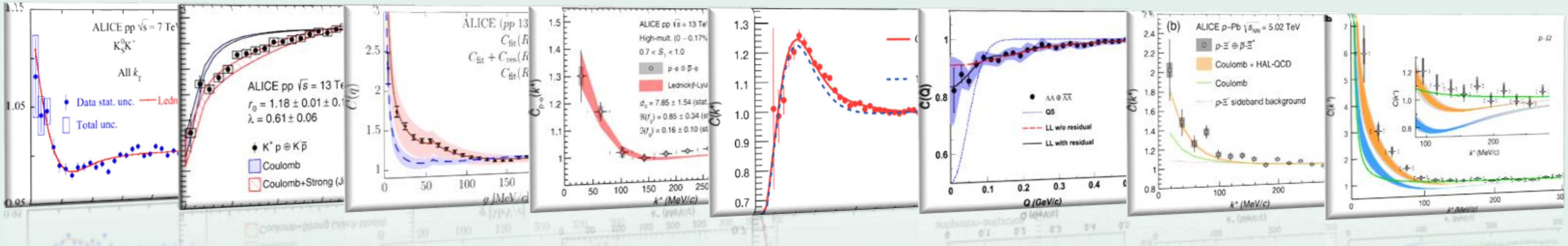
Ming-Zhu Liu, Ya-Wen Pan, Zhi-Wei Liu, Tian-Wei Wu, Jun-Xu Lu, **LSG***, Phys.Rept. 1108 (2025) 1-108

□ Direct measurement of the two-hadron interactions

The Large Hadron Collider

Brookhaven National Laboratory RHIC Relativistic Heavy Ion Collider

$K^0_S K^{\pm}$ $K^{\pm}p$ $K^- p$ ϕp $\bar{p}\bar{p}$ $\Lambda\Lambda$ $\Xi^- p$ $\Omega^- p$



PLB 2019 *PRL 2020* *PRL 2020* *PRL 2021* *Nature 2015* *PRL 2015* *PRL 2019* *Nature 2020*

$K^- d$
$\Sigma^0 p$
$\Lambda\Xi$
$\Xi\Xi$
$\Omega\Omega$
pD
ppp
$pp\Lambda$

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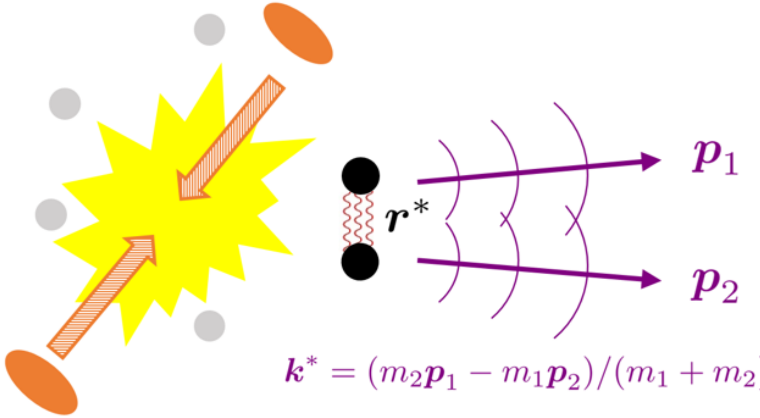
Femtoscopic correlation functions (CFs)

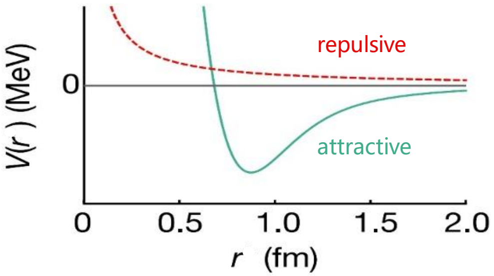
$$C(\boldsymbol{p}_1, \boldsymbol{p}_2) = \frac{\mathrm{P}(\boldsymbol{p}_1, \boldsymbol{p}_2)}{\mathrm{P}(\boldsymbol{p}_1) \cdot \mathrm{P}(\boldsymbol{p}_2)}$$



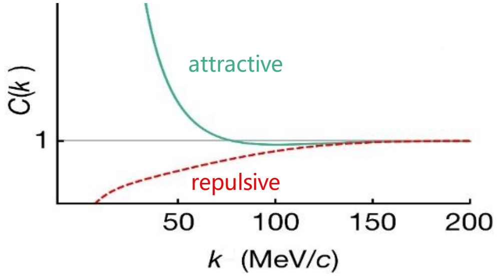
$$\boldsymbol{k}^* = (m_2\boldsymbol{p}_1 - m_1\boldsymbol{p}_2)/(m_1 + m_2)$$

Emission source $S_{12}(\boldsymbol{r}^*)$ Two-particle wavefunction $\psi(\boldsymbol{k}^*, \boldsymbol{r}^*)$

Interacting potential



Correlation function



Schrödinger equation

Two-particle wavefunction $\psi(\boldsymbol{k}, \boldsymbol{r})$

Exp. measurement

mixed-event technique

$$C(k) = \xi(k)\frac{N_{\mathrm{same}}(k)}{N_{\mathrm{mixed}}(k)}$$

N_{same}: the same event distributions

N_{mixed}: the mixed event distributions

ξ: the corrections for experimental effects

Theo. description

Koonin–Pratt formula

$$C(k) = \int S_{12}(\boldsymbol{r})\ |\psi(\boldsymbol{k}, \boldsymbol{r})|^2\ \mathrm{d}\boldsymbol{r}$$

spacial structure

final-state interactions

quantum statistics effects

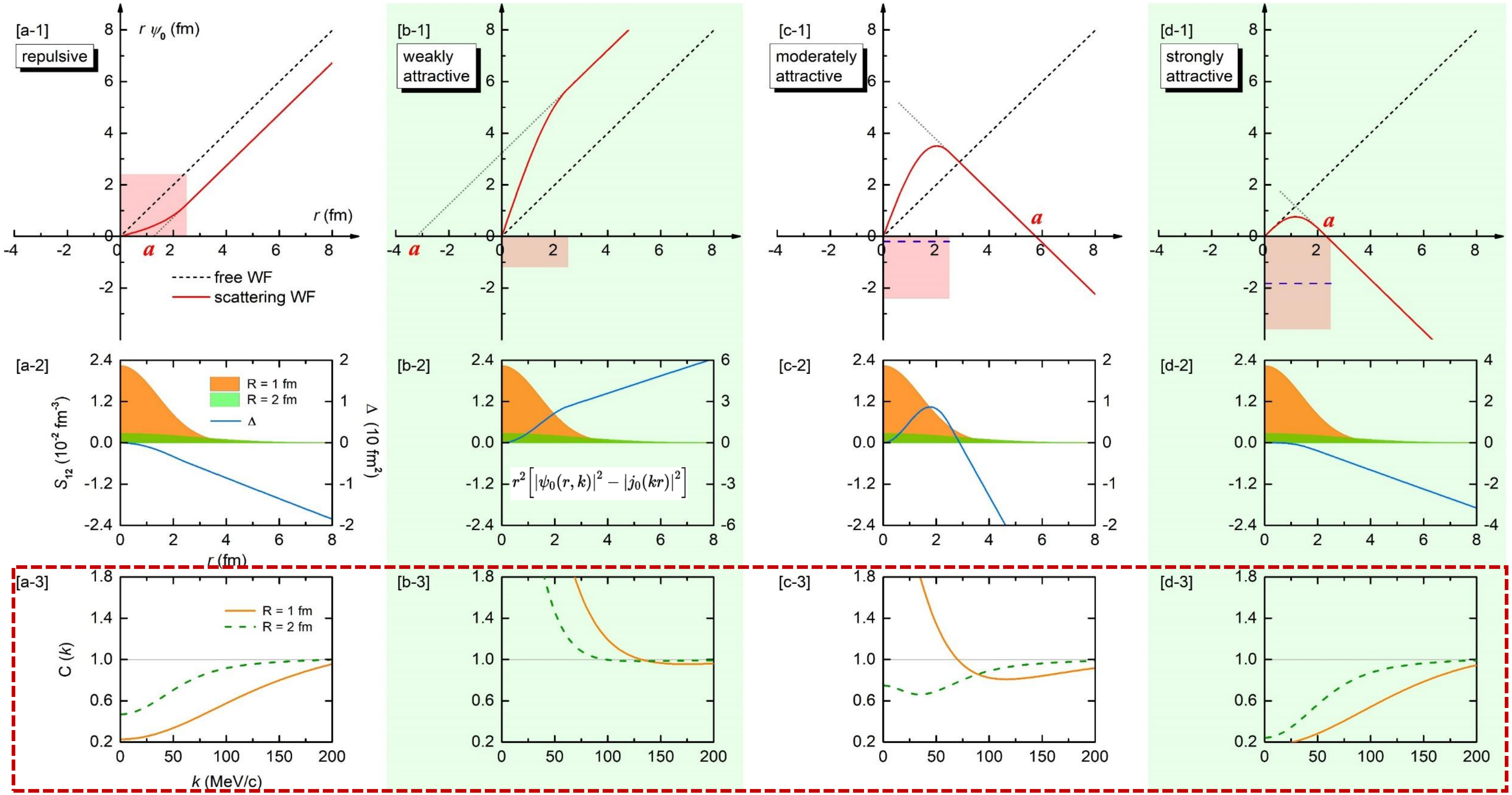
coupled-channel effects

Basic Properties

$C(k)$
- >1 if the interaction is attractive
- =1 if there is no interaction
- <1 if the interaction is repulsive

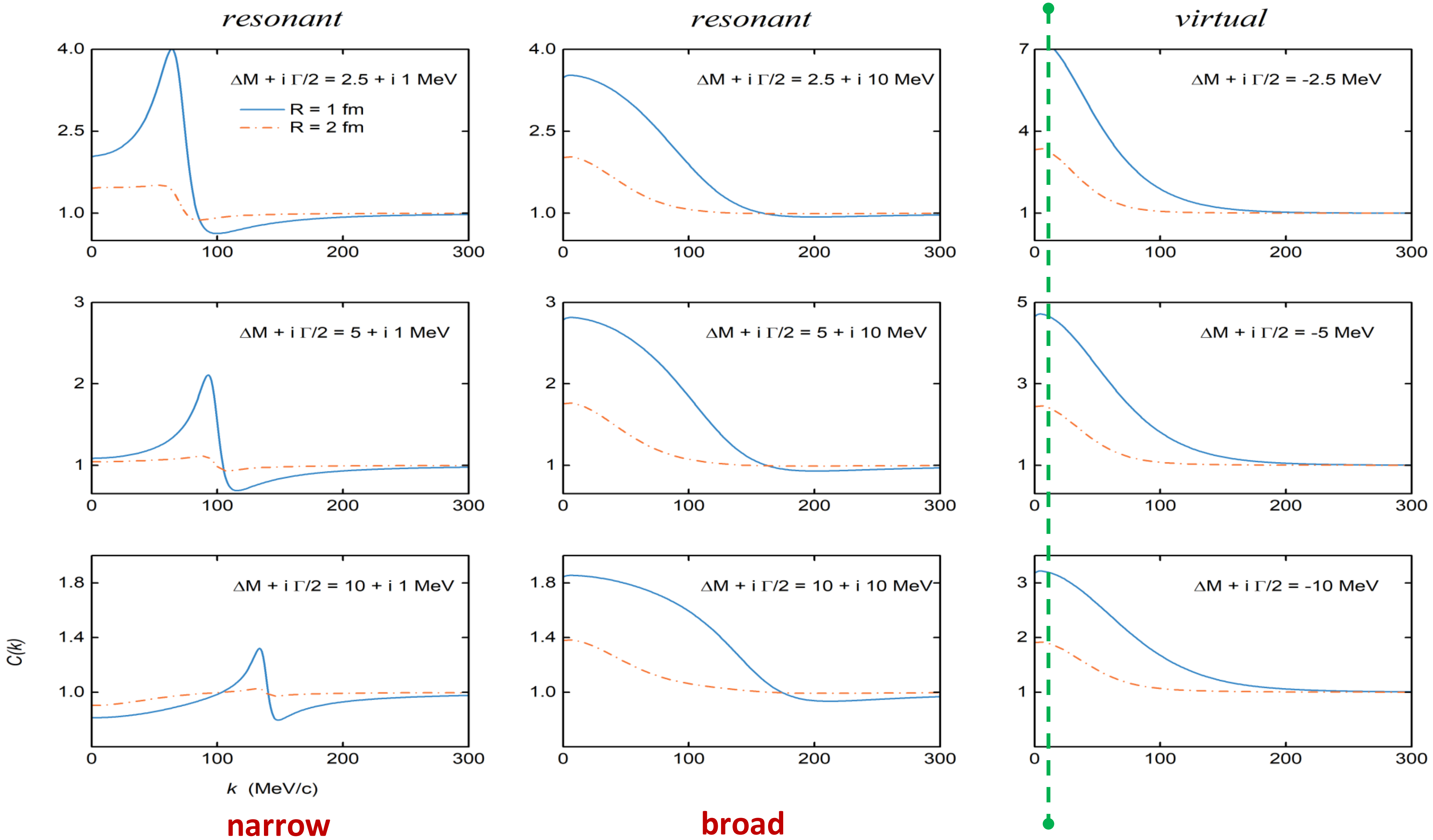
CFs for interactions capable of generating bound states

*Zhi-Wei Liu, Jun-Xu Lu and **LSG***, PRD 107, 074019 (2023)*



CFs for interactions capable of generating resonant/virtual states

Zhi-Wei Liu, Ming-Zhu Liu, Jun-Xu Lu and LSG, Sci.Bull. 70 (2025) 3515-3521*



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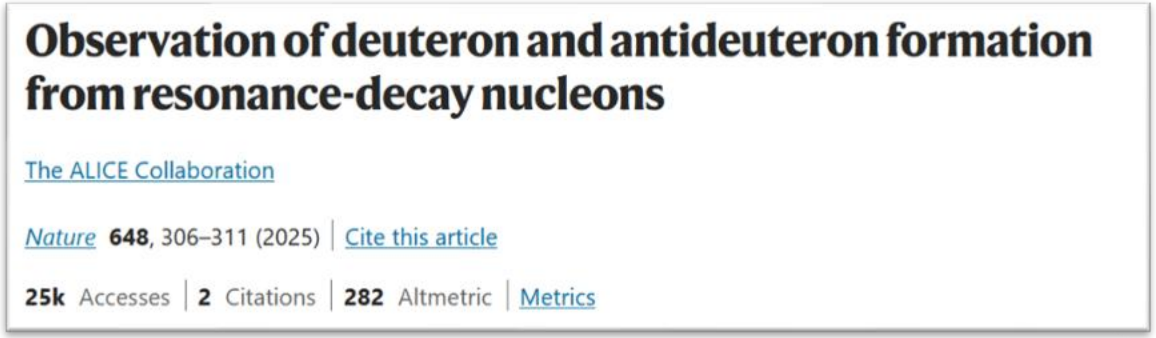
Why study the $\pi^+ p$ CF?

□ $\pi^+ p$ CF as critical input for $\pi^+ d$ femtoscopy studies

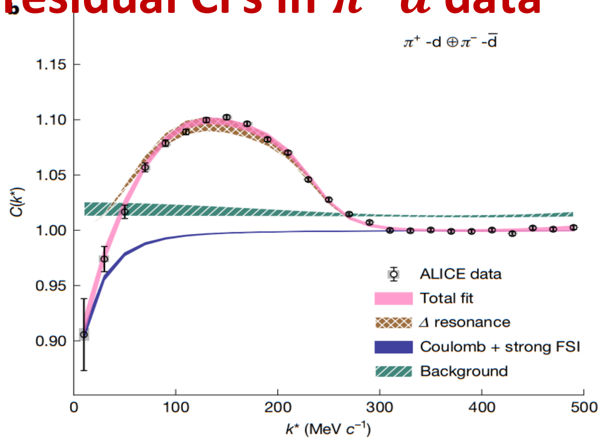
- Deuteron formation in high-energy hadronic collisions: a long-standing key question
- Deuteron formation follows resonance decays, evidenced by $\pi^+ p$ residual CFs in $\pi^+ d$ data


Observation of deuteron and antideuteron formation from resonance-decay nucleons

The ALICE Collaboration

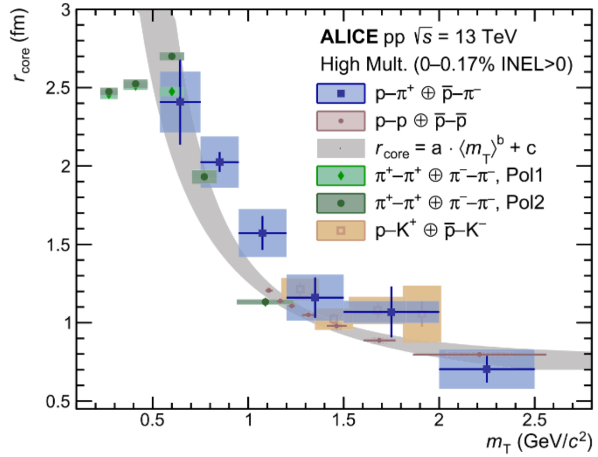
Nature **648**, 306–311 (2025) | Cite this article

25k Accesses | **2** Citations | **282** Altmetric | Metrics



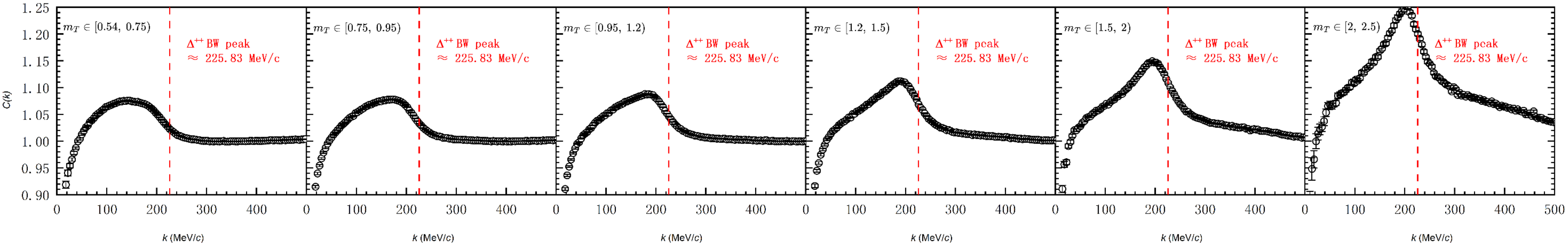
□ $\pi^+ p$ CF anchors emission-source calibration

- Abundant πN scattering data allow model-independent source calibration, which lays the groundwork for reliable extractions of other hadron pair interactions.



ALICE data difficult to understand

ALICE Collaboration, EPJA61 (2025) 194



- **Widely used resonance description in Femtoscopy seems to contradict the anomalous $\pi^+ p$ peak shift**

$$C_{\rm res}(k) = \frac{a\Gamma_R^2}{(\sqrt{s}-M_R)^2+\Gamma_R^2/4}$$

M_R: **Breit-Wigner mass;** Γ_R: **Breit-Wigner width**

- *Y. Kamiya, T. Hyodo, K. Morita, A. Ohnishi, and W. Weise, PRL 124 (2020) 132501* $[\Lambda(1520)]$
- *ALICE Collaboration, PRC 107 (2023) 054904* $[\phi(1020)]$
- *ALICE Collaboration, PLB 845 (2023) 138145* **[excited Ξ]**
- *ALICE Collaboration, PLB 856 (2024) 138915* $[K^*(892)]$

Exp.: temperature-dependent spectral function

➢ Basic formalism

ALICE Collaboration, EPJA61 (2025) 194

$$C_{\rm res}(k) = N_{\Delta^{++}} \times PS(p_{T,\Delta^{++}}, T) \times \mathrm{Sill}(M_{\Delta^{++}}, \Gamma_{\Delta^{++}})$$

$N_{\Delta^{++}}$: **overall normalization**

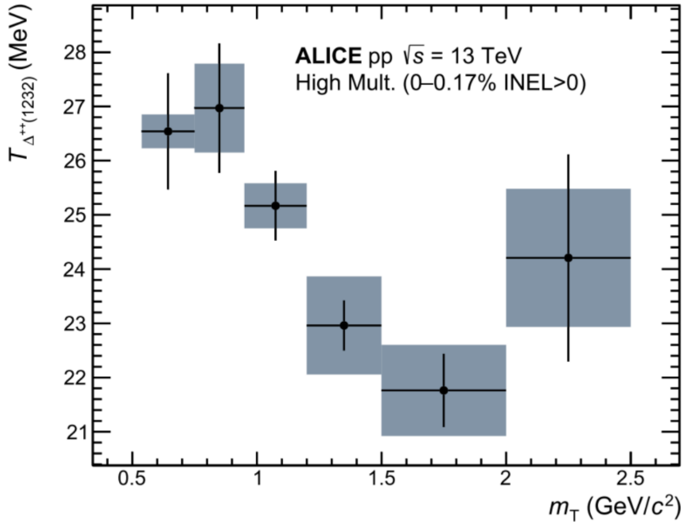
$$PS(p_{T,\Delta^{++}}, T) \propto \frac{M_{p\pi^+}}{\sqrt{M_{p\pi^+}^2 + p_{T,\Delta^{++}}^2}} \exp\left[-\frac{\sqrt{M_{p\pi^+}^2 + p_{T,\Delta^{++}}^2}}{T}\right]$$

$M_{p\pi^+}$: **invariant mass**

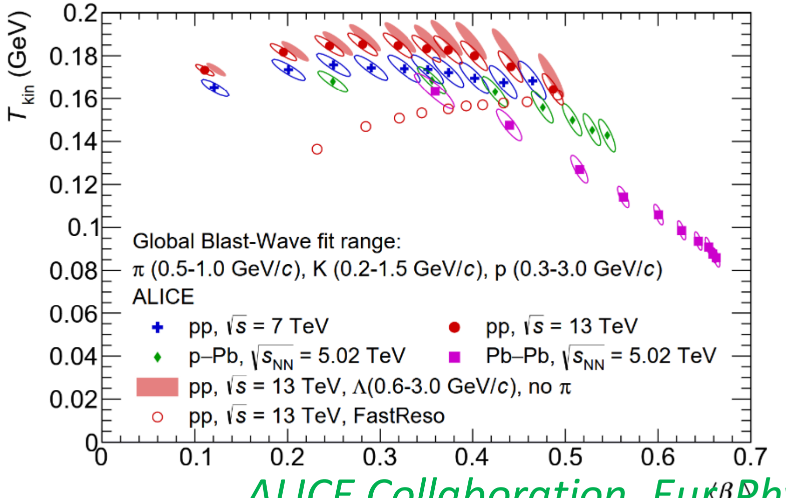
$p_{T,\Delta^{++}}$: **transverse momentum**

$$\mathrm{Sill}(M_{\Delta^{++}}, \Gamma_{\Delta^{++}}) = \frac{2\sqrt{s}}{\pi} \frac{\sqrt{s - m_{\rm th}^2}\,\tilde{\Gamma}_R}{(s - M_R^2)^2 + (\sqrt{s - m_{\rm th}^2}\,\tilde{\Gamma}_R)^2} \theta(\sqrt{s} - m_{\rm th}), \quad \tilde{\Gamma}_R = \frac{\Gamma_R M_R}{\sqrt{s - m_{th}^2}}, \quad m_{th} = M_p + m_{\pi^+}$$

➢ Extracted temperature contradicts blast-wave freeze-out



$\neq$



ALICE Collaboration, Eur. Phys. J. C 80 (2020) 693

Theo. 1: relativistic kinetic equations

- **Basic formalism** *L.-Y. Zhang, C. M. Ko, Y.-G. Ma, Q.-Y. Shou, K.-J. Sun, R. Wang, and S. Zhang, 2511.10298*

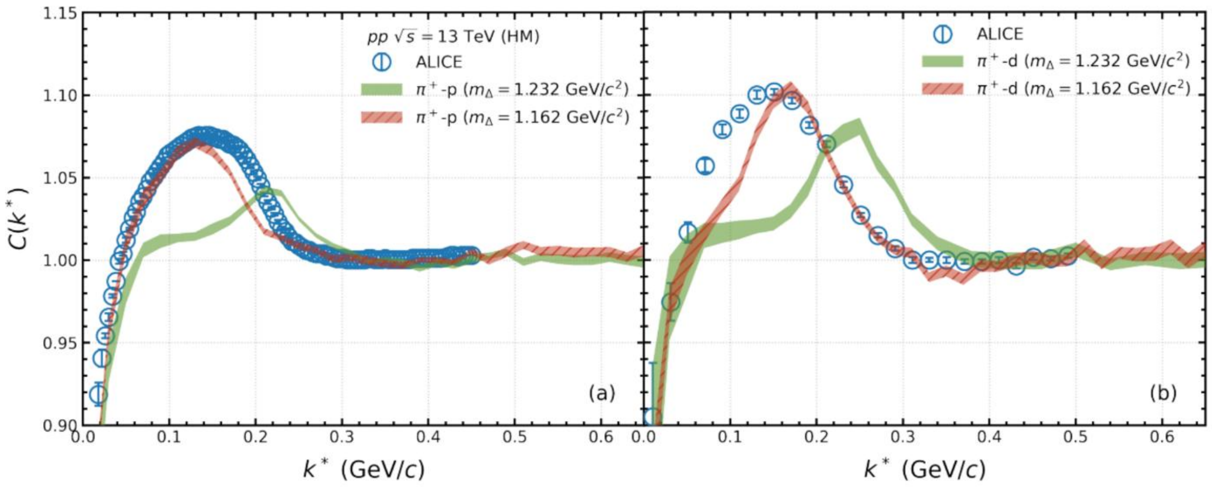
$$\frac{\partial f_d}{\partial t} + \frac{\boldsymbol{P}}{E_d} \cdot \frac{\partial f_d}{\partial \boldsymbol{R}} = -\mathcal{K}^{>} f_d + \mathcal{K}^{<}(1 + f_d)$$

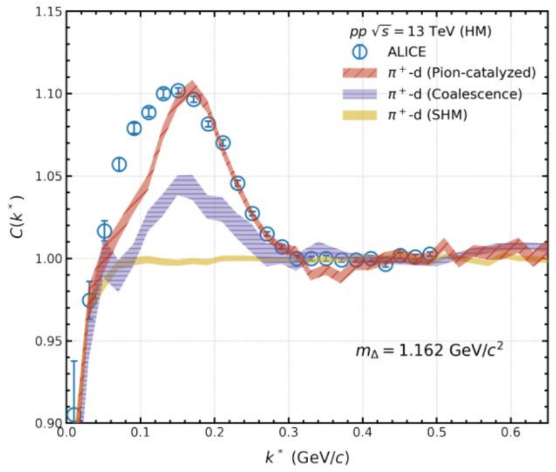
f_d: phase space distribution
$K^{>}$ ($K^{<}$): dissociation and regeneration rates

$$C(k) = \mathcal{N} \frac{\sum_{\text{pairs}}^{\text{same}} \delta(\boldsymbol{k}_{\text{pair}} - \boldsymbol{k}) |\Psi_k(\boldsymbol{r})|^2}{\sum_{\text{pairs}}^{\text{mixed}} \delta(\boldsymbol{k}_{\text{pair}} - \boldsymbol{k})}$$

$\boldsymbol{\Psi}_{\mathbf{k}}(\mathbf{r})$: Bethe-Salpeter amplitude

- **Freeze-out require a ≈70 MeV/c² mass reduction of the $\Delta(1232)$ resonance**





Theo. 2: Friedrichs-Lee model T-matrix

➢ Basic formalism

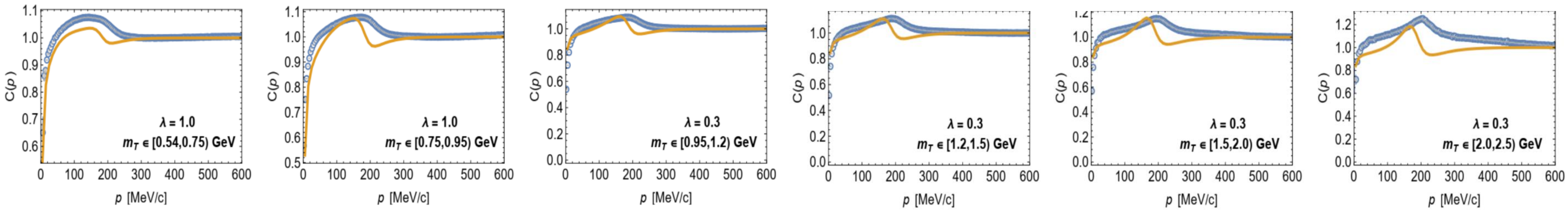
L. Zhang, T. Shao, S. Zhang, K.-J. Sun, and Y.-G. Ma, (2026), 2606.08230

$$C(p) = \int d^3k d^3k' \langle \psi_p^{(+)}|k\rangle\langle k|\hat{S}|k'\rangle\langle k'|\psi_p^{(+)}\rangle + \langle \psi_p^{(+)}|R\rangle\langle R|\hat{S}|R\rangle\langle R|\psi_p^{(+)}\rangle$$

$$\langle k|\psi_p^{(+)}\rangle = \delta^{(3)}(k-p) + G^{(+)}(k;E)T(k,p;E), \qquad T(k,p;E) = \frac{g(k)g^*(p)}{E - E_0 - \Sigma_0(E) + i\epsilon}$$

$$\langle \psi_p^{(+)}|R\rangle = g(p)\frac{E - E_0}{E - E_0 - \Sigma_0(E) + i\epsilon}$$

➢ Underestimates strength, predicts dip absent



A new attempt

Zhi-Wei Liu, Jun-Xu Lu, Duo-Lun Ge, LSG, 2607.04351*

Quantum interference effects enhanced in $\pi^+ p$ femtoscopic correlation functions

Zhi-Wei Liu (Michigan U., Math. Dept. and Shenzhen U.), Jun-Xu Lu (Beihang U.), Duo-Lun Ge (Beihang U.), Li-Sheng Geng (Beihang U. and Beijing, CSRC and Hui Zhou, Inst. Modern Phys.) (Jul 5, 2026)

e-Print: 2607.04351 [hep-ph]

1. **Model-independent treatment of the $\pi^+ p$ interaction**
2. **Consistent treatment of two mechanism: scattering and decay**
3. **Economical fitting—only four parameters**

Strong interaction

— Data Analysis Center —

Institute for Nuclear Studies

THE GEORGE WASHINGTON UNIVERSITY

WASHINGTON, DC

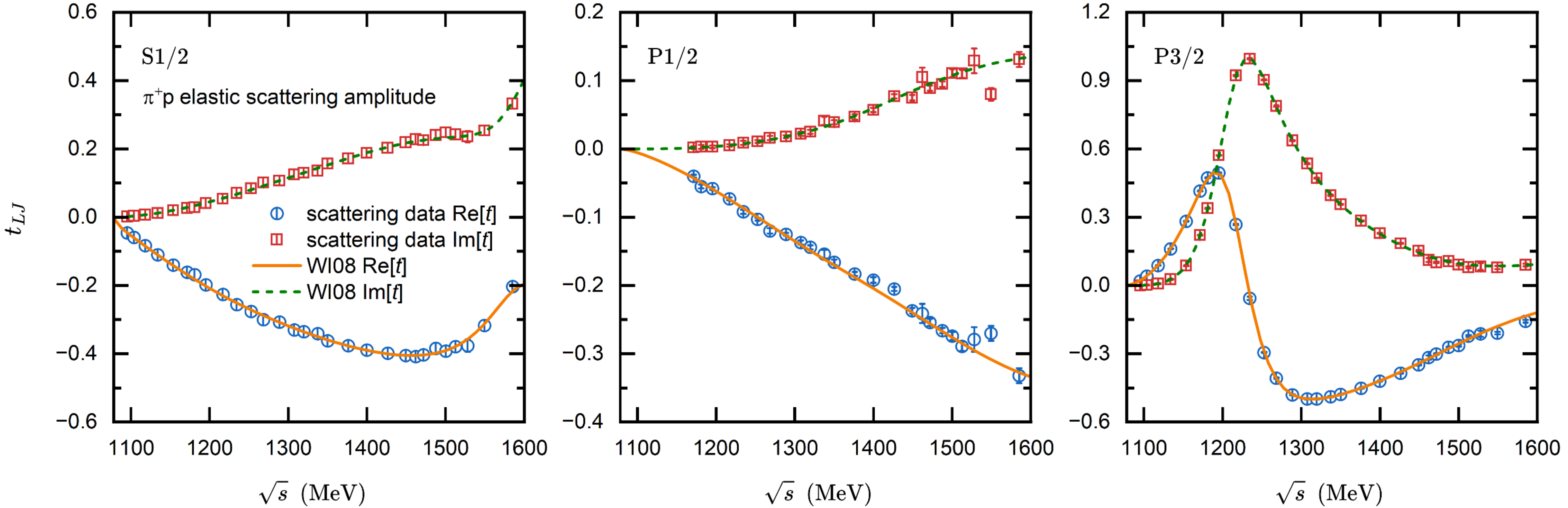
control off-shell behavior

$$T^{\rm str}_{LJ}(p,p') = -1/\rho \cdot t_{LJ} \cdot \boxed{\theta(\Lambda_F - p)\theta(\Lambda_F - p')}, \;\; \Lambda_F \in [0.6, 1.4]\text{GeV}/c$$

$$\rho = 2Mk/(8\pi\sqrt{s})$$

$$k = \lambda^{1/2}(s, m^2, M^2)/(2\sqrt{s})$$

$$\lambda(a,b,c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc$$



Coulomb interaction

Fourier-transforming the r-space Coulomb potential into p-space

$$V^{\text{cou}}(|\boldsymbol{p}'-\boldsymbol{p}|)=\int_0^{|\boldsymbol{r}|<R_c}\mathrm{d}\boldsymbol{r}\;e^{\mathrm{i}(\boldsymbol{p}'-\boldsymbol{p})\cdot\boldsymbol{r}}\frac{\epsilon\alpha}{r}=\frac{4\pi\epsilon\alpha}{|\boldsymbol{p}'-\boldsymbol{p}|^2}\left[1-\cos(|\boldsymbol{p}'-\boldsymbol{p}|R_c)\right], \qquad R_c=60\ \mathrm{fm}$$

Partial Wave Projection

$$V_L^{\text{cou}}(p,p')=\frac{1}{2}\int_{-1}^{1}\mathrm{d}\cos\theta\;V^{\text{cou}}(q)P_L(\cos\theta), \qquad q^2=p^2+p'^2-2pp'\cos\theta$$

Relativistic correction (relativistic propagator & relativistic normalization convention)

$$T_L^{\text{cou}}(p,p')=\sqrt{\frac{2E_M(p)2E_m(p)\xi(p)}{2M}}\sqrt{\frac{2E_M(p')2E_m(p')\xi(p')}{2M}}V_L^{\text{cou}}$$

where $\xi(p)=[\sqrt{s}-E_M(p)-E_m(p)]/[k^2/(2\mu)-p^2/(2\mu)]$

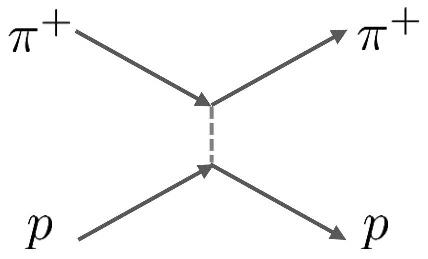
CF from the $\pi^+ p$ scattering

Outgoing wave function

$$\mathcal{R}_{LJ}(k,r) = \int_0^{|\boldsymbol{k'}|<\Lambda_F} \mathrm{d}\boldsymbol{k'}\ \mathcal{G}(k')\ T_{LJ}(k,k')\ j_L(k'r)$$

where $T_{LJ}(k,k') = T_{LJ}^{\mathrm{str}}(k,k') + T_L^{\mathrm{cou}}(k,k'),\qquad \mathcal{G}(k') = \dfrac{2M}{(2\pi)^3}\dfrac{E_M+E_m}{2E_M E_m}\dfrac{1}{s-(E_M+E_m)^2+\mathrm{i}\varepsilon}$

Correlation function from the $\pi^+ p$ scattering process



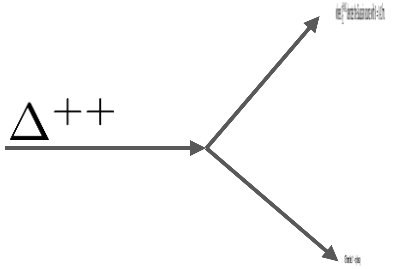
$$C_{\mathrm{sca}}(k) = \underbrace{C_{\mathrm{S1/2}} + C_{\mathrm{P1/2}} + C_{\mathrm{P3/2}}}_{\text{strong + coulomb}} + \underbrace{C_{\mathrm{hpw}}}_{\text{coulomb only}}$$

where $C_{LJ}(k) = \omega_{LJ}\displaystyle\int_0^{\infty} \mathrm{d}\boldsymbol{r}\ S_{12}\ (2L+1)|j_L + \mathcal{R}_{LJ}|^2,\qquad \omega_{LJ} = (2J+1)/[2(2L+1)]$

$$C_{\mathrm{hpw}}(k) = \int_0^{\infty} \mathrm{d}\boldsymbol{r}\ S_{12}\ \left(|\Phi^{\mathrm{cou}}|^2 - \sum_{L=0,1} |\Phi_L^{\mathrm{cou}}|^2\right)$$

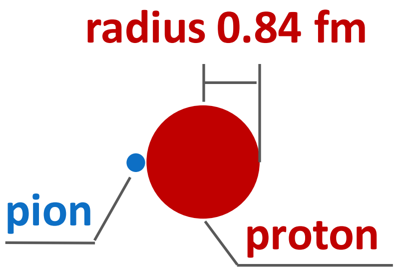
CF from the $\Delta^{++} \to \pi^+ p$ decay

Correlation function from the $\Delta^{++} \to \pi^+ p$ decay process



$$C_{\rm dec}(k) = \int_0^\infty {\rm d}\boldsymbol{r}\ S_{12}^{R=0.42}\ (2L+1)|\mathcal{R}_{LJ}|^2$$

where $S_{12}^{R=0.42}$ denotes the Gaussian source with $R = 0.42$ fm.



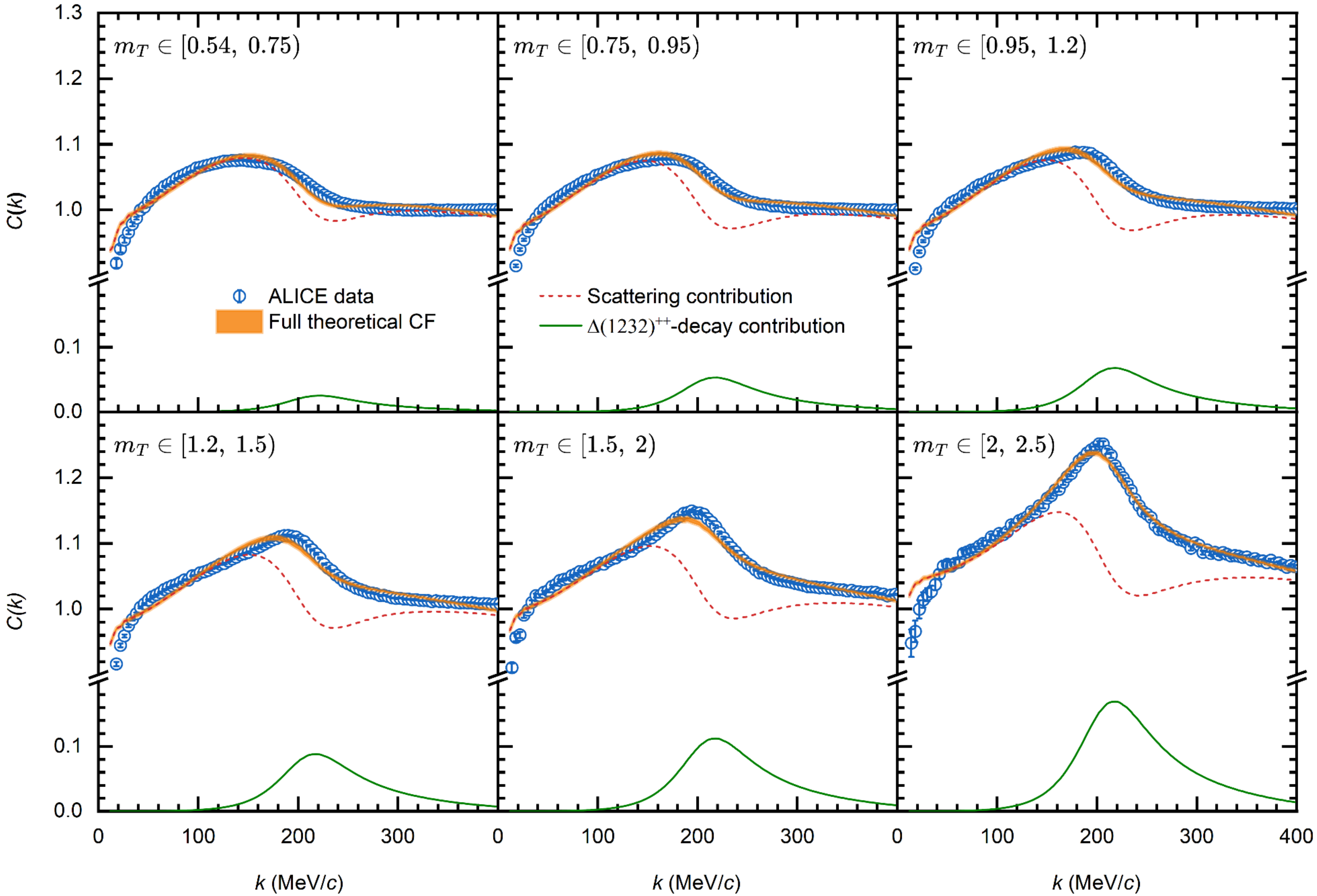
① **without incident wave j_L** ② **without weight factor ω_{LJ}** ③ **smaller source size**

Complete CF

$$C(k) = (a + bk)[1 + \lambda(1 - P)(C_{\mathrm{sca}} - 1)] + \lambda P C_{\mathrm{dec}}$$

- ✓ P: separable fraction of Δ decay
- ✓ λ: misidentification and feed-down effects
- ✓ $(a + bk^2)$: Correlation with mini-jet background

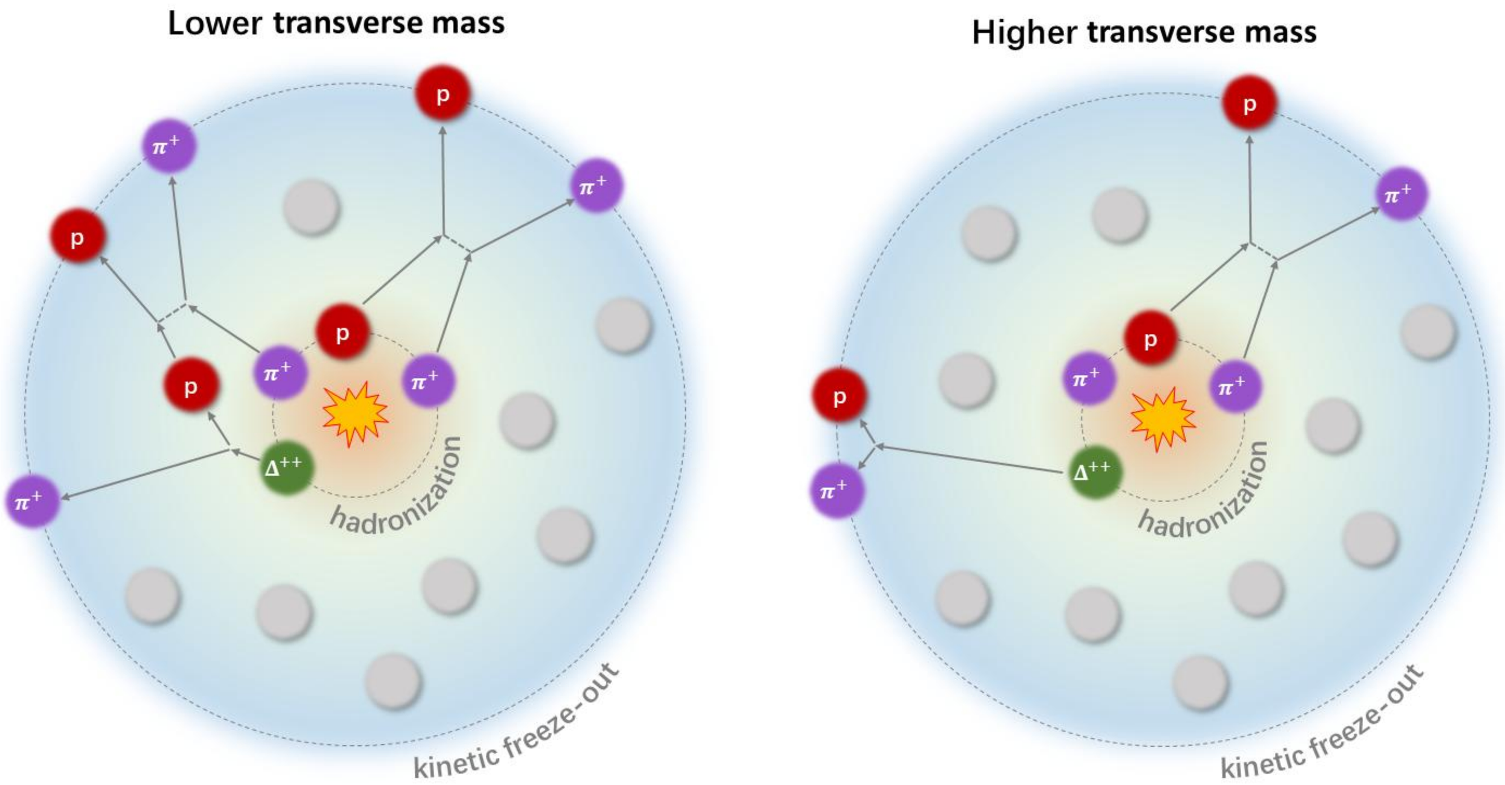
Comparison with the experimental data



- ✓ **The theory matches data and reproduces the peak shift from broad low-k to narrow high-k with increasing transverse-mass m_T.**
- ✓ **The observed peak is a weighted sum of the scattering and Δ-decay parts; their relative weights P drive the m_T-dependent shift.**

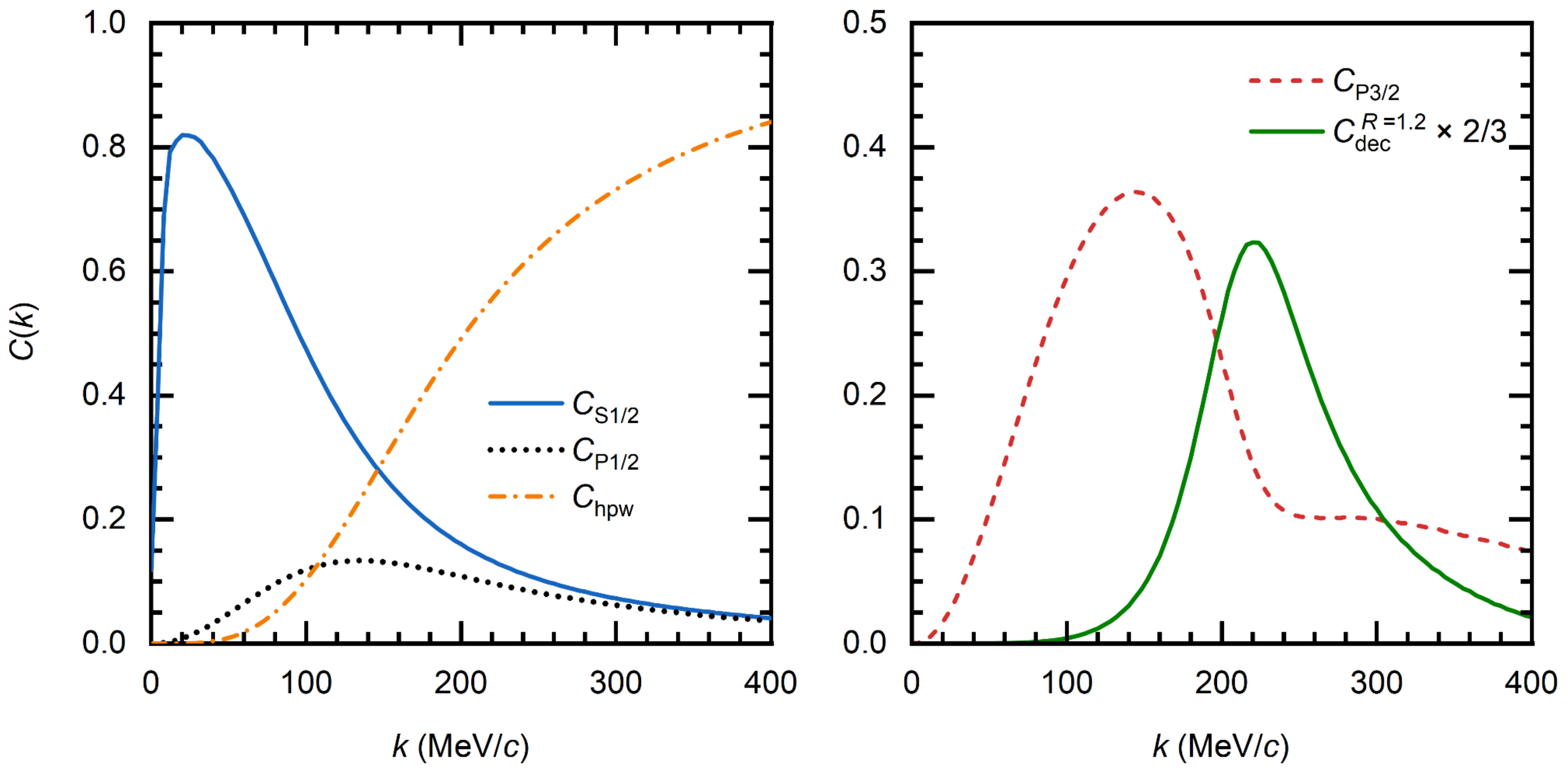
Physical picture of the peak shift

No.	m_T (GeV/c^2) [35]	λ [35]	P	R (fm)	a	b (10^{-4} MeV^{-1})
I	$[0.54, 0.75)$	0.677	0.03±0.00	1.32±0.02	1.08±0.00	-2.20±0.08
II	$[0.75, 0.95)$	0.687	0.06±0.00	1.24±0.02	1.06±0.00	-1.87±0.08
III	$[0.95, 1.20)$	0.689	0.08±0.00	1.21±0.02	1.06±0.00	-1.83±0.08
IV	$[1.20, 1.50)$	0.688	0.11±0.00	1.18±0.02	1.07±0.00	-1.85±0.08
V	$[1.50, 2.00)$	0.661	0.14±0.00	1.15±0.02	1.08±0.00	-1.87±0.08
VI	$[2.00, 2.50)$	0.616	0.23±0.00	1.02±0.01	1.13±0.00	-1.91±0.09



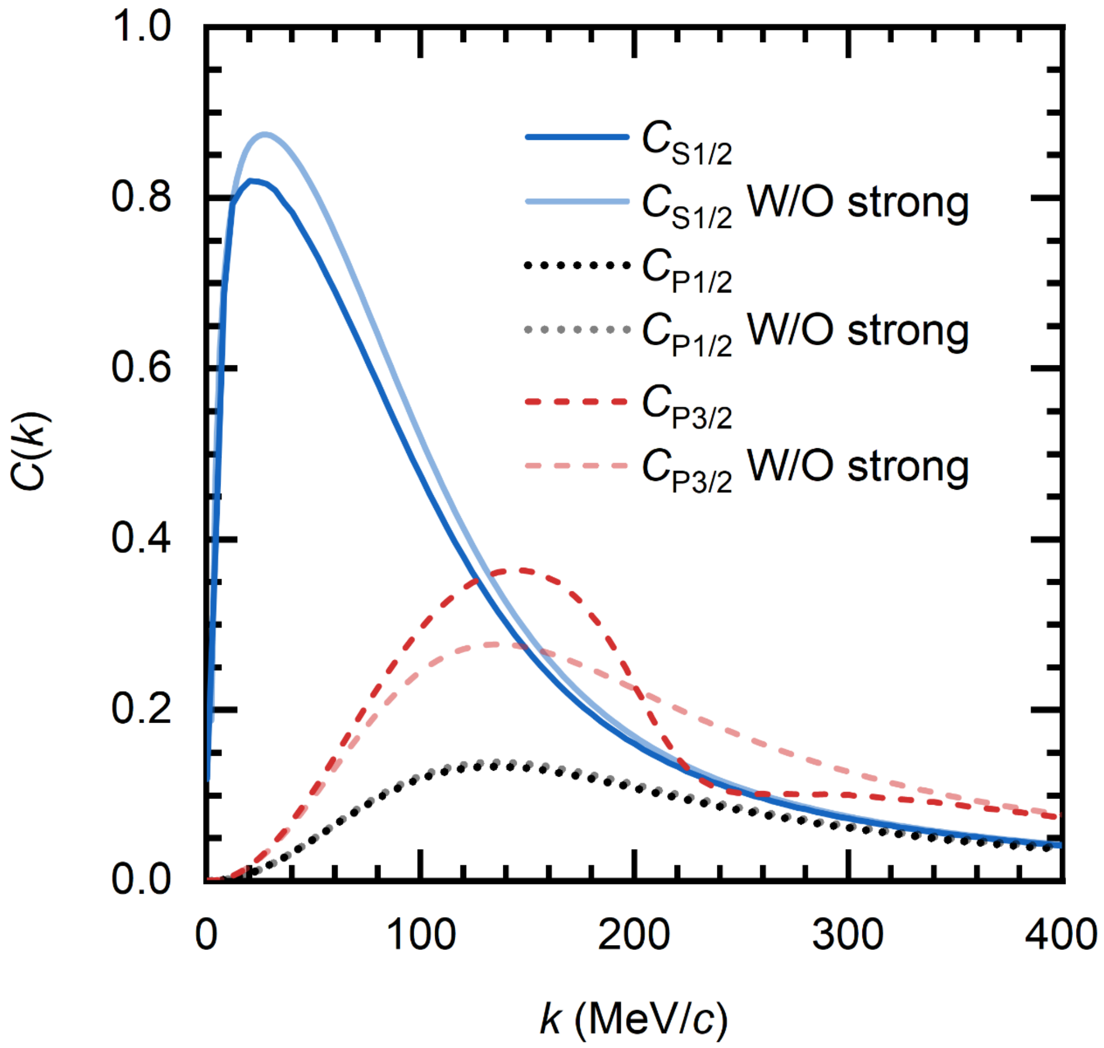
✓ **At low m_T, Δ decays inside source, mixing with direct pairs; at high m_T, escapes before Δ decay. Increasing m_T raises P (escape fraction) supporting the picture.**

Why scattering and decay peaks differ



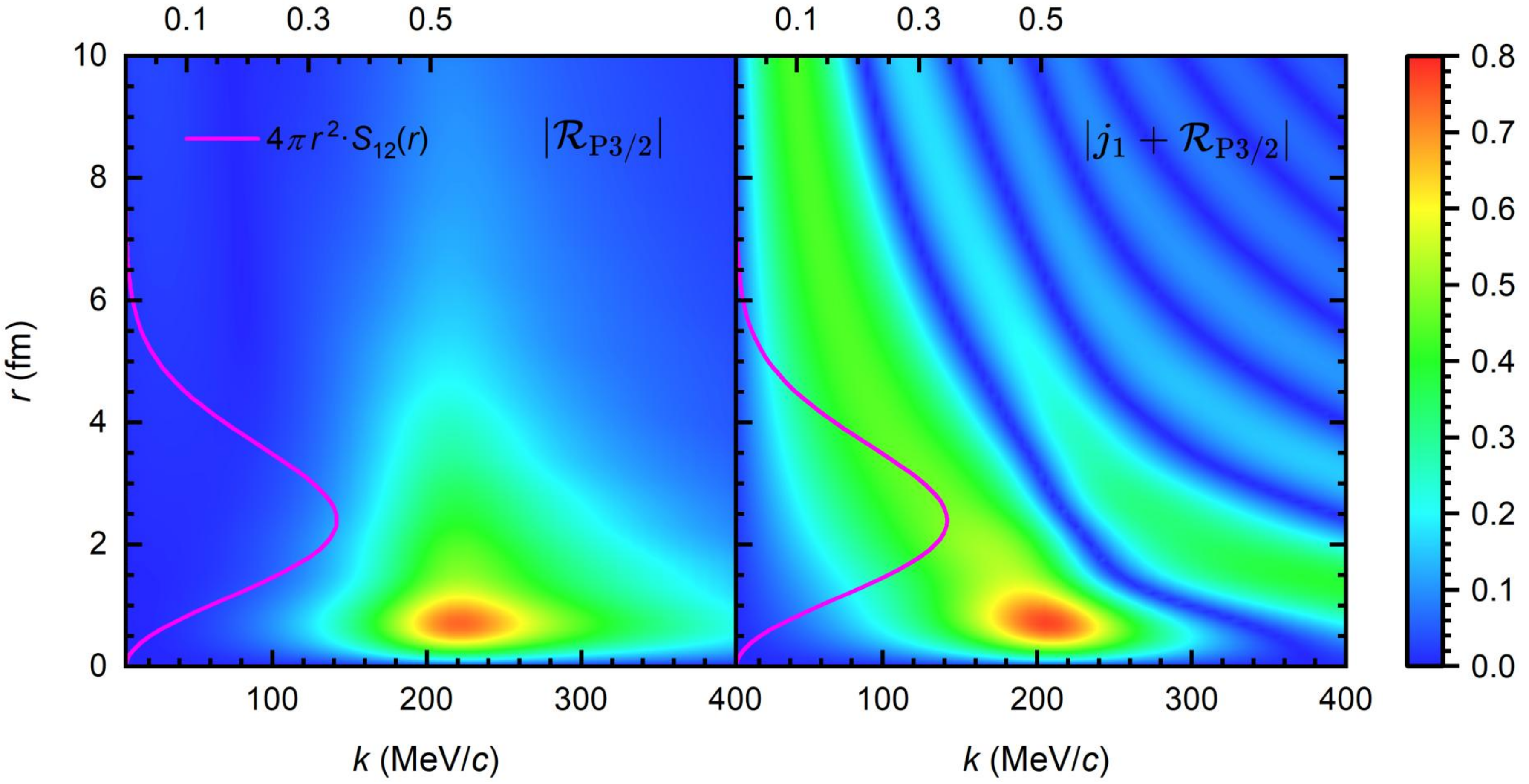
- ✓ **The scattering peak near 140 MeV/c originates from the P3/2 partial wave, while the decay peak near 220 MeV/c resembles the conventional Breit-Wigner form.**
- ✓ **The shift arises because scattering includes the incident wave j_1 in the outgoing wave function, while the decay does not.**

Influence of the strong interaction



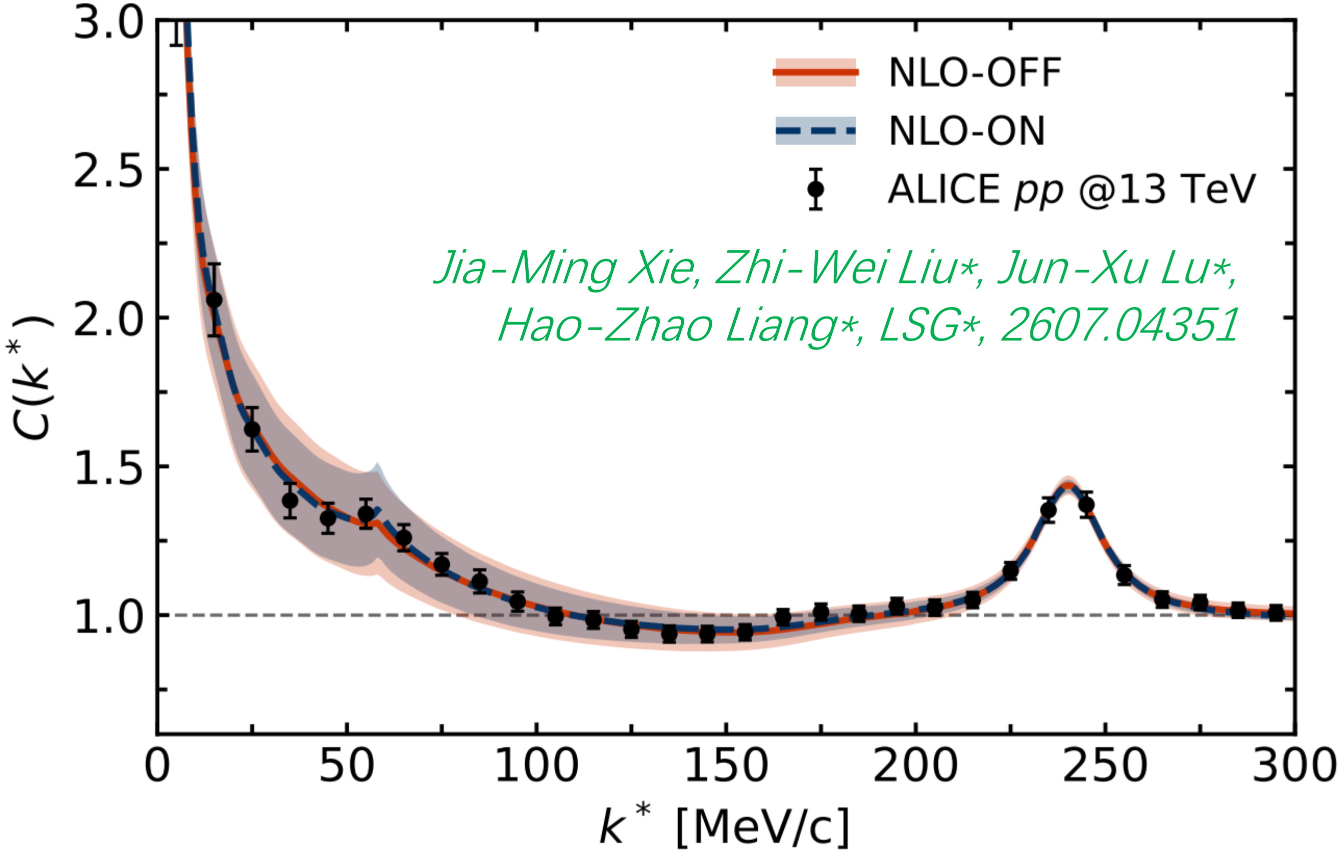
- ✓ **Comparison of partial-wave correlations with / without strong interaction shows strong suppression of S1/2, negligible effect on P1/2, and distortion of P3/2 with a peak at 140 and dip at 230 MeV/c.**

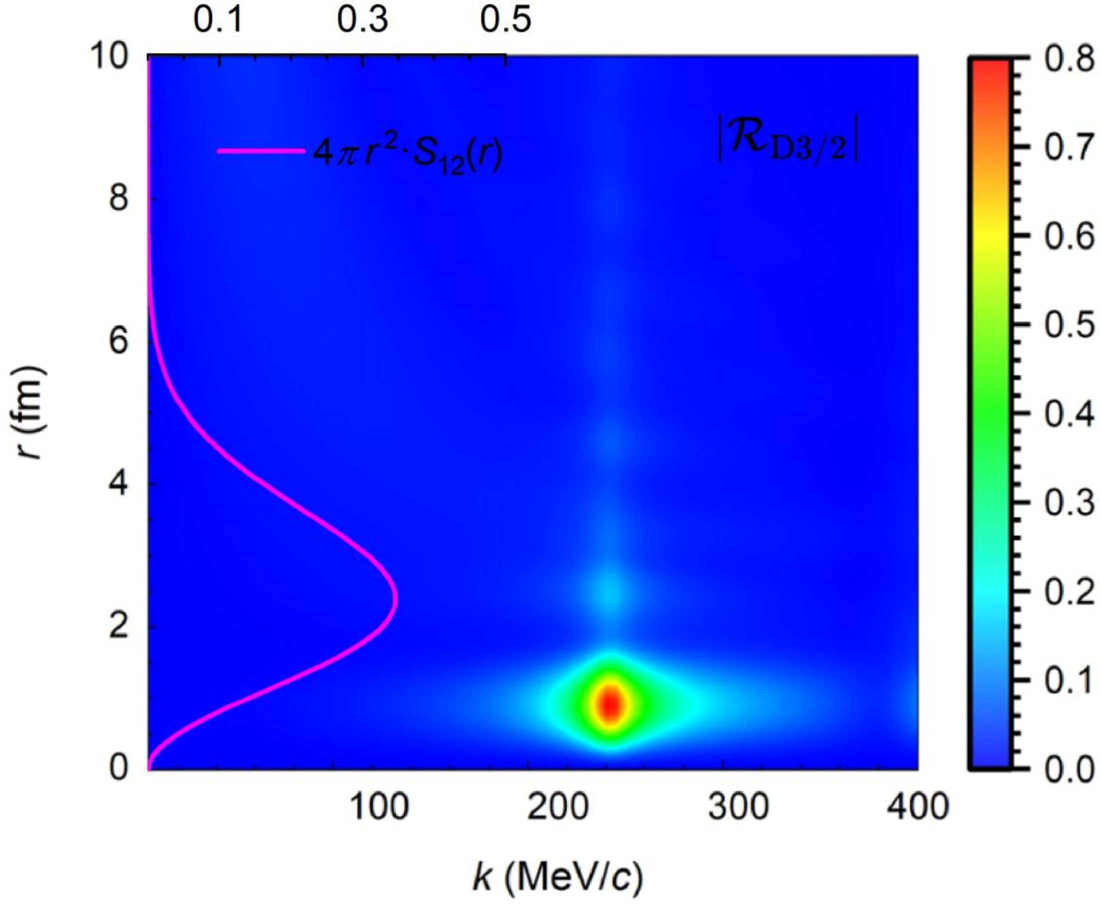
Interference-induced peak shift



- ✓ **Comparison of $|j_1 + R_{P3/2}|$ and $|R_{P3/2}|$ in phase space shows that interference shifts amplitude to low k.**

Interference from broad vs narrow resonances





✓ **Large $\Delta(1232)$ width enhances $\pi^+ p$ interference, while narrow $\Lambda(1520)$ width causes negligible $K^- p$ peak shift (only 2 MeV/c shift).**

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- ☞ **Summary and outlook**

Summary and outlook

1. **Femtoscopy offers high-precision tests of the strong interaction between pairs of (un)stable particles and can be valuable to decipher the nature of the many exotic hadrons discovered so far**

2. **First the first time, we successfully solved the peak-shift puzzle of the $\pi^+ p$ CF:**
   - ✓ There is a **nontrivial quantum interference** of scattering and incoming waves
   - ✓ There is also **a delicate interference between scattering and decay**, which can be modeled as a nontrivial source distribution (ongoing)

3. **With more data from LHC Run3, the future is very promising:**
   - ✓ Pp data taking: event count increase: **~3000×**
   - ✓ Pb-pb data taking: event count increase: **~70×**

Thanks a lot for your attention!

(Image: CERN)

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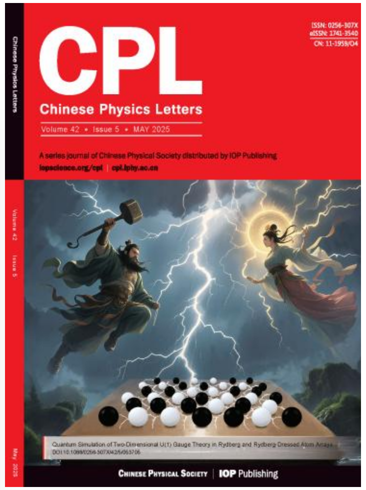

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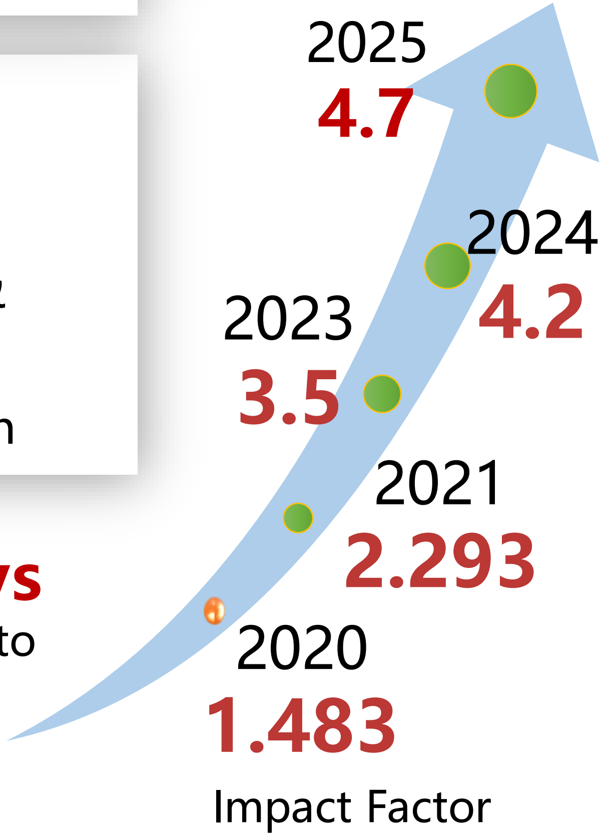

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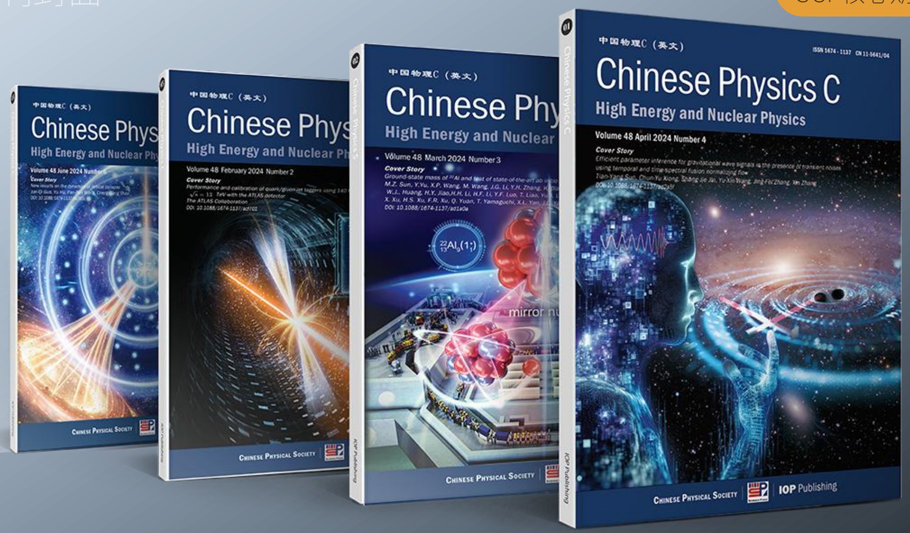

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  - *Y. Kamiya, T. Hyodo, K. Morita, A. Ohnishi, and W. Weise, PRL 124 (2020) 132501* [$\Lambda(1520)$]
  - *ALICE Collaboration, PRC 107 (2023) 054904* [$\phi(1020)$]
  - *ALICE Collaboration, PLB 845 (2023) 138145* [excited Ξ]
  - *ALICE Collaboration, PLB 856 (2024) 138915* [$K^*(892)$]
- **Temperature-dependent spectral function:** **extracted temperature contradicts blast-wave freeze-out**
  - *ALICE Collaboration, EPJA 61 (2025) 194*
- **Relativistic kinetic equations:** **require a ≈70 MeV/c² mass reduction of the $\Delta(1232)$ resonance**
  - *L.-Y. Zhang, C. M. Ko, Y.-G. Ma, Q.-Y. Shou, K.-J. Sun, R. Wang, and S. Zhang, arXiv.2511.10298*
- **Friedrichs-Lee model T-matrix:** **underestimates strength, predicts dip absent**
  - *L. Zhang, T. Shao, S. Zhang, K.-J. Sun, and Y.-G. Ma, (2026), arXiv:2606.08230*