Probing K-molecule interactions via correlation functions: three-body bound states in KX(3872) and K Ds0*(2317)

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Outline

- **Motivation:** why particle-resonance correlation functions?
- **Formalism:** fixed-center approximation + Lippmann-Schwinger unitarization
- **Results:** ***KX(3872) & KDs0*(2317)***
- **Summary and outlook**

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Motivation: using a kaon as an external probe

Central question

Can a kaon probe the inner structure of a hadronic molecule?

- Correlation functions probe low relative momentum.
- A kaon can interact with the constituents of a molecule.
- Strong three-body attraction may generate a bound state.
- This can change $C(p)$ at low momentum.
- It can also produce a peak in the invariant-mass spectrum.
- These are two complementary signals of molecular structure.

Molecular cluster
$X(3872) = D\bar{D}^*$ molecule
$D^*_{s0}(2317) = DK$ molecule

↓

Kaon + cluster

↓

Unitary three-body amplitude

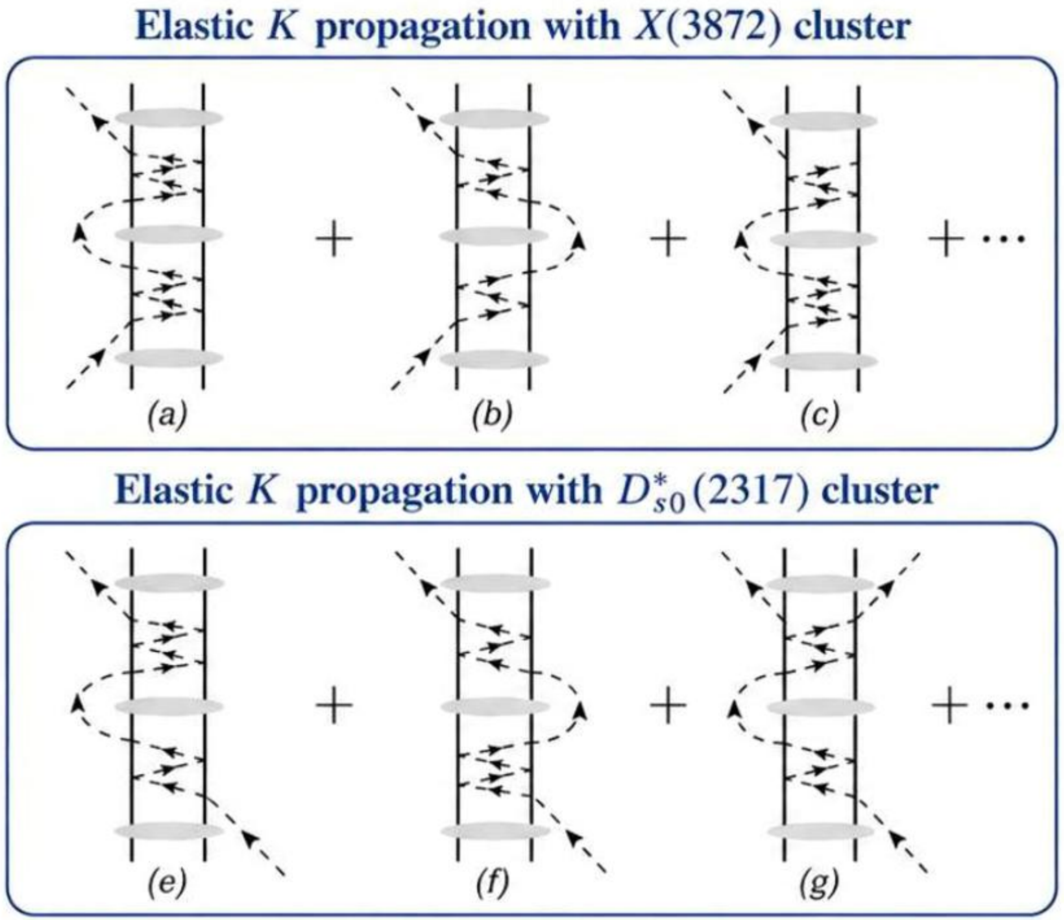
↓

Correlation function and mass spectrum

P. Encarnación et al., PRD 111, 114023 (2025); M.-Z. Liu et al., Phys. Rep. 1108, 1 (2025); F.-K. Guo et al., RMP 90, 015004 (2018).

A kaon can probe a molecule through *C(p)* and a mass spectrum.

Why unitarize the FCA amplitude?



Near the K + cluster threshold, elastic propagation of the full K-molecule system must be included explicitly.

- The standard FCA multiple scattering series provides the effective kernel (optical potential).
- An additional L-S resummation restores exact elastic unitarity.
- This is essential for extracting scattering parameters and correlation observables.

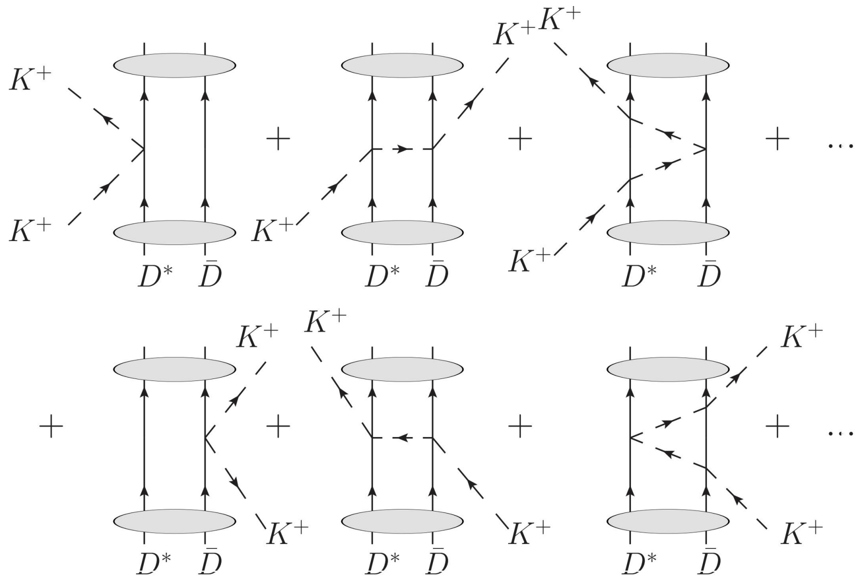
$$T' = \left[1 - \tilde{T} G_C\right]^{-1} \tilde{T}, \qquad T_{tot} = \sum_{ij} T'_{ij}$$

N. Ikeno and E. Oset, PRD 112, 094019 (2025); B. Agatão et al., EPJC 85, 1136 (2025).

Two systems

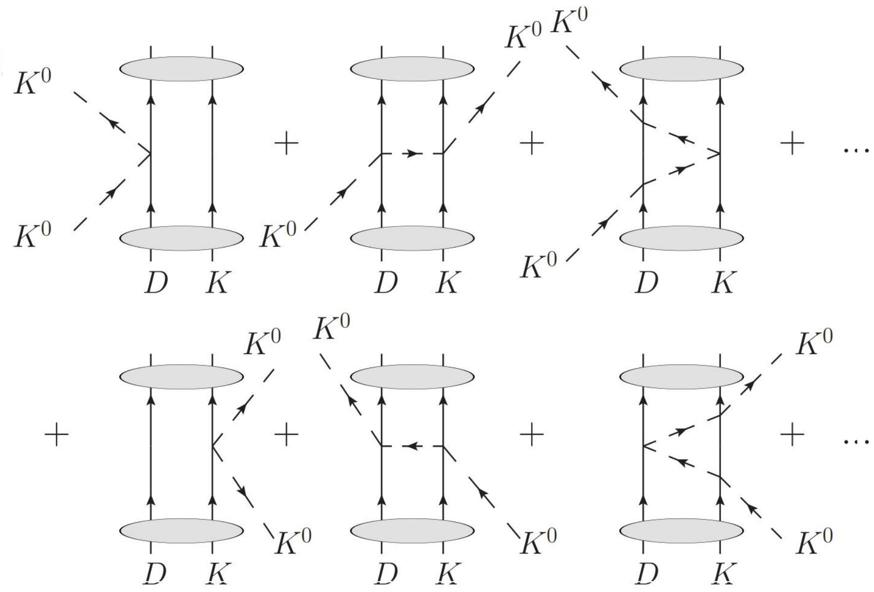
KX(3872)

- Assumption: $X(3872)$ is an $I = 0, C = +\ D\bar{D}^*$ molecule.
- The external kaon scatters from the D and D^* constituents.
- Attraction arises from KD and KD^* interactions.



$KD_{s0}^*(2317)$

- Assumption: $D_{s0}^*(2317)$ is dominantly an $I = 0$ DK molecule.
- The external kaon interacts with the D and K inside the cluster.
- DK attraction is strong, while KK interaction is repulsive.



In both cases, the kaon acts a probe of molecular constituens.

Gamermann et al., PRD 81, 014029 (2010); D. Mohler et al., PRL 111, 222001 (2013).

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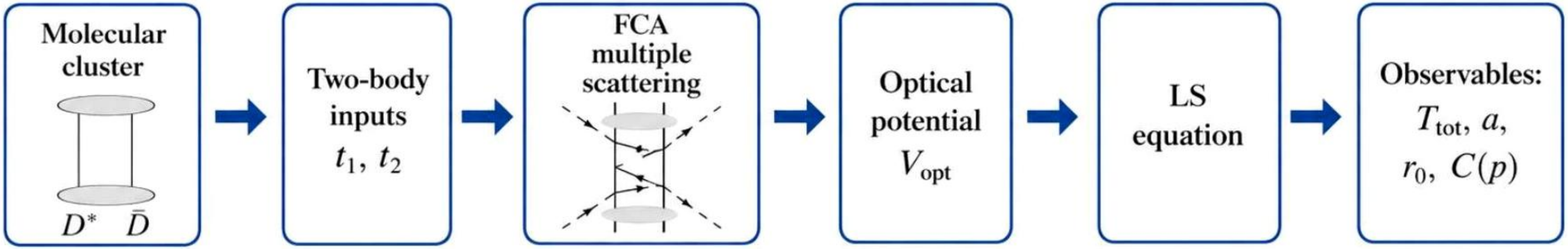
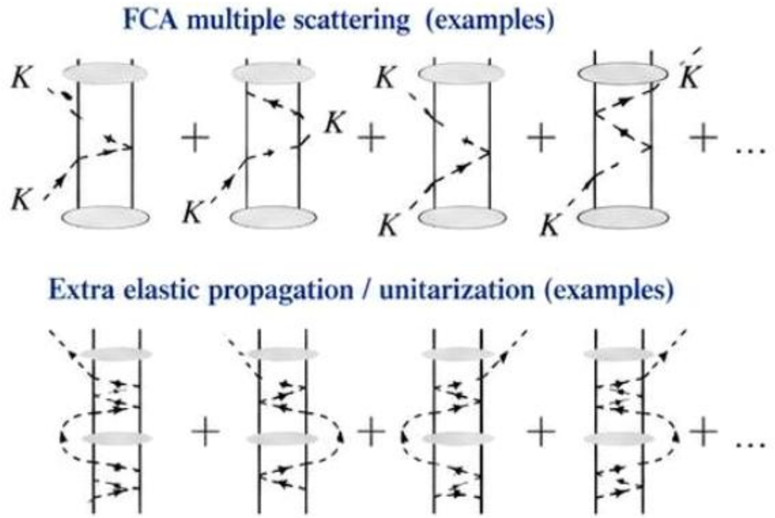
Theory framework: from FCA to a unitary three-body amplitude



$$T' = [1 - TG_C]^{-1}T \qquad T_{\mathrm{tot}} = \sum_{i,j} T'_{ij}$$

- The fixed-center approximation describes the kaon scattering from a bound cluster.
- The additional Lippmann-Schwinger resummation restores the correct elastic unitarity near threshold.
- This makes it possible to extract low-energy observables and correlation functions consistently.

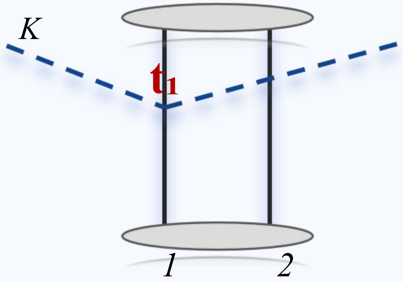


Key point: unitarization is essential for a reliable K + cluster amplitude.

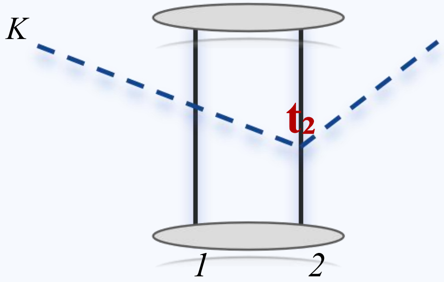
S. S. Kamalov et al., NPA 690, 494 (2001); N. Ikeno and E. Oset, PRD 112, 094019 (2025).

Formalism I: what are t_1, t_2, G_0 and Gc ?

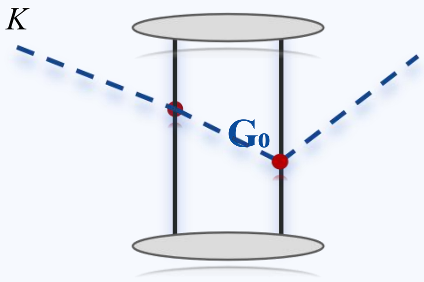
Two-body amplitude t_1



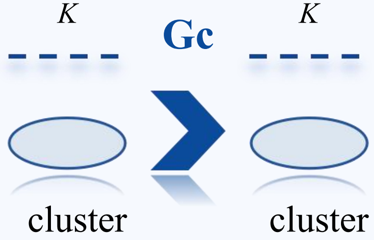
Two-body amplitude t_2



Propagation inside cluster G_0



Elastic K+ cluster loop Gc



elastic K + cluster propagation

- t_1, t_2: on-shell kaon–constituent amplitudes
- G_0: kaon propagates from one constituent to the other inside the fixed cluster (**form the FCA optical potential**)
- Gc: the whole elastic K + cluster intermediate state propagates in the LS series (**elastic K + cluster propagation**)

$$T_1 = t_1 + t_1 G_0 T_2 \qquad T_2 = t_2 + t_2 G_0 T_1 \qquad T_{tot} = T_1 + T_2$$

N. Ikeno and E. Oset, PRD 112, 094019 (2025); B. Agatão et al., EPJC 85, 1136 (2025).

Loop functions: explicit formulas for G_0 and G_C

G_0: propagation inside the fixed cluster

$$G_0(\sqrt{s}) = \int \frac{d^3q}{(2\pi)^3} \frac{F_C(q)}{2\omega_K(q)\,2\omega_C(q)} \frac{1}{\sqrt{s} - \omega_K(q) - \omega_C(q) + i\varepsilon}$$

$$\times\, \Theta(q^{(1)}_{\max} - q_1^*)\, \Theta(q^{(2)}_{\max} - q_2^*)$$

One cluster form factor
Kaon moves between constituent 1 and constituent 2

G_C: elastic K + cluster loop

$$G_C = \mathrm{diag}\left(G_C^{(1)}, G_C^{(2)}\right)$$

$$G_C^{(i)}(\sqrt{s}) = \int \frac{d^3q}{(2\pi)^3} \frac{[F_C^{(i)}(q)]^2}{2\omega_K(q)\,2\omega_C(q)} \frac{1}{\sqrt{s} - \omega_K(q) - \omega_C(q) + i\varepsilon}$$

$$\times\, \Theta(q^{(i)}_{\max} - q_i^*)$$

Squared channel form factor
The whole elastic K + cluster state propagates

Common ingredients

$$F_C^{(1)}(q) = F_C\left(\frac{m_2}{m_1+m_2}q\right), \qquad F_C^{(2)}(q) = F_C\left(\frac{m_1}{m_1+m_2}q\right)$$

$$\omega_K(q) = \sqrt{m_K^2 + q^2}, \qquad \omega_C(q) = \sqrt{M_C^2 + q^2}$$

- i = 1, 2 labels the two constituents.
- q_i^* is the momentum in the kaon-constituent channel.
- The Θ functions apply the same cutoffs used in t_1 and t_2.

Different role: G_0 builds the FCA kernel; G_C restores elastic K + cluster propagation.

Phys. Rev. D 114, 035042 (2026); arXiv:2604.07261 (2026).

Formalism II: calculating the total amplitude

1. FCA: build the kaon–constituent multiple-scattering kernel

$$\tilde{T}_{11} = \frac{\tilde{t}_1}{1-\tilde{t}_1\tilde{t}_2 G_0^2}, \qquad \tilde{T}_{22} = \frac{\tilde{t}_2}{1-\tilde{t}_1\tilde{t}_2 G_0^2}, \qquad \tilde{T}_{12} = \tilde{T}_{21} = \frac{\tilde{t}_1\tilde{t}_2 G_0}{1-\tilde{t}_1\tilde{t}_2 G_0^2}$$

2. Propagate the full K + cluster state

$$\tilde{T}' = \left[1-\tilde{T} G_C\right]^{-1}\tilde{T}, \qquad G_C = \mathrm{diag}\left(G_C^{(1)}, G_C^{(2)}\right)$$

Same structure for the two systems

KX(3872): $D^*\bar{D}$ and $D\bar{D}^*$ components

K $D_{s0}^*(2317)$: cluster = DK, $\tilde{t}_1$ = KD, $\tilde{t}_2$ = KK

3. Sum all unitarized partitions

$$T^{\mathrm{tot}}(\sqrt{s}) = \sum_{i,j}\tilde{T}'_{ij} = \frac{N}{D}$$

$$N = \tilde{t}_1 + \tilde{t}_2 + \left(2G_0 - G_C^{(1)} - G_C^{(2)}\right)\tilde{t}_1\tilde{t}_2$$

$$D = 1 - G_C^{(1)}\tilde{t}_1 - G_C^{(2)}\tilde{t}_2 - \left(G_0^2 - G_C^{(1)}G_C^{(2)}\right)\tilde{t}_1\tilde{t}_2$$

For X(3872): inverse-average of two components

$$(T_{KX}^{\mathrm{tot}})^{-1} = \frac{1}{2}\left[(T_{D\bar{D}^*}^{\mathrm{tot}})^{-1} + (T_{D^*\bar{D}}^{\mathrm{tot}})^{-1}\right]$$

Then use this Ttot for observables

structure/3-body → a, r_0 → C(p)

N. Ikeno and E. Oset, PRD 112, 094019 (2025); B. Agatão et al., EPJC 85, 1136 (2025).

Observable: particle-resonance correlation function

$$C(p) = 1 + 4\pi \int dr\, r^2 S_{12}(r) \left\{ \left| j_0(pr) + TG \right|^2 - j_0^2(pr) \right\}$$

Meaning

$C(p)$ compares the measured two-particle distribution with an uncorrelated reference.

Ingredients

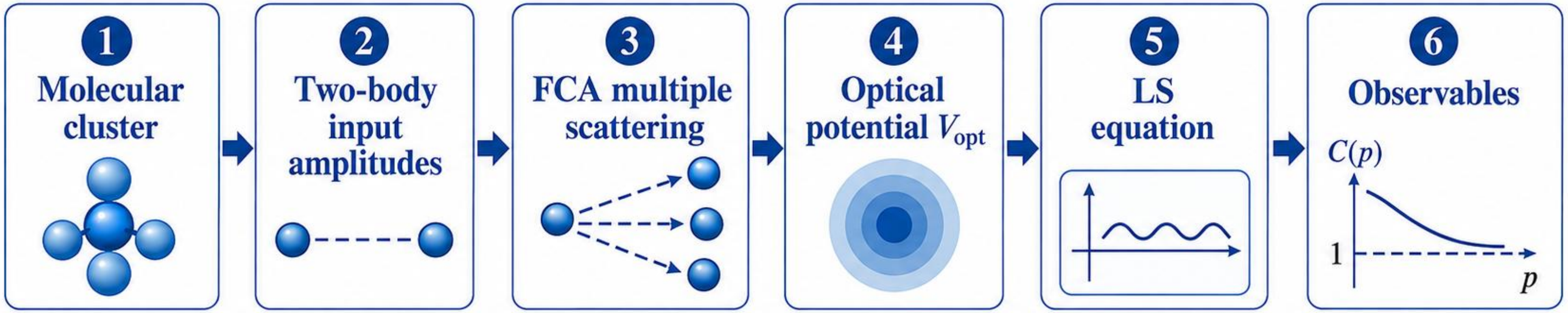
$S_{12}(r)$ is the source function and R is the source radius.
TG encodes the strong final-state interaction.
It is the scattering amplitude T times the propagator G.

Physical message

If the interaction is weak, $C(p)$ stays close to 1. Strong attraction or a nearby pole produces a clear low-p structure.

N. Ikeno et al., PLB 847, 138281 (2023); P. Encarnación et al., PRD 111, 114023 (2025).

Formalism: unitarization and observables



$$\tilde{T}' = \left[1 - \tilde{T} G_C\right]^{-1} \tilde{T} \qquad T_{\text{tot}} = \sum_{i,j} \tilde{T}'_{ij}$$

Low-energy parameters

$$-8\pi\sqrt{s}\,(T_{\text{tot}})^{-1} \approx -\frac{1}{a} + \frac{1}{2} r_0 q_{\text{cm}}^2 - i q_{\text{cm}}$$

- a: scattering length
- r_0: effective range

Correlation function

$$C(p) = 1 + 4\pi \int dr\, r^2\, S(r) \left\{ \left| j_0(pr) + T_G \right|^2 - j_0(pr)^2 \right\}$$

Gaussian source radius $R = 1.0,\ 1.5,\ 2.0$ fm

$C(p) \approx 1$: weak final-state interaction

Strong deviation at low p: femtoscopic imprint

Near-threshold pole: characteristic low-momentum signal

N. Ikeno and E. Oset, PRD 112, 094019 (2025); N. Ikeno et al., PLB 847, 138281 (2023).

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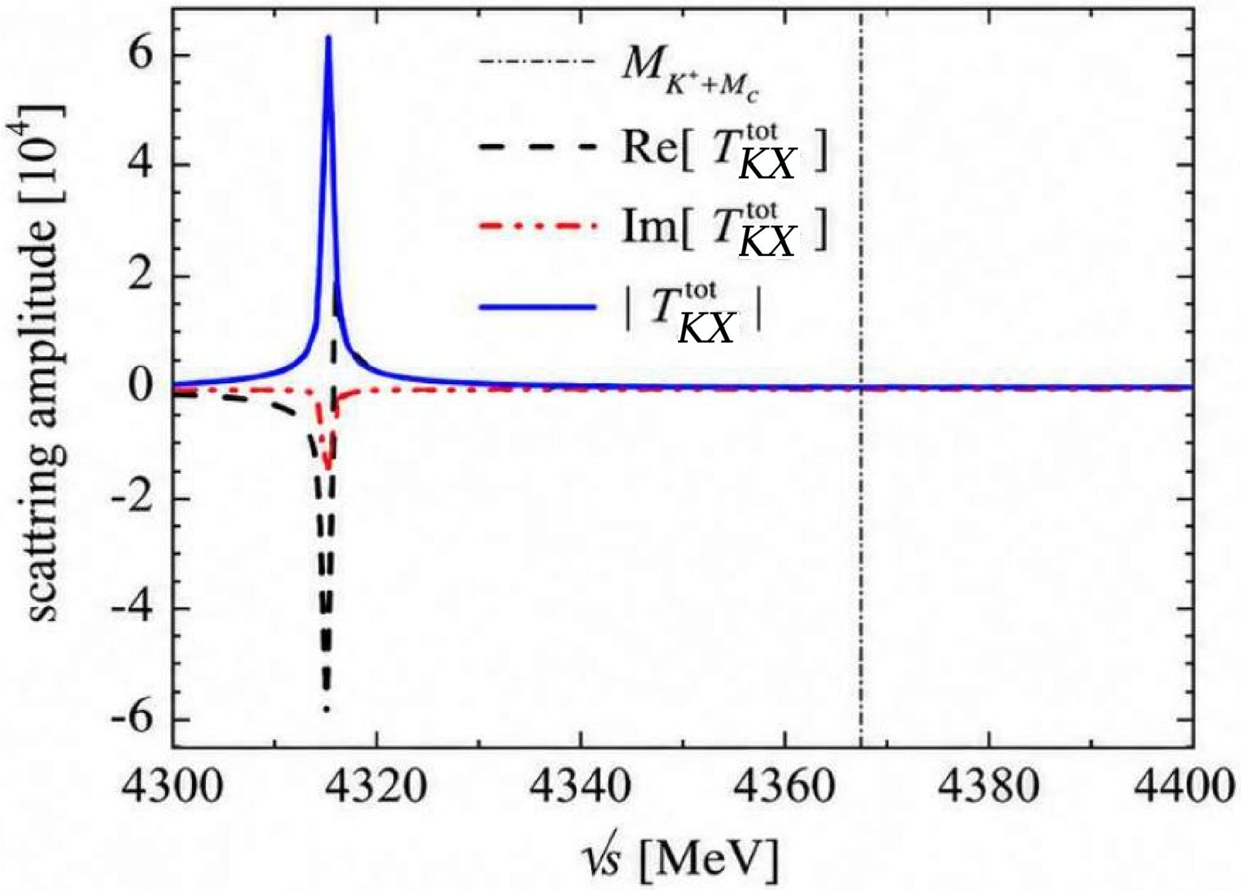
Result I: K X(3872) scattering amplitude

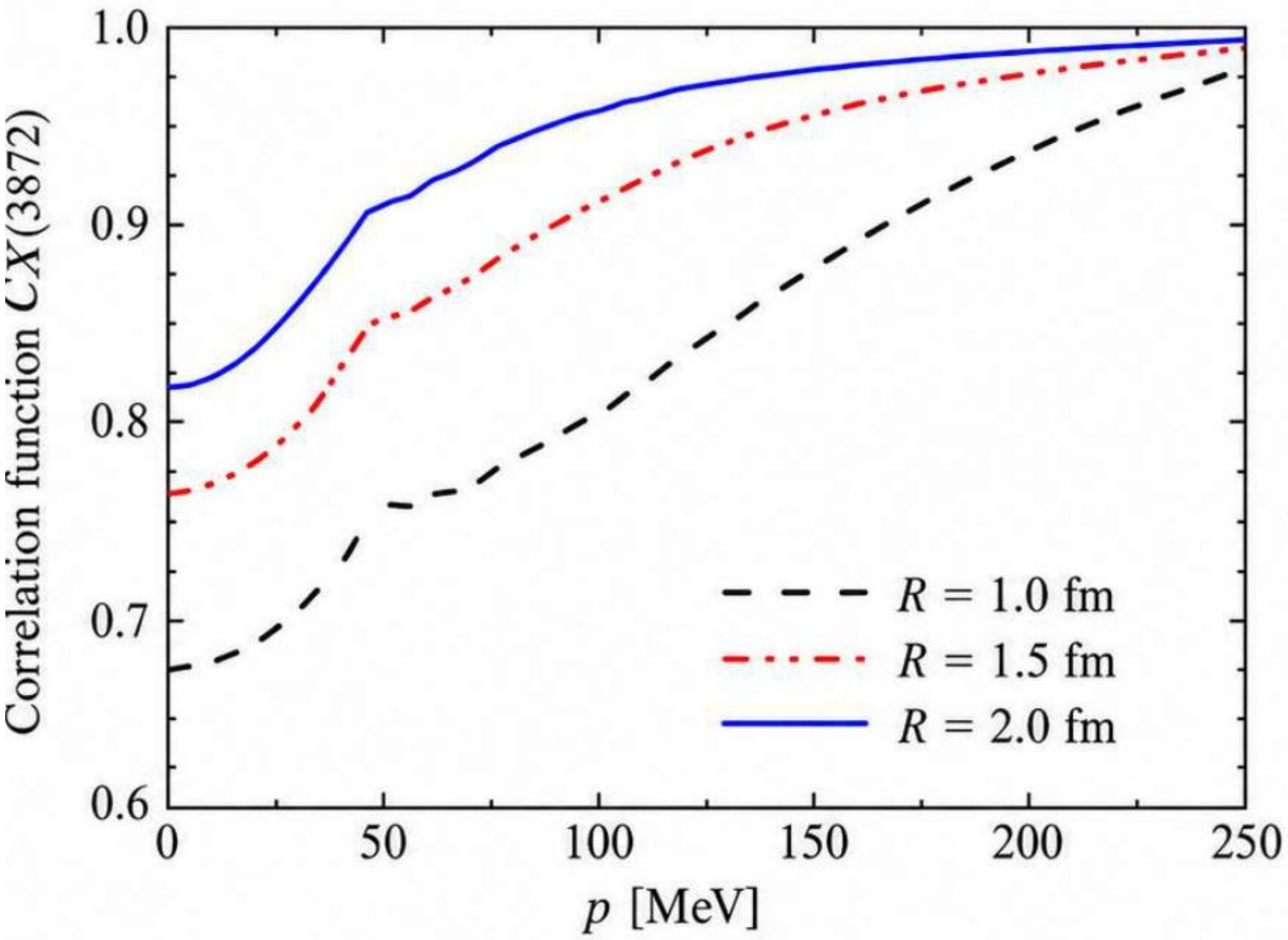


Predicted pole-like structure

- A narrow structure appears around 4315 MeV.
- It lies about 50 MeV below the KX(3872) threshold.
- The width is of order 1 MeV.
- The state is generated by combined KD and KD^* attraction.
- The relevant molecular subchannels are $D^*_{s0}(2317)$ and $D_{s1}(2460)$.

The sharp enhancement in $|T|$ together with the rapid behavior of $\mathrm{Re}(T)$ and $\mathrm{Im}(T)$ signals a three-body bound state below threshold.

Result I: K X(3872) correlation function



Main signal

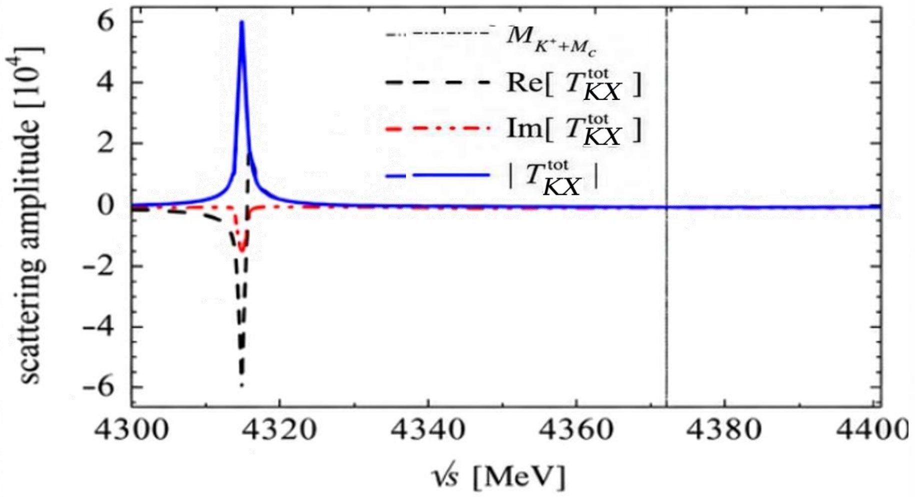
- $C(p)$ deviates clearly from unity at low momentum.
- The effect is visible for source radii $R = 1.0$, 1.5, and 2.0 fm.
- This behavior is the femtoscopic imprint of strong attraction.

Implication

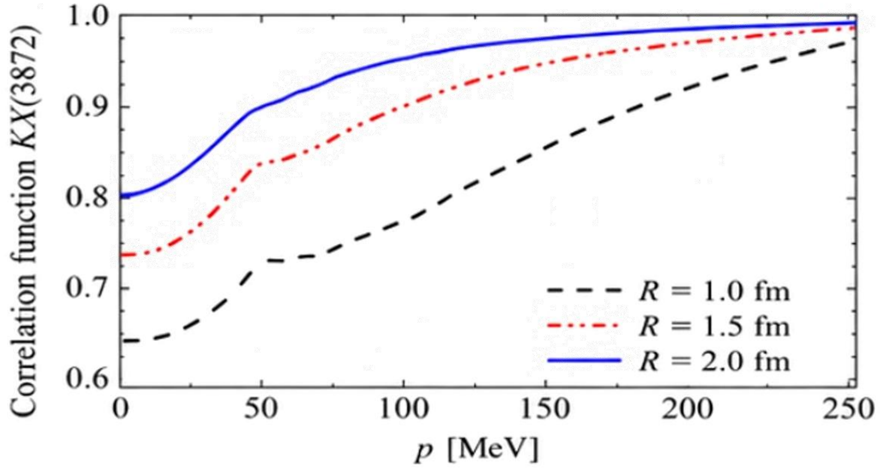
- A low-momentum correlation measurement can test the molecular interpretation of $X(3872)$.
- The correlation signal is complementary to the pole in the scattering amplitude.

Low-p correlation provides a direct experimental handle on the KX interaction.

Result I: K X(3872)



(a) Scattering amplitude



(b) Correlation function

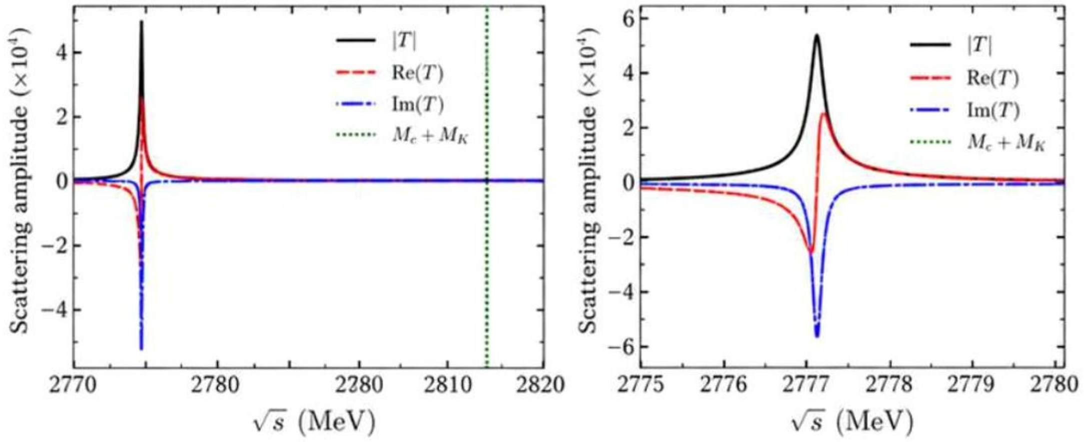
Main findings

- A narrow pole-like structure appears around 4315 MeV.
- It lies about 50 MeV below the $K + X(3872)$ threshold.
- Estimated width: about 1 MeV.
- Scattering length: $a = (0.39 - i\ 0.00)$ fm.
- Effective range: $r_0 = (1.16 - i\ 1.66)$ fm.
- The low-momentum correlation function shows a strong deviation from unity for $R = 1.0$–2.0 fm.

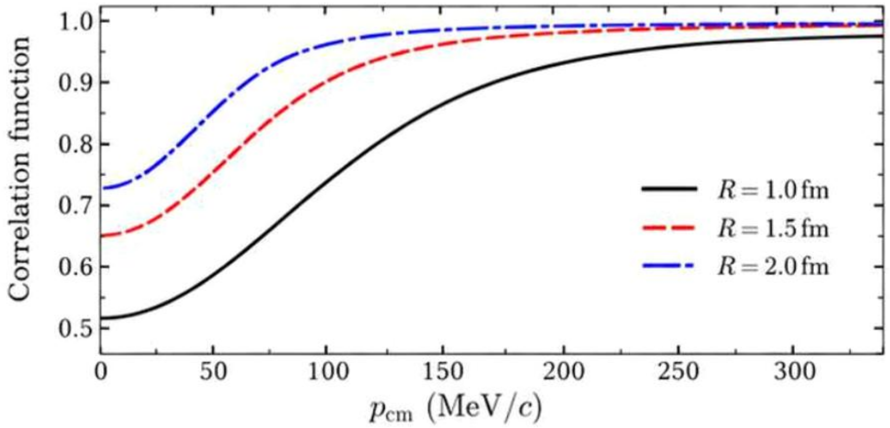
Physical message

Combined KD and KD^* attraction generates a narrow three-body bound state tied to the molecular nature of $X(3872)$.

Result II: K D_{s0}*(2317)



(a) (b)



Main findings

- A narrow structure appears around 2777 MeV.
- It lies about 40 MeV below the $KD^*_{s0}(2317)$ threshold.
- Estimated width: about 0.2 MeV (very narrow, sub-MeV).
- Scattering length: $a = (0.517 - 0.00016\,i)$ fm.
- Effective range: $r_0 = (-0.511 + 0.102\,i)$ fm.
- The correlation function shows a strong low-momentum deviation from unity.

Experimental signatures and unitary picture

Common message: if $X(3872)$ and $D^*_{s0}(2317)$ are hadronic molecules, kaon scattering shows their internal two-body attraction and gives a clear low-momentum correlation function.

System	Binding below threshold	Width	Correlation signal	Search channel
$K\,X(3872)$	~50 MeV	~1 MeV	strong low-p deviation in $C_{KX}(p)$	KX correlation; $K+X$ invariant mass; reconstruct $X(3872)$ via $J/\psi\,\pi^+\pi^-$
$K\,D_{s0}^*(2317)$	~40 MeV	~0.2 MeV	strong low-p deviation in $C_{KD_{s0}}(p)$	$K\,D_{s0}$ correlation; invariant mass below threshold; reconstruct $D_{s0}^*(2317)$ via $D_s^+\,\pi^0$

✓ **These observables are directly testable in future ALICE studies and complementary invariant-mass analyses.**

N. Ikeno and E. Oset, PRD 112, 094019 (2025); B. Agatão et al., EPJC 85, 1136 (2025).

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Experimental signatures, summary, and outlook

System	Binding below threshold	Width	Correlation signal	Search channel
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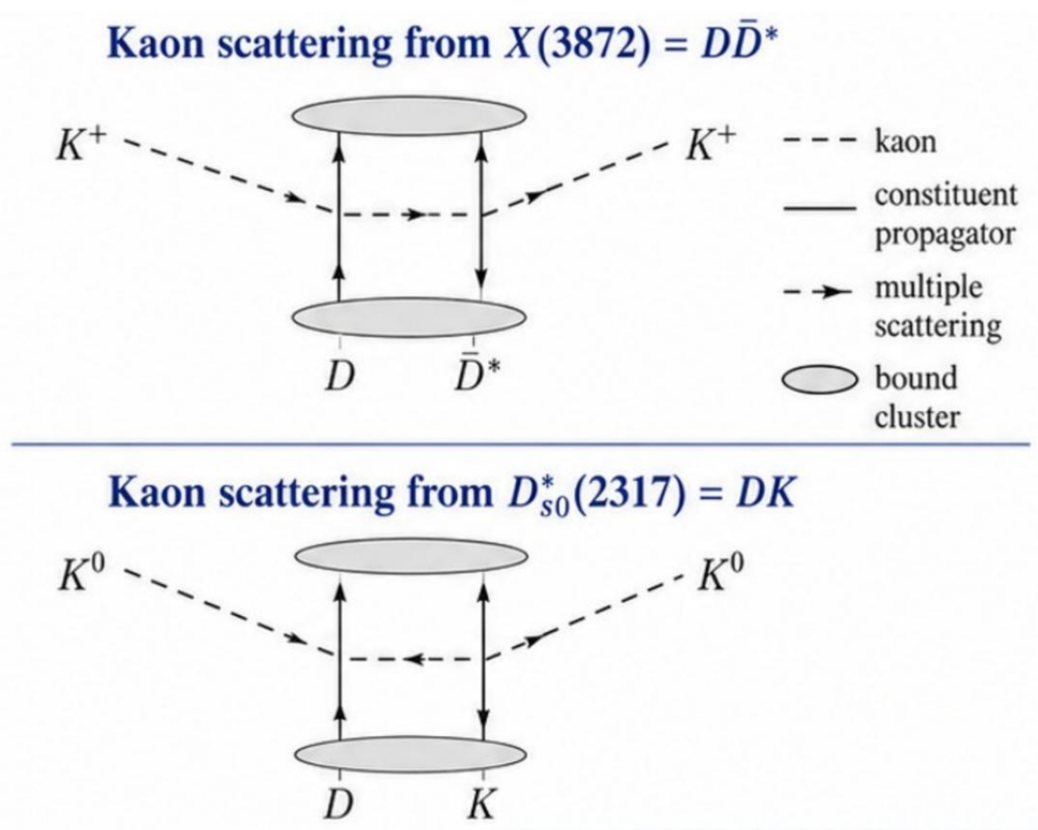
$K\,D_{s0}^*(2317)$	~40 MeV	~0.2 MeV	strong low-p deviation in $C_{KD_{s0}}(p)$	$K\,D_{s0}$ correlation; invariant mass below threshold; reconstruct $D_{s0}^*(2317)$ via $D_s^+\,\pi^0$

- Unitary FCA + LS description of kaon–molecule scattering.
- Narrow three-body bound states appear below threshold.
- Low-p correlations provide clear signatures of attraction.
- ALICE measurements can test the predicted dynamics and molecular picture.

Thank you for your attention!

Formalism I: FCA for kaon scattering from a bound cluster



- Treat the hadronic molecule as a preformed bound cluster.
- The external kaon undergoes repeated scattering from the cluster constituents.
- The multiple-scattering series is resummed within the fixed-center approximation.
- Two-body amplitudes t_1 and t_2 and the propagator G_0 enter the FCA equations.

$$T_1 = t_1 + t_1 G_0 T_2 \qquad T_2 = t_2 + t_2 G_0 T_1 \qquad T_{tot} = T_1 + T_2$$

Physical picture: repeated kaon scattering probes the molecular substructure of the target.

S. S. Kamalov et al., NPA 690, 494 (2001); L. Roca and E. Oset, PRD 82, 054013 (2010).