Effective field theory (and phenomenological) aspects of hadronic molecules: circling around the idea of compositeness

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Including content from FZ Peng, MJ Yan, ZY Yang, M Sánchez

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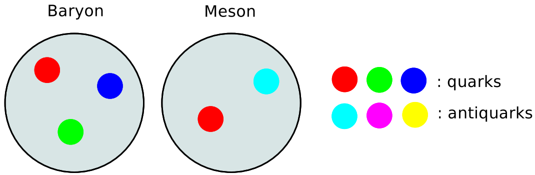
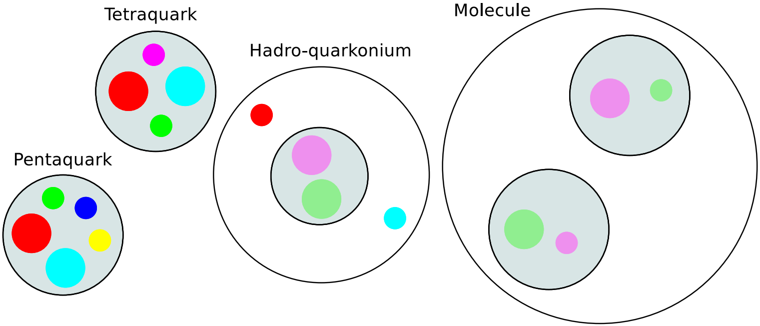
Exotic hadrons

Exotic hadrons

Standard hadrons come in two varieties

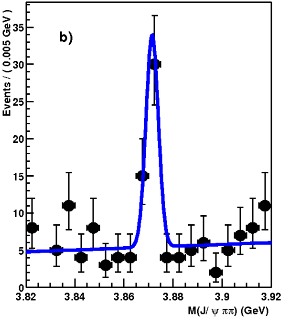


But there are more types of possible hadrons...



Exotic hadrons: the X(3872)

Exotic hadrons became extremely popular thanks to a discovery by the Belle collaboration in $B^\pm \to K^\pm J/\Psi\pi\pi$ (03):



Looks molecular, but no wide consensus about its nature yet!

Exotic hadrons: are you a fox or a hedgehog?

Phillip Tetlock: Expert political judgement, how good it is? (2005)
(hint: as good as dart-throwing chimps... except for the foxes)

- **Hedgehog**: knows one big idea (intellectual economy)
  Resistance to update priors Convergence Fav word: Moreover
- **Fox**: knows many little ideas (intellectual scavenger)
  Bayesian operators Zigzagging Fav word: However

A "thought ecosystem" (though selling stories = better life outcomes).
Yet, hadron physics is messy: better lean to the fox side.

Exotic hadrons: practicing your fox skills with the $X(3872)$

For $X(3872)$: contradictory/ambiguous information to be balanced

(i) Close to $D^*\bar{D}$ threshold: large coupling with it

Tornqvist hep-ph/0308277; Voloshin PLB 579, 316; Braaten, Kusunoki PRD 69, 074005

(ii) $X \to \psi(nS)\gamma$, $n = 1, 2$: $c\bar{c}$ core Guo et al. PLB 742 (2015) 394-398

(iii) $X \to J/\psi\, 2\pi$ and $X \to J/\psi\, 3\pi$ pattern easier to explain in molecular picture Gamermann, Oset PRD 80 (2009) 014003 ...but compact state can also have this branching ratio Swanson PLB 588 (2004) 189-195

(iv) $X(4014)$ by Belle (predicted mol partner, but poor statistics)

Often forgotten fact:
the wave function is not an observable

Though it seems to lean more to molecular than not...

But before dealing with the problem of molecularness:

EFT crash course in one slide

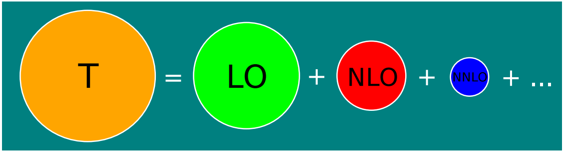
Effective field theories: a crash course

- Generic low energy descriptions of physical phenomena

$$\underbrace{\text{quarks \& gluons}}_{\text{high energy}} \quad \Longleftrightarrow \quad \underbrace{\text{hadrons } (D^{(*)},\ \Sigma_c^{(*)},\ \pi)}_{\text{low energy (EFT)}}$$

  (equivalent by virtue of the renormalization group)

- Everything within EFT is an expansion:



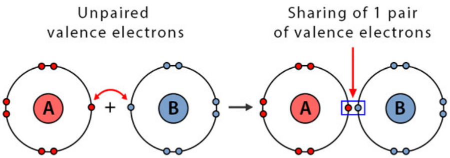
  Calculations are amenable to error estimations
  (that is, the expected size of the next blob)

- Hadronic molecules: scarcity of data, that is,
  usually leading order (LO) is enough in most cases

What is a molecule?

Exotic hadrons: what is a molecule?

Chemistry textbook molecules (a.k.a. actual molecules):



Hadronic molecules: definitely not two clearly separated heavy quarks sharing a pair (or a few pairs) of light-quarks

(Exception: Born-Oppenheimer approx. with exotics)

But the name is catchy! $\Rightarrow$ We adopted it ;)

Here: $|\text{molecule}\rangle = \sqrt{(1-\delta)}|H_1 H_2\rangle + \sqrt{\delta}|\text{other things}\rangle$, δ smallish

And... we obviate the evident lack of rigor with this, as usual.

(After all, we are physicists...)

EFT and Spectroscopy

Spectroscopy in EFT: Basic setup

Two-hadron interaction depends on EFT & counting choice:

(a) Are we using pionless or pionful EFT?

(b) Within pionful EFT: perturbative or non-perturbative pions?

Most common setup: pionful EFT w/ perturbative pions at LO $^{(*)}$
(which is indistinguishable from pionless EFT at this oder)

$$V^{\mathrm{LO}} = V_C^{\mathrm{LO}} = C_a + C_b\, \hat{\vec{S}}_{L1} \cdot \hat{\vec{S}}_{L2} + \dots$$

(i) Includes HQSS: dependence on spin of light quarks only

(ii) Contact w/ multipolar type expansion (E0, M1, E2, M3, etc.) (E1, M2, E3, etc. terms possible for $H_1 H_2 \to H_2 H_1$ non-diagonal transitions if H_1, H_2 have different parities$^{(**)}$)

(iii) Couplings are in principle unknown and have to be fitted

$(*)$ HQSS constraints on contacts common since AlFiky et al. (06); the LO EFT interpretation came later; $(**)$ Peng, Yan, Sanchez and myself (23)

Spectroscopy in EFT: No predictions without data

EFT couplings determined by existing data
$\Rightarrow$ this limits predictive power (probably for the best)

(a) X(3872) as 1^{++} $D^*\bar{D}$ has $V_C = C_a + C_b$ implies a 2^{++} $D^*\bar{D}^*$ partner because $V_C = C_a + C_b$ too (if assumptions behind this EFT description are correct)

(b) $P_c(4312)$, $P_c(4440)$, $P_c(4457)$ as $\bar{D}\Sigma_c$, $\bar{D}^*\Sigma_c$ bound states imply either a $1/2^-$ or a $5/2^-$ $\bar{D}^*\Sigma_c^*$, maybe both $^{(*)}$.

More rarely the LO finite-range potential might be of enough strength as to predict binding (without requiring hard cutoffs) $^{(**)}$.

(*) Most of the people within the audience has at least one paper about this.

(**) Examples might be: Manohar, Wise (93), Cohen, Gelman, Nussinov (03); for the EFT-style formulation check Lu, Geng, Ren and myself (18)

Spectroscopy in EFT: Phenomenology & saturation

One might circumvent this limitations by means of saturation

$$C = C_V + C_S + \dots$$

w/ C_V (C_S) the contribution from vector (scalar) mesons:

(a) C_a becomes very often attractive if scalars included

(b) Stablishes a hierarchy (grounded on multipolar expansion):

$$|C_a(E_0)| > |C_b(M_1)| > |C_c(E_2)| > \dots$$

Problem: most applications of this idea only take into account vector meson exchange $^{(*)}$. Fails a trivial base-rate check:

- S-wave nucleon-nucleon interaction attractive and strong, but vector meson saturation alone implies repulsive ${}^1S_0$ and ${}^3S_1$:

$$C_V^{E0}({}^1S_0) \propto +10\,\frac{g_V^2}{m_V^2} \quad , \quad C_V^{E0}({}^3S_1) \propto +6\,\frac{g_V^2}{m_V^2}$$

(*) Including people in the audience ;)

Spectroscopy in EFT: Subleading two-pion exchange

NN pionful EFT generates a really strong isoscalar central force from two-pion exchange at Q^3 (breaking counting expectations)

(a) Plausible reason for SU(4)-Wigner symmetry, or the fanciful information optimization conjecture from the Seattle group

(b) At the practical level, the EFT version of sigma exchange

If this were to be the case in most two hadron systems, then

(c) Effectively this would require $C_a < 0$ and strong at LO: the spectrum should be rather rich (e.g. $D\bar{D}$ expectable)

However, situation not clear: depends on the determination of certain LECs. A recent result indicates $D\bar{D}$ Q^3 TPE repulsive $^{(*)}$

(∗) However this depends on saturation assumptions plus sign inconsistencies in the literature, both affecting the relevant LEC, though couldn't check this thoroughly.

EFT: decays and production

Spectroscopy in EFT: decays and production

EFT applicable to decays (or production) for low momenta

(a) $X \to D\bar{D}\pi$, $X \to D\bar{D}\gamma$, same for T^+_{cc}, analogous calculations involving pentaquarks, etc.

The limitation is that this usually involve relatively narrow widths (wrt to the toal width of the state) for which there are rarely good experimental data.

(b) $\mathcal{A}_{\rm prod} = P + PGT(E)$, with P non-resonant background and $T(E)$ calculable within EFT in certain cases

Maybe more explicit examples than for decays ($X(3872)$ by Braaten, Lu; $Z_c(3900)$ by Albaladejo et al.; Pentaquaks by Du, Baru et al. ; T_{cc} by Albaladejo)

Compositeness and EFTS

Compositeness: Basic ideas

The state wave function is a mixture of a two-body and another (presumably compact) component:

$$|\text{state}\rangle = \sqrt{X}\,|H_1 H_2\rangle + \sqrt{Z}\,|\text{other things}\rangle$$

and we want to determine X and Z. But:

(a) **The wave function is not an observable**: forget about a model-independent determinations of X and Z...
I think this has already sunk in, or it seems so in recent compositeness literature, but not sure: testing the waters here.

(b) It might be better to talk about the part of the wave function inside and outside our Hilbert space, not necessarily about molecularness and compositeness.
e.g. two-body channels not included, D-wave probabilities, etc.

Compositeness: Basic QM considerations

Trivial observation: if the two-body part of the wf takes the form

$$\Psi_{2B}(r) = \frac{u(r)}{\sqrt{4\pi}\,r} \quad \text{with} \quad u(r \to \infty) \to A_S\, e^{-\gamma r} \,,$$

then the two-body probability will simply be

$$X = \int_0^\infty u^2(r) dr \,, \quad \text{where a pair of comments are in order:}$$

(a) If the Hilbert space contains only the two-body channel, for energy independent potentials X is always equal to one.

(b) But if the potential is genuinely energy dependent though, then one has to change the normalization condition to

$$\int_0^\infty u^2(r) \left(1 - \frac{d}{dE} V(r) \right) dr = 1$$

Compositeness: Weinberg criterion reinterpeted (I)

Usually we do not know the full potential, but might know the asymptotics of the wave function:

$$u(r \to \infty) \to u_0(r) \quad \text{with} \quad u_0(r) = A_S\, e^{-\gamma r}\,,$$

with A_S the asymptotic normalization coefficient (ANC), and then:

$$X_{\rm ANC} = \frac{A_S^2}{2\gamma} = \int_0^\infty u_0^2(r) dr \neq 1$$

simply because we do not know the full wave function.

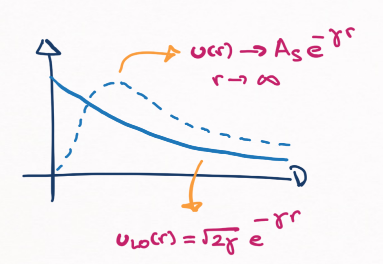
Deuteron: $A_S = 0.8846\,{\rm fm}^{-1/2}$ and $\gamma = 0.2316\,{\rm fm}^{-1}$, leading to

$$X_{\rm ANC} \approx 1.69 \quad \text{which is indeed larger than one!}$$

Weinberg's original X is equivalent to $X_{\rm ANC}$ and there is absolutely no problem with it being larger than one.

Compositeness: Weinberg criterion reinterpeted (II)

If one considers the typical form of the wave function:



(i.e. short distance growth, maximum, exponential tail),

then it is sensible to think:

(a) $X_{\rm ANC} > 1$ probably implies two-body description complete.

(b) $X_{\rm ANC} < 1$ probably implies the necessity of degrees of freedom not included in our Hilbert space.

Yet, $X_{\rm ANC}$ is not X (which is unobservable) but a proxy for it.

Compositeness: X is better thought as a guideline

X is preferably interpreted as a guideline: trying to refine it beyond the qualitative level may be more tricky than expected.

Though X is claimed to be close to model independence for $\gamma \to 0$

$$a_0 = \frac{2X}{1+X}\,\frac{1}{\gamma}\,\left[1 + \mathcal{O}(\frac{Q}{M})\right] \,,$$

by Kinugawa-Hyodo, the problem is the existence of a *threshold universality* $^{(*)}$ ($X \to 1$ for $\gamma \to 0$) which in EFT terms implies

$$1 - X = \mathcal{O}(\frac{Q}{M}) \,, \quad \text{for } \gamma \sim Q$$

and thus difficult to separate from the range corrections, unless the coefficient in front of Q/M is clearly non-natural.

$(*)$ appears in lots and lots of works (Hyodo & cia, the Sino-German paper-industrial complex, Dai/Song/Oset, already mentioned on Weinberg)

Compositeness: X and the effective range expansion

Reminder: there is a connection between the residue of the T-matrix and the ANC, which happens to be really useful: $^{(*)}$

$$T(E \to E_B) \to \frac{\phi^2}{E - E_B} \text{ with } \phi = \frac{\sqrt{4\pi} A_S}{2\mu} \text{ for contact theories.}$$

For T given by the effective range expansion (valid for $k < m/2$):

$$T(E) = -\frac{2\pi}{\mu} \frac{1}{k \cot\delta - ik} \quad , \quad k \cot\delta = -\frac{1}{a_0} + \frac{1}{2} r_0 k^2 + \sum_{n=2}^{\infty} v_n k^{2n}$$

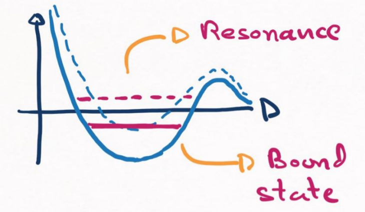
then one has: $$X_{\mathrm{ANC}} = \frac{1}{1 - \gamma r_0 + \sum_{n=2}^{\infty} 2n\,(-1)^n \gamma^{2n-1} v_n}$$

Thus $r_0 > 0$ implies composinetess & $r_0 < 0$ suggests elementarity

$(*)$ Basically derived in every other paper in the field

Compositeness: X depends on counting

However there are potentials with large, negative effective ranges:



but yet they produce states that are fully composite!

...where r_0 unrelated range of interaction. Solvable though if

(a) one redefines $X_{\rm ANC} = A_S^2/(A_S^{\rm LO})^2$

(b) one notices that this situation requires r_0 to be LO, leading to $(A_S^{\rm LO})^2 = 2\gamma/(1-\gamma r_0)$, $X_{\rm ANC} = (1-\gamma r0)/(1-\gamma r_0 + 4\gamma^3 v_2)$

$\Rightarrow$ Compositeness depends now on sign of v_2, apparently: $v_2 < 0$ leans molecular, $v_2 > 0$ leans composite

Relation between counting and compositeness previously discussed in van Kolck (22)

Compositeness: LO finite-range forces

If $V = V_S + V_L$ (decaying as m_S, m_L) with V_L known, there is the modified effective range expansion (MERE)[(*)]:

$$K_0^M(k) = \mathcal{C}^2(k)\, k\, \cot\delta_M + h(k) = -\frac{1}{a_{0M}} + \frac{1}{2}\, r_{0M} k^2 + \sum_{n=2}^{\infty} v_{nM} k^{2n}$$

with $\delta_M = \delta - \delta_L$ and valid for $k < m_S/2$ (instead of $k < m_L/2$).

The functions $C(k)$ and $h(k)$ are defined from the $r \to 0$ behavior of the V_L-only wave functions behaving asymptotically ($r \to \infty$) as

$$f_{L,k}(r) \to \sin(kr + \delta_L)\ ,\ g_{L,k}(r) \to \sin(kr + \delta_L)$$

$$\text{from which:}\quad \begin{aligned} \mathcal{C}(k) &= \frac{1}{g_{L,k}(0)} = \lim_{r\to 0} \frac{f_{L,k}(r)}{kr} \\ h(k) &= \frac{g'_{L,k}(0)}{g_{L,k}(0)} = \lim_{r\to 0} kr\, \frac{g'_{L,k}(r)}{f_{L,k}(r)} \end{aligned}$$

(*) Haeringen & Kok (82); notice though that here I used a custom derivation.

Compositeness: Residues with finite-range forces (I)

Again, the residue of the T-matrix is still proportional to A_S^2 when there are long-range forces. One might write:

$$T(E) = -\frac{2\pi}{\mu}\,\frac{1}{k\,\cot\delta_L - ik} - \frac{2\pi}{\mu}\,\frac{e^{2i\delta_L}}{k\,\cot\delta_M - ik}$$

insert the MERE in the second term

$$\frac{1}{k\,\cot\delta_M - ik} = \frac{\mathcal{C}^2(k)}{K_0^M(k) - h(k) - ik\,\mathcal{C}^2(k)}$$

and then calculate the ratio of the residues at the pole depending on the number of MERE parameters that one includes.

Compositeness: Residues with finite-range forces (II)

The bound state condition with the MERE is:

$$K_0^M(\gamma) + H(\gamma) = 0 \quad \text{with} \quad H(\gamma) = -\frac{f_L'(0,\gamma)}{f_L(\gamma)}$$

with $f_L(r,\gamma) \to e^{-\gamma r}$ solved with $V = V_L$, and $f_L(\gamma) = f_L(0,\gamma)$.

One obtains then:

$$X_{\rm ANC}^M = \frac{dH(\gamma)}{dH(\gamma) - \gamma\, r_{0M} + \sum_{n=2}^{\infty} 2n\,(-1)^n\,\gamma^{2n-1}\,v_{nM}}$$

where $dH = dH(\gamma)/d\gamma$ (notice the analogy with the recent Kinugawa and Hyodo analysis for Coulomb).

Again, compositeness depends on the sign and magnitude of r_{0M}

For a derivation in terms of formal scattering theory, loop functions, etc., GPT-5.6 can get you there in a couple of prompts. As a freebie, contemplate your own obsolescence.

Compositeness: Residues with finite-range forces (III)

All the previous caveats still apply.

Besides there is a very common situation yielding $X_{\mathrm{ANC}} < 1$: when the short-range potential V_S is repulsive.

For instance, the following two-exponential model:

$$V = V_L + V_S = \alpha_S e^{-m_S r} + \alpha_L e^{-m_L r} \,,$$

For $\alpha_S = 500\,\mathrm{MeV}$, $m_S = 750\,\mathrm{MeV}$, $\alpha_L = -150\,\mathrm{MeV}$, $m_L = 200\,\mathrm{MeV}$ and $2\mu = M_N$, $\exists$ bound state at $4.45\,\mathrm{MeV}$:

(a) One has $r_{0M} = -1.21\,\mathrm{fm}$, large ($|m_S r_{0M}| \sim 5$) and negative!

(b) But $dH(\gamma) = 33.7$, rather large!

This results in $X_{\mathrm{ANC}}^M \approx 0.988$ (to be compared with the exact ratio $X_{\mathrm{ANC}}^M = 0.986$), so this counterexample might not be that serious.

Compositeness: Perturbative Yukawa

Most common situation for two charmed hadrons:

(a) LO a non-derivative contact,

(b) NLO derivative contact & OPE, which is a Yukawa in S-wave:

Effective range fixed, plus the potential: $2\mu V_{\rm OPE} = + \dfrac{e^{-\mu_\pi r}}{R_\pi\, r}$

In this case $dH = 1 - \frac{2}{R_\pi (\mu_\pi + 2\gamma)}$, $\mathcal{C}^2(k) = 1 - \frac{1}{k R_\pi} \mathrm{atan} \frac{2\,k}{\mu_\pi}$,
$h(k) - h(0) = \frac{1}{2 R_\pi}\, \log\left(1 + \frac{4k^2}{\mu_\pi}\right)$, from which one may derive:

$$X_{\rm ANC}^{\rm OPE} = \frac{1}{1 - \gamma (r_0 + \delta r_0)} \quad \text{with} \quad \delta r_0 = \frac{2}{R_\pi \mu_\pi^2} + \dots$$

implying that if $V_{\rm OPE}$ is repulsive, $r_0 < (-\delta r_0)$ for $X_{\rm ANC}^{\rm OPE} < 1$

(that is, negative r_0 alone is no longer enough for $X < 1$)

Compositeness and the deuteron D-wave probability

The deuteron D-wave probability (P_D): **not an observable!**

(a) Deuteron wave function:

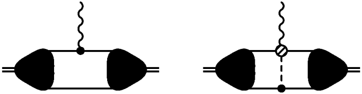
$$|d\rangle = \cos\theta_D |^3S_1\rangle + \sin\theta_D |^3D_1\rangle$$

(b) Deuteron magnetic moment: $\mu_{exp} = 0.86\,\mu_N$,
but $\mu(^3S_1) = 0.88\,\mu_N \Rightarrow \exists$ non S-wave component

(c) D-wave probability $P_D \sim (3-5)\%$, but with assumptions:

(c.1) No relativistic corrections included Gilman, Gross JPG 28, R37

(c.2) No two-body currents included D.R. Phillips, JPG 34, 365



$O(e)$ $O(eP^3)$

That is: within EFT P_D still makes sense at lower orders.

Conclusions (list)

- EFT for hadronic molecules
  - Generally a very simple formulation at LO
  - Great theoretical analysis tool, particularly to uncover implicit assumptions (e.g. regarding compositeness)
  - Data hungry though, not suited for *a priori* predictions
- How molecular is a state? Fundamentally not observable:
  - Thus compositeness X is model-dependent, admits multiple definitions, depends on assumptions about the system
  - But still potentially useful as a guideline
  - Easily expanded to include finite-range interactions
  - Useful to think beyond compositeness X: deuteron P_D as a template
  - Molecularness makes more sense in specific contexts (e.g. lowest orders within a given EFT), but not always!

Thanks For Your Attention!

Extra Slides

Compositeness: molecular or not? (the $T_{cc}^+(3875)$)

The T_{cc}^+ decay width into $DD\pi$ and $DD\gamma$:

(a) T_{cc}^+ wave function:

$$|T_{cc}\rangle = \cos\theta_C |D^* D\rangle + \sin\theta_C |cc\bar{u}\bar{d}\rangle$$

(b) T_{cc}^+ width: $\Gamma_{\rm exp}^{(*)} = 48 \pm 2^{+0}_{-12}\,\mathrm{KeV}$, but if $\Gamma_{\rm th}^{\rm mol} > \Gamma_{\rm exp}$
$\Rightarrow \exists$ non molecular component (provided $\Gamma_{\rm th}^{\rm tetra} \ll \Gamma_{\rm th}^{\rm mol}$)

(c) Same caveats as in the deuteron (also $\exists$ T_{cc}'s D-wave)

What do we have? Well... (hint: molecularness can be determined)

$$\Gamma_{\rm th}^{\rm LO} = 49 \pm 3 \pm 16\,\mathrm{KeV} \quad , \quad \Gamma_{\rm th}^{\rm NLO} = 58^{+5}_{-3} \pm 5\,\mathrm{KeV}$$

And this is with $\Lambda \to \infty$ (otherwise $\Gamma_{\rm th}^{\rm LO} > \Gamma_{\rm exp}^{(*)}$ already.)

If T_{cc} not highly molecular $\Rightarrow$ no T_{cc}^* ($D^* D^*$) partner

From arguments analogous to those in Cincioglu et al. EPJC76, 576

(*): not the actual T_{cc} experimental width, includes theoretical assumptions.

Compositeness: molecular or not? (the $P^{\Lambda}_{\psi s}(4338)$)

The $P^{\Lambda}_{\psi s}(4338)$ slightly above threshold: not describable with your usual single channel, energy- and momentum-independent contact.

How molecular is it then? Use $X_{\rm mol} = \sqrt{\frac{1}{1+2|\frac{r_0}{a_0}|}}$ Matuschek et al. EPJA 57, 101

(a) Energy-dependent: $V_C = d_a + 2\, d_{2a}\, k^2 \Rightarrow X_{\rm mol} = 0.33$

(b) Momentum-dependent:

$$V_C = d_a + d_{2a}\,(p^2 + p'^2) \Rightarrow X_{\rm mol} = 0.95$$

(c) Coupled-channel:

$$V_C = \begin{pmatrix} \frac{1}{2}(d_a + \tilde{d}_a) & \frac{1}{\sqrt{2}}(d_a - \tilde{d}_a) \\ \frac{1}{\sqrt{2}}(d_a - \tilde{d}_a) & d_a \end{pmatrix} \quad \Rightarrow \quad X_{\rm mol} = 0.77$$

(a) and (b) on-shell equivalent, but different $X_{\rm mol}$ (that is, a counterexample) $\Rightarrow$ non-observability of the wave function