Predicting $B^{(*)}\overline{D}^{(*)}$ states from a reanalysis of lattice QCD data

Pan-Pan Shi (IFIC)

Collaborators: M. Albaladejo, M.-L Du, F.-K Guo, V. Montesinos, and J. Nieves

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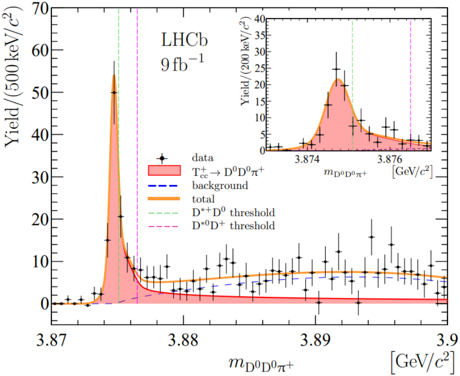
Outline

T_{cc}^+ and its flavor partners

- T_{cc}^+ state LHCb, Nature Commun.13(2022)3351, Nature Phys.18(2022)751



  - Pole position: this pole lies $-273 \pm 61 \pm 5^{+11}_{-14}$ keV (Breit-Winger fit) below the $D^{*+}D^0$ threshold with width $410 \pm 165 \pm 43^{+18}_{-38}$ keV
  - Minimal quark content: $cc\bar{u}\bar{d}$ without isospin partner
  - Compositeness: Z<0.52(0.58) at 90(95)% CL

- Based on the heavy quark flavor symmetry, T_{cc}^+ should has heavy quark partner with quark content $cb\bar{u}\bar{d}/bb\bar{u}\bar{d}$ S. Sakai, et al, PRD96(2017)054023; Luciano M. Abreu, NPB 985(2022)115994; M. Abolnikov, et al, PRD 113(2026)114041

Lattice results in the $B^{(*)}\overline{D}^{(*)}/D^{(*)}\overline{B}^{(*)}$ system

- Signal for $I(J^P) = 0(1^+)$: the ground-state EL below the $\overline{D}B^*$ threshold [-61,-15] MeV with $m_\pi \simeq 164$, 299, and 415 MeV ($[ud][\bar{c}\bar{b}]$ + local $\overline{D}B^*$ operators) A. Francis, et al, PRD99(2019)054505
- No signal for $0(0^+, 1^+)$ systems: the ground-states EL above the corresponding $\overline{D}B$ and $\overline{D}B^*$ thresholds with $m_\pi \simeq 192$ MeV ($[ud][\bar{c}\bar{b}]$ + local $\overline{D}B^*$ operators) R.J. Hudspith, et al, PRD102 (2020) 114506
- No signal for $0(1^+)$ system: the ground-state EL is compatible with the $\overline{D}B^*$ threshold with $m_\pi \simeq 139 - 431$ MeV ($[ud][\bar{c}\bar{b}]$ + local/nonlocal $\overline{D}B^*$ operators) S. Meinel, et al, PRD106(2022)034507
- Bound states in the $0(0^+, 1^+)$ systems: $B\overline{D}$ and $B^*\overline{D}$ bound states with binding energies $E_b = -0.5^{+0.4}_{-1.5}$ and $-2.4^{+2.0}_{-0.7}$ at $m_\pi \simeq 220$ MeV ($[ud][\bar{c}\bar{b}]$ + local/nonlocal $\overline{D}B^*$ operators) C. Alexandrou, et al, PRL132(2024)151902
- Extrapolation of $\bar{u}\bar{d}cb$ system with $0(0^+, 1^+)$: $\bar{B}D$ and $\bar{B}^*D$ bound states with binding energies $E_b = -39^{+4+8}_{-6-18}$ and -43^{+6+14}_{-6-24} MeV at the physical pion mass ($[ud][\bar{c}\bar{b}]$ + local $\overline{B}D/\overline{B}^*D$ operators at $m_\pi \sim 0.5 - 3.0$ GeV) M. Padmanath, et al, PRL132(2024)201902, PRD 110(2024)034506

Outline

T-matrix in the infinite volume

- Isoscalar $B^{(*)}\overline{D}^{(*)}$ systems

$$\begin{aligned} 0^+ &: \left\{B\overline{D},\ B^*\overline{D}^*\right\}, \\ 1^+ &: \left\{B\overline{D}^*,\ B^*\overline{D},\ B^*\overline{D}^*\right\}, \\ 2^+ &: \left\{B^*\overline{D}^*\right\}. \end{aligned}$$

- T-matrix in the infinite volume (Lippmann-Schwinger equation)

$$T_{\eta\delta}(E,\boldsymbol{p},\boldsymbol{k}) = \sum_{\alpha\beta} F^{\Lambda}_{\eta\alpha}(\boldsymbol{p})\, T'_{\alpha\beta}(E,\boldsymbol{p},\boldsymbol{k}) F^{\Lambda}_{\beta\delta}(\boldsymbol{k}),$$

with

$$T'_{\alpha\beta}(E,\boldsymbol{p},\boldsymbol{k}) = V_{\alpha\beta}(E,\boldsymbol{p},\boldsymbol{k}) + \sum_{\gamma}\int \frac{d\boldsymbol{q}^3}{(2\pi)^3} V_{\alpha\gamma}(E,\boldsymbol{p},\boldsymbol{q}) I^{\Lambda}_{\gamma\gamma}(E,\boldsymbol{q})\, T'_{\gamma\beta}(E,\boldsymbol{q},\boldsymbol{k}).$$

Here the Gaussian form factor $F_\Lambda(p_i) = \exp(-p_i^2/\Lambda^2)$ for heavy-meson channels is used to ensure unitarity of T-matrix. The Green's function is

$$I^{\Lambda}_{\alpha\beta}(E,\boldsymbol{q}) = \frac{\delta_{\alpha\beta}\, e^{-2\boldsymbol{q}^2/\Lambda^2}}{E - (\omega_1(\boldsymbol{q}) + \omega_2(\boldsymbol{q})) + i\varepsilon}.$$

Short-range interactions from HQSS

At LO, the interactions between heavy mesons are described by the contact term

- Isoscalar 0^+ systems: $\{B\overline{D},\ B^*\overline{D}^*\}$

$$V_C^0(B\overline{D}) = C_{0a},$$
$$V_C^0(B^*\overline{D}^*) = C_{0a} - 2C_{0b}.$$

- Isoscalar 1^+ systems: $\{B\overline{D}^*,\ B^*\overline{D},\ B^*\overline{D}^*\}$

$$V_C^1\{B\overline{D}^*, B^*\overline{D}\} = \begin{pmatrix} C_{0a} & -C_{0b} \\ -C_{0b} & C_{0a} \end{pmatrix},$$
$$V_C^1(B^*\overline{D}^*) = C_{0a} - C_{0b}.$$

- Isoscalar 2^+ system: $\{B^*\overline{D}^*\}$

$$V_C^2(B^*\overline{D}^*) = C_{0a} + C_{0b}.$$

C_{0a} and C_{0b} are low-energy constants fixed from the lattice data.

Long-range interaction in the ChPT

The one-pion-exchange potentials between the heavy mesons are obtained from ChPT

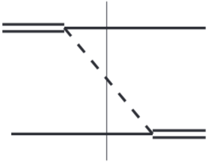
- For the isoscalar $B^*\overline{D}$–$B\overline{D}^*$ coupled-channel system

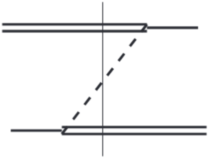
$$V_{\mathrm{OPE}}(B^*\overline{D}) = -\frac{3g^2}{4F_\pi^2} D_\pi(E) \begin{pmatrix} 0 & [\boldsymbol{p}_1 \cdot \varepsilon^*(\boldsymbol{k}_1)][\boldsymbol{k}_2 \cdot \varepsilon(\boldsymbol{p}_2)] \\ [\boldsymbol{p}_2 \cdot \varepsilon^*(\boldsymbol{k}_2)][\boldsymbol{k}_1 \cdot \varepsilon(\boldsymbol{p}_1)] & 0 \end{pmatrix}$$

- For the isospain $B^*\overline{D}^*$ system with $J^P = 0^+, 1^+, 2^+$

$$V_{\mathrm{OPE}}(B^*\overline{D}^*) = \frac{3g^2}{4F_\pi^2} D_\pi(E) [\boldsymbol{q} \cdot (\varepsilon^*(\boldsymbol{k}_1) \times \varepsilon(\boldsymbol{p}_1))] \, [\boldsymbol{q} \cdot (\varepsilon^*(\boldsymbol{k}_2) \times \varepsilon(\boldsymbol{p}_2))].$$

$D_\pi(E)$ is the pion propagator in the time-ordered perturbation theory (TOPT)





T-matrix in the finite-volume

- For the A_1^+ irrep ($B\overline{D}$ system), only the contact potential, which is the normal finite-volume framework
- In the T_1^+ irrep, the coupled-channel Lippmann-Schwinger equation for the $B^*\overline{D}$–$B\overline{D}^*$ system is

$$\tilde{T}'^{\Gamma}_{\alpha\lambda_1 r_1,\beta\lambda_2 r_2}(E,\boldsymbol{p},\boldsymbol{k}) = \tilde{V}^{\Gamma}_{\alpha\lambda_1 r_1,\beta\lambda_2 r_2}(E,\boldsymbol{p},\boldsymbol{k}) + \sum_{\gamma,\lambda_3,r_3,\boldsymbol{q}_n} \tilde{V}^{\Gamma}_{\alpha\lambda_1 r_1,\gamma\lambda_3 r_3}(E,\boldsymbol{p},\boldsymbol{q}_n)\,\tilde{I}^{\Lambda}_{\gamma\lambda_3 r_3,\gamma\lambda_3 r_3}(E,\boldsymbol{q}_n)\,\tilde{T}'^{\Gamma}_{\gamma\lambda_3 r_3,\beta\lambda_2 r_2}(E,\boldsymbol{q}_n,\boldsymbol{k}),$$

where λ_i is the helicity of vector mesons, and r_i are the row of irrep Γ. $\tilde{V}^{\Gamma}_{\alpha\lambda_1 r_1,\gamma\lambda_3 r_3}(E,\boldsymbol{p},\boldsymbol{q}_n)$ is the potential projected into the irrep Γ. The finite-volume Green's function is

$$\tilde{I}^{\Lambda}_{\alpha\lambda_1 r_1,\beta\lambda_2 r_2}(E,\boldsymbol{q}_n) = \frac{1}{L^3}\frac{\delta_{\alpha\beta}\delta_{\lambda_1\lambda_2}\delta_{r_1 r_2}\,e^{-2\boldsymbol{q}_n^2/\Lambda^2}}{E-\tilde{\omega}_{B^{(*)}}(\boldsymbol{q}_n)-\tilde{\omega}_{\overline{D}^{(*)}}(\boldsymbol{q}_n)}.$$

- The coupled-channel contact potentials

$$V_C^1\{B\overline{D}^*, B^*\overline{D}\} = \begin{pmatrix} C_{0a}\varepsilon^*(\boldsymbol{k}_2)\cdot\varepsilon(\boldsymbol{p}_2) & -C_{0b}\varepsilon^*(\boldsymbol{k}_1)\cdot\varepsilon(\boldsymbol{p}_2) \\ -C_{0b}\varepsilon^*(\boldsymbol{k}_2)\cdot\varepsilon(\boldsymbol{p}_1) & C_{0a}\varepsilon^*(\boldsymbol{k}_1)\cdot\varepsilon(\boldsymbol{p}_1) \end{pmatrix}.$$

Projection of potential into the irrep Γ

- The potentials are reduced to the specific irrep with the projection operator

$$\tilde{V}^{\Gamma}_{\alpha\lambda_1 r_1,\beta\lambda_2 r_2}(E,\boldsymbol{p},\boldsymbol{k}) = \sum_{\mu} \langle \boldsymbol{p},\Gamma,\lambda_1,r_1,\mu|\,\hat{V}_{\alpha\beta}(E,\boldsymbol{p},\boldsymbol{k})|\boldsymbol{k},\Gamma,\lambda_2,r_2,\mu\rangle,$$

where the orthonormal basis of the Γ irrep is

$$|\boldsymbol{p},\Gamma,\lambda,r,\mu\rangle = N^{\Gamma}_{\boldsymbol{p}\lambda} \sum_{\nu} \left[c^{\Gamma,r}_{\boldsymbol{p},\lambda}\right]_{\nu} P^{\Gamma}_{\mu\nu}|\boldsymbol{p},\lambda\rangle \quad \text{with} \quad P^{\Gamma}_{\mu\nu} = \frac{\dim(\Gamma)}{|G|} \sum_{g\in G} \overline{D}^{\Gamma}_{\mu\nu}(g)g.$$

Kang Yu, et al, JHEP04(2025)108

- The potential is reduced to

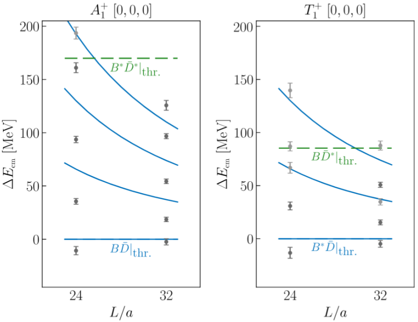
$$V^{\Gamma}_{\alpha\lambda_1 r_1,\beta\lambda_2 r_2}(E,\boldsymbol{p},\boldsymbol{k}) = \frac{N_{\boldsymbol{p},\lambda_1}}{N_{\boldsymbol{k},\lambda_2}} \sum_{g\in O^2_H} \left(\left[c^{\Gamma,r}_{\boldsymbol{p},\lambda_1}\right]^{\dagger} \overline{D}^{\Gamma} \left[c^{\Gamma,r}_{\boldsymbol{k},\lambda_2}\right] \langle \boldsymbol{p},\lambda_1|\,\hat{V}^{L} g|\boldsymbol{k},\lambda_2\rangle \right)$$

In the potential, the polarization vectors are in the helicity basis.

Outline

Determination of the LECs from the lattice data

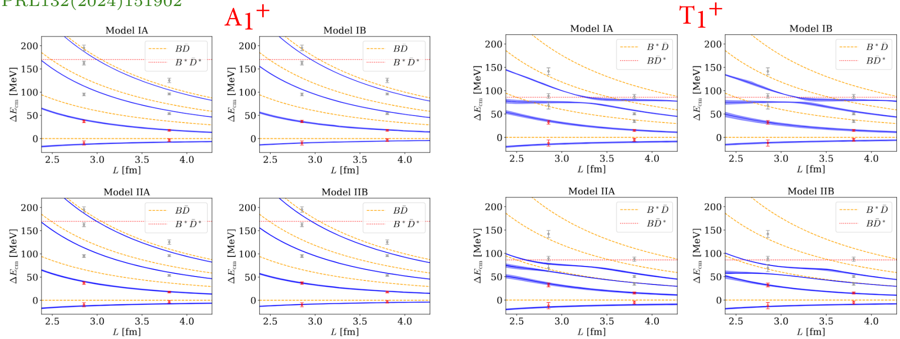
The LECs C_{0a}, C_{0b} are constrained by the Lattice data C. Alexandrou, et al, PRL132(2024)151902



Model IA/IB: Contact potential with $\Lambda = 0.8/1.2$ GeV;
Model IIA/IIB: Contact + OPE potentials with $\Lambda = 0.8/1.2$ GeV;

Fitting results

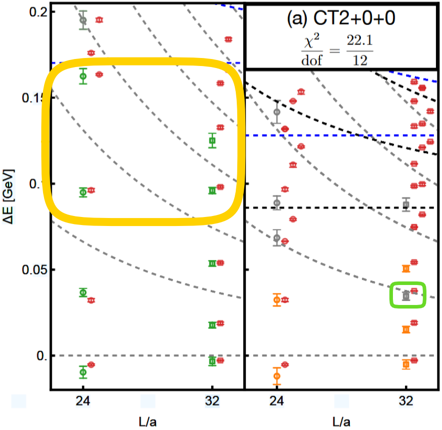
The LECs C_{0a}, C_{0b} are constrained by the Lattice data C. Alexandrou, et al, PRL132(2024)151902

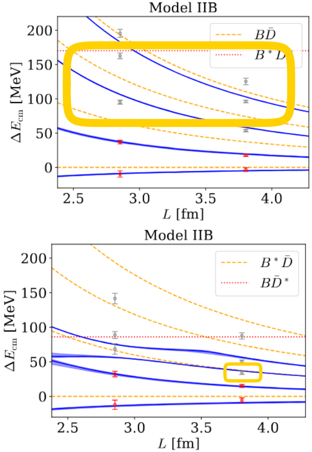


Model IA/IB: Contact potential with $\Lambda = 0.8/1.2$ GeV;

Model IIA/IIB: Contact + OPE potentials with $\Lambda = 0.8/1.2$ GeV;

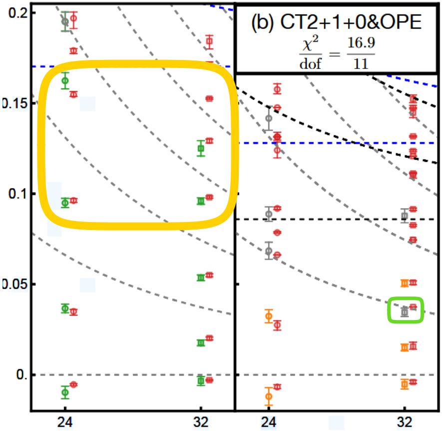
M. Abolnikov, et al, PRD 113(2026)114041

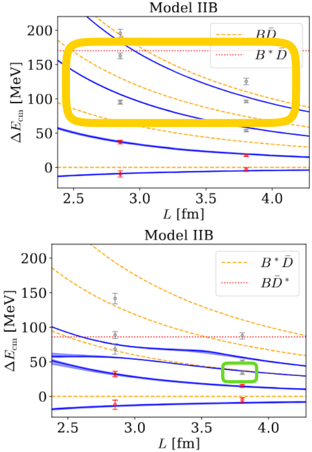




- Left panel: $B\overline{D}$–$B^*\overline{D}^*$ in A_1^+ irrep, $B^*\overline{D}$–$B\overline{D}^*$–$B^*\overline{D}^*$ in T_1^+ irrep
- NLO potential improves the description of high-lying ELs in A_1^+ irrep
- *D*-wave contributions are significantly different between two fits

M. Abolnikov, et al, PRD 113(2026)114041

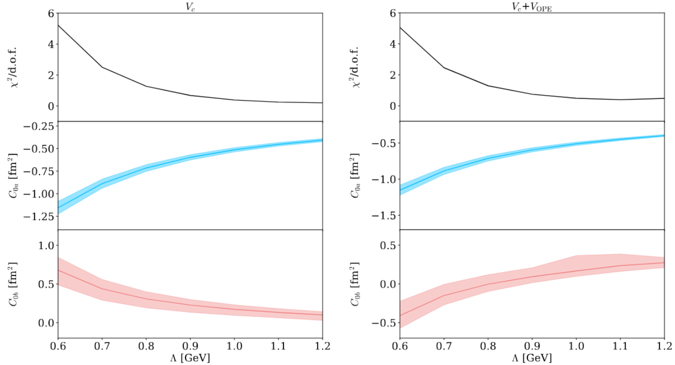




- Left panel: $B\overline{D}$–$B^*\overline{D}^*$ in A_1^+ irrep, $B^*\overline{D}$–$B\overline{D}^*$–$B^*\overline{D}^*$ in T_1^+ irrep
- NLO potential improves the description of high-lying ELs
- D-wave contributions are significantly different between two fits

Cutoff dependence of parameters

Varying the cutoff in the region of $[0.6, 1.2]$ GeV, we fit the C_{0a} and C_{0b} with the four energy levels both in A_1^+ and T_1^+ irreps



- The C_{0a} exhibits the same trend for both the Contact and Contact + OPE potentials, whereas C_{0b} exhibits the opposite trend
- The OPE potential is sensitive to the variation of the cutoff

Pole positions for the S-wave $B^{(*)}\bar{D}^{(*)}$ system

Using the fitted C_{0a} and C_{0b}, we calculate the physical pole positions

Table 1: Predicted pole position in units of MeV.

J^{PC}	IA	IB	IIA	IIB	Thr.
0^+	7144^{+1}_{-1}	7146^{+1}_{-1}	7144^{+1}_{-1}	7146^{+0}_{-1}	7147
1^+	7187^{+1}_{-2}	7190^{+1}_{-2}	7187^{+1}_{-2}	7191^{+0}_{-1}	7192
	$7287^{+3}_{-2} - 2^{+1}_{-1}i$	$7287^{+3}_{-2} - 1^{+1}_{-1}i$	$7287^{+3}_{-2} - 2^{+0}_{-1}i$	$7287^{+1}_{-1} - 0^{+1}_{-0}i$	7288
0^+	7293^{+14}_{-13}	7303^{+20}_{-24}	7296^{+17}_{-19}	7081^{+36}_{-37}	7333
1^+	7314^{+5}_{-5}	7320^{+7}_{-8}	7315^{+8}_{-9}	7224^{+17}_{-17}	7333
2^+	7331^{+2}_{-10}	7332^{+1}_{-11}	7333^{+1}_{-1}	7332^{+0}_{-0}	7333

- $J^P = 1^+$ pole in the $B\bar{D}^*$ system is located on the $(-,+)$ RS
- $J^P = 2^+$ pole in the $B^*\bar{D}^*$ system is located on the $(-)$ RS
- For IIB, the poles in the 0^+ and 1^+ $B^*\bar{D}^*$ systems have deep E_b
- The original LQCD results shows other two poles: $m - m_{B\bar{D}} = 138(13)$ MeV $\Leftrightarrow$ $146 - 156$ MeV for 0^+ $B^*\bar{D}^*$; $m - m_{B^*D} = 67(24)$ MeV $\Leftrightarrow$ 95 MeV for 1^+ $B\bar{D}^*$ C. Alexandrou, et al, PRL132(2024)151902

Outline

Summary

The lattice data of the 0^+ and 1^+ $bc\bar{q}\bar{q}$ systems are reanalyzed in the EFT framework with/without the long-range OPE potential; Possible structures in the $B^{(*)}\overline{D}^{(*)}$ systems are predicted in terms of HQSS

- 0^+: two bound states close to $B\overline{D}$ and $B^*\overline{D}^*$ thresholds; the mass of one state near $B^*\overline{D}^*$ is consistent with the one in original LQCD calculation in A_1^+ irrep
- 1^+: three states close to $B^*\overline{D}$, $B\overline{D}^*$, and $B^*\overline{D}^*$ thresholds; the mass of the one close to the $B\overline{D}^*$ threshold is consistent with the one calculated by LQCD in the T_1^+ irrep
- 2^+: a virtual state near $B^*\overline{D}^*$ threshold

Our results suggest that the off-diagonal OPE potential does not affect the S-wave scattering information when the $B\overline{D}\pi$ three-body cut outside the energy region, but changes the higher-wave information, like finite-volume spectrum

Thanks for your attention!

Backup

The r-orthonormal eigenvector $c^{\Gamma}_{\vec{n}_o,\underline{\lambda}}$ is calculated by solving the eigen value problem

$$\begin{aligned} g|\vec{n},\underline{\lambda}\rangle &= \sum_{\lambda} D^{s}_{\lambda\underline{\lambda}}\left(R_{\mathrm{st}}(\vec{n})\right) g|\vec{n},\lambda\rangle \\ &= \sum_{\lambda,\lambda'} D^{s}_{\lambda\lambda'}\left(R_{\mathrm{st}}(g\vec{n})\right) D^{s}_{\lambda'\underline{\lambda}}\left(R_{\mathrm{st}}^{-1}(g\vec{n})gR_{\mathrm{st}}(\vec{n})\right)|g\vec{n},\lambda\rangle \\ &= \sum_{\lambda} D^{s}_{\lambda\underline{\lambda}}\left(R_{\mathrm{st}}(g\vec{n})\right) e^{-i\underline{\lambda}\varphi_w(\vec{n},g)}|g\vec{n},\lambda\rangle \\ &= e^{-i\underline{\lambda}\varphi_w(\vec{n},g)}|g\vec{n},\underline{\lambda}\rangle \end{aligned}$$

$$I^{\Gamma}_{\underline{\lambda}}\left(\vec{n}_o\right) = \sum_{g\in\mathrm{LG}(\vec{n}_o)} \bar{D}^{\Gamma}(g)\left\langle \vec{n}_o,\underline{\lambda}\right| g \left|\vec{n}_o,\underline{\lambda}\right\rangle = \begin{cases} C^{\Gamma}_{\vec{n}_o}\sum_{g\in\mathrm{LG}(\vec{n}_o)^{+}} \bar{D}^{\Gamma}(g) & \text{if } \underline{\lambda}=0 \\ \sum_{g\in\mathrm{LG}(\vec{n}_o)^{+}} e^{-i\underline{\lambda}\varphi_w(\vec{n}_o,g)}\bar{D}^{\Gamma}(g) & \text{if } \underline{\lambda}\neq 0 \end{cases}$$

Here φ is the Wigner angle.

Backup

Projection of the effective potential Kang Yu, et al, JHEP04(2025)108

$$
\begin{aligned}
V^{\Gamma}_{\vec{n}_o' r', \vec{n}_o r} &= N_{\vec{n}_o} N_{\vec{n}_o'} \sum_{\nu,\nu'} \left[c^{r'}_{\vec{n}_o'}\right]^*_{\nu'} \left[c^{r}_{\vec{n}_o}\right]_{\nu} \left\langle \vec{n}_o' \right| P^{\Gamma\dagger}_{\mu\nu'} \hat{V}^L P^{\Gamma}_{\mu\nu} \left| \vec{n}_o \right\rangle \\
&= \sqrt{\frac{1}{|\mathrm{LG}(\vec{n}_o)|\,|\mathrm{LG}(\vec{n}_o')|}} \sum_{g\in G} \left(\left[c^{r'}_{\vec{n}_o'}\right]^\dagger \cdot \bar{D}^{\Gamma}(g) \cdot \left[c^{r}_{\vec{n}_o}\right] \right) \left\langle \vec{n}_o' \right| \hat{V}^L g \left| \vec{n}_o \right\rangle \\
&= \sqrt{\frac{1}{|\mathrm{LG}(\vec{n}_o)|\,|\mathrm{LG}(\vec{n}_o')|}} \sum_{\substack{g\in \frac{G}{\mathrm{LG}(\vec{n}_o)} \\ h\in \mathrm{LG}(\vec{n}_o)}} \left(\left[c^{r'}_{\vec{n}_o'}\right]^\dagger \cdot \bar{D}^{\Gamma}(g) \cdot \bar{D}^{\Gamma}(h) \cdot \left[c^{r}_{\vec{n}_o}\right] \right) \left\langle \vec{n}_o' \right| \hat{V}^L gh \left| \vec{n}_o \right\rangle \\
&= \frac{N_{\vec{n}_o'}}{N_{\vec{n}_o}} \sum_{g\in \mathrm{LC}(\vec{n}_o)} \left(\left[c^{r'}_{\vec{n}_o'}\right]^\dagger \cdot \bar{D}^{\Gamma}(g) \cdot \left[c^{r}_{\vec{n}_o}\right] \right) \left\langle \vec{n}_o' \right| \hat{V}^L g \left| \vec{n}_o \right\rangle \\
&= \frac{N_{\vec{n}_o'}}{N_{\vec{n}_o}} \sum_{g\in \mathrm{LC}(\vec{n}_o)} \left(\left[c^{r'}_{\vec{n}_o'}\right]^\dagger \cdot \bar{D}^{\Gamma\eta}(g) \cdot \left[c^{r}_{\vec{n}_o}\right] \right) V^L\left(\vec{n}_o', g\vec{n}_o\right) .
\end{aligned}
$$

Backup

Pion-mass dependence of f_π and g in the OPE potential V. Baru, et al, PLB726(2013)537

$$f_\pi(\xi) = f_\pi^{\rm ph}\left[1+\left(1-\frac{f_0}{f_\pi^{\rm ph}}\right)\left(\xi^2-1\right)-\frac{\left(m_\pi^{\rm ph}\right)^2}{8\pi^2 f_0^2}\xi^2\log\xi\right]$$

$$g(\xi) = g^{\rm ph}\left[1+C_1\left(\xi^2-1\right)+C_2\xi^2\log\xi\right],$$

with

$$C_1 = 1-\left[1-\frac{1+2g_0^2}{8\pi^2 f_0^2}\left(m_\pi^{\rm ph}\right)^2\log\frac{m_\pi^{\rm ph}}{\mu_{\rm lat}}+\alpha_{\rm lat}\left(m_\pi^{\rm ph}\right)^2\right]^{-1} \tag{1}$$

$$C_2 = -\frac{1+2g_0^2}{8\pi^2 f_0^2}\left(m_\pi^{\rm ph}\right)^2\left(1-C_1\right). \tag{2}$$

Patameters: $f_0 = 85$ MeV; $f_\pi^{\rm ph} = 92.1$ MeV; $\xi = m_\pi/m_\pi^{\rm ph}$; $g_0 = 0.46$, $\alpha_{\rm lat} = -0.16{\rm GeV}^{-2}$, $\mu_{\rm lat} = 1{\rm GeV}$. D. Becirevic, et al, PLB721(2013)94