Hyperon non-leptonic decays in relativistic ChPT with resonances

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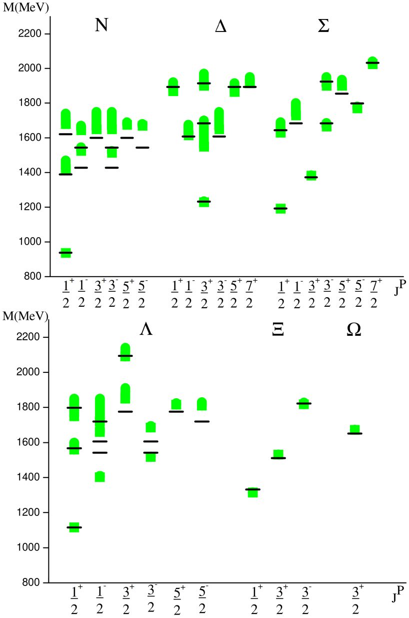

Figure 1: [Aslanzadeh, ChJP, 53 (2015)]

Baryon Resonances

- Baryon excitations give insight into QCD

  $1/2^+$: $N(1440)$, $\Lambda(1600)$, ...
  $1/2^-$: $\Lambda(1405)$, $N(1535)$, ...
  $3/2^+$: $\Delta(1232)$, $\Sigma(1385)$, ...

- Meson-baryon scattering
  - First way to study baryon excitations (since $\Delta(1232)$ in the '50s)
  - Approachable from first principles on the lattice with Lüscher method
  - Baryon Scattering Collaboration (BaSc)
  - We present preliminary results on $\pi\Xi - \bar{K}\Lambda - \bar{K}\Sigma$ channels and the $\Xi(1620)$ and $\Xi(1690)$ resonances in the lattice
- Resonances are crucial in many other processes
  - Weak reactions can make some resonances more prominent than strong processes
  - We investigate the resonances' role in hyperon non-leptonic decays in ChPT

Hyperon non-leptonic decays

Hyperon non-leptonic decays in relativistic ChPT with resonances

$Y \to B\pi$

$(\Lambda \to p\,\pi^-,\ \Lambda \to n\,\pi^0, \Sigma^+ \to n\,\pi^+,\ \Sigma^+ \to p\,\pi^0,\ \Sigma^- \to n\,\pi^-,\ \Xi^- \to \Lambda\,\pi^-,\ \Xi^0 \to \Lambda\,\pi^0)$

- Recent BESIII results [Nat Phys 15 (2019)], [Nature 606, (2022)]
- Long standing theoretical problem
- Very challenging on the lattice, due to four quark operator mixing (only $K \to \pi\pi$ so far)
- ChPT
  - Heavy baryon does not describe both s- and p-waves , e.g. [Jenkins NPB '92]
  - Borasoy & Holstein compute at tree level with $1/2^-$ & $1/2^+$ baryon excitated octets and call for further investigation [PRD 59 (1999)]
- Our work [2604.00646]
  - first relativistic NLO ChPT computation of the decays
  - explicit decuplet dofs
  - account for resonant octets at tree level, which approximate the counterterms

    $1/2^+$: $N(1440)$, $\Lambda(1600)$, $\Sigma(1660)$, $\Xi(1790)$

    $1/2^-$: $N(1535)$, $\Lambda(1670)$, $\Sigma(1680)$, $\Xi(1790)$

Hyperon non-leptonic decays
in relativistic Chiral Perturbation Theory with resonances

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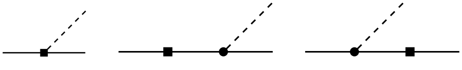
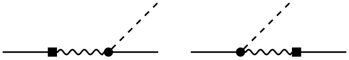
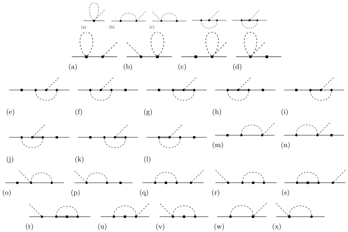
[4] *National Centre for Nuclear Research, Pasteura 7, 02-093 Warsaw, Poland*
(Dated: April 16, 2026)



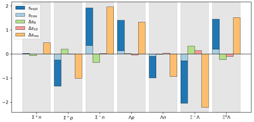
(a) s-wave **(b)** p-wave

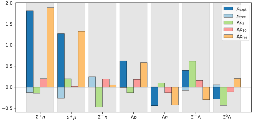


(c) Resonance contribution

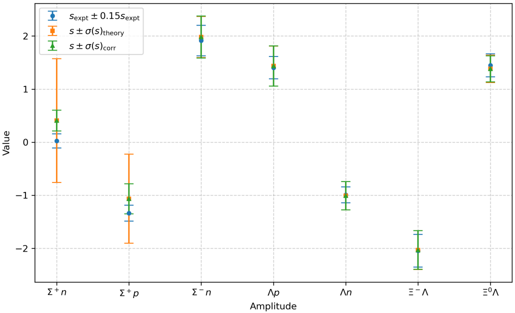


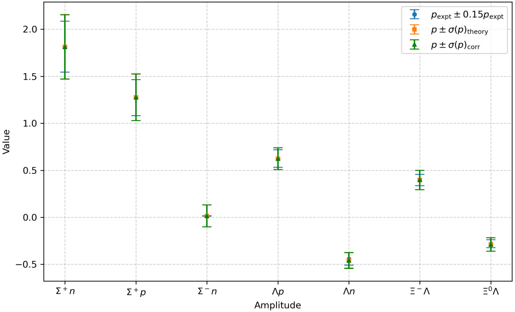
- $\Delta S = 1$ BB-vertex probes small mass splittings:
  - p-wave enhancement $(m_\Sigma - m_N)/\mathrm{GeV} \sim 0.25$
  - Resonances like the $N(1440)$ are relatively far in mass from N, $\Delta m \sim 0.5$ GeV
  - The same mechanism that enhances p-wave makes resonance contributions large
    $(m_{N(1440)} - m_\Sigma)/\mathrm{GeV} \sim 0.25$,
    $(m_{N(1535)} - m_\Sigma)/\mathrm{GeV} \sim 0.35$





- The weak LECs are fitted
- A good combined description of both s- and p-wave amplitudes is achieved
  - Our work is not the last word: slow ChPT convergence and not very tightly constrained fit



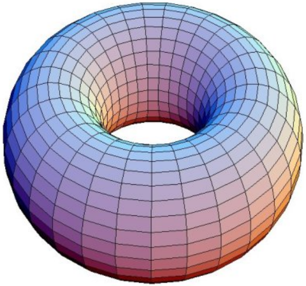


- Comments & Outlook:
  - Similar resonance impact observed in $Y \to B\gamma$ (LECs in common with $Y \to B\pi$) [Borasoy & Holstein, PRD 59(1999)] (see also recent [Shi, Li, Lu, Geng, Sci.Bull. 67 (2022)])
  - Investigate resonances role in the semi-leptonic decays
  - dynamical implementation of the resonances

Meson-baryon s-wave scattering on the Lattice

Lattice QCD approach

- Scattering in a finite volume thanks to Lüscher method [NPB,354(1991)][NPB,364(1991)]
- Necessity of multi-hadron operators already clear in meson-meson, otherwise the E spectrum is not resolved , e.g. [Mohler et al PRD 87 (2013), Wilson et al PRD 92 (2015)]



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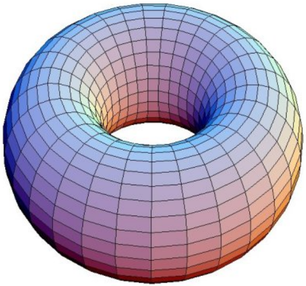
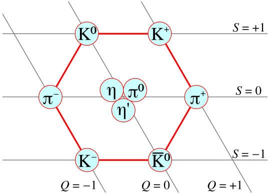
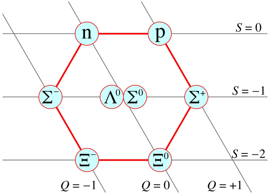


- In meson-baryon, πN operators, e.g.
  - $N(1535)$ [Lang & Verduci PRD 87 (2013)]
  - $N(1440)$ [Lang et al PRD 95(2017)]
  - $\Delta(1232)$ [Alexandrou et al PRD 96 (2017), Bulava et al NPB 987 (2023)]

Negative parity hyperons from coupled channel scattering





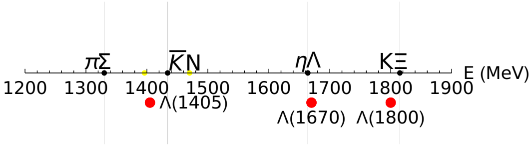
Meson-baryon scattering

- Meson-baryon scattering in s-wave $\Rightarrow J^P = 1/2^-$:
  N^*, Λ^*, Σ^*, Ξ^*
- strangeness improves the signal
- $\Lambda(1405)$
  - Past works have studied the $\Lambda(1405)$ only with single-hadron operators [Nemoto PRD 68 (2003) 094505, Burch PRD 74 (2006) 014504, Takahashi PRD 81 (2010) 034505, Menadue PRL 108 (2012) 112001, Engel PRD 87(2013), 034502 & 034504, Hall PRL 114 (2015), Gubler PRD 94 (2016) 114518]
  - The $\Lambda(1405)$ study of BaSc employs two-hadron operators [Bulava et al PRL 132 (2024), Bulava et al PRD 109 (2024)]
- Ξ^* channel (preliminary results)

Negative parity hyperons from coupled channel scattering

$\Lambda(1405)$ channel

- '60s-'80s bubble chamber exps. $K^- p$ [Alston et al PRL 6 (1961) 698]
- $J^P = 1/2^-$ confirmed in $\gamma p \to K^+ \Lambda(1405)$ polarisation by CLAS [Moriya et al., PRC 87 035206 (2013)]
- $\Lambda(1405)$ (****) & $\Lambda(1380)$ (**)?



Simplified illustration of m_q dep. of $\boldsymbol{\Lambda}\mathbf{(1405)}$

- SU(3): $8 \otimes 8 = 1 \oplus 8 \oplus 8 \oplus 10 \oplus \bar{1}0 \oplus 27$

Negative parity hyperons from coupled channel scattering

Simplified illustration of m_q dep. of $\boldsymbol{\Lambda(1405)}$

- SU(3): $8 \otimes 8 = 1 \oplus 8 \oplus 8 \oplus 10 \oplus \bar{1}0 \oplus 27$
- $\mathcal{L}_{\mathrm{WT}} \Longrightarrow$ only $1 \oplus 8 \oplus 8$ are attractive

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- $\mathcal{L}_{\mathrm{WT}} \Longrightarrow$ only $1 \oplus 8 \oplus 8$ are attractive
- If one observes one pole with $I = 0$, then one could expect three isosinglets
- $\sim$UChPT with SU(3) broken by m_q predicts only two poles arround 1400 MeV

  [Oller & Meissner PLB 500 (2001); Jido, Oller, Oset, Ramos, Meissner NPA 725 (2003)]

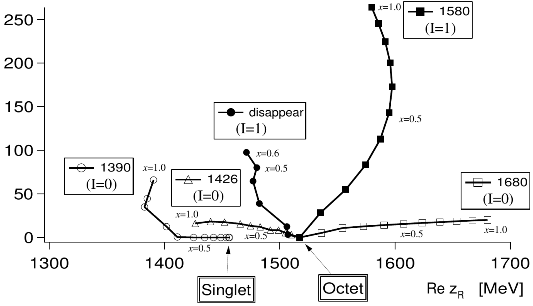

FIg. from [Jido, Oller, Oset, Ramos, Meissner NPA 725 (2003)]

Lattice setup: a pathway through CLS ensembles

joint effort of the **BaSc** collaboration

- $\Lambda(1405)$ already studied on D200 ($m_\pi = 200$ MeV) with CLS action [Bulava et al]

  (= tree-level improved Lüscher-Weisz gluons and nonperturbatively $\mathcal{O}(a)$ improved Wilson fermions)

- More ensembles under investigation

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1. Study m_q dependence along $\mathrm{Tr}\, M_\mathrm{q} \simeq \mathrm{phys}$
   - connect SU(3) and phys point ⇒ theoretical insight and physical relevance

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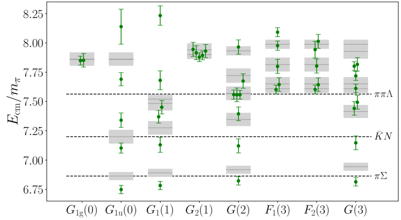
2. Lattice spacing dependence has to be corrected
3. More statistics and different E_{cm} improve amplitude extraction

- $\Rightarrow$ Motivates current analysis on X451: $\neq m_q, a, L$ (and next N451)
  - $m_{\mathrm{q}}^{\mathrm{phys}}$ on E250 is on the horizon
- We also study Ξ channel on X451 (N451 next)

ensembles	boundary	a	$T \times L^3$	m_π	m_K	$m_\pi L$	N_{cnfg}
D200	open	0.064 fm	128×64^3	200 MeV	480 MeV	4.3	2000
X451	periodic (better sampling)	0.075 fm	" $\times 40^3$	287 MeV	464 MeV	4.4	2000
N451	"	"	" $\times 48^3$	"	"	5.3	1000
E250	"	0.064 fm	192×96^3	131 MeV	493 MeV	4.1	1000

$\pi\Sigma - \bar{K}N$ and the $\Lambda(1405)$ resonance

$\pi\Sigma - \bar{K}N$ and the $\Lambda(1405)$ resonance



Lüscher method

- $\det\left[\tilde{K}^{-1}(E) - B^{\mathsf{P}}(E)\right] = 0$

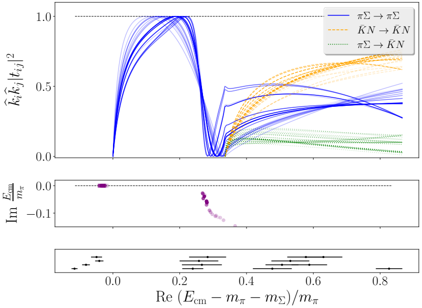
Amplitude analysis

- five $\tilde{K}_{ij}$ parametrisations, $t^{-1} = \tilde{K}^{-1} - i\hat{k}$

Two poles extracted

(at $a \simeq 0.064$ fm, $m_\pi \simeq 200$ MeV, $m_K \simeq 480$ MeV)

1. virtual state:
   $E_1 = 1392(9)_{\rm st}(2)_{\rm md}(16)_{\rm sc}$ MeV
2. resonance pole:
   $E_2 = [1455(13)_{\rm st}(2)_{\rm md}(17)_{\rm sc} - i\ 11.5(4.4)_{\rm st}(4)_{\rm md}(0.1)_{\rm sc}]$ MeV



- **Ongoing work: Moving the $\Lambda(1405)$ analysis to other m_q, a and L in CLS**

ensembles	boundary	a	$T \times L^3$	m_π	m_K	$m_\pi L$	$N_{\rm cnfg}$
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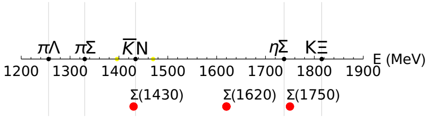
N451	"	"	" $\times\, 48^3$	"	"	5.3	1000

- Figs: free E_n in X451 and N451
- They only differ in volume $\Rightarrow$ constrain more $E_{\rm cm}$

Σ flavour channel

lattice results in this sector are highly desired [Mai et al 2507.14283]

- $\Sigma(1430)$
  - predicted by $\sim$UChPT, e.g. [Oller & Meissner PLB 500 (2001)]
  - experimental analyses are inconclusive
- $\Sigma(1620)$ (*) [Van Horn NPB ('75)]



$\pi\Xi - \bar{K}\Lambda - \bar{K}\Sigma$ channels and the $\Xi(1620)$ and $\Xi(1690)$ resonances

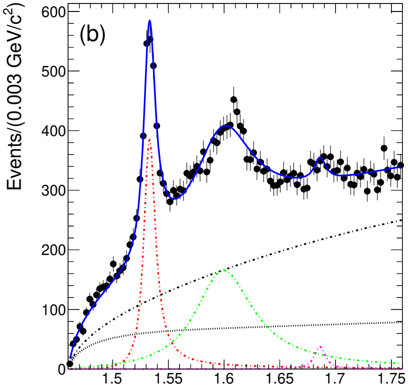


Figure 3: Events vs $M_{\Xi^-\pi^+}$ (GeV/c^2) ($\Xi(1530)+\Xi(1620)+\Xi(1690)$) from Belle $\Xi_c \to \Xi\pi\pi$ [Sumihama et al PRL (2019)].

$\Xi(1620)$ & $\Xi(1690)$, *a stranger counterpart of the* $\Lambda(1405)$

(**) & (***) on PDG

- Recent experimental results support both states
  - Belle $\Xi_c \to \Xi\pi\pi$

    $m = 1610.4 \pm 6.0^{+5.9}_{-3.5}$ MeV, $\Gamma = 59.9 \pm 4.8^{+2.8}_{-3.0}$ MeV

    [Sumihama PRL (2019)]
  - LHCb $\Xi_b \to J/\Psi\Lambda K$

    $m = 1692.0 \pm 1.3^{+1.2}_{-0.4}$, $\Gamma = 25.9 \pm 9.5^{+14.0}_{-13.5}$

    [Aaij, Sci. Bull. (2021)]
  - ALICE $\pi\Xi$ femtoscopy

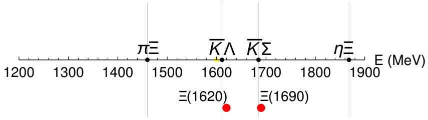
    $m^0 = 1608.16 \pm 1.5$ MeV, $\Gamma^0 = 47.12 \pm 5.63$ MeV;

    $m^0 = 1680.72 \pm 3.27$ MeV, $\Gamma^0 = 26.41 \pm 13.86$ MeV

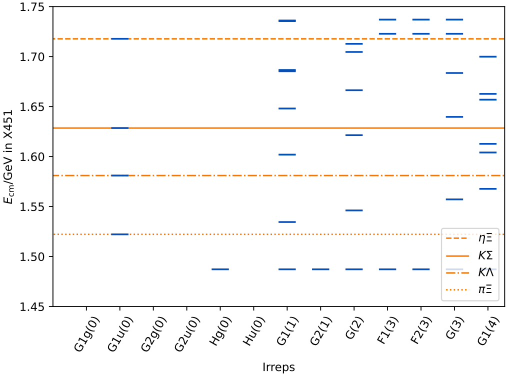
    [Abualrob, PLB (2026)]

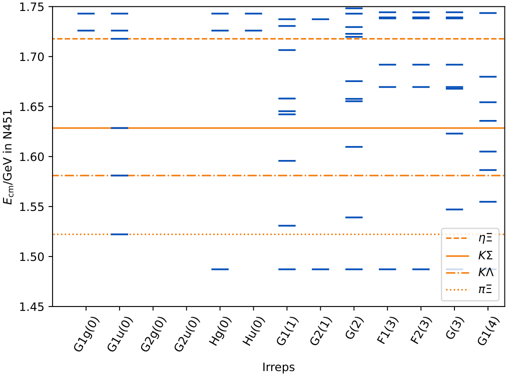
Ξ flavour channel

- $\Xi(1620)$
  - UChPT (Weinberg-Tomozawa), e.g. [Ramos, Oset & Bennhold, PRL 89 (2002)] & [García-Recio, Lutz & Nieves, PLB 582 (2004)]
  - NLO UChPT [Feijoo et al PLB 841 (2023)]
  - femtoscopy [Mantovani et al PRD 110 (2024)] argues that it is mostly $\eta\Xi$ in contrast with previous literature
- $\Xi(1690)$
  - UChPT: also predicted in García-Recio's and Feijoo's and many others
- Different approaches and resulting $m_R, \Gamma_R, |g_{ij}|$.

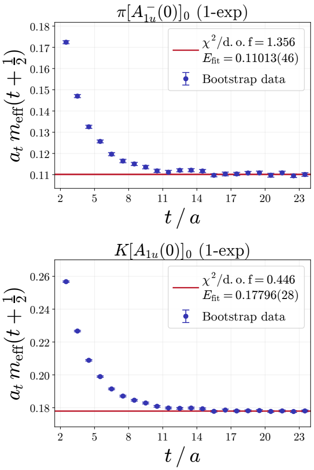


- Ξ channel expected free levels in X451 and N451
- double strangeness should improve signal with respect to Λ channel
- absence of nucleons should reduce the baryon noise



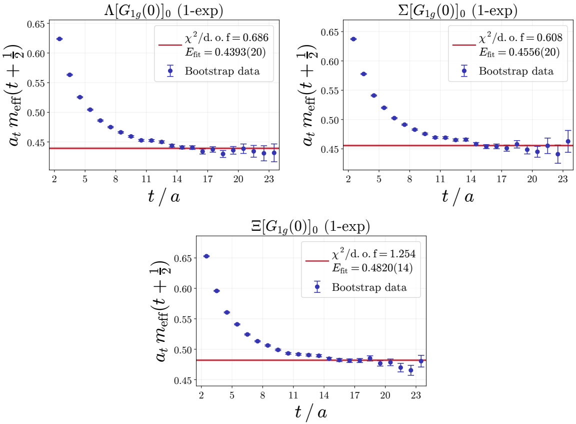


J^P	[000]	[00n]	[0nn]	[nnn]	
$1/2^+$	G_{1g}	G_1	G	G	Ξ
$1/2^-$	G_{1u}	G_1	G	G	$\Xi(1620)$, $\Xi(1690)$
$3/2^+$	H_g	G_1, G_2	$2G$	F_1, F_2, G	$\Xi(1530)$
$3/2^-$	H_u	G_1, G_2	$2G$	F_1, F_2, G	$\Xi(1820)$



Production on X451:

- in Figs.: single hadron m_{eff} on 1683/2026 configs, one source and direction (from tot four, 1683/8104 meas)



Production on X451:

- in Figs.: single hadron m_{eff} on 1683/2026 configs, one source and direction (from tot four, 1683/8104 meas)
- periodic boundary conditions and source time varied randomly: reduce baryon noise
- consistency with expectations and $\sim$ ideal scaling
- $m_\pi \simeq 286$ MeV, $m_K \simeq 462$ MeV, $m_\Lambda \simeq 1141$ MeV, $m_\Sigma \simeq 1183$ MeV, $m_\Xi \simeq 1255$ MeV

- 3161(/8000) meas. in 1592(/2026) cfg.
- $\pi\Xi - \bar{K}\Lambda - \bar{K}\Sigma$ operators + all local 3-quarks operators
- GEVP with 9/8 operators

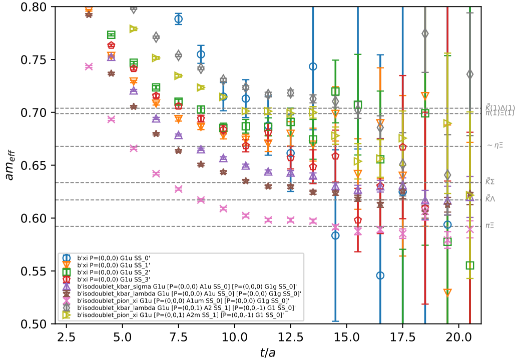


Figure 4: Effective masses of the 9 G1u ops.

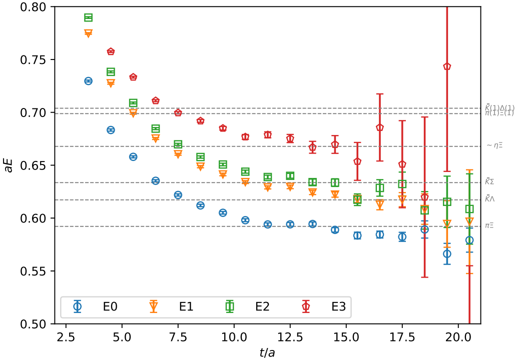
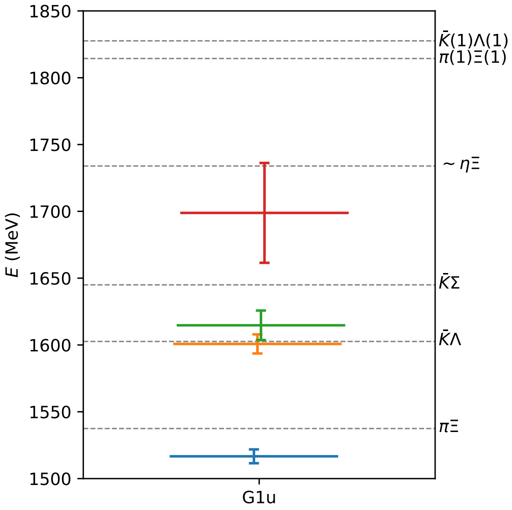


Figure 5: GEVP E_{0-4}.



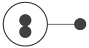
- Fits with $\chi_i^2/\mathrm{dof} \simeq 1$
- In the physical world $\eta\Xi$ opens ~ 1870 MeV, significantly higher than $\Xi(1620)$, $\Xi(1690)$.
- $\pi\Xi$ attraction, $\Delta E_{\pi\Xi} = 21 \pm 6$ MeV
  - attraction also observed by ALICE 2026 (also in line with LO Weinberg-Tomozawa)

Displaced Operators

Displaced Operators

- We are studying displaced operators in order to improve the description of hadrons in general

SS: $\epsilon_{abc}\tilde{\psi}^{A}_{a\alpha}\tilde{\psi}^{B}_{b\beta}\tilde{\psi}^{C}_{c\gamma}$ 

SD: $\epsilon_{abc}\tilde{\psi}^{A}_{a\alpha}\tilde{\psi}^{B}_{b\beta}\left(\tilde{D}^{(p)}_{j}\tilde{\psi}\right)^{C}_{c\gamma}$ 

  - We have started studying Λ ops. on X451
    - displaced ops. should help to disentangle states
    - drawback: noisier than local ones
  - theoretical input is beneficial: $\Lambda(1520)$ case

- **Displaced operators: $\Lambda(1520)$**, a $J^P = \frac{3}{2}^-$ state
  - $\sim$UChPT meson-decuplet scatt. [Kolomeitsev & Lutz PLB 585 (2004)]
  - quark model: $L = 1$, $S = 1/2$, flav. singlet [Gromes ZPC 18 (1983)]

$$(O_{\Lambda^*})_{j\gamma} = \epsilon^{abc}(C\gamma_5)_{\alpha\beta}\left(\frac{1+\gamma_0}{2}\right)_{\gamma\delta}[\tilde{s}^a_\alpha \tilde{d}^b_\beta (\tilde{\nabla}_j \tilde{u})^c_\delta - \tilde{s}^a_\alpha \tilde{u}^b_\beta (\tilde{\nabla}_j \tilde{d})^c_\delta + \tilde{u}^a_\alpha (\tilde{\nabla}_j \tilde{d})^b_\beta \tilde{s}^c_\delta - \tilde{d}^a_\alpha (\tilde{\nabla}_j \tilde{u})^b_\beta \tilde{s}^c_\delta]$$

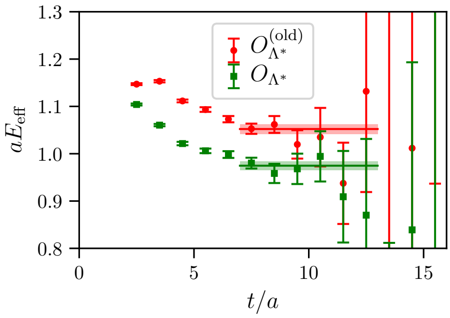


Figure 6: $\Lambda(1520)$ from [Meinel et al. PRD 103 (2021) 074505]

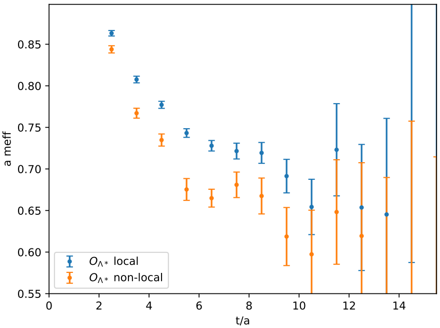


Figure 7: $\Lambda(1520)$ on X451

- A comprehensive set of single- and multi-hadron operators is indispensable to study scattering
- $\Lambda(1405)$ analysis:
  - Investigation of m_q dependence by computing at $m_\pi \simeq 287$ MeV, $m_K = 464$ MeV (X451 box)
  - This and other ensembles will allow us to obtain a very robust amplitude parametrization and extrapolate to the physical world
  - Displaced operators will improve analysis

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- We have embarked in a route along several m_q, a and L, looking for the lowest lying resonances

Appendix I

Using the above definition, it is straightforward to show that the unitarity of S implies the Hermiticity of K. Time reversal invariance of the S-matrix implies that the K-matrix must be symmetric. Given its Hermiticity, this means that K is real and symmetric. Rotational invariance implies that the K-matrix must have the form

- $t^{-1} = \tilde{K}^{-1} - i\hat{k}$, with $m_\pi \hat{k} = \text{diag}\ (k_{\pi\Sigma}, k_{\bar{K}N})$ and $\tilde{K}$ hermitic due to unitarity
- $\det[\widetilde{K}^{-1}(E_{\text{cm}}) - B^P(E_{\text{cm}})] + \mathrm{O}(\mathrm{e}^{-ML}) = 0$
- $\langle J'm'_J\ell'S'a'|K|Jm_J\ell Sa\rangle = \delta_{J'J}\delta_{m'_Jm_J}K^{(J)}_{\ell'S'a';\ell Sa}(E_{\text{cm}})$
- all models with $\text{AIC} - \text{AIC}_{\text{min}} < 1$, and taking half the difference in the maximal spread of values as the model uncertainty. In future work, when a full extrapolation to the physical point is performed, a more thorough determination of the model-average and systematic uncertainty will be performed.
- AIC_{min} for ERE (and $\chi^2/\text{dof} = 10.52/11$), values
  - ERE: $\tilde{K}_{ij} = \frac{m_\pi}{E_{cm}}(A_{ij} + B_{ij}\Delta(E_{cm}))$, with $\Delta(E_{cm}) = (E_cm^2 - (m_\pi + m_\Sigma)^2)/(m_\pi + m_\Sigma)^2$

Appendix II

- link variables are stout smeared, $\rho = 0.1$, $n_\rho = 36$
- An advantage feature of LapH is source-sink factorization of the correlators which greatly facilitates the evaluation of large Hermitian correlation matrices via optimized tensor contractions
- Low-lying eigenvalues of the Dirac matrix of either the light quark doublet or the strange quark can cause instabilities in the HMC an issue which is sufficiently ameliorated by light- and strange-quark reweighting.
- relative quark lines only at the sink

D200

- $N_{\mathrm{ev}} = 448$
- $N_{\mathrm{bin}} = 10$, jackknife and bootstrap ($N_B = 800$)
- single-pivot with $t_0 = 4a$ and $t_{\mathrm{diag}} = 16a$
  - $t^{-1} = \tilde{K}^{-1} - i\hat{k}$
  - $\det[\widetilde{K}^{-1}(E_{\mathrm{cm}}) - B^P(E_{\mathrm{cm}})] + \mathrm{O}(\mathrm{e}^{-ML}) = 0$
  - several $\tilde{K}_{ij}$ parametrisations (ERE, ERE of $\tilde{K}_{ij}^{-1}$, Blatt-Biederharn, W-T inspired, lin. around $\pi\Sigma_{\mathrm{thr}}$)
- sixteen energies are used, which include the fifteen energies from the s-wave fits and one G1g(0) level, and we incorporate a 1/2+ wave in the K-matrix parametrization. In the second additional fit, seventeen energies are used, which include the fifteen energies from the s-wave fits and one level each in the F1(2), F2(3) irreps, and we add a 3/2+ wave in the K-matrix parametrization.
- coupling from residues of poles 1 and 2:
  $\left|\frac{c^{(1)}_{\pi\Sigma}}{c^{(1)}_{\bar{K}N}}\right| = 1.9(4)_{\mathsf{st}}(6)_{\mathsf{md}}$, $\left|\frac{c^{(2)}_{\pi\Sigma}}{c^{(2)}_{\bar{K}N}}\right| = 0.53(9)_{\mathsf{st}}(10)_{\mathsf{md}}$

Appendix III

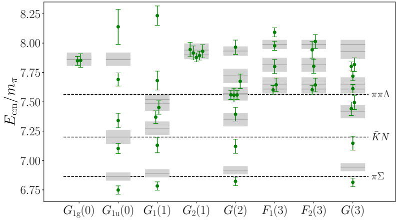
X451

- $N_{\text{ev}} = 180$

Appendix IV

Single- & two-hadron ops. are necessary to extract levels:

$$\Lambda[G_{1u}(0)]_0$$
$$\Lambda[G_{1u}(0)]_1$$
$$\Lambda[G_{1u}(0)]_2$$
$$\Lambda[G_{1u}(0)]_3$$
$$\bar{K}[A_{1u}(0)]_0 \; N[G_{1g}(0)]_0$$
$$\pi[A_{1u}^{-}(0)]_0 \; \Sigma[G_{1g}(0)]_0$$
$$\bar{K}[A_2(1)]_1 \; N[G_1(1)]_0$$
$$\pi[A_2^{-}(1)]_1 \; \Sigma[G_1(1)]_0$$



D200 extracted E levels

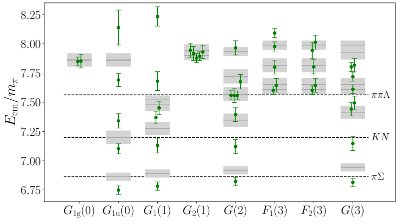
Rest Frame Irreps

J^P	[000]	[00n]	[0nn]	[nnn]	
$1/2^+$	G_{1g}	G_1	G	G	Λ, $\Lambda(1600)$
$1/2^-$	G_{1u}	G_1	G	G	$\Lambda(1405)$, $\Lambda(1670)$
$3/2^+$	H_g	G_1, G_2	$2G$	F_1, F_2, G	-
$3/2^-$	H_u	G_1, G_2	$2G$	F_1, F_2, G	$\Lambda(1520)$, $\Lambda(1690)$

- $G_{1u} \simeq 1/2^-$ (only contaminated by $\ell = 4$)
- moving frames: mixing of quantum numbers (more operators)
  - $J^P = 1/2^+, 3/2^+$ contamination seems negligible close to lowest threshold, but more data will improve analysis

Appendix V

- Correlation matrix
  - $C_{ij}(t) = \langle \mathcal{O}_i(t) \bar{\mathcal{O}}_j(0) \rangle = \sum_k A_k^{(ij)} e^{-tE_k}$
  - Stochastic Laplacian Heaviside distillation [Peardon et al PRD 80 (2009)], [Morningstar et al PRD 83 (2011)]
    - source-sink factorization allows to use optimised tensor contractions to evaluate correlation matrices containing large sets of multi-hadron ops
    - only low modes of the Laplacian are kept (smearing) (quark perambulators can be stored)

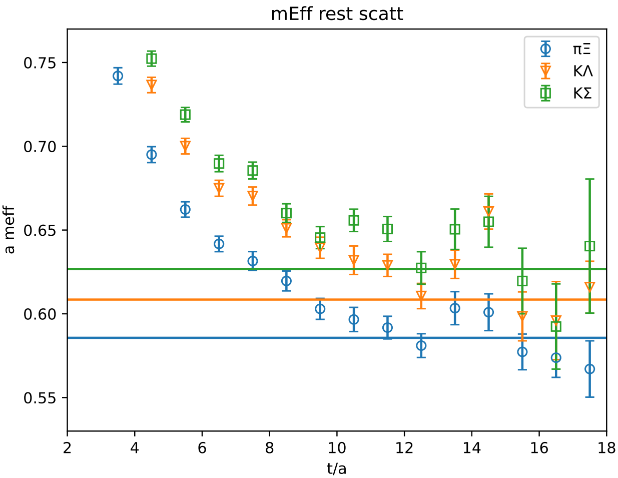


D200 extracted E levels

- GEVP (increases $\Delta E_{\rm eff}$) and ratio fits to extract lowest lying E_k from earlier plateaus
  - $C(t)\,\vec{v} = \lambda(t)\,C(t_0)\,\vec{v}$

Production on X451:

- three coupled channels $\mathcal{O}(50\mathrm{MeV})$ separation
- 135 cnfgs on one source and direction ($\sim$1% data)
  - consistent with noninteracting thresholds



Appendix VII

- Single hadron operators in the $J^P = 1/2^-$ channel

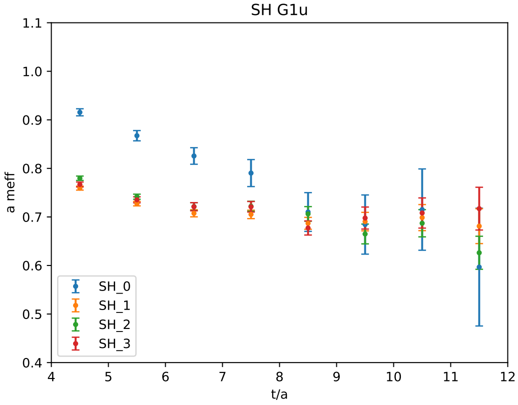


Figure 8: before GEVP

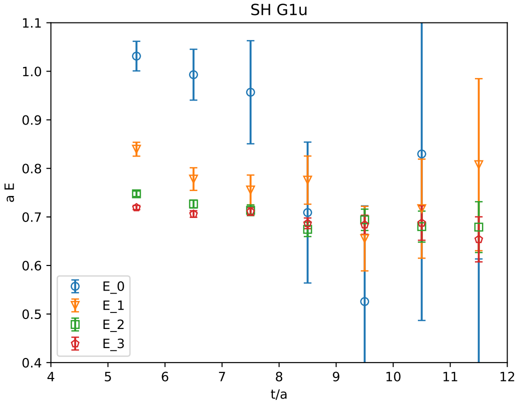


Figure 9: after GEVP

- Lutz's, Oset's and Feijoo's papers: $\sim \Xi(1620)$ couples to $|g_{\pi\Xi}|^2 > |g_{\bar{K}\Lambda}|^2 >> |g_{\bar{K}\Sigma}|^2 > |g_{\eta\Xi}|^2 \sim 0$.

Appendix VIII

LEC $[G_F m_{\pi^+}^2 \sqrt{2} F_\pi]$	h_D	h_F	h_C	w_d	w_f	d^*	f^*
Value	-0.181 ± 0.252	0.171 ± 0.041	-0.090 ± 0.472	-7.99 ± 3.82	7.91 ± 1.32	1.10 ± 1.55	-2.33 ± 1.50

$$\tilde{\chi}^2 \equiv \chi^2/(n_{\mathrm{dat}} - n_{\mathrm{pars}}) = 0.11/(8-7) = 0.11$$

	s_{expt}	s_{theory}	s_{tree}	Δs_8	Δs_{10}	Δs_{res}	p_{expt}	p_{theory}	p_{tree}	Δp_8	Δp_{10}	Δp_{res}
L_{11}^{Σ}	-0.936	-0.342	-0.173	0.076	-0.006	-0.238	2.714	2.715	-0.317	-0.008	0.213	2.828
L_{13}^{Σ}	1.948	1.947	0.346	-0.341	0.018	1.924	0.015	0.015	0.246	-0.475	0.192	0.053
L_{11}^{Λ}	1.412	1.424	0.135	0.015	-0.044	1.319	0.625	0.624	-0.006	-0.134	0.183	0.581
L_{12}^{Ξ}	-2.046	-2.032	-0.284	0.327	0.140	-2.216	0.397	0.397	-0.078	0.616	0.157	-0.298

	s_{expt}	s_{theory}	s_{tree}	Δs_8	Δs_{10}	Δs_{res}	p_{expt}	p_{theory}	p_{tree}	Δp_8	Δp_{10}	Δp_{res}
$\Sigma^+ \to n\pi^+$	0.023	0.407	0	-0.065	0	0.472	1.815	1.813	-0.129	-0.148	0.201	1.889
$\Sigma^+ \to p\pi^0$	-1.337	-1.066	-0.249	0.201	-0.008	-1.010	1.272	1.277	-0.265	0.197	0.017	1.328
$\Sigma^- \to n\pi^-$	1.914	1.981	0.352	-0.348	0.018	1.958	0.015	0.015	0.246	-0.476	0.192	0.053
$\Lambda \to p\pi^-$	1.403	1.434	0.136	0.015	-0.045	1.327	0.625	0.624	-0.006	-0.134	0.183	0.581
$\Lambda \to n\pi^0$	-0.992	-1.009	-0.096	-0.011	0.031	-0.933	-0.442	-0.459	0.005	0.096	-0.135	-0.425
$\Xi^- \to \Lambda\pi^-$	-2.046	-2.032	-0.284	0.327	0.140	-2.216	0.397	0.397	-0.078	0.616	0.157	-0.298
$\Xi^0 \to \Lambda\pi^0$	1.447	1.382	0.200	-0.232	-0.101	1.515	-0.281	-0.289	0.056	-0.434	-0.113	0.202

TABLE VII: Results of the combined fit to s- and p-waves including tree-level diagrams with resonances. The total theory result (label "theory") is split up into the contributions from LO chPT (tree), the loop contributions with only octet baryons in the loop (8), the loop contributions involving decuplet states (10), and the contributions from resonances as a proxy for the NLO counterterms (res). The uncertainties of the theory results are shown in Figs. 4 and 7.