# Revealing the puzzle of the dynamical nature of the two $D_0^*$ states from an analysis of LQCD results

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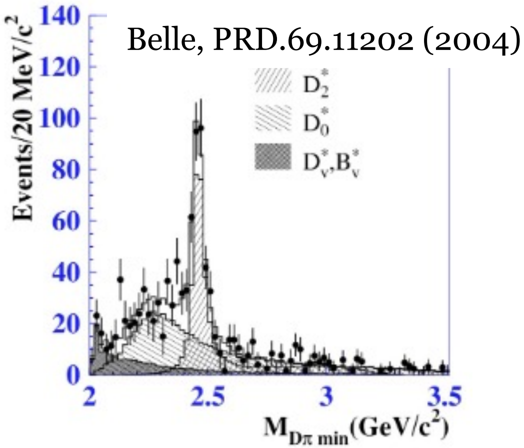
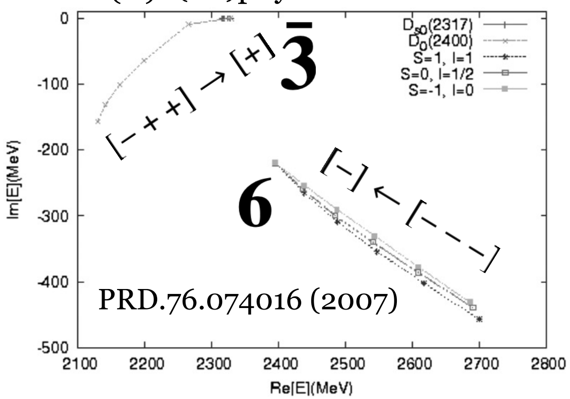
**In collaboration with Fernando Gil & R. Molina**

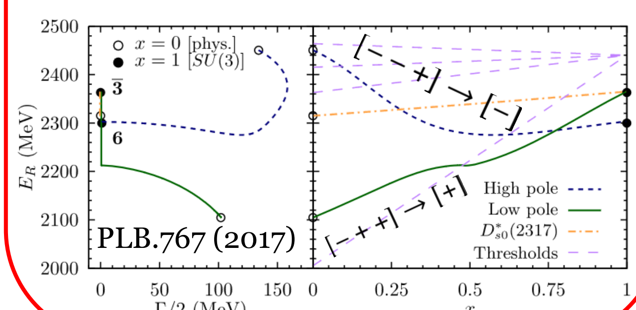
**Based on arXiv: 2606.03826**

# Background




+(−): (un)physical sheet




- A broader resonance discovered by Belle Coll.
  - $J^P = 0^+$
  - $M_{D_0^*} = 2308(17)(15)(28)$ (MeV) $\quad (S, I) = \left(0, \frac{1}{2}\right)$
  - $\Gamma = 276(21)(18)(60)$ (MeV)
- Two-pole picture predicted by UChPT

$$D\pi - D\eta - D_s\bar{K}$$

F.-K. Guo et al, PLB.647 (2007)

G. D. et al, PRD.76.074016 (2007)

M. A. et al, PRD.89.014026 (2014)

Z.-H. Guo et al, PRD.92.094008 (2015)

M. A. et al, PLB.767 (2017) …

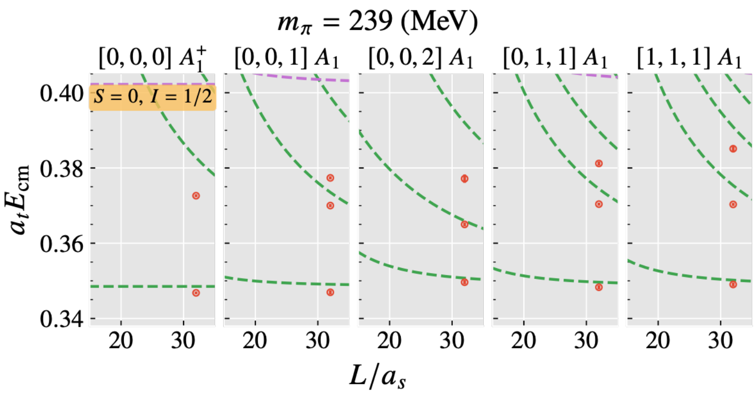
- SU3 multiplet $\quad \bar{\mathbf{3}} \otimes \mathbf{8} = \bar{\mathbf{3}} \oplus \mathbf{6} \oplus \overline{\mathbf{15}}$

**No chiral trajectories of the hadron masses or LQCD results as input**

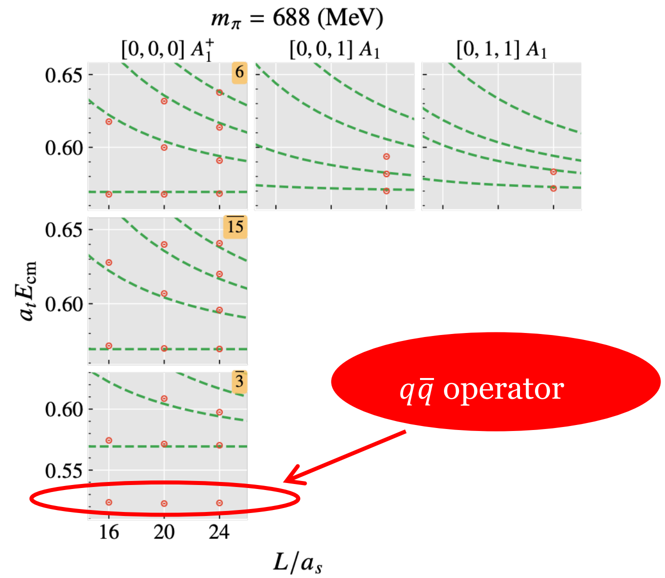
**The interpretations of two $D_0^*$ states remains elusive!**

# $D_0^*$ in LQCD

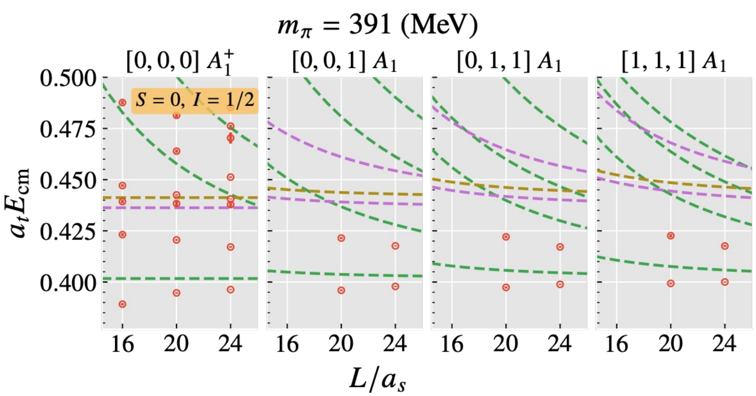
$m_s = m_{s,\text{phy}}$




$D\pi$ simulation JHEP07(2021)123




JHEP07(2024)012




Coupled-channel simulation

JHEP10(2016)011

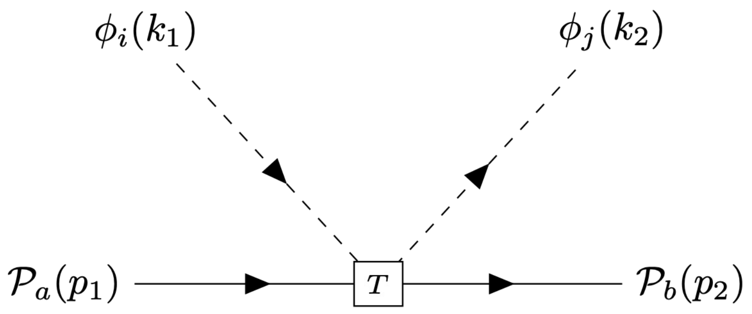
- $z = 2196(64) - i212(112)$ (MeV) [239 MeV]
- $z = 2276(1)$ (MeV) [391 MeV]
- No second pole reported in $m_\pi = 239$ or 391 MeV
- SU3 result
  - $z_{\bar{3}} = 2435(4)$ (MeV) [CDD pole] Bound state
  - Compositeness $X_{\bar{3}} = 0.56(2)(10)$
  - $z_6 = 2560(50)$ (MeV) Virtual state
  - ✗ Connections between the SU3 and the $D_0^*$ states not done

# LO interaction

- Weinberg-Tomozawa interaction

$$V_{\mathrm{WT}} = -\frac{C_{\mathrm{LO}}^{(ia,jb)}}{4 f_i f_j}(s-u)$$

$$C_{\mathrm{WT}}^{(ia,jb)} = \begin{pmatrix} 2 & 0 & \sqrt{\frac{3}{2}} \\ 0 & 0 & \sqrt{\frac{3}{2}} \\ \sqrt{\frac{3}{2}} & \sqrt{\frac{3}{2}} & 1 \end{pmatrix}$$




$$\mathcal{P}\phi = \{D\pi, D\eta, D_s\bar{K}\}$$

- SU3 flavor symmetry $\bar{\mathbf{3}} \otimes \mathbf{8} = \bar{\mathbf{3}} \oplus \mathbf{6} \oplus \overline{\mathbf{15}}$
  - Attractive interaction in $\bar{\mathbf{3}}$ and $\mathbf{6}$
  - Repulsive interaction in $\overline{\mathbf{15}}$
  - Decoupled interaction in the SU3 limit

$$V_{\mathrm{SU3}}^{\mathrm{WT}} = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \times \left[-\frac{1}{4f^2}(s-u)\right]$$

# NLO interaction

$$V_{\mathrm{NLO}} = \frac{-1}{f_i f_j}\left(C^{(ia,jb)}_{h_0} h_0 + C^{(ia,jb)}_{h_1} h_1 + H_{24}(s,t,u) + H_{35}(s,t,u)\right)$$

$$H_{24}(s,t,u) = C^{(ia,jb)}_{h_2} h_2 (k_1 \cdot k_2) + C^{(ia,jb)}_{h_4} h_4 [(k_1 \cdot p_2)(k_2 \cdot p_1) + (k_1 \cdot p_1)(k_2 \cdot p_2)]$$

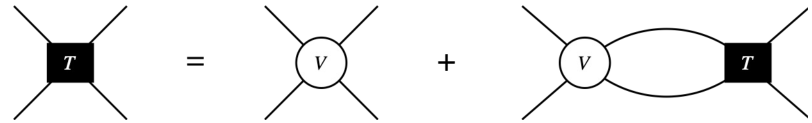
$$H_{35}(s,t,u) = C^{(ia,jb)}_{h_{3(5)}} [h_3 (k_1 \cdot k_2) + h_5 (k_1 \cdot p_2)(k_2 \cdot p_1) + (k_1 \cdot p_1)(k_2 \cdot p_2)]$$

- $h_l, l = 0{,}3$ are dimensionless, $h_{4(5)}$ have unit Mass$^{-2}$
- $h_0 = 0.014$ and $h_1 = 0.45$ determined by the masses of the $D$ mesons
- $h_l, l = 2{,}5$ constrained by the energy levels in LQCD

# Bethe-Salpeter equation

- Coupled channels scattering matrix

$$T = V + VGT = [\mathbb{I} - VG]^{-1}V$$




$$G_j = i\int \frac{d^4q}{(2\pi)^4}\frac{1}{(P-q)^2 - m_j^2 + i\epsilon}\frac{1}{q^2 - M_j^2 + i\epsilon}$$

$$G = G^{\mathrm{DR}}\left(\sqrt{s}, a(\mu = 1\,\mathrm{GeV}, \Lambda)\right) + \begin{cases} \frac{ik}{4\pi\sqrt{s}}, & \mathrm{Re}\sqrt{s} > m + M \\ 0, & \text{others} \end{cases}$$

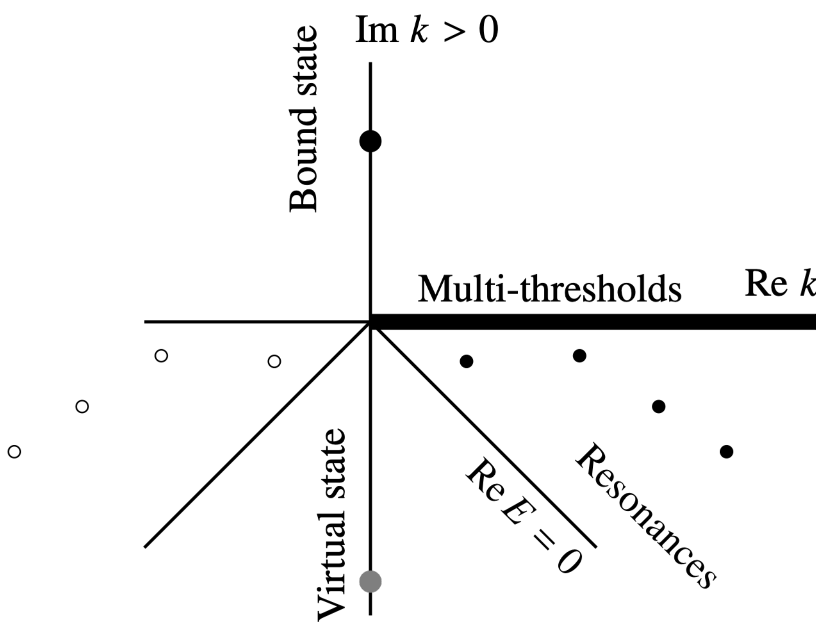
J. A. Oller et al, PRD.60.074023

J. A. Oller, Prog. Part. Nucl. Phys. 110.103728 (2020)

$\Lambda$ as free parameter

- Particle as pole in $\det[\mathbb{I} - VG] = 0$

## Complex $k$-plane




- Scattering matrix in a finite box

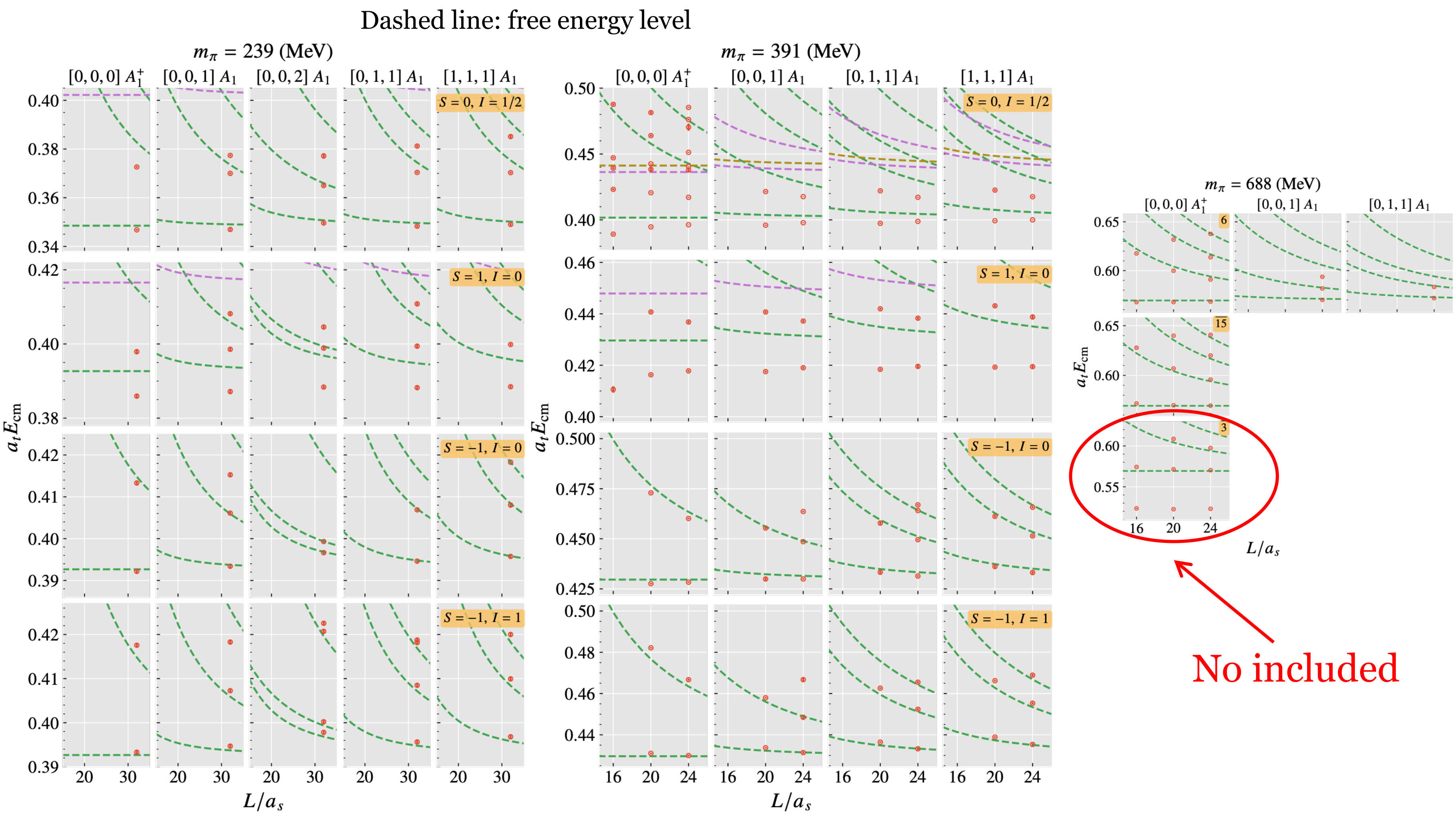
$$\tilde{T} = \left[\mathbb{I} - V\tilde{G}\right]^{-1}V$$

  - $\tilde{G}_j = G_j^{\mathrm{DR}}(P) + \lim_{q_{\max}\to\infty}\Delta\tilde{G}_j(P, q_{\max})$
  - $\Delta\tilde{G}_j = \frac{1}{L^3}\sum_{\vec{n}}^{q_{\max}}\frac{E}{P_0}I_j(|\vec{q}^{\,*}(\vec{q})|) - G^{\mathrm{cutoff}}(P)$
  - $I(|\vec{q}^{\,*}|) = \frac{1}{2\omega_{1j}\omega_{2j}}\frac{\omega_{1j}\omega_{2j}}{s - (\omega_{1j}+\omega_{2j})^2 + i\epsilon}$ EPJA.2012.48.114

- The energy levels of a finite volume

$$\det\left[\mathbb{I} - V\tilde{G}\right] = 0$$

# Data considered




- We perform a global fit to the energy levels (~ 100 points) in different sectors $(S, I)$
- The energy levels in $\overline{\mathbf{3}}$ sector are not included, which will be analyzed independently

# LECs constrained by the energy levels

$$h_{45} = \frac{h_4}{h_5} \quad h'_5 = m_{H,\mathrm{avg}} h_5 \quad m_{H,\mathrm{avg}} = 1929\,\mathrm{MeV}$$

| $h_2$ | $h_3$ | $h_{45}$ | $h'_5$ |
|---|---|---|---|
| 0.56(26)(22) | -5.39(5)(35) | 0.17(1)(12) | 2.6(2)(1) |

- ❑ $\Lambda = 677(45)_{\mathrm{st}}(50)_a$ (MeV)
- ❑ Non-negligible correlation between the LECs

$\chi_{\mathrm{d.o.f}} = 1.5$

| | $\Lambda$ | $h_2$ | $h_3$ | $h_{45}$ | $h'_5$ |
|---|---|---|---|---|---|
| $\Lambda$ | 1.000 | -0.281 | 0.343 | 0.189 | 0.078 |
| $h_2$ | -0.281 | 1.000 | 0.229 | -0.795 | -0.037 |
| $h_3$ | 0.343 | 0.229 | 1.000 | 0.163 | 0.029 |
| $h_{45}$ | 0.189 | -0.795 | 0.163 | 1.000 | 0.063 |
| $h'_5$ | 0.078 | -0.037 | 0.029 | 0.063 | 1.000 |

➢ A bare $Q\bar{q}$ pole included in the interaction to fit the energy levels in $\bar{\mathbf{3}}$ sector

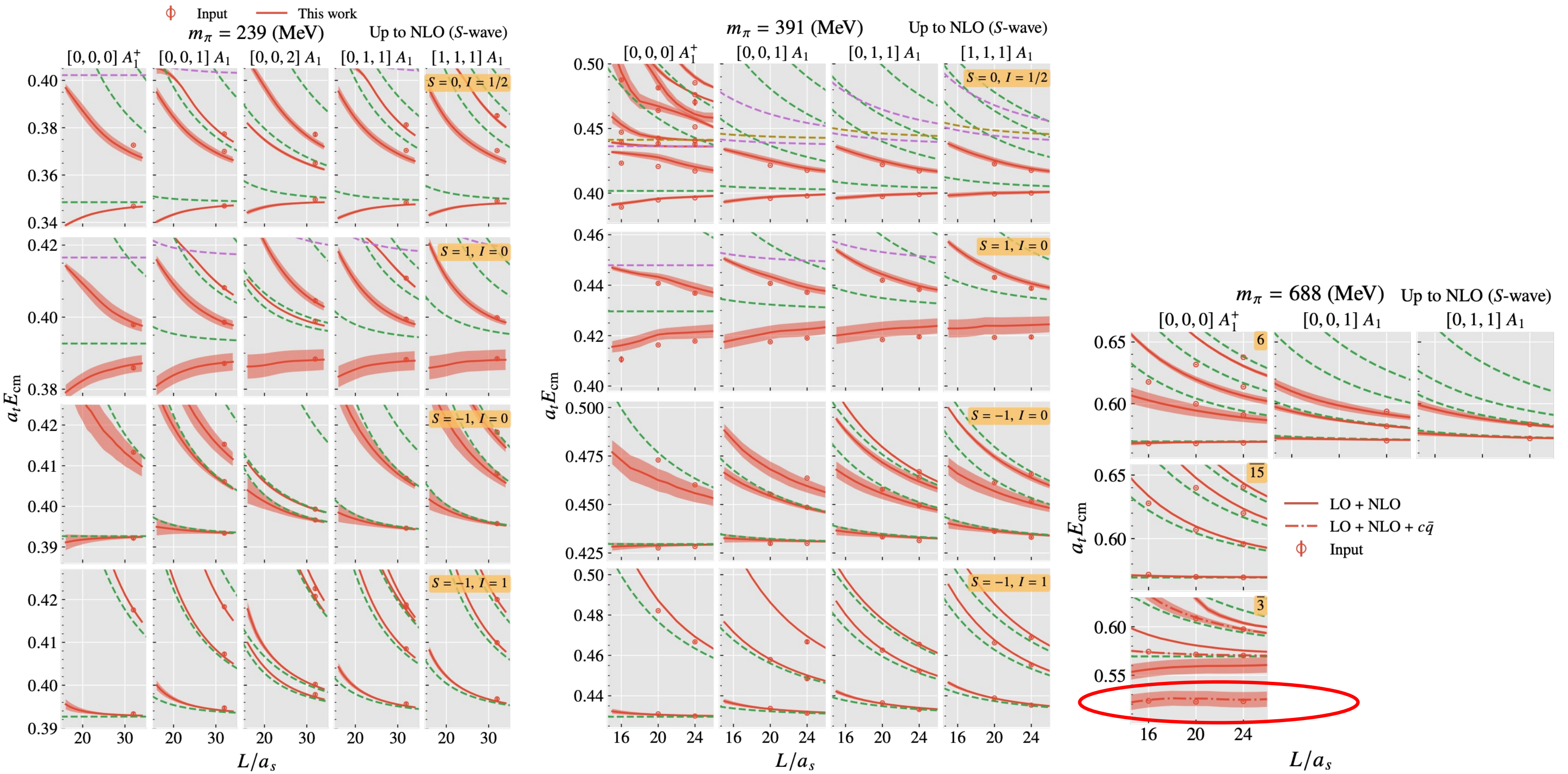
$$V_{\mathrm{tot}} = V_{\mathrm{ChPT}} + V_{c\bar{q}}$$

$$V_{c\bar{q}} = \frac{g^2}{s - m_{c\bar{q}}^2}$$

$$g = g_0 + (m_\pi - m_{\pi,\mathrm{phy}}) g_1$$

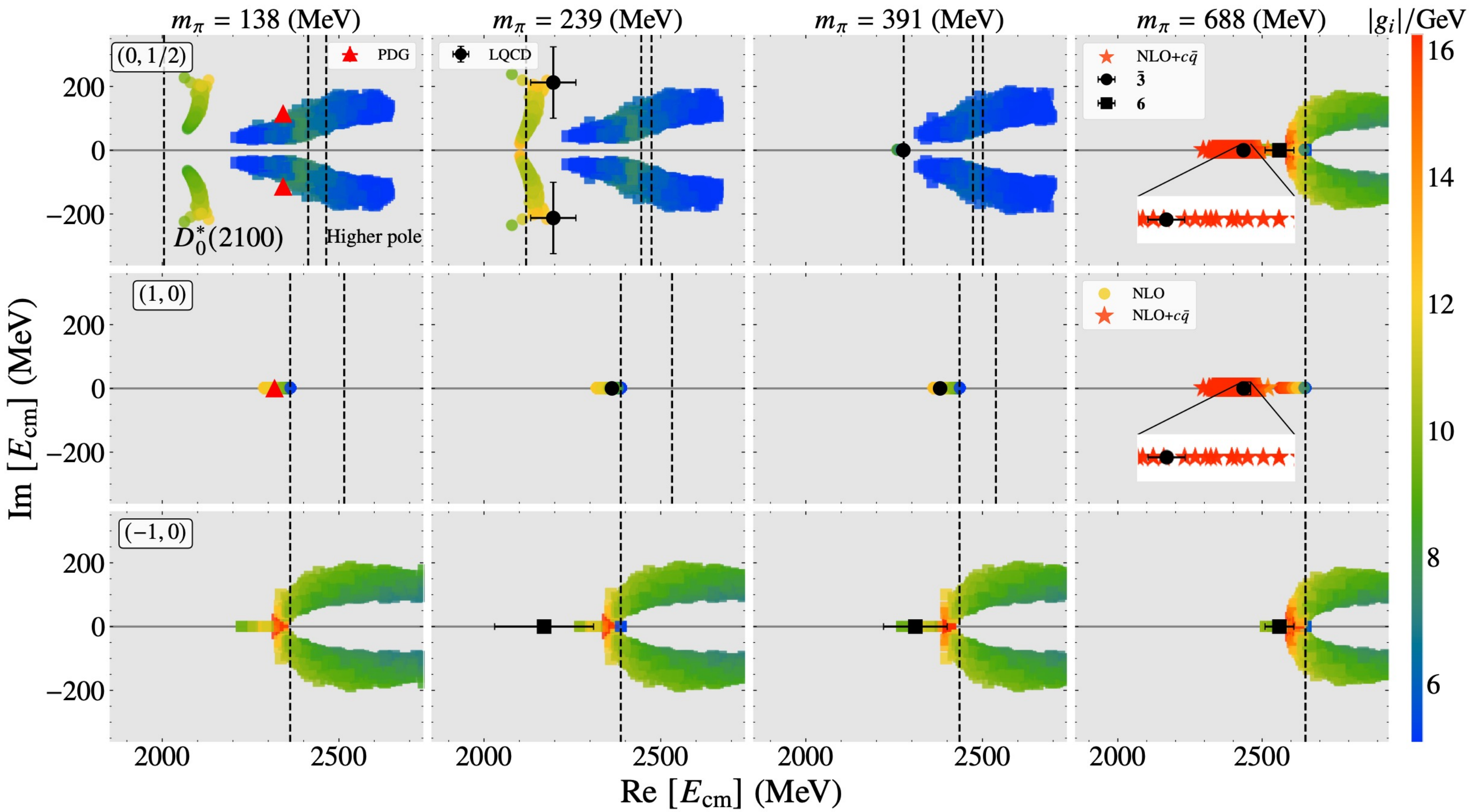
| | $g_0$ (GeV) | $g_1$ | $m_{c\bar{s}}$ (MeV) |
|---|---|---|---|
| LO + $c\bar{q}$ | 4.9(10) | 40(13) | 2550(96) |
| NLO + $c\bar{q}$ | 2.4(5)(5) | 40(6)(7) | 2550 [Fixed] |

# Fitting results



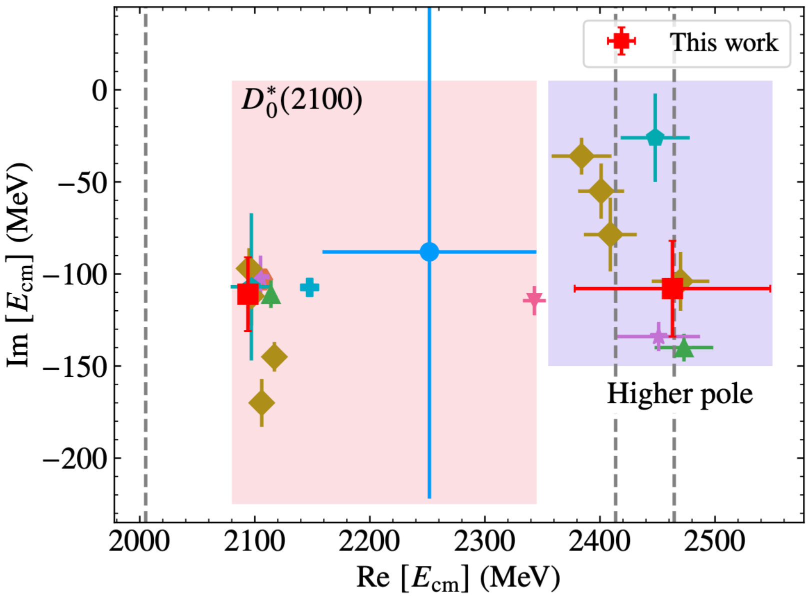

- The energy levels are reproduced well in UChPT model
- It is indispensable to include the bare term as input

# Two-pole structure of $D_0^*$




- Two-pole structure of $D_0^*$ observed within $1\sigma$ region from $m_\pi$ = 138 MeV to SU3 limit
- A molecule $\bar{3}$ pole ($X\sim1$) [**ChPT**] (**Model 1**) and a CDD $\bar{3}$ pole ($X = 0.63(11)(26)$) [**ChPT+$c\bar{q}$**] (**Model 2**) obtained in SU3 limit
- The lower pole consists with HadSpec results at various pion-mass setups
- $D_{s0}(2317)$ as a bound state at various pion-mass setups
- A pole in sector $(S, I) = (-1,0)$ is a resonance or a virtual state within $1\sigma$ region JHEP02(2021)100

# Comparsion with other work




[18] PRD.76.074016 (2007)

[57] PRD.89.014026(2014)

[26] PLB.767.465 (2017)

[56] EPJC.79.13 (2019)

[58] PLB.641.278 (2006)

[2] PDG

[32] PRD.111.014503 (2025). One of the simulations at $m_\pi = 133$ MeV

[55] PRD.92.094008 (2015)

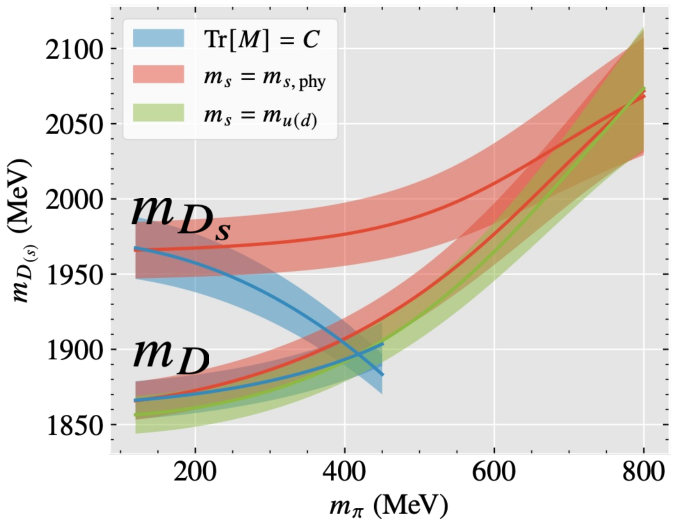
- We do find two poles

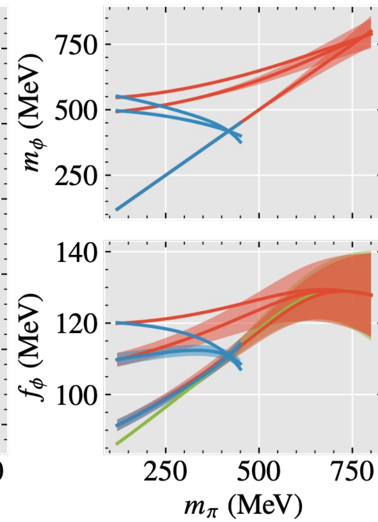
**+(−): (un)physical sheet**

  - $D_0^*(2100)$: 2094(7)(1)-$i$111(7)(13) (MeV) $[-++]$
  - Higher pole: 2463(60)(30)-$i$108(14)(12) (MeV) $[---]$
  - Disagreement with PDG average

- The pole position of the $D_0^*(2100)$ compatible with other ChPT results
- The higher pole reported by other chiral based results is located in RS $[--+]$

# Pion mass dependence of hadron mass



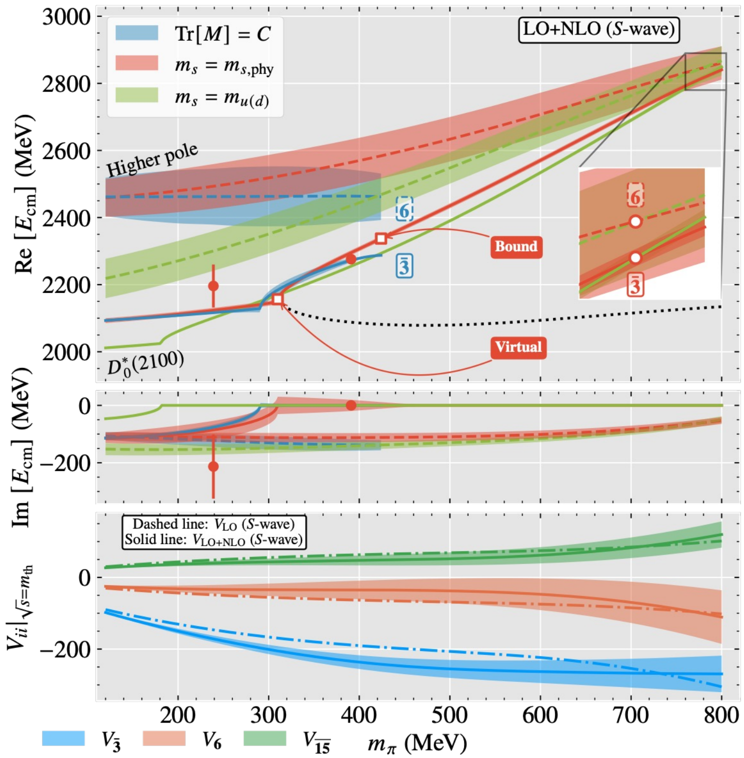

$m_c = m_{c,\mathrm{phy}}$

**Z. Zhuang et al, arXiv:2606.03826** **Z. Zhuang et al, arXiv: 2606.08609**

- Three trajectories: $\mathrm{Tr}[M] = C$, $m_s = m_{s,\mathrm{phy}}$, and $m_s = m_{u(d)}$ $M = \mathrm{diag}\{m_u, m_d, m_s\}$

| | $m_\phi$ (MeV) | $f_\phi$ (MeV) | $m_H$ (MeV) |
|---|---|---|---|
| $\mathrm{Tr}[M] = C$ | 420(3)(1) | 110(1)(12) | 1897(14) |
| $m_s = m_{s,\mathrm{phy}}$ | 780(60)(22) | 128(4)(10) | 2062(38) |

# Pole trajectories over various trajectories




- Two poles in multiples $\bar{\mathbf{3}}$ and $\mathbf{6}$

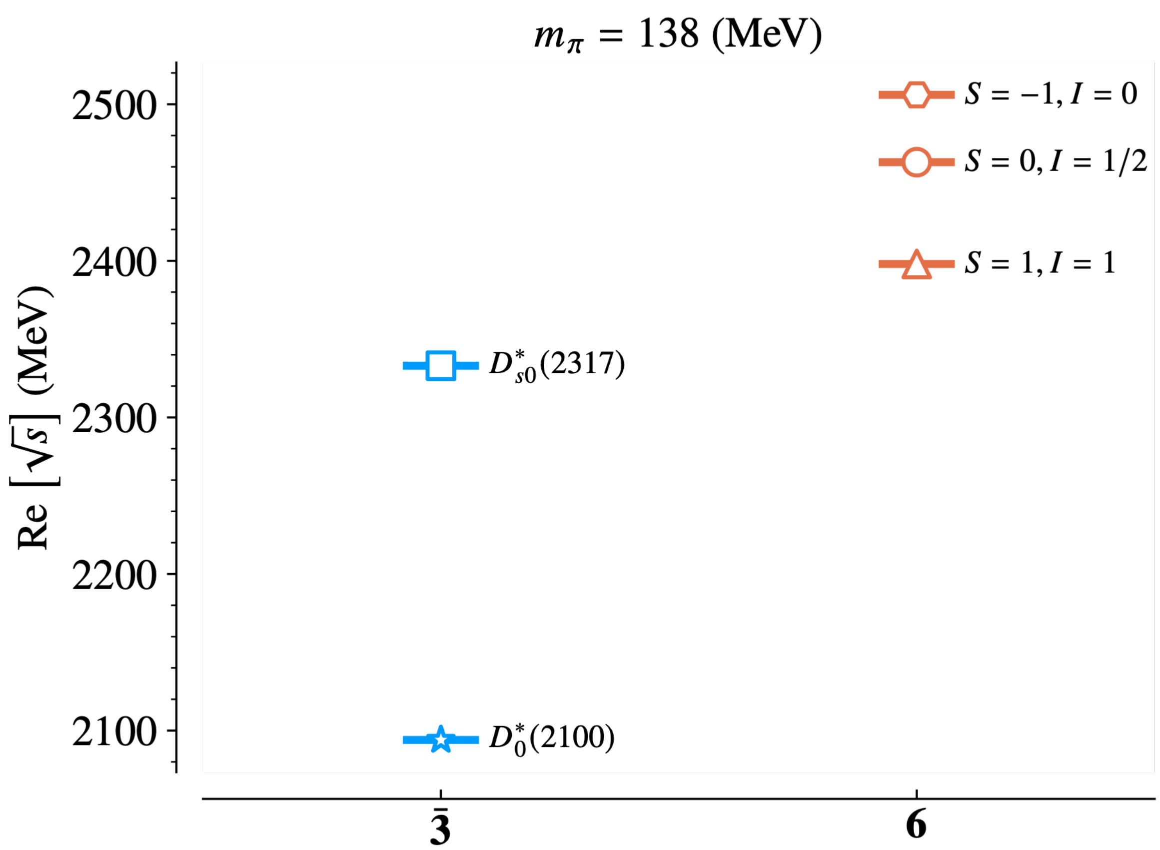
$m_s = m_{s,\text{phy}}$

| | $\bar{\mathbf{3}}$ [+] | $\mathbf{6}$ [−] |
|---|---|---|
| Pole (MeV) | 2816(7)(11) | $2846(50)(5) - i61^{(10)(30)}_{(30)(30)}$ |
| $\lvert g_{\bar{3}}\rvert$ (GeV) | 13.2(5)(20) | 0 |
| $\lvert g_6\rvert$ (GeV) | 0 | 9.2(3)(4) |

- No pole in $\overline{\mathbf{15}}$ due to its repulsive interaction

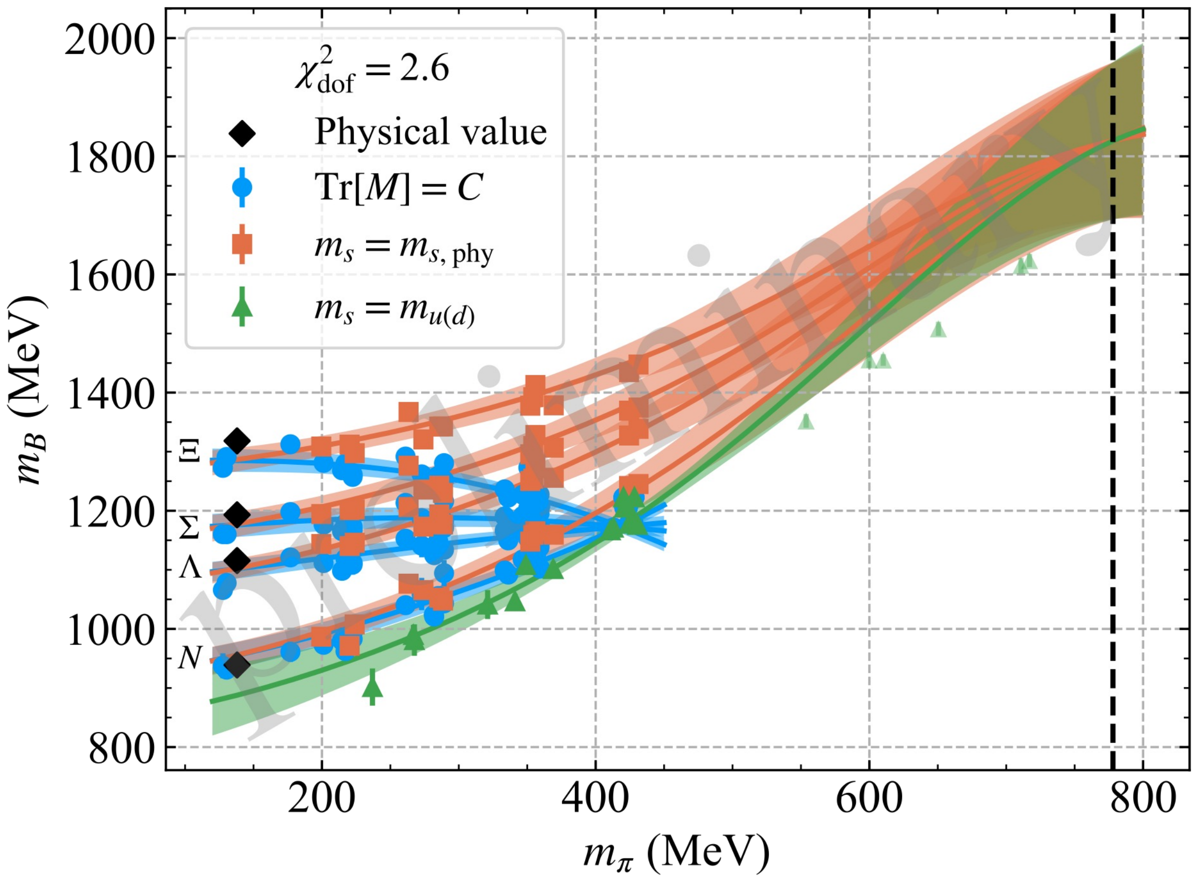
- The $D_0^*(2100)$ separated to two poles at $m_\pi = 310$ MeV and one of them being a bound state at $m_\pi \simeq 420$ MeV
- **Smaller $s\bar{s}$ component** in $D_0^*(2100)$ due to the consistence of the pole in various trajectories
- Different trends of the trajectories indicating that the higher pole has **essential $s\bar{s}$ content**
- The $D_0^*(2100)$ connected to $\bar{\mathbf{3}}$, while the higher pole coupled to $\mathbf{6}$

# Mass spectrum




- Classifying the poles into the flavor multiplets to which they couple in the SU3 limit

# Pion mass dependence of the baryon-octet mass



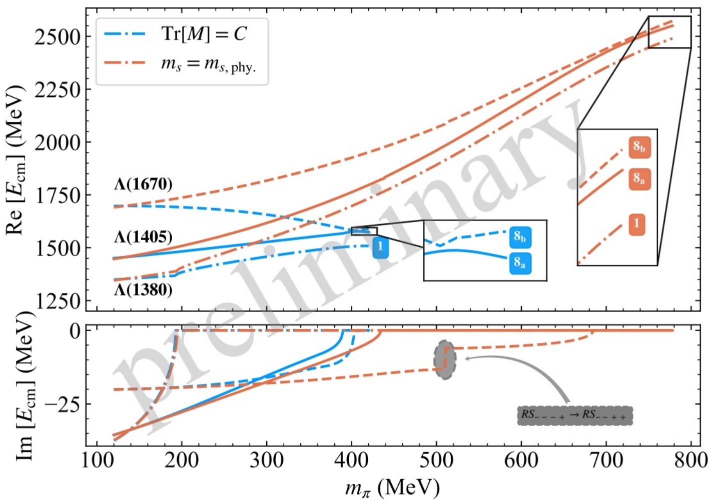

- No $s$ quark component in $N(qqq)$ $\quad q = u, d$

- The trajectories of the $\Lambda(sqq)$, $\Sigma(sqq)$, and $\Xi(ssq)$ **dramatically influenced** by $s$ quark

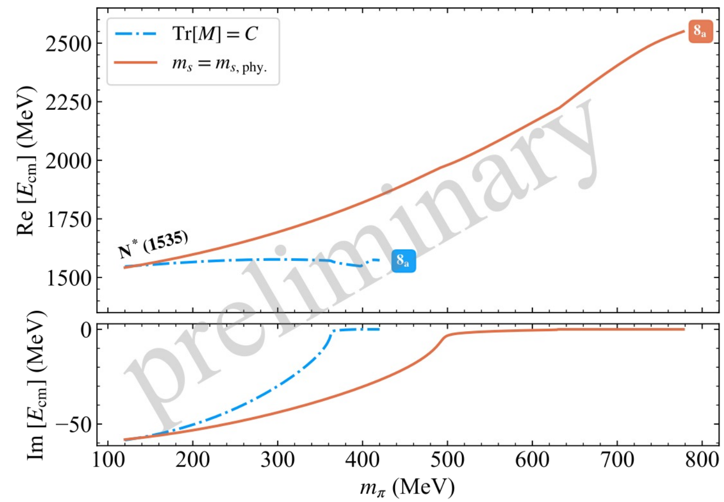
# Chiral trajectories of the $\Lambda^*$ and $S_{11}(N(1535))$ (LO)

In collaboration with R. Molina, J.-X. Lu, and L.-S. Geng

$(S, I) = (-1, 0)$




$(S, I) = (0, 1/2)$




- The $\Lambda^*$ states have definitively $s$ quark component
- The widely divergent trajectories of the $N(1535)$ exhibit its **hidden $s$-quark component**

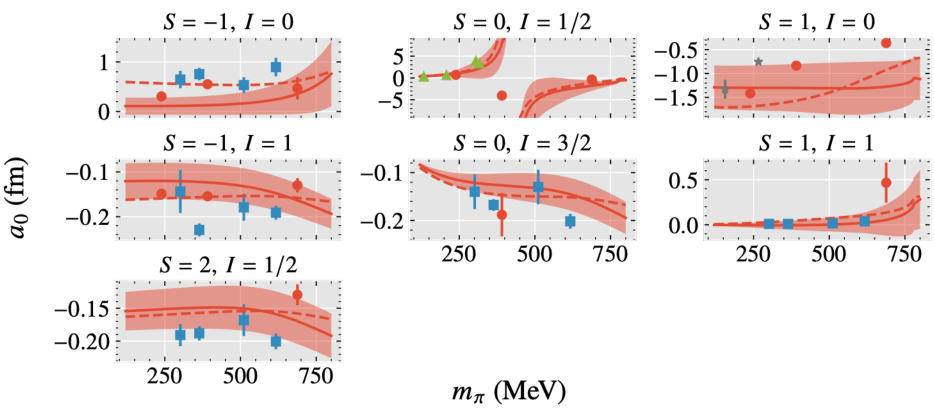
# Conclusion

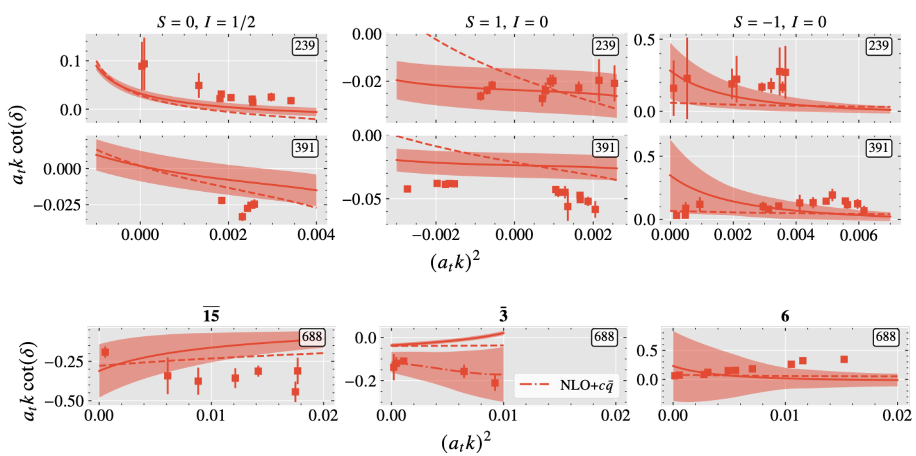
- We performed an exhaustive analysis of the scattering LQCD data with NLO UChPT
- The pion mass dependence of two $D_0^*$ poles for various chiral trajectories is investigated for the first time
- The $D_0^*(2100)$ is separated to a virtual bound and a bound state as $m_\pi$ increasing, coupling to the $\overline{\mathbf{3}}$ representation
- The UChPT analysis provides a compositeness $X \simeq 1$ in the SU3 limit, while the SU3 data demands a $Q\bar{q}$ state with $X = 0.63(11)(26)$
- The higher pole connected to the **6** is either a resonance or a virtual state within $1\sigma$ region
- The hidden strange content in the higher pole is manifested as distinctly different chiral trajectories over $m_s = m_{s,\,\mathrm{phy}}$ and $\mathrm{Tr}[M] = C$

***Thank you for your attention !!!***

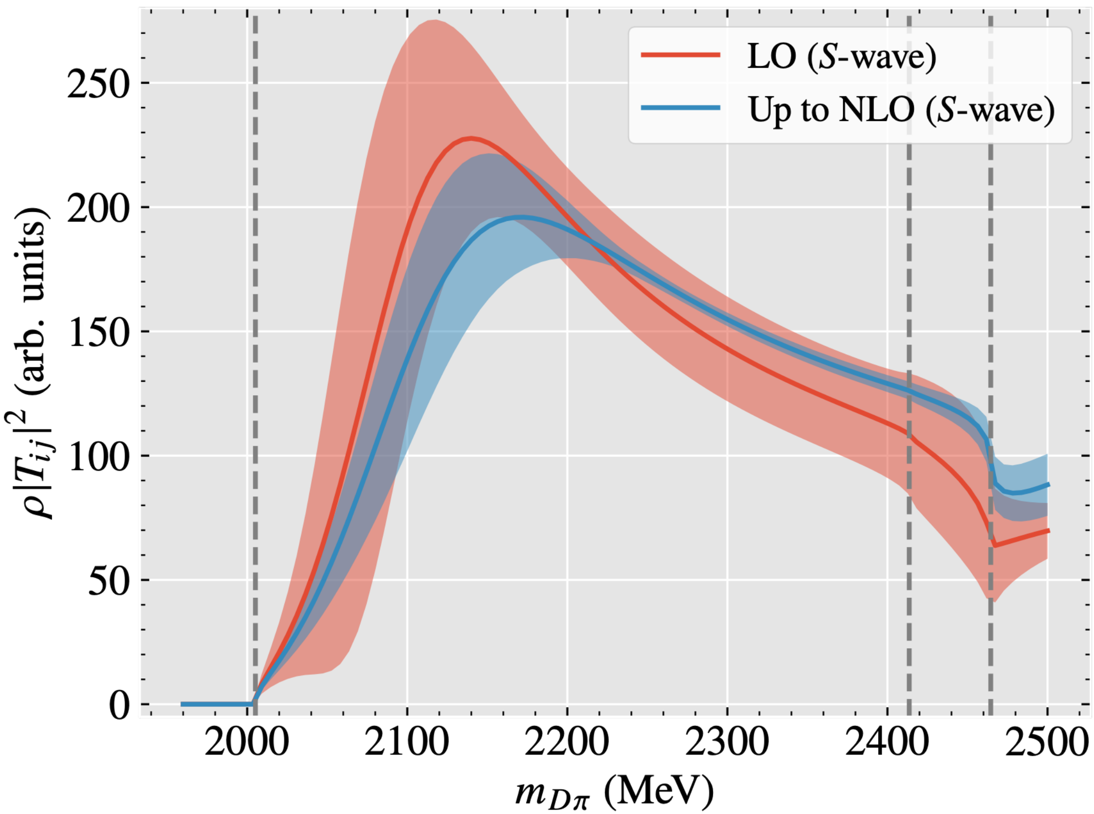
# Backup

# Scattering length and phase shift



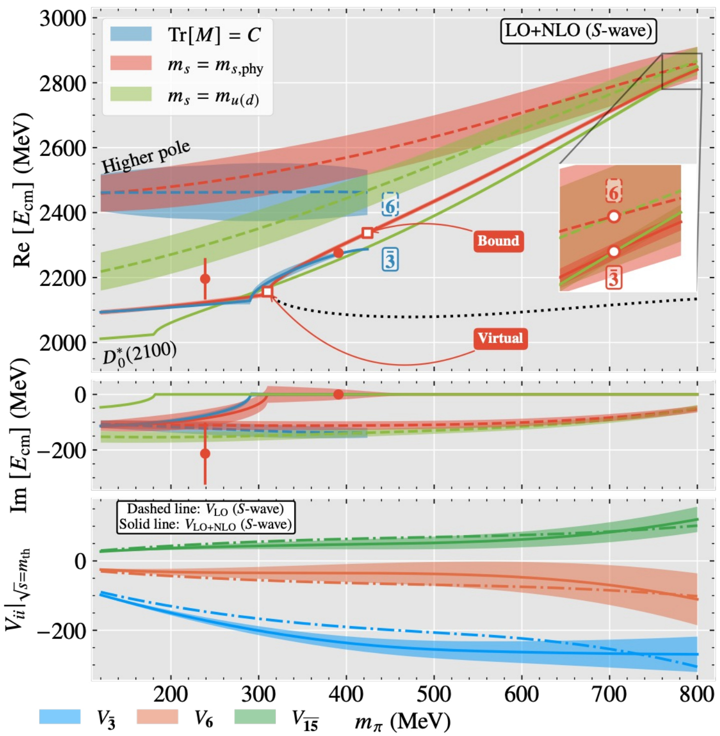



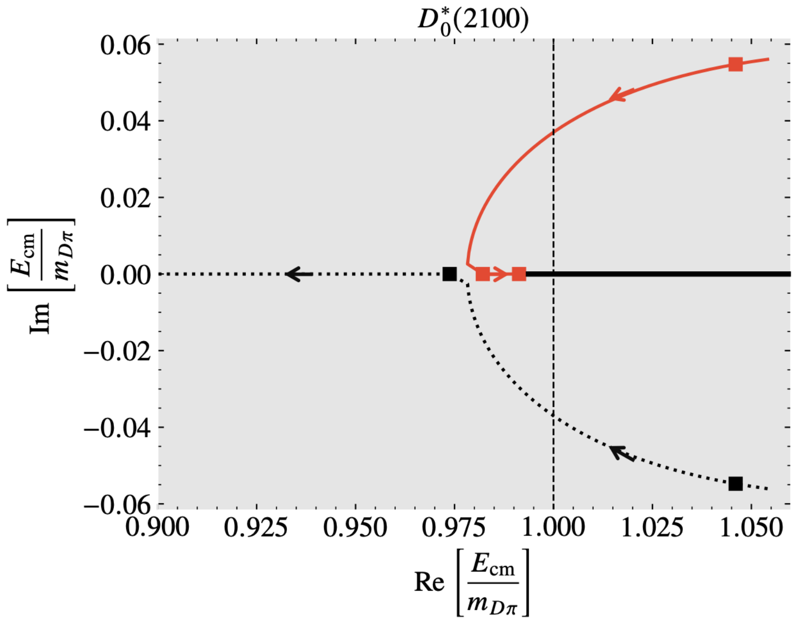
# Predicted invariant mass spectrum




- The $D_0^*(2100)$ is clearly a peak, the higher pole appears as a bump close to the $D\eta$ and $D_s\bar{K}$ thresholds

# Pole movement of the $D_0^*(2100)$





- The $D_0^*(2100)$ separated into two poles at $m_\pi = 310$ MeV
- The one of them being a bound state around $m_\pi \simeq 420$ MeV moving in the $m_s = m_{s,\text{phy}}$ trajectory
- Another pole as virtual state moving away from the threshold

# Pole position of two $D_0^*$ states

The pole positions of the two $D_0^*$ resonances and their couplings to the channels.

| $m_\pi$ (MeV) | R.S. | Pole (MeV) | $\lvert g_{D\pi}\rvert$ (GeV) | $\lvert g_{D\eta}\rvert$ (GeV) | $\lvert g_{D_s\bar{K}}\rvert$ (GeV) |
|---|---|---|---|---|---|
| 138 | $[-++]$ | $2094(7)(1) - i111(7)(13)$ | 9.6(2)(2) | 2.0(2)(3) | 6.8(5)(3) |
| | $[---]$ | $2463(58)(27) - i108(14)(12)$ | 4.4(1)(6) | 6.3(1)(5) | 6.1(4)(3) |
| 239 | $[-++]$ | $2125(7)(1) - i84(9)(17)$ | 10.6(1)(1) | 2.5(2)(4) | 7.8(5)(3) |
| | $[---]$ | $2492(66)(19) - i113(16)(9)$ | 4.7(2)(5) | 6.1(2)(3) | 5.9(2)(4) |
| 391 | $[+++]$ | 2277(7)(4) | 2.3(1)(16) | 0.6(4)(1) | 1.8(8)(5) |
| | $[---]$ | $2549(68)(15) - i121(16)(7)$ | 4.9(1)(4) | 5.8(1)(3) | 5.3(2)(2) |

The poles (MeV) and the couplings (GeV) in the SU(3).

| $m_\pi$ (MeV) | R.S. | Threshold (MeV) | Pole (MeV) | $\lvert g_{\bar{3}}\rvert$ (GeV) | $\lvert g_6\rvert$ (GeV) |
|---|---|---|---|---|---|
| 420 ($\mathrm{Tr}[M] = C$) | $[+]$ | 2317 | 2286(2)(13) | 10.7(3)(10) | 0 |
| | $[-]$ | | $2462(70)(10) - i137(18)(3)$ | 0 | 9.1(2)(2) |
| 688 | $[+]$ | 2650 | 2612(6)(14) (NLO) | 13.2(5)(20) | 0 |
| | | | $2443(25)(1)$ (NLO + $c\bar{q}$) | 16.8(4)(5) | 0 |
| | $[-]$ | | $2735^{(60)(120)}_{(40)(180)} - i107^{(60)(10)}_{(50)(60)}$ | 0 | 9.2(3)(4) |
| 780 | $[+]$ | 2841 | 2816(7)(11) | 13.0(5)(27) | 0 |
| | $[-]$ | | $2846(50)(5) - i61^{(10)(30)}_{(30)(30)}$ | 0 | 10.5(3)(10) |

# Poles in other sectors $(S, I)$

| | Up to NLO | | |
|---|---|---|---|
| | $(1,0)$ | | |
| R.S. | $[++]$ | $[++]$ | $[++]$ |
| Pole | $2333(18)$ | $2362(17)$ | $2406(18)$ |
| $\lvert g_{DK}\rvert$ | $10.3(16)$ | $10.0(17)$ | $10.3(18)$ |
| $\lvert g_{D_s\eta}\rvert$ | $6.3(10)$ | $6.3(11)$ | $6.5(11)$ |
| | $(-1,0)$ | | |
| R.S. | $[-]$ | $[-]$ | $[-]$ |
| Pole | $2506^{(60)(2)}_{(42)(266)} - i126^{(60)(7)}_{(63)(63)}$ | $2526^{(180)(112)}_{(42)(264)} - i126^{(62)(7)}_{(63)(63)}$ | $2574^{(180)(112)}_{(42)(287)} - i126^{(61)(7)}_{(63)(63}$ |
| $\lvert g_{D\bar{K}}\rvert$ | $8.7(6)$ | $8.8(6)$ | $8.8(6)$ |
| | $(1,1)$ | | |
| R.S. | $[--]$ | $\cdots$ | $\cdots$ |
| Pole | $2398(41) - i123(12)$ | $\cdots$ | $\cdots$ |
| $\lvert g_{D_s\pi}\rvert$ | $6.9(2)$ | $\cdots$ | $\cdots$ |
| $\lvert g_{DK}\rvert$ | $6.1(1)$ | $\cdots$ | $\cdots$ |

# Lattice setups

Table 1: A summary of lattice setups for $m_\pi = 239, 284, 327, 391$ MeV, and 688 MeV. The errors of the $a_s$ and $a_t$ are propagated from the $a_s = a_t\xi$ and $a_t^{-1} = \frac{m_{\Omega,\mathrm{phy}}}{a_t m_\Omega}$. The anisotropy used in the SU(3) favor symmetry is the mean of $\xi_{\eta_8} = 3.446(3)$, $\xi_{D_{\bar{3}}} = 3.468(2)$, $\xi_{D^*_{\bar{3}}|_{\lambda=0}} = 3.492(4)$, $\xi_{D^*_{\bar{3}}||\lambda|=1} = 3.476(4)$.

| $(L/a_s)^3 \times T/a_t$ | $m_\pi$/MeV | $a_t^{-1}$/MeV | $a_t m_\pi$ | $a_t m_K$ | $a_t m_\eta$ | $a_t m_\Omega$ | $\xi$ | $a_s$/fm |
|---|---|---|---|---|---|---|---|---|
| $32^3 \times 256$ | 239 | 6079(13) | 0.03928(18) | 0.08344(7) | 0.09299(56) | 0.2751(6) | 3.453(6) | 0.1121(3) |
| $24^3 \times 256$ | 284 | 5988(17) | 0.04735(22) | 0.08659(14) | 0.09602(70) | 0.2793(8) | 3.455(6) | 0.1139(4) |
| $24^3 \times 256$ | 327 | 5854(16) | 0.05593(28) | 0.09027(15) | 0.09790(100) | 0.2857(8) | 3.456(9) | 0.1165(4) |
| $\{16^3, 20^3, 24^3\} \times 128$ | 391 | 5667(42) | 0.06906(13) | 0.09698(9) | 0.10364(19) | 0.2951(22) | 3.444(6) | 0.1199(9) |
| $\{16^3, 20^3, 24^3\} \times 128$ | 688 | 4655(9) | 0.1478(1) | 0.1478(1) | 0.1478(1) | 0.3593(7) | 3.471(28) | 0.1471(12) |

| $m_\pi$ | $m_K$ | $m_\eta$ | $m_D$ | $m_{D_s}$ | $m_\Omega$ |
|---|---|---|---|---|---|
| 239(1) | 507(1) | 565(3) | 1880(1) | 1967(1) | 1672(4) |
| 391(1) | 550(1) | 587(1) | 1885(1) | 1951(1) | 1672(12) |
| 688(1) | $\cdots$ | $\cdots$ | $\cdots$ | 1962(1) | 1672(3) |