Correlation functions for coupled $K\Lambda$ interaction and $N^*(1535)$ resonance

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Outline

- Introduction

➢ chiral unitary approach for the $K\Lambda$ interaction and $N^*(1535)$

➢ correlation functions for hadron-hadron final state interactions

- Numerical results

➢ $N^*(1535)$ in the $\Lambda_c^+ \to \bar{K}^0 \eta p$ decay

- Summary

The Review of Particle Physics (2026)

F. Takahashi *et al.* (Particle Data Group), Int. J. Mod. Phys. A 41, 2630011 (2026)

$N(1535)\ 1/2^-$

$I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$ Status: ∗∗∗∗

- Pole position $z_R = [(1500\sim1520) - i(40\sim65)]$ MeV

- Breit-Wigner parameterization

$$(M_R, \Gamma_R) = (1525 \sim 1545, 125 \sim 175)\mathrm{MeV} = (\simeq 1535, \simeq 150)\mathrm{MeV}$$

N(1535) DECAY MODES

	Mode	Fraction (Γ_i/Γ)
Γ_1	$N\pi$	32–52 %
Γ_2	$N\eta$	30–55 %
Γ_3	$N\pi\pi$	4–31 %

$N(1440)$ DECAY MODES

	Mode $I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$ Status: $****$	Fraction (Γ_i/Γ)
Γ_1	$N\pi$	55–75 %
Γ_2	$N\eta$	$<$ 1 %
Γ_3	$N\pi\pi$	17–50 %

$N(1520)$ DECAY MODES

	Mode $I(J^P) = \frac{1}{2}(\frac{3}{2}^-)$ Status: $****$	Fraction (Γ_i/Γ)
Γ_1	$N\pi$	55–65 %
Γ_2	$N\eta$	0.07–0.09 %
Γ_3	$N\pi\pi$	25–35 %

$N(1650)$ DECAY MODES

	Mode $I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$ Status: $****$	Fraction (Γ_i/Γ)
Γ_1	$N\pi$	50–70 %
Γ_2	$N\eta$	15–35 %
Γ_3	ΛK	5–15 %
Γ_4	$N\pi\pi$	20–58 %

$N^*(1535)$: a dynamically generated state

Chiral dynamics and the $S_{11}(1535)$ nucleon resonance

N. Kaiser, P.B. Siegel[1], W. Weise

Physics Letters B 362 (1995) 23–28

The $SU(3)$ chiral effective lagrangian at next-to-leading order is applied to the S-wave meson-baryon interaction in the energy range around the ηN threshold. A potential is derived from this lagrangian and used in a coupled channel calculation of the πN, ηN, $K\Lambda$, $K\Sigma$ system in the isospin-$\frac{1}{2}$, $l = 0$ partial wave. Using the same parameters as obtained previously from a fit to the low energy $\overline{K}N$ data it is found that a quasi-bound $K\Sigma$-$K\Lambda$-state is formed, with properties remarkably similar to the $S_{11}(1535)$ nucleon resonance. In particular, we find a large partial decay width into ηN consistent with the empirical data.

- The direct interaction between the η meson and the nucleon is very weak and there is a small direct coupling between the πN and ηN channels (C_{22} and C_{12} are small).
- However, there is a strong coupling of both the πN and ηN channels to the $K\Sigma$ channel (C_{14} and C_{24} are sizeable).
- Thus *the resonance formed will strongly connect the πN and ηN channels through the coupled channel dynamics*.

the possibility that the $S_{11}(1535)$ resonance is a quasi-bound $K\Sigma$-$K\Lambda$ state. Using the same parameters as obtained from fitting the low energy $\overline{K}N$ data and two finite range parameters, a resonance can be formed at 1557 MeV with the characteristic properties of the $S_{11}(1535)$. The πN S_{11} phase shifts and inelasticities as well as the ηN-production cross section are remarkably well reproduced. The dynamically generated resonance has a full width of 179 MeV and branching ratios extracted from the shape of the resonance of 69% into πN and 31% into ηN final states. Furthermore,

PHYSICAL REVIEW C, VOLUME 65, 035204

Chiral unitary approach to S-wave meson baryon scattering in the strangeness $S=0$ sector

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(Received 31 October 2001; published 14 February 2002)

We study the S-wave interaction of mesons with baryons in the strangeness $S=0$ sector in a coupled channel unitary approach. The basic dynamics is drawn from the lowest order meson-baryon chiral Lagrangians. Small modifications inspired by models with explicit vector meson exchange in the t channel are also considered. In addition the $\pi\pi N$ channel is included and shown to have an important repercussion in the results, particularly in the isospin 3/2 sector. The $N^*(1535)$ resonance is dynamically generated and appears as a pole in the second Riemann sheet with its mass, width, and branching ratios in fair agreement with experiment. A $\Delta(1620)$ resonance also appears as a pole at the right position although with a very large width, coming essentially from the coupling to the $\pi\pi N$ channel, in qualitative agreement with experiment.

$$P_0^R = 1543 - i46 \ \ \text{MeV}$$

$$|g_{\pi N}| < |g_{K\Lambda}| < |g_{\eta n}| \sim |g_{K\Sigma}|$$

Chiral dynamics of the $S_{11}(1535)$ and $S_{11}(1650)$ resonances revisited

Peter C. Bruns[a], Maxim Mai[b,*], Ulf-G. Meißner[b,c] Physics Letters B 697 (2011) 254–259

We analyze s-wave pion-nucleon scattering in a unitarized chiral effective Lagrangian including all dimension two contact terms. We find that both the $S_{11}(1535)$ and the $S_{11}(1650)$ are dynamically generated, but the $S_{31}(1620)$ is not. We further discuss the structure of these dynamically generated resonances.

$$W_{1535} = (1.506 - 0.140i)\ \text{GeV}$$

$$W_{1650} = (1.692 - 0.046i)\ \text{GeV}$$

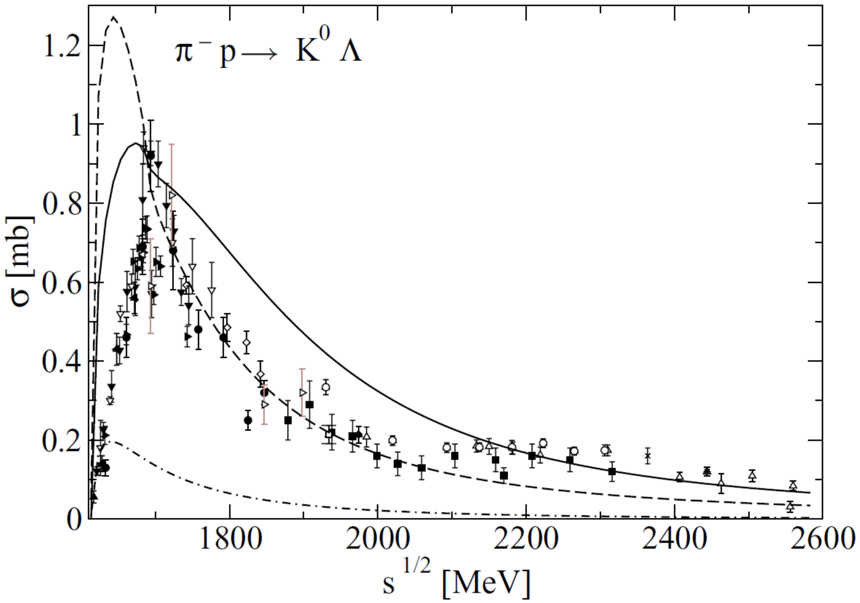
$$|g_{\Lambda K^+}|^2 > |g_{p\eta}|^2 > |g_{\Sigma^+ K^0}|^2 \simeq |g_{n\pi^+}|^2 > |g_{\Sigma^0 K^+}|^2 \simeq |g_{p\pi^0}|^2$$

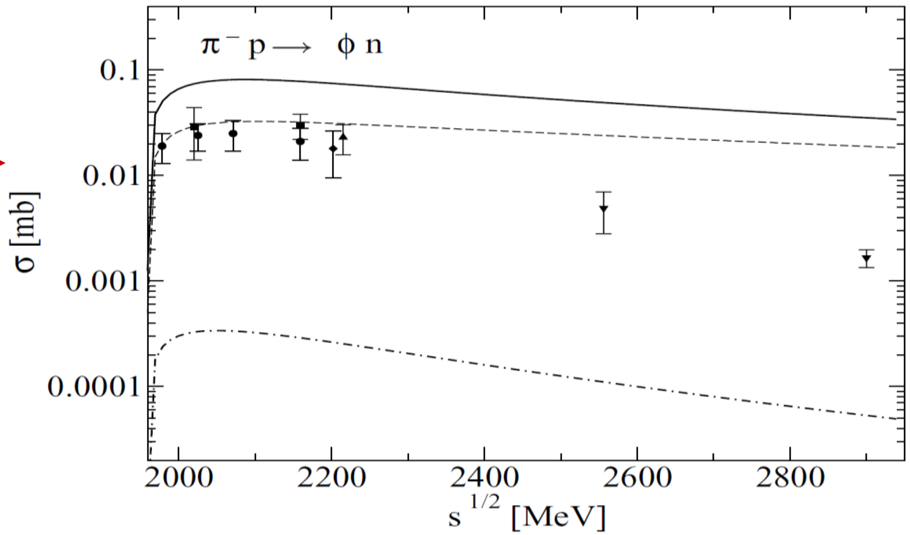
$\longrightarrow$ $N^*(1535)$ *couples strongly to the* ΛK *channel.*

PHYSICAL REVIEW C **78**, 025207 (2008)

Role of the $N^*(1535)$ resonance and the $\pi^- p \to KY$ amplitudes in the OZI forbidden $\pi N \to \phi N$ reaction

M. Döring,[1] E. Oset,[2] and B. S. Zou[2,3]





- The authors study the πN→ϕN reaction near the ϕN threshold using the chiral unitary approach. They combine the reaction amplitudes of πN→KΛ and πN→KΣ, and introduce the ϕ meson coupling to the kaon meson final states through loop contributions.
- When the dominant πN→KΛ amplitude is constrained by experimental cross section data, their results agree well with the experimental measurements.

PHYSICAL REVIEW C **77**, 015206 (2008)

Role of the $N^*(1535)$ in $pp \to pp\phi$ and $\pi^- p \to n\phi$ reactions

Ju-Jun Xie,[1,2,*] Bing-Song Zou,[1,2,3,†] and Huan-Ching Chiang[1,3,4]

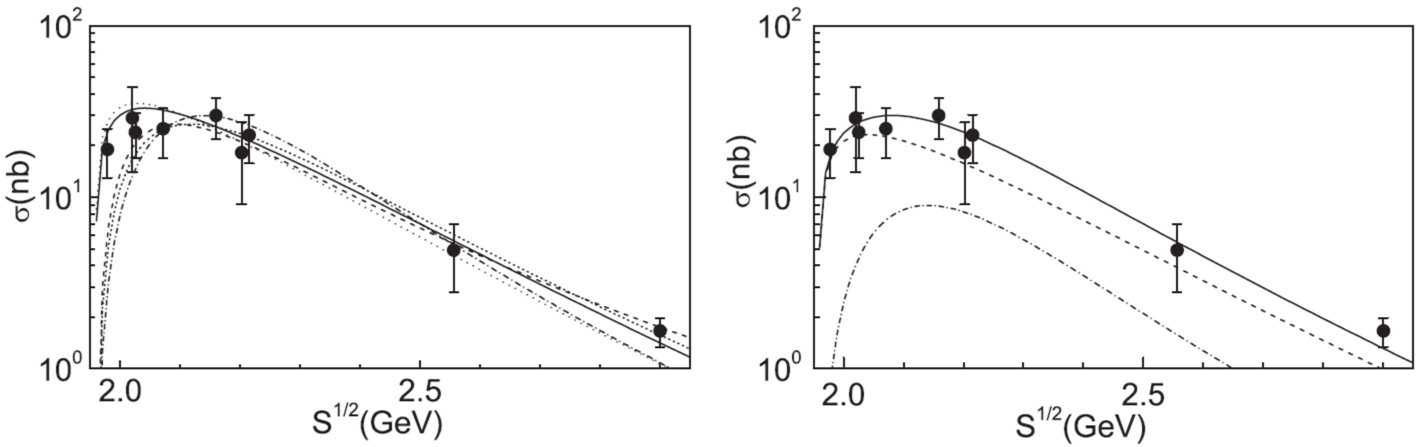


FIG. 3. Total cross sections vs the center-of-mass (c.m.) energy $S^{\frac{1}{2}}$ for $\pi^- p \to n\phi$ reactions with data from Ref. [29]. (Left) The best fits with $N^*(1535)$ (solid), $N^*(1650)$ (dotted), $N^*(1710)$ (dashed), $N^*(1720)$ (short-dashed), and $N^*(1900)$ (dot-dashed), respectively; (right) fit (solid) with $N^*(1535)$ (dashed) plus $N^*(1900)$ (dot-dashed).

- The significant coupling of the $N^*(1535)$ resonance to the ϕN channel together with the earlier findings of large couplings of the $N^*(1535)$ resonance to the $K\Lambda$ and ηN channels gives a coherent picture that there is a large component of strangeness in the $N^*(1535)$ resonance.
- It also gives a natural explanation for the significant enhancement of the ϕ production from πp and pp reactions over the naïve OZI rule predictions.

$N^*(1535)$: strongly couples to strangeness channels

	Mode	Fraction (Γ_i/Γ)
Γ_1	$N\pi$	32–52 %
Γ_2	$N\eta$	30–55 %

PRL **96**, 042002 (2006) PHYSICAL REVIEW LETTERS week ending 3 FEBRUARY 2006

Mass and $K\Lambda$ Coupling of the $N^*(1535)$

B. C. Liu[1,3] and B. S. Zou[2,1]

$$R \equiv \left| \frac{g_{N^*(1535)K\Lambda}}{g_{N^*(1535)N\eta}} \right| \approx 1.3 \pm 0.3.$$

$J/\psi \to \bar{p}N^* \to \bar{p}\,(K\Lambda)\,/\,\bar{p}\,(p\eta) \to$ large $g_{N^*K\Lambda}$

Liu&Zou, PRL96 (2006) 042002; Geng,Oset,Zou&Doring, PRC79 (2009) 025203

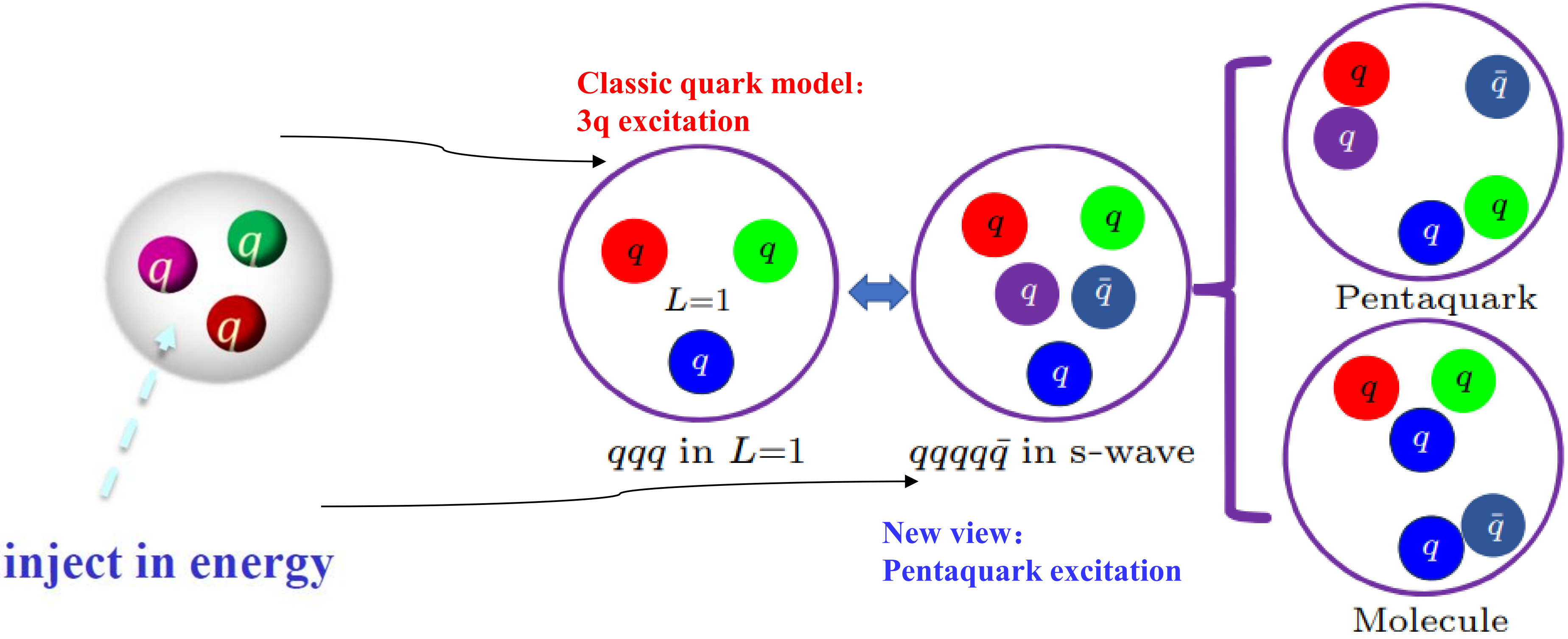
$\gamma p \to p\eta'$ & $pp \to pp\eta'$ $\to$ large $g_{N^*N\eta'}$

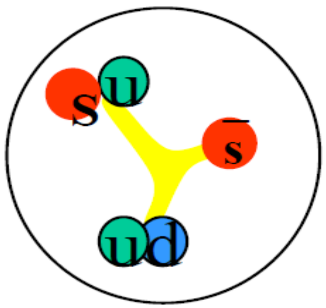
M.Dugger et al., PRL96 (2006) 062001; Cao&Lee, PRC78(2008) 035207

$\pi^- p \to n\phi$ & $pp \to pp\phi$ & $pn \to d\phi$ $\to$ large $g_{N^*N\phi}$

Xie, Zou & Chiang, PRC77(2008)015206; Cao, Xie, Zou & Xu, PRC80(2009)025203

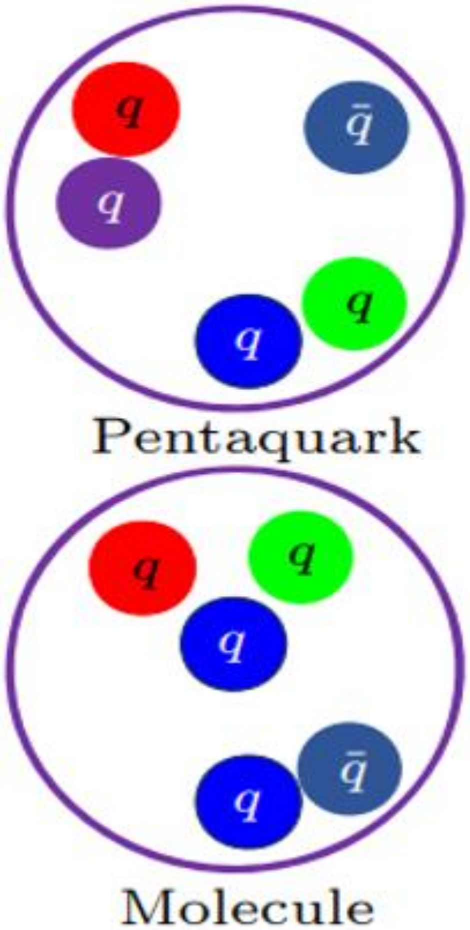
Configurations for $N^*(1535)\ J^P = \frac{1}{2}^-$





Larger $[ud][us]\bar{s}$ component in N*(1535) makes it coupling strong to Nη & KΛ.

uud (L=1) 1/2⁻ ~ N*(1535) ~ [ud][us] $\bar{s}$



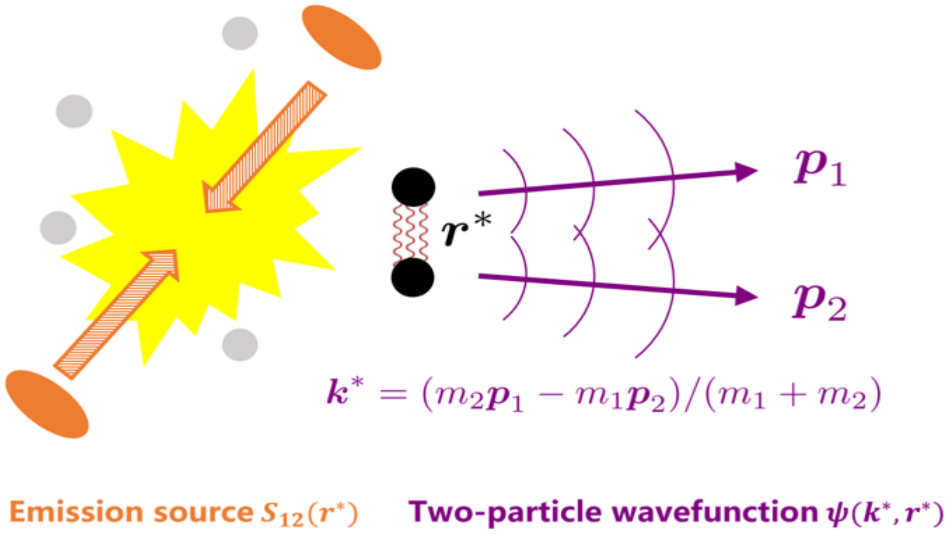
BUT:

Difficult to distinguish them.

Measure the direct two-hadron (components) interactions.

Correlation functions (experimental side)

Annu. Rev. Nucl. Part. Sci. 2005. 55:357–402



Definition:

$$C(\vec{p}_1, \vec{p}_2) = \frac{P(\vec{p}_1, \vec{p}_2)}{P(\vec{p}_1)P(\vec{p}_2)}$$

Assumption: The emission process is initially uncorrelated.

Experimental measurement (mixed-event technique)

$$C(p) = \mathcal{N} \frac{N_{\text{same}}(p)}{N_{\text{mixed}}(p)}$$

N_{same}: the p distribution of pairs measured in the same event

N_{mixed}: the reference distribution from mixed (different) events

N: a normalization constant determined by the physical expectation that particles are uncorrelated at large relative momenta, which ensures $C(p) = 1$.

Correlation functions
(theoretical side)

Koonin–Pratt (KP) formula

S. E. Koonin, Phys. Lett. B ***70*** *(1) (1977) 43*

A. Ohnishi, Nucl. Phys. A ***954*** *(2016) 294*

$$C_i(\vec{p}) = \int_0^\infty d^3\vec{r}\, S(\vec{r})\, |\Psi_i(\vec{r},\vec{p})|^2$$

Emitting source function**:** usually parametrized as a normalized Gaussian form

$$S(\vec{r}) = \frac{e^{-\frac{|\vec{r}|^2}{4R^2}}}{(4\pi R^2)^{3/2}}$$

Only S-wave

$$C_i(\vec{p}) = 1 + \int_0^\infty d^3\vec{r}\, S(\vec{r}) \left(2\mathrm{Re}\left[\Psi'_{ii}(\vec{r},\vec{p}) j_0(|\vec{p}|\,|\vec{r}|)\right] + \sum_j \left|\Psi'_{ij}(\vec{r},\vec{p})\right|^2 \right)$$

Scattering wave function: derived from Lippmann-Schwinger equation

$$T = V + VGT \quad \Longrightarrow \quad |\psi\rangle = |\phi\rangle + GT|\phi\rangle$$

Correlation function
(coupled channels)

$$\Psi_{ij}(\vec{r},\vec{p}) = \delta_{ij} e^{i\vec{p}\vec{r}} + T_{ij}(\vec{r},\vec{p}) \int_0^\infty \frac{d^4 q}{(2\pi)^4} \frac{i(2M_j)^n e^{i\vec{q}\vec{r}}}{(q^2 - M_j^2 + i\epsilon)((P-q)^2 - m_j^2 + i\epsilon)}$$
$$= \delta_{ij} e^{i\vec{p}\vec{r}} + T_{ij}(\vec{r},\vec{p}) \tilde{G}_j(\vec{r},\vec{p}),$$

$$\tilde{G}_j(\vec{r},\vec{p}) = (2M_j)^n \int_0^\infty \frac{d^3\vec{q}}{(2\pi)^3} \frac{(\omega_{M_j} + \omega_{m_j}) j_0(|\vec{q}|\,|\vec{r}|)}{2\omega_{M_j}\omega_{m_j}\left[s - (\omega_{M_j} + \omega_{m_j})^2 + i\epsilon\right]}$$
$$= (2M_j)^n \left[\int_0^\infty \frac{d^3\vec{q}}{(2\pi)^3} \frac{(\omega_{M_j} + \omega_{m_j}) j_0(|\vec{q}|\,|\vec{r}|) - j_0(|\vec{q}_{j,\mathrm{on}}|\,|\vec{r}|)}{2\omega_{M_j}\omega_{m_j}\left[s - (\omega_{M_j} + \omega_{m_j})^2 + i\epsilon\right]} \right.$$
$$\left. + j_0(|\vec{q}_{j,\mathrm{on}}|\,|\vec{r}|) \int_0^\infty \frac{d^3\vec{q}}{(2\pi)^3} \frac{\omega_{M_j} + \omega_{m_j}}{2\omega_{M_j}\omega_{m_j}\left[s - (\omega_{M_j} + \omega_{m_j})^2 + i\epsilon\right]} \right]$$

$$\boxed{C_i(\vec{p}) = 1 + \int_0^\infty d^3\vec{r}\, S(\vec{r}) \left(2\mathrm{Re}\left[T_{ii}(\vec{r},\vec{p}) \tilde{G}_i(\vec{r},\vec{p}) j_0\left(|\vec{p}|\,|\vec{r}|\right)\right] + \sum_j \left| T_{ij}(\vec{r},\vec{p}) \tilde{G}_j(\vec{r},\vec{p}) \right|^2 \right)}$$

The T_{ij} is the two-body scattering amplitude that can be calculated by the chiral unitary approach.

$K\Lambda$ interactions in coupled channels

6 channels: $K^0\Sigma^+, K^+\Sigma^0, K^+\Lambda, \pi^+ n, \pi^0 p,\ \eta p$

C_{ij}	$K^0\Sigma^+$	$K^+\Sigma^0$	$K^+\Lambda$	$\pi^+ n$	$\pi^0 p$	ηp
$K^0\Sigma^+$	1	$\sqrt{2}$	0	0	$\frac{1}{\sqrt{2}}$	$-\sqrt{\frac{3}{2}}$
$K^+\Sigma^0$		0	0	$\frac{1}{\sqrt{2}}$	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$
$K^+\Lambda$			0	$-\sqrt{\frac{3}{2}}$	$-\frac{\sqrt{3}}{2}$	$-\frac{3}{2}$
$\pi^+ n$				1	$\sqrt{2}$	0
$\pi^0 p$					0	0
ηp						0

Transition potential:

$$V_{ij} = -\frac{C_{ij}}{4f_i f_j}(2\sqrt{s} - M_i - M_j)\sqrt{\frac{(M_i+E_i)(M_j+E_j)}{4M_iM_j}}$$

Loop function:

$$G_j(s) = \frac{2M_j}{16\pi^2}\Big\{\alpha_j(\mu) + \ln\frac{M_j^2}{\mu^2} + \frac{s+\Delta}{2s}\ln\frac{m_j^2}{M_j^2} + \frac{p_j}{\sqrt{s}}\Big[\ln\left(s-\Delta+2p_j\sqrt{s}\right)$$
$$-\ln\left(-s-\Delta+2p_j\sqrt{s}\right) + \ln\left(s+\Delta+2p_j\sqrt{s}\right) - \ln\left(-s+\Delta+2p_j\sqrt{s}\right)\Big]\Big\}$$

Bethe-Salpeter equation:

$$T = (1 - VG)^{-1}V$$

One can calculate the correlation functions.

R. Molina, C.W. Xiao, W.H. Liang and E. Oset, Phys. Rev. D 109, 054002 (2024).

PHYSICAL REVIEW D **109**, 054002 (2024)

Correlation functions for the $N^*(1535)$ and the inverse problem

Raquel Molina,[1,2,†] Chu-Wen Xiao,[1,3,4,‡] Wei-Hong Liang,[1,3,*] and Eulogio Oset[1,2,§]

$$C_{K^+\Lambda}(p_{K^+}) = 1 + 4\pi\theta(q_{\max} - p_{K^+}) \int dr r^2 S_{12}(r) \cdot \{|j_0(p_{K^+}r) + T_{K^+\Lambda,K^+\Lambda}(E)\tilde{G}^{(K^+\Lambda)}(r;E)|^2$$
$$+ |T_{K^0\Sigma^+,K^+\Lambda}(E)\tilde{G}^{(K^0\Sigma^+)}(r;E)|^2 + |T_{K^+\Sigma^0,K^+\Lambda}(E)\tilde{G}^{(K^+\Sigma^0)}(r;E)|^2$$
$$+ |T_{\eta p,K^+\Lambda}(E)\tilde{G}^{(\eta p)}(r;E)|^2 - j_0^2(p_{K^+}r)\},$$

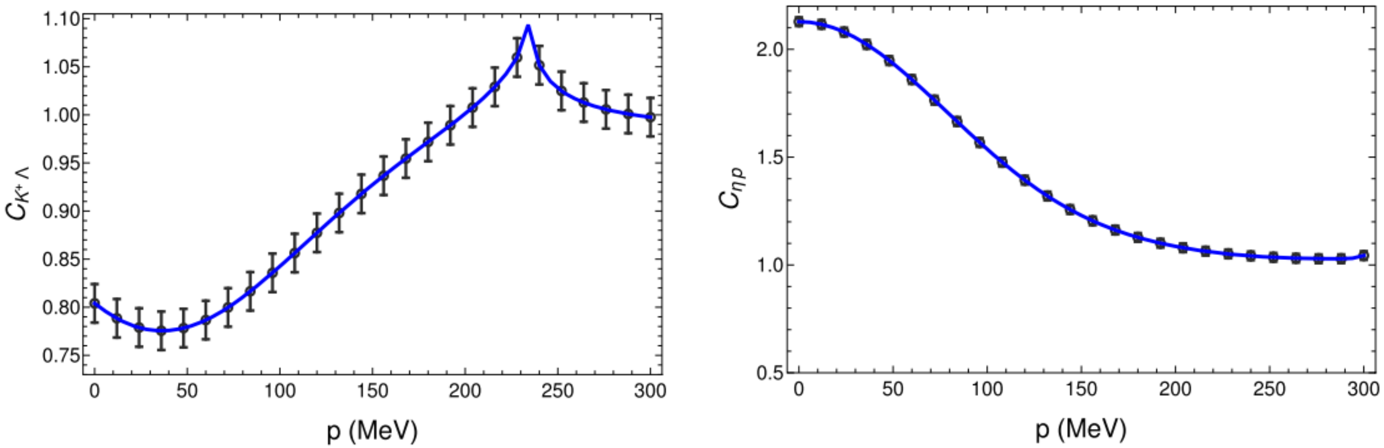


FIG. 2. Correlation functions for $K^+\Lambda$ and ηp channels.

Without contributions from the $\pi^+ n$ and $\pi^0 p$ channels.

Correlation functions: experimental measurements

Inverse

Scattering amplitude

$$|1 - VG| = 0$$

The scattering length and the effective range:

$$a_j = \frac{2M_j T_{jj}}{8\pi\sqrt{s}}\bigg|_{s=s_{th,j}}, \qquad r_j = 2\frac{\partial}{\partial p^2}\left[-\frac{8\pi\sqrt{s}}{2M_j T_{jj}} + ip_j\right]\bigg|_{s=s_{th,j}}$$

Pole position: $\sqrt{s_{pole}} = M_R - i\,\frac{\Gamma_R}{2}$

Coupling constants:

$$g_i g_j = \frac{r}{2\pi}\int_0^{2\pi} T_{ij}(\sqrt{s})e^{i\theta}d\theta\bigg|_{r\to 0}$$

$$\sqrt{s} = \sqrt{s_{pole}} + re^{i\theta}$$

Partial decay widths:

$$\Gamma_j = \frac{|g_j|^2}{4\pi}\frac{(M_R^2 + M_j^2 - m_j^2)}{4M_R^2}\lambda^{1/2}(M_R^2, M_j^2, m_j^2)$$

$K\Lambda$ correlation functions

Accessing the strong interaction between Λ baryons and charged kaons with the femtoscopy technique at the LHC

ALICE Collaboration *

Physics Letters B 845 (2023) 138145

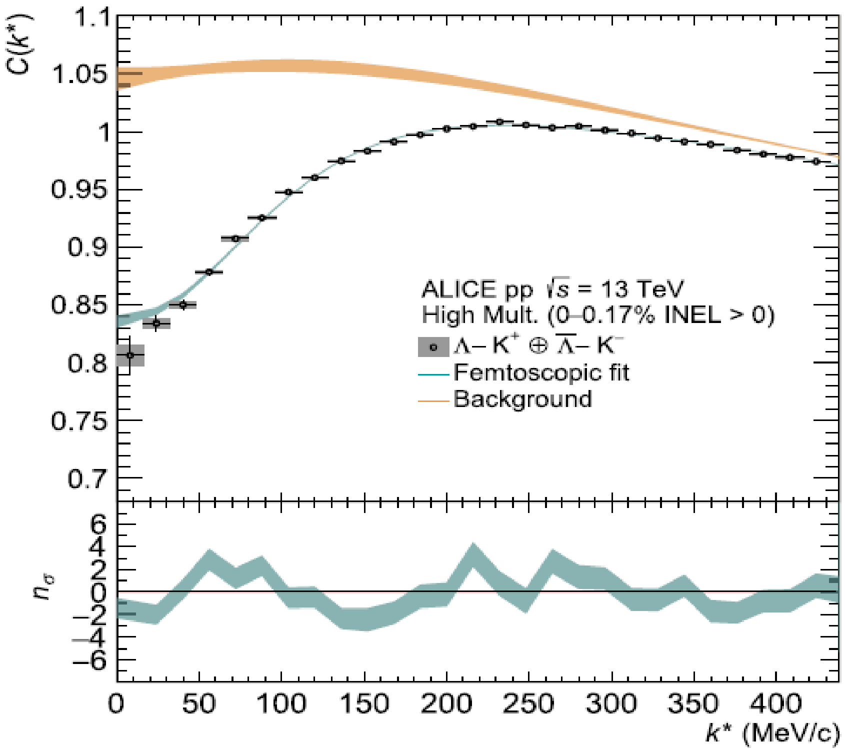


Fig. 2. Measured correlation function of Λ–K$^+$ pairs. Statistical (bars) and systematic (boxes) uncertainties are shown separately. The light cyan band represents the total fit obtained using Eq. (2) from which the normalization N_D, and the scattering parameters ($\Re f_0$, $\Im f_0$, and d_0) are extracted. The orange band represents the $C_{\text{background}}(k^*)$ contribution, modeled as described in Section 3, and multiplied by the constant N_D. Lower panel: n_σ deviation between data and model in terms of numbers of standard deviations.

The measured correlations are fitted with a correlation function:

$$\boxed{C_{\text{tot}}(k^*) = N_D \times C_{\text{model}}(k^*) \times C_{\text{background}}(k^*)}$$

N_D: a normalization constant;

$C_{\text{background}}$: possible residual background;

$C_{\text{model}} = 1 + \sum_i \lambda_i \times (C_i - 1)$: includes the genuine correlation (i=gen);

Each of different contributions is weighted by λ_i parameter, which can be evaluated from the experimental side.

Example: λ_i parameter

PHYSICAL REVIEW C **99**, 024001 (2019)

p-p, p-Λ, and Λ-Λ correlations studied via femtoscopy in pp reactions at $\sqrt{s} = 7$ TeV

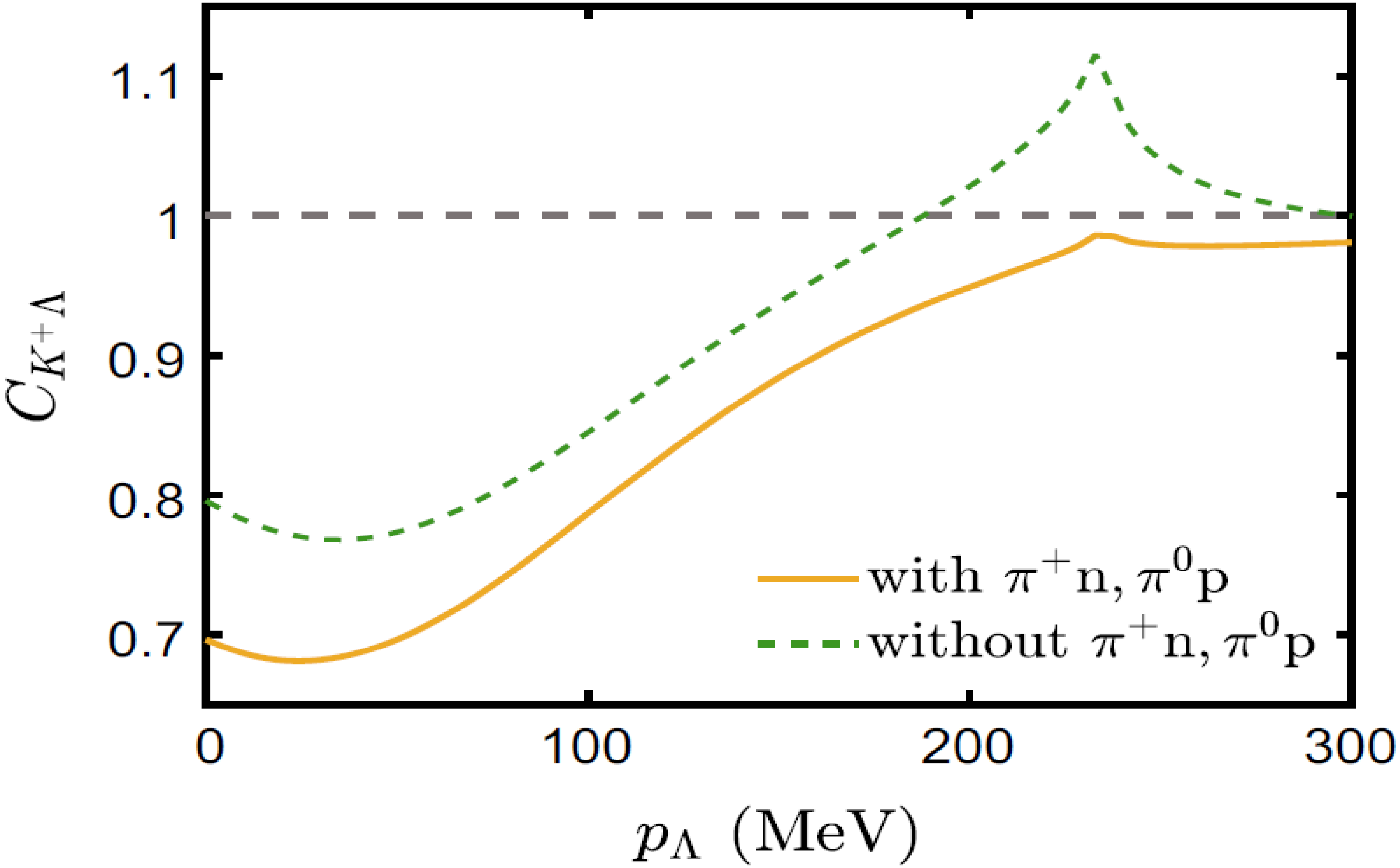
S. Acharya *et al.**
(ALICE Collaboration)

TABLE II. Weight parameters of the individual components of the p-p, p-Λ, and Λ-Λ correlation function.

p-p		p-Λ		Λ-Λ	
Pair	λ parameter [%]	Pair	λ parameter [%]	Pair	λ parameter [%]
pp	74.18	$p\Lambda$	47.13	$\Lambda\Lambda$	29.94
$p_\Lambda p$	15.52	$p\Lambda_{\Xi^-}$	9.92	$\Lambda\Lambda_{\Sigma^0}$	19.96
$p_\Lambda p_\Lambda$	0.81	$p\Lambda_{\Xi^0}$	9.92	$\Lambda_{\Sigma^0}\Lambda_{\Sigma^0}$	3.33
$p_{\Sigma^+}p$	6.65	$p\Lambda_{\Sigma^0}$	15.71	$\Lambda\Lambda_{\Xi^0}$	12.61
$p_{\Sigma^+}p_{\Sigma^+}$	0.15	$p_\Lambda\Lambda$	4.93	$\Lambda_{\Xi^0}\Lambda_{\Xi^0}$	1.33
$p_\Lambda p_{\Sigma^+}$	0.70	$p_\Lambda\Lambda_{\Xi^-}$	1.04	$\Lambda\Lambda_{\Xi^-}$	12.61
$\tilde{p}p$	1.72	$p_\Lambda\Lambda_{\Xi^0}$	1.04	$\Lambda_{\Xi^-}\Lambda_{\Xi^-}$	1.33
$\tilde{p}p_\Lambda$	0.18	$p_\Lambda\Lambda_{\Sigma^0}$	1.64	$\Lambda_{\Sigma^0}\Lambda_{\Xi^0}$	4.20
$\tilde{p}p_{\Sigma^+}$	0.08	$p_{\Sigma^+}\Lambda$	2.11	$\Lambda_{\Sigma^0}\Lambda_{\Xi^-}$	4.20
$\tilde{p}\tilde{p}$	0.01	$p_{\Sigma^+}\Lambda_{\Xi^-}$	0.44	$\Lambda_{\Xi^0}\Lambda_{\Xi^-}$	2.65
		$p_{\Sigma^+}\Lambda_{\Xi^0}$	0.44	$\tilde{\Lambda}\Lambda$	4.38
		$p_{\Sigma^+}\Lambda_{\Sigma^0}$	0.70	$\tilde{\Lambda}\Lambda_{\Sigma^0}$	1.46
		$\tilde{p}\Lambda$	0.55	$\tilde{\Lambda}\Lambda_{\Xi^0}$	0.92
		$\tilde{p}\Lambda_{\Xi^-}$	0.18	$\tilde{\Lambda}\Lambda_{\Xi^-}$	0.92
		$\tilde{p}\Lambda_{\Xi^0}$	0.12	$\tilde{\Lambda}\tilde{\Lambda}$	0.16
		$\tilde{p}\Lambda_{\Sigma^0}$	0.12		
		$p\tilde{\Lambda}$	3.45		
		$p_\Lambda\tilde{\Lambda}$	0.36		
		$p_{\Sigma^+}\tilde{\Lambda}$	0.15		
		$\tilde{p}\tilde{\Lambda}$	0.04		

The λ_i parameter can be evaluated from the experimental side.

$K\Lambda$ correlation functions $C_{K^+\Lambda}$
(πN channel)



Fitting process

$$C_{\text{fit}} = N' \times C_{\text{tot}} \times C_{\text{background}} \qquad C_{\text{tot}} = 1 + \lambda_{\text{gen}}(C_{\text{gen}} - 1) + \sum_n \lambda_n (C_n - 1)$$

$$C_{\text{fit}} = N \times (\lambda C_{\text{gen}} + 1 - \lambda) \times C_{\text{background}}$$

$C_{\text{background}}$: taken it as the experiment.

$$\lambda = \frac{\lambda_{\text{gen}}}{\lambda_{\text{gen}} + X} \qquad \mathrm{X} = \sum_n \lambda_n C_n \qquad \lambda_{\text{gen}} + \sum_n \lambda_n = 1$$

The nongenuine contributions are taken as a flat constant term X.

Fitting strategy

Scenario I: subtraction constants $a(\mu)$ are fixed; N and λ are free parameters.

There are a total of 28 data and 2 free parameters.

Scenario II: subtraction constants $a_{K^0\Sigma^+} = a_{K^0\Sigma^+} = a_{K\Sigma}$, $a_{K\Lambda}$, $a_{\eta p}$, $a_{\pi^+ n} = a_{\pi^0 p} = a_{\pi N}$, N and λ are free parameters.

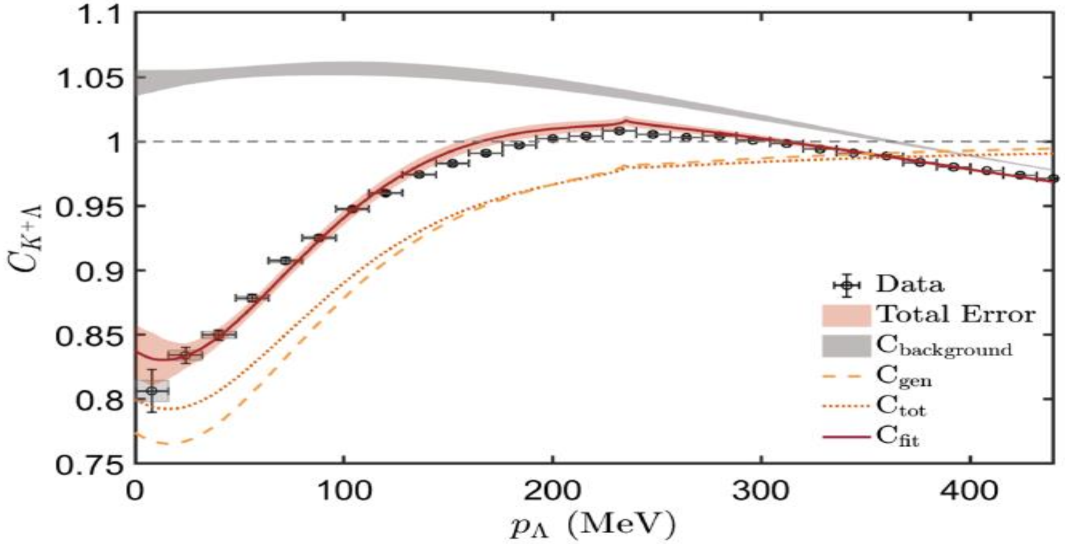
Pole position of $N^*(1535)$, $M_R = 1510 \pm 10$ MeV, $\Gamma_R = 105 \pm 25$ MeV, are also fitted.

There are a total of 30 data and 6 free parameters.

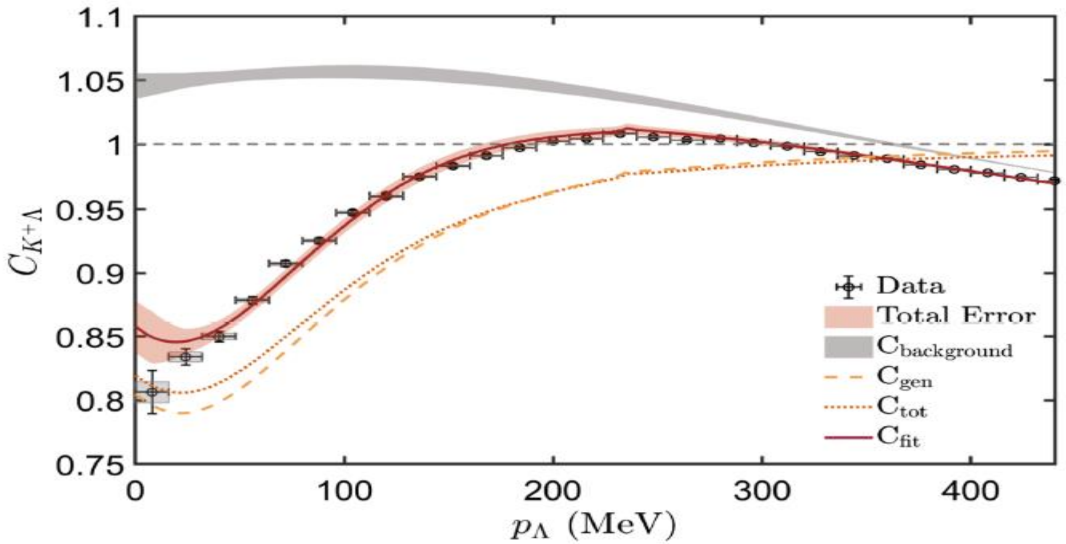
S.W. Liu and J.J. Xie, Phys. Rev. D 112, 034027 (2025). [arXiv:2503.22453 [hep-ph]].

Fitted results

Scenario I:



Scenario II:



Fitted parameters and the $N^*(1535)$

TABLE I. The fitted model parameters. Note that the subtraction constants for scenario I were fixed as in Ref. [7].

Scenario	I	II
N	0.99554 ± 0.00045	0.99633 ± 0.00032
λ	0.87 ± 0.01	0.91 ± 0.02
$\alpha_{K\Sigma}$	-2.8	-3.17 ± 0.06
$\alpha_{K\Lambda}$	1.6	1.86 ± 0.06
$\alpha_{\eta N}$	0.2	0.76 ± 0.09
$\alpha_{\pi N}$	2.0	1.91 ± 0.04
χ/d.o.f.	1.9	1.2

[7] T. Inoue, E. Oset, and M. J. Vicente Vacas, Chiral unitary approach to S wave meson baryon scattering in the strangeness $S = 0$ sector, Phys. Rev. C **65**, 035204 (2002).

TABLE II. The obtained properties for the $N^*(1535)$ resonance.

Scenario		I	II
$\sqrt{s_{\text{pole}}}$ (MeV)		$1531 - i36.5$	$1512 - i39.1$
Coupling	$g_{K^0\Sigma^+}$	$2.21 - i0.18$	$2.14 - i0.34$
	$g_{K^+\Sigma^0}$	$1.56 - i0.13$	$1.51 - i0.24$
	$g_{K^+\Lambda}$	$1.37 - i0.10$	$1.13 - i0.16$
	$g_{\pi^+ n}$	$0.55 + i0.32$	$0.62 + i0.30$
	$g_{\pi^0 p}$	$0.40 + i0.22$	$0.44 + i0.20$
	$g_{\eta p}$	$-1.47 + i0.43$	$-1.63 + i0.58$
Partial Width (MeV)	$\Gamma_{\pi^+ n}$	19.7	21.9
	$\Gamma_{\pi^0 p}$	9.9	11.1
	$\Gamma_{\eta p}$	41.2	39.6
Scattering Length (MeV^{-1})	$a_{K^0\Sigma^+}$	$0.28 - i0.14$	$0.26 - i0.12$
	$a_{K^+\Sigma^0}$	$0.19 - i0.13$	$0.18 - i0.11$
	$a_{K^+\Lambda}$	$0.33 - i0.14$	$0.29 - i0.17$
	$a_{\pi^+ n}$	-0.08	-0.08
	$a_{\pi^0 p}$	0.02	0.02
	$a_{\eta p}$	$-0.25 - i0.17$	$-0.54 - i0.38$
Effective Range (MeV^{-1})	$r_{K^0\Sigma^+}$	$1.05 - i1.55$	$0.83 - i1.35$
	$r_{K^+\Sigma^0}$	$-1.46 - i3.69$	$-1.41 - i3.01$
	$r_{K^+\Lambda}$	$-0.52 - i0.38$	$0.47 - i0.89$
	$r_{\pi^+ n}$	$-13.39 - i18.04$	$-13.39 - i17.29$
	$r_{\pi^0 p}$	30.34	21.26
	$r_{\eta p}$	$-8.60 + i6.77$	$-4.66 + i3.02$

S.W. Liu and J.J. Xie, Phys. Rev. D 112, 034027 (2025). [arXiv:2503.22453 [hep-ph]].

Branching rations of the $N^*(1535)$

Scenario I:

$$\mathrm{Br}[N^*(1535) \to \pi N] = \frac{\Gamma_{\pi^+ n} + \Gamma_{\pi^0 p}}{\Gamma_{N^*(1535)}} = 41\%,$$

$$\mathrm{Br}[N^*(1535) \to \eta N] = \frac{\Gamma_{\eta p}}{\Gamma_{N^*(1535)}} = 56\%,$$

Scenario II:

$$\mathrm{Br}[N^*(1535) \to \pi N] = \frac{\Gamma_{\pi^+ n} + \Gamma_{\pi^0 p}}{\Gamma_{N^*(1535)}} = 42\%,$$

$$\mathrm{Br}[N^*(1535) \to \eta N] = \frac{\Gamma_{\eta p}}{\Gamma_{N^*(1535)}} = 51\%,$$

$$\frac{\Gamma_{\eta N}}{\Gamma_{\pi N}} = \frac{\Gamma_{\eta p}}{\Gamma_{\pi^+ n} + \Gamma_{\pi^0 p}} = \begin{cases} 1.4, \text{ for scenario I} \\ 1.2, \text{ for scenario II} \end{cases}$$

The Review of Particle Physics (2026)

F. Takahashi *et al.* (Particle Data Group), Int. J. Mod. Phys. A 41, 2630011 (2026)

In agreement with these values:

	Mode	Fraction (Γ_i/Γ)
Γ_1	$N\pi$	32–52 %
Γ_2	$N\eta$	30–55 %

Partial wave analyses of $\psi(3686) \to p\bar{p}\pi^0$ and $\psi(3686) \to p\bar{p}\eta$

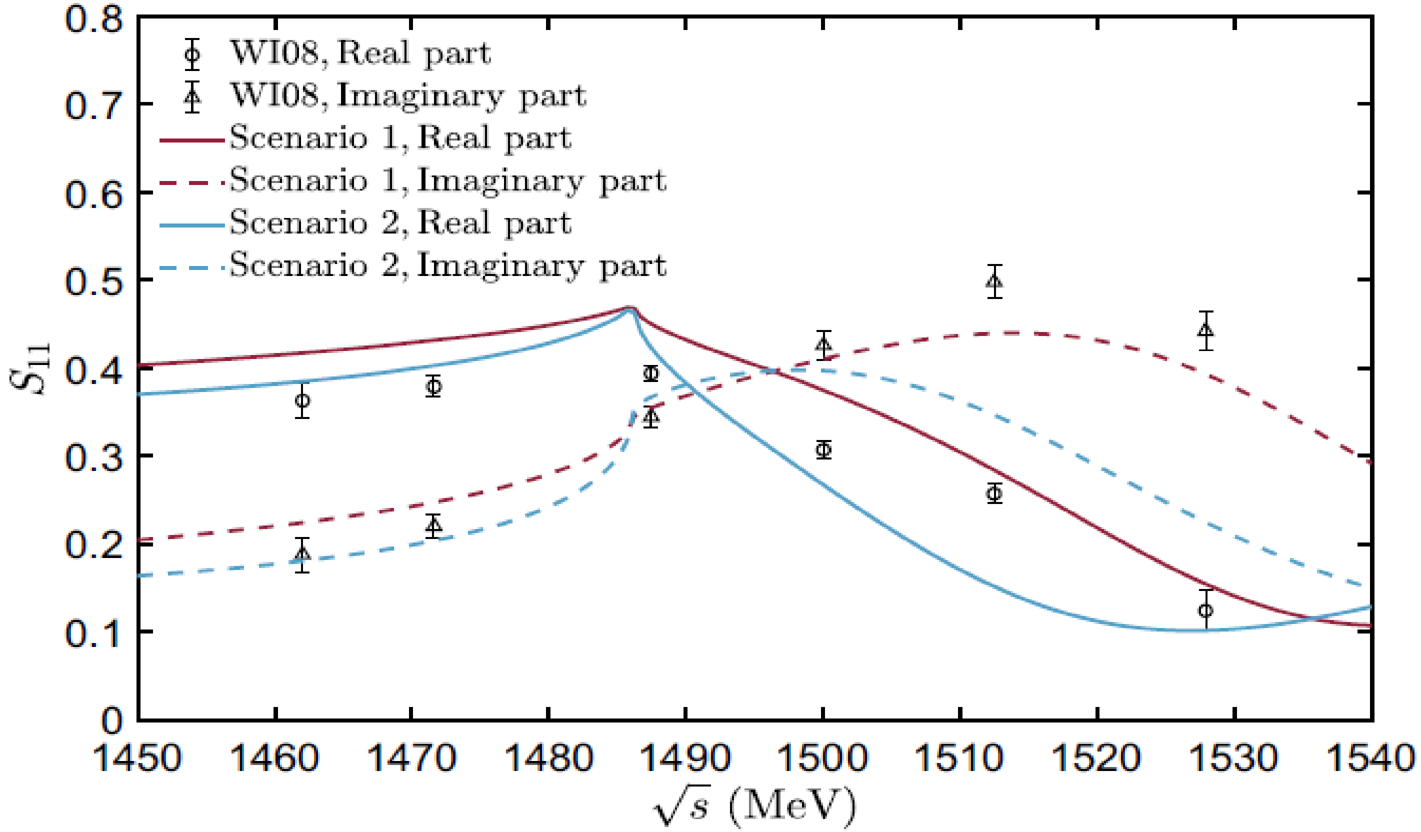
M. Ablikim *et al.*[*]

(BESIII Collaboration)

PHYSICAL REVIEW D **111**, 032011 (2025)

$$\frac{Br[N^*(1535) \to N\eta]}{Br[N^*(1535) \to N\eta]} = 0.99 \pm 0.05 \pm 0.19$$

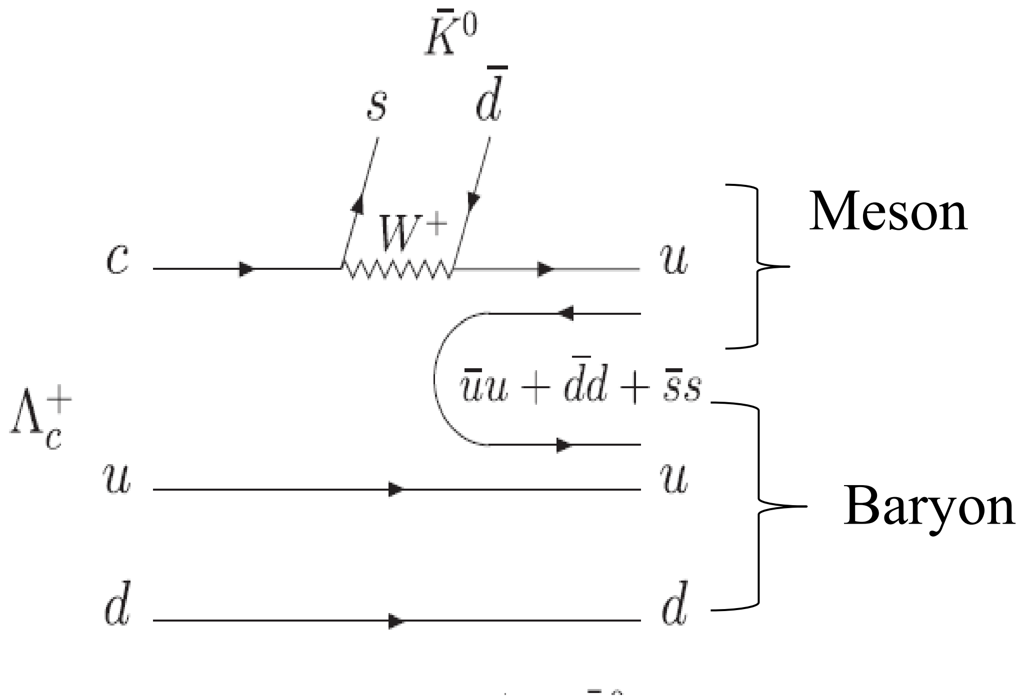
πN scattering amplitude S_{11}



Both scenario I and II can reproduce the real part of the πN scattering amplitude S_{11} in the energies around 1450–1540 MeV.

However, the imaginary part above 1510 MeV can not be well described for scenario II.

$N^*(1535)$ in the $\Lambda_c^+ \to \bar{K}^0 \eta p$ decay

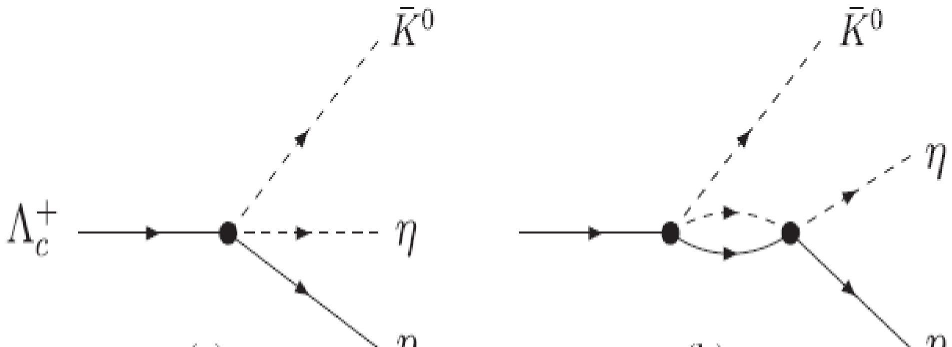


$$|uud> \to \frac{1}{\sqrt{2}}|u(ud - du)>$$

$$\bar{u}u + \bar{d}d + \bar{s}s$$

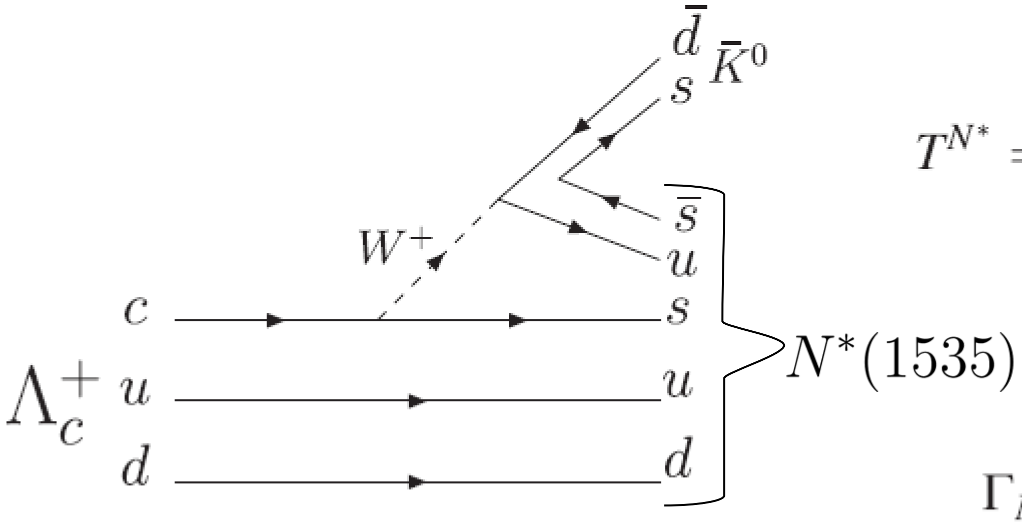
$$|MB\rangle = \frac{\sqrt{3}}{3}|\eta p\rangle + \frac{\sqrt{2}}{2}|\pi^0 p\rangle + |\pi^+ n\rangle - \frac{\sqrt{6}}{3}|K^+\Lambda\rangle,$$

$$\begin{aligned} T^{MB} = V_P \Bigg(& \frac{\sqrt{3}}{3} + \frac{\sqrt{3}}{3} G_{\eta p}(M_{\eta p}) t_{\eta p \to \eta p}(M_{\eta p}) \\ & + \frac{\sqrt{2}}{2} G_{\pi^0 p}(M_{\eta p}) t_{\pi^0 p \to \eta p}(M_{\eta p}) \\ & + G_{\pi^+ n}(M_{\eta p}) t_{\pi^+ n \to \eta p}(M_{\eta p}) \\ & - \frac{\sqrt{6}}{3} G_{K^+\Lambda}(M_{\eta p}) t_{K^+\Lambda \to \eta p}(M_{\eta p}) \Bigg), \end{aligned}$$



(a) (b)

Effective Lagrangian approach and the N*(1535) resonance as a Breit-Wigner resonance



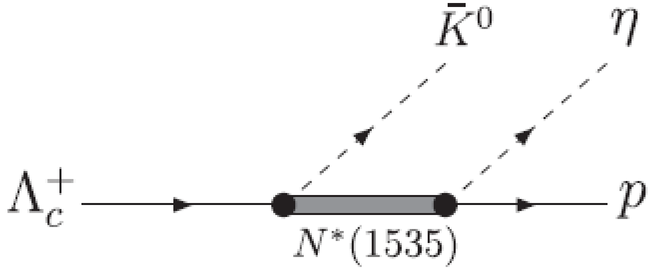
(a)

$$T^{N^*} = i g_{N^* N \eta} \bar{u}(p_3, s_p) G_{N^*}(q)(A + B\gamma_5) u(p, s_{\Lambda_c^+}),$$

$$G_{N^*}(q) = i \frac{\not{q} + M_{N^*}}{q^2 - M_{N^*}^2 + i M_{N^*} \Gamma_{N^*}(q^2)},$$

$$\Gamma_{N^*}(q^2) = \Gamma_{N^* \to \pi N}(q^2) + \Gamma_{N^* \to \eta N}(q^2) + \Gamma_0,$$

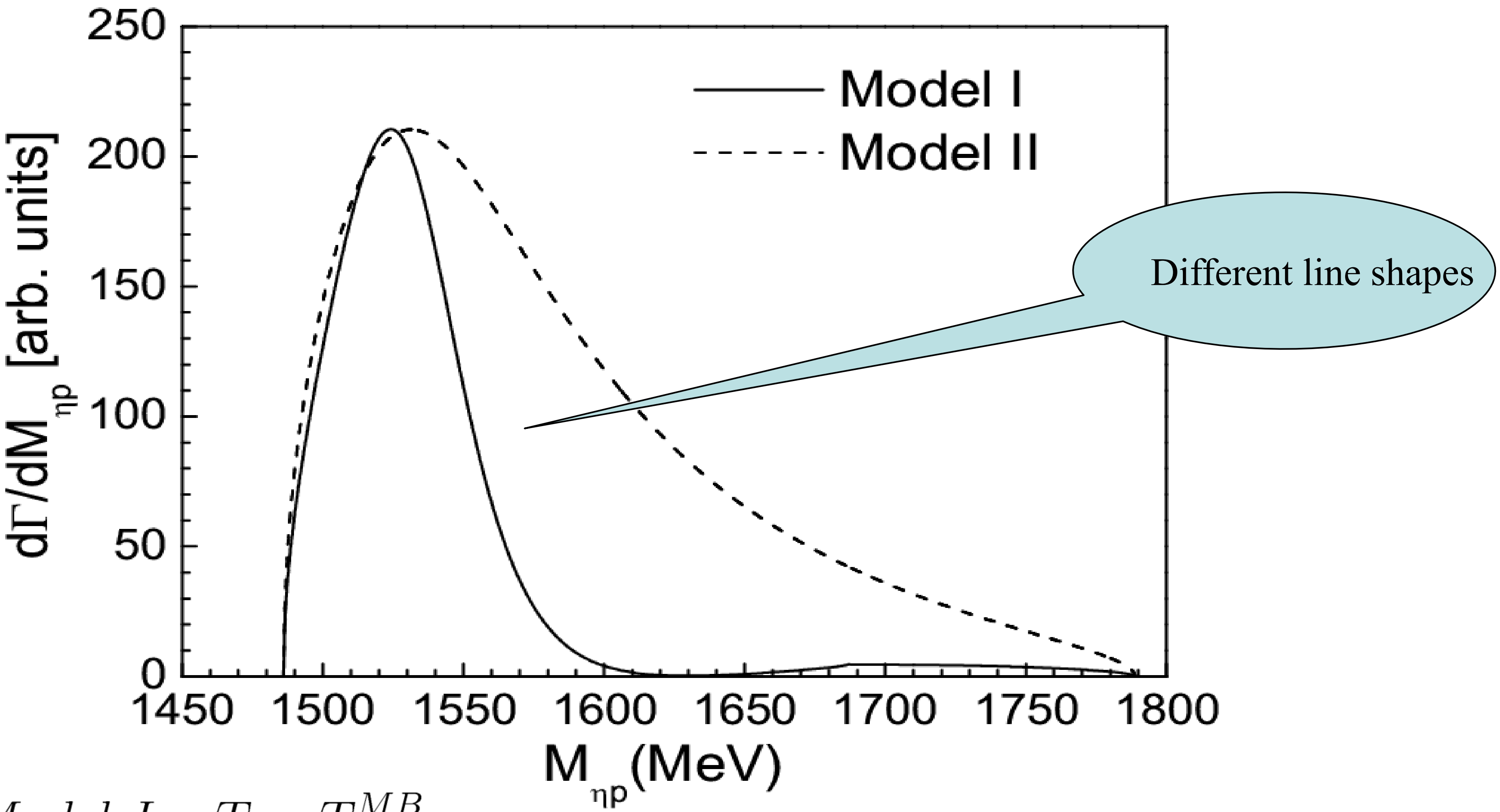
$$\Gamma_0 = 19.5 \text{ MeV} \quad \text{for} \quad \Gamma_{N^*}(\sqrt{q^2} = 1535 \text{ MeV}) = 150 \text{ MeV}.$$



(b)

Ju-Jun Xie and Li-Sheng Geng, PRD 96, 054009 (2017).

Invariant ηp mass distributions



$$Model\ I:\ T = T^{MB}$$

$$Model\ II:\ T = T^{N^*},\ M_{N^*} = 1535\ \mathrm{MeV},\ \Gamma_{N^*} = \Gamma_{N^*}(q^2)$$

Other contributions

$$N^*(1650) \to \eta p$$

$$\Sigma^* \ resonances$$

$$\to \bar{K}^0 p$$

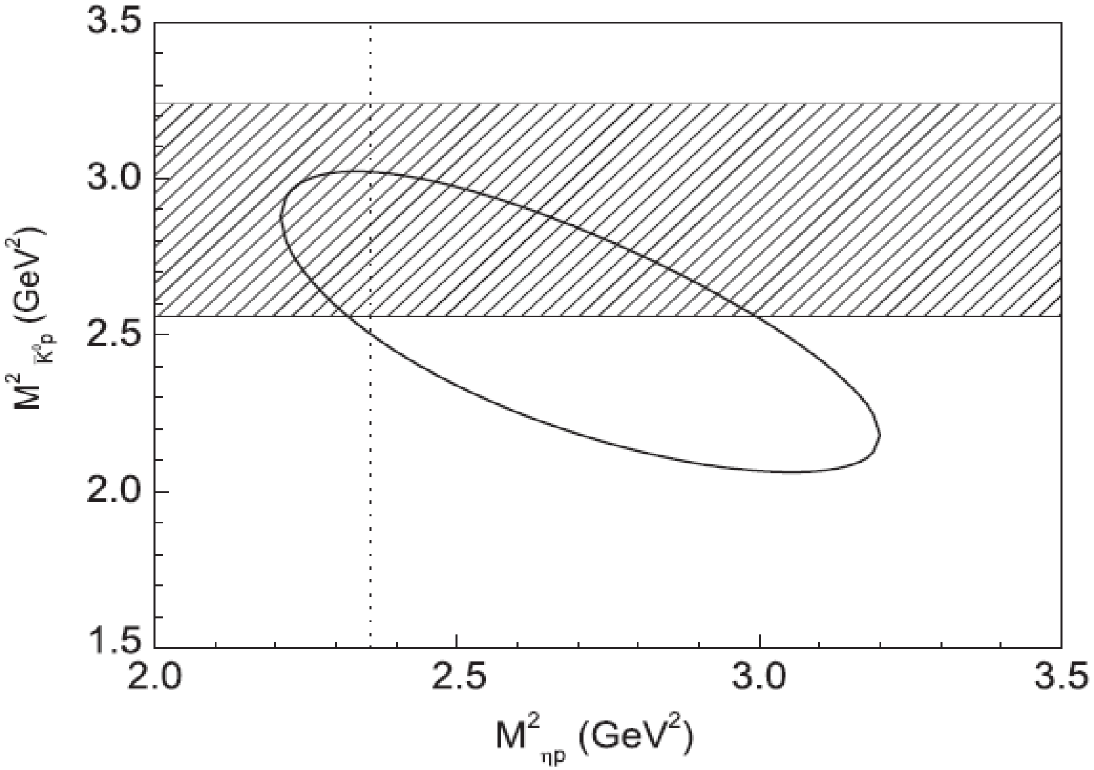


FIG. 7. Dalitz plot for $M^2_{\eta p}$ and $M^2_{\bar{K}^0 p}$ in the $\Lambda_c^+ \to \bar{K}^0 \eta p$ decay. The $N^*(1535)$ energy is shown by the vertical dotted line, and the horizontal band represents the masses of Σ^* states from 1600 to 1800 MeV.

Production of $N^*(1535)$ and $N^*(1650)$ in $\Lambda_c \to \overline{K}^0 \eta p$ (πN) decay

R. Pavao,* S. Sakai, and E. Oset

PRC 98, 015201 (2018).

PHYSICAL REVIEW D **107,** 032004 (2023)

Measurement of branching fractions of $\Lambda_c^+ \to pK_S^0K_S^0$ and $\Lambda_c^+ \to pK_S^0\eta$ at Belle

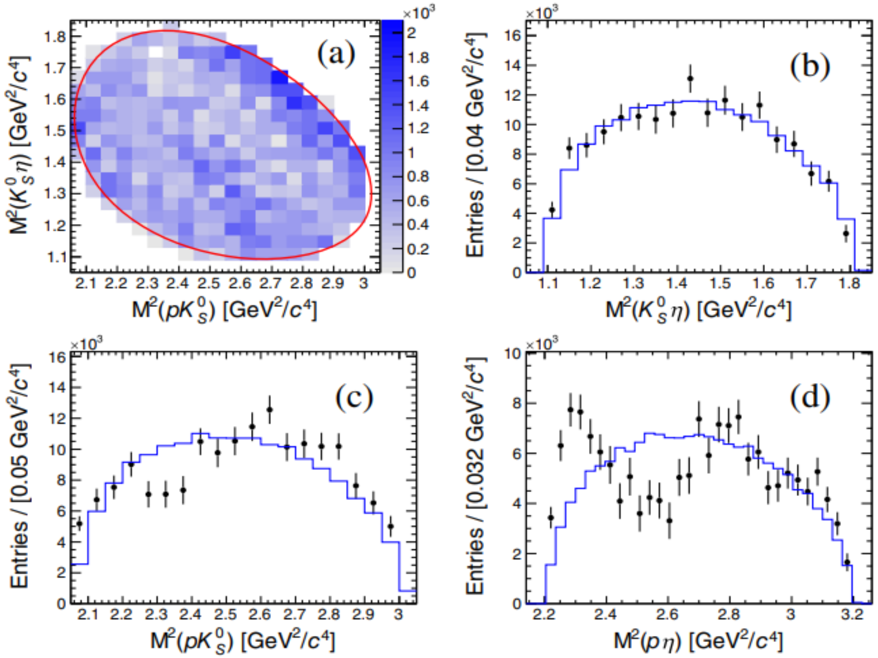
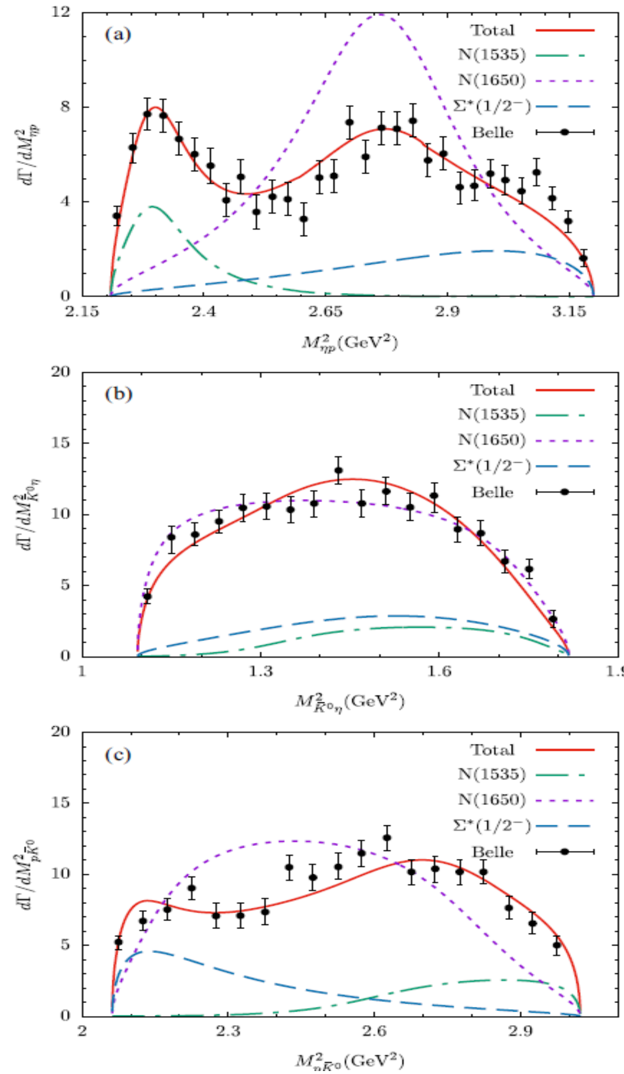


FIG. 4. For $\Lambda_c^+ \to pK_S^0\eta$, the Dalitz plot after background subtraction and efficiency correction bin-by-bin and its projections superimposing with signal MC produced by phase space mode (blue histograms). A significant structure of $N^*(1535)$ near the $p\eta$ threshold is found.

PHYSICAL REVIEW D **110,** 074010 (2024)

Theoretical study of $N(1535)$ and $\Sigma^*(1/2^-)$ in the Cabibbo-favored process $\Lambda_c^+ \to p\bar{K}^0\eta$

Ying Li,[1] Si-Wei Liu,[2,3] En Wang,[1,4,*] De-Min Li,[1,†] Li-Sheng Geng,[5,6,7,8,‡] and Ju-Jun Xie[2,3,8,§]



Summary

- ✓ Correlation function offers, from one side, new test of the strong interaction between pairs of hadrons
- ✓ Correlation function can be valuable to investigate the nature of exotic hadrons
- ✓ The $\Lambda_c^+ \to \bar{K}^0 \eta p$ decay can be used to study the $N^*(1535)$ resonance

Exotic hadron states are made of *hadrons.*

Thank you very much for your attention!