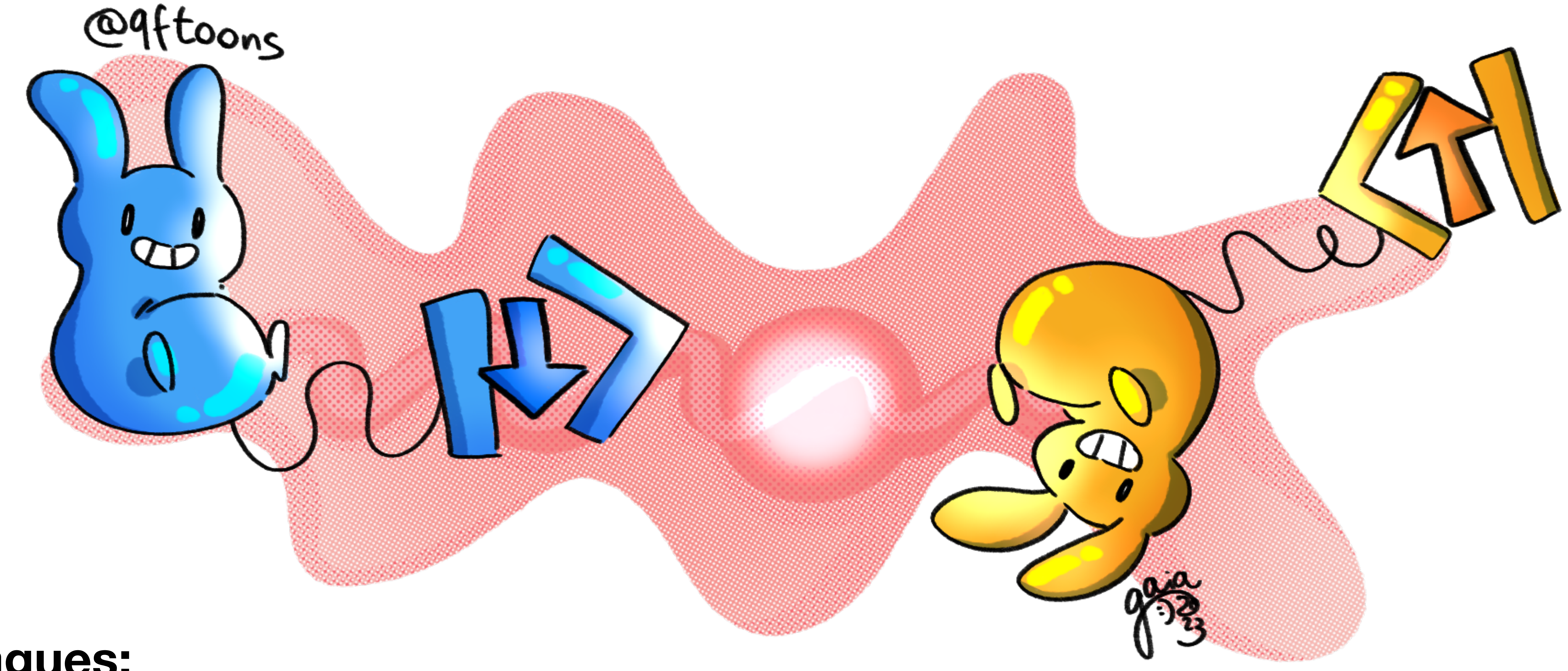
Quantum Observables for Collider Physics: Exploring Higher-Order Effects



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Thanks to the work of many enthusiastic colleagues:
Mohammad Mahdi Altakach, Rafael Aoude, Alan Barr, José Manuel Camacho, Valentin Durupt, Cristian degli Esposti, Federica Fabbri, Morgan del Gratta, Guillermo García Mir, Michele Grossi, Priyanka Lamba, Luca Mantani, Olivier Mattelaer, Kentarou Mawatari, Olimpia Miniati, Maria Moreno Llácer, Davide Pagani, Giovanni Pellicioli, Kazuki Sakurai, Claudio Severi, Max Sioli, Simone Tentori, Alessandro Vicini, Marcel Vos, Eleni Vryonidou,…

References:

- **Quantum properties of heavy-fermion pairs at a lepton collider with polarised beams**
  Altakach, Lamba, Maltoni, Sakurai — arXiv:2601.09558
- **Automated computation of spin-density matrices and quantum observables**
  Durupt, Maltoni, Mattelaer — arXiv:2510.17730
- **Z-boson quantum tomography at next-to-leading order**
  Del Gratta, Fabbri, Grossi, Maltoni, Pagani et al. — arXiv:2509.20456
- **Decoherence effects in entangled fermion pairs at colliders**
  Aoude, Barr, Maltoni, Satrioni — arXiv:2504.07030
- **Quantum properties of H → VV*: SM predictions and new physics sensitivity**
  Del Gratta, Fabbri, Lamba, Maltoni, Pagani — arXiv:2504.03841
- **Quantum tops at circular lepton colliders**
  Maltoni, Severi, Tentori, Vryonidou — arXiv:2404.08049
- **Quantum detection of new physics in $t\bar{t}$ production**
  Maltoni, Severi, Tentori, Vryonidou — arXiv:2401.08751
- **Probing new physics through entanglement in diboson production**
  Aoude, Madge, Maltoni, Mantani — arXiv:2307.09675
- **Quantum information and measurement at future lepton colliders**
  Altakach, Lamba, Maltoni, Mawatari, Sakurai — arXiv:2211.10513
- **Quantum SMEFT tomography: $t\bar{t}$ production at the LHC**
  Aoude, Madge, Maltoni, Mantani — arXiv:2203.05619
- **Quantum tops at the LHC: from entanglement to Bell inequalities**
  Severi, Degli Esposti Boschi, Maltoni, Sioli — arXiv:2110.10112

Work in Progress:

- **Quantum detection of CP-violation in $t\bar{t}$ production at the LHC**
  Priyanka Lamba, Fabio Maltoni, Olimpia Miniati, Eleni Vryonidou
- **Decoherence of elementary-fermion qubit pairs at colliders**
  Rafael Aoude, José Manuel Camacho, Valentin Durupt, Guillermo García Mir, Fabio Maltoni, Maria Moreno Llácer, Leonardo Satrioni, Marcel Vos
- **Theory of decoherence at colliders**
  Rafael Aoude, Alan Barr, Valentin Durupt, Fabio Maltoni, Leonardo Satrioni

Introduction

The quantum nature of the SM is so pervasive in the daily life of the particle physicist, that we often forget about it.

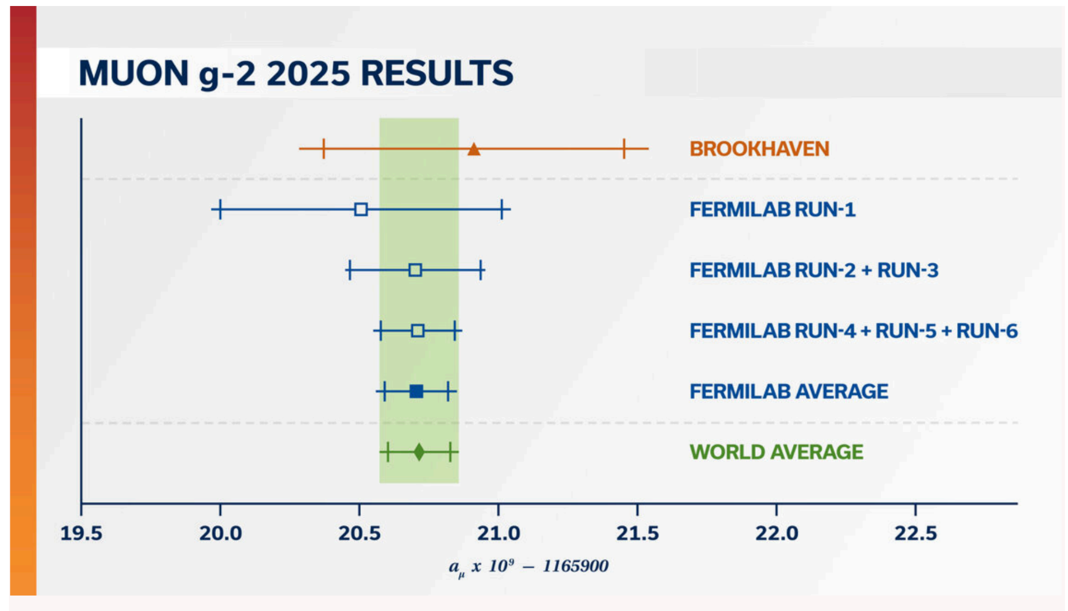
<u>Naively:</u>

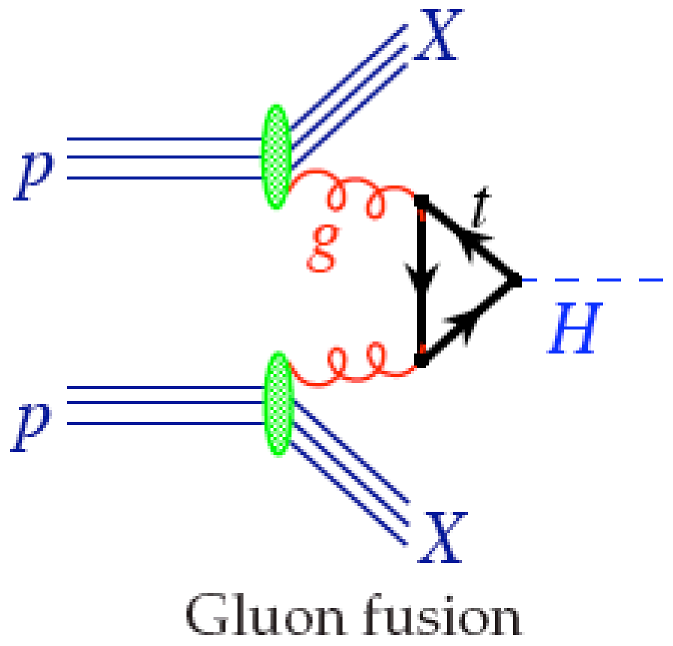
$$Z = \int D\phi \, e^{\frac{i}{\hbar} S[\phi]}$$

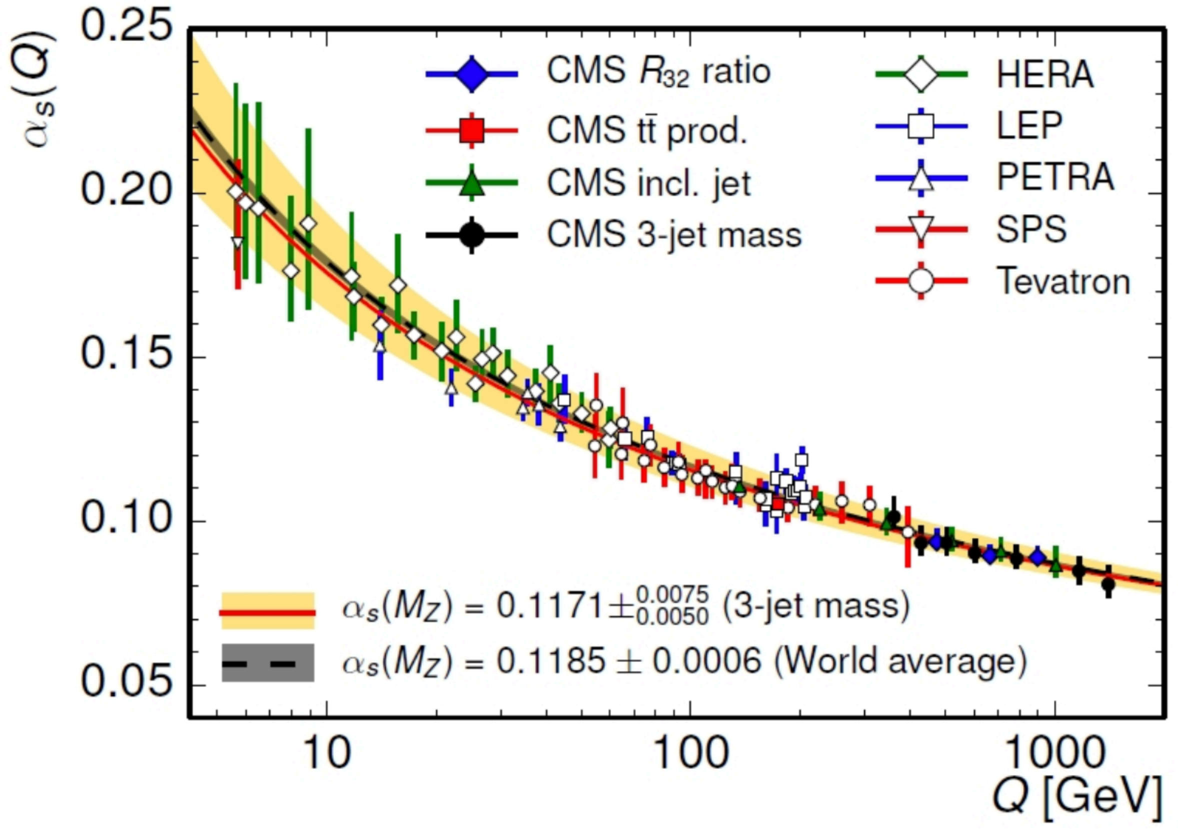
- **tree level** → order $\hbar^0$
- **1-loop** → order $\hbar^1$
- **2-loop** → order $\hbar^2$
- etc.

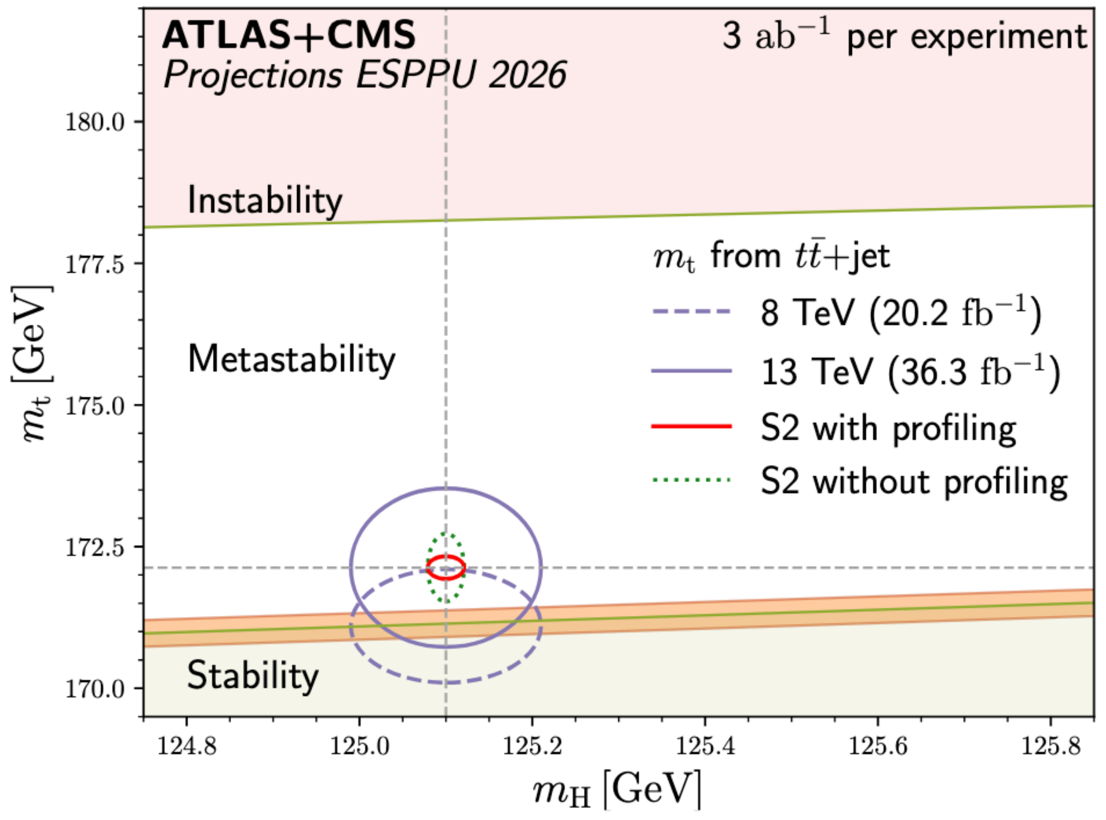
We have extensively validated QFT at different scales, and always found it a consistent way of making predictions in HEP.

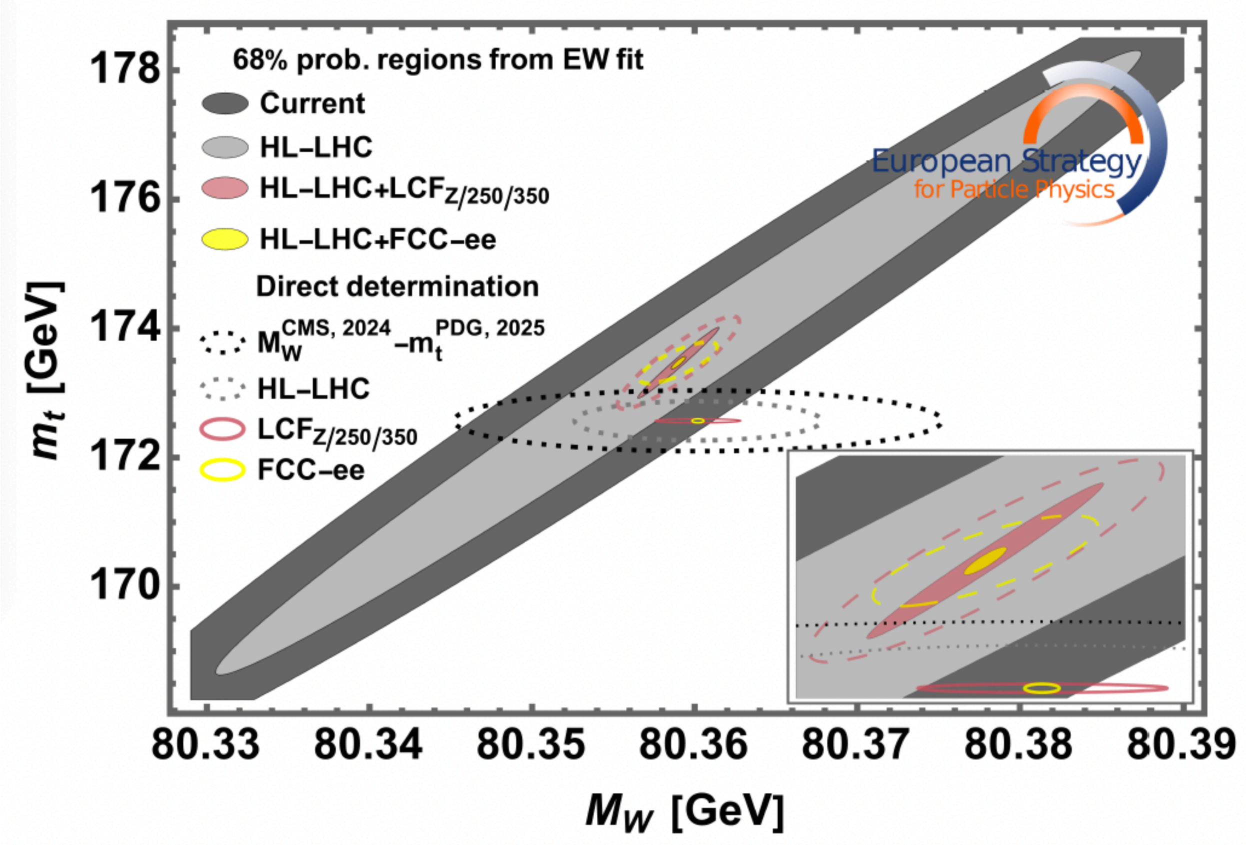
Are there other features predicted by QFT that we can study/exploit?











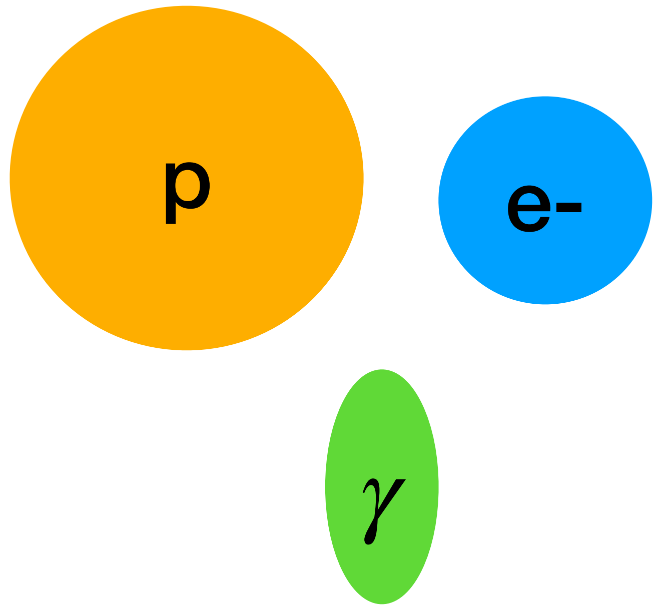
Introduction

Entanglement is the potential of quantum states to exhibit correlations that cannot be accounted for classically. For decades, entanglement has been the focus of much work in the foundations of quantum mechanics, being associated particularly with quantum nonseparability and the violation of Bell's inequalities [1]. In recent years, however, it has begun to be viewed also as a potentially useful resource. The predicted capabilities of a quantum computer, for example, rely crucially on entanglement [2], and a proposed quantum cryptographic scheme achieves security by converting shared entanglement into a shared secret key [3].

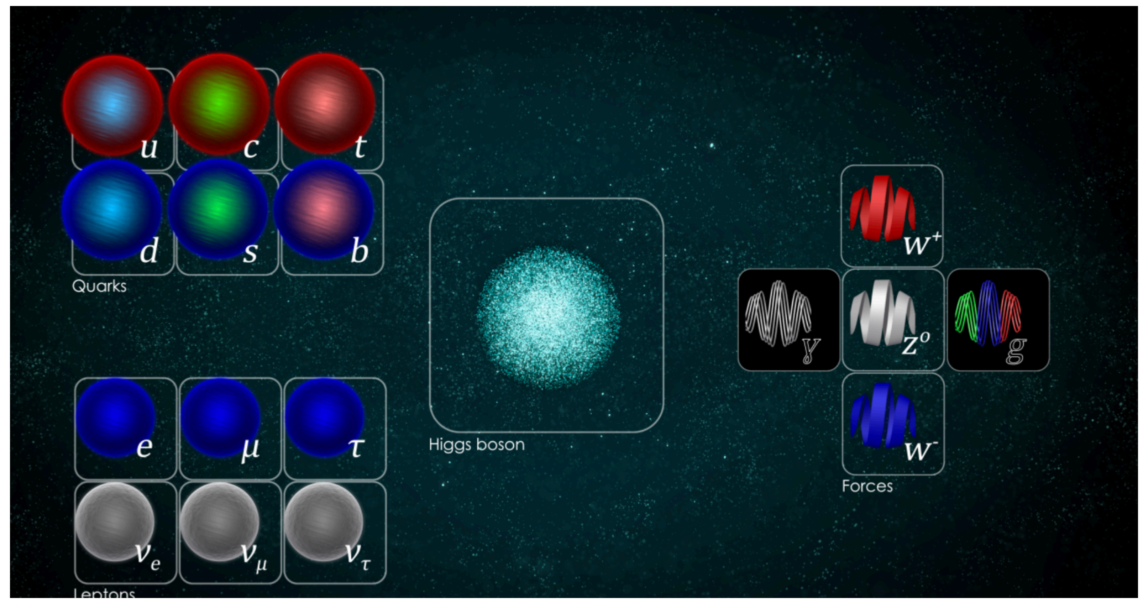
Theory of Resources ⇒ Entanglement => communication resource
Magic => computational resource

Don't ask what you can do for QM, but what QM can do for you!

Introduction



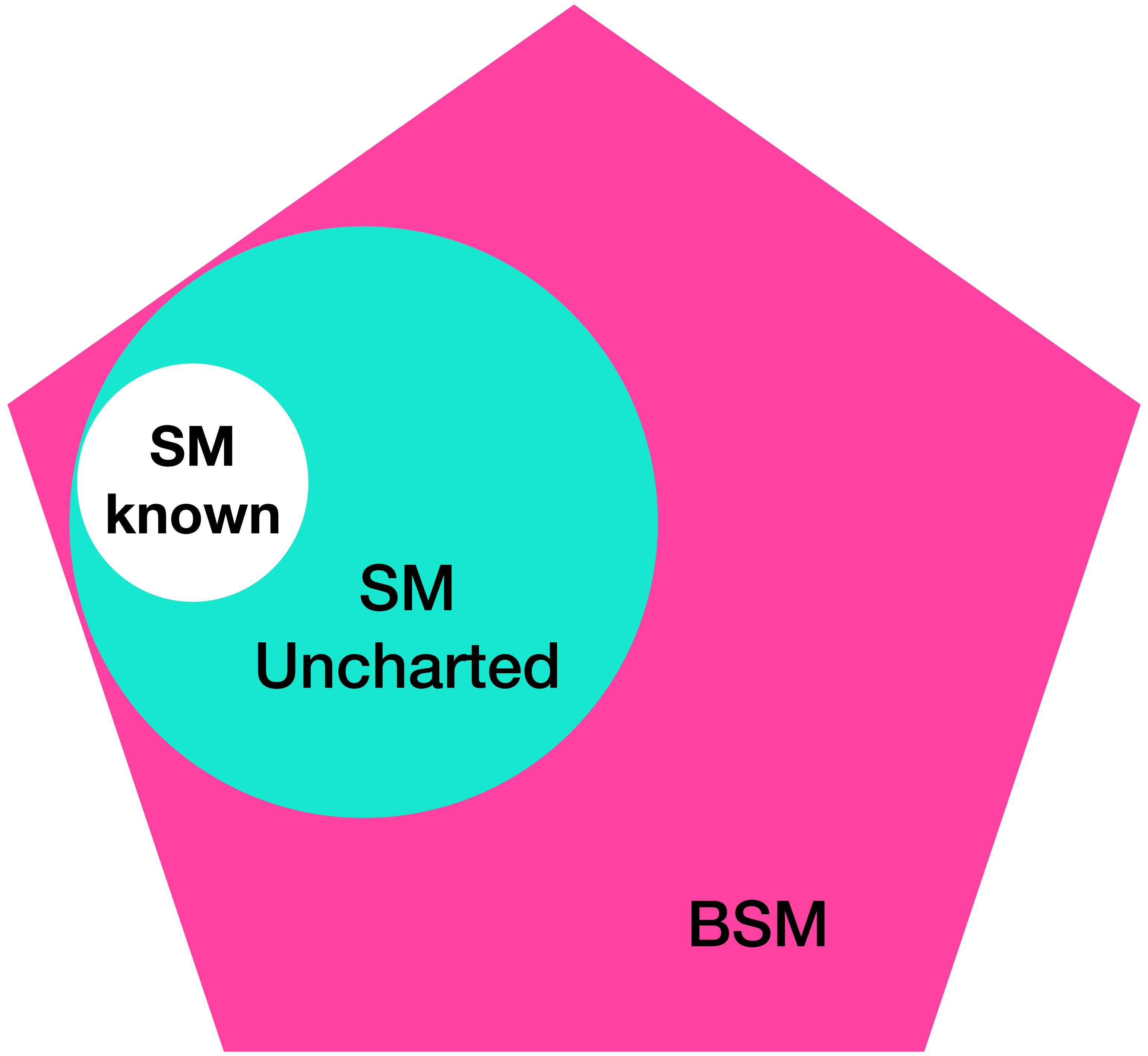
Vs



What can we learn on Fundamental Interactions from quantum information ideas/methods/techniques/results?

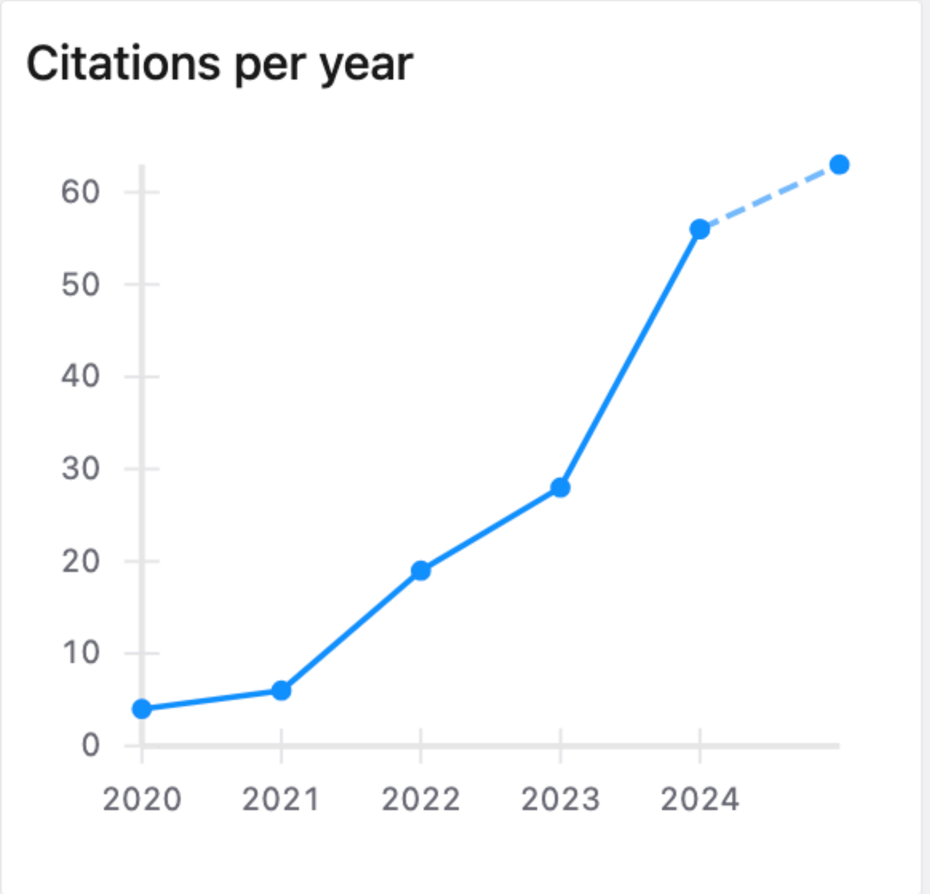
- **Community pride:** $\mathbb{QM} \subset \mathbb{QFT}$
- **Meaning:** A plethora of fundamental results (theorems!) for QI : what do they mean for HEP?
- **Impact:** And viceversa, what can be learnt on QI from particle physics?
- **Value:** Opportunity to elaborate (and communicate) what is important/interesting in our field:

New Physics $\equiv$ Uncharted SM physics + BSM



Introduction

- Is there a relation between symmetries and entanglement? 1812.03138, 2210.12085
- What is the best frame for making quantum measurements? 2311.09166
- How is decoherence happening for collider final states? What about NLO? 2504.07030, 2510.13951
- When is the actual measurement of spin really happening? 2401.06854
- Is there an optimal way to do quantum tomography? 2311.09166
- Is there a general approach to quantum measurements at colliders? 2201.03159
- Are there quantities in colliders that can be entangled beyond spin and flavour? Color?
- Is the information entropy a useful quantity in collider physics?
- Are SM interactions minimal in with respect to alternative theories? 2307.08112
- Can multi-partite systems be studied at colliders? 2310.01477
- Is entanglement conserved/augmented/lost in SM interactions? 1703.02989,2209.01405
- Is there a relation between scattering in QFT and computing in IS? 2312.02242, 2310.10838
- Can entanglement be used to do model building? 2307.08112
- What is the analogue of purification at collider processes?
- Can Bell-inequalities be tested at colliders ? 2507.15949, 2507.15947
- What is the analogue of distillation? 2401.06854
- What is the most general constraint on non-locality from scattering processes? 2401.01162
- Entanglement in neutrino oscillations? Many papers, see 2305.06095
- Entanglement and Bell in B0/B0-mixing? Several papers, see 2106.07399
- How should we think about virtual particles? 2211.05782
- Maximal or minimal entanglement as a guiding principle? 1703.02989 vs 2307.08112 and 2410.23343
- Magic : 2406.07321, 2508.14967

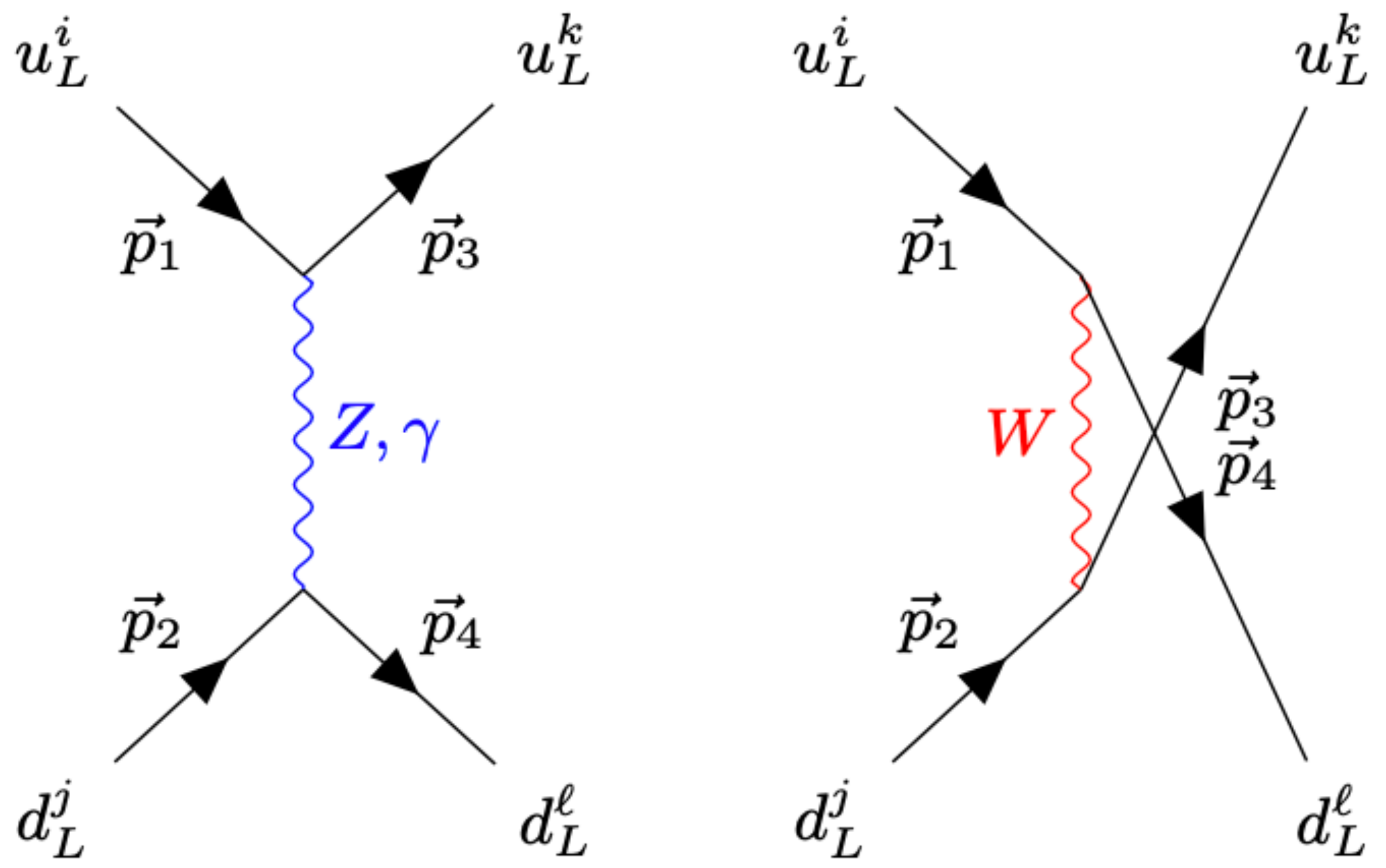


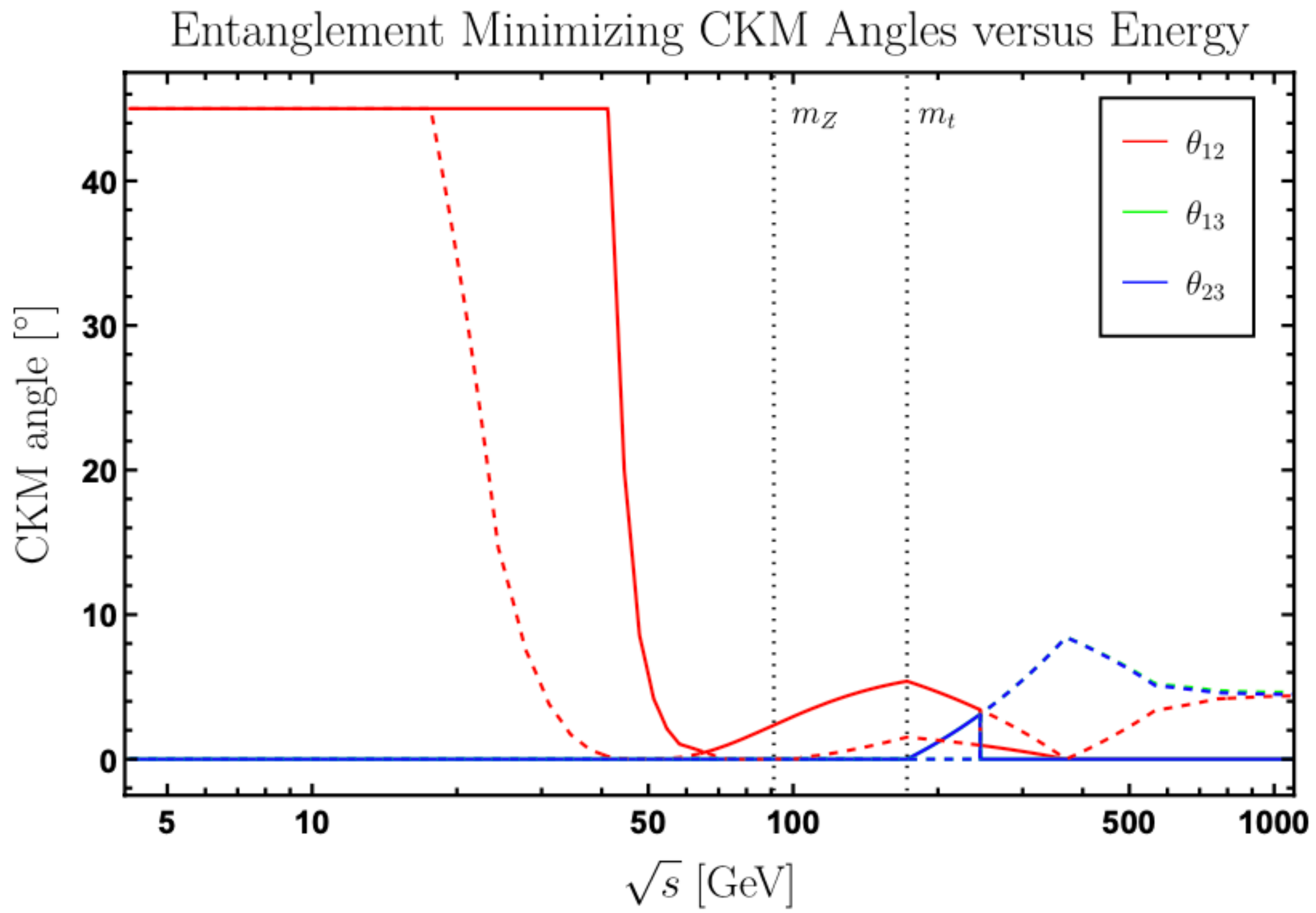
Citations to the Afik & de Nova paper

Entanglement as a fundamental principle?

Cervera-Lierta, Latorre, Rojo, Rottoli,1703.02989
Carena, Low, Wagner, Xiao 2307.08112
Thaler, Trifinopoulos. 2410.23343
Liu, Low, Yin, 2509.18251
Nunez, Pardina, Asorey, Latorre, Cervera-Lierta, 2511.04358
McGinnis 2511.10559

⇒QED as minimal theory
⇒ symmetry enhancement
⇒CKM/PMNS matrix structure
⇒Value of sin(thetaW)
⇒Gauge invariance
⇒SU(N) always generates entanglement





Ent. generated by scattering is minimized when the CKM matrix is almost (but not exactly) diagonal and when the PMNS matrix features two large angles and a smaller one,

Quantum Observables for Collider Physics 2026

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CERN
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TH workshop secretariat

thworkshops.secretariat...

The workshop aims at gathering theorists and experimentalists interested in measuring quantum information observables, such as magic and entanglement, on particles created at colliders and using these new observables as an innovative direction to probe fundamental interactions.

The programme includes discussion on the recent experimental results obtained in this field, on the implications of these new observables for new physics searches, and feasibility studies for multiple final states in current and future colliders. On the theoretical side, the workshop also aims to investigate the possible integration of quantum information–theoretic concepts in quantum field theory and particle physics.

It will bring together experts not only from particle physics, but also from quantum information science and nuclear theory, and will include panel discussions and overview talks to explore the broader links between quantum information, technologies, and high-energy physics.

This is the third edition of a workshop series previously held at the Galileo Galilei Institute (GGI) in Florence in 2023 and 2025. It builds on a growing number of international meetings on quantum-information applications to high-energy physics organized in recent years, including those in Oxford in 2023 and 2024, Pittsburgh in 2024, and at the WQC in Shanghai 2025.

This event is sponsored by the Department of Theoretical Physics at CERN as well as the CERN-Korea collaboration program.

Alan Barr
Fabio Maltoni
Federica Fabbri
Hyun Min Lee
Michele Grossi
Myeonghun Park
Regina Demina
Sokratis Trifinopoulos
Yoav Afik

3rd Edition at CERN!

Plan

- Review of the basics and recent results
- Higher-order effects in H $\rightarrow 4$ fermions
- Quantum decoherence and radiation

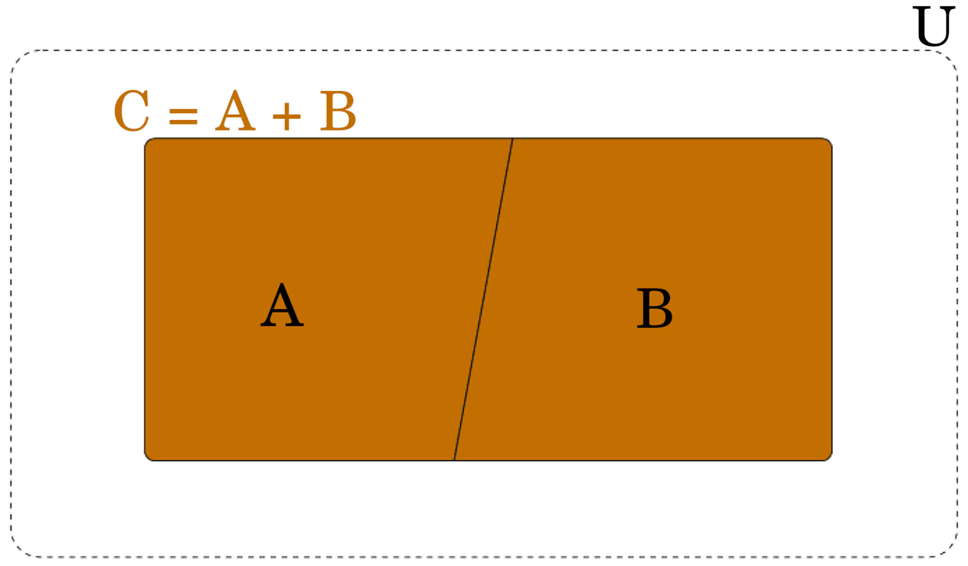
Basics

Density matrix : pure versus mixed

Schrödinger wave function (pure)	Pure	Generic (mixed)
$\vert\psi\rangle = \sum_n \alpha_n \vert\phi_n\rangle$	$\rho = \vert\psi\rangle\langle\psi\vert$	$\rho = \sum_j p_j \vert\psi_j\rangle\langle\psi_j\vert \quad (\sum_j p_j = 1, p_j \geq 0)$
$i\hbar \frac{d}{dt}\vert\psi(t)\rangle = H\vert\psi(t)\rangle$	$i\hbar \frac{d\rho}{dt} = [H, \rho]$	
$\langle A \rangle = \langle\psi\vert A \vert\psi\rangle$	$\langle A \rangle = \mathrm{Tr}[A\rho]$	
$\langle\psi\vert\psi\rangle = 1$	$\mathrm{Tr}[\rho] = 1$	
$\vert\langle\phi\vert\psi\rangle\vert^2 \geq 0$	$\mathrm{Tr}[\rho^2] = 1 \quad \rho = \rho^2$	$\mathrm{Tr}[\rho^2] < 1 \quad \rho \neq \rho^2$

Basics

Composite systems



Pure	$\|\psi\rangle = \|a\rangle \otimes \|b\rangle$	$\|\psi\rangle = \sum_{ij} p_{ij} \|a_i\rangle \otimes \|b_j\rangle \quad p_{ij} \in \mathbb{C}\,,\ \sum_{ij} p_{ij} p_{ij}^* = 1$ $\|a_i\rangle, \|b_j\rangle$ orthonormal bases
	Separable	Non-separable=ENTANGLED
Mixed	$\rho = \sum_i p_i\, \rho_A^i \otimes \rho_B^i$	$\rho \neq \sum_i p_i\, \rho_A^i \otimes \rho_B^i$

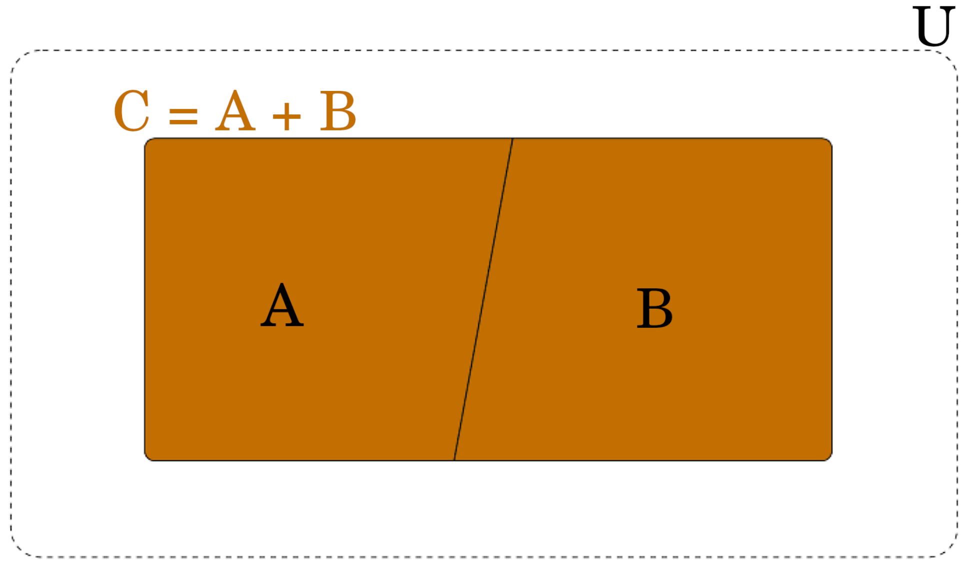
$p_i \geq 0\,, \sum_i p_i = 1$

Here one can use pure states instead of ρ_A, ρ_B

The properties are different for pure and mixed states

Entanglement

Theorem



If $|\psi\rangle$ is a **pure state** of the AB system, then two (orthonormal) bases (in A and B) exist such that

$$|\psi\rangle = \sum_i \sqrt{\lambda_i}\,|w_i\rangle_A \otimes |z_i\rangle_B \qquad \text{with} \qquad \sum_i \lambda_i = 1, \quad \lambda_i \geq 0$$

$$\rho_A = \sum_i \lambda_i |w_i\rangle_A\, {}_A\langle w_i|$$

$$\rho_B = \sum_i \lambda_i |z_i\rangle_B\, {}_B\langle z_i|$$

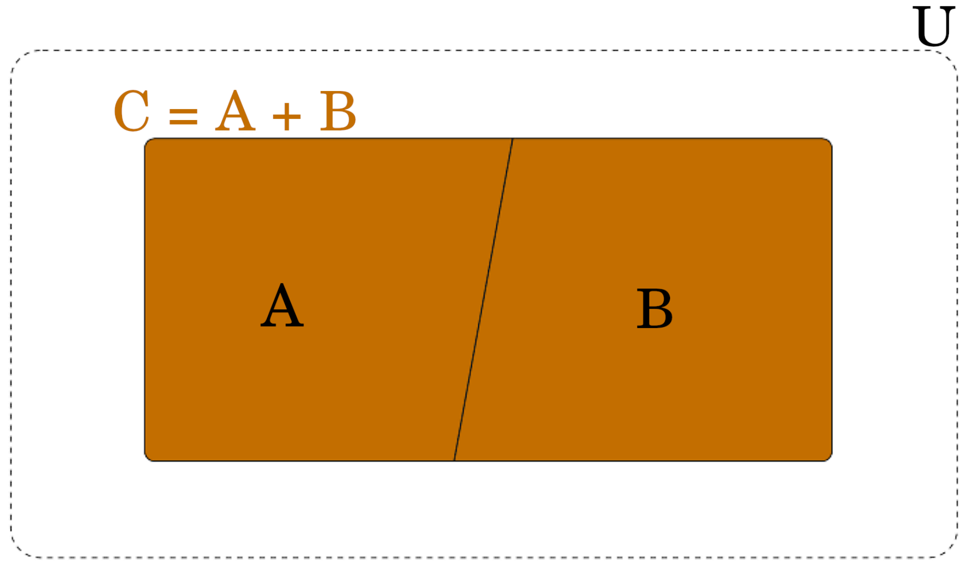
$\Rightarrow$

- The states of the subsystems are mixed-states!
- They have the same eigenvalues => they are equally impure

Consequences:

1. One can always think of a mixed state as the trace out a subsystem of a larger system (purification).
2. Two subsystems that partition a pure state are entangled IFF their reduced states are mixed.

Basics
Concurrence



Take an entangled **pure** state between the two subsystems A and B. $\mathscr{H} = \mathscr{H}_A \otimes \mathscr{H}_B$

As a result, the states in A and B must be mixed and

$\mathrm{Tr}\left[\rho_A^2\right] \leq 1$ and $\mathrm{Tr}\left[\rho_B^2\right] \leq 1$

The concurrence $C_{A|B}$ is defined as

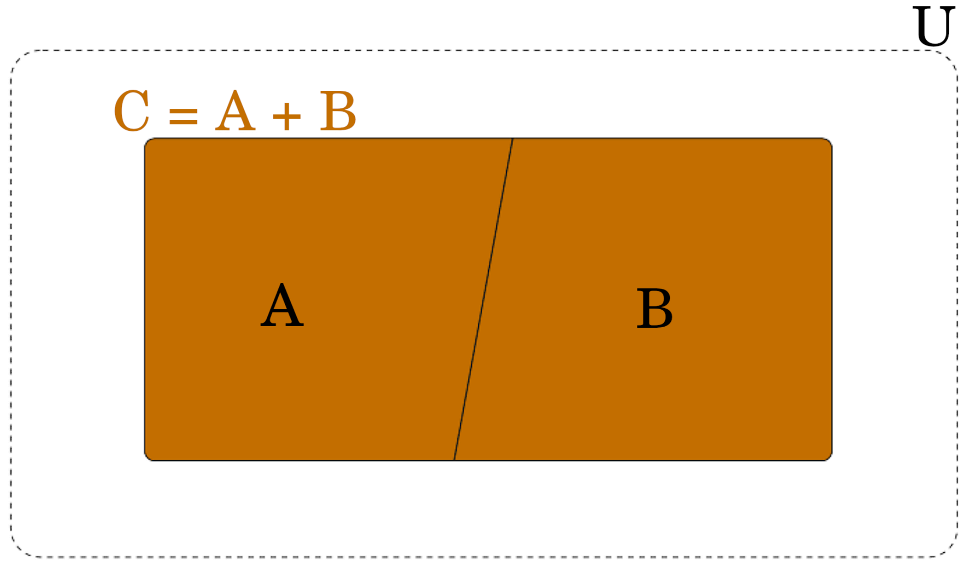
$$0 \leq C_{A|B}^2 = 2(1 - \mathrm{Tr}[\rho_A^2]) = C_{B|A}^2 \leq 1$$

$$C_{A|B}^2 = 2S_2(\rho_A)$$ Tsallis-2 linear entropy

For mixed states, things are in general more complicated.

Basics

Peres-Horodecki criterium



This is a necessary (and for two qubits sufficient) criterium for separability of a mixed state of two subsystems A and B. Consider a generic state:

$$\rho = \sum_{ijkl} p_{ij} p^*_{kl} |a_i\rangle \otimes |b_j\rangle \langle a_k| \otimes \langle b_l|$$

$$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$$

And the partial transpose on B

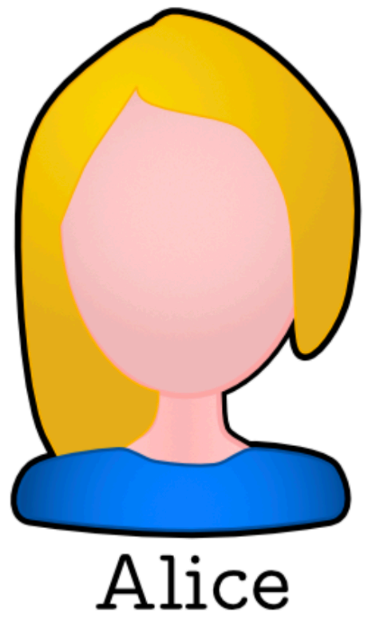
$$\rho^{T_B} = (I \otimes T)[\rho] = \sum_{ijkl} p_{ij} p^*_{kl} |a_i\rangle\langle a_k| \otimes (|b_j\rangle\langle b_l|)^T = \sum_{ijkl} p_{ij} p^*_{kl} |a_i\rangle\langle a_k| \otimes |b_l\rangle\langle b_j| = \sum_{ijkl} p_{il} p^*_{kj} |a_i\rangle\langle a_k| \otimes |b_j\rangle\langle b_l|$$

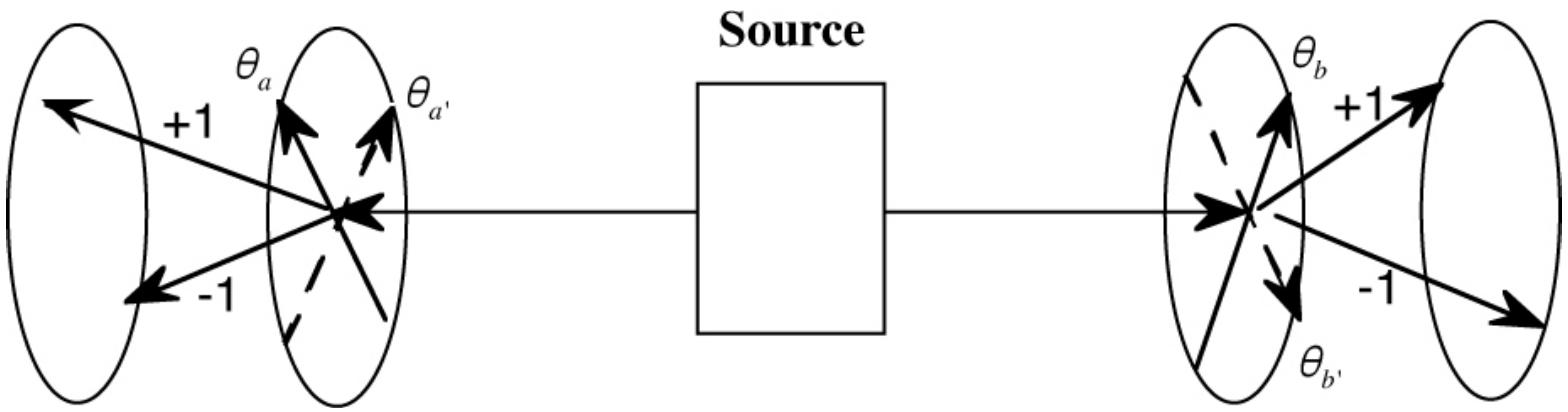
The criterion states that if ρ is separable then all the eigenvalues of ρ^{T_B} are non-negative.
In other words, if ρ^{T_B} has a negative eigenvalue, then the system is guaranteed to be entangled.

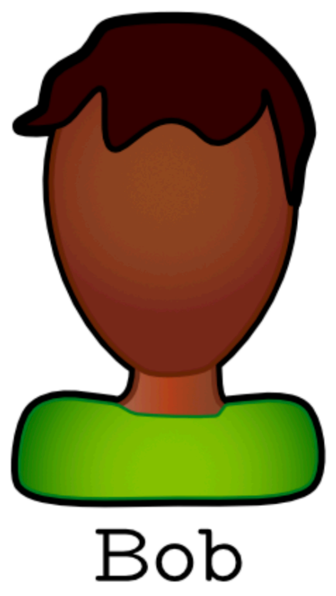
For 2 qubits or 1 qubit x 1 qutrit is a IFF

Basics

Bell (Clauser, Horne, Shimony, and Holt) inequalities







Assuming:
1] Measurements reveal element of reality, physical properties present beforehand.
2] Alice and Bob are separated by a space-like distance

$$A = \pm 1$$
$$A' = \pm 1$$

Local reality: $E(AB) + E(AB') + E(A'B) - E(A'B') \leq 2$

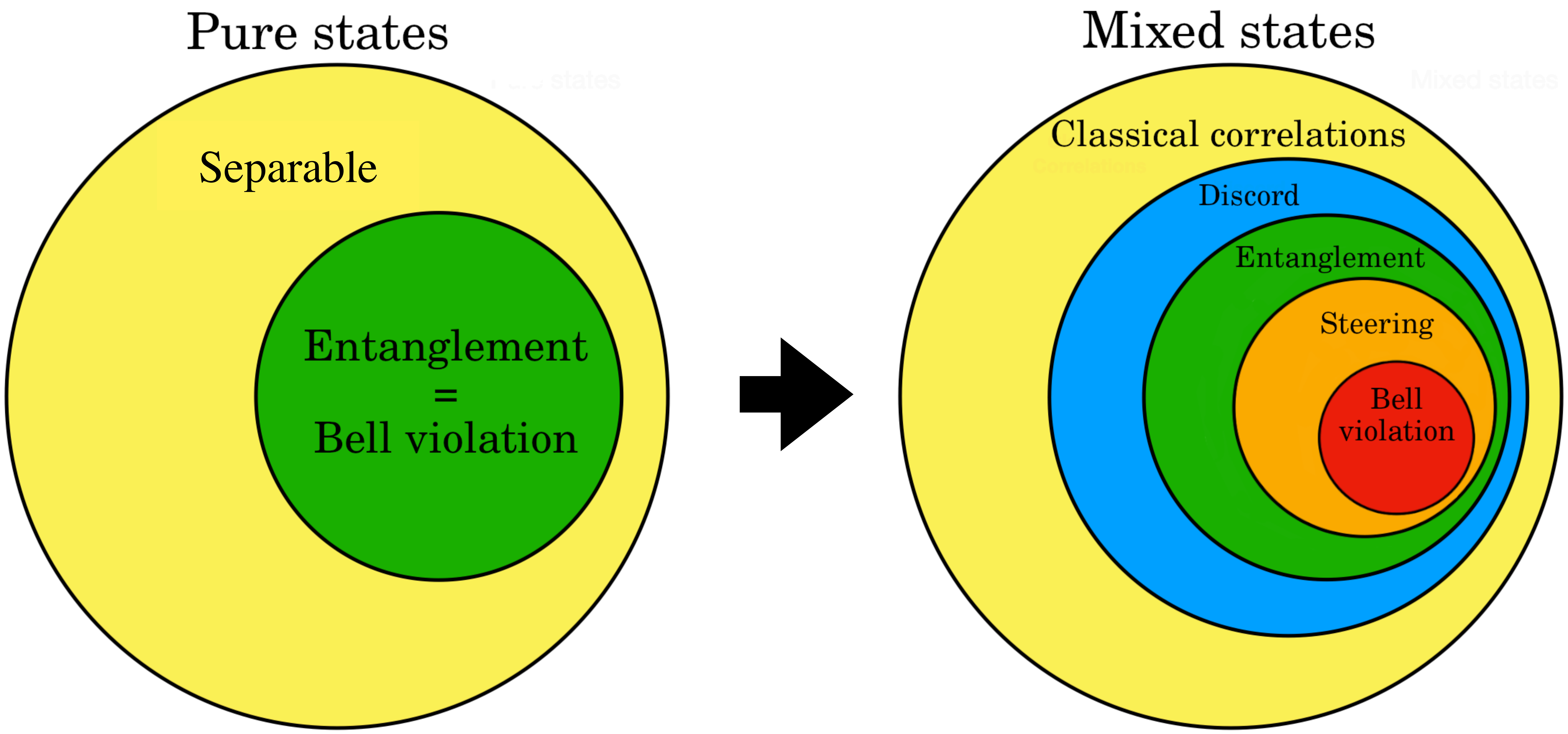
QM: $E(AB) + E(AB') + E(A'B) - E(A'B') \leq 2\sqrt{2}$

$$B = \pm 1$$
$$B' = \pm 1$$

In principle, CHSH can be tested independently of QM. However, we will use it as a measure of a strong entanglement.

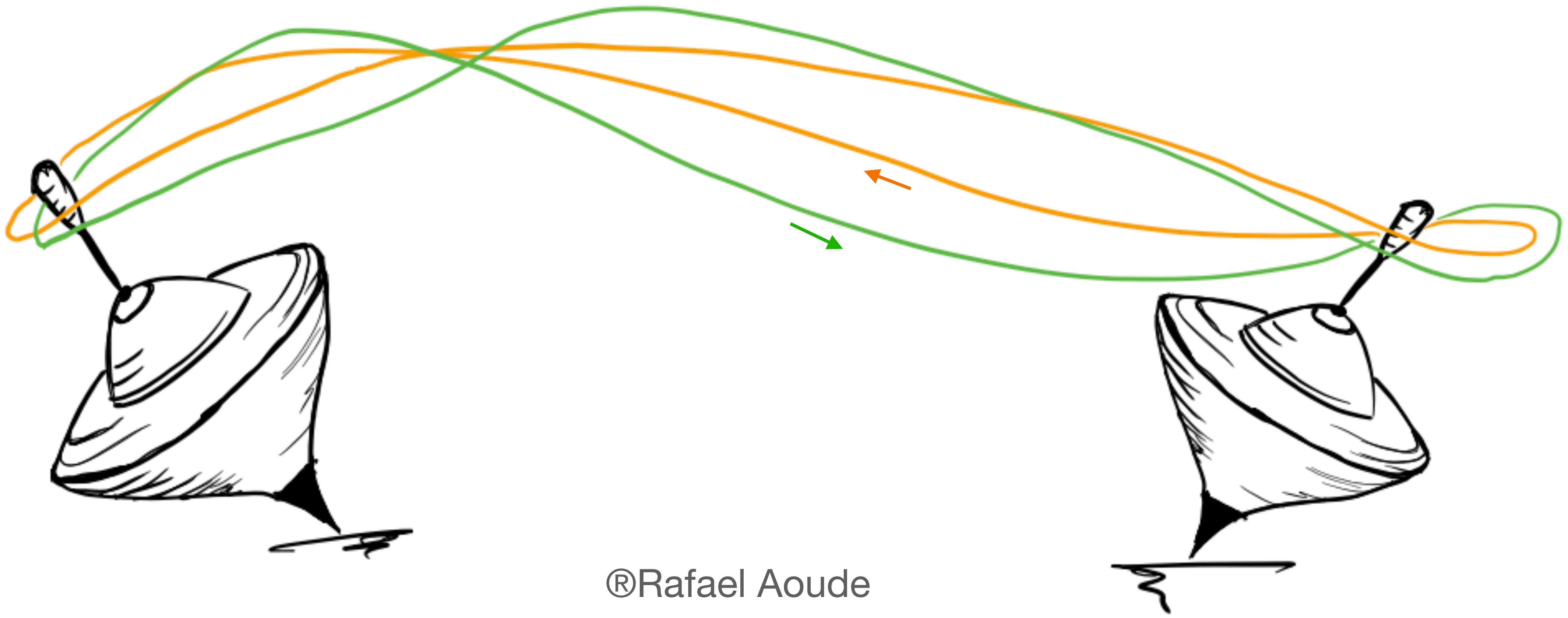
Basics

Grading QM correlations



$\mathrm{Tr}[\rho^2] = 1$ $\qquad$ $\mathrm{Tr}[\rho^2] < 1$

Quantum tops

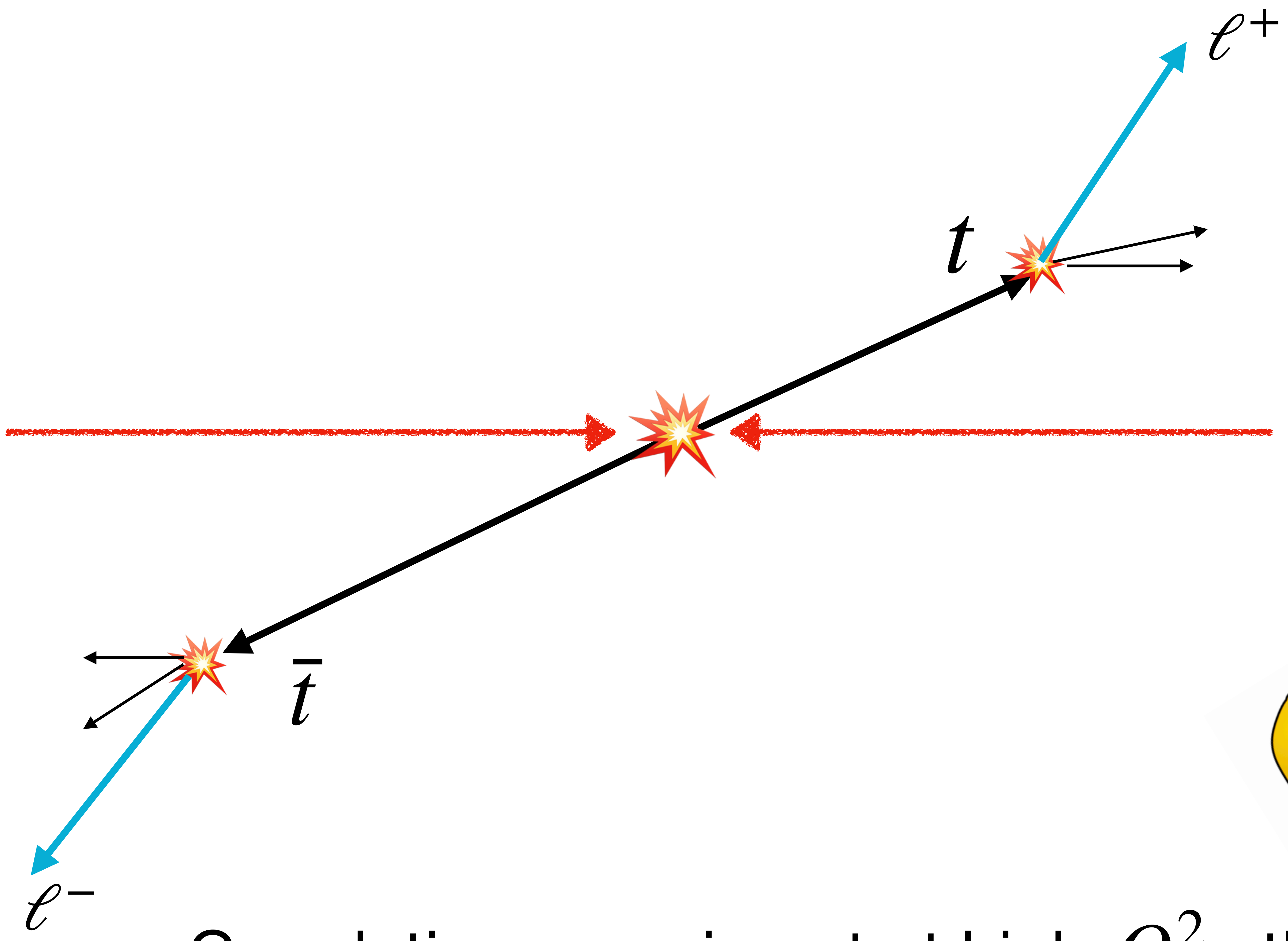


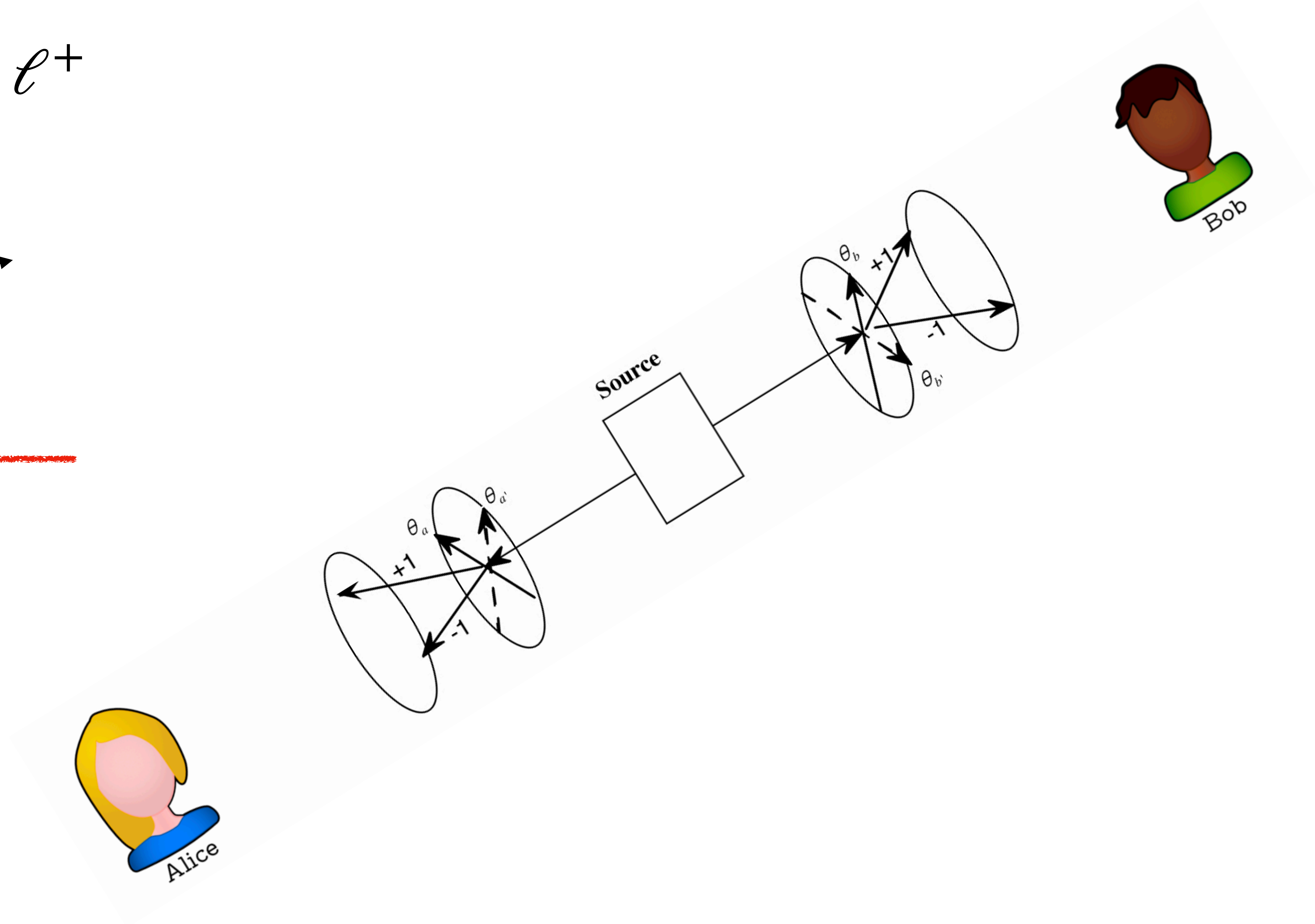
Qubit **Qubit**

Y. Afik and JRM de Nova: 2003.02280
M. Fabbrichesi, R. Floreanini. G. Panizzo: 2102.11883
C. Severi, C. Boschi, FM, M. Sioli : 2110.10112
Y. Afik and JRM de Nova: 2203.05582
R. Aoude, E. Madge, FM, L. Mantani: 2203.05619
J.A. Aguilar-Saavedra, J.A. Casas: 2205.00542
Y. Afik and JRM de Nova: 2209.03969
C. Severi, E. Vryonidou: 2210.09330
Z. Dong, D. Gonçalves, K. Kong, A. Navarro: 2305.07075
J.A. Aguilar-Saavedra : 2307.06991
T. Han, M. Low, TA Wu: 2310.17696
J.A. Aguilar-Saavedra, J.A. Casas: 2401.06854
J.A. Aguilar-Saavedra : 2402.14725
C. Severi, FM, S. Tentori, E. Vryonidou: 2401.08751
C. Severi, FM, S. Tentori, E. Vryonidou: 2404.08049
White, White : : 2406.07321
K. Cheng,T. Han, M. Low: 2407.01672
Z. Dong, D. Gonçalves, K. Kong, Larkowski, A. Navarro: 2407.07147
R. Aoude, Banks Whjite and White: 2505.12522
M. Altakach, P. Lambda, FM, K. Sakurai : 2601.09558
L. Antozzi et al. : 2602.23426
Y. Afik et al. : 2602.15115
Y.C. Guo, T. Han, M. Low, Y. Su : 2602.02719

Caution: possibly incomplete list!

Quantum tops





Correlation experiment at high-Q^2 : the lepton is correlated with the spin.

Why looking at tops?

- **LHC:** a top factory.

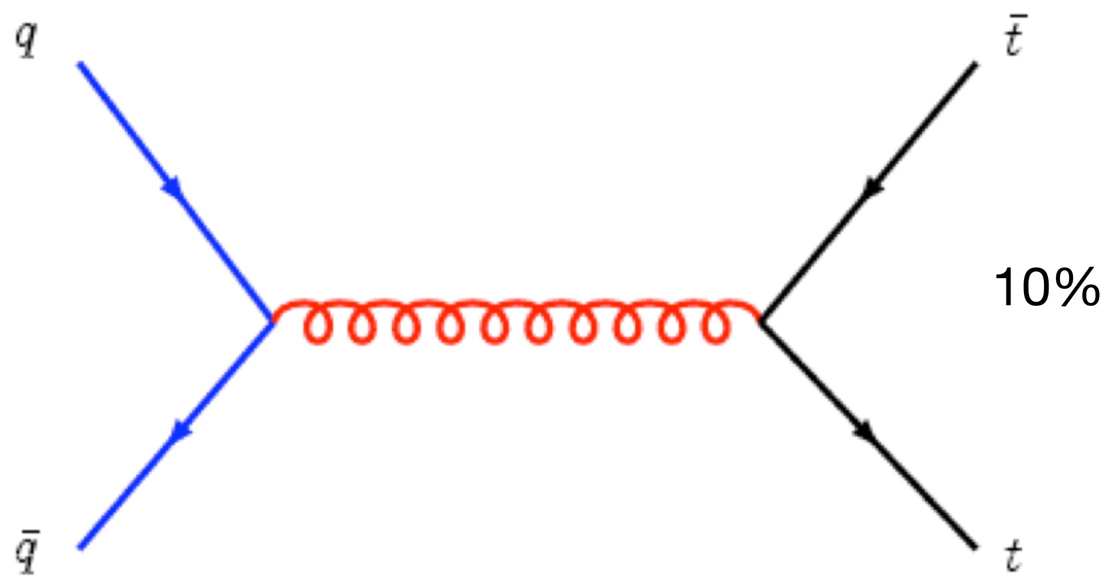
- **Top decay:** The decay occurs in two steps, t→Wb is the first one:

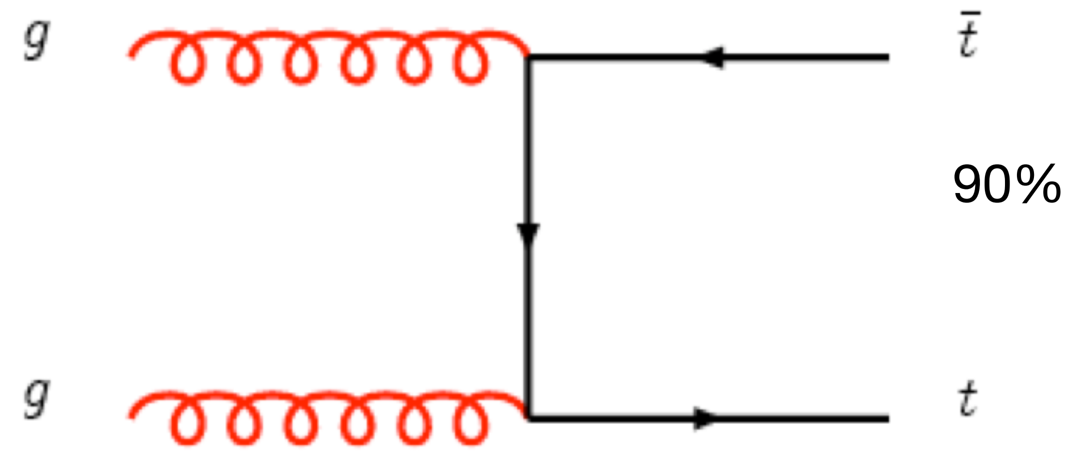
  $\tau_{had} \approx h/\Lambda_{QCD} \approx 2 \cdot 10^{-24}$ s

  $\tau_{top} \approx h/\Gamma_{top} = 1/(G_F m_t^3 |V_{tb}|^2/8\pi\sqrt{2}) \approx 5 \cdot 10^{-25}$ s (with h=6.6 10^{-25} GeV s)

  $$\tau_{spin\text{-}flip} \approx \left(\frac{\Lambda^2_{QCD}}{m_t}\right)^{-1} \gg \tau_{had}$$

- Due to the structure of weak interactions, it "magically" turns out that the direction of the lepton is 100% correlated with that of the spin of the top.

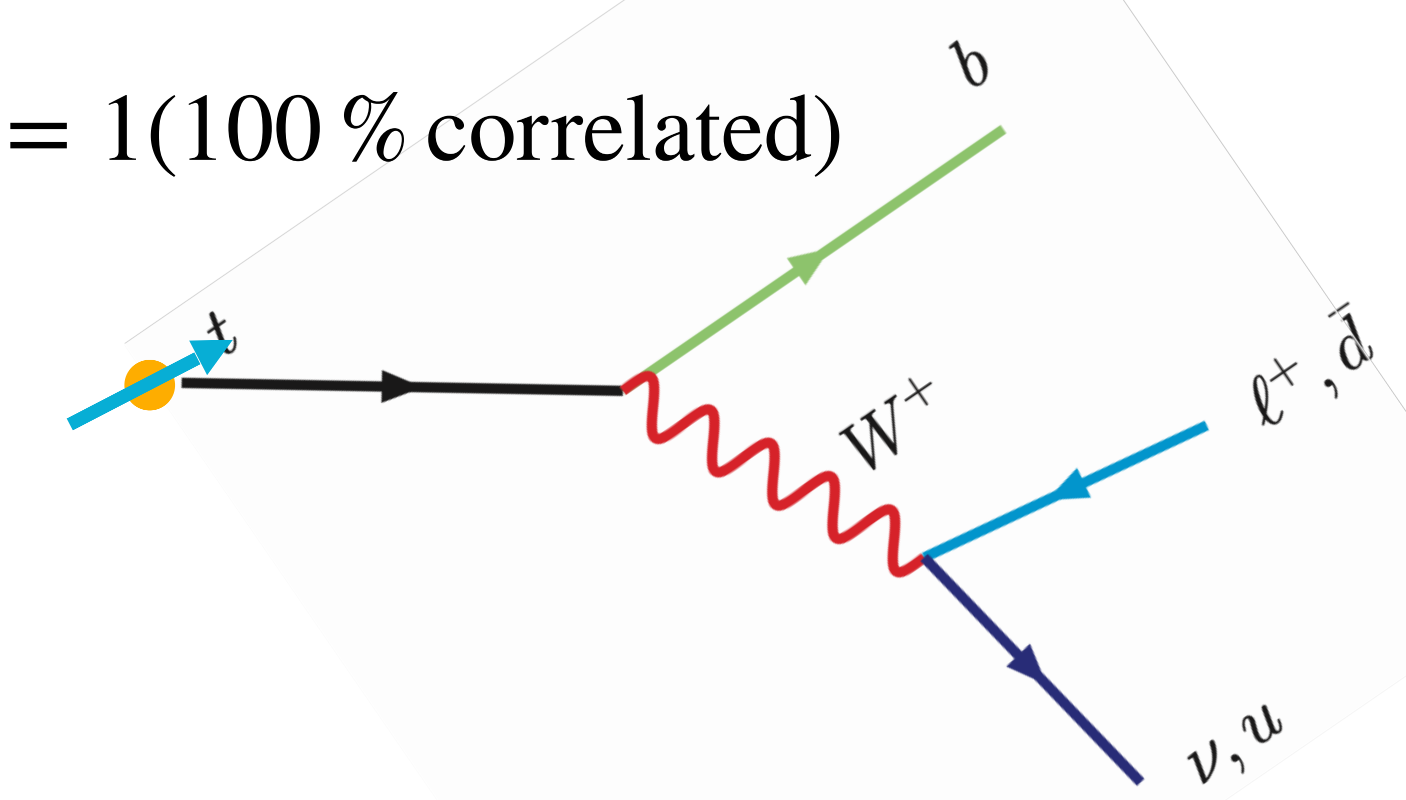




$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\varphi} = \frac{1+\alpha\cos\varphi}{2}$$

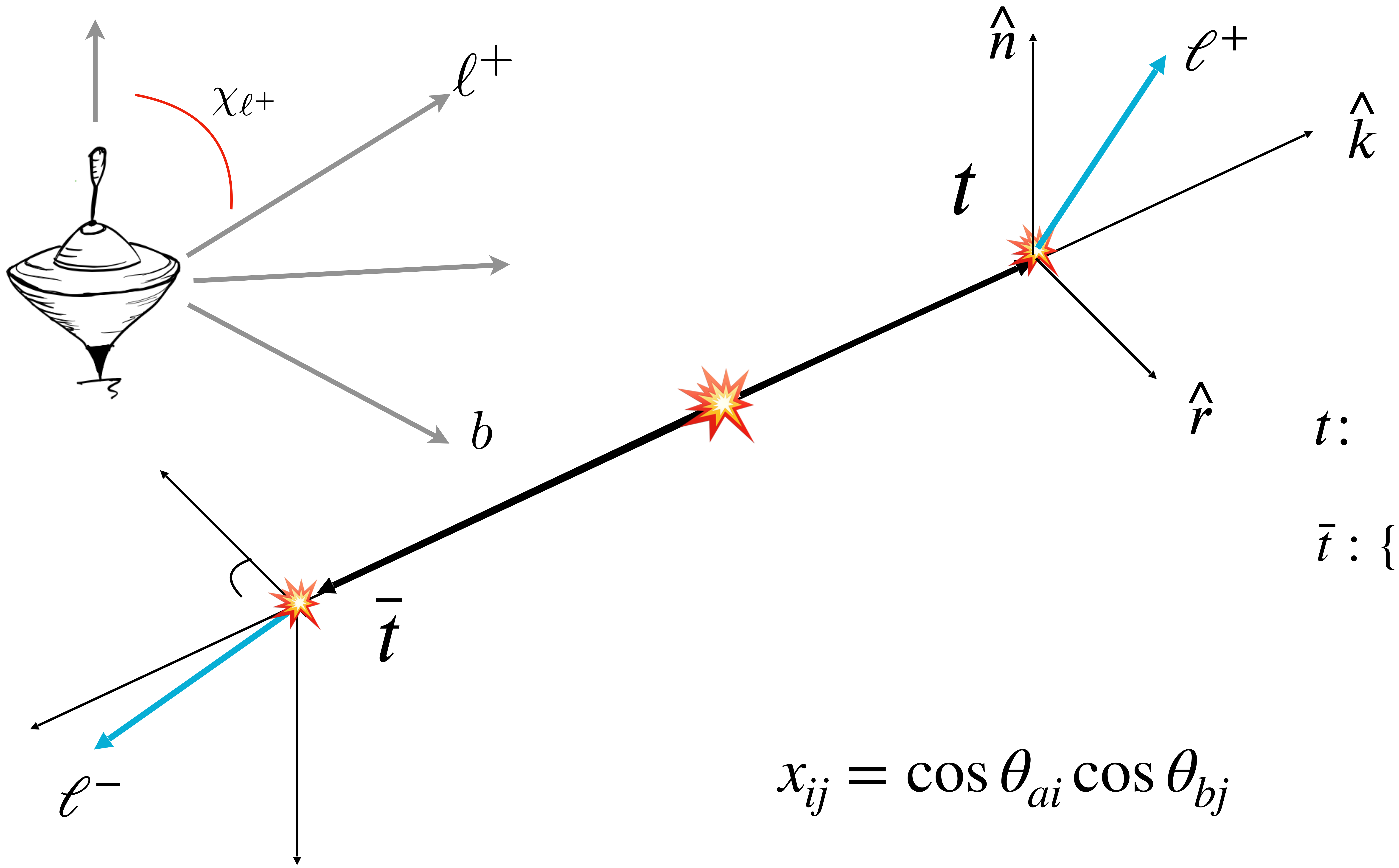
$$\alpha_d = 1, \alpha_u = -0.3, \alpha_b = -0.4, \alpha_W = 0.4$$

$$\alpha_\ell = 1(100\,\%\ \text{correlated})$$



⇒ The charged lepton is the best proxy for the spin

$t\bar{t}$ tomography



$t: \quad \hat{k} = \text{top direction}, \quad \hat{r} = \frac{\hat{p} - \hat{k}\cos\theta}{\sin\theta}, \quad \hat{n} = \frac{\hat{p} \times \hat{k}}{\sin\theta}$

$\bar{t}: \{-\hat{k}, -\hat{r}, -\hat{n}\}$

$$x_{ij} = \cos\theta_{ai}\cos\theta_{bj}$$

$$\frac{1}{\sigma}\frac{d\sigma}{dx_{ij}} = \frac{C_{ij}x_{ij} - 1}{2}\log|x_{ij}|$$

Quantum tops

"The devil is in the details" : $t\bar{t}$ pair is not in a pure state.

- The qubit-qubit system is described by the following density matrix

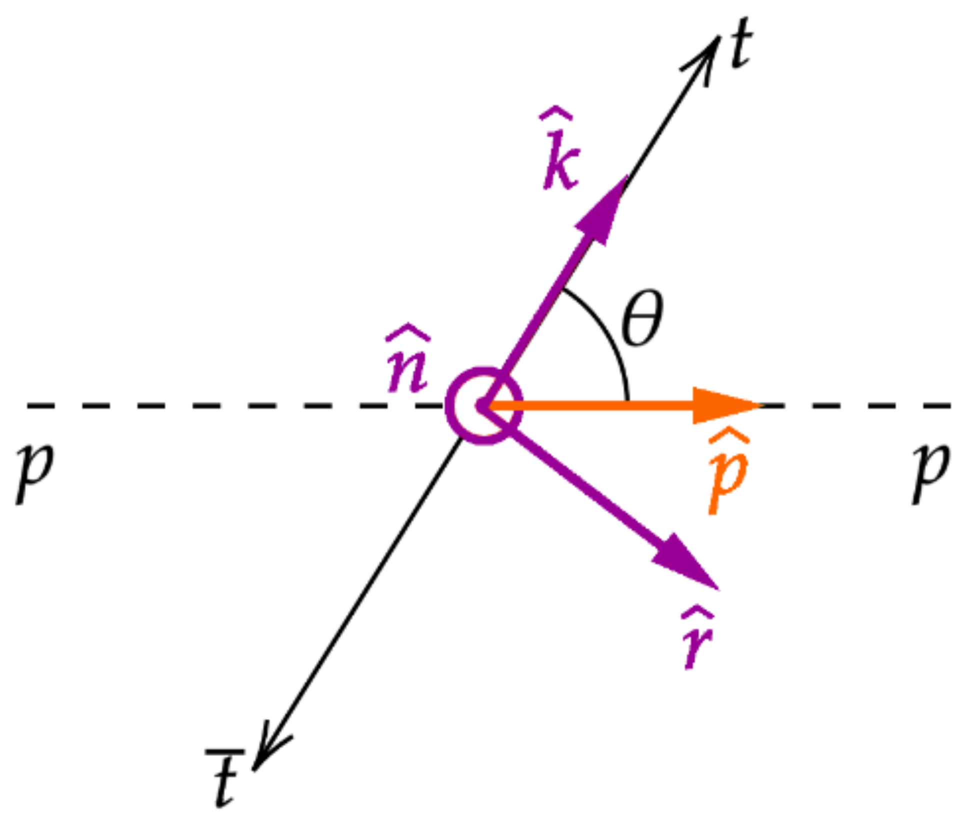
$$\rho = \frac{1}{4}\left(\mathbf{1}\otimes\mathbf{1} + \mathcal{B}\_ \cdot \boldsymbol{\sigma}\otimes\mathbf{1} + \bar{\mathcal{B}}\_ \cdot \mathbf{1}\otimes\boldsymbol{\sigma} + \mathcal{C}\cdot\boldsymbol{\sigma}\otimes\boldsymbol{\sigma}\right)$$

which, for $t\bar{t}$ can be approximated by $B_1 = B_2 = 0$, and C is symmetric (CP conservation) and almost diagonal in the helicity basis.

$$\langle S_i\rangle = B_i, \quad \langle \bar{S}_i\rangle = \bar{B}_j, \quad \langle S_i\bar{S}_j\rangle = C_{ij}$$

Quantum tops

$$\hat{k} = \text{top direction}, \quad \hat{r} = \frac{\hat{p} - \hat{k}\cos\theta}{\sin\theta}, \quad \hat{n} = \frac{\hat{p} \times \hat{k}}{\sin\theta}$$



We can write four sufficient conditions for entanglement:

$$-C_{kk} - C_{rr} - C_{nn} \equiv -3D^{(1)} > 1$$

$$-C_{kk} + C_{rr} + C_{nn} \equiv -3\,D^{(k)} > 1$$

$$+C_{kk} + C_{rr} - C_{nn} \equiv -3\,D^{(n)} > 1$$

$$+C_{kk} - C_{rr} + C_{nn} \equiv -3\,D^{(r)} > 1$$

$$D < -1/3 \qquad \Rightarrow \text{Entanglement}$$

$$\frac{1}{\sigma}\frac{d\sigma}{d\cos\varphi} = \frac{1}{2}(1 - D\cos\varphi)$$

$$\mathcal{C}^{(\text{singlet})} = \begin{pmatrix} -\eta & 0 & 0 \\ 0 & -\eta & 0 \\ 0 & 0 & -\eta \end{pmatrix}, \; 0 < \eta \leq 1$$

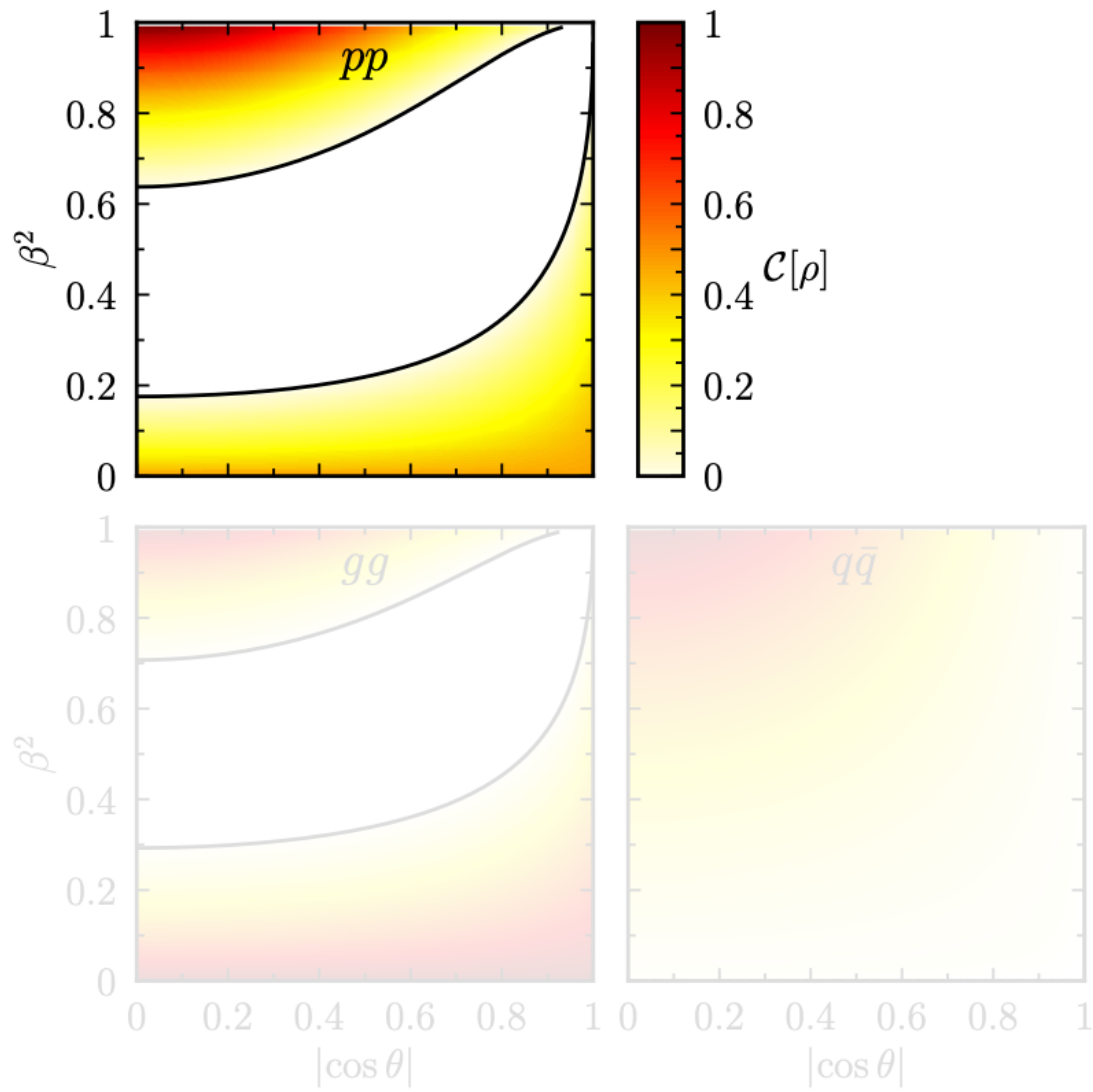
$$\mathcal{C}^{(\text{triplet})} = \begin{pmatrix} \eta & 0 & 0 \\ 0 & \eta & 0 \\ 0 & 0 & -\eta \end{pmatrix}, \qquad 0 < \eta \leq 1$$

Where D=-η and in the limiting case of $\eta = 1$ we have the four Bell states:

$$|\Phi^\pm\rangle = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle \pm |\downarrow\downarrow\rangle),$$

$$|\Psi^\pm\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle \pm |\downarrow\uparrow\rangle).$$

SM Entanglement



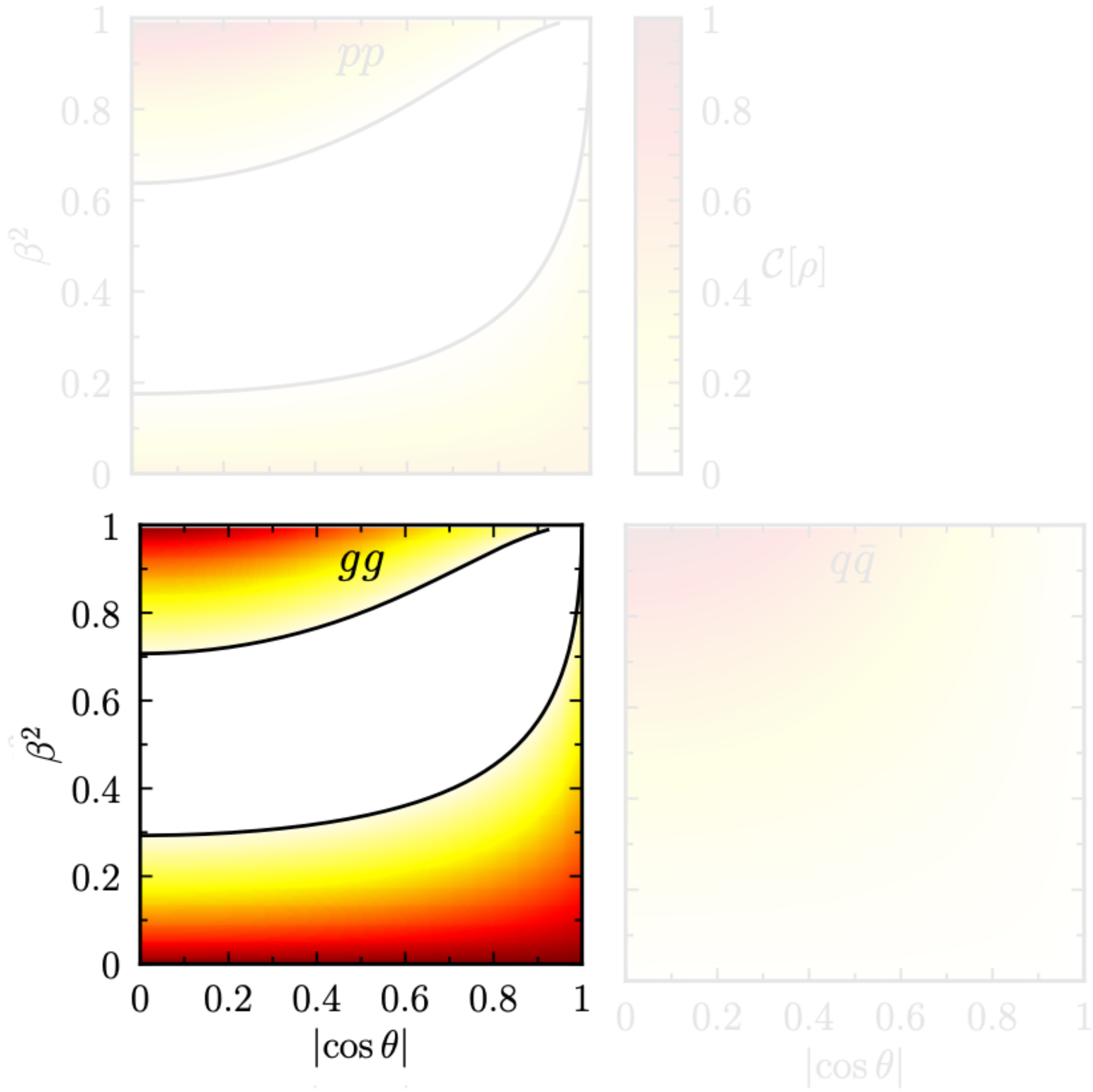
pp

White regions: zero-entanglement

Maximal entanglement points/regions:

- At threshold: $\beta^2 = 0, \forall\theta$
- high-E: $\beta^2 \to 1, \cos\theta = 0$

SM Entanglement



$$gg$$

Maximal entanglement points/regions:

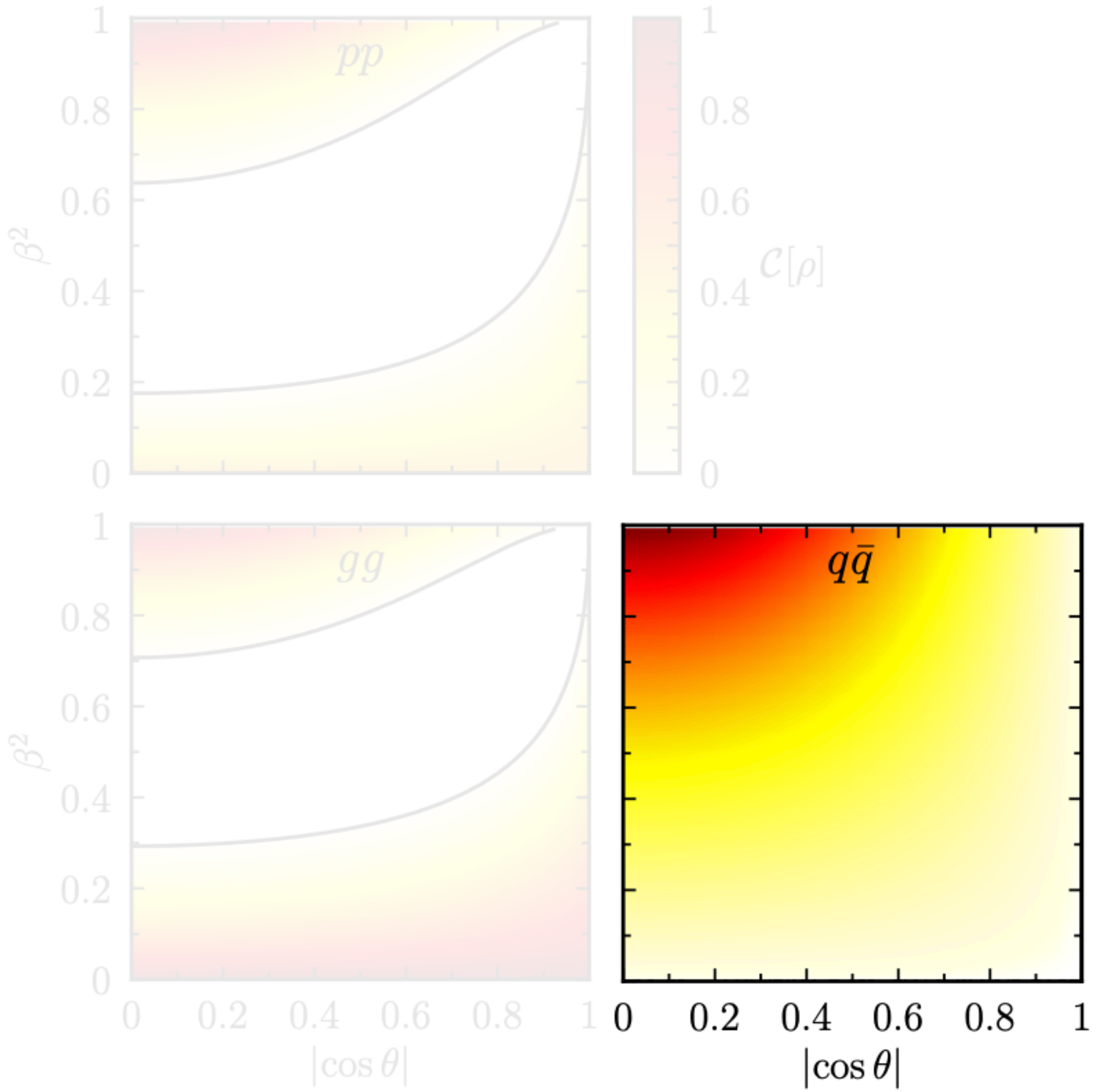
- At threshold: $\beta^2 = 0, \forall\theta$

$$\rho_{gg}^{\mathrm{SM}}(0,z) = |\Psi^-\rangle_{\boldsymbol{n}}\langle\Psi^-|_{\boldsymbol{n}} \qquad \text{(singlet)}$$

- High-E: $\beta^2 \to 1, \cos\theta = 0$

$$\rho_{gg}^{\mathrm{SM}}(1,0) = |\Psi^+\rangle_{\boldsymbol{n}}\langle\Psi^+|_{\boldsymbol{n}} \qquad \text{(triplet)}$$

SM Entanglement



$q\bar{q}$

Maximal entanglement points/regions:

- At threshold: $\beta^2 = 0, \forall\theta$

  mixed but separable

- High-E: $\beta^2 \to 1, \cos\theta = 0$

$\rho^{\rm SM}_{q\bar{q}}(1,0) = |\Psi^+\rangle_{\boldsymbol{n}}\langle\Psi^+|_{\boldsymbol{n}}.$ (triplet)

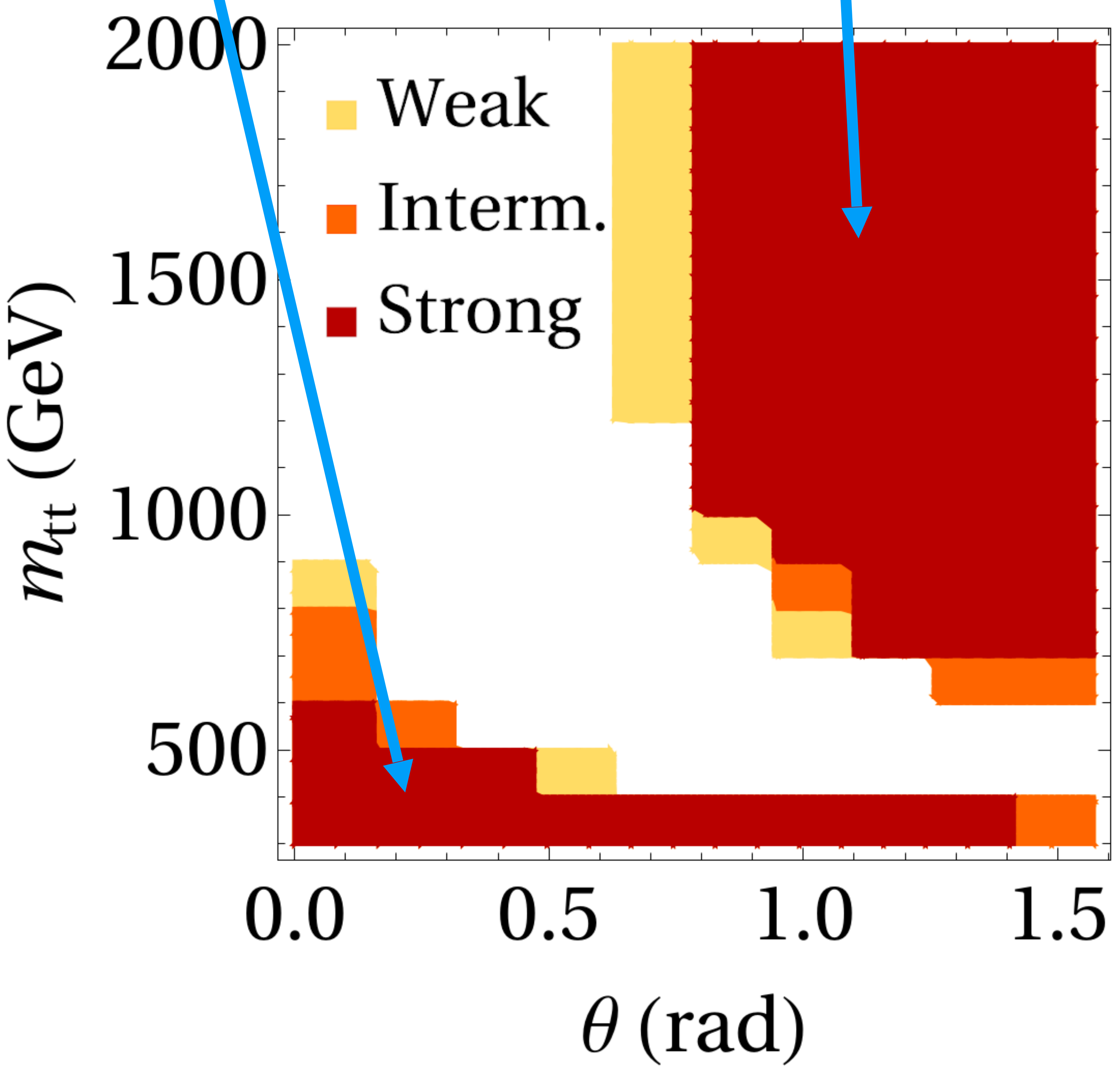
SM Entanglement

$C_{kk} + C_{rr} - C_{nn} > 1$

$-C_{kk} - C_{rr} - C_{nn} > 1$

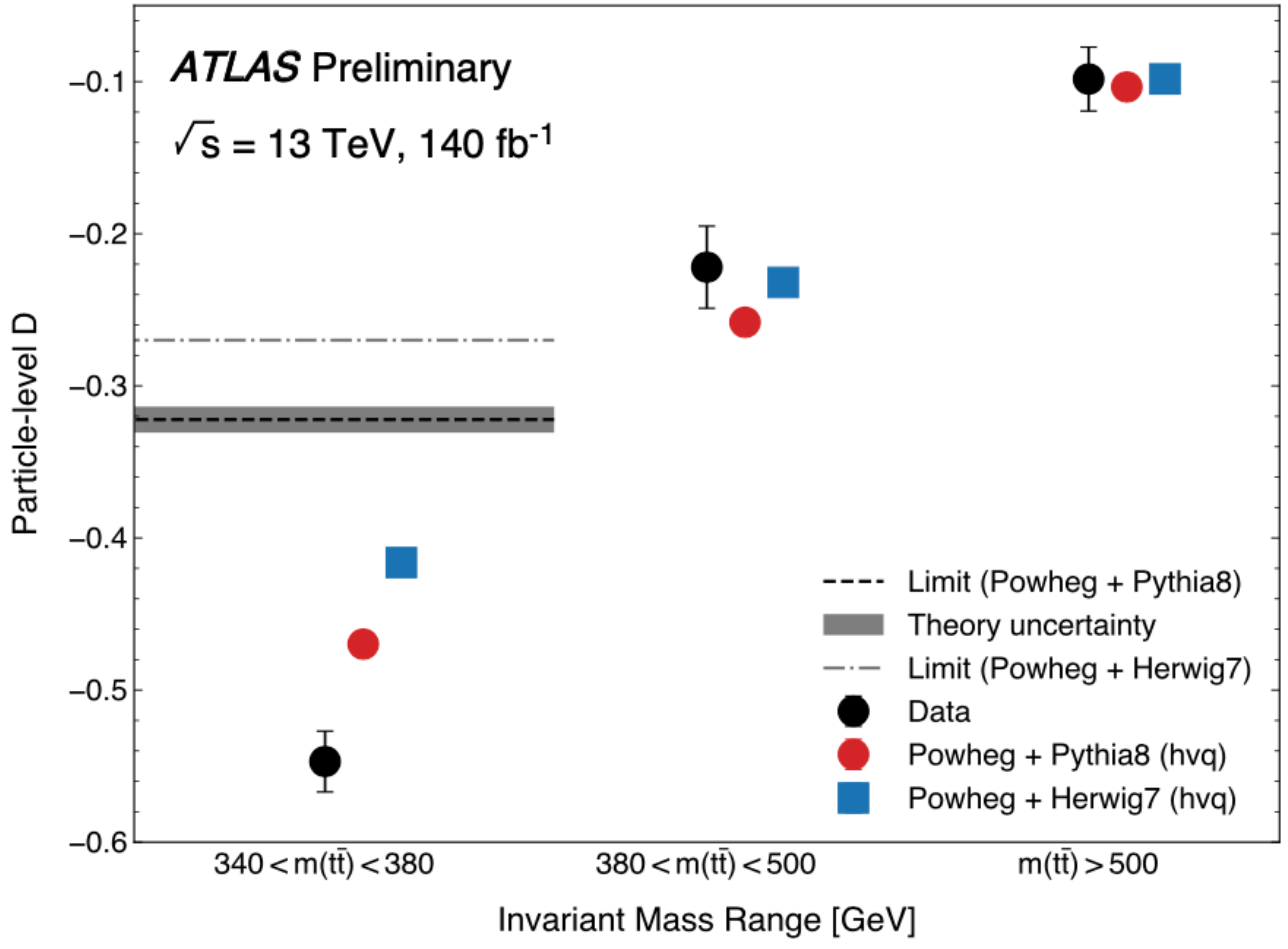
Region	Selection	Cross section	$\lvert C_{kk}+C_{rr}\rvert - C_{nn}$ Reconstructed	$\lvert C_{kk}+C_{rr}\rvert - C_{nn}$ Significance for > 1
Threshold	Weak	14 pb	1.31 ± 0.02	$\gg 5\sigma$
	Intermediate	12 pb	1.34 ± 0.02	$\gg 5\sigma$
	Strong	10 pb	1.38 ± 0.02	$\gg 5\sigma$
High-p_T	Weak	1.9 pb	1.32 ± 0.07	5σ
	Intermediate	1.5 pb	1.36 ± 0.08	4σ
	Strong	1.0 pb	1.42 ± 0.13	3σ

C. Severi, C. Boschi, **FM**, M. Sioli : 2110.10112

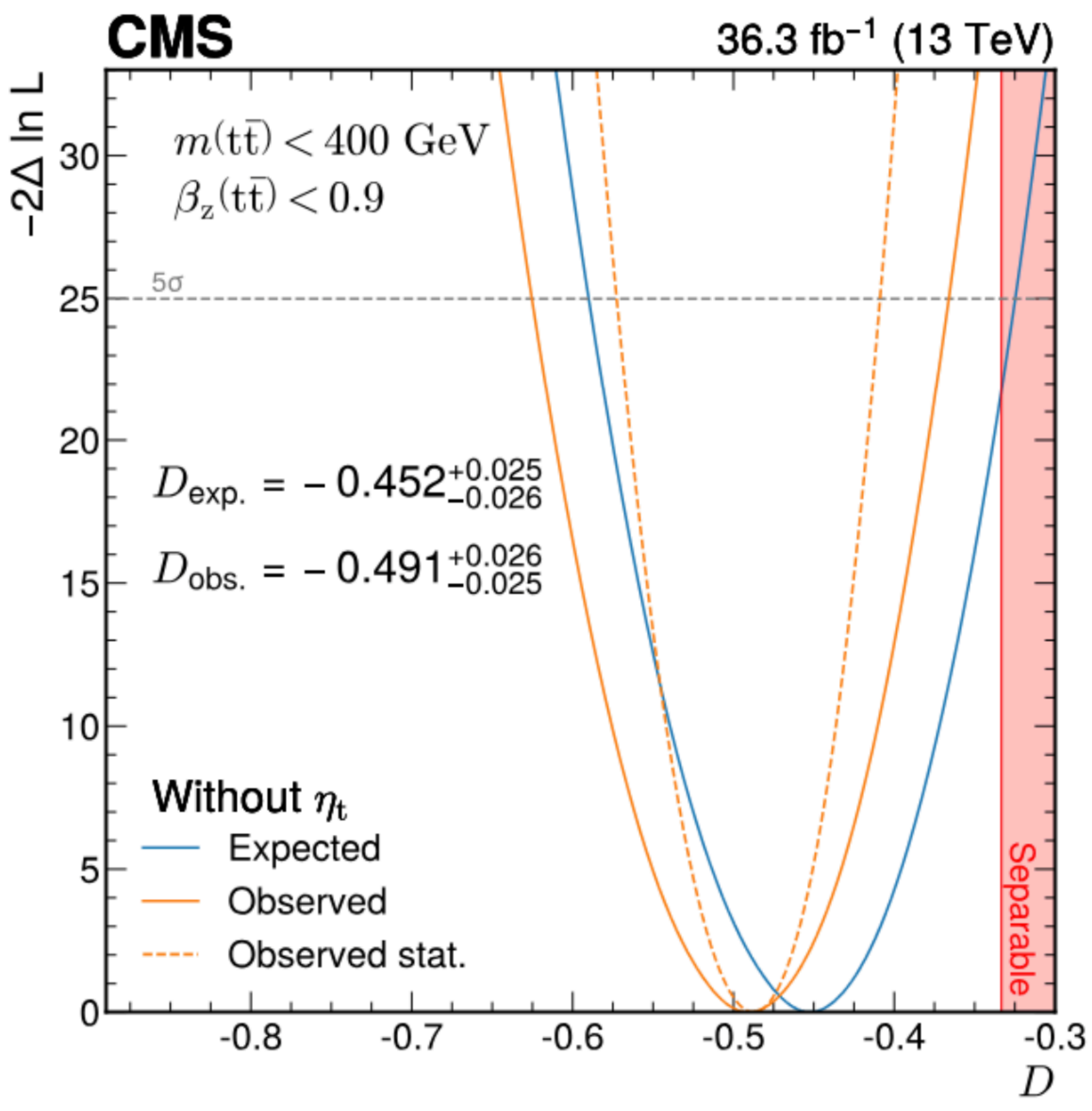


First measurements

Dilepton channel at threshold



ATLAS-CONF-2023-069



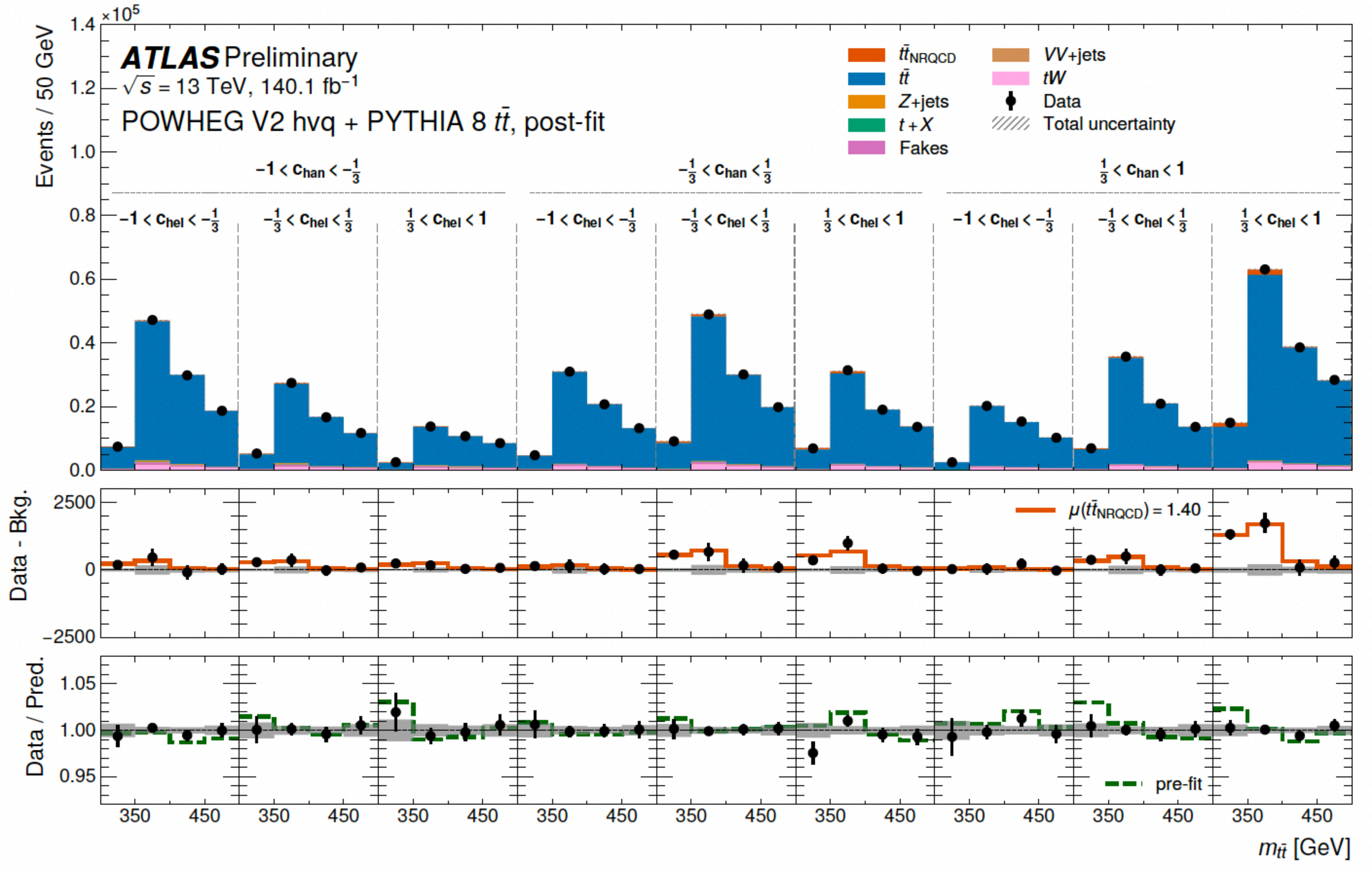
TOP-23-001-pas

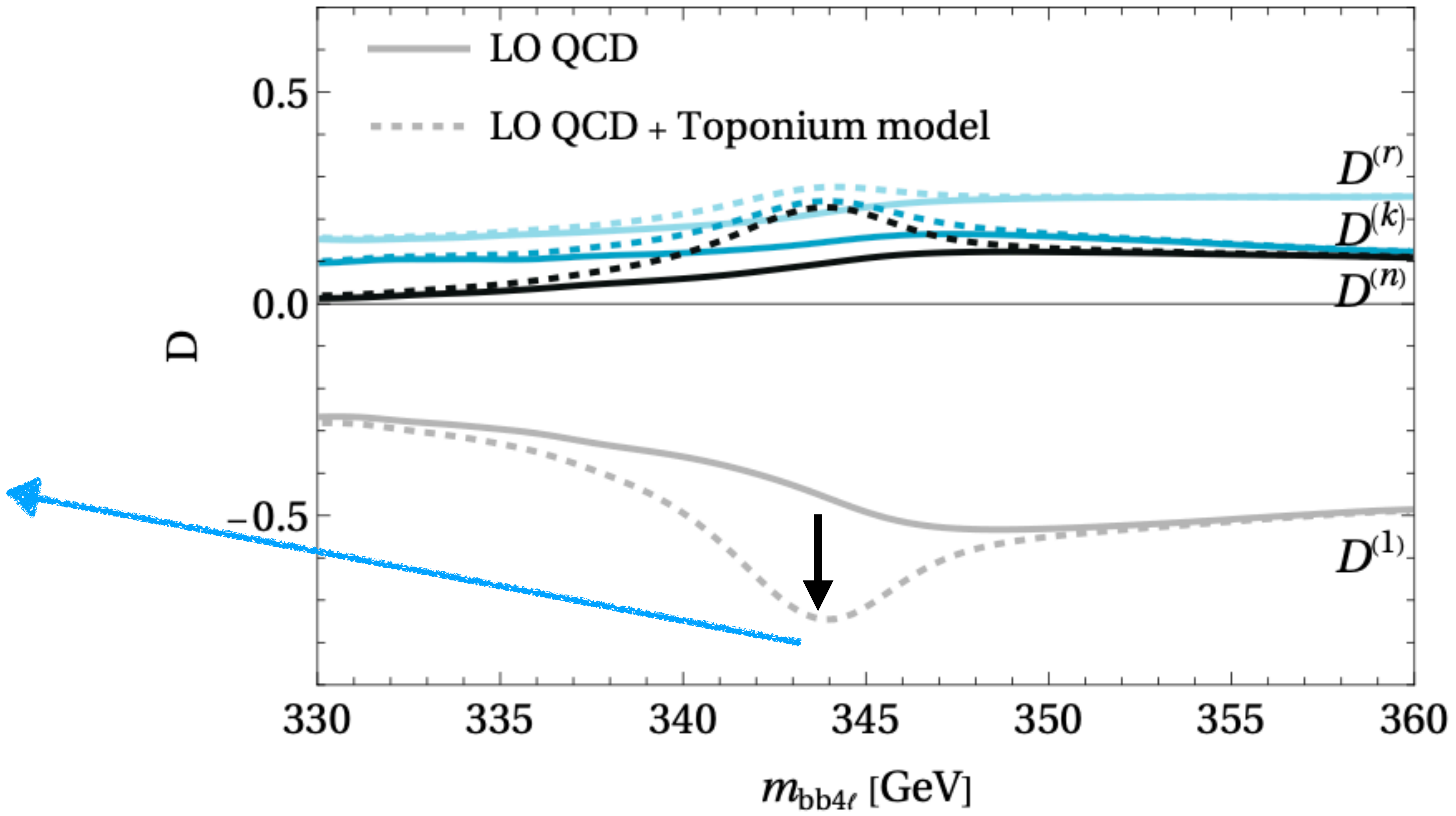
Entanglement observation by ATLAS

Entanglement observation by CMS

First measurements

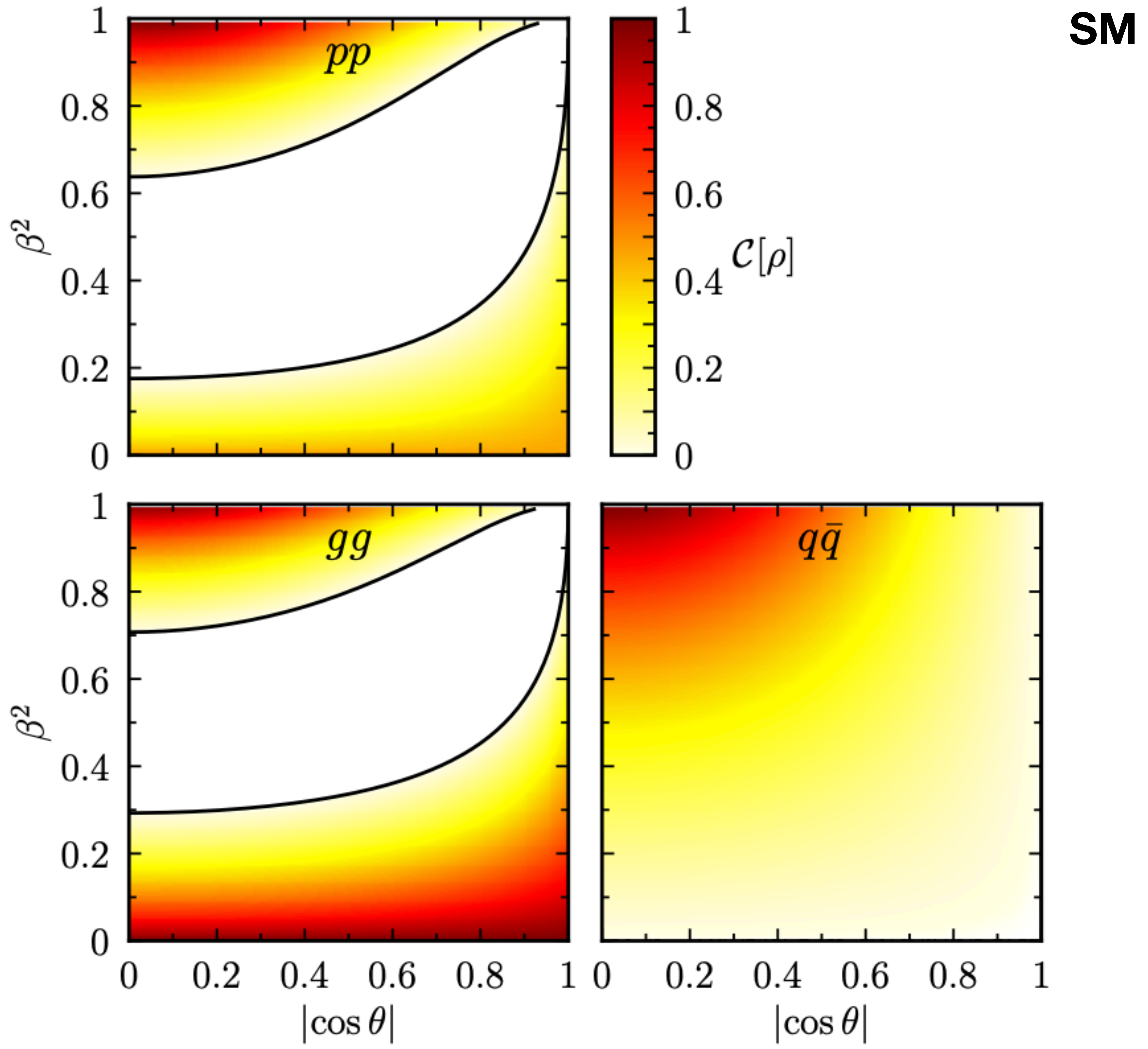
Threshold enhancement (aka toponium)

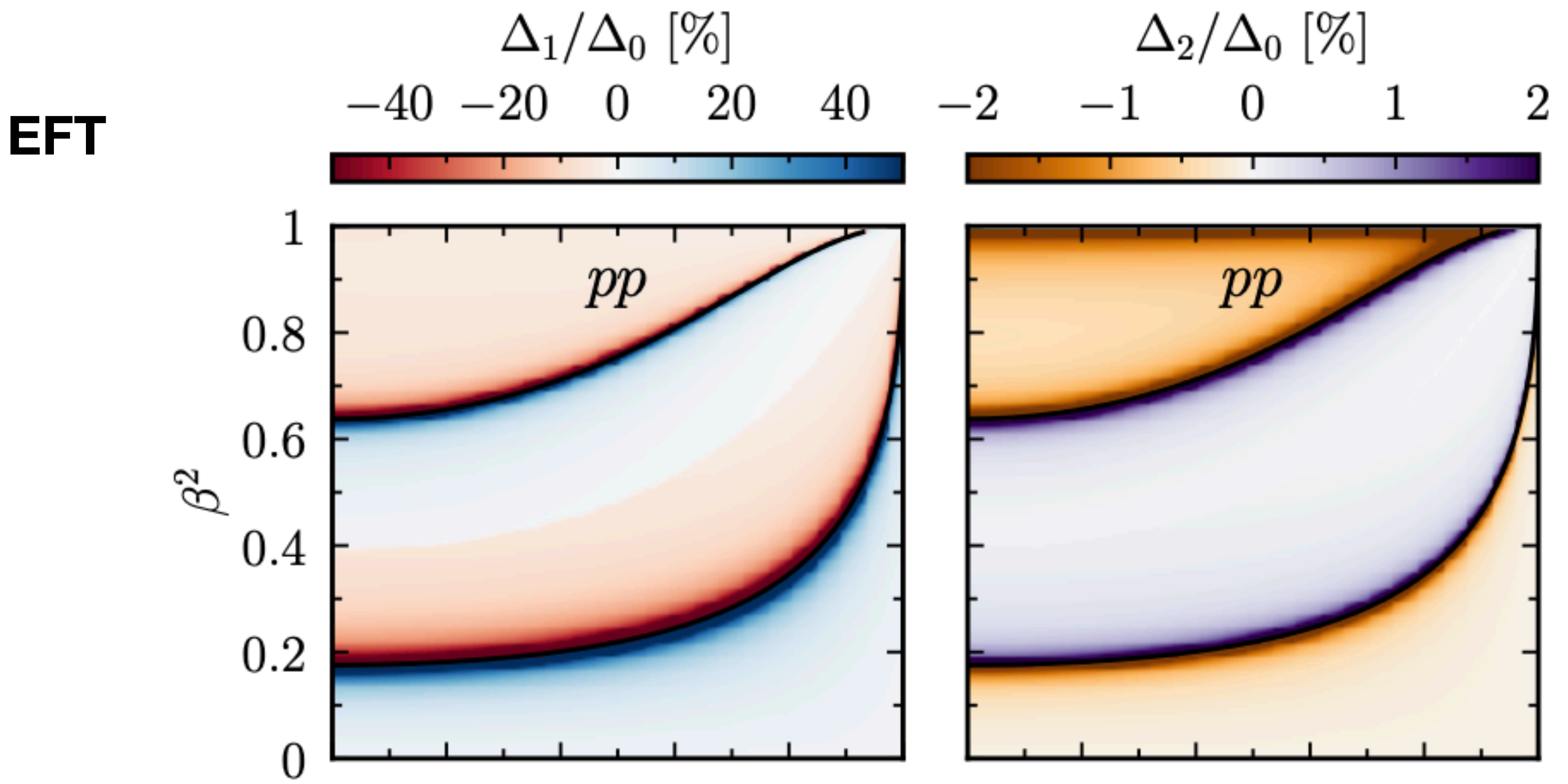




As the story unfolded, this can be considered a spin-off result of using quantum observables to study uncharted SM physics.

Quantum Advantage for SMEFT : Entanglement





- New interactions modify both conventional and quantum observables
- Dimension-6 operators can modify the degree of entanglement between top quarks
- SMEFT introduce new structures, thus probing new linear combinations between coefficients
- QI observables can break degeneracies between operators when combined with standard observables

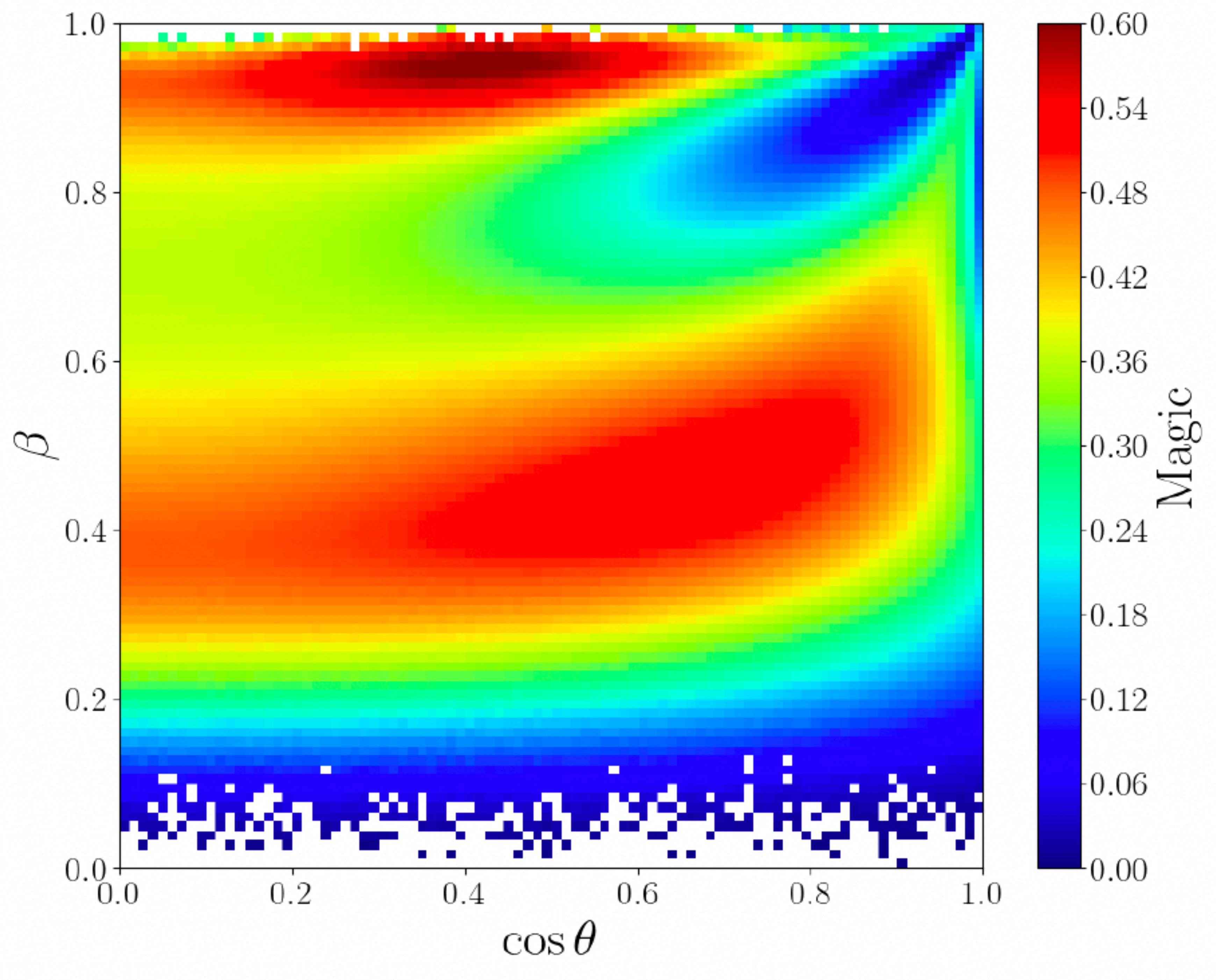
White^2 2406.07321

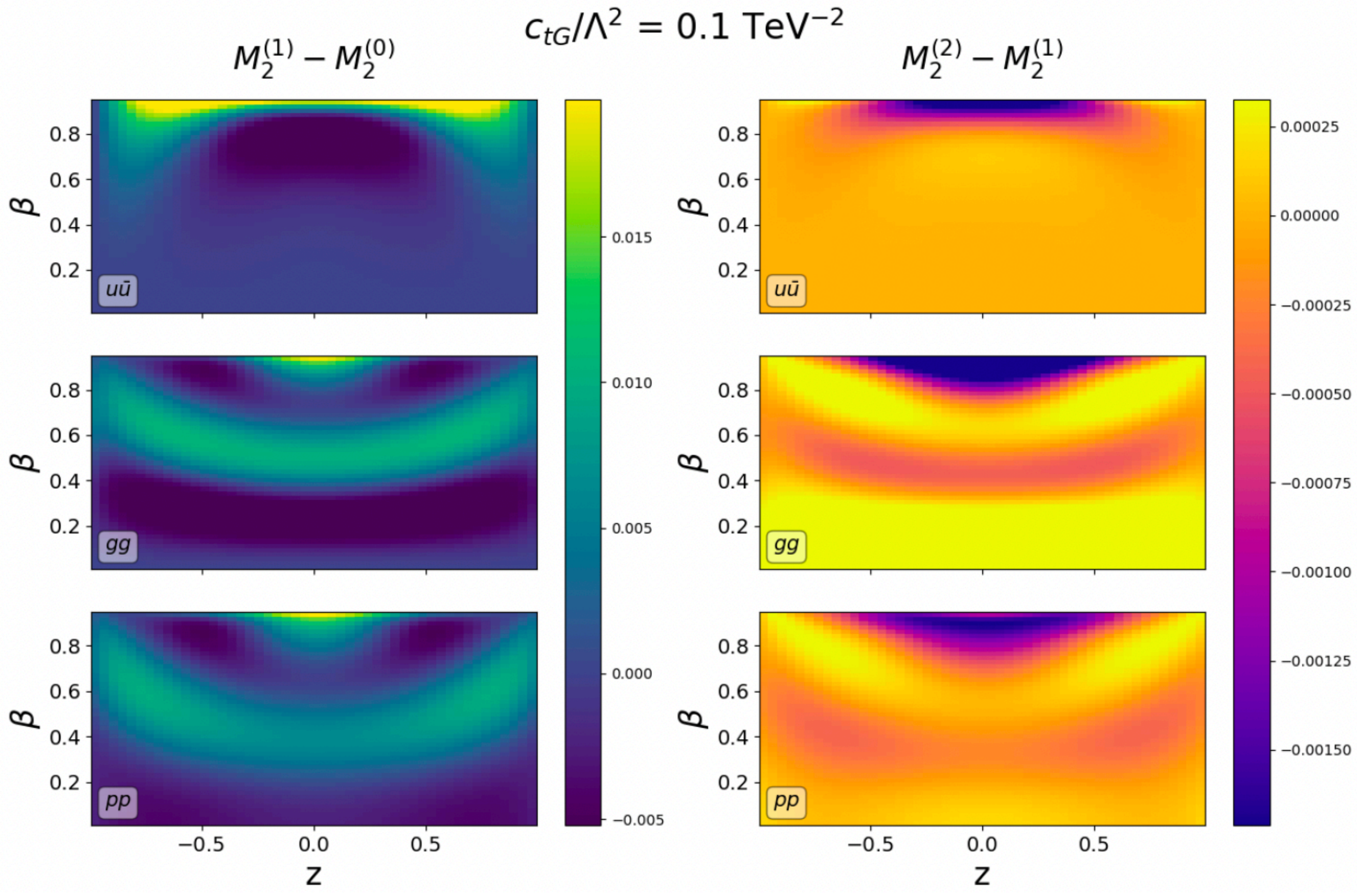
Aoude, Banks, White^2 2505.12522

Quantum Advantage for SMEFT : Magic

In quantum information, **magic** refers to the amount of *non-Clifford, non-stabilizer* structure present in a quantum state or operation. It quantifies **how far** a state is from the set of states that can be efficiently simulated classically via stabilizer methods (Gottesman–Knill).

SM | **EFT**



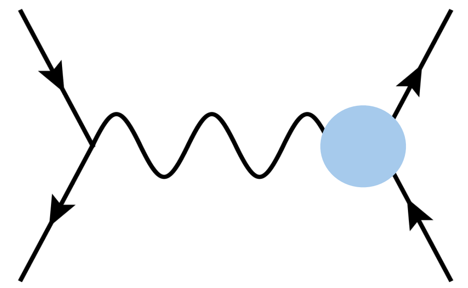


Lamba, FM, Miniati, Vryonidou , WIP

Quantum Advantage for SMEFT : Magic

Production at a lepton collider

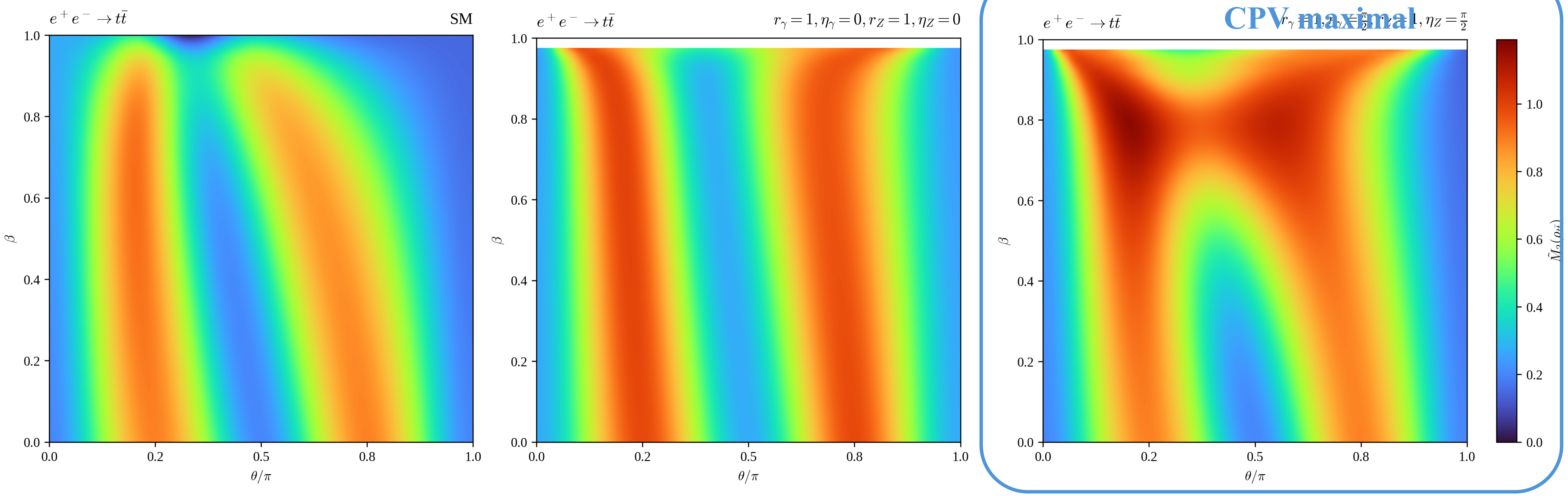
- 4-fermion CP-odd operators → no CPV effects.
- O_{uW}, O_{uB} → magnetic and electric dipole moment form factors:

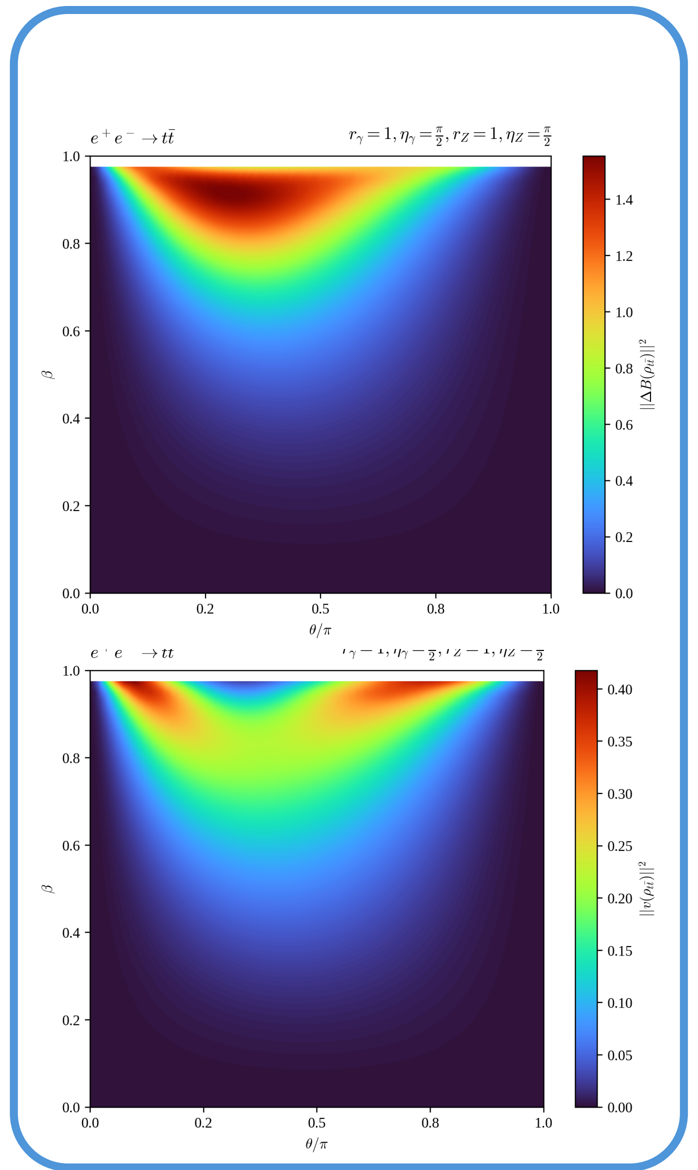


$$d_Z = r_Z(\cos\eta_Z + i\sin\eta_Z)$$
$$d_\gamma = r_\gamma(\cos\eta_\gamma + i\sin\eta_\gamma)$$

$$t\bar{t}V^\mu = ie\gamma^\mu(\bar{g}_V^V - \bar{g}_V^A\gamma^5) - \frac{v}{\Lambda^2}\sigma^{\mu\nu}q_\nu(d_V P_R + d_V^* P_L)$$

- $t\bar{t}$ individually polarized and correlated.

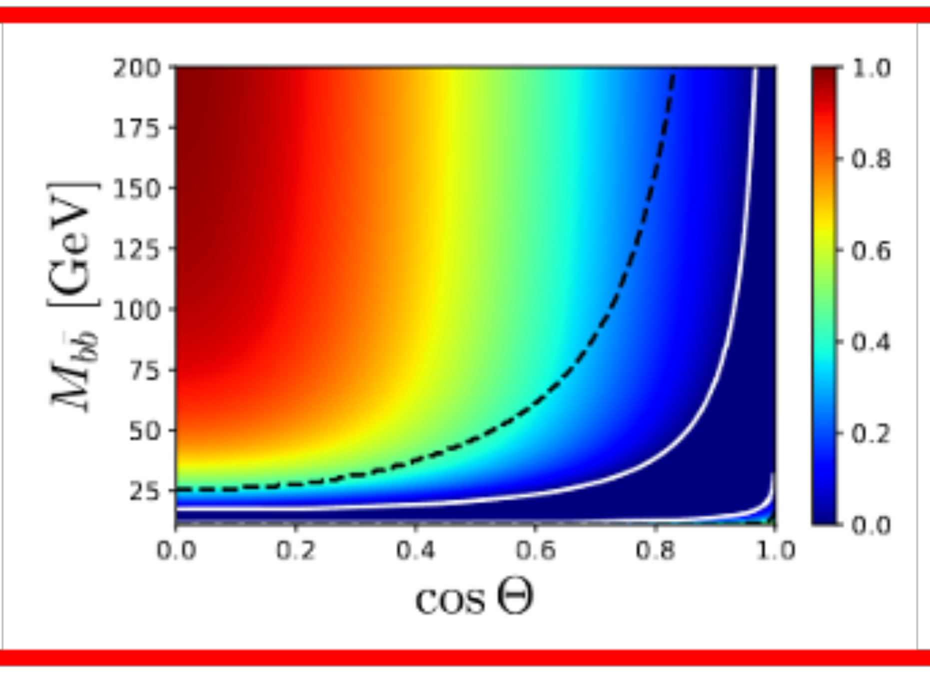




Many on-going studies…

Several proposal exist for measurement at HL-LHC:

- Measurement of entanglement between two quark b from Z decay
  - Exploit the propagation of polarization to baryon final state
  - Reconstruct the b spin from the charged lepton in the Λ_b decay chain

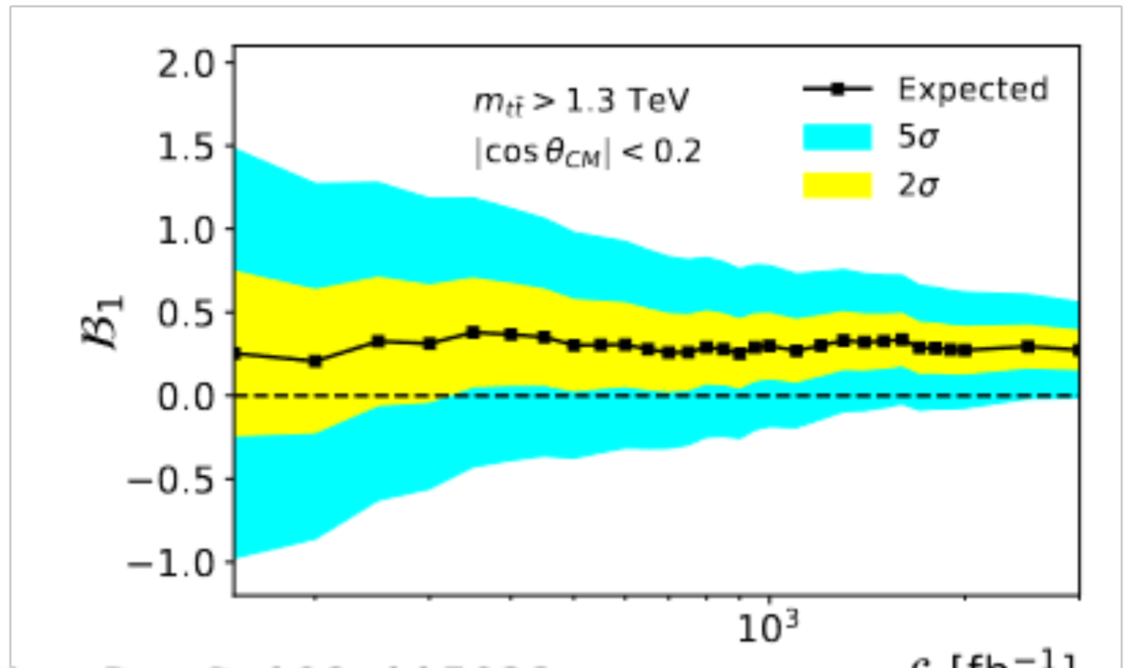


- Observation of entanglement and Bell's inequality violation in H→VV
  - Orthogonal final state compared to top-pair
  - Qutrit highly entangled across the whole phase space
  - Current main limitation is stat in ZZ final state and neutrino reconstruction in WW final state

- Measurement of entanglement in non-resonant di-boson final states
  - High potential for new physics constraints

- Measurement of the post-decay entanglement *Phys. Rev. D 109, 096027*
  - Study the evolution of entanglement after the decay of one of the particles
  - Proposed in top pair production, between the top and the W
  - **In lepton colliders with polarised beams could also be possible to observe an increase in the entanglement compared to top-pair**

- Observation of BIV violation in top-pair production



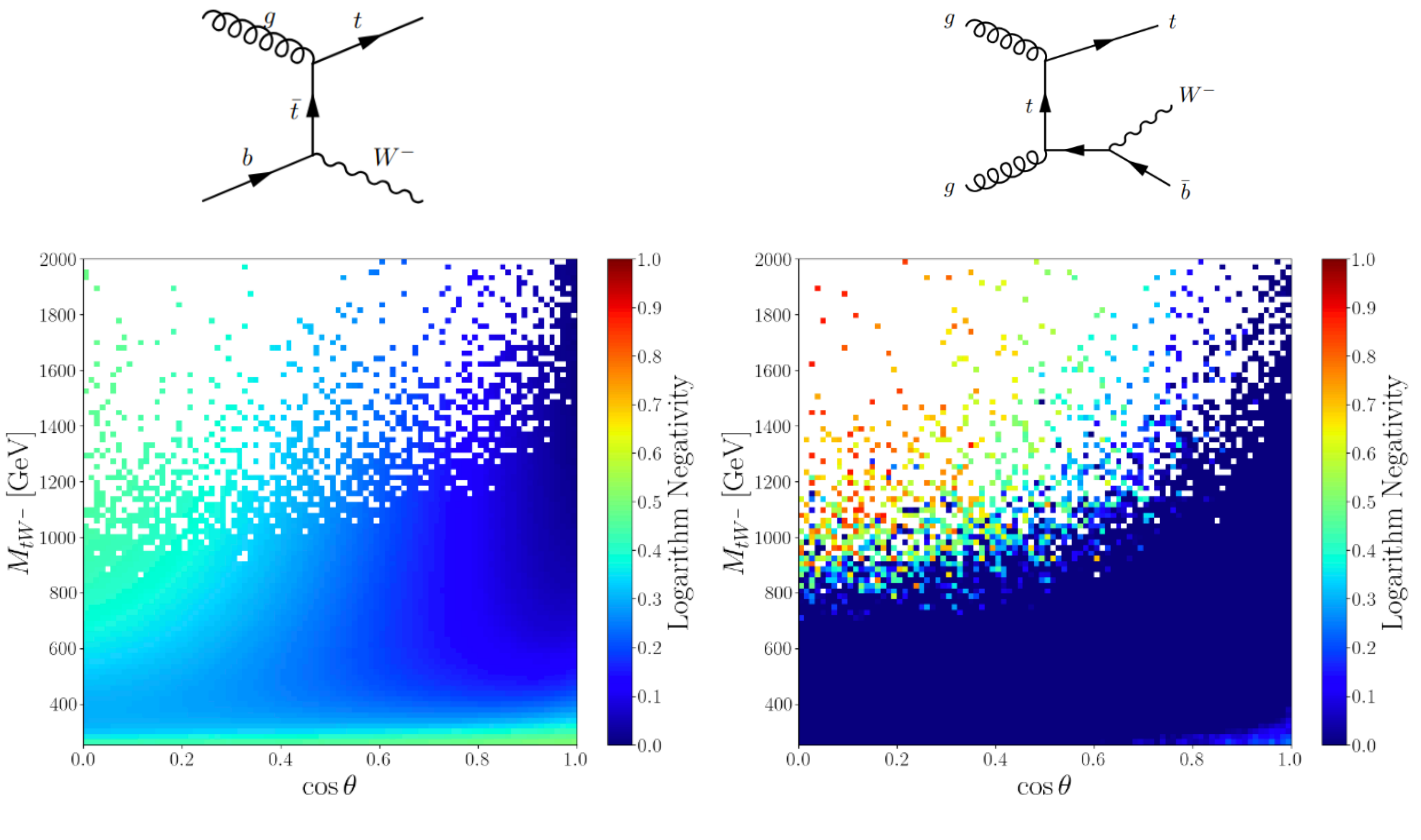
Phys. Rev. D 109, 115023

- Multibody entanglement:
  - Across multiple qubit in the final state
  - Between particles and the momentum of the system

- Several proposal based on mesons entanglement
  - Both in terms of flavour and spin
  - Flavour studies suggested to test different decoherence models

Durupt, FM, Mattelaer 2510.17730

Automatic ρ computation for colliders



(a) (b)

Recent release of MadSpinQ, that allows to compute spin density matrices even-by-event at the tree level. Wide set of sample applications.

Quantum observables can be used to discriminate among different production mechanism.

Quantum observables offer an additional discriminating power.

Today's question

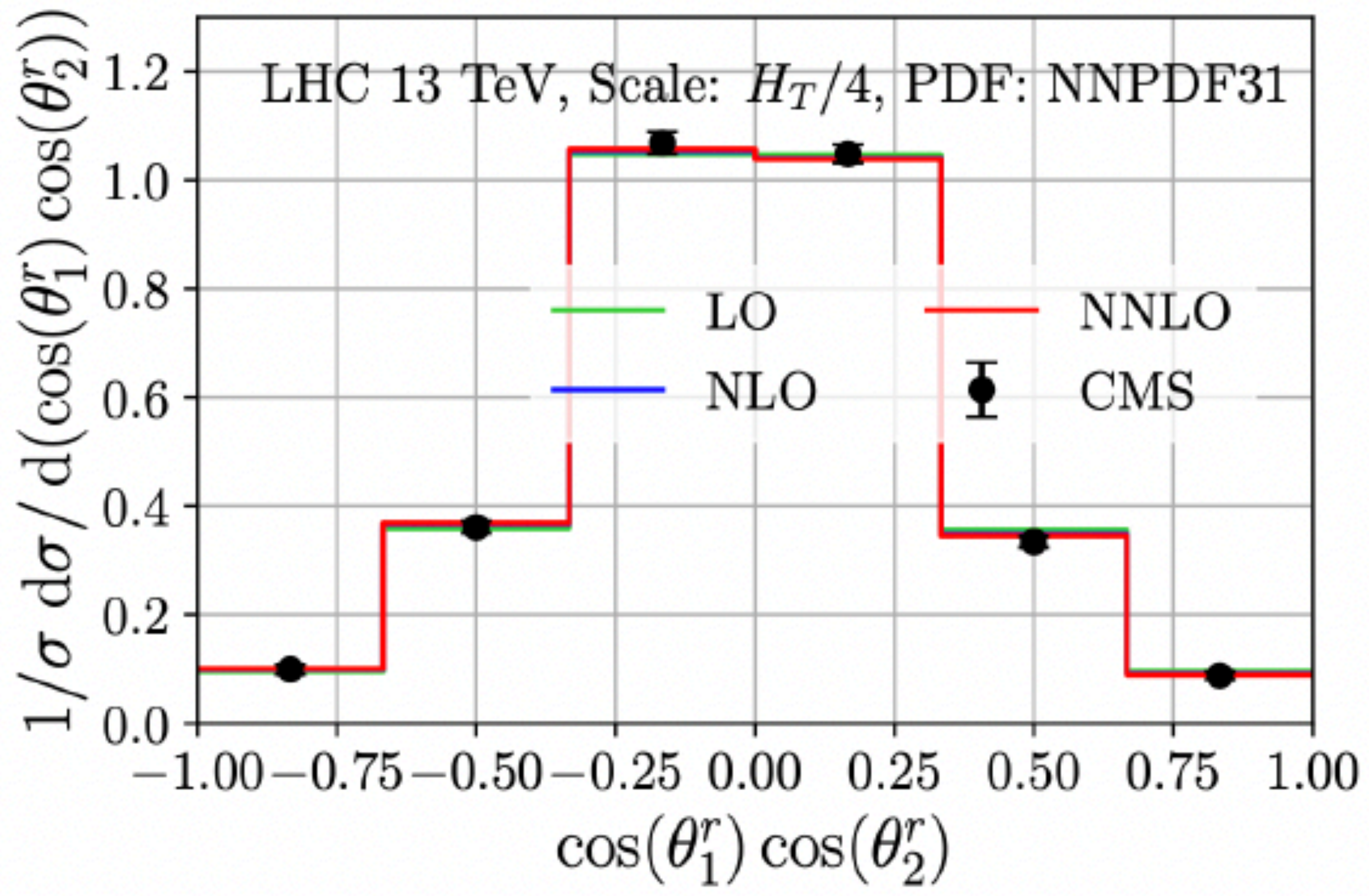
How do higher-order corrections change the naive leading-order picture?

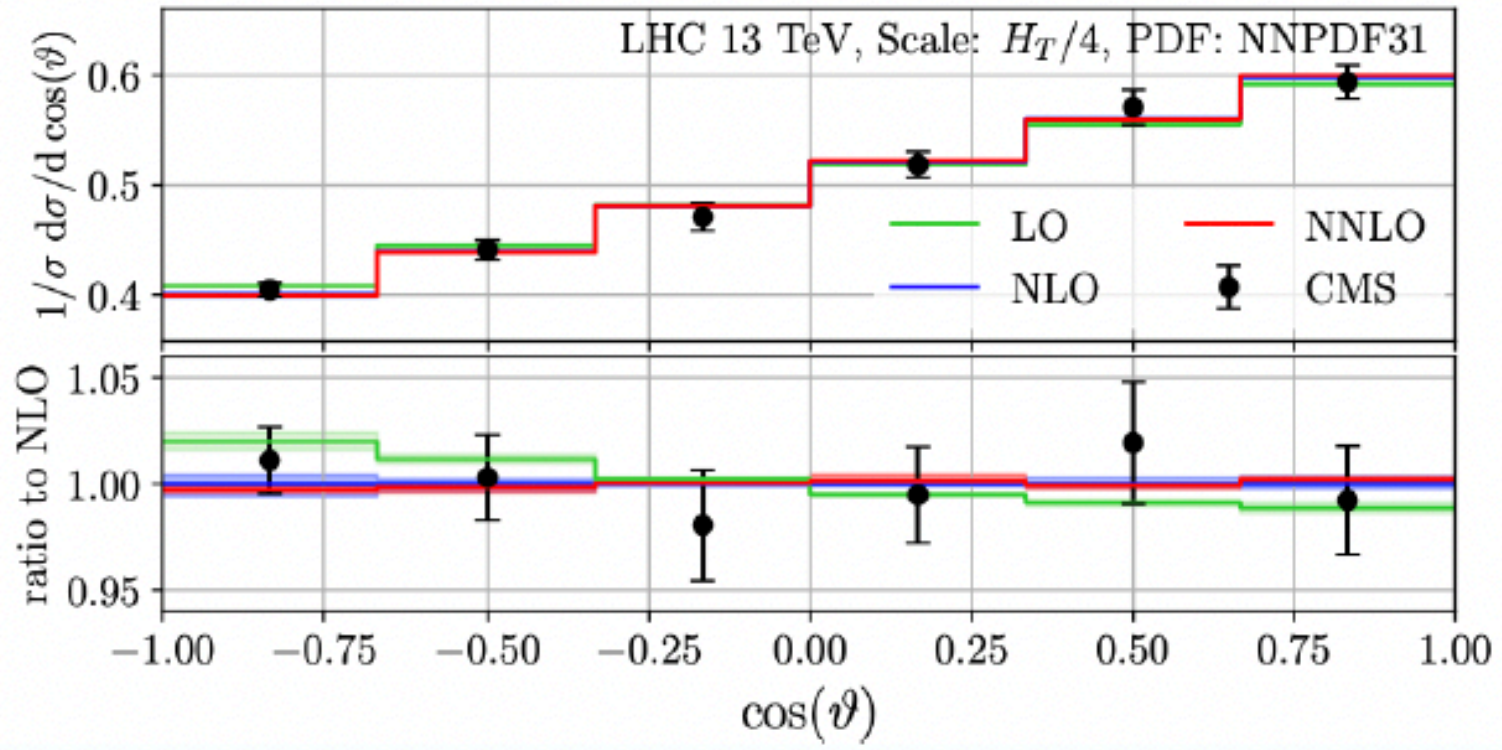
Czakon, Mitov, Poncelet, 2008.11133

The “simplest" case

$t\bar{t}$ production at NNLO in QCD

Coefficient	LO ($\times 10^3$)	NLO ($\times 10^3$)	NNLO ($\times 10^3$)	CMS ($\times 10^3$)
B_1^k	1^{+0}_{-0} [sc] ± 1 [mc]	1^{+0}_{-1} [sc] ± 2 [mc]	-1^{+0}_{-1} [sc] ± 4 [mc]	5 ± 23
B_1^r	0^{+0}_{-0} [sc] ± 1 [mc]	0^{+1}_{-0} [sc] ± 2 [mc]	0^{+1}_{-2} [sc] ± 2 [mc]	-23 ± 17
B_1^n	0^{+0}_{-0} [sc] ± 1 [mc]	3^{+1}_{-1} [sc] ± 1 [mc]	4^{+1}_{-0} [sc] ± 3 [mc]	6 ± 13
B_2^k	0^{+0}_{-0} [sc] ± 1 [mc]	0^{+0}_{-1} [sc] ± 1 [mc]	-5^{+2}_{-3} [sc] ± 3 [mc]	7 ± 23
B_2^r	0^{+0}_{-0} [sc] ± 1 [mc]	0^{+2}_{-0} [sc] ± 1 [mc]	-2^{+0}_{-1} [sc] ± 2 [mc]	-10 ± 20
B_2^n	0^{+0}_{-0} [sc] ± 1 [mc]	-2^{+0}_{-1} [sc] ± 1 [mc]	-3^{+1}_{-0} [sc] ± 3 [mc]	17 ± 13
C_{kk}	324^{+7}_{-7} [sc] ± 1 [mc]	330^{+2}_{-2} [sc] ± 3 [mc]	323^{+2}_{-5} [sc] ± 6 [mc]	300 ± 38
C_{rr}	6^{+5}_{-5} [sc] ± 1 [mc]	58^{+18}_{-12} [sc] ± 2 [mc]	69^{+8}_{-7} [sc] ± 3 [mc]	81 ± 32
C_{nn}	332^{+1}_{-0} [sc] ± 1 [mc]	330^{+1}_{-1} [sc] ± 2 [mc]	326^{+1}_{-1} [sc] ± 4 [mc]	329 ± 20
$C_{nr} + C_{rn}$	1^{+0}_{-0} [sc] ± 1 [mc]	-1^{+1}_{-0} [sc] ± 3 [mc]	-4^{+4}_{-0} [sc] ± 6 [mc]	-4 ± 37
$C_{nr} - C_{rn}$	0^{+0}_{-1} [sc] ± 1 [mc]	-1^{+1}_{-0} [sc] ± 2 [mc]	2^{+4}_{-2} [sc] ± 8 [mc]	-1 ± 38
$C_{nk} + C_{kn}$	0^{+0}_{-0} [sc] ± 1 [mc]	2^{+1}_{-0} [sc] ± 1 [mc]	3^{+4}_{-1} [sc] ± 3 [mc]	-43 ± 41
$C_{nk} - C_{kn}$	1^{+0}_{-0} [sc] ± 1 [mc]	1^{+1}_{-1} [sc] ± 2 [mc]	6^{+0}_{-2} [sc] ± 7 [mc]	40 ± 29
$C_{rk} + C_{kr}$	-229^{+4}_{-4} [sc] ± 1 [mc]	-203^{+9}_{-7} [sc] ± 2 [mc]	-194^{+8}_{-6} [sc] ± 7 [mc]	-193 ± 64
$C_{rk} - C_{kr}$	1^{+0}_{-0} [sc] ± 1 [mc]	1^{+0}_{-1} [sc] ± 4 [mc]	-1^{+1}_{-3} [sc] ± 5 [mc]	57 ± 46





- Perturbative series (QCD) under control
- Very small corrections at the inclusive level
- NNLO results within scale uncertainties of NLO results
- Agreement with CMS measurement

Today's question

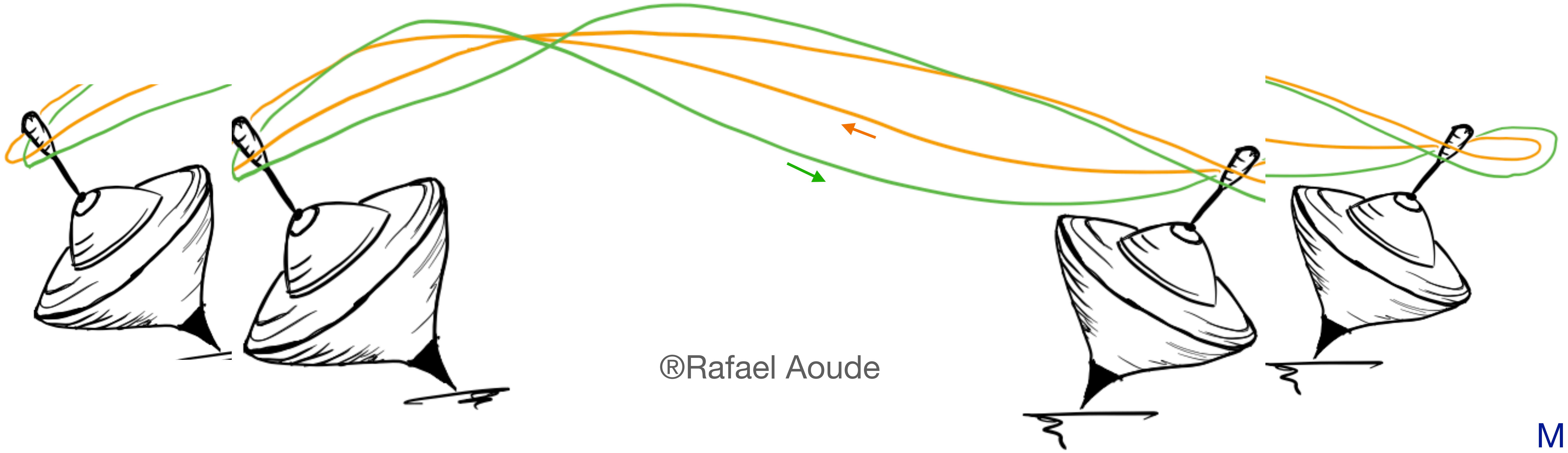
How do higher-order corrections change the naive leading-order picture?

1. Are quantum correlations modified? How is the tomography affected? Is the reconstructed ρ well-defined? $H \to VV$

2. Is there a way to re-formulate higher-order effects in terms of QI concepts? $H \to t\bar{t}$

Quantum W's and Z's

A. J. Barr, 2106.01377
A. J. Barr, P. Caban, J. Rembieliński, 2204.11063
R. Ashby-Pickering, A. J. Barr, A. Wierzchucka, 2209.13990
J. A. Aguilar-Saavedra, A. Bernal, J. A. Casas, J. M. Moreno, 2209.13441
R. A. Morales, 2306.17247
J. A. Aguilar-Saavedra, 2307.06991
R. Aoude, E. Madge, FM, L. Mantani, 2307.09675
M. Fabbrichesi, R. Floreanini, E. Gabrielli, L. Marzola, 2302.00683
M. Fabbrichesi, R. Floreanini, E. Gabrielli, L. Marzola, 2304.02403
A. Subba, R. K. Singh, R. M. Godbole, 2411.19171
A. Bernal, P. Caban, J. Rembieliński, 2405.16525
M. Sullivan, 2410.10980
Q. Bi, Q.-H. Cao, K. Cheng, H. Zhang, 2307.14895
A. Bernal, P. Caban, J. Rembieliński, 2307.13496
F. Fabbri, J. Howarth, T. Maurin, 2307.13783
M. Grossi, G. Pelliccioli, A. Vicini, 2409.16731
J. A. Aguilar-Saavedra, 2411.13464
Y. Wu, R. Jiang, A. Ruzi, Y. Ban, X. Yan, Q. Li, 2410.17025
A. Ruzi, Y. Wu, R. Ding, S. Qian, A. M. Levin, Q. Li, 2408.05429
R. Ding, A. Ruzi, S. Qian, A. Levin, Y. Wu, Q. Li, 2504.09832
M. Fabbrichesi, R. Floreanini, E. Gabrielli, L. Marzola, 2503.14587
Del Gratta, Fabbri, Lamba, FM, Pagani, 2504.03841
Goncalves, Kaladharan, Krauss, Navarro, 2505.12125
Goncalves, Kaladharan, Navarro 2506.19951
Del Gratta, Fabbri, Grossi, FM, Pelliccioli, Pagani, Vicini, 2509.20456



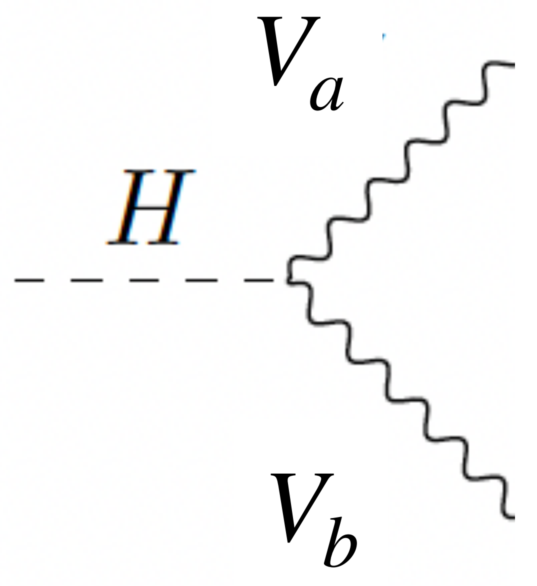
Qutrit **Qutrit**

Caution: possibly incomplete list!

$H \to V_a V_b$

Qutrit-qutrit $H \to V_a V_b$ production at present and future colliders studies in the SM and BSM, many papers in the last years. In-depth study of the process $H \to V_a V_b$ performed by several groups.

$$|\psi\rangle = a_L |0\,0\rangle + a_T \frac{|+-\rangle + |-+\rangle}{\sqrt{2}}$$



$$a_L = \frac{-\beta}{\sqrt{2+\beta^2}}, \qquad a_T = \frac{\sqrt{2}}{\sqrt{2+\beta^2}}, \qquad \beta = 1 + \frac{m_H^2 - (m_a + m_b)^2}{2 m_a m_b}$$

$H \to V_a V_b$

Qutrit-qutrit $H \to V_a V_b$ production at present and future colliders studies in the SM and BSM, many papers in the last years. In-depth study of the process $H \to V_a V_b$ performed by several groups.

$$\rho = \frac{1}{9}\left[\mathbf{1}_3 \otimes \mathbf{1}_3 + A^{(1)}_{lm}(T_{lm} \otimes \mathbf{1}_3) + A^{(2)}_{lm}(\mathbf{1}_3 \otimes T_{lm}) + C_{lml'm'}(T_{lm} \otimes T_{l'm'})\right].$$

$$\rho_{\rm LO}(\beta) = \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \frac{a_T^2}{2} & \cdot & \frac{a_L a_T}{\sqrt{2}} & \cdot & \frac{a_T^2}{2} & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \frac{a_L a_T}{\sqrt{2}} & \cdot & a_L^2 & \cdot & \frac{a_L a_T}{\sqrt{2}} & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \frac{a_T^2}{2} & \cdot & \frac{a_L a_T}{\sqrt{2}} & \cdot & \frac{a_T^2}{2} & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix}$$

$$\rho_{\rm LO} = \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & x & \cdot & y & \cdot & x & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & y & \cdot & 1-2x & \cdot & y & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & x & \cdot & y & \cdot & x & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix}$$

$$\rho_{\rm LO} = \int \rho_{\rm LO}(\beta) w(\beta) d\beta$$

$V \to 2f$

In practice, the qutrits are reconstructed via the tomography. Let's assume fermions can be detected and identified and first consider one vector boson:

$$\frac{4\pi}{3}\frac{1}{\sigma}\frac{\mathrm{d}^2\sigma}{\mathrm{d}\cos\theta\,\mathrm{d}\phi} = \sum_{\lambda,\lambda'} \rho_{\lambda,\lambda'}\Gamma_{\lambda,\lambda'}(\theta,\phi) \quad \hat{=} \quad \frac{1}{4\pi} + \sum_{l=1}^{2}\sum_{m=-l}^{l} \alpha_{lm} Y_{lm}(\theta,\phi)$$

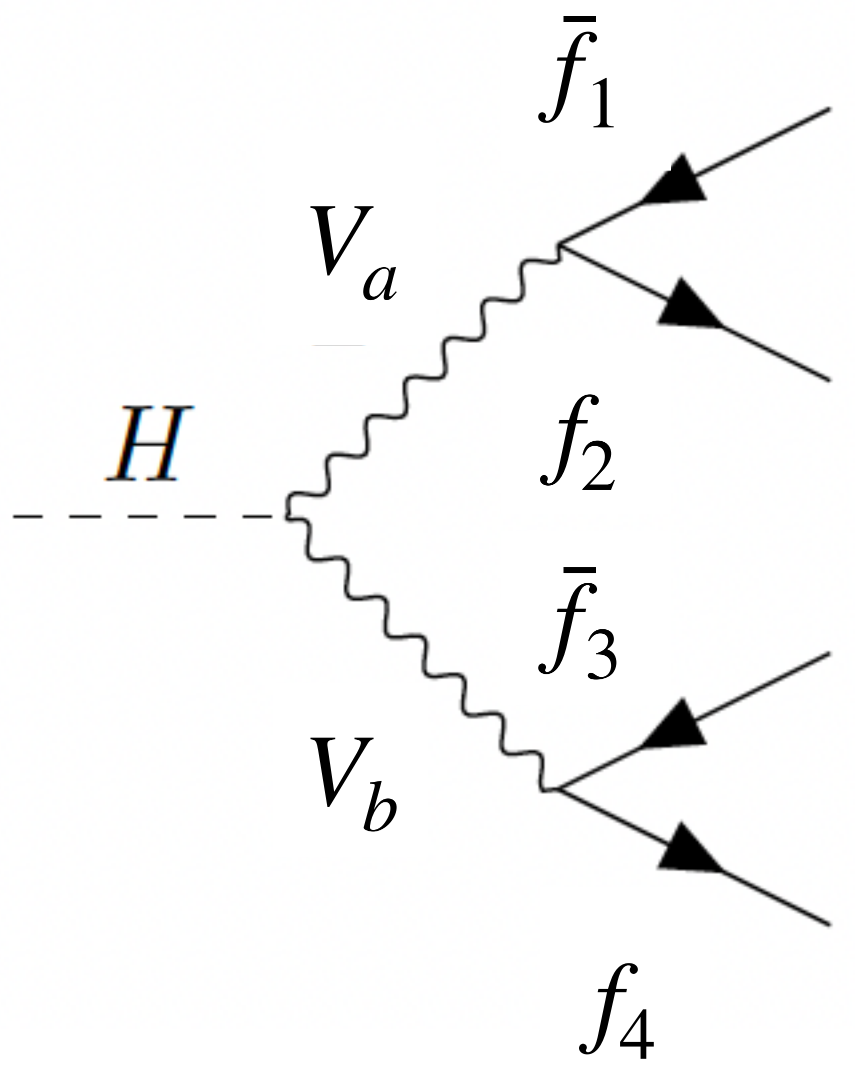
$$\rho = \frac{1}{3}\left[\mathbf{1}_3 + A_{lm}T_{lm}\right] \qquad \Gamma = \frac{1}{3}\left[\mathbf{1}_3 + \sqrt{2\pi}\,\eta_\ell\, T_{1m}Y_{1m} + \sqrt{\frac{2\pi}{5}}\,T_{2m}Y_{2m}\right]$$

$$\sqrt{8\pi}\alpha_{1m} = \eta_\ell\, A_{1m}$$

$$\sqrt{40\pi}\alpha_{2m} = A_{2m}$$

$$\eta_\ell = \frac{2\, g_{V,\ell}\, g_{A,\ell}}{g_{V,\ell}^2 + g_{A,\ell}^2} = \frac{1 - 4\sin^2\theta_{\mathrm{w}}}{1 - 4\sin^2\theta_{\mathrm{w}} + 8\sin^4\theta_{\mathrm{w}}} = 0.2131\ldots$$

$H \to V_a V_b \to 4f$



For two vector bosons, we can build a correlation matrix:

$$\frac{16\pi^2}{9}\frac{1}{\sigma}\frac{\mathrm{d}^4\sigma}{\mathrm{d}\cos\theta_1\,\mathrm{d}\phi_1\,\mathrm{d}\cos\theta_2\,\mathrm{d}\phi_2} = \sum_{\lambda_1,\lambda_1',\lambda_2,\lambda_2'} \rho_{\lambda_1,\lambda_1',\lambda_2,\lambda_2'}\Gamma_{\lambda_1,\lambda_1'}(\theta_1,\phi_1)\Gamma_{\lambda_2,\lambda_2'}(\theta_2,\phi_2)$$

$$\hat{=} \quad \frac{1}{(4\pi)^2} + \frac{1}{4\pi}\sum_{l=1}^{2}\sum_{m=-l}^{l}\alpha_{lm}^{(1)}Y_{lm}(\theta_1,\phi_1) + \frac{1}{4\pi}\sum_{l=1}^{2}\sum_{m=-l}^{l}\alpha_{lm}^{(2)}Y_{lm}(\theta_2,\phi_2)$$

$$+\sum_{l=1}^{2}\sum_{l'=1}^{2}\sum_{m=-l}^{l}\sum_{m'=-l'}^{l'}\gamma_{lml'm'}Y_{lm}(\theta_1,\phi_1)\,Y_{l'm'}(\theta_2,\phi_2)$$

$$x \equiv \frac{C_{2,2,2,-2}}{3}, \qquad y \equiv \frac{C_{2,1,2,-1}}{3}.$$

$$\rho = \frac{1}{9}\left[\mathbf{1}_3\otimes\mathbf{1}_3 + A_{lm}^{(1)}(T_{lm}\otimes\mathbf{1}_3) + A_{lm}^{(2)}(\mathbf{1}_3\otimes T_{lm}) + C_{lml'm'}(T_{lm}\otimes T_{l'm'})\right].$$

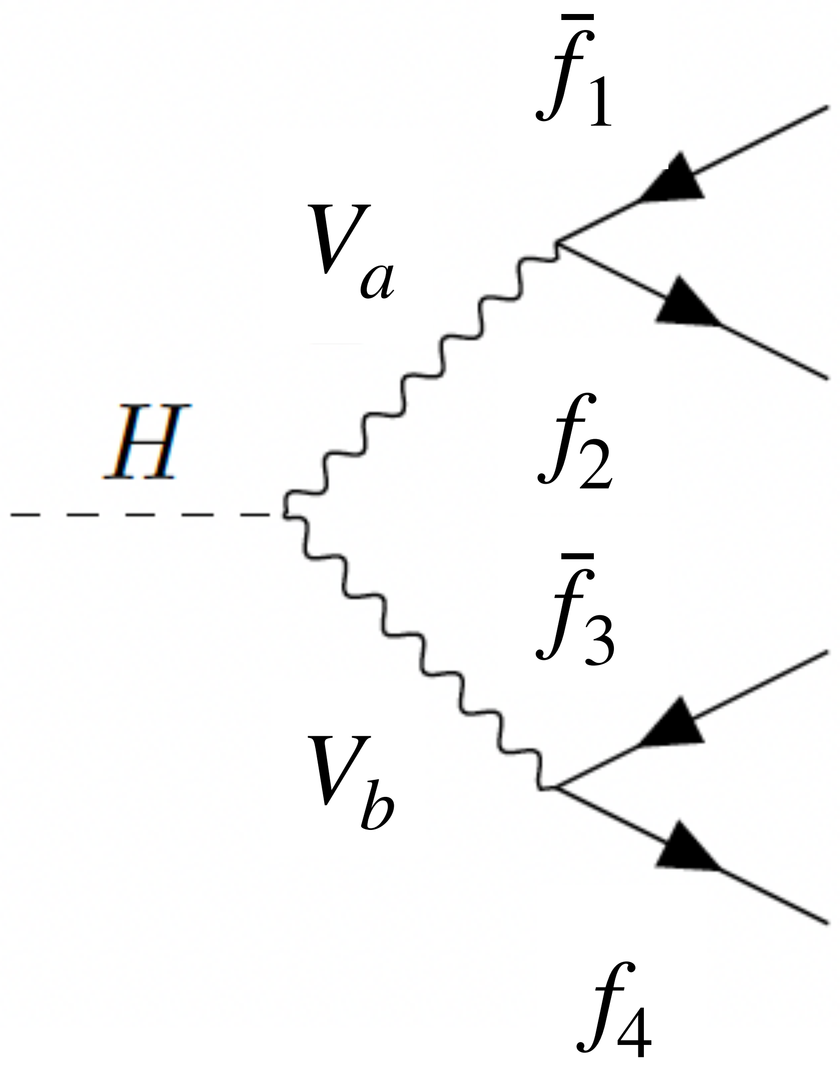
$$C_{2,1,2,-1}\neq 0 \qquad \text{or} \qquad C_{2,2,2,-2}\neq 0\,.$$

$$\sqrt{8\pi}\alpha_{1m} = \eta_\ell\,A_{1m}, \qquad \sqrt{40\pi}\alpha_{2m} = A_{2m}, \qquad 8\pi\gamma_{1m1m'} = \eta_\ell^2\,C_{1m1m'},$$

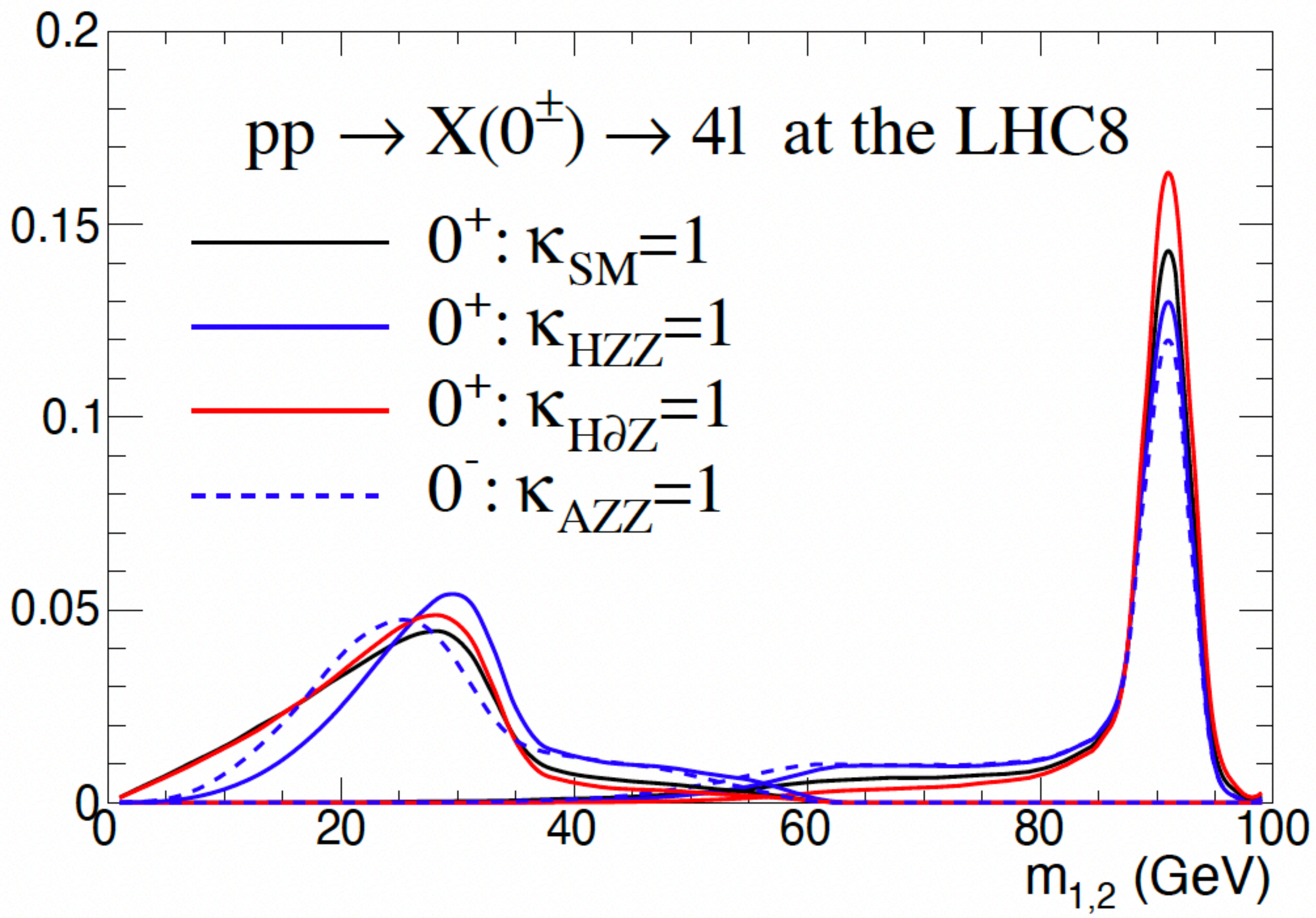
$$40\pi\gamma_{2m2m'} = C_{2m2m'}, \qquad 8\pi\sqrt{5}\gamma_{2m1m'} = \eta_\ell\,C_{2m1m'}, \qquad 8\pi\sqrt{5}\gamma_{1m2m'} = \eta_\ell\,C_{1m2m'}\,.$$

See also J. A. Aguilar-Saavedra, 2411.13464

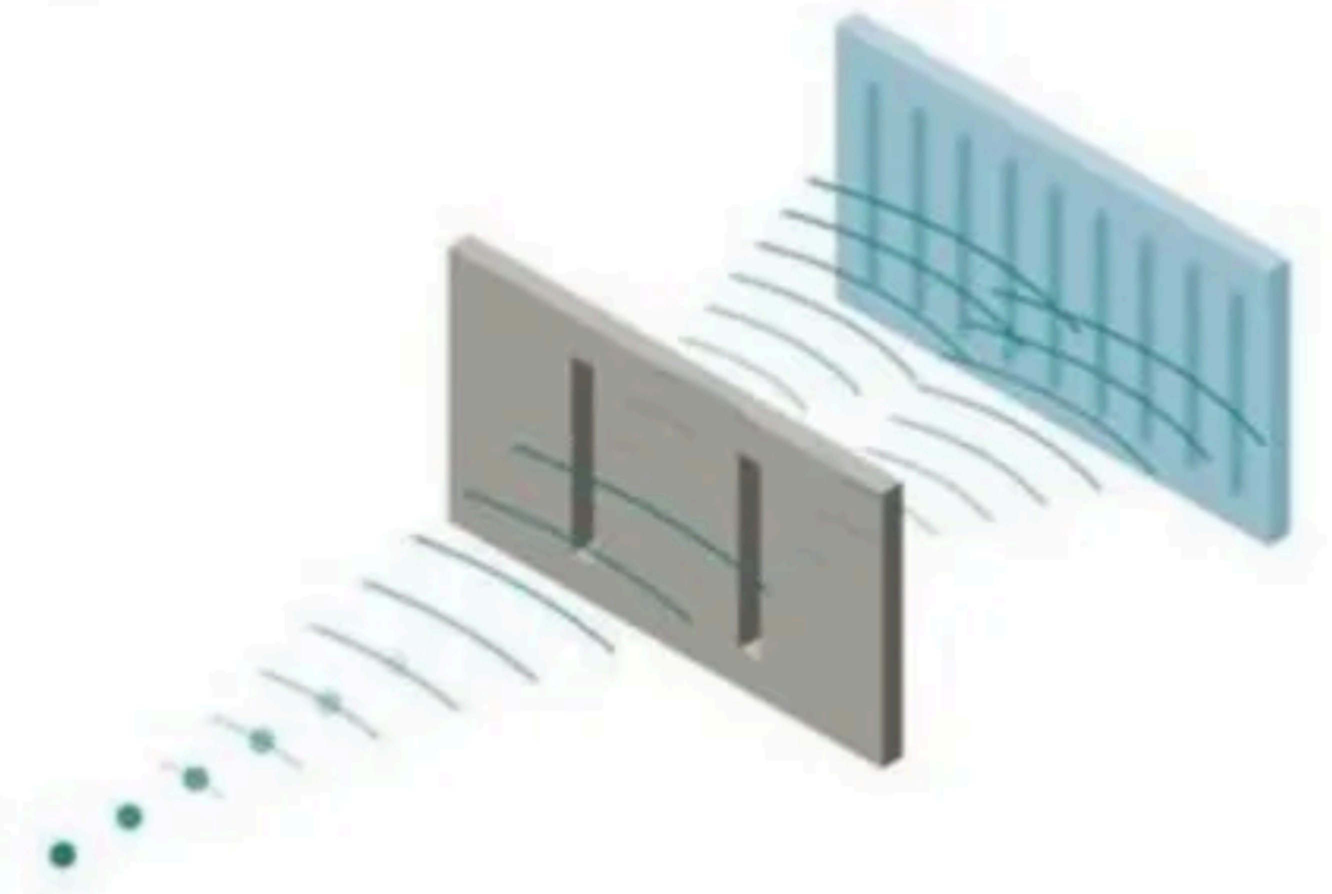
$$H \to V_a^{(*)} V_b^{*} \to 4f$$



In the Higgs decay one pair is always off-shell. The other is typically on shell and this is enough to say who's who. When both are off-shell and close to the maximum value, for identical fermions the system behaves as in a double-slit experiment.



$1/m_a \simeq 1/m_b \simeq 2/m_H$



M. Grossi, G. Pelliccioli, A. Vicini, 2409.16731
Del Gratta, Fabbri, Lamba, FM, Pagani, 2504.03841
Goncalves, Kaladharan, Krauss, Navarro, 2505.12125
Goncalves, Kaladharan, Navarro 2506.19951
Del Gratta, Fabbri, Grossi, FM, Pellicciolli, Pagani, Vicini, 2509.20456

$H \to V_a^{(*)} V_b^{*} \to 4f$ at NLO EW

Automatic tools are available to compute NLO EW. One can just use them and obtain:

	LO	NLO	NLO / LO
$A^1_{2,0}$	$-0.592(1)$	$-0.509(2)$	$0.860(2)$
$A^2_{2,0}$	$-0.591(1)$	$-0.565(2)$	$0.956(2)$
$C_{2,1,2,-1}$	$-0.937(2)$	$-0.943(4)$	$1.006(3)$
$-C_{1,1,1,-1}$	$-0.94(1)$	$-0.16(2)$	$0.17(2)$
$A^1_{2,0}/\sqrt{2}+1$	$0.5817(7)$	$0.640(1)$	$1.101(2)$
$C_{2,2,2,-2}$	$0.581(3)$	$0.568(4)$	$0.977(6)$
$-C_{1,0,1,0}$	$0.59(1)$	$0.03(2)$	$0.06(4)$
$C_{2,0,2,0}$	$1.418(3)$	$1.400(5)$	$0.987(3)$
$C_{1,0,1,0}+2$	$1.41(1)$	$1.97(2)$	$1.39(1)$

$$\rho_{\rm NLO} = \begin{pmatrix} 0.099(4) & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & 0.004(2) & \cdot & 0.131(4) & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & 0.111(4) & \cdot & -0.183(4) & \cdot & 0.189(1) & \cdot & \cdot \\ \cdot & 0.131(4) & \cdot & -0.009(2) & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & -0.183(4) & \cdot & 0.591(1) & \cdot & -0.183(4) & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & -0.009(2) & \cdot & 0.131(4) & \cdot \\ \cdot & \cdot & 0.189(1) & \cdot & -0.183(4) & \cdot & 0.110(3) & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & 0.131(4) & \cdot & 0.004(2) & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0.099(3) \end{pmatrix}.$$

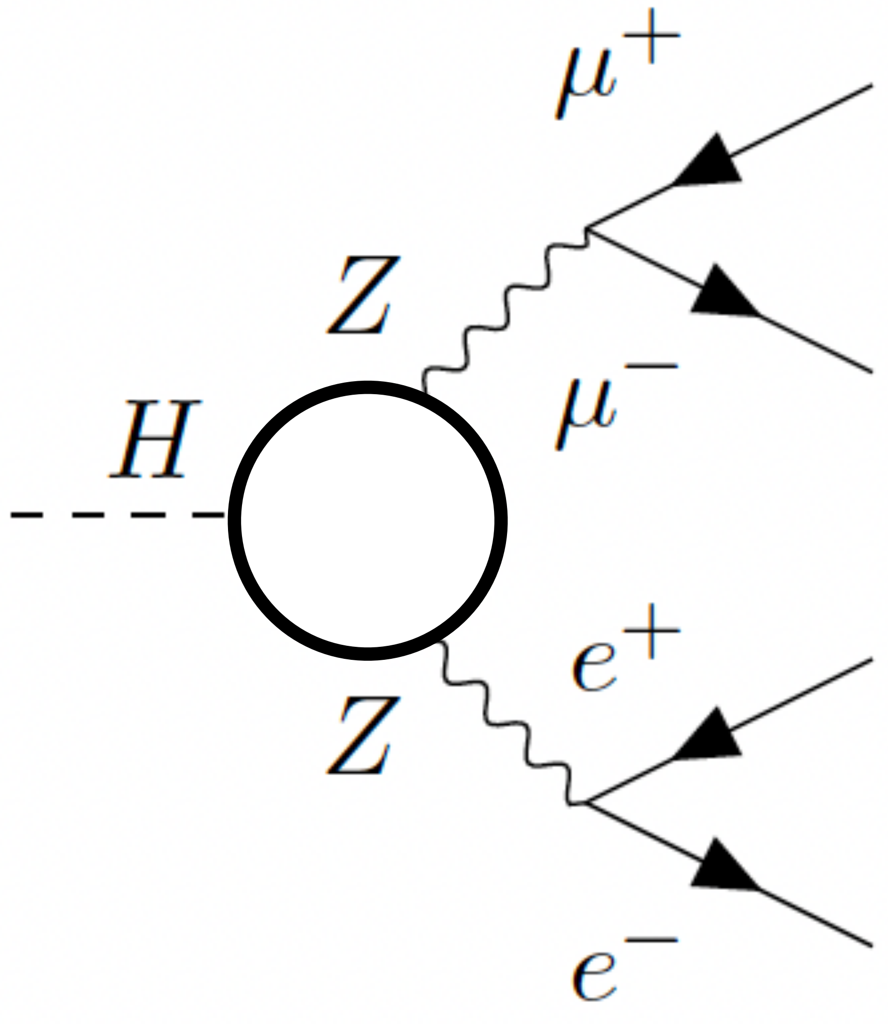
$$|\rho_{ij}|^2 \leq \rho_{ii}\rho_{jj}, \quad i \neq j$$

The reconstructed ρ is not positive definite !

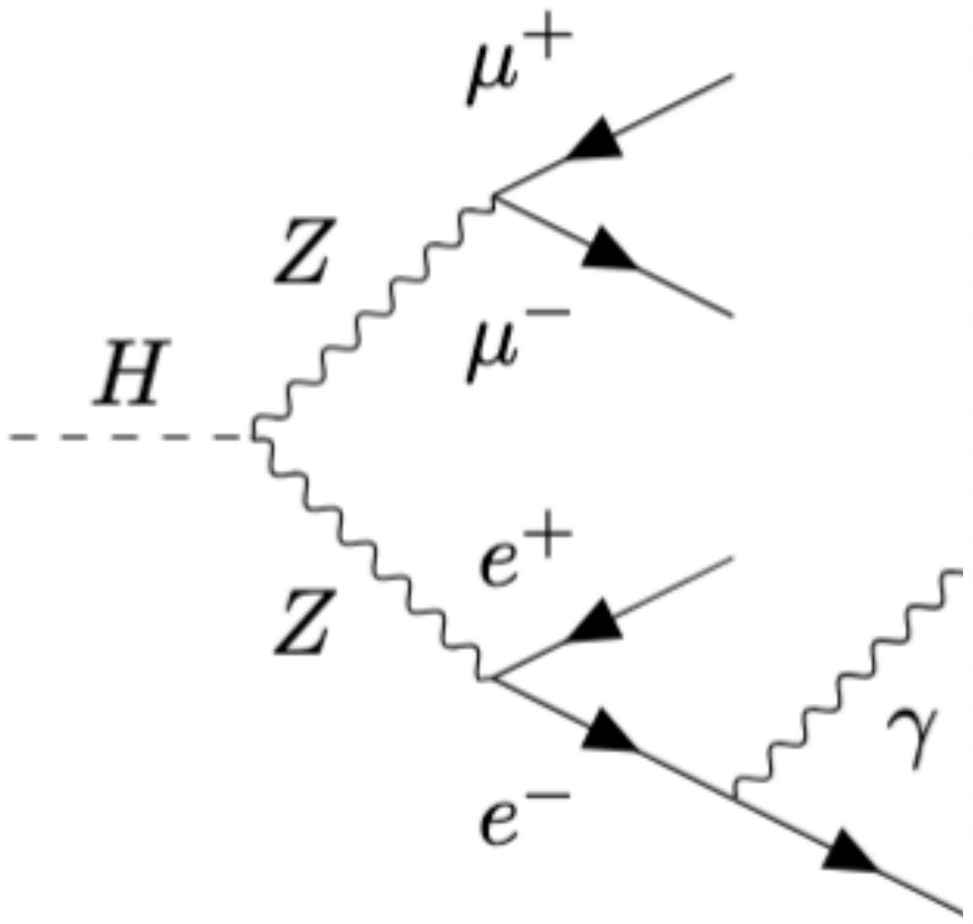
How can than be??? Better think it over….

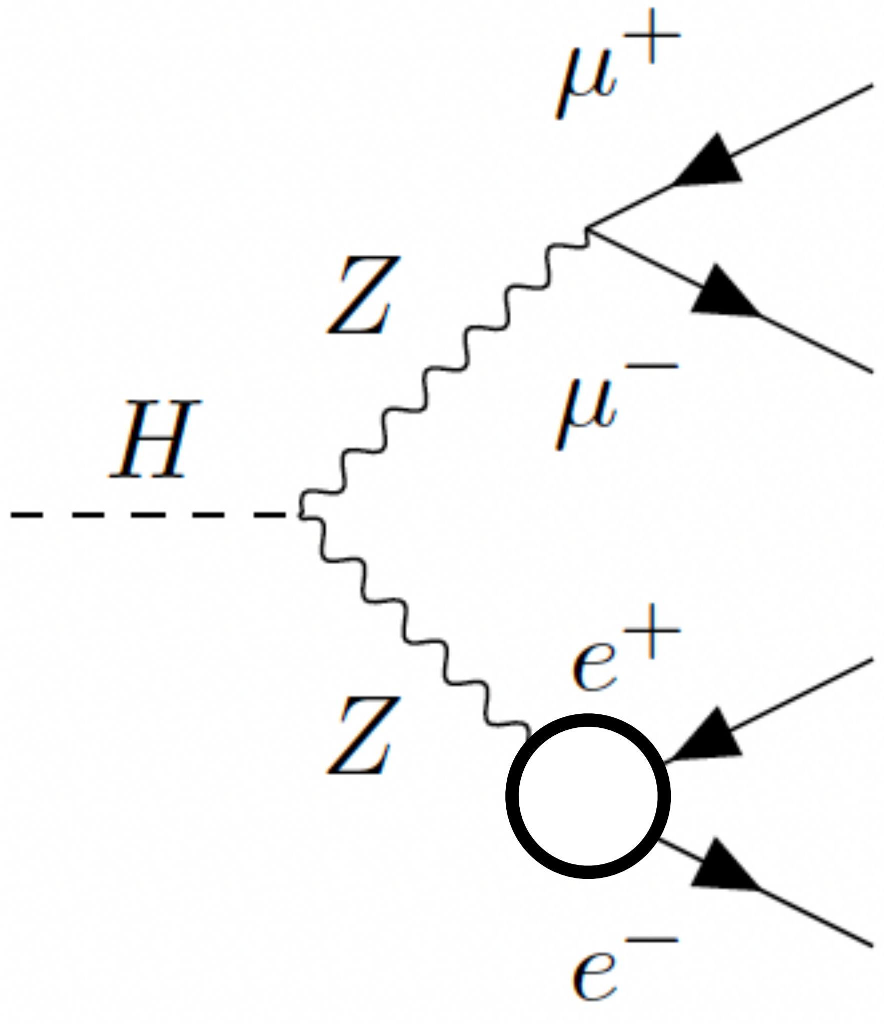
$H \to V_a^{(*)} V_b^{*} \to 4f$ at NLO

At NLO EW several new things happen:

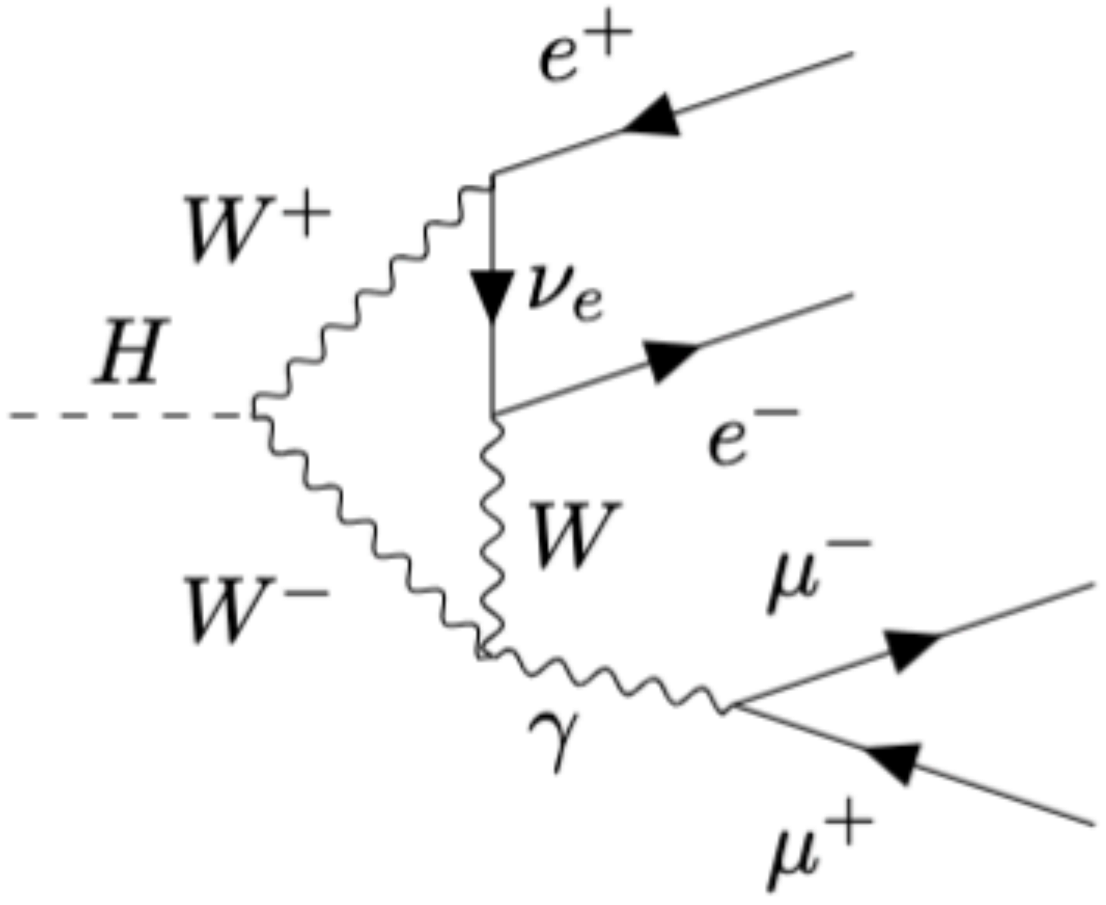


“Genuine” EW corrections to ρ





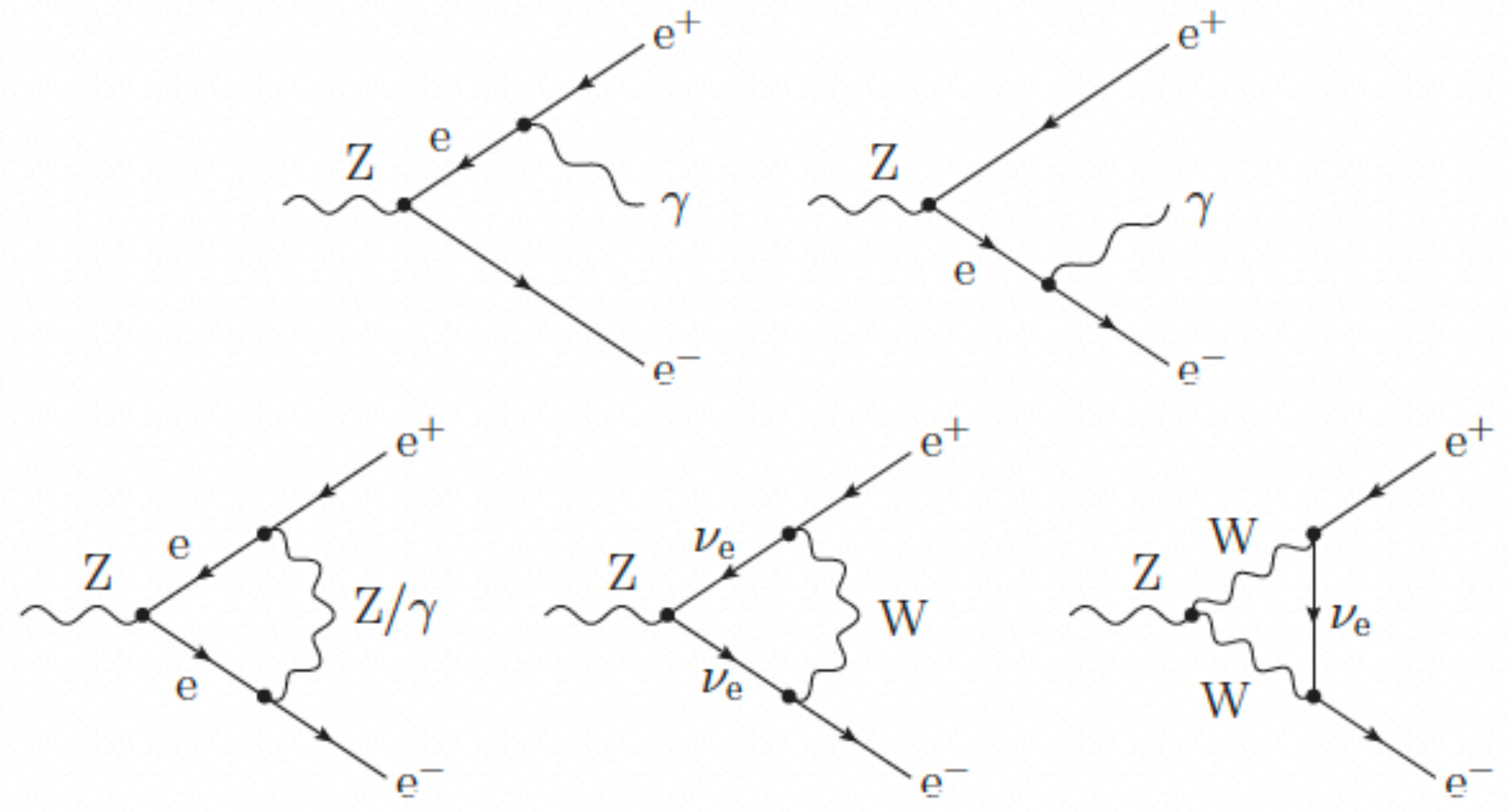
Real and virtual corrections to
to $V_a^{(*)} \to 2f$:
affect the tomography



Terms with 1 or 0
intermediate Z bosons

$H \to V_a^{(*)} V_b^* \to 4f$ at NLO

A careful look at the table shows the large K-factors are associated to l=1 components which depend on the spin analysing power which receives huge corrections



$$\eta_\ell^{\mathrm{LO}} = 0.2131(1)\,, \qquad \eta_\ell^{\mathrm{EW,virt}} = 0.1409(1)\,, \qquad \eta_\ell^{\mathrm{NLO}} = 0.1405(8)\,,$$

That can be effectively taken into account by using:

$$\eta_\ell^{\mathrm{eff}} \equiv \eta_\ell^{\mathrm{LO}}\Big|_{\sin^2\theta_{\mathrm{w}} \to \sin^2\theta_{\mathrm{w}}^{\mathrm{eff}}} = \frac{1 - 4\sin^2\theta_{\mathrm{w}}^{\mathrm{eff}}}{1 - 4\sin^2\theta_{\mathrm{w}}^{\mathrm{eff}} + 8\sin^4\theta_{\mathrm{w}}^{\mathrm{eff}}}\,.$$

M_{H} [GeV]	η_ℓ^{NLO}	f_{--}^{LO}	f_{--}^{NLO}
183	0.1420(4)	0.3303	0.3304
200	0.1423(4)	0.2516	0.2537
225	0.1432(4)	0.1619	0.1644
250	0.1439(4)	0.1041	0.1059

Real radiation can be accounted for using rather generous recombination ΔR (but it is a systematics). Optimal procedure proposed for on-shell Z's. Works for heavy Higgs. For the SM Higgs…for the moment use even correlations. WIP.

Today's question

How do higher-order corrections change the naive leading-order picture?

1. Are quantum correlations modified? How is the tomography affected? Is the reconstructed ρ well-defined?

$$H \to VV$$

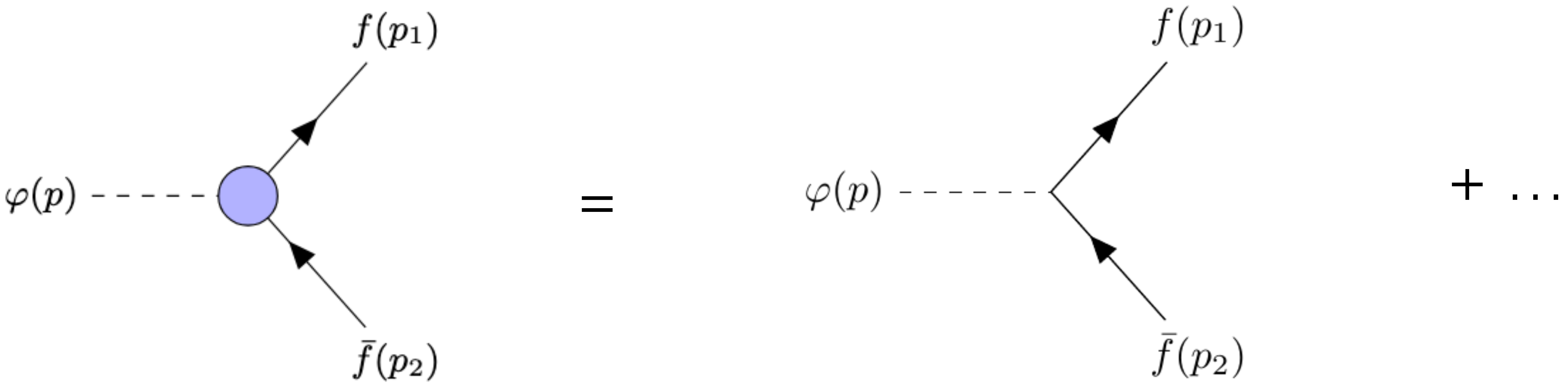
2. Is there a way to re-formulate higher-order effects in terms of QI concepts?

$$H \to t\bar{t}$$

Aoude, Barr, FM, Satrioni, arXiv:2504.07030

A simple case

Fermion pair from a scalar decay



A simple case

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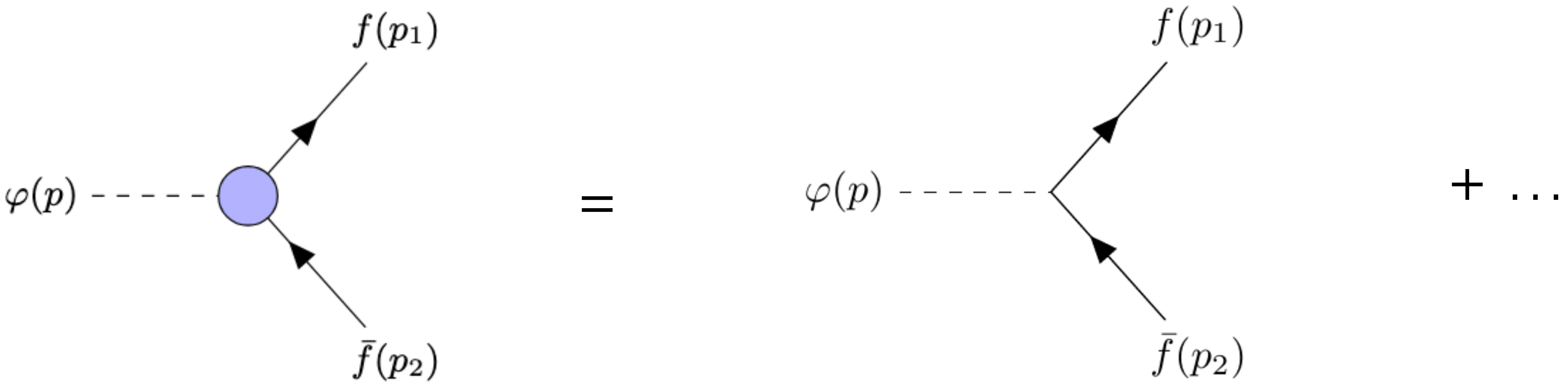


At tree-level: $R_{\mathrm{LO}} = \frac{4N_C y_f^2 m_f^2 \beta^2}{1-\beta^2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ $\quad \rho_{\mathrm{LO}} = \frac{1}{\mathrm{tr}[R_{\mathrm{LO}}]} R_{\mathrm{LO}} = |\Psi^+\rangle\langle\Psi^+|$

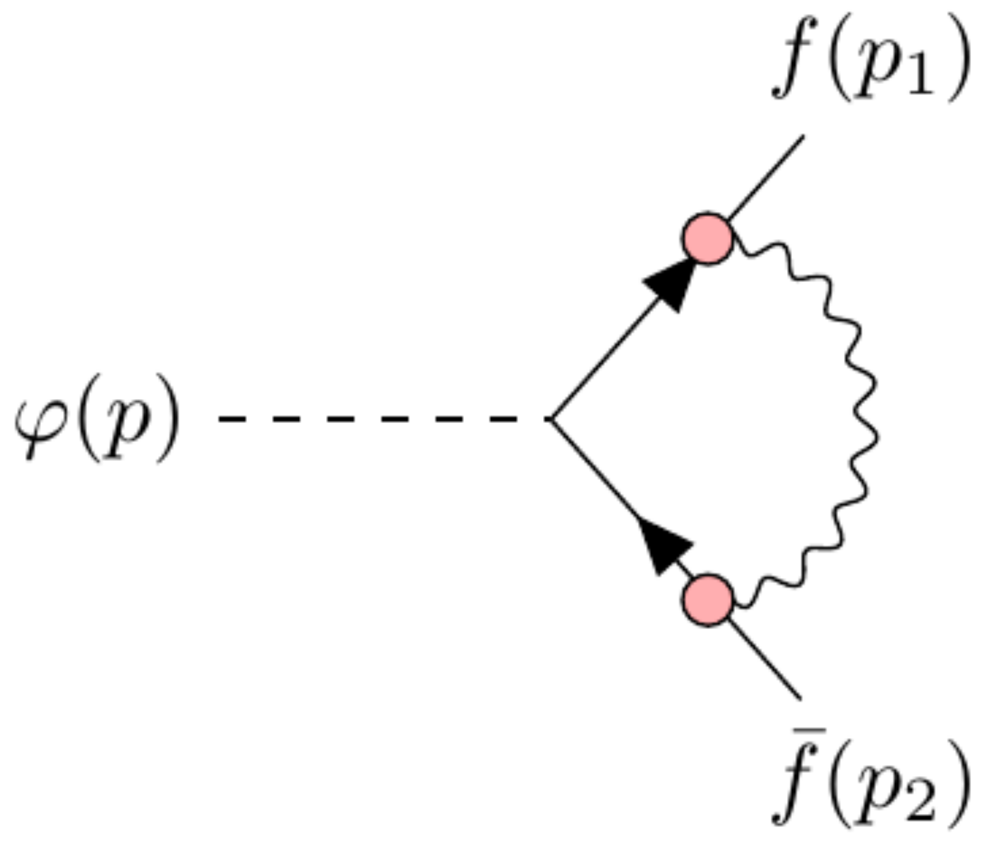
Maximally entangled: controlled place to study entanglement decrease

NLO corrections

General interaction:

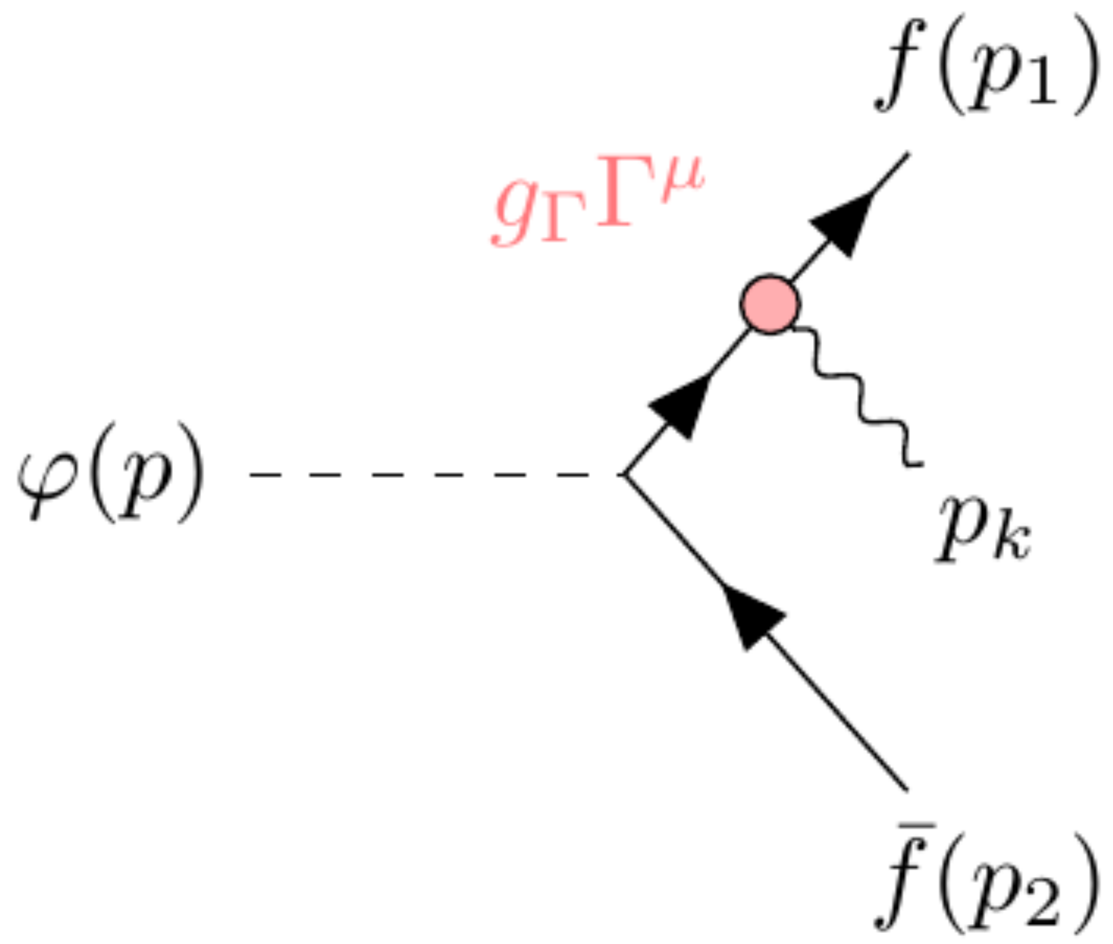
Scalar, pseudo scalar, vector and axial $$g_\Gamma \Gamma^\mu = \{g_S 1, g_P \gamma^5, g_V \gamma^\mu, g_A \gamma^\mu \gamma^5\}$$

Virtual correction: one-loop



$+$

Real emission



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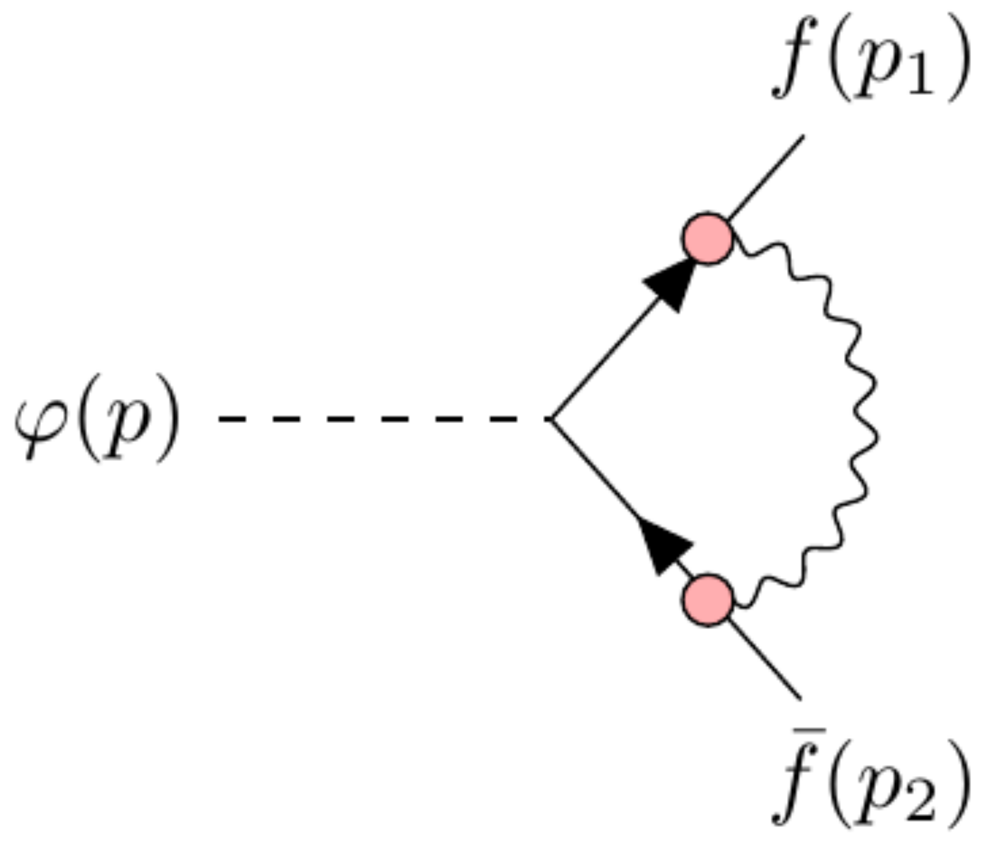
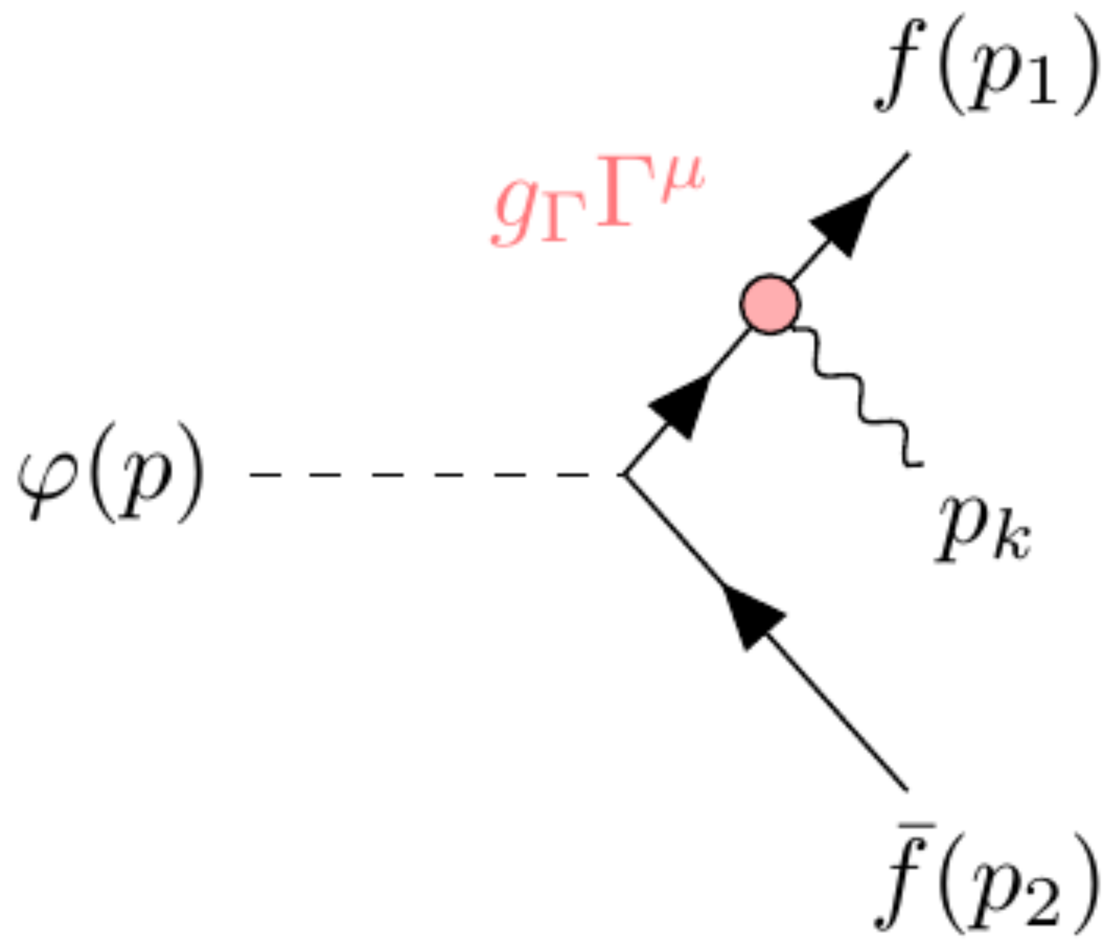


+

Real emission



Trace over the extra d.o.f (environment)

↓

Same Hilbert space and no change.

Quantum Map.
Open Quantum system

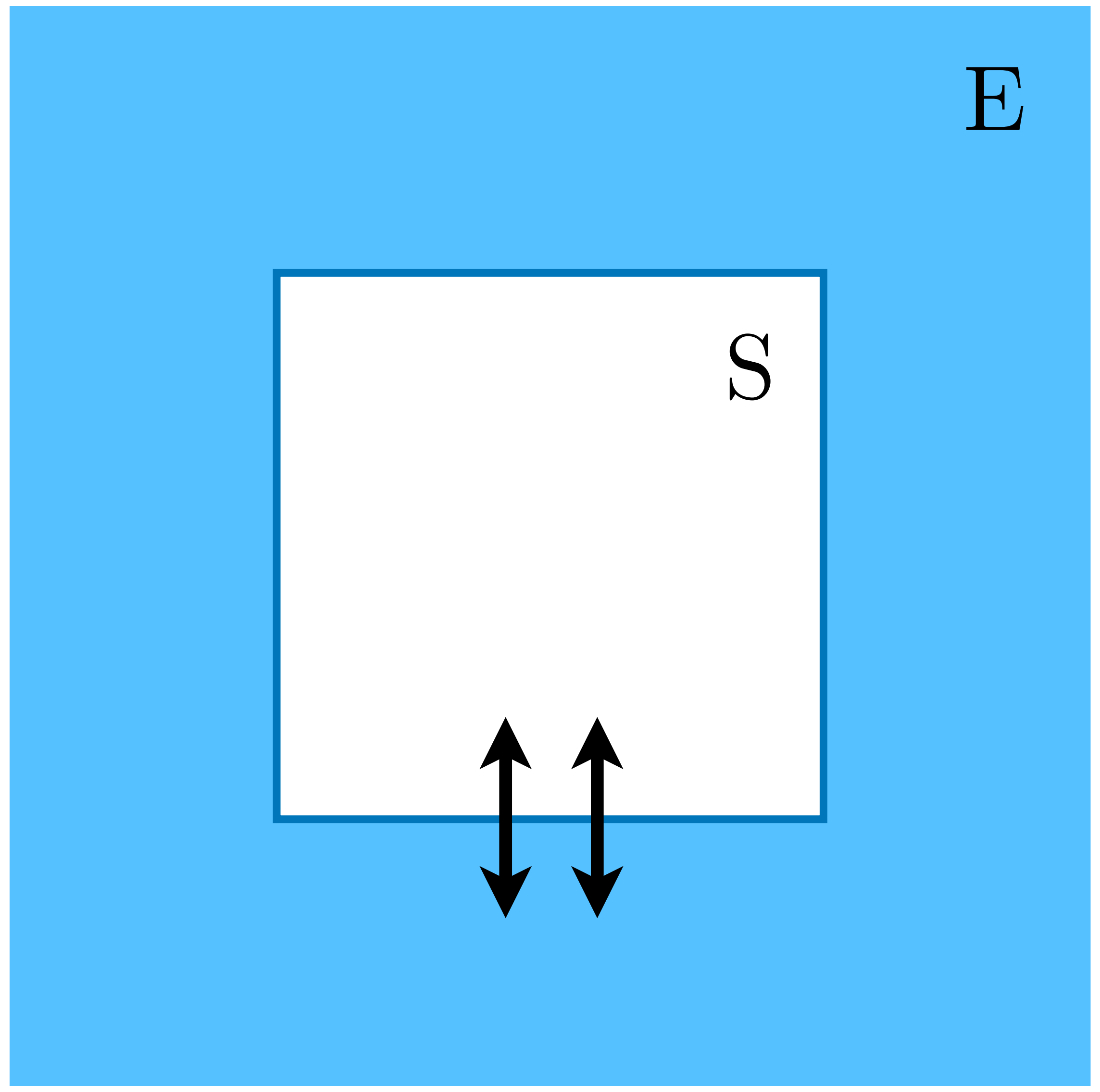
Quantum Maps

The evolution of a system+environment is unitary

$$\rho'(t) = U(t)\rho_{\mathrm{S}}(0) \otimes \rho_E(0)U^\dagger(t)$$

Tracing over the environment subsystem

$$\rho_{\mathrm{S}}(t) = \mathrm{tr}_E\left[U(t)\rho_{\mathrm{S}}(0) \otimes \rho_E(0)U^\dagger(t)\right]$$



Quantum Maps

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$$\rho'(t) = U(t)\rho_{\rm S}(0) \otimes \rho_E(0) U^\dagger(t)$$

Tracing over the environment subsystem

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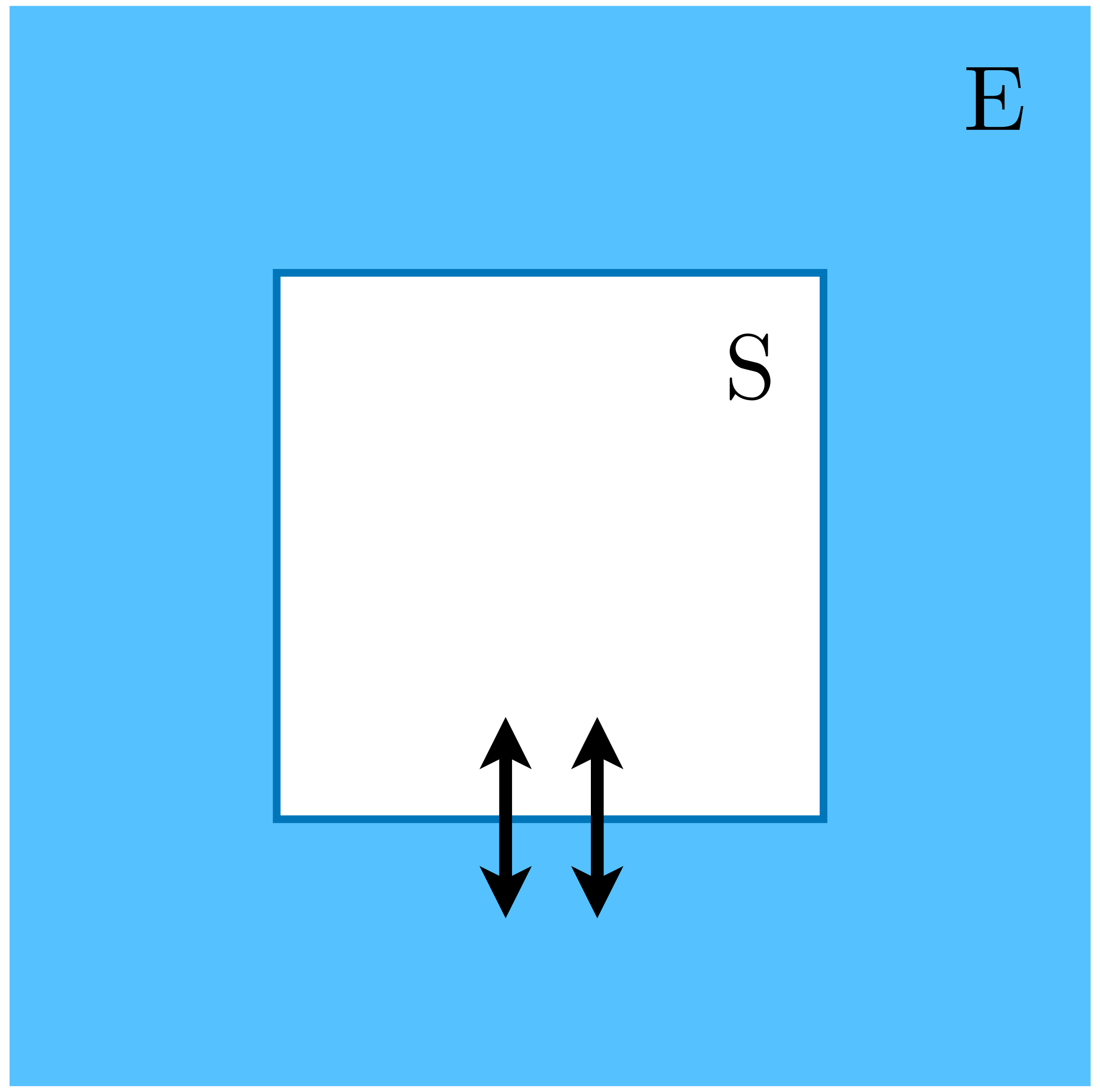
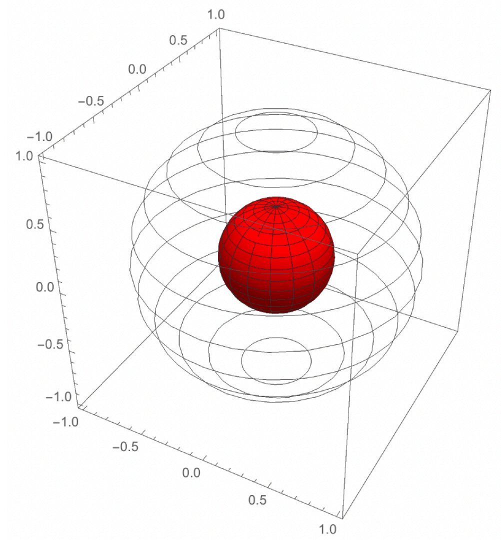


which we can write as a operator-sum representation (Kraus operators)

$$\rho_{\rm S}(t) = \sum_j K_j \rho_{\rm S}(0) K_j^\dagger =: \mathcal{E}[\rho_{\rm S}(0)] \quad \text{s.t} \quad \sum_j K_j^\dagger K_j = 1$$

For bipartite qubits: K_j Tensor product of Pauli's and Id.

Decoherence channels

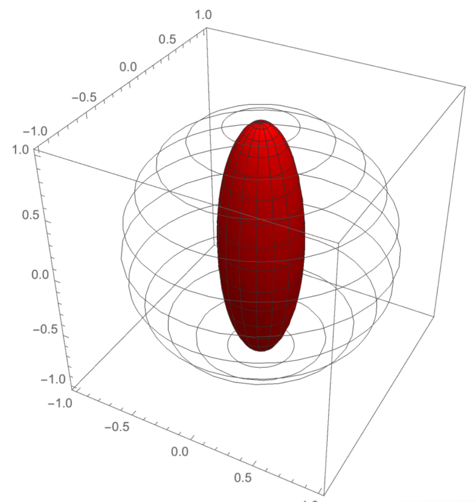


$$K_1 = \sqrt{1 - \frac{3p}{4}}\mathbb{1} \quad , \quad K_2 = \sqrt{\frac{p}{4}}\sigma_1, \quad K_3 = \sqrt{\frac{p}{4}}\sigma_2 \quad , \quad K_4 = \sqrt{\frac{p}{4}}\sigma_3.$$

Depolarizing

Typical random noise

$$\mathcal{E}(\rho) = (1-p)\rho + p\frac{I}{2}$$

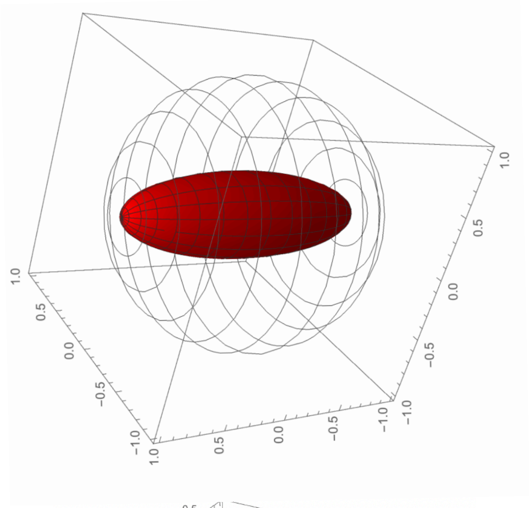


$$K_1 = \sqrt{1 - \frac{p}{2}}\mathbb{1} \qquad K_2 = \frac{p}{2}\sigma_3$$

Phase damping

This is the **pure decoherence channel**: it destroys **off-diagonal elements** but leaves populations unchanged.

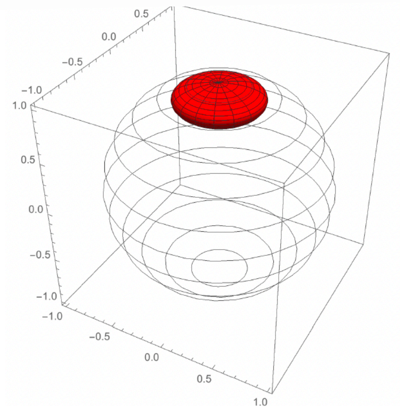
$$\mathcal{E}(\rho) = \left(1 - \frac{p}{2}\right)\rho + \frac{p}{2} Z\rho Z$$



$$K_0 = \sqrt{1-p}\, I, \qquad K_1 = \sqrt{p}\, X$$

Spin flip

$$\mathcal{E}(\rho) = (1-p)\rho + p\, X\rho X$$



$$K_1 = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-p} \end{pmatrix}, \quad K_2 = \begin{pmatrix} 0 & \sqrt{p} \\ 0 & 0 \end{pmatrix}$$

Amp damping

$$\mathcal{E}(\rho) = \rho + \gamma \left(\sigma_- \rho \sigma_+ - \frac{1}{2}\{\sigma_+\sigma_-, \rho\} \right)$$

Environment as unresolved radiation

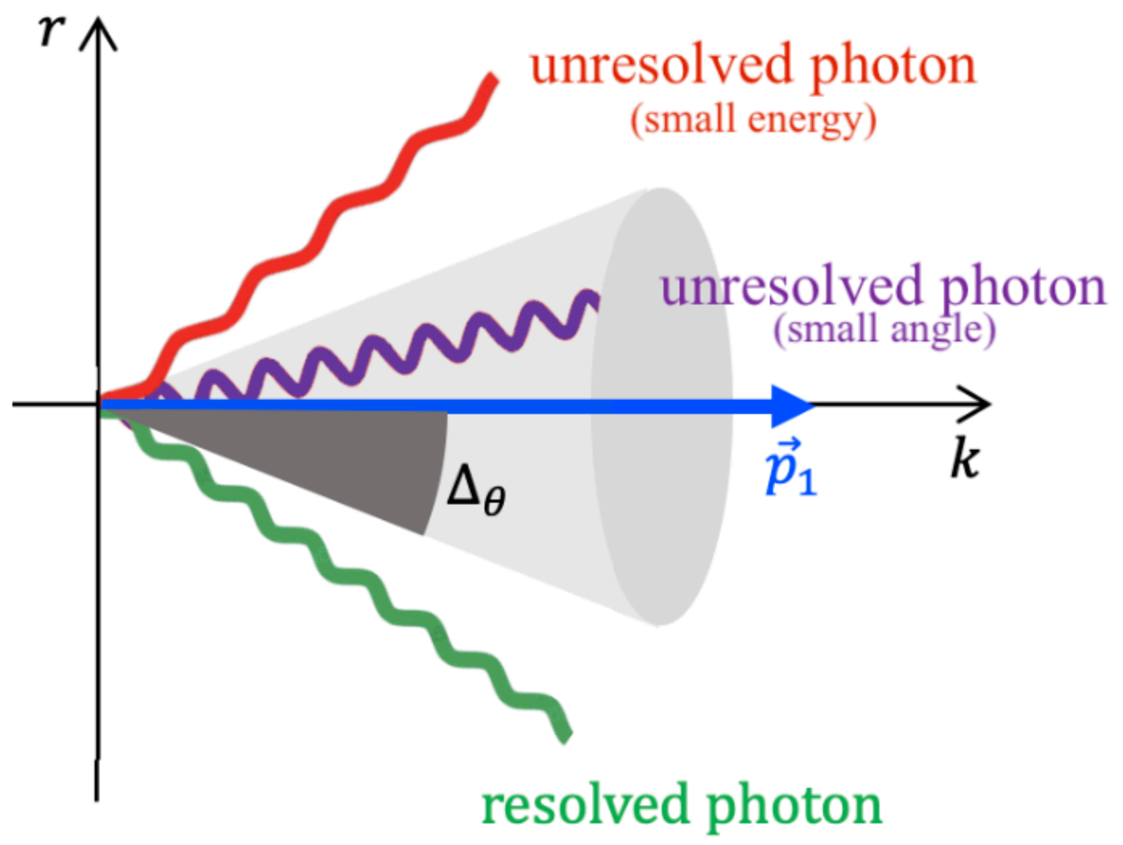
Tree-level $\mathcal{A}_{\alpha\beta}(\varphi \to t\bar{t})[\mathcal{A}_{\alpha'\beta'}(\varphi \to t\bar{t})]^\dagger \sim \mathcal{O}(y_t^2)$

One-loop $\mathcal{A}^{(1)}_{\alpha\beta}(\varphi \to t\bar{t})\left[\mathcal{A}^{(0)}_{\alpha'\beta'}(\varphi \to t\bar{t})\right]^\dagger + \mathrm{h.c} \quad \sim \mathcal{O}(y_t^2\, g^2)$

Real emission contribution to the R-matrix

$$\mathcal{A}^h_{\alpha\beta}(\varphi \to t\bar{t} + k)[\mathcal{A}^h_{\alpha'\beta'}(\varphi \to t\bar{t} + k)]^\dagger \sim \mathcal{O}(y_t^2\, g^2)$$

Environment as unresolved radiation



©L.Satrioni

Tree-level: $\mathcal{A}_{\alpha\beta}(\varphi \to t\bar{t})[\mathcal{A}_{\alpha'\beta'}(\varphi \to t\bar{t})]^\dagger \sim \mathcal{O}(y_t^2)$

One-loop: $\mathcal{A}^{(1)}_{\alpha\beta}(\varphi \to t\bar{t})\left[\mathcal{A}^{(0)}_{\alpha'\beta'}(\varphi \to t\bar{t})\right]^\dagger + \mathrm{h.c} \sim \mathcal{O}(y_t^2\, g^2)$

Real emission: $\mathcal{A}^{h}_{\alpha\beta}(\varphi \to t\bar{t} + k)[\mathcal{A}^{h}_{\alpha'\beta'}(\varphi \to t\bar{t} + k)]^\dagger \sim \mathcal{O}(y_t^2\, g^2)$

We trace out the **unresolved** interaction: soft or collinear

$$\mathrm{tr}_{\mathcal{H}_k}[\cdot] = \int \mathrm{d}\Phi(k) \sum_{\sigma=\pm} \langle k,\sigma| \cdot |k,\sigma\rangle$$

If it’s resolved: three-body decay

NLO reduced density matrix

$$\rho^{\mathrm{red}}_{\mathrm{LO+NLO}} = \sum_j K_j \rho_{\mathrm{LO}} K_j^\dagger$$

$$= \mathrm{p}_{\mathrm{LO}}\, \mathbb{1} \rho_{\mathrm{LO}} \mathbb{1} + \bar{\mathcal{E}}_{\mathrm{V}}[\rho_{\mathrm{LO}}] + \bar{\mathcal{E}}_{\mathrm{R}}[\rho_{\mathrm{LO}}]$$

LO contribution

“Map” of virtual emission

UV and IR divergent

“Map” of real emission

IR divergent

Virtual and Soft Maps

Virtual current factorizes

$$\bar{u}(p_1,h_1)\mathbb{T}_{\text{virt.}}v(p_2,h_2) = \tilde{T}_{\text{virt.}}\boxed{\bar{u}(p_1,h_1)v(p_2,h_2)}$$

integral (w/.gamma's)

same as tree current

$$\bar{\mathcal{E}}_{\rm V}[\rho_{\rm LO}] = \mathsf{p}_{\rm V}\mathbb{1}\rho_{\rm LO}\mathbb{1}$$

* only for scalar decay!!!

Virtual and Soft Maps

Virtual current factorizes $\bar{u}(p_1,h_1)\mathbb{T}_{\text{virt.}}v(p_2,h_2) = \tilde{T}_{\text{virt.}}\bar{u}(p_1,h_1)v(p_2,h_2)$

integral (w/.gamma's)

same as tree current

$$\bar{\mathcal{E}}_{\mathrm{V}}[\rho_{\mathrm{LO}}] = \mathsf{p}_{\mathrm{V}}\mathbb{1}\rho_{\mathrm{LO}}\mathbb{1}$$

* only for scalar decay!!

Real emissions are different ⟶ change the LO spin-structure

$$\bar{\mathcal{E}}_{\mathrm{R}}[\rho_{\mathrm{LO}}] = \bar{\mathcal{E}}_{\mathrm{R}}^{\mathrm{soft}}[\rho_{\mathrm{LO}}] + \bar{\mathcal{E}}_{\mathrm{R}}^{\mathrm{col.}}[\rho_{\mathrm{LO}}]$$

both as operator-sum representation (Kraus)

$$\bar{\mathcal{E}}_{\mathrm{R}}[\rho_{\mathrm{LO}}] = \sum_j K_j \rho_{\mathrm{LO}} K_j^\dagger$$

Built of Pauli matrices

Soft part

We can use the soft theorem

$$\mathcal{M}_{n+1} = \sum_{i=1}^{n} \left[\frac{p_i \cdot \varepsilon_h(k)}{p_i \cdot k} + \cdots \right] \mathcal{M}_n$$

Scalar function

Next-to-leading soft
(change structures)

Leading-soft map

$$\bar{\mathcal{E}}_{\mathrm{R}}^{\mathrm{soft}}[\rho_{\mathrm{LO}}] = \underbrace{\mathsf{p}_{\mathrm{R}}^{\mathrm{soft}} \mathbb{1} \rho_{\mathrm{LO}} \mathbb{1}}_{\mathrm{scalar,vector}} .$$

p's cancel the IR divergence of virtual: KLN theorem

‘Hard’ (collinear) emission part

Now, this has a non-trivial Kraus operator part

$$\bar{\mathcal{E}}_{\mathrm{R}}^{\mathrm{col.}}[\rho_{\mathrm{LO}}] = \mathsf{p}_{\mathrm{R}}^{\mathrm{col.}} \mathbb{1} \rho_{\mathrm{LO}} \mathbb{1} + \mathsf{q}^{\mathrm{col}} \sum_{j \neq \mathrm{id}} K_j \rho_{\mathrm{LO}} K_j^\dagger$$

non-zero q = decoherence

without the identity

Change in spin-structure: dipole-like interaction (IR finite)

Taking the collinear limit for the emission …

Full NLO map

$$\mathcal{E}_{\text{full}}[\rho_{\text{LO}}] = \mathsf{p}_{\text{id}}\, \mathbb{1} \rho_{\text{LO}} \mathbb{1} + \mathsf{q} \sum_{j \neq \text{id}} K_j \rho_{\text{LO}} K_j^\dagger$$

$$\mathsf{p}_{\text{id}} = (\text{p}_{\text{LO}} + \text{p}_{\text{V}} + \text{p}_{\text{R}}^{\text{soft}} + \text{p}_{\text{R}}^{\text{col.}})$$

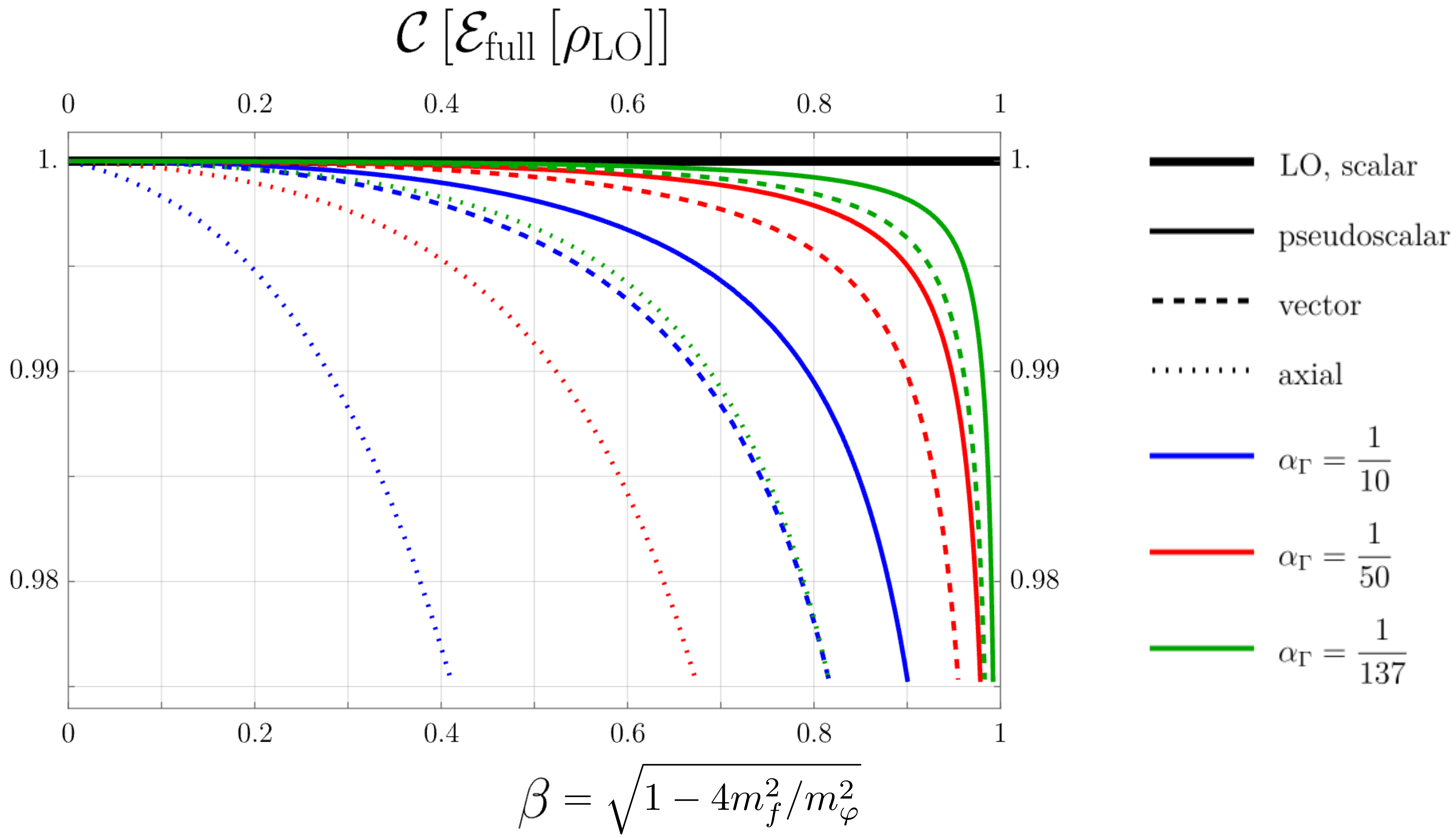
Identity part:
does not change entanglement

$$\mathsf{q} = \mathsf{q}^{\text{col.}}$$

Non-trivial Kraus part: Decoherence!

*in the leading soft/collinear limit

Decoherence

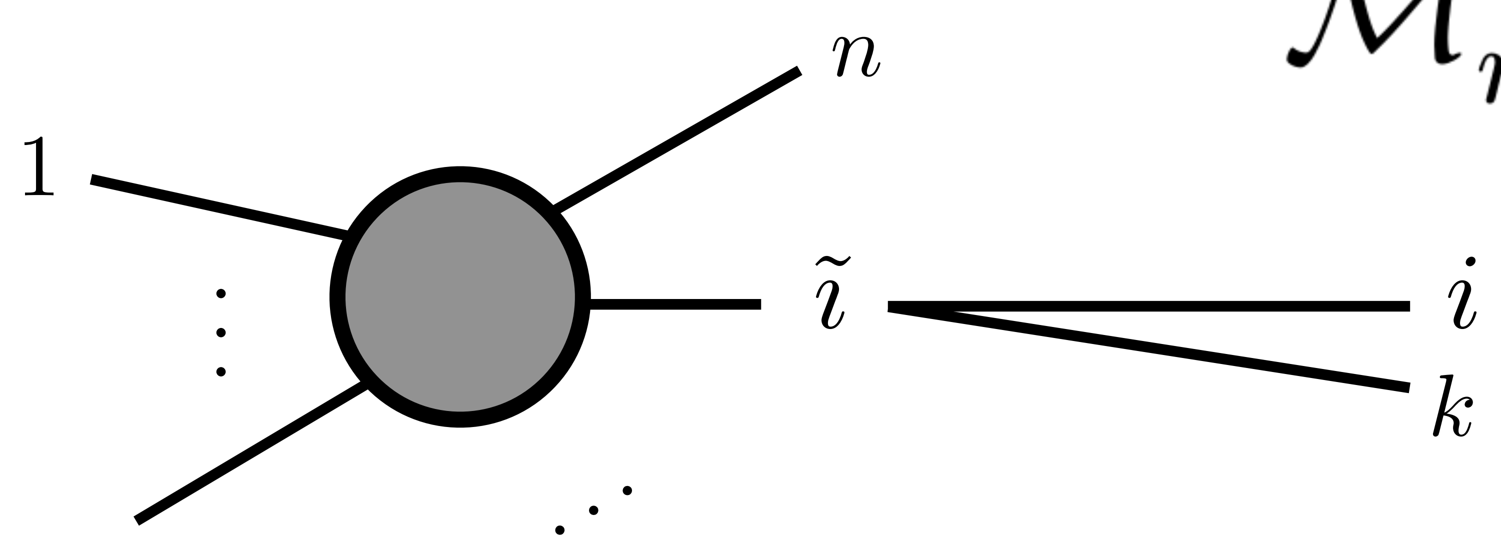


Entanglement is lost mainly due to collinear emission $\Rightarrow$ Small effects ~ 1%

See also:
Richardson, Webster 1807.01955,
Hamilton, Karlberg, Salam, Scyboz, Verheyen 2111.01161

Splitting functions as Kraus operators

In the collinear limit, a n-parton system undergoes a splitting $\tilde{\imath} \to ik$



$$\mathcal{M}_{n+1}^{\lambda_i \lambda_k}(\cdots, p_i, p_k, \cdots) = \mathcal{S}_{\tilde{\imath}\to ik}^{\lambda_{\tilde{\imath}} \lambda_i \lambda_k} \; \mathcal{M}_{n}^{\lambda_{\tilde{\imath}}}(\cdots, p_{\tilde{\imath}}, \cdots)$$

Helicity dependent AP splitting function

Splitting functions as Kraus operators

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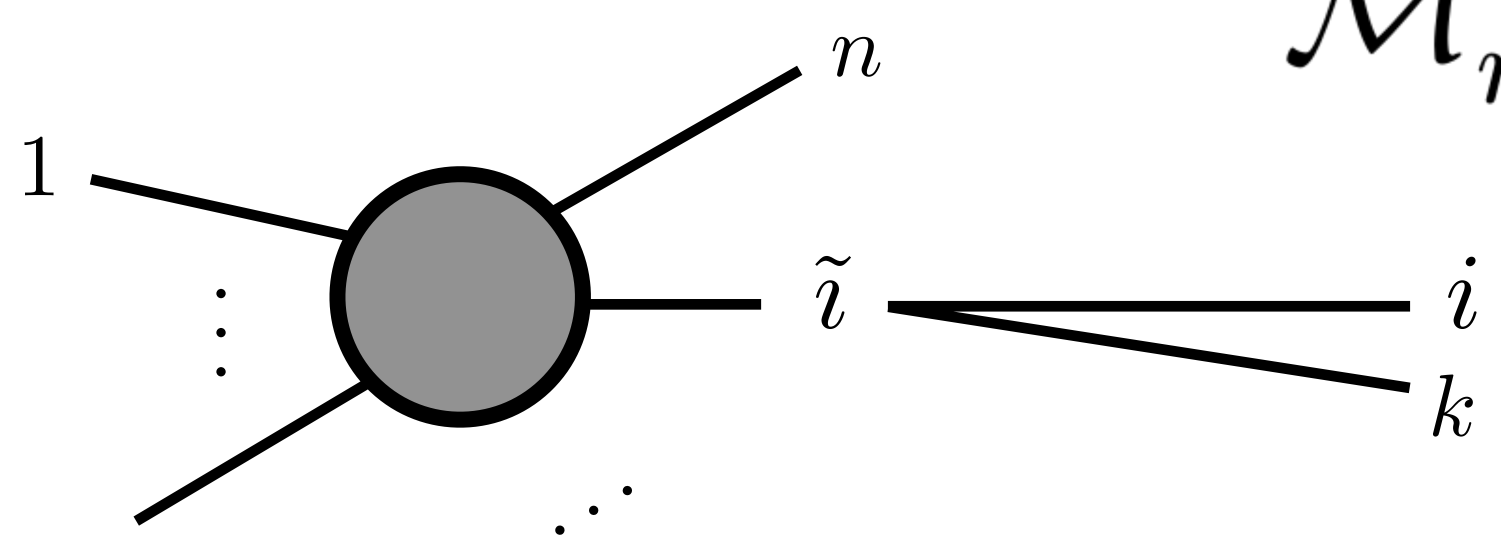


$$\mathcal{M}_{n+1}^{\lambda_i\lambda_k}(\cdots, p_i, p_k, \cdots) = \mathcal{S}_{\tilde{\imath}\to ik}^{\lambda_{\tilde{\imath}}\lambda_i\lambda_k} \, \mathcal{M}_n^{\lambda_{\tilde{\imath}}}(\cdots, p_{\tilde{\imath}}, \cdots)$$

Helicity dependent AP splitting function

Tracing over the unresolved d.o.f

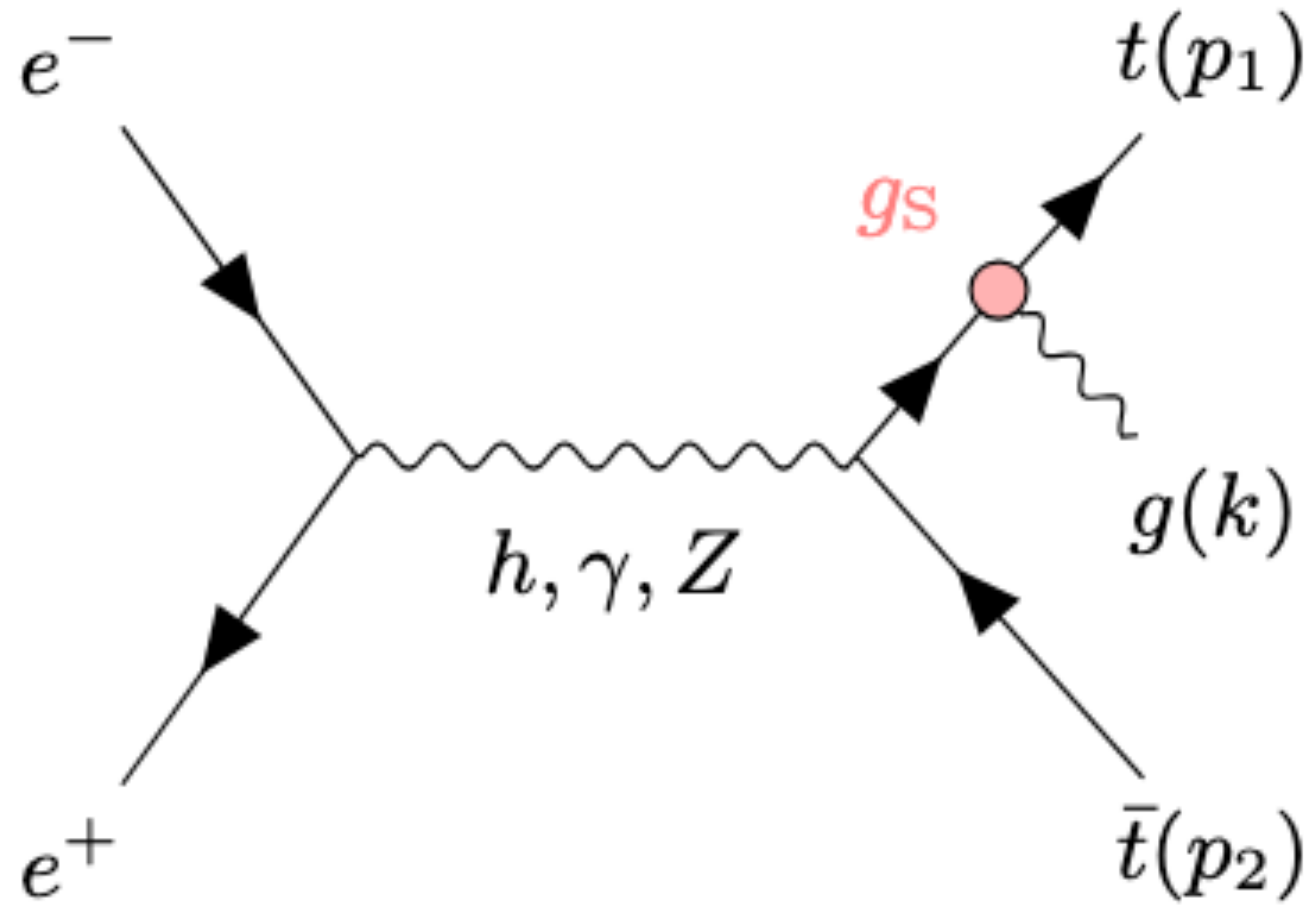
$$\bar{\mathcal{E}}_{\mathrm{col}}[\rho] = \rho_{\mathrm{red}}^{\lambda_i\lambda_i'} = \sum_{\sigma=\pm}\int_{p_k} \mathcal{S}_{\tilde{\imath}\to ik}^{\lambda_{\tilde{\imath}}\lambda_i\sigma} \cdot \rho^{\lambda_{\tilde{\imath}}\lambda_{\tilde{\imath}}'} \cdot \mathcal{S}_{\tilde{\imath}\to ik}^{\lambda_{\tilde{\imath}}'\lambda_i'\sigma} = \mathsf{q}^{\mathrm{col}} \sum_{j\neq \mathrm{id}} K_j \rho_{\mathrm{LO}} K_j^\dagger$$

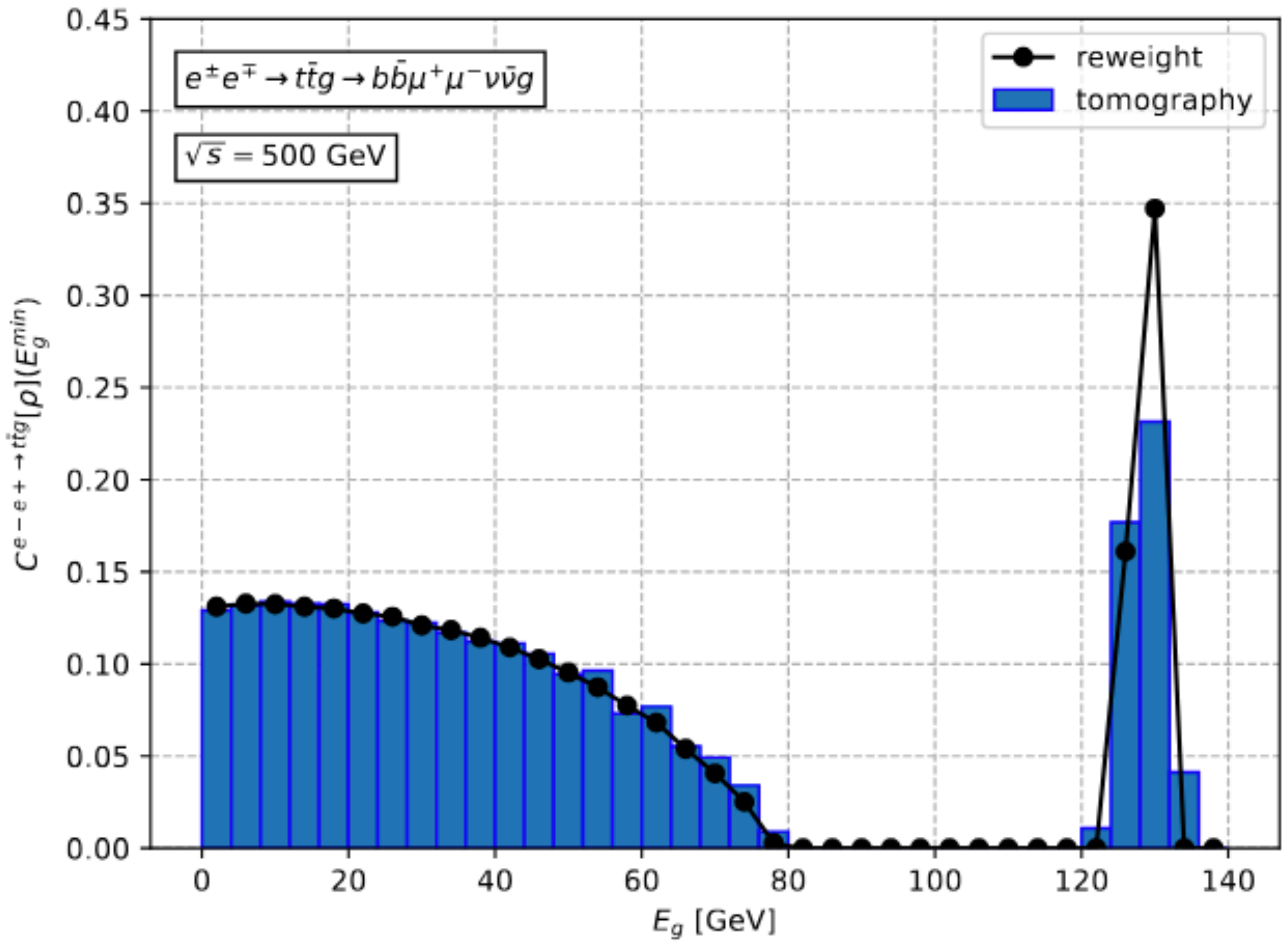
Splitting functions as Kraus (here: one emission for more see

Gu, Lin, Shao, Wang, Yang, 2510.13951

Application: $e^+e^- \to t\bar{t}g$

Rafael Aoude, José Manuel Camacho, Valentin Durupt, Guillermo García Mir,
Fabio Maltoni, Maria Moreno Llácer, Leonardo Satrioni, Marcel Vos
Work in PROGRESS





In addition, the virtual also gives a finite non-trivial spin-spin modification…

Conclusions

❖ Our current description of fundamental interactions, based on QFT, has QM at its core. Theoretically, it is embedded in our formalism so deeply that (sometimes) we do not even notice. Experimentally, however, most of our measurements are not (quantum) correlations, but just counting experiments.

❖ **Looking at fundamental interactions at TeV scale with QI glasses** is leading to a renewed interest in a variety of phenomena with novel ideas, studies, results…

❖ We have just started studying the effects of higher-order corrections to spin observables and interpreting them in terms of decoherence. More to come…

Tomography in QI

- The state of a quantum system can be reconstructed by a procedure called tomography.
- Imagine to have a source of qubits whose ρ is unknown, i.e.

$$\rho = \frac{1}{2}\begin{pmatrix} 1+n_z & n_x - in_y \\ n_x + in_y & 1-n_z \end{pmatrix}, \text{with} \; , \; n_x^2 + n_y^2 + n_z^2 \leq 1 \, ,$$

With n_x, n_y, n_z unknown. What measurements should be conducted on the system to infer the n?

- To infer three parameters we need three types of measurements conducted many times.

Tomography in QI

- We use measurement projectors in the computational basis

$$P_0 = |0\rangle \langle 0| = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad P_1 = |1\rangle \langle 1| = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

And perform them many times, to determine n_z

$$p_0 = \mathrm{Tr}[\rho\, P_0] = \frac{1}{2}(1 + n_z), \quad p_1 = \mathrm{Tr}[\rho\, P_1] = \frac{1}{2}(1 - n_z). \qquad n_z = \frac{N_0 - N_1}{N_0 + N_1},$$

To determine n_x ans n_y, we first "rotate ρ" with $\rho' = U \rho U^\dagger$

$$U_x \equiv \frac{1}{\sqrt{2}}(\mathbb{I}_2 + i\sigma_y) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix},$$

$$U_y \equiv \frac{1}{\sqrt{2}}(\mathbb{I}_2 - i\sigma_x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix},$$

And then

$$\begin{aligned} p_0 &= \mathrm{Tr}[\rho'\, P_0] \\ &= \mathrm{Tr}[\rho\, U_x^\dagger P_0 U_x] \\ &= \frac{1}{2}\mathrm{Tr}[\rho \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}] = \frac{1}{2}(1 + n_x), \end{aligned}$$

And similarly for n_y

Quantum random tomography

- Now imagine that we cannot do that, as for some reason we are unable to exactly choose U, which is distributed randomly:

$$U = U(\theta, \phi) = \begin{pmatrix} \cos\theta & e^{+i\phi}\sin\theta \\ -e^{-i\phi}\sin\theta & \cos\theta \end{pmatrix}, \; U^\dagger = U^{-1}.$$

- I can still perform it, applying the rnd U and measuring in one direction. The result would be:

$$p_0 = \mathrm{Tr}[U\rho U^\dagger P_0] = \frac{1}{2} + \frac{n_x}{2}\cos\phi\sin 2\theta + \frac{n_y}{2}\sin\phi\sin 2\theta + \frac{n_z}{2}\cos 2\theta\,.$$

- So now I can fit the coefficients of the angular functions and determine n_x, n_y, n_z.

Quantum random tomography

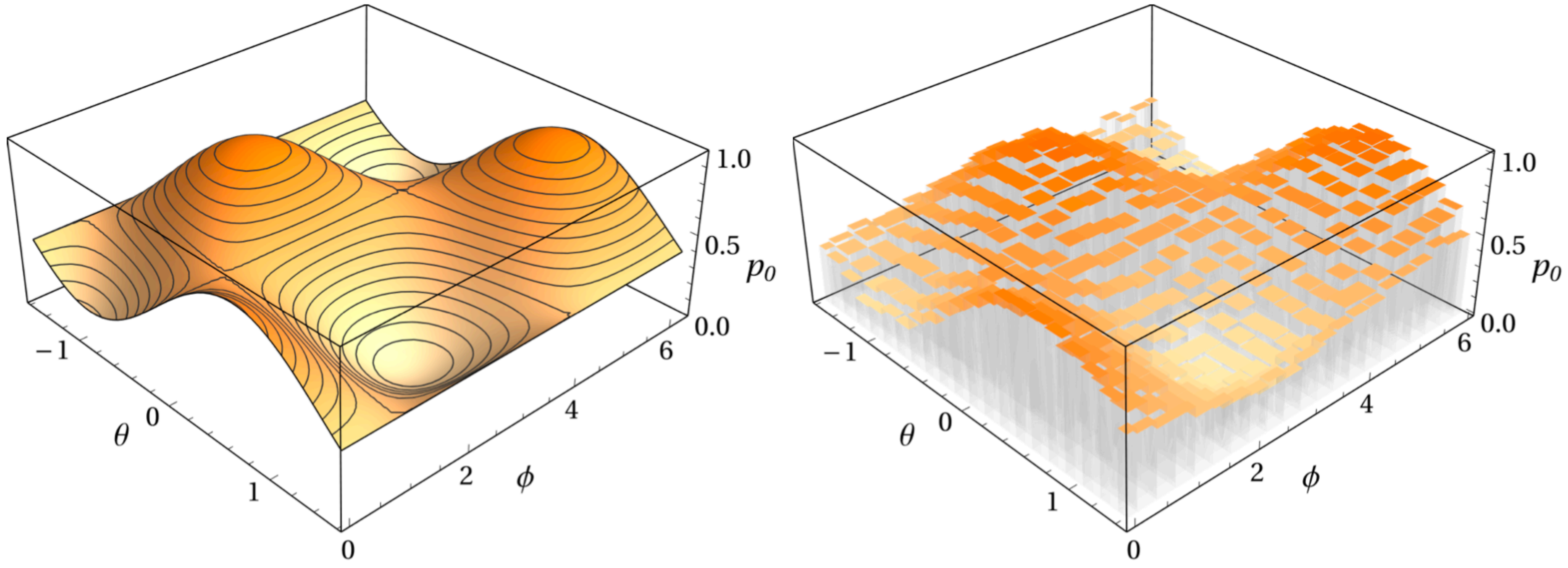


Figure 3: Random tomography of a qubit with $n_x = 0.6, n_y = -0.5, n_z = 0.2$. On the left, probability p_0 for measuring 0 after the transformation $U(\theta, \phi)$ has been applied to ρ, on the right, same probability extracted from with $N = 300\,000$ random measurements, binned in θ and ϕ.

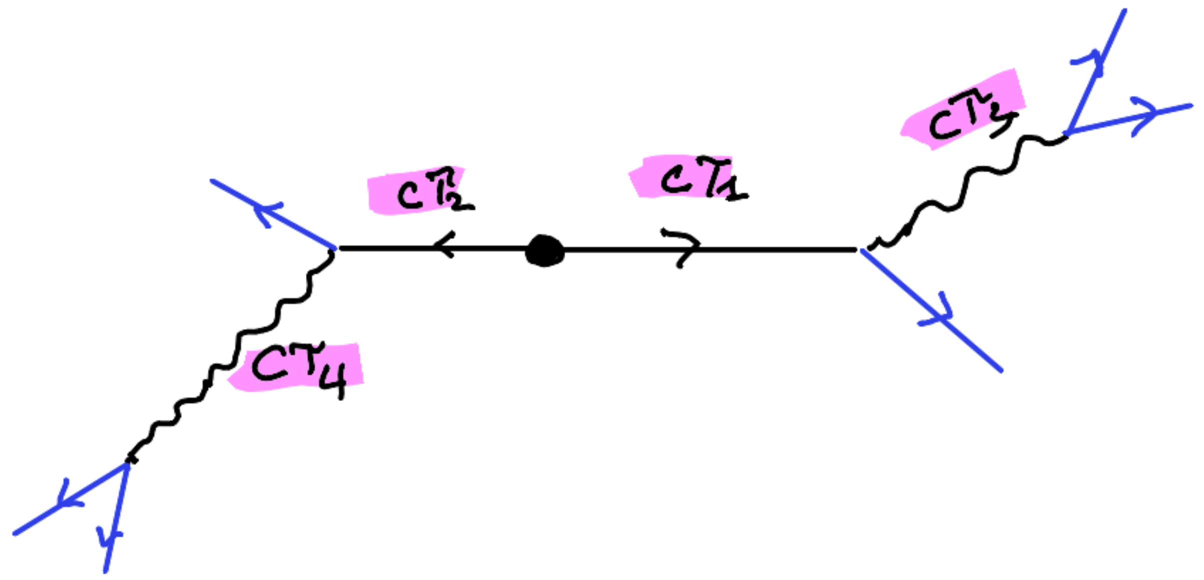
This is what were are actually doing at colliders with the decays of the t,W,Z,τ,..

Underlying space-time picture

- At the end we are not measuring spin, and amplitudes 2 → 6 depend only on momenta.

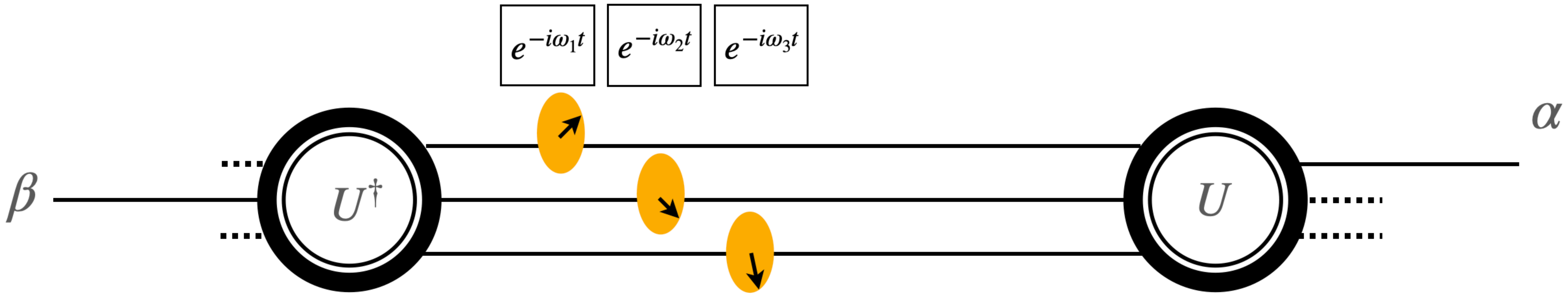
- In general, if amplitudes have (screened) poles, then contain 1-particle "classical propagation" and then provide space/time information:

$$G(t) = \int_{-\infty}^{\infty} \frac{e^{-iEt}\, dE}{E^2 - m_0^2 + i m_0 \Gamma_0} \propto e^{-i m_0 t} e^{-\frac{\Gamma_0}{2} t} \theta(t)$$



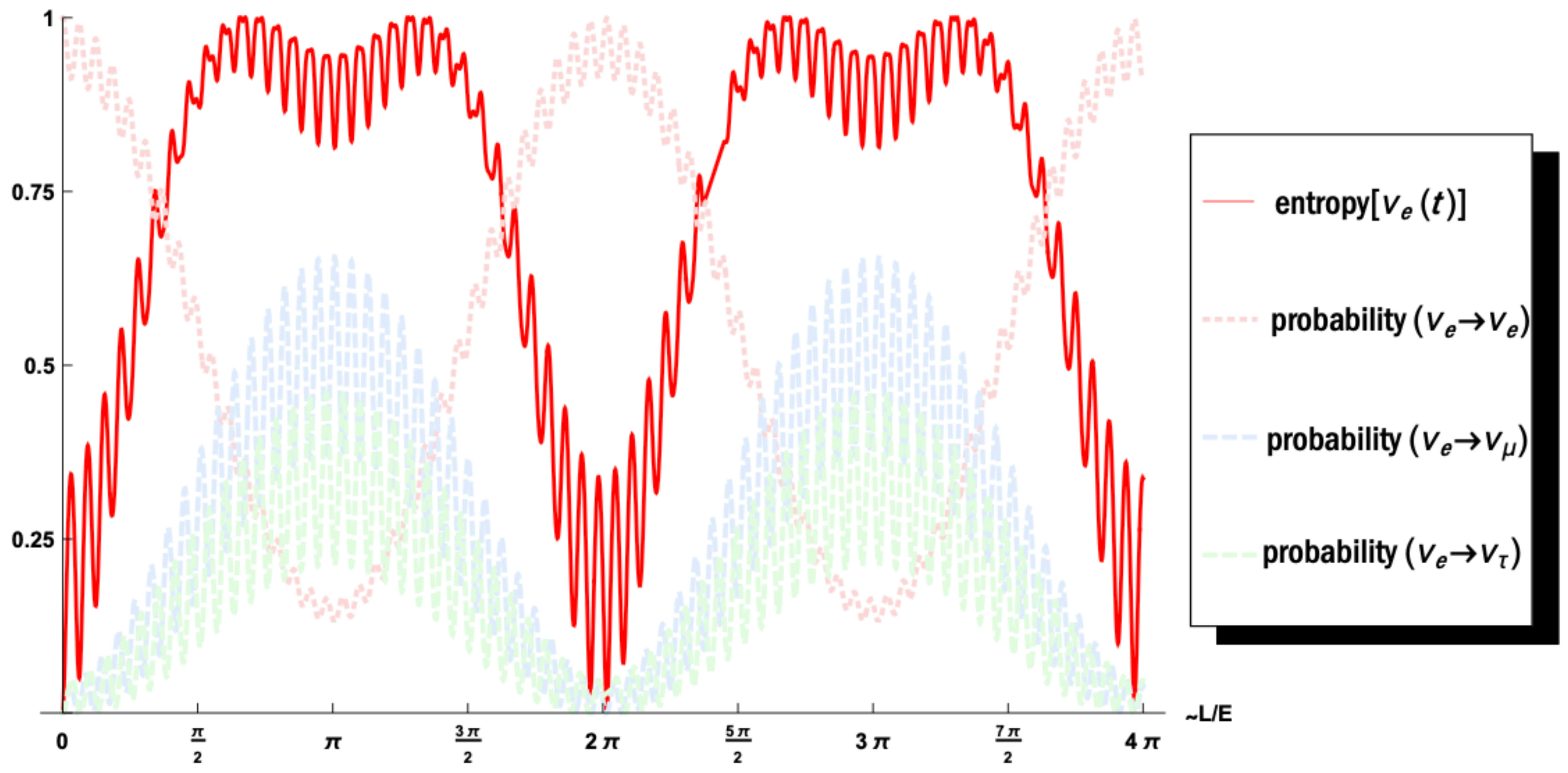
- Related to this, see picture of decays in QM as an open system (Bertelsmann et al., 2006) and also recent paper by Barr on decays as weak measurements 2511.10197.

Neutrino oscillations

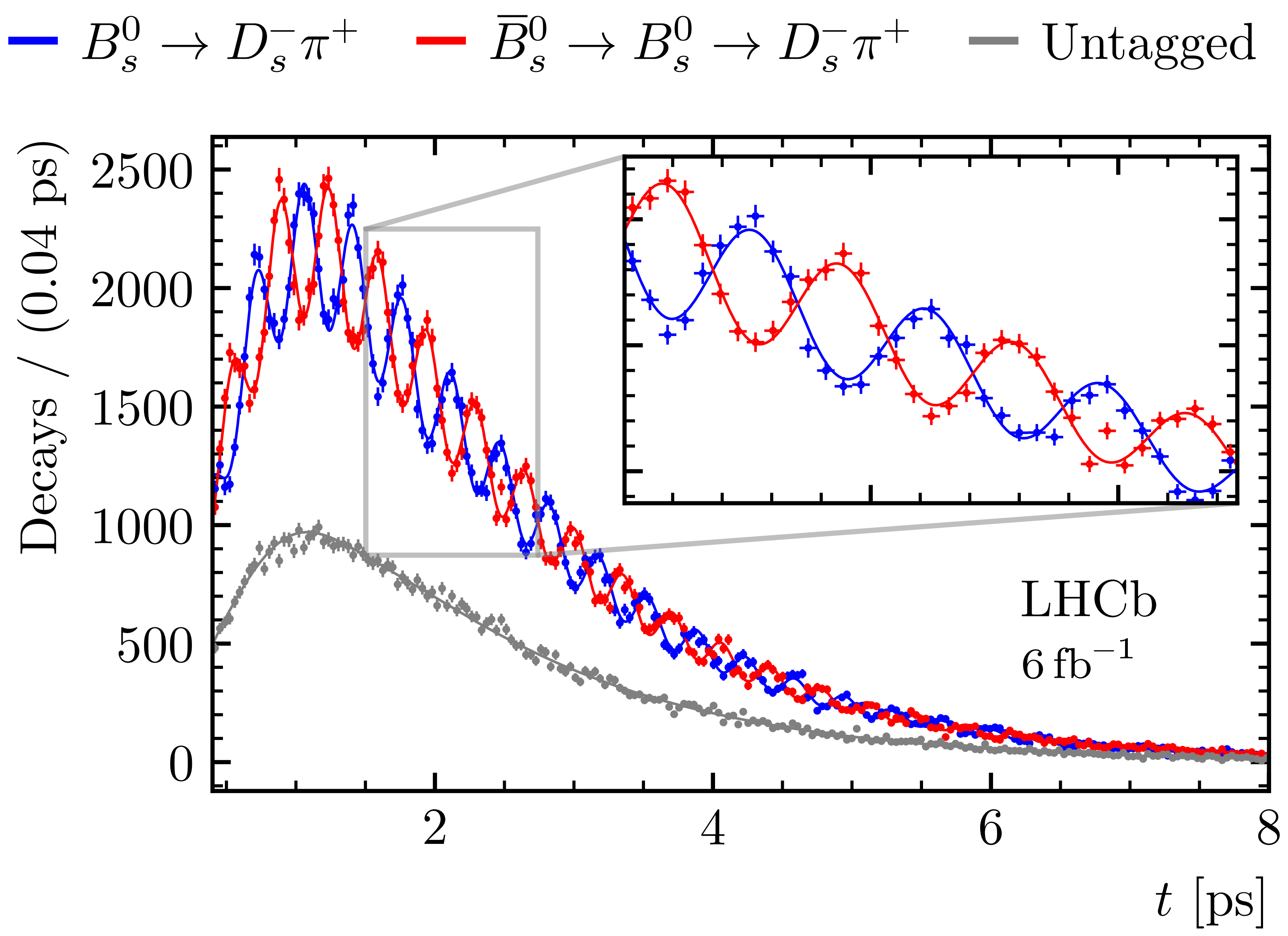


$$|\Psi(t)\rangle_\alpha = a_{\alpha e}(t)\ |100\rangle + a_{\alpha\mu}(t)\ |010\rangle + a_{\alpha\tau}(t)\ |001\rangle$$

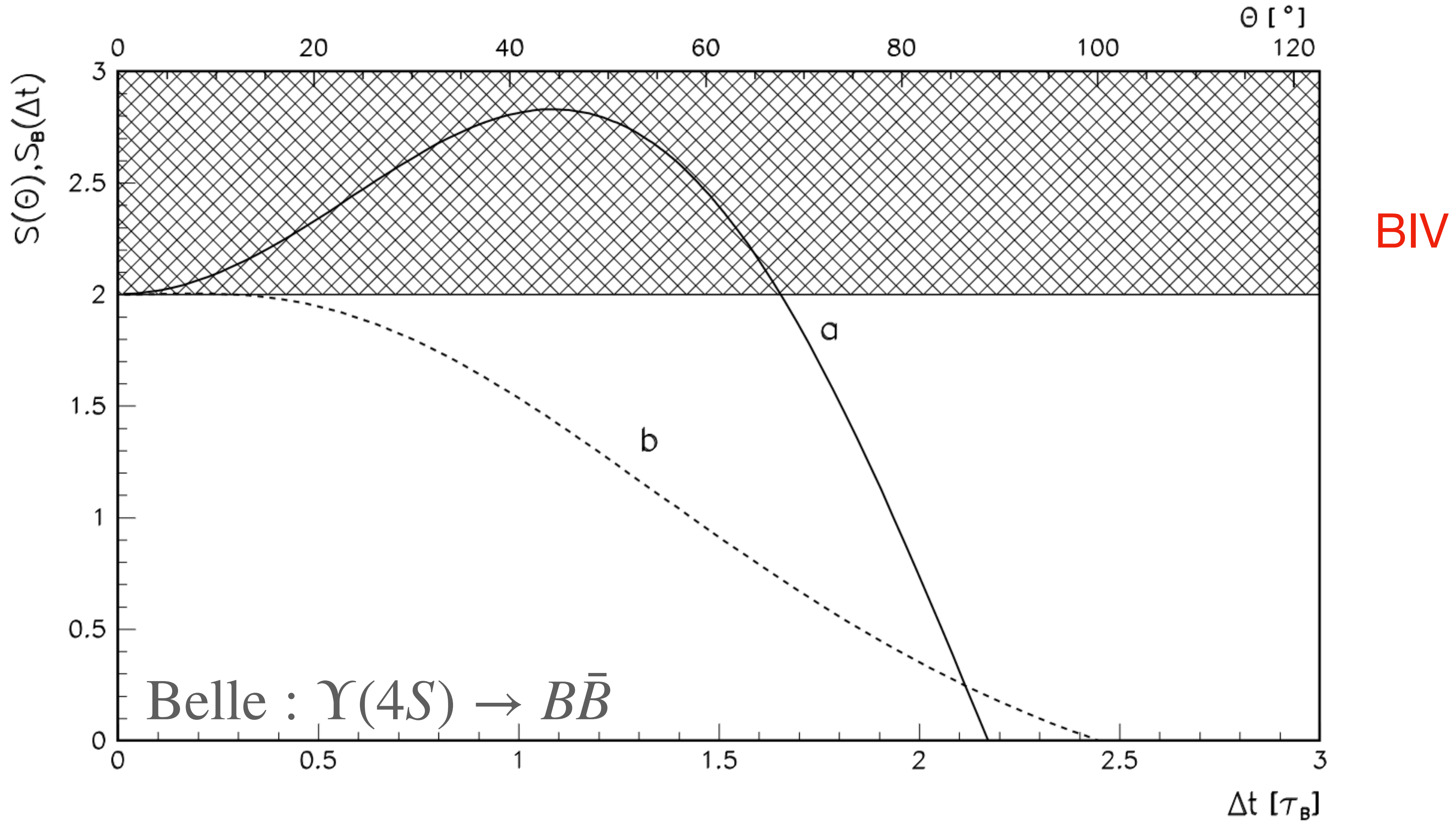
Neutrino oscillations as bi- and tri-partite systems [Blasone et al.] to more recent analyses of neutrino oscillations [Banerjee et al.]. See also [Kumar et al.]. Possibility of using quantum observables to access the mass hierarchy [Dixit et al.], distinguishing between Majorana vs Dirac [Richter et al.]. For a very interesting proposal to use Leggett-Garg violations at different energies was made [Formaggio et al.].



B-flavour oscillations

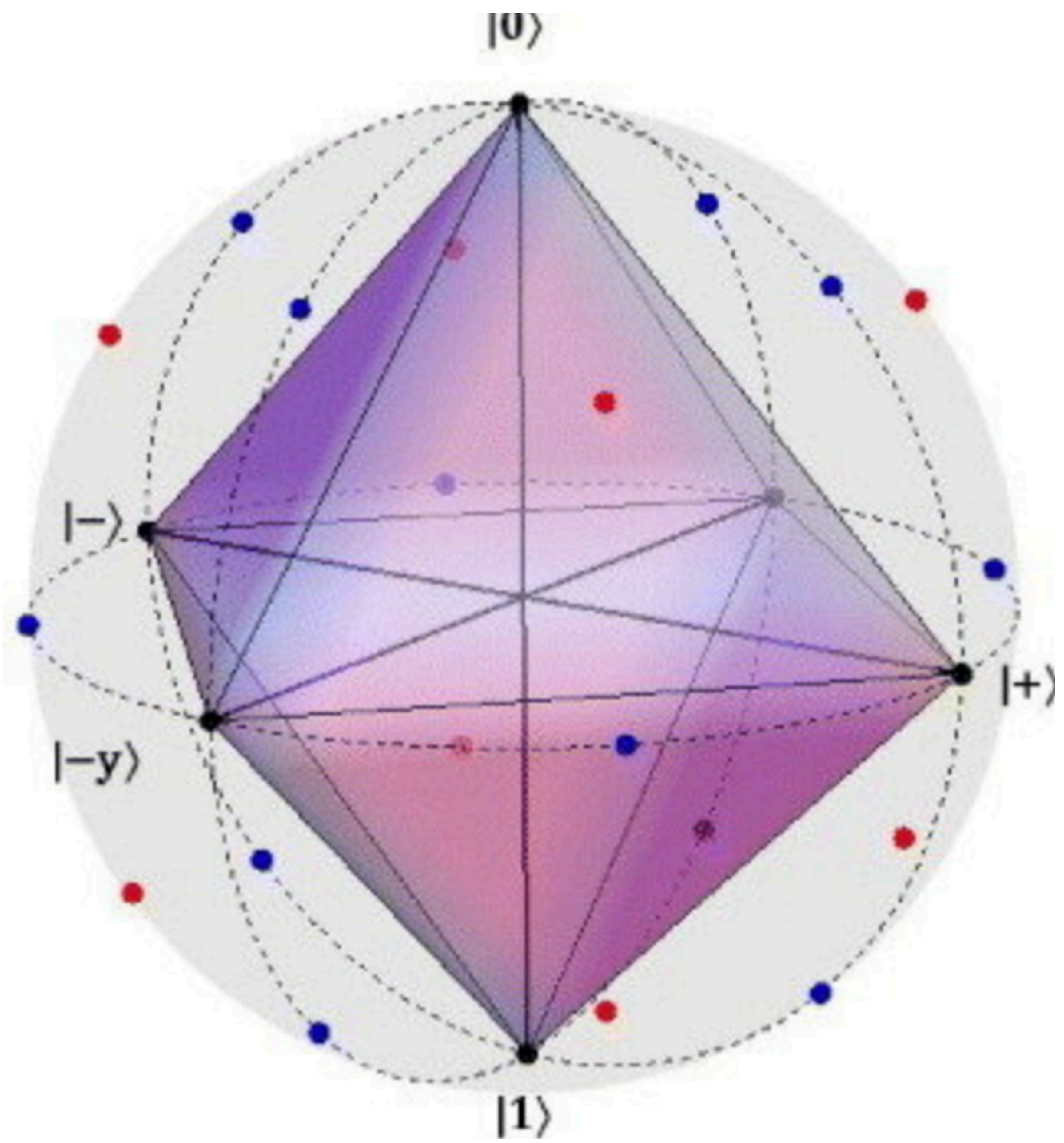


1-leg: Leggett-Garg K3



2-leg : Leggett-Garg K4 (like Bell)

Quantum Magic



•Quantum magic quantifies how much a quantum state lies outside the stabilizer (Clifford) sector.

•Clifford operations (H, S, CNOT, Pauli) acting on stabilizer states are classically simulable (Gottesman–Knill theorem).

•Universal quantum computation requires non-Clifford resources, e.g. the T gate.

•States generated by non-Clifford operations are called magic states.

•Magic is treated as a resource theory: stabilizer states are free, non-stabilizer states carry magic.

•Magic can be quantified by measures such as mana, robustness of magic, or stabilizer Rényi entropy.