The strong coupling and the top-quark mass

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Based on work done in collaboration with:

- *Determination of the strong coupling from high-energy data*
  S. Alekhin, M.V. Garzelli, S. M. and O. Zenaiev arXiv:2510.21435
- *NNLO PDFs driven by top-quark data*
  S. Alekhin, M.V. Garzelli, S. M. and O. Zenaiev arXiv:2407.00545
- *Top-quark pole mass extraction at NNLO accuracy, from total, single- and double-differential cross sections for $t\bar{t} + X$ production at the LHC*
  M.V. Garzelli, J. Mazzitelli, S. M. and O. Zenaiev arXiv:2311.05509
- *NLO PDFs from the ABMP16 fit*
  S. Alekhin, J. Blümlein and S. M. arXiv:1803.07537
- *Parton distribution functions, α_s, and heavy-quark masses for LHC Run II*
  S. Alekhin, J. Blümlein, S. M. and R. Plačakytė arXiv:1701.05838
- Many more papers of ABM and friends ...
  2008 - ...

Top-quark mass

What is the value of the top-quark mass ?

$$m_t = ?$$

Introduction (I)

Classical mechanics

- Measurement of mass through interaction with gravitational force

- Mass is defined as product of density and volume of matter
  - classical concept

Introduction (II)

- Subatomic physics needs different concepts

Special relativity

- Equivalence principle
  $E = mc^2$ Einstein

Standard Model

- Higgs boson gives mass to matter fields via Higgs-Yukawa coupling
  - large top-quark mass m_t

Quantum field theory

- Classical part of QCD Lagrangian

$$\mathcal{L} \;=\; -\frac{1}{4} F^a_{\mu\nu} F_b^{\mu\nu} + \sum_{\text{flavors}} \bar{q}_i \left(\mathrm{i}\not{D} - m_q\right)_{ij} q_j$$

  - field strength tensor $F^a_{\mu\nu}$ and matter fields $q_i, \bar{q}_j$
  - covariant derivative $D_{\mu,ij} = \partial_\mu \delta_{ij} + \mathrm{i} g_s \left(t_a\right)_{ij} A^a_\mu$

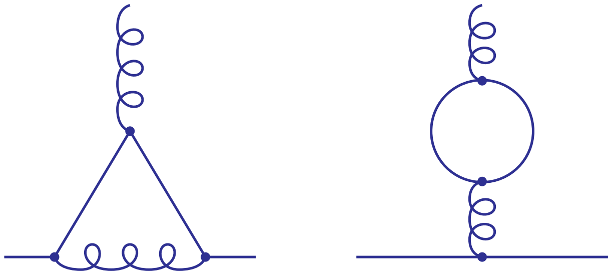
- Formal parameters of the theory (no observables)
  - strong coupling $\alpha_s = g_s^2/(4\pi)$, quark masses m_q
- Parameters of Lagrangian have no unique physical interpretation
  - radiative corrections require definition of renormalization scheme

Challenge

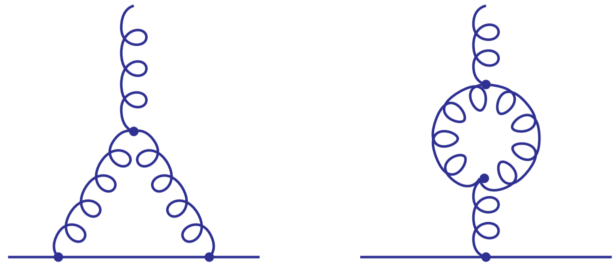
- Suitable observables for measurements of α_s, m_q, ...
  - comparison of theory predictions and experimental data

Coupling renormalization

- Effective coupling α_s depends on resolution
- QCD distinguished by self-interaction of gluons; e.g. vertex corrections



– screening (like in QED)



– anti-screening (color charge of g)

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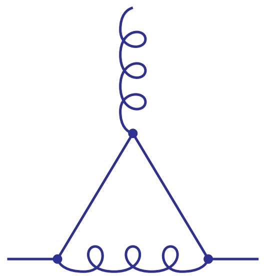

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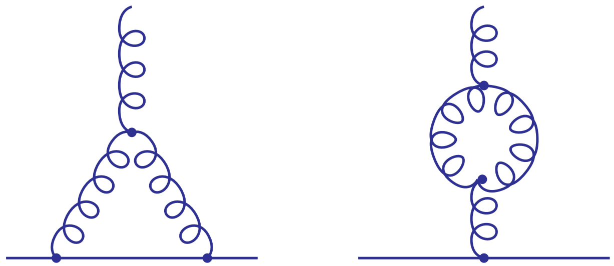


– anti-screening (color charge of g)

- Scale dependence governed by β-function of QCD

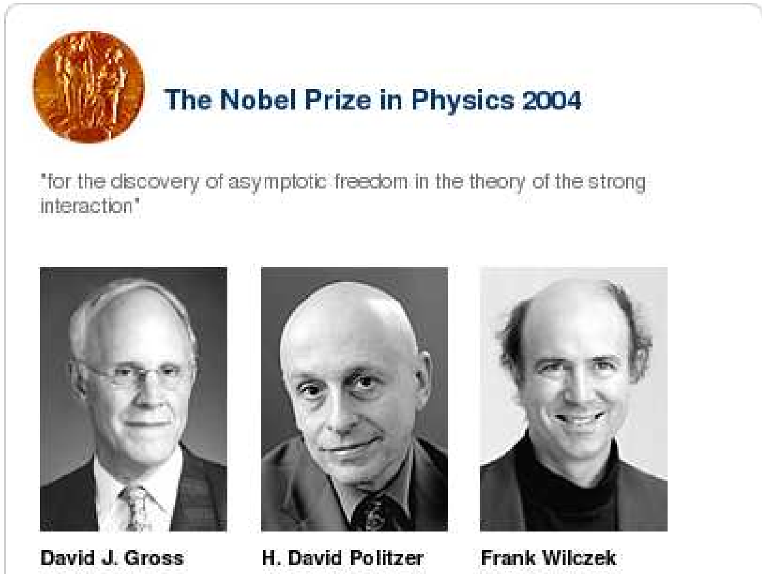
$$\mu^2 \frac{d}{d\mu^2} \alpha_s(\mu) = \beta\left(\alpha_s\right) = -\beta_0 \alpha_s^2 - \beta_1 \alpha_s^3 - \beta_2 \alpha_s^4 - \beta_3 \alpha_s^5 - \dots$$

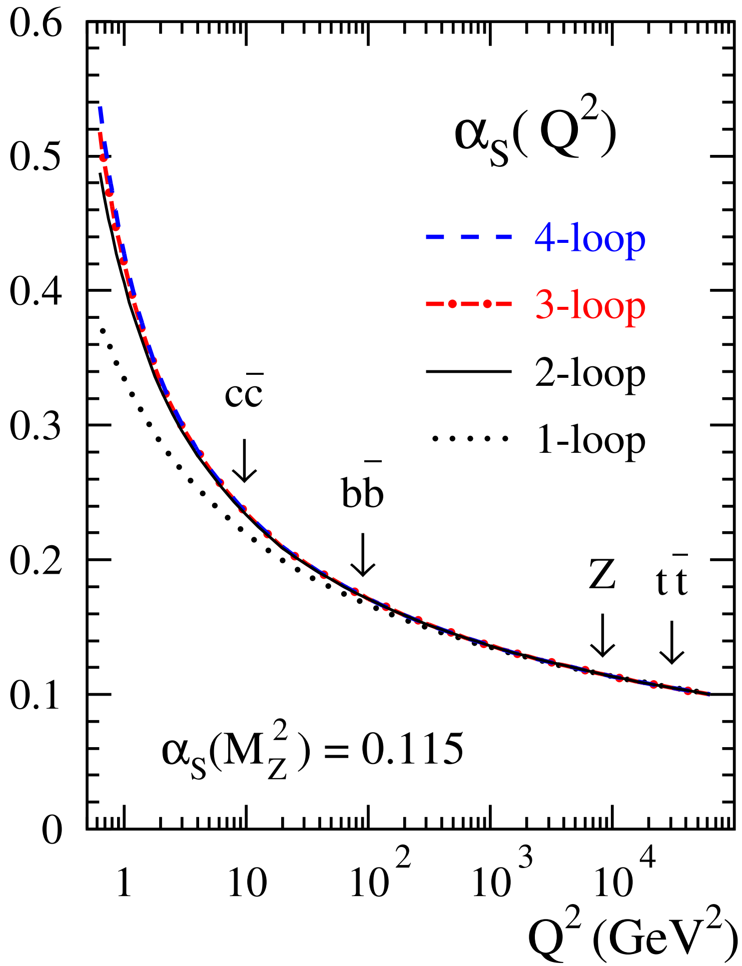
  - QCD β-function has negative sign
  - perturbative expansion with coefficients $\beta_0, \beta_1, \beta_2, \dots$

$$\beta_0 = \frac{1}{4\pi} \left(\frac{11}{3} C_A - \frac{2}{3} n_f \right) = \frac{1}{4\pi} \left(7 \right) \qquad (\text{for } n_f = 6)$$

Asymptotic freedom

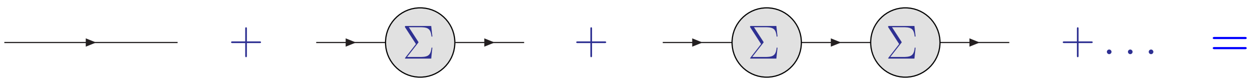
- Perturbative solution of QCD β-function to five loops
  Baikov, Chetyrkin, Kühn '16; Herzog, Ruijl, Ueda, Vermaseren, Vogt '17; Luthe, Maier, Marquard, Schröder '17; Chetyrkin, Falcioni, Herzog, Vermaseren '17
  - very good convergence of perturbative series even at low scales (but $\alpha_s \gg \alpha_{\text{QED}}$)





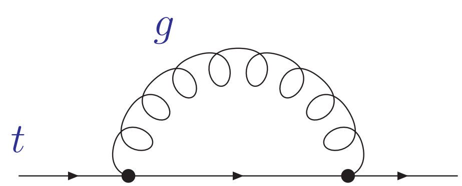
Quark mass renormalization

- Heavy-quark self-energy $\Sigma(p, m_q)$



$$= \frac{\mathrm{i}}{\not{p} - m_q - \Sigma(p, m_q)}$$

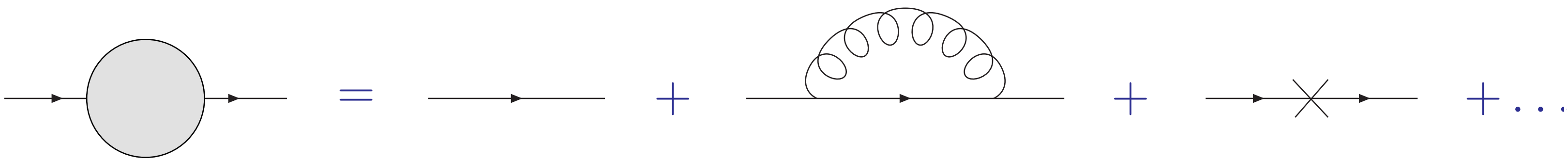
QCD



- QCD corrections to self-energy $\Sigma(p, m_q)$
  - dimensional regularization $D = 4 - 2\epsilon$
  - one-loop: UV divergence $1/\epsilon$ (Laurent expansion)

$$\Sigma^{(1),\mathrm{bare}}(p, m_q) = \frac{\alpha_s}{4\pi} \left(\frac{\mu^2}{m_q^2}\right)^{\epsilon} \left\{ (\not{p} - m_q) \left(-C_F \frac{1}{\epsilon} + \text{fin.} \right) + m_q \left(3C_F \frac{1}{\epsilon} + \text{fin.} \right) \right\}$$

- Relate bare and renormalized mass parameter $m_q^{\mathrm{bare}} = m_q^{\mathrm{ren}} + \delta m_q$



$\Sigma^{\mathrm{ren}}(p, m_q)$ $\quad\quad$ $(Z_\psi - 1)\not{p} - (Z_m - 1) m_q$

Mass renormalization scheme

Pole mass

- Based on (unphysical) concept of top-quark being a free parton
  - $m_q^{\rm ren}$ coincides with pole of propagator at each order

$$\not p - m_q - \Sigma(p, m_q)\Big|_{\not p = m_q} \to \not p - m_q^{\rm pole}$$

- Definition of pole mass ambiguous up to corrections $\mathcal{O}(\Lambda_{QCD})$
  - heavy-quark self-energy $\Sigma(p, m_q)$ receives contributions from regions of all loop momenta – also from momenta of $\mathcal{O}(\Lambda_{QCD})$

$\overline{MS}$ *scheme*

- $\overline{MS}$ mass definition
  - one-loop minimal subtraction

$$\delta m_q^{(1)} = m_q \frac{\alpha_s}{4\pi} 3C_F \left(\frac{1}{\epsilon} - \gamma_E + \ln 4\pi\right)$$

- $\overline{MS}$ scheme induces scale dependence: $m(\mu)$

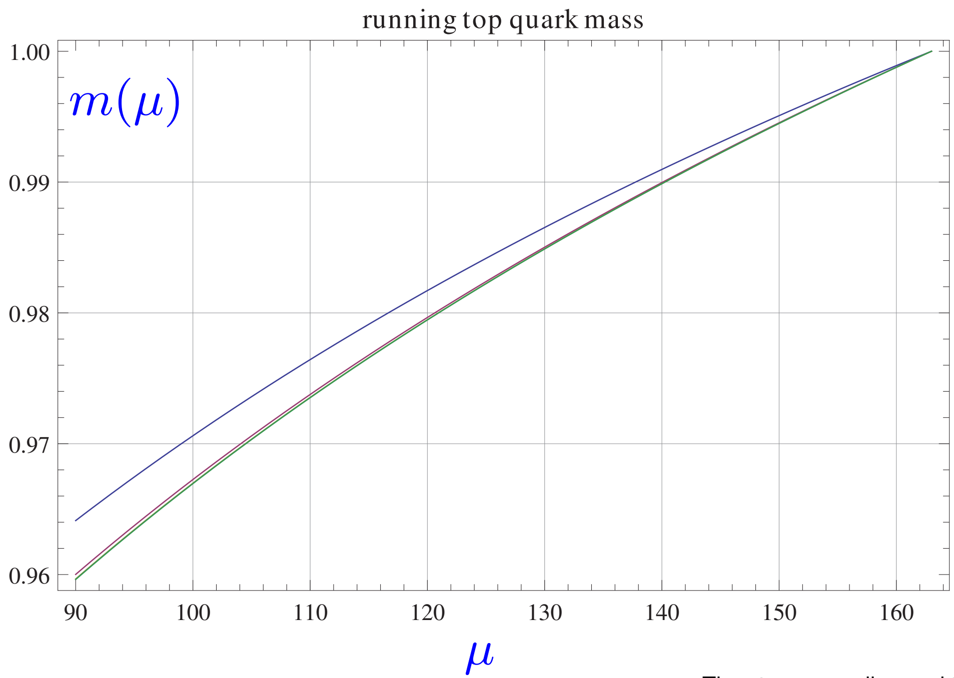
Running quark mass

Scale dependence

- Renormalization group equation for scale dependence
  - mass anomalous dimension γ known to five loops
    Baikov, Chetyrkin, Kühn '14, Luthe, Maier, Marquard, Schröder '17

$$\left(\mu^2 \frac{\partial}{\partial \mu^2} + \beta(\alpha_s)\frac{\partial}{\partial \alpha_s}\right) m(\mu) \;=\; \gamma(\alpha_s) m(\mu)$$

- Plot mass ratio $m_t(163\text{GeV})/m_t(\mu)$



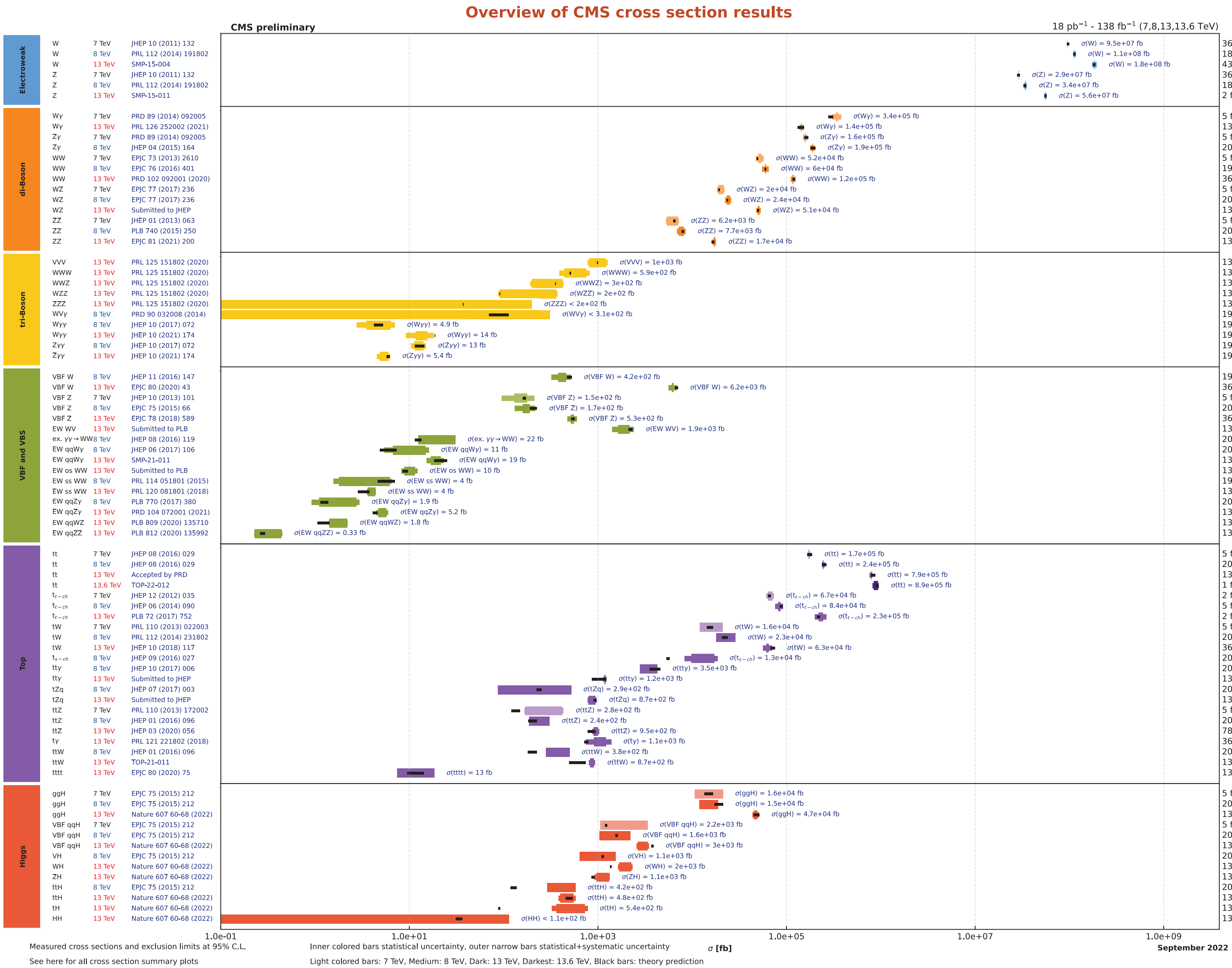
Top-quark measurements

The LHC

- Highest energies at colliders until 203x

Standard Model cross sections

- Standard Model cross sections and predictions at the LHC CMS coll. '22



Data on top-quark cross sections

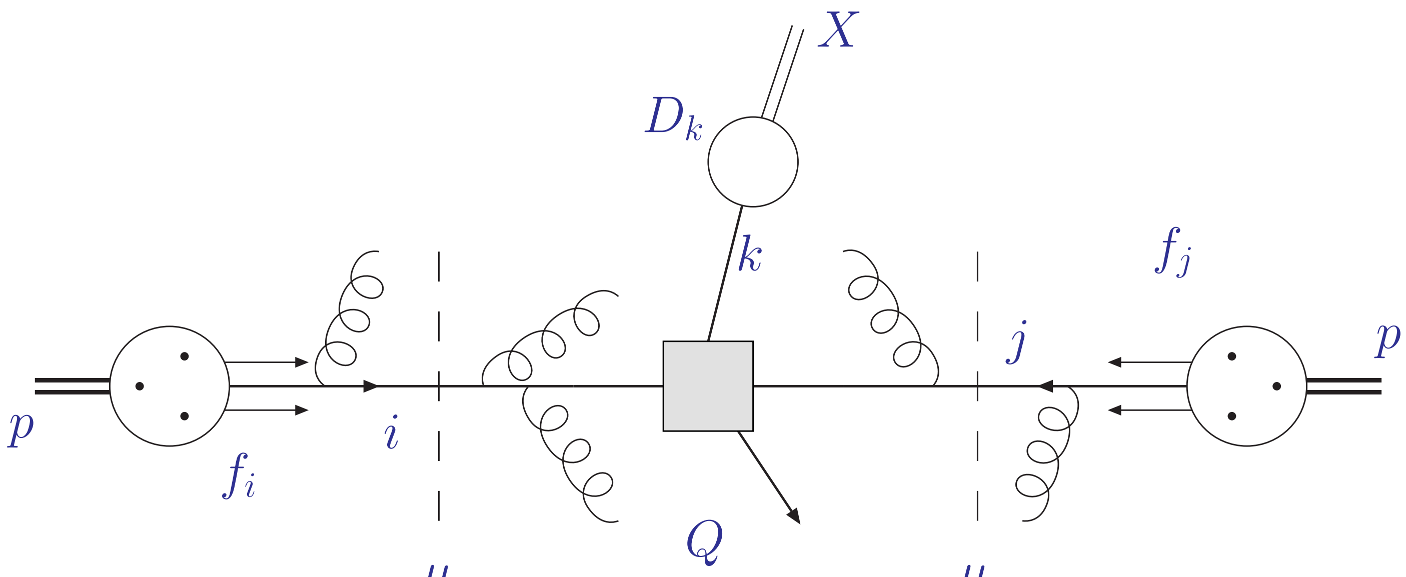
experiment	decay channel	dataset	luminosity	$\sqrt{s}$	ref.
ATLAS & CMS	combined	2011	5 fb^{-1}	7 TeV	2205.13830
ATLAS & CMS	combined	2012	20 fb^{-1}	8 TeV	2205.13830
ATLAS	dileptonic, semileptonic	2011	257 pb^{-1}	5.02 TeV	2207.01354
CMS	dileptonic	2011	302 pb^{-1}	5.02 TeV	2112.09114
ATLAS	dileptonic	2015-2018	140 fb^{-1}	13 TeV	2303.15340
ATLAS	semileptonic	2015-2018	139 fb^{-1}	13 TeV	2006.13076
CMS	dileptonic	2016	35.9 fb^{-1}	13 TeV	1812.10505
CMS	semileptonic	2016-2018	137 fb^{-1}	13 TeV	2108.02803
ATLAS	dileptonic	2022	11.3 fb^{-1}	13.6 TeV	ATLAS-CONF-2023-006
CMS	dileptonic, semileptonic	2022	1.21 fb^{-1}	13.6 TeV	2303.10680

Experiment	decay channel	dataset	luminosity	$\sqrt{s}$	observable(s)	n	ref.
CMS	semileptonic	2016–2018	137 fb^{-1}	13 TeV	$M(t\bar{t})$, $\lvert y(t\bar{t})\rvert$	34	2108.02803
CMS	dileptonic	2016	35.9 fb^{-1}	13 TeV	$M(t\bar{t})$, $\lvert y(t\bar{t})\rvert$	15	1904.05237
ATLAS	semileptonic	2015–2016	36 fb^{-1}	13 TeV	$M(t\bar{t})$, $\lvert y(t\bar{t})\rvert$	19	1908.07305
ATLAS	all-hadronic	2015–2016	36.1 fb^{-1}	13 TeV	$M(t\bar{t})$, $\lvert y(t\bar{t})\rvert$	10	2006.09274
CMS	dileptonic	2012	19.7 fb^{-1}	8 TeV	$M(t\bar{t})$, $\lvert y(t\bar{t})\rvert$	15	1703.01630
ATLAS	semileptonic	2012	20.3 fb^{-1}	8 TeV	$M(t\bar{t})$	6	1511.04716
ATLAS	dileptonic	2012	20.2 fb^{-1}	8 TeV	$M(t\bar{t})$	5	1607.07281
ATLAS	dileptonic	2011	4.6 fb^{-1}	7 TeV	$M(t\bar{t})$	4	1607.07281
ATLAS	semileptonic	2011	4.6 fb^{-1}	7 TeV	$M(t\bar{t})$	4	1407.0371

- Measurements of top-quark hadro-production ATLAS, CMS
  - total inclusive $t\bar{t} + X$ cross sections $(NDP = 10)$
  - differential $t\bar{t} + X$ cross sections in $M(t\bar{t})$, $y(t\bar{t})$ $(NDP = 112)$

QCD factorization

QCD factorization

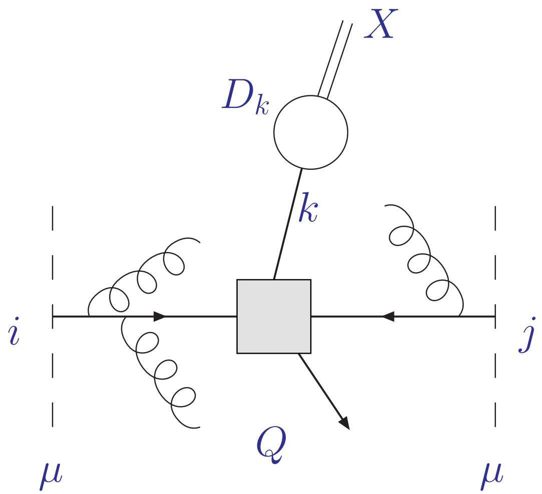


$$\sigma_{pp\to X} = \sum_{ij} f_i(\mu^2) \otimes f_j(\mu^2) \otimes \hat{\sigma}_{ij\to X}\left(\alpha_s(\mu^2), Q^2, \mu^2, m_X^2\right)$$

- Factorization at scale μ
  - separation of sensitivity to dynamics from long and short distances
- Hard parton cross section $\hat{\sigma}_{ij\to X}$ calculable in perturbation theory
  - cross section $\hat{\sigma}_{ij\to k}$ for parton types i, j and hadronic final state X
- Non-perturbative parameters: parton distribution functions f_i, strong coupling α_s, particle masses m_X
  - known from global fits to exp. data, lattice computations, . . .

Hard scattering cross section

- Parton cross section $\hat{\sigma}_{ij\to k}$ calculable pertubatively in powers of α_s
  - known to NLO, NNLO, . . . ($\mathcal{O}(\text{few}\%)$ theory uncertainty)



- Accuracy of perturbative predictions
  - LO (leading order) ($\mathcal{O}(50-100\%)$ unc.)
  - NLO (next-to-leading order) ($\mathcal{O}(10-30\%)$ unc.)
  - NNLO (next-to-next-to-leading order) ($\lesssim$ $\mathcal{O}(10\%)$ unc.)
  - $\mathrm{N^3LO}$ (next-to-next-to-next-to-leading order)
  - . . .

Parton luminosity

- Long distance dynamics due to proton structure

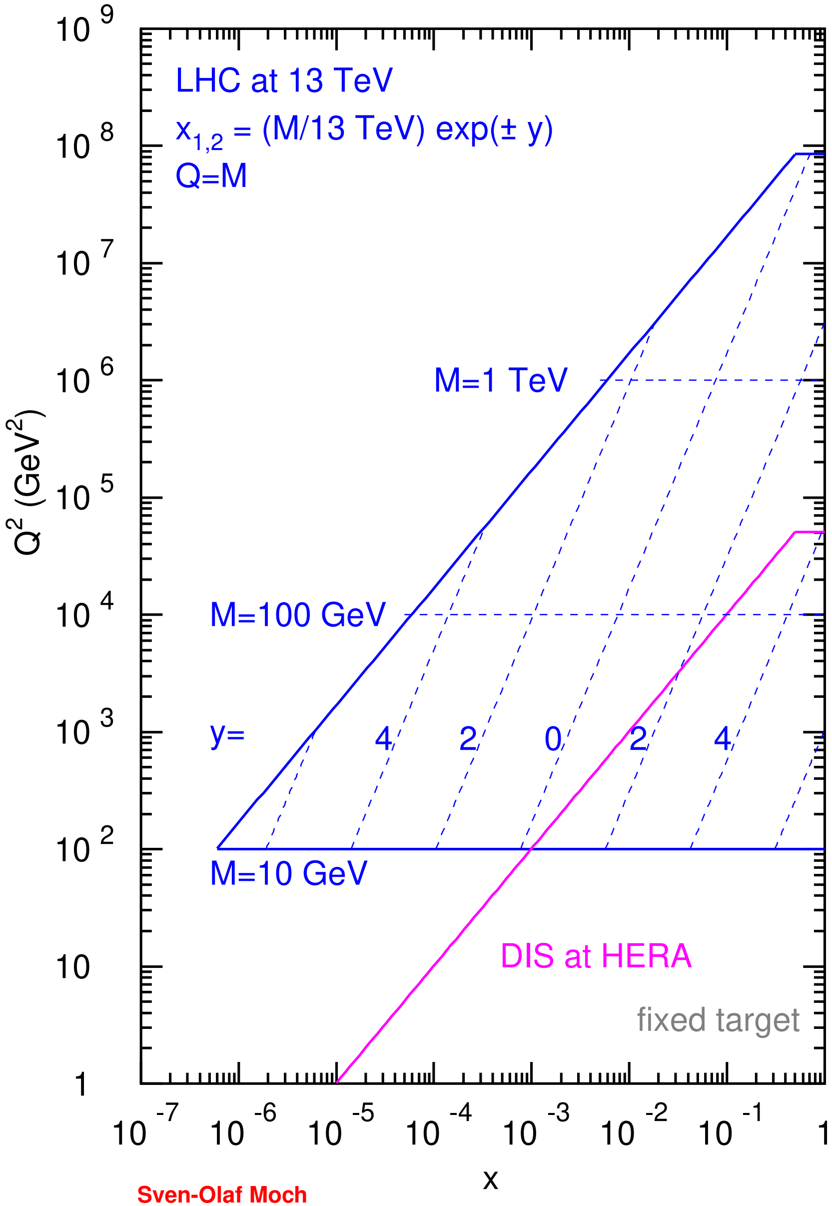
- Cross section depends on parton distributions f_i

$$\sigma_{pp\to X} = \sum_{ij} f_i(\mu^2) \otimes f_j(\mu^2) \otimes \left[\ldots \right]$$

- Parton distributions known from global fits to exp. data
  - available fits accurate to NNLO
  - information on proton structure depends on kinematic coverage

Parton kinematics at LHC

- Information on proton structure depends on kinematic coverage

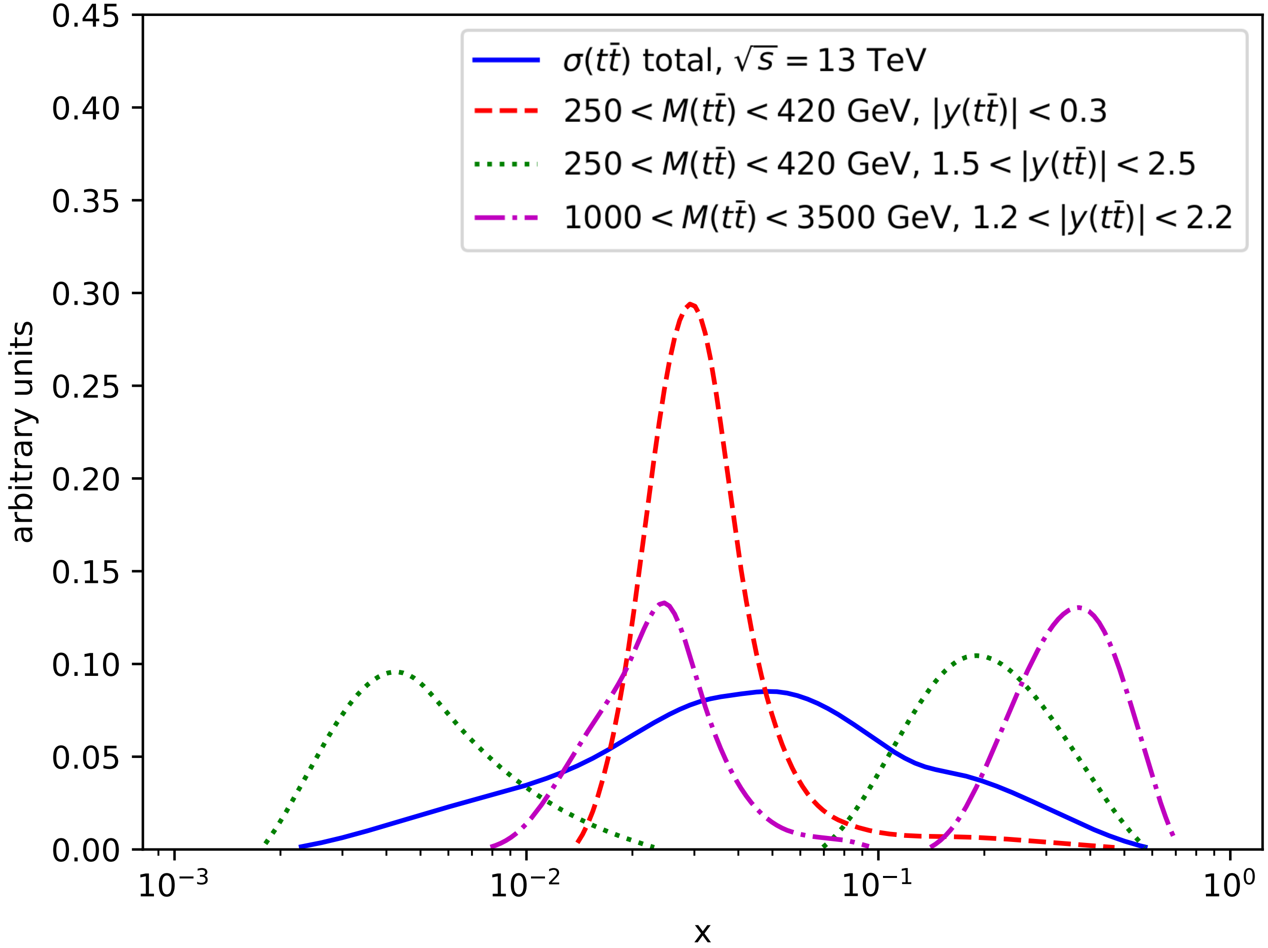


- LHC run at $\sqrt{s}$ = 13 TeV
  - parton kinematics well covered by HERA and fixed target experiments
- Parton kinematics with $x_{1,2} = M/\sqrt{S}\mathrm{e}^{\pm y}$
  - forward rapidities sensitive to small-x
- Cross section depends on convolution of parton distributions
  - small-x part of f_i and large-x PDFs f_j

$$\sigma_{pp\to X} = \sum_{ij} f_i(\mu^2) \otimes f_j(\mu^2) \otimes \left[\ldots\right]$$

Parton kinematics of top quark data

- Parton x values for rapidity bins $y(t\bar{t})$ Alekhin, Garzelli, SM, Zenaiev '24
  - local probes of x values in parton luminosity $f_i(\mu^2) \otimes f_j(\mu^2)$



Theory status

- NNLO QCD differential predictions for top-quark pairs at the LHC
  Czakon, Heymes, Mitov ‘15
- Top-quark pair hadroproduction at NNLO in QCD
  Catani, Devoto, Grazzini, Kallweit, Mazzitelli, Sargsyan ‘19
  - to be implemented in future public release of MATRIX code
    Catani, Devoto, Grazzini, Kallweit, Mazzitelli ‘19
- NNLO event generation for top-quark pair production
  Mazzitelli, Monni, Nason, Re, Wiesemann and Zanderighi ‘20
- Top-pair production at the LHC with MiNNLO_PS
  Mazzitelli, Monni, Nason, Re, Wiesemann and Zanderighi ‘21
- Narrow-width-approximation at NNLO
  - NNLO QCD corrections to leptonic observables in top-quark pair production and decay
    - implemented in private STRIPPER code
      Czakon, Mitov, Poncelet ‘20

Differential cross sections (I)

Challenges

- NNLO codes not easily publicly usable/accessible
- Very long run times (few CPU years) for distributions with fixed input parameters (m_t, PDFs, . . .)
- Accuracy of NNLO subtraction schemes
  - local sector subtraction (STRIPPER)
  - phase space slicing with q_T^{cut} (MATRIX)

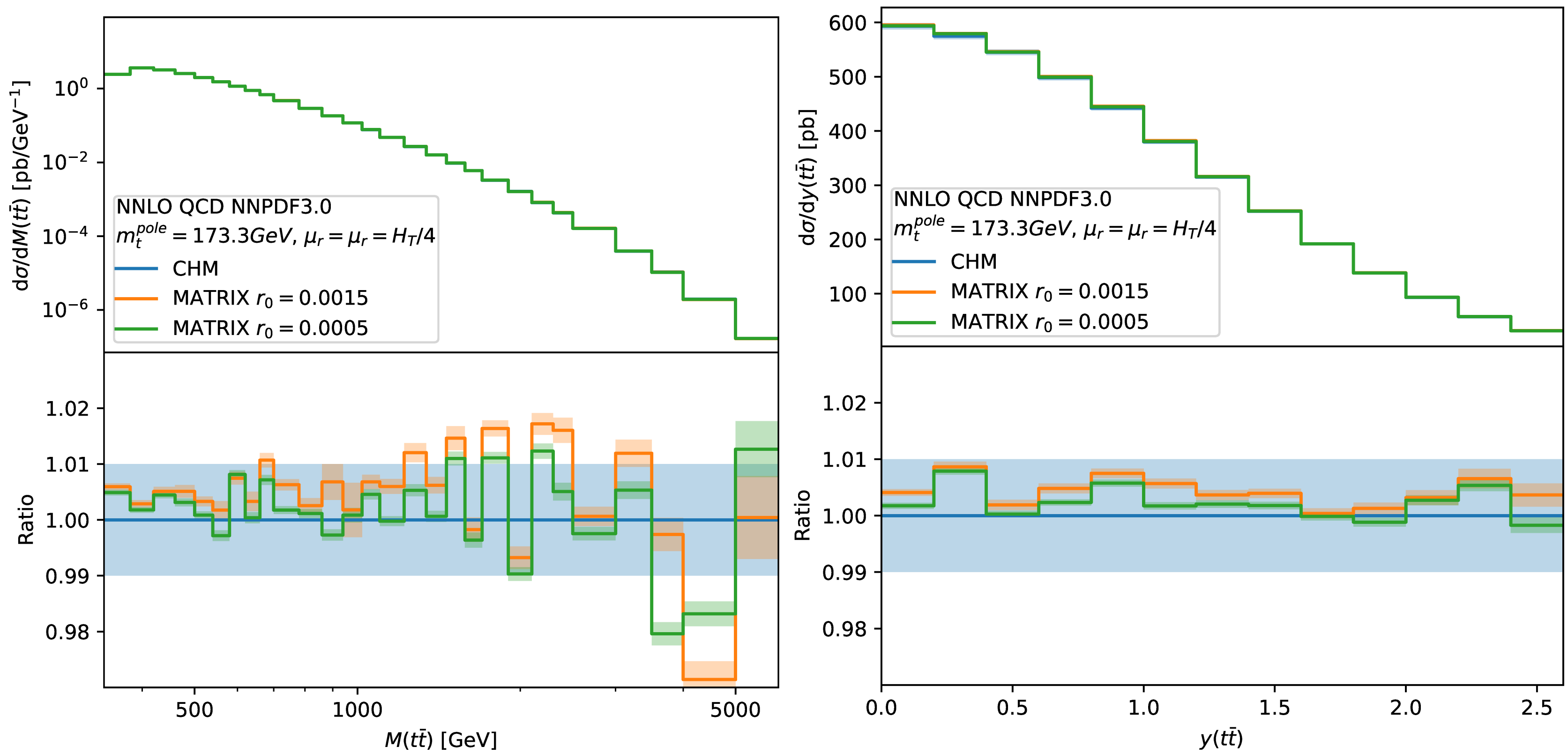
Needs

- NNLO QCD predictions for range of m_t values
- Variation of PDFs (complete set of eigenvectors)

Solution

- Customized version of MATRIX — Garzelli, Mazzitelli, SM, Zenaiev ‘23
  - interface to PineAPPL library for storage of grids

Differential cross sections (II)



- Validation of MATRIX using cuts $r_0 = 0.0015$ and $r_0 = 0.0005$ with results from Czakon, Heymes, Mitov '17 with their numerical uncertainties
- NNLO differential cross sections Garzelli, Mazzitelli, SM, Zenaiev '23
  - left: for invariant mass of $t\bar{t}$-pair
  - right: the rapidity of $t\bar{t}$-pair

Differential cross sections (III)

Grids and tables

- Grids interfaced to PineAPPL library available on
  `https://ploughshare.web.cern.ch/ploughshare/#table1-w`

  ```
  xfitter-cms-ttbar-mt1650-arxiv-2108.02803
  xfitter-cms-ttbar-mt1675-arxiv-2108.02803
  xfitter-cms-ttbar-mt1700-arxiv-2108.02803
  xfitter-cms-ttbar-mt1720-arxiv-2108.02803
  xfitter-cms-ttbar-mt1750-arxiv-2108.02803
  xfitter-cms-ttbar-mt1775-arxiv-2108.02803
  ```

- Tables with experimental $t\bar{t} + X$ data available through xFitter project
  `https://gitlab.cern.ch/fitters/xfitter`

Ultimate goal

- Implementation of complete framework for ABMP PDF fit in xFitter
  - PDFs public available and reproducible

Data in global PDF fits (I)

Data sets considered in ABMP16 analysis

Alekhin, Blümlein, S.M., Placakyte '17

- Analysis of world data for deep-inelastic scattering, fixed-target data for Drell-Yan process and collider data ($W^{\pm}$-, Z-bosons, top-quarks)
  - inclusive DIS data HERA, BCDMS, NMC, SLAC ($NDP = 2155$)
  - semi-inclusive DIS charm-, bottom-quark data HERA ($NDP = 81$)
  - Drell-Yan data (fixed target) E-605, E-866 ($NDP = 158$)
  - neutrino-nucleon DIS (di-muon data) CCFR/NuTeV, CHORUS, NOMAD ($NDP = 232$)
  - $W^{\pm}$-, Z-boson production data D0, ATLAS, CMS, LHCb ($NDP = 172$)
  - inclusive top-quark hadro-production CDF&D0, ATLAS, CMS ($NDP = 24$)

Iterative cycle of PDF fits

i) check of compatibility of new data set with available world data

ii) study of potential constraints due to addition of new data set to fit

iii) perform high precision measurement of PDFs, strong coupling $\alpha_s(M_Z)$ and heavy quark masses m_c, m_b, m_t,

Top-quark data in ABMP fit (I)

ABMPtt PDF fit

Alekhin, Garzelli, S.M., Zenaiev '24

- New PDF fit with differential LHC top-quark data in ABMP framework
  - differential $t\bar{t} + X$ cross sections in $M(t\bar{t})$, $y(t\bar{t})$ ($NDP = 112$)
- Results
  - top-quark $\overline{\text{MS}}$ mass $m_t(m) = 160.6 \pm 0.6$ GeV
  - pole mass $m_t(m) = 170.2 \pm 0.7$ GeV
  - strong coupling $\alpha_s(M_Z) = 0.1150 \pm 0.0009$

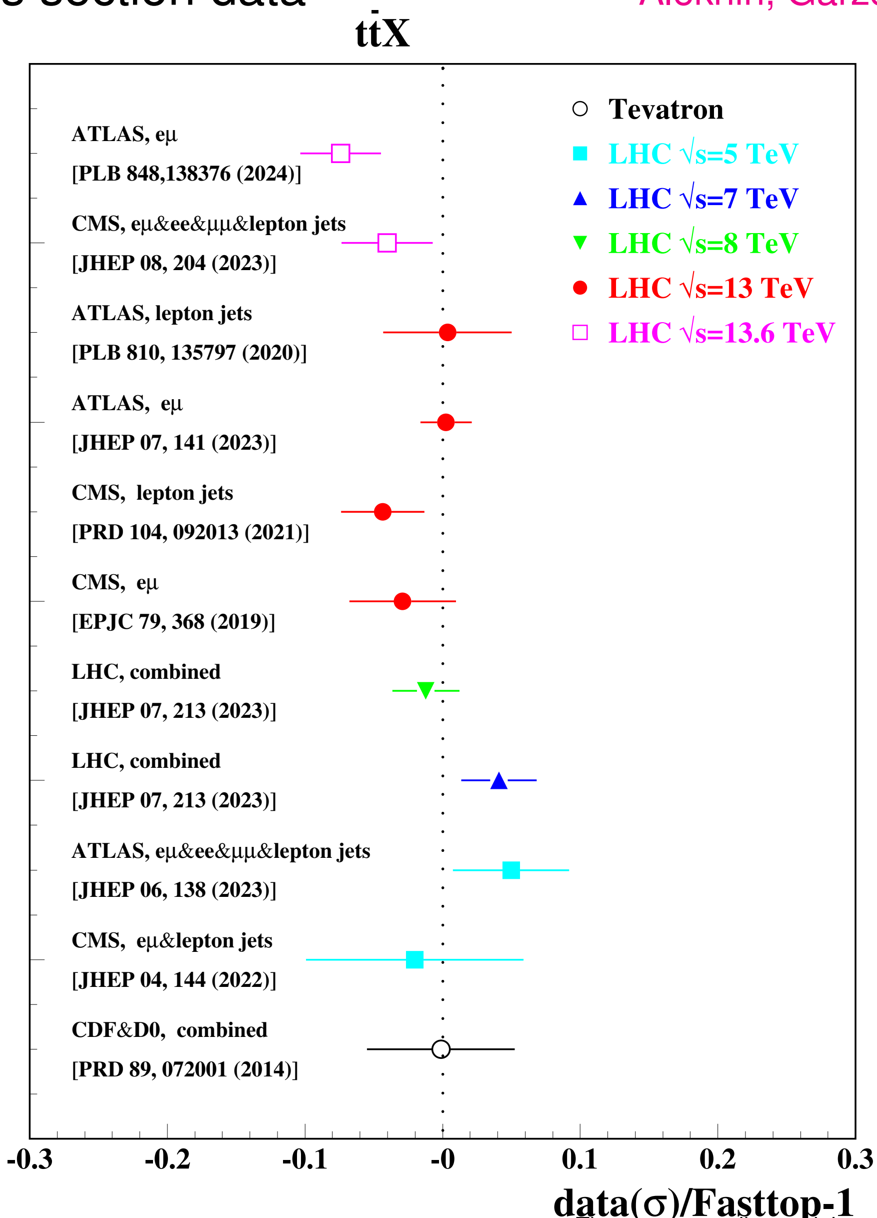
Upshot

- Good compatibilty of PDFs from ABMP16tt and ABMP16
- Confirmation of gluon PDF in range $x \simeq 10^{-2}$ with reduced uncertainties compared to ABMP16
- PDFs sets available in LHAPDF for fixed number of flavors, $n_f = 3, 4, 5$
  ABMPtt_3_nnlo (0+29)
  ABMPtt_4_nnlo (0+29)
  ABMPtt_5_nnlo (0+29)

Top-quark data in ABMPtt fit (II)

- Pulls for total cross section data

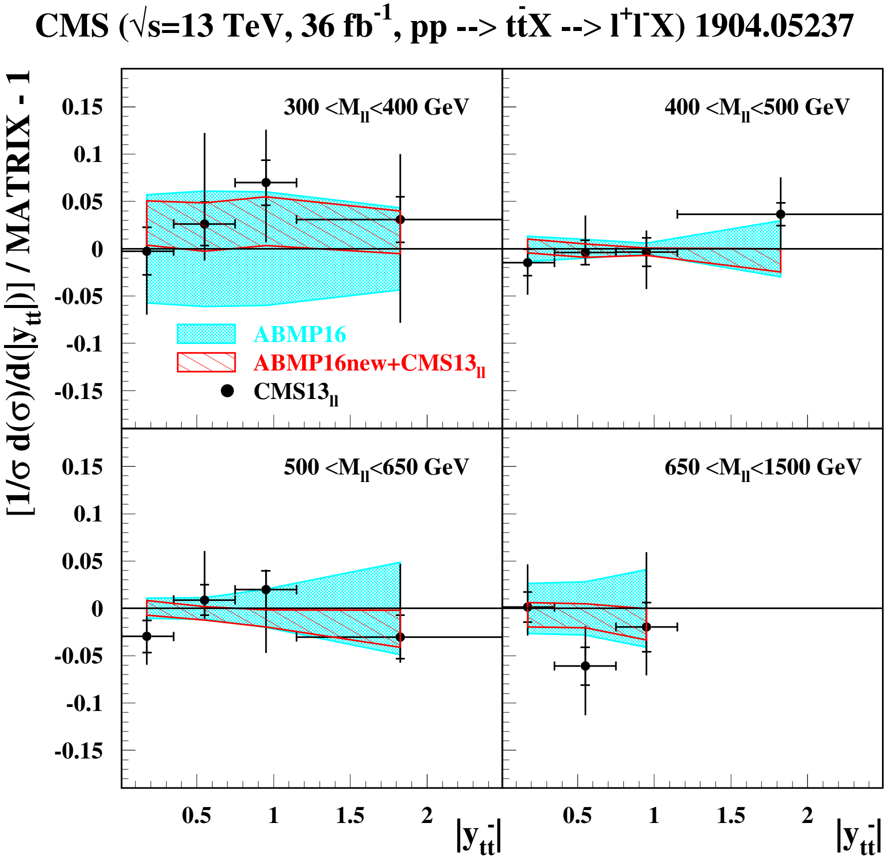
Alekhin, Garzelli, SM, Zenaiev '24

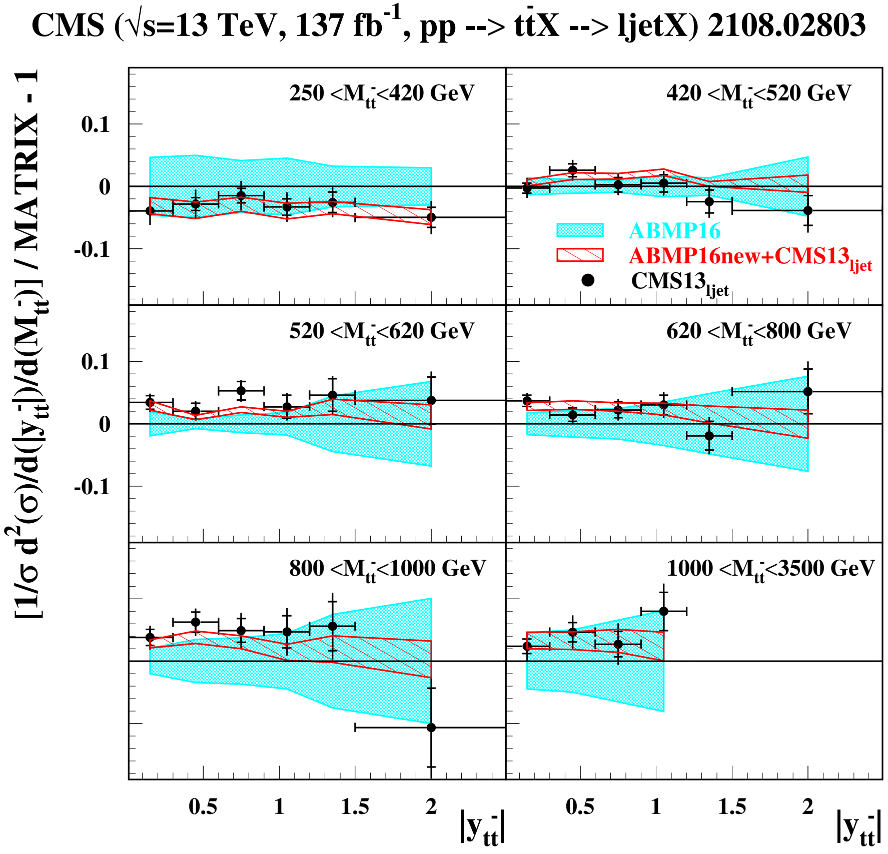


Top-quark data in ABMPtt fit (III)

- Data from CMS

Alekhin, Garzelli, SM, Zenaiev '24

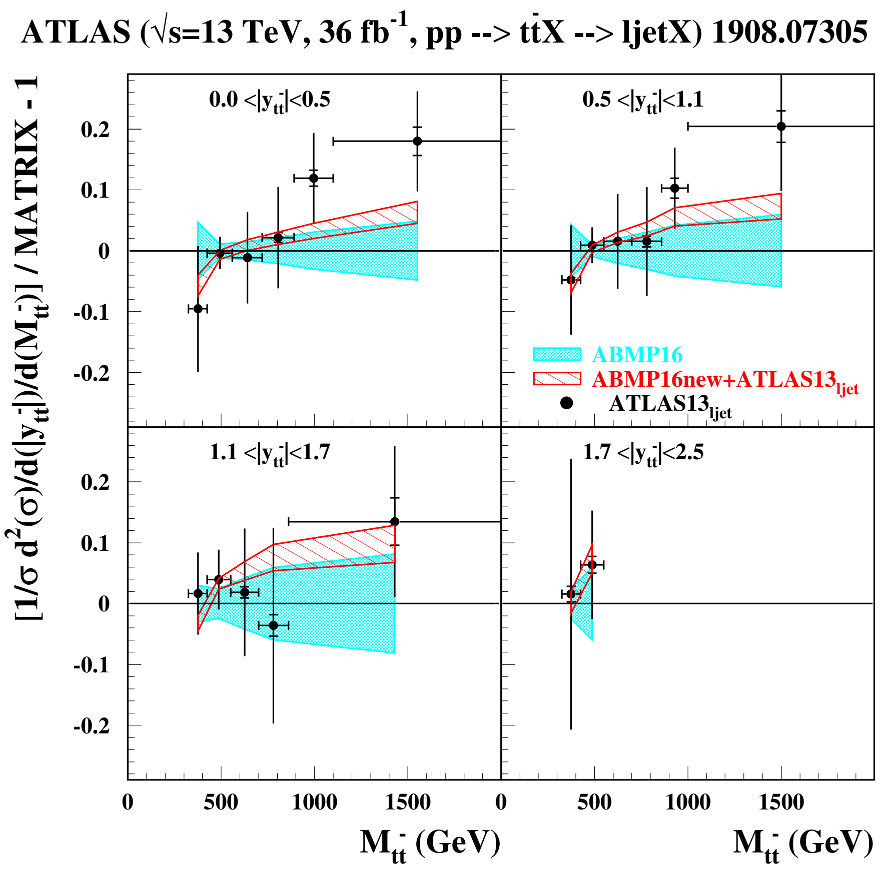


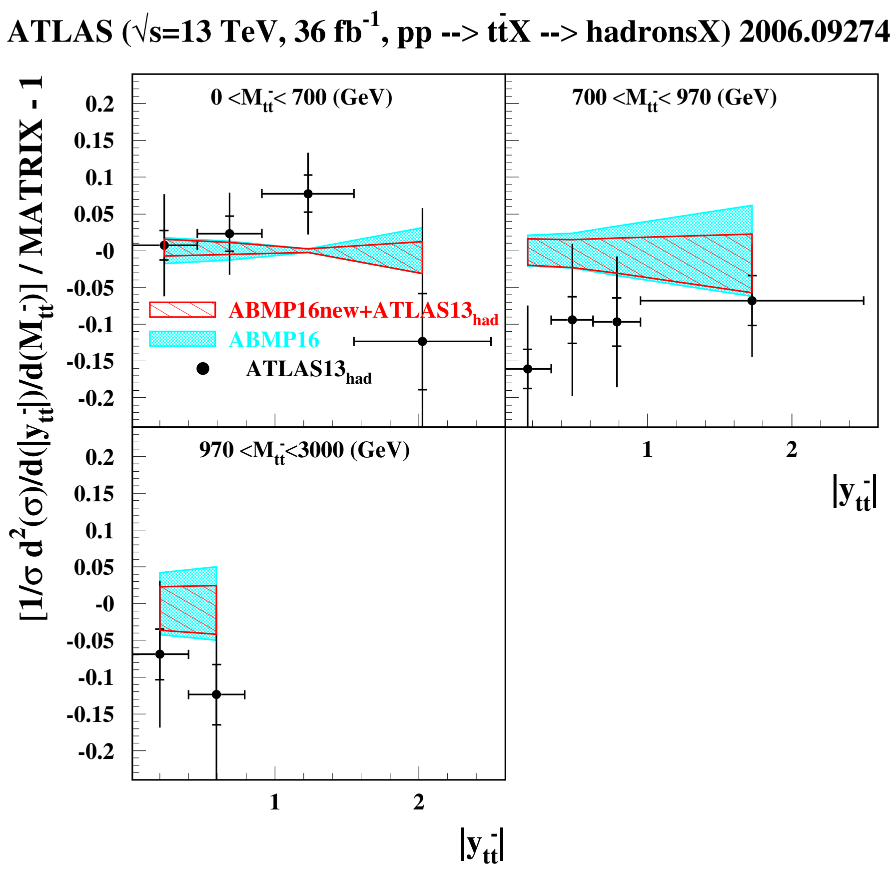


Top-quark data in ABMPtt fit (IV)

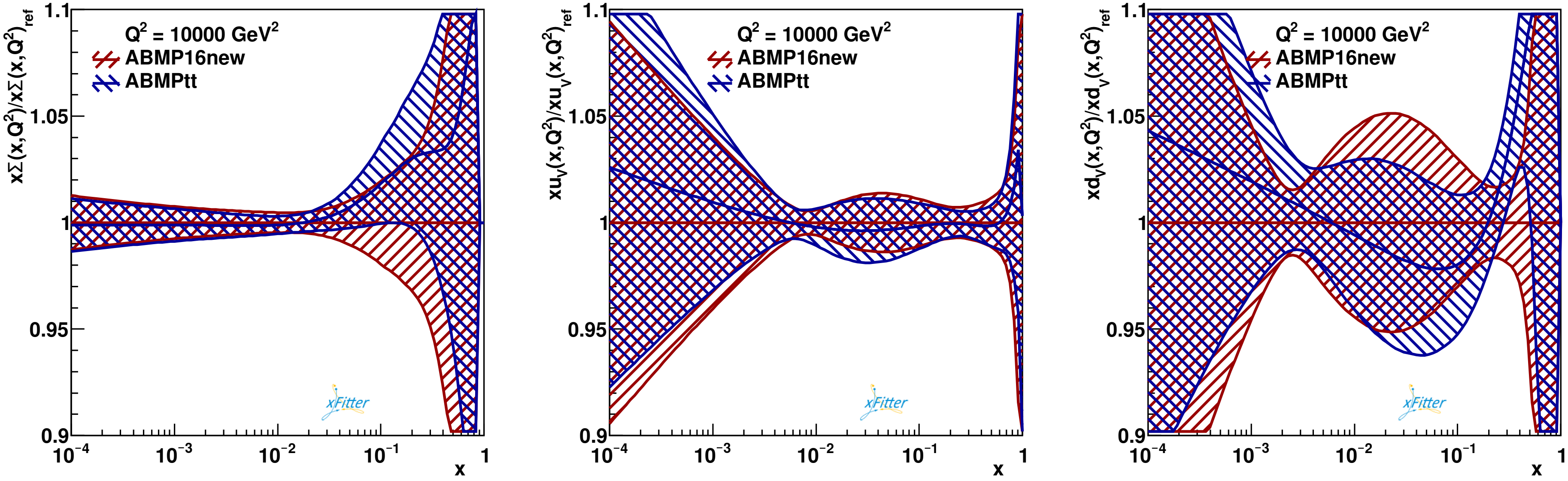
- Data from ATLAS

Alekhin, Garzelli, SM, Zenaiev ‘24



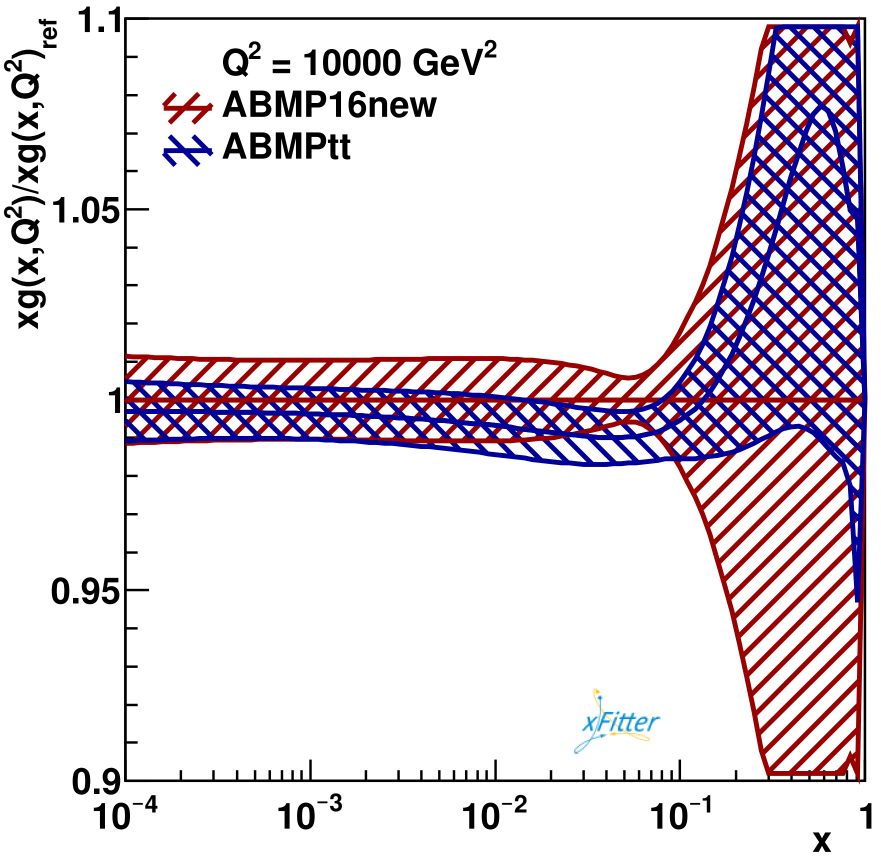


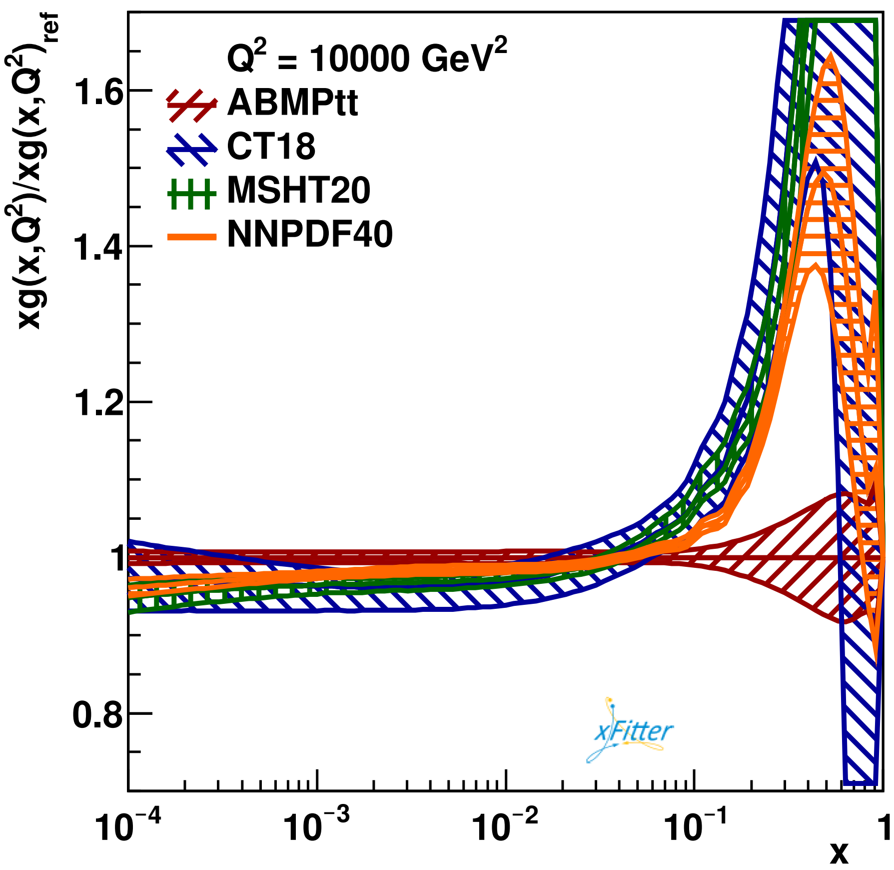
ABMPtt PDFs (I)



- Ratio of PDFs (sea-quark, valence up- and down-quark) at scale $Q = 100$ GeV for $n_f = 5$
  - fit ABMP16tt vs. ABMP16

ABMPtt PDFs (II)





- Ratio of gluon PDF at scale $Q = 100$ GeV for $n_f = 5$
  - fit ABMP16tt vs. ABMP16 $\longrightarrow$ confirmation of gluon PDF
  - comparison to different PDF sets CT18, MHST20, NNPDF4.0

Strong coupling

Strong coupling (I)

Essential facts

- $\alpha_s(M_Z)$ from e^+e^- data high
- $\alpha_s(M_Z)$ from DIS data low
- NLO QCD theory
- World average 1992
  $\alpha_s(M_Z) = 0.117 \pm 0.004$

Bethke, Catani CERN TH-6484/92

	Process	Ref.	Q [GeV]	$\alpha_s(Q)$	$\alpha_s(M_{Z^0})$	$\Delta\alpha_s(M_{Z^0})$ exp.	$\Delta\alpha_s(M_{Z^0})$ theor.	order of perturb.
1	R_τ [LEP]	[7-10]	1.78	$0.318\ ^{+\,0.048}_{-\,0.039}$	$0.117\ ^{+\,0.006}_{-\,0.005}$	$^{+\,0.003}_{-\,0.004}$	$^{+\,0.005}_{-\,0.004}$	NNLO
2	R_τ [world]	[2]	1.78	0.32 ± 0.04	$0.118\ ^{+\,0.004}_{-\,0.006}$	–	–	NNLO
3	DIS [ν]	[3]	5.0	$0.193\ ^{+\,0.019}_{-\,0.018}$	$0.111\ ^{+\,0.006}_{-\,0.007}$	$^{+\,0.004}_{-\,0.006}$	0.004	NLO
4	DIS [μ]	[12]	7.1	0.180 ± 0.014	0.113 ± 0.005	0.003	0.004	NLO
5	$J/\Psi, \Upsilon$ decay	[4]	10.0	$0.167\ ^{+\,0.015}_{-\,0.011}$	$0.113\ ^{+\,0.007}_{-\,0.005}$	–	–	NLO
6	e^+e^- [σ_{had}]	[14]	34.0	0.163 ± 0.022	0.135 ± 0.015	–	–	NNLO
7	e^+e^- [shapes]	[15]	35.0	0.14 ± 0.02	0.119 ± 0.014	–	–	NLO
8	$p\bar{p} \to b\bar{b}X$	[11]	20.0	$0.136\ ^{+\,0.025}_{-\,0.024}$	$0.108\ ^{+\,0.015}_{-\,0.014}$	0.006	$^{+\,0.014}_{-\,0.013}$	NLO
9	$p\bar{p} \to W$ *jets*	[13]	80.6	0.123 ± 0.027	0.121 ± 0.026	0.018	0.020	NLO
10	$\Gamma(Z^0 \to \text{had.})$	[5]	91.2	0.133 ± 0.012	0.133 ± 0.012	0.012	$^{+\,0.003}_{-\,0.001}$	NNLO
11	Z^0 ev. shapes							
	ALEPH	[7]	91.2	$0.119\ ^{+\,0.008}_{-\,0.010}$		–	–	NLO
	DELPHI	[8]	91.2	0.113 ± 0.007		0.002	0.007	NLO
	L3	[9]	91.2	0.118 ± 0.010		–	–	NLO
	OPAL	[10]	91.2	$0.122\ ^{+\,0.006}_{-\,0.005}$		0.001	$^{+\,0.006}_{-\,0.005}$	NLO
	SLD	[6]	91.2	$0.120\ ^{+\,0.015}_{-\,0.013}$		0.009	$^{+\,0.012}_{-\,0.009}$	NLO
	Average	[6-10]	91.2		0.119 ± 0.006	0.001	0.006	NLO
12	Z^0 ev. shapes							
	ALEPH	[7]	91.2	0.125 ± 0.005		0.002	0.004	resum.
	DELPHI	[8]	91.2	0.122 ± 0.006		0.002	0.006	resum.
	L3	[9]	91.2	0.126 ± 0.009		0.003	0.008	resum.
	OPAL	[10]	91.2	$0.122\ ^{+\,0.003}_{-\,0.006}$		0.001	$^{+\,0.003}_{-\,0.006}$	resum.
	Average	[7-10]	91.2		0.123 ± 0.005	0.001	0.005	resum.

Table 1: Summary of measurements of α_s. For details see text.

2

Strong coupling (II)

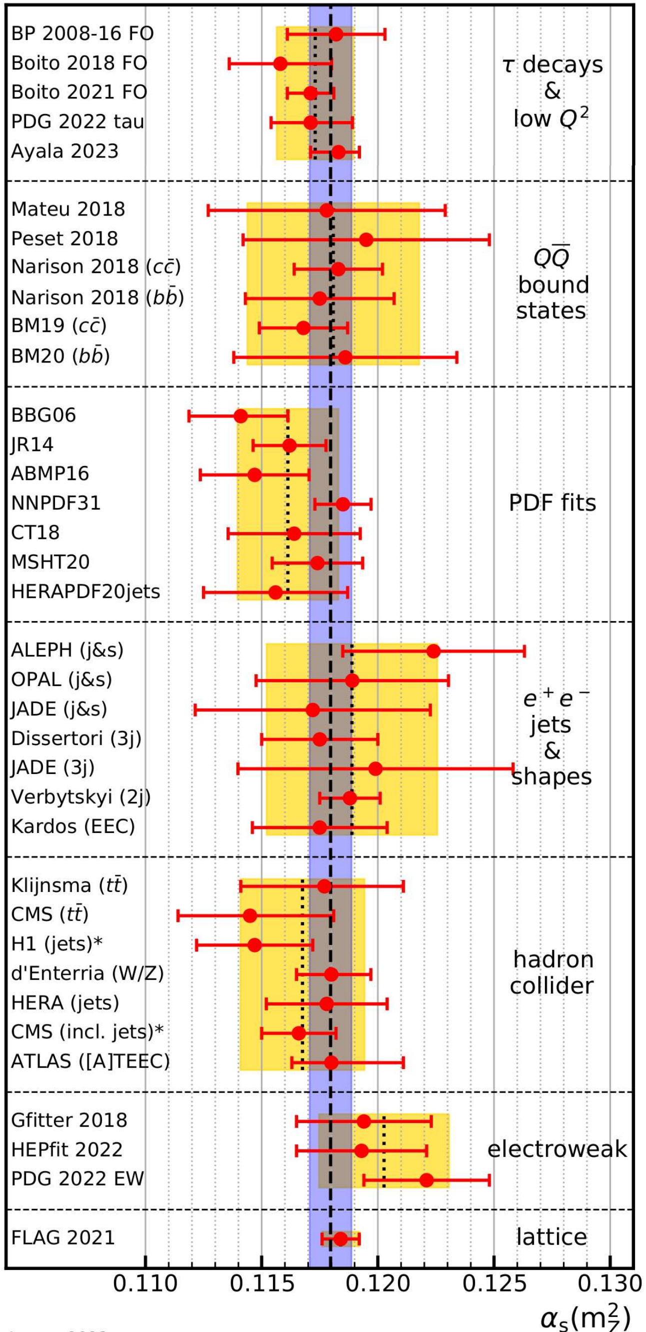
α_s *2024*

- $\alpha_s(M_Z)$ from e^+e^- data high
- $\alpha_s(M_Z)$ from DIS data low
- NNLO QCD theory + lattice

FCC-ee

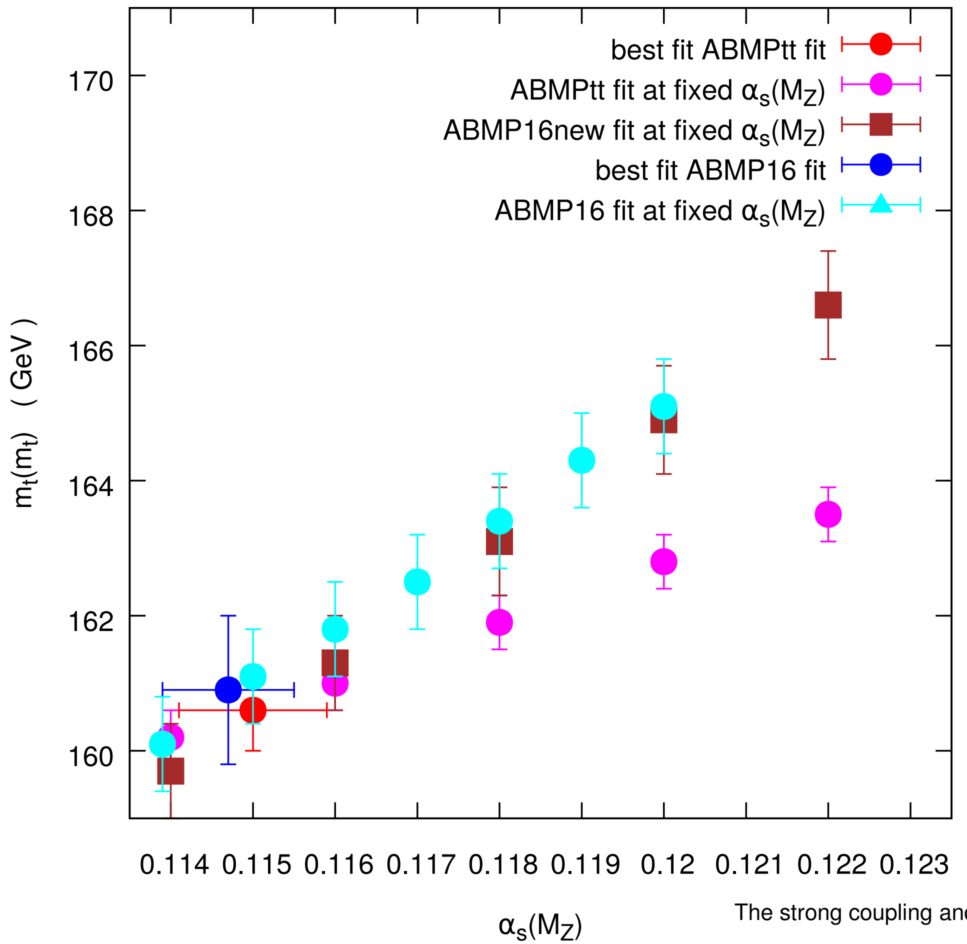
- Potential of FCC-ee: $\alpha_s(M_Z)$ at 1% precision

PDG 2024

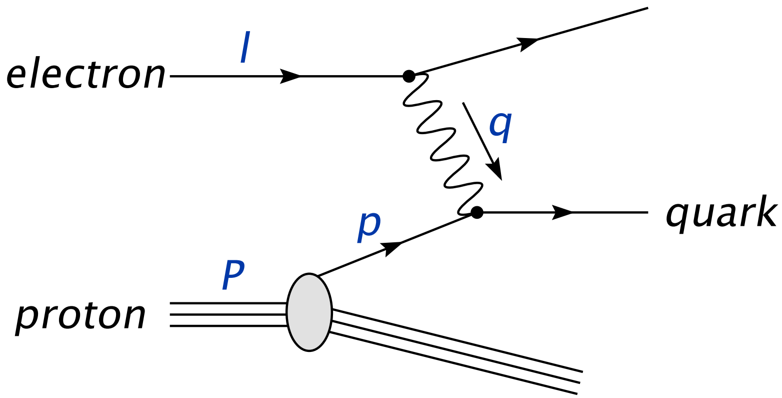


Strong coupling (III)

- Correlation of $\alpha_s(M_Z)$ and m_t values — Alekhin, Garzelli, SM, Zenaiev '24
  - full correlations with PDFs need to be taken into account
  - data on differential $t\bar{t} + X$ cross sections decreases correlation
  - best fit in ABMPtt:
    $\alpha_s(M_Z) = 0.1150 \pm 0.0009$ and $m_t = 170.2 \pm 0.7$ GeV



Theory description of deep-inelastic scattering



Kinematic variables

- momentum transfer $Q^2 = -q^2$
- Bjorken variable $x = Q^2/(2p \cdot q)$

- Structure functions (up to order $\mathcal{O}(1/Q^2)$)

$$F_a\left(x, Q^2\right) = \sum_i \left[C_{a,i}\left(\alpha_s(\mu^2), \mu^2/Q^2\right) \otimes PDF\left(\mu^2\right)\right](x)$$

- Perturbative expansion of coefficient functions up to N^{3}LO

$$C_{a,i} = \alpha_s^n \left(c_{a,i}^{(0)} + \alpha_s c_{a,i}^{(1)} + \alpha_s^2 c_{a,i}^{(2)} + \alpha_s^3 c_{a,i}^{(3)} + \ldots\right)$$

- Application to DIS data requires careful consideration of kinematic region in Q^2 and x
  - invariant mass of the hadronic system $W^2 = M_P^2 + Q^2(1-x)/x$
  - cuts $W^2 \geq 12.5\ \mathrm{GeV}^2$ and $Q^2 \geq 2.5 \div 10\ \mathrm{GeV}^2$

- Additional corrections for $F_a\,(x,Q^2)$ necessary dependent on cuts
  - higher twist and target mass corrections

Higher twist

- Operator product expansion predicts inifinte tower of $(1/Q^2)^n$ of power corrections (higher twist terms)
- Physical interpretation as multi-parton correlations
- Higher twist terms modify structure functions (up to order $\mathcal{O}(1/Q^4)$)

$$F_i^{\rm ht}(x,Q^2) \;=\; F_i^{\rm TMC}(x,Q^2) + \frac{H_i^{\tau=4}(x)}{Q^2}\,, \qquad\qquad i \;=\; 2,T$$

Target mass corrections

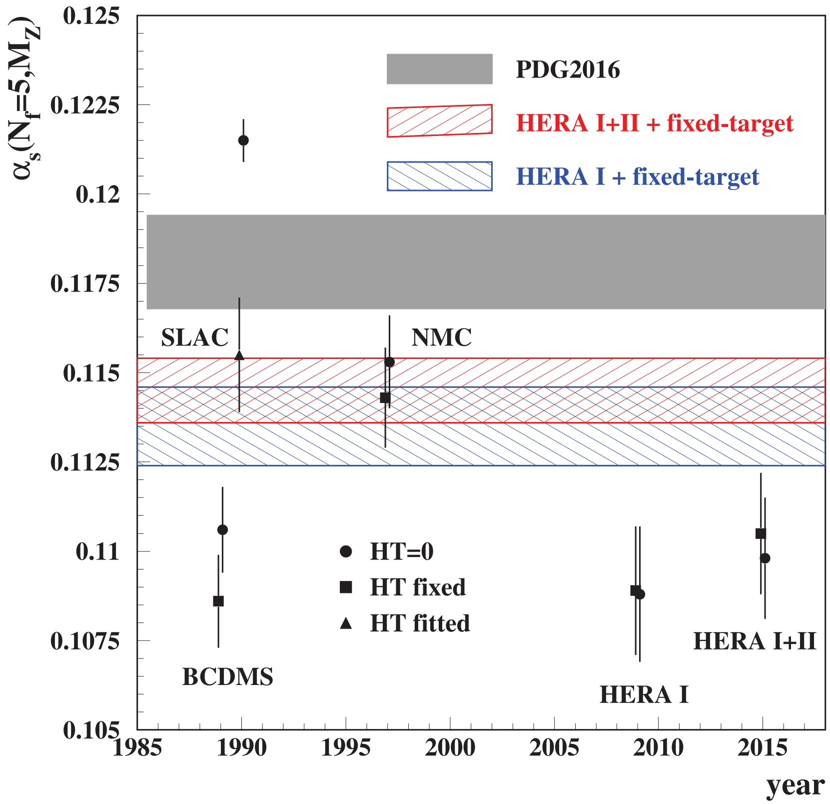
- Finite nucleon mass leads to target mass corrections up to $\mathcal{O}(M_N^2/Q^2)$

$$F_2^{\rm TMC}(x,Q^2) \;=\; \frac{x^2}{\xi^2\gamma^3}\,F_2(\xi,Q^2) + 6\,\frac{x^3 M_N^2}{Q^2\gamma^4}\int\limits_{\xi}^{1}\frac{d\xi'}{\xi'^2}\,F_2(\xi',Q^2)$$

  - kinematic variable $\xi = 2x/(1+\gamma)$
  - Nachtmann variable $\gamma = (1+4x^2M_N^2/Q^2)^{1/2}$

Impact on α_s determinations

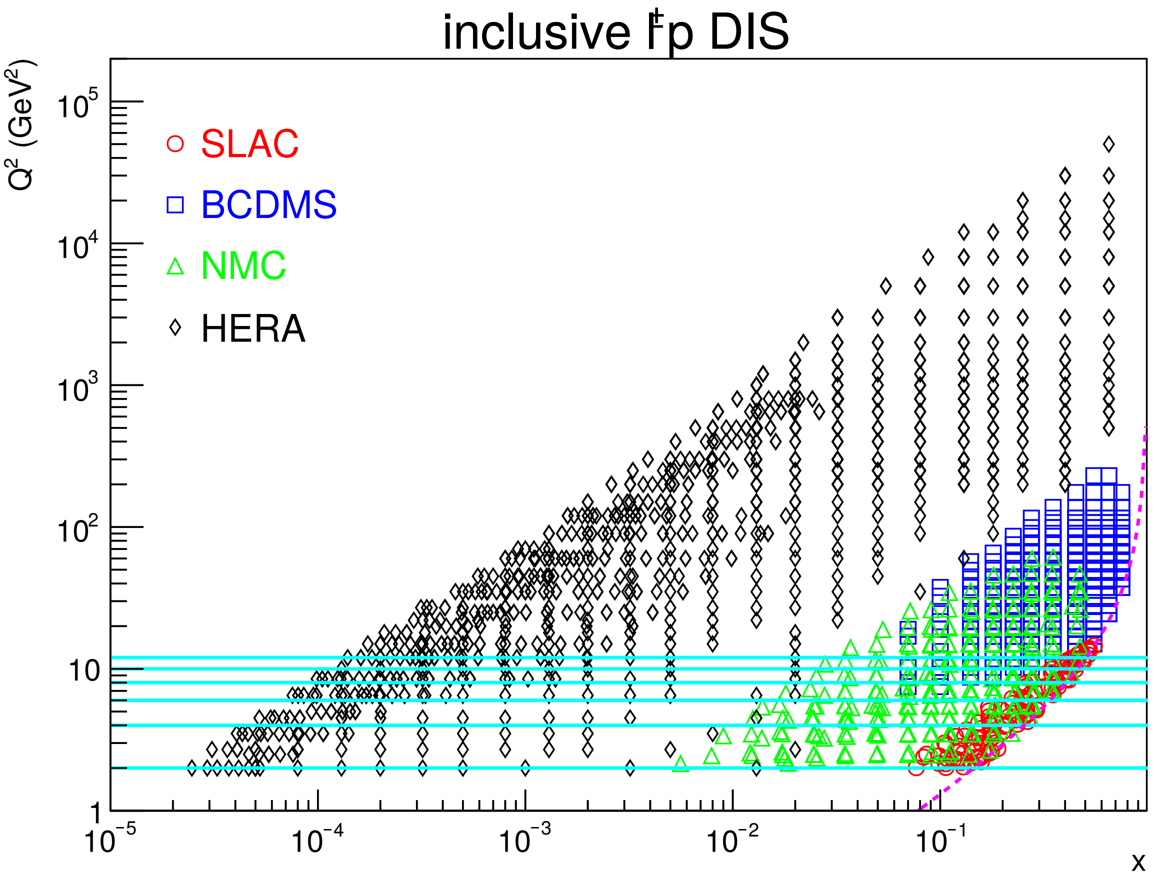
- Correlation of errors among different data DIS sets
- Target mass corrections (powers of nucleon mass M_N^2/Q^2)
- Variants with no higher twist give larger α_s values Alekhin, Blümlein, S.M. '17



- Theoretical uncertainty of α_s at NNLO from DIS data $\gtrsim \mathcal{O}(1\ldots2)\%$

Kinematics of world DIS data

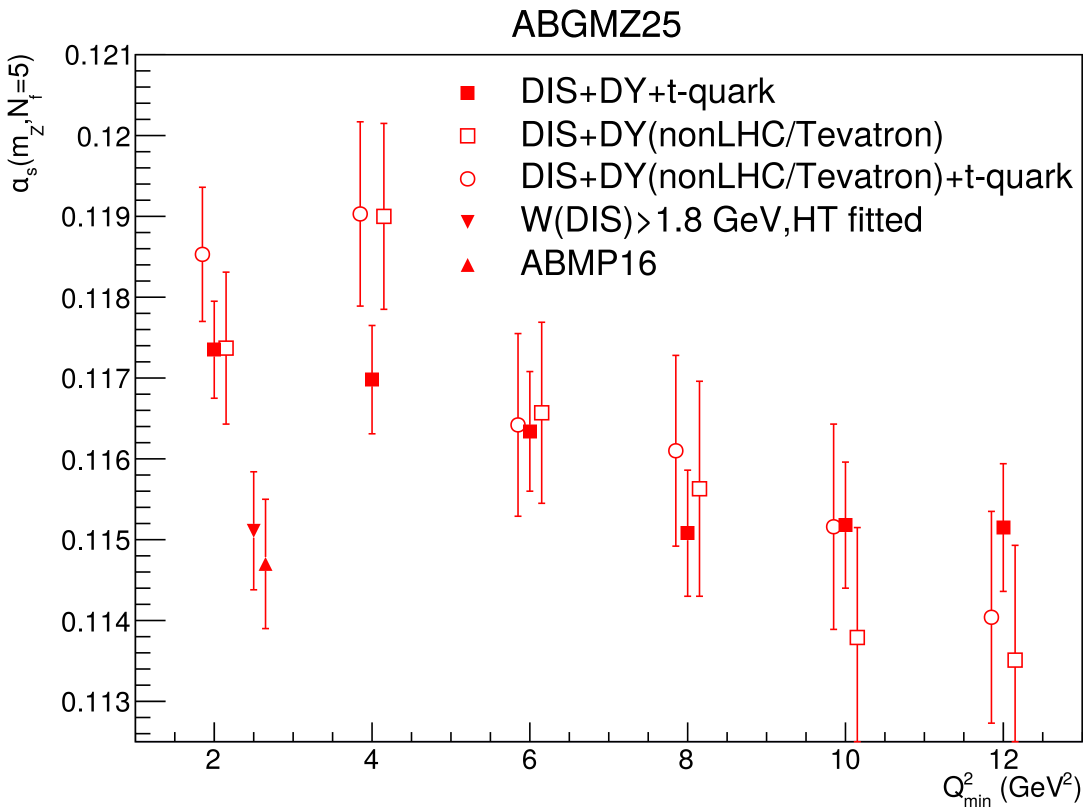
- World DIS data with cuts on kinematics



- cuts $W^2 \geq 12.5\ \mathrm{GeV}^2$ and $Q^2 \geq 2 \div 12\ \mathrm{GeV}^2$

Strong coupling from high energy data

- Strong coupling as a function of cut on Q^2
  - fit of $\alpha_s(M_Z)$ without higher twist corrections in DIS



- Sensitivity to higher twist terms reduced for larger cuts $Q^2 \geq 10 \text{ GeV}^2$
- Strong coupling $\alpha_s(M_Z) = 0.1152 \pm 0.0008$ with cut $Q^2 \geq 10 \text{ GeV}^2$

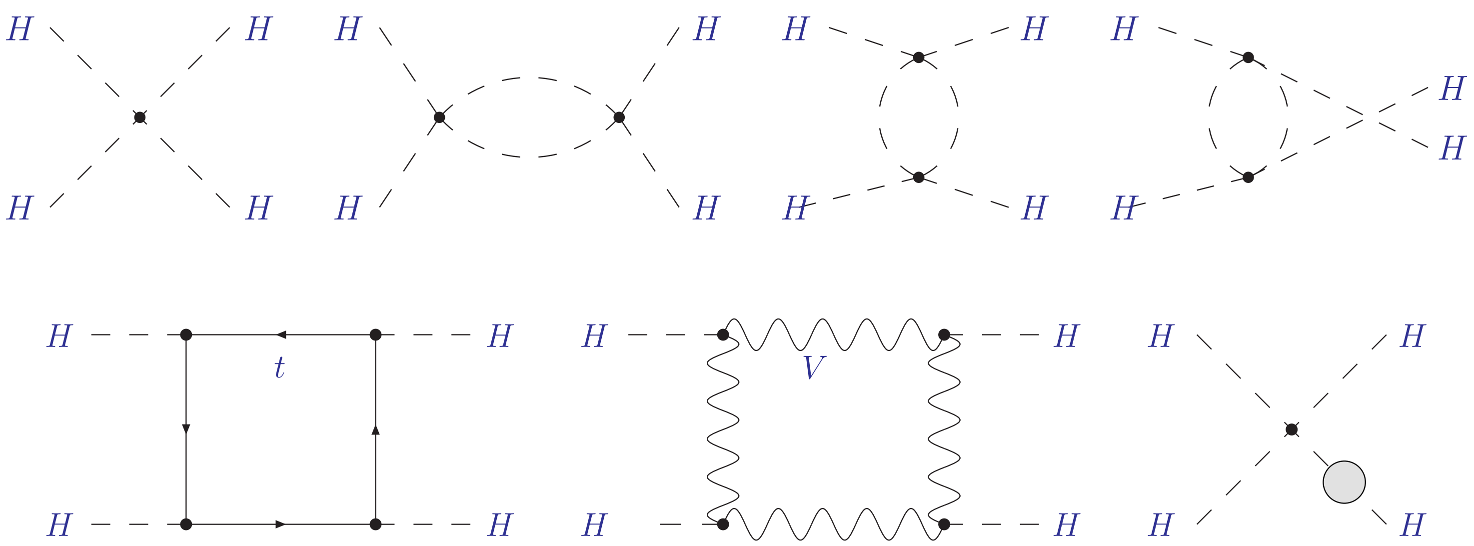
Physics implications

Higgs potential

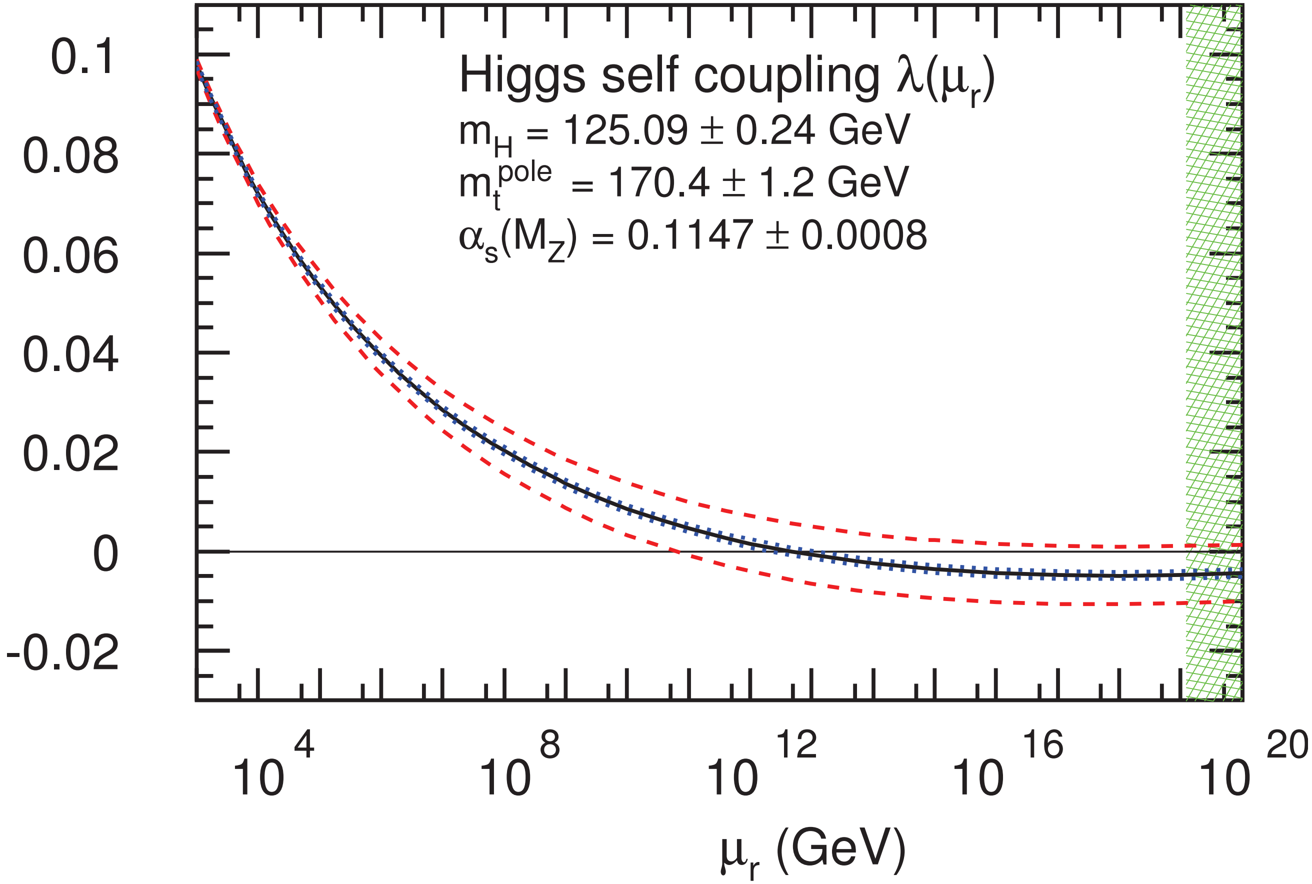
Renormalization group equation

- Quantum corrections to Higgs potential $V(\Phi) = \lambda \left|\Phi^\dagger \Phi - \frac{v}{2}\right|^2$
- Radiative corrections to Higgs self-coupling λ
  - electro-weak couplings g and g' of $SU(2)$ and $U(1)$
  - top-Yukawa coupling y_t

$$16\pi^2 \frac{d\lambda}{dQ} = 24\lambda^2 - \left(3g'^2 + 9g^2 - 12y_t^2\right)\lambda + \frac{3}{8}g'^4 + \frac{3}{4}g'^2 g^2 + \frac{9}{8}g^4 - 6y_t^4 + \dots$$



Higgs self-coupling



- Renormalization group evolution of λ with uncertainties in m_H, m_t and α_s up to $\mu_r = M_{\rm Planck}$ (using program mr Kniehl, Pikelner, Veretin '16)
  - top-quark mass least precise parameter
- $\lambda(\mu_r = M_{\rm Planck}) \simeq 0$ implies "fate of universe" may not be fatal, after all

Summary

- Experimental precision of $\lesssim 1\%$ makes theoretical predictions at NNLO in QCD mandatory
  - theoretical predictions at NNLO in QCD nowadays standard
- Need public NNLO QCD codes for Standard Model processes in hadro-production (incl. benchmarking)
- Precision studies of hadron structure
  - dedicated analysis of experimental data
  - correlations of PDFs with $\alpha_s(M_Z)$ and top-quark mass extraction
- New ABMPtt PDF fit
  - confirmation of gluon PDF in x-range of LHC kinematics
- New $\alpha_s(M_Z)$ extraction with cuts on Q^2
  - kinematic cuts eliminate sensitivity to higher twist terms

Future tasks

- Joint effort theory and experiment