# Determination of the pole top-quark mass from LHC experimental data for single- and double-differential cross sections of top-quark pair production

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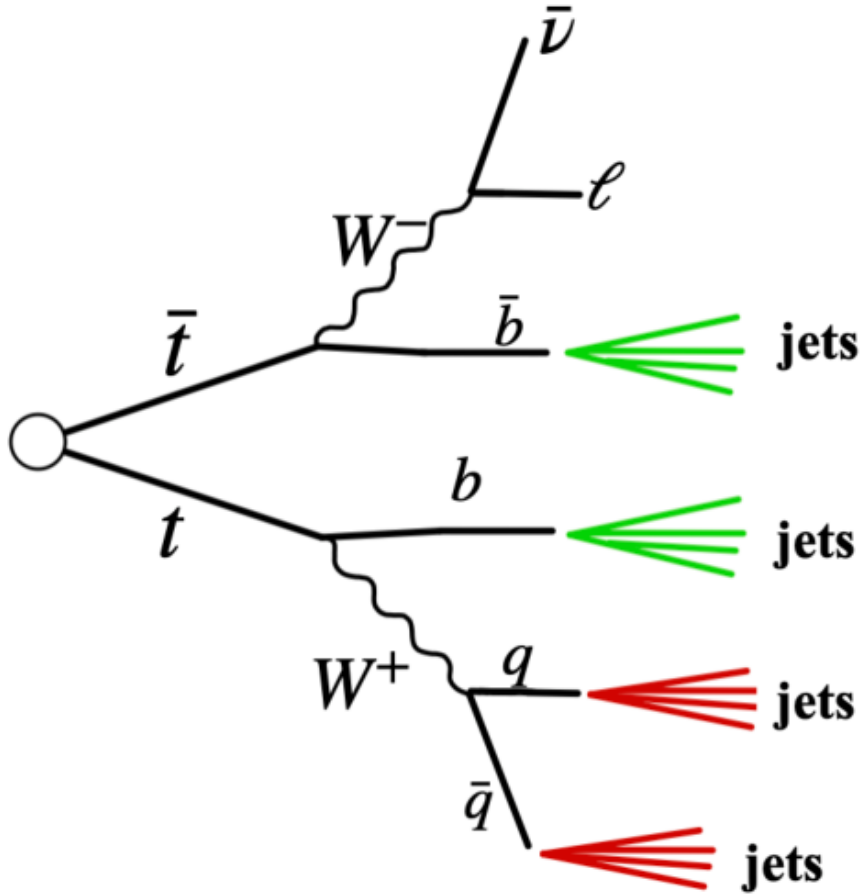
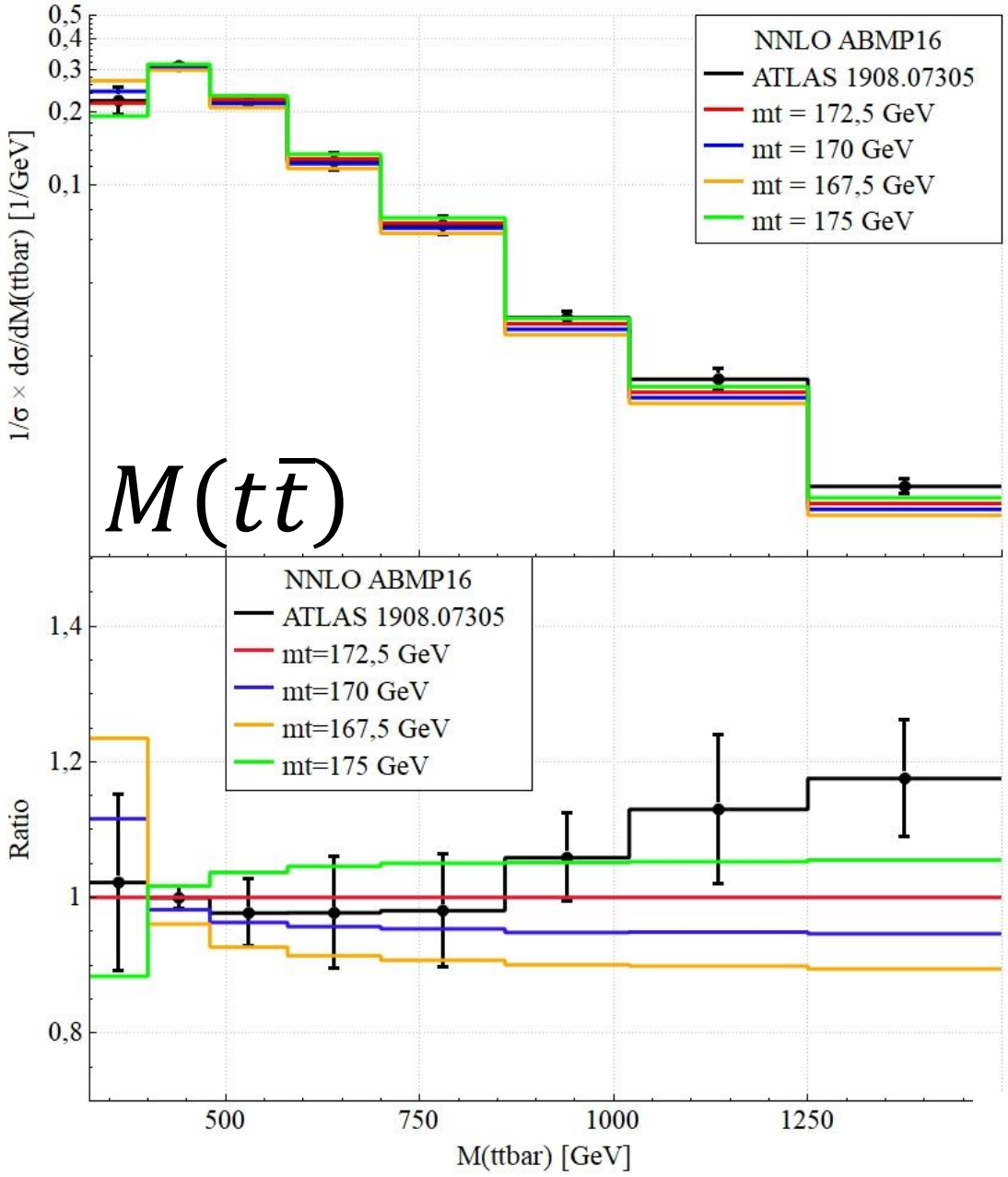
# Introduction

The aim of this work is to investigate and compare two methods for determining the pole top-quark mass based on the analysis of differential cross sections, as well as to evaluate the accuracy that they allow to achieve.




Top anti-top quark decay in the semi-leptonic channel.



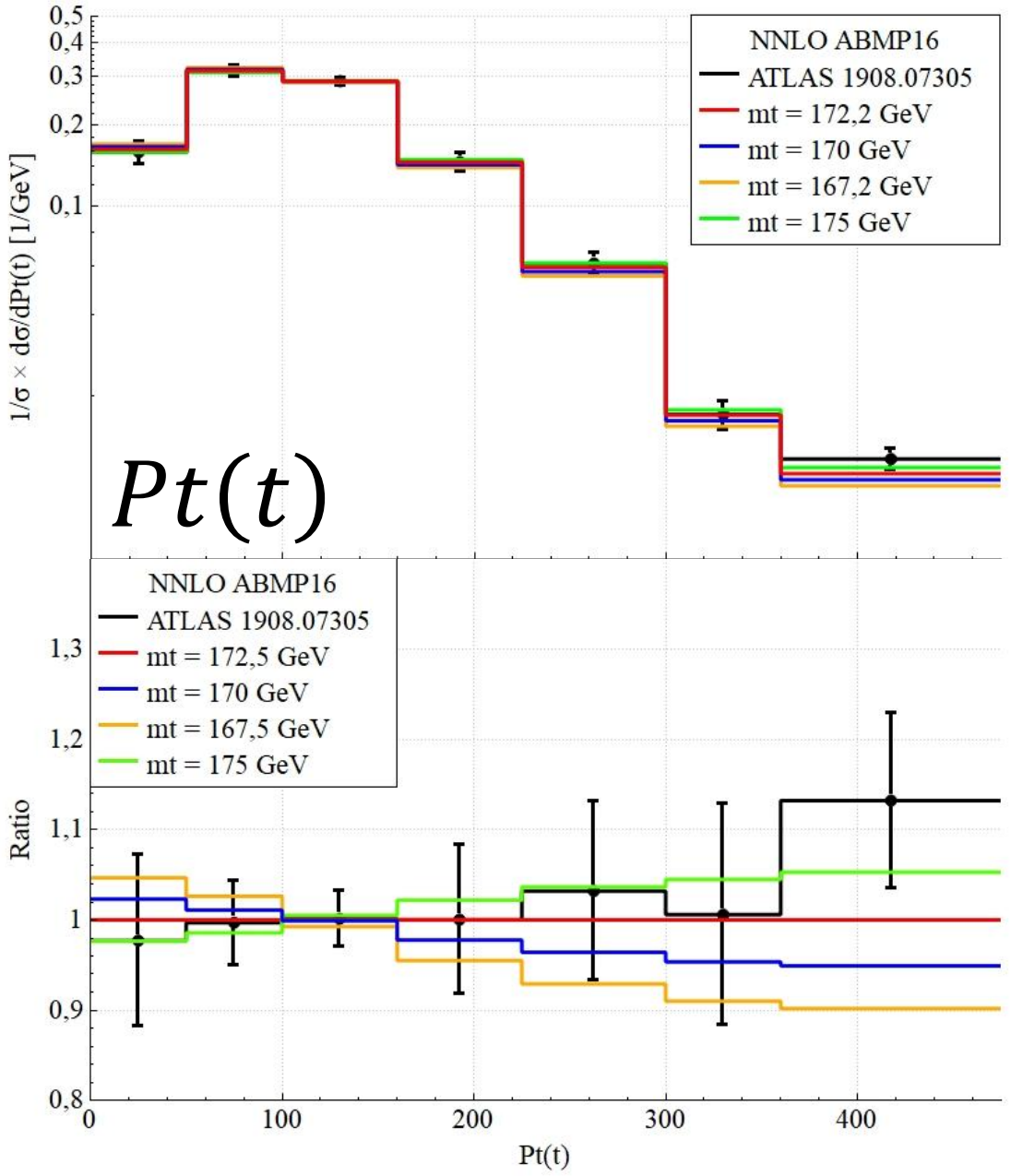

# Data and theory

- The theoretical predictions are calculated at NNLO using the MATRIX framework interfaced with PineAPPL (stored on the *ploughshare* resource and used for JHEP 05 (2024) 321) and the ABMP16 proton PDFs. (See talk by O. Zenaiev for more details)

- Experimental data obtained from the hepdata resource – normalized differential cross sections for the $t\bar{t}$ pairs production in the semilepton channel in proton-proton collisions at $\sqrt{s}$= 13 TeV, published by the ATLAS collaboration. *Eur. Phys. J. C* 79 (2019) 1028 [1908.07305]).

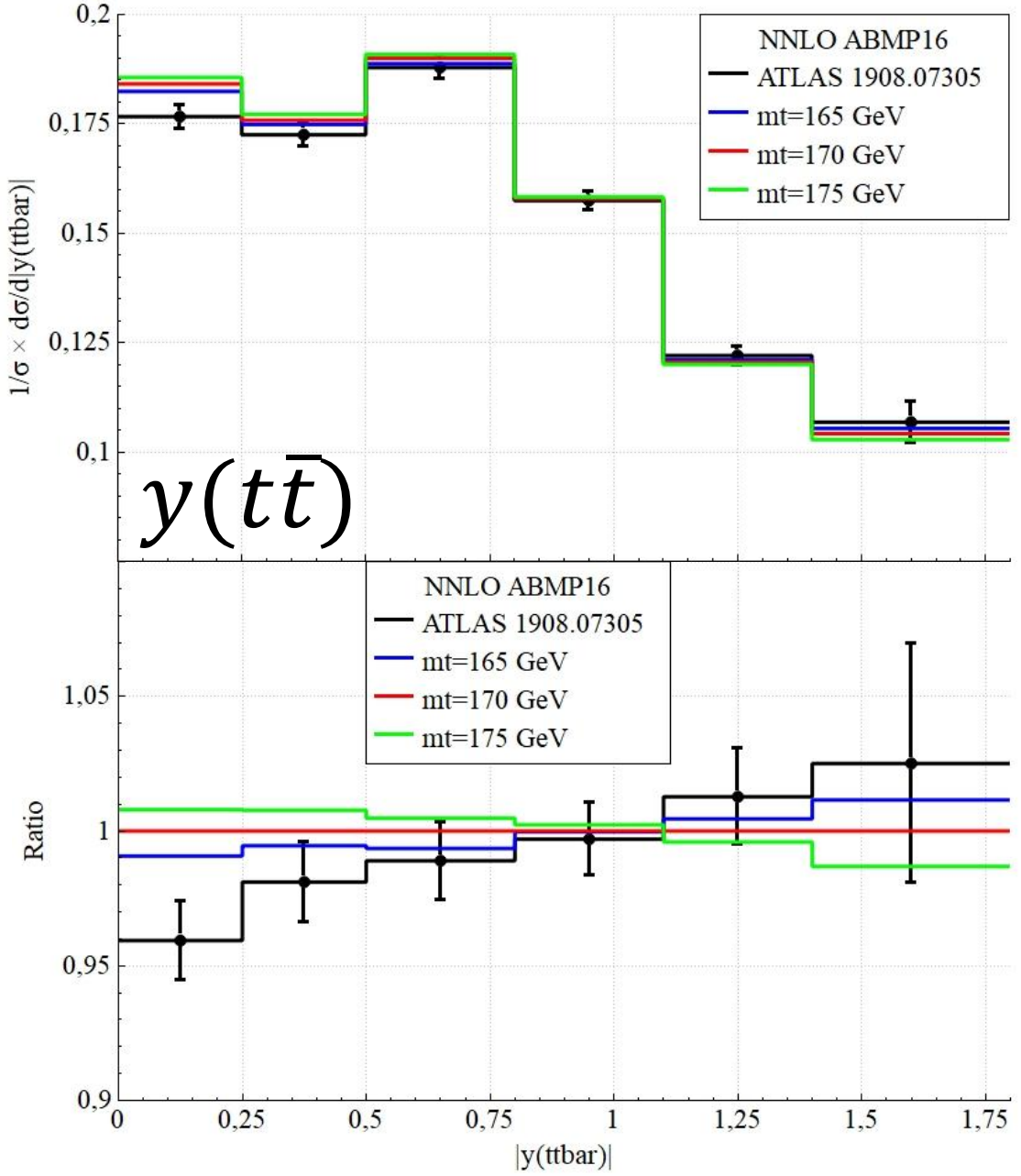
Comparison of the normalized single-differential cross section of $t\bar{t}$ production as a function of transverse momentum of $t$ quark with theoretical predictions for different $m_t$.

Ratio – all data divided by the theoretical prediction for $m_t$ = 172.5 GeV.



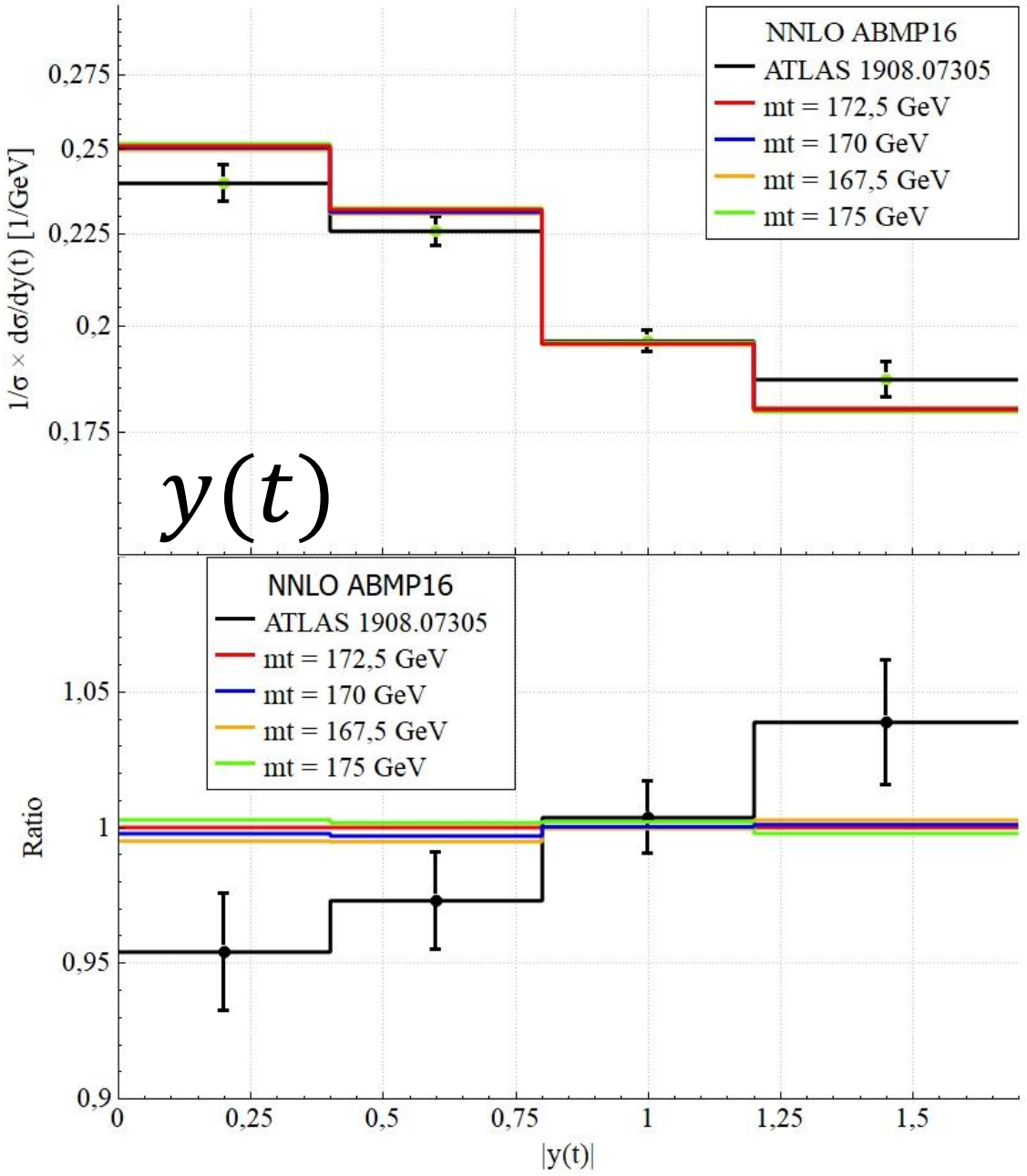
Comparison of the normalized single-differential cross section of $t\bar{t}$ production as a function of rapidity of $t\bar{t}$ pair with theoretical predictions for different $m_t$.

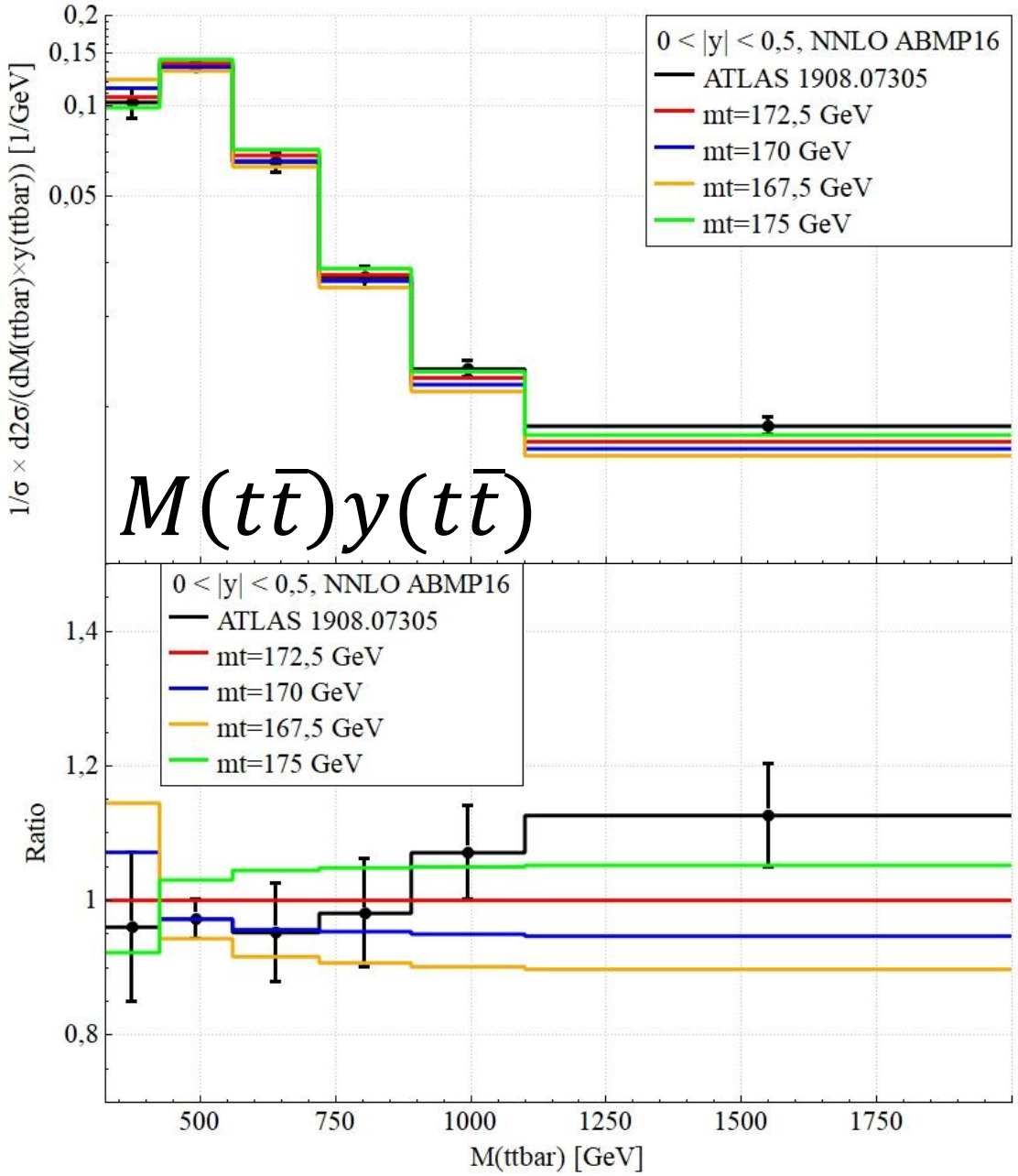
Ratio – all data divided by the theoretical prediction for $m_t$ = 170 GeV.

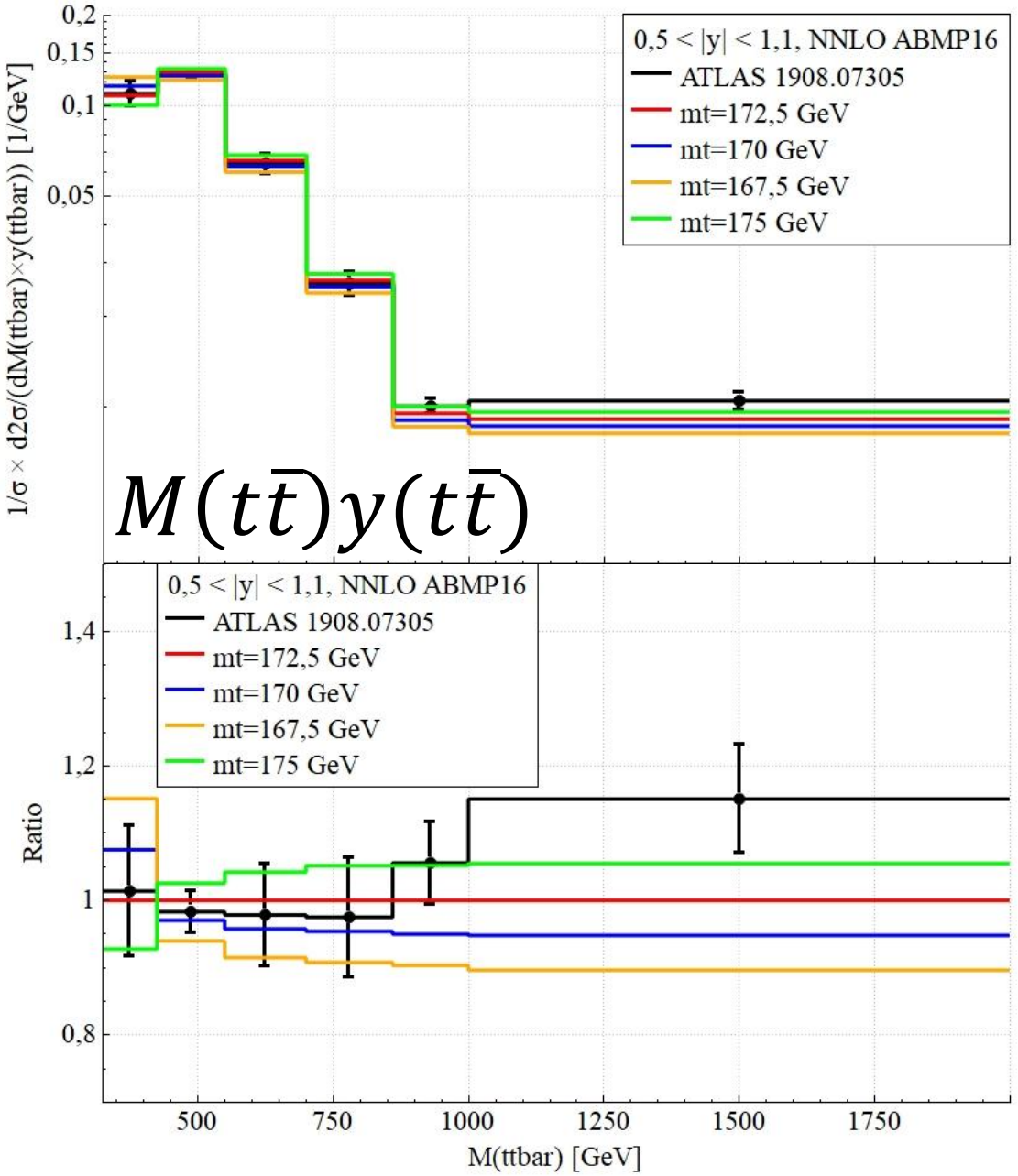


Comparison of the normalized single-differential cross section of $t\bar{t}$ production as a function of rapidity of $t$ quark with theoretical predictions for different $m_t$.

Ratio – all data divided by the theoretical prediction for $m_t$ = 172.5 GeV.




0 < |y| < 0,5, NNLO ABMP16
ATLAS 1908.07305
mt=172,5 GeV
mt=170 GeV
mt=167,5 GeV
mt=175 GeV
1/σ × d2σ/(dM(ttbar)×y(ttbar)) [1/GeV]
0,2
0,15
0,1
0,05
$M(t\bar{t})y(t\bar{t})$
Ratio
1,4
1,2
1
0,8
500
750
1000
1250
1500
1750
M(ttbar) [GeV]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ATLAS 1908.07305
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mt=170 GeV
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1/σ × d2σ/(dM(ttbar)×y(ttbar)) [1/GeV]
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Ratio
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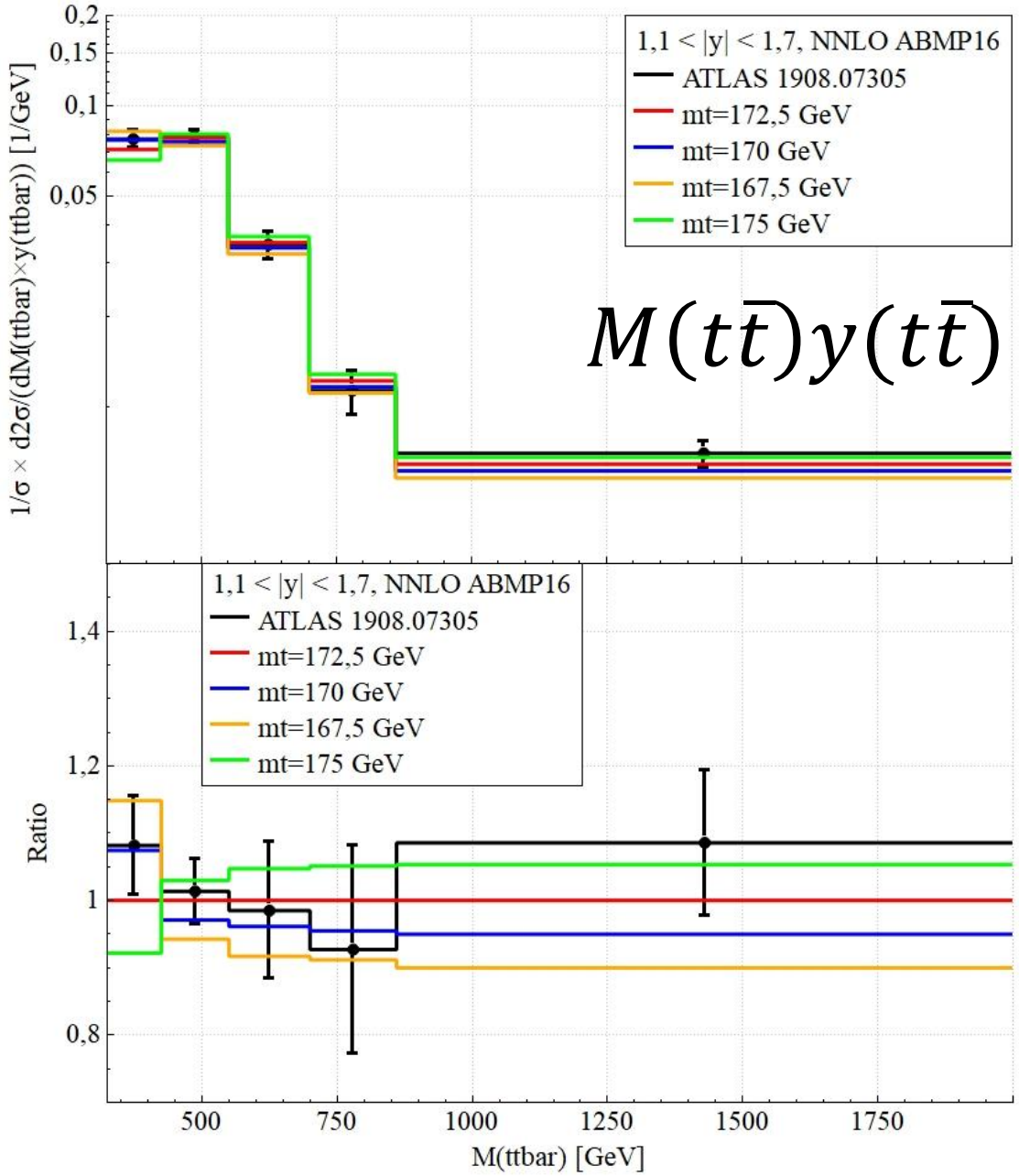
M(ttbar) [GeV]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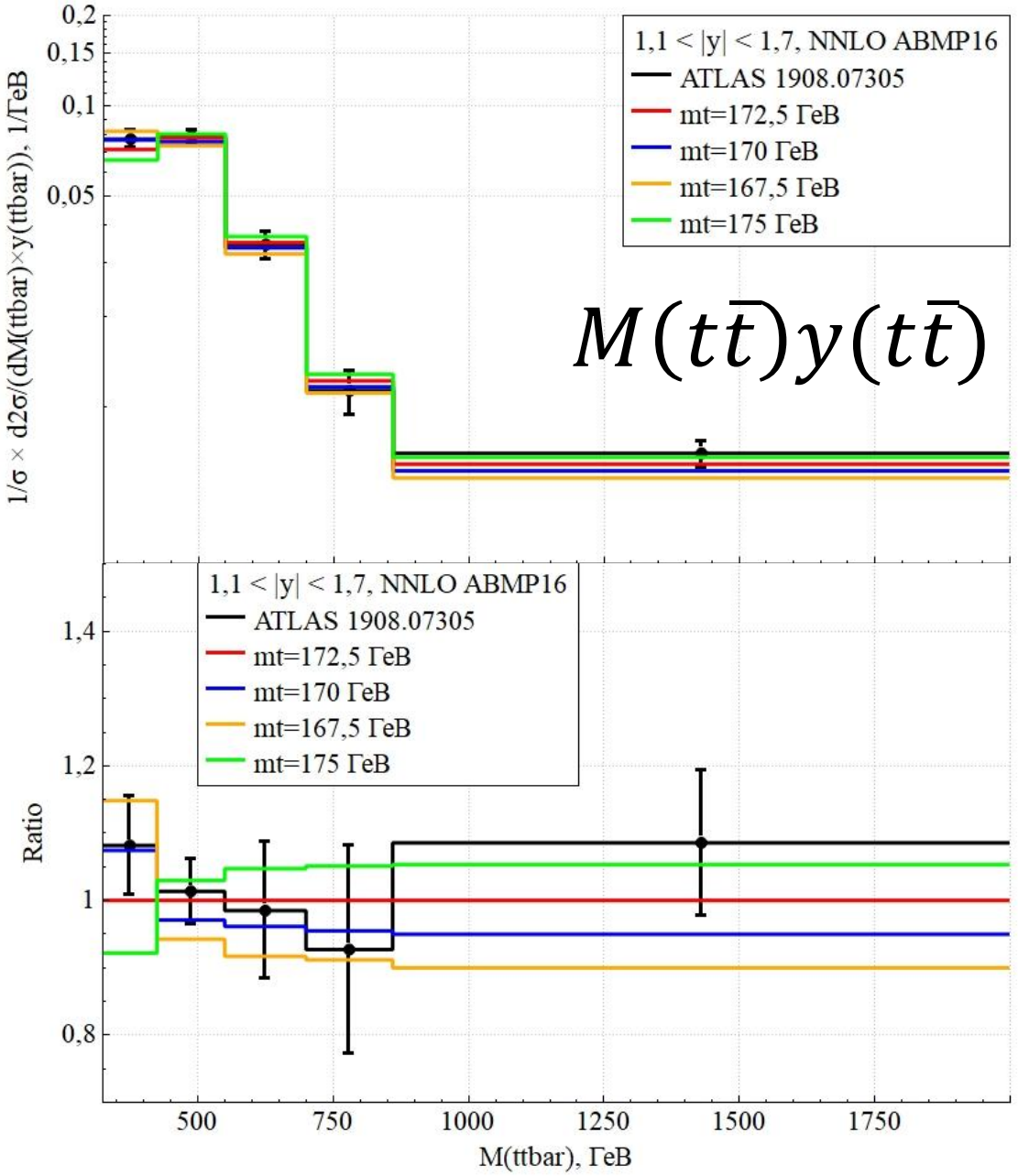
mt=170 GeV
mt=167,5 GeV
mt=175 GeV
$M(t\bar{t})y(t\bar{t})$
1/σ × d2σ/(dM(ttbar)×y(ttbar)) [1/GeV]
Ratio
M(ttbar) [GeV]


1,1 < |y| < 1,7, NNLO ABMP16
ATLAS 1908.07305
mt=172,5 ГеВ
mt=170 ГеВ
mt=167,5 ГеВ
mt=175 ГеВ
$M(t\bar{t})y(t\bar{t})$
1/σ × d2σ/(dM(ttbar)×y(ttbar)), 1/ГеВ
Ratio
M(ttbar), ГеВ

# Determination of the top-quark mass using the $\chi^2$ minimization method

$$\chi^2 = V_{N-1}^T \times COV_{N-1}^{-1} \times V_{N-1}$$

$V_{N-1}$ – vector column of differences between measured and theoretically predicted normalized cross section

$COV_{N-1}$ – submatrix of the covariance matrix of size $(N-1) \times (N-1)$ obtained by discarding the corresponding row and column, where N is the number of bins.

Errors in determining the top-quark mass are obtained by determining the values of the top quark masses corresponding to the values $\chi^2 = \chi^2_{min} + 1$.

The covariance matrices for the cross sections used in this work were non-degenerate. This may result from excessive rounding or another error present in the ATLAS data published on HEPData.

# xFitter software package

The xFitter software package was used to calculate the values of $\chi^2$.
https://gitlab.cern.ch/fitters/xfitter

Key features of xFitter:

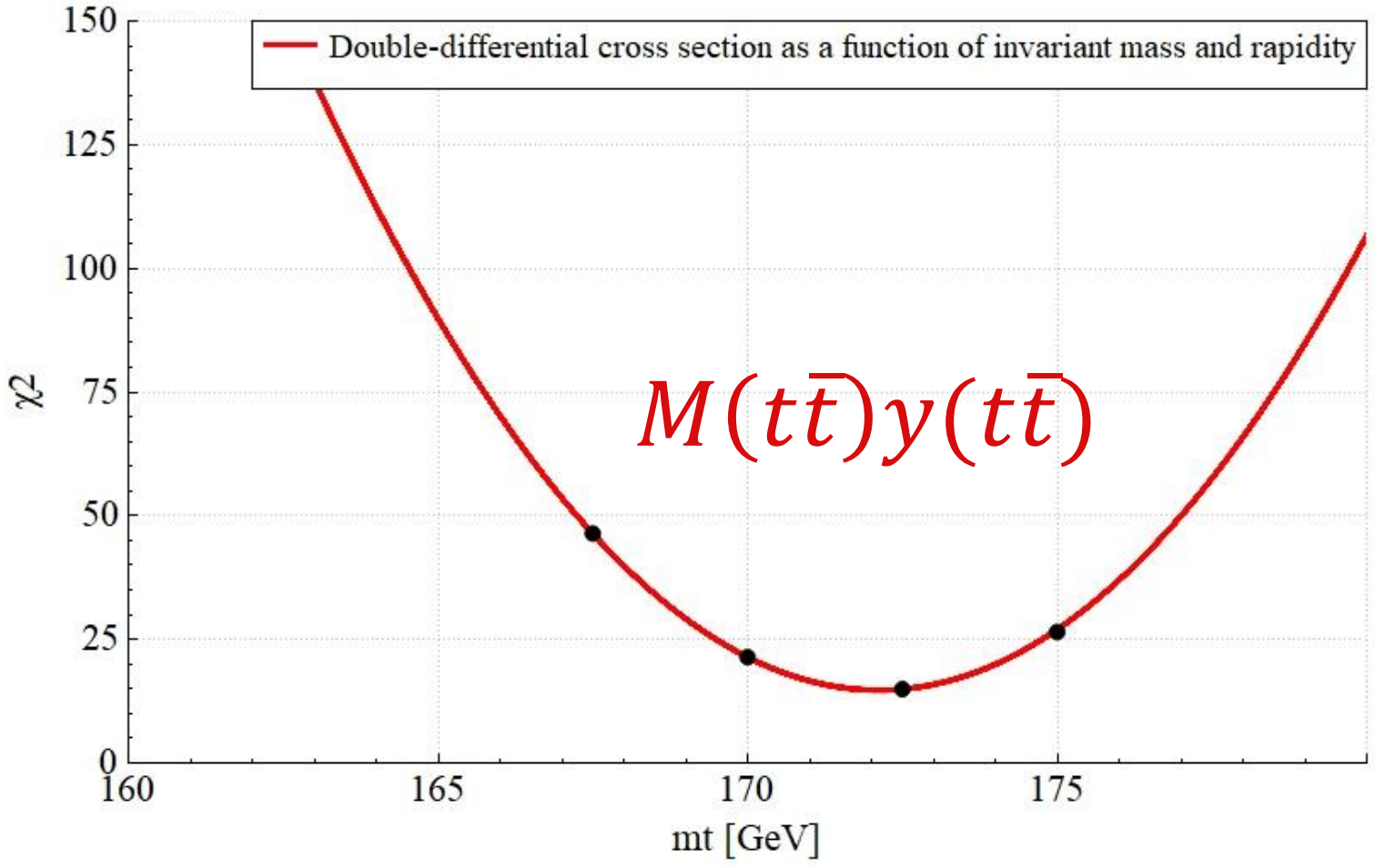
- Flexibility. This package supports a large number of existing methods and schemes used to define PDF.
- Support for various theoretical models.
- Analysis of new data. Can be used to study the impact of precise measurements from hadron colliders.

# Double-differential cross section

| $m_t$ [GeV] | $\chi^2$ |
|---|---|
| 167.5 | 46.37 |
| 170 | 21.38 |
| 172.5 | 14.91 |
| 175 | 26.46 |

$m_t$ = 172.12 ± 0.82 GeV

$\chi^2_{min}/dof = 14.7/19$
(p = 0.74).

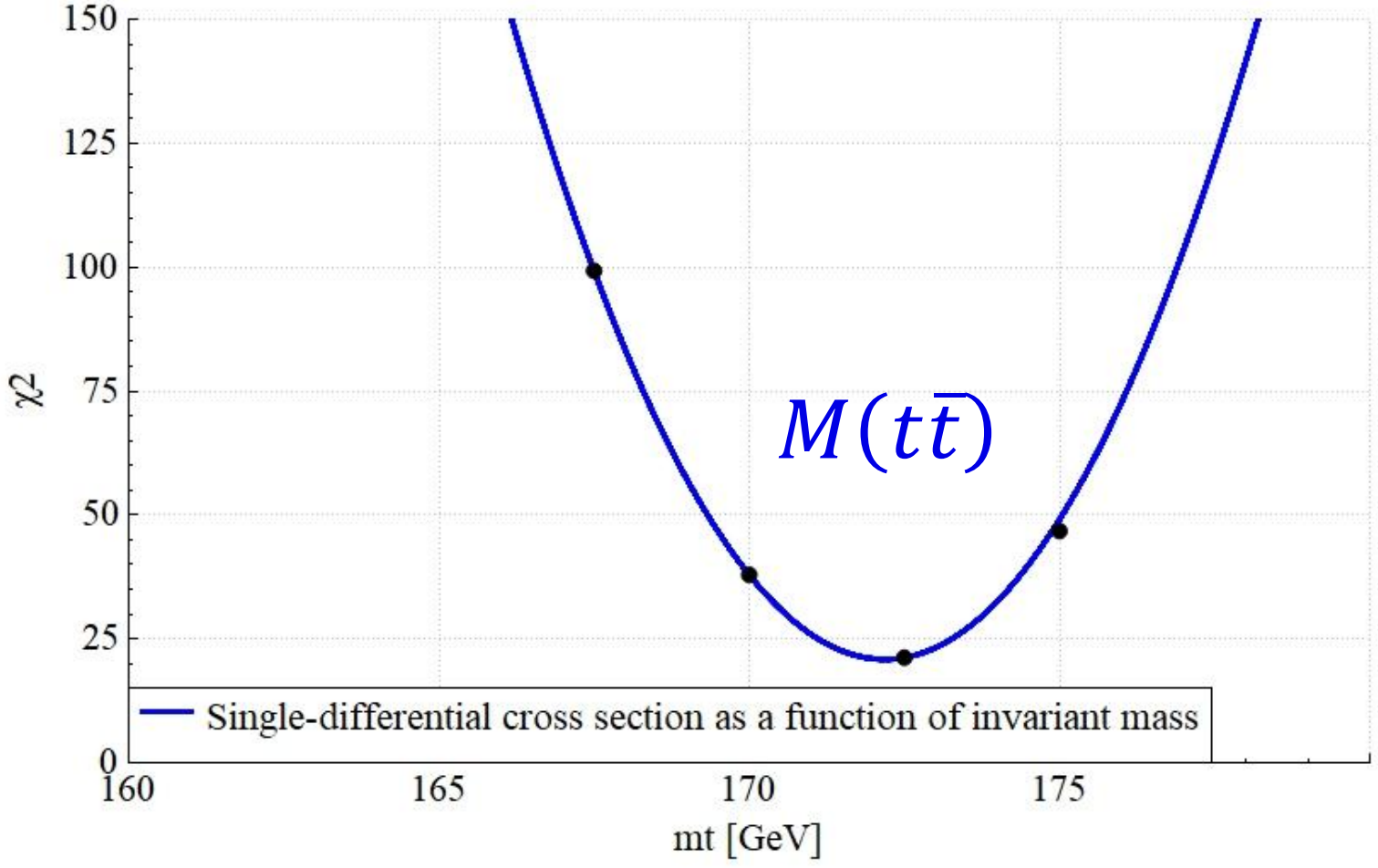


# Single-differential cross section as a function of invariant mass of $t\bar{t}$ pair

| $m_t$ [GeV] | $\chi^2$ |
|---|---|
| 167.5 | 99.23 |
| 170 | 37.91 |
| 172.5 | 21.19 |
| 175 | 46.73 |

$m_t$ = 172.19 ± 0.53 GeV

$\chi^2_{min}/dof = 20.8/8$
(p = 0.008).

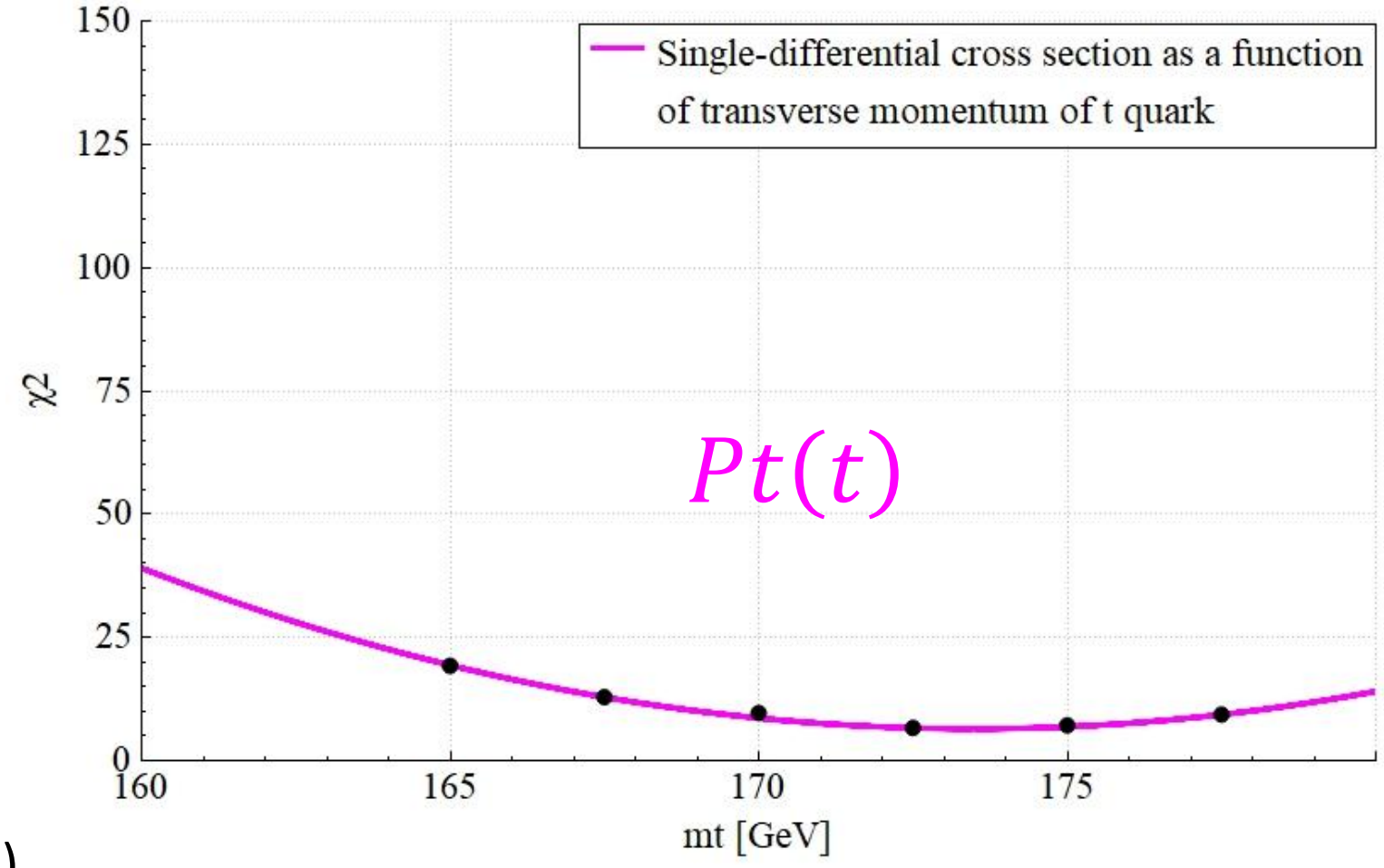


# Single-differential cross section as a function of transverse momentum of $t$ quark

| $m_t$ [GeV] | $\chi^2$ |
|---|---|
| 165 | 19.2 |
| 167.5 | 12.85 |
| 170 | 9.63 |
| 172.5 | 6.6 |
| 175 | 7.13 |
| 177.5 | 9.31 |

$m_t$ = 173.5 ± 2.4 GeV

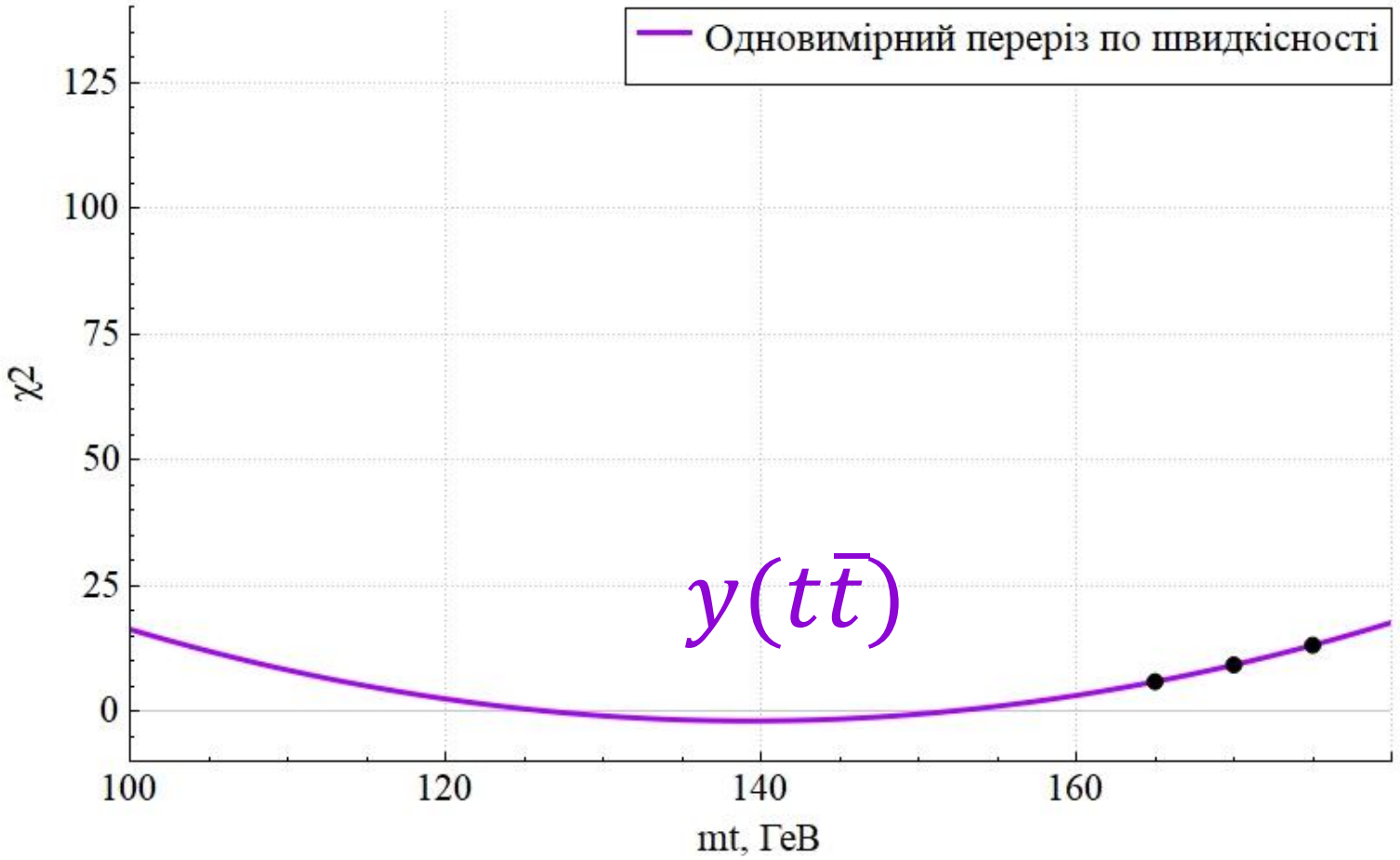
$\chi^2_{min}/dof$ = 6.4/7 (p = 0.49).



# Single-differential cross section as a function of rapidity of $t\bar{t}$ pair

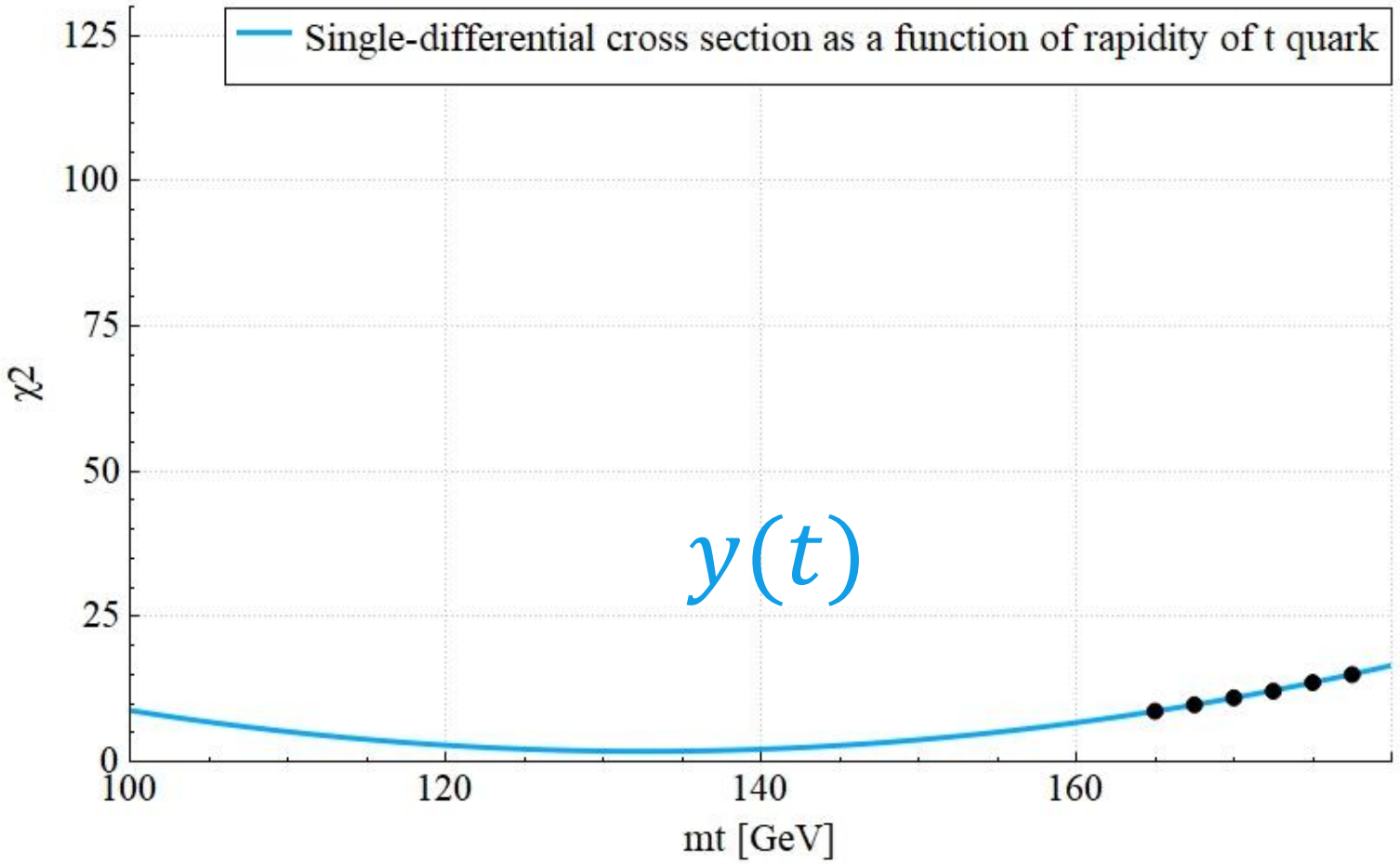
| $m_t$ [GeV] | $\chi^2$ |
|---|---|
| 165 | 5.81 |
| 170 | 9.14 |
| 175 | 13.06 |

$m_t$ = 143 ± 13 GeV.



# Single-differential cross section as a function of rapidity of $t$ quark

| $m_t$ [GeV] | $\chi^2$ |
|---|---|
| 165 | 8.65 |
| 167.5 | 9.75 |
| 170 | 10.95 |
| 172.5 | 12.1 |
| 175 | 13.58 |
| 177.5 | 14.96 |




$m_t$ = 133 ± 12 GeV.

$\chi^2_{min}/dof = 1.74/4$ (p = 0.78).

# $\chi^2$ calculation using two single-differential cross sections

$COV_{mY}$ – covariance matrix between the bins of the cross section in invariant mass and the bins of the cross section in rapidity

$COV_{mm}$ – covariance matrix between the bins of the cross section in invariant mass

$COV_{YY}$ – covariance matrix between the bins of the cross section in rapidity

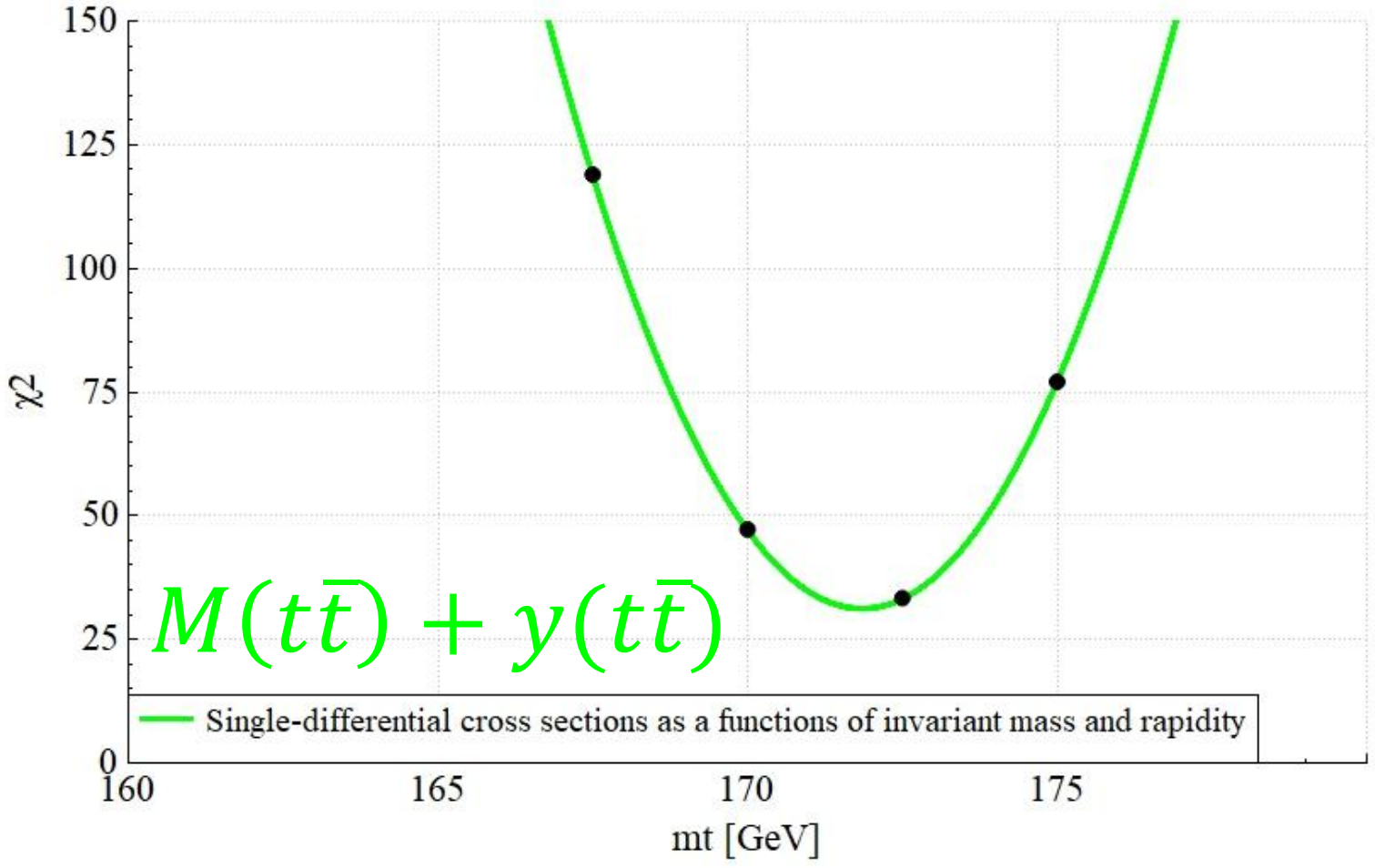
$$\left(\begin{array}{c|c} \left(COV_{mm}\right) & \left(COV_{mY}\right) \\ \hline \left(COV_{mY}^{T}\right) & \left(COV_{YY}\right) \end{array}\right)$$

# Two single-differential cross sections as a functions of invariant mass of $t\bar{t}$ pair and rapidity of $t\bar{t}$ pair

| $m_t$ [GeV] | $\chi^2$ |
|---|---|
| 167.5 | 118.93 |
| 170 | 47.2 |
| 172.5 | 33.31 |
| 175 | 77.05 |

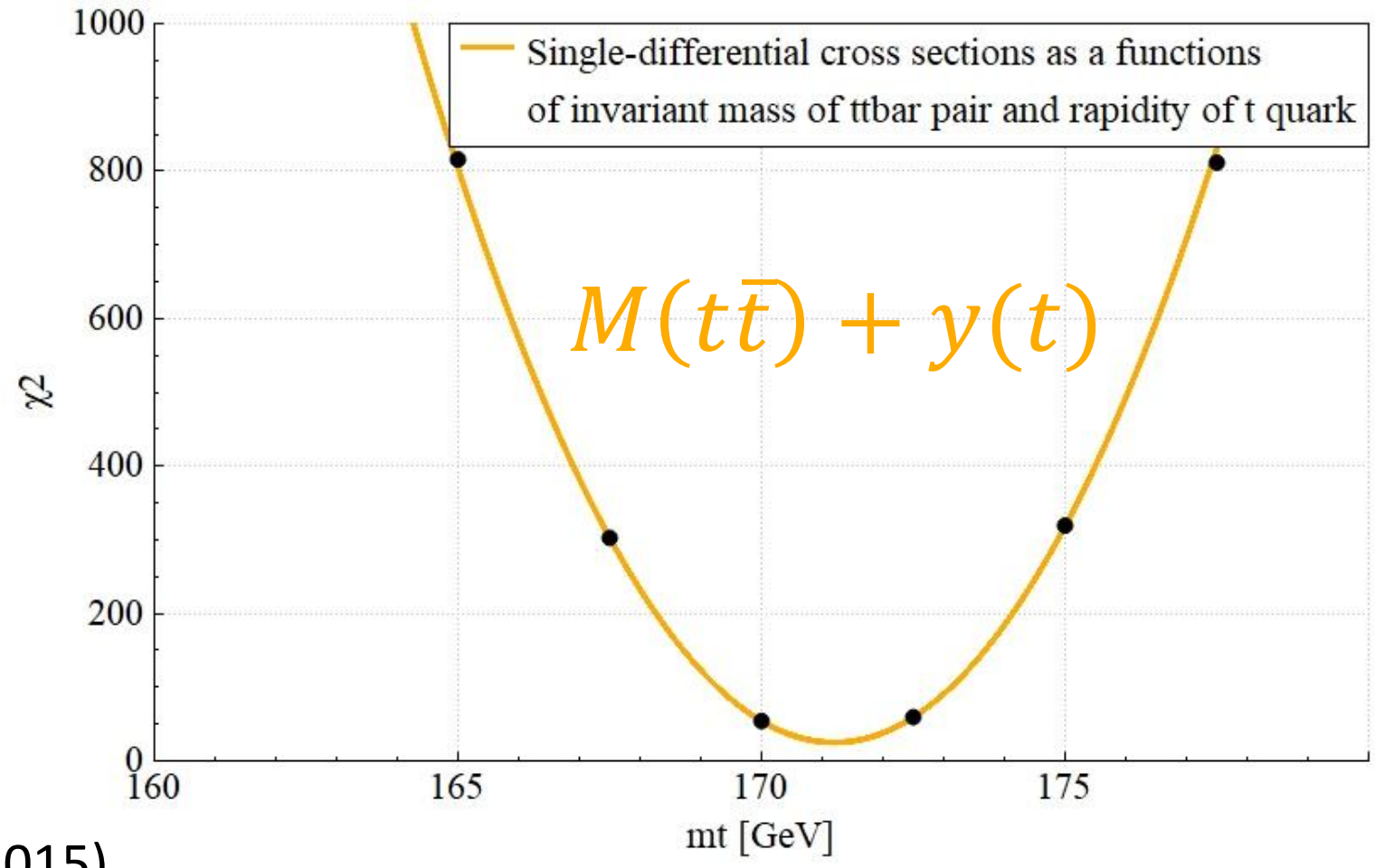
$m_t$ = 171.85 ± 0.46 GeV

$\chi^2_{min}/dof = 31.4/14$
(p = 0.005).



# Two single-differential cross sections as a functions of invariant mass of $t\bar{t}$ pair and rapidity of $t$ quark

| $m_t$ [GeV] | $\chi^2$ |
|---|---|
| 165 | 815.62 |
| 167.5 | 302.35 |
| 170 | 54.01 |
| 172.5 | 59.28 |
| 175 | 319.05 |
| 177.5 | 811.26 |




$m_t$ = 171.20 ± 0.22 GeV

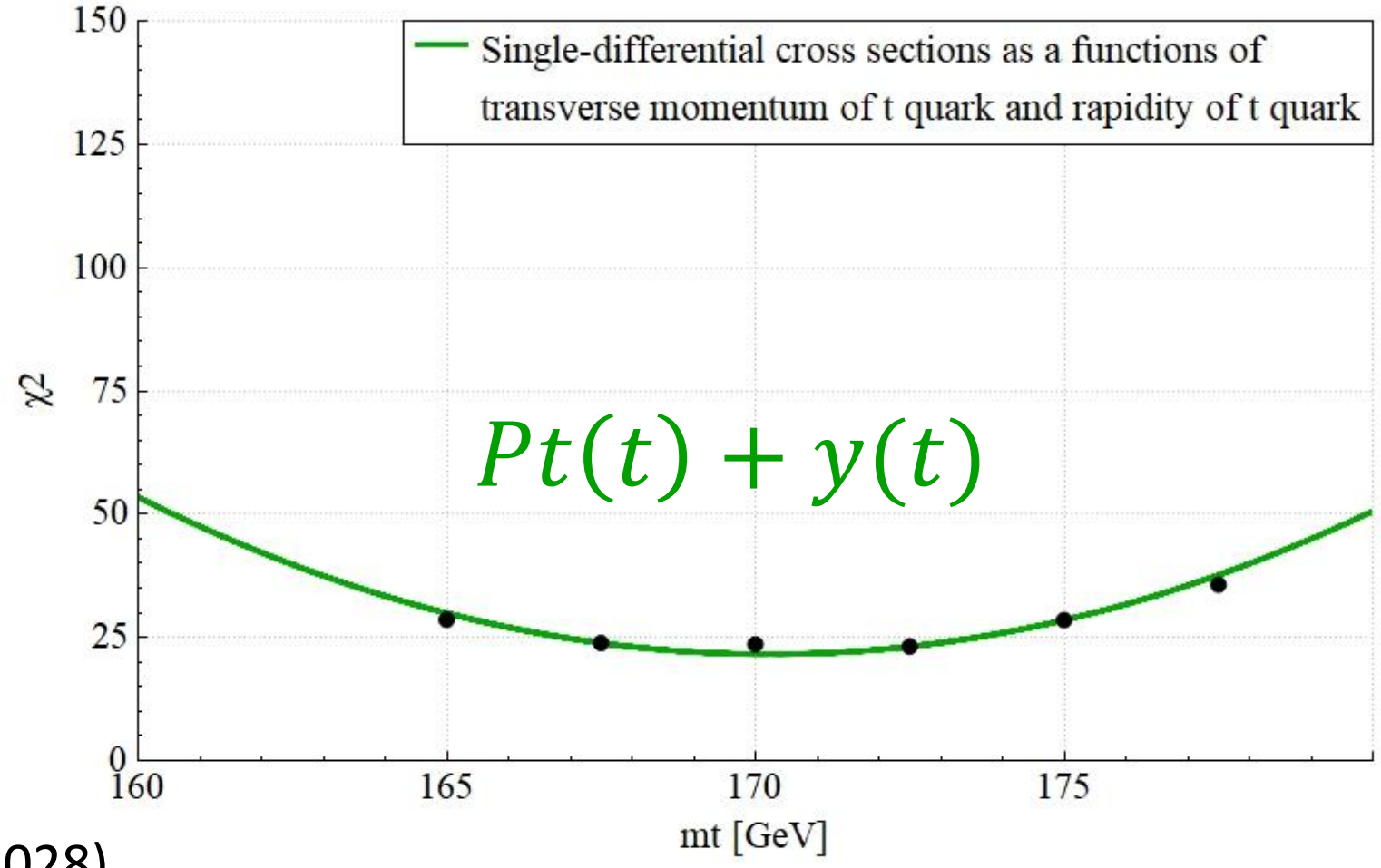
$\chi^2_{min}/dof = 24.9/12$ (p = 0.015).

# Two single-differential cross sections as a functions of transverse momentum of $t$ quark and rapidity of $t$ quark

| $m_t$ [GeV] | $\chi^2$ |
|---|---|
| 165 | 28.54 |
| 167.5 | 23.83 |
| 170 | 23.57 |
| 172.5 | 23.12 |
| 175 | 28.47 |
| 177.5 | 35.66 |

$m_t$ = 170.2 ± 1.8 GeV

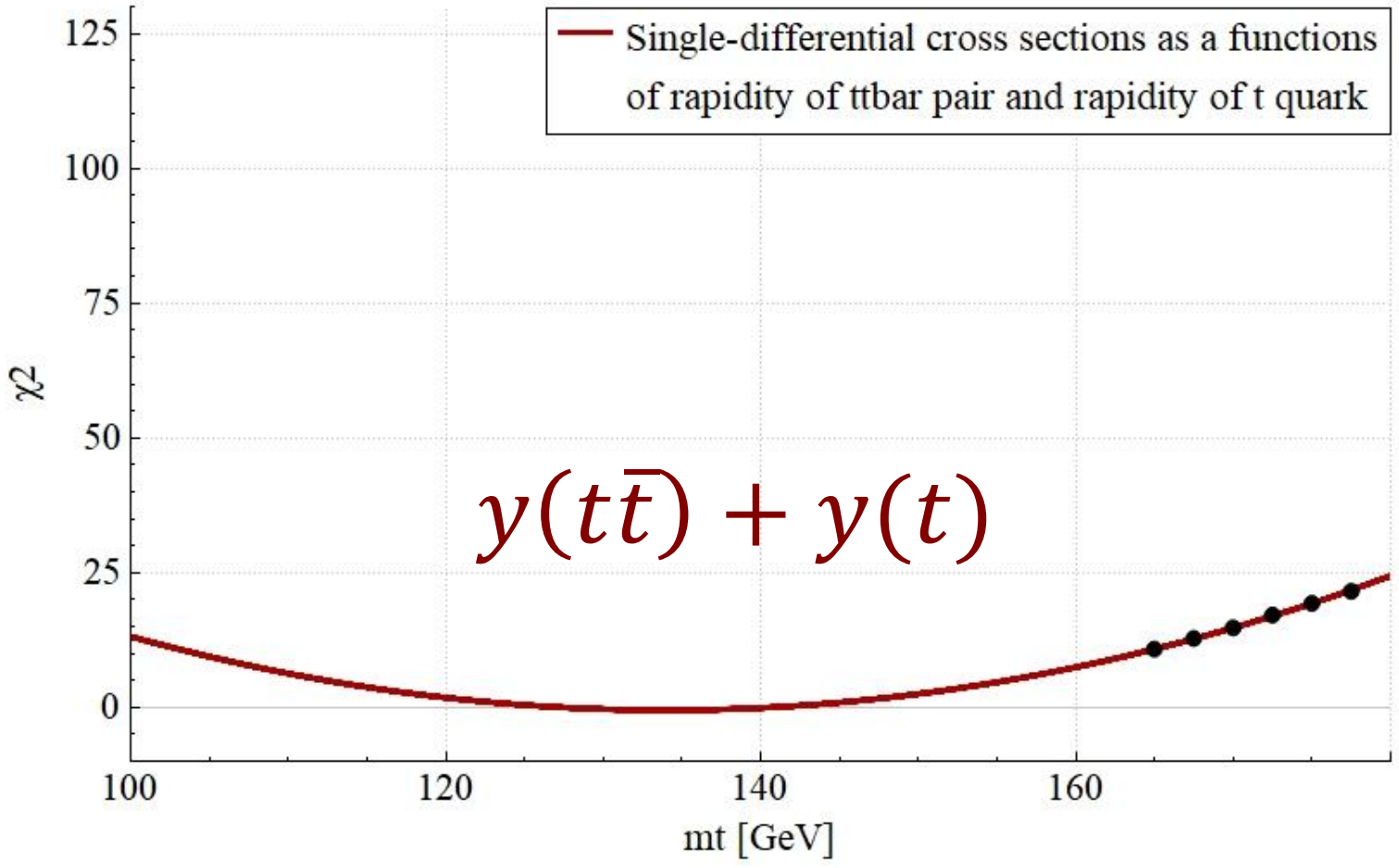
$\chi^2_{min}/dof = 21.6/11$ (p = 0.028).



# Two single-differential cross sections as a functions of rapidity of $t\bar{t}$ pair and rapidity of $t$ quark

| $m_t$ [GeV] | $\chi^2$ |
|---|---|
| 165 | 10.77 |
| 167.5 | 12.75 |
| 170 | 14.72 |
| 172.5 | 17.09 |
| 175 | 19.26 |
| 177.5 | 21.5 |

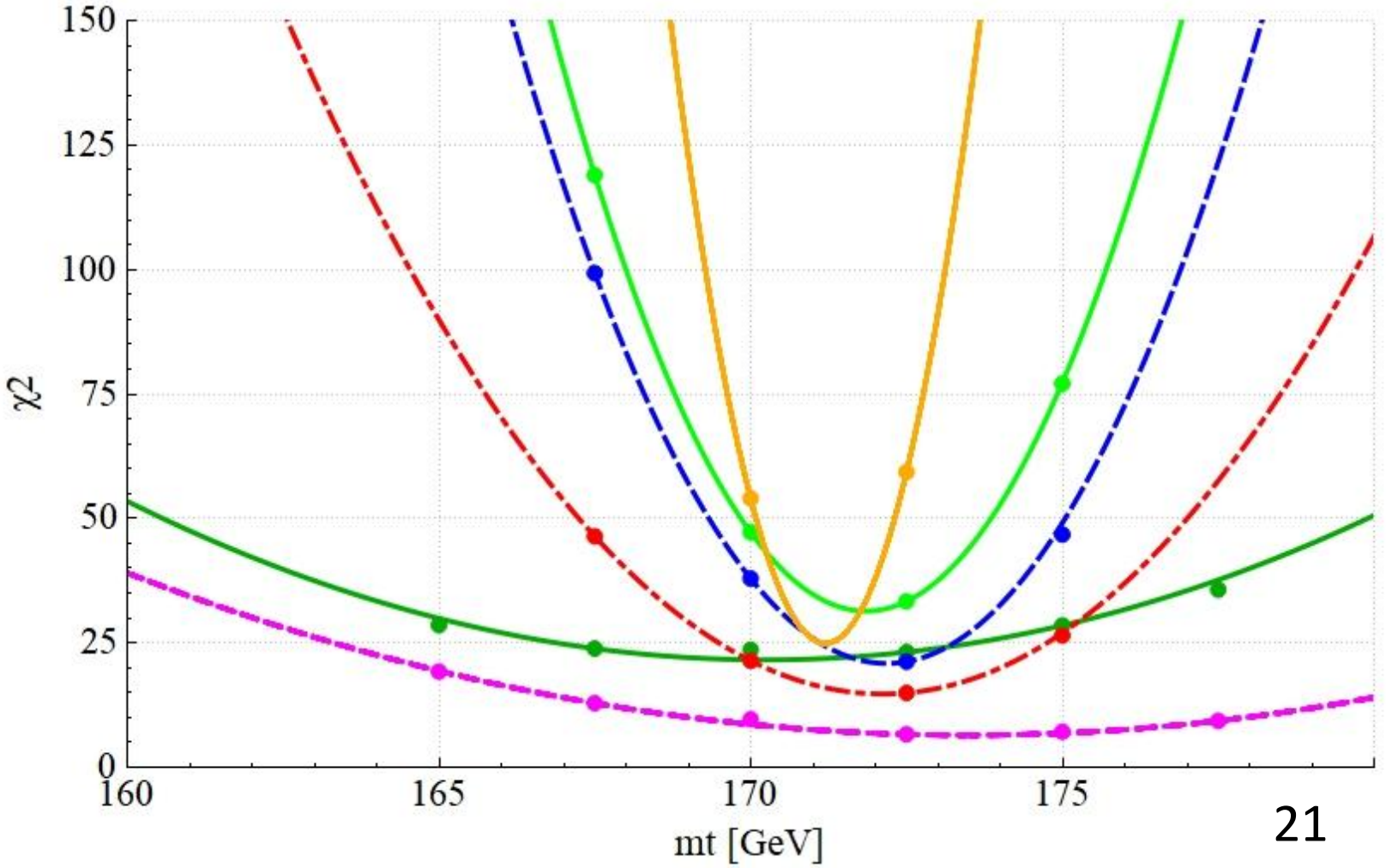
$m_t$ = 134.0 ± 9.2 GeV



# Comparison of the obtained results

- Double-differential cross section as a function of invariant mass of ttbar pair and rapidity of ttbar pair
- Single-differential cross section as a function of invariant mass of ttbar pair
- Single-differential cross section as a function of transverse momentum of t quark
- Single-differential cross sections as a functions of invariant mass of ttbar pair and rapidity of ttbar pair
- Single-differential cross sections as a functions of invariant mass of ttbar pair and rapidity of t quark
- Single-differential cross sections as a functions of transverse momentum of t quark and rapidity of t quark

| Cross sections used | Obtained $m_t$ [GeV] |
|---|---|
| | 172.12 ± 0.82 |
| | 172.19 ± 0.53 |
| | 173.5 ± 2.4 |
| | 171.85 ± 0.46 |
| | 171.20 ± 0.22 |
| | 170.2 ± 1.8 |
| PDG mass | 172.4 ± 0.7 |



# Theoretical Uncertainties

For two single-differential cross sections as a functions of invariant mass of $t\bar{t}$ pair and rapidity of $t\bar{t}$ pair:
Scale Uncertainties: $^{+0.14}_{-0.09}$ PDF Uncertainties: $\pm 0.14$ =>

=> $m_t = 171.85 \pm 0.46\ ^{+0.14}_{-0.09} \pm 0.14$ GeV

For two single-differential cross sections as a functions of invariant mass of $t\bar{t}$ pair and rapidity of $t$ quark:
Scale Uncertainties: $^{+0.03}_{-0.03}$ PDF Uncertainties: $\pm 0.03$ =>

=> $m_t = 171.20 \pm 0.22\ ^{+0.03}_{-0.03} \pm 0.03$ GeV

# Conclusions

- For the first time, the top-quark mass was determined taking into account the correlation between two single-differential cross sections at parton level.

- The obtained results are consistent, within uncertainties, with the PDG value of 172.4 ± 0.7 GeV. All cross sections are reasonably well described by the NNLO predictions.

- The value of $\chi^2$ calculated using the two single-differential ATLAS cross sections was found to be the most sensitive to variations in the top-quark mass. This is explained by the larger number of bins at low $t\bar{t}$ invariant mass, which are most sensitive to variations in the top-quark mass.

- We encounter a problem that the total covariance matrix is not positive definite when we use more than two cross sections, as well as certain combinations of two cross sections: under investigation.

- Future steps of this study could include increasing the number of single-differential cross sections used in the determination of the top-quark mass. We aim to include all 4 single-differential ATLAS cross sections.

# Thank you for your attention