Endpoint Factorization for Semileptonic, Boosted and Off-shell Top Decays

Based on work with Oliver Jin, Simon Plätzer, Daniel Samiz and Christoph Regner

arXiv:2507.17872 arXiv:2511.xxxx

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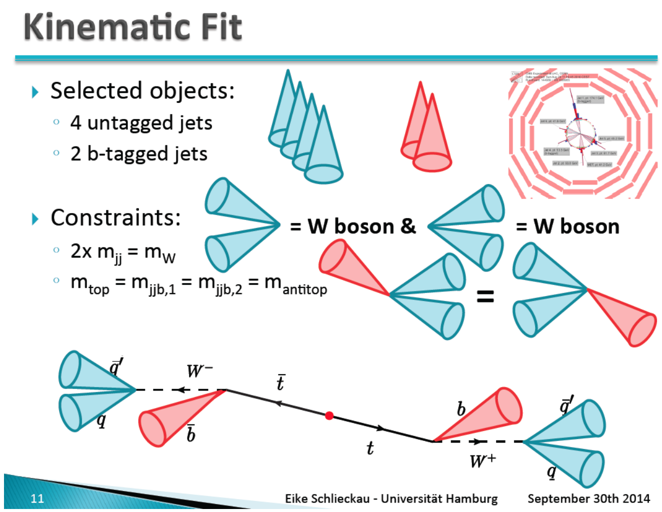
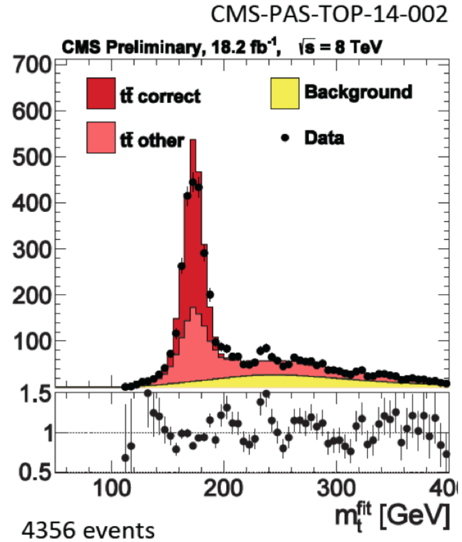
Content

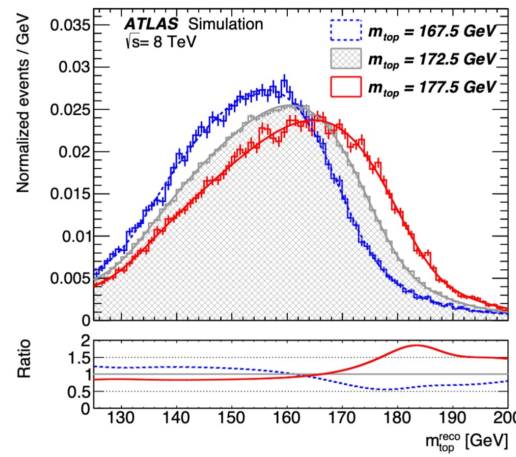
- Motivation: MC top quark mass parameter m_t^{MC}
- Current status on the understanding of m_t^{MC}
  based on inclusive jet mass observables for boosted top quarks
- Factorization for semileptonic, boosted and off-shell top decays
- Conclusions

Most Precise Top Mass Measurements Method

LHC+Tevatron: Direct top mass measurements







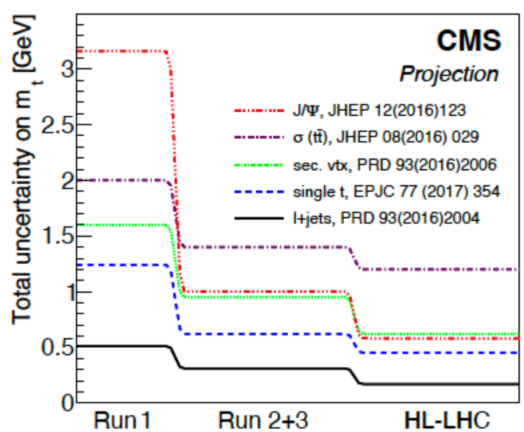
$$m_t^{\mathrm{MC}} = 171.77 \pm 0.37\ \mathrm{GeV}$$

CMS collaboration. arXiv: 2302.01967

kinematic mass determination

based on the picture of a top quark particle

Determination of the best-fit value of the Monte-Carlo top quark mass parameter



← Δm_t ~ 200 MeV (projection)

⊕ High top mass sensitivity

⊖ Precision of MC ?

⊖ Meaning of m_t^{MC} ?

What is m_t^{MC} ? (Answer 1.0)

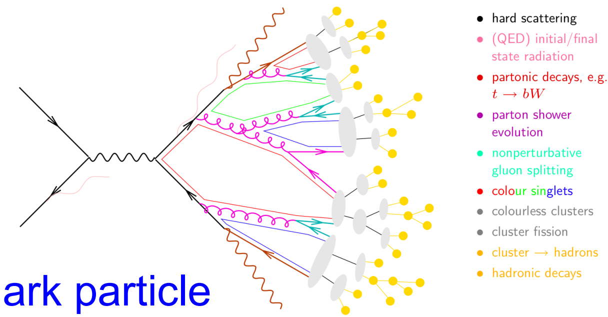
What does the question mean in the first place?

→ It means that we can provide the relation $$m_t^{\rm MC} = m_t^{\rm scheme}(\mu) + \frac{\alpha_s(\mu)}{\pi}\,\delta m^{\rm scheme} + \ldots$$

where $\delta m^{\rm scheme}$ can be **computed in pQCD to (at least) NLO**

The issue is complicated as we must understand and control the interplay of the different components of MC event generators

- Parton shower
- Matching
- Hadronization model



Direct measurements are based on the picture of a top quark particle

- Direct measurements are based on reconstructed top quarks on the top quark resonance
- Employed MCs (Pythia, Herwig) are based on the narrow width limit
- MCs model QCD, hadronization and unstable particle effects & m_t^{MC} = mass in propagator

⇨ m_t^{MC} is close to the top quark pole mass m_t^{pole}

Conservative statement: $$m_t^{\rm MC} = m_t^{\rm pole} + \mathcal{O}(\Gamma_t, \Lambda_{\rm QCD})$$

“Within a precision of about 1 GeV we can consider the top quark as a physical particle with a rest mass which is close to the pole mass.”

What is m_t^{MC} ? (Answer 2.0)

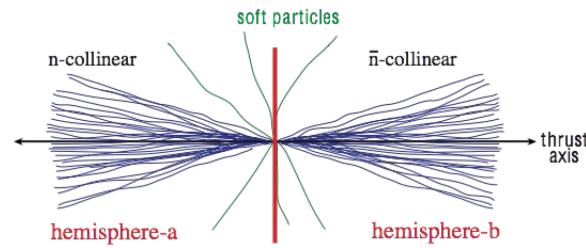
There are 3 essential ingredients to resolve the problem from first principles:

1) At least NLL precise parton shower
2) Factorization compatible hadronization model
3) Observable with factorization of non-perturbative and perturbative contributions and summation of large logarithms

Currently there is <u>only 1 observable class</u> where all 3 ingredients are available.

Jet-mass based <u>event-shape observables</u> <u>in e^+e^- collisions</u> for <u>boosted</u> top pair production in <u>dijet region</u>:

→ 2-jettiness, thrust, … (<u>decay insensitive, global</u>)



Hadron level:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\tau}(\tau, Q, m, \delta m) = \int_0^{Q\tau} \mathrm{d}\ell\, \underbrace{\frac{\mathrm{d}\hat{\sigma}_s}{\mathrm{d}\tau}\Big(\tau - \frac{\ell}{Q}, Q, m, \delta m\Big)}_{\text{Parton cross section}}\, S_{\mathrm{mod}}(\ell)$$

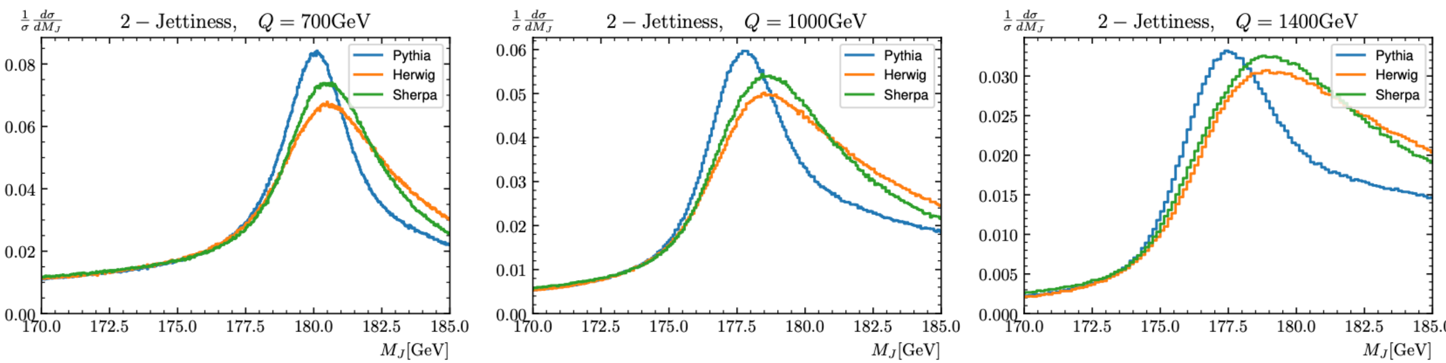
Shape function

$$S_{\mathrm{mod}}(\ell) = \frac{1}{N_c}\,\mathrm{tr_c}\big[\langle 0|[(\overline{\mathrm{Y}}_{\bar{\mathrm{n}},-})^{\mathrm{T}}\,\mathrm{Y}_{\mathrm{n},-}](0)\,\delta(\ell - \mathrm{n}\cdot\hat{\mathrm{P}}_{\mathrm{a}} - \bar{\mathrm{n}}.\hat{\mathrm{P}}_{\mathrm{b}})[\mathrm{Y}^{\dagger}_{\mathrm{n},+}\,(\overline{\mathrm{Y}}^{\dagger}_{\bar{\mathrm{n}},+})^{\mathrm{T}}](0)|0\rangle\big]$$

Boosted Top Eventshapes

$$\frac{d\sigma}{d\tau}(\tau, Q)\bigg|_{\mathrm{had}} = \frac{d\hat{\sigma}}{d\tau}\left(\tau - \frac{2\Omega_1}{Q}, Q\right)\left[1 + \mathcal{O}\left(\frac{\Lambda^2_{\mathrm{QCD}}}{\Gamma_t^2}\right)\right]$$

$$\Omega_1 = \frac{1}{2}\int d\ell\, \ell\, S_{\mathrm{had}}(\ell) = \frac{1}{N_c}\langle 0|\mathrm{tr}\; \overline{Y}^T_{\bar{n}}(0) Y_n(0) \hat{\mathcal{E}}_T(0) Y_n^\dagger(0) \overline{Y}^*_{\bar{n}}(0)|0\rangle$$



$$m_t^{\mathrm{MC}} = 173\,\mathrm{GeV}$$

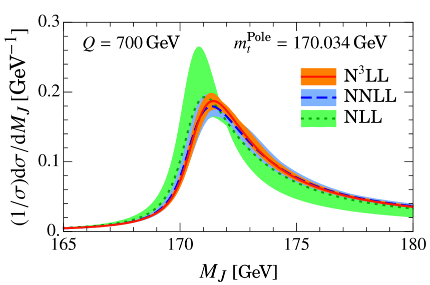
$$M_J = Q\sqrt{\frac{\tau}{2}}$$

Partonic cross section (uses effective theories SCET, bHQET):

$$\left(\frac{d^2\sigma}{dM_t^2\, dM_{\bar{t}}^2}\right)_{\mathrm{hemi}} = \sigma_0\, H_Q(Q, \mu_m) H_m\left(m, \frac{Q}{m}, \mu_m, \mu\right) \times \int_{-\infty}^{\infty} d\ell^+ d\ell^-\, B_+\left(\hat{s}_t - \frac{Q\ell^+}{m}, \Gamma, \mu\right) B_-\left(\hat{s}_{\bar{t}} - \frac{Q\ell^-}{m}, \Gamma, \mu\right) S_{\mathrm{hemi}}(\ell^+, \ell^-, \mu)$$

pert. ultra-collinear soft

pert. large-angle soft



Known at NNNLL order

Fleming, Mantry, Stewart, AHH (2007)

Bachu, Mateu, Pathak, Stewart, AHH (2022)

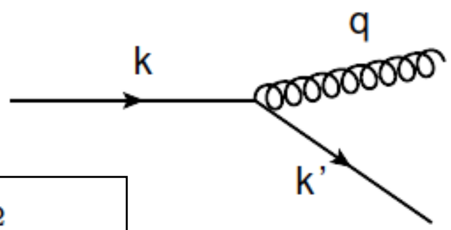
Angular ordered parton shower (Herwig)

→ Coherent Branching algorithm (default Herwig shower):

Dokshitzer, Fadin, Khoze (1982)
Bassetto, Ciafaloni, Marchesini (1983)
Catani, Marchesini, Webber (1991)
Gieseke, Stephens, Webber (2003)

$$\text{evolution variables: } z\,,\ \tilde{q} = \frac{q_\perp^2 + (1-z)^2 m_t^2}{z^2(1-z)^2}$$

probabilities from splitting functions
and Sudakov form factors



color coherence of soft gluon emissions → angular ordering: $z_i^2 \tilde{q}_i^2 > \tilde{q}_{i+1}^2$

→ NLL precise for jet mass based global observables

→ analytic jet mass distribution (inv. mass generated from CB from one boosted quark)

$k^2 \approx$ hemisphere mass (does not account for out of cone radiation)

$$\begin{aligned} J(Q^2, k^2 - m^2, m^2) &= \delta(k^2 - m^2) \\ &+ \int_0^{Q^2} \frac{\mathrm{d}\tilde{q}^2}{\tilde{q}^2} \int_0^1 \mathrm{d}z\, P_{QQ}\big[\alpha_s(z(1-z)\tilde{q}), z, m\big]\theta\Big(\tilde{q}^2 - \frac{Q_0^2 + m^2(1-z)^2}{z^2(1-z)^2}\Big) \\ &\times \Big[zJ\big(z^2\tilde{q}^2, z(k^2-m^2) - z^2(1-z)\tilde{q}^2\big) - J\big(\tilde{q}^2, k^2 - m^2\big)\Big] \end{aligned}$$

$$\boxed{q_\perp > Q_0}$$

Shower Cut Dependence

Dependence of the parton-level peak position from CB on the shower cut Q_0

$$\frac{d\sigma^{\mathrm{cb}}}{d\tau}(\tau, Q, m, Q_0) = \frac{d\sigma^{\mathrm{cb}}}{d\tau}\Big(\tau + \Big[16\frac{Q_0}{Q} - 8\pi\frac{Q_0 m}{Q^2}\Big]\frac{C_F\alpha_s(Q_0)}{4\pi}, Q, m, Q_0 = 0\Big)$$

large-angle soft

should be compensated by hadronization corrections

ultra-collinear

(self-energy absorbed into generator mass)

Comparison to factorization formula with the same cutoff implemented shows:

- Coherent branching compatible with analytic factorization with Q_0 cutoff
- Ultra-collinear term implies that the generator mass is

$$m_t^{\mathrm{CB}}(Q_0) = m_t^{\mathrm{pole}} - \frac{2}{3}\alpha_s(Q_0)Q_0$$

Ultra-collinear linear Q_0-dependence cancels in the observable.

→ Herwig's default cluster hadronization did not have the proper Q_0 dependence.

→ An improved cluster model was developed

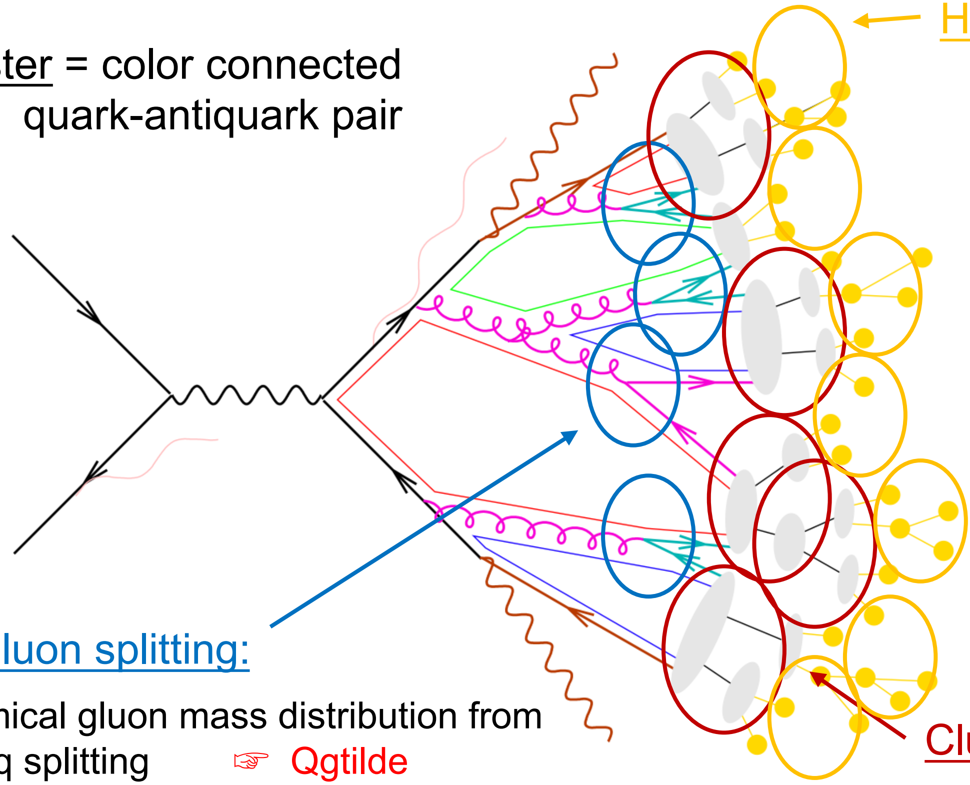
(D) Factorization compatible hadronization model

AHH, Jin. Plätzer, Samitz 2024.09856

Modified cluster hadronization mimics aspects of parton shower dynamics:

Cluster = color connected quark-antiquark pair

Hadron formation and decays



Forced gluon splitting:

- Dynamical gluon mass distribution from g → qq splitting ☞ Qgtilde
- Kinematics in analogy to parton shower

Cluster fission:

- Cluster splitting from branching q → q g and splitting g → qq ☞ Qqtilde
- Kinematics in analogy to the parton shower

➡ Model parameters can consistently adapt to changes of Q_0 in PS

See also : Gieseke etal. arXiv: 2505.14542

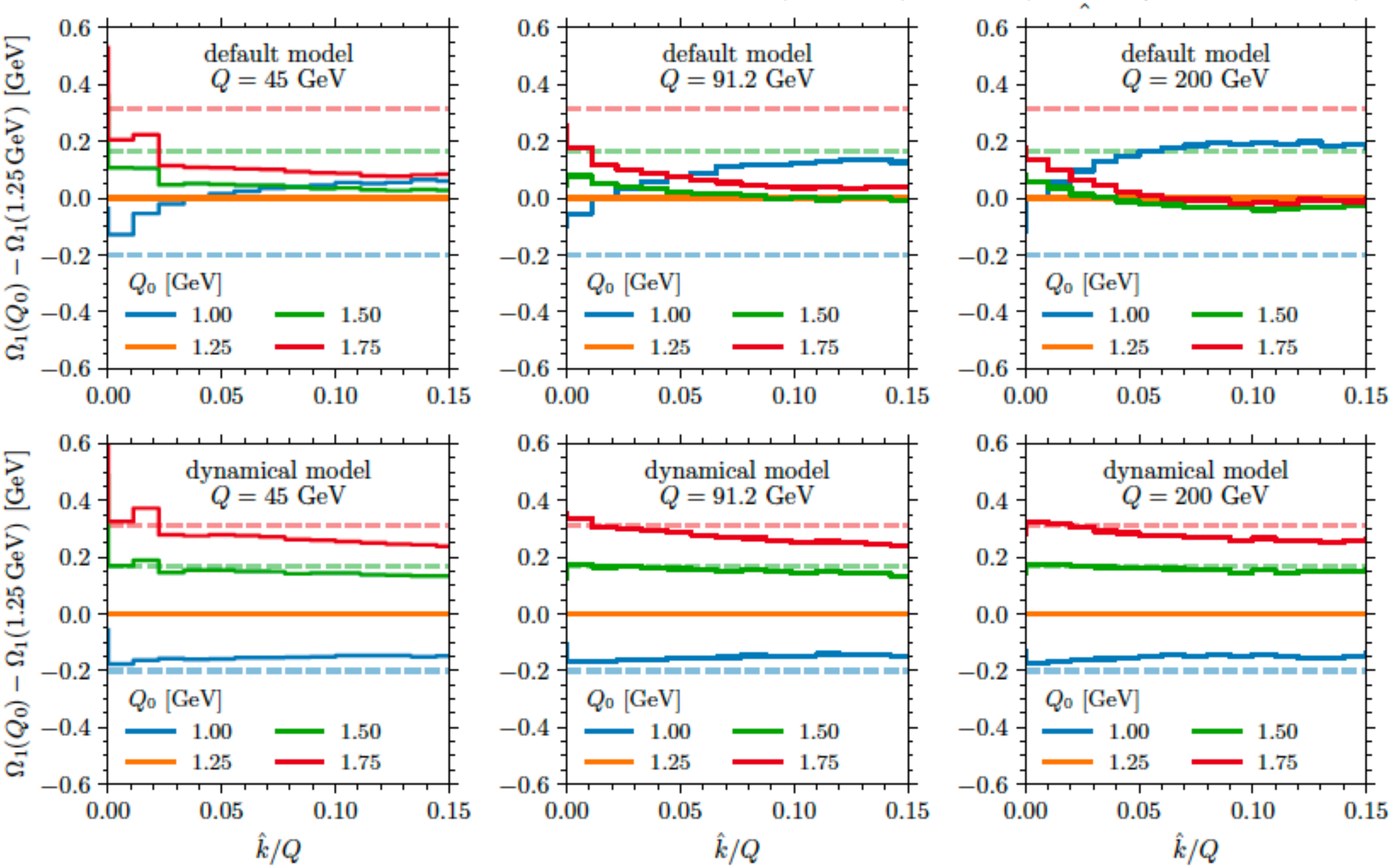
Old Default Model vs. New Dynamical Model

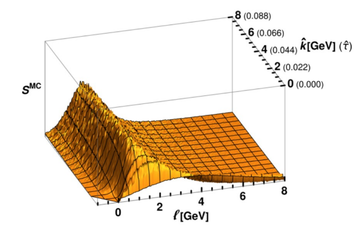
AHH, Jin. Plätzer, Samitz 2404.09856

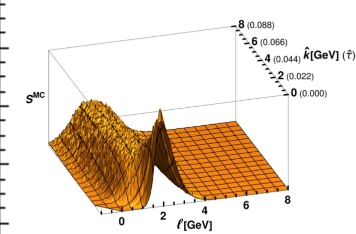
Massless quark production:

Shower cutoff dependence of first moment Ω_1 of migration matrix from simulations for 2-jettiness $\rightarrow$ ”MC scheme for hadronization correction“

$$\Omega_1^{\rm MC}(\hat{k}, Q, Q_0) - \Omega_1^{\rm MC}(\hat{k}, Q, Q_{0,\rm ref} = 1.25\,\mathrm{GeV})$$



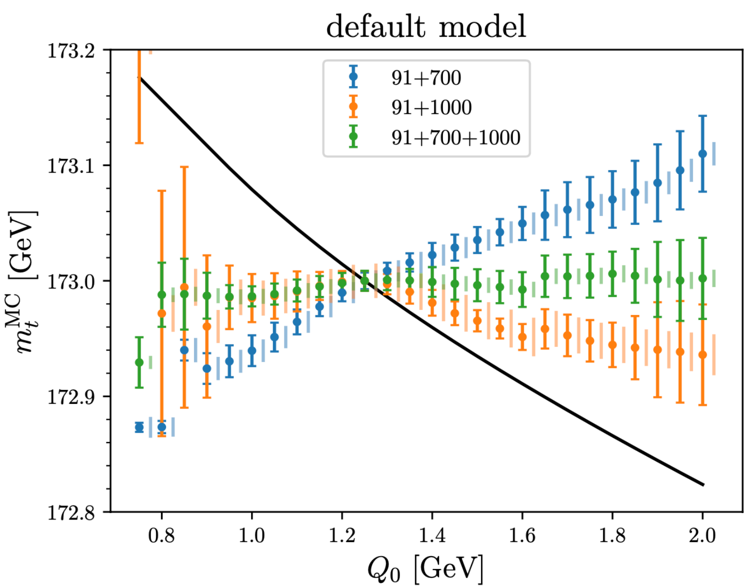




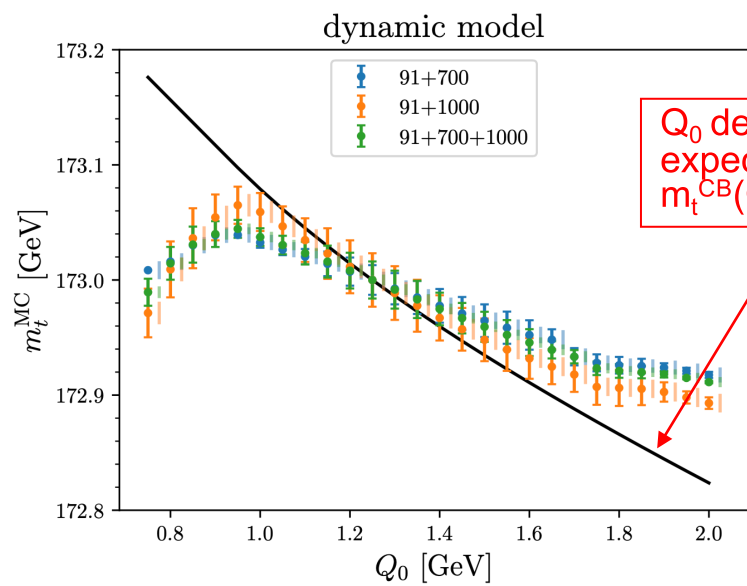
Old Default Model vs. New Dynamical Model

AHH, Jin, Plätzer, Samitz
to appear

Default model



Dynamical model



Q_0 dependence expected from $m_t^{CB}(Q_0)$

Agreement within 50 MeV !

$$m_t^{\rm CB}(Q_0) = m_t^{\rm pole} - \frac{2}{3}\alpha_s(Q_0)Q_0$$

$$\frac{\rm d}{{\rm d}Q_0} m_t^{\rm CB}(Q_0) = -\frac{2}{3}\alpha_s(Q_0)$$

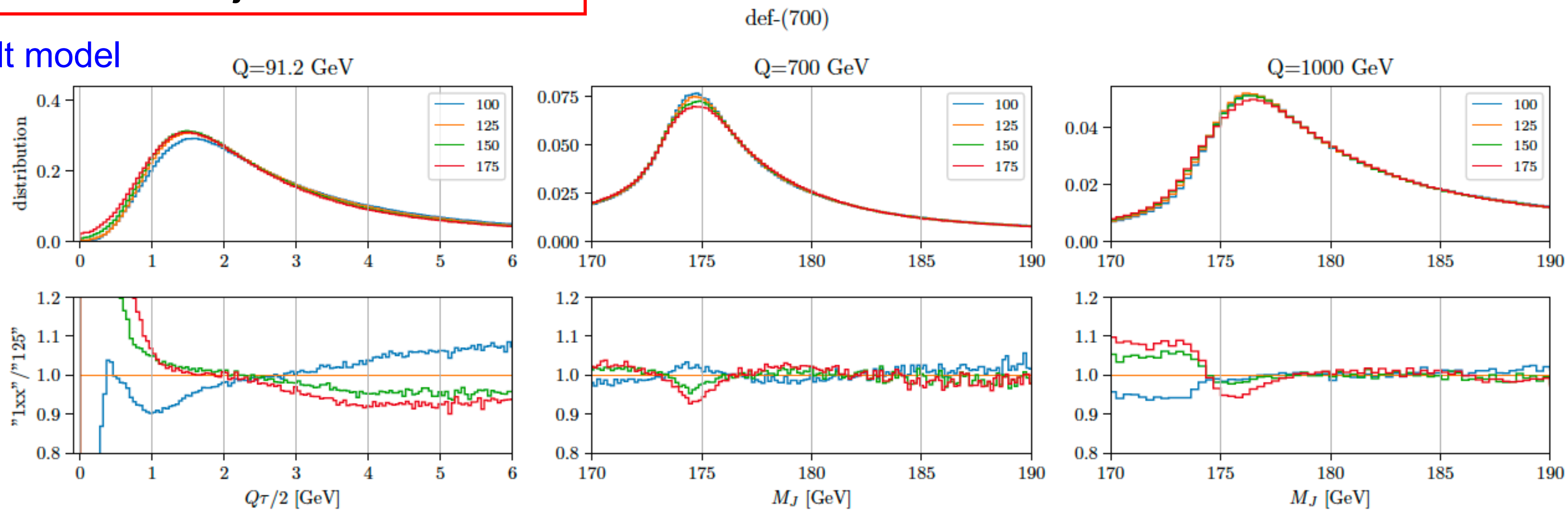
Improved independence on the input top data used for tuning.

Old Default Model vs. New Dynamical Model

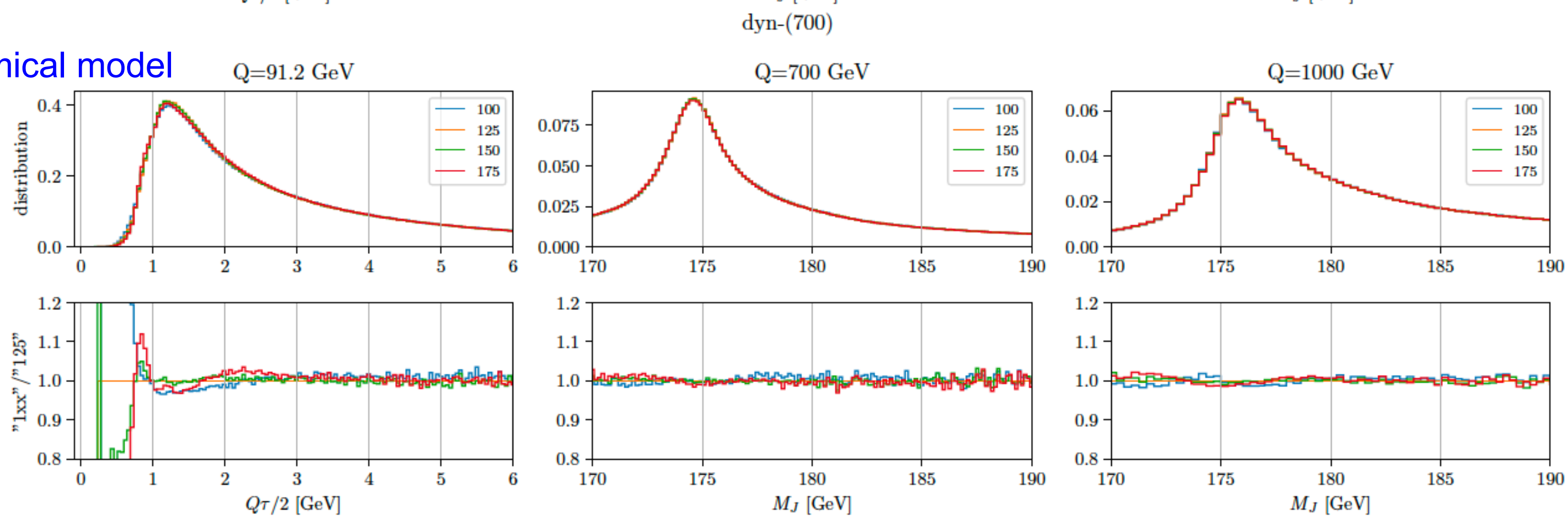
AHH, Jin, Plätzer, Samitz to appear

Predictions from Q_0-tuned MC simulations: 2-jettiness

Default model



Dynamical model



Intermediate Summary

→ Requirements to elevate m_t^{MC} to a scheme (potentially generator dependent)

- (at least) NLL- precise parton shower
- shower cut Q_0 elevated to a factorization scale (RG flow)
- hadronization model consistent with factorization

→ For global inclusive top jet-mass observables (e^+e^-) and Herwig (coherent branching)

$$\boxed{m_t^{\mathrm{Herwig}} = m_t^{\mathrm{CB}}(Q_0) = m_t^{\mathrm{pole}} - \frac{2}{3}\alpha_s(Q_0)Q_0 + \ldots}$$

Depends on p_T cut used for coherent branching !

→ This result should in principle be universal for all observables because the self-energy correction from the ultra-collinear radiation is the same for all observables. But this should also be demonstrated explicitly.

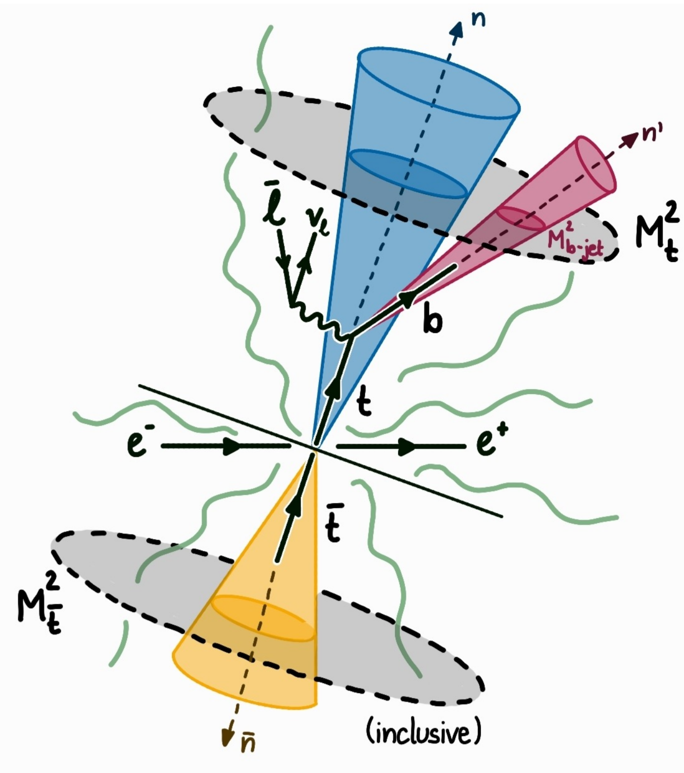
- top decay sensitive observables ←
- non-global results
- other parton showers

Next step to be explored:
Beyond the narrow width limit

Including Semileptonic Top Quark Decay

Regner, AHH, 2507.17672

$$e^- e^+ \to t^\star \, \bar{t}^\star, \quad t^\star \longrightarrow b\ell^+\nu_\ell + X'$$



- Boosted top-antitop production
- semileptonic top quark decay, antitop inclusive
- b-jet: all hadrons in top-hemisphere
- Measurement of hemisphere masses M_t and $M_{\bar{t}}$
  Double resonant kinematics i.e. $M_{t/\bar{t}} \approx m_t$

$$\frac{d\sigma}{dM_t^2 dM_{\bar{t}}^2} \longrightarrow \frac{d\sigma}{dM_t^2 dM_{\bar{t}}^2 dX} \quad \text{@ hadron level}$$

- X constructed from lepton and b-jet momenta
- Hierarchy of scales

$$E_{\rm cm} = Q \gg m_t \gg M_{b\text{-jet}} \gg M_{t/\bar{t}} - m_t \sim \Gamma_t$$

- Endpoint region: $M_{b\text{-jet}} \sim m_t \Gamma_t \ll m_t$

Four distinct QCD radiation modes (no interference)

- ultracollinear radiation (top)
- ultracollinear radiation (antitop)
- large-angle soft radiation
- hard-collinear radiation (b-jet)

Global context of the new factorization formula

Inclusive dijet factorization (top resonance)

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}M_t^2\mathrm{d}M_{\bar t}^2} = \sigma_0\, H_Q(Q)\, H_m\Big(m_t, \frac{Q}{m_t}\Big) \int \mathrm{d}\ell^+\, \mathrm{d}\ell^- \times J_{B_t}^{\Gamma_t}\Big(s_t^* - v_n^-\,\ell^+\Big)\, J_{B_{\bar t}}^{\Gamma_t}\Big(s_{\bar t}^* - v_{\bar n}^+\,\ell^-\Big)\, S_{\rm hemi}(\ell^+, \ell^-)$$

Inclusive semileptonic B-decay (endpoint region)

$$\mathrm{d}\Gamma(\bar B \to X_u \ell \bar\nu_\ell) = \mathrm{d}H^2\, \mathrm{d}\Pi_3(p_B; p_\ell, p_{\nu_\ell}, H)\, \Big(\frac{4G_F}{\sqrt 2}\Big)^2 |V_{ub}|^2\, \tilde L_{\mu\nu}(p_\ell, p_{\nu_\ell})\, W^{\mu\nu}(p_B, q)$$

$$W^{\mu\nu}(v,\, p_B, q) = H_d^{\mu\nu}(q, v, m_b) \int \mathrm{d}\hat\ell^+\, J_{n'}(\hat H^-(m_b\hat v^+ + \overline\Lambda \hat v^+ - \hat q^+ - \hat\ell^+))\, S_{\rm shape}(\hat v^+, \hat\ell^+)$$

Inclusive semileptonic B-decay (endpoint region)

$$\frac{\mathrm{d}^3\sigma}{\mathrm{d}M_t^2\mathrm{d}M_{\bar t}^2\mathrm{d}X} = \sigma_0 \int \mathrm{d}H^2\, \mathrm{d}\Pi_3(M_t = m_t v_n + k^*; p_\ell, p_{\nu_\ell}, H)\, \delta(X - \mathcal{X}(M_t, p_\ell, p_{\nu_\ell}, H))$$

$$\times\ \Big(\frac{e}{\sqrt 2\, s_w}\Big)^4 |V_{tb}|^2\, \tilde L^{\sigma\nu}(p_\ell, p_{\nu_\ell})\, \frac{\tilde g_{\rho\sigma}\, \tilde g_{\mu\nu}}{|q^2 - M_W^2 + iM_W\Gamma_W|^2}\, W^{\rho\mu}(n, v, k, q)$$

$$W^{\rho\mu}(n, v, k, q) = H_Q(Q)\, H_m\Big(m_t, \frac{Q}{m_t}\Big)\, H_d^{\rho\mu}(q = p_\ell + p_{\nu_\ell}, v, m_t)$$

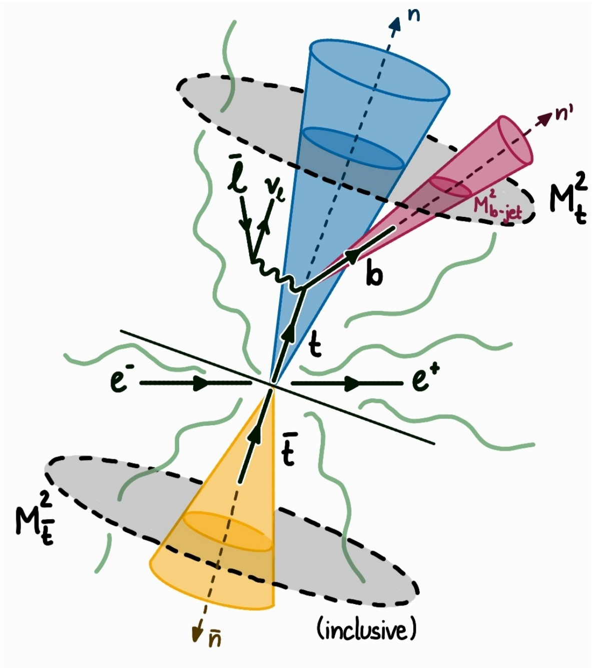
$$\times \int \mathrm{d}\ell^+\, \mathrm{d}\ell^- \mathrm{d}\hat\ell^+\, J_{B_{\bar t}}^{\Gamma_t}\Big(s_{\bar t}^* - \frac{Q}{m_t}\ell^-\Big)\, S_{\rm hemi}(\ell^+, \ell^-)$$

$$\times\ J_{n'}\Big(\hat H^-(m_t \hat v_n^+ - \hat q^+ - \hat\ell^+)\Big)\, S_{ucs}\Big(\hat v_n^+, v_n^-, \gamma^2, \frac{s_t^*}{2} - \frac{Q}{m_t}\ell^+, \frac{s_t^*}{2}, \hat\ell^+\Big)$$

Global context of the new factorization formula

Inclusive dijet factorization (top resonance)

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}M_t^2\mathrm{d}M_{\bar{t}}^2} \sim H_Q \times H_m \times J_{B_{\bar{t}}}^{\Gamma_t} \otimes S_{\mathrm{hemi}} \otimes J_{B_t}^{\Gamma_t}$$



Inclusive semileptonic B-decay (endpoint region)

$$\mathrm{d}\Gamma(\bar{B} \to X_u \ell \bar{\nu}_\ell) = \mathrm{dPS} \times G_F^2 \times L_{\mu\nu} \times W^{\mu\nu}$$

$$W^{\mu\nu} \sim H_d \times S_{\mathrm{shape}} \otimes J_{n'}$$

Semileptonic decay of boosted resonant top (endpoint)

$$\frac{\mathrm{d}^3\sigma}{\mathrm{d}M_t^2\mathrm{d}M_{\bar{t}}^2\mathrm{d}X} \sim \mathrm{dPS} \times L^{\sigma\nu} \times \frac{\tilde{g}_{\rho\sigma}\tilde{g}_{\mu\nu}}{|q^2 - M_W^2 + iM_W\Gamma_W|^2} \times W^{\rho\mu}$$

$$W^{\rho\mu} \sim H_Q \times H_m \times H_d^{\rho\mu} \times J_{B_{\bar{t}}}^{\Gamma_t} \otimes S_{\mathrm{hemi}} \,\tilde{\otimes}\, S_{ucs} \otimes J_{n'}$$

- Merges dijet and inclusive semileptonic meson decay factorization
- “Fermi-motion” of decaying top fixed by hemisphere mass measurement
- Leading power hadronization effects & summation of large logs (Q, m_t, Γ_t)

Global context of the new factorization formula

Inclusive dijet factorization (top resonance)

$$\frac{d^2\sigma}{dM_t^2 dM_{\bar t}^2} \sim H_Q \times H_m \times J_{B_{\bar t}}^{\Gamma_t} \otimes S_{\rm hemi} \otimes J_{B_t}^{\Gamma_t}$$

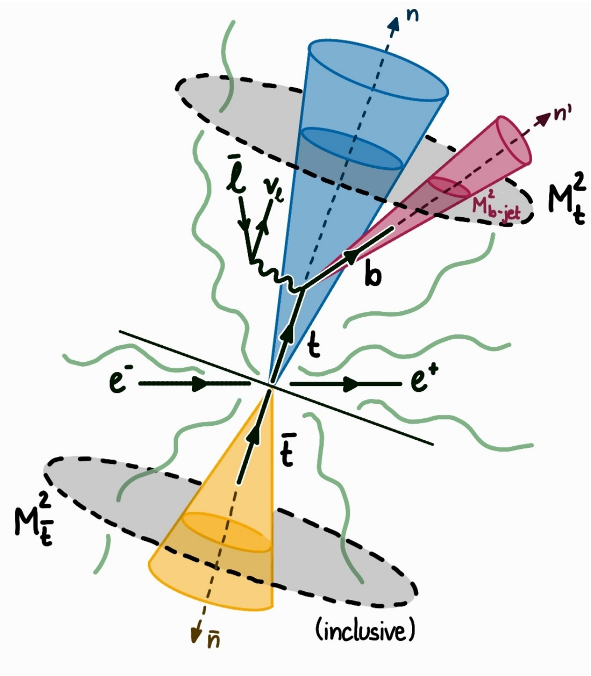
Inclusive semileptonic B-decay (endpoint region)

$$d\Gamma(\bar B \to X_u \ell \bar\nu_\ell) = dPS \times G_F^2 \times L_{\mu\nu} \times W^{\mu\nu}$$

$$W^{\mu\nu} \sim H_d \times S_{\rm shape} \otimes J_{n'}$$

Semileptonic decay of boosted resonant top (endpoint)

$$\frac{d^3\sigma}{dM_t^2 dM_{\bar t}^2 dX} \sim dPS \times L^{\sigma\nu} \times \frac{\tilde g_{\rho\sigma}\tilde g_{\mu\nu}}{|q^2 - M_W^2 + iM_W\Gamma_W|^2} \times W^{\rho\mu}$$



$$W^{\rho\mu} \sim H_Q \times H_m \times H_d^{\rho\mu} \times J_{B_{\bar t}}^{\Gamma_t} \otimes S_{\rm hemi} \tilde\otimes S_{ucs} \otimes J_{n'}$$

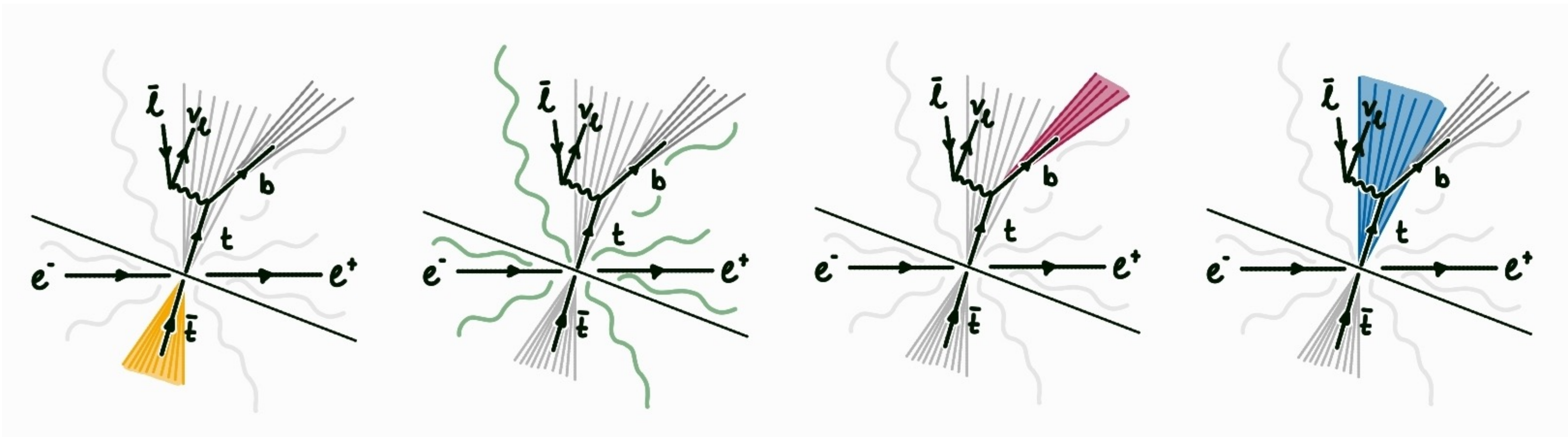
- Merges dijet and inclusive semileptonic meson decay factorization
- “Fermi-motion” of decaying top fixed by hemisphere mass measurement
- Leading power hadronization effects & summation of large logs (Q, m_t, Γ_t)

Bases of Factorization

Four distinct QCD radiation modes

- ultracollinear radiation (top)
- ultracollinear radiation (antitop)
- large-angle soft radiation
- hard-collinear radiation (b-jet)

different modes do not interfere between each other



→ Separation of modes in QFT

- Soft-Collinear Effective Theory – SCET (hard-colinear and soft)
- Boosted Heavy Quark Effective Theory – bHQET (ultra-collinear)

Factorization Derivation

→ Step 1: Factorization as for $\dfrac{d\sigma}{dM_t^2 dM_{\bar t}^2}$

$$\mathrm{d}\sigma = \sum_X^{\text{res.}} (2\pi)^4 \delta^{(4)}(q_{ee} - P_X) \sum_{i=v,a} L^i_{\mu\nu} \langle 0| \mathcal{J}^{\mu\dagger}_{t\bar t,i}(0) |X\rangle \langle X| \mathcal{J}^{\nu}_{t\bar t,i}(0) |0\rangle$$

QCD → SCET: $\mathcal{J}^{\mu}_{t\bar t,i}(0) = C_Q(Q)\, \mathcal{J}^{\mu}_{i,\mathrm{SCET}}(0)\,, \qquad \mathcal{J}^{\mu}_{i,\mathrm{SCET}}(0) = [\bar\chi_n\, S^\dagger_{n,+}\, \Gamma^\mu_i\, S_{\bar n,-}\, \chi_{\bar n}](0)$

SCET → bHQET: $\mathcal{J}^{\mu}_{i,\mathrm{SCET}}(0) = C_m\Big(m_t, \dfrac{Q}{m_t}\Big)\, \mathcal{J}^{\mu}_{i,\mathrm{bHQET}}(0)\,, \quad \mathcal{J}^{\mu}_{i,\mathrm{bHQET}}(0) = [\bar h_{v_n} W^{\mathrm{uc}}_{n,-}\, Y^\dagger_{n,+}\, \Gamma^\mu_i\, Y_{\bar n,-}\, W^{\mathrm{uc}\,\dagger}_{\bar n,+} h_{v_{\bar n}}](0)$

Fierz +
measurement function

$$M_t^2 = (m_t v_n + k_n + k_{s,a})^2$$

$$M_{\bar t}^2 = (m_t v_{\bar n} + k_{\bar n} + k_{s,b})^2$$

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}M_t^2 \mathrm{d}M_{\bar t}^2} = \sigma_0\, H_Q(Q)\, H_m\Big(m_t, \frac{Q}{m_t}\Big) \int \mathrm{d}\ell^+\, \mathrm{d}\ell^- \times J^{\Gamma_t}_{B_t}\Big(s_t^* - \frac{Q}{m_t}\ell^+\Big)\, J^{\Gamma_t}_{B_{\bar t}}\Big(s_{\bar t}^* - \frac{Q}{m_t}\ell^-\Big)\, S_{\mathrm{hemi}}(\ell^+, \ell^-)$$

Only top ultra-collinear jet function needs to be considered further

Large-angle soft radiation insensitive to top decay.

Including Top Quark Decay

→ Step 2: Generalization of inclusive jet function adding the top decay (top rest frame)

$$J_{B_t}^{\Gamma_t}(k^+,k^-) = \frac{1}{8\pi\, N_c\, m_t} \sum_X (2\pi)^4\, \delta^{(4)}(m_t\, v + k - P_X)\,\mathrm{Tr}\Big[\langle 0|\, \overline{T}\, (W_{n,+}^{\mathrm{uc},\dagger}\, h_v)(0)\, |X\rangle\, \langle X|\, T(\bar{h}_v\, W_{n,-}^{\mathrm{uc}})(0)\, |0\rangle\Big]$$

Implement top decay final states and SM decay operator via T-product

$$|X\rangle \to |\ell^+\nu_\ell\rangle\, |X_b\rangle$$

$$T[\bar{h}_v W_{n,-}^{\mathrm{uc}}](0) \to \int \mathrm{d}^4 z_1\, T\, \tilde{\mathcal{O}}_\Gamma(z_1)\, [\bar{h}_v W_{n,-}^{\mathrm{uc}}](0)$$

$$\tilde{\mathcal{O}}_\Gamma(z) = -i\,\frac{4G_F}{\sqrt{2}} V_{tb}\, \frac{M_W^2\, \tilde{g}_{\mu\sigma}}{M_W^2 - q^2 - iM_W\Gamma_W}\, \big[(\bar{b}\,\gamma^\mu P_L\, t)\, (\bar{\nu}_\ell\, \gamma^\sigma P_L\, \ell)\big](z)$$

Match SM decay operator on (b)HQET

$$(\bar{b}\gamma^\mu P_L t)(0) = \sum_{j=1}^{3} C_j(m_t, \hat{v}^+\hat{H}^-)\, \big[\bar{\chi}_{n'}\, Y_{n',+}^{\mathrm{uc},\dagger}\, \Gamma_j^\mu\, h_v\big](z)$$

$$\mathrm{d}J_{B_t}^{\Gamma_t}(k^+,k^-) = \left(\frac{e}{\sqrt{2}\, s_w}\right)^4 |V_{tb}|^2\, \mathrm{d}H^2\, \mathrm{d}\Pi_3(M_t; p_\ell, p_{\nu_\ell}, H)\, \tilde{L}^{\sigma\nu}(p_\ell, p_{\nu_\ell})\, \frac{\tilde{g}_{\rho\sigma}\, \tilde{g}_{\mu\nu}}{|q^2 - M_W^2 + iM_W\Gamma_W|^2}\, W^{\rho\mu}(n,v,k,q)$$

$$W^{\rho\mu}(n,v,k,q) = H_d^{\rho\mu}(q,v,m_t) \int \mathrm{d}\hat{k}_{n'}^+\, \mathrm{d}\hat{k}_{uc}^+\, \delta(m_t + \hat{k}^+ - \hat{q}^+ - \hat{k}_{n'}^+ - \hat{k}_{uc}^+)\, J_{n'}(\hat{H}^- \hat{k}_{n'}^+)\, \tilde{S}_{ucs}(\gamma^2, k^+ + k^-, \hat{k}_{uc}^+)$$

novel factorization function
(soft radiation in top rest frame)

The Ultra-Collinear-Soft-Function

Describes **coherent soft radiation in boosted top's rest frame**

- Gauge invariant
- Non-local
- Can be computed perturbatively

- Sensitive to top production, propagation and decay
  → contains non-factorizable contributions
- Depends on $\bar n^\mu, n'^\mu, v^\mu$
  → dependence on frame-dependent b-jet emission angle

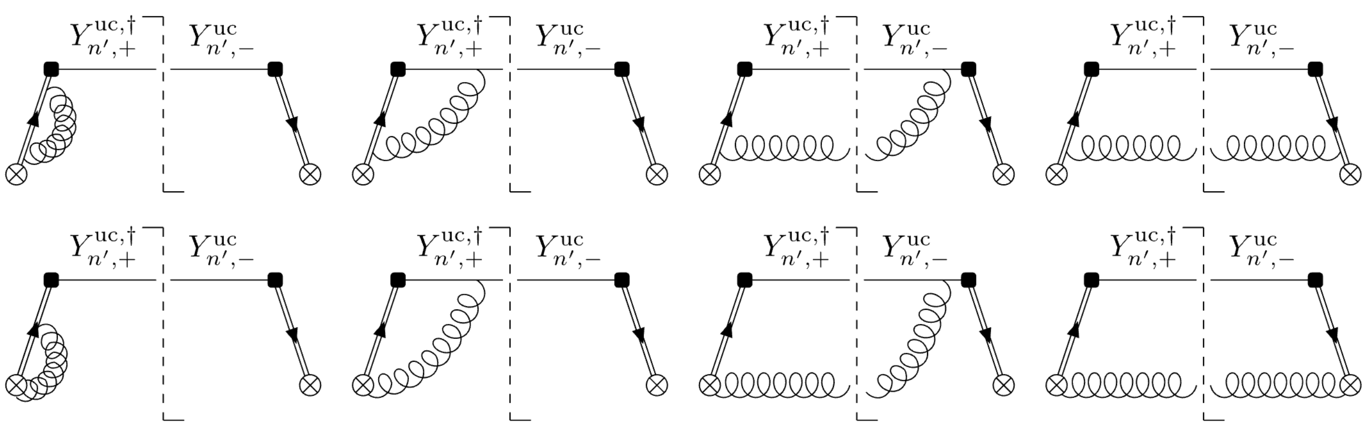
$k^+ = k.n$

$\hat a^+ = a.n'$ (b-jet direction)

$\gamma^2 = \dfrac{2}{\bar n \cdot n'}$

$$\tilde{S}_{\rm ucs}(2v\cdot k,\hat a^+) = \frac{1}{8\pi N_C m_t}\sum_{X_{\rm uc}} \delta(\hat a^+ - \hat K^+_{X_{\rm uc}}) \int \mathrm{d}^4 z_1\, \mathrm{d}^4 z_1\, e^{ik\cdot(z_2-z_1)} \times$$

$$\times\ \langle 0|\overline{\mathbf T}\Big\{[(W^{{\rm uc},\dagger}_{n,+})^{bc}(h_{v_n})^c_\beta](z_2)[(\bar h_{v_n})^d_\alpha (Y^{\rm uc}_{n',-})^{de}](0)\Big\}|X_{\rm uc}\rangle\langle X_{\rm uc}|\,\mathbf T\Big\{\underbrace{[(Y^{{\rm uc},\dagger}_{n',+})^{ef}(\overbrace{h_{v_n})^f_\alpha](0)[(\bar h_{v_n}}^{\text{top propagation}}}_{\text{top decay}}\underbrace{)^a_\beta (W^{\rm uc}_{n,-})^{ab}](z_1)}_{\text{top production}}\Big\}|0\rangle$$



shape-function type diagrams (but off-shell)

$+$ c.c.

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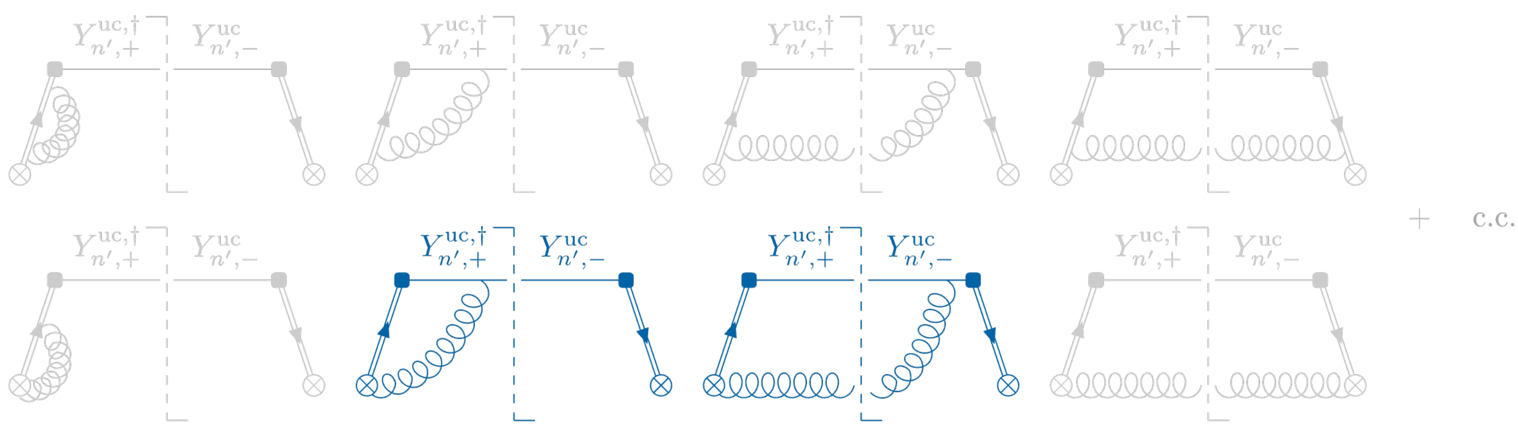
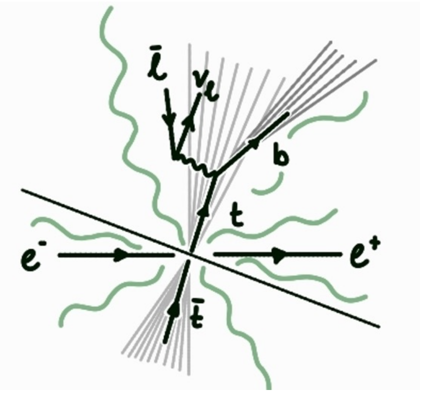


non-factorizable Feynman diagrams

Hadronization corrections

The large-angle soft momenta contribute to

- hemisphere mass
- b-jet



This implies an entangled and angle dependent form of the convolution involving the large-angle soft radiation function for the top hemisphere.

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}M_t^2\,\mathrm{d}M_{\bar t}^2} = \sigma_0\, H_Q(Q)\, H_m\Big(m_t, \frac{Q}{m_t}\Big) \int \mathrm{d}\ell^+\, \mathrm{d}\ell^- \times J_{B_t}^{\Gamma_t}\Big(s_t^* - v_n^- \ell^+\Big)\, J_{B_{\bar t}}^{\Gamma_t}\Big(s_{\bar t}^* - v_{\bar n}^+ \ell^-\Big)\, S_{\mathrm{hemi}}(\ell^+, \ell^-)$$

$$\sim H_d^{\rho\mu} \int \mathrm{d}\hat\ell^+\, J_{n'}(\hat H^-(m_t - \hat q^+ - \hat\ell^+))\, S_{ucs}\Big(\gamma^2, s_t^* - \frac{Q}{m_t}\ell^+, \hat\ell^+ + \frac{s_t^+}{2} - \frac{Q}{\gamma^2 m_t}\ell^+\Big)$$

- The hadronization corrections from the hemisphere soft function S_{hemi} affect the hemisphere mass and the b-jet
- We have calculated NLO corrections to the ucs function. All other FO ingredients are known to at least 2 loops, all anomalous dimensions to 3 loops.

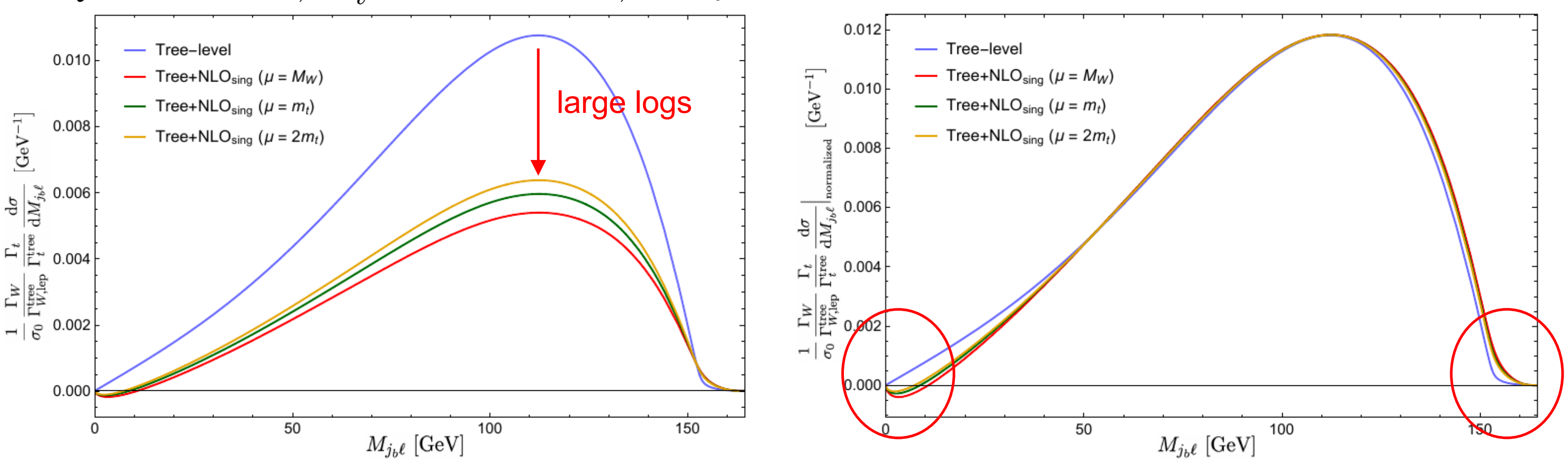
First Numerical Results

- Resummation of logarithms and implementation of hadronization effects are very involved due to the complexity of the convolutions.

→ NLO fixed-order results for the b-jet lepton invariant mass

$$\frac{\mathrm{d}\sigma}{\mathrm{d}M_{j_b\ell}}(\Delta M_t)\bigg|_{\mathrm{NLO,sing}} \equiv \int_{(m_t-\Delta M_t)^2}^{(m_t+\Delta M_t)^2} \mathrm{d}M_t^2 \int_{(m_t-\Delta M_t)^2}^{(m_t+\Delta M_t)^2} \mathrm{d}M_{\bar{t}}^2 \, \frac{\mathrm{d}^3\sigma}{\mathrm{d}M_t^2 \mathrm{d}M_{\bar{t}}^2 \mathrm{d}M_{j_b\ell}}\bigg|_{\mathrm{NLO,sing}}$$

$$Q = 700 \text{ GeV},\ m_t^{\mathrm{pole}} = 173 \text{ GeV},\ \Delta M_t = 10 \text{ GeV}$$



- Resummation of large logarithms is essential
- NLO effects in the endpoint regions due to corrections of the hemisphere resonance mass (M_t) and the b-jet mass ($M_{b\text{-jet}}$).

Summary and Outlook

- The top mass in MC generators can be elevated to a renormalization scheme if
  - parton shower is (at least) NLL precise
  - shower cutoff Q_0 is elevated to a factorization scale
  - hadronization model is compatible with this factorization
  - hadron-level factorization for the observables available for scrutinize MC
- For Herwig (angular ordered PS + cluster hadronization model) this blueprint has been worked out for fat top jet invariant mass distribution and top dijet event shapes

$$m_t^{\mathrm{Herwig}} = m_t^{\mathrm{CB}}(Q_0) = m_t^{\mathrm{pole}} - \frac{2}{3}\alpha_s(Q_0)Q_0 + \ldots$$

- We have extended the factorization for fat top jets adding inclusive semileptonic top decay which allows us to extend these studies to observables such as decay $M_{\text{b-jet lepton}}$
- Upcoming work:
  - analysis of non-factorizable QCD corrections
  - summation of large logs
  - analysis of shower cut dependence
  - tests of Herwig shower and hadronization model
  - spin-observables (→ principles applicable also to toponium)
- Future directions: towards observables suitable for LHC (e.g. non-global)

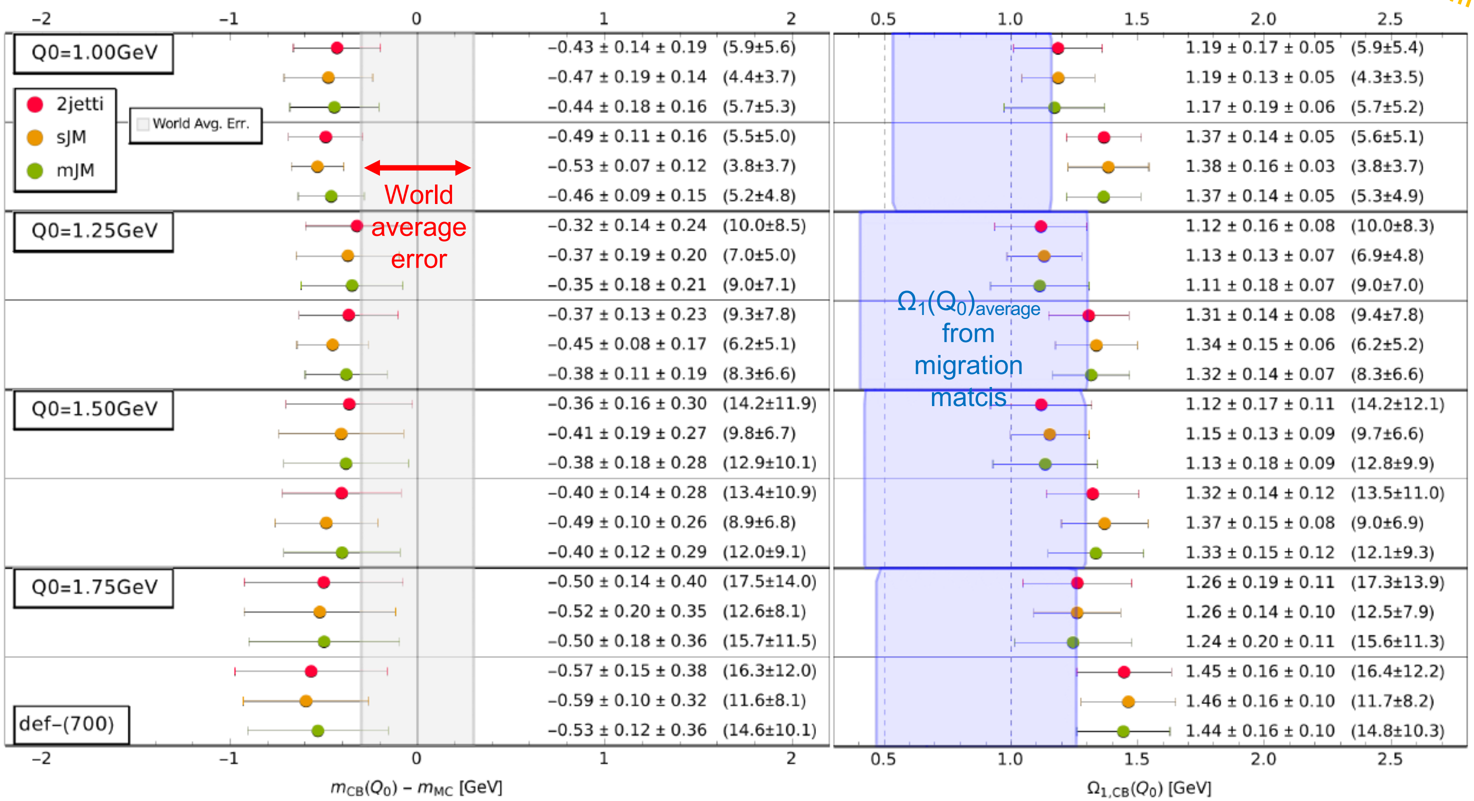
Old Default Model vs. New Dynamical Model

AHH, Jin. Plätzer, Samitz to appear

Cross check: apply top mass calibration to determine $m_t^{CB}(Q_0)$

preliminary

default model



Default: m_t^{MC} incompatible with $m_t^{CB}(Q_0)$

First moment of migration matrix with large variations, Q_0-evolution not visible

Dynamical: m_t^{MC} compatible with $m_t^{CB}(Q_0)$

First moment of migration matrix with smaller variations, Q_0-evolution clearly visible

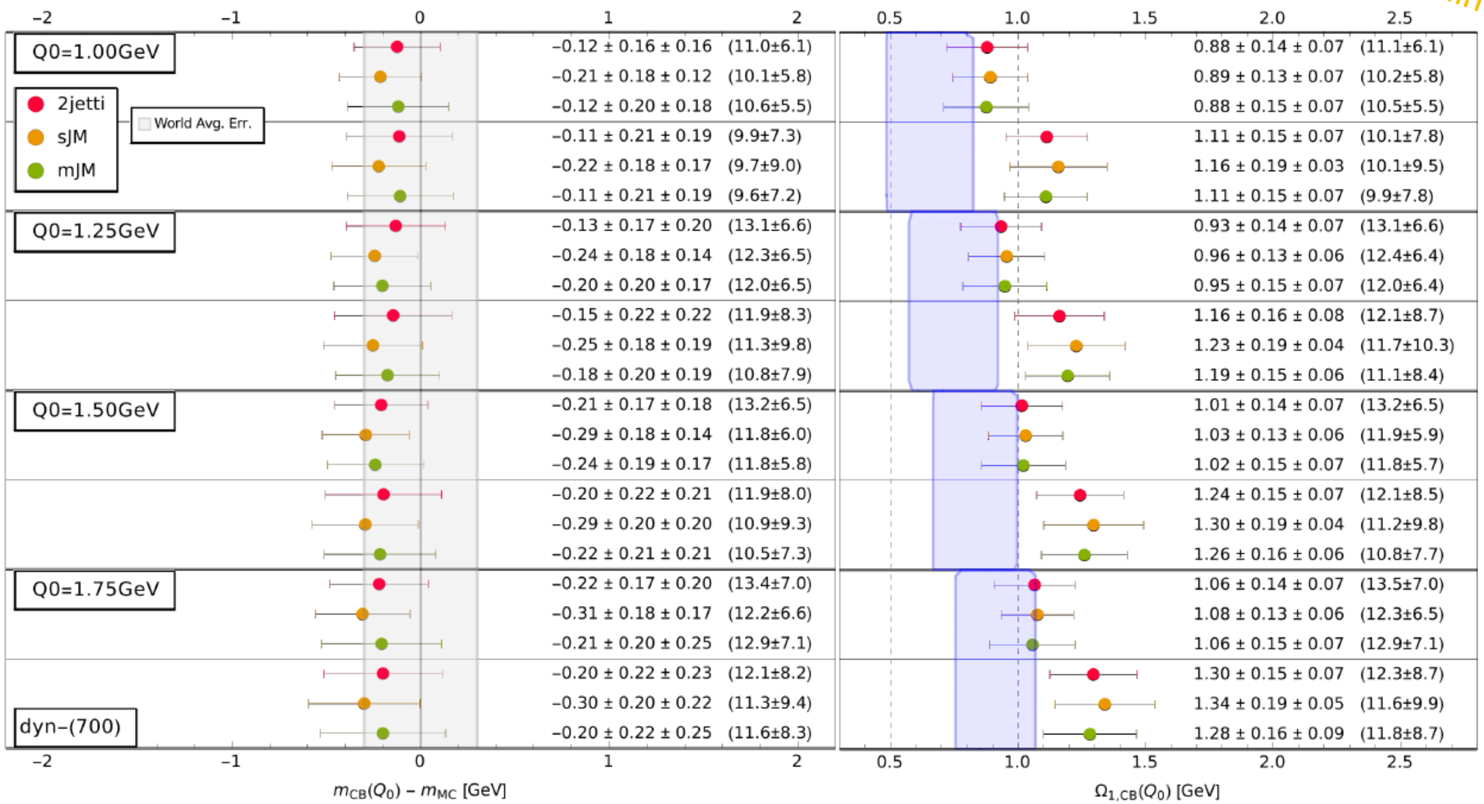
Old Default Model vs. New Dynamical Model

AHH, Jin. Plätzer, Samitz to appear

Cross check: apply top mass calibration to determine $m_t^{CB}(Q_0)$

preliminary

dynamical model



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First moment of migration matrix with large variations, Q_0-evolution not visible

Dynamical: m_t^{MC} compatible with $m_t^{CB}(Q_0)$

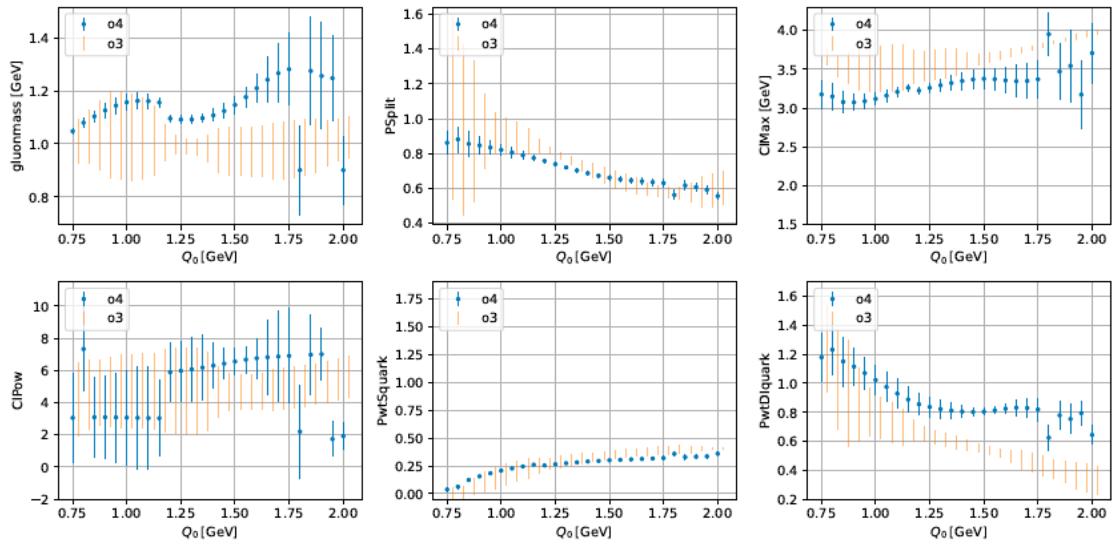
First moment of migration matrix with smaller uncertainties, Q_0-evolution clearly visible

Old Default Model vs. New Dynamical Model

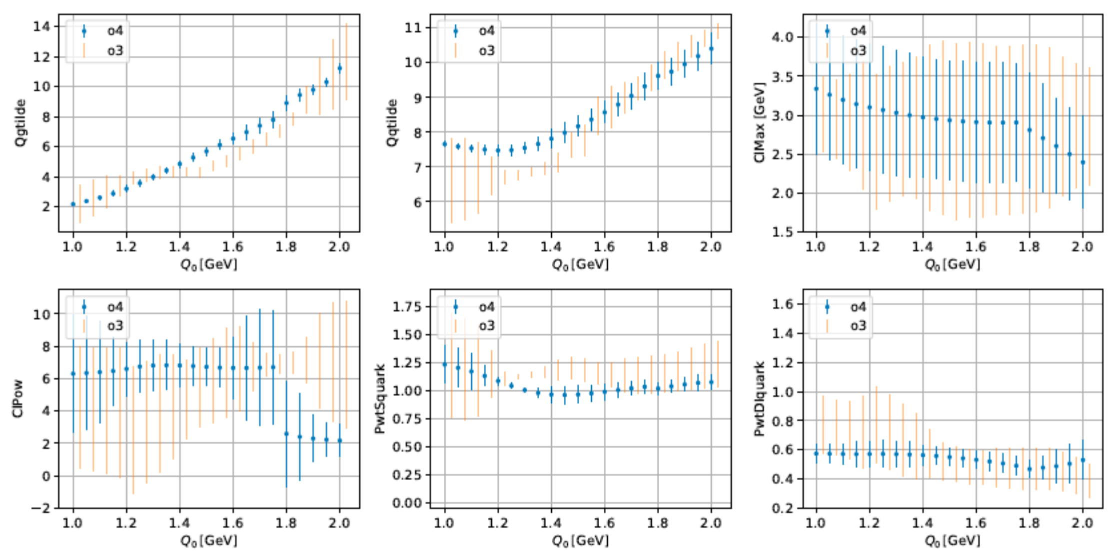
Tuned parameters for Q_0-dependent tuning analyses (apart from m_t^{MC})

AHH, Jin, Plätzer, Samitz
arXiv:2404.09856

Default model



Dynamical model



Old Default Model vs. New Dynamical Model

AHH, Jin, Plätzer, Samitz arXiv:2404.09856

Shower cutoff Q_0 minimal χ^2-values obtained in the tuning fits