



*CosmoLattice*

— School **2025** —



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## Lecture 5

# Non-minimally coupled scalar fields

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Based on: Ben Stefanek's lecture notes

# Most general action of scalar field in curved Space-time

$$\mathcal{S} = \mathcal{S}_{EH} + \mathcal{S}_{\chi} = \int d^4x \sqrt{-g} \left[ \frac{M_p^2}{2} R - \frac{1}{2} g^{\mu\nu} \partial_{\mu} \chi \partial_{\nu} \chi - \frac{1}{2} \xi \chi^2 R - V(\chi) \right]$$

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Why consider NMC?

$R$  = Ricci Scalar Curvature

$\xi$  = Non-minimal coupling (NMC)

- Consistent with all symmetries from GR and SM (and free of scale).

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Why consider NMC?

$R$  = Ricci Scalar Curvature

$\xi$  = Non-minimal coupling (NMC)

- In Conformal Field Theories,  
Scalar fields in curved space-  
time requires  $\xi \neq 0$ .

# Most general action of scalar field in curved Space-time

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Why consider NMC?

$R$  = Ricci Scalar Curvature  
 $\xi$  = Non-minimal coupling (NMC)

- $\xi = 0$  not quantum mechanically stable.
- Non-zero  $\xi$  will be generated via renormalization group in curved space-time.



# Jordan Frame

Scalar-tensor theory of gravity

$$\mathcal{S} = \int d^4x \sqrt{-g} \left[ \frac{M_p^2}{2} f(\chi) R - \frac{1}{2} g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - V(\chi) \right]$$

With

$$f(\chi) = 1 - \frac{\xi \chi^2}{M_p^2}$$

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It is common to study this in Einstein frame

# Einstein Frame

Conformal transformation

$$g_{\mu\nu} \rightarrow \tilde{g}_{\mu\nu} = \Omega^2(x)g_{\mu\nu}$$



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$$g_{\mu\nu} \rightarrow \tilde{g}_{\mu\nu} = \Omega^2(x)g_{\mu\nu}$$

$\Omega^2(x)$  continuous, non-vanishing, finite, real  
arbitrary function of  $x$ .

# Einstein Frame

Conformal transformation

$$g_{\mu\nu} \rightarrow \tilde{g}_{\mu\nu} = \Omega^2(x)g_{\mu\nu}$$

Rules of transformation in d-dimensions

$$g_{\mu\nu} \rightarrow \Omega^{-2}\tilde{g}_{\mu\nu} \qquad g^{\mu\nu} \rightarrow \Omega^2\tilde{g}^{\mu\nu} \qquad g = \det(g^{\mu\nu}) \rightarrow \Omega^{-2d}\tilde{g}$$



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In 4-D transformation of  $R$  is

$$R = \Omega^2\tilde{R} + \Omega^{-1}\Box\Omega - 6\tilde{g}^{\mu\nu}(\partial_\mu\Omega)(\partial_\nu\Omega) = \Omega^2[\tilde{R} + 6\Box\ln\Omega - 6\tilde{g}^{\mu\nu}(\partial_\mu\ln\Omega)(\partial_\nu\ln\Omega)]$$

# Einstein Frame

Action becomes

$$\tilde{\mathcal{S}} = \int d^4x \sqrt{-\tilde{g}} \left[ \frac{M_p^2}{2} f(\chi) \Omega^{2-d} (\tilde{R} + 6 \tilde{\square} \ln \Omega - 6 \tilde{g}^{\mu\nu} (\partial_\mu \ln \Omega) (\partial_\nu \ln \Omega)) - \frac{1}{2} \Omega^{2-d} \tilde{g}^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - \Omega^{-d} V(\chi) \right]$$

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In 4-D we want  $f(\chi) \Omega^{-2} = 1$

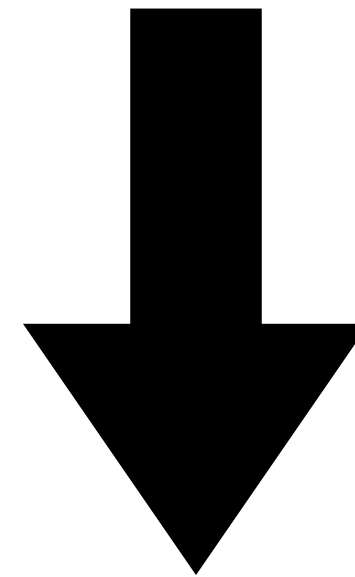


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$4 - D$



$$f(\chi) \Omega^{-2} = 1$$

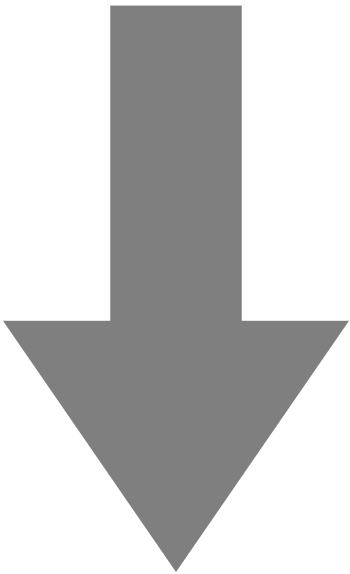
$$\tilde{\mathcal{S}}_E = \int d^4x \sqrt{-\tilde{g}} \left[ \frac{M_p^2}{2} \tilde{R} - \frac{3}{4f^2} M_p^2 \tilde{g}^{\mu\nu} \partial_\mu f \partial_\nu f - \frac{1}{2f} \tilde{g}^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - \frac{1}{f^2} V(\chi) \right]$$

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$$f(\chi) \Omega^{-2} = 1$$

$$\tilde{\mathcal{S}}_E = \int d^4x \sqrt{-\tilde{g}} \left[ \frac{M_p^2}{2} \tilde{R} - \frac{3}{4} \Omega^{2-d} \tilde{g}^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - \Omega^{-d} V(\chi) \right]$$

we have thrown a total derivative term  
 $\# \times \tilde{\Box} \ln \Omega$

# Einstein Frame

$$\tilde{\mathcal{S}}_E = \int d^4x \sqrt{-\tilde{g}} \left[ \frac{M_p^2}{2} \tilde{R} - \frac{1}{2} \mathcal{K}^2(f) \tilde{g}^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - \frac{1}{f^2} V(\chi) \right]$$

With

$$\mathcal{K}(f) = \frac{f + 6\xi(1-f)}{f^2} = \frac{1 + \xi(6\xi - 1)\chi^2/M_p^2}{(1 - \xi\chi^2/M_p^2)^2}$$



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We want a canonically normalized kinetic term  $\mathcal{K} \partial_\mu \chi = \partial_\mu \tilde{\chi}$

$$\tilde{\mathcal{S}}_E = \int d^4x \sqrt{-\tilde{g}} \left[ \frac{M_p^2}{2} \tilde{R} - \frac{1}{2} \tilde{g}^{\mu\nu} \partial_\mu \tilde{\chi} \partial_\nu \tilde{\chi} - \tilde{V}(\tilde{\chi}) \right]$$

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One scalar degree of freedom with Einstein gravity

$$\tilde{\square} \tilde{\chi} - \frac{\partial \tilde{V}}{\partial \tilde{\chi}} = \frac{1}{\sqrt{-\tilde{g}}} \partial_\mu (\sqrt{-\tilde{g}} \tilde{g}^{\mu\nu} \partial_\nu \tilde{\chi}) - \frac{\partial \tilde{V}}{\partial \tilde{\chi}} = 0$$

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We can obtain  $\tilde{\chi}$  in terms of  $\chi$

$$\pm \frac{\tilde{\chi}}{M_p} = \sqrt{6} \left[ \frac{\xi \chi / M_p}{\sqrt{(1/6 + (\xi - 1/6) \xi \chi^2 / M_p^2)}} \right] - \sqrt{\frac{6\xi - 1}{\xi}} \sinh^{-1} \left[ \sqrt{\frac{6\xi - 1}{\xi}} \frac{\xi \chi}{M_p} \right]$$

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- $\xi = 1/6$ : In this case we get conformally invariant theory:

$$\chi = \sqrt{6} \tanh \left( \frac{\tilde{\chi}}{\sqrt{6} M_p} \right) \Rightarrow \tilde{V}(\tilde{\chi}) = V(\chi(\tilde{\chi})) \cosh^4 \left( \frac{\tilde{\chi}}{\sqrt{6} M_p} \right)$$

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$\chi = \sqrt{\frac{2}{3}} \tilde{\chi}$   $\tilde{\chi} = \sqrt{\frac{3}{2}} \chi$

This mean EOM for conformal amplitude in conformal time reduces to free field in FLRW.

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- $\xi \chi^2 / M_p^2 \gg 1$ : In this case  $\chi = \frac{M_p}{\sqrt{\xi}} \exp \left( \frac{\tilde{\chi}}{\sqrt{6} M_p} \right)$



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- $\xi$  Inverting the formula for  $\chi/\tilde{\chi}$  is non-linear!

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- $\xi$  Fields are quantum, so not clear the meaning of the conformal mapping.

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We will remain in JORDAN FRAME.

- $\xi \chi^2 / M_p^2 \gg 1$ : In this case  $\chi = \frac{M_p}{\sqrt{\xi}} \exp \left( \frac{\tilde{\chi}}{\sqrt{6} M_p} \right)$

# NMC fields in Jordan Frame

$$\mathcal{S}_{\mathcal{J}} = \mathcal{S}_{EH} + \mathcal{S}_{\chi} = \int d^4x \sqrt{-g} \left[ \frac{M_p^2}{2} R - \frac{1}{2} g^{\mu\nu} \partial_{\mu} \chi \partial_{\nu} \chi - \frac{1}{2} \xi \chi^2 R - V(\chi) + \mathcal{L}_m \right]$$

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FLRW universe with the “ $\alpha$ -time” metric

$$ds^2 = - a(\eta)^{2\alpha} + a(\eta)^2 \delta_{ij} dx^i dx^j$$



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FLRW universe with the “ $\alpha$ -time” metric

$$ds^2 = -a(\eta)^{2\alpha} d\eta^2 + a(\eta)^2 \delta_{ij} dx^i dx^j$$

With  $\sqrt{-g} = a(\eta)^{3+\alpha}$  and Ricci scalar:

$$R = \frac{6}{a^{2\alpha}} \left[ \frac{a''}{a} - (1 - \alpha) \left( \frac{a'}{a} \right)^2 \right] = \frac{6}{a^{2\alpha}} \left[ \frac{a''}{a} - (1 - \alpha) \mathcal{H}^2 \right]$$

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FLRW universe with the “ $\alpha$ -time” metric

Notice (') are time-derivatives respect to  $\alpha$ -time.

$$\text{Therefore } \mathcal{H} = H a^{\alpha}$$

$$R = \frac{6}{a^{2\alpha}} \left[ \frac{a''}{a} - (1 - \alpha) \left( \frac{a'}{a} \right)^2 \right] = \frac{6}{a^{2\alpha}} \left[ \frac{a''}{a} - (1 - \alpha) \mathcal{H}^2 \right]$$

# Equation of motion for NMC fields in Jordan Frame

Variating  $\mathcal{S}_\chi$  w.r.t  $\chi$  we obtain

$$\square\chi - \xi R\chi - \frac{\partial V}{\partial\chi} = 0$$



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Variating  $\mathcal{S}_\chi$  w.r.t  $\chi$  we obtain

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Which in  $\alpha$  -time and FLRW metric reads as

$$\chi'' + (3 - \alpha)\frac{a'}{a}\chi' - a^{-2(1-\alpha)}\nabla^2\chi + a^{2\alpha}\left(\xi R\chi + \frac{\partial V}{\partial\chi}\right) = 0$$

# Friedmann Equations

To be consistent with FLRW metric we need

$$T_{\nu}^{\mu} = \text{diag}\{-\rho(\eta), p(\eta), p(\eta), p(\eta)\}$$

$p(\eta)$  and  $\rho(\eta)$  depend on volume averages



# Friedmann Equations

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Friedmann equations take the following form

$$\mathcal{H}^2 \equiv \left(\frac{a'}{a}\right)^2 = \frac{a^{2\alpha}}{3m_p^2} \rho(\eta)$$

$$\frac{a''}{a} = -\frac{a^{2\alpha}}{6m_p^2} [(1 - 2\alpha)\rho(\eta) + 3p(\eta)]$$

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$$\frac{a''}{a} = -\frac{a^{2\alpha}}{6m_p^2} [(1 - 2\alpha)\rho(\eta) + 3p(\eta)]$$

$p(\eta)$  and  $\rho(\eta)$  are given by

$$\rho(\eta) = \rho(\eta)_\chi + \rho(\eta)_m \equiv a^{-2\alpha} \langle T_{00}^\chi \rangle + a^{-2\alpha} \langle T_{00}^m \rangle$$

$$p(\eta) = p(\eta)_\chi + p(\eta)_m \equiv \frac{1}{3} a^{-2} \delta^{ij} \langle T_{ij}^\chi \rangle + \frac{1}{3} a^{-2} \delta^{ij} \langle T_{ij}^m \rangle$$

# Energy-momentum tensor for NMC scalar

Energy-momentum tensor for  $\chi$  is defined as

$$T_{\mu\nu}^{\chi} = -\frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g} \mathcal{L}_{\chi})}{\delta g^{\mu\nu}} = -2 \frac{\delta(\mathcal{L}_{\chi})}{\delta g^{\mu\nu}} + g_{\mu\nu} \mathcal{L}_{\chi}$$



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We get for  $T_{\mu\nu}^{\chi}$

$$T_{\mu\nu}^{\chi} = \partial_{\mu}\chi\partial_{\nu}\chi - g^{\mu\nu} \left( \frac{1}{2}g^{\rho\sigma}\partial_{\rho}\chi\partial_{\sigma}\chi + V(\chi) \right) + \xi(G_{\mu\nu} + g_{\mu\nu}\square - \nabla_{\mu}\nabla_{\nu})\chi^2$$

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*minimally coupled*

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Minimally coupled

Non-minimally coupled



# Energy-momentum tensor for NMC scalar

Energy-momentum tensor for  $\chi$  is defined as

Non-minimally coupled part contains curvature terms!

We get for  $T_{\mu\nu}^\chi$

$$T_{\mu\nu}^\chi = \underbrace{\partial_\mu \chi \partial_\nu \chi - g^{\mu\nu} \left( \frac{1}{2} g^{\rho\sigma} \partial_\rho \chi \partial_\sigma \chi + V(\chi) \right)}_{\text{Minimally coupled}} + \underbrace{\xi (G_{\mu\nu} + g_{\mu\nu} \square - \nabla_\mu \nabla_\nu) \chi^2}_{\text{Non-minimally coupled}}$$

Minimally coupled

Non-minimally coupled

# Energy-momentum tensor for NMC scalar

Energy density  $\rho_\chi = -T_0^0 = -g^{00}T_{00}$

$$\rho(\eta) = \frac{1}{2a^{2\alpha}}\langle\chi'^2\rangle + \frac{1}{2a^2}\langle(\nabla\chi)^2\rangle + \langle V\rangle + \frac{3\xi}{a^{2\alpha}}\mathcal{H}^2\langle\chi^2\rangle + \frac{6\xi}{a^{2\alpha}}\mathcal{H}\langle\chi\chi'\rangle - \frac{\xi}{a^2}\langle\nabla^2\chi^2\rangle$$



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Pressure  $p = g^{ij} T_{ij} / 3$

$$\begin{aligned} p(\eta) = & \frac{(1 - 4\xi)}{2a^{2\alpha}} \langle \chi'^2 \rangle - \frac{(1 - 12\xi)}{6a^2} \langle (\nabla \chi)^2 \rangle - \langle V \rangle + \frac{2\xi}{a^{2\alpha}} \mathcal{H} \langle \chi \chi' \rangle - \frac{\xi}{3a^2} \langle \nabla^2 \chi^2 \rangle \\ & + 2\xi \langle \chi V_{,\chi} \rangle + \frac{\xi}{a^{2\alpha}} \left[ \mathcal{H}^2 + 12 \left( \xi - \frac{1}{6} \right) \left( \frac{a''}{a} + (1 - \alpha) \mathcal{H}^2 \right) \right] \langle \chi^2 \rangle \end{aligned}$$

# Energy-momentum tensor for NMC scalar

$$\rho(\eta) = \frac{1}{2a^{2\alpha}} \langle \chi'^2 \rangle + \frac{1}{2a^2} \langle (\nabla \chi)^2 \rangle + \langle V \rangle + \frac{3\xi}{a^{2\alpha}} \mathcal{H}^2 \langle \chi^2 \rangle + \frac{6\xi}{a^{2\alpha}} \mathcal{H} \langle \chi \chi' \rangle - \frac{\xi}{a^2} \langle \nabla^2 \chi^2 \rangle$$

$$p(\eta) = \frac{(1-4\xi)}{2a^{2\alpha}} \langle \chi'^2 \rangle - \frac{(1-12\xi)}{6a^2} \langle (\nabla \chi)^2 \rangle - \langle V \rangle + \frac{2\xi}{a^{2\alpha}} \mathcal{H} \langle \chi \chi' \rangle - \frac{\xi}{3a^2} \langle \nabla^2 \chi^2 \rangle$$
$$+ 2\xi \langle \chi V_{,\chi} \rangle + \frac{\xi}{a^{2\alpha}} \left[ \mathcal{H}^2 + 12 \left( \xi - \frac{1}{6} \right) \left( \frac{a''}{a} + (1-\alpha) \mathcal{H}^2 \right) \right] \langle \chi^2 \rangle$$

# Equation of motion for $a(\eta)$

Acceleration equation

$$\frac{a''}{a} = -\frac{a^{2\alpha}}{6M_p^2}[(1 - 2\alpha)\rho(\eta) + 3p(\eta)]$$

Enough to evolve  $a(\eta)$ , but it contains non-linear derivatives of  $a(\eta)$  and surface terms.

Is there a better way?



# Equation of motion for $a(\eta)$

Acceleration equation

$$\frac{a''}{a} = -\frac{a^{2\alpha}}{6M_p^2}[(1 - 2\alpha)\rho(\eta) + 3p(\eta)]$$

Let us recall definition of  $R$

$$\frac{a''}{a} - (1 - \alpha)\left(\frac{a'}{a}\right)^2 = \frac{a^{2\alpha}}{6}R$$

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Let us recall definition of  $R$

$$\frac{a''}{a} - (1 - \alpha) \left( \frac{a'}{a} \right)^2 = \frac{a^{2\alpha}}{6} R$$

and trace of Einstein equation

$$R = -\frac{T}{M_p^2} = \frac{1}{M_p^2}(\rho - 3p)$$



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Trace is a linear operator so

$$T = T_\chi + T_m$$

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$$-M_p^2 R = T_\chi + T_m$$



# Equation of motion for $a(\eta)$

$$-M_p^2 R = T_\chi + T_m$$

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$$-M_p^2 R = T_\chi + T_m$$

Trace of  $T_\chi$  (in 4-D) is given by

$$T_\chi = (6\xi - 1)(\partial^\mu \chi \partial_\mu \chi + \xi \chi^2 R) - 4V(\chi) + 6\xi \chi V_{,\chi}$$

# Equation of motion for $a(\eta)$

$$-M_p^2 R = T_\chi + T_m$$

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$$T_\chi = (6\xi - 1)(\partial^\mu \chi \partial_\mu \chi + \xi \chi^2 R) - 4V(\chi) + 6\xi \chi V_{,\chi}$$

We can solve for  $R$

$$R = \frac{1}{M_p^2} \frac{(1 - 6\xi)\langle \partial^\mu \chi \partial_\mu \chi \rangle + 4\langle V \rangle - 6\langle \chi V_{,\chi} \rangle - \langle T_m \rangle}{1 + (6\xi - 1)\xi \langle \chi^2 \rangle / M_p^2}$$



# Equation of motion for $a(\eta)$

$$R = \frac{1}{2} \frac{(1 - 6\xi) \langle \partial^\mu \chi \partial_\mu \chi \rangle + 4 \langle V \rangle - 6 \langle \chi V_{,\chi} \rangle - \langle T_m \rangle}{a^4}$$

Everything set to write  
lattice formulation.

$$\frac{a''}{a} - (1 - \alpha) \left( \frac{a'}{a} \right)^2 = \frac{a^{2\alpha}}{6} R$$

# Lattice implementation for NMC scalars

$$\frac{a''}{a} - (1 - \alpha) \left( \frac{a'}{a} \right)^2 = \frac{a^{2\alpha}}{6} R$$

EoM for  $a(\eta)$

$$R = \frac{1}{M_p^2} \frac{(1 - 6\xi) \langle \partial^\mu \chi \partial_\mu \chi \rangle + 4 \langle V \rangle - 6 \langle \chi V_{,\chi} \rangle - \langle T_m \rangle}{1 + (6\xi - 1) \xi \langle \chi^2 \rangle / M_p^2}$$

$$\chi'' + (3 - \alpha) \frac{a'}{a} \chi' - a^{-2(1-\alpha)} \nabla^2 \chi + a^{2\alpha} \left( \xi R \chi + \frac{\partial V}{\partial \chi} \right) = 0$$

EoM for  $\chi$

# Lattice implementation for NMC scalars

Program variables

$$\tilde{\chi} = \frac{\chi}{f_*}, \quad d\tilde{\eta} = \omega_* d\eta, \quad d\tilde{x}_i = \omega_* dx_i$$



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$$\tilde{\chi} = \frac{\chi}{f_*}, \quad d\tilde{\eta} = \omega_* d\eta, \quad d\tilde{x}_i = \omega_* dx_i$$

$$\tilde{R} = \omega_*^{-2} R \quad \tilde{V} = \frac{V}{f_*^2 \omega_*^2}$$

# Lattice implementation for NMC scalars

Program variables

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$$\tilde{R} = \omega_*^{-2} R \quad \tilde{V} = \frac{V}{f_*^2 \omega_*^2}$$

In particular we chose

$$f_* = M_p \quad \omega_* = H_i$$



# Lattice implementation for NMC scalars

We choose  $\tilde{\pi}_\chi = a^{3-\alpha}\tilde{\chi}'$

$$\left\{ \begin{array}{l} \tilde{\chi}' = a^{\alpha-3}\tilde{\pi}_\chi \\ \tilde{\pi}_\chi' = \tilde{K}_\chi[a, \tilde{\chi}, \{\tilde{\phi}_m\}, \tilde{R}] \end{array} \right.$$
$$\tilde{K}_\chi[a, \tilde{\chi}, \{\tilde{\phi}_m\}, \tilde{R}] \equiv a^{1+\alpha} \tilde{\nabla}^2 \tilde{\chi} - a^{3+\alpha} \left( \xi \tilde{R} \tilde{\chi} + \frac{\partial \tilde{V}}{\partial \tilde{\chi}} \right)$$

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$$\tilde{K}_\chi[a, \tilde{\chi}, \{\tilde{\phi}_m\}, \tilde{R}] \equiv a^{1+\alpha} \tilde{\nabla}^2 \tilde{\chi} - a^{3+\alpha} \left( \xi \tilde{R} \tilde{\chi} + \frac{\partial \tilde{V}}{\partial \tilde{\chi}} \right)$$

# Lattice implementation for NMC scalars

We choose  $\tilde{\pi}_a = a^{1-\alpha} a'$

$$\begin{cases} a' = a^{\alpha-1} \tilde{\pi}_a \\ \tilde{\pi}_a' = \tilde{K}_a[a, \tilde{R}] \end{cases} \quad \tilde{K}_a[a, \tilde{R}] \equiv \frac{a^{2+\alpha}}{6} \tilde{R}$$

$$\tilde{R} = \frac{f_*^2}{M_P^2} \left[ \frac{2(1-6\xi)(\tilde{E}_G^{\tilde{\chi}} - \tilde{E}_K^{\tilde{\chi}}) + 4\langle \tilde{V} \rangle - 6\xi\langle \tilde{\chi} \tilde{V}_{,\tilde{\chi}} \rangle + (\tilde{\rho}_m - 3\tilde{p}_m)}{1 + (6\xi - 1)\xi\langle \tilde{\chi}^2 \rangle f_*^2 / M_P^2} \right]$$



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$$\tilde{R} = \frac{f_*^2}{M_P^2} \left[ \frac{2(1 - 6\xi)(\tilde{E}_G^{\tilde{\chi}} - \tilde{E}_K^{\tilde{\chi}}) + 4\langle\tilde{V}\rangle - 6\xi\langle\tilde{\chi}\tilde{V}_{,\tilde{\chi}}\rangle + (\tilde{\rho}_m - 3\tilde{p}_m)}{1 + (6\xi - 1)\xi\langle\tilde{\chi}^2\rangle f_*^2/M_P^2} \right]$$

$$\tilde{E}_K^{\tilde{\chi}} = \frac{1}{2a^6} \langle \tilde{\pi}_{\tilde{\chi}}^2 \rangle$$

Kinetic Energy of NMC field

$$\tilde{E}_G^{\tilde{\chi}} = \frac{1}{2a^2} \sum_i \langle \tilde{\partial}_i \tilde{\chi} \partial_i \tilde{\chi} \rangle$$

Gradient Energy of NMC field

# Non symplectic evolver

$$\tilde{K}_a[a, \tilde{R}] \equiv \frac{a^{2+\alpha}}{6} \tilde{R}$$

$$\tilde{R} = \frac{f_*^2}{M_P^2} \left[ \frac{2(1 - 6\xi)(\tilde{E}_G^{\tilde{\chi}} - \tilde{E}_K^{\tilde{\chi}}) + 4\langle \tilde{V} \rangle - 6\xi\langle \tilde{\chi} \tilde{V}_{,\tilde{\chi}} \rangle + (\tilde{\rho}_m - 3\tilde{p}_m)}{1 + (6\xi - 1)\xi\langle \tilde{\chi}^2 \rangle f_*^2 / M_P^2} \right]$$

$\tilde{R}$  depends on all field amplitudes and conjugate momenta

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$\tilde{R}$  depends on all field amplitudes and conjugate momenta

$K_{\tilde{\chi}}[\tilde{R}[\tilde{\pi}_{\tilde{\chi}}]]$  = kernel of NMC field depends on its conjugate momenta



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We need non-symplectic evolver like Runge-Kutta!

$\tilde{R}$  depends on all field amplitudes and conjugate momenta

$K_{\tilde{\chi}}[\tilde{R}[\tilde{\pi}_{\tilde{\chi}}]]$  = kernel of NMC field depends on its conjugate momenta

**working example**

**Based on: Toby Opferkuch's presentation**



# What are NMC Scalars good for?

$$\mathcal{L} \supset \xi R \phi^2 \sim \xi H^2 \phi^2$$

*Inflation & reheating:*

Dominates only in the  
very early Universe

Higgs inflation & reheating

*Bezrukov & Shaposhnikov 0710.3755*

*Bezrukov et al 0812.3622 Garcia-Bellido et al 0812.4624*

$f(\phi)$  inflation & reheating

*Watanabe & Komatsu gr-qc/0612120*

$f(R)$  inflation & reheating

*De Felice & Tsujikawa 1002.4928*

Geometric preheating

*Bassett & Liberati hep-ph/9709417*

Ricci reheating

*Figueroa & Byrnes 1604.03905 Opferkuch et al 1905.06823*

NMC reheating

*Series of papers by DeCross et al 1510.08553, 1610.08868 & 1610.08916*

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Example

Ricci reheating

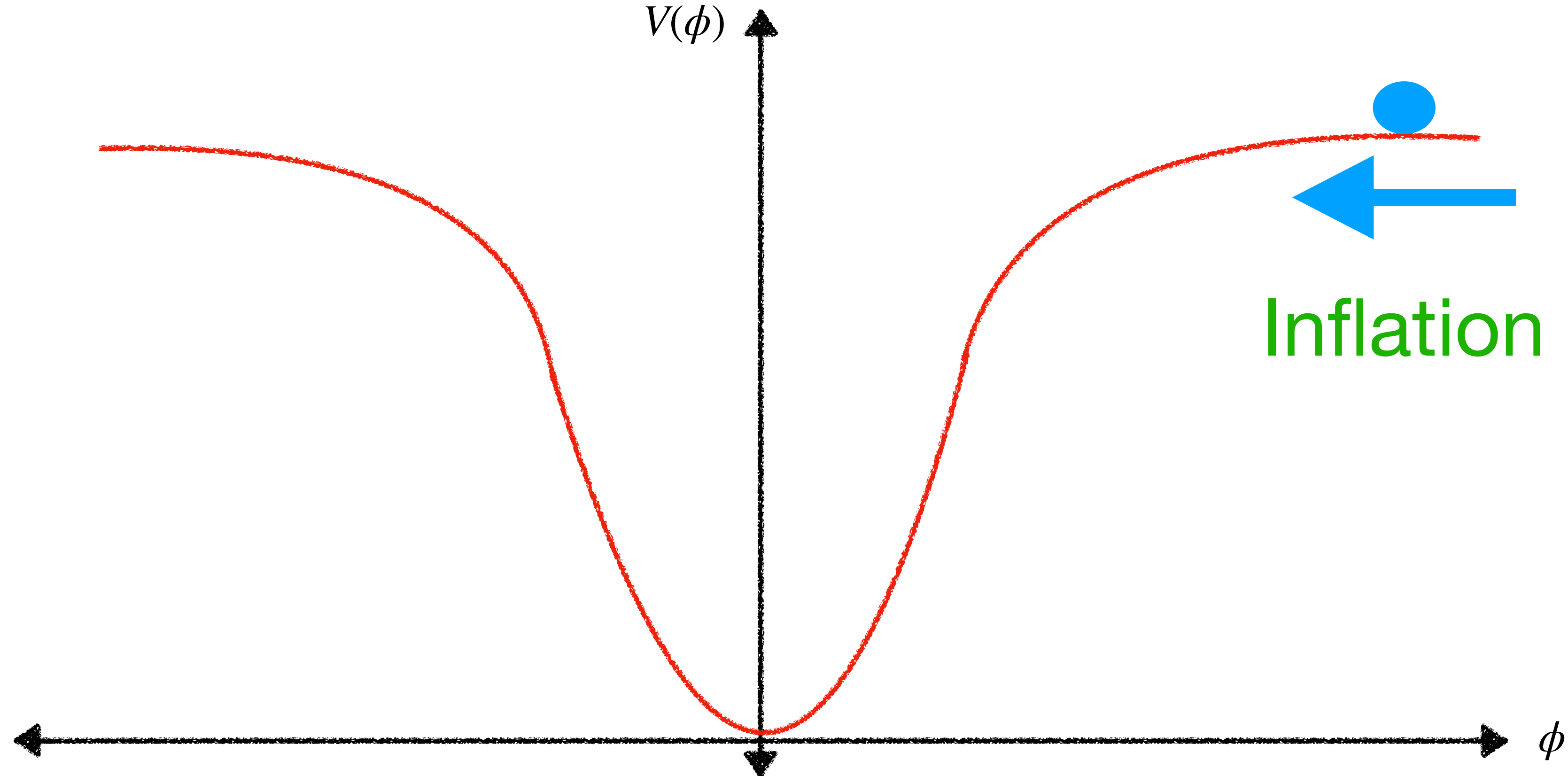
*Figueroa & Byrnes 1604.03905 Opferkuch et al 1905.06823*

NMC reheating

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# Inflationary Model

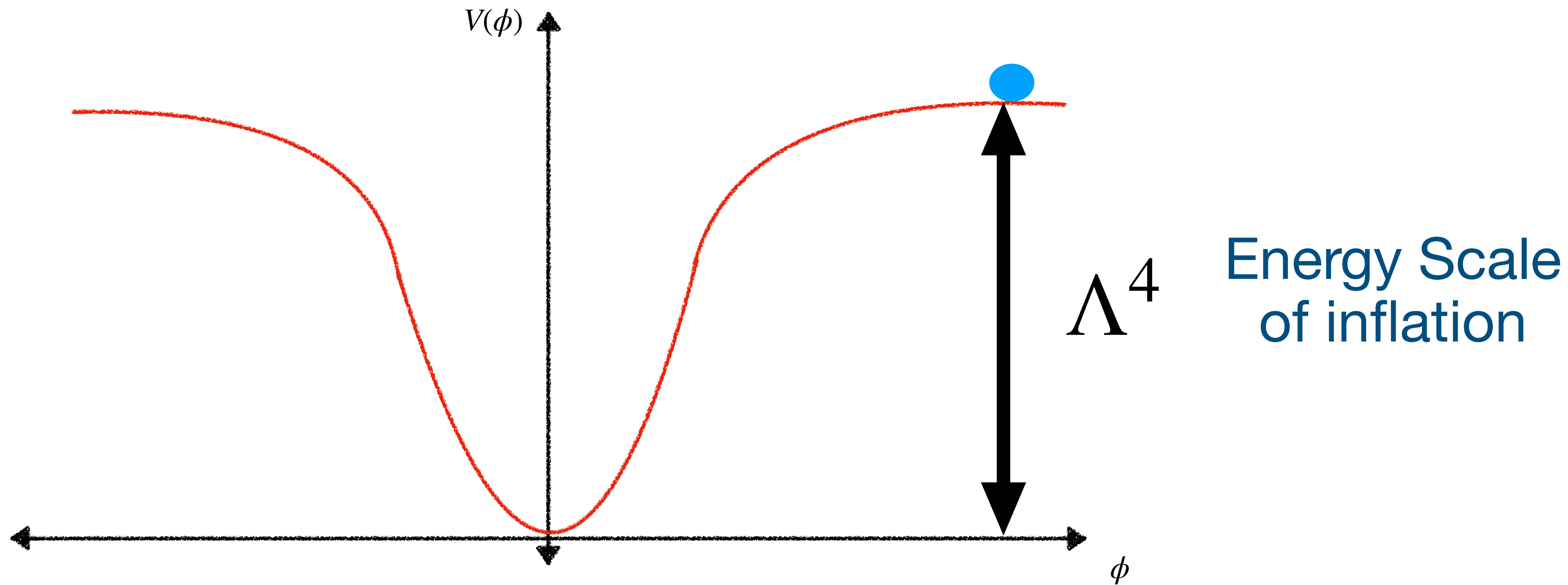
$$V(\phi) = \frac{\Lambda^4}{p} \tanh^p \left( \frac{\phi}{M} \right)$$



**Inflationary model:  $\alpha$ -attractor**



$$V(\phi) = \frac{\Lambda^4}{p} \tanh^p \left( \frac{\phi}{M} \right)$$



# CMB constraints for $\alpha$ -attractors

They look better in the Asymptotic limit  $\mathcal{M} \ll 1$

$$n_s(M, p) \rightarrow \frac{29}{30} \simeq 0.967$$

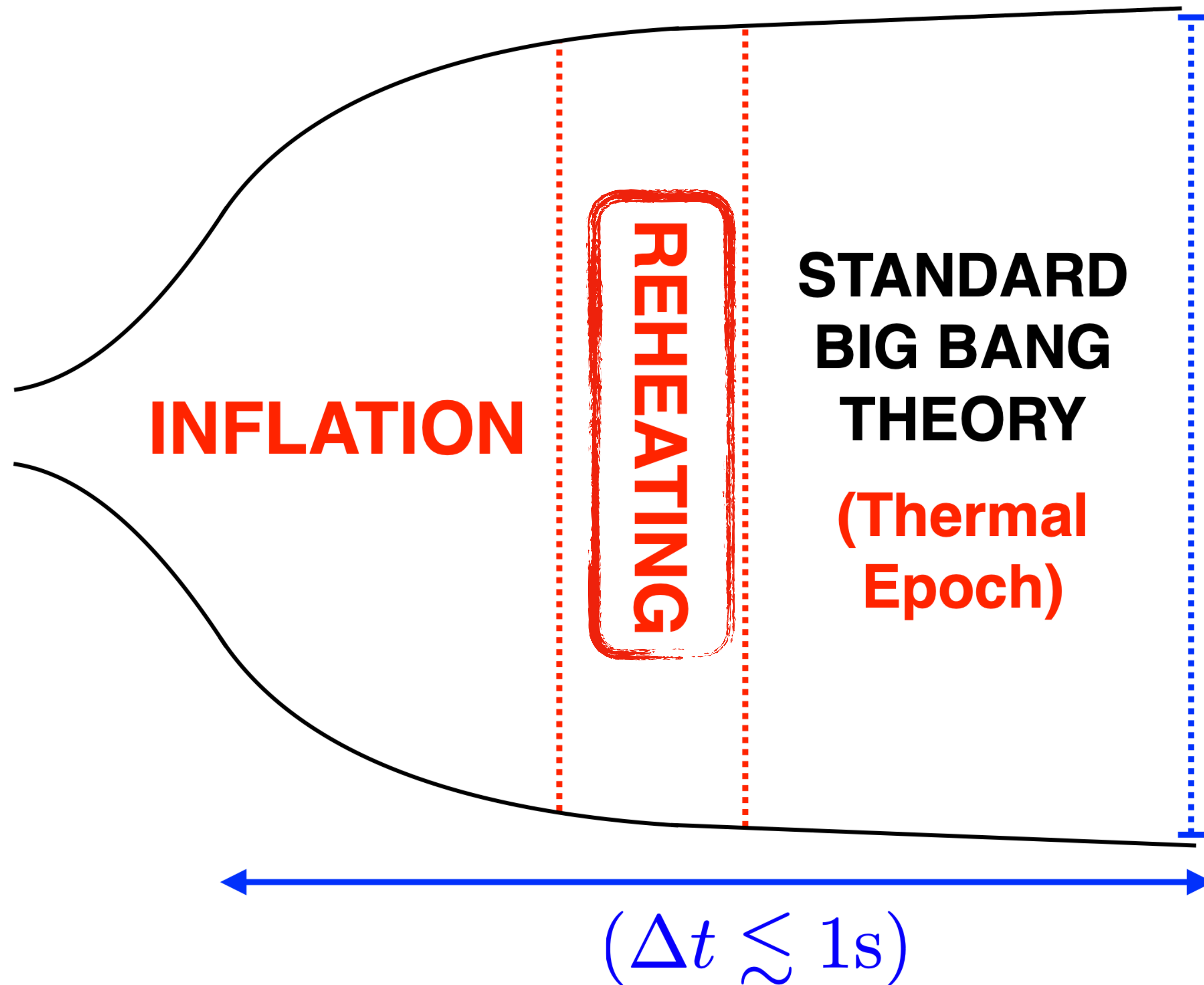
$$r(M, p) \rightarrow \frac{(M/m_p)^2}{1800} = 5.56 \cdot 10^{-4} \left( \frac{M}{m_p} \right)^2$$

$$\frac{\Lambda(M, p)}{m_p} \rightarrow \left( \frac{24\pi^2 p A_s}{28800} \right)^{1/4} \sqrt{\frac{M}{m_p}} \simeq 2.04 \cdot 10^{-3} p^{1/4} \sqrt{\frac{M}{m_p}}$$

For which we choose  $N_k = 60$

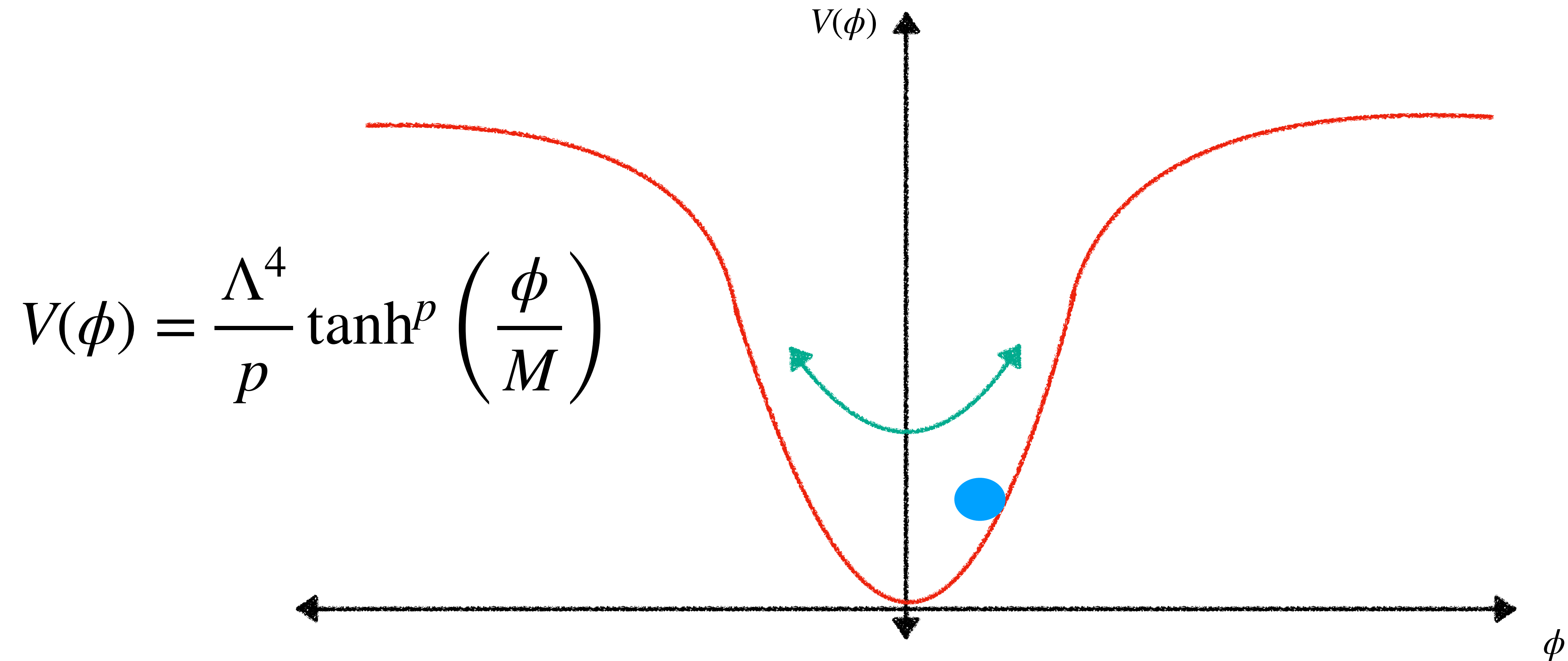
Energy Scale  
of inflation

# Early Universe



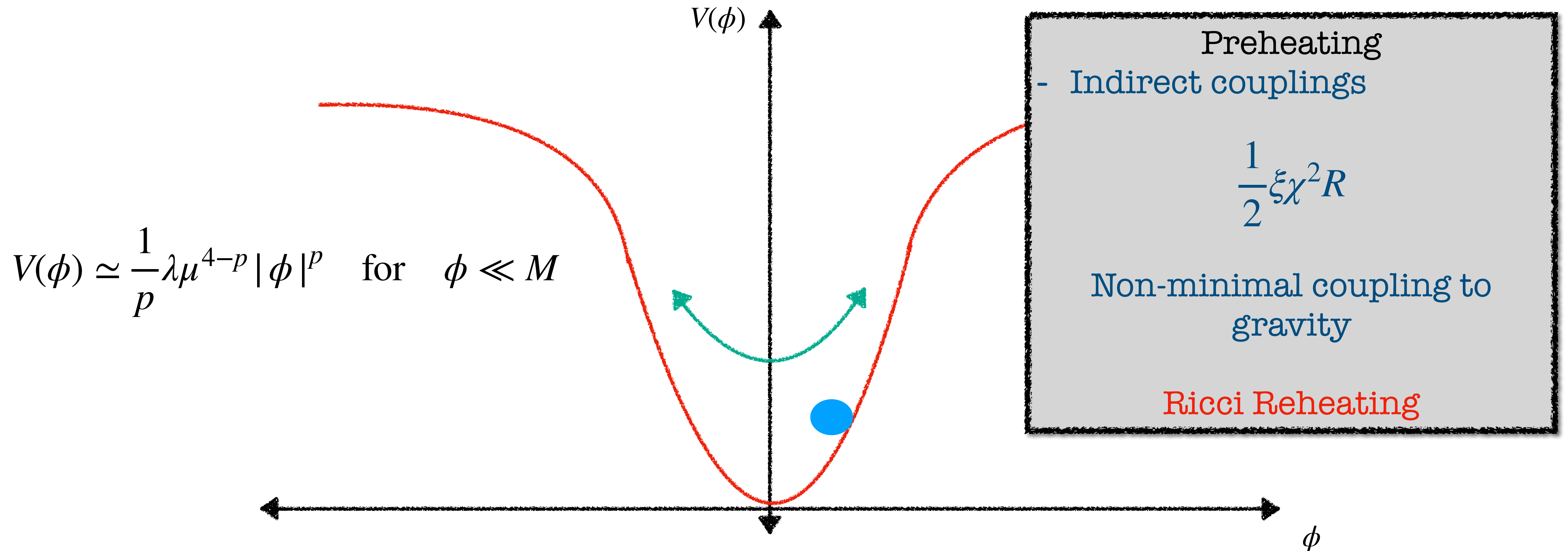


# Inflaton oscillations



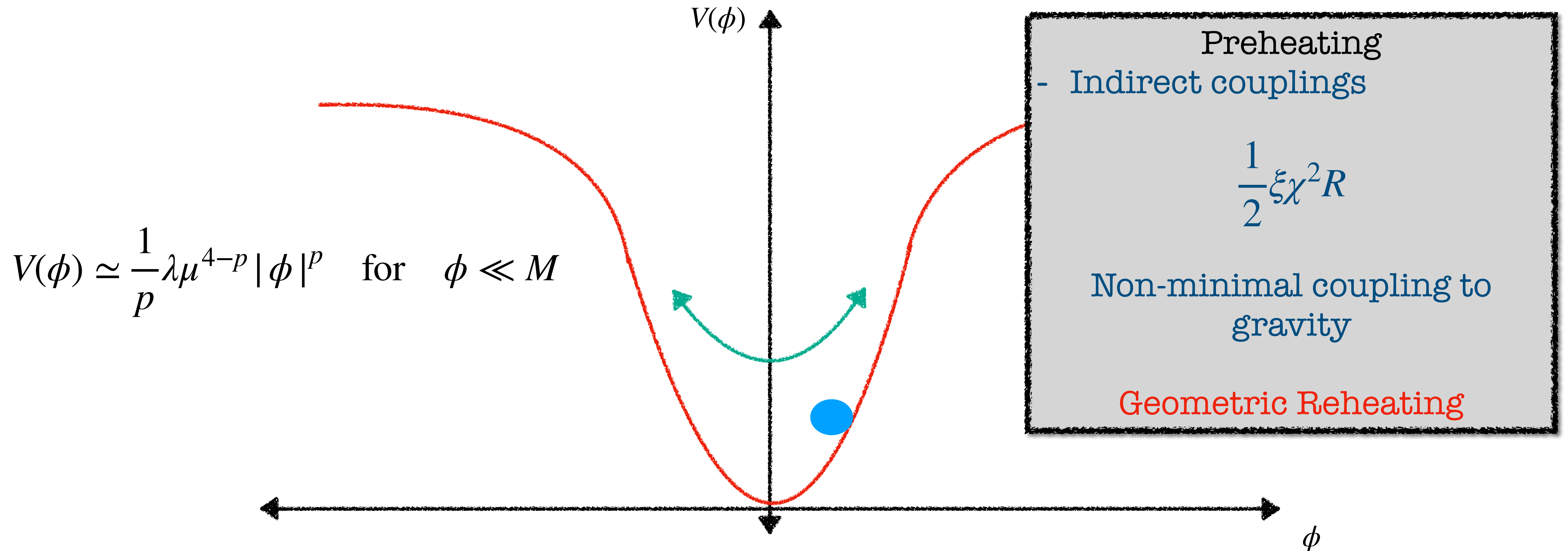


# Inflaton oscillations

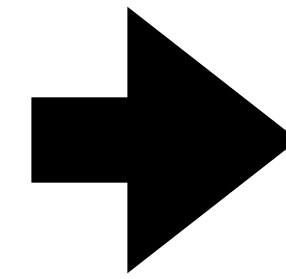
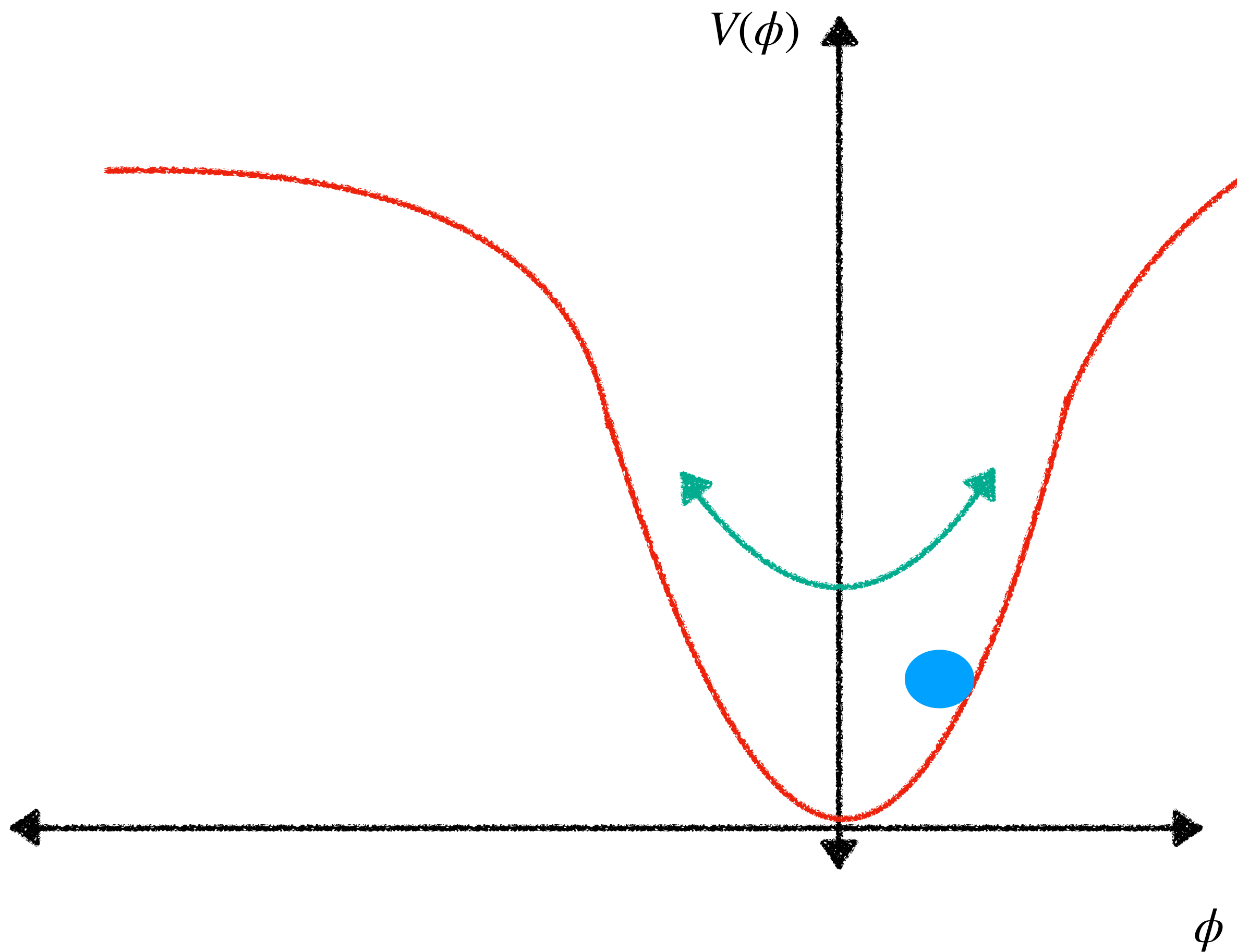




# Inflaton oscillations



# Inflaton oscillations



Oscillations of the Ricci Scalar

Lets consider the metric

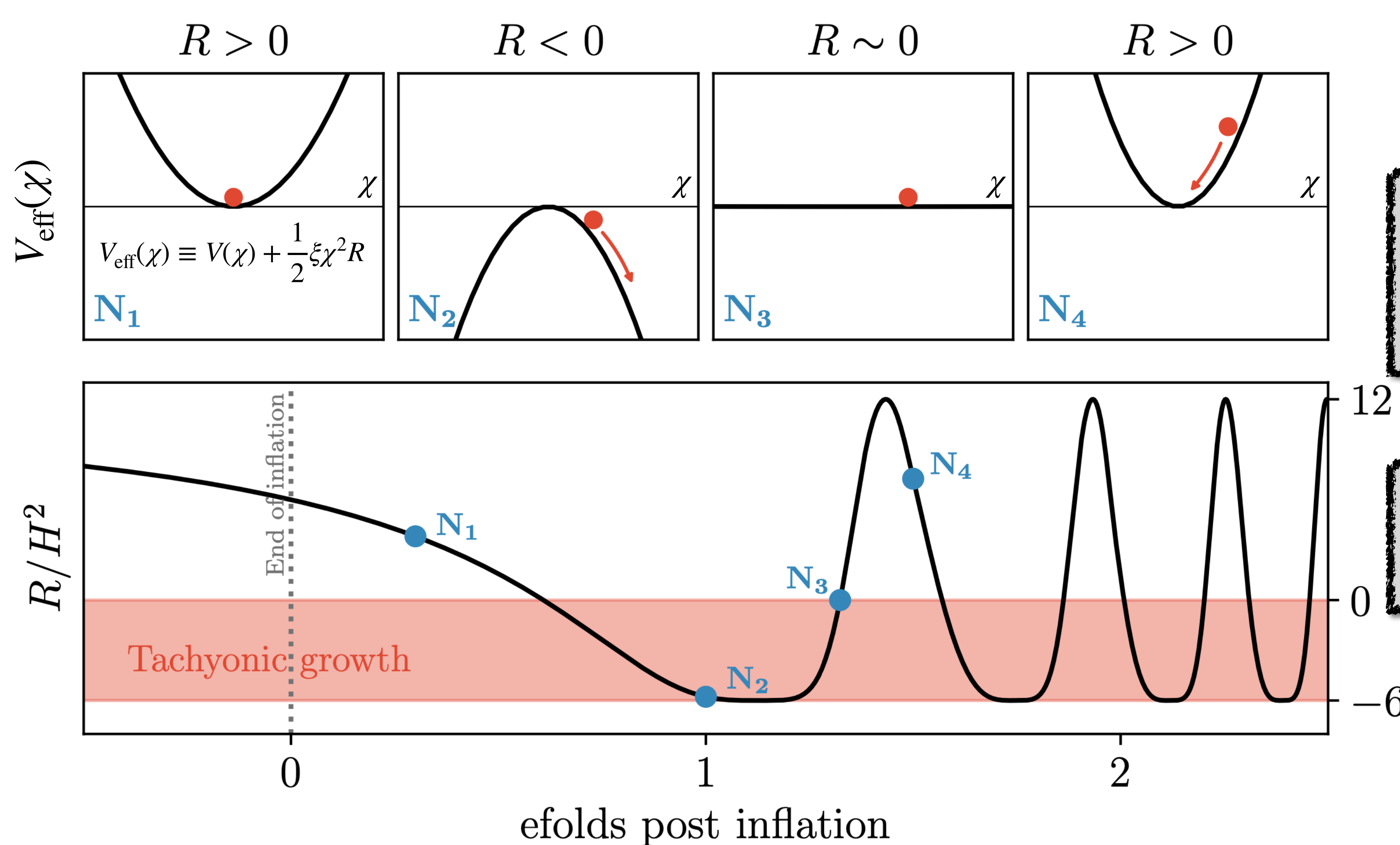
$$g_{\mu\nu} \equiv \text{diag}(-a(\eta)^{2\alpha}, a(\eta)^2, a(\eta)^2, a(\eta)^2)$$

$$R = \frac{6}{a^{2\alpha}} \left[ \frac{a''}{a} - (\alpha - 1) \left( \frac{a'}{a} \right)^2 \right]$$

If is driven purely by the  
homogeneous oscillations of  
the inflation

$$R = \frac{2}{m_p^2} \left( 2\langle V_{\text{inf}}(\phi) \rangle - \langle K(\phi) \rangle \right)$$

# Brief description of Geometric (p)reheating



$$\frac{1}{2}\xi\chi^2 R$$

Whenever  $R < 0$   
Effective mass of  $\chi$  becomes  
tachyonic

Whenever  $R > 0$   
 $\chi$  oscillates as free field



# Inflaton oscillations

We can relate Ricci oscillations  
to the Equation of State (EoS)

$$w$$

For potentials of the kind  $\sim \phi^p$

$$w_{\text{osc}} \equiv \frac{\overline{p}_\phi}{\overline{\rho}_\phi} \simeq \frac{p-2}{p+2}$$

Ricci scalar normalized by  
Hubble square is

$$\frac{\overline{R}}{H^2} = 3(1 - 3w_{\text{osc}}) \simeq 6 \frac{(4-p)}{(p+2)}$$



# Inflaton oscillations

We can relate Ricci oscillations to the Equation of State (EoS)  $w$

For potential  $\alpha$ -attractors

$$w_{\text{osc}} \simeq \frac{p-2}{p+2} \quad \text{for} \quad M \gtrsim m_p$$

$$w_{\text{osc}} \simeq (\text{increasing function}) \rightarrow \frac{p-2}{p+2} \quad \text{for} \quad M \lesssim m_p$$

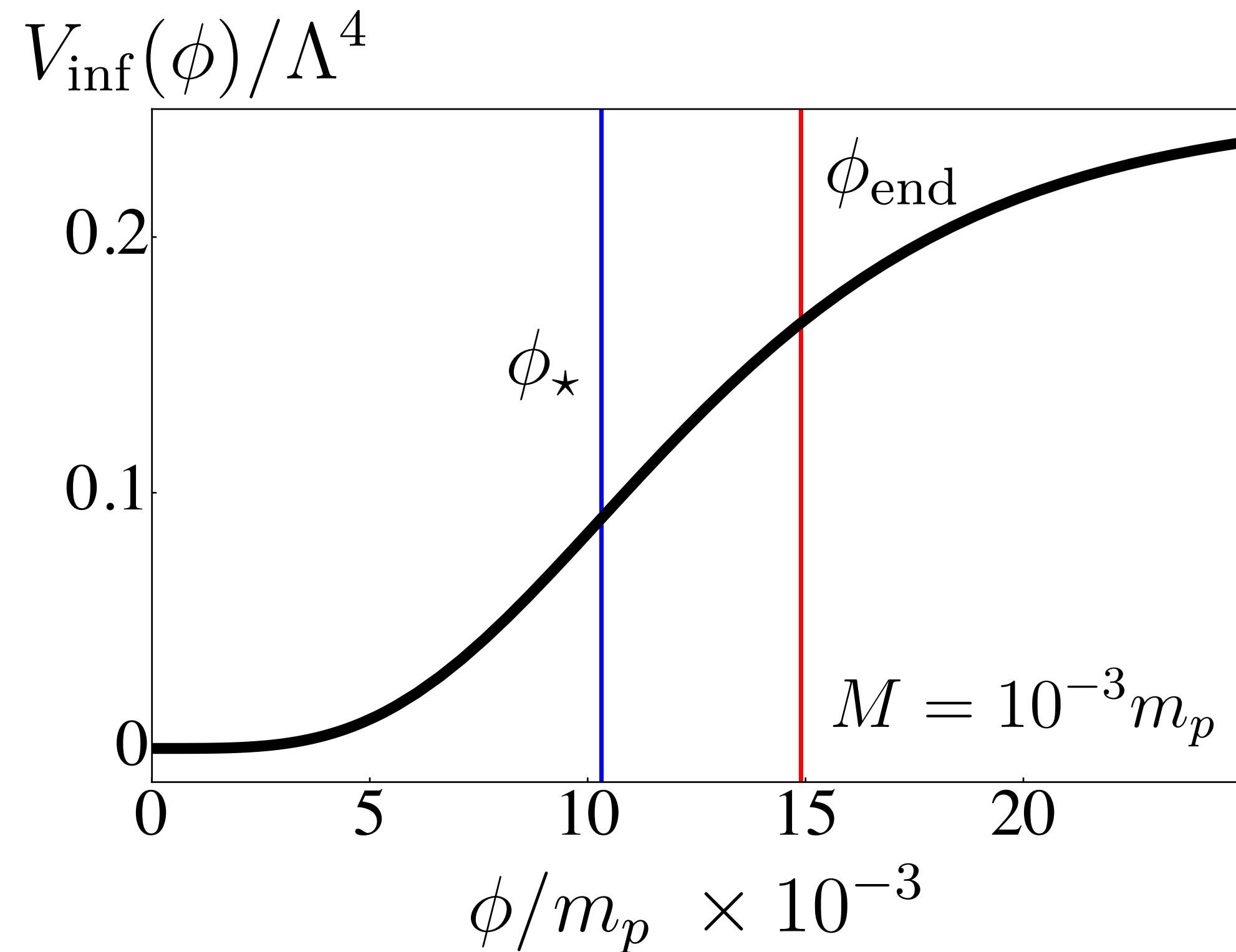
Ricci scalar normalized by Hubble square is

$$\frac{\bar{R}}{H^2} \simeq 6 \frac{(4-p)}{(p+2)} \quad \text{for} \quad M \gtrsim m_p$$

$$\frac{\bar{R}}{H^2} \simeq (\text{decreasing function}) \rightarrow 6 \frac{(4-p)}{(p+2)} \quad \text{for} \quad M \lesssim m_p$$

# Inflaton oscillations

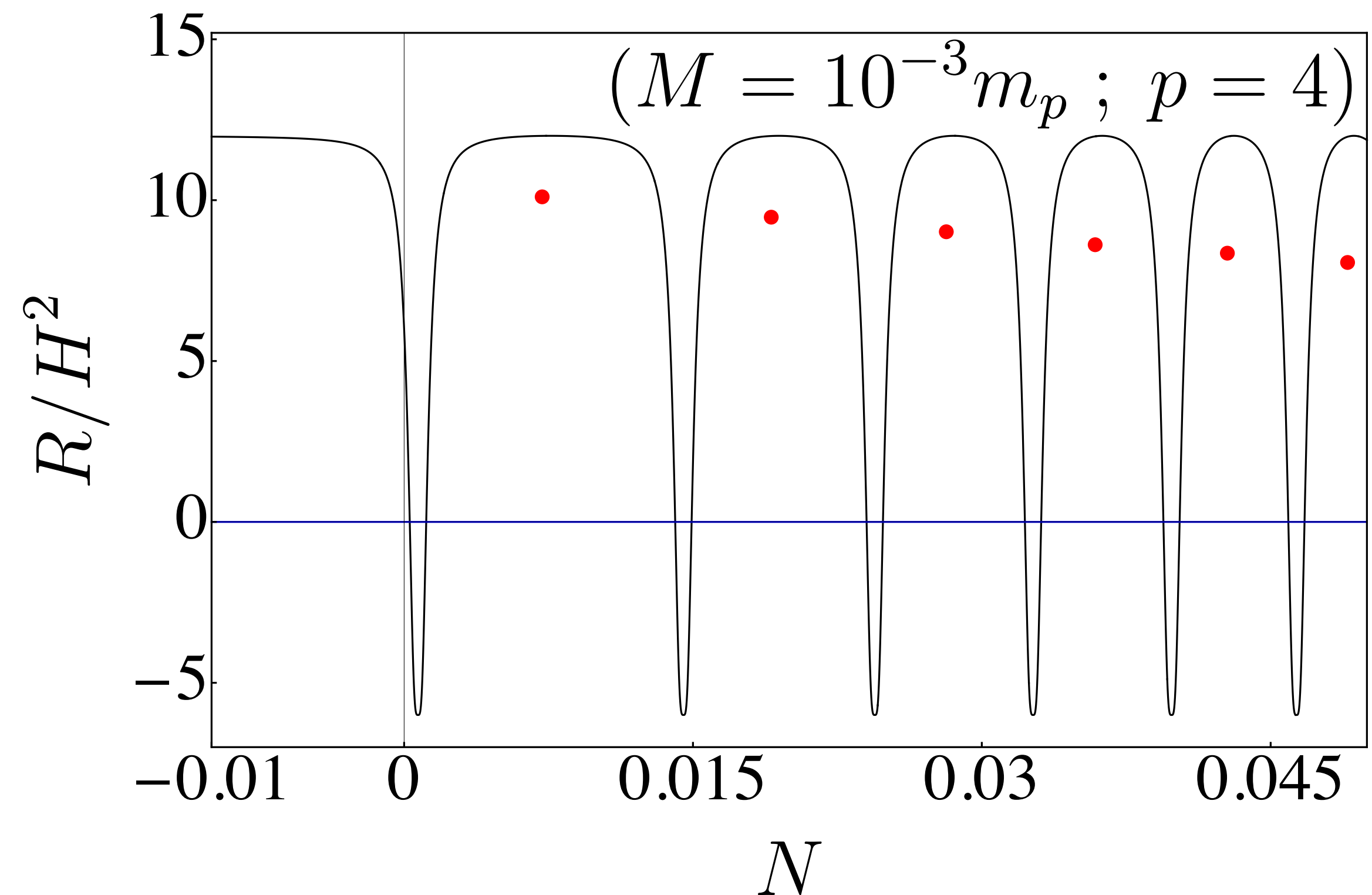
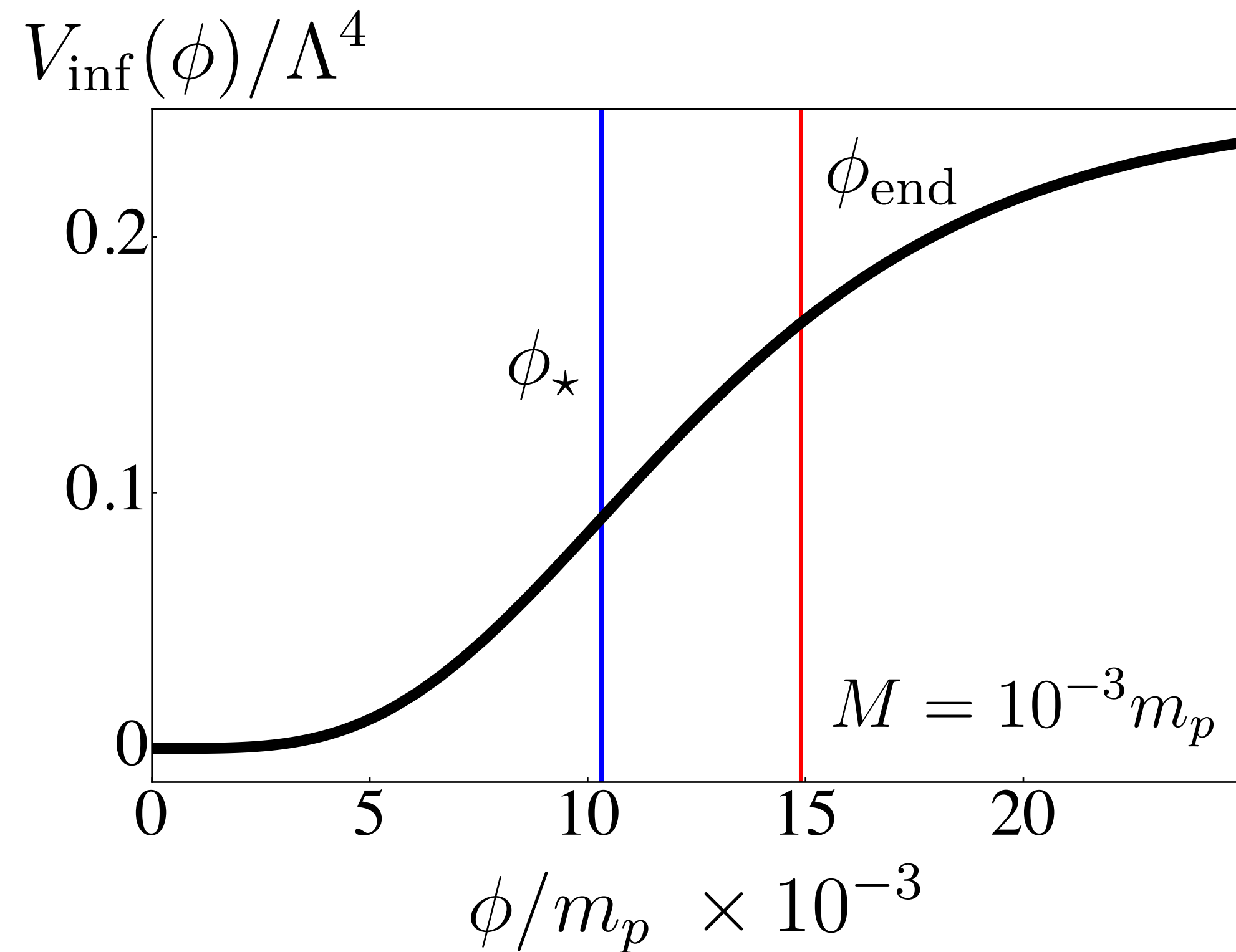
$$\frac{\phi_\star}{M} = \operatorname{arcsinh} \left\{ \sqrt{\frac{p-1}{2}} \right\}$$



# Inflaton oscillations

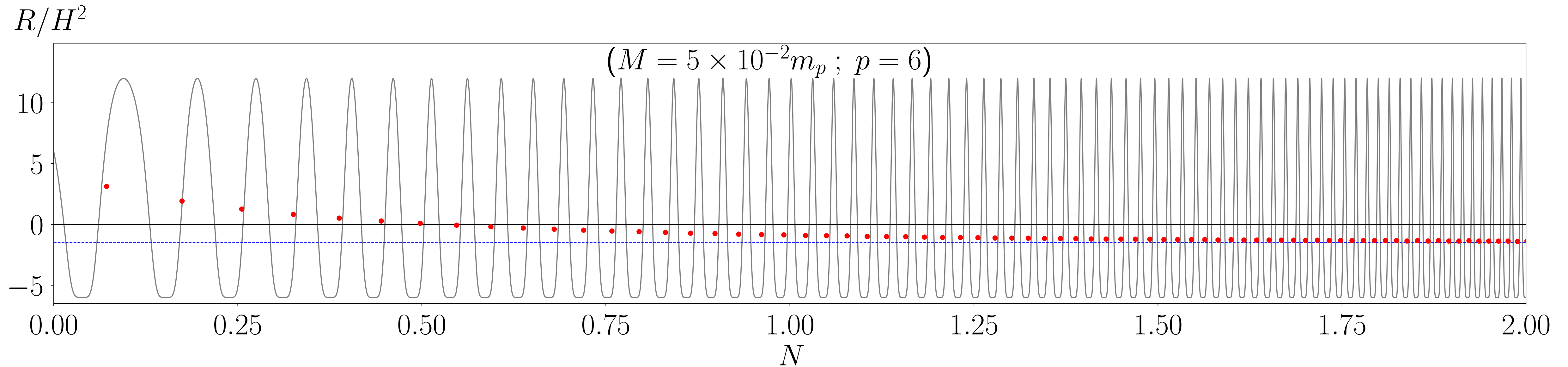
$$\frac{\phi_\star}{M} = \operatorname{arcsinh} \left\{ \sqrt{\frac{p-1}{2}} \right\}$$

Oscillations get affected by  
lowering  $M$

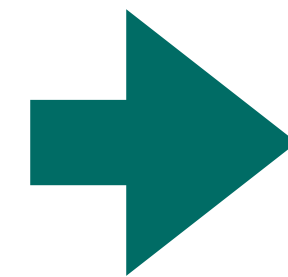




# Inflaton oscillations



We are interested in those cases that lead to **NEGATIVE** Ricci oscillations



$$\frac{\bar{R}}{\bar{H}^2} < 0 \implies p > 4$$



# Geometric preheating: Linear regime

Let us consider the action

$$S = \int d^4x \sqrt{-g} \left( \frac{1}{2} m_p^2 R - \frac{1}{2} \xi R \chi^2 - \frac{1}{2} g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi, \chi) \right)$$

Non-minimal coupled field EoM in  $\alpha$ -time

$$\chi'' + (3 - \alpha) \mathcal{H} \chi' - a^{-2(1-\alpha)} \nabla^2 \chi + a^{2\alpha} \left( \xi R \chi + V_{,\chi} \right) = 0$$

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$$\chi'' + (3 - \alpha) \mathcal{H} \chi' - a^{-2(1-\alpha)} \nabla^2 \chi + a^{2\alpha} \left( \xi R \chi + V_{,\chi} \right) = 0$$

In the linear regime we can focus on the  $\chi_k$  modes

$$\tilde{\chi}_k'' + \omega_k^2 \tilde{\chi}_k = 0, \quad \text{with} \quad \omega_k^2 \equiv k^2 + a^2 \left( \xi - \frac{1}{6} \right) R$$

# Geometric preheating: Linear regime

$$\tilde{\chi}_k'' + \omega_k^2 \tilde{\chi}_k = 0, \quad \text{with } \omega_k^2 \equiv k^2 + a^2 \left( \xi - \frac{1}{6} \right) R$$

Whenever  $\omega_k^2 < 0$  modes grow exponentially

$$\frac{k_*}{aH} < \sqrt{6\xi - 1}$$

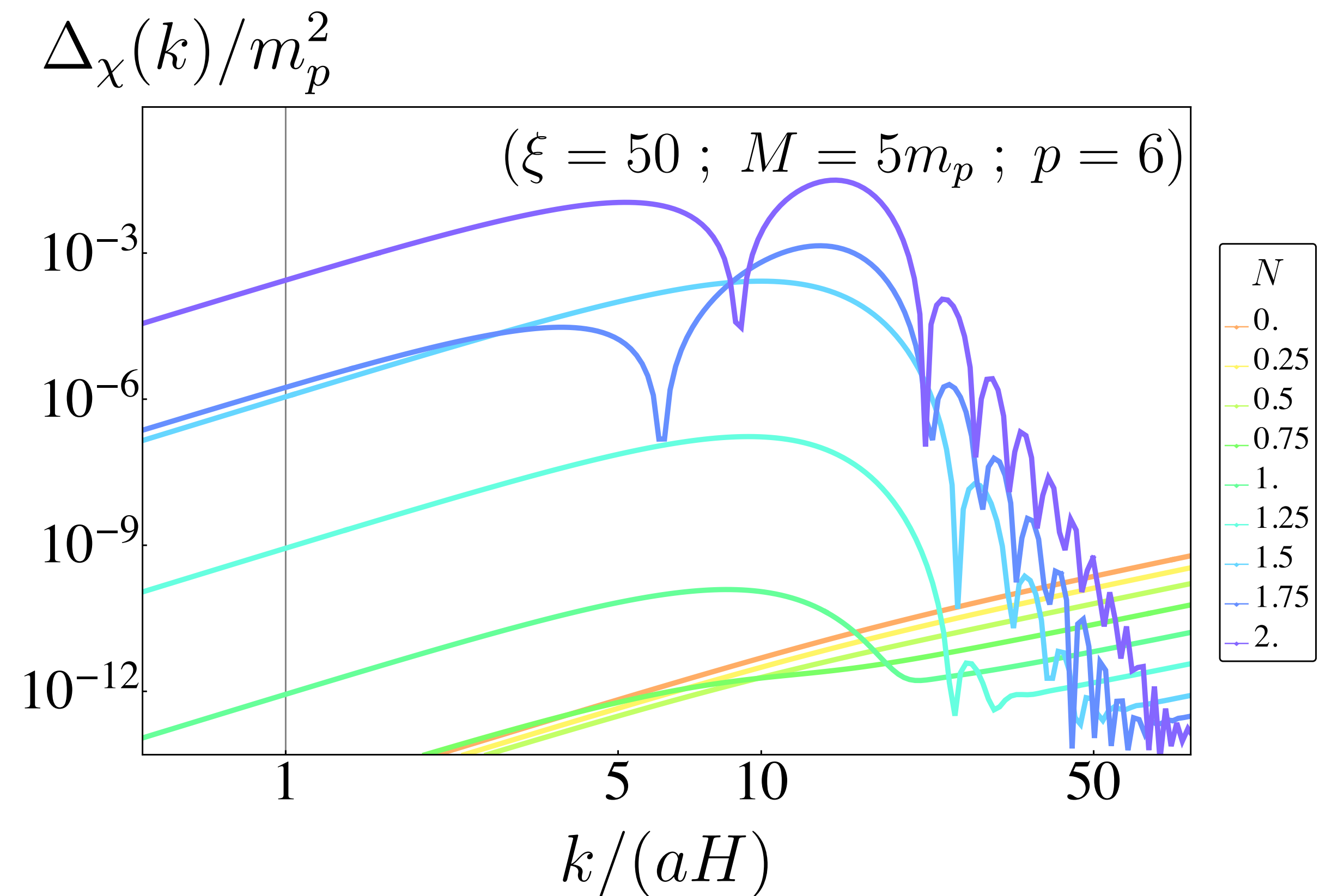
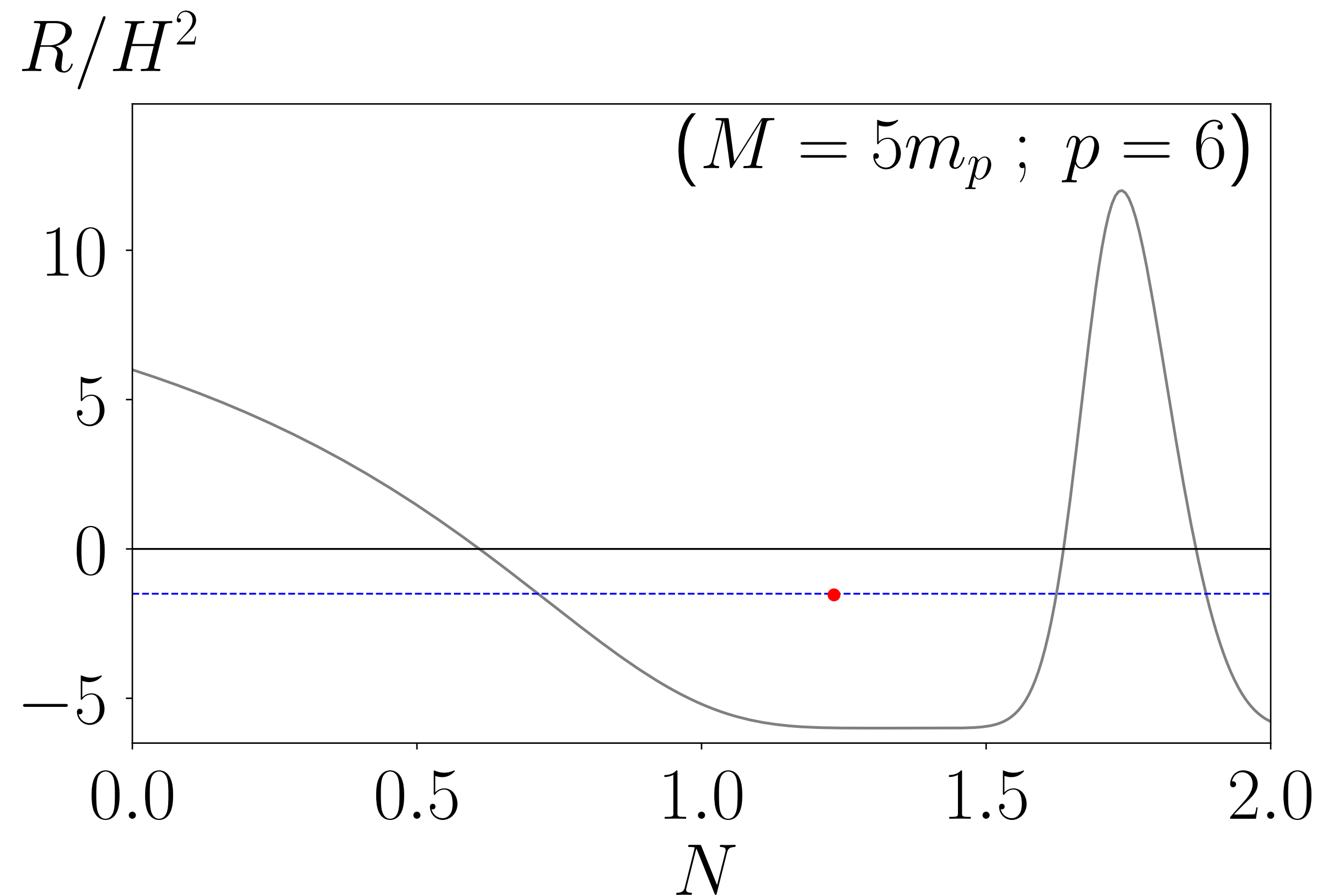
$k_*$  is a threshold momenta



# Geometric preheating: Linear regime

We can solve linear equation for  $\chi_k$  modes

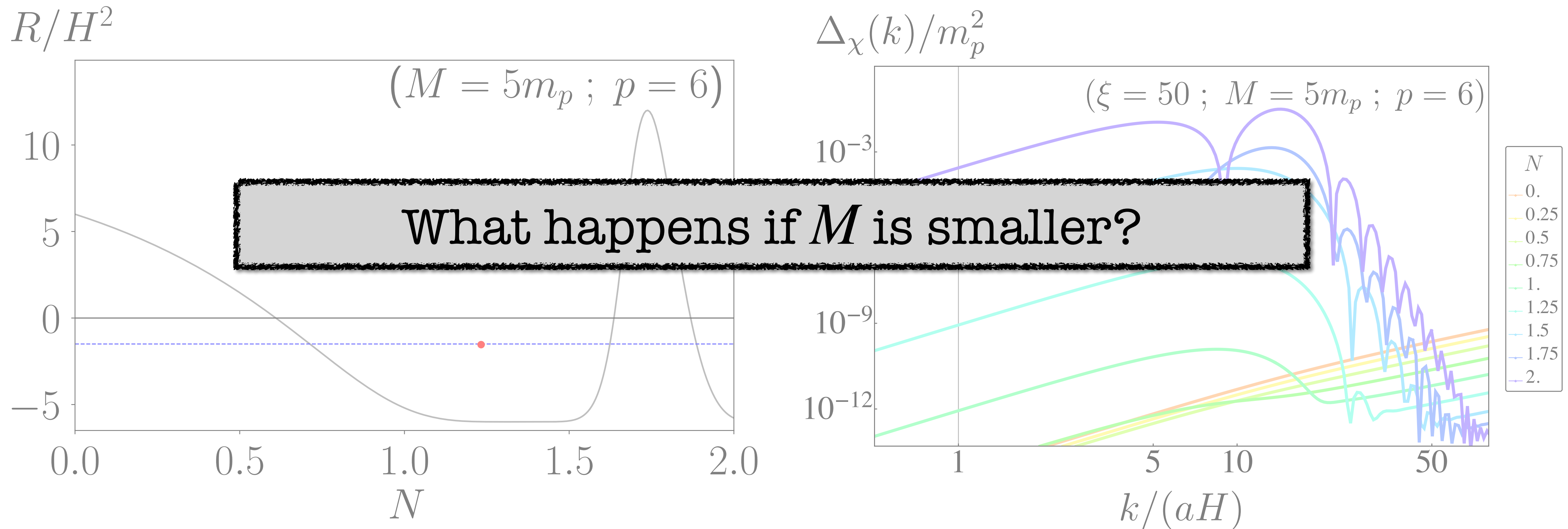
$$p = 6$$



# Geometric preheating: Linear regime

We can solve linear equation for  $\chi_k$  modes

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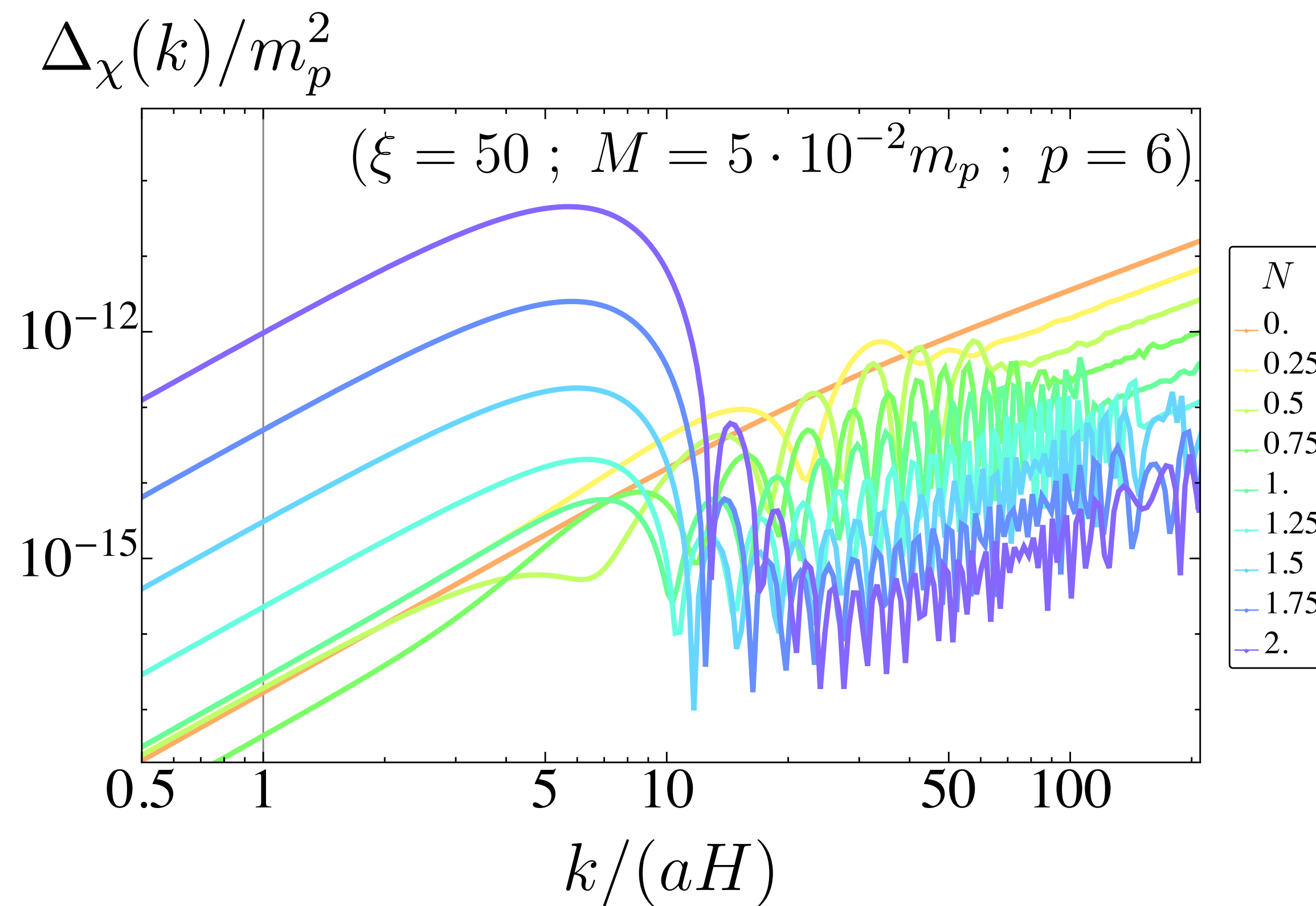


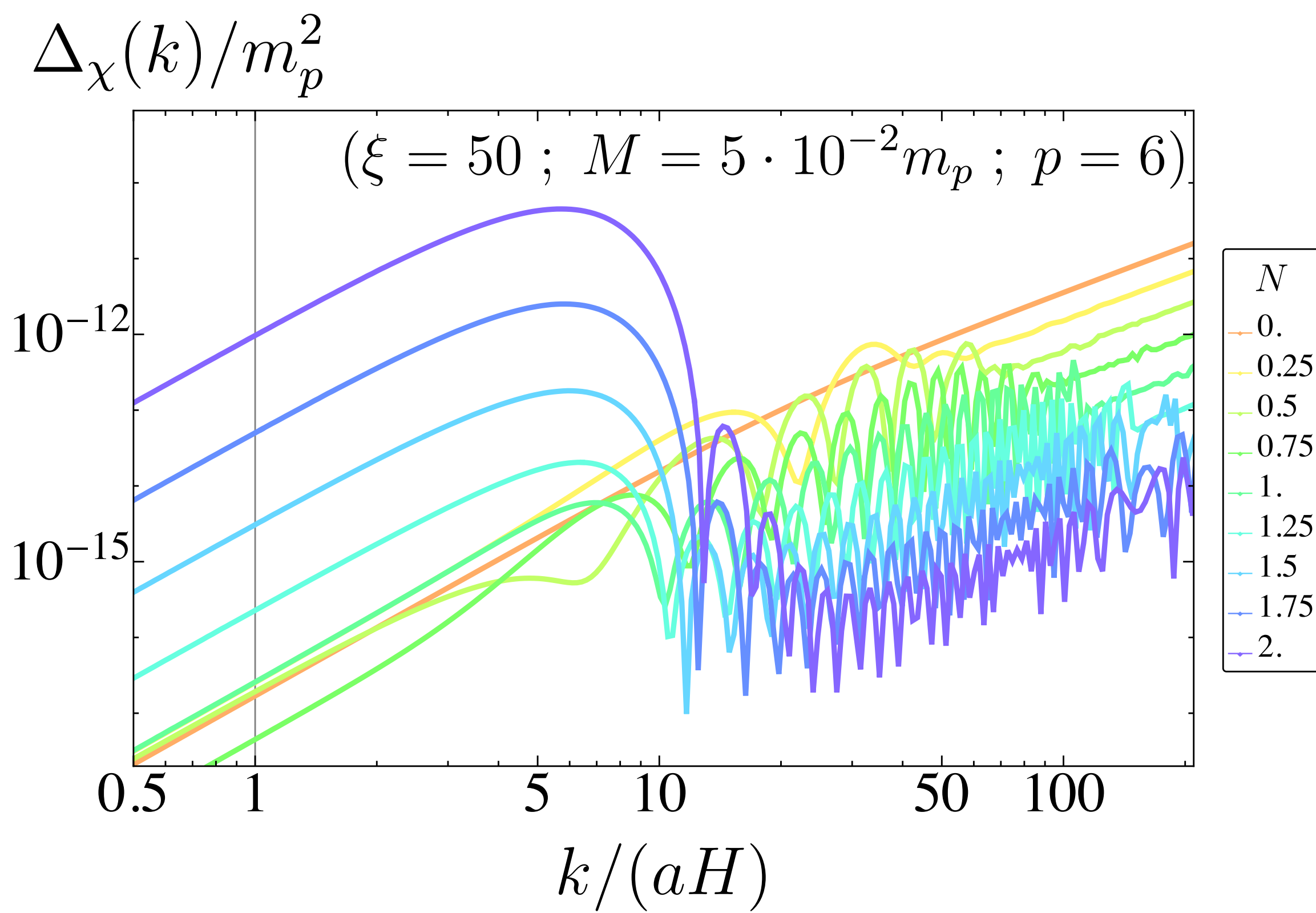
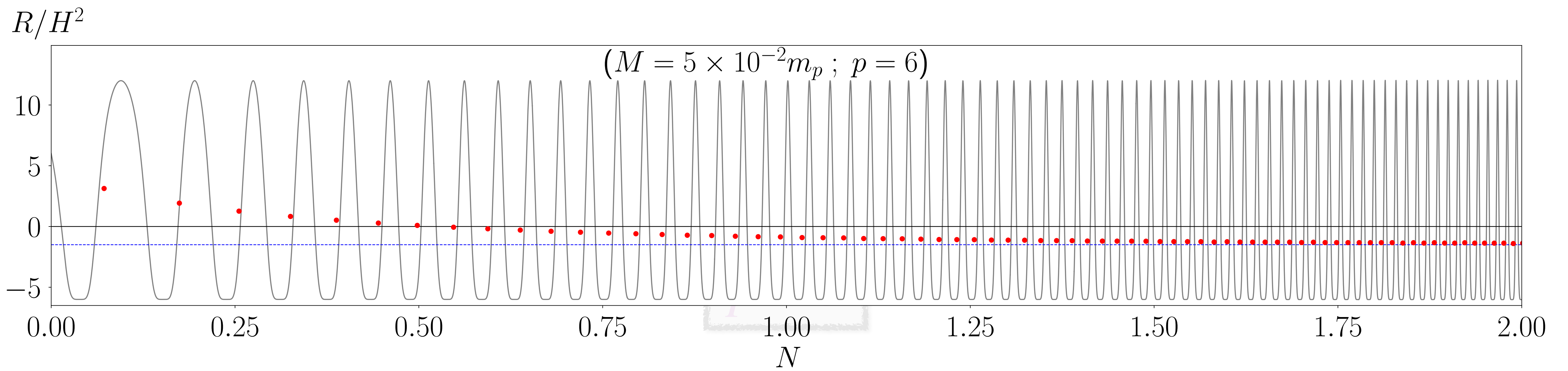


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We can solve linear equation for  $\chi_k$  modes

$$p = 6$$



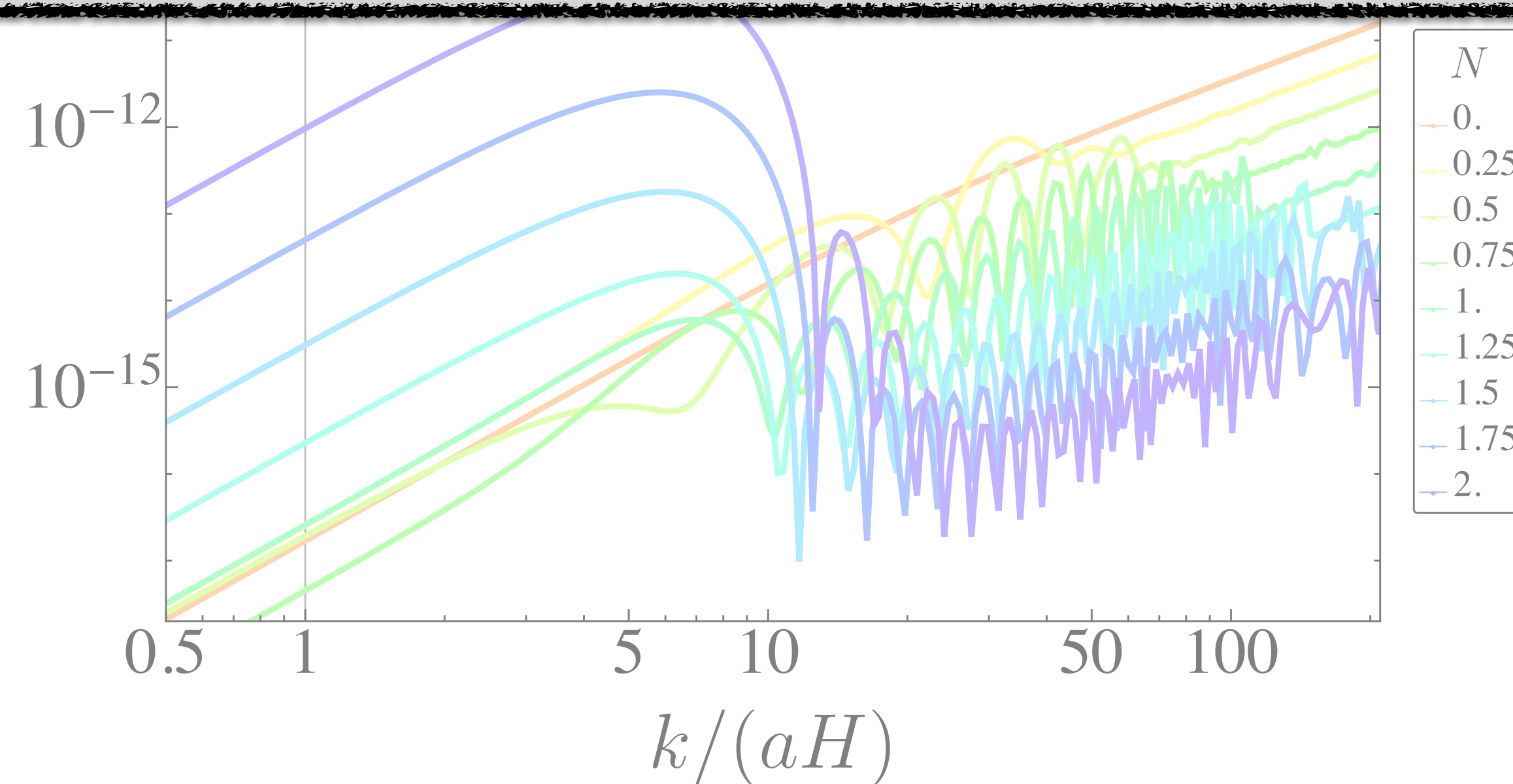


# Geometric preheating: Linear regime

We can solve linear equation for  $\chi_k$  modes

$p = 6$

$p=6$  shows to be the best candidate to achieve reheating for low energy scales.





# Geometric (p)reheating: Non-linear regime

In linear regime  $\chi_k$  modes grow unbounded

Let us go back to the action

$$S = \int d^4x \sqrt{-g} \left( \frac{1}{2} m_p^2 R - \frac{1}{2} \xi R \chi^2 - \frac{1}{2} g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi, \chi) \right)$$

and the NMC-field EoM

$$\chi'' + (3 - \alpha) \mathcal{H} \chi' - a^{-2(1-\alpha)} \nabla^2 \chi + a^{2\alpha} \left( \xi R \chi + V_{,\chi} \right) = 0$$

can we get the effect of the  $\chi$  growth on  $R$  dynamics?

# Geometric (p)reheating: Non-linear regime

We have now a full system of equations that characterize the system

$$\chi'' + (3 - \alpha) \left( \frac{a'}{a} \right) \chi' - a^{-2(1-\alpha)} \nabla^2 \chi = -a^{2\alpha} \left( \xi R \chi + V_{,\chi} \right)$$

$$\phi'' + (3 - \alpha) \left( \frac{a'}{a} \right) \phi' - a^{-2(1-\alpha)} \nabla^2 \phi = -a^{2\alpha} V_{,\phi}$$

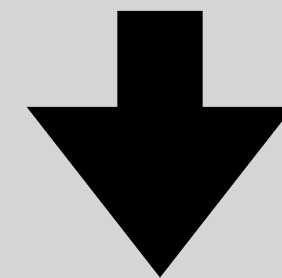
$$\frac{a''}{a} + (1 - \alpha) \left( \frac{a'}{a} \right)^2 = \frac{a^{2\alpha}}{6} R$$

with

$$R = \frac{\left[ (6\xi - 1) \left( \frac{\langle \chi'^2 \rangle}{a^{2\alpha}} - \frac{\langle (\nabla \chi)^2 \rangle}{a^2} \right) - 6\xi \langle \chi V_{,\chi} \rangle + 4 \langle V \rangle - \frac{\langle \phi'^2 \rangle}{a^{2\alpha}} + \frac{\langle (\nabla \phi)^2 \rangle}{a^2} \right]}{m_p^2 + (6\xi - 1)\xi \langle \chi^2 \rangle}$$

# Geometric (p)reheating: Non-linear regime

Full non-linear system  
impossible to solve analytically



Lattice simulations

*CosmoLattice*



# CosmoLattice input

Model: Tanh4NMC.h

Parameter file:  
Tanh4NMC.in

```
struct ModelPars : public TempLat::DefaultModelPars {
    static constexpr size_t NScalars = 2;
    // In this example, we only want 2 scalar fields.
    static constexpr size_t NPotTerms = 2;
    // Only the inflaton has a potential

    // All the numbers of fields are 0 by default, so we need only
    // to specify that we want two scalar fields.
    // See the model with gauge fields to have an example of how to turn
    // them on and specify interactions.

    typedef CouplingsManager<NScalars, 1, false, true> NonMinimalCouplings;
```

```
#IC
#kCutOff = 1
ext_PS = none xi_50 Nafter_1 Nbefore_14 N256 M_5 tp6.dat
initial_amplitudes = 3.58973e16 0 # homogeneous amplitudes in GeV
initial_momenta = -2.0602e29 0 # homogeneous amplitudes in (GeV)^2

#Model Parameters
M = 5 # M scale in mp units
xis = 0
```



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One of the fields is NMC

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```

```
#TC  
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ext_PS = none xi_50 Nafter 1 Nbefore 14 N256 M 5 tp6.dat  
initial_amplitudes = 3.58973e16 0 # homogeneous amplitudes in GeV  
initial_momenta = -2.0602e29 0 # homogeneous amplitudes in (GeV)^2  
  
#Model Parameters  
M = 5 # M scale in mp units  
xis = 0
```

External Power spectrum as  
input



# CosmoLattice input

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struct ModelPars : public TempLat::DefaultModelPars {
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Parameter file:  
Tanh4NMC.in

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```

NMC parameter

# CosmoLattice output

average\_scale\_factor.txt

$a, a', a'/a, R$



# CosmoLattice output

average\_scale\_factor.txt

$a, a', a'/a, R$

average\_energy\_densities.txt

$\tilde{\eta}, \tilde{K}_\phi, \dots, 3\xi/a^{2\alpha}\mathcal{H}\langle\phi^2\rangle, 6\xi/a^{2\alpha}\langle\phi\phi'\rangle, E_{\text{tot}}$

# CosmoLattice output

average\_scale\_factor.txt

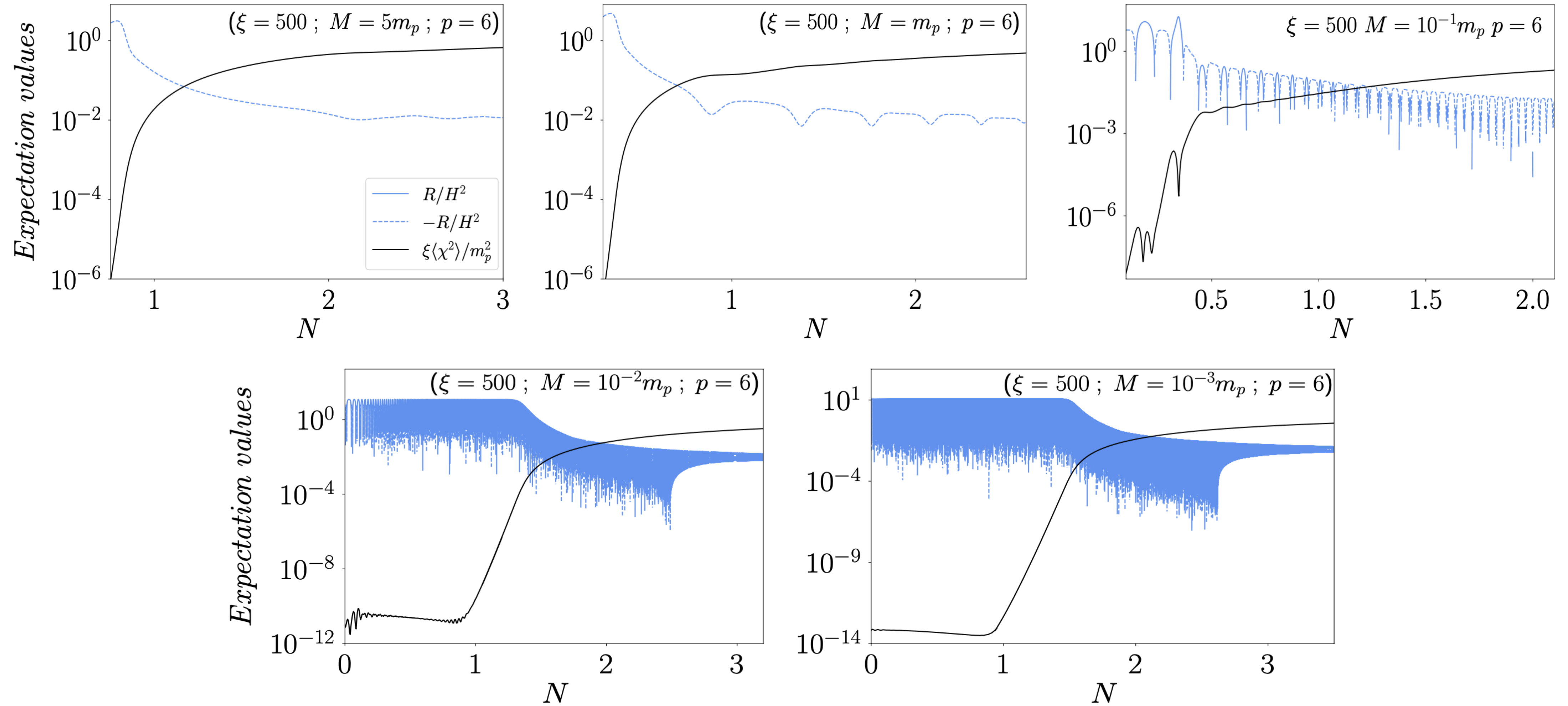
$a, a', a'/a, R$

average\_energy\_densities.txt

$\tilde{\eta}, \tilde{K}_\phi, \dots, 3\xi/a^{2\alpha}\mathcal{H}\langle\phi^2\rangle, 6\xi/a^{2\alpha}\langle\phi\phi'\rangle, E_{\text{tot}}$

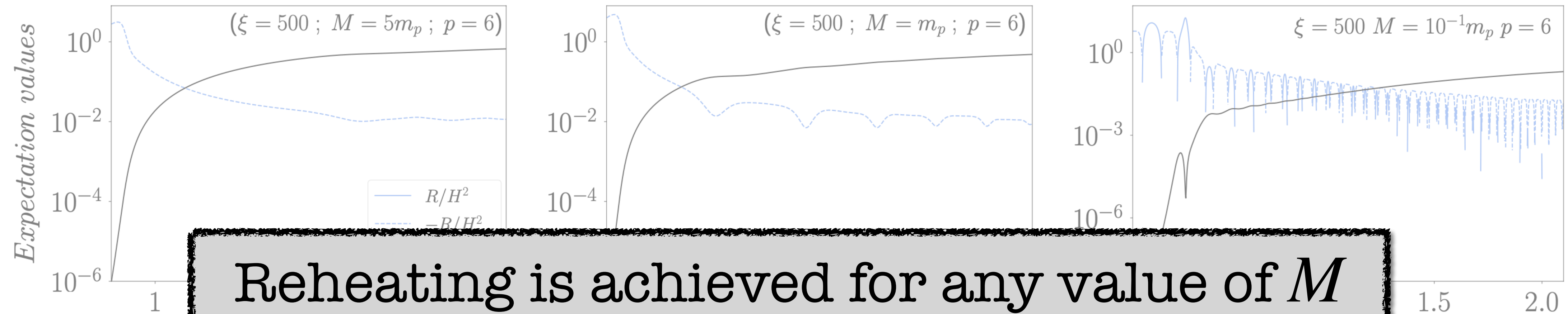
spectra\_scalar\_1.txt

# Geometric (p)reheating: Lattice results $p=6$



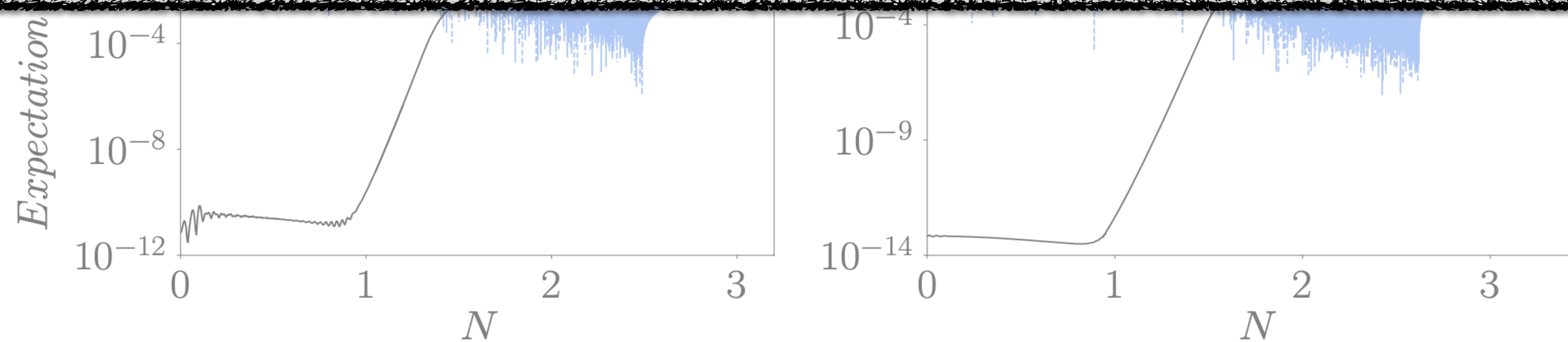


# Geometric (p)reheating: Lattice results $p=6$



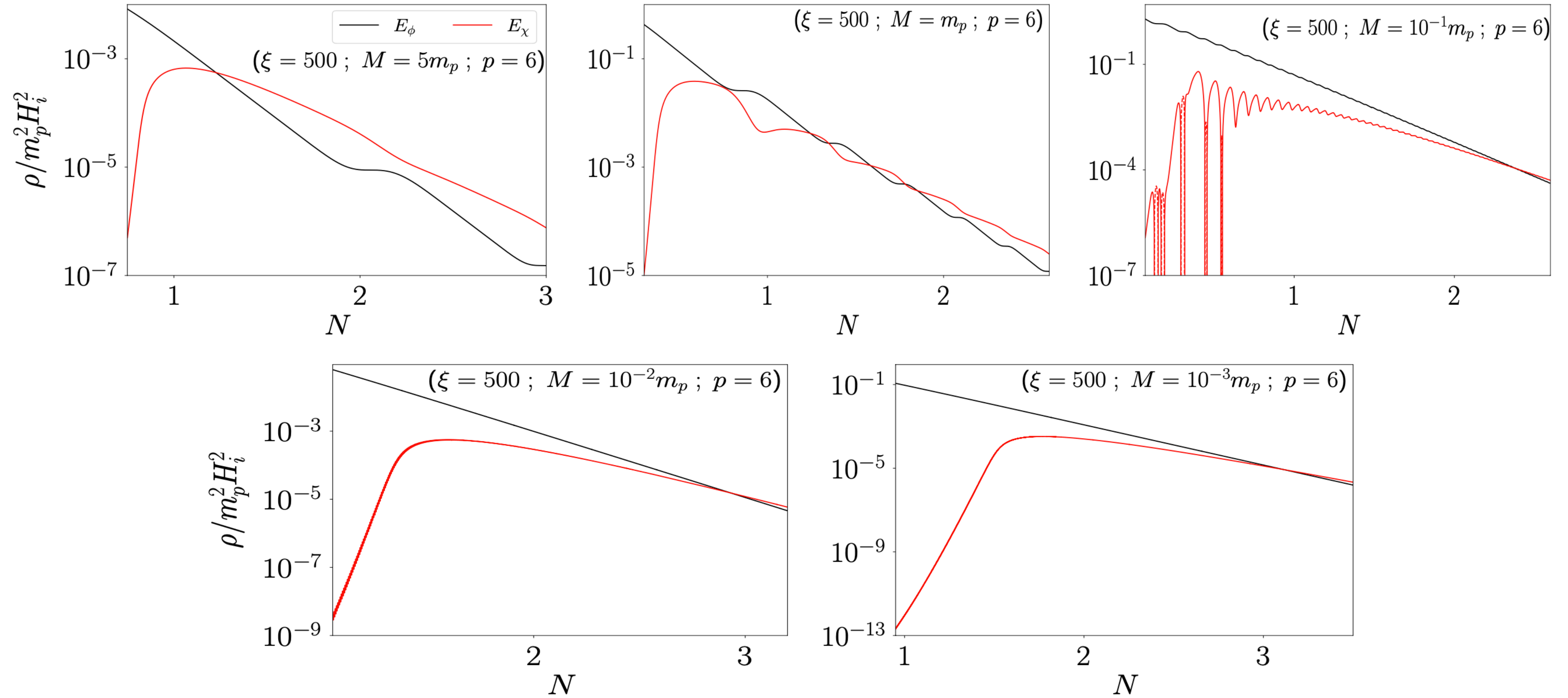
Reheating is achieved for any value of  $M$

For any  $M$  eventually  $\bar{R} < 0$

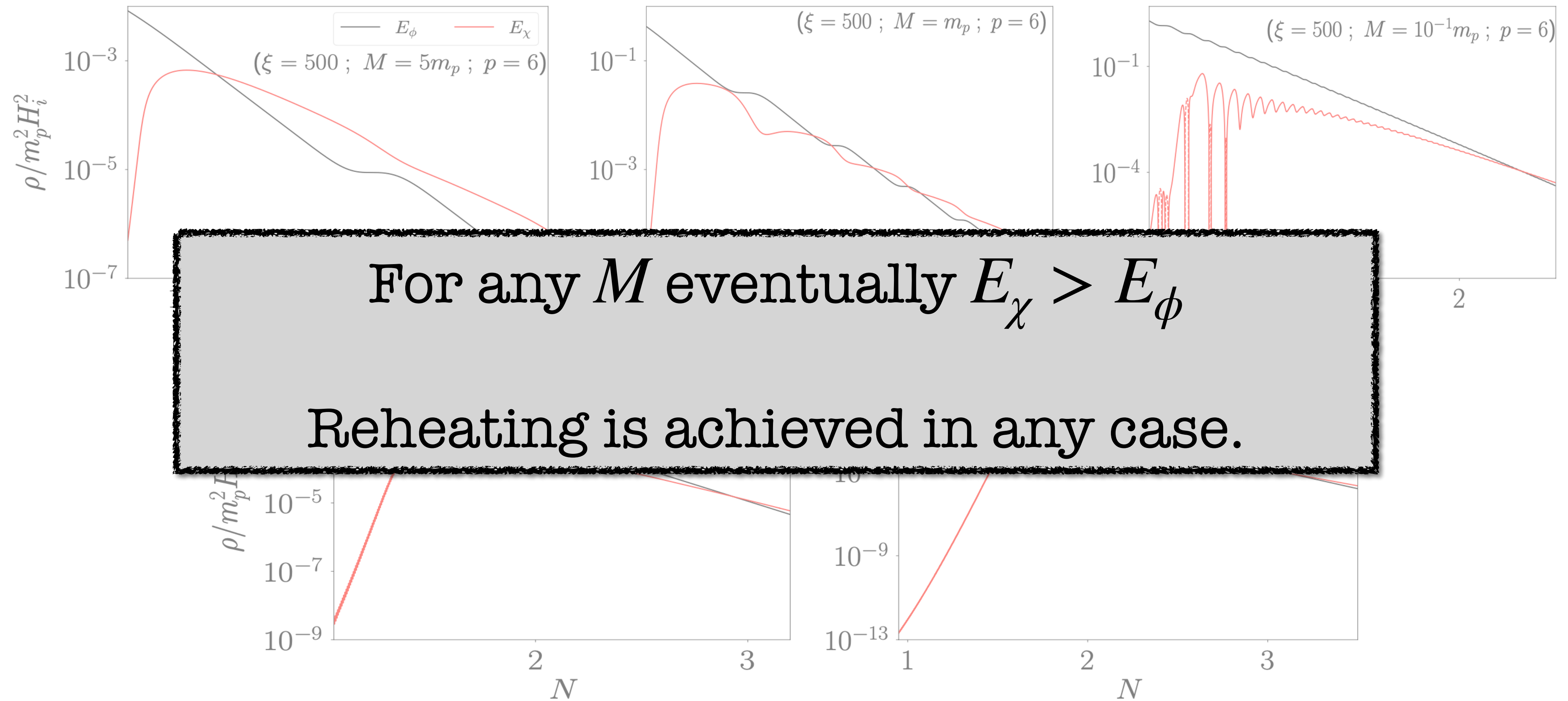




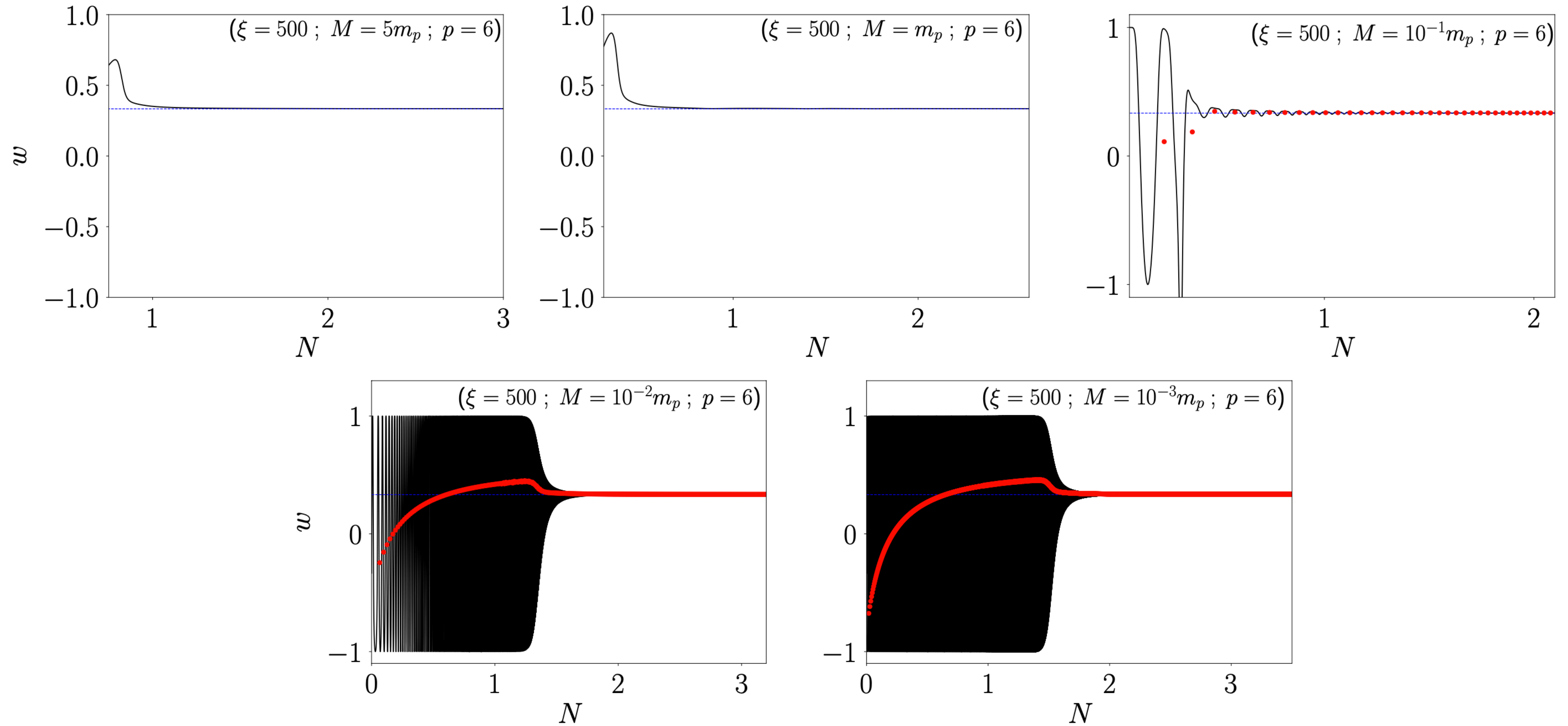
# Geometric (p)reheating: Lattice results $p=6$



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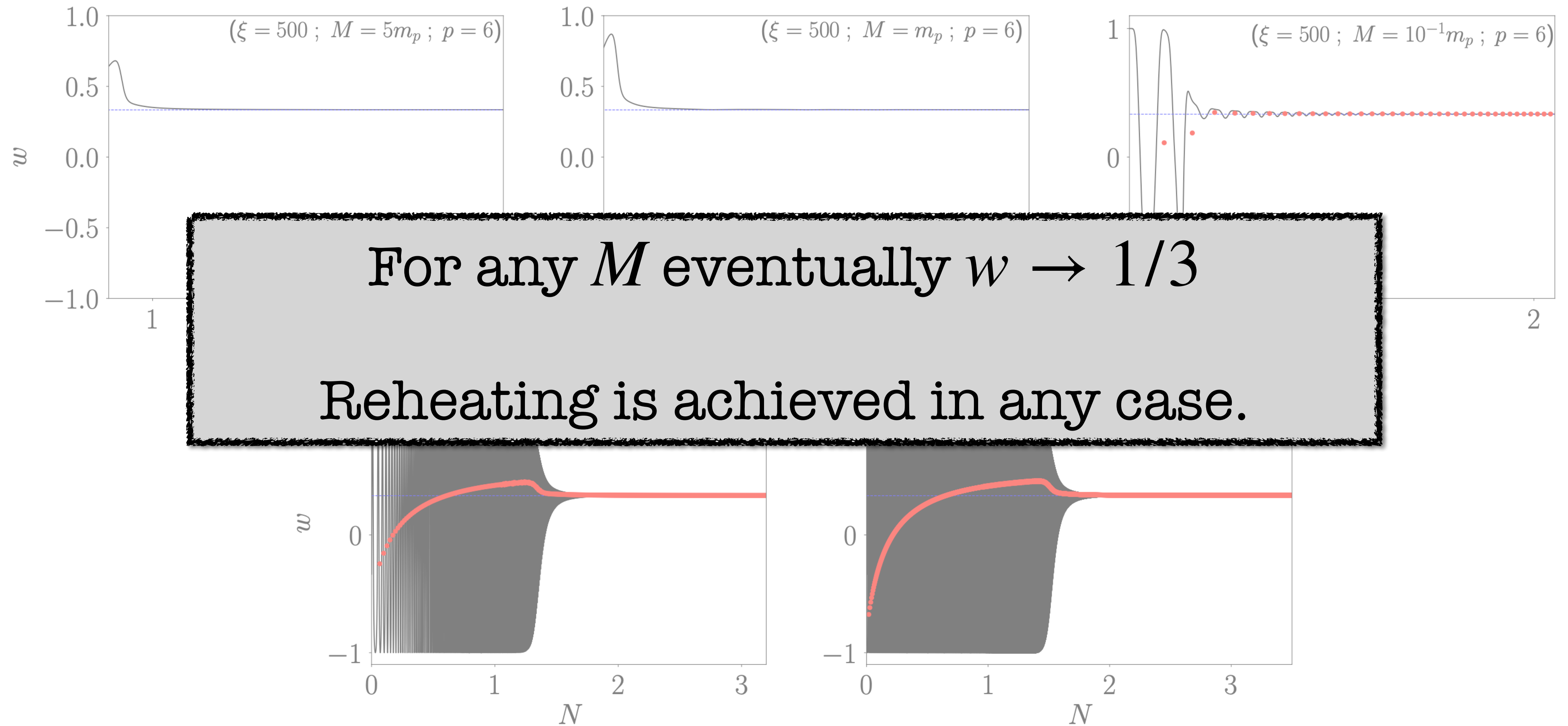


# Geometric (p)reheating: Lattice results $p=6$



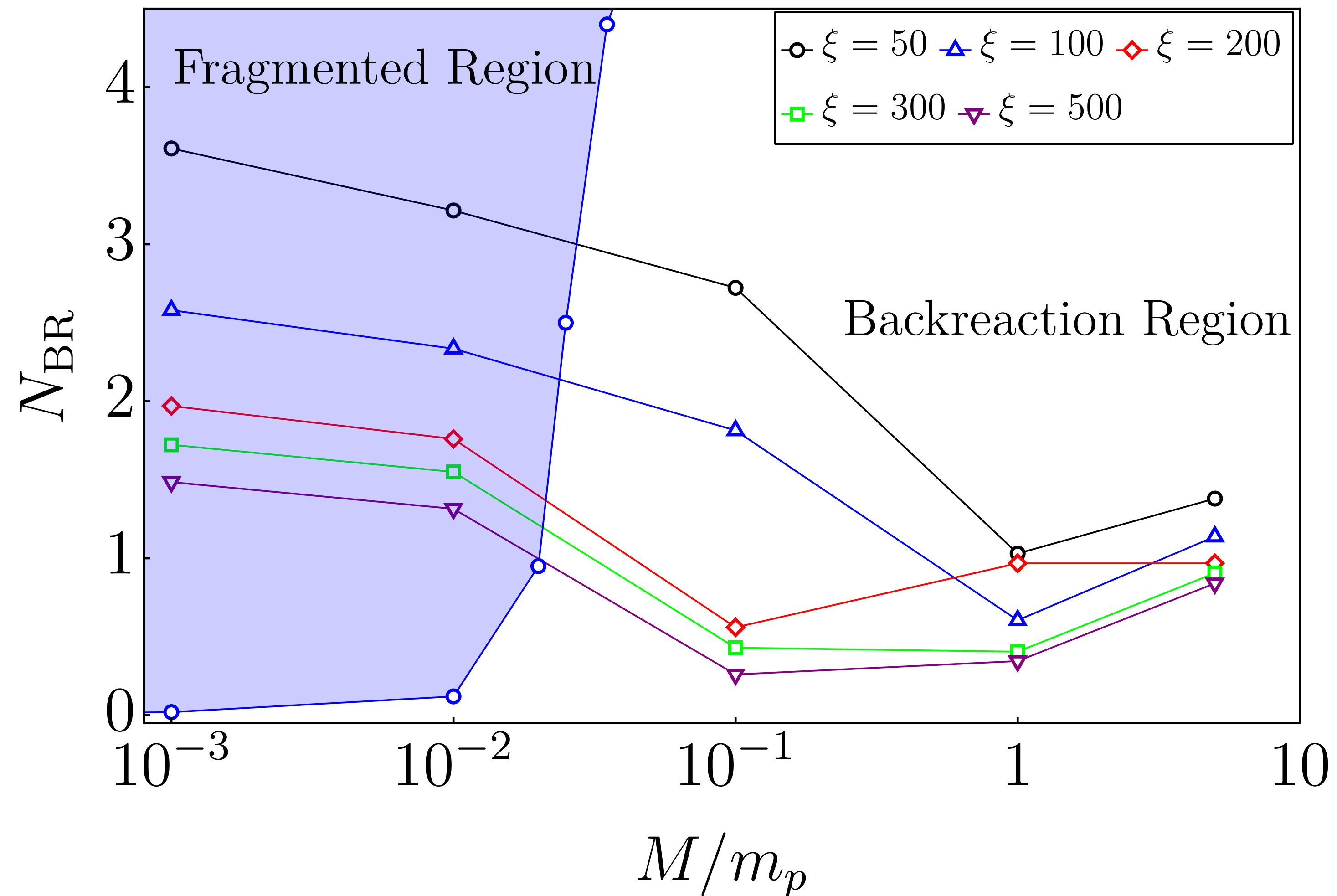


# Geometric (p)reheating: Lattice results $p=6$



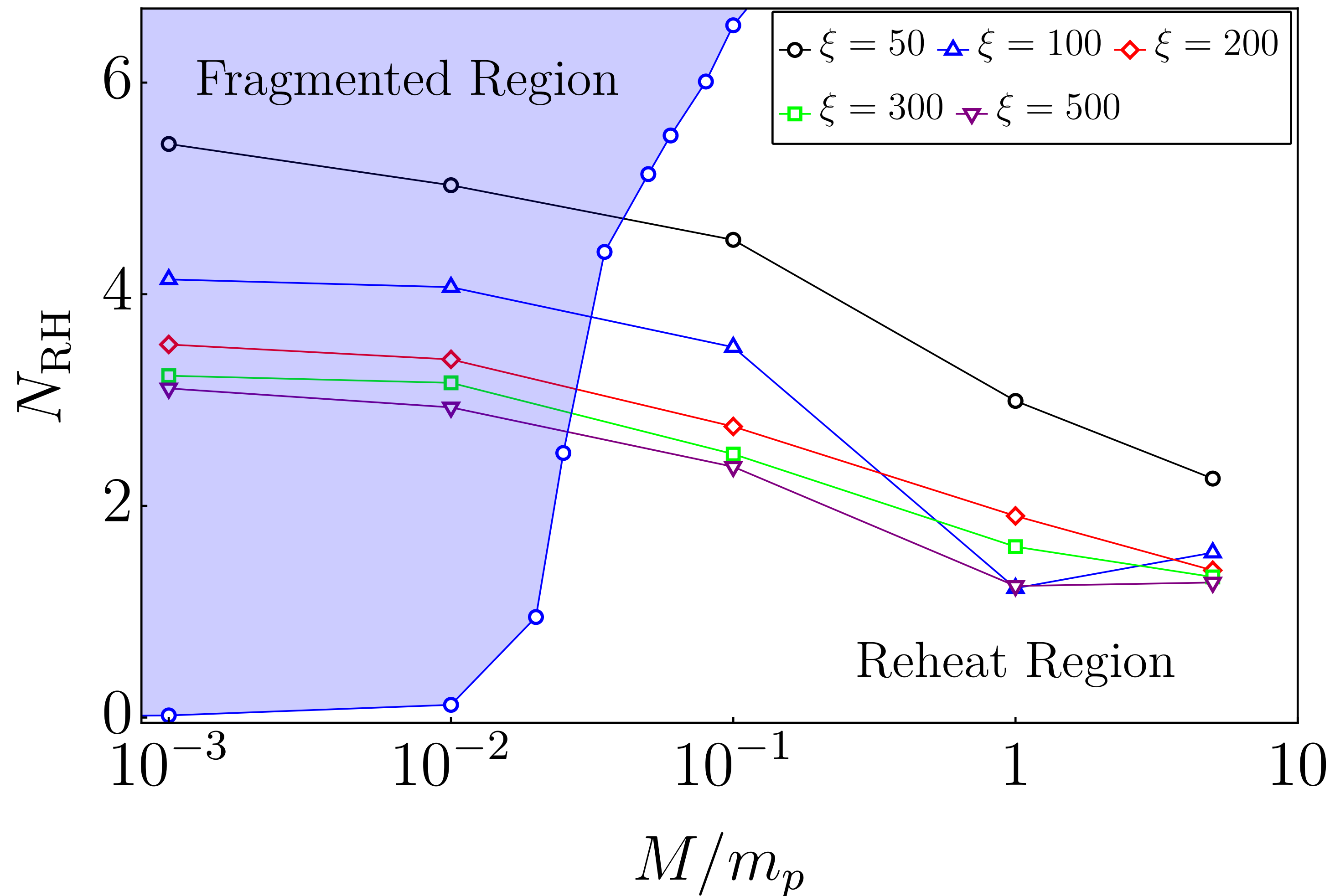
# Geometric (p)reheating: Lattice results $p=6$

$N_{\text{BR}} \equiv$  when  $w$  (EoS) envelope falls below 90% its Max value



# Geometric (p)reheating: Lattice results $p=6$

$$N_{\text{RH}} \equiv \text{moment when } E_\chi = E_\phi$$



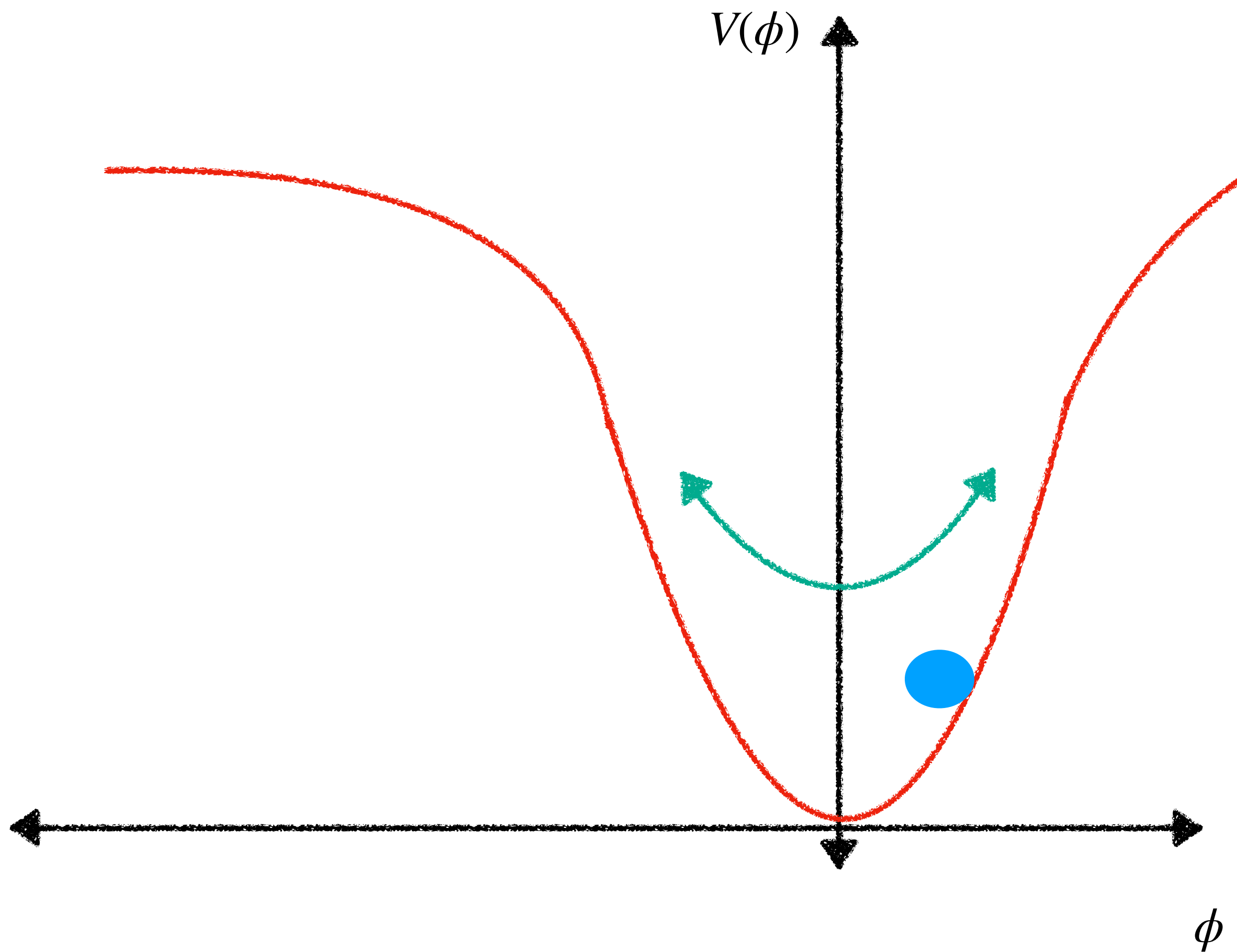


Thanks!

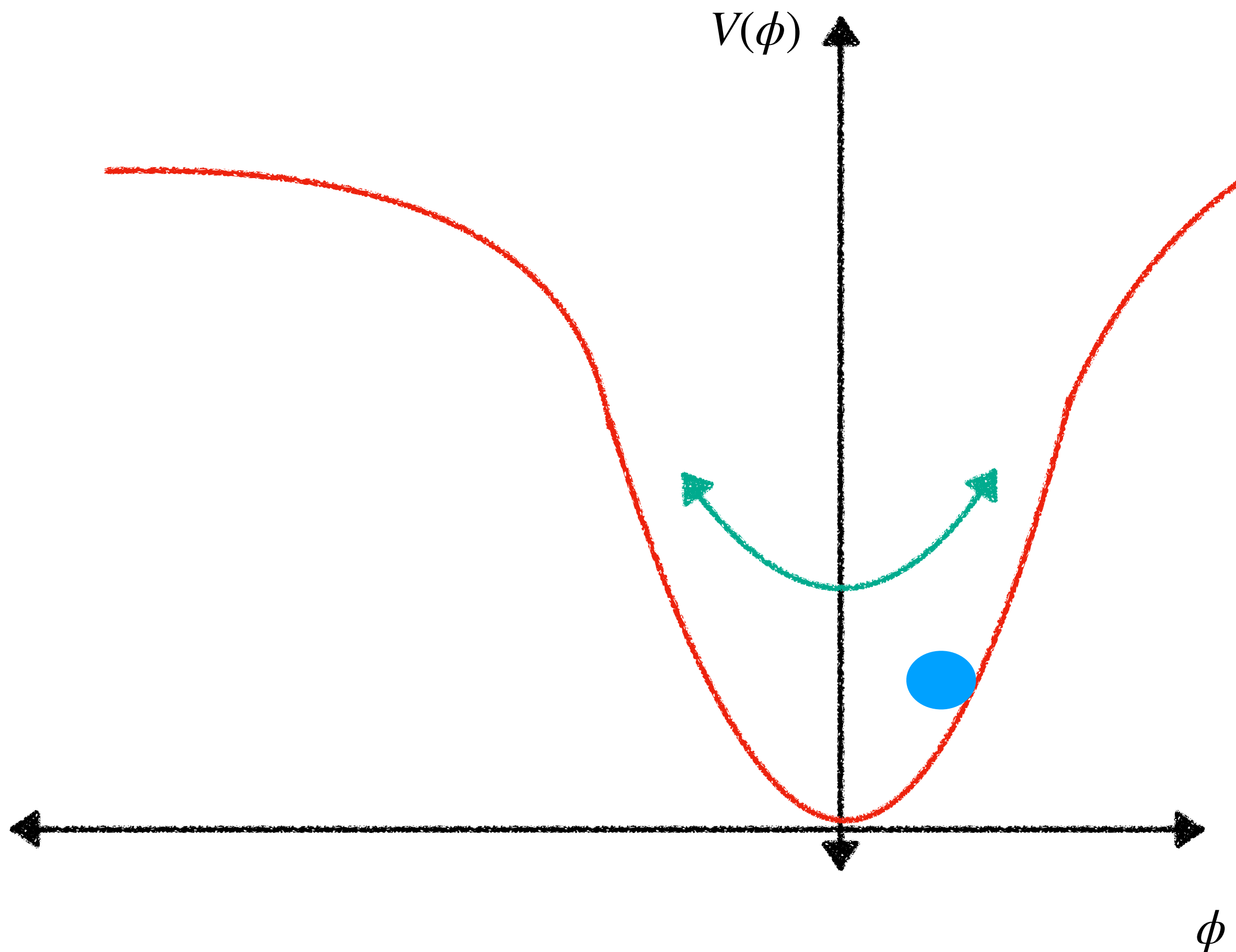
# Inflaton fragmentation

Inflaton inhomogeneous perturbations  $\delta\phi(\mathbf{x}, t)$  couple to the background

$$\delta\ddot{\phi}_k + \left[ \kappa^2 + (p-1) |\phi|^{p-2} \right] \delta\phi_k = 0$$



# Inflaton fragmentation

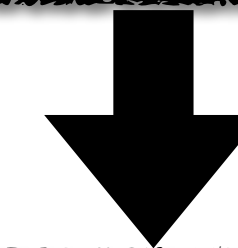


Fragmentation  $\implies w = 1/3$

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$$\delta\ddot{\phi}_k + \left[ \kappa^2 + (p-1) |\phi|^{p-2} \right] \delta\phi_k = 0$$

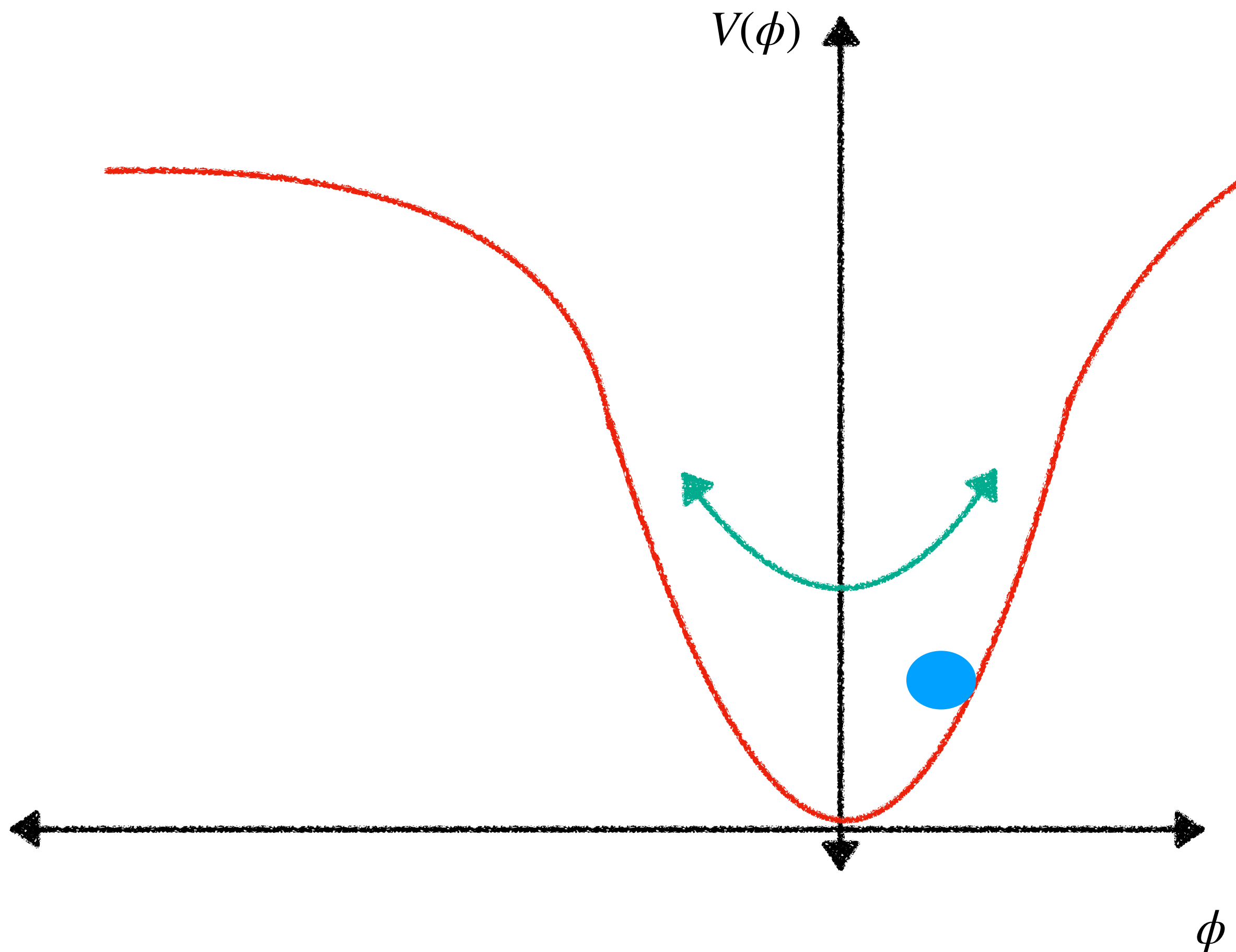
For  $M \gtrsim m_p$   
Inhomogeneities grow due to parametric resonance



Leading to the fragmentation of the homogeneous background



# Inflaton fragmentation



Inflaton inhomogeneous perturbations  $\delta\phi(\mathbf{x}, t)$  couple to the background

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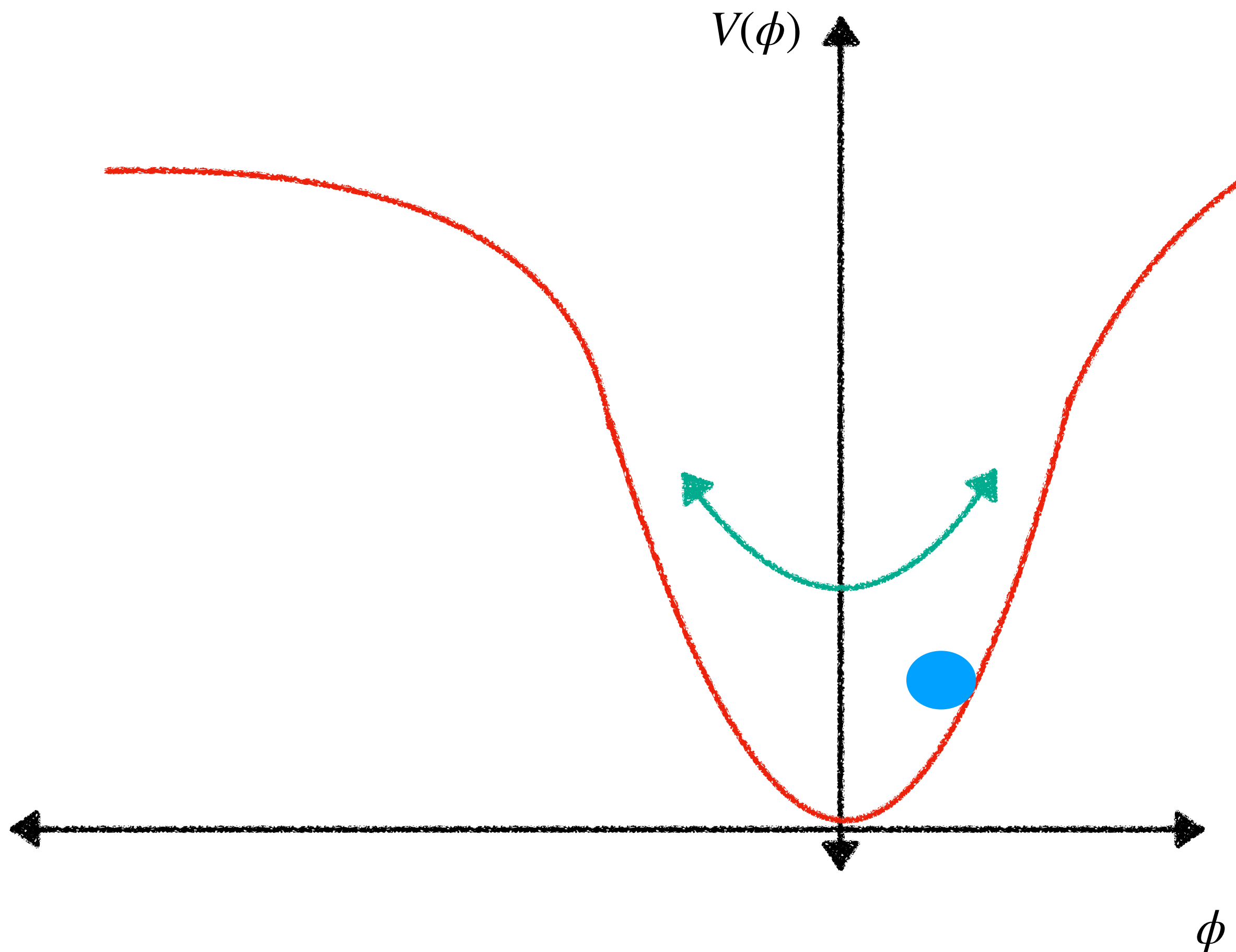
For  $M \gtrsim m_p$   
Inhomogeneities grow due to parametric resonance

**Inefficient effect**  
Fragmentation time scale very long  $N_{\text{frag}} \sim \mathcal{O}(1 - 10)$

Fragmentation  $\implies w = 1/3$



# Inflaton fragmentation



Inflaton inhomogeneous perturbations  $\delta\phi(\mathbf{x}, t)$  couple to the background

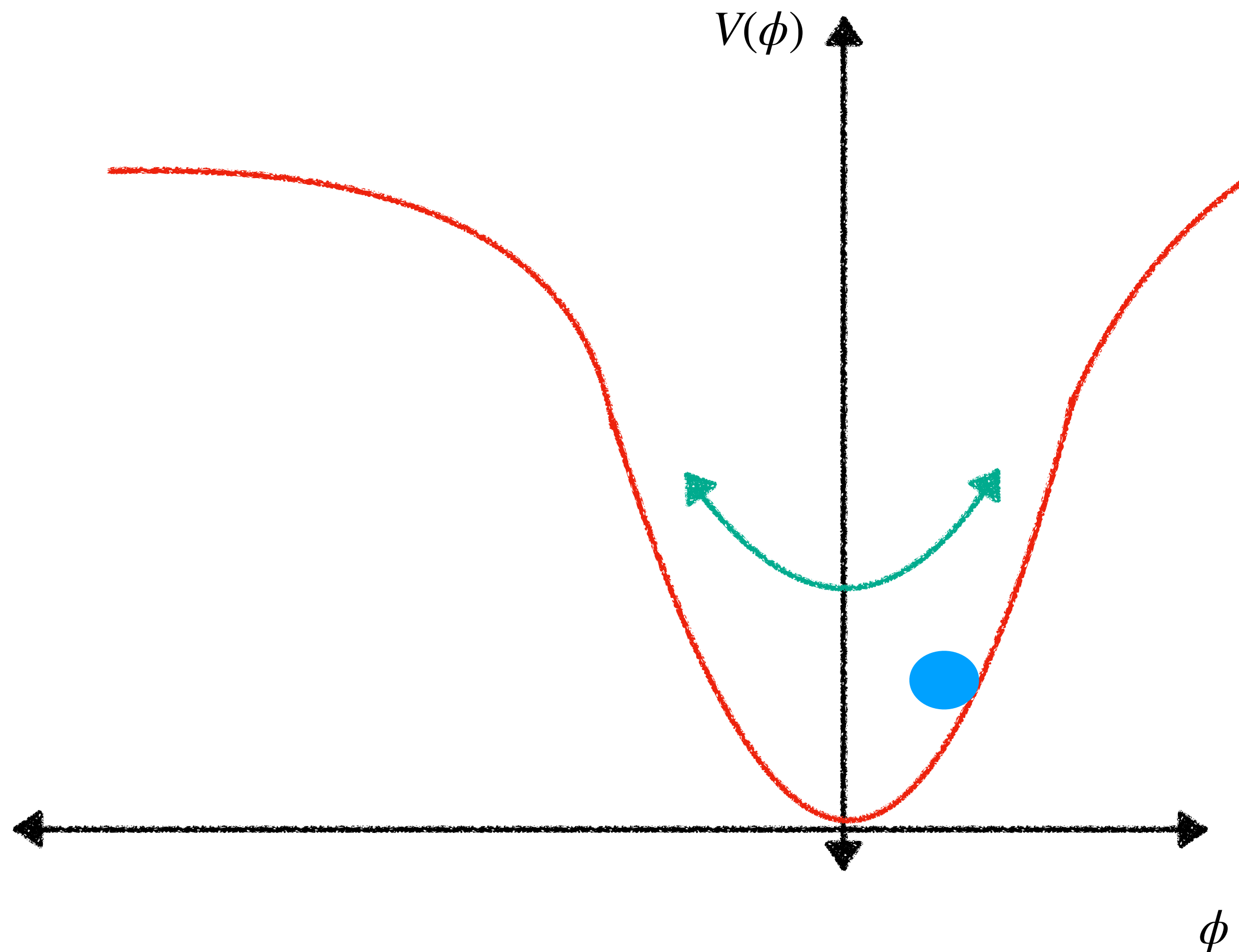
$$\delta\ddot{\phi}_k + \left[ \kappa^2 + \frac{\partial^2 V(\phi)}{\partial\phi^2} \Big|_{\phi_{\text{end}}} \right] \delta\phi_k = 0$$

For  $M \ll m_p$  initial effective mass of modes is negative

$$\frac{\partial^2 V(\phi)}{\partial\phi^2} \Big|_{\phi_{\text{end}}} < 0$$

Fragmentation  $\Rightarrow w = 1/3$

# Inflaton fragmentation



Fragmentation  $\implies w = 1/3$

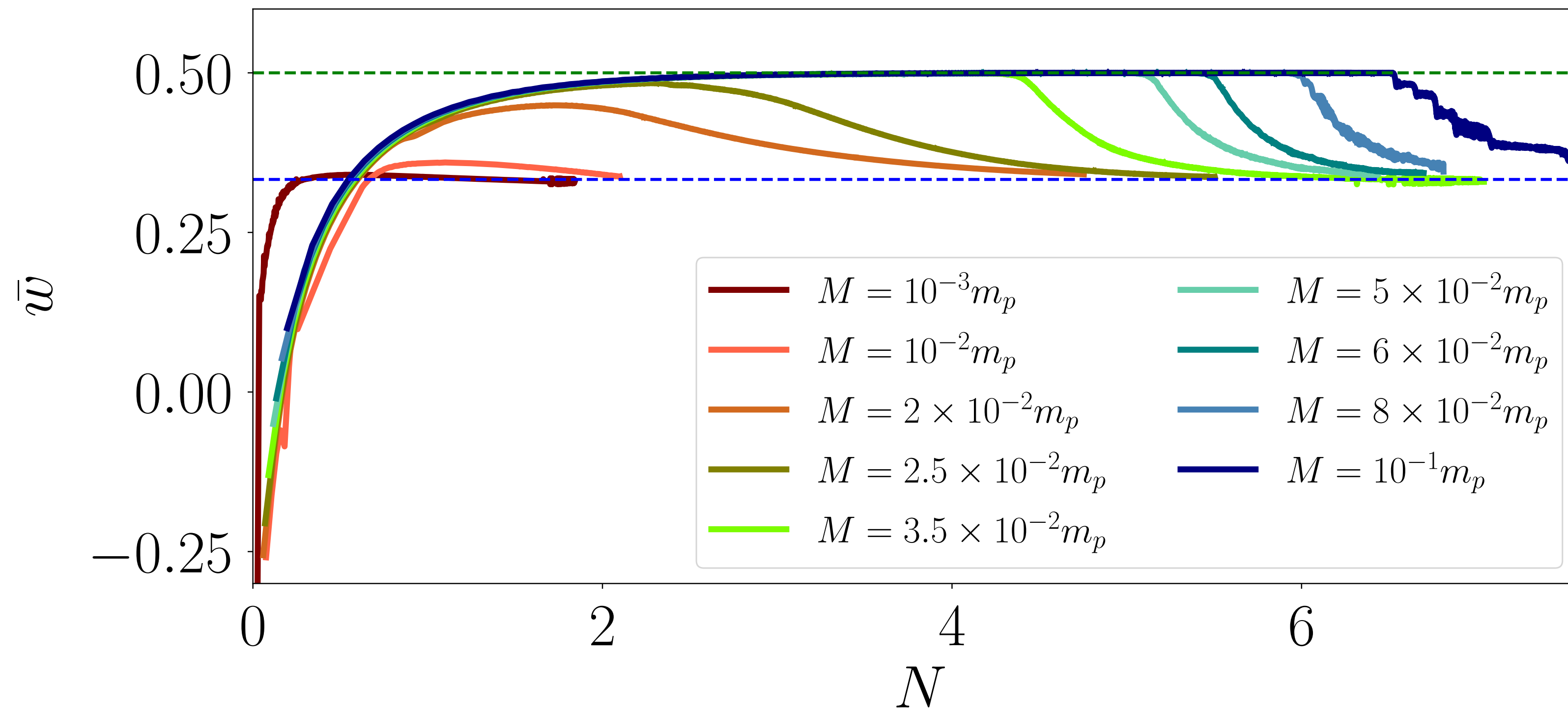
Inflaton inhomogeneous perturbations  $\delta\phi(\mathbf{x}, t)$  couple to the background

$$\delta\ddot{\phi}_k + \left[ \kappa^2 + \frac{\partial^2 V(\phi)}{\partial \phi^2} \Big|_{\phi_{\text{end}}} \right] \delta\phi_k = 0$$

A very large range of modes  
 $\kappa \lesssim (M/m_p)^{-1}$   
grow exponentially (tachyonic excitation)

**Efficient effect**  
Fragmentation time scale very short  $N_{\text{frag}} \sim \mathcal{O}(10^{-2})$

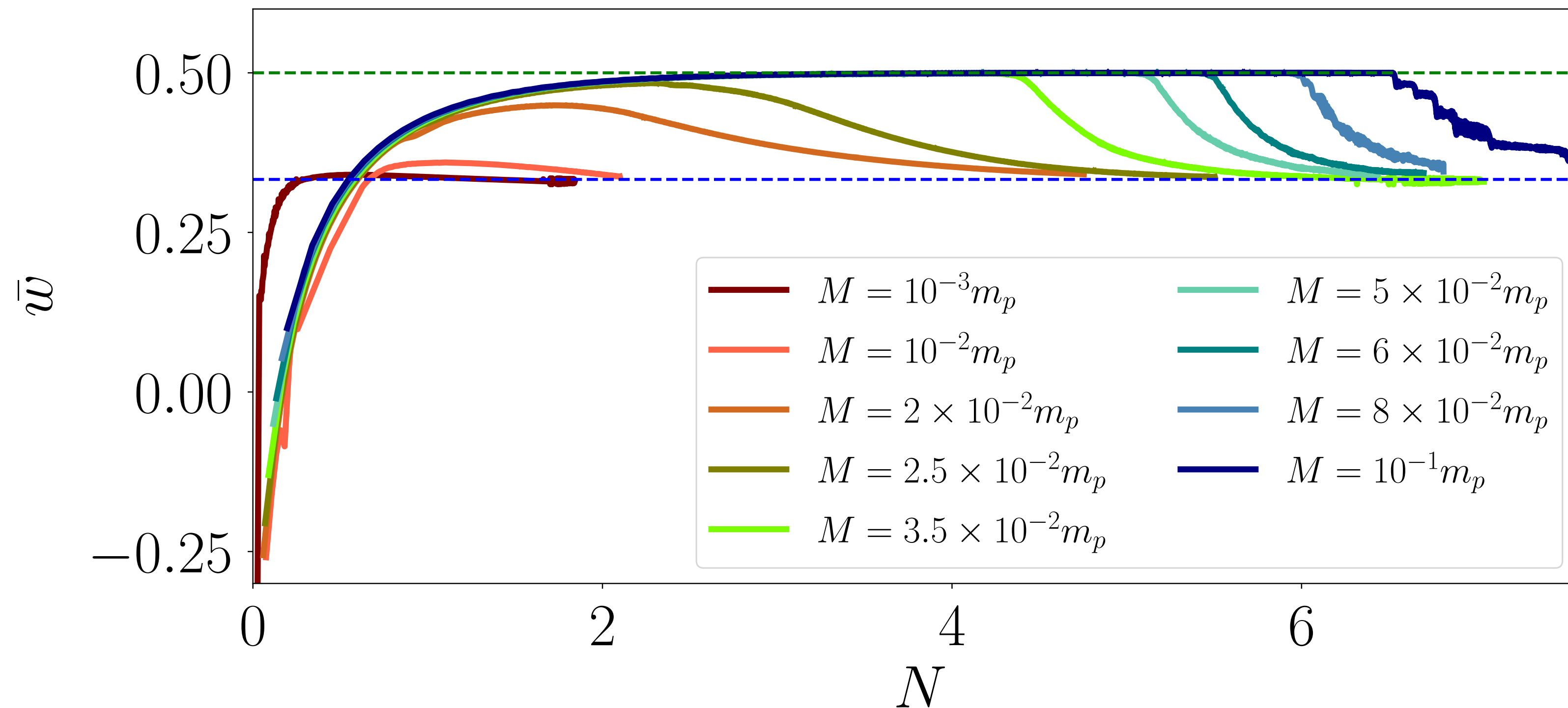
# Inflaton fragmentation



Lattice simulations for  $p = 6$



# Inflaton fragmentation



Lattice simulations for  $p = 6$

For  $M \gtrsim 10^{-2}m_p$  competition  
between tachyonic and  
parametric resonance

Long  $N_{\text{frag}}$

For  $M \lesssim 10^{-2}m_p$  dominated  
tachyonic resonance

Short  $N_{\text{frag}}$



# Inflaton fragmentation

