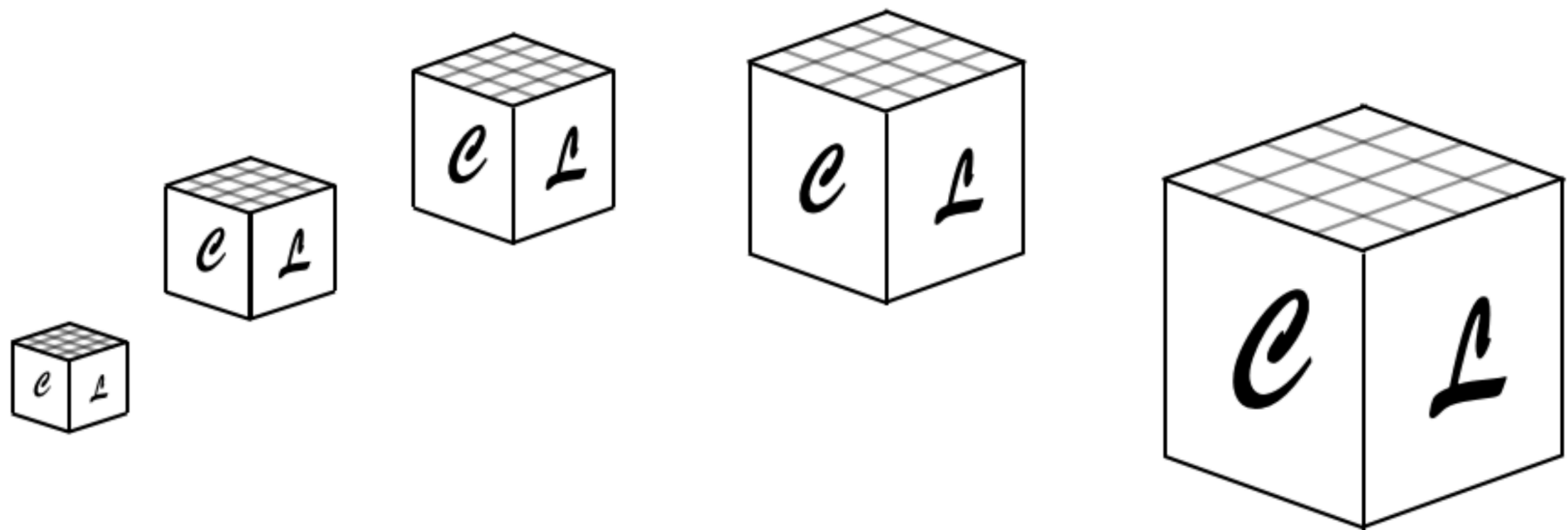


CosmoLattice

— School 2025 —

Local Organizers: Mohammad Ali Gorji, Masahide Yamaguchi, Dong-Won Jung



IBS, Daejeon, South Korea, 22-26 September 2025

INSTITUT DE
FÍSICA
CORPUSCULAR



EXCELENCIA
SEVERO
OCHOA

VNIVERSITAT
DE VALÈNCIA

525
anys



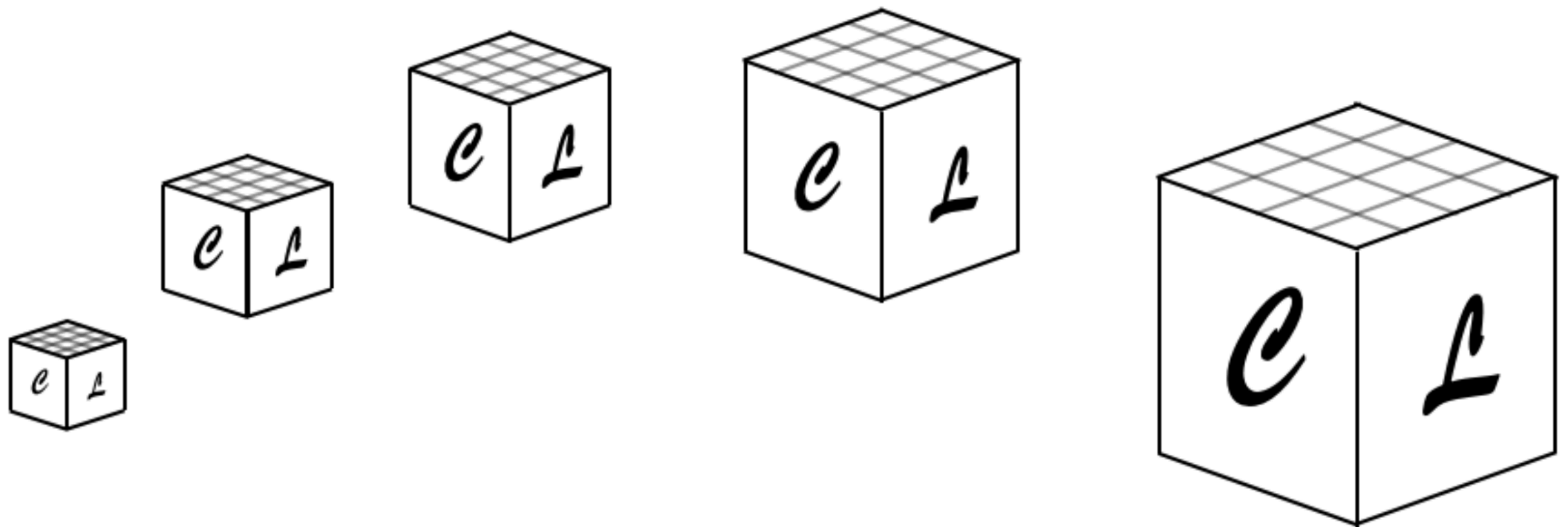
CSIC
CONSEJO SUPERIOR DE INVESTIGACIONES CIENTÍFICAS



GENERALITAT
VALENCIANA

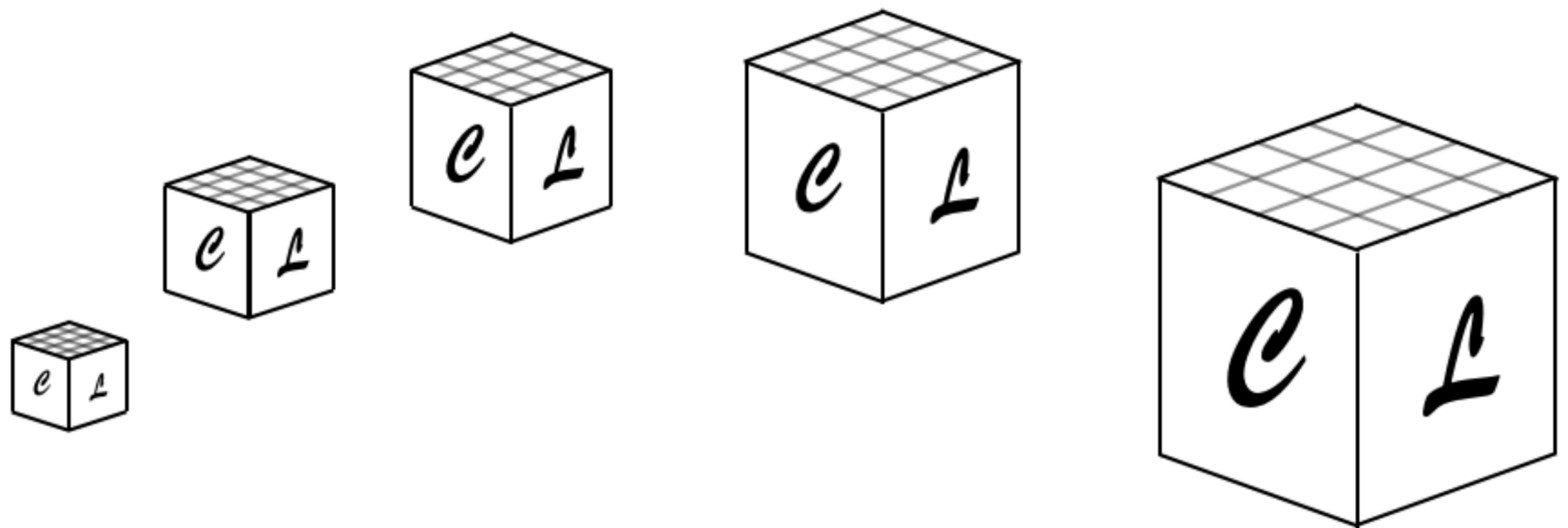
LATTICE-COSMOLOGY

* *TECHNIQUES* *



IBS, Daejeon, South Korea, 22-26 September 2025

The Art of Simulating the Early Universe



IBS, Daejeon, South Korea, 22-26 September 2025

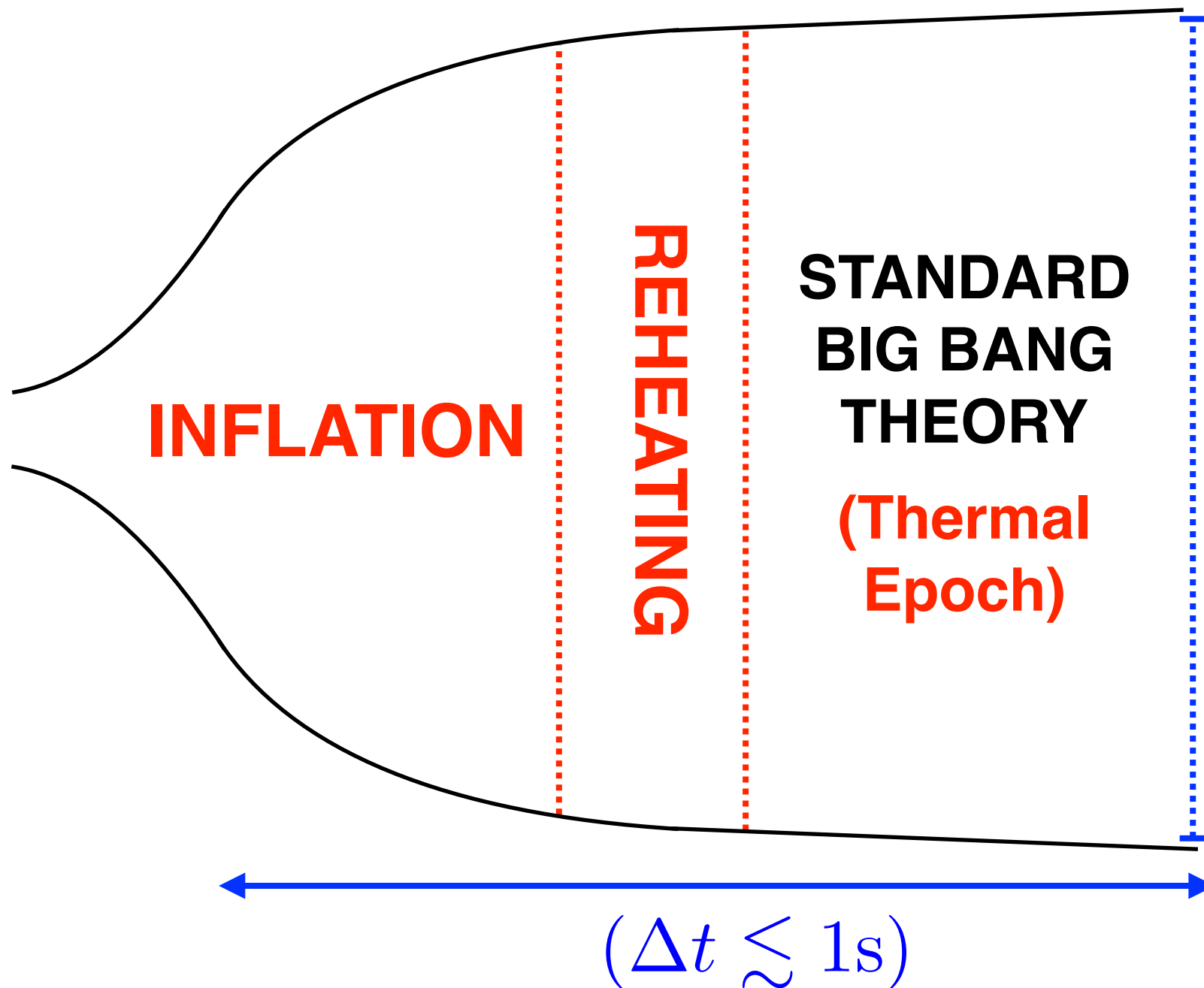
The Art of Simulating the Early Universe

MOTIVATION

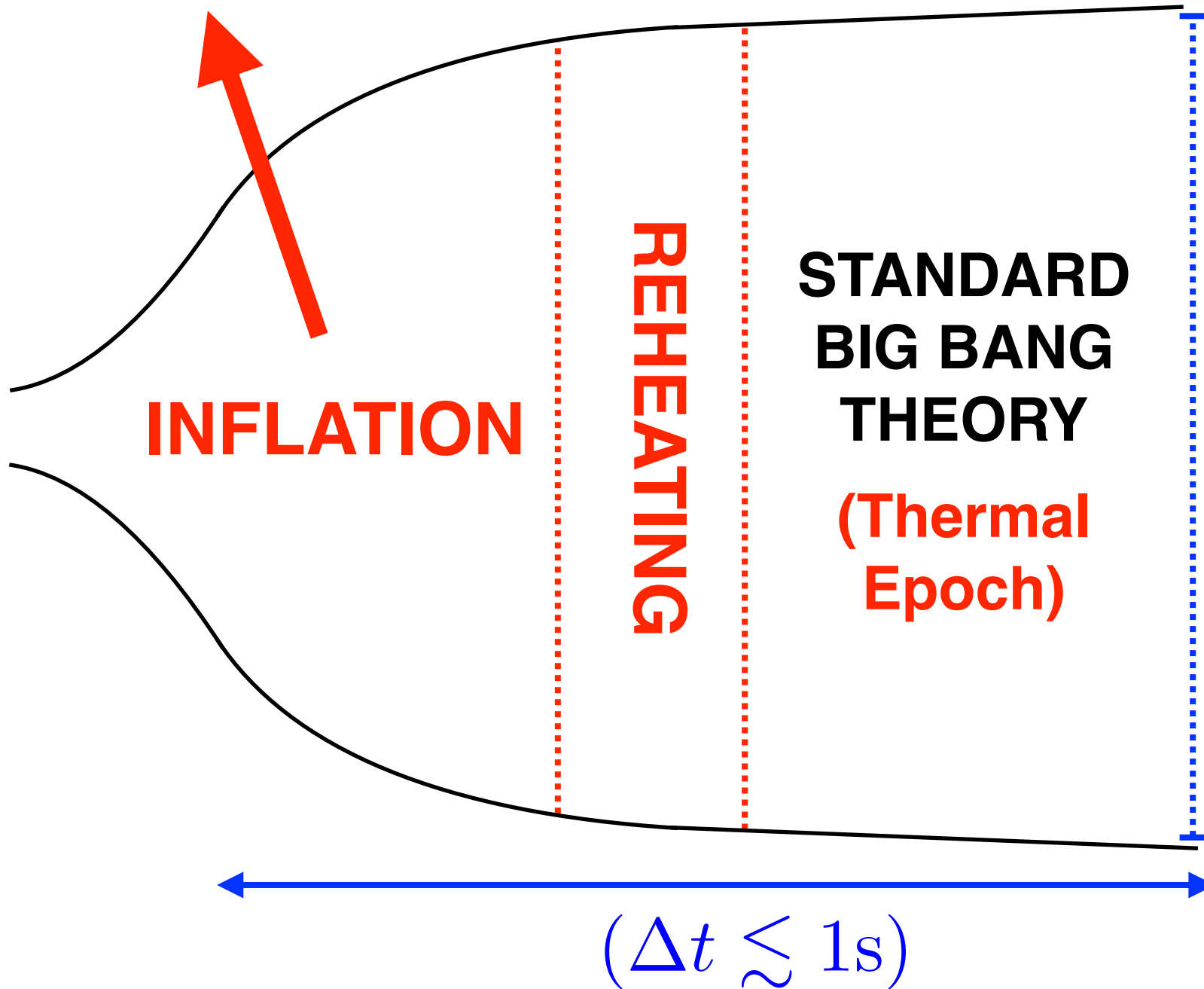
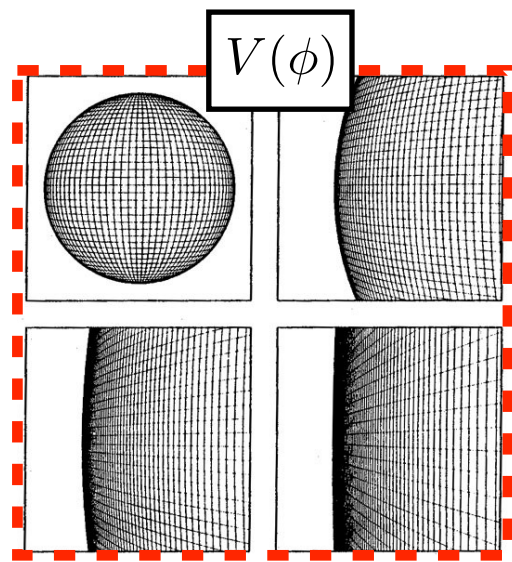
The Art of Simulating the Early Universe

MOTIVATION

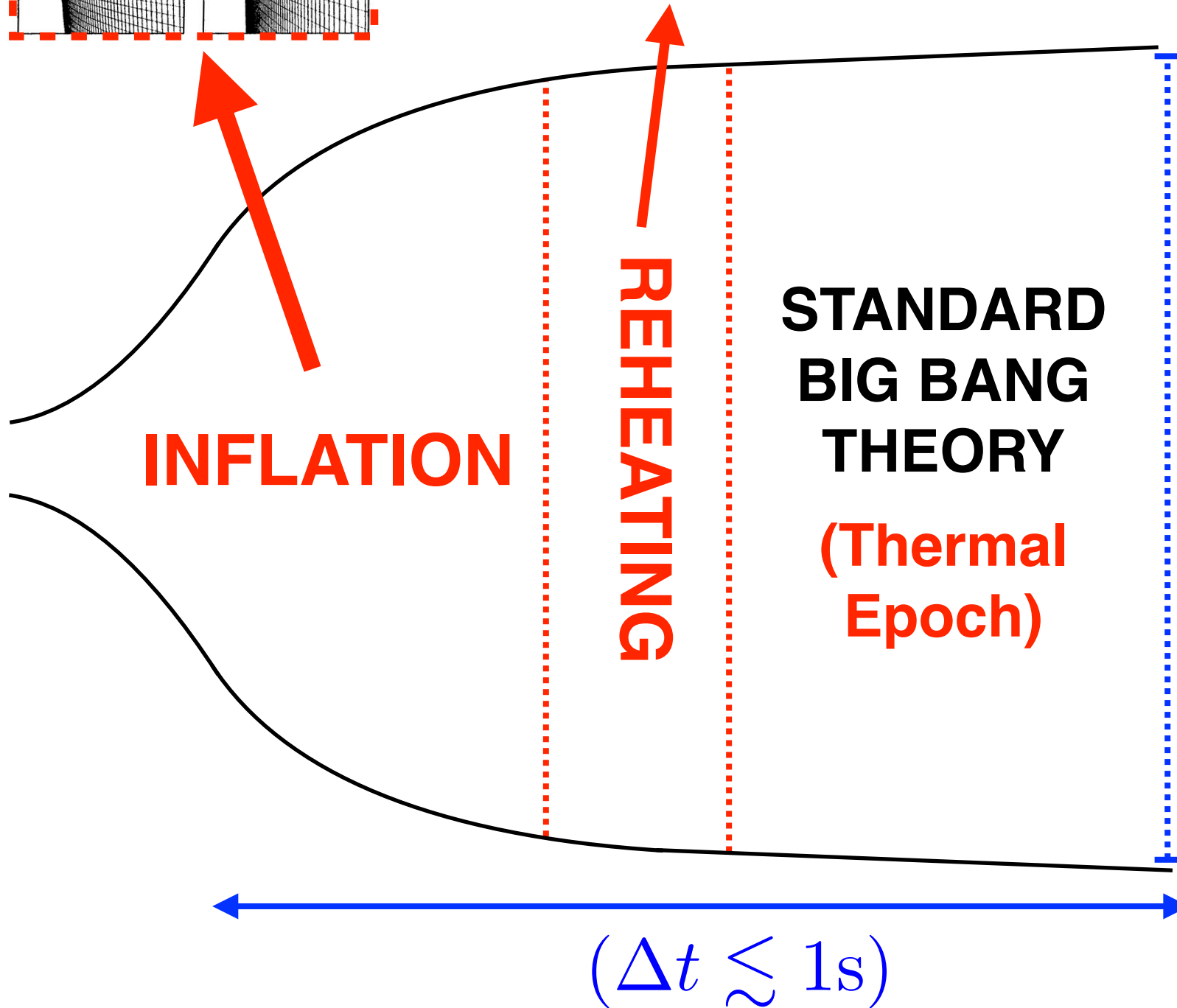
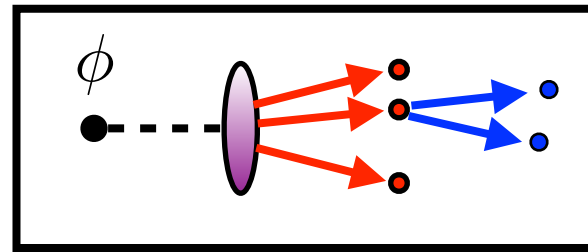
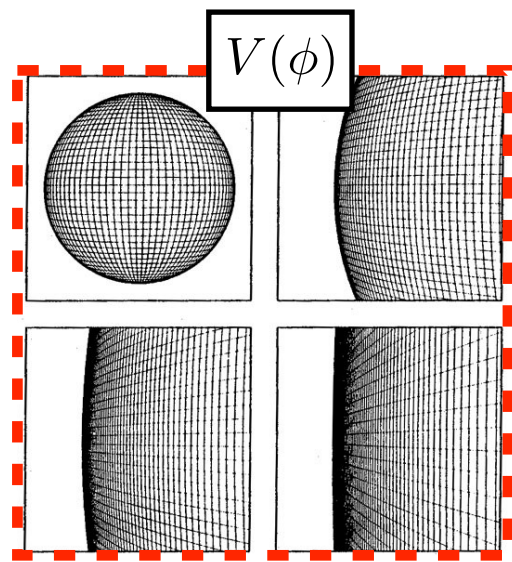
The Early Universe



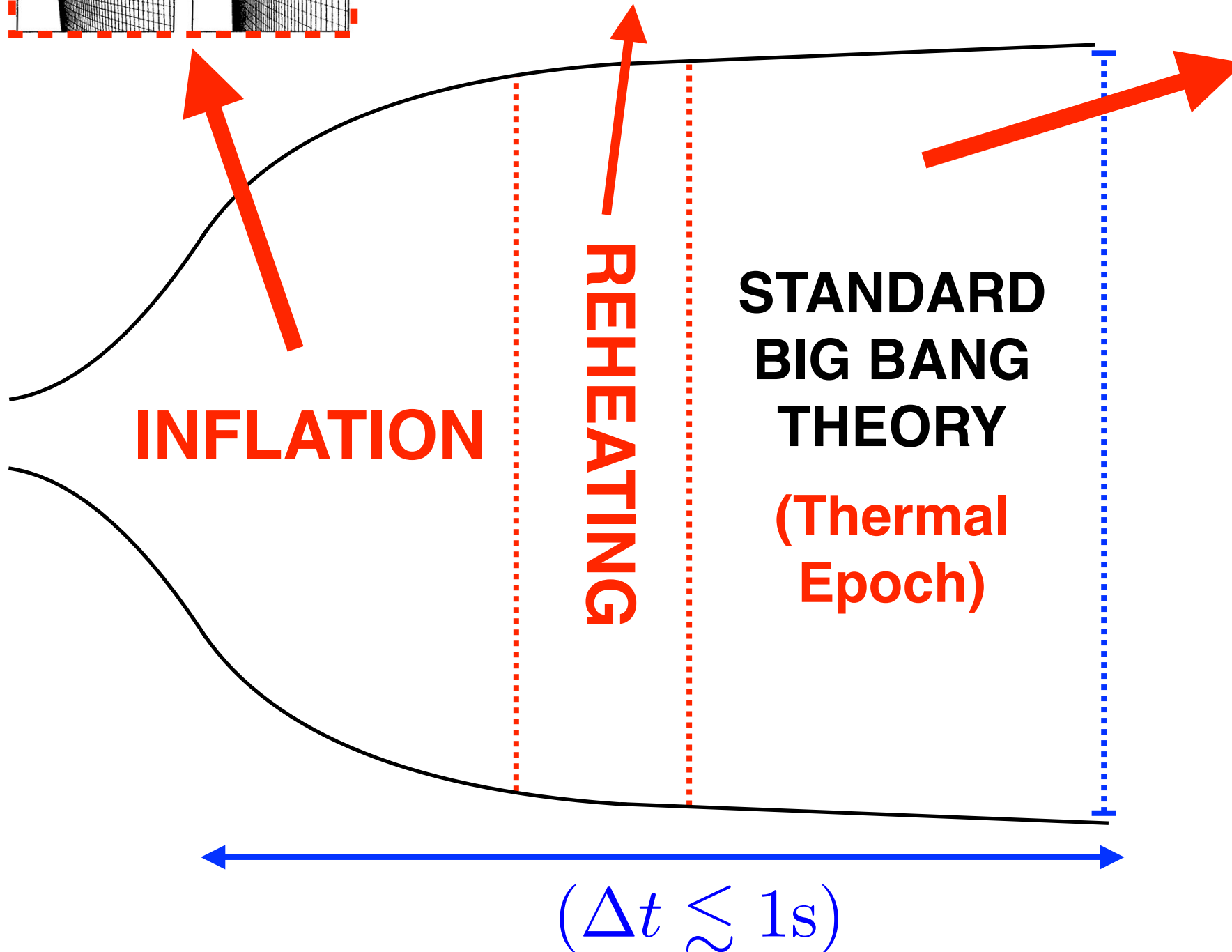
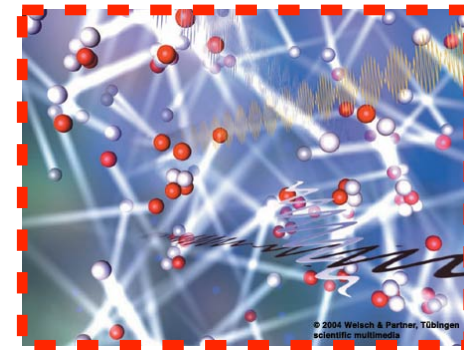
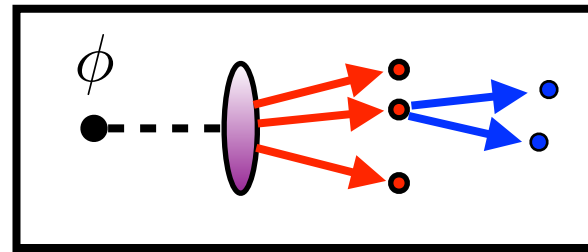
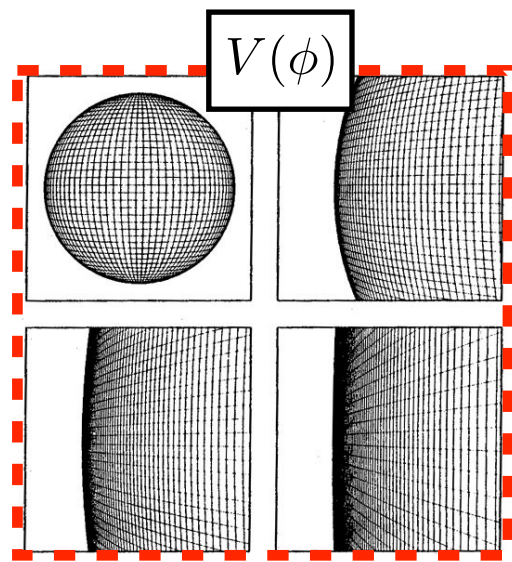
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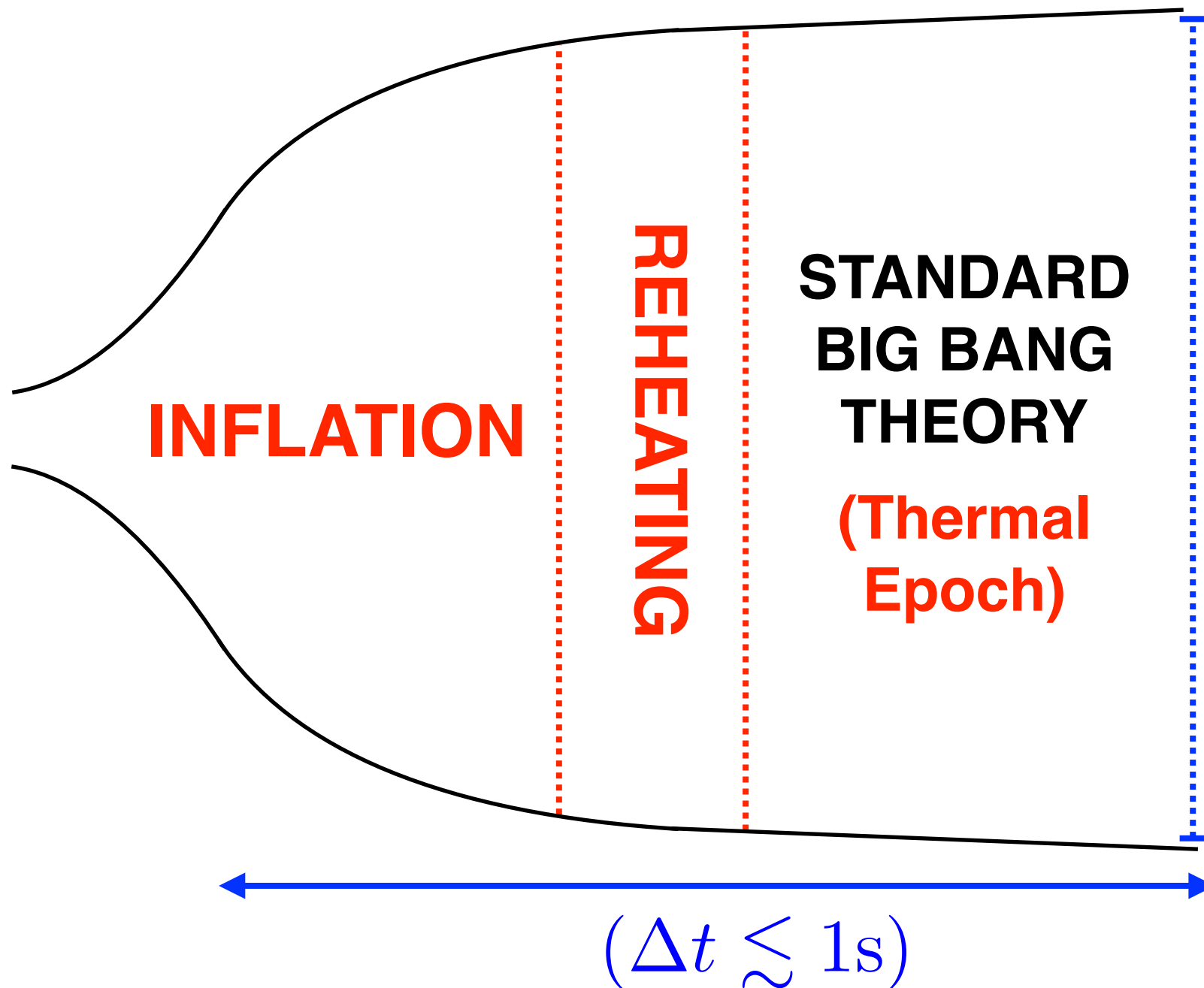
The Early Universe

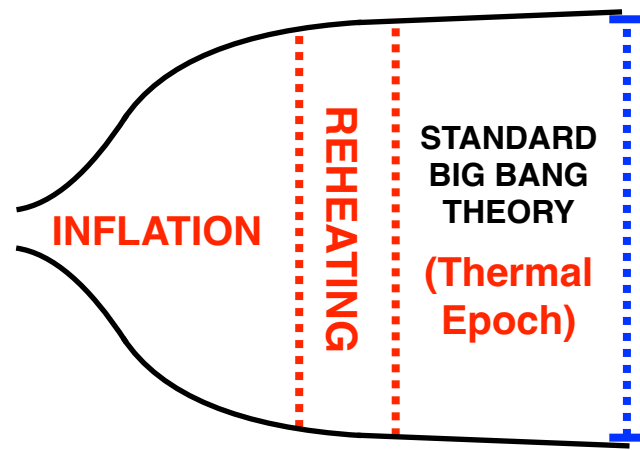


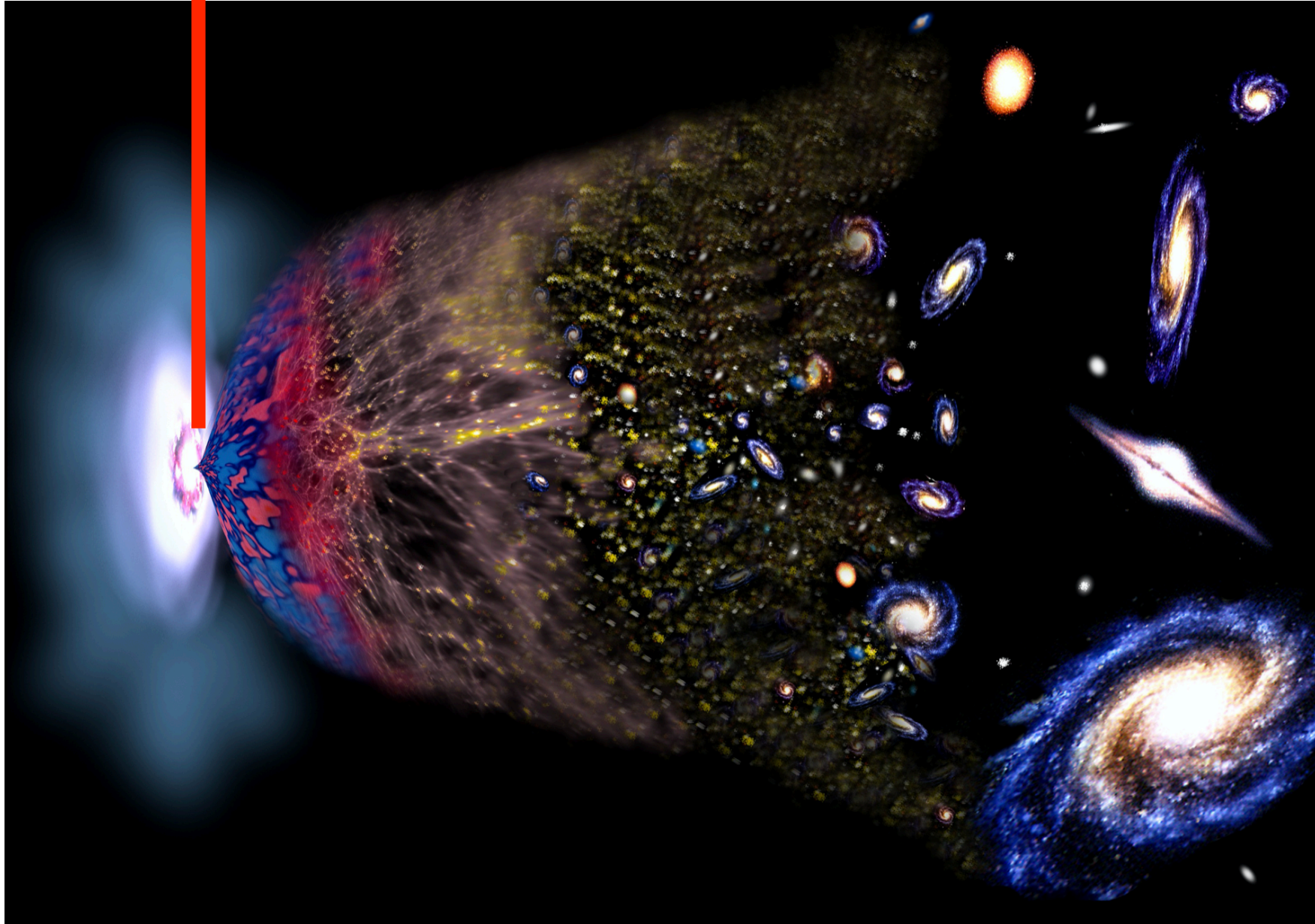
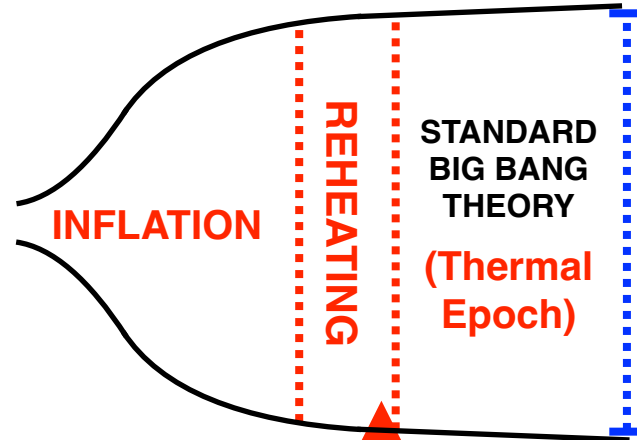
The Early Universe



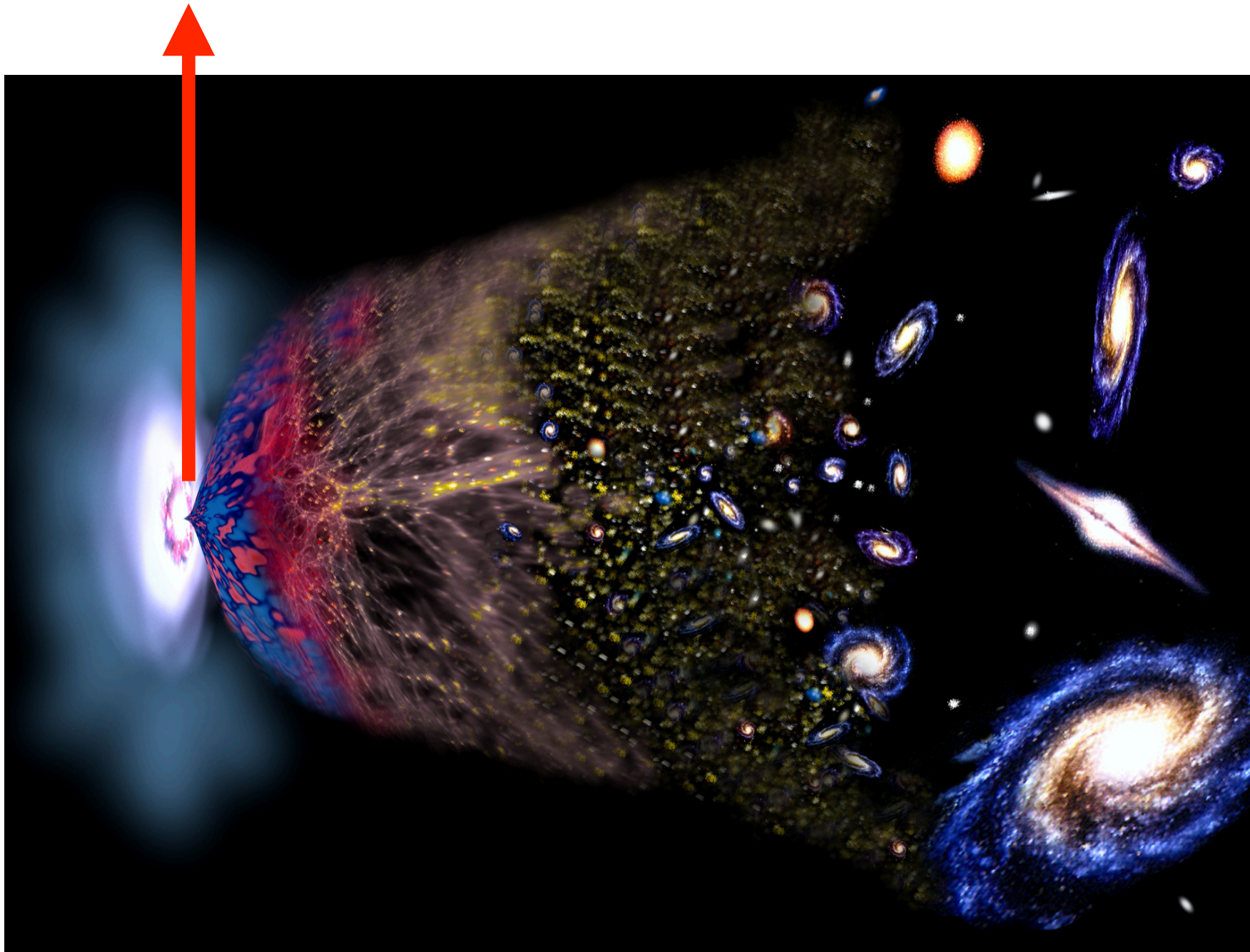
The Early Universe



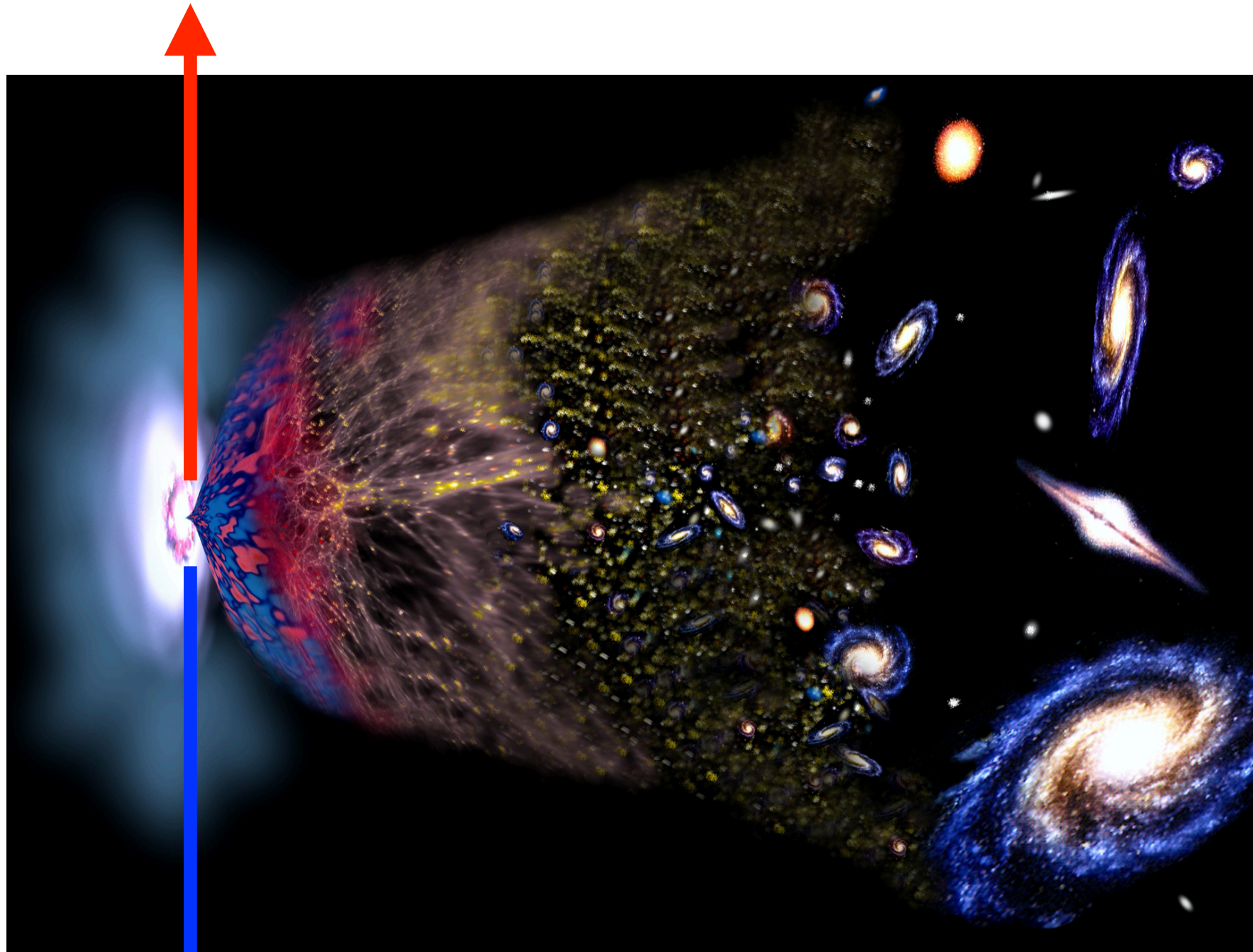




**Can we simulate the very first
instants of the Universe ?**

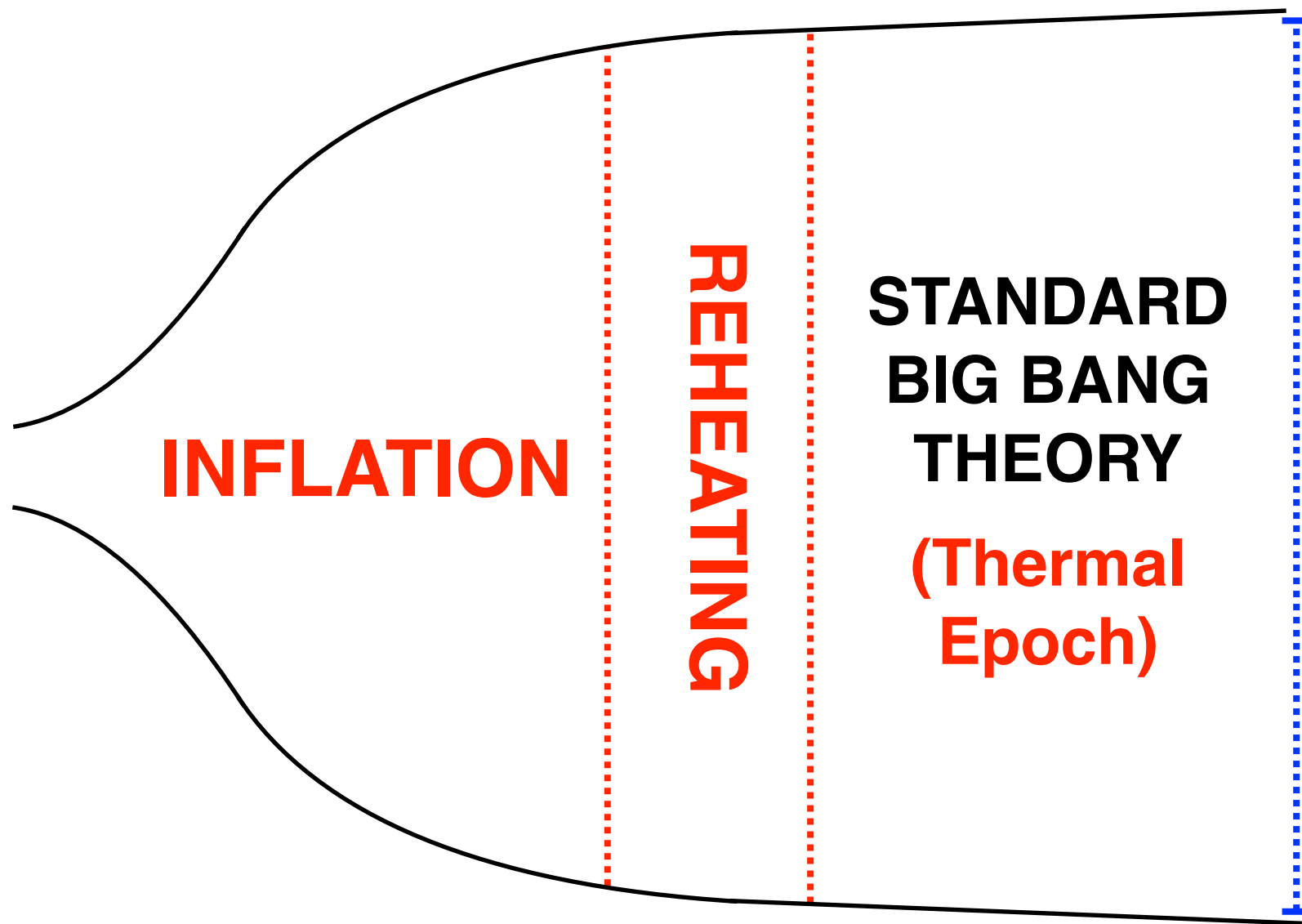


**Can we simulate the very first
instants of the Universe ?**

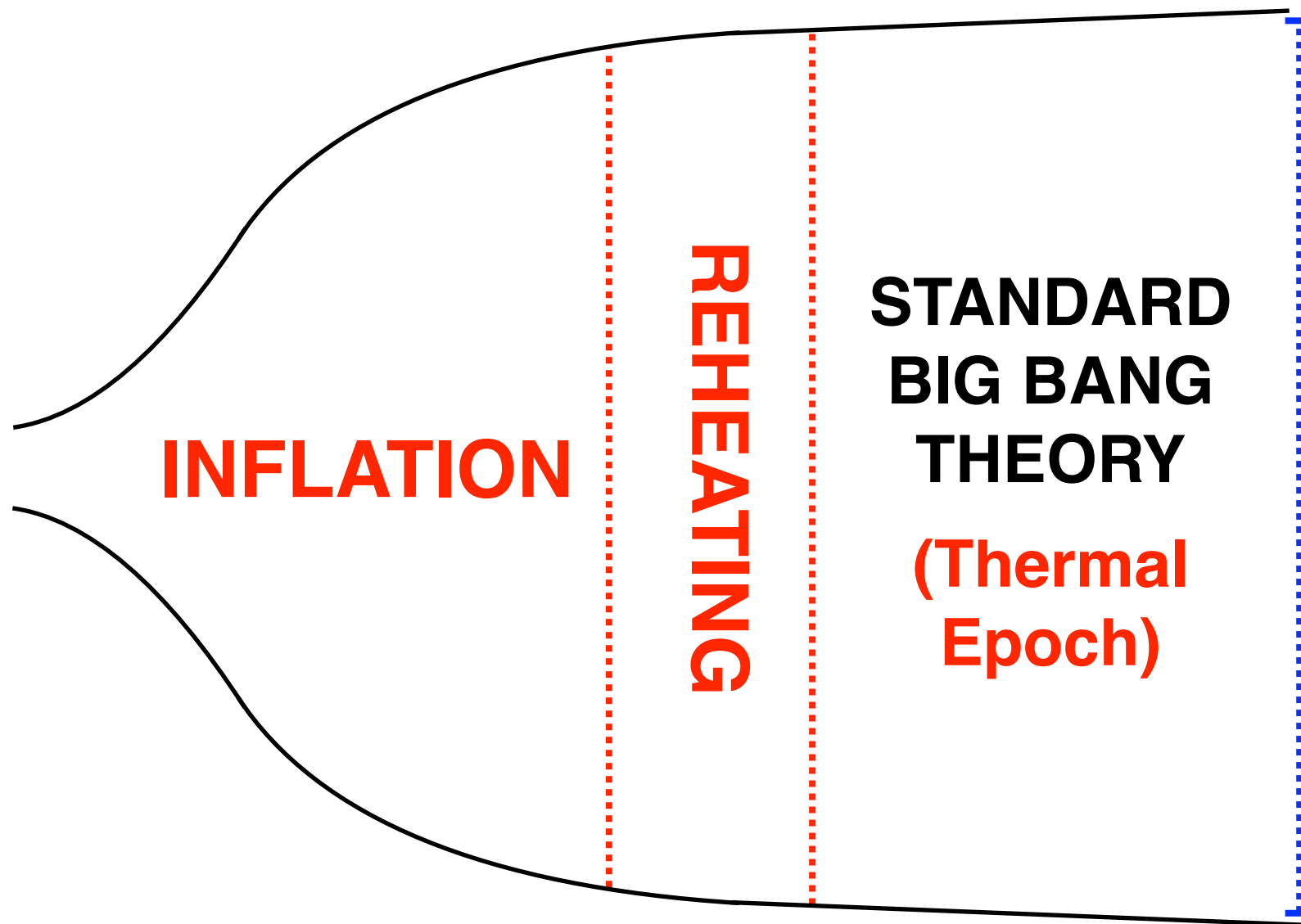


If so, How? What can we learn?

The Early Universe

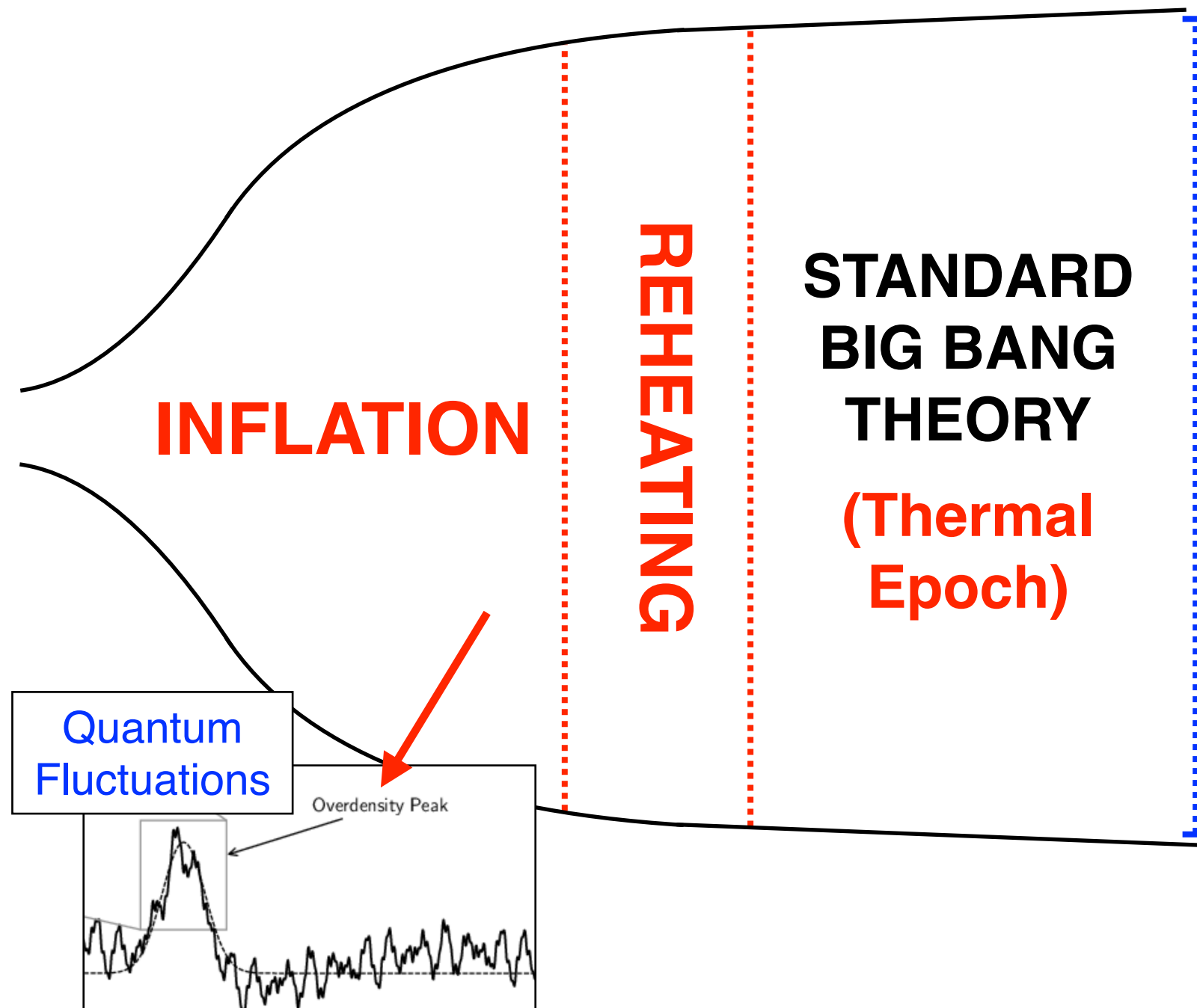


The Early Universe

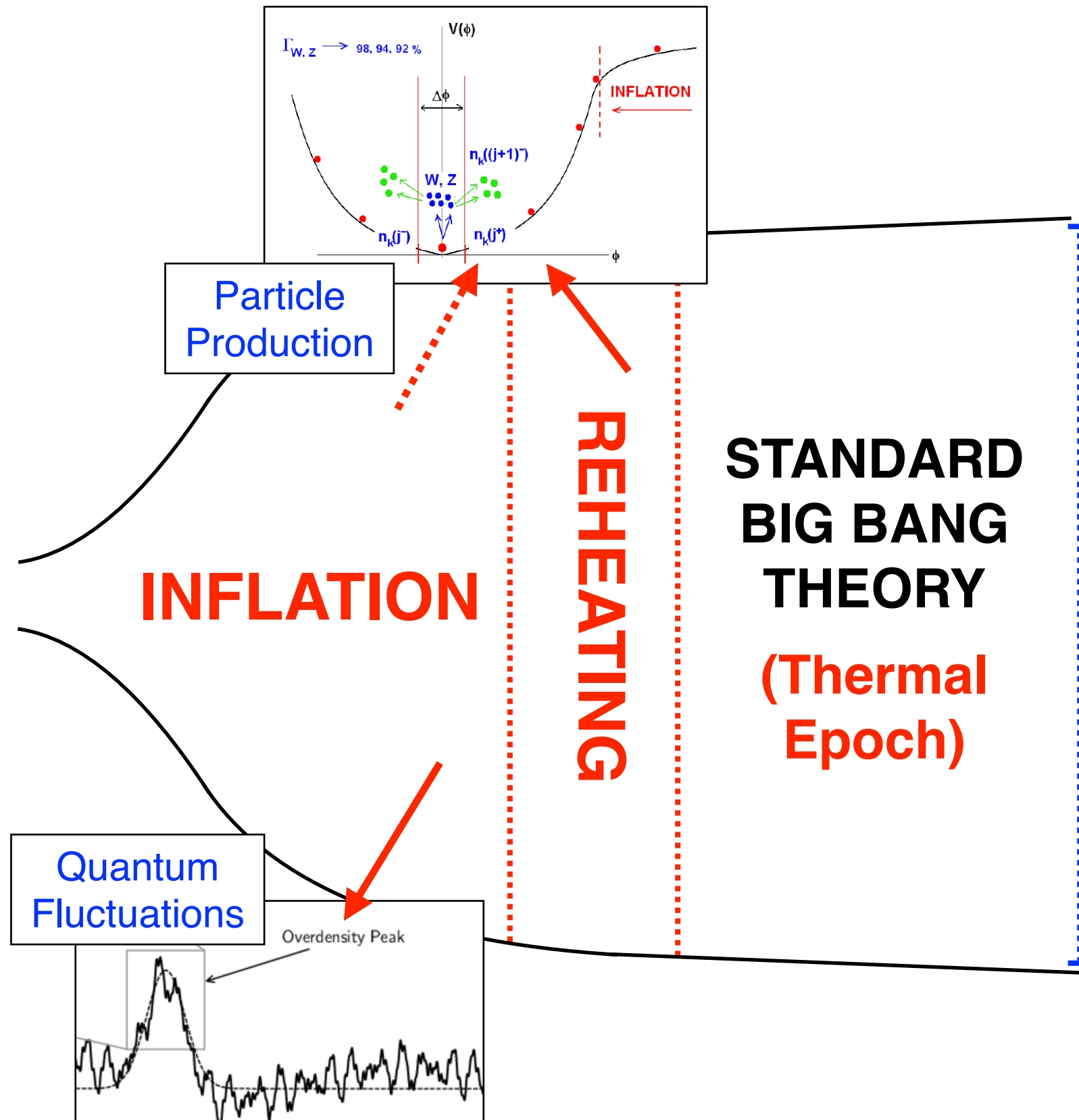


**What
interesting
phenomena?**

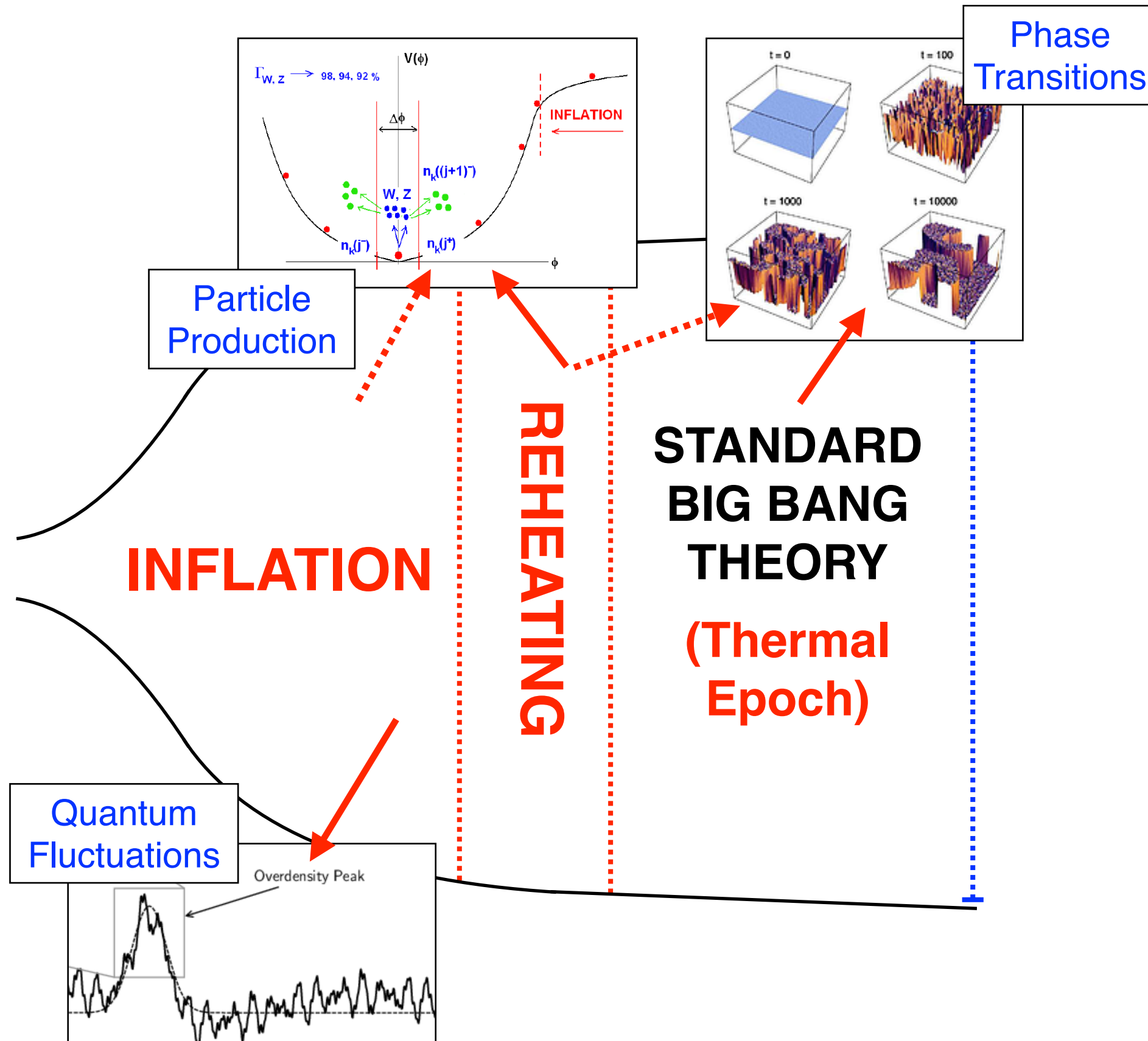
The Early Universe



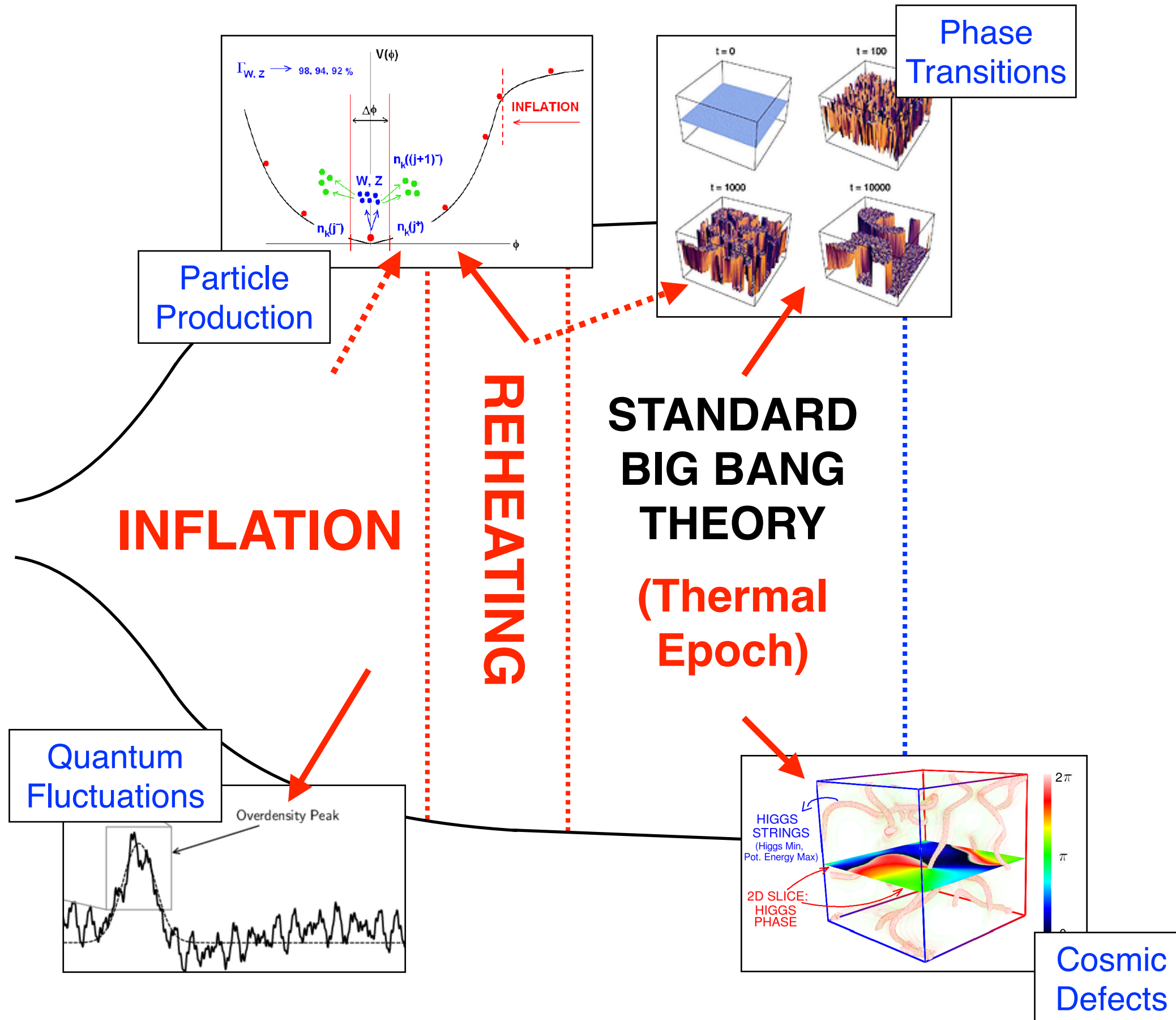
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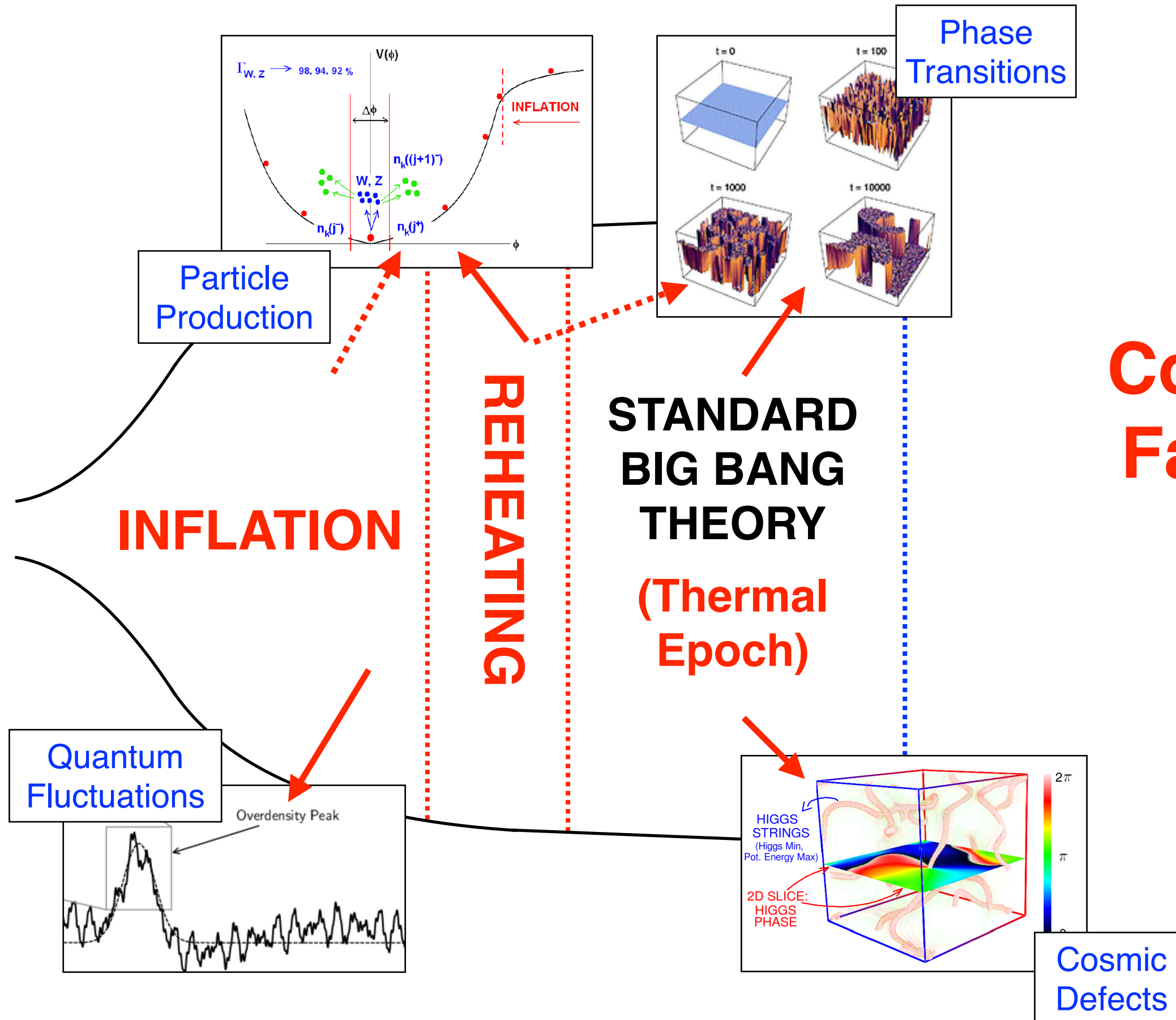
The Early Universe



The Early Universe

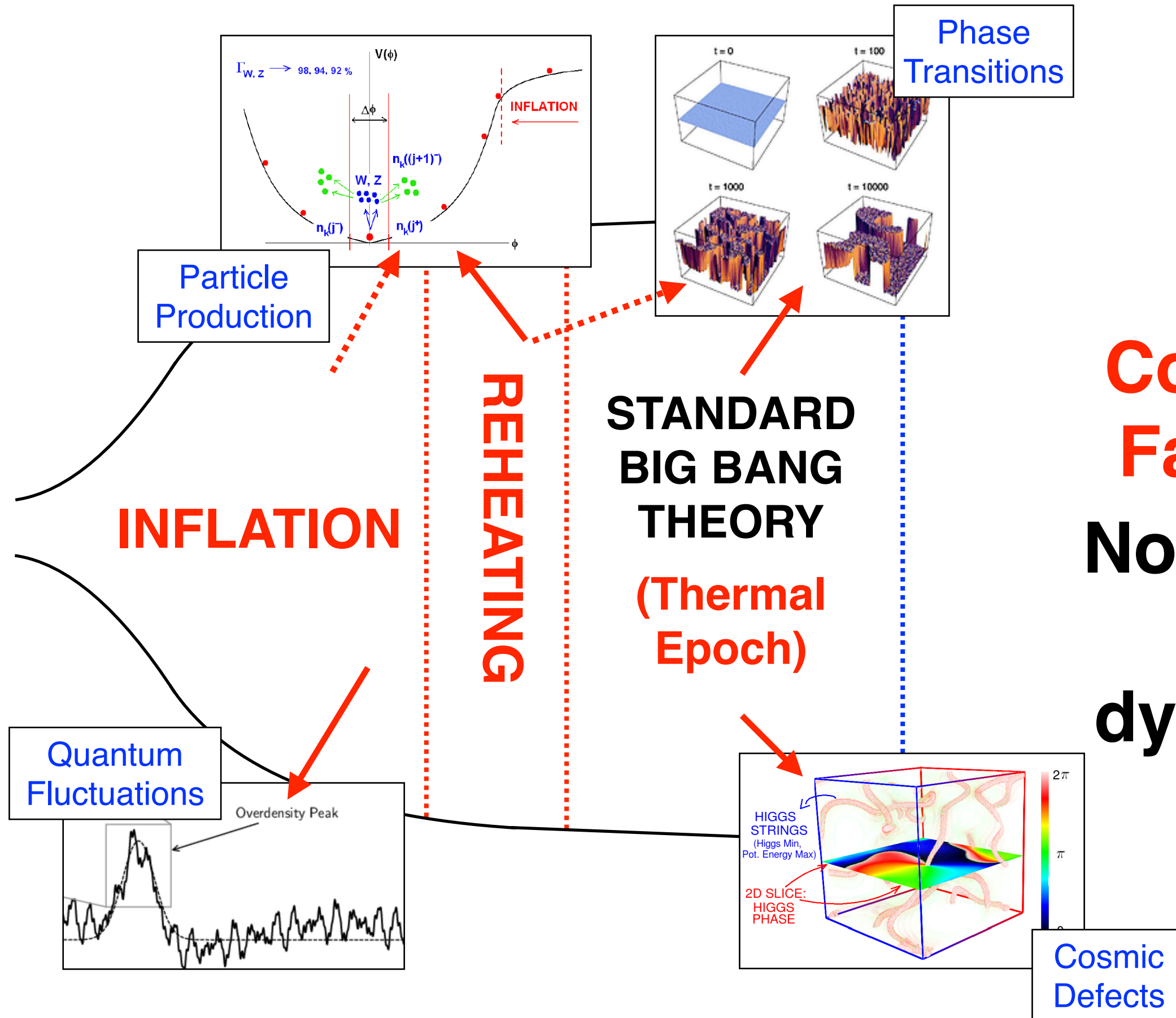


The Early Universe



**Common
Factor ?**

The Early Universe



The Early Universe

Particle
Production

Phase
Transitions

Curvature
Fluctuations

Cosmic
Defects

**Common
Factor ?**

**Non-linear
field
dynamics**

The Early Universe

Particle
Production

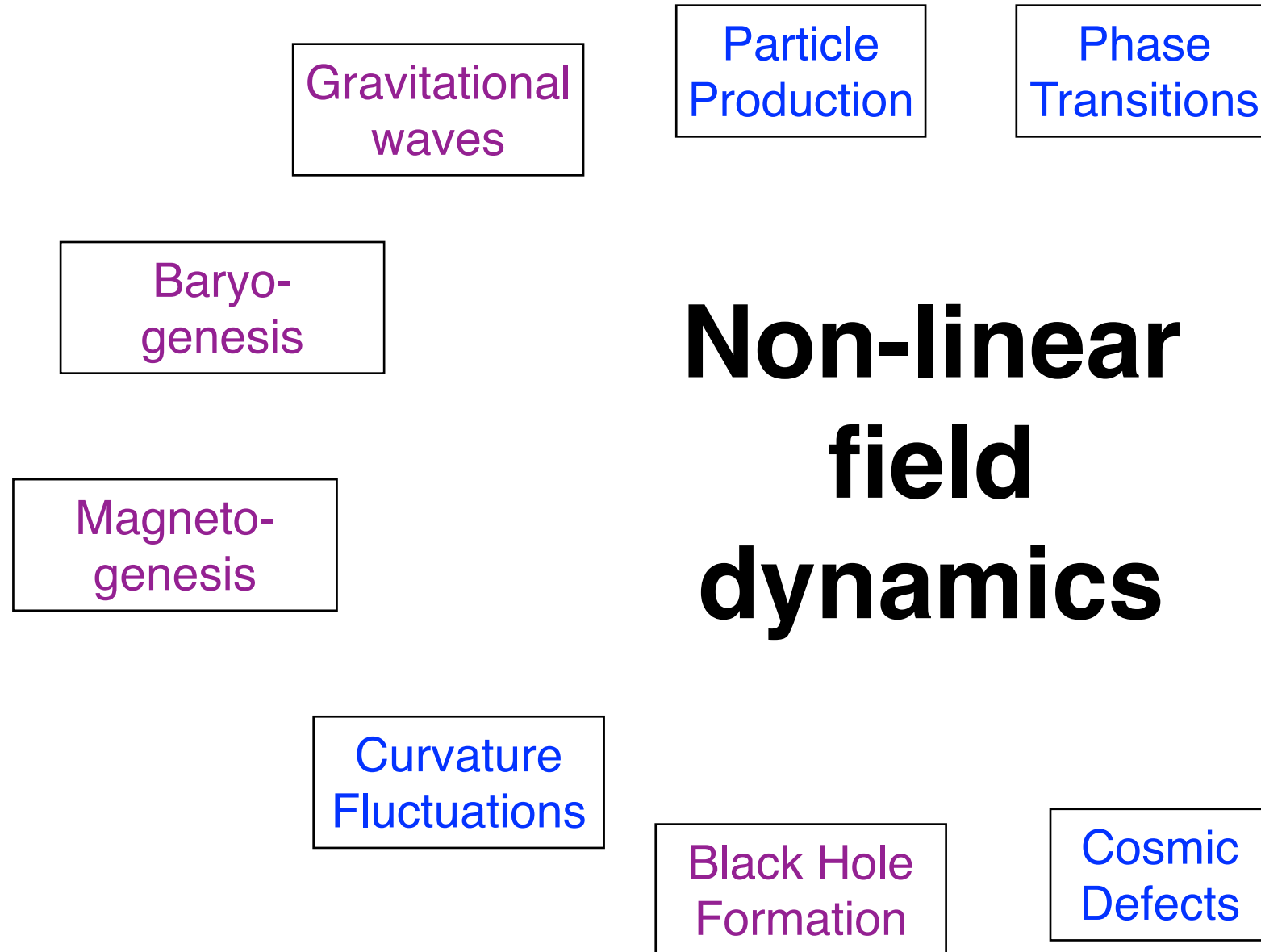
Phase
Transitions

**Non-linear
field
dynamics**

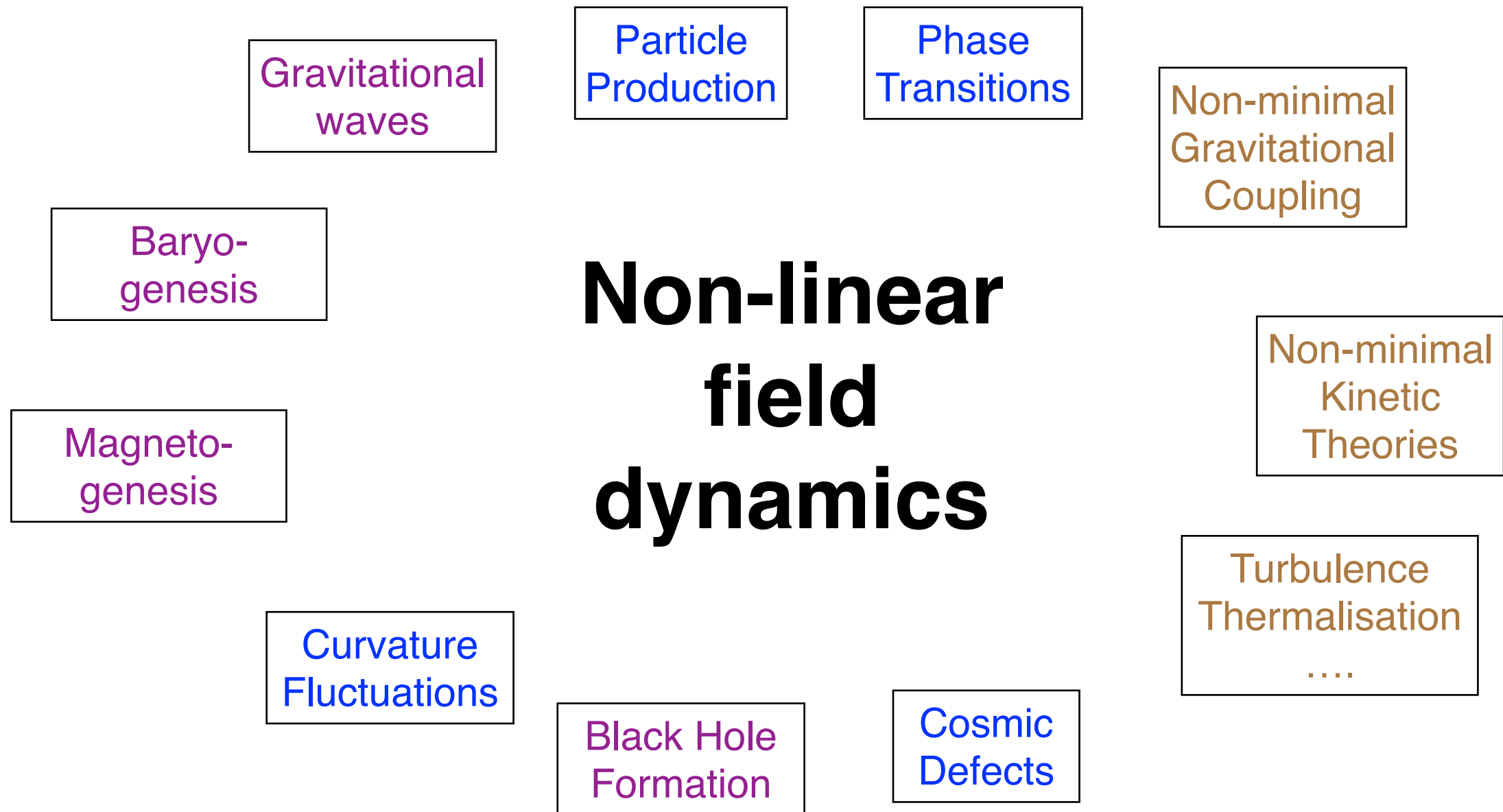
Curvature
Fluctuations

Cosmic
Defects

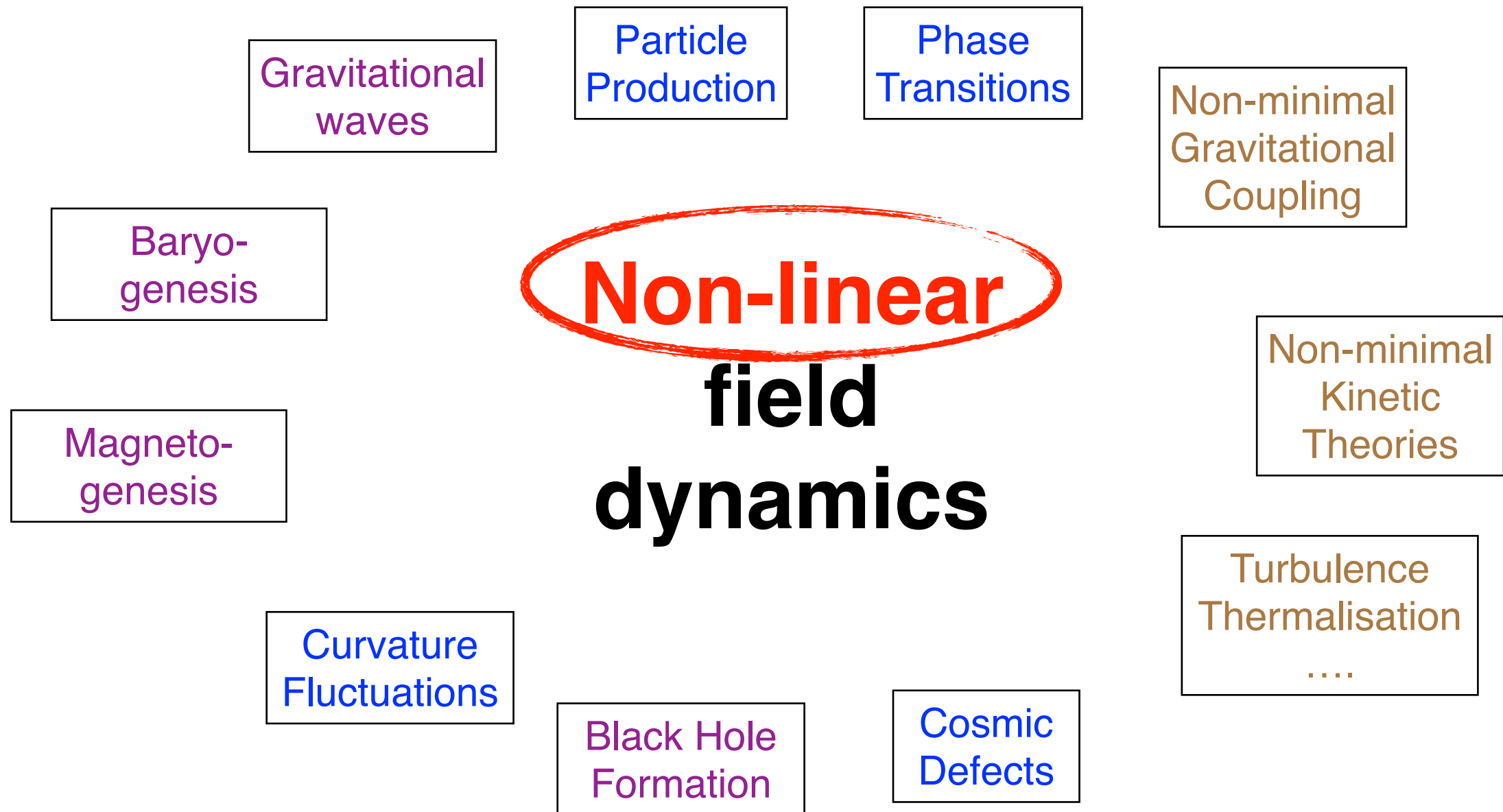
The Early Universe



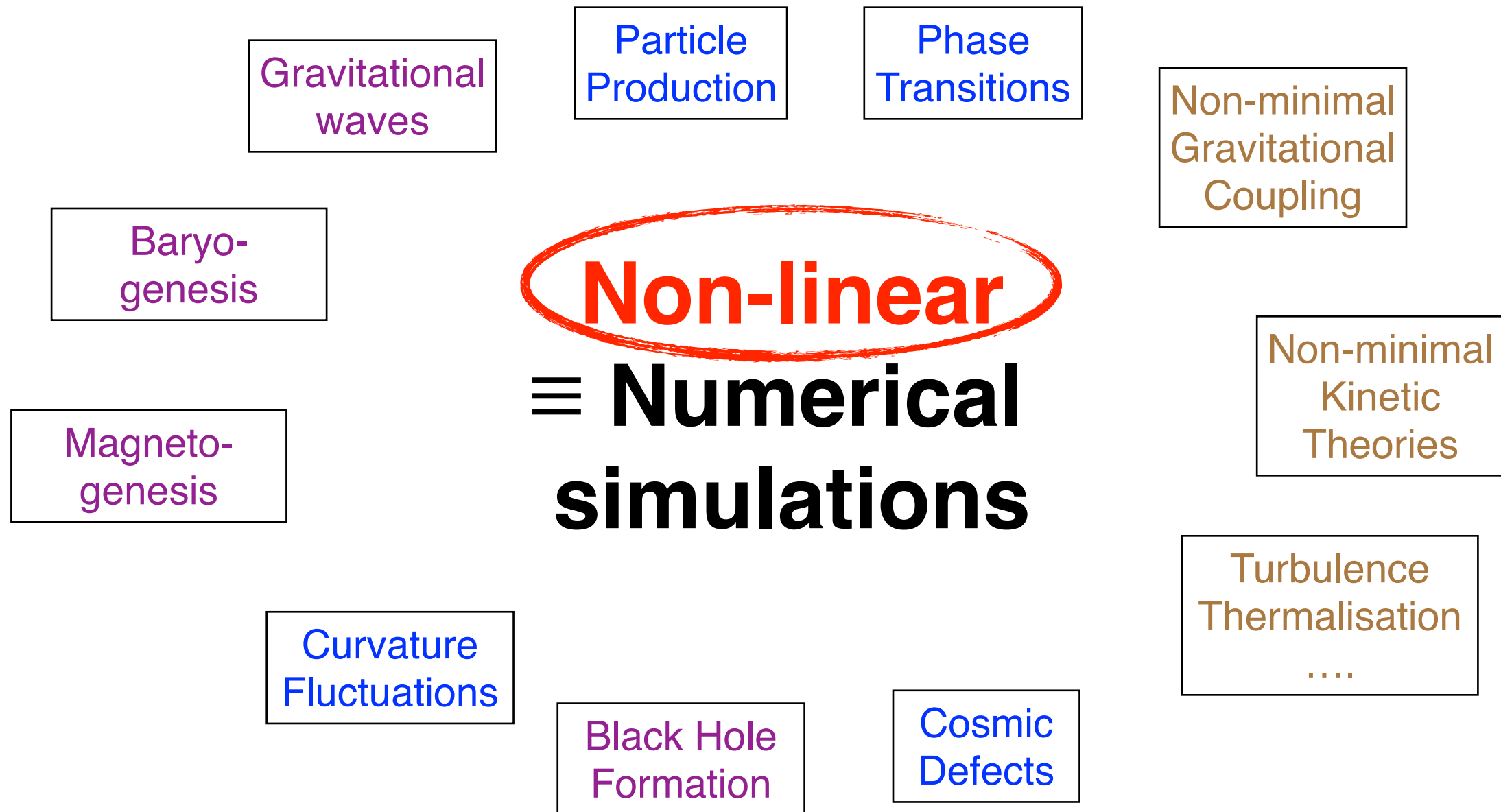
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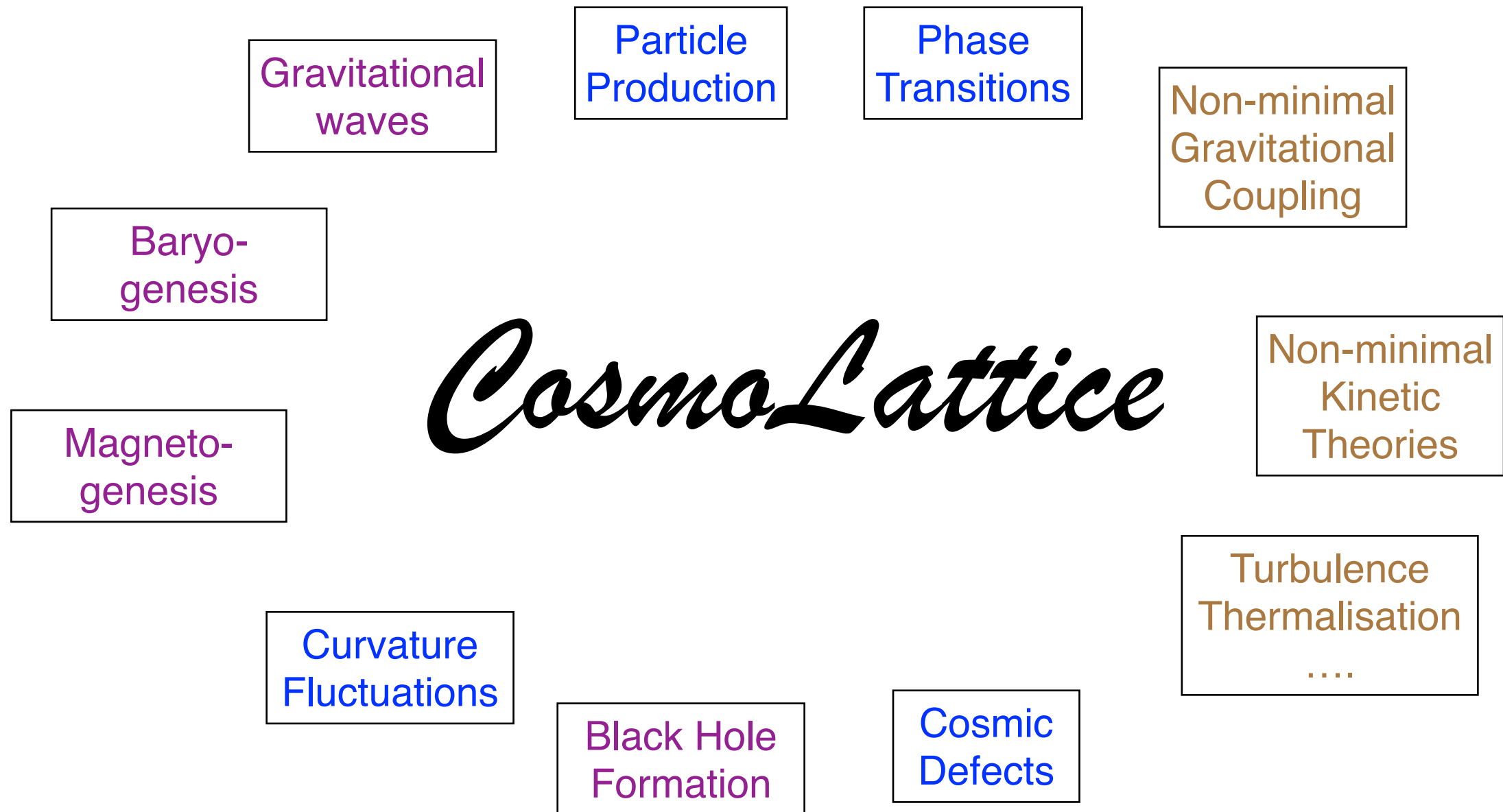
The Early Universe



The Early Universe



The Early Universe



CosmoLattice – School 2025

School aimed to provide a *pedagogical* introduction to lattice field theory techniques

+

their adaptation to simulate the *dynamics* of interacting fields in an expanding background

CosmoLattice – School 2025

We will introduce *CosmoLattice*, our public code for lattice simulations of early Universe scenarios

... so you learn how to simulate *non-linear scalar + gauge field* dynamics in an expanding universe

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... so you learn how to simulate *non-linear scalar + gauge field* dynamics in an expanding universe

+ fluid dynamics !
(*~ fermions*)

CosmoLattice – School 2025

Day 1
(Monday)



Day 2
(Tuesday)



Day 3
(Wednesday)



Day 4
(Thursday)



Day 5
(Friday)



CosmoLattice – School 2025

Day 1
(Monday)

Lecture 1: What is a Lattice?

Lecture 2: Inflation and post-inflationary dynamics

Practice 1: My first run! fun with ϕ^4

Lecture 3: Evolution algorithms

Day 2
(Tuesday)

Day 3
(Wednesday)

Day 4
(Thursday)

Day 5
(Friday)

CosmoLattice – School 2025

Day 1
(Monday)

Lecture 1: What is a Lattice? (1h15')

Lecture 2: Inflation and post-inflationary dynamics (1h15')

Practice 1: My first run! fun with ϕ^4 (1h15')

Lecture 3: Evolution algorithms (1h15')

Day 2
(Tuesday)

Day 3
(Wednesday)

Day 4
(Thursday)

Day 5
(Friday)

CosmoLattice – School 2025

Day 1
(Monday)

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Practice 1: My first run! fun with ϕ^4 (1h15')

Lecture 3: Evolution algorithms (1h15')

Morning

Afternoon

Day 2
(Tuesday)

Day 3
(Wednesday)

Day 4
(Thursday)

Day 5
(Friday)

CosmoLattice – School 2025

Day 1
(Monday)

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Lecture 3: Evolution algorithms (1h15')

Morning

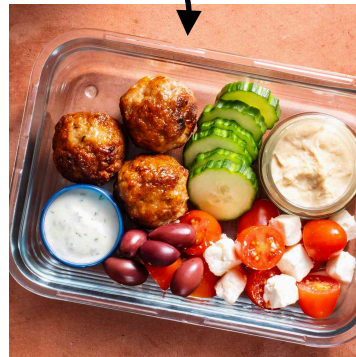
Afternoon

Day 2
(Tuesday)

Day 3
(Wednesday)

Day 4
(Thursday)

Day 5
(Friday)



(1h30')

CosmoLattice – School 2025



(30')

Day 1
(Monday)

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Practice 1: My first run! fun with ϕ^4 (1h15')

Lecture 3: Evolution algorithms (1h15')

Day 2
(Tuesday)



(30')

Day 3
(Wednesday)

Day 4
(Thursday)

Day 5
(Friday)

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Day 5
(Friday)

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(Tuesday)

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Practice 2: Playing with scalar fields (choose your potential!)
Lecture 5: Non-minimally coupled scalar fields
Lecture 6: U(1) & SU(2) Lattice Gauge Theories

Day 3
(Wednesday)

Day 4
(Thursday)

Day 5
(Friday)

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Day 3
(Wednesday)

Lecture 7: Axion coupling
Practice 3: Beyond scalars... simulating gauge flds !
Lecture 8: Gravitational wave (GW) creation and propagation
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Day 4
(Thursday)

Day 5
(Friday)

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Day 4
(Thursday)

Lecture 9: (Relativistic) fluid dynamics (MHD, ...)
Practice 5: Playing with fluid dynamics
Lecture 10: CL technical features (plotting, parallelization, ...)
Practice 6: GWs from hydrodynamic turbulence

Day 5
(Friday)

CosmoLattice – School 2025

Day 1 (Monday)	- Lecture 1: What is a Lattice? - Lecture 2: Inflation and post-inflationary dynamics - Practice 1: My first run! fun with ϕ^4 - Lecture 3: Evolution algorithms
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Day 5 (Friday)	- Lecture 11: Simulating Cosmic Defects - Lecture 12: Advanced CL features (GPU, ...) - Final Overview / Discussion

Basic

Advanced

CosmoLattice – School 2025

Day 1
(Monday)

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- Practice 1: My first run! fun with ϕ^4
- Lecture 3: Evolution algorithms

Day 2
(Tuesday)

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- Lecture 12: Advanced CL features (GPU, ...)
- Final Overview / Discussion

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CosmoLattice – School 2025

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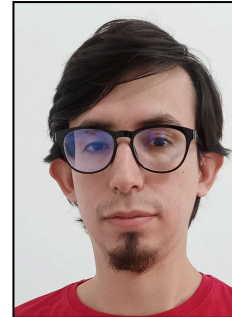
CosmoLattice – School 2025



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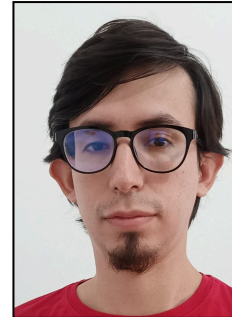


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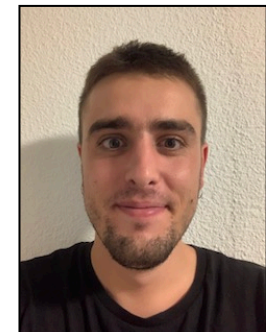
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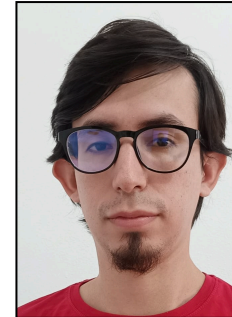
Faculty



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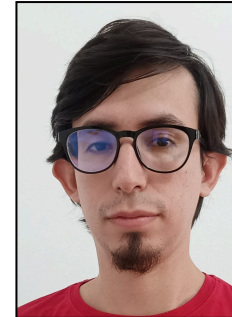
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Senior Postdocs

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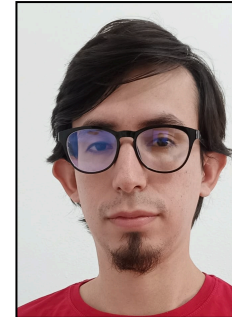
Jr. Postdocs



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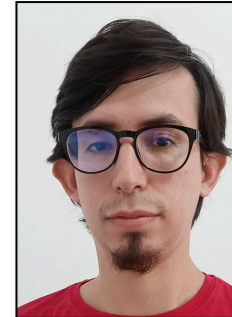
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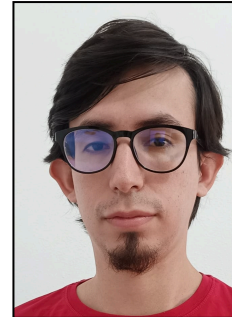
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PhD → Jr. Postdoc



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Franz SATTLER
(Bielefeld Univ., Germany)



Francisco TORRENTI
(UB, Barcelona, Spain)



Ander URIO
(UPV/EHU, Bilbao, Spain)

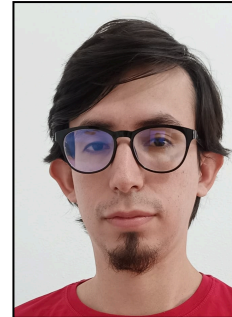
CosmoLattice – School 2025



Jorge BAEZA-BALLESTEROS
(DESY, Zeuthen, Germany)



Daniel FIGUEROA
(IFIC, Valencia, Spain)



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CosmoLattice – School 2025

Day 1
(Monday)

- Lecture 1: What is a Lattice?
- Lecture 2: Inflation and post-inflationary dynamics
- Practice 1: My first run! fun with ϕ^4
- Lecture 3: Evolution algorithms

Day 2
(Tuesday)

- Lecture 4: Interacting scalar fields in an expanding background
- Practice 2: Playing with scalar fields (choose your potential!)
- Lecture 5: Non-minimally coupled scalar fields
- Lecture 6: U(1) & SU(2) Lattice Gauge Theories

Day 3
(Wednesday)

- Lecture 7: Axion coupling
- Practice 3: Beyond scalars... simulating gauge flds !
- Lecture 8: Gravitational wave (GW) creation and propagation
- Practice 4: GWs from scalars and gauge flds !

Day 4
(Thursday)

- Lecture 9: (Relativistic) fluid dynamics (MHD, ...)
- Practice 5: Playing with fluid dynamics
- Lecture 10: CL technical features (plotting, parallelization, ...)
- Practice 6: GWs from hydrodynamic turbulence

Day 5
(Friday)

- Lecture 11: Simulating Cosmic Defects
- Lecture 12: Advanced CL features (GPU, ...)
- Final Overview / Discussion

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Lecture 3: Evolution algorithms

CosmoLattice – School 2025

Day 1
(Monday)

Lecture 1: What is a Lattice? - Francisco

Lecture 2: Inflation and post-inflationary dynamics - Francisco

Practice 1: My first run! fun with ϕ^4 - Nico, Ken, Franz

Lecture 3: Evolution algorithms - Ander, Antonino

CosmoLattice – School 2025

Day 1
(Monday)

Intro - Dani

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– Intro –

- * **Intro.a: Motivation / School / Speakers** ✓
- * **Intro.b: Overview of CosmoLattice**

CosmoLattice – School 2025

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*** Intro.b: Overview of CosmoLattice**

CosmoLattice – School 2025

Overview of CosmoLattice (CL)

CosmoLattice

Figuerola, Florio, Torrenti, Valkenburg

CasinoLattice

(Industry)



Figueroa, Florio, Torrenti, Valkenburg

CosmoLattice

Figuerola, Florio, Torrenti, Valkenburg

CosmoLattice

Figueroa, Florio, Torrenti, Valkenburg



Daniel FIGUEROA
(IFIC, Valencia, Spain)



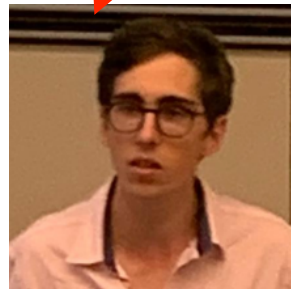
Francisco TORRENTI
(Barcelone U, Spain)

CosmoLattice

Figueroa, Florio, Torrenti, Valkenburg



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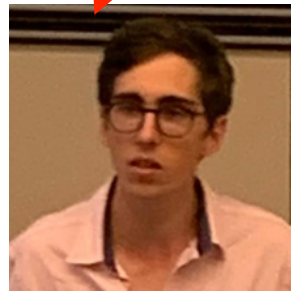
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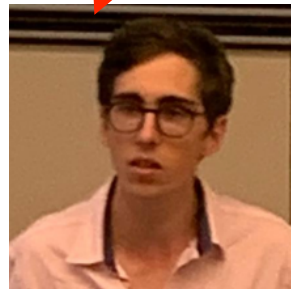
CosmoLattice Original Creators

CosmoLattice

Figueroa, Florio, Torrenti, Valkenburg



Daniel FIGUEROA
(IFIC, Valencia, Spain)



Adrien FLORIO
(Bielefeld, Germany)



Francisco TORRENTI
(Barcelone U, Spain)

CosmoLattice Original Creators

Could not make it
to the school...
but says Hello !

CosmoLattice

Figuerola, Florio, Torrenti, Valkenburg

Lattice Theory Review: [arXiv: 2006.15122](#) (+100 pages)

Code Manual: [arXiv: 2102.01031](#) (+100 pages)

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<http://www.cosmolattice.net/>

CosmoLattice

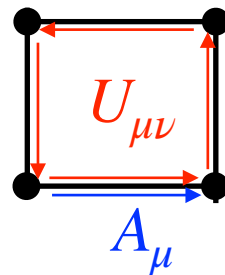
Figuerola, Florio, Torrenti, Valkenburg

Lattice Theory Review: [arXiv: 2006.15122](#)

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- Simulates **scalar-gauge** field dynamics [including **U(1)** & **SU(2)** interactions]
[$U(1) \times SU(2)$]

Links & plaquettes
(~ **lattice-QCD**)



CosmoLattice

Figuerola, Florio, Torrenti, Valkenburg

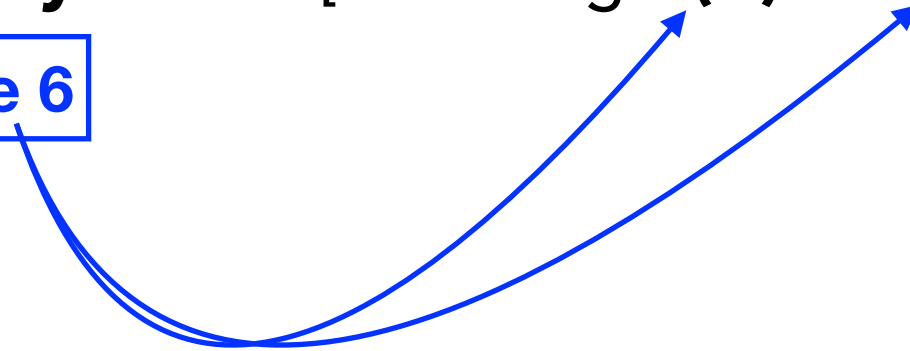
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- Simulates **scalar** and **gauge field dynamics** [including **U(1)** & **SU(2)** interactions]

Lecture 4

Lecture 6



CosmoLattice

Figueroa, Florio, Torrenti, Valkenburg

Lattice Theory Review: [arXiv: 2006.15122](#)

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- Simulates **scalar** and **gauge field dynamics** [including **U(1)** & **SU(2)** interactions]

Lecture 4

Lecture 6

- Nowadays **Gravitational Waves**

Lecture 8

CosmoLattice

Figuerola, Florio, Torrenti, Valkenburg

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- Written in **C++**, with **modular structure** separating physics ([CosmoInterface](#) library) and technical details ([TempLat](#) library).

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**Practice
Sessions**

CosmoLattice

Figuerola, Florio, Torrenti, Valkenburg

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- **Parallelized** in multiple spatial dimensions (**but you write in serial**)

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Lectures 10,12

CosmoLattice

Figuerola, Florio, Torrenti, Valkenburg

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- **Family** of evolution **algorithms**, accuracy ranging from $\delta\mathcal{O}(\delta t^2) - \delta\mathcal{O}(\delta t^{10})$

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Figuerola, Florio, Torrenti, Valkenburg

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[Lecture 3](#) → [Lectures 4, 5, 6, 7, 8, 9](#)

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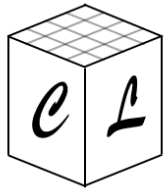
EVENTS ▼

PUBLICATIONS



CosmoLattice

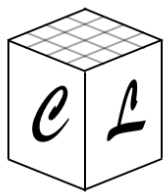
A modern code for lattice simulations of scalar and gauge
field dynamics in an expanding universe



What Field theory ?

► Matter content:

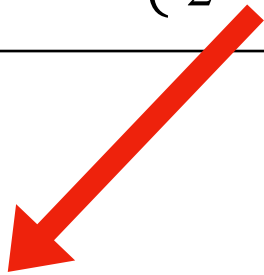
$$S = - \int d^4x \sqrt{-g} \left\{ \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + (D_\mu^A \varphi)^* (D_A^\mu \varphi) + \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + (D_\mu \Phi)^\dagger (D^\mu \Phi) + \frac{1}{2} \text{Tr}\{G_{\mu\nu} G^{\mu\nu}\} + V(\phi, |\varphi|, |\Phi|) \right\}$$



What Field theory ?

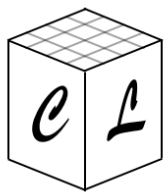
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$$\phi \in \mathcal{R}e$$

Scalar
sector



What Field theory ?

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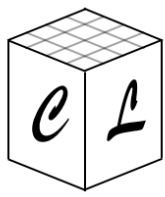
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$$\phi \in \mathcal{R}e$$

**Scalar
sector**

$$\begin{aligned} \varphi &\equiv \frac{1}{\sqrt{2}}(\varphi_0 + i\varphi_1) \\ D_\mu^A &\equiv \partial_\mu - iQ_A^{(\varphi)} g_A A_\mu \\ F_{\mu\nu} &\equiv \partial_\mu A_\nu - \partial_\nu A_\mu \end{aligned}$$

U(1) gauge sector



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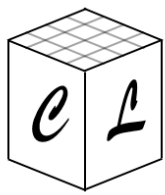
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SU(2) gauge sector



What Field theory ?

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$$\phi \in \mathcal{R}e$$

Scalar
sector

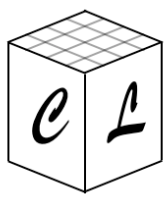
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SU(2) gauge sector

Scalar
potential



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U(1) gauge sector

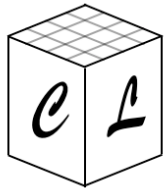
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SU(2) gauge sector

Scalar
potential

► Background Metric:

$$ds^2 = -dt^2 + a^2(t) \delta_{ij} dx^i dx^j \left\{ \begin{array}{l} \text{► Self-consistent expansion (Friedmann equations)} \\ \text{► Fixed power-law background } a(t) \sim t^{\frac{2}{3(1+w)}} \end{array} \right.$$

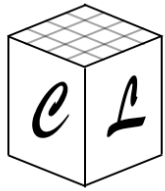


Lattice Equations

- Hamiltonian scheme: coupled first-order differential equations

- **Scalar fld example**

$$\frac{d^2\phi}{dt^2} - \frac{1}{a^2} \nabla^2 \phi + \frac{3}{a} \frac{da}{dt} \frac{d\phi}{dt} = - \frac{\partial V}{\partial \phi} \xrightarrow{\pi_\phi \equiv \phi' a^{3-\alpha}} \begin{array}{ll} \textbf{KICK:} & (\pi_\phi)' = - a^{3+\alpha} \frac{\partial V}{\partial \phi} + a^{1+\alpha} \nabla^2 \phi \\ \textbf{DRIFT:} & \phi' \equiv \pi_\phi a^{\alpha-3} \end{array}$$



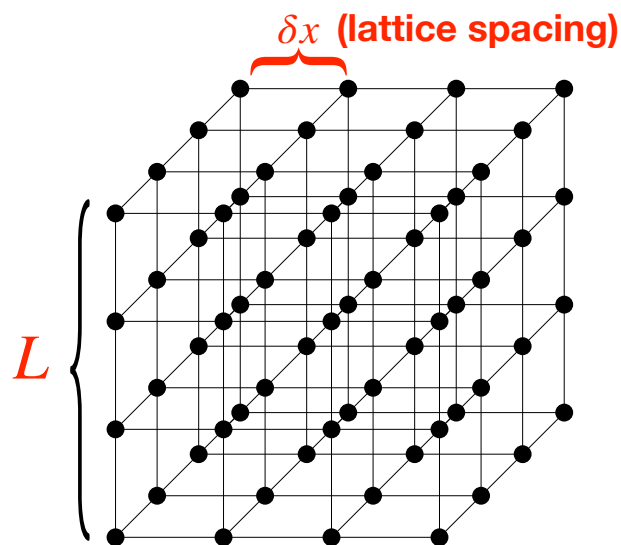
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- **Scalar Fields and momenta** are defined in the **lattice sites**



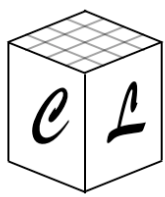
N : number of points/dimension

$L = N \cdot \delta x$: length side

δt : time step

Minimum and maximum momenta:

$$k_{\min} = \frac{2\pi}{L} \quad k_{\max} = \frac{\sqrt{3}}{2} N k_{\min}$$



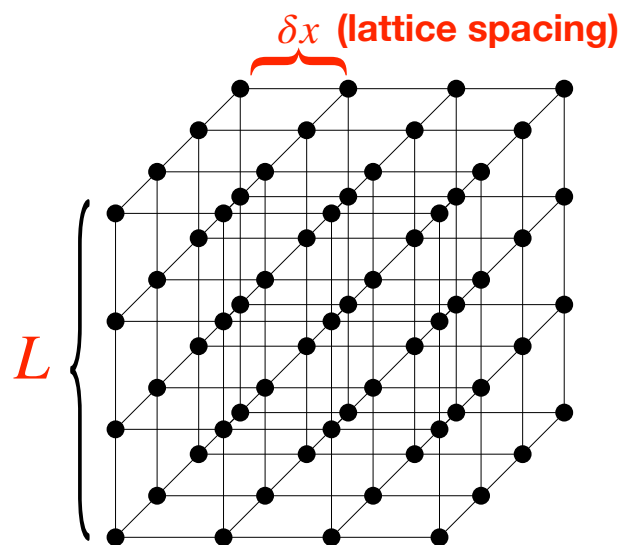
Lattice Equations

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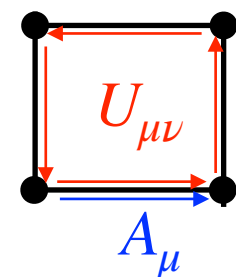
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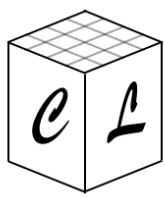
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- **Gauge fields** introduced via **links** and **plaquettes** (like in **lattice-QCD**)





Writing a model

- Equations solved in (dimensionless) **program variables**:

Choose:
 $\{\alpha, \omega_*, f_*\}$



$$\begin{aligned} d\tilde{\eta} &\equiv a^{-\alpha} \omega_* dt \\ d\tilde{x}^i &\equiv \omega_* dx^i \end{aligned}$$

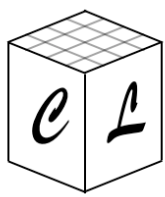
Space and time

$$\tilde{\phi} = \frac{\phi}{f_*} \quad \tilde{\varphi} = \frac{\varphi}{f_*} \quad \tilde{\Phi} = \frac{\Phi}{f_*}$$

Scalar
fields

$$\widetilde{A}_\mu = \frac{A_\mu}{\omega_*} \quad \widetilde{B}_\mu^a = \frac{B_\mu^a}{\omega_*}$$

Gauge
fields



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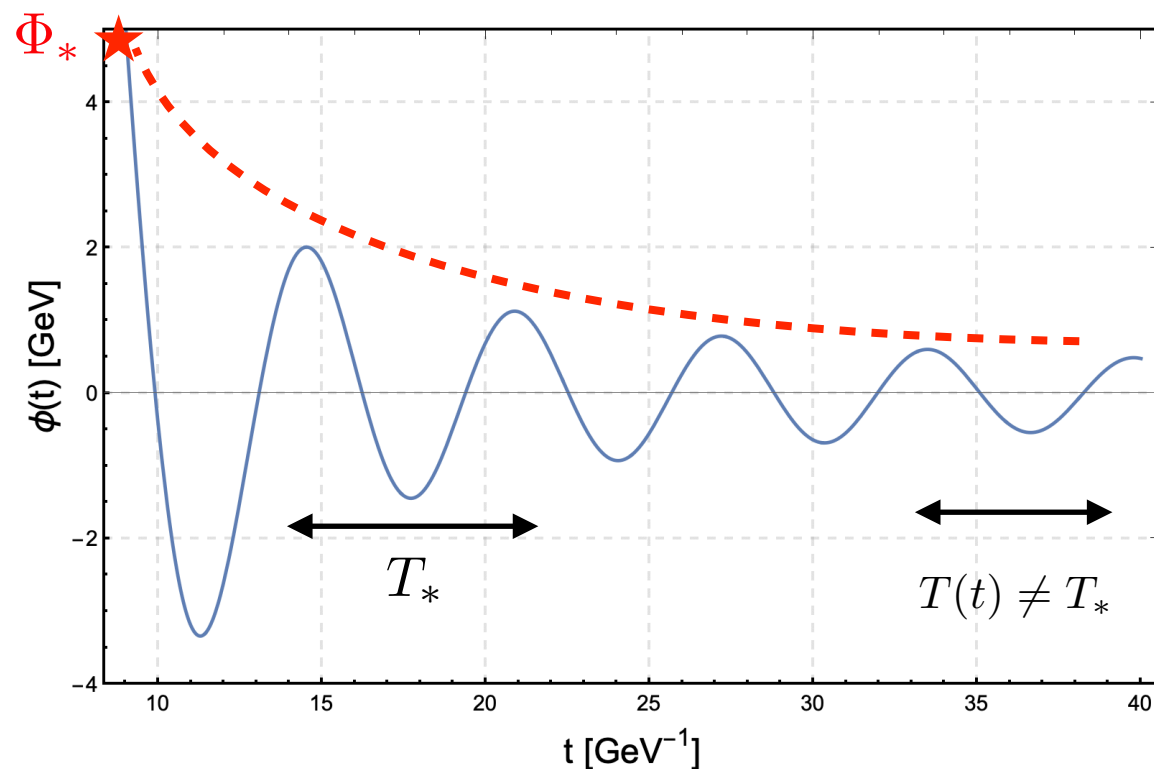
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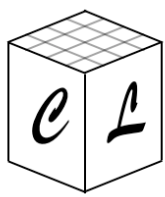
Scalar
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$$\widetilde{A}_\mu = \frac{A_\mu}{\omega_*} \quad \widetilde{B}_\mu^a = \frac{B_\mu^a}{\omega_*}$$

Gauge
fields

Example: $\phi(t) \simeq \Phi_* \times f_{\text{osc}}(t)$





Writing a model

► Equations solved in (dimensionless) **program variables**:

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Space and time

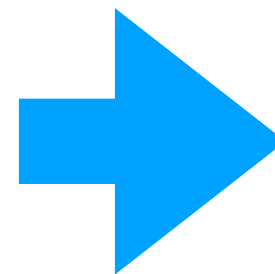
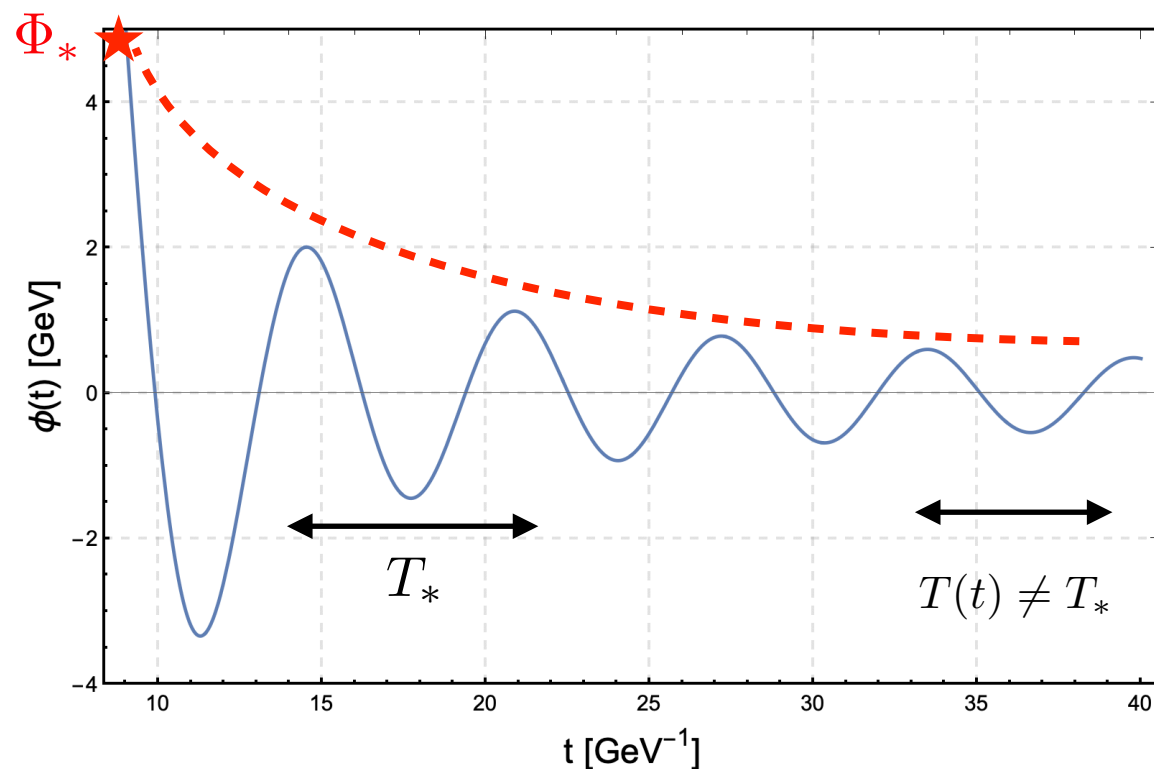
$$\tilde{\phi} = \frac{\phi}{f_*} \quad \tilde{\varphi} = \frac{\varphi}{f_*} \quad \tilde{\Phi} = \frac{\Phi}{f_*}$$

Scalar fields

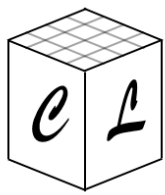
$$\widetilde{A}_\mu = \frac{A_\mu}{\omega_*} \quad \widetilde{B}_\mu^a = \frac{B_\mu^a}{\omega_*}$$

Gauge fields

Example: $\phi(t) \simeq \Phi_* \times f_{\text{osc}}(t)$



$$\left\{ \begin{aligned} f_* &= \Phi_* \\ \omega_* &= 1/T_* \\ \alpha &\longrightarrow \text{Make period constant in } \tilde{\eta} \end{aligned} \right.$$



Writing a model

- Equations solved in (dimensionless) **program variables**:

Choose:
 $\{\alpha, \omega_*, f_*\}$



$$\begin{aligned} d\tilde{\eta} &\equiv a^{-\alpha} \omega_* dt \\ d\tilde{x}^i &\equiv \omega_* dx^i \end{aligned}$$

Space and time

$$\tilde{\phi} = \frac{\phi}{f_*} \quad \tilde{\varphi} = \frac{\varphi}{f_*} \quad \tilde{\Phi} = \frac{\Phi}{f_*}$$

Scalar fields

$$\widetilde{A}_\mu = \frac{A_\mu}{\omega_*} \quad \widetilde{B}_\mu^a = \frac{B_\mu^a}{\omega_*}$$

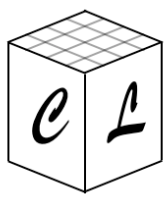
Gauge fields

- Write **scalar potential** and first and second derivatives in **one file** (*model.h*)

$$\tilde{V}(\tilde{\phi}, |\tilde{\varphi}|, |\tilde{\Phi}|) \equiv \frac{1}{f_*^2 \omega_*^2} V(f_* \tilde{\phi}, f_* |\tilde{\varphi}|, f_* |\tilde{\Phi}|)$$



$$\frac{\partial \tilde{V}}{\partial \tilde{\phi}}, \quad \frac{\partial \tilde{V}}{\partial |\tilde{\varphi}|}, \quad \frac{\partial \tilde{V}}{\partial |\tilde{\Phi}|}, \quad \frac{\partial^2 \tilde{V}}{\partial \tilde{\phi}^2}, \quad \frac{\partial^2 \tilde{V}}{\partial |\tilde{\varphi}|^2}, \quad \frac{\partial^2 \tilde{V}}{\partial |\tilde{\Phi}|^2},$$



Writing a model

- Equations solved in (dimensionless) **program variables**:

Choose:
 $\{\alpha, \omega_*, f_*\}$



$$\begin{aligned} d\tilde{\eta} &\equiv a^{-\alpha} \omega_* dt \\ d\tilde{x}^i &\equiv \omega_* dx^i \end{aligned}$$

Space and time

$$\tilde{\phi} = \frac{\phi}{f_*} \quad \tilde{\varphi} = \frac{\varphi}{f_*} \quad \tilde{\Phi} = \frac{\Phi}{f_*}$$

Scalar fields

$$\widetilde{A}_\mu = \frac{A_\mu}{\omega_*} \quad \widetilde{B}_\mu^a = \frac{B_\mu^a}{\omega_*}$$

Gauge fields

- Write **scalar potential** and first and second derivatives in **one file** (*model.h*)

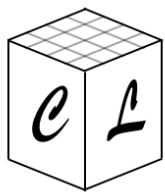
$$\tilde{V}(\tilde{\phi}, |\tilde{\varphi}|, |\tilde{\Phi}|) \equiv \frac{1}{f_*^2 \omega_*^2} V(f_* \tilde{\phi}, f_* |\tilde{\varphi}|, f_* |\tilde{\Phi}|)$$

$$\rightarrow \frac{\partial \tilde{V}}{\partial \tilde{\phi}}, \frac{\partial \tilde{V}}{\partial |\tilde{\varphi}|}, \frac{\partial \tilde{V}}{\partial |\tilde{\Phi}|}, \frac{\partial^2 \tilde{V}}{\partial \tilde{\phi}^2}, \frac{\partial^2 \tilde{V}}{\partial |\tilde{\varphi}|^2}, \frac{\partial^2 \tilde{V}}{\partial |\tilde{\Phi}|^2},$$

- **Parameters** passed via **one file** (input.txt)
(no need to re-compile !)

```

1 #Output
2 outputfile = ./
3
4 #Evolution
5 expansion = true
6 evolver = VV2
7
8 #Lattice
9 N = 32
10 dt = 0.01
11 KIR = 0.75
12 nBinsSpectra = 55
13
14 #Times
15 tOutputFreq = 0.1
16 tOutputInfreq = 1
17 tMax = 300
18
19 #IC
20 kCutOff = 1.75
21 initial_amplitudes = 7.42675e18 0 # homogeneous amplitudes in GeV
22 initial_momenta = -6.2969e30 0 # homogeneous amplitudes in GeV2
23
24 #Model Parameters
25 lambda = 9e-14
26 q = 100
  
```



Self-consistent Expansion

- Algorithms use second Friedmann equation to evolve the scale factor.

$$\frac{a''}{a} = \frac{a^{2\alpha}}{3m_p^2} \langle (\alpha - 2)(K_\phi + K_\varphi + K_\Phi) + \alpha(G_\phi + G_\varphi + G_\Phi) + (\alpha + 1)V + (\alpha - 1)(K_{U(1)} + G_{U(1)} + K_{SU(2)} + G_{SU(2)}) \rangle$$

$\langle \dots \rangle$ represents volume averaging

$$K_\phi = \frac{1}{2a^{2\alpha}} \phi'^2$$

$$K_\varphi = \frac{1}{a^{2\alpha}} (D_0^A \varphi)^* (D_0^A \varphi) ;$$

$$K_\Phi = \frac{1}{a^{2\alpha}} (D_0 \Phi)^\dagger (D_0 \Phi)$$

(Kinetic-Scalar)

$$G_\phi = \frac{1}{2a^2} \sum_i (\partial_i \phi)^2$$

$$G_\varphi = \frac{1}{a^2} \sum_i (D_i^A \varphi)^* (D_i^A \varphi) ;$$

$$G_\Phi = \frac{1}{a^2} \sum_i (D_i \Phi)^\dagger (D_i \Phi)$$

(Gradient-Scalar)

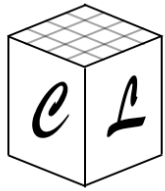
$$K_{U(1)} = \frac{1}{2a^{2+2\alpha}} \sum_i F_{0i}^2$$

$$K_{SU(2)} = \frac{1}{2a^{2+2\alpha}} \sum_{a,i} (G_{0i}^a)^2$$

$$G_{U(1)} = \frac{1}{2a^4} \sum_{i,j < i} F_{ij}^2$$

$$G_{SU(2)} = \frac{1}{2a^4} \sum_{a,i,j < i} (G_{ij}^a)^2$$

(Electric & Magnetic)



Self-consistent Expansion

- Algorithms use second Friedmann equation to **evolve the scale factor**.
- The first Friedmann equation is used to check the accuracy of the simulation.

$$\frac{a''}{a} = \frac{a^{2\alpha}}{3m_p^2} \langle (\alpha - 2)(K_\phi + K_\varphi + K_\Phi) + \alpha(G_\phi + G_\varphi + G_\Phi) + (\alpha + 1)V + (\alpha - 1)(K_{U(1)} + G_{U(1)} + K_{SU(2)} + G_{SU(2)}) \rangle$$

$$\left(\frac{a'}{a}\right)^2 = \frac{a^{2\alpha}}{3m_p^2} \langle K_\phi + K_\varphi + K_\Phi + G_\phi + G_\varphi + G_\Phi + K_{U(1)} + G_{U(1)} + K_{SU(2)} + G_{SU(2)} + V \rangle$$

$\langle \dots \rangle$ represents volume averaging

$$\begin{aligned} K_\phi &= \frac{1}{2a^{2\alpha}} \phi'^2 \\ K_\varphi &= \frac{1}{a^{2\alpha}} (D_0^A \varphi)^* (D_0^A \varphi) ; \\ K_\Phi &= \frac{1}{a^{2\alpha}} (D_0 \Phi)^\dagger (D_0 \Phi) \end{aligned}$$

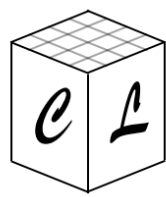
(Kinetic-Scalar)

$$\begin{aligned} G_\phi &= \frac{1}{2a^2} \sum_i (\partial_i \phi)^2 \\ G_\varphi &= \frac{1}{a^2} \sum_i (D_i^A \varphi)^* (D_i^A \varphi) ; \\ G_\Phi &= \frac{1}{a^2} \sum_i (D_i \Phi)^\dagger (D_i \Phi) \end{aligned}$$

(Gradient-Scalar)

$$\begin{aligned} K_{U(1)} &= \frac{1}{2a^{2+2\alpha}} \sum_i F_{0i}^2 \\ K_{SU(2)} &= \frac{1}{2a^{2+2\alpha}} \sum_{a,i} (G_{0i}^a)^2 \\ G_{U(1)} &= \frac{1}{2a^4} \sum_{i,j < i} F_{ij}^2 \\ G_{SU(2)} &= \frac{1}{2a^4} \sum_{a,i,j < i} (G_{ij}^a)^2 \end{aligned}$$

(Electric & Magnetic)

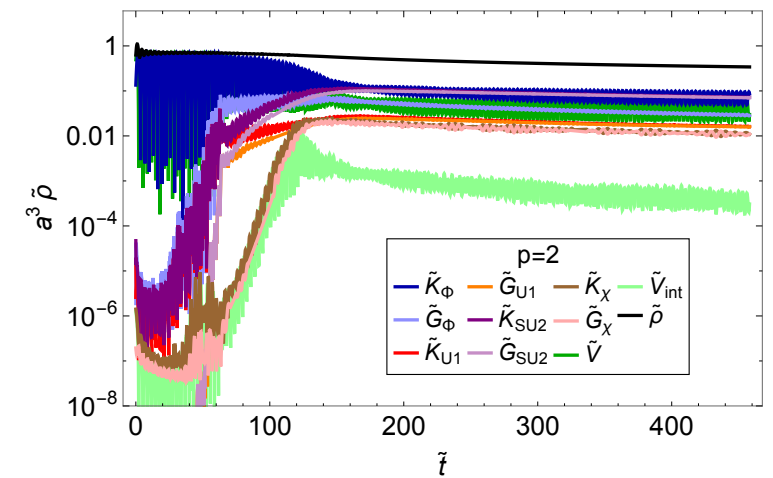
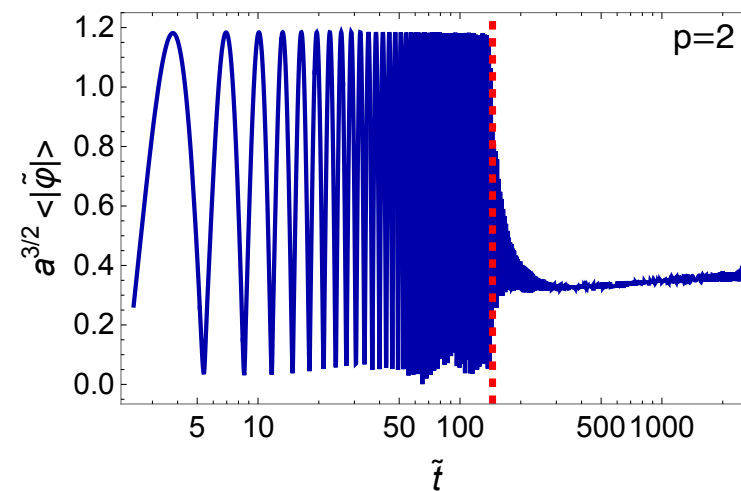


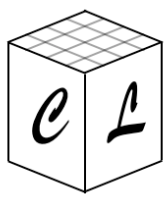
Output from your Run

**Output
Types**



Volume averages: variance, energies, etc



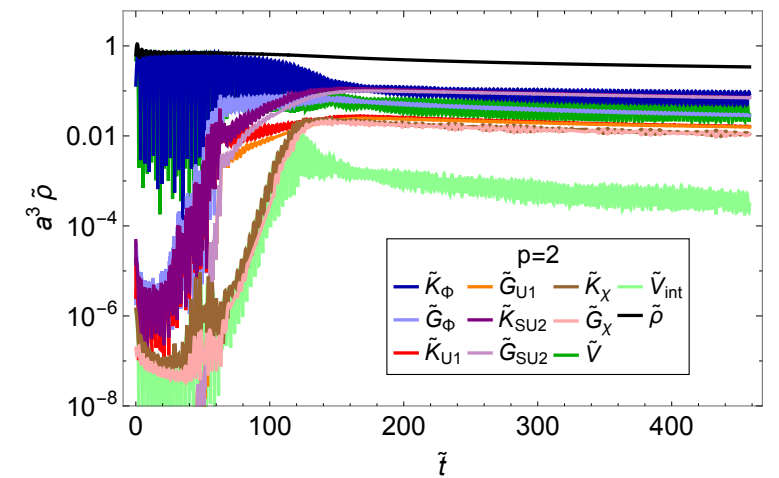
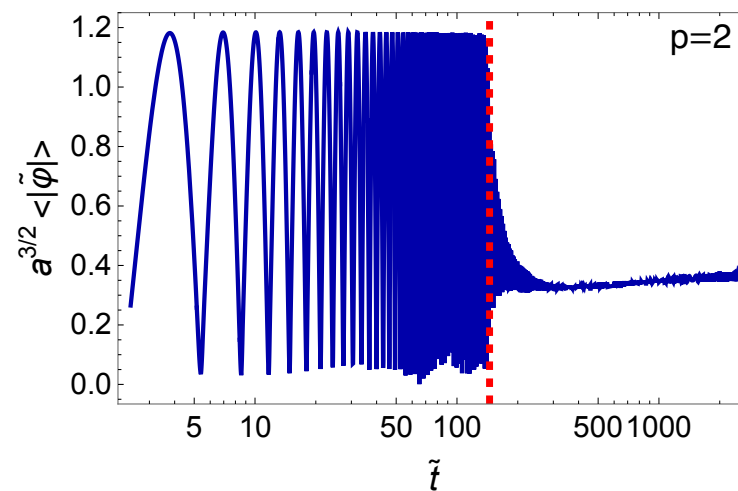


Output from your Run

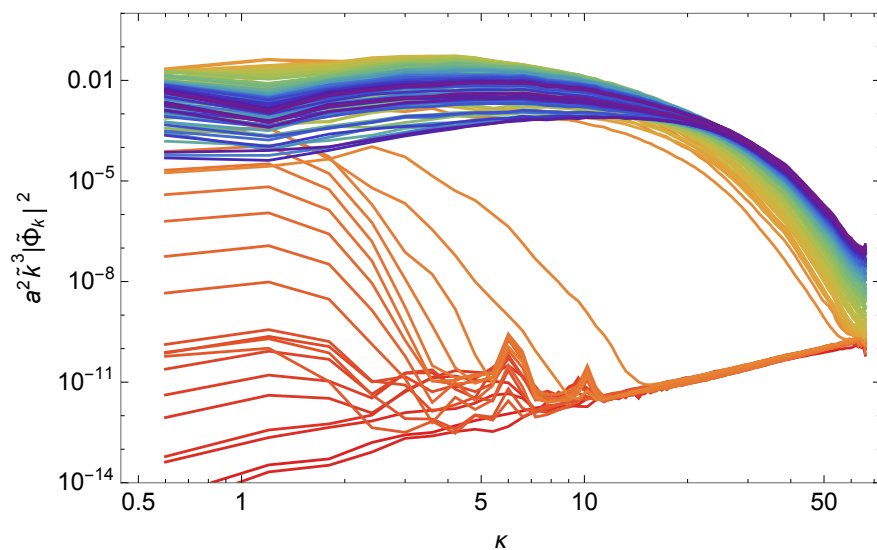
**Output
Types**

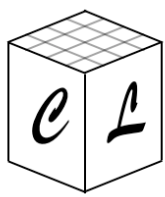


Volume averages: variance, energies, etc



Fld Spectra: Raw/Binned

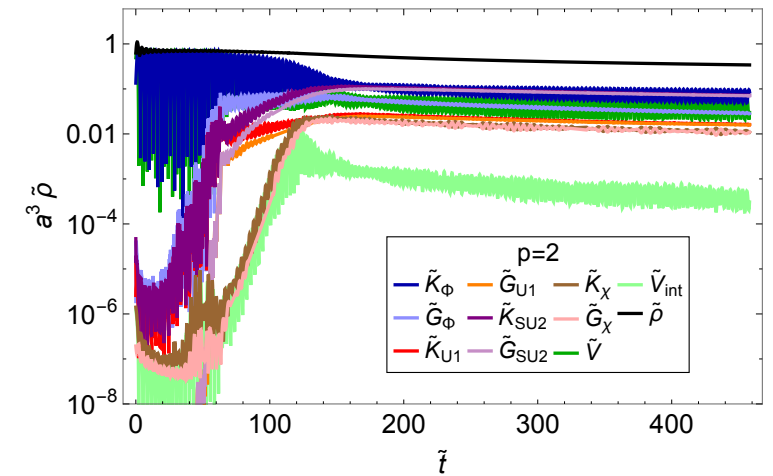
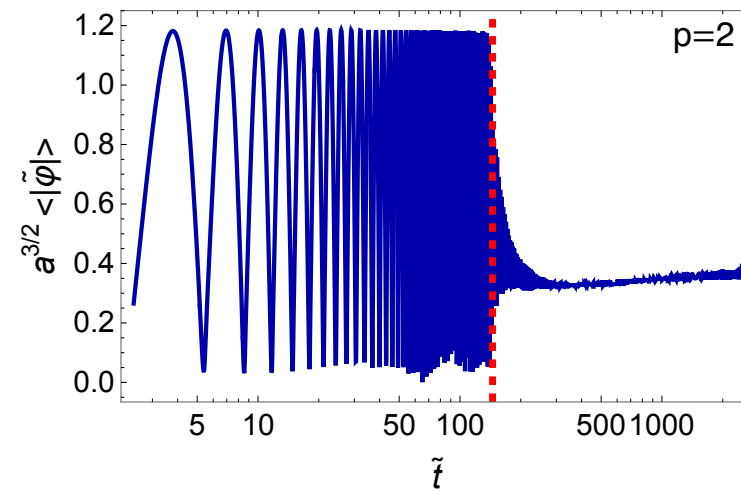




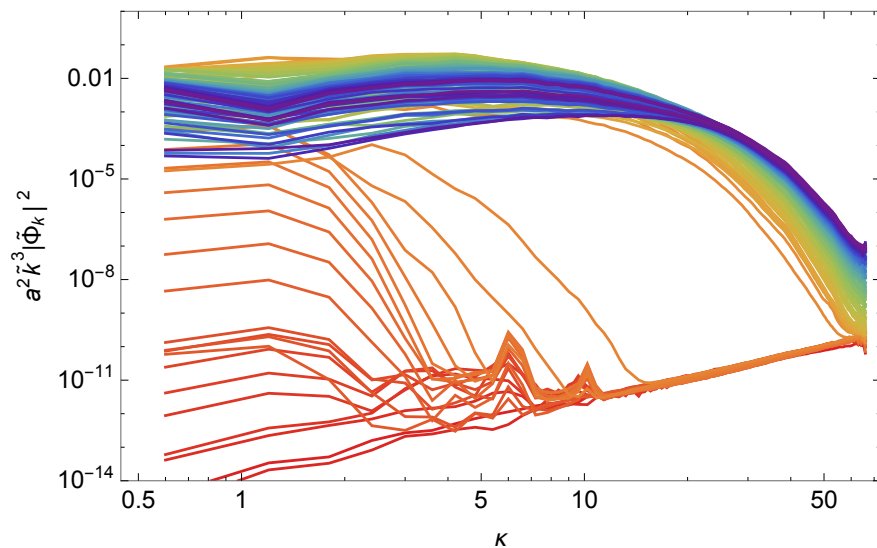
Output from your Run

**Output
Types**

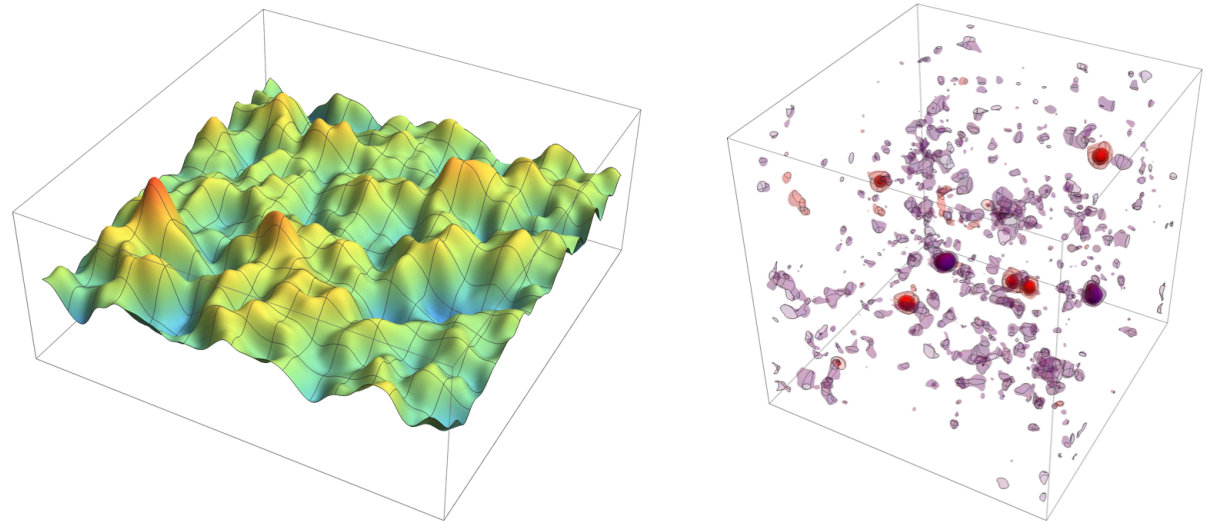
Volume averages: variance, energies, etc



Fld Spectra: Raw/Binned



Snapshots: 2D/3D distribution



CosmoLattice

Theory Review
arXiv: [2006.15122](#)

<http://www.cosmolattice.net/>

Code Manual
arXiv: [2102.01031](#)

In summary ...

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Field Th. Problem

- * Init Conditions
- * Eqs. of Motion

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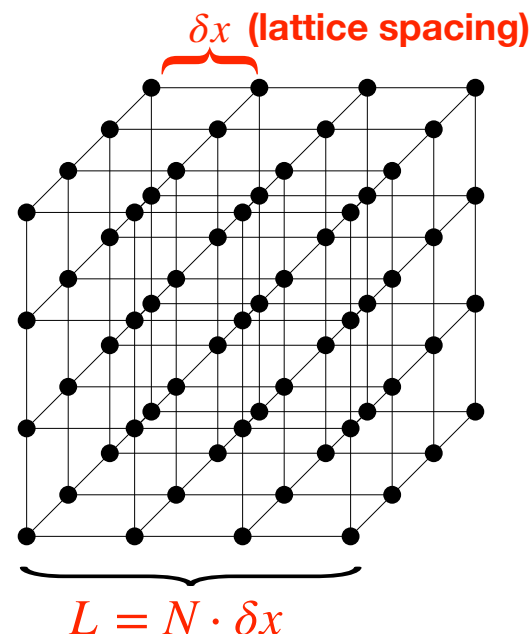
CosmoLattice

Field Th. Problem

- * Init Conditions
- * Eqs. of Motion



- * Choose Lattice: $\delta t, N, \delta x$
- * Choose Algorithm $\mathcal{O}(\delta t^n)$
- * Choose Param: $g, m, \dots$
- * Choose Observables



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Field Th. Problem

- * Init Conditions
- * Eqs. of Motion



- * Choose Lattice: Δt , N , Δx
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- * Choose Param: g , m , ...
- * Choose Observables

Algorithms

- Staggered LeapFrog (LF)
- Position-Verlet (PV2)
- Velocity-Verlet (VV2)
- Runge-Kutta (RK2, RK3, RK4)
- Yoshida (VV4, VV6, VV8, VV10)

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CosmoLattice

- * Choose Lattice: dt, N, dx
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- * Choose Param: $g, m, \dots$
- * Choose Observables

$$\lambda_1, \lambda_2, \dots, g_1^2, g_2^2, \dots$$

$$m_\phi^2, m_\psi^2, \dots, v^2, \Phi_*, \dots$$

```
1 #Output
2 outputfile = ./
3
4 #Evolution
5 expansion = true
6 evolver = VV2
7
8 #Lattice
9 N = 32
10 dt = 0.01
11 kIR = 0.75
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21 initial_amplitudes = 7.42675e18 0 # homogeneous amplitudes in GeV
22 initial_momenta = -6.2969e30 0 # homogeneous amplitudes in GeV2
23
24 #Model Parameters
25 lambda = 9e-14
26 q = 100
```

➤ **Parameters via input file**
(no need to re-compile !)

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Code Manual
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Field Th. Problem

- * Init Conditions
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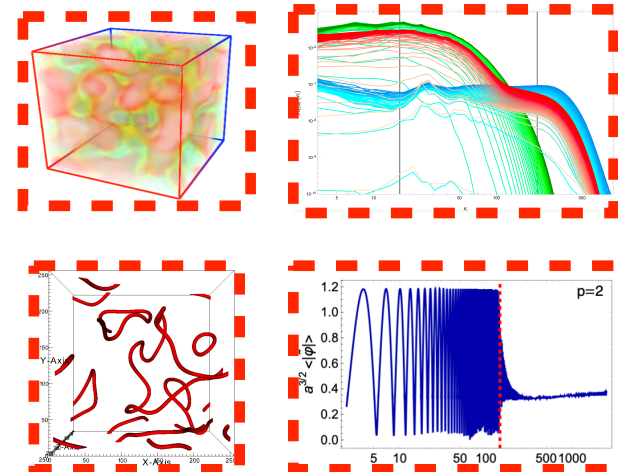


CosmoLattice

- * Choose Lattice: dt, N, dx
- * Choose Algorithm $\mathcal{O}(\delta t^n)$
- * Choose Param: $g, m, \dots$
- * Choose Observables



Output



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Field Th. Problem

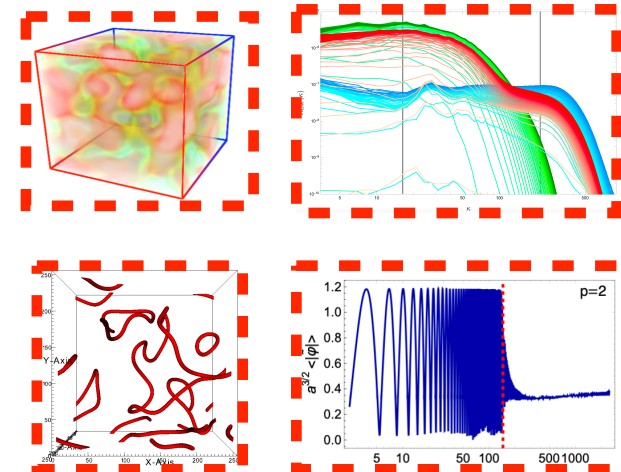
- * Init Conditions
- * Eqs. of Motion



- * Choose Lattice: dt, N, dx
- * Choose Algorithm $\mathcal{O}(\delta t^n)$
- * Choose Param: $g, m, \dots$
- * Choose Observables



Output



CL is a **platform** for field theories
You **choose the problem** to solve !

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Field Th. Problem

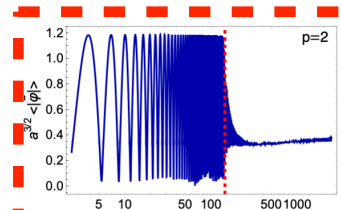
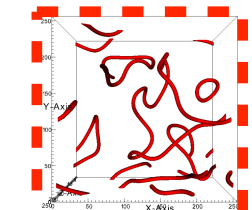
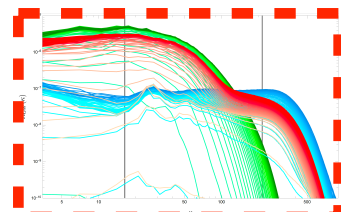
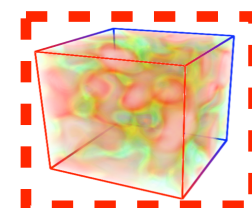
* Init Co

New Problem

(Field Objects
Field Algebra)

- * Choose Lattice: dt, N, dx
- * Choose Algorithm $\mathcal{O}(\delta t^n)$
- * Choose Param: $g, m, \dots$
- * Choose Observables

Output



CL is a **platform** for field theories
You **choose the problem** to solve !

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Code Manual
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CosmoLattice

Field Th. Problem

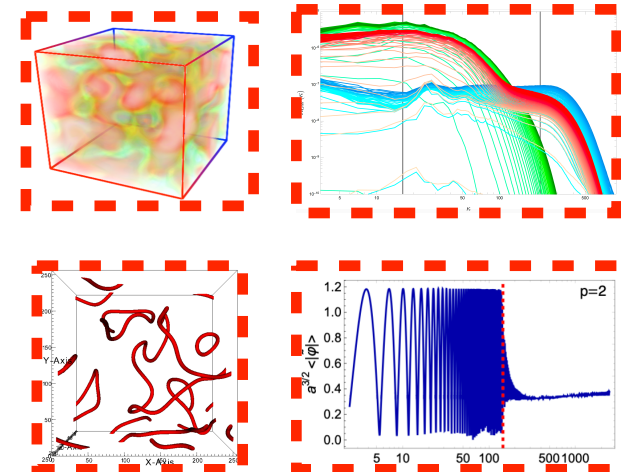
- * Init Conditions
- * Eqs. of Motion



- * Choose Lattice: dt, N, dx
- * Choose Algorithm $\mathcal{O}(\delta t^n)$
- * Choose Param: $g, m, \dots$
- * Choose Observables



Output



CL is a **platform** for field theories
You **choose the problem** to solve !

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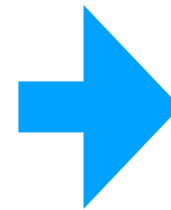
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Field Th. Problem

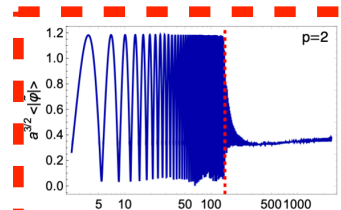
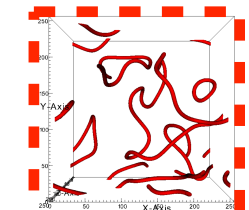
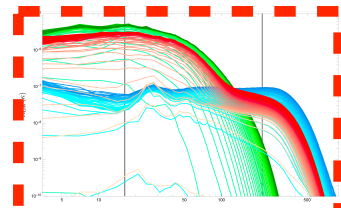
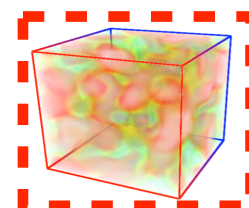
- * Init Conditions
- * Eqs. of Motion



CosmoLattice



Output



**Basic use of CL:
Black Box**

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Theory Review
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<http://www.cosmolattice.net/>

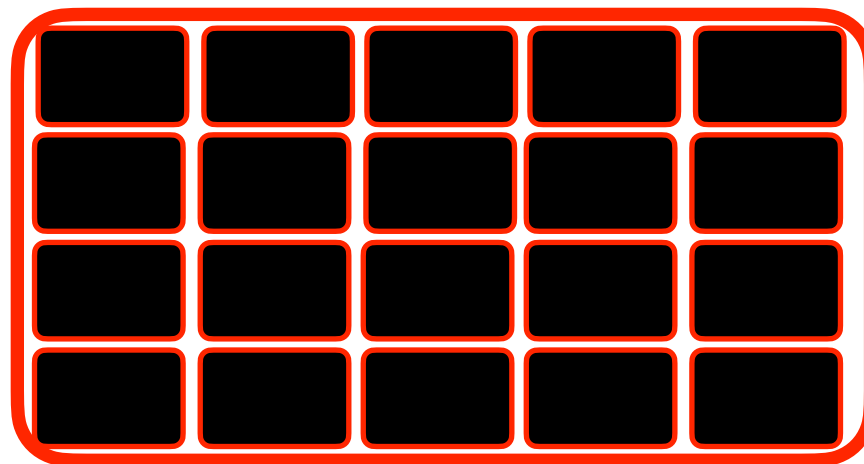
Code Manual
arXiv: [2102.01031](https://arxiv.org/abs/2102.01031)

Field Th. Problem

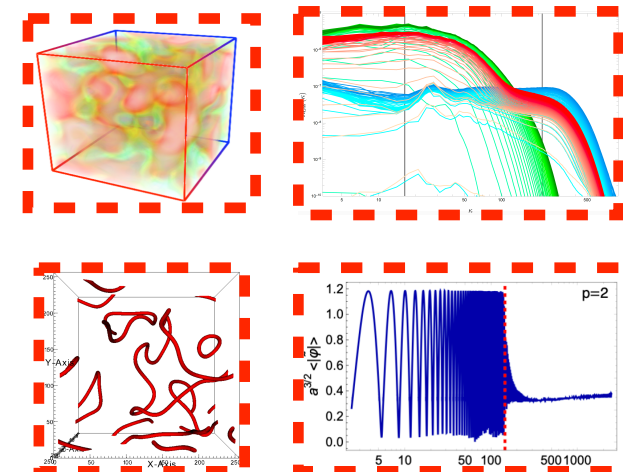
- * Init Conditions
- * Eqs. of Motion



CosmoLattice



Output



**Basic use of CL:
Set of Black Boxes**

CosmoLattice

Theory Review
arXiv: [2006.15122](https://arxiv.org/abs/2006.15122)

<http://www.cosmolattice.net/>

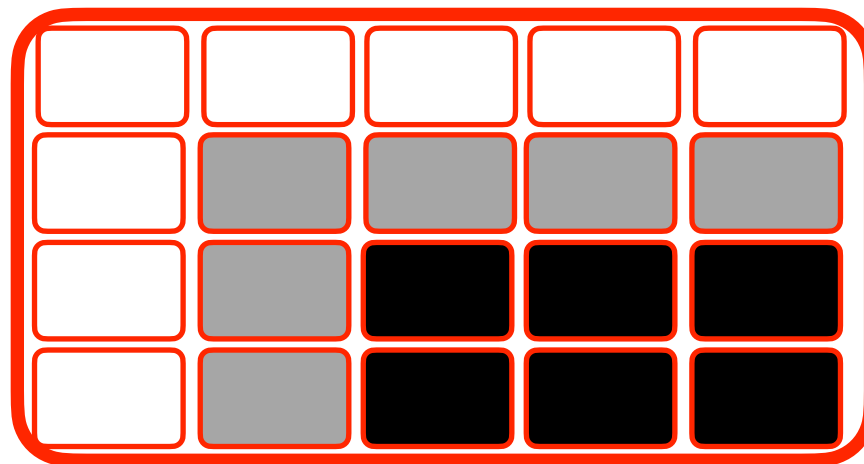
Code Manual
arXiv: [2102.01031](https://arxiv.org/abs/2102.01031)

Field Th. Problem

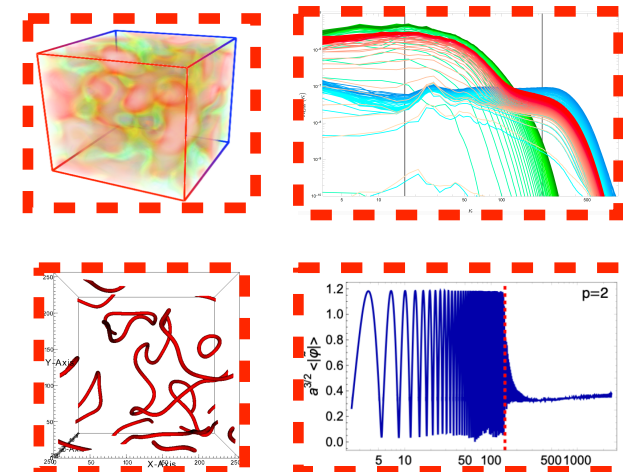
- * Init Conditions
- * Eqs. of Motion



CosmoLattice



Output



Proper use of CL:
White, Grey & Black Boxes

CosmoLattice

Theory Review
arXiv: [2006.15122](https://arxiv.org/abs/2006.15122)

<http://www.cosmolattice.net/>

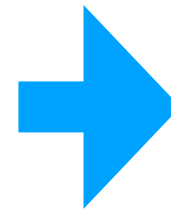
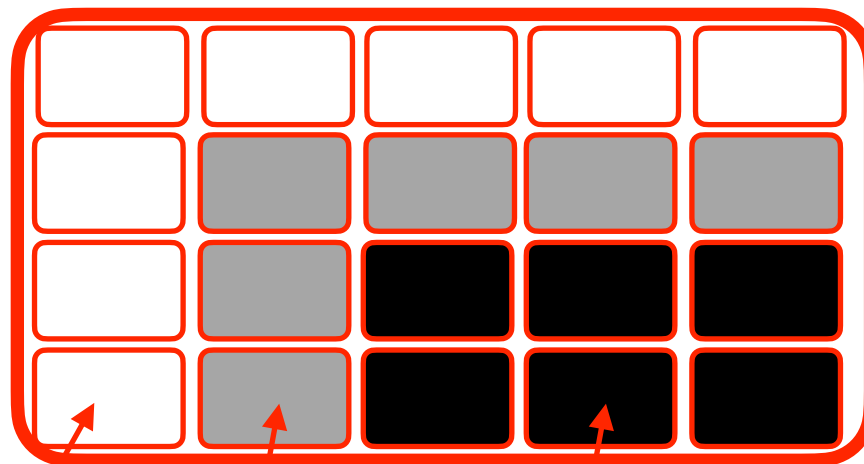
Code Manual
arXiv: [2102.01031](https://arxiv.org/abs/2102.01031)

Field Th. Problem

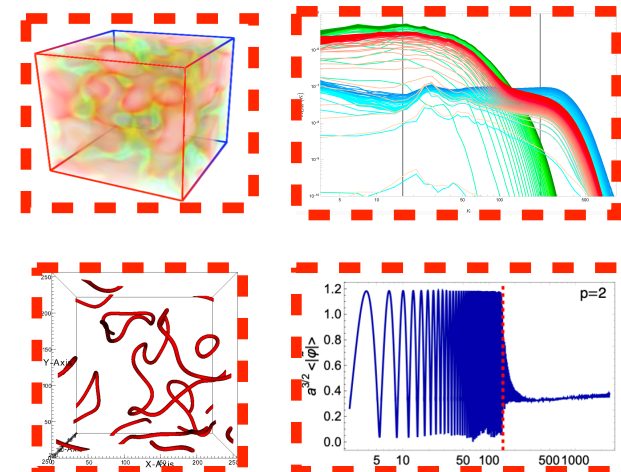
- * Init Conditions
- * Eqs. of Motion



CosmoLattice



Output



White, Grey & Black Boxes

(good
level)

(advance
level)

(expert
level)

CosmoLattice

Theory Review
arXiv: [2006.15122](https://arxiv.org/abs/2006.15122)

<http://www.cosmolattice.net/>

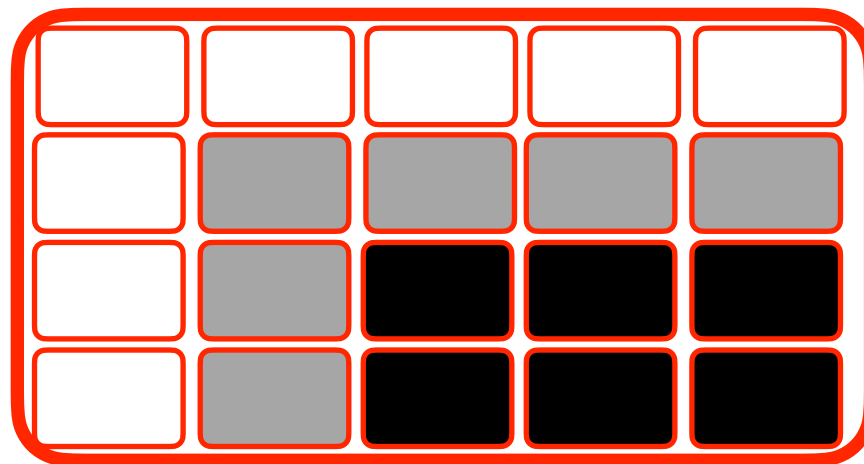
Code Manual
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Field Th. Problem

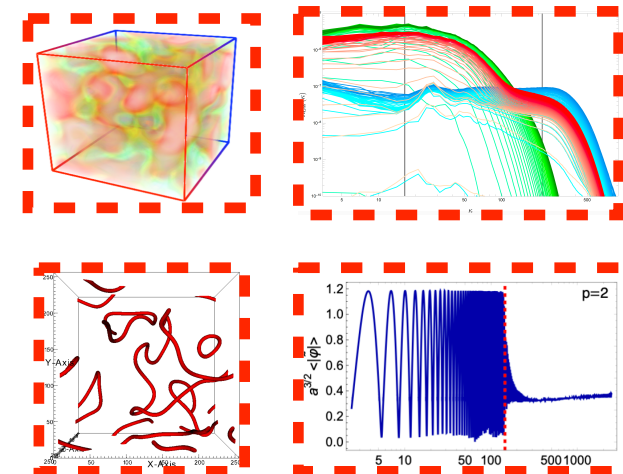
- * Init Conditions
- * Eqs. of Motion



CosmoLattice



Output



➤ CL so far (v1.0, Public):

- Global scalar field dynamics
- U(1) scalar-gauge dynamics
- SU(2) scalar-gauge dynamics

CosmoLattice

Theory Review
arXiv: [2006.15122](https://arxiv.org/abs/2006.15122)

<http://www.cosmolattice.net/>

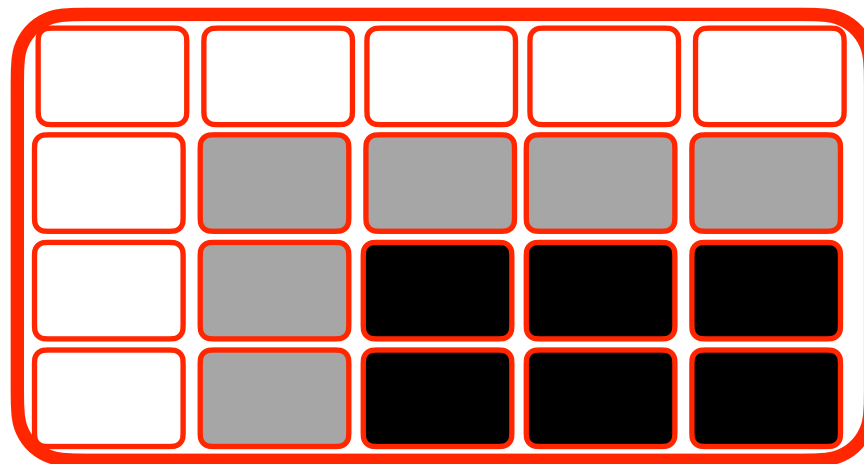
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Field Th. Problem

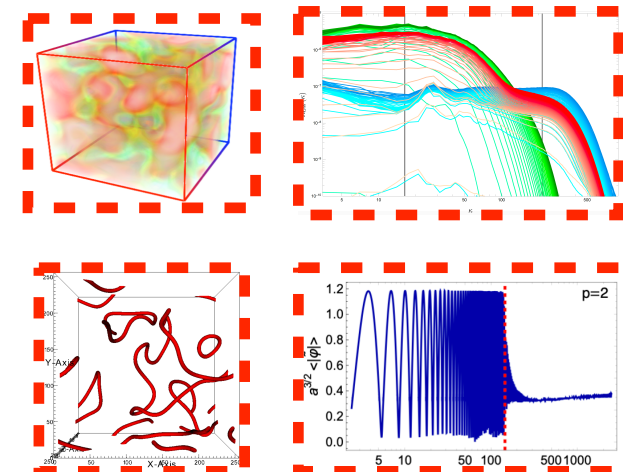
- * Init Conditions
- * Eqs. of Motion



CosmoLattice



Output



➤ CL so far (v1.x, Public):

- Global scalar field dynamics
- U(1) scalar-gauge dynamics
- SU(2) scalar-gauge dynamics
- GWs from Global/Abelian sectors

CosmoLattice

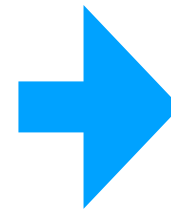
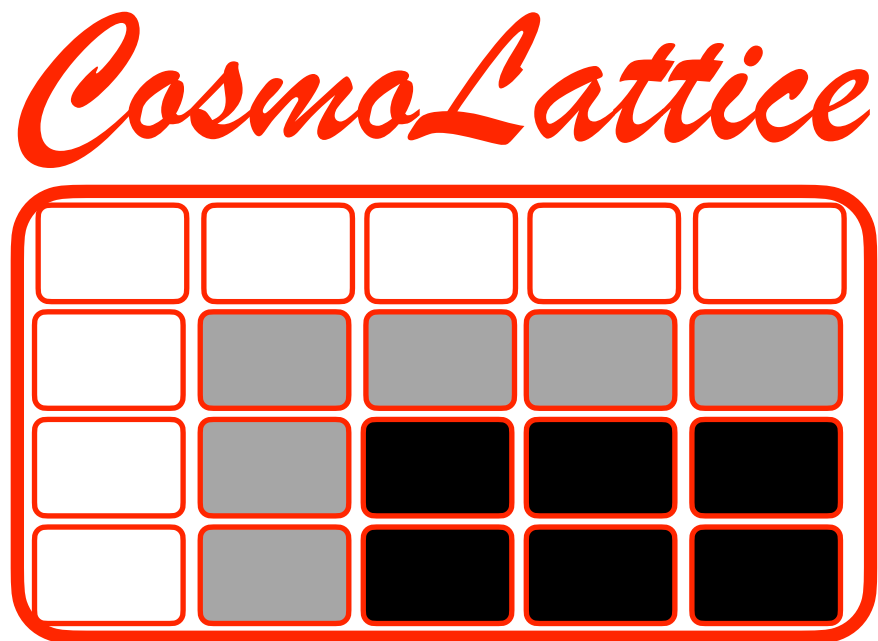
Theory Review
arXiv: [2006.15122](https://arxiv.org/abs/2006.15122)

<http://www.cosmolattice.net/>

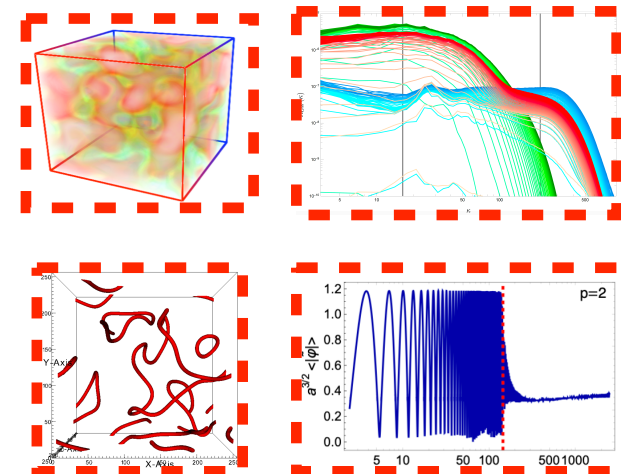
Code Manual
arXiv: [2102.01031](https://arxiv.org/abs/2102.01031)

Field Th. Problem

- * Init Conditions
- * Eqs. of Motion



Output



➤ CL so far (v1.x, Public):

- Global scalar field dynamics
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**One can add
New Modules !**
(Interactions, fields, EOM,
observables, features, ...)

New Modules

- * **Magneto Hydro-dynamics (MHD)**
- * **Axion-gauge interactions**
- * **Cosmic string networks**
- * **Non-minimal Grav. coupling**

New Modules

- * **Magneto Hydro-dynamics (MHD)**
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Magneto Hydro-dynamics (MHD)

$$T^{\mu\nu} = (p + \rho)U^\mu U^\nu - p g^{\mu\nu}$$

$$D_\nu T^{\mu\nu} = \partial_\nu T^{\mu\nu} + \Gamma_{\sigma\nu}^\mu T^{\sigma\nu} + \Gamma_{\nu\sigma}^\nu T^{\mu\sigma} = \frac{1}{\sqrt{-g}} \partial_\nu (\sqrt{-g} T^{\mu\nu}) = 0,$$

$$\begin{aligned}\partial_\eta \tilde{T}^{00} + \partial_i \tilde{T}^{0i} &= \tilde{S}^0[\phi, A_k, \{\tilde{T}_{lk}\}], \\ \partial_\eta \tilde{T}^{0i} + \partial_j \tilde{T}^{ij} &= \tilde{S}^i[\phi, A_k, \{\tilde{T}_{lk}\}],\end{aligned}$$

Work in progress ... key to GWs from PhT's !

(w/ K. Marschall, A. Midiri and A. Roper Pol)

Magneto Hydro-dynamics (MHD)

Lecture 9

$$T^{\mu\nu} = (p + \rho)U^\mu U^\nu - p g^{\mu\nu}$$

$$D_\nu T^{\mu\nu} = \partial_\nu T^{\mu\nu} + \Gamma_{\sigma\nu}^\mu T^{\sigma\nu} + \Gamma_{\nu\sigma}^\nu T^{\mu\sigma} = \frac{1}{\sqrt{-g}} \partial_\nu (\sqrt{-g} T^{\mu\nu}) = 0,$$

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Work in progress ... key to GWs from PhT's !

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New Physics

- * Magneto Hydro-dynamics (MHD)
- * **Axion-gauge interactions**
- * Cosmic string networks
- * Non-minimal Grav. coupling

Axion-gauge interactions

Lecture 7

$$\mathcal{S}_{\text{ax}} = - \int d^4x \sqrt{-g} \left\{ \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + V(\phi) + \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\alpha_\Lambda}{4 m_p} F_{\mu\nu} \tilde{F}^{\mu\nu} \right\}$$

$$\left. \begin{aligned} \ddot{\phi} &= -3H\dot{\phi} + \frac{1}{a^2} \vec{\nabla}^2 \phi - V_{,\phi} + \frac{\alpha_\Lambda}{a^3 m_p} \vec{E} \cdot \vec{B}, \\ \dot{\vec{E}} &= -H\vec{E} - \frac{1}{a^2} \vec{\nabla} \times \vec{B} - \frac{\alpha_\Lambda}{a m_p} \left(\dot{\phi} \vec{B} + \vec{\nabla} \phi \cdot \vec{E} \right), \\ \ddot{a} &= -\frac{a}{3m_p^2} (2\rho_K - \rho_V + \rho_{\text{EM}}), \\ \vec{\nabla} \cdot \vec{E} &= -\frac{\alpha_\Lambda}{a m_p} \vec{\nabla} \phi \cdot \vec{B}, \quad [\text{Gauss law}] \\ H^2 &= \frac{1}{3m_p^2} (\rho_K + \rho_G + \rho_V + \rho_{\text{EM}}), \quad [\text{Hubble law}] \end{aligned} \right\}$$

(w/ J. Lizarraga, N. Loayza, and A. Urrio)

Phys. Rev. Lett. 131 (2023) 15, 151003 ; *Phys.Rev.D* 111 (2025) 6, 063545

Axion-gauge interactions

$$\mathcal{S}_{\text{ax}} = - \int d^4x \sqrt{-g} \left\{ \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + V(\phi) + \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\alpha_\Lambda}{4} \frac{\phi}{m_p} F_{\mu\nu} \tilde{F}^{\mu\nu} \right\}$$

$$\left. \begin{aligned} \ddot{\phi} &= -3H\dot{\phi} + \frac{1}{a^2} \vec{\nabla}^2 \phi - V_{,\phi} + \frac{\alpha_\Lambda}{a^3 m_p} \vec{E} \cdot \vec{B}, \\ \dot{\vec{E}} &= -H\vec{E} - \frac{1}{a^2} \vec{\nabla} \times \vec{B} - \frac{\alpha_\Lambda}{a m_p} \left(\dot{\phi} \vec{B} + \vec{\nabla} \phi \cdot \vec{E} \right), \\ \ddot{a} &= -\frac{a}{3m_p^2} (2\rho_K - \rho_V + \rho_{\text{EM}}), \\ \vec{\nabla} \cdot \vec{E} &= -\frac{\alpha_\Lambda}{a m_p} \vec{\nabla} \phi \cdot \vec{B}, \quad [\text{Gauss law}] \\ H^2 &= \frac{1}{3m_p^2} (\rho_K + \rho_G + \rho_V + \rho_{\text{EM}}), \quad [\text{Hubble law}] \end{aligned} \right\}$$

(w/ J. Lizarraaga, N. Loayza, and A. Urrio)

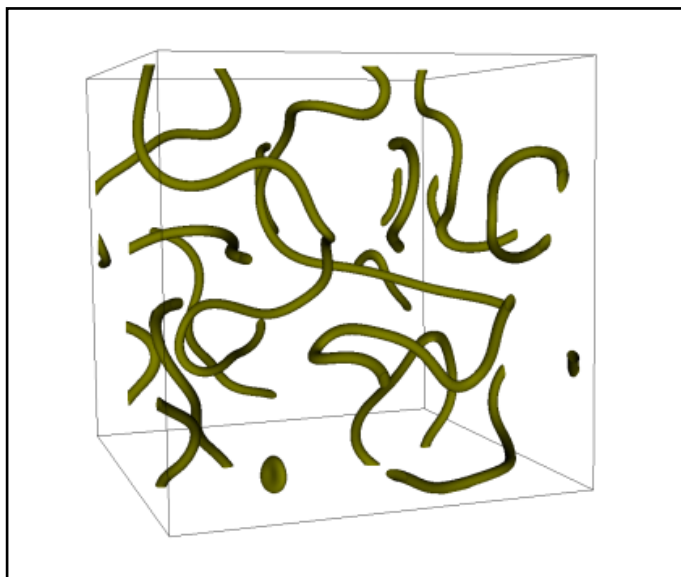
Phys. Rev. Lett. 131 (2023) 15, 151003 ; *Phys.Rev.D* 111 (2025) 6, 063545

New Physics

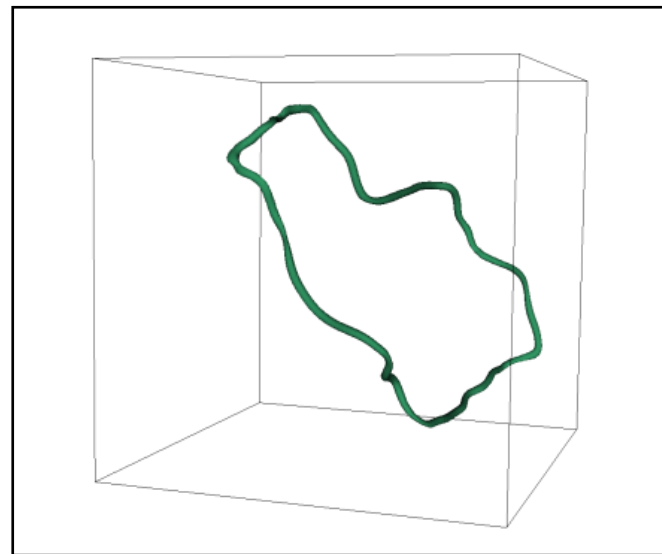
- * Magneto Hydro-dynamics (MHD)
- * Axion-gauge interactions
- * **Cosmic defect networks**
- * Non-minimal Grav. coupling

Cosmic (Topological) Defects

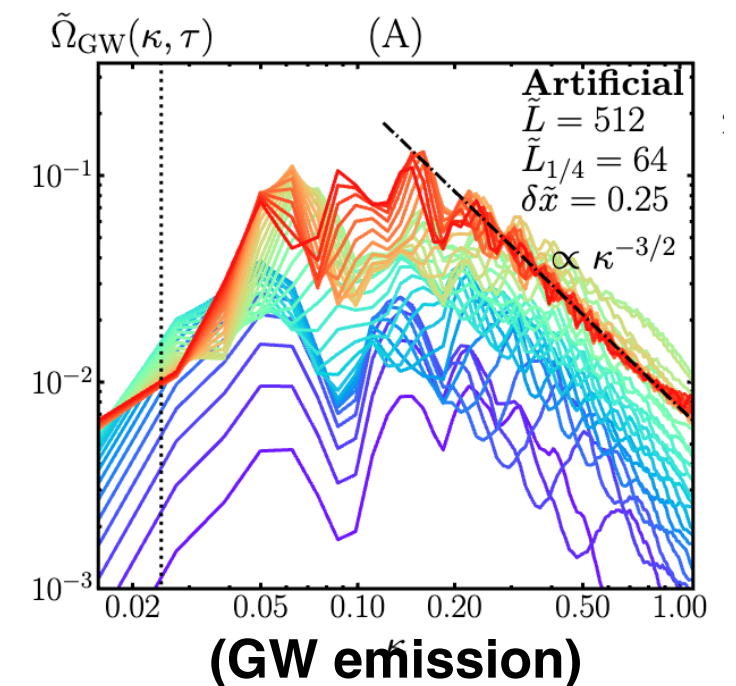
Particle & GW Emission



(String Networks)



(Isolated loops)



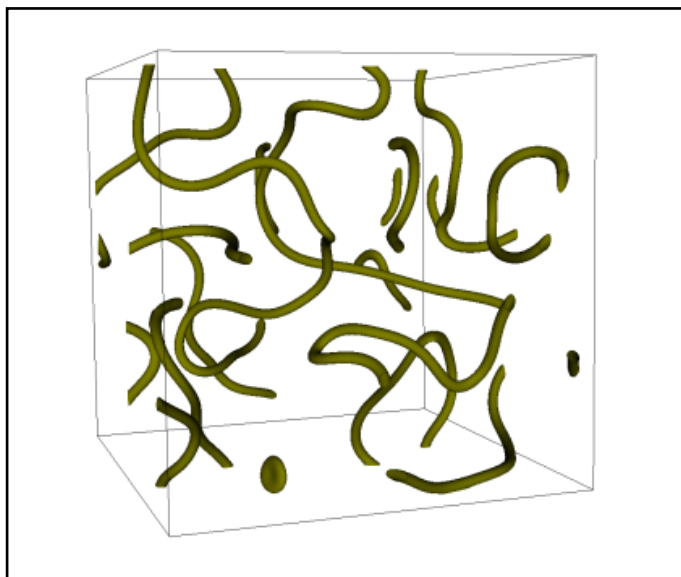
(w/ J. Baeza-Ballesteros, E. Copeland, J. Lizarraga)

ArXiv:2308.08456 (Global) ; ArXiv:2408.02364 (Local)

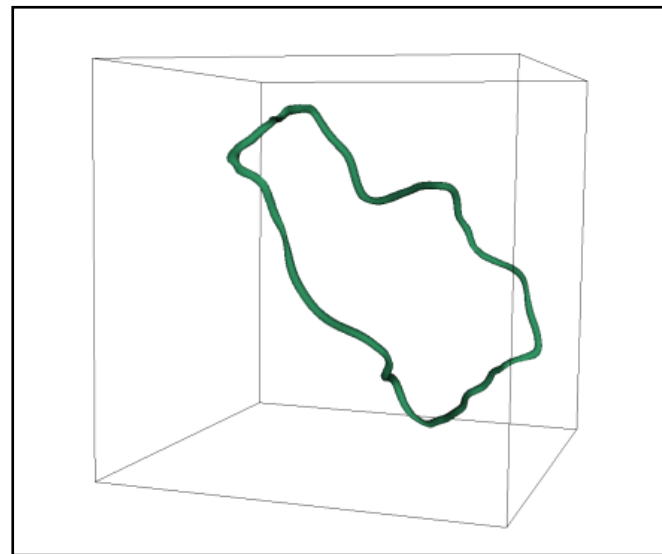
Cosmic (Topological) Defects

Lecture 11

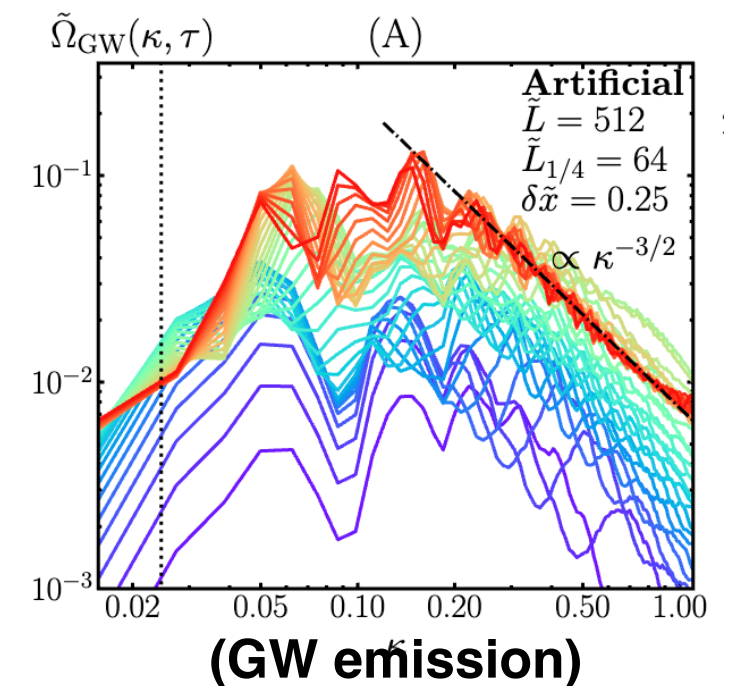
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New Physics

- * Magneto Hydro-dynamics (MHD)
- * Axion-gauge interactions
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- * **Non-minimal Grav. coupling**

Non-minimal Grav. coupling

$$\mathcal{S}_{\text{NMC}} = - \int d^4x \sqrt{-g} \left(\frac{1}{2} \xi R \phi^2 + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + V(\phi, \{\varphi_m\}) \right)$$

$$\begin{aligned} \text{[Non-minimally coupled]} & \begin{cases} \phi' = a^{\alpha-3} \pi_\phi, \\ \pi'_\phi = a^{1+\alpha} \nabla^2 \phi - a^{3+\alpha} (\xi R \phi + V_{,\phi}), \end{cases} \\ \text{[Expanding background]} & \begin{cases} a' = a^{\alpha-1} \pi_a, \\ \pi'_a = \frac{a^{2+\alpha}}{6} R, \end{cases} \end{aligned}$$

with

$$R = \frac{1}{m_p^2} \left[\frac{2(1 - 6\xi)(E_G^\phi - E_K^\phi) + 4\langle V \rangle - 6\xi\langle \phi V_{,\phi} \rangle + (\rho_m - 3p_m)}{1 + (6\xi - 1)\xi\langle \phi^2 \rangle/m_p^2} \right],$$

(w/ B. Stefanek, T. Opferkuch, A. Florio, N. Loayza)

ArXiv:2112.08388; ArXiv:2406.02689; ArXiv:2404.17654

Non-minimal Grav. coupling

Lecture 5

$$\mathcal{S}_{\text{NMC}} = - \int d^4x \sqrt{-g} \left(\frac{1}{2} \xi R \phi^2 + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + V(\phi, \{\varphi_m\}) \right)$$

$$\begin{aligned} \text{[Non-minimally coupled]} & \begin{cases} \phi' = a^{\alpha-3} \pi_\phi, \\ \pi'_\phi = a^{1+\alpha} \nabla^2 \phi - a^{3+\alpha} (\xi R \phi + V_{,\phi}), \end{cases} \\ \text{[Expanding background]} & \begin{cases} a' = a^{\alpha-1} \pi_a, \\ \pi'_a = \frac{a^{2+\alpha}}{6} R, \end{cases} \end{aligned}$$

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New Physics

- * **Magneto Hydro-dynamics (MHD)**
- * **Axion-gauge interactions**
- * **Cosmic string networks**
- * **Non-minimal Grav. coupling**

To be released in 2025/26 !

New Physics

- * **Magneto Hydro-dynamics (MHD)**
- * **Axion-gauge interactions**
- * **Cosmic string networks**
- * **Non-minimal Grav. coupling**
- * **Grav. Pert. Th / Full GR**
(w/ N. Loayza & R. Flauger)

To be released in 202X?

CosmoLattice – School 2025

Day 1
(Monday)

Intro - Dani ✓

Lecture 1: What is a Lattice? - Paco

Lecture 2: Inflation and post-inflationary dynamics - Paco

Practice 1: My first run! fun with ϕ^4 - Nico, Ken, Franz

Lecture 3: Evolution algorithms - Ander, Antonino

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– Lecture 1 –

Welcome to the Lattice

Francisco Torrenti

UB, Barcelona, Spain