CosmoLattice

— School 2025 —

CosmoLattice School 2025 (IBS, Korea), Sept 22-26

Practice 3

Running gauge preheating

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Abelian-Higgs model

Abelian Gauge model

$$\boxed{\mathcal{S} = -\int d^4x \sqrt{-g}\left[(D_\mu^A \varphi)^*(D_A^\mu \varphi) + \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + V(|\varphi|)\right]}$$

$$\boxed{V(|\varphi|) = \lambda|\varphi|^4 = \frac{1}{4}|\varphi_0^2 + \varphi_1^2|}$$

Abelian-Higgs model

Abelian Gauge model

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$$V(|\varphi|) = \lambda|\varphi|^4 = \frac{1}{4}|\varphi_0^2 + \varphi_1^2|$$

We choose program variables

$$f_* = |\varphi_*| \qquad \omega_* = \sqrt{\lambda}\,|\varphi_*|$$

$$\alpha = 1$$

$$\tilde{V}(\tilde{\varphi}) \equiv \frac{1}{f_*^2\omega_*^2}V(f_*|\varphi_*|) = |\tilde{\varphi}|^4$$

Abelian-Higgs model

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Initial amplitude is distributed equally between its components

$$\varphi_0 = \varphi_1 = \varphi_*/\sqrt{2} \qquad \dot{\varphi}_0 = \dot{\varphi}_1 = \dot{\varphi}_*/\sqrt{2}$$

Abelian-Higgs model

Let us take a look to the model file: lphi4U1.h

```
struct ModelPars : public TempLat::DefaultModelPars{
    static constexpr size_t NScalars = 0;
    static constexpr size_t NCScalars = 1;
    static constexpr size_t NU1Flds = 1;
    static constexpr size_t NSU2Doublet = 0;
    static constexpr size_t NSU2Flds = 0;
    static constexpr size_t NPotTerms = 1;

    // Coupling managers:  they deal with the different couplings between the gauge f
    //  --> If a type of interaction is not present, comment the corresponding line
    typedef TempLat::CouplingsManager<NCScalars, NU1Flds,true> CsU1Couplings; // acti
   // typedef TempLat::CouplingsManager<NSU2Doublet, NU1Flds, true> SU2DoubletU1Coupl
//  typedef TempLat::CouplingsManager<NSU2Doublet, NSU2Flds,true> SU2DoubletSU2Coupli
};

#define MODELNAME lphi4U1
// Here we define the name of the model. This should match the name of your file.
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Abelian-Higgs model

Let us take a look to the parameters file: lphi4U1.h

```
#IC
kCutOff = 3
initial_amplitudes = 0
initial_momenta = 0
cmplx_field_initial_norm = 5.6964e18      # homogeneous amplitudes in GeV
cmplx_momentum_initial_norm = -6.88343e30 # homogeneous amplitudes in GeV2

#Gauge couplings and effectiveCharges
CSU1Charges = 1
gU1s = 1.11037e-05

#Model Parameters
lambda = 9e-14
```

$|\varphi_*|$

$|\dot{\varphi}_*|$

Abelian-Higgs model

Let us take a look to the parameters file: lphi4U1.in

```
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kCutOff = 3
initial_amplitudes = 0
initial_momenta = 0
cmplx_field_initial_norm = 5.6964e18      # homogeneous amplitudes in GeV
cmplx_momentum_initial_norm = -6.88343e30 # homogeneous amplitudes in GeV2

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#Model Parameters
lambda = 9e-14
```

Q_A

g_A

λ

Abelian-Higgs model

Let us take a deeper look at the model file: lphi4U1.h

```
// COMPLEX SCALAR NORM: initial homogeneous amplitude and derivative
double normCmplx0 = parser.get<double>("cmplx_field_initial_norm");
double normPiCmplx0 = parser.get<double>("cmplx_momentum_initial_norm");

    // We distribute the norm equally between the two components using the "Complex
    fldCS0(0_c) = Complexify(normCmplx0 / sqrt(2.0), normCmplx0 / sqrt(2.0));
    piCS0(0_c)  = Complexify(normPiCmplx0 / sqrt(2.0), normPiCmplx0 / sqrt(2.0));
```

$$\varphi = \left(\frac{\varphi_*}{\sqrt{2}}, \frac{\varphi_*}{\sqrt{2}}\right)$$

```
alpha = 1;
fStar = normCmplx0;
omegaStar = sqrt(lambda) * normCmplx0;
```

$f_* = |\varphi_*|$ $\quad\omega_* = \sqrt{\lambda}\,|\varphi_*|$ $\quad\alpha = 1$

```
auto potentialTerms(Tag<0>) // Term 0:
{
    return  pow<4>(norm(fldCS(0_c)));
}
```

$$\tilde{V}(\tilde{\varphi}) = |\tilde{\varphi}|^4$$

```
auto potDeriv2NormCS(Tag<0>) // 2nd derivati
{
    return  12 * pow<2>(norm(fldCS(0_c))) ;
}
```

```
auto potDerivNormCS(Tag<0>) // Derivative
{
    return 4 * pow<3>(norm(fldCS(0_c)));
}
```

$$\tilde{V}'(\tilde{\varphi}) = 4|\tilde{\varphi}|^3$$

$$\tilde{V}'(\tilde{\varphi}) = 12|\tilde{\varphi}|^2$$

Abelian-Higgs model

Output

- average_norm_cmplx_scalar_[nfld].txt: $\tilde{\eta}$, $\langle|\tilde{\varphi}|\rangle$, $\langle|\tilde{\varphi}'|\rangle$, $\langle|\tilde{\varphi}|^2\rangle$, $\langle|\tilde{\varphi}'|^2\rangle$, $\mathrm{rms}(|\tilde{\varphi}|)$, $\mathrm{rms}(|\tilde{\varphi}'|)$
- average_[Re/Im]_cmplx_scalar_[nfld].txt: $\tilde{\eta}$, $\langle\tilde{\varphi}_n\rangle$, $\langle\tilde{\varphi}'_n\rangle$, $\langle\tilde{\varphi}_n^2\rangle$, $\langle\tilde{\varphi}_n'^2\rangle$, $\mathrm{rms}(\tilde{\varphi}_n)$, $\mathrm{rms}(\tilde{\varphi}'_n)$
- average_norm_[U1]_[nfld].txt: $\tilde{\eta}$, $\langle|\vec{\tilde{\mathcal{E}}}|\rangle$, $\langle|\vec{\tilde{\mathcal{B}}}|\rangle$, $\langle|\vec{\tilde{\mathcal{E}}}|^2\rangle$, $\langle|\vec{\tilde{\mathcal{B}}}|^2\rangle$, $\mathrm{rms}(|\vec{\tilde{\mathcal{E}}}|)$, $\mathrm{rms}(|\vec{\tilde{\mathcal{B}}}|)$

- average_energies.txt:

$\tilde{\eta}$, $\tilde{E}_K^{(\phi,0)}$, $\tilde{E}_G^{(\phi,0)}$, ... , $\tilde{E}_K^{(\phi,N_s-1)}$, $\tilde{E}_G^{(\phi,N_s-1)}$, $\tilde{E}_K^{(\varphi,0)}$, $\tilde{E}_G^{(\varphi,0)}$, ... , $\tilde{E}_K^{(\varphi,N_c-1)}$, $\tilde{E}_G^{(\varphi,N_c-1)}$,
$\tilde{E}_K^{(\Phi,0)}$, $\tilde{E}_G^{(\Phi,0)}$, ... , $\tilde{E}_K^{(\Phi,N_d-1)}$, $\tilde{E}_G^{(\Phi,N_d-1)}$, $\tilde{E}_K^{(A,0)}$, $\tilde{E}_G^{(A,0)}$, ... , $\tilde{E}_K^{(A,N_{u1}-1)}$, $\tilde{E}_G^{(A,N_{u1}-1)}$,
$\tilde{E}_V^{(0)}$, ..., $\tilde{E}_V^{(N_p-1)}$, $\langle\tilde{\rho}\rangle$

- average_gauss_[U1/SU2]_[nfld].txt: $\tilde{\eta}$, $\frac{\langle\sqrt{(\mathrm{LHS}-\mathrm{RHS})^2}\rangle}{\langle\sqrt{(\mathrm{LHS}+\mathrm{RHS})^2}\rangle}$, $\langle\sqrt{(\mathrm{LHS}-\mathrm{RHS})^2}\rangle$, $\langle\sqrt{(\mathrm{LHS}+\mathrm{RHS})^2}\rangle$.
- spectra_norm_cmplx_scalar_[nfld].txt: $\tilde{k}$, $\widetilde{\Delta}_{\tilde{\varphi}}(\tilde{k})$, $\widetilde{\Delta}_{\tilde{\varphi}'}(\tilde{k})$, $\tilde{n}_{\tilde{k}}$, Δn_{bin}
- spectra_norm_[U1/SU2]_[nfld].txt: $\tilde{k}$, $\widetilde{\Delta}_{\tilde{\mathcal{E}}}(\tilde{k})$ $\widetilde{\Delta}_{\tilde{\mathcal{B}}}(\tilde{k})$, Δn_{bin}

Abelian-Higgs model

Gauge fields eom

$$A_i'' - a^{-2(1-\alpha)}\nabla^2 A_i + \partial_i\partial_j A_j + (1-\alpha)\frac{a'}{a}A_i' = a^{2\alpha} J_i^A$$

Abelian-Higgs model

Gauge fields eom

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Current

$$J_i^A \equiv 2g_A Q_A \mathfrak{Im}[\varphi^*(\varphi - ig_A Q_A A_i \varphi)]$$

Abelian-Higgs model

Gauge fields eom

$$A_i'' - a^{-2(1-\alpha)}\nabla^2 A_i + \partial_i\partial_j A_j + (1-\alpha)\frac{a'}{a}A_i' = a^{2\alpha}J_i^A$$

Current

$$J_i^A \equiv 2g_A Q_A \mathfrak{Im}[\varphi^*(\varphi - \boxed{ig_A Q_A A_i \varphi})] \Rightarrow g_A Q_A |\varphi|^2 A_i$$

Similar term to the one of parametric resonance

Abelian-Higgs model

Exercise

Run Model: lphi4U1.h

Use parameters file: lphi4U1.in

Analyze output using Mathematica notebook