



CosmoLattice

— School **2025** —



CosmoLattice School 2025 (IBS, Korea), Sept 22-26

Practice 3

Running gauge preheating

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Abelian-Higgs model

Abelian Gauge model

$$\mathcal{S} = - \int d^4x \sqrt{-g} \left[(D_\mu^A \varphi)^* (D_A^\mu \varphi) + \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + V(|\varphi|) \right]$$

$$V(|\varphi|) = \lambda |\varphi|^4 = \frac{1}{4} |\varphi_0^2 + \varphi_1^2|$$

Abelian-Higgs model

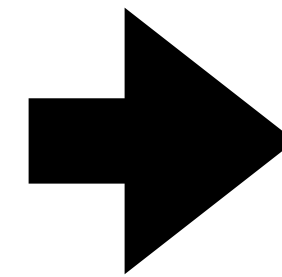
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We choose program variables

$$f_* = |\varphi_*| \quad \omega_* = \sqrt{\lambda} |\varphi_*|$$
$$\alpha = 1$$



$$\tilde{V}(\tilde{\varphi}) \equiv \frac{1}{f_*^2 \omega_*^2} V(f_* |\varphi_*|) = |\tilde{\varphi}|^4$$

Abelian-Higgs model

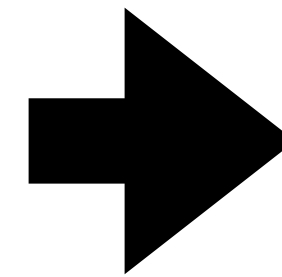
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Initial amplitude is distributed equally between its components

$$\varphi_0 = \varphi_1 = \varphi_*/\sqrt{2}$$

$$\dot{\varphi}_0 = \dot{\varphi}_1 = \dot{\varphi}_*/\sqrt{2}$$

Abelian-Higgs model

Let us take a look to the model file: [lphi4U1.h](#)

```
struct ModelPars : public TempLat::DefaultModelPars{
    static constexpr size_t NScalars = 0;
    static constexpr size_t NCSalars = 1;
    static constexpr size_t NU1Flds = 1;
    static constexpr size_t NSU2Doublet = 0;
    static constexpr size_t NSU2Flds = 0;
    static constexpr size_t NPotTerms = 1;

    // Coupling managers: they deal with the different couplings between the gauge f
    // --> If a type of interaction is not present, comment the corresponding line
    typedef TempLat::CouplingsManager<NCSalars, NU1Flds,true> CsU1Couplings; // acti
    // typedef TempLat::CouplingsManager<NSU2Doublet, NU1Flds, true> SU2DoubletU1Coupl
    // typedef TempLat::CouplingsManager<NSU2Doublet, NSU2Flds,true> SU2DoubletSU2Coupli
};

#define MODELNAME lphi4U1
// Here we define the name of the model. This should match the name of your file.
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Abelian-Higgs model

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Abelian-Higgs model

Let us take a look to the parameters file: [lphi4U1.h](#)

```
#IC
kCutOff = 3
initial_amplitudes = 0
initial_momenta = 0
cmplx_field_initial_norm = 5.6964e18      # homogeneous amplitudes in GeV
cmplx_momentum_initial_norm = -6.88343e30 # homogeneous amplitudes in GeV2
#Gauge couplings and effectiveCharges
CSU1Charges = 1
gU1s = 1.11037e-05
#Model Parameters
lambda = 9e-14
```

$$\begin{array}{c} |\varphi^*| \\ |\dot{\varphi}^*| \end{array}$$

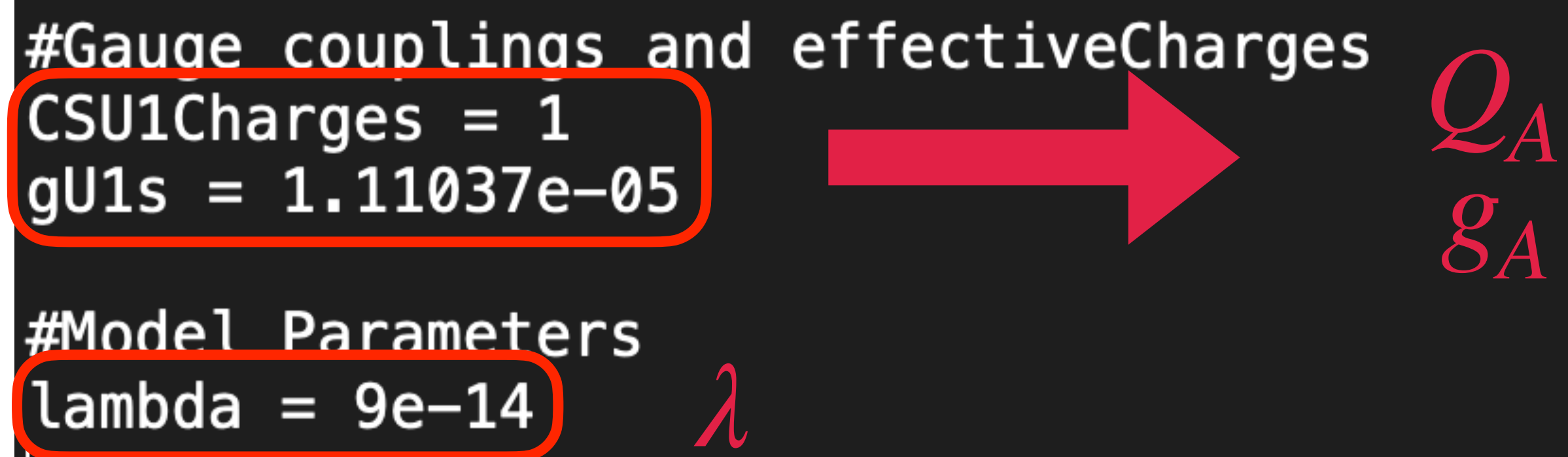
Abelian-Higgs model

Let us take a look to the parameters file: [lphi4U1.in](#)

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#Gauge couplings and effectiveCharges
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#Model Parameters
lambda = 9e-14
```



Q_A
 g_A

λ

Abelian-Higgs model

Let us take a deeper look at the model file: [lphi4U1.h](#)

```
// COMPLEX SCALAR NORM: initial homogeneous amplitude and derivative
double normCmplx0 = parser.get<double>("cmplx_field_initial_norm");
double normPiCmplx0 = parser.get<double>("cmplx_momentum_initial_norm");

// We distribute the norm equally between the two components using the "Complexify"
fldCS0(0_c) = Complexify(normCmplx0 / sqrt(2.0), normCmplx0 / sqrt(2.0));
piCS0(0_c) = Complexify(normPiCmplx0 / sqrt(2.0), normPiCmplx0 / sqrt(2.0));
```

$$\varphi = \left(\frac{\varphi_*}{\sqrt{2}}, \frac{\varphi_*}{\sqrt{2}} \right)$$

```
alpha = 1;
fStar = normCmplx0;
omegaStar = sqrt(lambda) * normCmplx0;
```

$$f_* = |\varphi_*| \quad \omega_* = \sqrt{\lambda} |\varphi_*| \quad \alpha = 1$$

```
auto potentialTerms(Tag<0>) // Term 0:
{
    return pow<4>(norm(fldCS(0_c)));
}
```

$$\tilde{V}(\tilde{\varphi}) = |\tilde{\varphi}|^4$$

```
auto potDeriv2NormCS(Tag<0>) // 2nd derivative
{
    return 12 * pow<2>(norm(fldCS(0_c))) ;
}
```

```
auto potDerivNormCS(Tag<0>) // Derivative
{
    return 4 * pow<3>(norm(fldCS(0_c)));
}
```

$$\tilde{V}'(\tilde{\varphi}) = 4 |\tilde{\varphi}|^3$$

$$\tilde{V}'(\tilde{\varphi}) = 12 |\tilde{\varphi}|^2$$

Abelian-Higgs model

Output

- `average_norm_cmplx_scalar_[nfld].txt`: $\tilde{\eta}$, $\langle |\tilde{\varphi}| \rangle$, $\langle |\tilde{\varphi}'| \rangle$, $\langle |\tilde{\varphi}|^2 \rangle$, $\langle |\tilde{\varphi}'|^2 \rangle$, $\text{rms}(|\tilde{\varphi}|)$, $\text{rms}(|\tilde{\varphi}'|)$
- `average_[Re/Im]_cmplx_scalar_[nfld].txt`: $\tilde{\eta}$, $\langle \tilde{\varphi}_n \rangle$, $\langle \tilde{\varphi}'_n \rangle$, $\langle \tilde{\varphi}_n^2 \rangle$, $\langle \tilde{\varphi}'_n{}^2 \rangle$, $\text{rms}(\tilde{\varphi}_n)$, $\text{rms}(\tilde{\varphi}'_n)$
- `average_norm_[U1]_[nfld].txt`: $\tilde{\eta}$, $\langle |\vec{\tilde{\mathcal{E}}}| \rangle$, $\langle |\vec{\tilde{\mathcal{B}}}| \rangle$, $\langle |\vec{\tilde{\mathcal{E}}}|^2 \rangle$, $\langle |\vec{\tilde{\mathcal{B}}}|^2 \rangle$, $\text{rms}(|\vec{\tilde{\mathcal{E}}}|)$, $\text{rms}(|\vec{\tilde{\mathcal{B}}}|)$
- `average_energies.txt`:
$$\tilde{\eta}, \tilde{E}_K^{(\phi,0)}, \tilde{E}_G^{(\phi,0)}, \dots, \tilde{E}_K^{(\phi,N_s-1)}, \tilde{E}_G^{(\phi,N_s-1)}, \tilde{E}_K^{(\varphi,0)}, \tilde{E}_G^{(\varphi,0)}, \dots, \tilde{E}_K^{(\varphi,N_c-1)}, \tilde{E}_G^{(\varphi,N_c-1)},$$
$$\tilde{E}_K^{(\Phi,0)}, \tilde{E}_G^{(\Phi,0)}, \dots, \tilde{E}_K^{(\Phi,N_d-1)}, \tilde{E}_G^{(\Phi,N_d-1)}, \tilde{E}_K^{(A,0)}, \tilde{E}_G^{(A,0)}, \dots, \tilde{E}_K^{(A,N_{u1}-1)}, \tilde{E}_G^{(A,N_{u1}-1)},$$
$$\tilde{E}_V^{(0)}, \dots, \tilde{E}_V^{(N_p-1)}, \langle \tilde{\rho} \rangle$$
- `average_gauss_[U1/SU2]_[nfld].txt`: $\tilde{\eta}$, $\frac{\langle \sqrt{(\text{LHS}-\text{RHS})^2} \rangle}{\langle \sqrt{(\text{LHS}+\text{RHS})^2} \rangle}$, $\langle \sqrt{(\text{LHS}-\text{RHS})^2} \rangle$, $\langle \sqrt{(\text{LHS}+\text{RHS})^2} \rangle$.
- `spectra_norm_cmplx_scalar_[nfld].txt`: $\tilde{k}$, $\tilde{\Delta}_{\tilde{\varphi}}(\tilde{k})$, $\tilde{\Delta}_{\tilde{\varphi}'}(\tilde{k})$, $\tilde{n}_{\tilde{k}}$, Δn_{bin}
- `spectra_norm_[U1/SU2]_[nfld].txt`: $\tilde{k}$, $\tilde{\Delta}_{\tilde{\mathcal{E}}}(\tilde{k})$, $\tilde{\Delta}_{\tilde{\mathcal{B}}}(\tilde{k})$, Δn_{bin}

Abelian-Higgs model

Gauge fields eom

$$A_i'' - a^{-2(1-\alpha)} \nabla^2 A_i + \partial_i \partial_j A_j + (1 - \alpha) \frac{a'}{a} A_i' = a^{2\alpha} J_i^A$$

Abelian-Higgs model

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Current

$$J_i^A \equiv 2g_A Q_A \Im \mathfrak{m}[\varphi^*(\varphi - ig_A Q_A A_i \varphi)]$$

Abelian-Higgs model

Gauge fields eom

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Current

$$J_i^A \equiv 2g_A Q_A \Im \mathfrak{m}[\varphi^*(\varphi - ig_A Q_A A_i \varphi)] \quad \longrightarrow \quad g_A Q_A |\varphi|^2 A_i$$

**Similar term to the one of
parametric resonance**

Abelian-Higgs model

Exercise

Run Model: lphi4U1.h

Use parameters file: lphi4U1.in

Analyze output using Mathematica notebook