Lecture 11: Lattice simulations of cosmic defects

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$\mathcal{C}$osmo$\mathcal{L}$attice school 2025 - 26th September 2025

Deutsches Elektronen
-Synchroton Zeuthen

Overview

1. Cosmic topological defects
2. Global and local cosmic strings
   2.1. Cosmic strings in the continuum
   2.2. Lattice simulations of cosmic strings
   2.3. Particle and GW emission from cosmic string loops
3. Domain walls [F. Torrenti]

Overview

Topological defects

Qualitative definition: Stable structures that *may* form in field theories with a spontaneously-broken symmetry, in which the field is trapped far from the true vacuum

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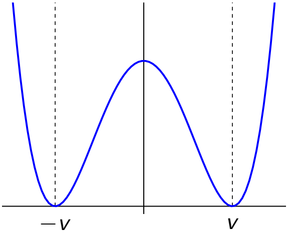

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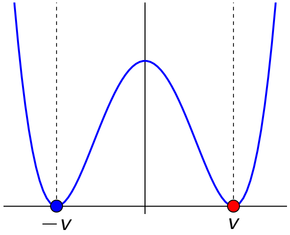

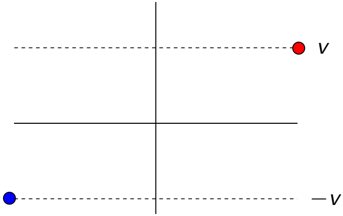

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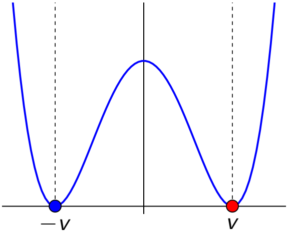

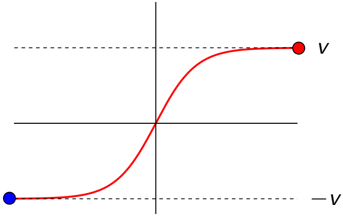

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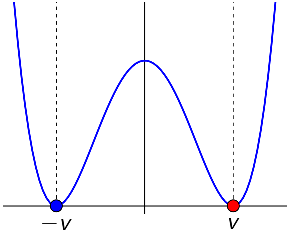
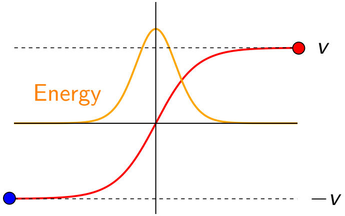
Energy is localized at the defects!





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➤ Stable due to a **conserved current** and **topological charge**

Current : $J_\mu = \varepsilon_{\mu\nu}\partial_\nu\phi$

Topological charge : $Q = \int dx J_0 = \frac{1}{2}\left[\phi(x=+\infty) - \phi(x=-\infty)\right]$

Given by **boundary values** only

Topological defects

Mathematical definition: Stable field configurations that *may* originate in field theories with a non-trivial vacuum manifold

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Same example: scalar theory in 1+1 dimensions

$$S = -\int d^2x \left[\frac{1}{2}\partial_\mu\phi\partial_\mu\phi + \frac{\lambda}{4}(\phi^2 - v^2)^2\right]$$

Symmetry of $\mathcal{L}$

$G = \mathbb{Z}_2 \quad (\phi \to -\phi)$

$\longrightarrow$

Symmetry of $|0\rangle$

$H = \mathbb{1}$

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Vacuum manifold

$\mathcal{M} = \{\pm v\} \cong G/H = \mathbb{Z}_2$

Non-trivial homotopy group of $\mathcal{M}$

$\pi_0(\mathcal{M}) = \mathbb{Z}_2$

Topological defects:

Example in 3+1 dimensions: **Domain walls**

$$S_{\mathrm{DW}} = -\int \mathrm{d}^4 x \left[\frac{1}{2}\partial_\mu \phi \partial^\mu \phi + \frac{\lambda}{4}(\phi^2 - v^2)^2 - \frac{\lambda v^4}{4} \right]$$

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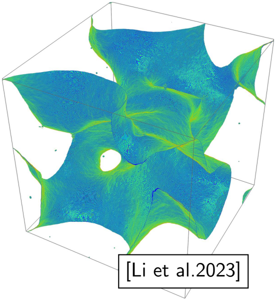
  Domain walls
- DWs are 2-dimensional surfaces embedded in 3-dimensional space
- DWs **separate inequivalent vacua**



[Li et al.2023]

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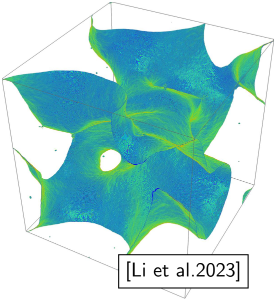

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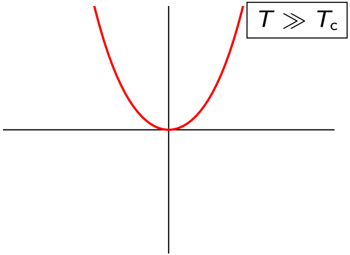
Domain walls **overclose the universe**, $\rho_{\rm DW} \sim H$ ⟶ Include a **bias**!

How do domain walls form? Kibble mechanism

DWs (and other topological defects) form after spontaneous symmetry breaking

- The **effective potential** depends on temperature



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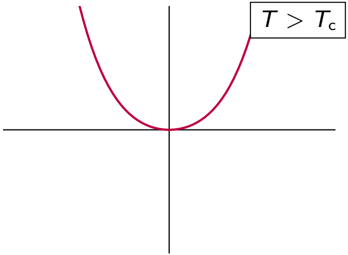

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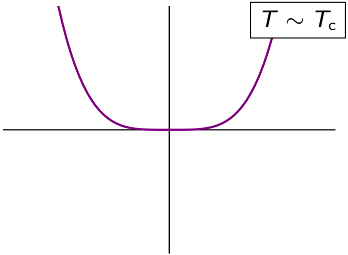

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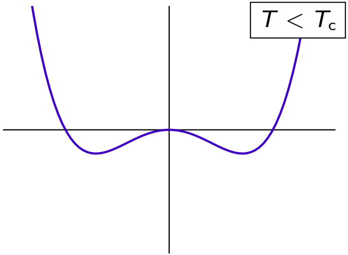

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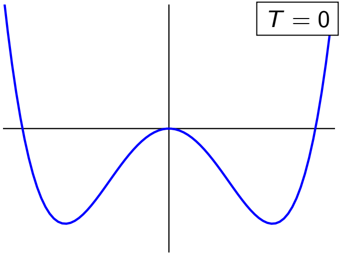

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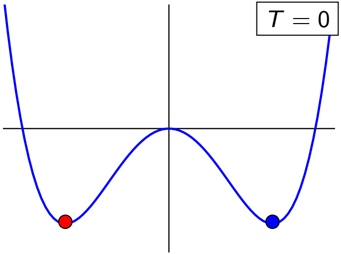

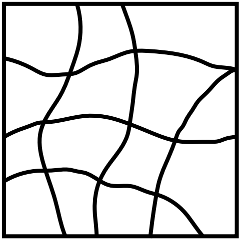

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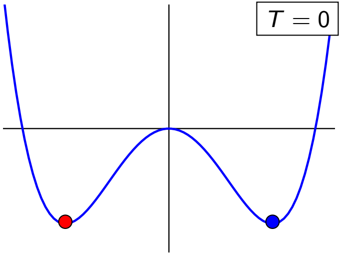

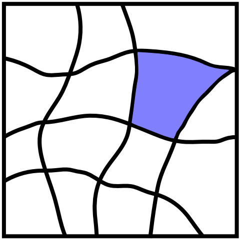

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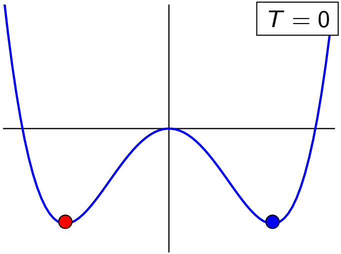

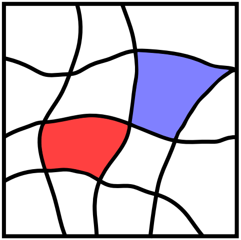

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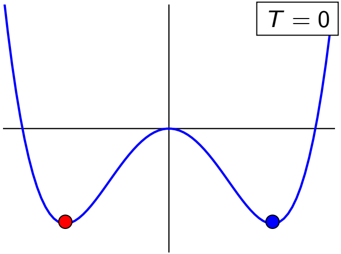

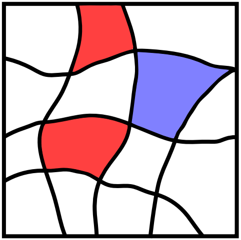

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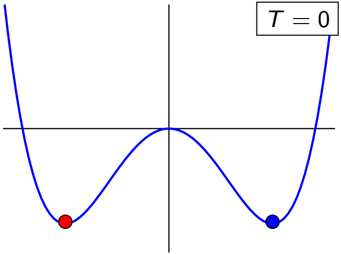

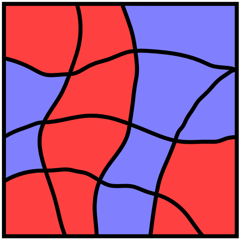

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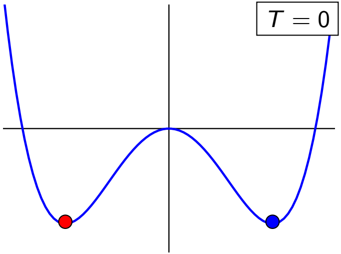

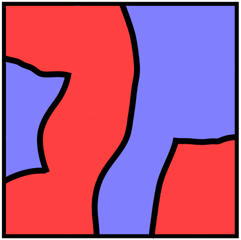

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Topological defects: cosmic strings

Another example:

$$S_{\text{global}} = -\int \mathrm{d}^4x \left[(\partial_\mu \varphi)^* (\partial^\mu \varphi) + \lambda \left(|\varphi|^2 - \frac{v^2}{2} \right)^2 \right]$$

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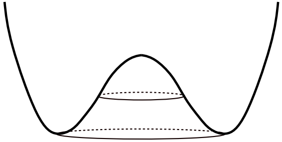

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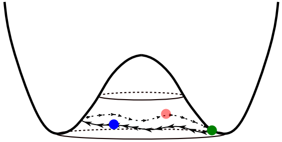

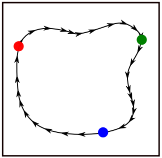

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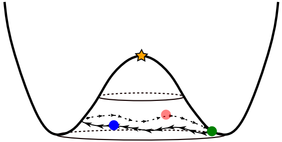

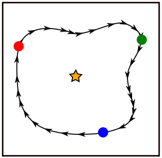

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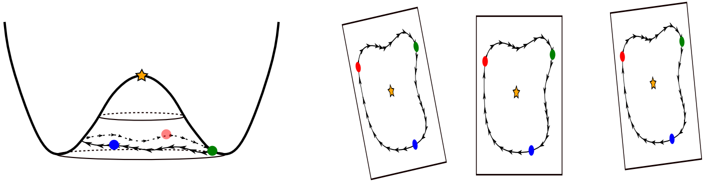

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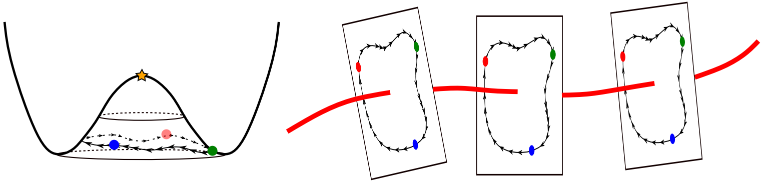

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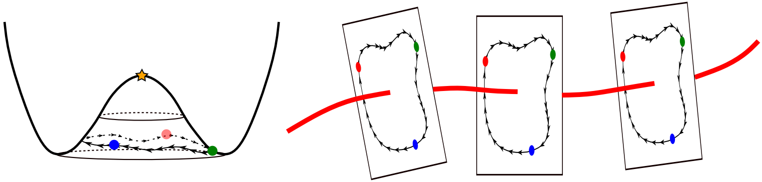
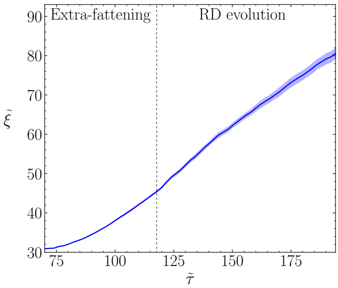


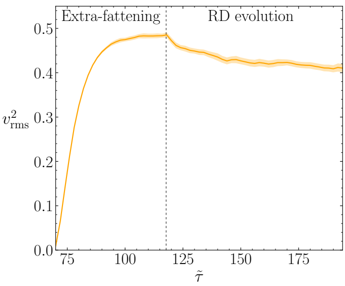
➤ **Many more types of strings**: **local strings**, current-carrying strings...

Scaling of topological defects

Topological defects are expected to reach a **scaling regime**: constant density of defects per Hubble patch

- **Domain walls**: $\mathcal{A} \propto \tau$ (area parameter)
- **Cosmic strings**: $\xi \propto \tau$ (length per patch) $v_{\rm rms}^2 \sim$ const





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Global cosmic strings

Global cosmic string arise in theories with a broken **global** U(1) symmetry

- **Example**: Axion theories
- Captured by a theory with a complex scalar field + mexican-hat potential

$$S_{\text{global}} = -\int \mathrm{d}^4x \left[\partial_\mu\varphi\partial^\mu\varphi + \lambda\left(|\varphi|^2 - \frac{v^2}{2}\right)^2\right]$$

- Equations of motion identical than any other scalar theory

$$(a^2\varphi')' - a^2\partial_i\partial_i\varphi = -2\lambda a^4\left(|\varphi|^2 - \frac{v^2}{2}\right)\varphi$$

  - We use **conformal time** ($\alpha = 1$)
  - Both v^2 and λ can be reabsorbed

Global cosmic strings

- Contains **massive and massless particles**

$$\textbf{Massive}: \quad m_\chi = \sqrt{2\lambda} v \sim r_c^{-1}$$

$$\textbf{Massless}: \quad m_\theta = 0$$

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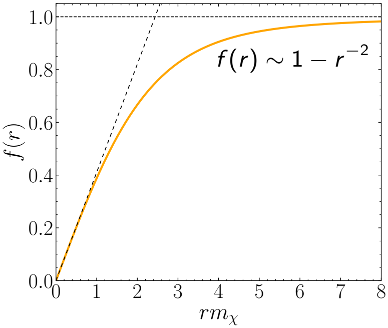
- Analytical solution for **infinite straight string**



- Ansatz: $\varphi = f(r) \frac{v}{\sqrt{2}} e^{i\theta}$
- **Logarithmic-growing tension**:

$$\mu \sim 2\pi v^2 \log\left(\frac{R}{r_c}\right)$$

- Long range interactions

Local cosmic strings

Local cosmic string arise in theory with a broken **local** U(1) symmetry

- **Example**: Grand unified theories (different than fundamental strings!)
- Captured by a theory with a complex scalar and fields + mexican-hat potential

$$S_{\text{local}} = -\int d^4x \left[(D_\mu \varphi)^* (D^\mu \varphi) + \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \lambda \left(|\varphi|^2 - \frac{v^2}{2} \right)^2 \right]$$

- Equations of motion identical than for standard U(1) theories

$$\begin{aligned} (a^2 \varphi')' - a^2 D_i D_i \varphi &= -2\lambda a^4 \left(|\varphi|^2 - \frac{v^2}{2} \right) \varphi \\ F'_{0i} - \partial_j F_{ji} &= 2a^2 e \, \mathrm{Im}[\varphi^* D_i \varphi] \end{aligned}$$

  - Use **conformal time** ($\alpha = 1$) and **temporal gauge** ($A_0 = 0$)
  - Dynamics depend on a single parameter, $\beta = e^2/2\lambda$ ($\beta = 1$, **critical case**)

Local cosmic strings

- Contains **massive particles**

$$m_\chi = \sqrt{2\lambda}v \qquad m_A = ve \qquad \text{(equal for } \beta = 1\text{)}$$

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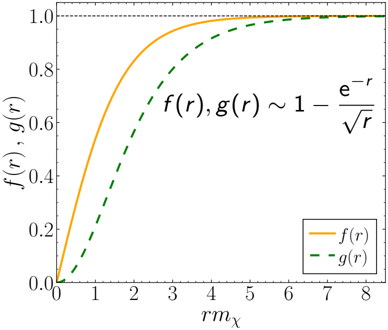
$$m_\chi = \sqrt{2\lambda} v \qquad m_A = ve \qquad \text{(equal for } \beta = 1\text{)}$$

➤ Analytical solution for **infinite straight string**



- Ansatz:

$$\varphi = f(r)\frac{v}{\sqrt{2}}e^{i\theta}$$
$$A_i = -\frac{g(r)}{er}\hat{\theta}_i$$

- **Finite tension**:

$$\mu = 2\pi v^2$$

- Short range interactions

Nambu-Goto approximation

Traditionally strings have been studied in the **Nambu-Goto** limit
(**infinitely-thin string approximation**)

- **Infinite strings** only decay via the **emission of loops**
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Accurate predictions need to incorporate the **field-theory nature** of strings → **Lattice simulations**

Overview

Cosmic strings on the lattice

Simulations of cosmic strings and other topological defects follow the **same basics as standard simulations of scalar and gauge theories**

Cosmic strings on the lattice

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Some **new additions**:

- **Resolution-preserving techniques**
- **Initial conditions** that generate a network of defects
- **Specialized observables** to keep track of the dynamics of the defects

Available in ***CosmoLattice* v2.0**

Loss of resolution on the lattice

Lattice simulations of cosmic defects suffer from **limited dynamical range**

Early universe

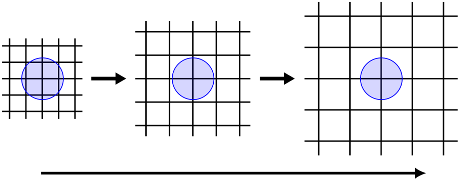
$$R/r_c \sim \begin{array}{l} 10^{30} \text{ (axions)} \\ 10^{50} \text{ (GUTs)} \end{array}$$

$\updownarrow$

Lattice

$$R/r_c \lesssim 10^4 - 10^5$$

Loss of resolution:



Simulation time

Resolution-preserving techniques

- **Fattening**: keep resolution constant

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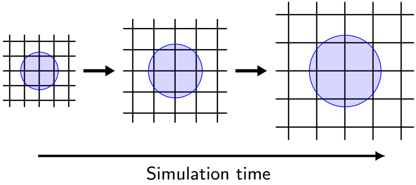

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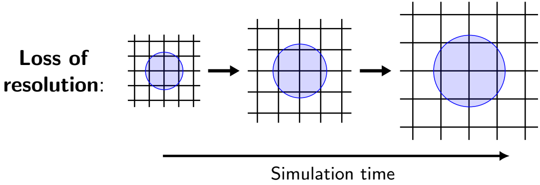
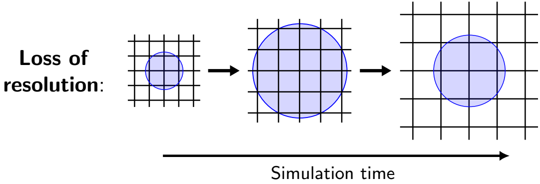


- **Extra-fattening**: grow core first, followed by physical evolution



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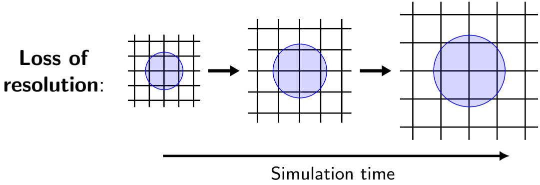

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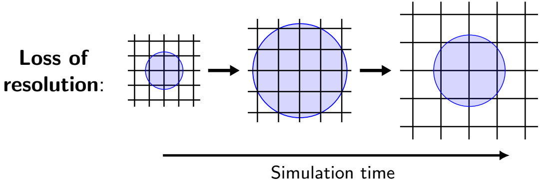

- **Other approaches**: Adaptative mesh refinement, this is, locally increase resolution at the defect (not in $\mathcal{C}$osmo$\mathcal{L}$attice v2.0)

Resolution-preserving techniques

Generalized equations of motion:

$$\begin{aligned}(a^2\varphi')' - a^2 D_i D_i &= -2a^{2(s+1)}\lambda\left(|\varphi|^2 - \frac{v^2}{2}\right)\varphi \\ (a^{2(1-s)}F'_{0i})' - a^{2(1-s)}\partial_j F_{ji} &= 2a^2 e\,\mathrm{Im}\left[\varphi^* D_i\varphi\right]\end{aligned}$$

- Different s correspond to different evolution:
  - **Physical**: $s = 1 \longrightarrow w_c \sim a^{-1}$
  - **Fattening**: $s = 0 \longrightarrow w_c \sim \text{const}$
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➤ Equivalent to promote λ and e to time-dependent variables

$$\lambda(\tau) = \left(\frac{a}{a_0}\right)^{2(s-1)}\lambda \qquad e^2(\tau) = \left(\frac{a}{a_0}\right)^{2(s-1)}e^2$$

after redefining $A_\mu \longrightarrow A_\mu/e$

Initial conditions: phase transition vs diffusion

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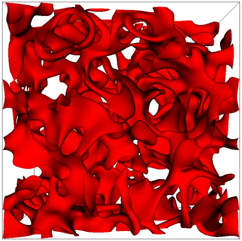

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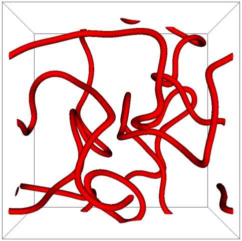

Cosmic string observables

New observables allow to get information about the strings

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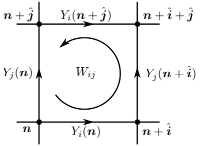
➤ Measure the **pierced plaquettes** ⟶ Location of the **string core**

- Non-zero winding on pierced plaquettes

$$W_{ij}(\boldsymbol{n}) = \frac{1}{2\pi}\left[Y_i(\boldsymbol{n}) + Y_j(\boldsymbol{n}+\hat{\boldsymbol{i}}) - Y_i(\boldsymbol{n}+\hat{\boldsymbol{j}}) - Y_j(\boldsymbol{n})\right]$$

- **Total length** of strings: $L_\mathrm{w} = \frac{2}{3}\delta x \#_\mathrm{pierced}$

Manhattan effect



$$Y_i(\boldsymbol{n}) = [\theta(\boldsymbol{n}) - \theta(\boldsymbol{n}-\hat{\boldsymbol{i}})]_\pi$$

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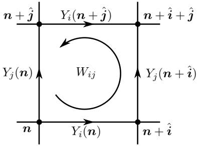
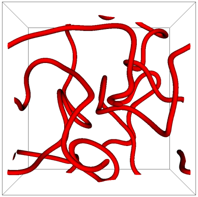
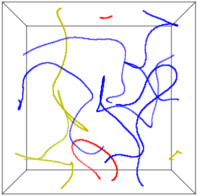


- **Total length** of strings: $L_{\rm w} = \frac{2}{3}\delta x \#_{\rm pierced}$

- Real-time quantitative information of network structure





Cosmic string observables

New observables allow to get information about the strings

- String energy components using **weight functions**

$$E_{\rm str} = \int d^3x W[\varphi]\rho(\boldsymbol{x}) \qquad \text{with} \qquad W[\varphi] \propto V[\varphi]\Theta\left(|\varphi|^2 - \frac{1}{2}\right)$$

- **Estimates of total length and mean-squared velocity**

Global string :

$$L_{\rm str} = \frac{1}{\mu}\frac{E_{\rm str}+f_V L_{\rm str}}{1-f_V}$$

$$v^2_{\rm rms} = \frac{E_{\rm str}+L_{\rm str}}{E_{\rm str}+f_V L_{\rm str}}$$

where μ and f_V are computed from the infinite string solution and

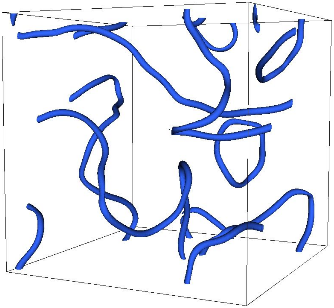
$$L_{\rm str} = K_{\rm str} - G_{\rm str} - V_{\rm str}$$

Overview

Decay of isolated loops

We have estudied the **decay of loops into particles and GWs**
[JBB, Copeland, Figueroa and Lizarraga, 2023 and 2024]

Local string



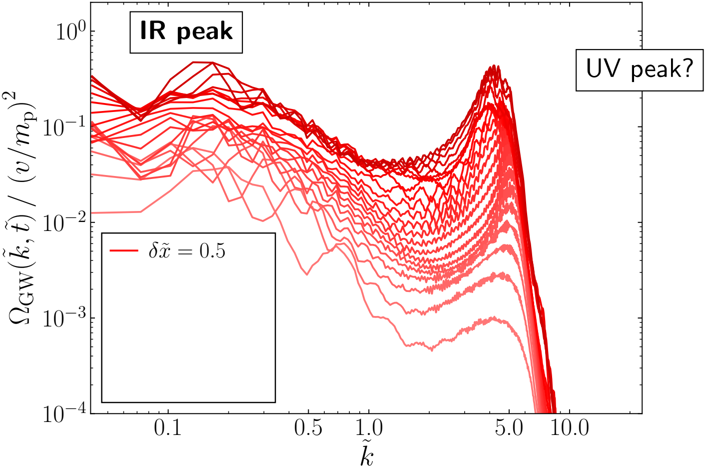
- We studied **isolated global and local loops in flat background** to avoid loss of resolution
- Analize emission of **particles and GWs** simultaneously
- Reached separation of scales

$$L_w/r_c \sim 10^4$$

- **Goal**: combine results for the GW emission with history of loop number density to predict GWB

GW emission from loops: UV resolution

Global loop: Fixed loop and lattice size + vary δx



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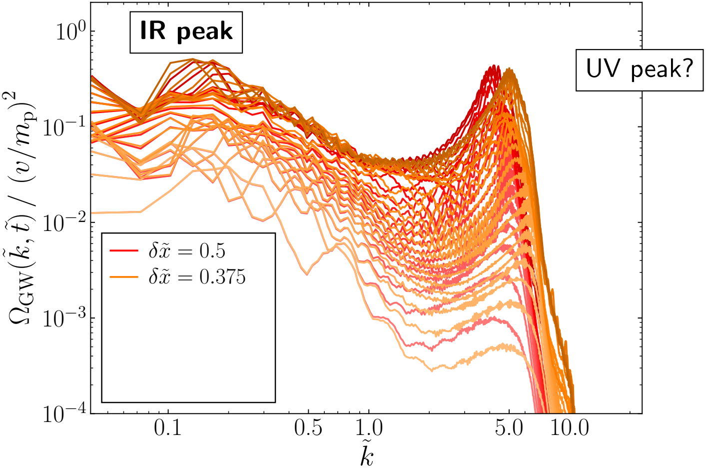

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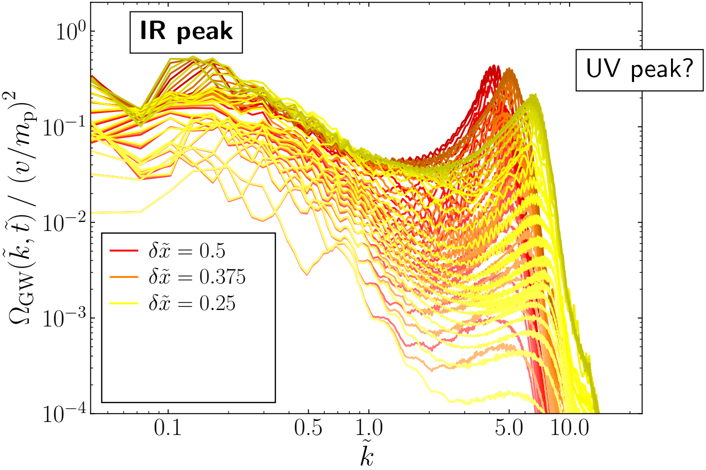

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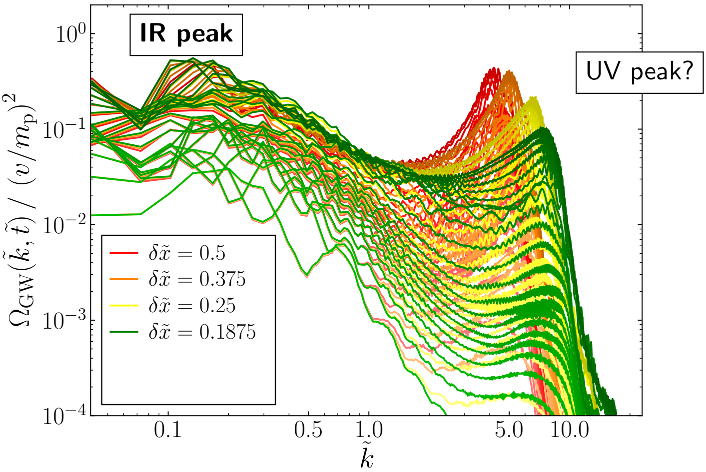

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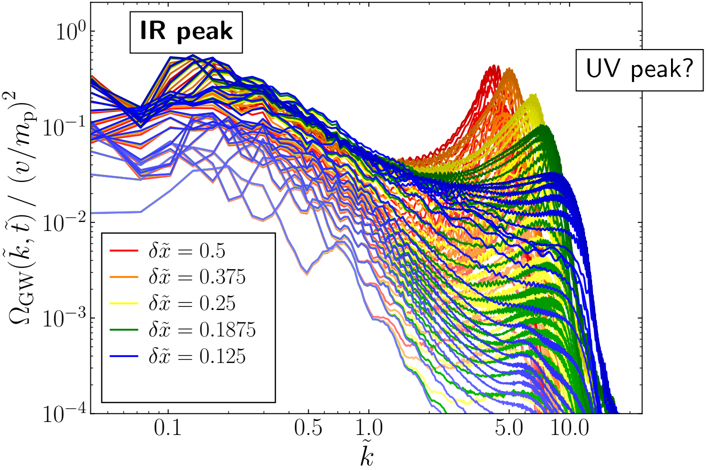

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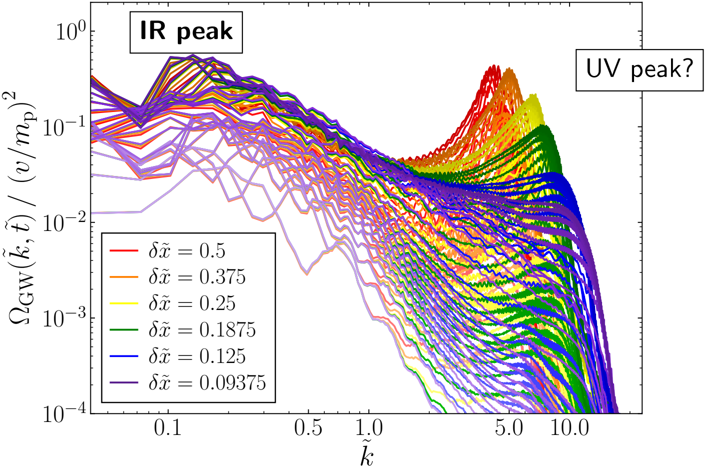

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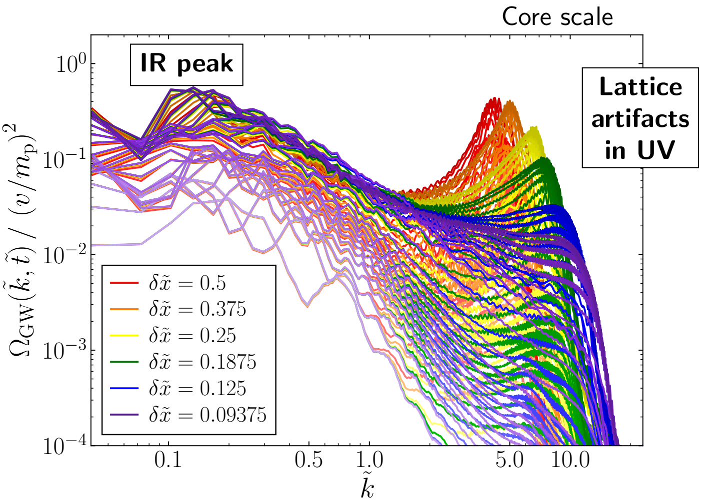

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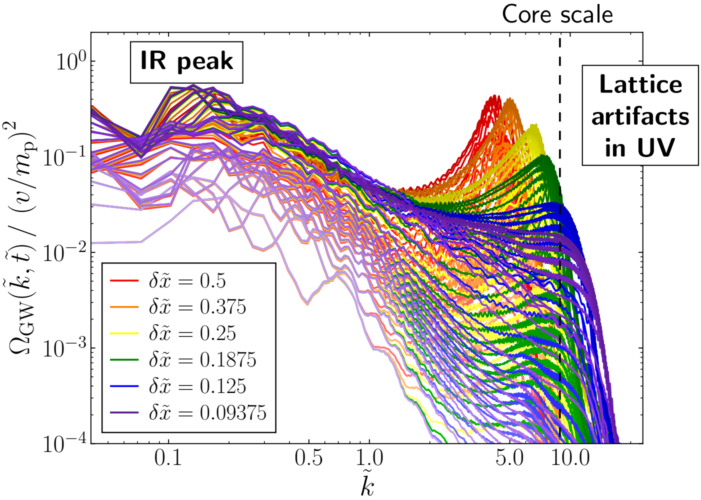

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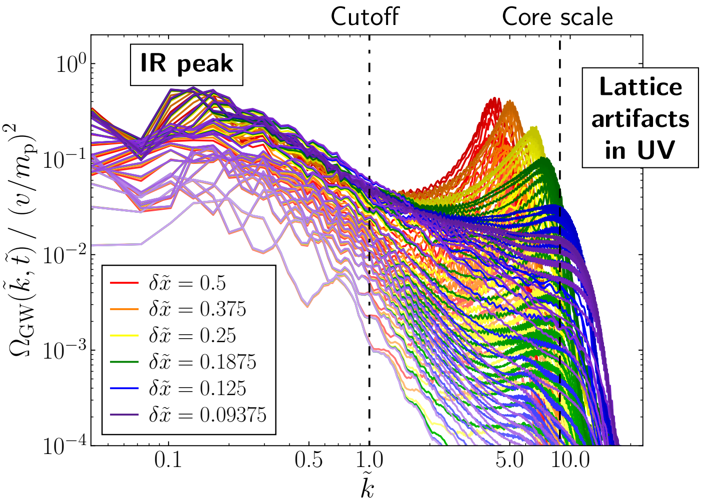

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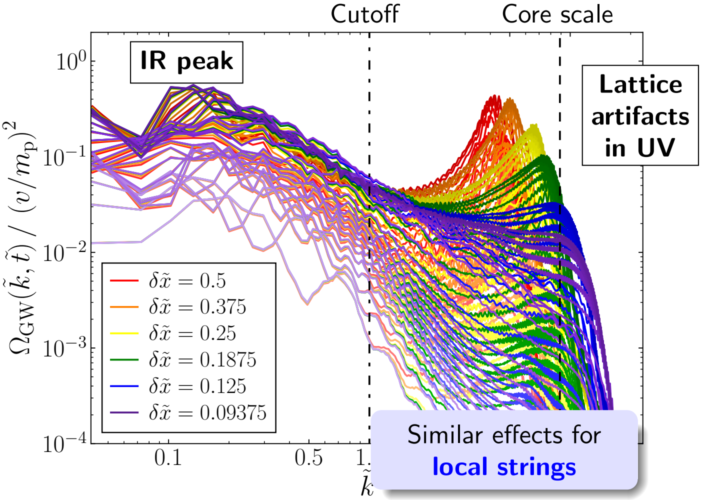
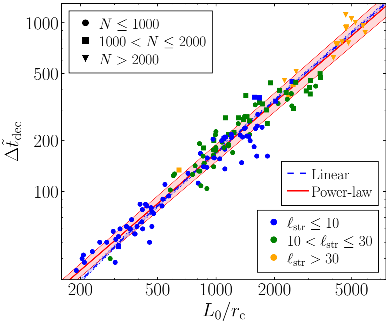


Similar effects for **local strings**

Particle and GW emission from local loops

We have simulated 100+ **local loops**, reaching $L_0/r_c \lesssim 7000$ [JBB et al. 2024]

Particle emission



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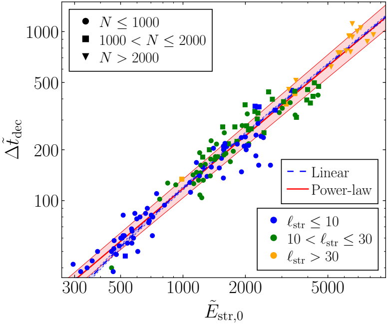

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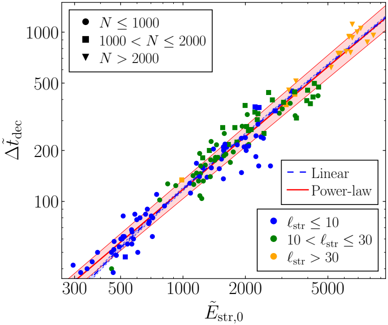

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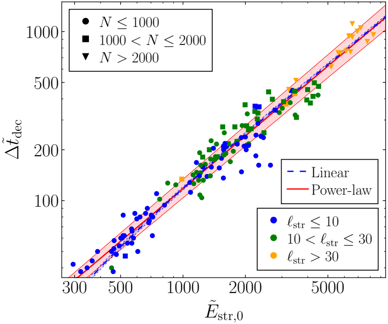

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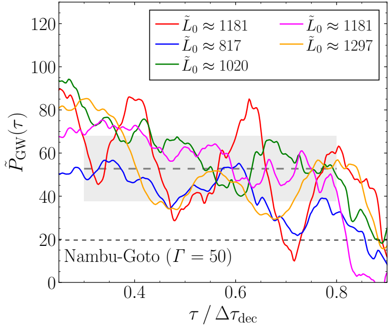

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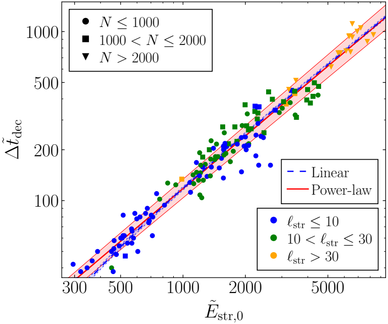

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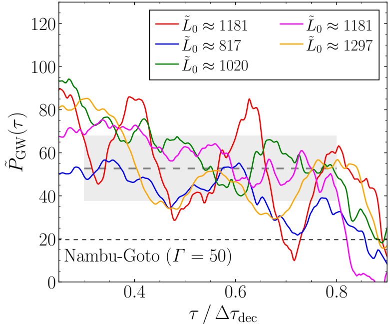

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GW emission is **very suppressed** compared to NG predictions

Overview