The shape of the nucleus: shape invariants and connection to $2\nu\beta\beta$

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EXCELENCIA MARÍA DE MAEZTU
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Deformation is everywhere:

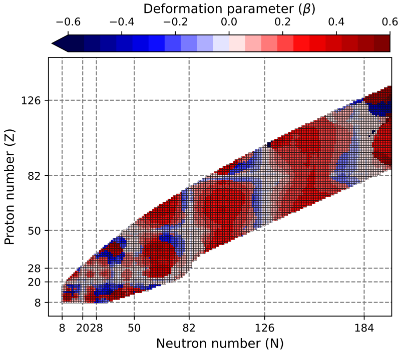
- Magic nuclei are spherical
- Deformation is enhanced mid-shell (most nuclei)

Quadrupole deformation:

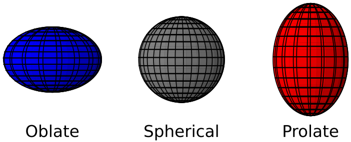
- Most extended shape
- Prolate: elongated sphere
- Oblate: flattened sphere

Shape coexistence:

- Different shapes coexist within the **same nucleus** at different energies
- Present in most nuclei



P. Möller, et al. Atom. Data Nucl. Data Tabl. 109-110 (2016)



Deformation is characterized in the intrinsic frame: (β, γ)

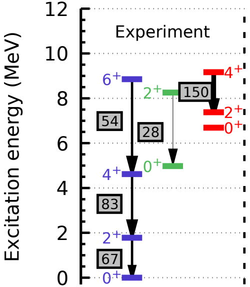
However, measurements are done in lab frame: $(B(E2), Q_s)$

Conversion from lab to intrinsic frame:

$$Q_s(J) \sim Q_{0,s}, \quad J \geq 1/2$$

$$B(E2, J_i \rightarrow J_f) \sim Q_{0,t}^2$$

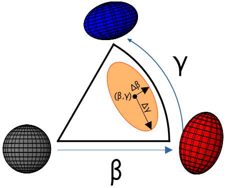
$\rightarrow$ Perfect axial rotor assumption: $Q_{0,s} \simeq Q_{0,s}$



Shape invariants $\langle Q^n \rangle$: model independent

Couple Q_2 (tensor) to obtain a scalar:

- $\langle Q^2 \rangle = \langle [Q_2 \times Q_2]_0 \rangle \rightarrow \beta$
- $\langle Q^3 \rangle = \langle [[Q_2 \times Q_2]_2 \times Q_2]_0 \rangle \rightarrow \gamma$
- All $\langle Q^n \rangle$ up to $\langle Q^6 \rangle$ are needed for fluctuations of (β, γ)



K. Kumar, Phys. Rev. Lett. 28, 249 (1972)

D. Cline, Nuclear Structure 313–326 (1985)

Shape invariants: Intermediate-state expansion

Intermediate-state expansion:

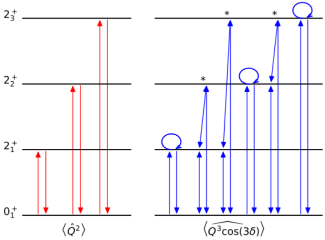
- (Infinite) expansion of $M_{if} = \langle i||Q||f\rangle \leftarrow Q_s$ or $B(E2)$
- Harder convergence for higher $\langle Q^n\rangle$
  $$\langle Q^2\rangle_i \sim \sum M_{it}M_{ti} \to \beta$$
  $$\langle Q^3\rangle_i \sim \sum M_{iu}M_{ut}M_{ti} \to \gamma$$

Sum-rule method:

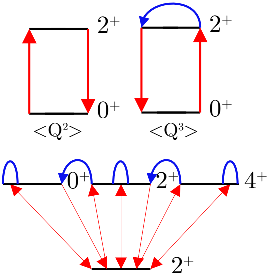
- Apply the $\hat{Q}_{20}$ operator to a state:
  $\hat{Q}_{20}|0^+\rangle = \mathrm{SR}(2^+)^{1/2}\overline{|2^+\rangle}$
  $\langle Q^2\rangle \sim \mathrm{SR}(2^+)$, $\langle Q^3\rangle \sim \overline{\langle 2^+|}Q\overline{|2^+\rangle}$
- Apply $\hat{Q}_{20}$ twice $\to \langle Q^4\rangle$, $\langle Q^6\rangle$

  Poves, A., Nowacki, F. & Alhassid, Y. Phys. Rev. C 101, 054307 (2020)

- **NEW!:** Extension for $J \neq 0$



J. Henderson Phys Rev C 102, 054306 (2020)

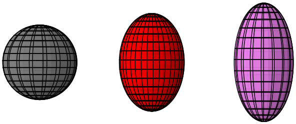


DF, A. Poves, J. Menéndez (in preparation)

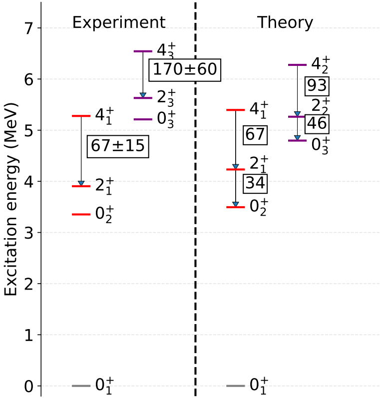
Doubly magical nucleus ($Z = N = 20$)

Shape coexistence in ^{40}Ca:

1. 0_1^+ (0.0 MeV): Spherical 0p-0h
2. 0_2^+ (3.4 MeV): Deformed 4p-4h
3. 0_3^+ (5.2 MeV): Superdeformed 8p-8h



Spherical Normal Superdeformed

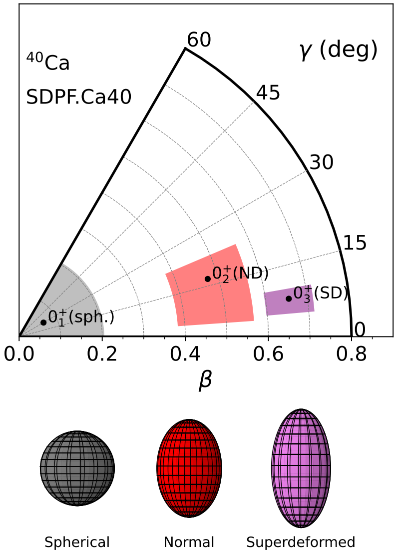


Accurate description of ^{40}Ca within the nuclear shell model

E. Caurier, J. Menéndez, F. Nowacki, and A. Poves
Phys. Rev. C **75**, 054317 (2007)

^{40}Ca shape coexistence:

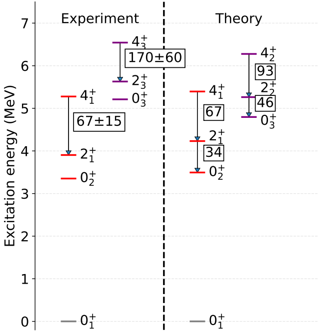
- "Spherical":
  $\beta = 0.07 \pm 0.14$
  $\gamma = -\ (0 - 60)$
  Compatible with $\beta = 0$ but reaches up to $\beta = 0.2$
- Normal deformed:
  $\beta = 0.47 \pm 0.09$
  $\gamma = 17\ (4 - 23)$
  Considerable fluctuations
- Superdeformed:
  $\beta = 0.66 \pm 0.06$
  $\gamma = 8\ (5 - 10)$
  Lower fluctuations for SD

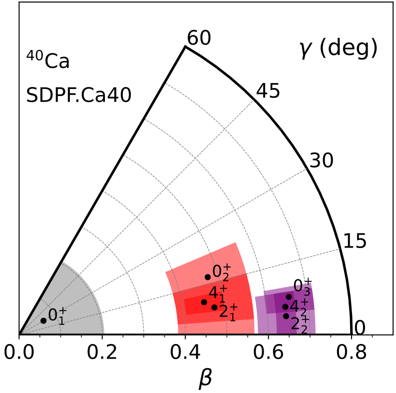


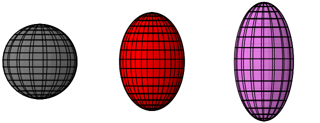
Band evolution in ^{40}Ca

Band evolution of (β, γ):

- (β, γ) are consistent across the band (with fluctuations)
- Robust method to identify states with the same intrinsic shape







Deformation impact on $2\nu\beta\beta$

- $2\nu\beta\beta$ (D. Castillo talk)

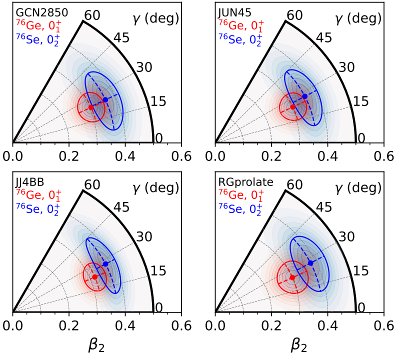
$$^{A}_{Z}X \rightarrow ^{A}_{Z+2}Y + 2e^{-} + 2\bar{\nu}$$

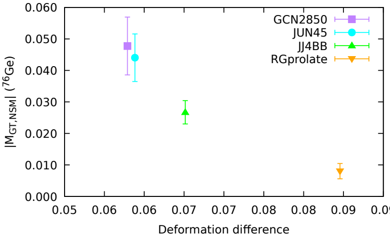
$$M_{GT}{}^{2} = \langle \Psi_f | \hat{O}_{GT} | \Psi_i \rangle^{2} \sim T_{1/2}^{-1}$$

D. Castillo, D. Frycz, B. Benavente & J. Menéndez
arXiv:2507.21868

Impact of deformation on $2\nu\beta\beta$:

- **Uncertainty** of M_{GT} stems from **interactions**
  $\rightarrow$ Structural differences
- Deformation differences of $|\Psi_f\rangle$ and $|\Psi_i\rangle$ **reduce** M_{GT}
- δ: polar plane distance
- Correlation: M_{GT} with δ
- Large impact of triaxiality γ





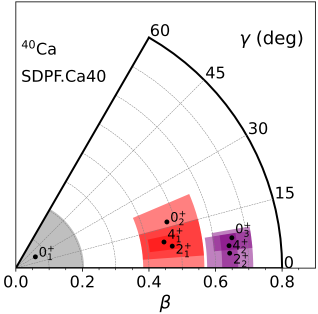
Conclusions

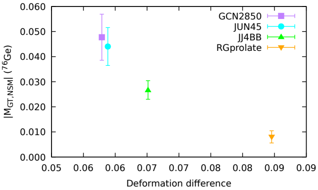
Shape invariants:

- General method to **identify shapes**: $\langle Q^n \rangle \to (\beta, \gamma)$
- (β, γ) fluctuations are large
- **Extension** of sum rule method for any J (up to $\langle Q^4 \rangle$)
- (β, γ) are **constant** across the (rotational) bands

Impact of deformation in $2\nu\beta\beta$:

- Deformation plays a key role
- **Correlation** of M_{GT} with δ
- Relevance of triaxiality (γ)





D. Castillo, D. Frycz, B. Benavente & J. Menéndez arXiv:2507.21868

THANK YOU!!!

Feel free to ask any questions

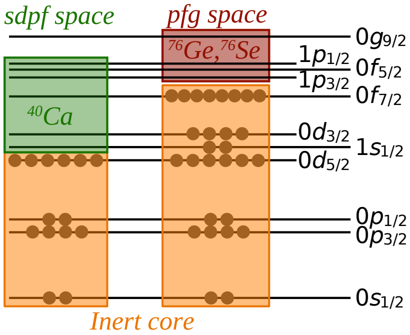
Shell model valence space

Schrödinger equation

$$\mathcal{H}|\Psi\rangle = E|\Psi\rangle$$

- **Interacting shell model**:
  $\mathcal{H}_{\text{eff}} = \mathcal{H}_0 + \mathcal{H}_{\text{res}}$
  $\mathcal{H}_{\text{res}}$: valence space
- Slater determinant basis $\{\Phi_i\}$
- Configuration mixing:
  $|\Psi_{\text{NSM}}\rangle = \sum_i C_i|\Phi_i\rangle$

Caurier E., et al. Rev. Mod. Phys. **77**, 427 (2005)



- Phenomenological interactions:
  ^{40}Ca: SDPF-Ca40
  ^{76}Ge and ^{76}Se: JUN45, JJ4BB, GCN2850, and RG-prolate

Intrinsic vs laboratory frame

Deformation is characterized in the intrinsic frame: (β, γ)

However, measurements are done in lab frame: $(B(E2), Q_s)$

Relation between lab and intrinsic frame:

$$Q_s(J) = \frac{3K^2 - J(J+1)}{(J+1)(2J+3)} Q_{0,s},$$

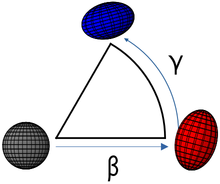
$$B(E2, J_i \to J_f) = \frac{5{Q_{0,t}}^2}{16\pi} \langle J_i K 20 | J_f K \rangle$$

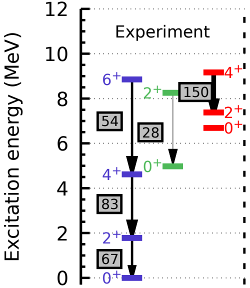
- If $Q_{0,s} \simeq Q_{0,t}$: well deformed rotor

1. For even-even nuclei (J=0) $\to$ $Q_s = 0$
2. Triaxial shapes diminish Q_s
3. $B(E2)$: scattered across out-band states

$\to$ Perfect axial rotor assumption!

Shape invariants $\langle Q^n \rangle$: model independent measure of deformation





Shape invariants: Sum rule method

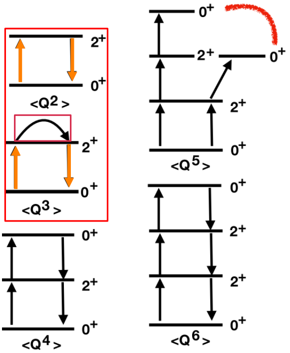
1). Apply the $\hat{Q}_{20}$ operator to a state:
$\hat{Q}_{20}|0^+\rangle = \text{SR1}(2^+)^{1/2}\overline{|2^+(1)\rangle}$
$\rightarrow \langle Q^2\rangle = 5\text{SR1}(2^+)$

- $\overline{|2^+(1)\rangle}$ not an eigenstate but contains the whole quadrupole strenght

2). Evaluate $\langle\langle Q\rangle\rangle = \langle 2^+(1)|Q|2^+(1)\rangle/\sqrt{5}$
$\rightarrow \langle Q^3\rangle = \langle Q^2\rangle\langle\langle Q\rangle\rangle$

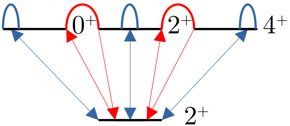
3). Apply $\hat{Q}_{20}$ a second and third time to obtain $\langle Q^4\rangle$, $\langle Q^5\rangle$, and $\langle Q^6\rangle$

- **Exact** calculation (up to numeric)
- Computationally cheap
- Currently only for $J = 0$
- **New!:** extension for any J up to $\langle Q^4\rangle$



Poves, A., Nowacki, F. & Alhassid, Y. Phys. Rev. C 101, 054307 (2020)

To be published:

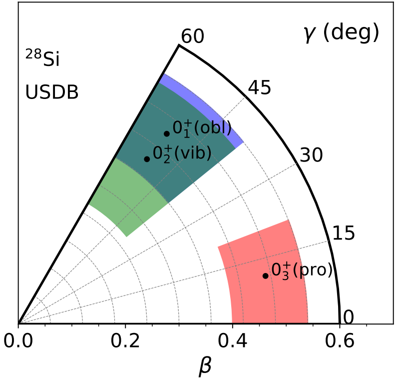


Shape invariants for ^{28}Si

^{28}Si shape coexistence (USDB):

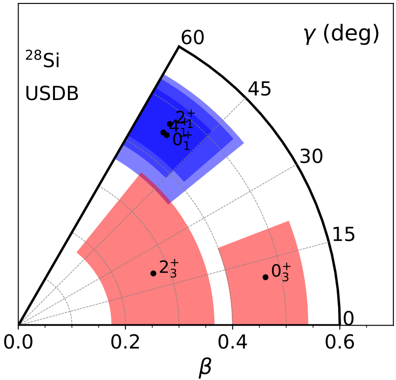
- Oblate: $\beta = 0.45 \pm 0.09$
  $\gamma = 53\ (39 - 60)$
- Vibration: $\beta = 0.39 \pm 0.13$
  $\gamma = 53\ (39 - 60)$
- Prolate: $\beta = 0.47 \pm 0.07$
  $\gamma = 11\ (0 - 21)$

- Shapes are γ **and** $\boldsymbol{\beta}$ **soft**
- Vibration is **similar** in deformation to oblate GS
- Prolate shape has similar deformation as GS



Band evolution of (β, γ):

^{28}Si	$\beta \pm \Delta\beta$	γ ($\Delta\gamma$)
0_1^+	0.45±0.09	52 (39-60)
2_1^+	0.47±0.06	53 (41-60)
4_1^+	0.45±0.06	53 (45-60)
0_3^+	0.47±0.07	11 (0-21)
2_3^+	0.27±0.10	21 (0-51)



Oblate (good rotational band):

- (β, γ) are consistent
- **Band states overlap**

Prolate (not really a band):

- 0_3^+ and 2_3^+ **different shapes**

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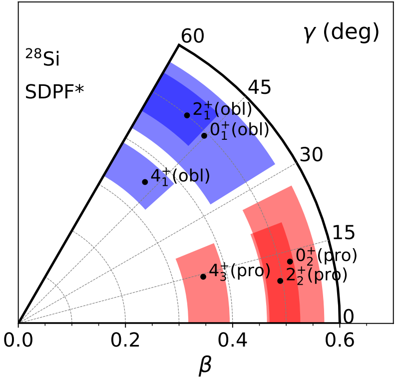

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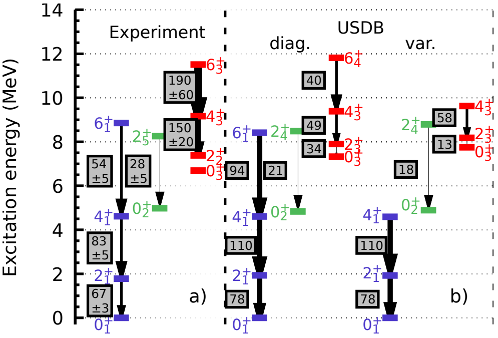
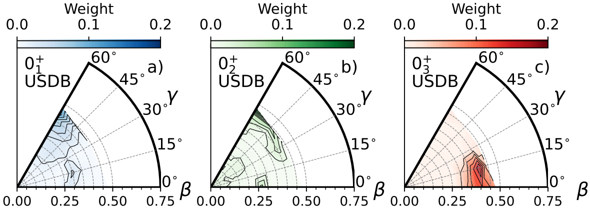
Spectrum of ^{28}Si (USDB)

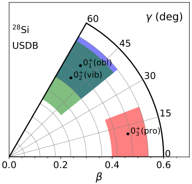
Oblate rotational band: **well described**, slightly more deformed

Vibrational band based on the ground state is also **well described**

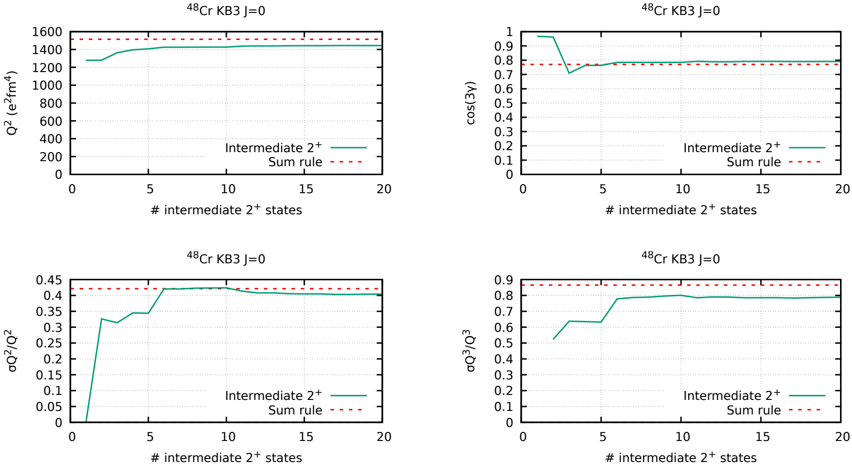
Prolate rotational band has too weak $B(E2)$







A word of caution:



The convergence is worse for more complex nuclei and other J

Shape invariants