MINISTERIO
DE CIENCIA, INNOVACIÓN
Y UNIVERSIDADES

Cofinanciado por
la Unión Europea

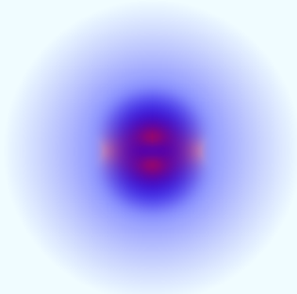
AGENCIA
ESTATAL DE
INVESTIGACIÓN

# The continuum of $^{17}$C and $^{19}$C studied via direct reactions

P. Punta, J. A. Lay and A. M. Moro
Departamento de Física Atómica Molecular y Nuclear,
Universidad de Sevilla

# Weakly-bound exotic nuclei

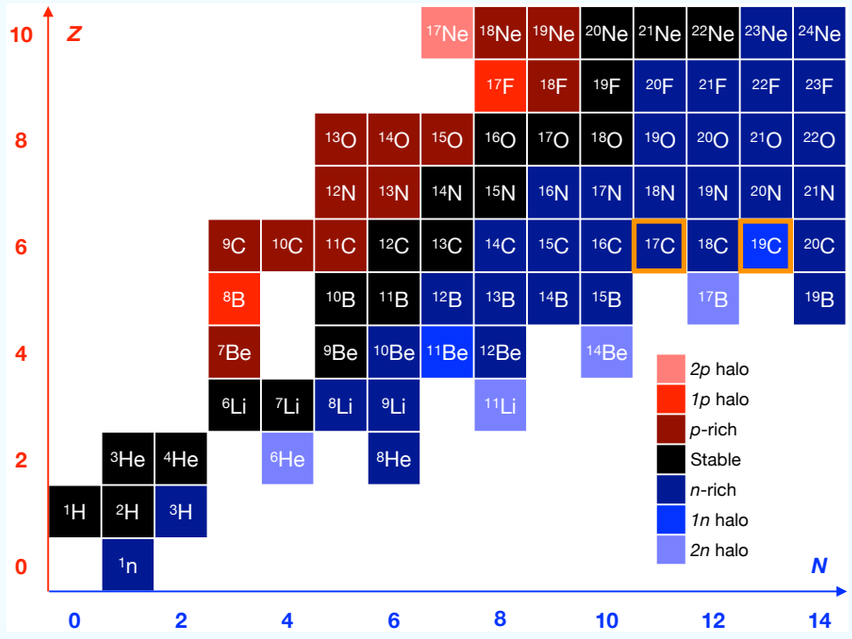
- Different properties
  - Halo nature
  - Shell closure

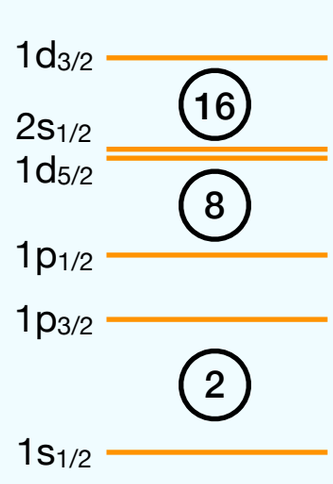


$^{19}C$

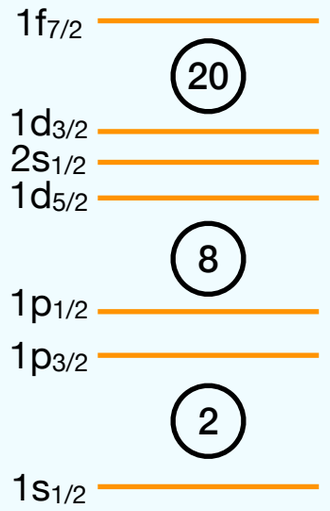
- Direct Reactions involving them
  - Advances in radioactive beam facilities
  - Realistic but manageable models

- Few-body models
  - Significant effect of deformation
  - Deformed core + neutron structure



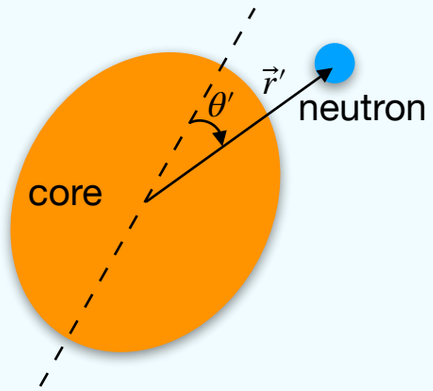

$^{17}C$




Stable Nuclei

# Nilsson+AMD model (NAMD)

P. Punta *et al.* PRC**111** (2025) 064614




- Deformed two-body model
- Nilsson Hamiltonian

$$H_N = T + V_0(r) + V_{\ell s}(r)(\overrightarrow{\ell} \cdot \vec{s}) + V_2(r)Y_{20}(\theta')$$

- Semimicroscopic coupling potentials with AMD transition densities
  [J. A. Lay *et al.* PRC**89** (2014) 014333]

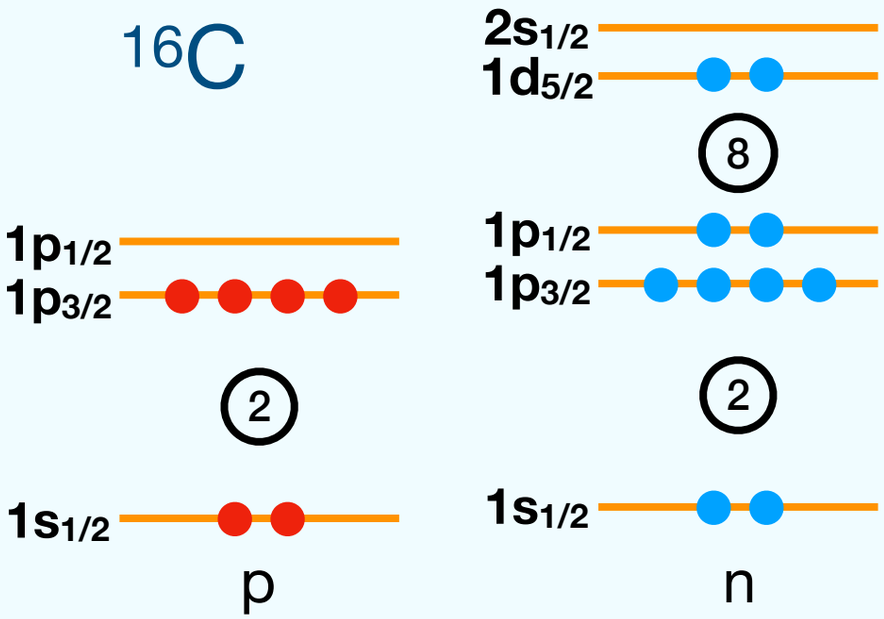
$$V_\lambda(r) = \int dr_1 r_1^2 \left\{ v_{IS}^\lambda(r_1, r)[\rho_{\lambda^+\to 0^+}^{n\lambda}(r_1) + \rho_{\lambda^+\to 0^+}^{p\lambda}(r_1)] + v_{IV}^\lambda(r_1, r)[\rho_{\lambda^+\to 0^+}^{n\lambda}(r_1) - \rho_{\lambda^+\to 0^+}^{p\lambda}(r_1)] \right\}$$

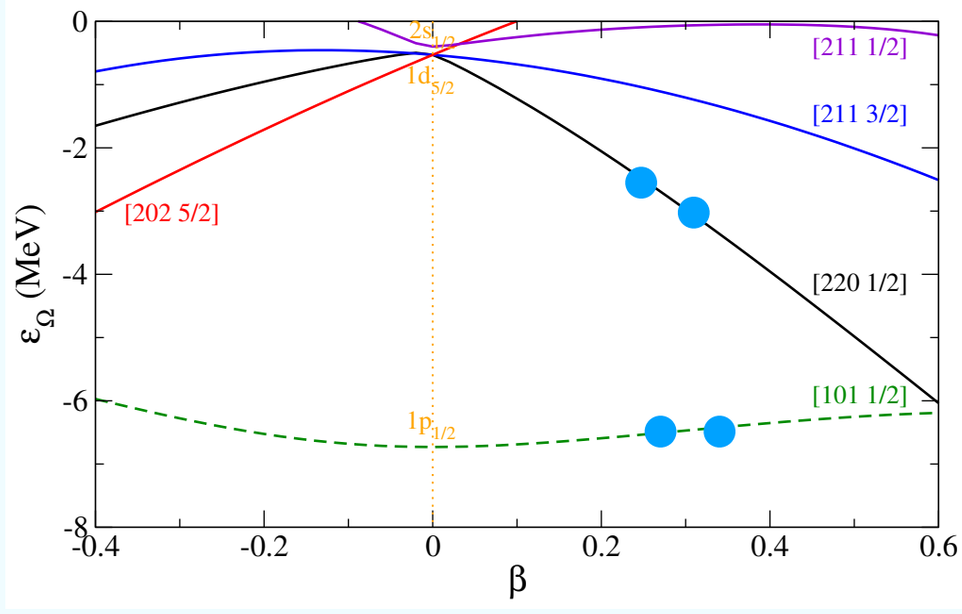
- Collective rotational term $H = H_N + \frac{\hbar^2}{2\mathcal{J}}\vec{I}^2$

- Eigenstates from diagonalization in THO basis
  [S. Karataglidis *et al.* PRC**71** (2005) 064601]

$$\phi_{nl}^{THO}(r) = \sqrt{\frac{ds}{dr}}\phi_{nl}^{HO}[s(r)]$$

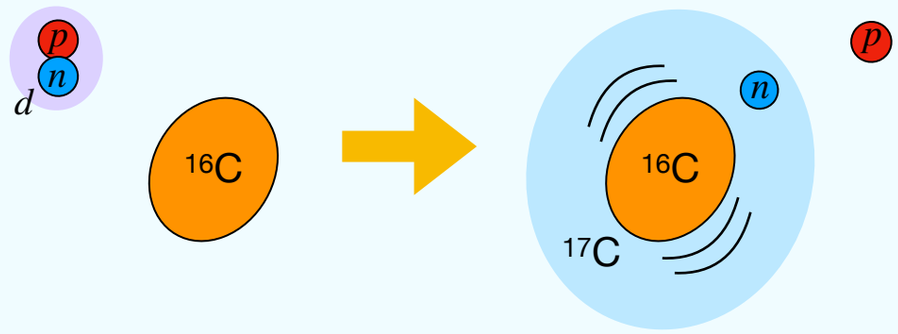
# Pauli-blocking effects




- Factorization of the system does not allow complete antisymmetrization
- States occupied by core neutrons should be blocked for the valence neutron
- **Total Blocking method (TB)**: The occupied Nilsson states are excluded in the diagonalization of the complete Hamiltonian

# Application to neutron transfer reactions




- $^{16}$C($d$,$p$)$^{17}$C reaction is studied applying the adiabatic distorted wave Approximation (ADWA)

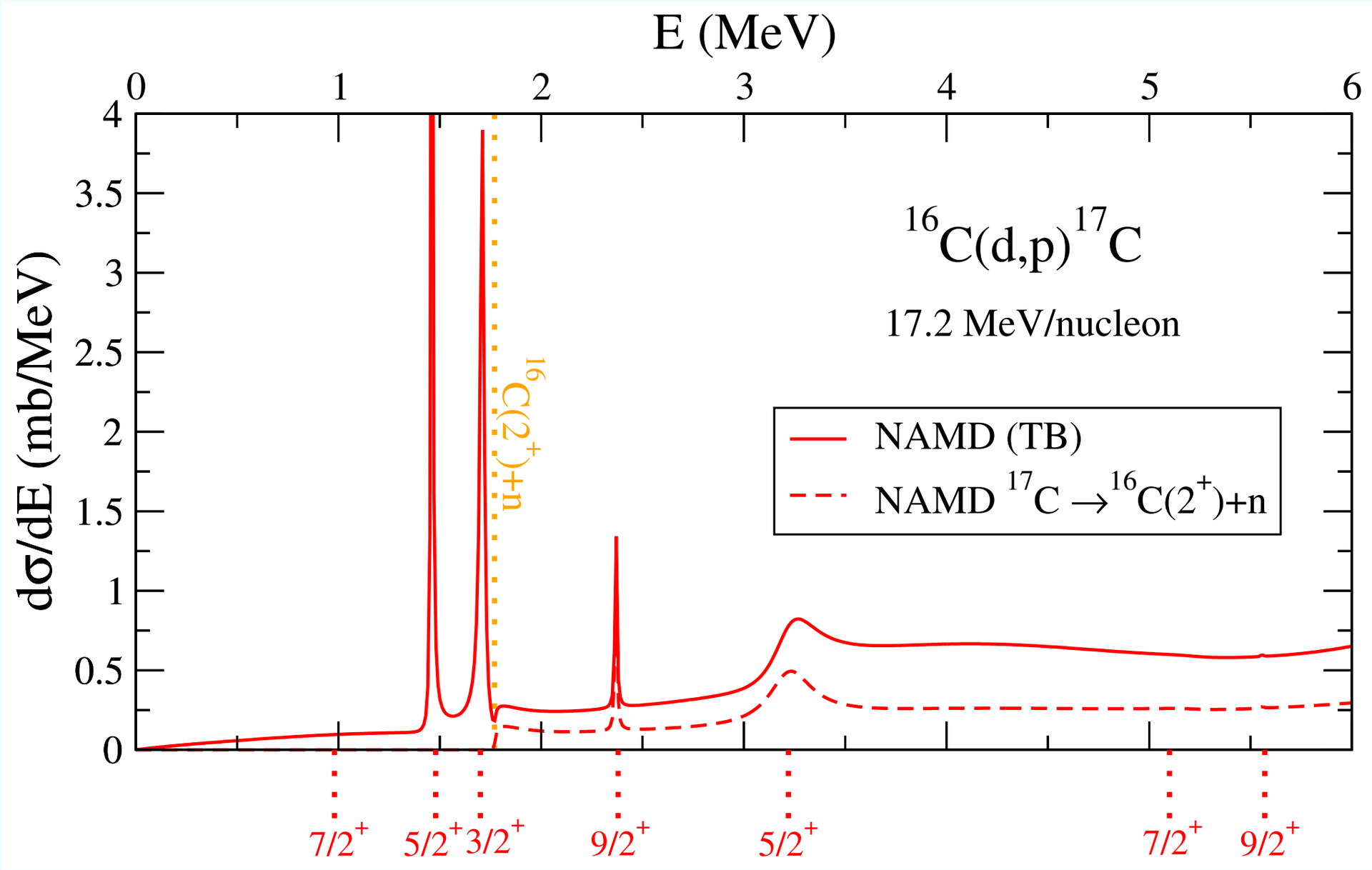
- Transfer to bound states: in the *post* form, the calculations only require the $\langle {}^{17}\mathrm{C} | {}^{16}\mathrm{C}(0^+) \rangle$ overlaps

- Transfer to the continuum: In the *prior* form, the calculations involves the vertex functions $\langle {}^{17}\mathrm{C} | V_{n-{}^{16}\mathrm{C}} | {}^{16}\mathrm{C}(0^+) \rangle$.

- Calculations are compared with experimental data from GANIL

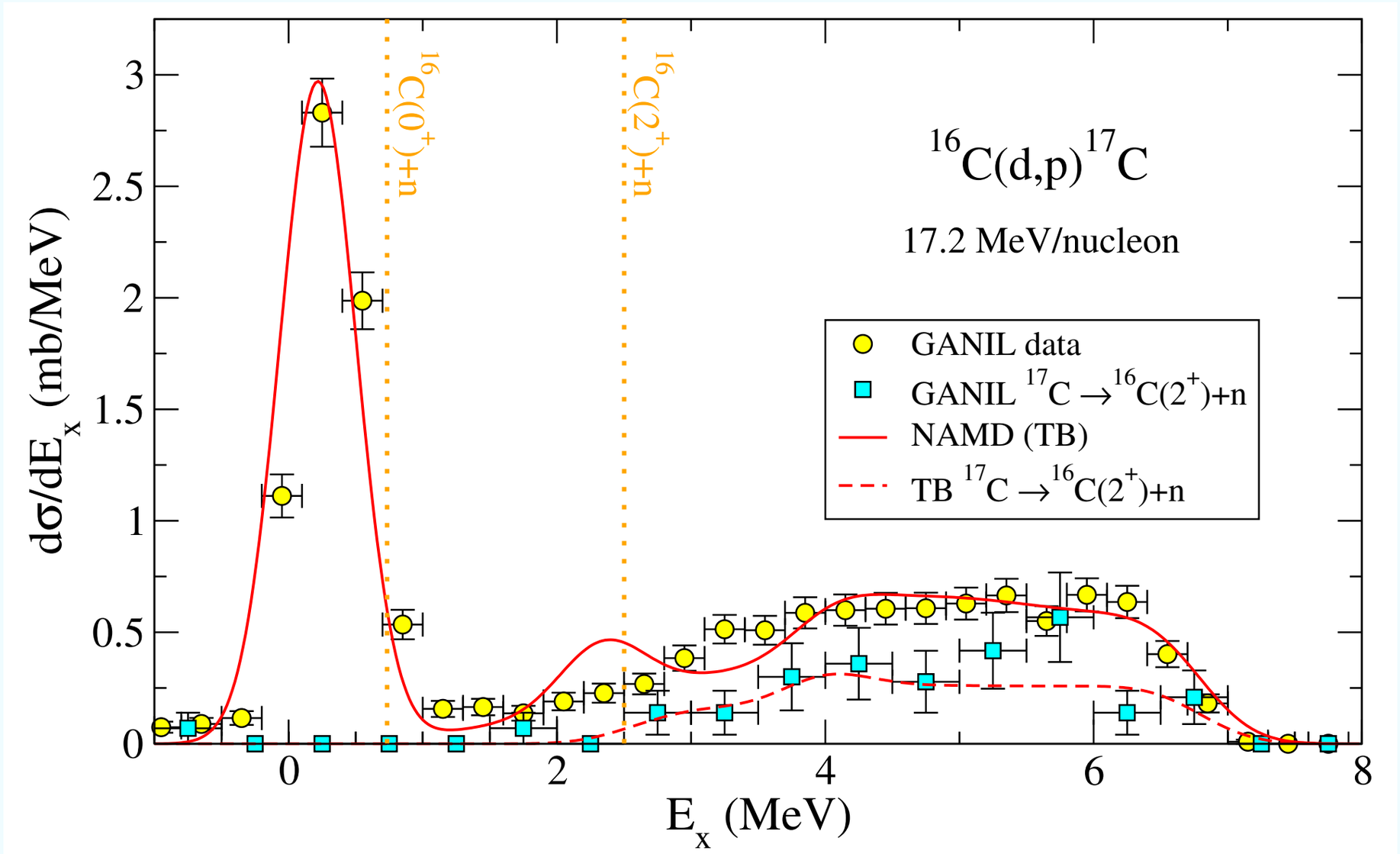
  - J. Lois-Fuentes *et al.* PLB**867** (2025) 139600

# $^{16}$C(*d*,*p*)$^{17}$C: transfer to the continuum



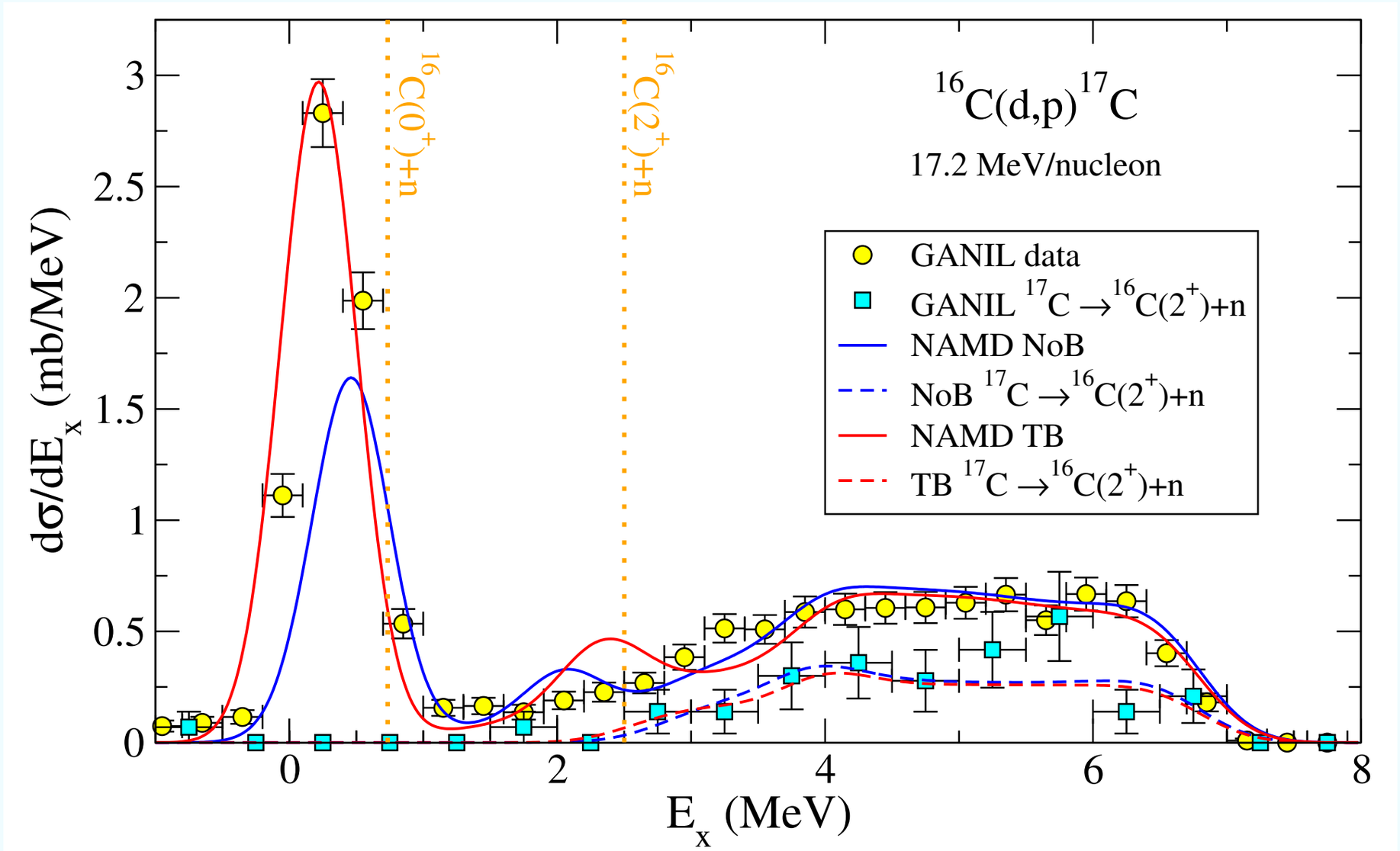

$E$: Energy with respect to $^{16}$C(0$^+$)+n threshold

# $^{16}$C(*d*,*p*)$^{17}$C: transfer to the continuum




$E_x$: Excitation energy of $^{17}$C

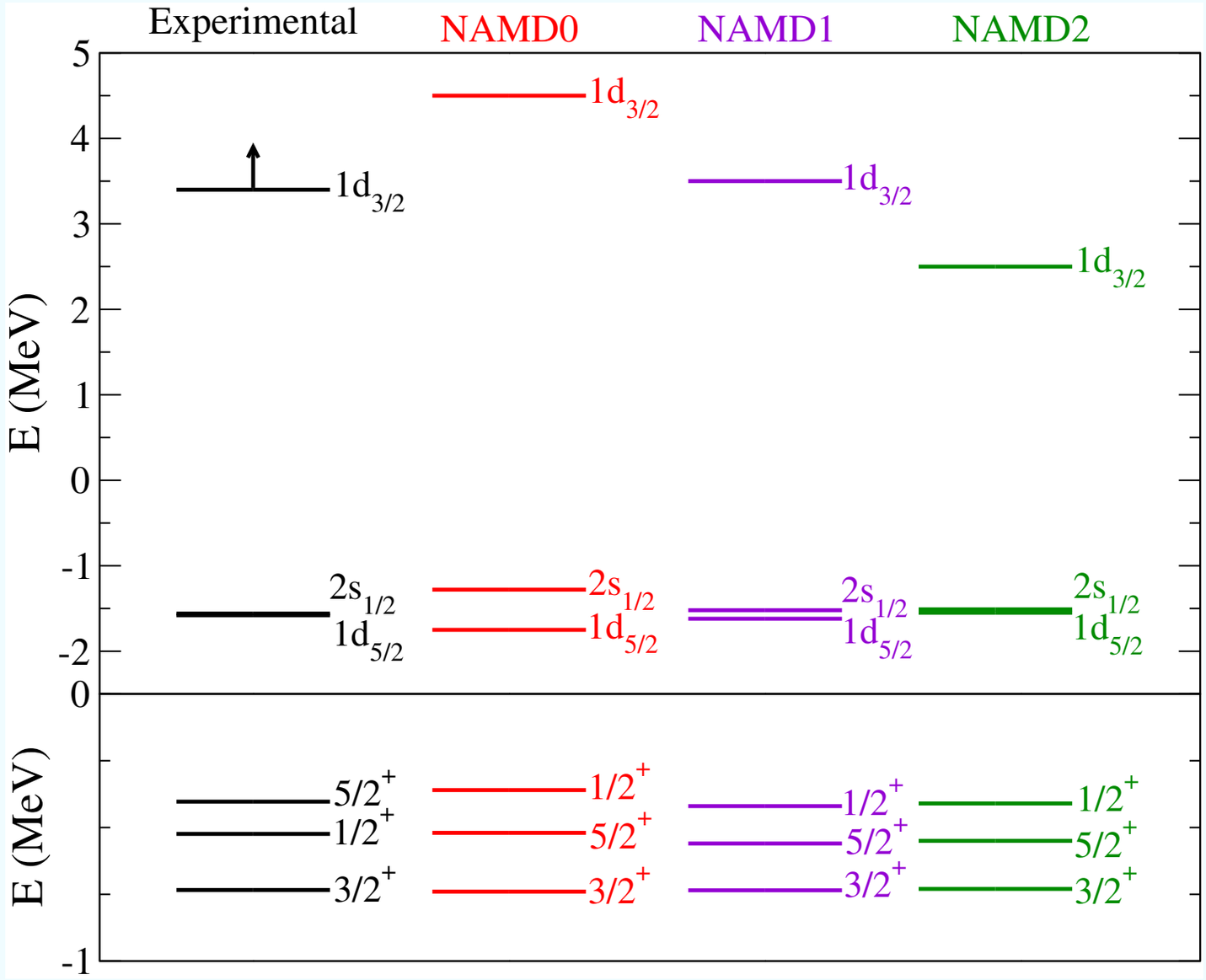
# $^{16}$C(*d*,*p*)$^{17}$C: transfer to the continuum



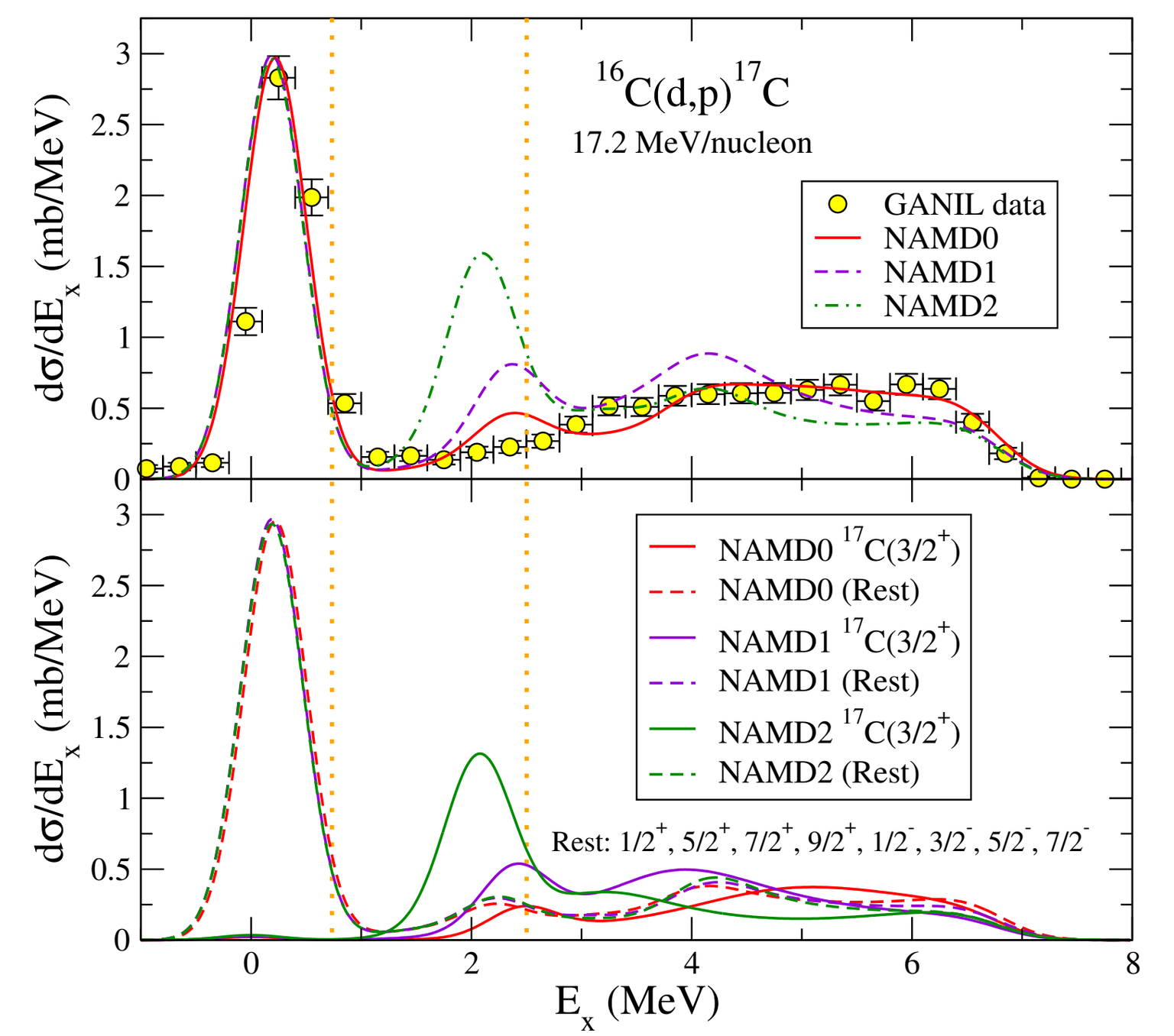

NoB: Non-blocking method

# N=16 shell gap

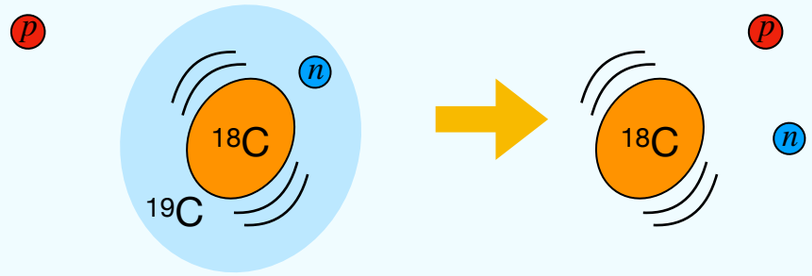
Spin-orbit splitting variations on the NAMD model (TB)



# N=16 shell gap



# Application to breakup reactions




- Breakup reactions $^{17}$C($p$,$p$')$^{16}$C+$n$ and $^{19}$C($p$,$p$')$^{18}$C+$n$ studied using extended continuum-discretized coupled-channel method (XCDCC) [PRC**95** (2017) 044611]

- $^{17}$C and $^{19}$C described using the NAMD model

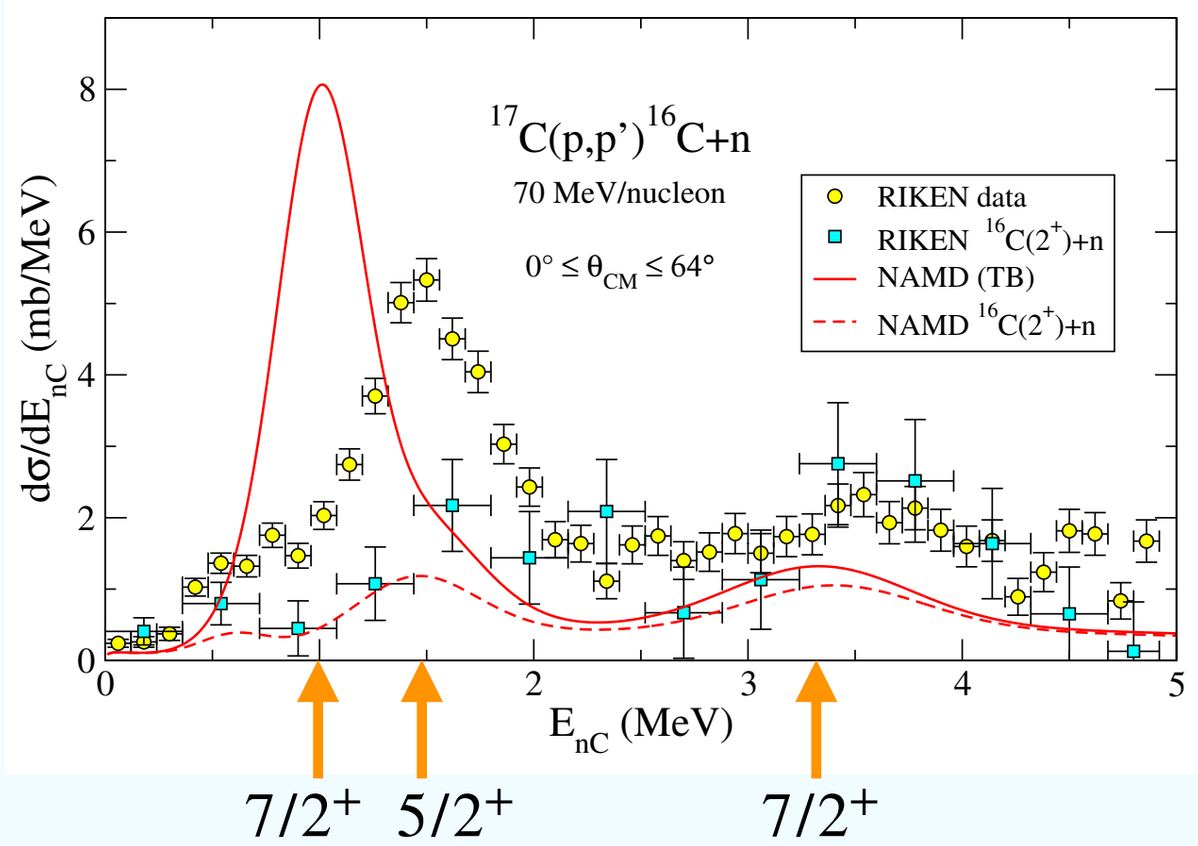
- $p$-$^{16}$C and $p$-$^{18}$C interactions calculated by convoluting an effective $N$-$N$ interaction with the AMD densities

- Gaussian $n$-$p$ potential [A. M. Moro and R. Crespo, PRC**85** (2012) 054613]

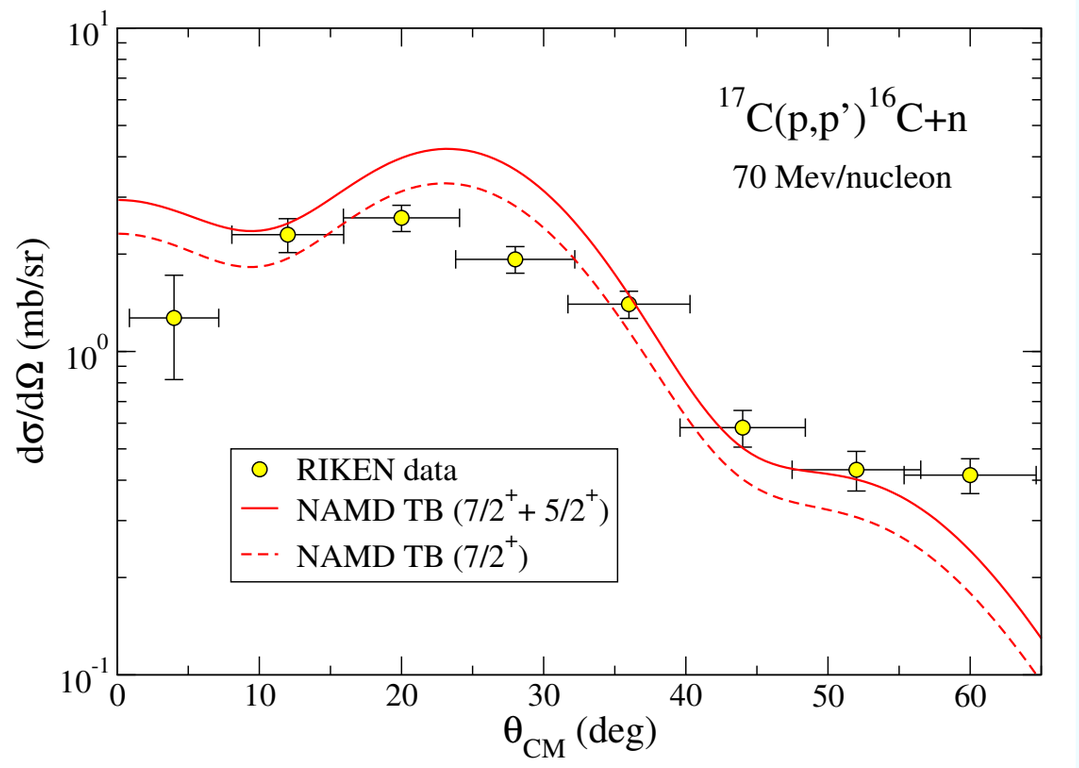
- Calculations are compared with experimental data from RIKEN [Y. Satou *et al.* PLB**660** (2008) 320]

# Application to $^{17}C(p,p')^{16}C+n$

## Energy distribution

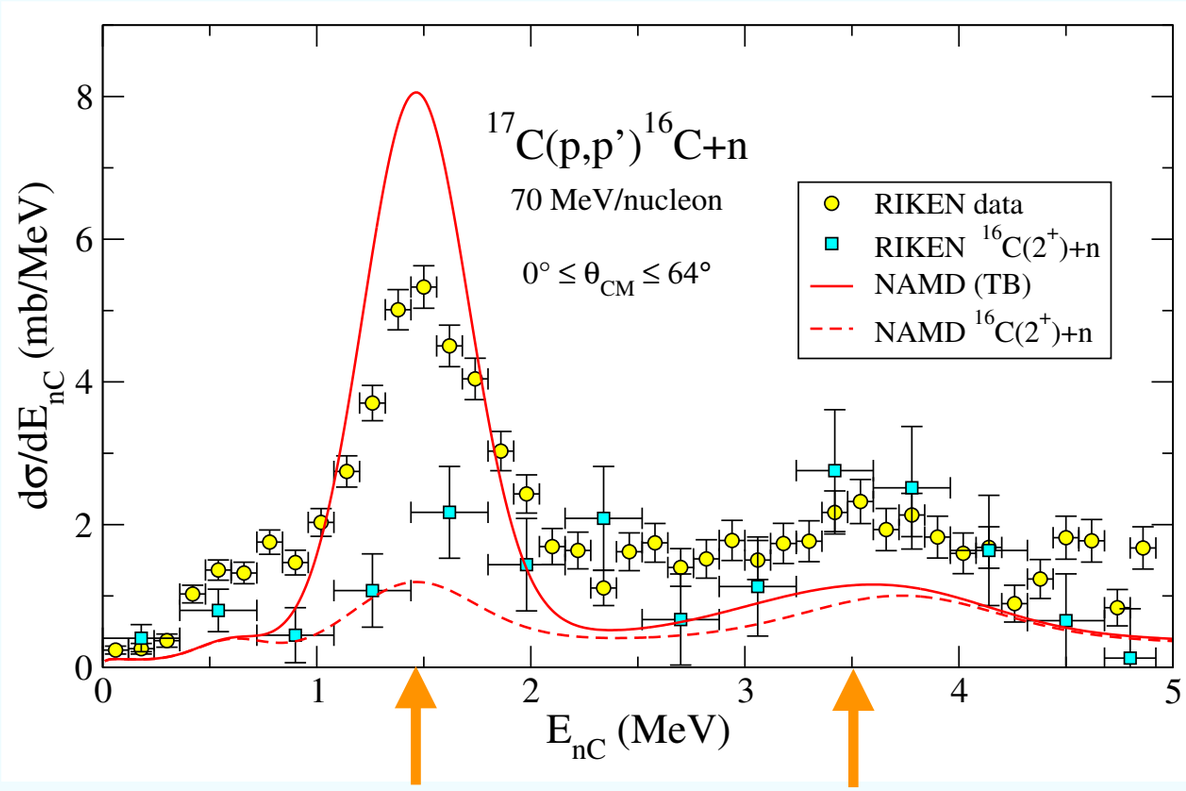



## Resonant Breakup

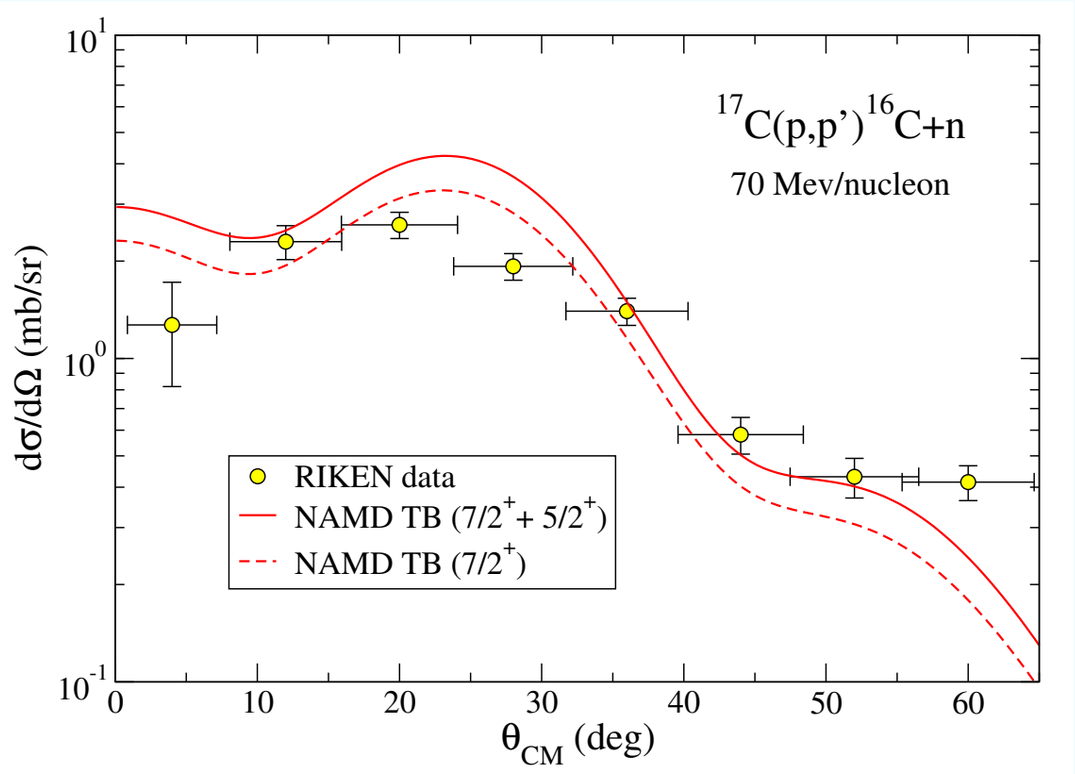


# Application to $^{17}C(p,p')^{16}C+n$

## Energy distribution
## (States $7/2^+$ shifted)



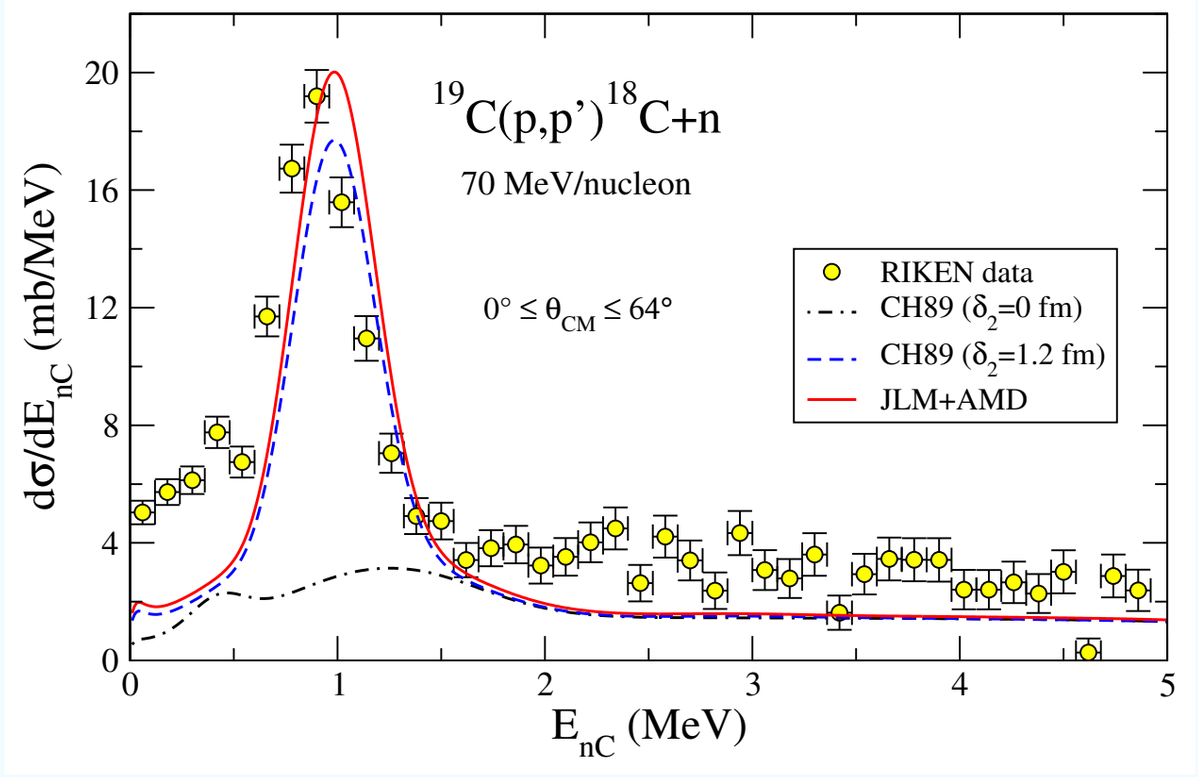

## Resonant Breakup



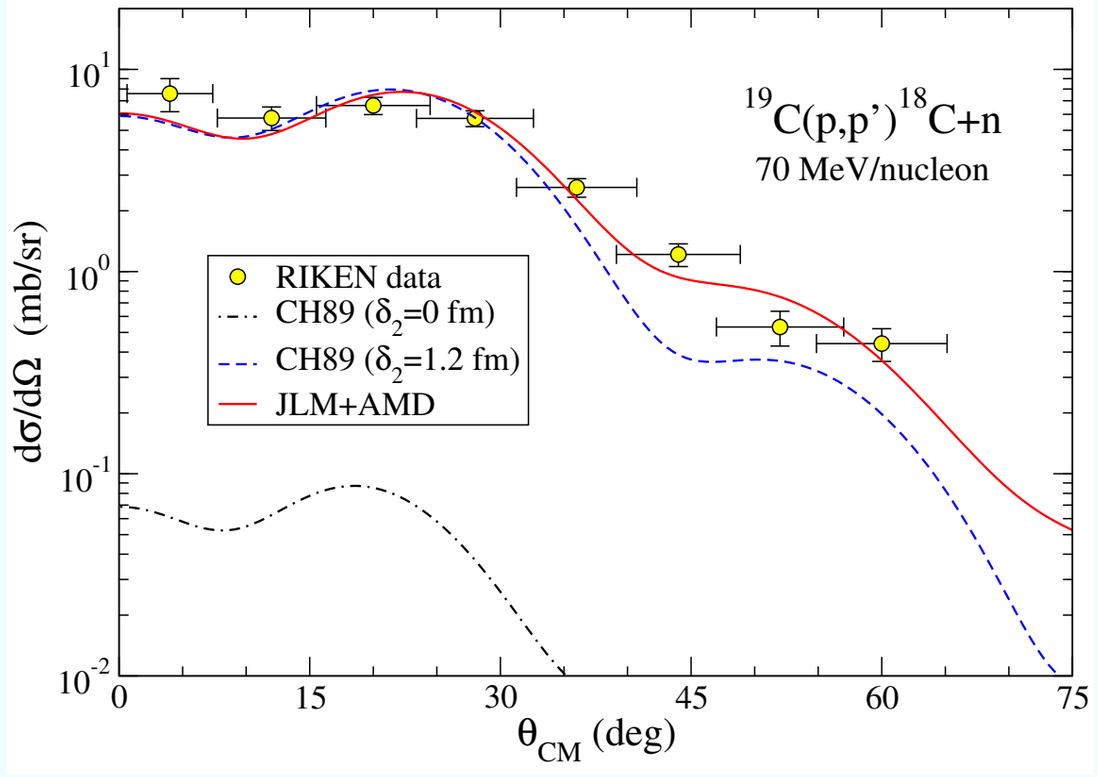
# Application to $^{19}C(p,p')^{18}C+n$

XCDCC calculations using different *p*-$^{18}C$ *interactions*

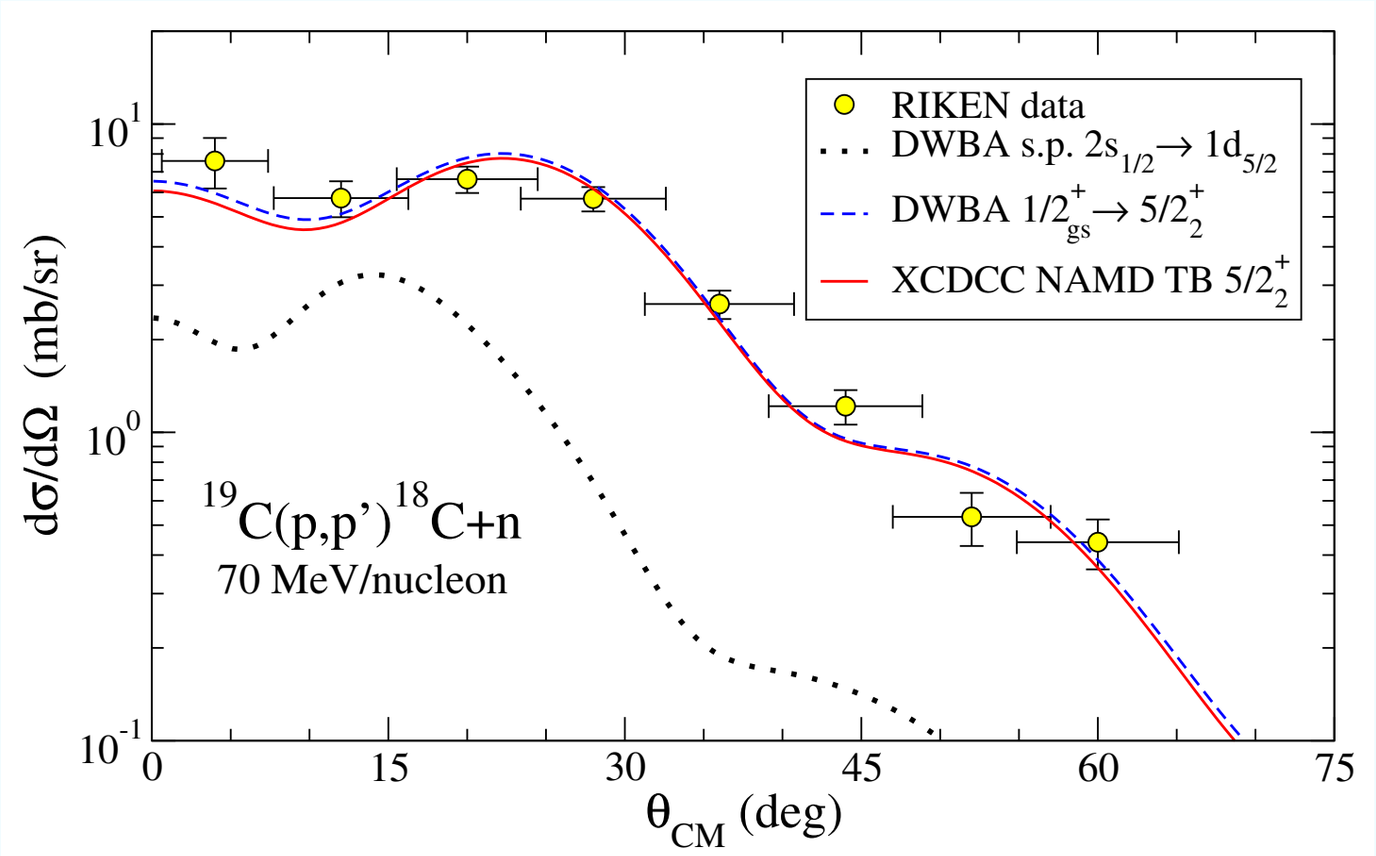
## Energy distribution




## Resonant Breakup



# $^{19}$C(*p*,*p*')$^{18}$C+*n*: resonant breakup




- Good agreement between experimental data and XCDCC calculations using the NAMD model with TB
- Similar results using the distorted wave Born approximation (DWBA)
- Single-particle calculations are not consistent with the data

# Conclusions

- The **NAMD** model shows **promising results** in the description of **$^{17}$C** and **$^{19}$C**

- Transfer reaction **$^{16}$C(*d*,*p*)$^{17}$C** populating **bound** and **unbound** **states** has been studied

    - **Pauli blocking** methods are **needed** to reproduce the **transfer to bound states**

    - The results for the **transfer to the continuum** suggest a ***N*=16 shell gap** larger than 5 MeV

- **Successful results** for breakup reactions **$^{17}$C(*p*,*p'*)$^{16}$C+*n*** and **$^{19}$C(*p*,*p'*)$^{18}$C+*n***

    - The population of **7/2$^+$ resonances** plays a **key role** in the **$^{17}$C breakup**.

    - The excitation to the **5/2$^+$ resonance** of **$^{19}$C** is clearly **dominated** by the **core excitation** mechanism

# Thanks for your attention

Proyecto PID2023-146401NB-I00 financiado por

MINISTERIO
DE CIENCIA, INNOVACIÓN
Y UNIVERSIDADES

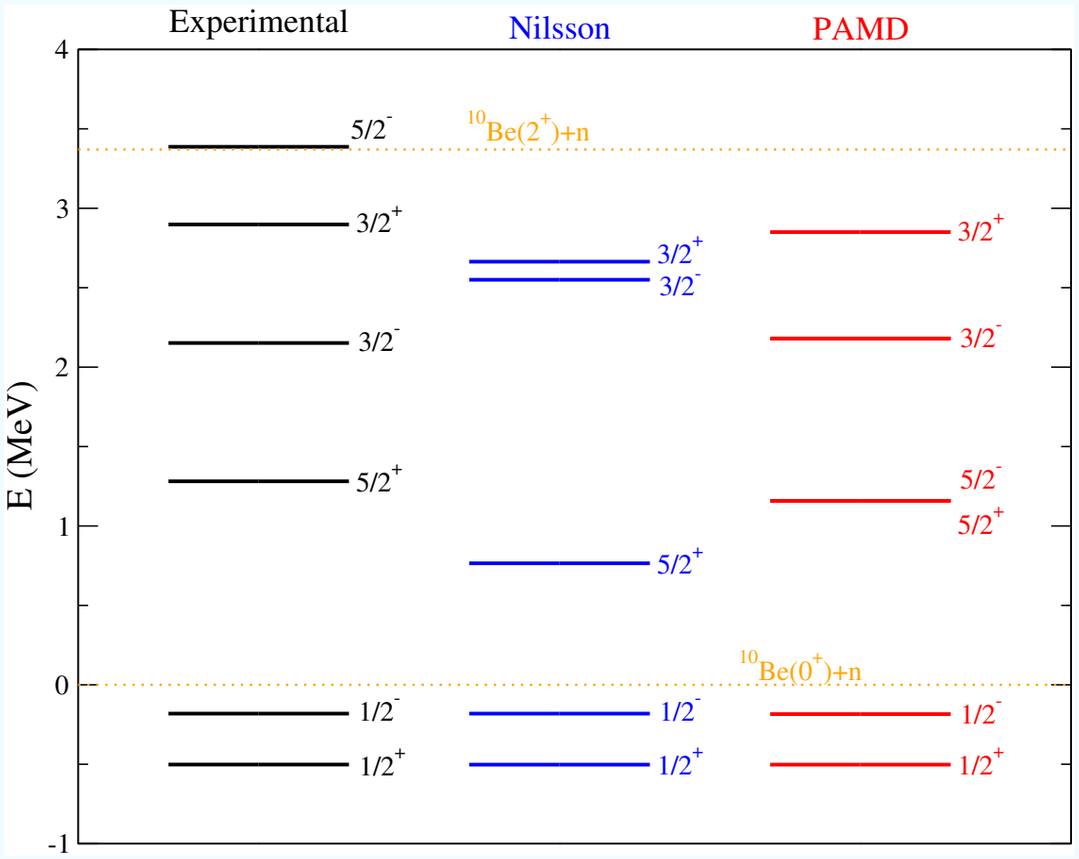
Cofinanciado por
la Unión Europea

AGENCIA
ESTATAL DE
INVESTIGACIÓN

Ph.D. grant FPU21/03931

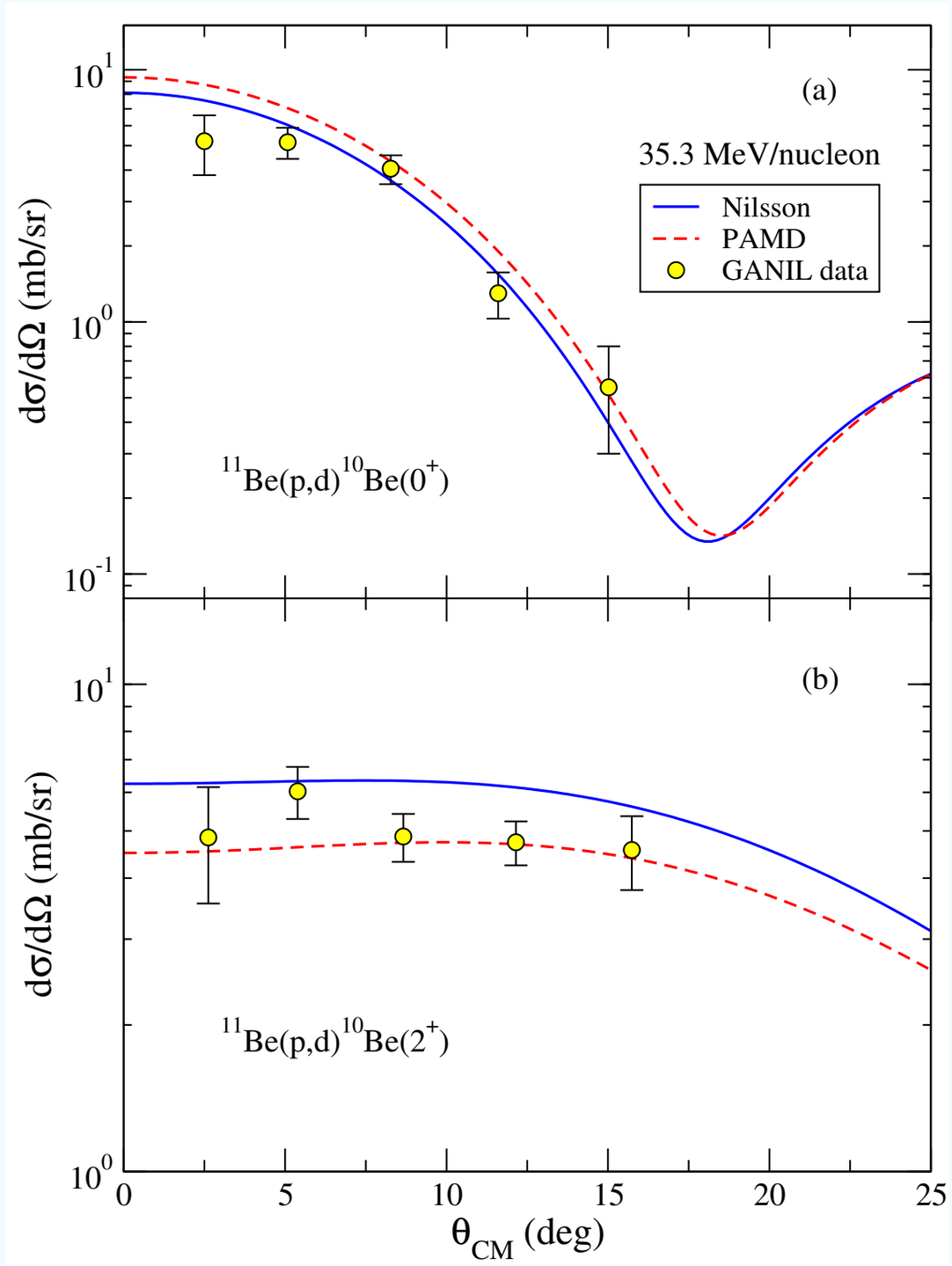
# Backup

# Application to $^{11}$Be




P. Punta, *et* al. PRC**108** (2023) 024613

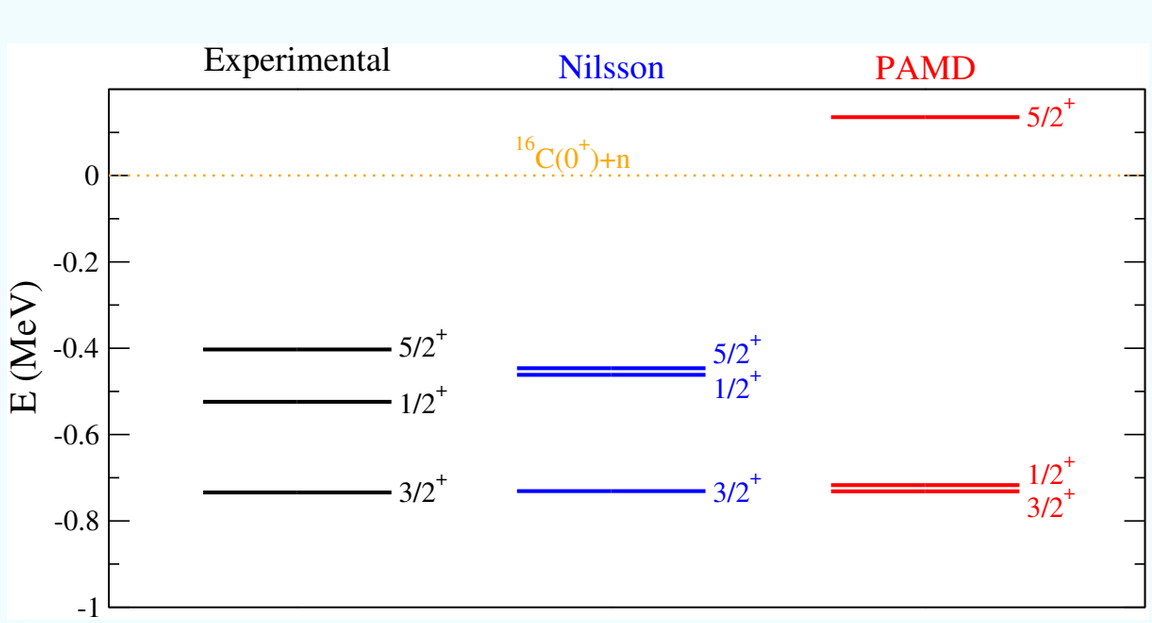
GANIL data [NPA**683** (2001) 48]



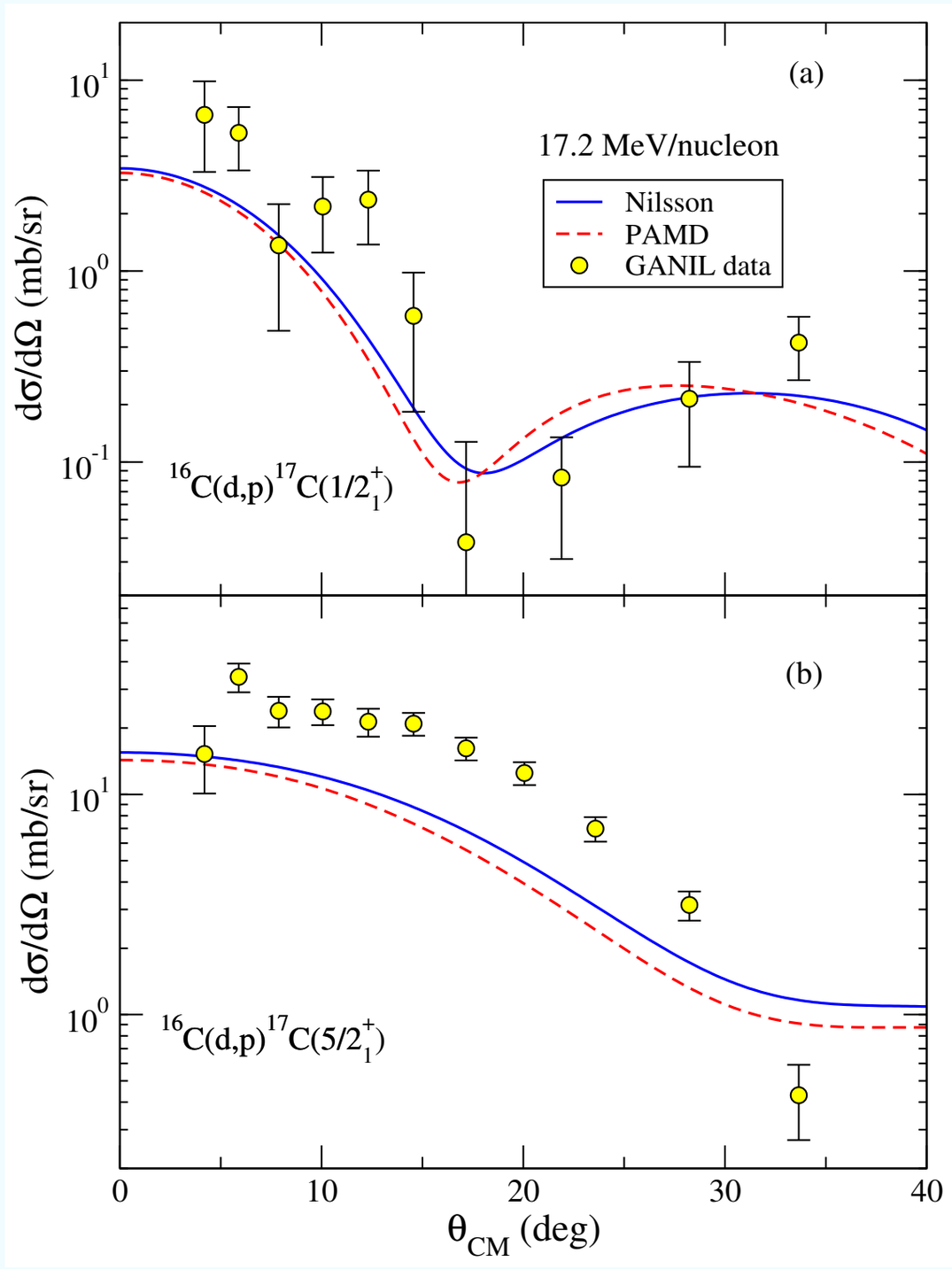
# Application to $^{17}$C

- Deformed two-body models
  - Nilsson
  - PAMD

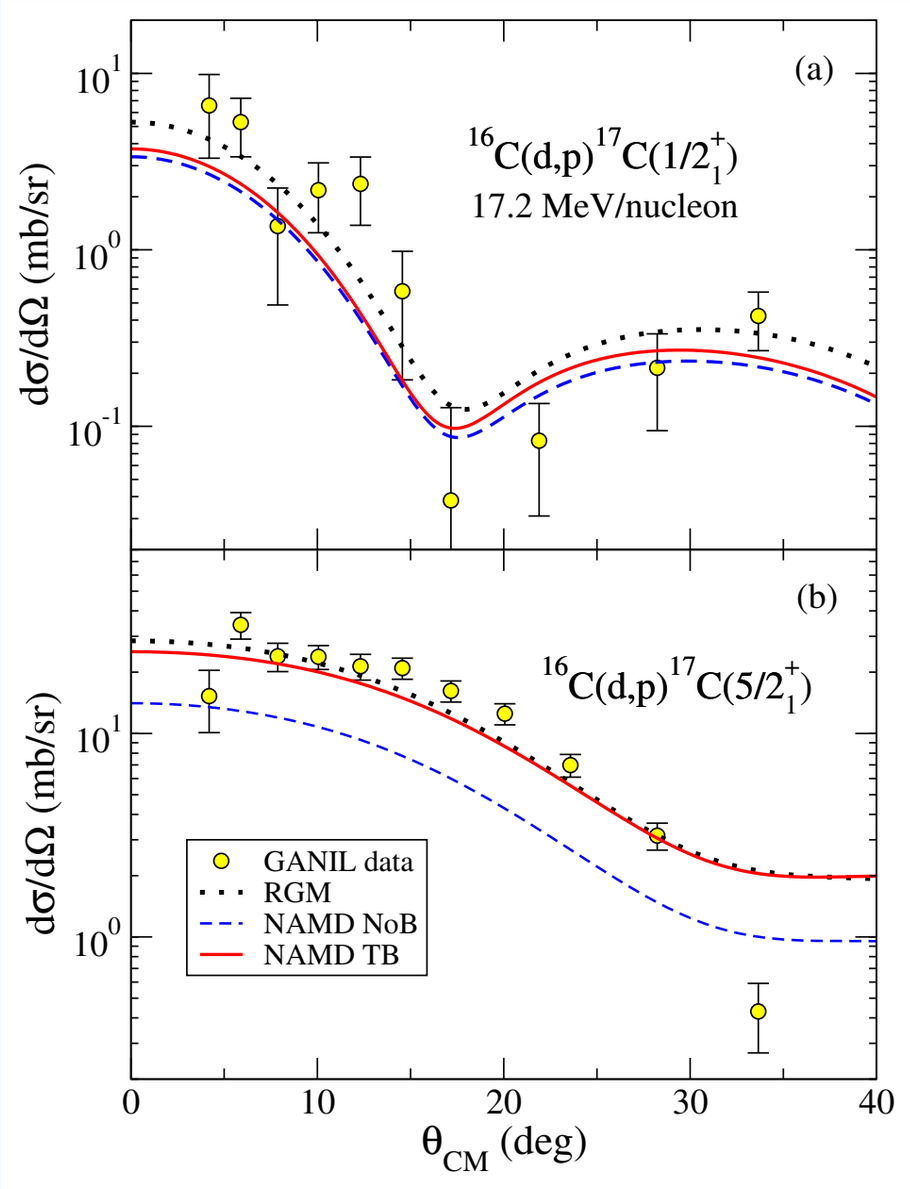
P. Punta, *et* al. PRC**108** (2023) 024613

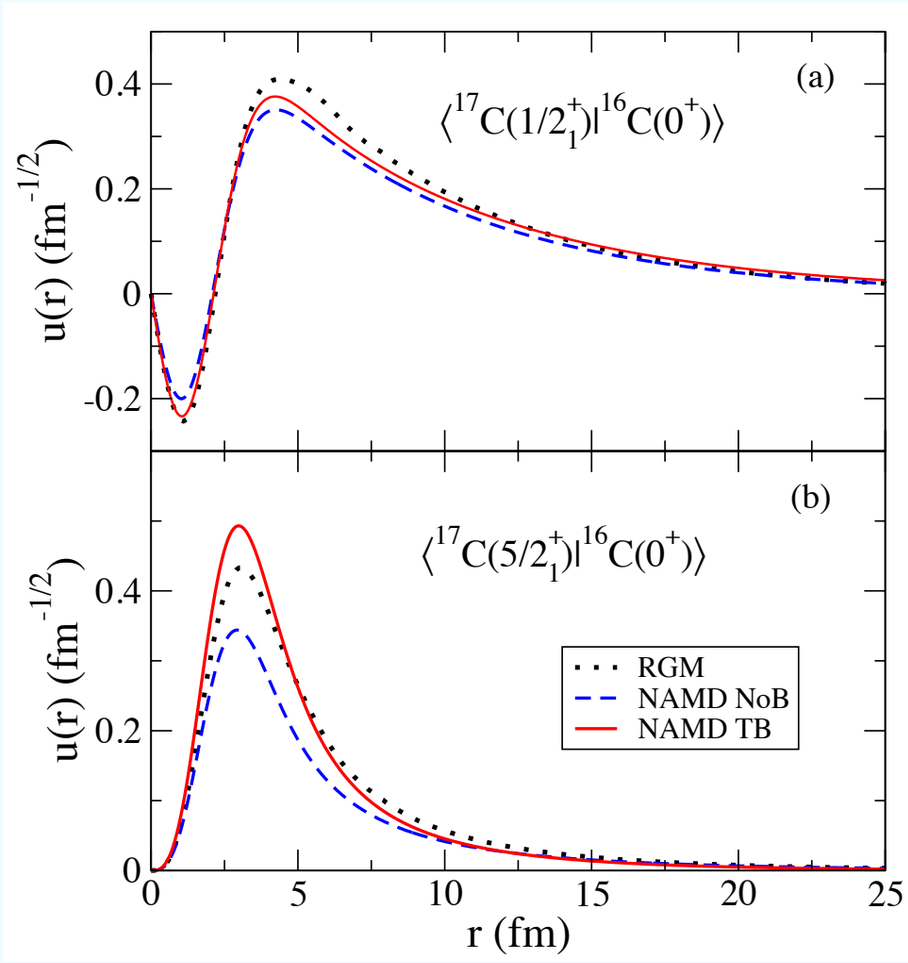



GANIL data [PLB**811** (2020) 135939]



# Application to $^{16}$C(*d*,*p*)$^{17}$C (bound states)

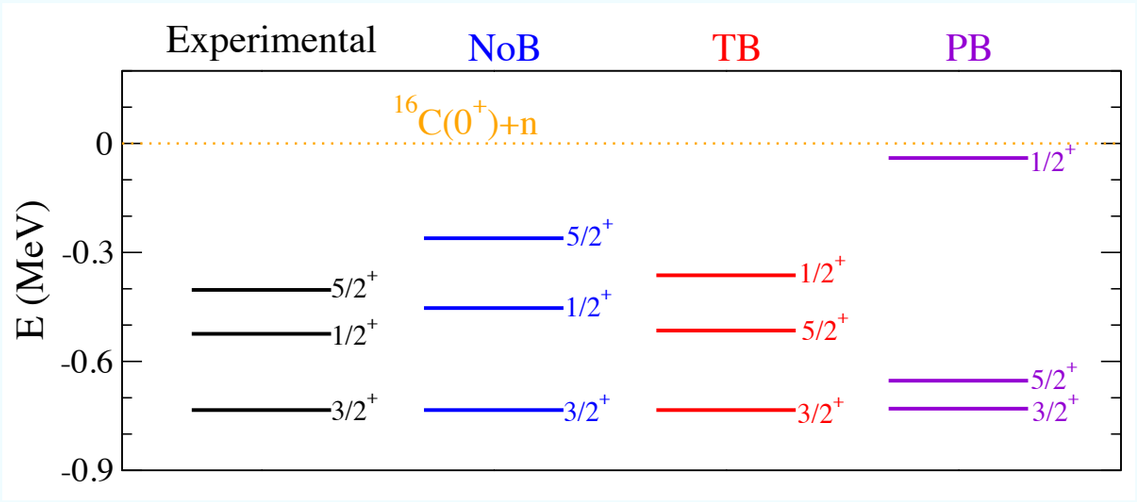



Resonating Group Method calculations (RGM):

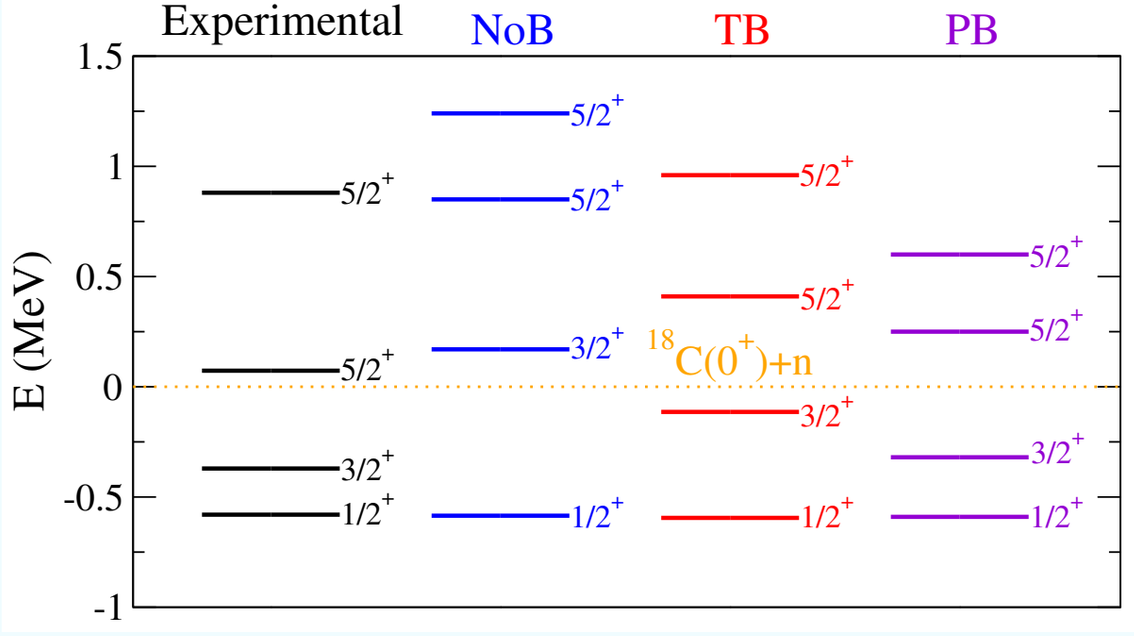
L. H. Chien and P. Descouvemont PRC**108** (2023) 044605

# Application to $^{17}$C and $^{19}$C$^{17}$C



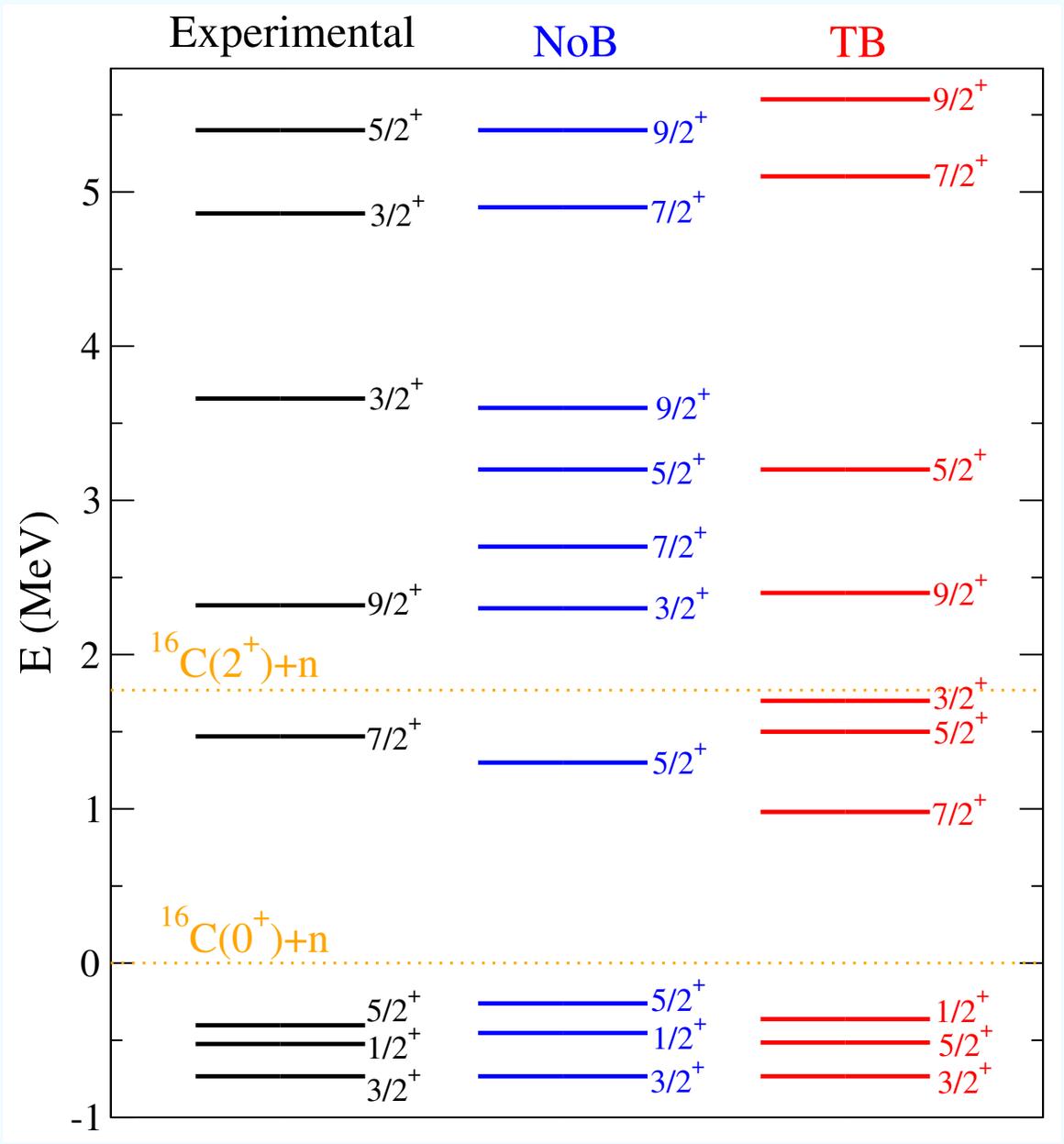

$^{17}$C

$^{19}$C




P. Punta *et al.* PRC**111** (2025) 064614

# Application to $^{17}$C



# BCS implementation

- Nilsson eigenstates

$$H_N|\nu\rangle = \varepsilon_\nu|\nu\rangle \qquad \psi_\nu(\vec{r}) = \sum_j R_j^\nu(r)\mathcal{Y}_{ls}^{j\Omega}(\hat{r}')$$

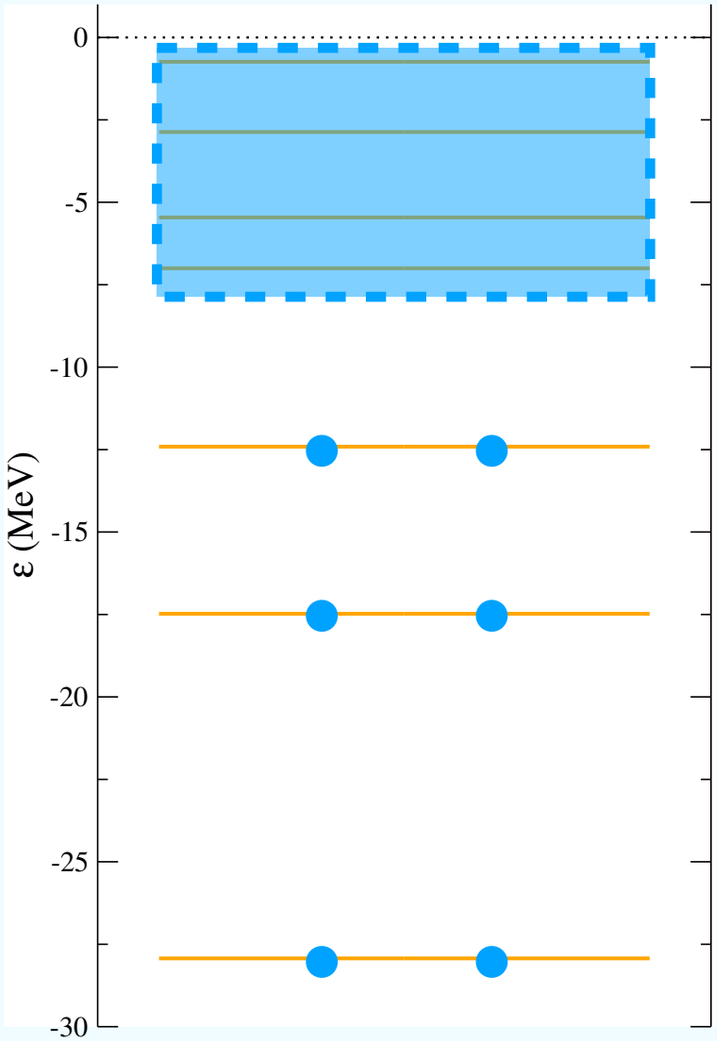
- BCS calculation

$$H_{BCS} = \sum_\nu (\varepsilon_\nu - \lambda)(a_\nu^\dagger a_\nu + a_{\bar{\nu}}^\dagger a_{\bar{\nu}}) - G\sum_{\nu\nu'} a_\nu^\dagger a_{\bar{\nu}}^\dagger a_{\bar{\nu}'} a_{\nu'}$$

- Complete Hamiltonian

$$H' = H_{BCS} + h_{\text{core}} - \langle \text{BCS}|H_{BCS} + h_{core}|\text{BCS}\rangle$$

- Diagonalization with one quasi-particle states

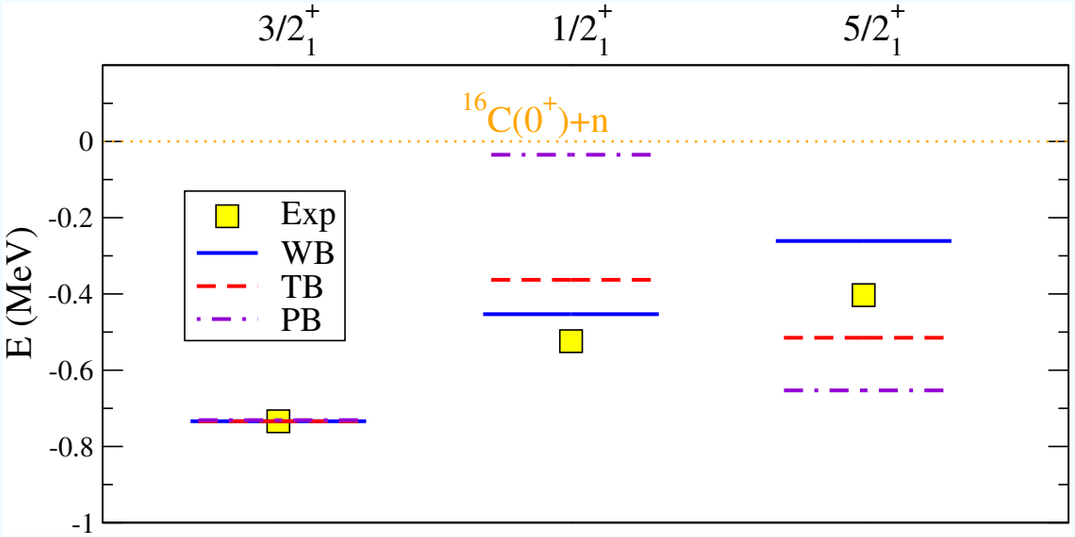



$$\langle BCS, \nu J|H'|BCS, \nu' J\rangle = E_\nu \delta_{\nu\nu'} + \frac{\hbar^2}{2\mathcal{J}}\langle \nu J|\vec{I}^2|\nu' J\rangle(u_{\nu'}u_\nu - v_{\nu'}v_\nu)$$

$$\Psi_{\nu J}^M(\vec{r'},\omega) = \frac{\sqrt{2J+1}}{4\pi}\left[\psi_\nu(\vec{r}')\mathcal{D}_{M\Omega}^J(\omega)^* + (-1)^{J-\Omega}\psi_{\bar{\nu}}(\vec{r}')\mathcal{D}_{M-\Omega}^J(\omega)^*\right]$$

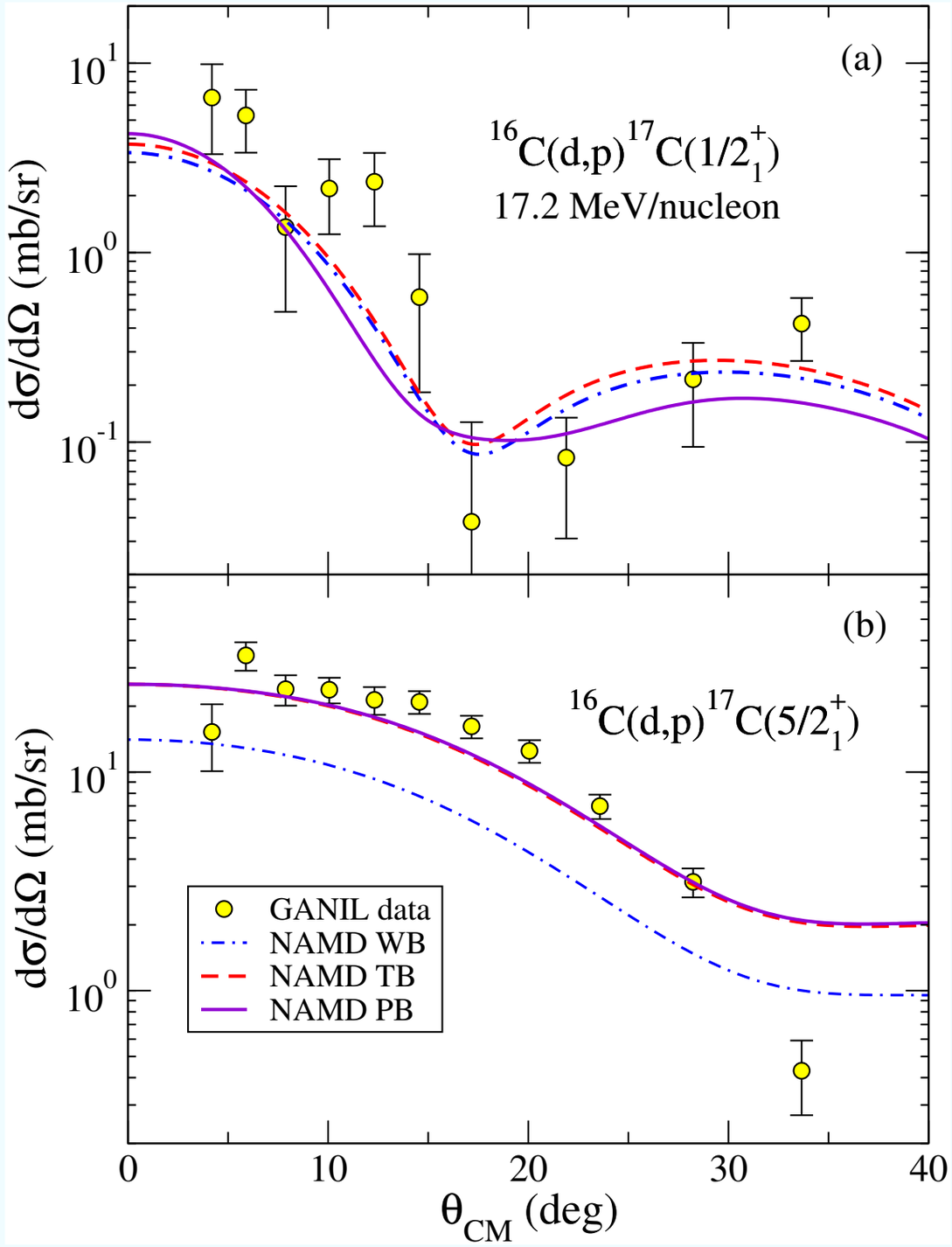
# Application to $^{17}$C and $^{16}$C(*d*,*p*)$^{17}$C

- Without Blocking (WB)
- Total Blocking (TB)
- Partial Blocking (PB)




Pairing Gap

| | |
|---|---|
| **Three-point formula** | 1.76 MeV |
| **NAMD PB** | 1.58 MeV |

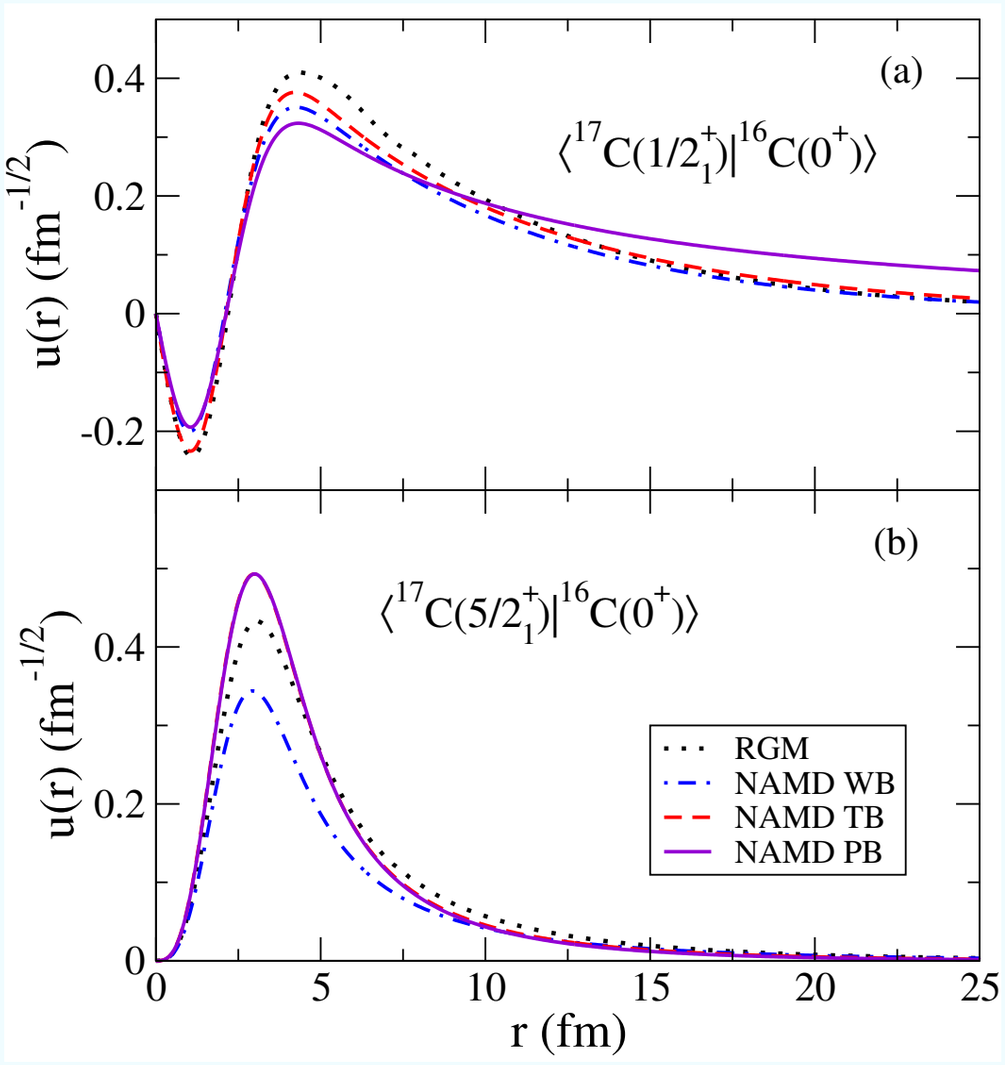


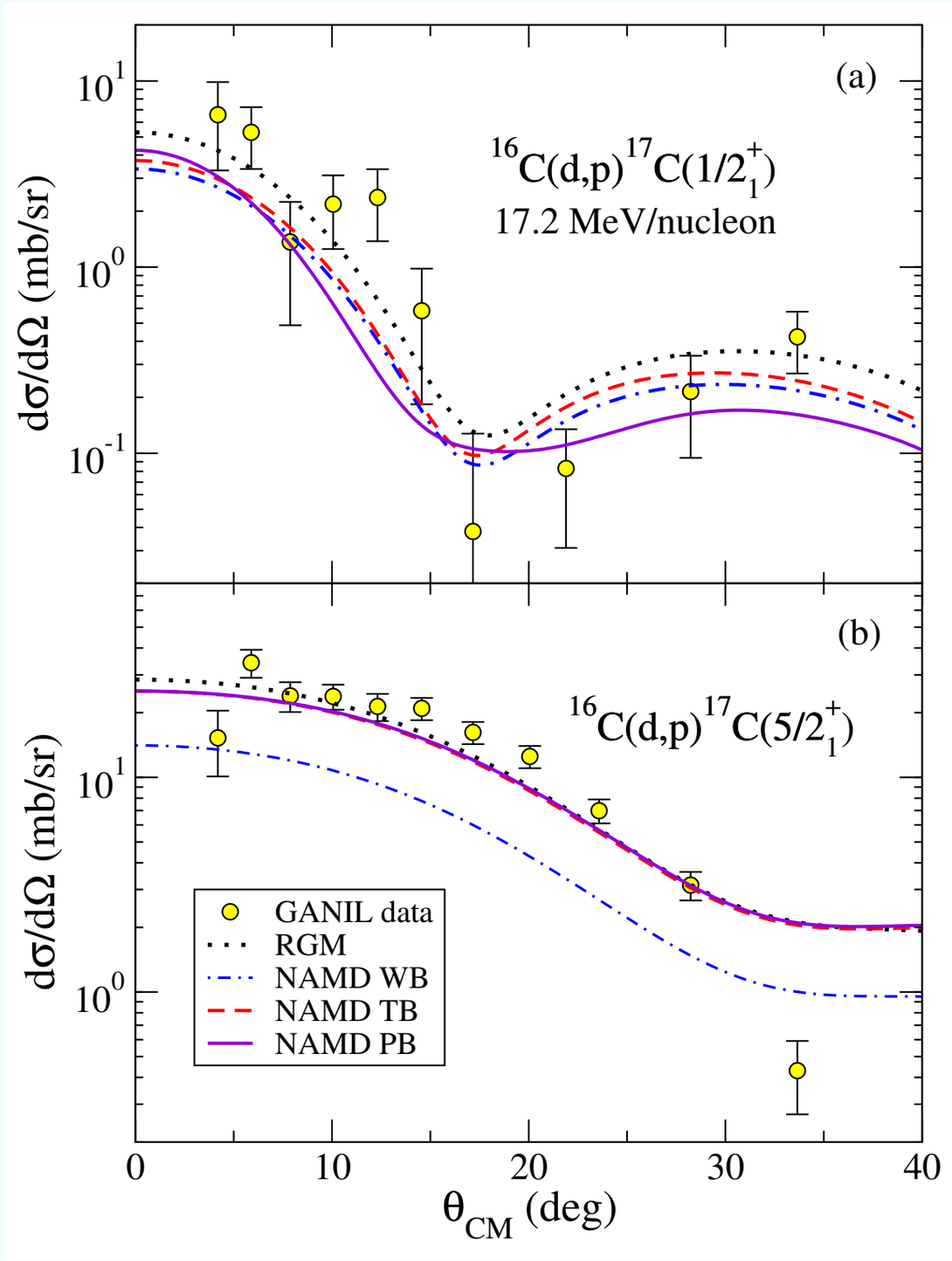
# Comparison with RGM calculations

## Resonating Group Method (RGM)

[Le Hoang Chien and P. Descouvemont

PRC**108** (2023) 044605]

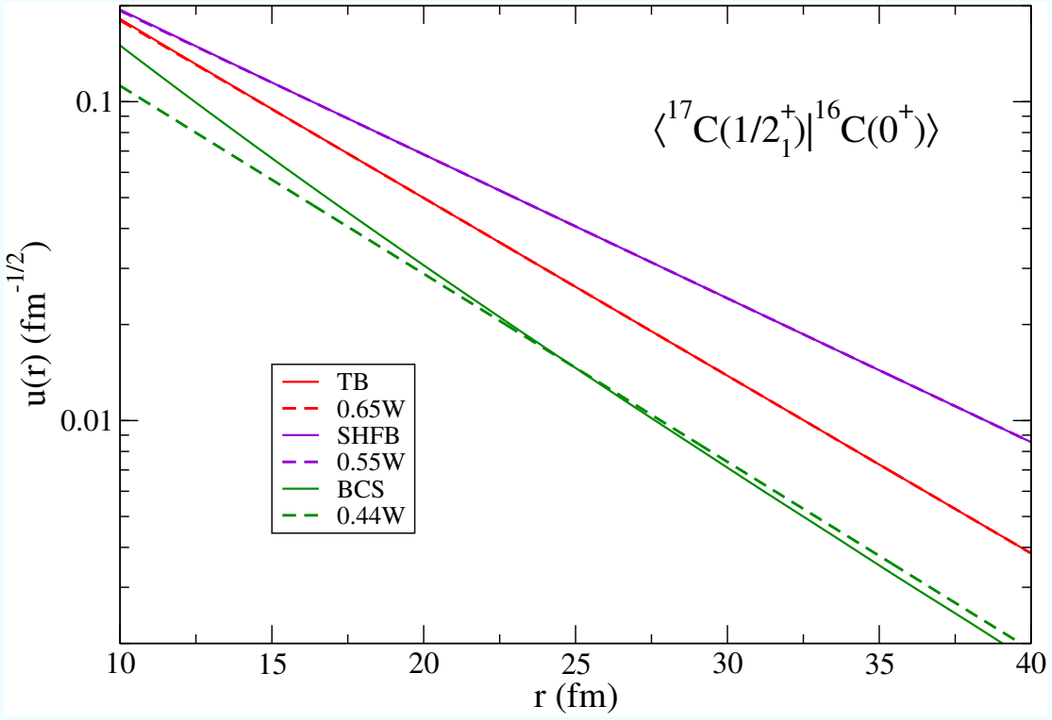


# Asymptotic behavior

- The NAMD+SHFB formalism is a variation of the NAMD+BCS formalism from P. Punta, *et al.* PRC**111** (2025) 064614

- The radial pairing interaction maintains the expected asymptotic behavior



# Simplified HFB formalism

- SHFB equations [I. Hamamoto, *et al.* PRC**73** (2006) 044317]

$$\begin{cases} (H_N - \lambda)\psi^u(\vec{r}') + \Delta(\vec{r}')\psi^v(\vec{r}') = E\psi^u(\vec{r}') \\ \Delta(\vec{r}')\psi^u(\vec{r}') + (\lambda - H_N)\psi^v(\vec{r}') = E\psi^v(\vec{r}') \end{cases}$$

- Pairing interaction derived from AMD densities

$$\Delta(\vec{r}') = \Delta_0(r) + \Delta_2(r)Y_{20}(\theta') \qquad \Delta_\lambda(r) \propto \rho^{n\lambda}_{\lambda^+\to 0^+}(r_1)$$

- Nilsson eigenstates as basis

$$H_N\psi_\nu(\vec{r}') = \epsilon_\nu\psi_\nu(\vec{r}') \qquad \begin{pmatrix} \psi^u_\mu \\ \psi^v_\mu \end{pmatrix} = \sum_\nu \begin{pmatrix} u^\mu_\nu \\ v^\mu_\nu \end{pmatrix} \psi_\nu$$

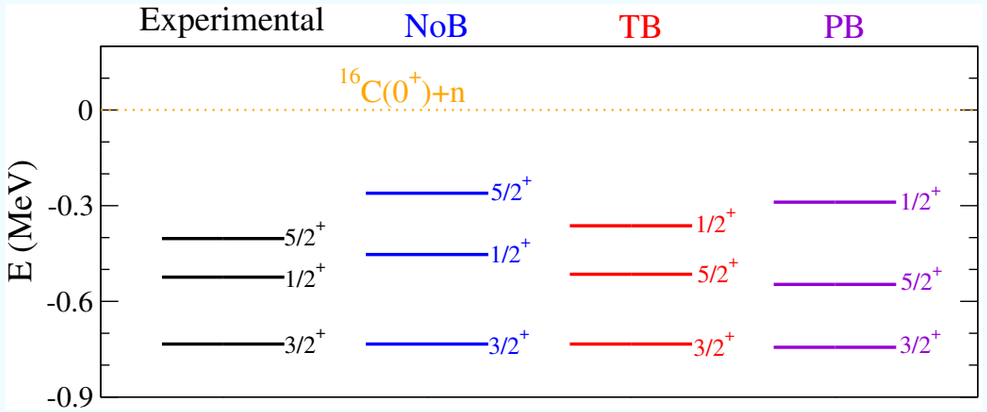
- One quasi-particles states

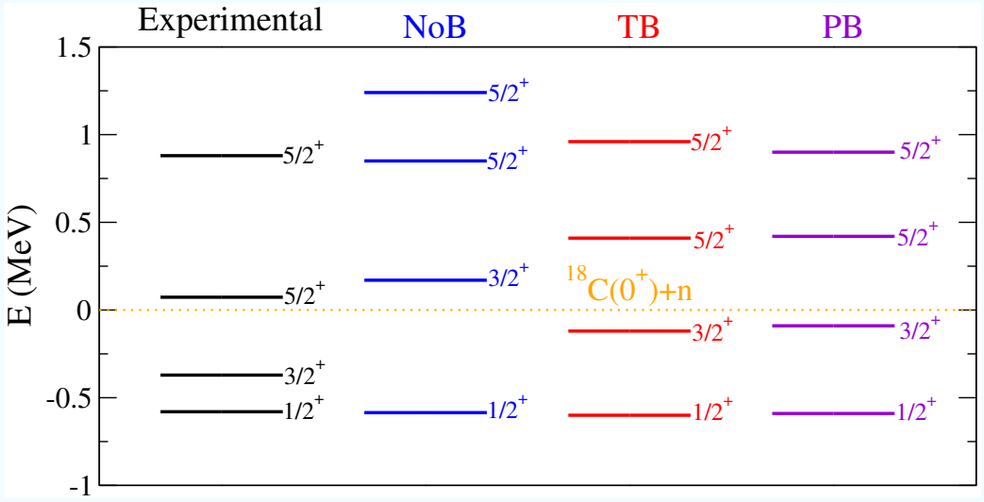
$$H'_N\,|\,\mathrm{SHFB},\mu\rangle = E_\mu\,|\,\mathrm{SHFB},\mu\rangle \qquad \sum_{\mu\nu} |\,v^\mu_\nu\,|^2 = \frac{N}{2}$$

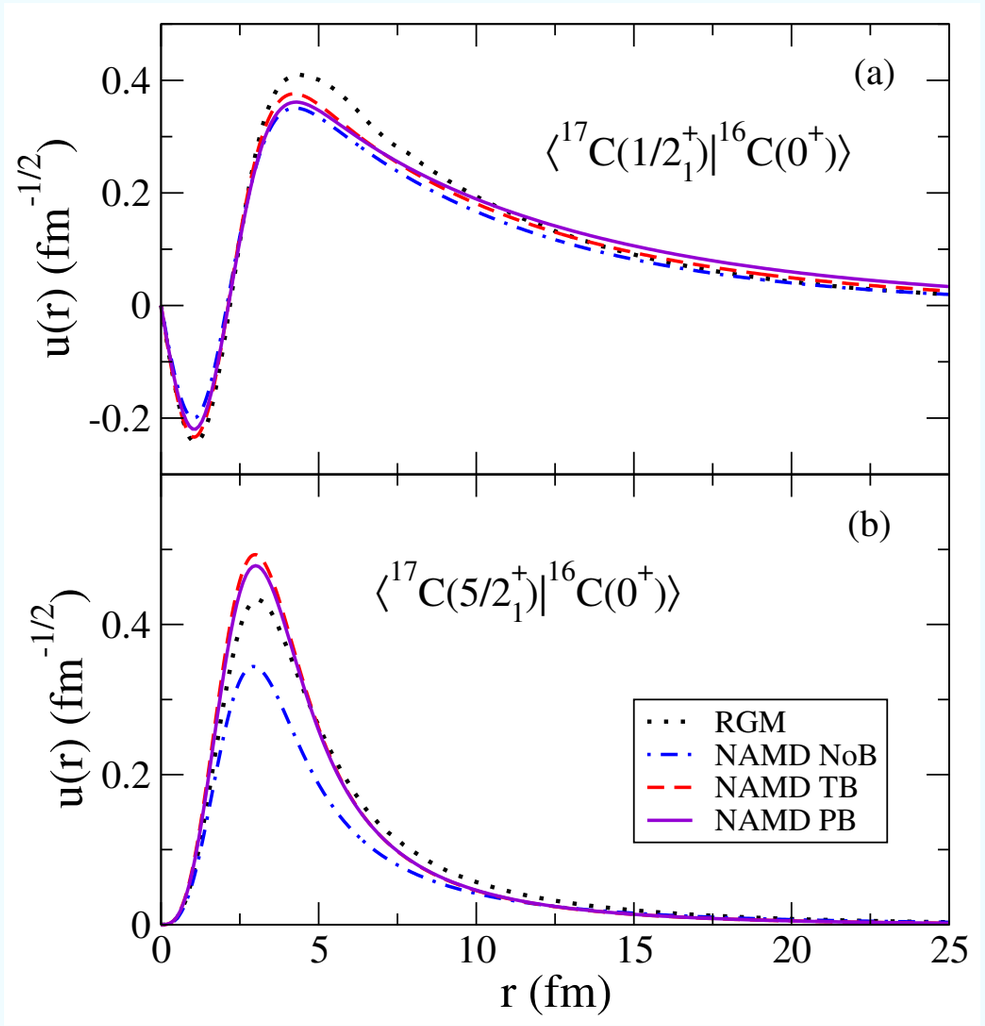
- Rotational term

$$\langle \mathrm{SHFB},\mu\,|\,H'\,|\,\mathrm{SHFB},\mu'\rangle = E_\mu\delta_{\mu\mu'} + \frac{\hbar^2}{2\mathcal{J}}\sum_{\nu\nu'}\langle\nu\,|\,\vec{I}^2\,|\,\nu'\rangle(u^\mu_\nu u^{\mu'}_{\nu'} - v^\mu_\nu v^{\mu'}_{\nu'})$$

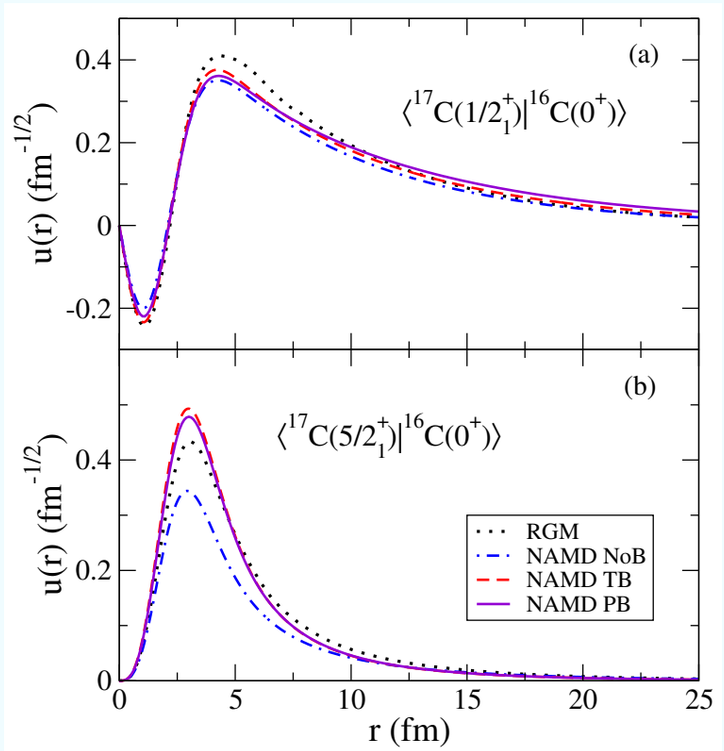
# Partial blocking with SHFB formalism



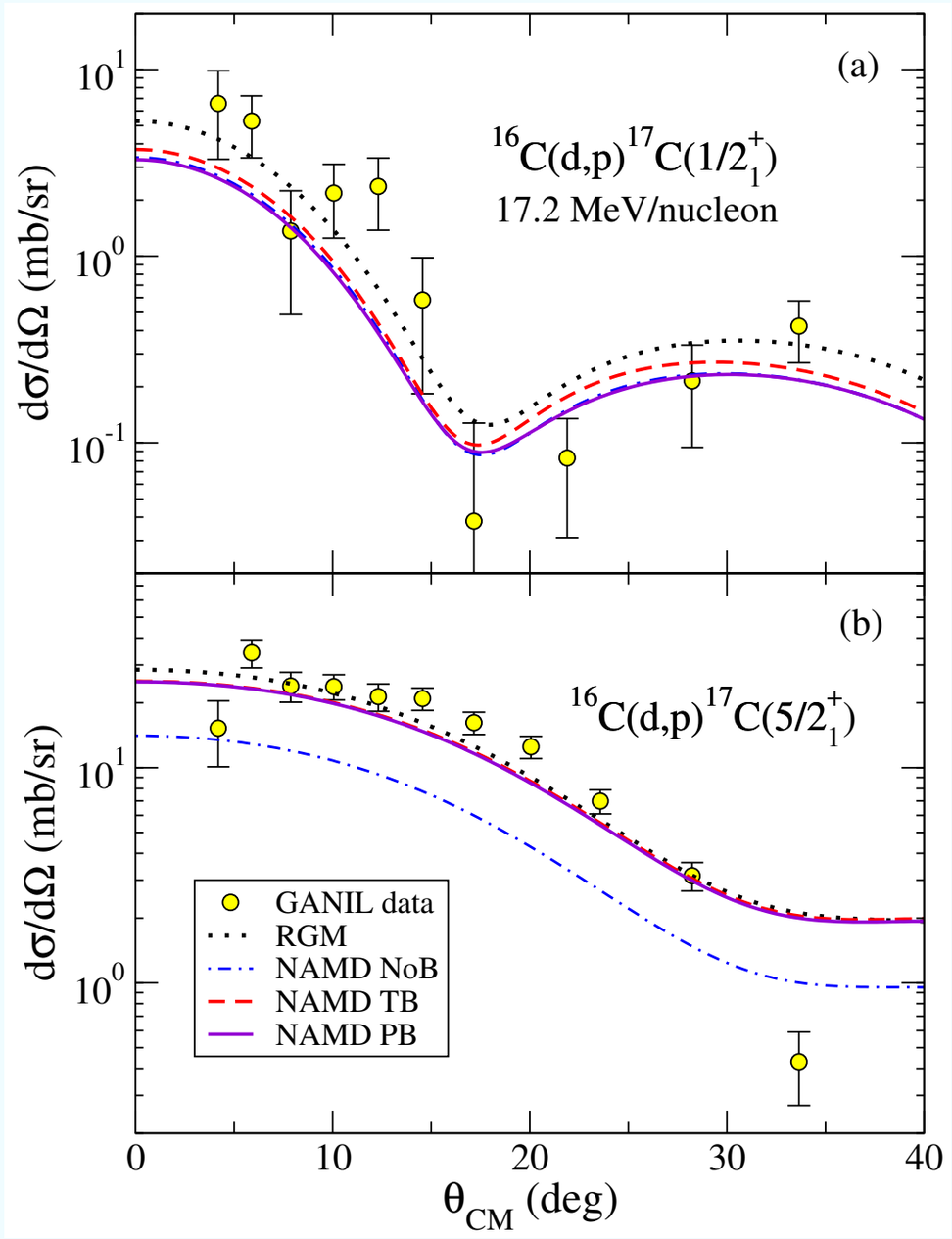



# Application to $^{16}$C($d$,$p$)$^{17}$C (bound states)



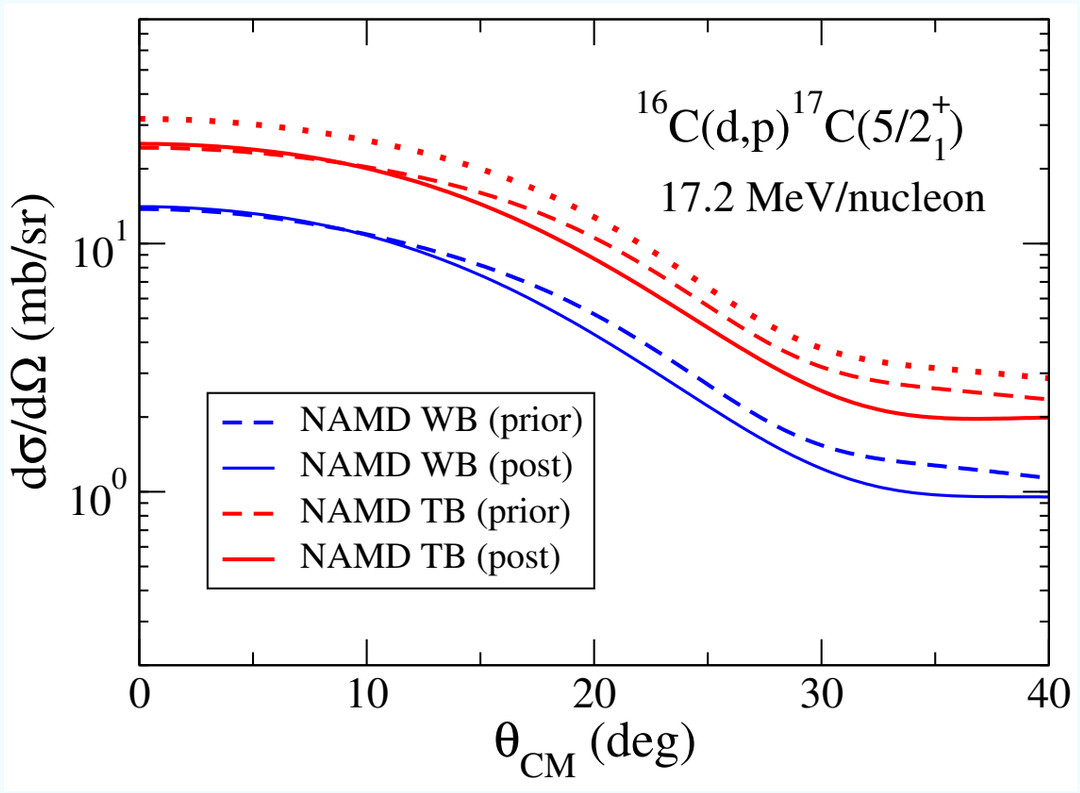

Spectroscopic Factors for $(\ell s)j \otimes 0^+$

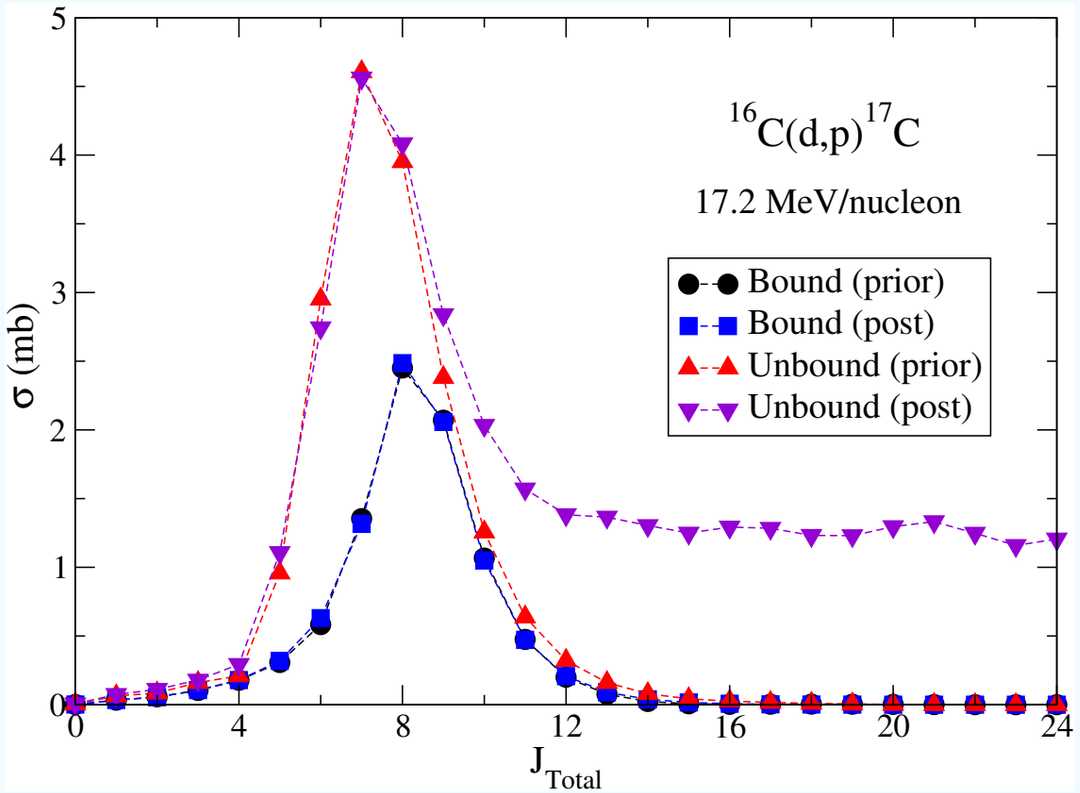
| | $3/2^+_1$ | $1/2^+_1$ | $5/2^+_1$ |
|---|---|---|---|
| **Exp** | 0.03 $^{+0.05}_{-0.03}$ | 0.64±0.18 | 0.62±0.13 |
| **RGM** | 0.01 | 0.94 | 0.56 |
| **NoB** | 0.00 | 0.68 | 0.33 |
| **TB** | 0.01 | 0.80 | 0.66 |
| **PB** | 0.01 | 0.80 | 0.63 |




GANIL data: X. Pereira-López *et al.* PLB**811** (2020) 135939

# Prior/Post


$^{16}$C(d,p)$^{17}$C(5/2$^+_1$)
17.2 MeV/nucleon
dσ/dΩ (mb/sr)
$10^1$
$10^0$
NAMD WB (prior)
NAMD WB (post)
NAMD TB (prior)
NAMD TB (post)
0
10
20
30
40
θ$_{CM}$ (deg)


$^{16}$C(d,p)$^{17}$C
17.2 MeV/nucleon
Bound (prior)
Bound (post)
Unbound (prior)
Unbound (post)
σ (mb)
0
1
2
3
4
5
0
4
8
12
16
20
24
J$_{Total}$

# Transfer to the continuum

- The *prior* form of the ADWA approximation is used

  X $\langle {}^{17}\mathrm{C} \,|\, {}^{16}\mathrm{C} \rangle$ → Too much spatial extension

  ✓ $\langle {}^{17}\mathrm{C} \,|\, V_{n-{}^{16}C} \,|\, {}^{16}\mathrm{C} \rangle$ → Little spatial extension

- The continuum is discretized with the pseudo-states method

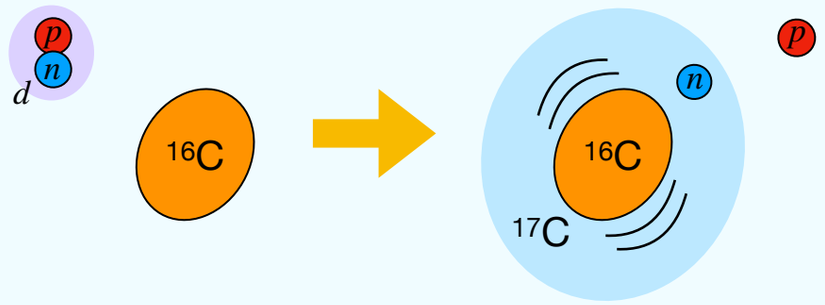
  - Scattering amplitudes for a discrete number of pseudo-states

  - Convolution with the *exact n*+core scattering states

$$\mathscr{F}(k,\theta) \approx \sum_n \langle \Psi(k,r) \,|\, \Psi_n^{THO}(r) \rangle \mathscr{F}_n^{THO}(\theta)$$

# Pseudo-states discretization method

$^{16}$C($d$,$p$)$^{17}$C




$$\mathscr{F}(k,\theta) = \sqrt{\frac{\mu_i \mu_f k_f}{(2\pi\hbar^2)^2 k_i}} \langle \chi_f^{(-)} \varphi_p \Psi_{^{17}C} \,|\, U \,|\, \chi_i^{(+)} \psi_d \phi_{^{16}C} \rangle$$

- Scattering amplitudes for a discrete number of pseudo-states

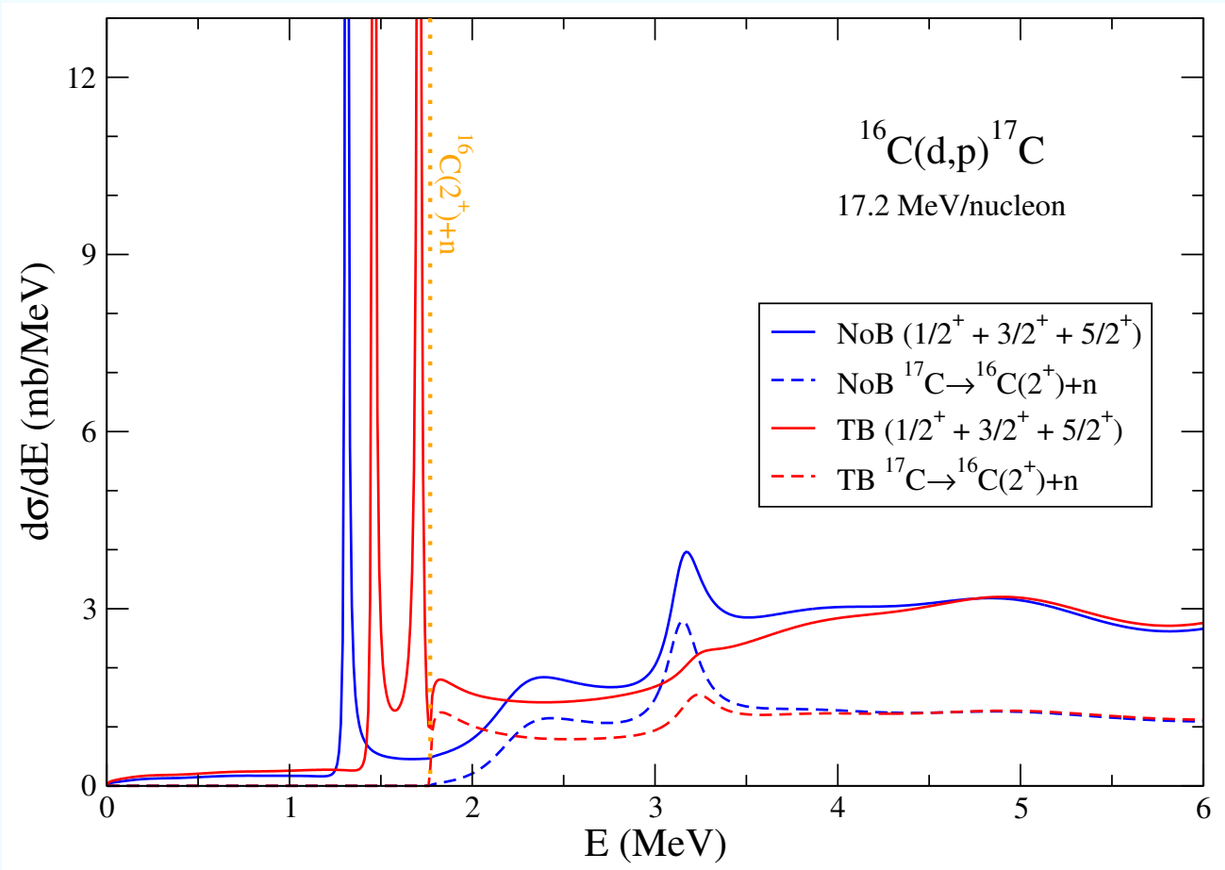
$$\mathscr{F}_n^{THO}(\theta) = \sqrt{\frac{\mu_i \mu_f k_n}{(2\pi\hbar^2)^2 k_i}} \langle \chi_f^{(-)} \varphi_p \Psi_n^{THO} \,|\, U \,|\, \chi_i^{(+)} \psi_d \phi_{^{16}C} \rangle$$

- Convolution with the *exact n*+core scattering states

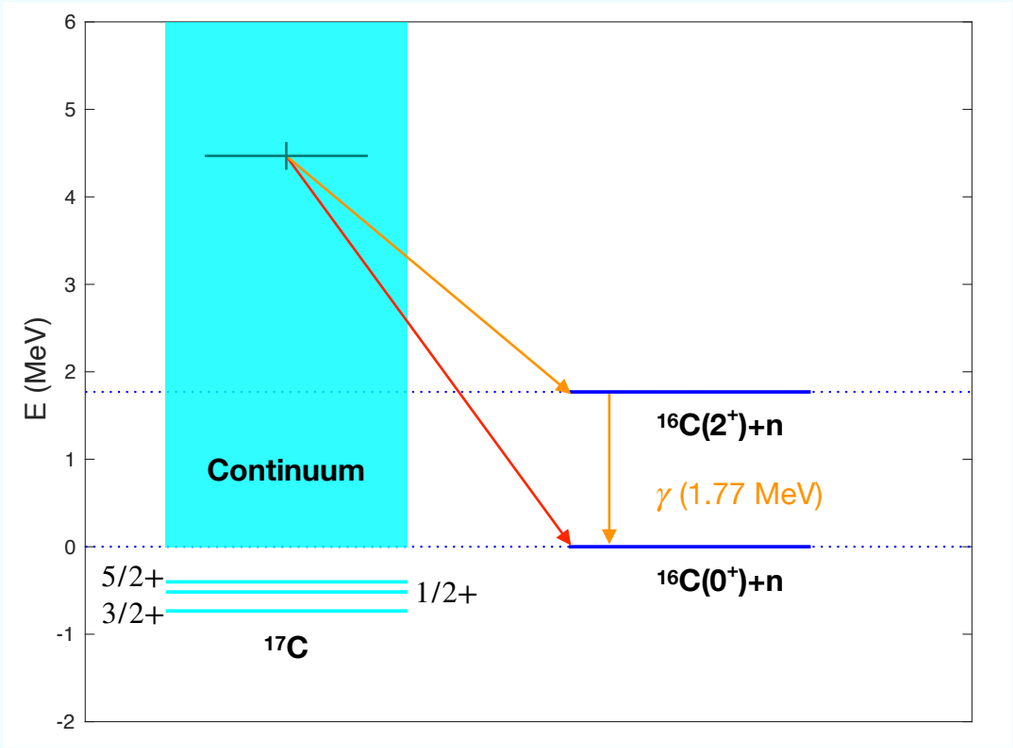
$$\mathscr{F}(k,\theta) \approx \sum_n \langle \Psi_{^{17}C}(k,r) \,|\, \Psi_n^{THO}(r) \rangle \mathscr{F}_n^{THO}(\theta)$$

# Transfer to the continuum: Decay mode

- The part of the cross section that involves a decay from an unbound state of $^{17}$C to $^{16}$C(2$^+$)+*n* is calculated

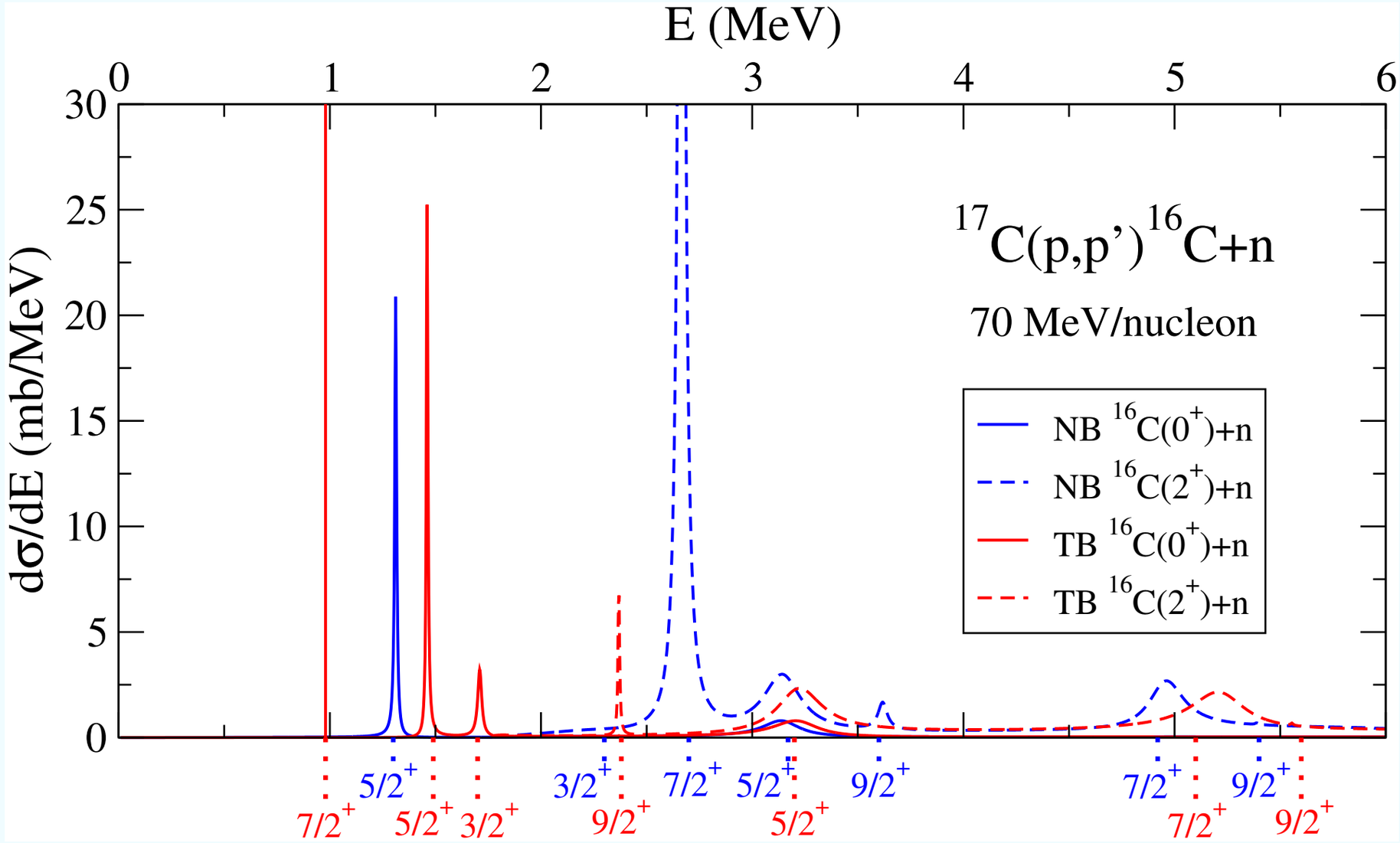



$^{17}$C → $^{16}$C+*n*



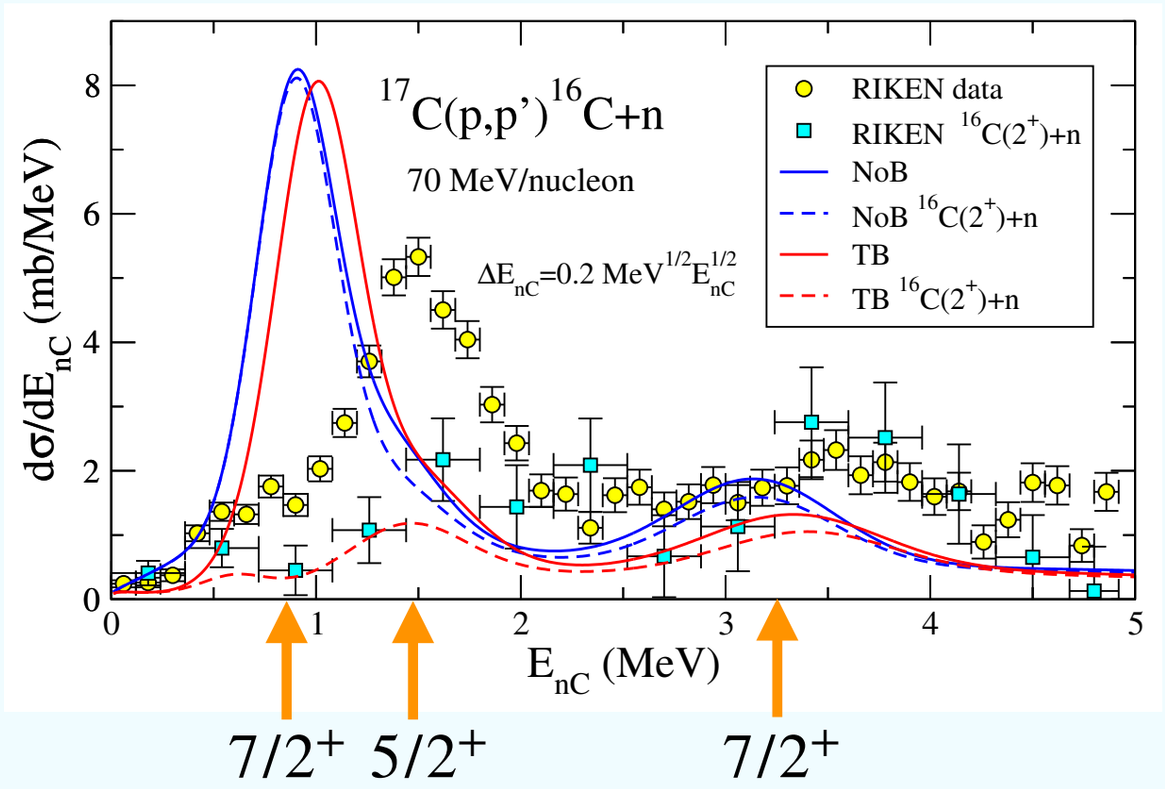
# Application to $^{17}$C(p,p')$^{16}$C+n

Energy with respect to $^{16}$C(0$^+$)+*n* threshold

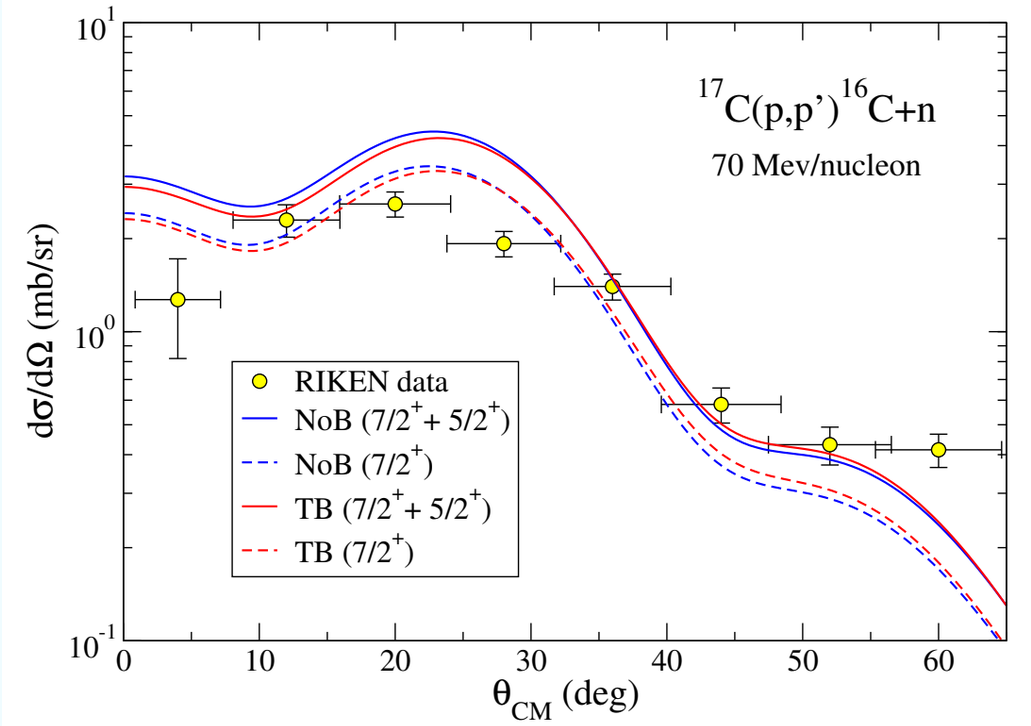


# Application to $^{17}C(p,p')^{16}C+n$

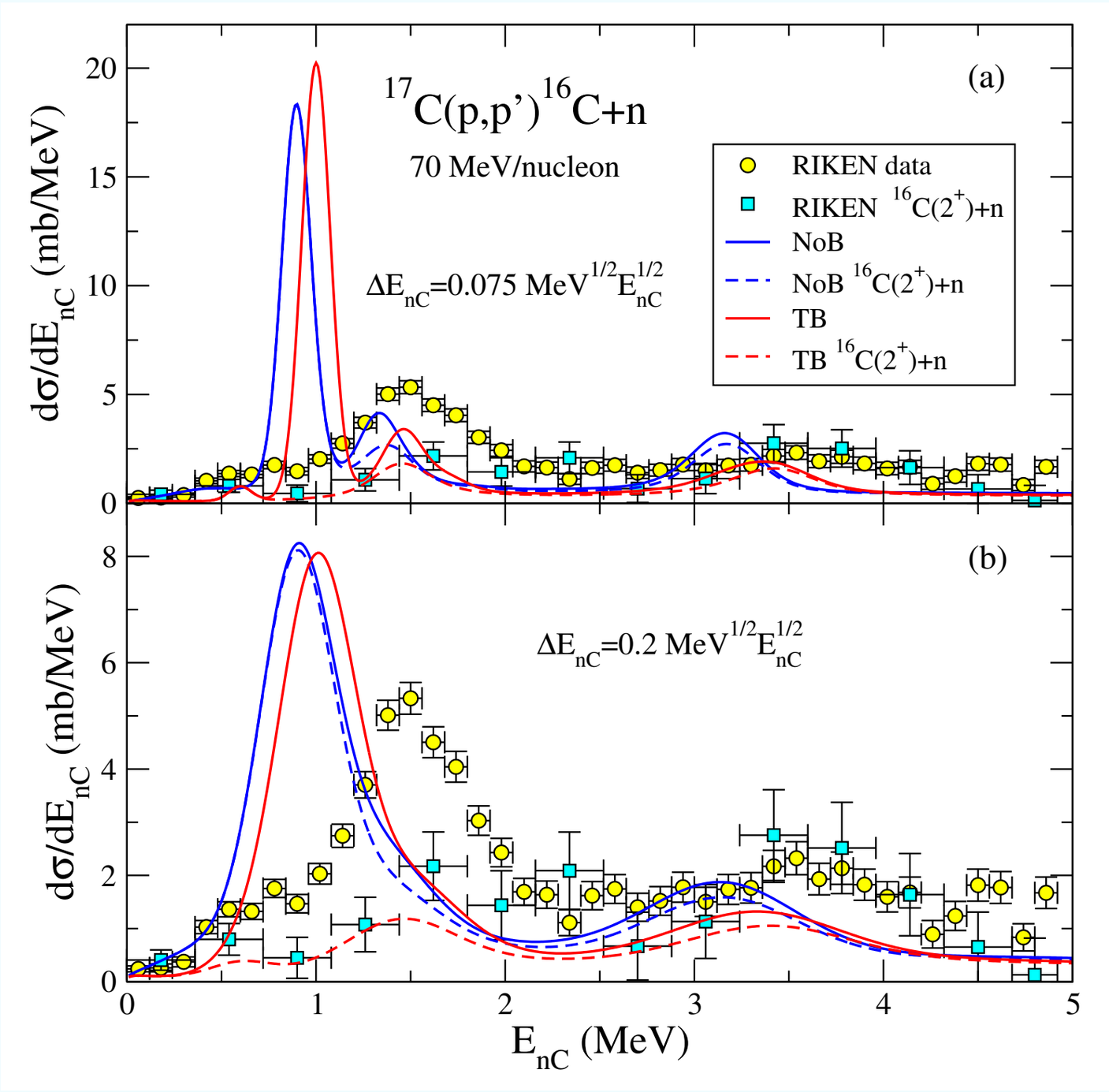
## Energy distribution




## Resonant Breakup



# Application to $^{17}$C(p,p')$^{16}$C+n



# Application to $^{19}C(p,p')^{18}C+n$