November 19th, 2025

XVII CPAN DAYS

Centro Nacional de Física de Partículas, Astropartículas y Nuclear.

$2\nu\beta\beta$ DECAYS TO EXCITED STATES

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Institut de Ciències del Cosmos

UNIVERSITAT DE BARCELONA

EXCELENCIA MARÍA DE MAEZTU 04/2025–03/2031

MINISTERIO DE CIENCIA, INNOVACIÓN Y UNIVERSIDADES

AGENCIA ESTATAL DE INVESTIGACIÓN

$\beta\beta$ DECAY

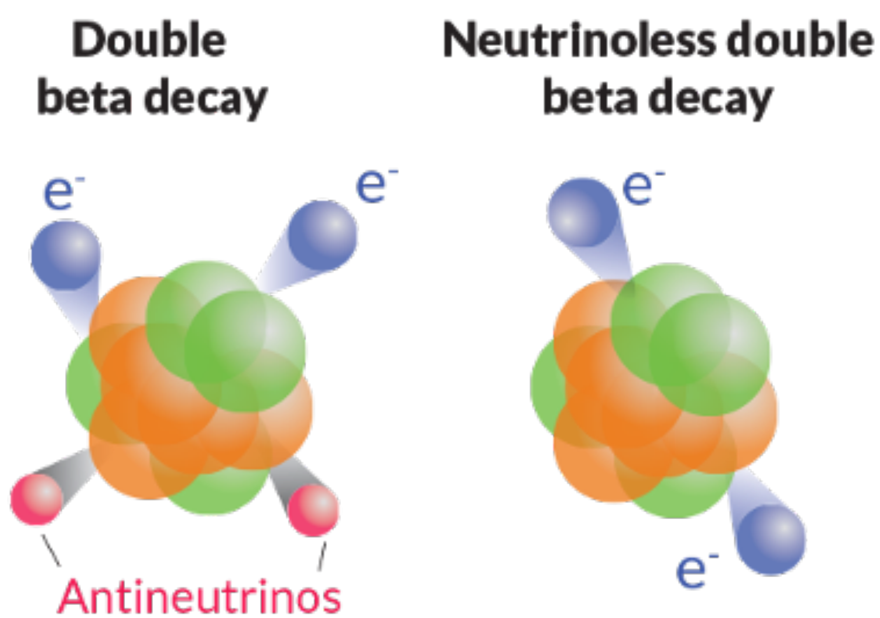
$2\nu\beta\beta : 2n \rightarrow 2p + 2e^{-} + 2\bar{\nu}_e$

Standard Model allowed

Lepton Number Conservation

Transferred momentum limited by Q-value (no virtual neutrinos)

Already measured



$0\nu\beta\beta : 2n \rightarrow 2p + 2e^{-}$

Beyond Standard Model

Lepton Number Violation

Transferred momentum can go higher than the Q-value (virtual neutrinos)

Hypothetical decay

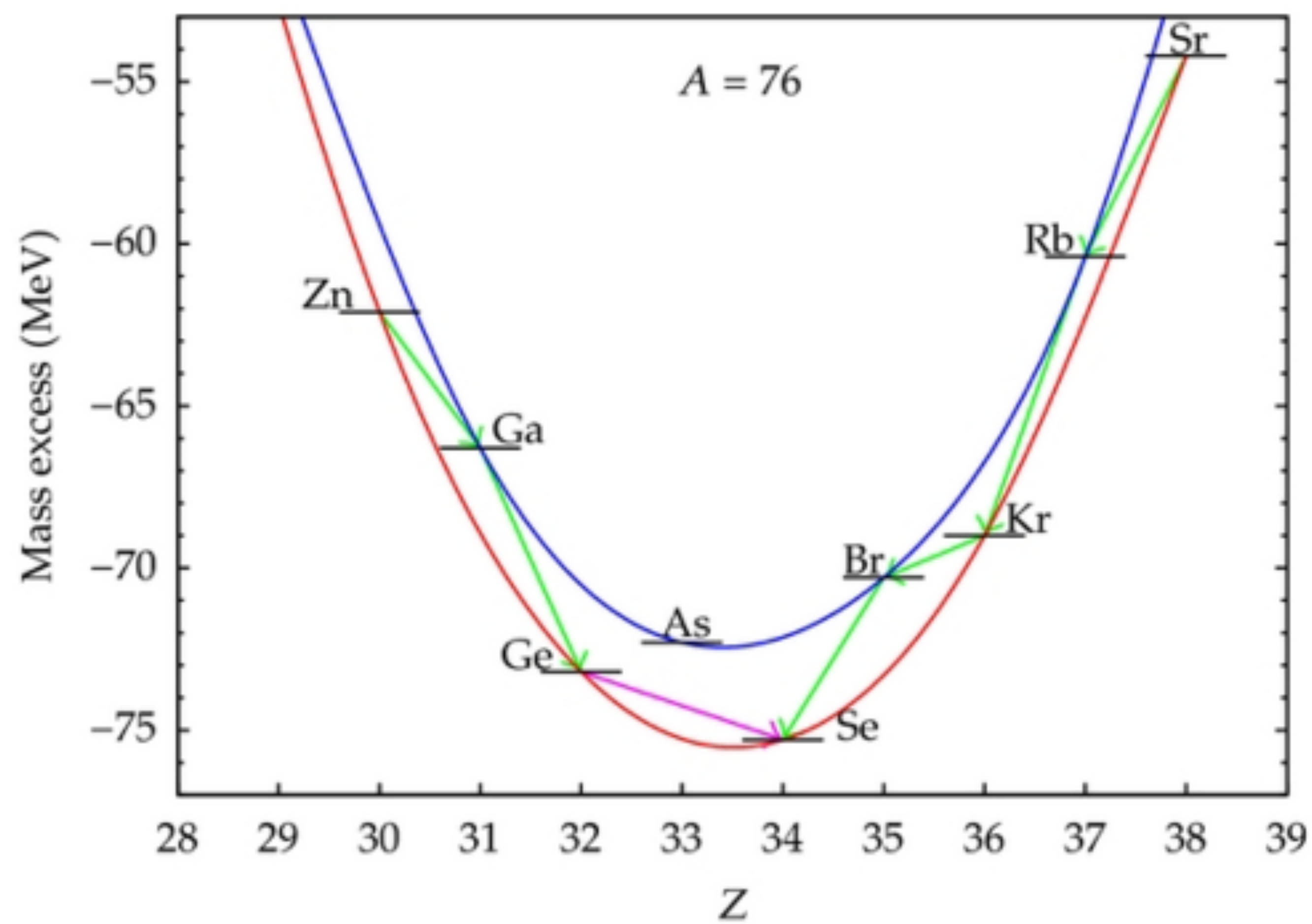
www.sciencenews.org/article/quest-identify-nature-neutrins-alter-ego-heating

$\beta\beta$ DECAY

^{76}Ge : β **decay forbidden**

↓

Energy difference between **odd-odd nuclei** and **even-even nuclei** ⟶ Pairing interaction



Giuliani, et al. Adv. High Energy Phys, 2012, 857016, 2012.

$\beta\beta$ **decay allowed** : $^{76}\text{Ge} \rightarrow {}^{76}\text{Se} \;\; (Q_{\beta\beta} > 0)$

$$Q_{\beta\beta} = E_i - E_f - 2m_e$$

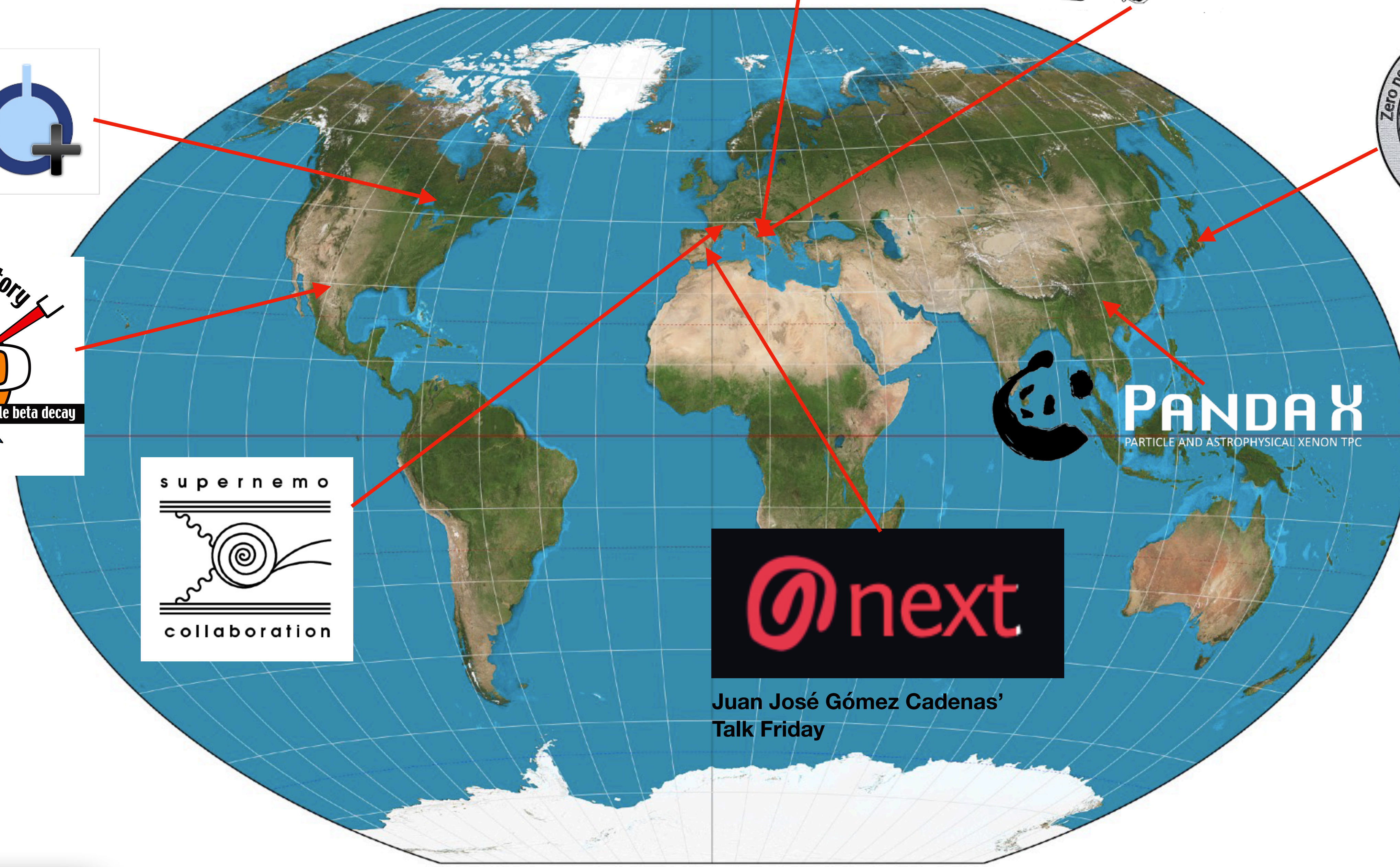
$Q_{\beta\beta} \equiv Q$ value for $\beta\beta$ decay

$m_e \equiv$ electron mass

$E_i \equiv$ initial energy state

$E_f \equiv$ final energy state

EXPERIMENTS

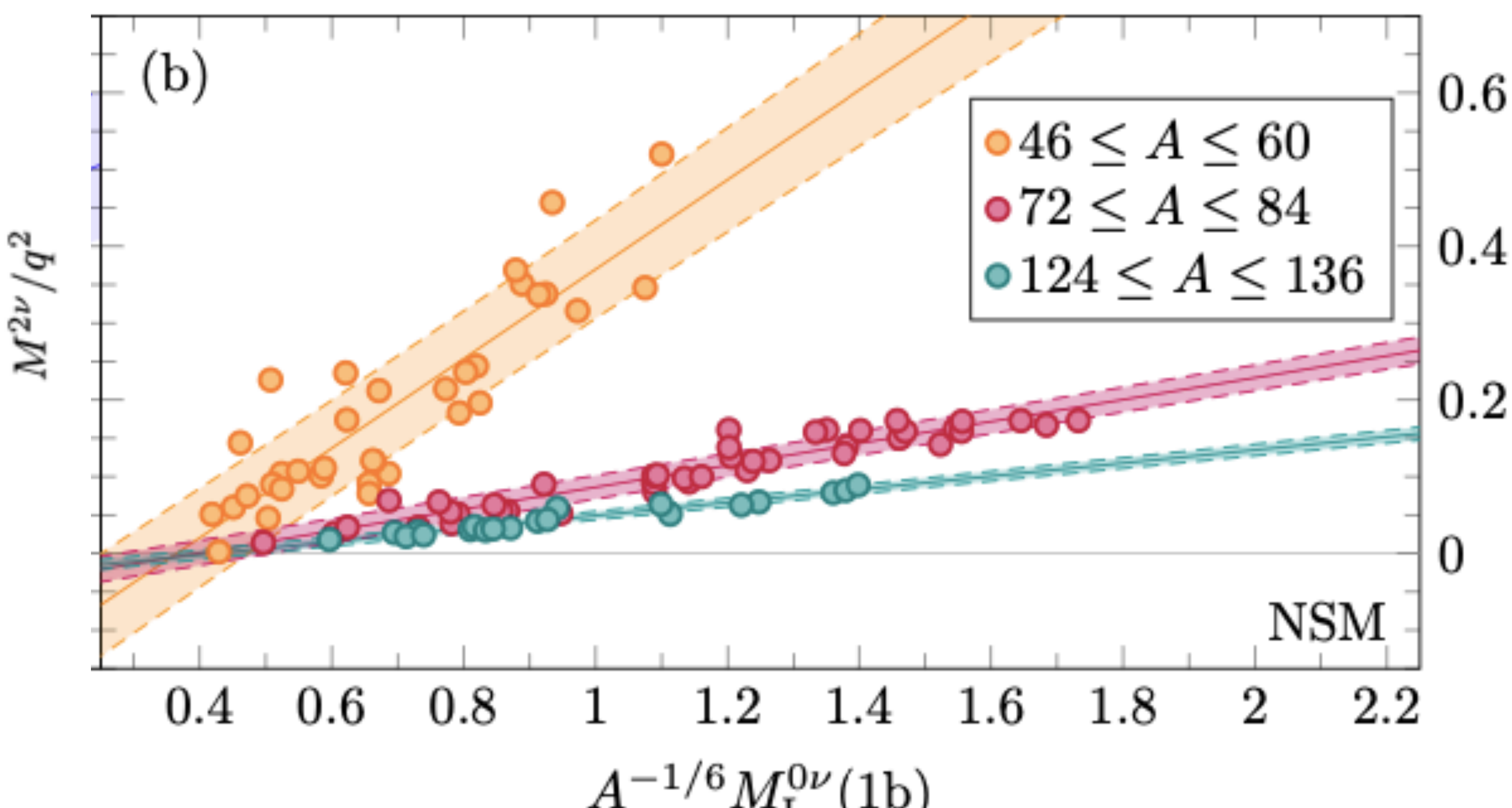


Juan José Gómez Cadenas' Talk Friday

MOTIVATION

Make predictions → **theoretical predictions** must reproduce the **experimental measurements**

→ To **unveil features** about $0\nu\beta\beta$ which is yet **unmeasured**



Jokiniemi, Romeo et al, PRC 107 044305 (2023)

Good correlation : systematic calculation of different nuclei

$M_L^{0\nu} \equiv 0\nu\beta\beta$ Nuclear Matrix Elements (NMEs)

$M^{2\nu} \equiv 2\nu\beta\beta$ NMEs

NUCLEAR MATRIX ELEMENTS

$$(T^{\beta\beta}_{1/2})^{-1} \propto G_0 \cdot g_A^4 \cdot M^{\beta\beta}$$

$T^{\beta\beta}_{1/2} \equiv$ Half-life

$g_A \equiv$ Axial coupling

$G_0 \equiv$ Phase-space factors

$M^{\beta\beta} \equiv$ **NME**

$$M^{\beta\beta} = \langle 0^+_f \,|\, \sum_{a,b} \hat{O}_k \tau^-_a \tau^-_b \,|\, 0^+_i \rangle$$

Solve the many-body Schrödinger equation

↓

Nuclear Shell Model (NSM)
Phenomenological Hamiltonian

$|0^+_{i(f)}\rangle \equiv$ Initial (final) wavefunctions

Obtain decay operators

↓

From weak currents

$\hat{O}_k \equiv$ Spin-space operators

$2\nu\beta\beta : \hat{O}_k = \mathrm{GT}$ (Gamow-Teller)

$\tau^- \equiv$ Ladder isospin operator

$2\nu\beta\beta$ DECAY TO EXCITED STATES

$$0^+_{GS} \rightarrow 0^+_{EXC}$$

$$(T^{2\nu}_{1/2})^{-1} \propto G^{2\nu}_0 \cdot g_A^4 \cdot (M^{2\nu})^2$$

↓

$M^{2\nu} \equiv$ **Leading-order NME**

We include **higher-order corrections**:

- Lepton energies expansion (Additional Phase-space factors and NMEs)
- Chiral effective field theory corrections (up to next-to leading-order)

DC, Frycz, Menéndez, Benavente, arXiv:2507.21868

$0^+_{GS} \rightarrow 0^+_{GS}$ have been **measured** for several nuclei with $T^{2\nu}_{1/2}$ that can exceed 10^{21} yr

↓

^{48}Ca, ^{76}Ge, ^{82}Se, ^{130}Te , ^{136}Xe . . .

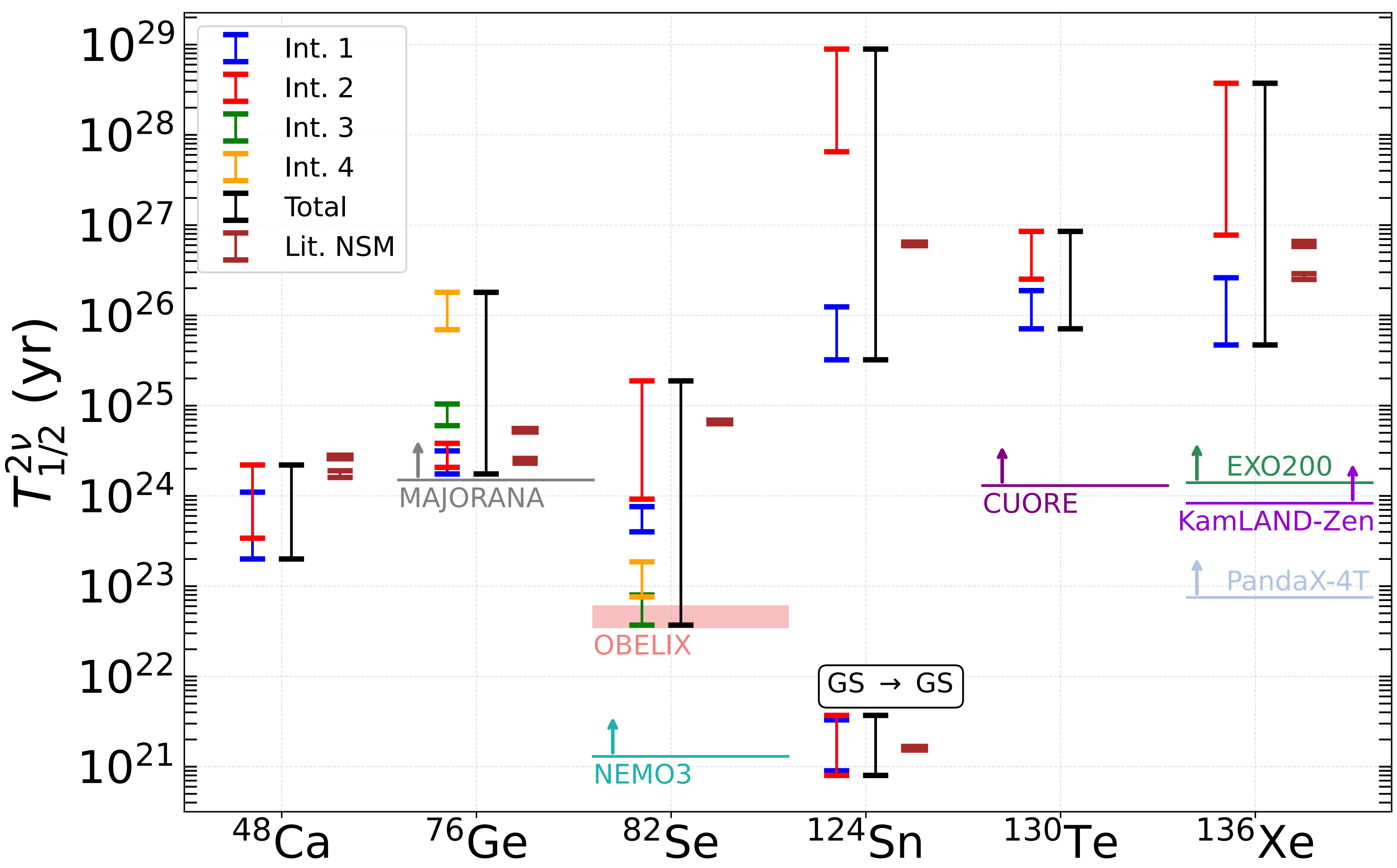
^{124}Sn : $0^+_{GS} \rightarrow 0^+_{GS}$ not yet

We use **two different GT operators for each Hamiltonian**:

- Bare
- Renormalized within the valence space

Coraggio et al. , Phys. Rev. C, 100, 014316 (2019)

$2\nu\beta\beta$ DECAY TO EXCITED STATES



DC, Frycz, Menéndez, Benavente, arXiv:2507.21868

Main uncertainty comes from the **different Hamiltonians**

↓

Deformations of the initial and final states play a key role

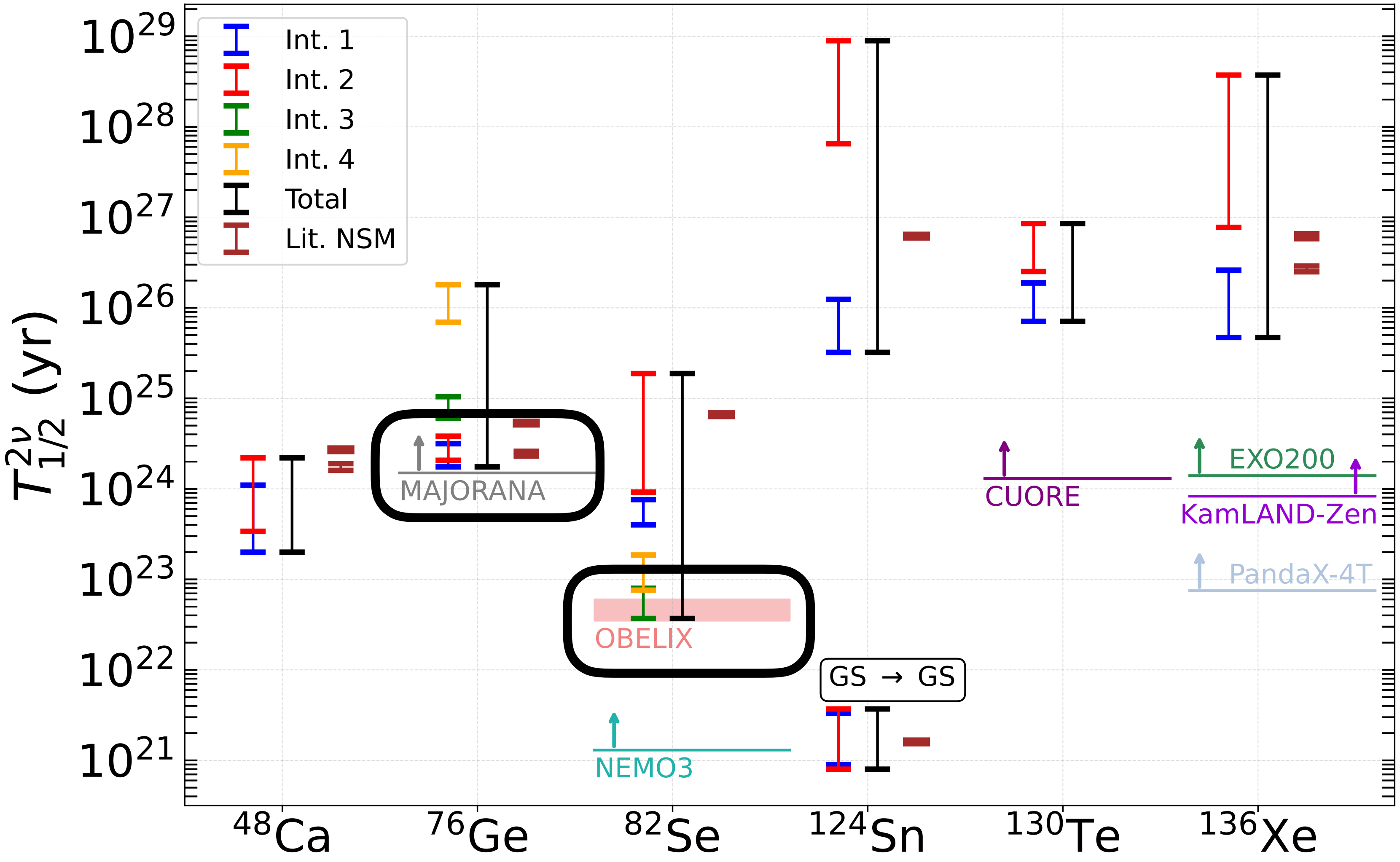
^{76}Ge, ^{82}Se :

Experimental evidence close to theoretical predictions

^{76}Ge :

Most **realible prediction** (Hamiltonian) → **Longest $T^{2\nu}_{1/2}$**

$2\nu\beta\beta$ DECAY TO EXCITED STATES



DC, Frycz, Menéndez, Benavente, arXiv:2507.21868

Main uncertainty comes from the **different Hamiltonians**

↓

Deformations of the initial and final states play a key role

^{76}Ge, ^{82}Se :

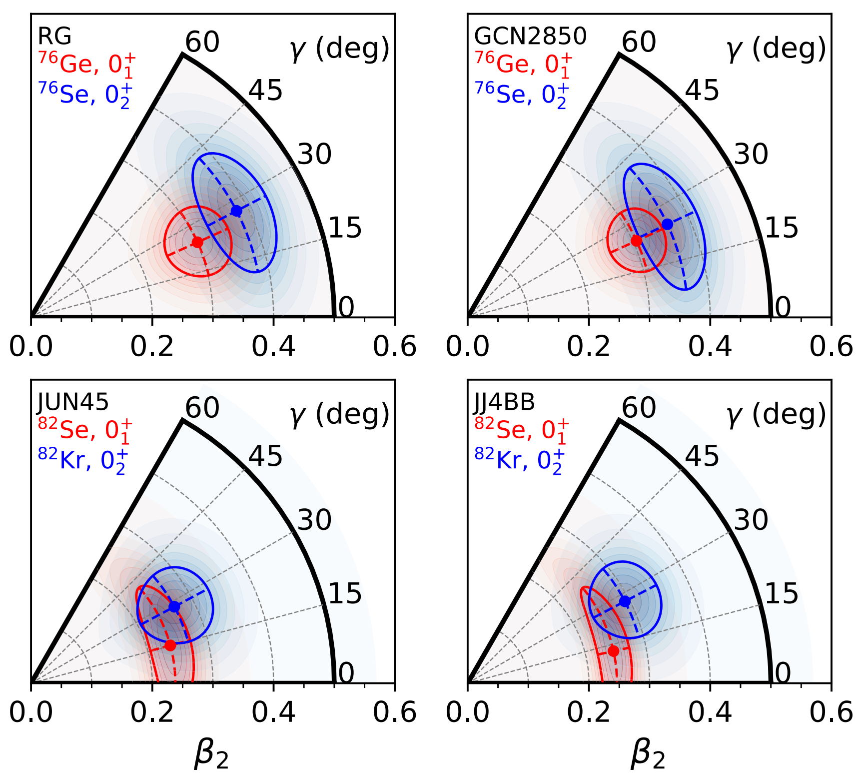
Experimental evidence close to theoretical predictions

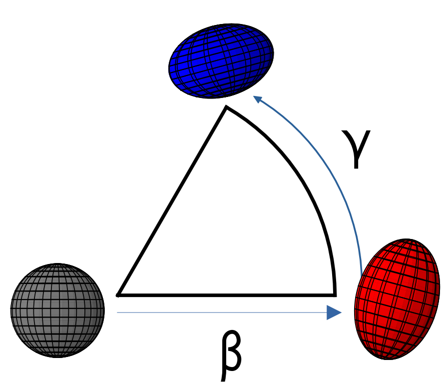
^{76}Ge :

Most **realible prediction** (Hamiltonian) → **Longest $T_{1/2}^{2\nu}$**

$2\nu\beta\beta$ DECAY: DEFORMATION

Dorian Frycz's Talk Tomorrow morning (9:45h)

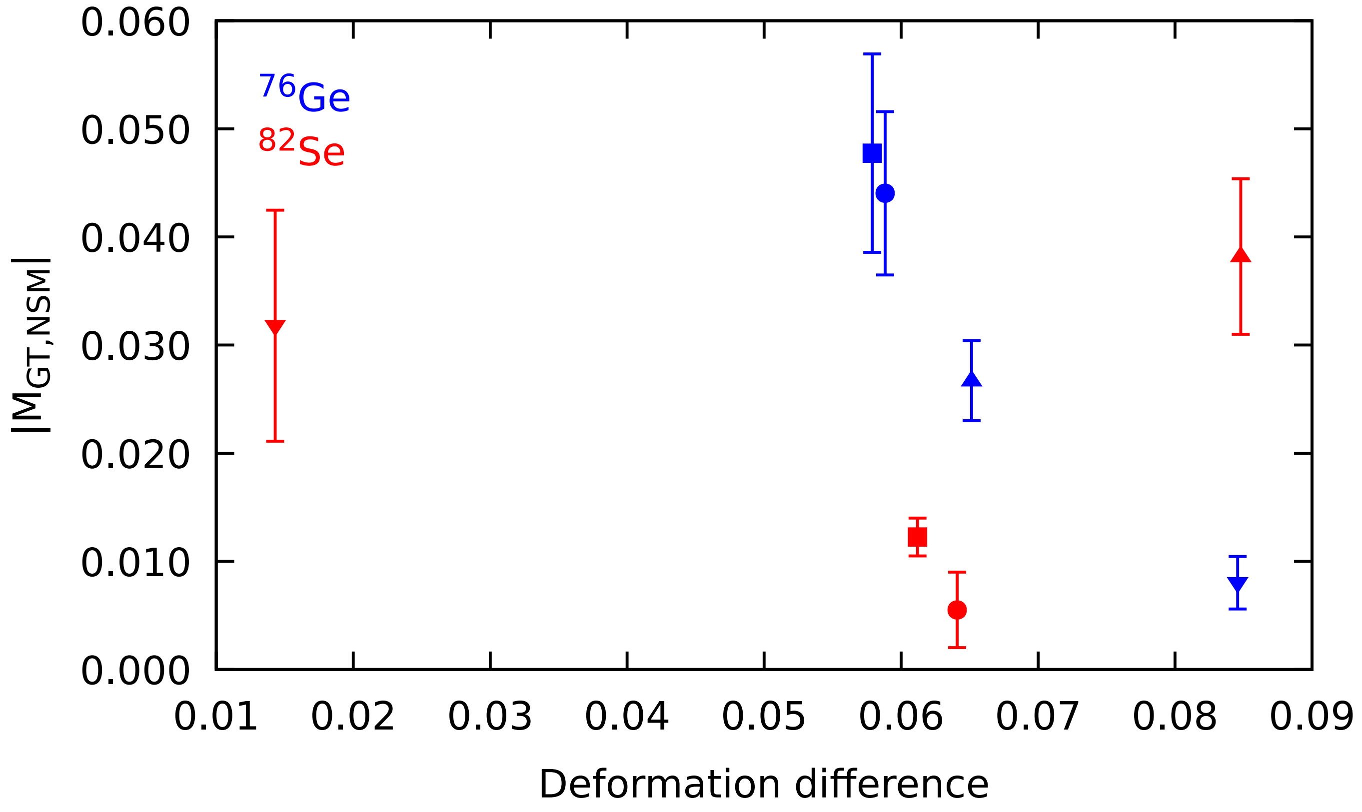




$\beta_2 \equiv$ Magnitude of deformation

$\gamma \equiv$ Type of deformation

Deformation difference ⟷ $M^{2\nu}$



DC, Frycz, Menéndez, Benavente, arXiv:2507.21868

SUMMARY AND OUTLOOK

$T^{2\nu}_{1/2}(0^+_{GS} \rightarrow 0^+_{EXC})$ predictions: ^{48}Ca, ^{76}Ge, ^{82}Se, ^{124}Sn, ^{130}Te and ^{136}Xe

Large uncertainty mainly due to the **Hamiltonian** choice (**deformation**)

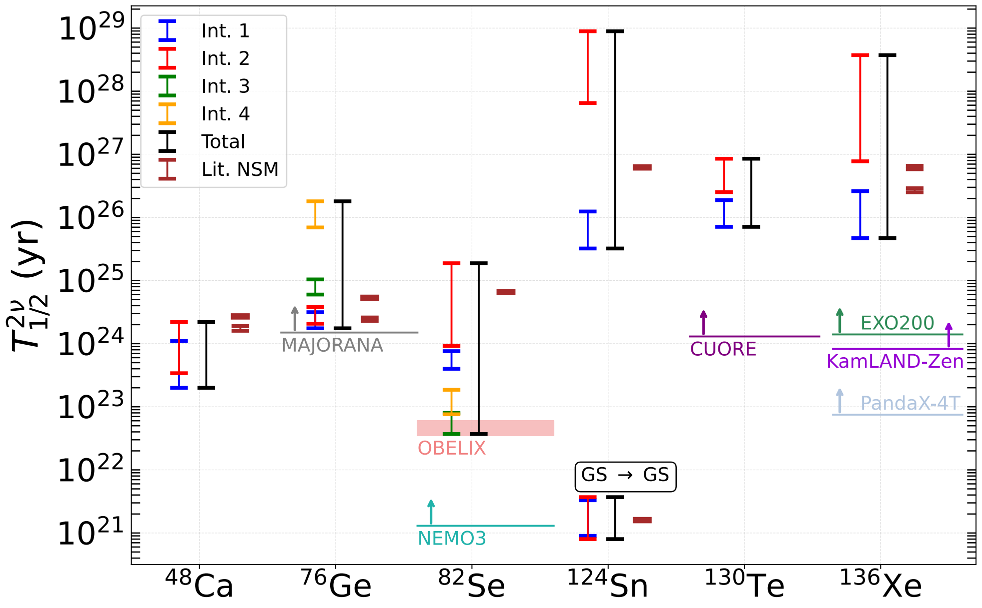
^{76}Ge predictions **near** to the **experimental limit**

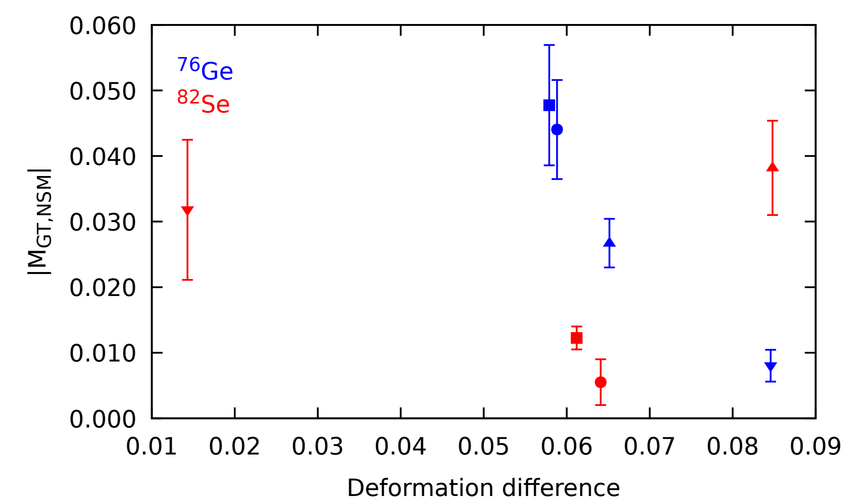
^{82}Se predictions **compatible** with **measurement claim**

Use $\chi -$ **Hamiltonians** to compute the wavefunctions

Extend the **decay operator** study to **two-body currents**

Looking forward to **experimental measurements**





THANK YOU FOR YOUR ATTENTION!

Two-neutrino $\beta\beta$ decay to excited states at next-to-leading order

Daniel Castillo[a,b,c,*], Dorian Frycz[a,b,c,*], Beatriz Benavente[c], Javier Menéndez[a,b]

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[b] *Institut de Ciències del Cosmos, Universitat de Barcelona, c. Martí i Franquès, 1, 08028 Barcelona, Spain*
[c] *Facultat de Física, Universitat de Barcelona, c. Martí i Franquès, 1, 08028 Barcelona, Spain*

BACKUP SLIDES

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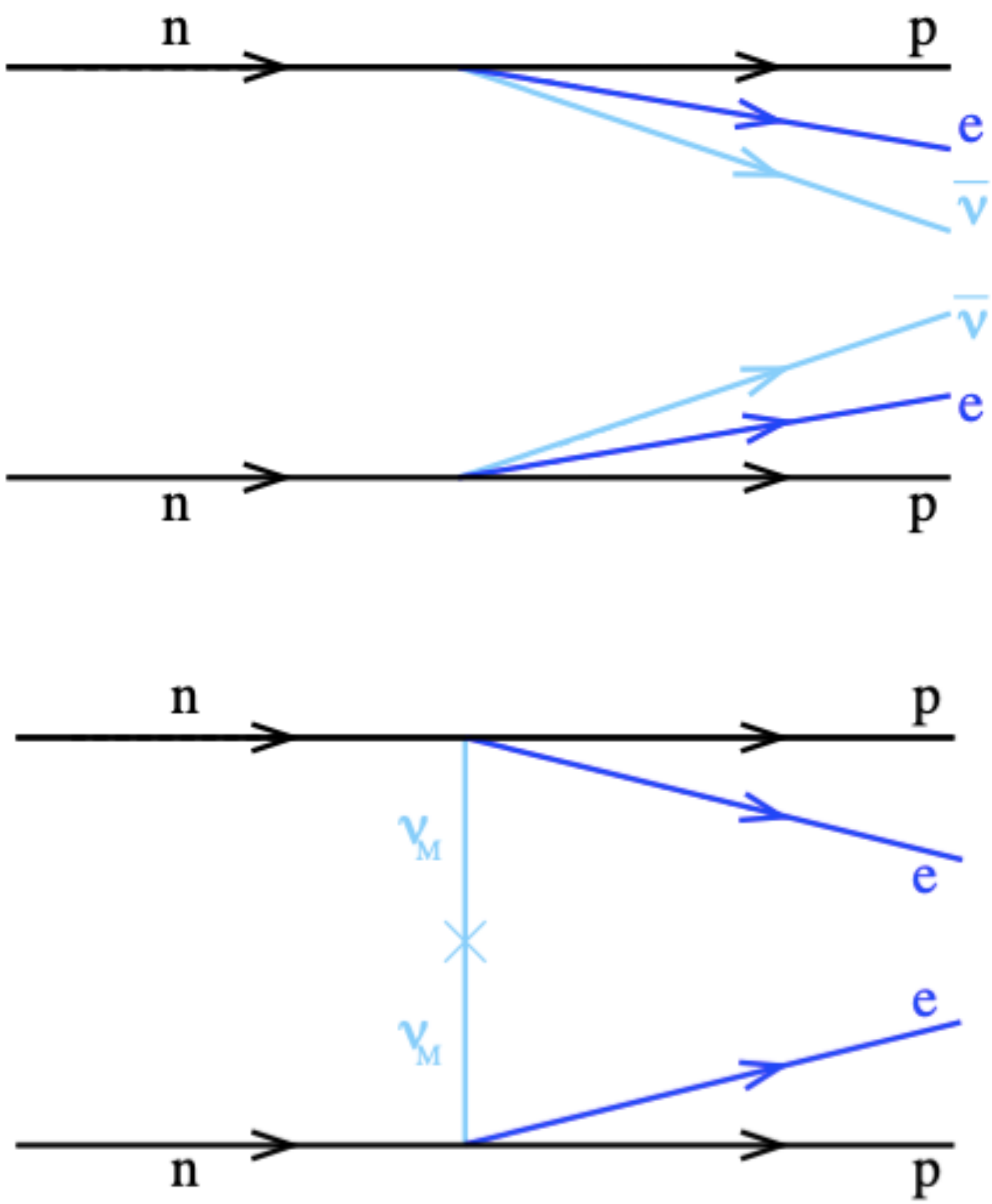


Engel, et al. Rep. Prog. Phys. 80, 046301, 2017

NUCLEAR SHELL MODEL

We want to solve the **Schrödinger equation** ⟶ $H_{eff}|0^+\rangle = E|0^+\rangle$

$H_{eff} \rightarrow$ **Phenomenological** effective interactions

Fitted to NN scattering and renormalized within each specific valence space

Very **good description** of **nuclear spectroscopy**

Orbitals –
- EMPTY ORBITALS
- **VALENCE SPACE**
- CORE

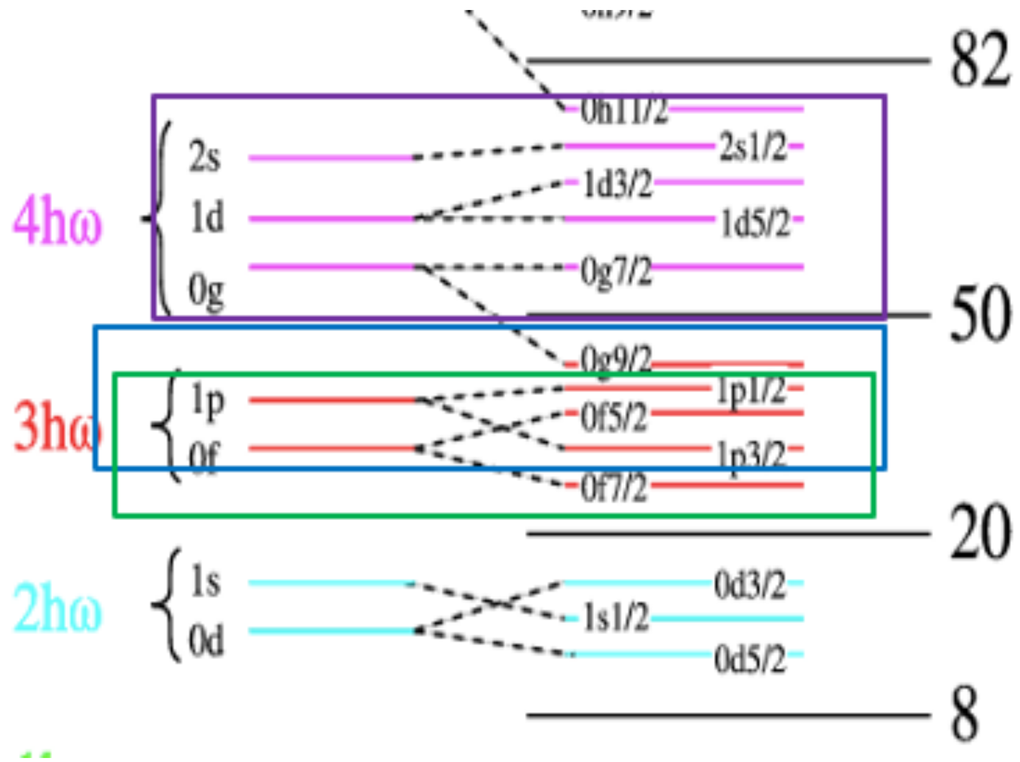
⟶ **Effective interactions**

Caurier et al. Rev. Mod. Phys. 77 427 (2005)

Valence Space (Magic Numbers):

^{48}Ca : 20 – 40 ^{76}Ge, ^{82}Se : 28 – 50

^{124}Sn, ^{130}Te, ^{136}Xe : 50 – 82



https://oer.physics.manchester.ac.uk/NP/Notes/Notes/Notesse23.xht

MOTIVATION

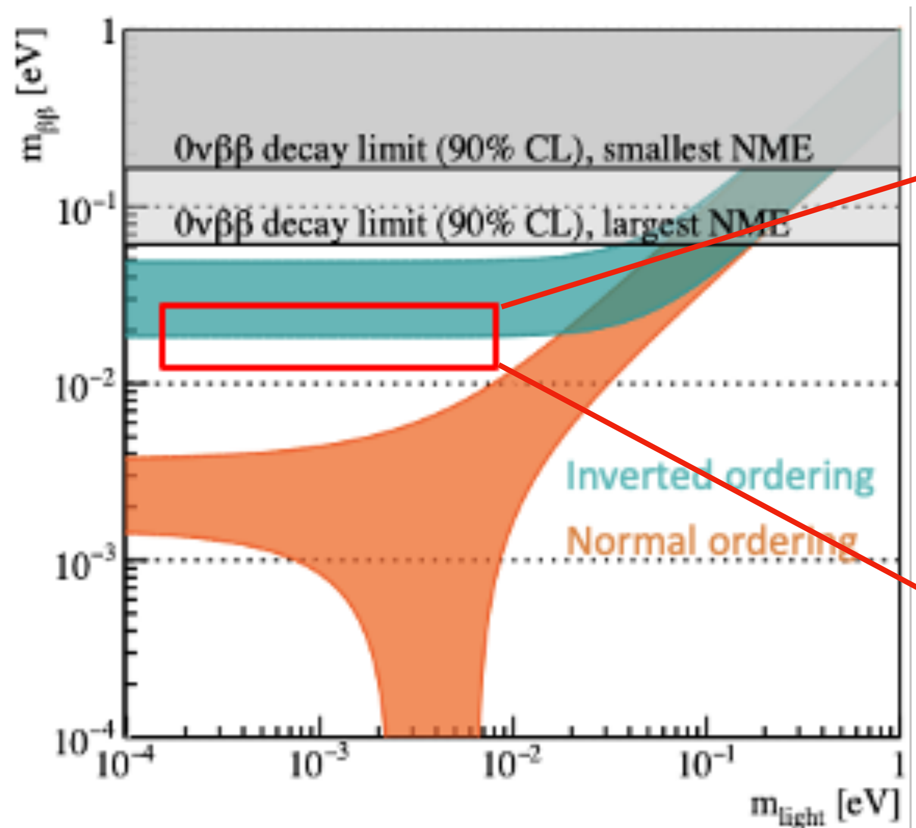
$$(T^{0\nu}_{1/2})^{-1} = g_A^4 G_{0\nu} \, |M^{0\nu}|^2 m_{\beta\beta}^2$$

$G_{0\nu} \equiv$ Phase-space factor

$M^{0\nu} \equiv$ **Nuclear matrix elements**

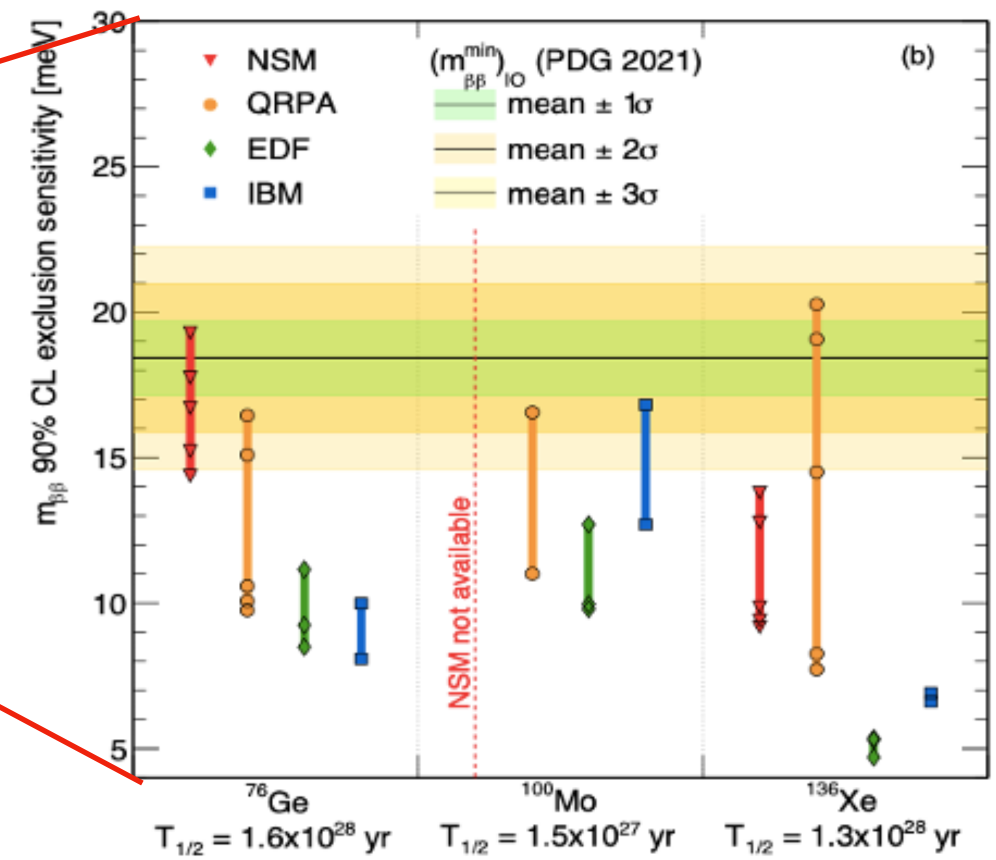
$g_A \equiv$ Axial coupling

$m_{\beta\beta} = \sum_{j=light} U_{ej} m_j \equiv$ Effective neutrino mass



Agostini et al. Rev. Mod. Phys. 95, 025002, 2023

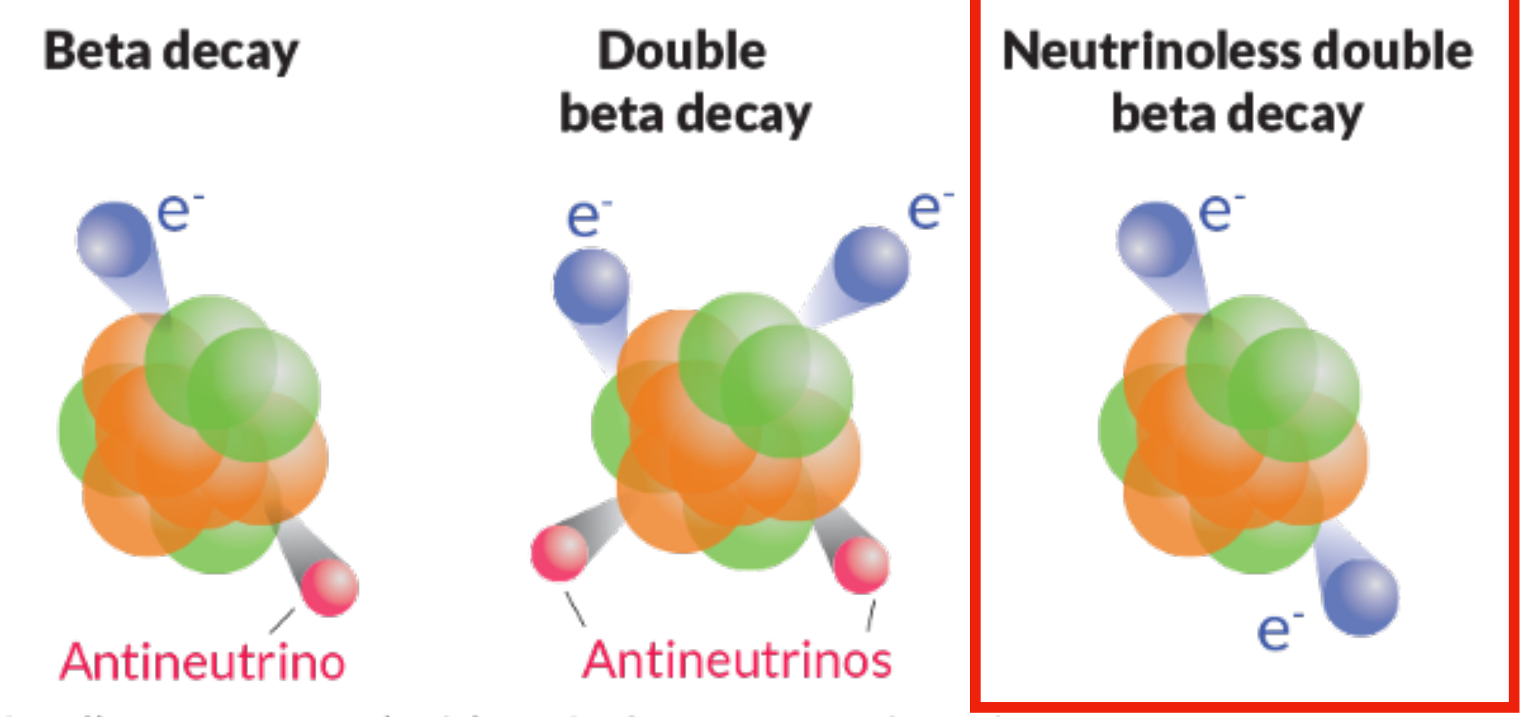
Next decade experiments



Agostini et al. Phys. Rev. C 104, L042501, 2021

NEUTRINOLESS $\beta\beta$ DECAY

$$0\nu\beta\beta : 2n \rightarrow 2p + 2e^-$$



https://www.sciencenews.org/article/quest-identify-nature-neutrinos-alter-ego-heating

New Information:

Absolute **mass of neutrino**

Neutrino as **Majorana fermion**

Lepton Number Violation
(Beyond Standard Model Physics)

Baryon-Antibaryon assymetry

NUCLEAR MATRIX ELEMENTS

$$M^{0\nu} = \langle 0_f^+ || \sum_{a,b} H_k \hat{O}_k \tau_a^- \tau_b^- || 0_i^+ \rangle$$

Solve the many-body Schrödinger equation

Nuclear Shell Model (NSM)
Quasi-particle random phase approximation (QRPA)

$|0^+_{i(f)}\rangle \equiv$ Initial (final) wavefunctions

Obtain decay operators

Chiral Effective Field Theory (χEFT)

$H_k(r) \equiv$ Neutrino potentials

$\hat{O}_k \equiv$ Spin-space operators

$k \equiv \mathrm{GT, F, T}$ (Gamow-Teller, Fermi, Tensor)

$\tau^- \equiv$ Ladder isospin operator

QUASI-PARTICLE RANDOM PHASE APPROXIMATION (QRPA)

$|QRPA\rangle = |0^+\rangle \longrightarrow$ Reference state

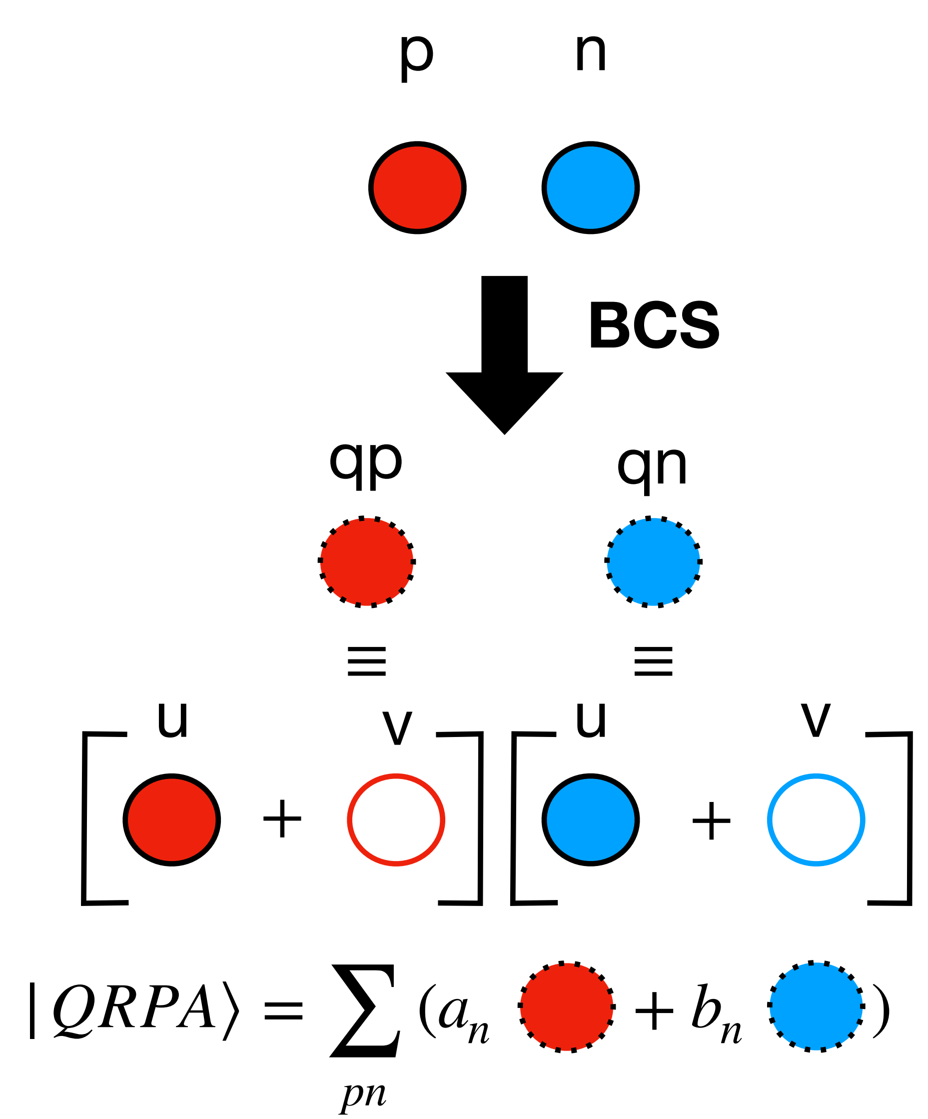
$n = 0 \rightarrow 2$ harmonic oscillators shells **above** the **Fermi level**

$\downarrow$

^{76}Ge, ^{82}Se : 18 orbitals

^{96}Zr, ^{100}Mo : 25 orbitals

^{116}Cd, ^{124}Sn, ^{130}Te, ^{136}Xe : 26 orbitals



Credit, Jokiniemi

More orbitals within the **valence space**
Less complex correlations between nucleons

J. Suhonen, Springer-Verlag, Berlin Heidelberg, 2007

$0\nu\beta\beta$ χEFT NMEs

χEFT DIAGRAMS

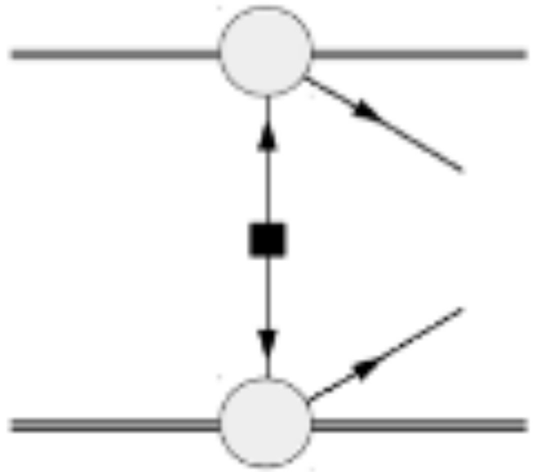
$q \equiv$ Transferred momentum

$$M^{0\nu} = M^{0\nu}_{\mathrm{L}} + M^{0\nu}_{\mathrm{S}} + M^{0\nu}_{\mathrm{usoft}} + M^{0\nu}_{\mathrm{loops}}$$

Leading Order (LO)

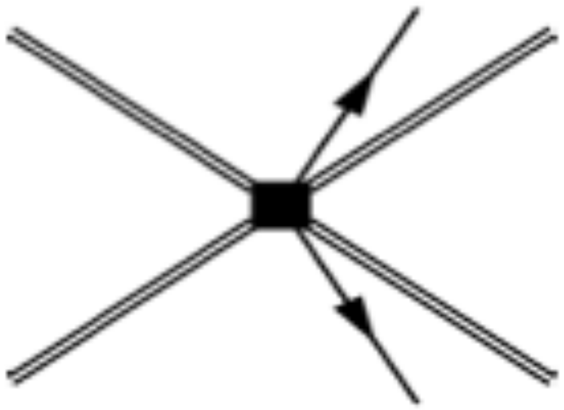
Long-range (L)

$(q \sim 100\,\mathrm{MeV} + q - \mathrm{dependent\ N^2LO})$



Short-range (S)

$(q \gg 100\,\mathrm{MeV})$



already computed

Jokiniemi et al. Phys. Let. B 823, 136720, 2021

Cirigliano, et al. Phys. Rev. C 97, 065501, 2018

LEADING ORDER

$$M_{\rm L}^{0\nu} = M_{\rm L,GT}^{0\nu} + M_{\rm L,F}^{0\nu} + M_{\rm L,T}^{0\nu}$$

$q-$ dependent N^{2}LO → **Finite Size Corrections**

Dipole form factors: $g_A^2(q^2)$ and $g_V^2(q^2)$

$$M_{\rm S}^{0\nu} \propto g_\nu^{NN} e^{-q^2/\Lambda^2}$$

$g_\nu^{NN} \equiv$ Nucleon-Nucleon coupling

$\Lambda \equiv$ Cutoff

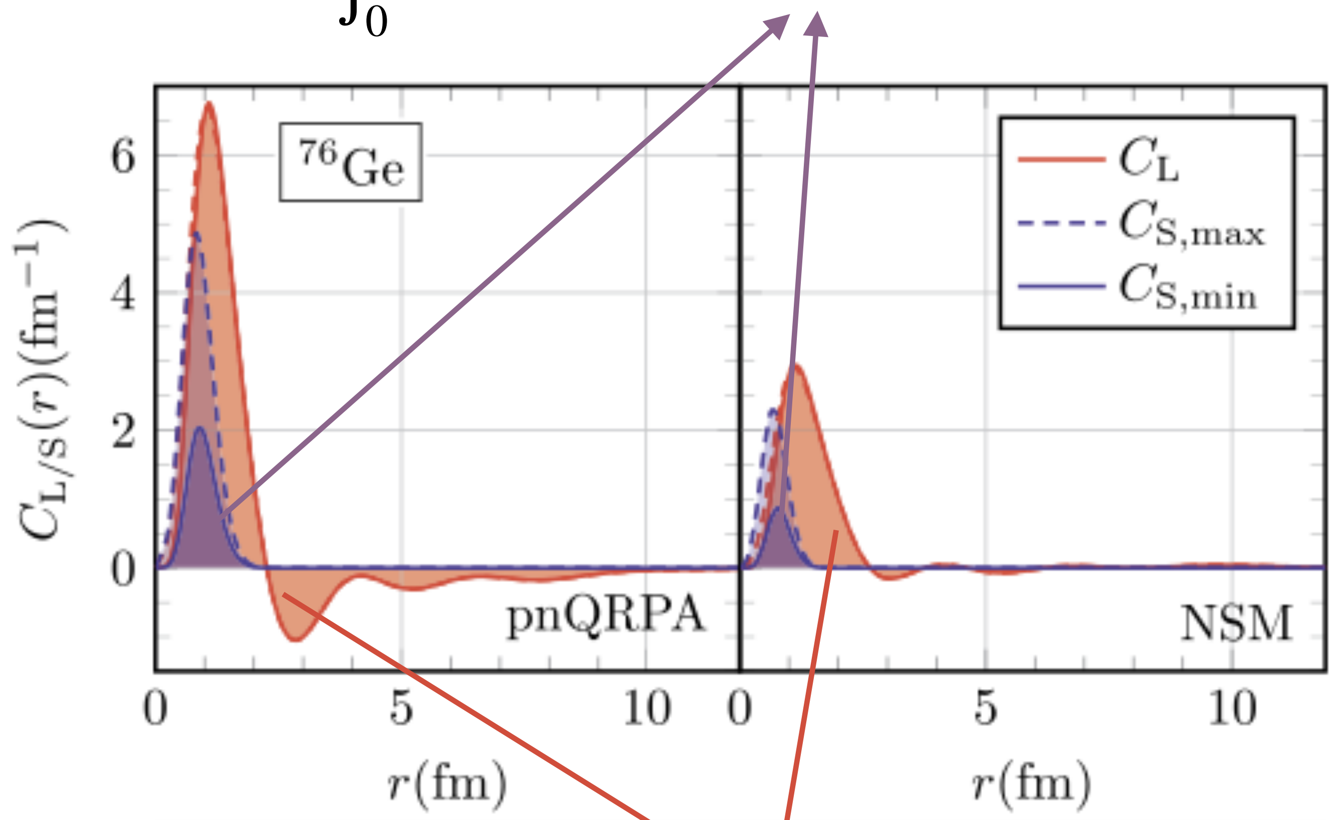
NSM: $M_{\rm S}^{0\nu}/M_{\rm L}^{0\nu} = +(15-50)\,\%$

QRPA: $M_{\rm S}^{0\nu}/M_{\rm L}^{0\nu} = +(30-80)\,\%$

Jokiniemi et al. Phys. Let. B 823, 136720, 2021

$$M_{\rm L/S}^{0\nu} = \int_0^\infty C_{\rm L/S}(r)dr$$

Hard neutrinos



Jokiniemi et al. Phys. Let. B 823, 136720, 2021

Soft neutrinos

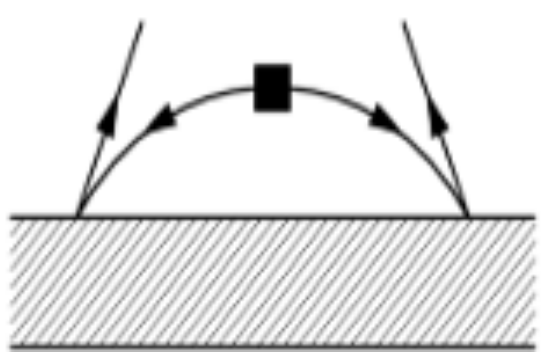
χEFT DIAGRAMS

$$M^{0\nu} = M_{\mathrm{L}}^{0\nu} + M_{\mathrm{S}}^{0\nu} + M_{\mathrm{usoft}}^{0\nu} + M_{\mathrm{loops}}^{0\nu}$$

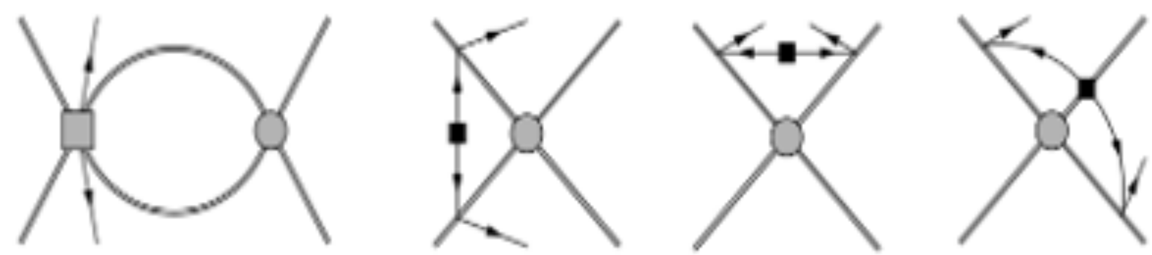
New terms

Next-to-Next-to-Leading Order (N^{2}LO)

Ultrasoft $(q \ll 100\,\mathrm{MeV})$



Loop terms



Cirigliano, et al. Phys. Rev. C 97, 065501, 2018

ULTRASOFT NME

Study of the **intermediate states** dependence of the **total ultrasoft NMEs**

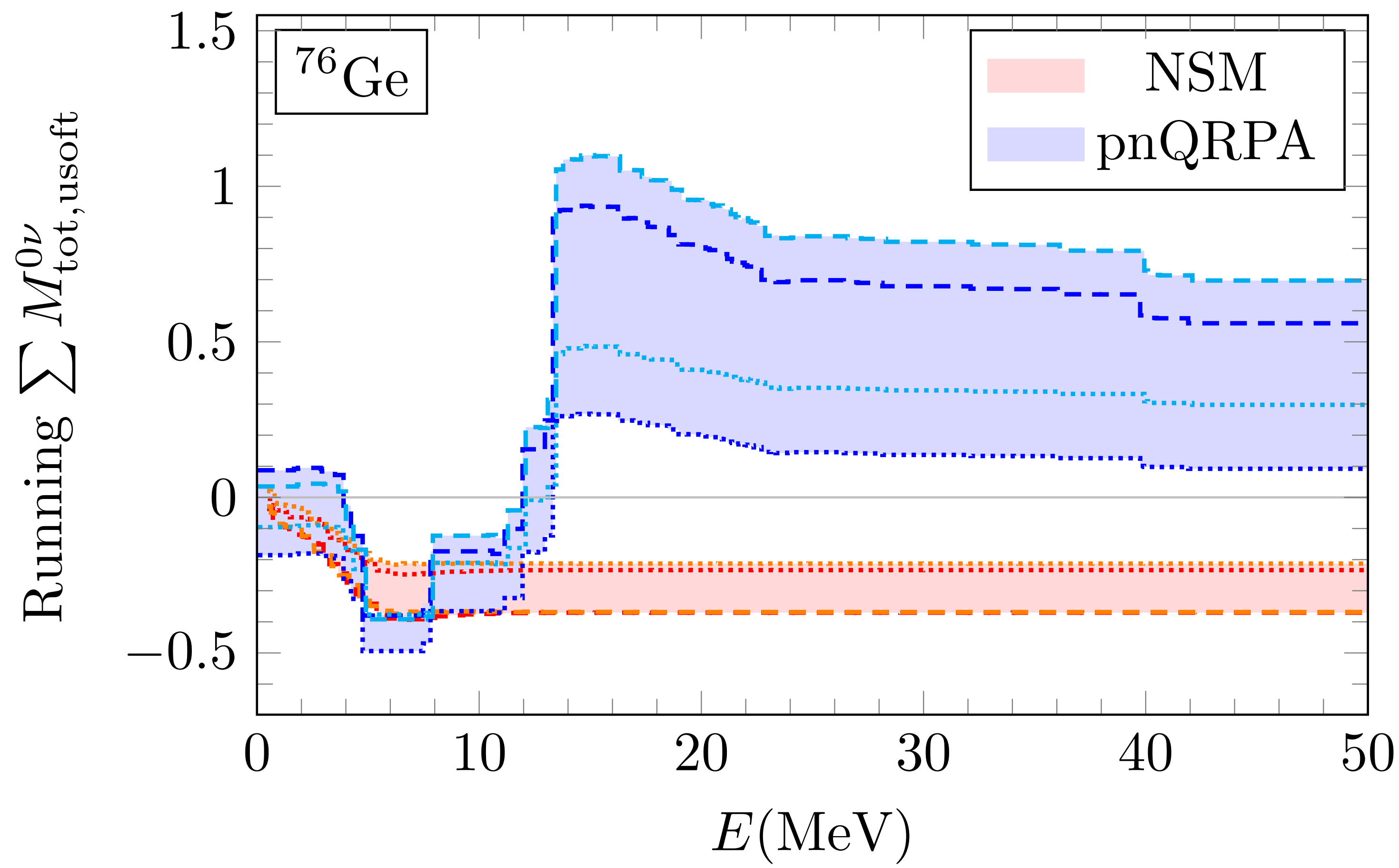
$$M^{0\nu}_{\text{tot,usoft}} = M^{0\nu}_{\text{usoft}} + M^{0\nu}_{\text{loops,usoft}}$$

$$M^{0\nu}_{\text{usoft}} \propto \langle 0^+_f \,|\, \text{GT} \,|\, 1^+_n \rangle \langle 1^+_n \,|\, \text{GT} \,|\, 0^+_i \rangle \cdot \ln(\mu_{\text{us}})$$

$\mu_{us} \equiv$ Ultrasoft scale

Different behaviour between models around **10 MeV**

→ **Different sign**

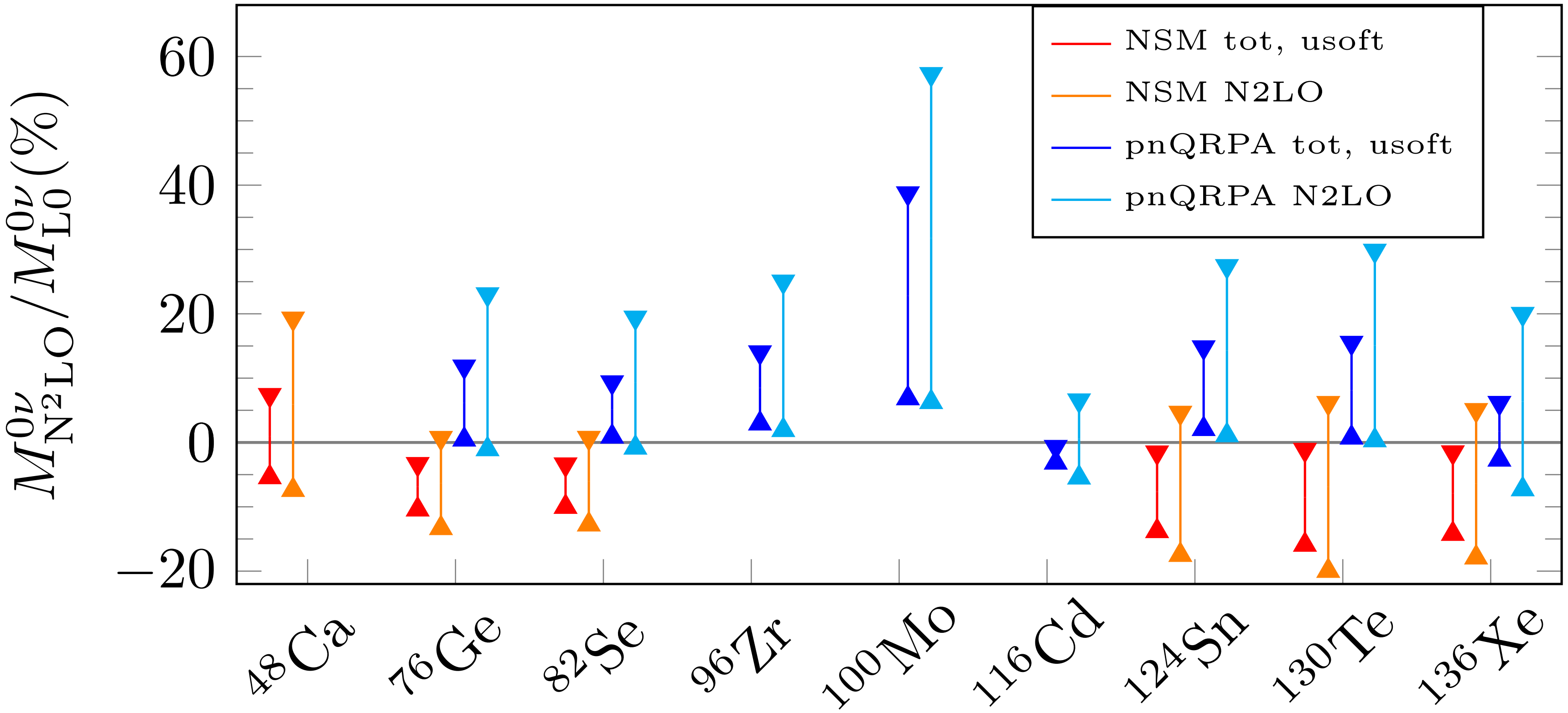


DC, Jokiniemi, Menéndez, Phys.Lett.B 860 (2025)

Main uncertainty ⟶ $\mu_{\text{us}} = \dfrac{m_\pi}{2} \ldots 2m_\pi$

TOTAL N²LO NMEs

$$M^{0\nu}_{\mathrm{N^2LO}} = M^{0\nu}_{\mathrm{tot,usoft}} + M^{0\nu}_{\mathrm{loops,soft}}$$



DC, Jokiniemi, Menéndez, Phys.Lett.B 860 (2025)

χEFT expectations $\sim$ (5-10) %

$M^{0\nu}_{\mathrm{N^2LO}}/M^{0\nu}_{\mathrm{LO}}$:

NON NEGLIGIBLE CONTRIBUTIONS

NSM $\leq$ 20 %

Central values:

Total ultrasoft: $-$(5-10) %

Total N²LO: $-$(5-10) %

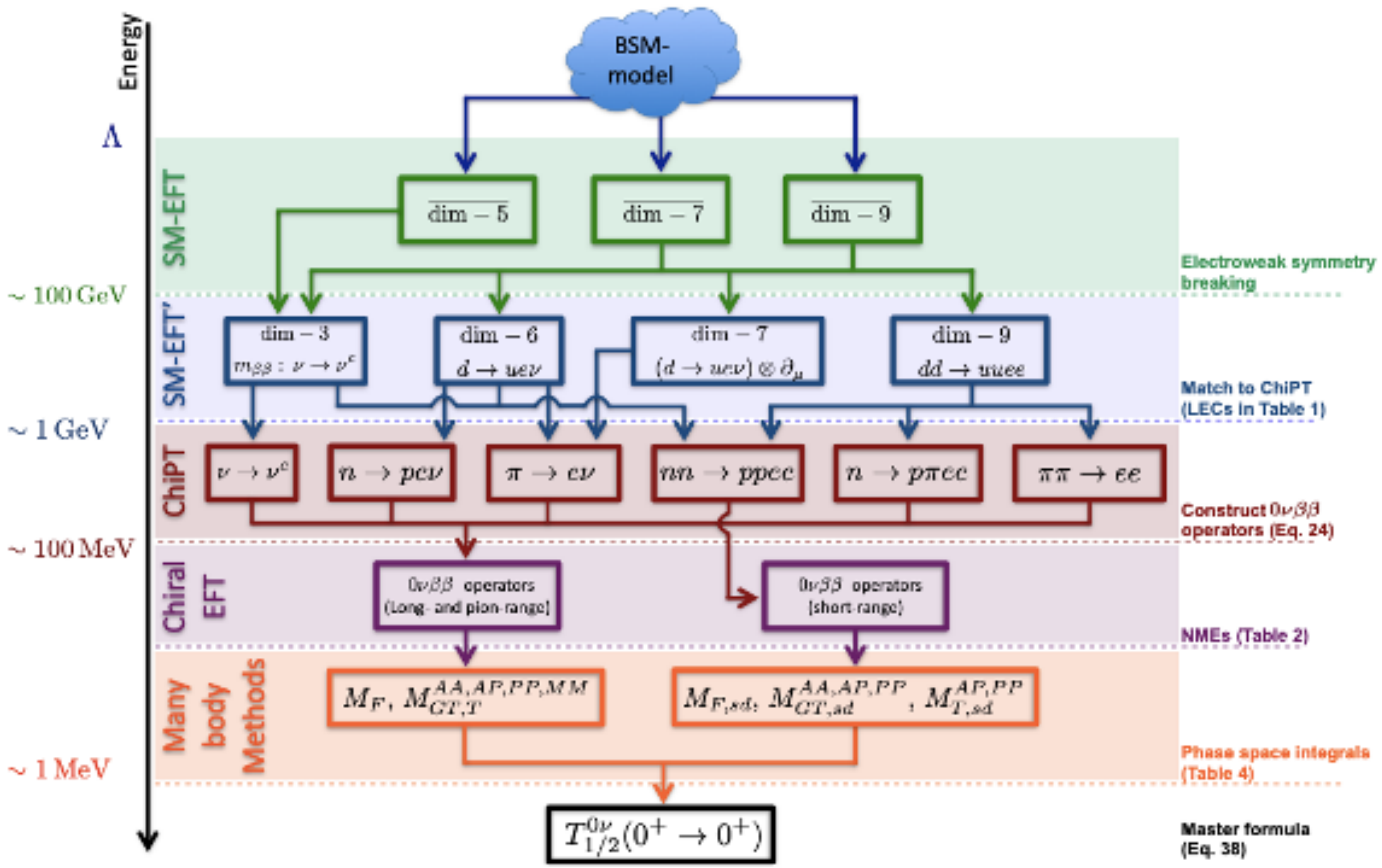
QRPA $\leq$ 30 %

Central values:

Total ultrasoft: $+$(5-10) %

Total N²LO: $+$(10-15) %

χEFT



Cirigliano et al. JHEP, **12**, 097, 2018

CLOSURE VS NON-CLOSURE

Non-closure: $|J_n^\pi\rangle \equiv$ Intermediate states

$$M^{0\nu}_{\text{non-cl}} \propto \frac{\langle 0_f^+ | J_\mu(x) | J_n^\pi \rangle \langle J_n^\pi | J^\mu(y) | 0_i^+ \rangle}{q(q + E_n - \frac{1}{2}(E_f + E_i))}$$

Closure:

$$M^{0\nu}_{\text{L}} \propto \frac{\langle 0_f^+ | J_\mu(x) J^\mu(y) | 0_i^+ \rangle}{q^2}$$

$$q \simeq k_F \simeq 100\,\text{MeV}$$

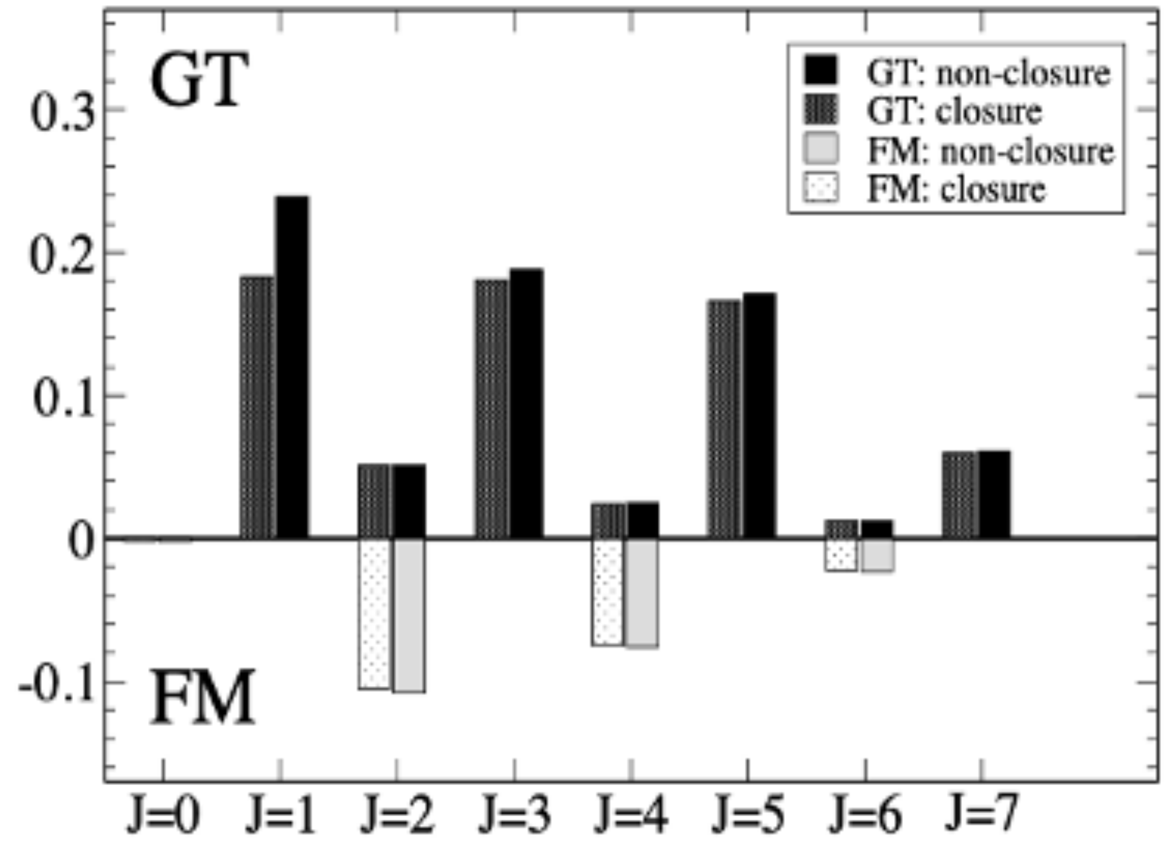
$$E_n - \frac{1}{2}(E_i + E_f) \to 0 \to (\chi\text{EFT})$$

$$J_\mu J^\mu = h_{\text{GT}}(q^2) + h_F(q^2) + h_T(q^2)$$

Main difference from $\mathbf{GT\,|1^+\rangle}$ contributions

↓

Same dependence as $M^{0\nu}_{\text{usoft}}$



Sen'kov and Horoi Phys. Rev. C 88, 064312, 2013

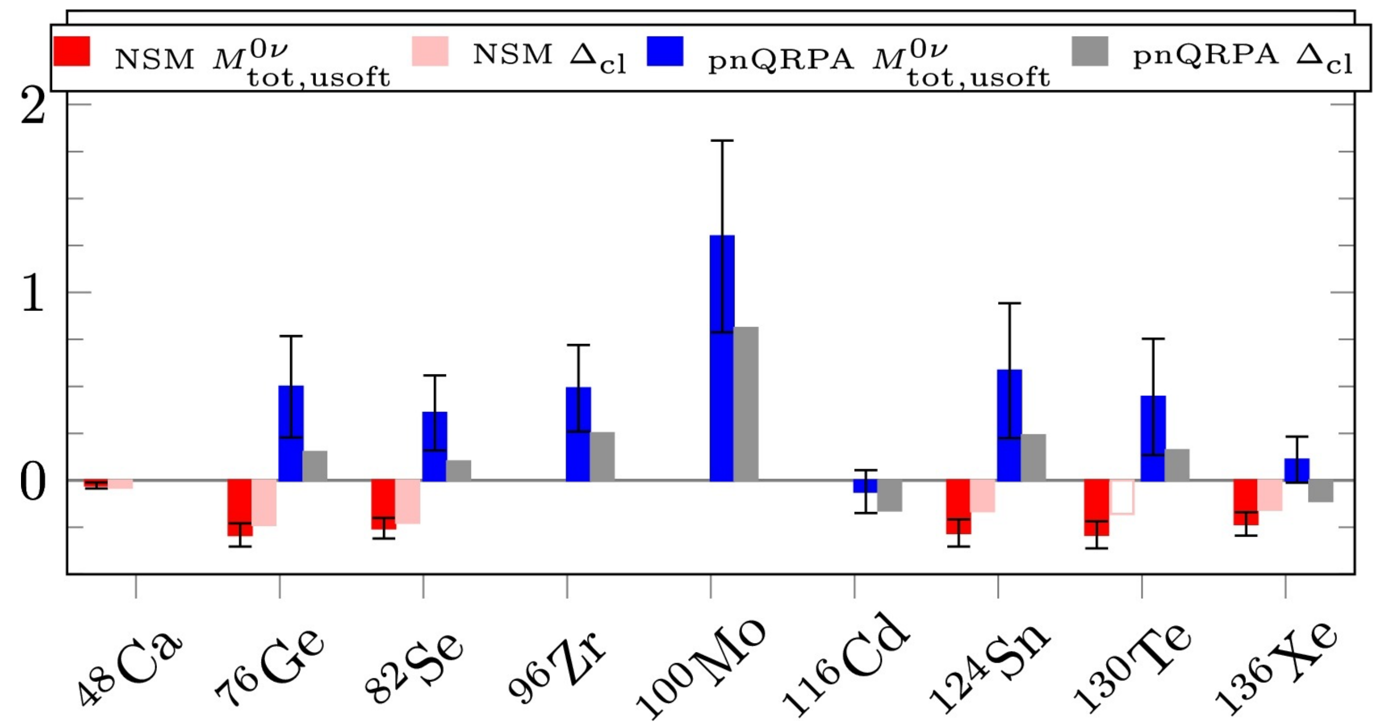
CLOSURE VS NON-CLOSURE

According to χEFT the **ultrasoft term** must be the **main contribution beyond the closure approximation**

$$M^{0\nu}_{\text{non-cl}} - M^{0\nu}_{\text{L}} = \Delta_{\text{cl}} \sim M^{0\nu}_{\text{tot,usoft}}$$

Cirigliano, et al. Phys. Rev. C 97, 065501, 2018

Agreement in the **sign** between $\boldsymbol{\Delta_{\text{cl}}}$ and $\boldsymbol{M^{0\nu}_{\text{tot,usoft}}}$



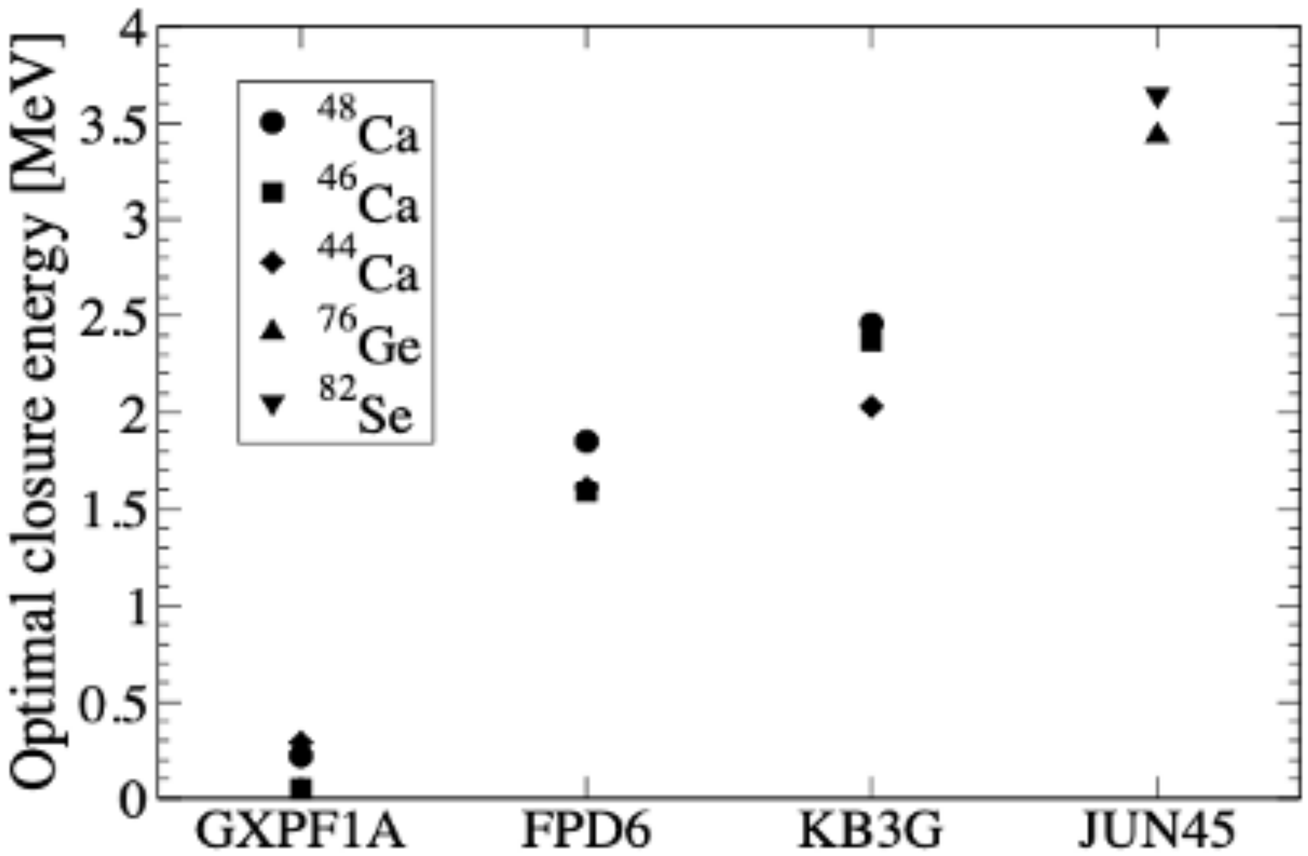
DC, Jokiniemi, Menéndez, Phys.Lett.B 860 (2025)

NSM: Good agreement with the χEFT prediction

QRPA: Milder agreement with the χEFT prediction

→ Important **uncertainties** due to μ_{us} dependence

CLOSURE ENERGIES



A. Neacsu and M. Horoi Phys. Rev. C **91**, 024309, 2015

Nucleus	Interaction	$\langle E_n \rangle$ (MeV)
^{48}Ca	KB3G	2.5
^{48}Ca	GXPF1A	0.23
^{76}Ge	JUN45	3.5
^{82}Se	JUN45	3.6
^{136}Xe	GCN5082	3.7

S. Sarkar et al. arXiv:2406.13417v1. 2024

LOOP NME

$$M^{0\nu}_{\text{loops,soft}} = M^{0\nu}_{AA,\text{loops}} + M^{0\nu}_{VV,\text{loops}} + M^{0\nu}_{CT,\text{loops}}$$

Important contributions in **short-range distances** (high momenta)

Similar behaviour between models

Main dependence ⟶ **Λ**

$\Lambda = 349$ MeV $\Lambda = 550$ MeV

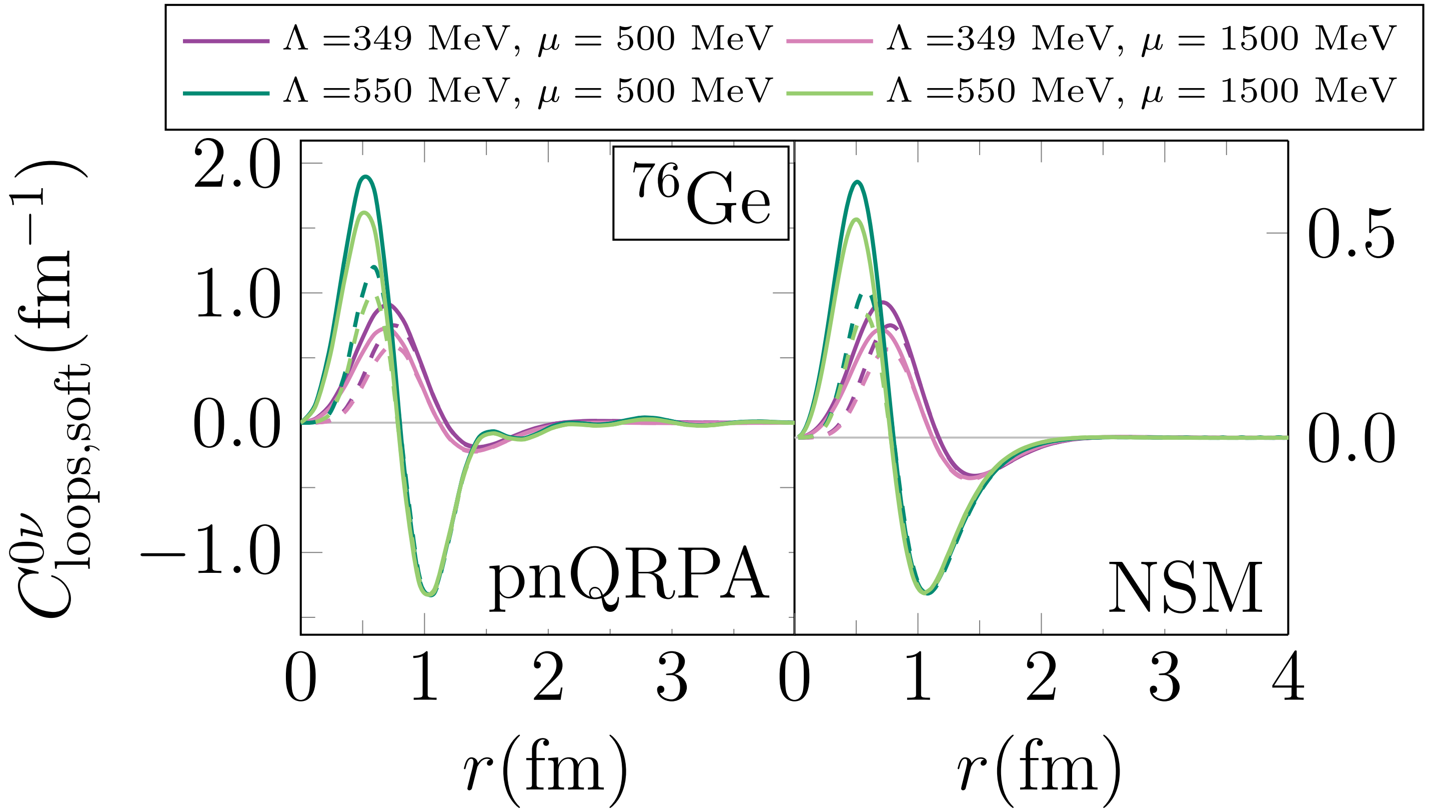
Most reliable: $\Lambda = 349$ MeV

Dependences: $H_{eff}, \mu, \Lambda \ldots$

$\Lambda \equiv$ Cutoff

$\mu \equiv$ Renormalization scale

$$M^{0\nu}_{\text{loops,soft}} = \int_0^\infty C^{0\nu}_{\text{loops,soft}}(r)dr$$



DC, Jokiniemi, Menéndez, Phys.Lett.B 860 (2025)

HEAVY-ν EXCHANGE: MASS DEPENDENCE

$$M^{0\nu}_{\rm sterile}(m_i)\ \begin{cases} M^{0\nu}_{\rm L,<}(m_i) + M^{0\nu}_{\rm S}(m_i) + M^{0\nu}_{\rm usoft}(m_i), & m_i < 100\,\text{MeV} \\ M^{0\nu}_{\rm L}(m_i) + M^{0\nu}_{\rm S}(m_i), & 100\,\text{MeV} \le m_i < 2\,\text{GeV} \\ M^{0\nu}_{9}(m_i), & 2\,\text{GeV} \le m_i \end{cases}$$

$m_i \equiv$ Sterile ν mass

LO

m_i dependence :

Change propagator

$$M^{0\nu}_{\rm L} \propto \frac{1}{q^2+m_i^2} \to M^{0\nu}_{\rm L,<}(m_i) = M^{0\nu}_{\rm L}(m_i) - m_i\Big[\frac{d}{dm_i} M^{0\nu}_{\rm L}(m_i)\Big]_{m_i=0}$$

To avoid double counting when treating different momentum regions

$$M^{0\nu}_{\rm S}(m_i) \propto g^{NN}_{\nu}(m_i)$$

Cirigliano, Dekens, Urrutia, arXiv:2412.10497

$$M^{0\nu}_{\rm usoft}(m_i) = m_i \cdot M^{0\nu}_{\rm usoft} + \mathcal{O}(\Delta E/m_i)$$

$$M^{0\nu}_{9} \propto \frac{1}{m_i^2} \to$$ **3 LECs:** $g_1^{\pi\pi}, g_1^{\pi N}$ and g_1^{NN} $\longrightarrow$ **The sterile ν are integrated out at the quark level**

Dekens, de Vries, DC et al, JHEP09(2024)201

PHENOMENOLOGICAL INTERACTIONS

$A = 48$

- KB3G
- GXPF1A

KUO-BROWN interaction

→ Mass dependence
Monopole modifications

BONN-C potential

→ Two-body matrix elements from $A = 47 - 66$

$A = 76 - 82$

- JUN45
- GCN2850

BONN-C potential

→ 133 two-body matrix elements
4 single-particle energies with $A = 63 - 96$

G-matrix
Fit to 300 energy levels

$A = 130 - 136$

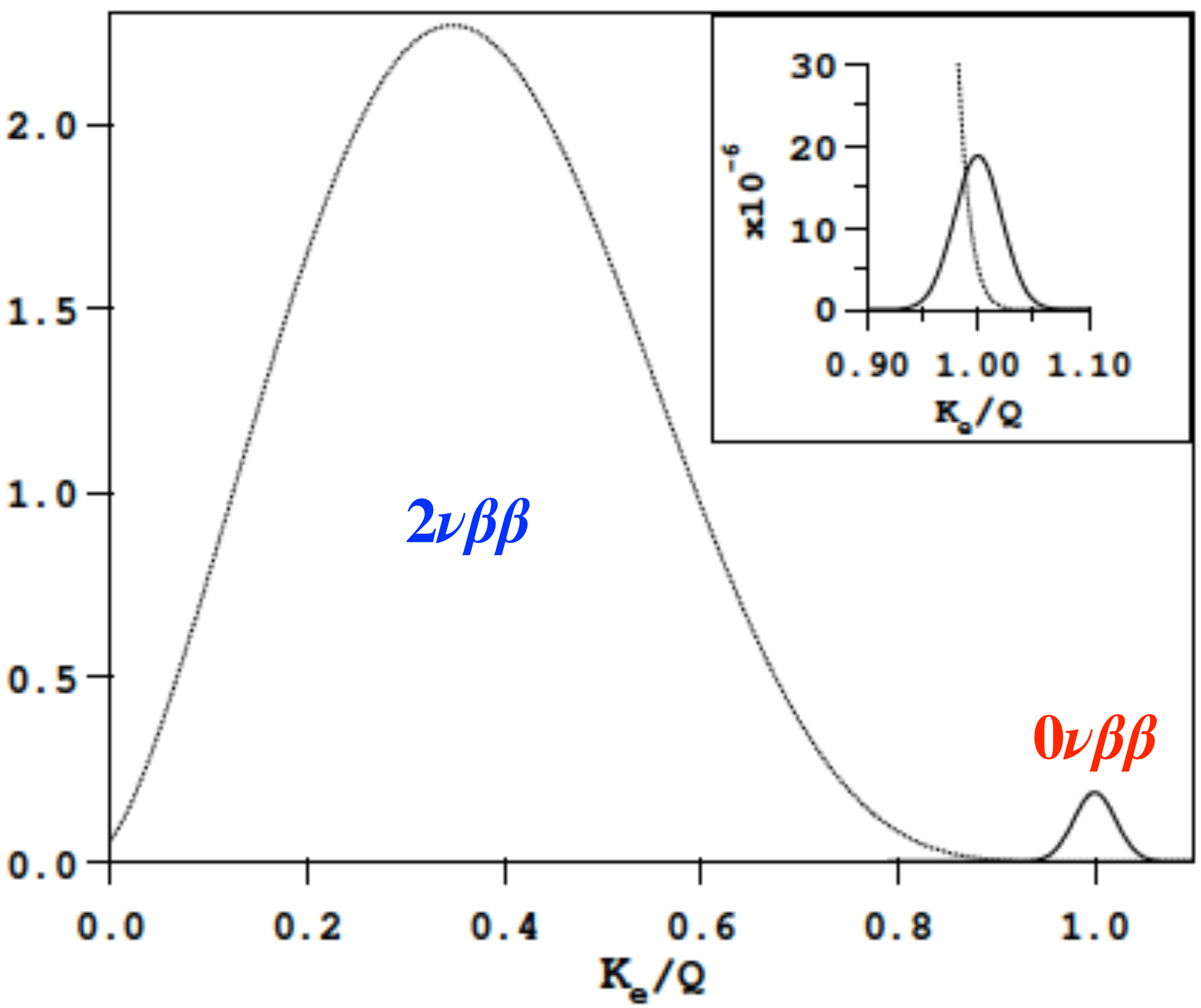
- GCN5082
- QX5082

Same as GCN2850

BONN-C potential

→ Binding energies pf 157 low-lying yrast states from $^{102\text{-}132}Sn$

MOTIVATION



Cardani Sci. Post. Phys. Proc 1 024 (2019)

$2\nu\beta\beta$

$0\nu\beta\beta$ Irreducible background

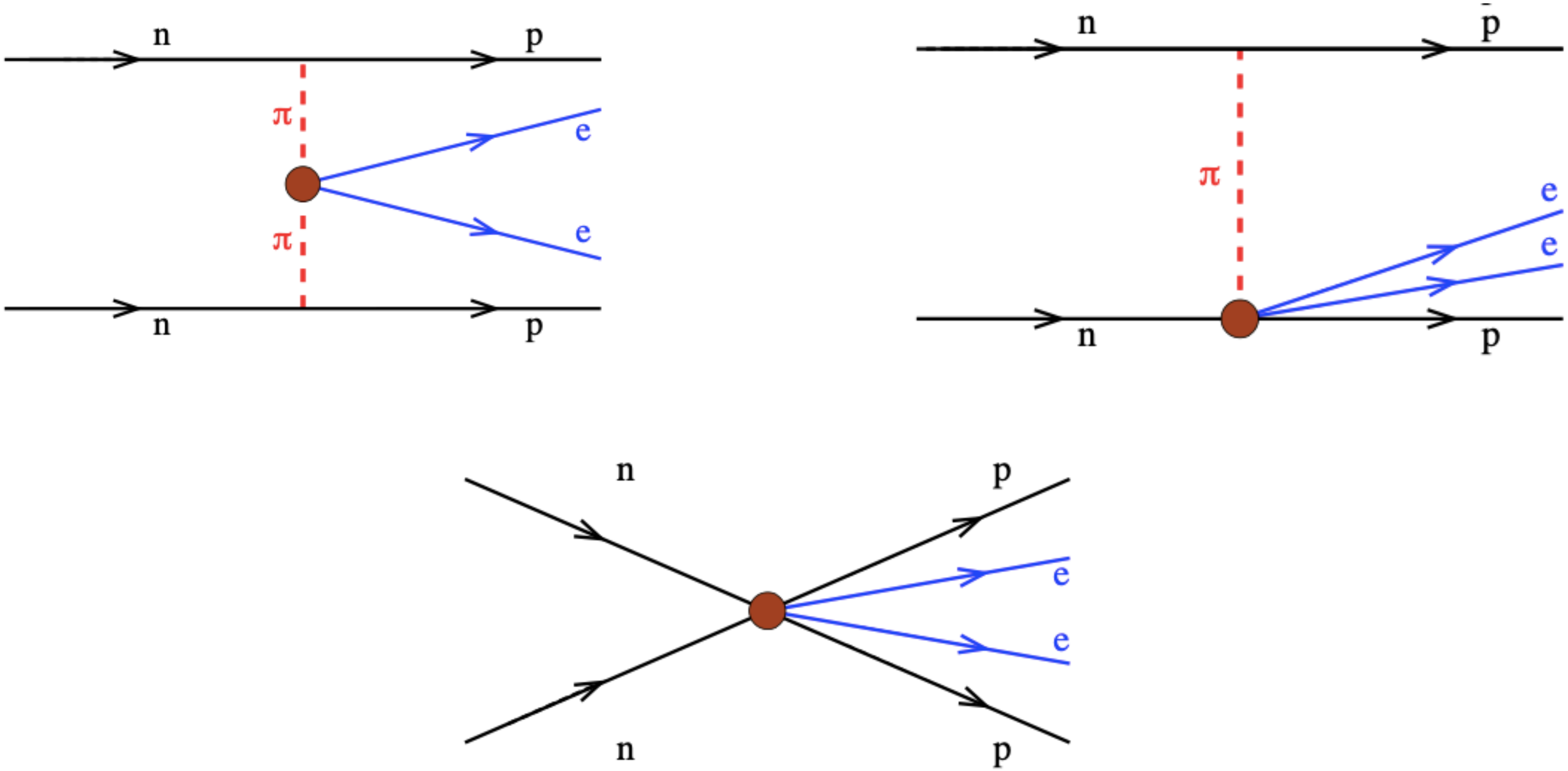
Experimentally accessible

Measured $T_{1/2}^{2\nu} \sim 10^{20}$ yr

$(0^+_{gs} \rightarrow 0^+_{gs})$

New nuclear structure information

HEAVY-ν EXCHANGE

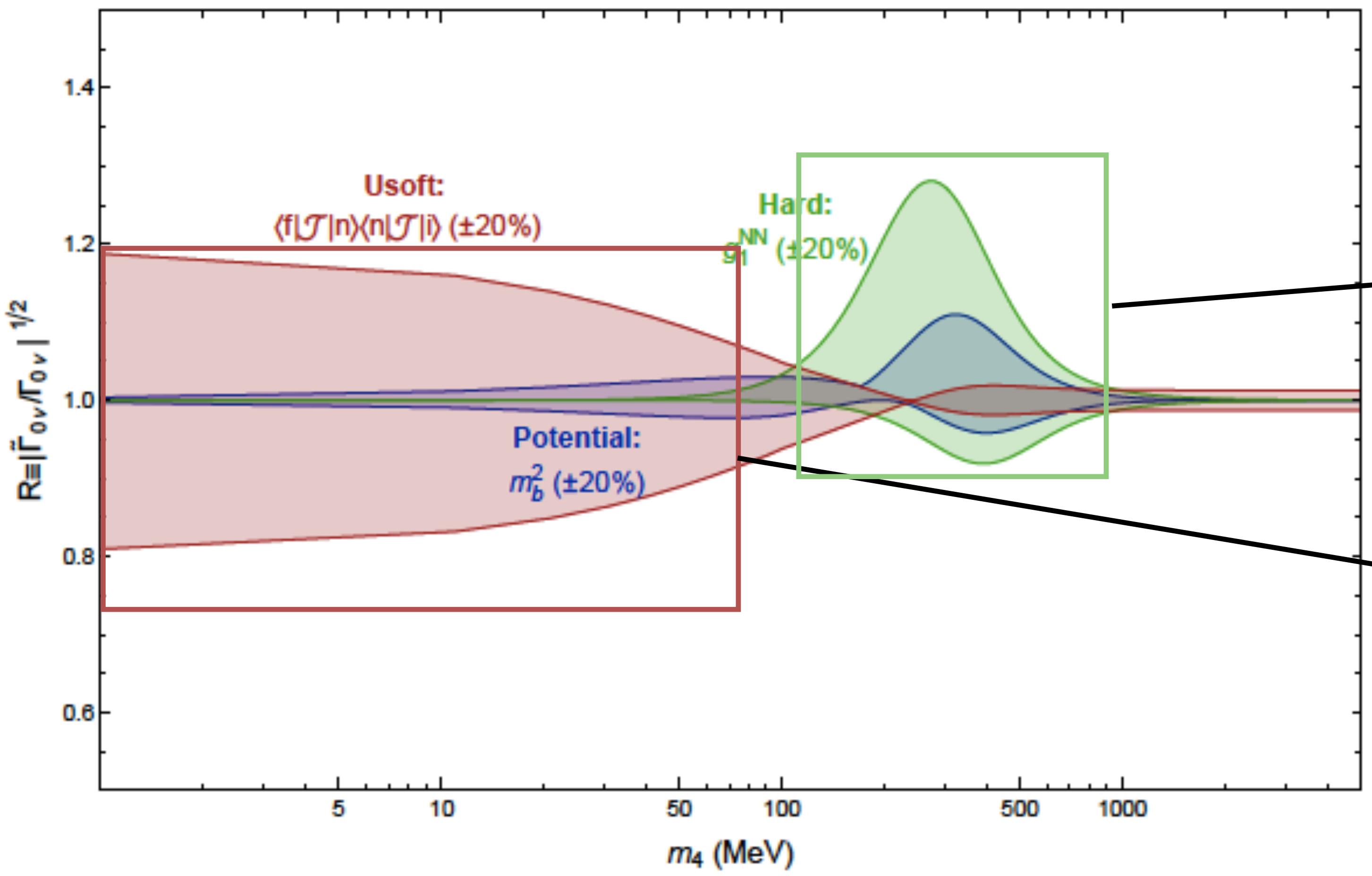


Engel, et al. Rep. Prog. Phys. 80, 046301, 2017

HEAVY-ν EXCHANGE: ULTRASOFT ENHANCEMENT

Potential: $M_{\rm L}^{0\nu}(m_i)$

Hard: $M_{\rm S}^{0\nu}(m_i)$ **Usoft:** $M_{\rm usoft}^{0\nu}(m_i)$



$M_{\rm S}^{0\nu}(m_i)$ represents the main contribution together with $M_{\rm L}^{0\nu}(m_i)$

$M_{\rm usoft}^{0\nu}(m_i)$ **is the main contribution at LO**

Dekens, de Vries, DC et al, JHEP09(2024)201

NMEs for sterile neutrinos

$$M_{\mathrm{L}}^{0\nu}(m_i) = M_{\mathrm{GT}}(m_i) - \frac{M_{\mathrm{F}}(m_i)}{g_A^2} + M_{\mathrm{T}}(m_i)$$

$$M_{\mathrm{S}}^{0\nu}(m_i) = 2 g_\nu^{NN}(m_i) m_\pi^2 \frac{M_{\mathrm{F,sd}}}{g_A^2}$$

$$M_9^{0\nu}(m_i) = 2\eta(\mu_0, m_i)\frac{m_\pi^2}{m_i^2}\left[\frac{5 g_1^{\pi\pi}}{6}(M_{\mathrm{GT}}^{PP} + M_{\mathrm{T}}^{PP}) + \frac{g_1^{\pi N}}{2}(M_{\mathrm{GT}}^{AP} + M_{\mathrm{T}}^{AP}) - \frac{2 g_1^{NN}}{g_A^2} M_{\mathrm{F,sd}}\right]$$

$$M_{\mathrm{usoft}}^{0\nu}(m_i) = R_A m_i \sum_n \langle 0_f^+ | \tau^- \boldsymbol{\sigma} | 1_n^+ \rangle \langle 1_n^+ | \tau^- \boldsymbol{\sigma} | 0_i^+ \rangle + \mathcal{O}(\Delta E / m_i)$$

$2\nu\beta\beta\ T_{1/2}^{2\nu}$ UP TO NLO

$$T_{1/2}^{2\nu} = g_A^4 (M_{\rm GT}^{-1})^2 \Delta_0 [G_0^{2\nu} + \xi_{31} \frac{\Delta_2}{\Delta_0} G_2^{2\nu} + G_4^{2\nu} (\xi_{51} \frac{\Delta_2}{\Delta_0} + \frac{1}{3} \xi_{31}^2) + G_{22}^{2\nu} + G_M^{2\nu}]$$

$$\xi_{31} = \frac{M_{\rm GT}^{-3}}{M_{\rm GT}^{-1}}, \quad \xi_{51} = \frac{M_{\rm GT}^{-5}}{M_{\rm GT}^{-1}}$$

$$M_{\rm GT}^{-2m-1} \propto \frac{\langle 0_f^+ | \mathrm{GT} | 1_n^+ \rangle \langle 1_n^+ | \mathrm{GT} | 0_i^+ \rangle}{(E_n - \frac{1}{2}(E_i + E_f))^{2m+1}}$$

$G_M^{2\nu} \equiv$ New Weak Magnetism contribution

$\Delta_0, \Delta_2 \equiv$ one-pion exchange terms and WM insertion
(same NMEs than $M_{\rm L}^{0\nu}(m_\pi)$)

Taylor expansion

Šimkovic et al Phys. Rev. C 97 034315 (2018)

NLO corrections

Morabit et al JHEP 06 082 (2025)