

Holographic entanglement entropy at finite temperature

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Outlook

Why *holography*?

Dualities → formal equivalence between different theories

AdS/CFT correspondence

AdS gravity
in $d + 1$ dimensions

=

CFT
in d dimensions

Why *holography*?

Dualities $\rightarrow$ formal equivalence between different theories

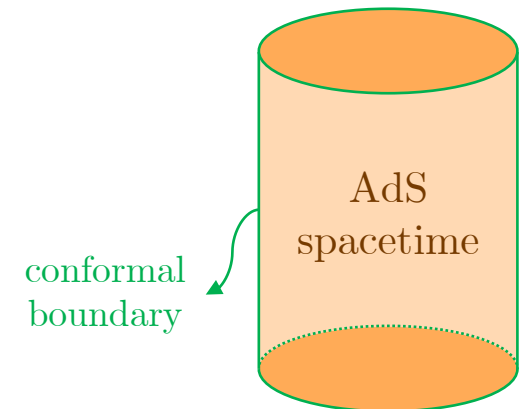
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*Holographic
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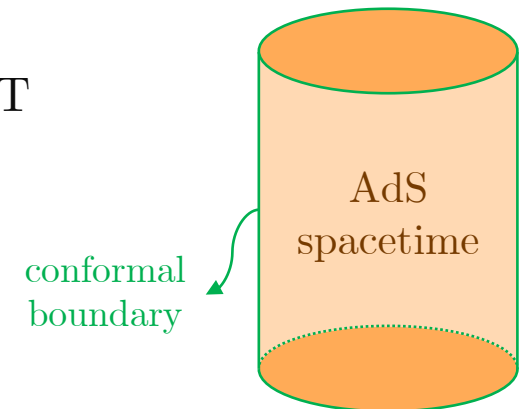
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CFT
in d dimensions

Theory of Quantum Gravity → QFT in flat spacetime

Classical supergravity ← Complex computations QFT

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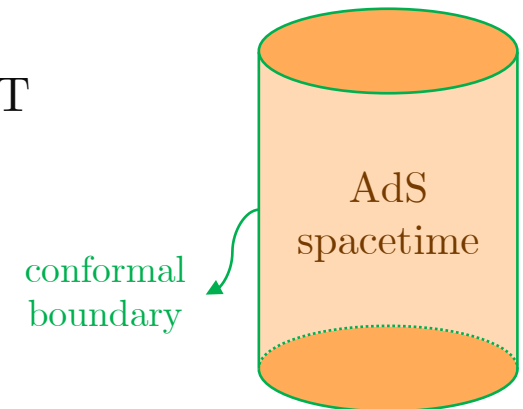
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AdS/CFT dictionary

Holographic principle



Why *entanglement entropy*?

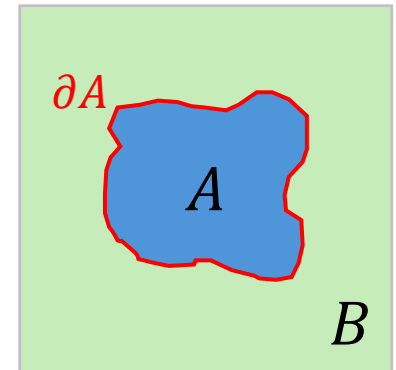
Formulate holography through **universal quantities**

Why *entanglement entropy*?

Formulate holography through **universal quantities**

Entanglement entropy

- Non-local quantity, defined in any QM system
- Quantum entanglement complementary spatial regions ($\overline{A} = B$) at a fixed time



Why *entanglement entropy*?

Formulate holography through **universal quantities**

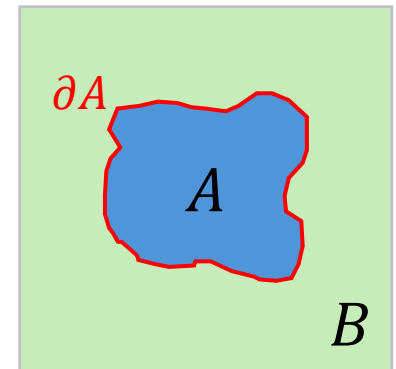
Entanglement entropy

- Non-local quantity, defined in any QM system
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Applications:

- Quantum critical phenomena
- Phase transitions (confinement/deconfinement, ...)
- Characterize quantum phases of matter



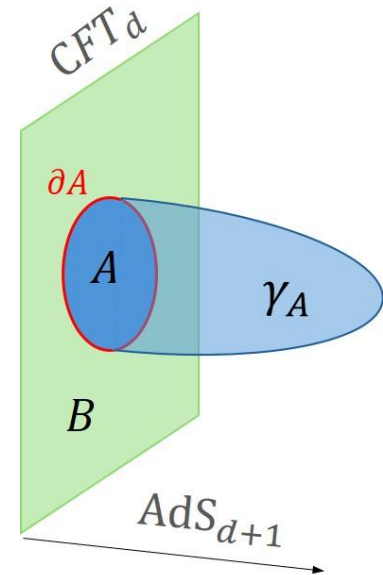
Holographic entanglement entropy

► Ryu-Takayanagi formula

Static geometries

CFTs dual to Einstein gravity

$$S_{\text{HEE}}^E(A) = \frac{\text{Area of } \gamma_A}{4G} \rightarrow \gamma_A \left\{ \begin{array}{l} \text{Surface anchored to } \partial A \\ \text{Minimizes the area functional} \end{array} \right.$$



Holographic entanglement entropy

► Ryu-Takayanagi formula

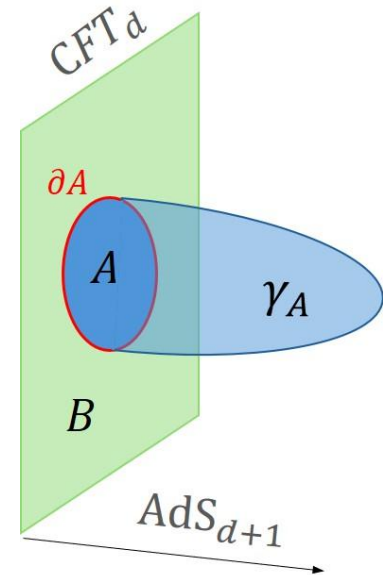
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► Generalized formula

Higher-curvature gravities = Broaden spectrum of CFTs



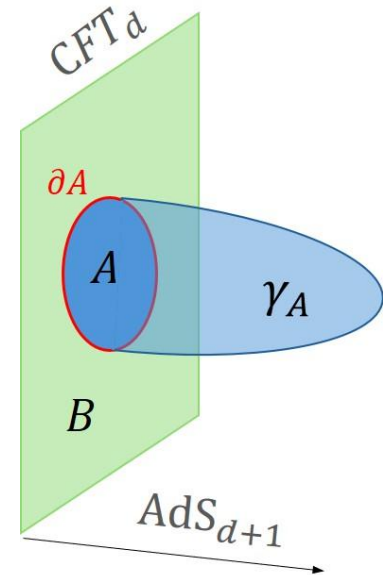
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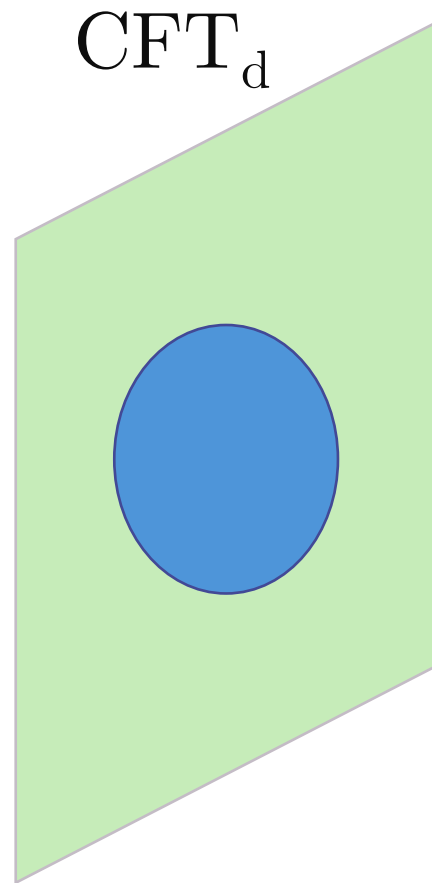
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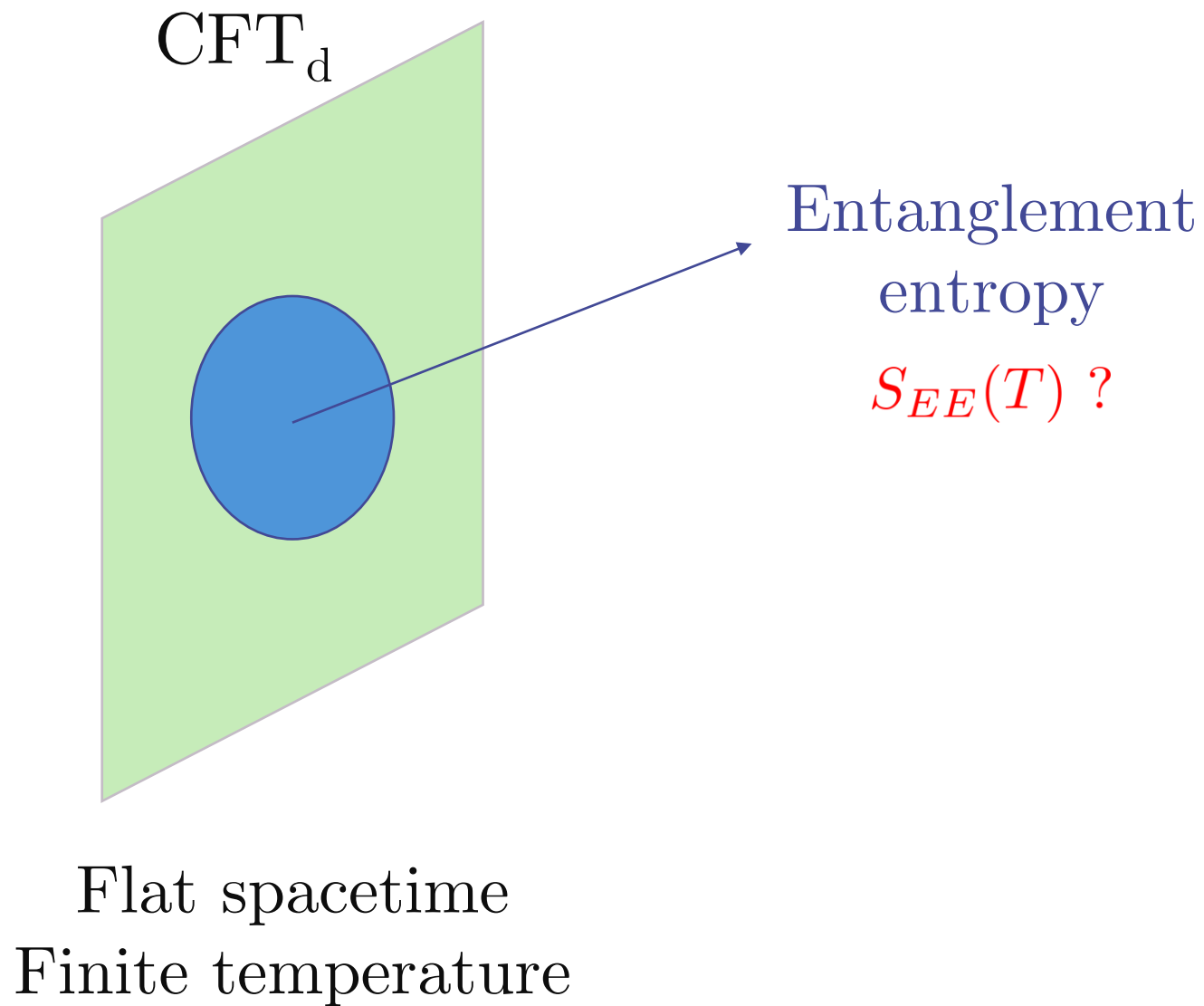
CFTs dual to Gauss-Bonnet (GB) gravity

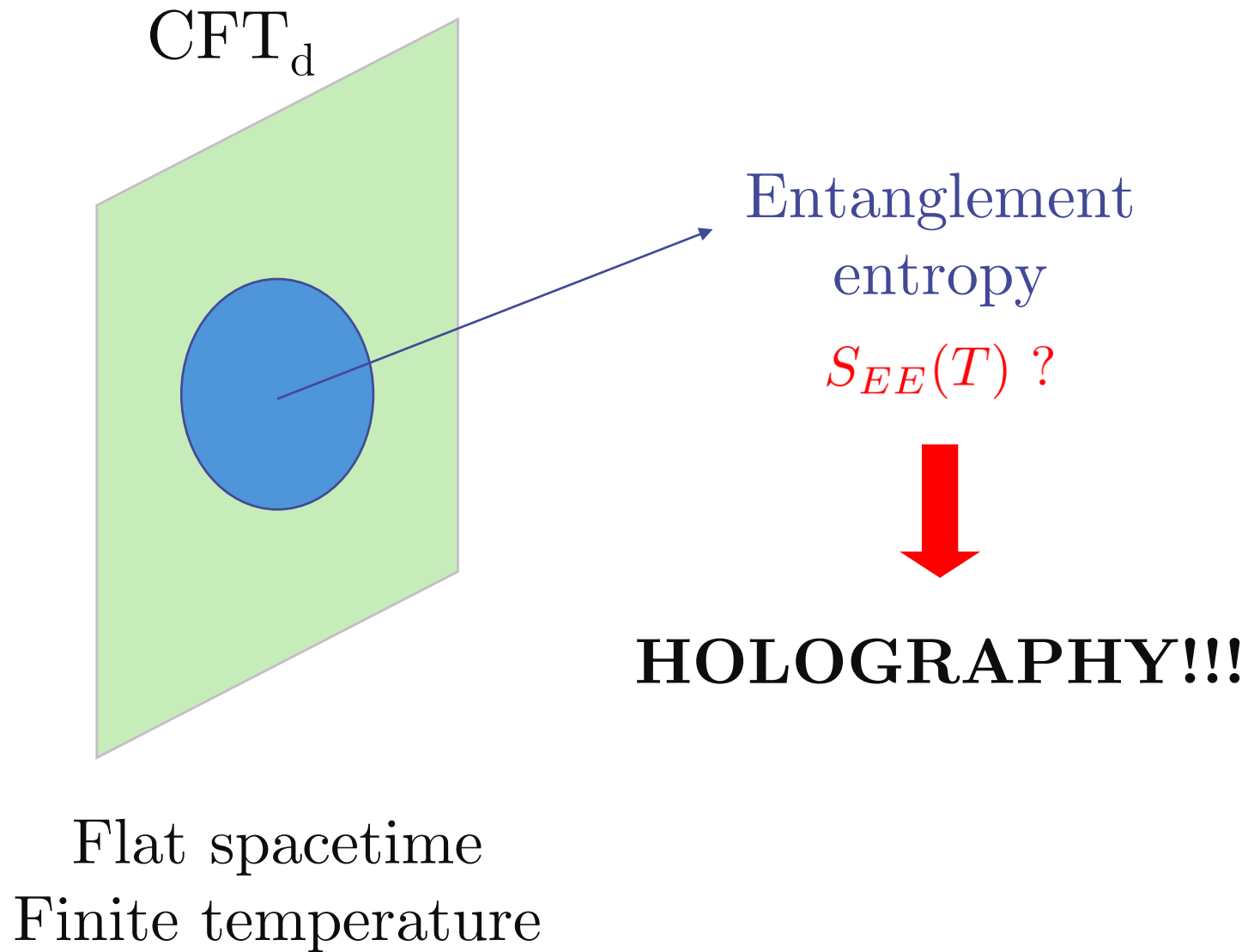
$$S_{\text{HEE}}^{\text{GB}}(A) = \frac{\text{Area of } \gamma_A}{4G} + \frac{1}{4G} \frac{2L^2\lambda}{(d-2)(d-3)} \int_{\gamma_A} d^{d-1}x \sqrt{|h|} R \rightarrow \gamma_A$$

- λ : GB coupling
- h : determinant induced metric on γ_A
- R : Ricci scalar on γ_A



Flat spacetime
Finite temperature





How?

1

AdS/CFT correspondence

- **Zero T CFT_d flat spacetime** = **Empty** AdS_{d+1} in Poincaré coordinates
- **Finite T CFT_d flat spacetime** = **Black hole** AdS_{d+1} planar horizon

How?

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- **Zero T** CFT_d flat spacetime = **Empty** AdS_{d+1} in Poincaré coordinates
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HEE formula

- RT formula → BH in **Einstein** gravity
- Generalized formula → BH in **GB** gravity

How?

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HEE formula

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Low T expansion

- Perturbative expansion around vacuum solution
⇒ Universal character of the coefficients?

Zero temperature

Empty AdS_{d+1}

$$ds^2 = \frac{L^2}{z^2} \left(-dt^2 + \sum_{i=1}^{d-1} dx_i^2 + dz^2 \right) \quad z=0 \rightarrow \text{boundary}$$

- L : AdS curvature radius

Zero temperature

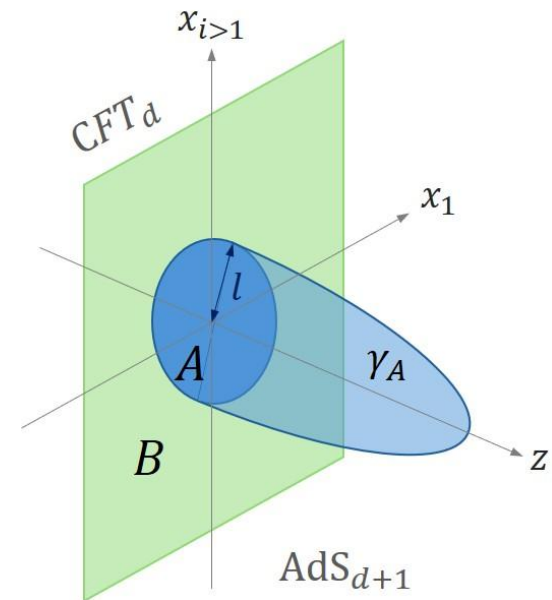
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Disk of radius l in a time slice

- Induced metric
- RT functional: $S_{\text{HEE}}^{\text{E, vac}}[z(\rho)]$



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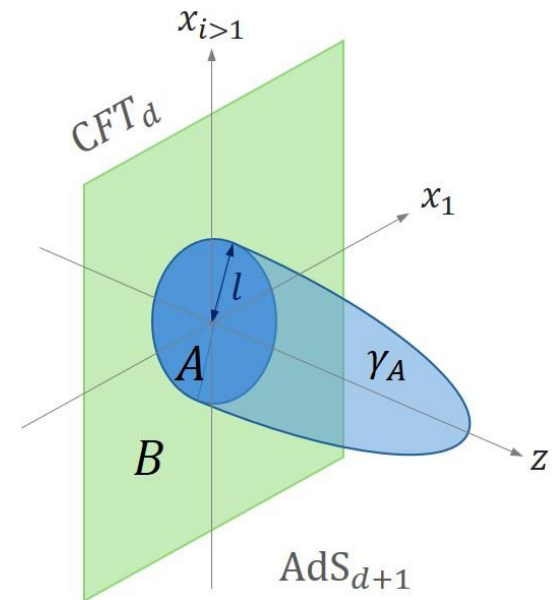
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Find the holographic surface γ_A

- Minimizing the functional: $z_0(\rho) = \sqrt{l^2 - \rho^2} \rightarrow \text{half } (d-1)\text{-dimensional sphere}$



Finite temperature

AdS_{d+1} planar BH

$$ds^2 = \frac{L^2}{z^2} \left[-f(z) dt^2 + \sum_{i=1}^{d-1} dx_i^2 + \frac{1}{f(z)} dz^2 \right], \quad f(z) \equiv 1 - \left(\frac{r_h z}{L^2} \right)^d$$

- r_h : horizon

Same procedure as before $\rightarrow$ RT functional: $S_{\text{HEE}}^{\text{E,T}}[z(\rho)]$

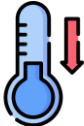
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Find $\gamma_A \rightarrow$ **Not analytically** $\Rightarrow$  **Low T:** $lT \ll 1 \iff r_h/L \ll 1$

- Metric approaches the empty AdS_{d+1} $\rightarrow$ Main contribution = zero T result
- Corrections $\rightarrow$ perturbatively, up to 2nd order

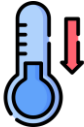
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 **Divergent** $\Rightarrow$ Finite quantity = **relative entropy**: $\Delta S_{\text{HEE}}^{\text{E}} \equiv S_{\text{HEE}}^{\text{E,T}} - S_{\text{HEE}}^{\text{E,vac}}$

Finite temperature

► Relative entropy result

$$\Delta S_{\text{HEE}}^{\text{E}} = \frac{2^{2d+1} \pi^{\frac{3d+1}{2}}}{d^d (d+1) \Gamma\left(\frac{d-1}{2}\right)} \left(\frac{L}{l_p}\right)^{d-1} (lT)^d + \frac{8^{d-1} \pi^{\frac{5d}{2}+1} [d^2(7d+12) - 3] \Gamma(d)}{d^{2d} (d+1) \Gamma\left(\frac{d-1}{2}\right) \Gamma\left(d + \frac{5}{2}\right)} \left(\frac{L}{l_p}\right)^{d-1} (lT)^{2d}$$

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► Central charges

- 2-point function: $\langle T_{ab}(x) T_{cd}(0) \rangle = \frac{C_T}{x^{2d}} \mathcal{I}_{ab,cd}(x)$

$$C_T^{\text{E}} = \frac{d+1}{d-1} \frac{\Gamma(d+1)}{\pi^{\frac{d}{2}} \Gamma\left(\frac{d}{2}\right)} \left(\frac{L}{l_p}\right)^{d-1}$$

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- **Entropy density:** $s = C_s T^{d-1}$

$$C_s^{\text{E}} = 2\pi \left(\frac{4\pi}{d}\right)^{d-1} \left(\frac{L}{l_p}\right)^{d-1}$$

Finite temperature

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$$C_T^{\text{E}} \propto \frac{L}{l_p}, \quad C_s^{\text{E}} \propto \frac{L}{l_p}$$

1 dimensionless parameter in
Einstein gravity



Expansion coefficients are fixed
 $\Rightarrow C_T^{\text{E}}$ and C_s^{E} indistinguishable

Zero temperature

Empty AdS_{d+1}

$$ds^2 = \frac{L^2}{z^2} \left(-dt^2 + \sum_{i=1}^{d-1} dx_i^2 + \frac{1}{f_\infty} dz^2 \right), \quad f_\infty = \frac{1 - \sqrt{1 - 4\lambda}}{2\lambda}$$

- $f_\infty \equiv f(z=0)$
- $\tilde{L} = L/\sqrt{f_\infty}$: AdS curvature radius

Generalized HEE formula

- Functional: $S_{\text{HEE}}^{\text{GB, vac}}[z(\rho)]$

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Find the holographic surface γ_A

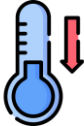
- $z_0(\rho) = \sqrt{f_\infty(l^2 - \rho^2)}$ $\rightarrow$ half $(d-1)$ -dimensional ellipsoid

Finite temperature

GB planar BH

$$ds^2 = \frac{L^2}{z^2} \left(-\frac{f(z)}{f_\infty} dt^2 + \sum_{i=1}^{d-1} dx_i^2 + \frac{1}{f(z)} dz^2 \right), \quad f(z) = \frac{1}{2\lambda} \left[1 - \sqrt{1 - 4\lambda \left(1 - \left(\frac{r_h z}{L^2} \right)^d \right)} \right]$$

- r_h : horizon

Find $\gamma_A \rightarrow$ **Not analytically** $\Rightarrow$  **Low T:** $lT \ll 1$

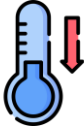
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Finite temperature

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- Finite quantity: $\Delta S_{\text{HEE}}^{\text{GB}} \equiv S_{\text{HEE}}^{\text{GB},T} - S_{\text{HEE}}^{\text{GB,vac}}$

► Relative entropy result

$$\Delta S_{\text{HEE}}^{\text{GB}} = \frac{4^d \pi^{\frac{3d}{2}+1} (f_\infty)^{d-1}}{d^d (d+1) \Gamma\left(\frac{d}{2}\right)} \left(\frac{\tilde{L}}{l_p} \right)^{d-1} (lT)^d + \frac{2^{4d-5} \pi^{\frac{5d}{2}+1} \Gamma\left(\frac{d+1}{2}\right) K(d, f_\infty)}{d^{2d} (d-3)(d+1) \Gamma\left(d + \frac{5}{2}\right)} \frac{(f_\infty)^{2d-1}}{(f_\infty - 2)^2} \left(\frac{\tilde{L}}{l_p} \right)^{d-1} (lT)^{2d}$$

$$K(d, f_\infty) \equiv -\left[d\left(d(d(7d+51)+52) - 23 \right) - 15 \right] f_\infty + 2d\left[d(7d(d+3)+8) - 13 \right] - 6$$

Finite temperature

► Central charges

- 2-point function:

$$C_T^{\text{GB}} = \frac{d+1}{d-1} \frac{\Gamma(d+1)}{\pi^{\frac{d}{2}} \Gamma(\frac{d}{2})} \left(\frac{\tilde{L}}{l_p} \right)^{d-1} (1 - 2f_\infty \lambda)$$

- Entropy density:

$$C_s^{\text{GB}} = 2\pi \left(\frac{4\pi f_\infty}{d} \right)^{d-1} \left(\frac{\tilde{L}}{l_p} \right)^{d-1}$$

Finite temperature

► Central charges

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2 dimensionless parameters in
GB gravity

$\Rightarrow C_T^{\text{GB}}$ and C_s^{GB} independent

Finite temperature

► Central charges

- 2-point function:

$$C_T^{\text{GB}} = \frac{d+1}{d-1} \frac{\Gamma(d+1)}{\pi^{\frac{d}{2}} \Gamma(\frac{d}{2})} \left(\frac{\tilde{L}}{l_p} \right)^{d-1} (1 - 2f_\infty \lambda)$$

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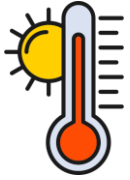
$$C_s^{\text{GB}} = 2\pi \left(\frac{4\pi f_\infty}{d} \right)^{d-1} \left(\frac{\tilde{L}}{l_p} \right)^{d-1}$$

$\Rightarrow C_T^{\text{GB}}$ and C_s^{GB} independent

► Results

- $C_T^{\text{GB}} \rightarrow$ expansion coefficients depend non-trivially on f_∞

- $C_s^{\text{GB}} \rightarrow \left. \Delta S_{\text{HEE}}^{\text{GB}, C_s} = \frac{2\pi^{\frac{d}{2}+1}}{d(d+1)\Gamma(\frac{d}{2})} C_s^{\text{GB}} (lT)^d + \mathcal{O}((lT)^{2d}) \right\}$ Universal character of the 1st order term??



High temperature

Leading contribution = thermal entropy



Incomplete AdS/CFT dictionary

Simple expression for the expansion coefficients



Other gravitational theories

Same behavior?

Thank you!!

Backup

Gravitational action (d+1 dimensions)

- **Einstein-Hilbert action + cosmological constant term**

$$I_E = \frac{1}{16\pi G} \int d^{d+1}x \sqrt{|g|} \left[\frac{d(d-1)}{L^2} + R \right]$$

- **+ Gauss-Bonnet term**

Quadratic correction to General Relativity

$$I_{GB} = \frac{1}{16\pi G} \int d^{d+1}x \sqrt{|g|} \left[\frac{d(d-1)}{L^2} + R + \frac{L^2 \lambda}{(d-2)(d-3)} \mathcal{X}_4 \right]$$

- $\mathcal{X}_4 = R_{\mu\nu\gamma\delta} R^{\mu\nu\gamma\delta} - 4R_{\mu\nu} R^{\mu\nu} + R^2$
- λ : GB coupling

Low temperature expansion

Expansion parameter: $\varepsilon = (r_h/L)^d$

- RT functional

$$S_{\text{HEE}}^{\text{E},T}[z] = S_0[z] + \varepsilon S_1[z] + \varepsilon^2 S_2[z] + \mathcal{O}(\varepsilon^3) \rightarrow S_0 \equiv S_{\text{HEE}}^{\text{E},\text{vac}}: \text{zero T functional}$$

- Holographic surface

$$z(\rho) = z_0(\rho) + \varepsilon z_1(\rho) + \varepsilon^2 z_2(\rho) + \mathcal{O}(\varepsilon^3) \rightarrow z_0(\rho): \text{zero T surface}$$



$$S_{\text{HEE}}^{\text{E},T} = S_{\text{HEE}}^{\text{E},\text{vac}}[z_0] + \varepsilon S_1[z_0] + \varepsilon^2 \left(S_2[z_0] + \frac{1}{2} \int d\rho' \frac{\delta S_1[z_0]}{\delta z(\rho')} z_1(\rho') \right) + \mathcal{O}(\varepsilon^3)$$

$$\left\{ \begin{array}{l} S_1, S_2 \rightarrow \text{known} \\ z_0 \rightarrow \text{known} \\ z_1 \rightarrow \text{1st order solution of the Euler-Lagrange equation} \end{array} \right.$$

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