Predictions for multiple H production: comparing HEFT and SMEFT

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IPARCOS

UNIVERSIDAD COMPLUTENSE MADRID

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AGENCIA ESTATAL DE INVESTIGACIÓN

COMETA

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In collaboration with:

R.L. Delgado, R. Gómez-Ambrosio, F.J. Llanes-Estrada, J. Martínez-Martín, A. Salas-Bernárdez,

JHEP 03 (2024) 037; **PRD** 106 (2022) 5, 5; **Commun.Theor.Phys.** 75 (2023) 9, 095202

A. Nasoni (Master Thesis 2025), S. Martellini (Master Thesis 2025) ,

A. Dobado, C. Quezada Calonge (PhD Thesis 2024)

A. Pich, I. Rosell, PRD 112 (2025) 3, 035009

Outline

1. SM, SMEFT and HEFT

2. $W_L W_L \to n \times h$ VBS: theory predictions

3. $W_L W_L \to n \times h$ VBS: SMEFT pheno suppression

- **SM:**
  - Complex doublet H
  - Renormalizable (canonical dim. $D \leq 4$)

$$\boxed{\mathcal{L}_{SM} = \mathcal{L}_{D\leq 4}}$$

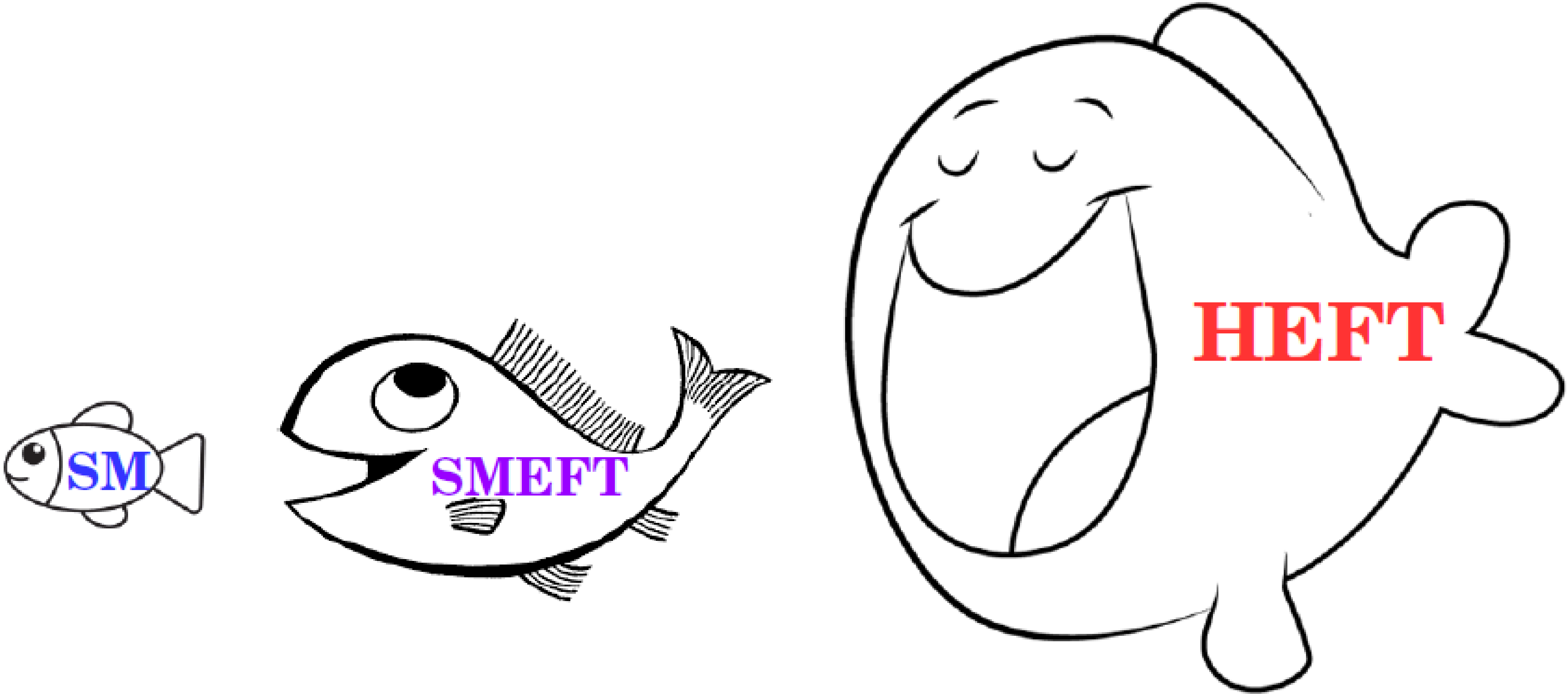
- **SMEFT:**
  - Complex doublet H around H=0
  - Non-renormalizable (canonical dim. expan.)

$$\boxed{\mathcal{L}_{SMEFT} = \mathcal{L}_{SM}^{(D\leq 4)} + \sum_{D>4} \frac{c_i^{(D)}}{\Lambda^{D-4}} O_i^{(D)}}$$

- **HEFT**
  (= EWChL = EWET)
  - 3 EW Goldstones + 1 singlet Higgs h (indep.)
  - Non-renormalizable (chiral expan.)

$$\boxed{\mathcal{L}_{HEFT} = \mathcal{L}_{p^2} + \mathcal{L}_{p^4} + \ldots}$$

[w/ $\mathcal{L}_{SM} \subset \mathcal{L}_{p^2}$]



(x) See, e.g., Alonso,Jenkins,Manohar, PLB 754 (2016) 335-342; PLB 756 (2016) 358-364; JHEP 08 (2016) 101; Cohen,Craig,Lu,Sutherland, JHEP 03 (2021) 237; JHEP 12 (2021) 003; Brivio,Corbett,Éboli,Gavela,González-Fraile,González-García,Merlo,Rigolin, JHEP 03 (2014) 024; Agrawal,Saha,Xu,Yu,Yuan, PRD 101 (2020) 7, 075023; Gómez-Ambrosio,Llanes-Estrada,Salas-Bernárdez,SC, PRD 106 (2022) 5, 5; Commun.Theor.Phys. 75 (2023) 9, 095202; Dawson,Fontes,Quezada-Calonge,SC, 2311.16897 [hep-ph]; PRD 108 (2023) 5, 055034; Arco,Domenech,Herrero,Morales, PRD 108 (2023) 9, 095013;

- Relevant HEFT Lagrangian at LO, O(p^2) in this work *:

- kinematics well over WW threshold: $s \gg m_W^2 \sim m_h^2$
- Mass corrections neglected
- Chiral LO: only $O(\partial^2)$ derivative operators
- Equivalence theorem appr.: $W_L \approx \omega$

$$\mathcal{L}_{\mathrm{HEFT}} = \frac{1}{2}(\partial_\mu h)^2 + \frac{v^2}{4}\mathcal{F}(h)\,\mathrm{Tr}\left\{\partial_\mu U^\dagger \partial^\mu U\right\}$$

w/ the SU(2)-singlet "Flare" function,

$$\mathcal{F}(h) = 1 + a_1\frac{h}{v} + a_2\left(\frac{h}{v}\right)^2 + a_3\left(\frac{h}{v}\right)^3 + a_4\left(\frac{h}{v}\right)^4 + \ldots$$

Other notation: $\boldsymbol{\kappa_V} \equiv a \equiv a_1/2\,,\quad \boldsymbol{\kappa_{2V}} \equiv b \equiv a_2$

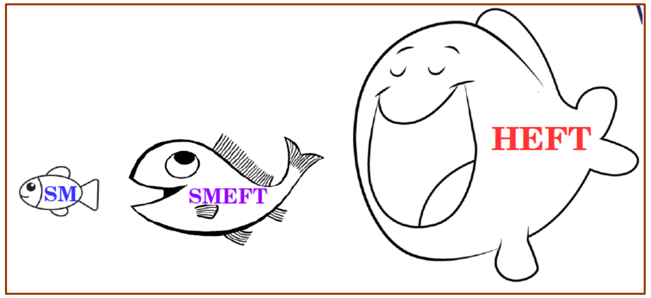
- Non-linear Goldstone realization: $U(\omega) = 1 + i\sigma^a\omega^a/v + \mathcal{O}(\omega^2)$

* Delgado,Gómez-Ambrosio,Martínez-Martín,Salas-Bernárdez,SC, JHEP 03 (2024) 037

SMEFT: $\omega\omega \to 2h, 3h, 4h \ldots$
multi-H VERTEX suppression

- SMEFT correlations among the Higgs couplings:

$$
\begin{aligned}
a_1/2 &= a = 1 + \frac{d}{2} + \frac{d^2}{2}\left(\frac{3}{4} + \rho\right) + \mathcal{O}\left(d^3\right) \\
a_2 &= b = 1 + 2d + 3d^2\left(1 + \rho\right) + \mathcal{O}\left(d^3\right) \\
a_3 &= \frac{4}{3}d + d^2\left(\frac{14}{3} + 4\rho\right) + \mathcal{O}\left(d^3\right) \\
a_4 &= \frac{1}{3}d + d^2\left(\frac{11}{3} + 3\rho\right) + \mathcal{O}\left(d^3\right)
\end{aligned}
$$



a_5 , a_6 are also obtained at D=8. *
The remaining a_n vanish for $n \geq 7$ at $1/\Lambda^4$ order .

$$
d = \frac{2v^2 c^{(6)}_{H\Box}}{\Lambda^2} \quad , \quad \rho = \frac{c^{(8)}_{H\Box}}{2(c^{(6)}_{H\Box})^2}
$$

* Gómez-Ambrosio,Llanes-Estrada,Salas-Bernárdez,SC,
PRD 106 (2022) 5, 5; Commun.Theor.Phys. 75 (2023) 9, 095202

* Delgado,Gómez-Ambrosio,Martínez-Martín,Salas-Bernárdez,SC, JHEP 03 (2024) 037

(x) Custodial breaking still keeps this structure:
See Domenech,Herrero,Morales,Salas-Bernárdez, 2506.21716 [hep-ph]

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a_2 &= b = \boxed{1} + 2d + 3d^2\left(1+\rho\right) + \mathcal{O}\left(d^3\right) \\
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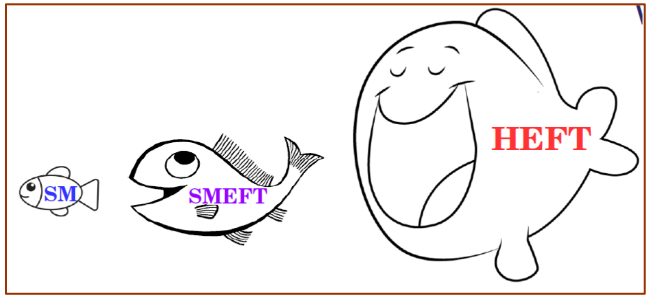

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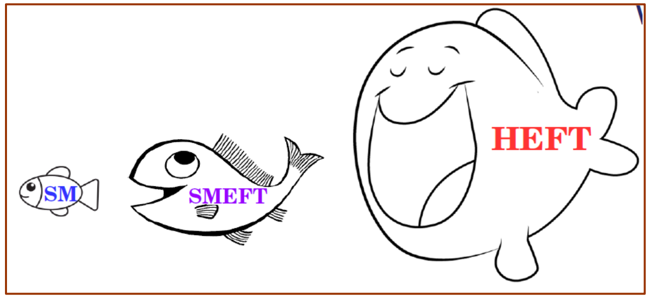
$\mathbf{1/\Lambda^2}$



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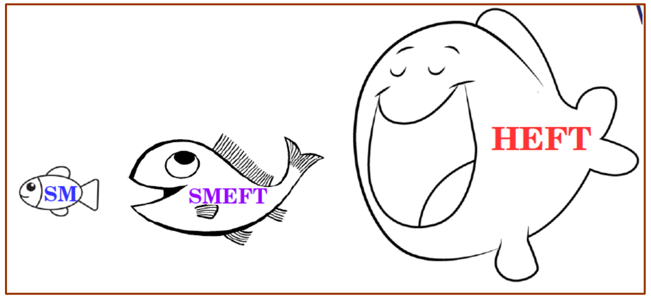
$\mathbf{1/\Lambda^4}$



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PRD 106 (2022) 5, 5; Commun.Theor.Phys. 75 (2023) 9, 095202

* Delgado,Gómez-Ambrosio,Martínez-Martín,Salas-Bernárdez,SC, JHEP 03 (2024) 037

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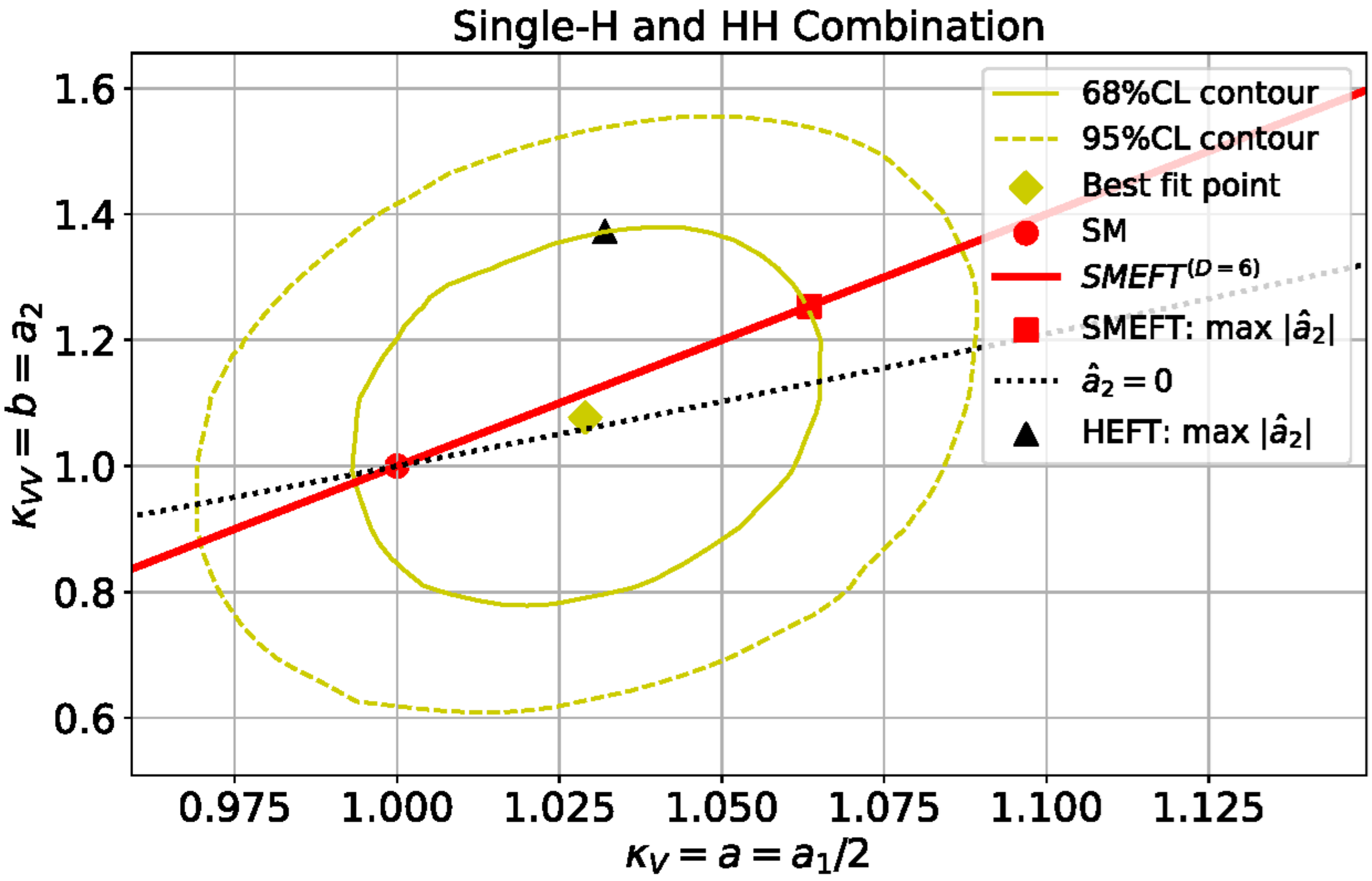
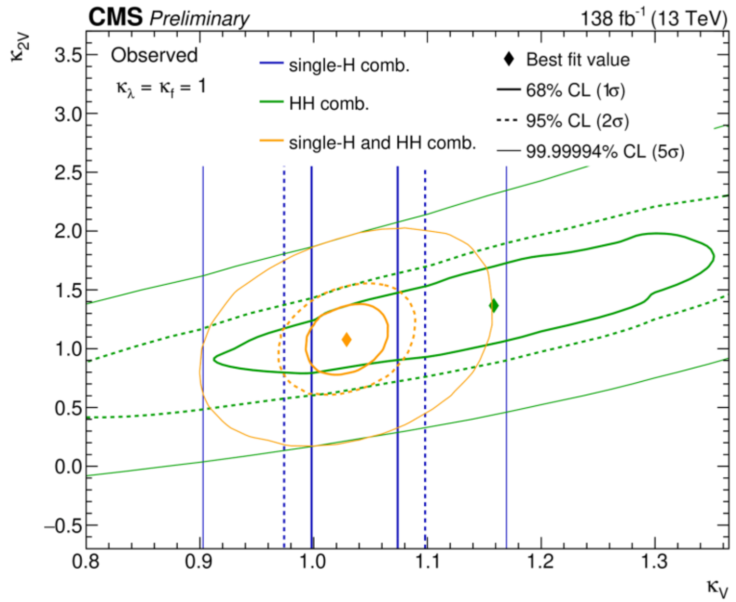
See Domenech,Herrero,Morales,Salas-Bernárdez, 2506.21716 [hep-ph]



This SMEFT (D=6) line is actually a hyper-line in the (a_1, a_2, a_3, a_4) space

Data from CMS-PAS-HIG-23-006:



[$\hat{a}_2 = 0$ is the line with vanishing $\omega\omega \to hh$ scattering]

$$\boxed{\omega\omega \to 2h}$$

$$T_{\omega\omega\to 2h} = -\frac{\hat{a}_2 s}{v^2}$$

$$\sigma_{\omega\omega\to 2h} = \frac{8\pi^3\,\hat{a}_2^2}{s}\left(\frac{s}{16\pi^2 v^2}\right)^2$$

- Relevant combination: $\boxed{\hat{a}_2 = a_2 - a_1^2/4 = b - a^2}$

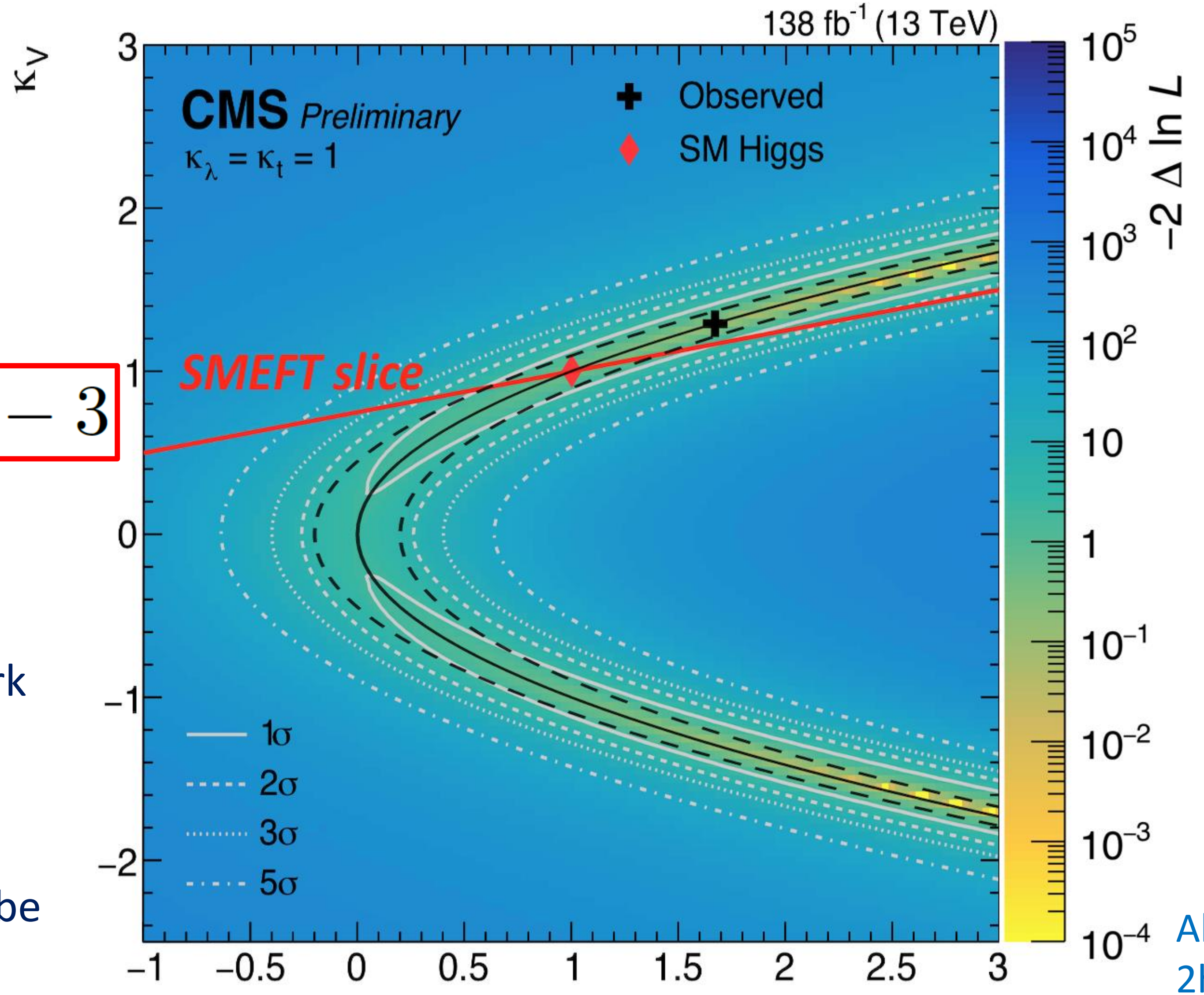
- Pure s-wave (J=0) ➔ critical angular information

* Delgado,Gómez-Ambrosio,Martínez-Martín,Salas-Bernárdez,SC, JHEP 03 (2024) 037

(x) Gonzalez-Lopez,Herrero,Martinez-Suarez, EPJ C 81 (2021) 3, 260

(a)

$$a_2 = 2a_1 - 3$$



$$\boxed{\hat{a}_2 = \pm 0.2}$$

Figure 2. a) CMS experimental confidence regions for the hWW coupling $\kappa_V = a = a_1/2$ and for the $hhWW$ coupling $\kappa_{2V} = b = a_2$, from a non-resonant hh production search with each Higgs boson decaying into a highly boosted $b\bar{b}$ pair [77] (white lines and colour map in figure 11 from the additional material in CMS-B2G-22-003). b) CMS confidence regions from processes with one Higgs boson

The equivalence theorem approximations in this work seem to be in agreement with hh-production data

Indications that we might be O(10%) close to the SM in (a, b)

Also previous theoretical 2h-production for LHC* noted important correlations between (κ_V, κ_{2V})

(a) Phys. Rev. Lett. 131 (2023) 041803 [2205.06667]

* Delgado,Gómez-Ambrosio,Martínez-Martín,Salas-Bernárdez,SC, JHEP 03 (2024) 037

* Anisha, Atkinson, Bhardwaj, Englert, Stylianou, JHEP 10 (2022) 172

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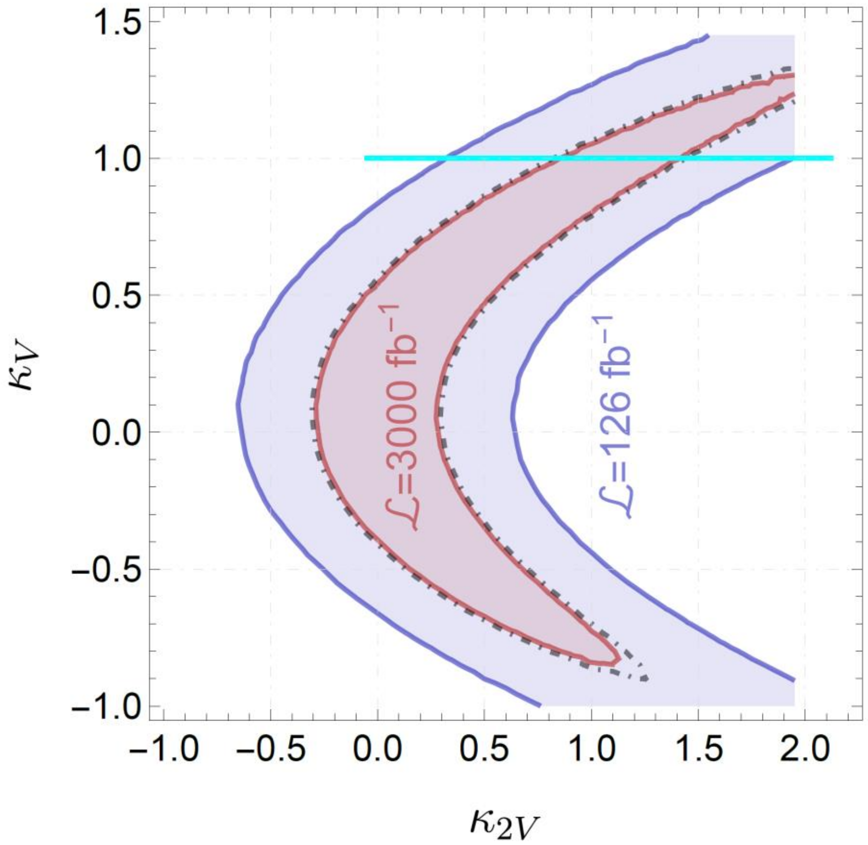
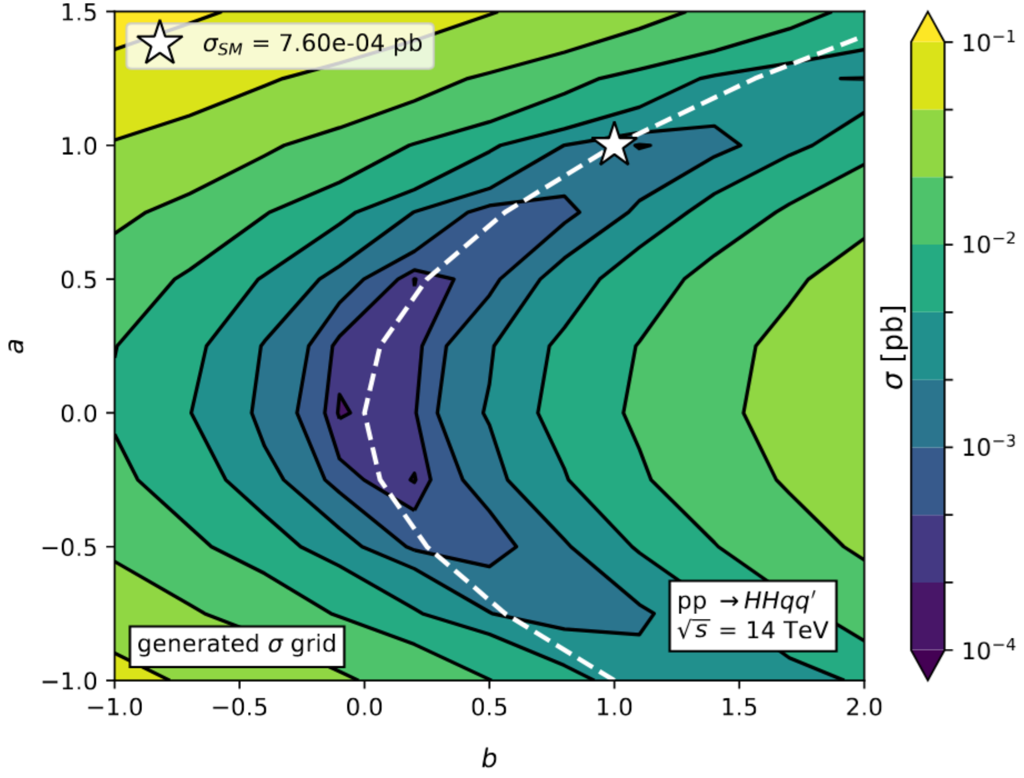


- Also previous and posterious theoretical hh-production simulations for LHC* noted an important correlation between (a, b)

** Domenech,García-Mir,Herrero, 2507.20988 [hep-ph]



$$\boxed{\omega\omega \to 3h}$$

$$T_{\omega\omega\to 3h} = -\frac{3\hat{a}_3 s}{v^3}$$

Already in consideration for future analyses:
V. Brigljevic et al.,
“HHH Whitepaper”
2407.03015 [hep-ph]

$$\sigma_{\omega\omega\to 3h} = \frac{12\pi^3\,\hat{a}_3^2}{s}\left(\frac{s}{16\pi^2 v^2}\right)^3$$

- Relevant combination: $\boxed{\hat{a}_3 = a_3 - \frac{2}{3}a_1\left(a_2 - a_1^2/4\right) = a_3 - \frac{4}{3}a\left(b - a^2\right)}$

- Pure s-wave (J=0) ➔ critical angular information

++ MCHM: Contino,Grojean,Pappadopulo,Rattazzi,Thamm, JHEP 02 (2014) 006
** LHC analysis: Anisha,Domenech,Englert,Herrero,Morales, PRD 111 (2025) 5, 055004

(*) Delgado,Gómez-Ambrosio,Martínez-Martín,Salas-Bernárdez,SC, JHEP 03 (2024) 037
(*) 1 loop: Sara Martellini, Master Thesis 2025; Martellini,Nasoni,SC, forthcoming

$$\boxed{\omega\omega \to 4h}$$

1-crossed-propagator
dimensionless angular function

[BACKUP SLIDES]

$$T_{\omega\omega\to 4h} = -\frac{4s}{v^4}\left(3\hat{a}_4 + \hat{a}_2^2\left(B-1\right)\right)$$

$$\sigma_{\omega\omega\to 4h} = \frac{8\pi^3}{9s}\left(\frac{s}{16\pi^2 v^2}\right)^4\left[\left(3\hat{a}_4-\hat{a}_2^2\right)^2 + 2\left(3\hat{a}_4-\hat{a}_2^2\right)\hat{a}_2^2\chi_1 + \hat{a}_2^4\chi_2\right]$$

numerical integration constants $\chi_{1,2}$: MaMuPaXS [link]

- Relevant combination:

$$\boxed{\hat{a}_4 = a_4 - \frac{3}{4}a_1 a_3 + \frac{5}{12}a_1^2\left(a_2 - a_1^2/4\right) = a_4 - \frac{3}{2}a\, a_3 + \frac{5}{3}a^2\left(b-a^2\right)}$$

$$\boxed{\hat{a}_2 = a_2 - a_1^2/4 = b - a^2}$$

[exactly same combination as in $\omega\omega \to 2h$]

- Almost s-wave (J=0) [$\chi_1 = -0.12,\ \chi_2 = 0.019$] ➔ critical angular information

* Delgado,Gómez-Ambrosio,Martínez-Martín,Salas-Bernárdez,SC, JHEP 03 (2024) 037

SMEFT theory: $\omega\omega \to 2h, 3h, 4h \ldots$ suppression

- SMEFT⇔HEFT relations for the relevant combinations:

$$\hat{a}_2 = d + 2d^2(1+\rho) + \mathcal{O}(d^3)$$
$$\hat{a}_3 = \frac{4}{3}d^2(1+\rho) + \mathcal{O}(d^3)$$
$$\hat{a}_4 = \frac{1}{3}d^2(1+\rho) + \mathcal{O}(d^3)$$

$$d = \frac{2v^2 c_{H\Box}^{(6)}}{\Lambda^2} \quad , \quad \rho = \frac{c_{H\Box}^{(8)}}{2(c_{H\Box}^{(6)})^2}$$

- Multi-Higgs fine-tuned suppression in SMEFT

* Delgado,Gómez-Ambrosio,Martínez-Martín,Salas-Bernárdez,SC, JHEP 03 (2024) 037

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$$\mathbf{1/\Lambda^4}$$

$$\begin{aligned}
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SMEFT pheno: $\omega\omega \to 2h, 3h, 4h$ ... suppression

- $\omega\omega \to 2h$

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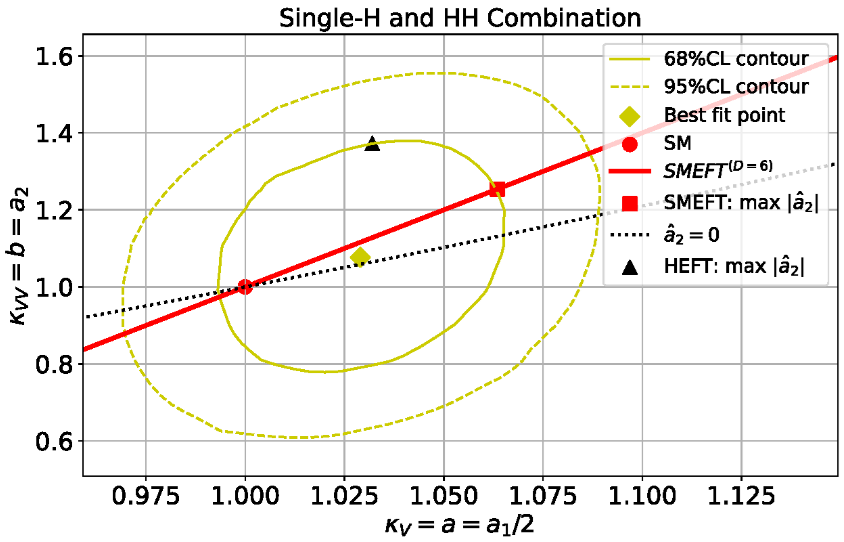

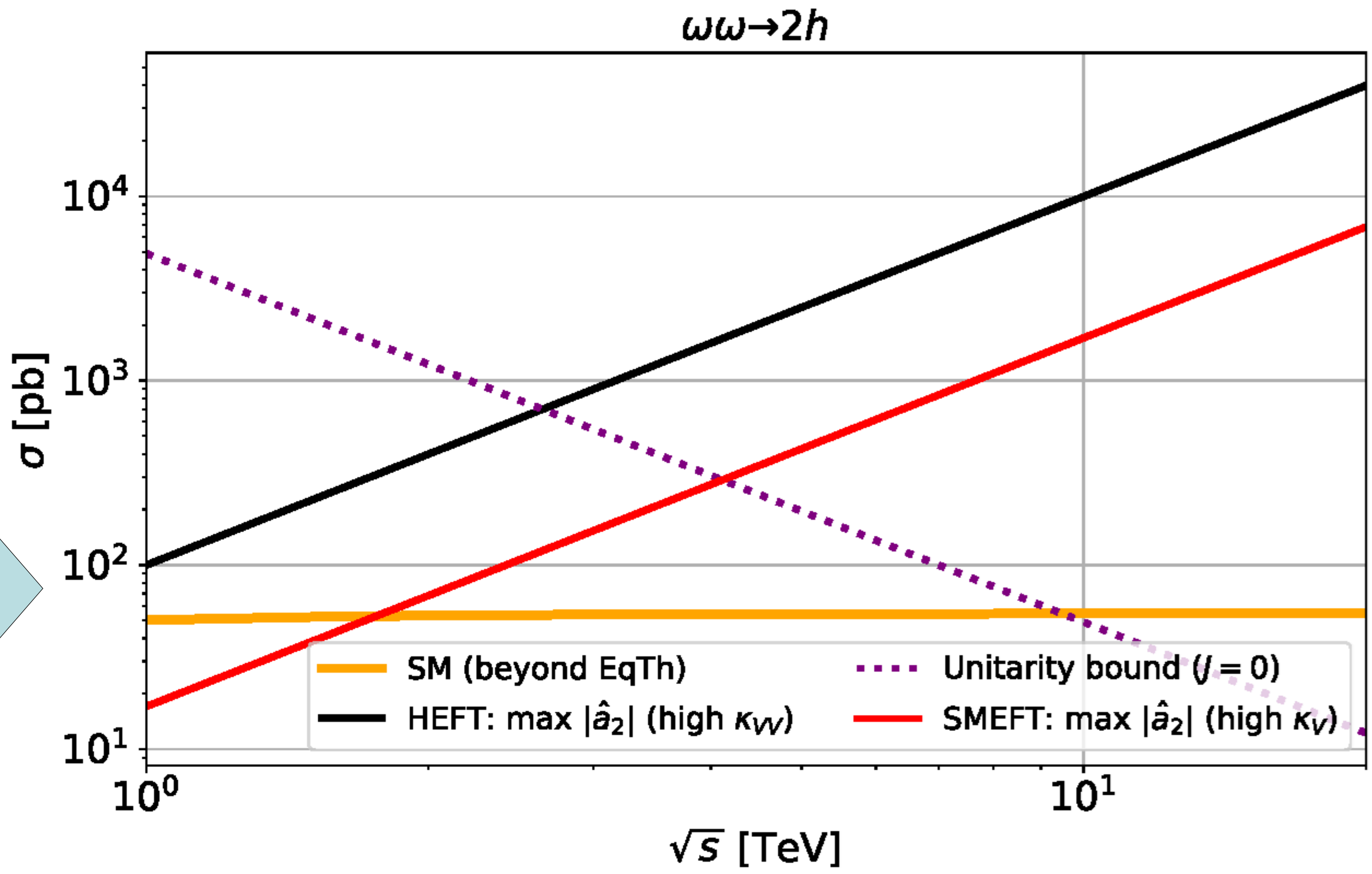

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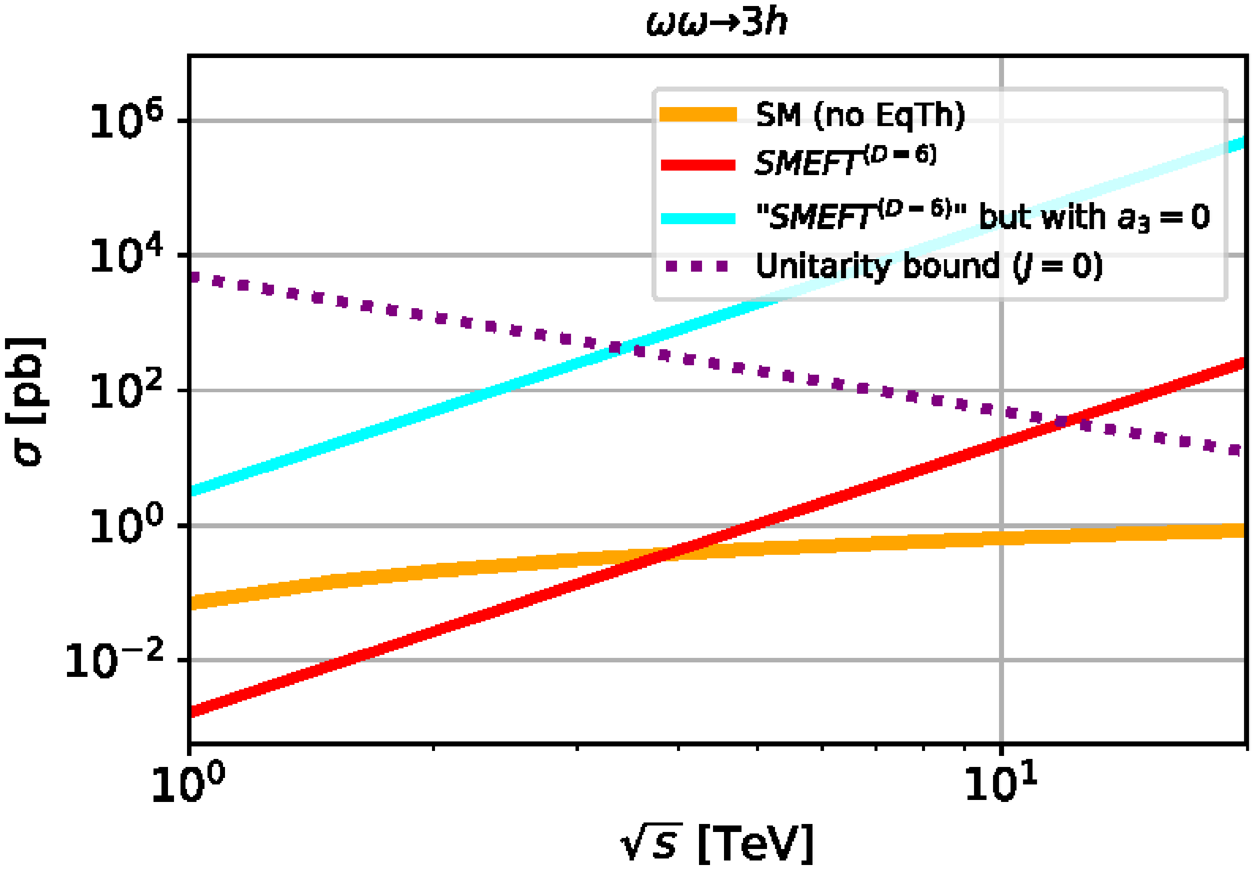

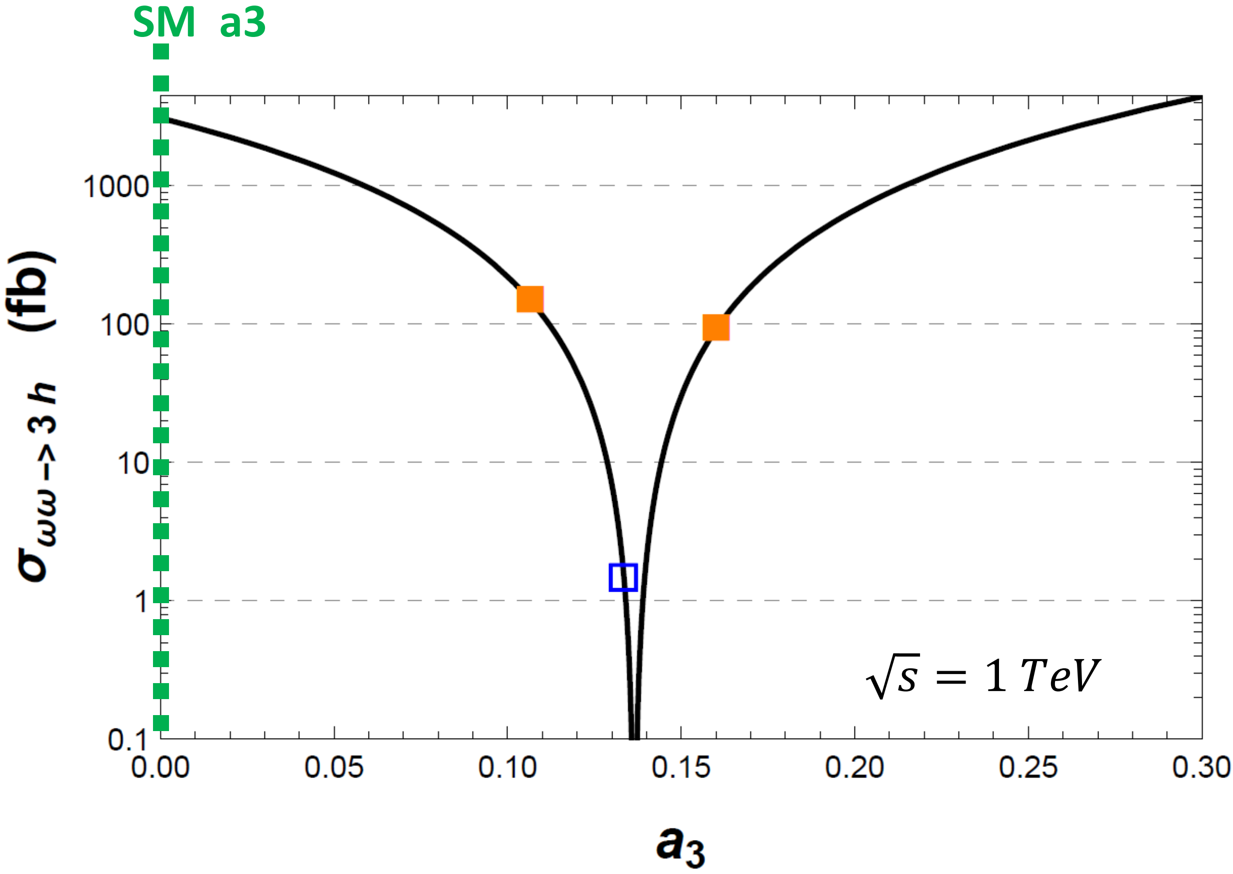

(*) Delgado,Gómez-Ambrosio,Martínez-Martín,Salas-Bernárdez,SC, JHEP 03 (2024) 037
(*) 1 loop: Sara Martellini, Master Thesis 2025; Martellini,Nasoni,SC, forthcoming

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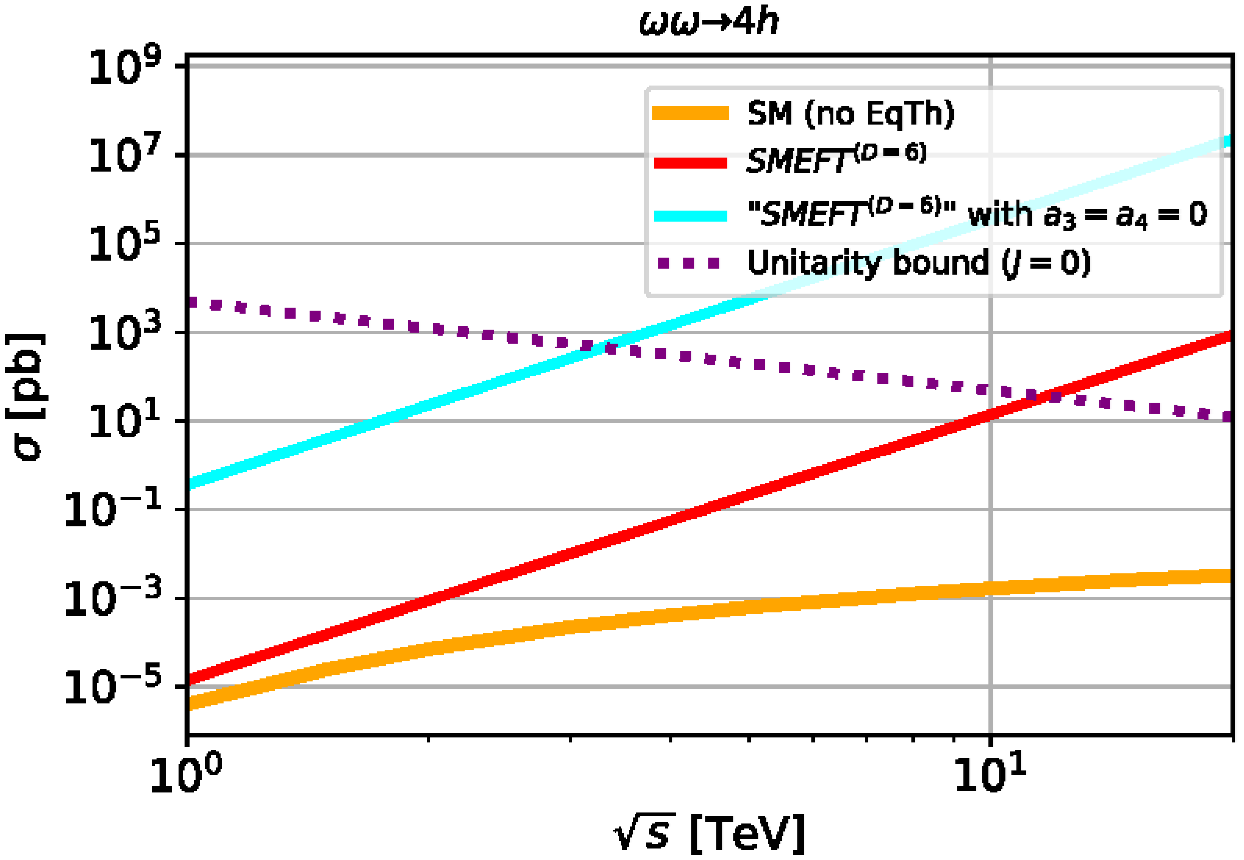

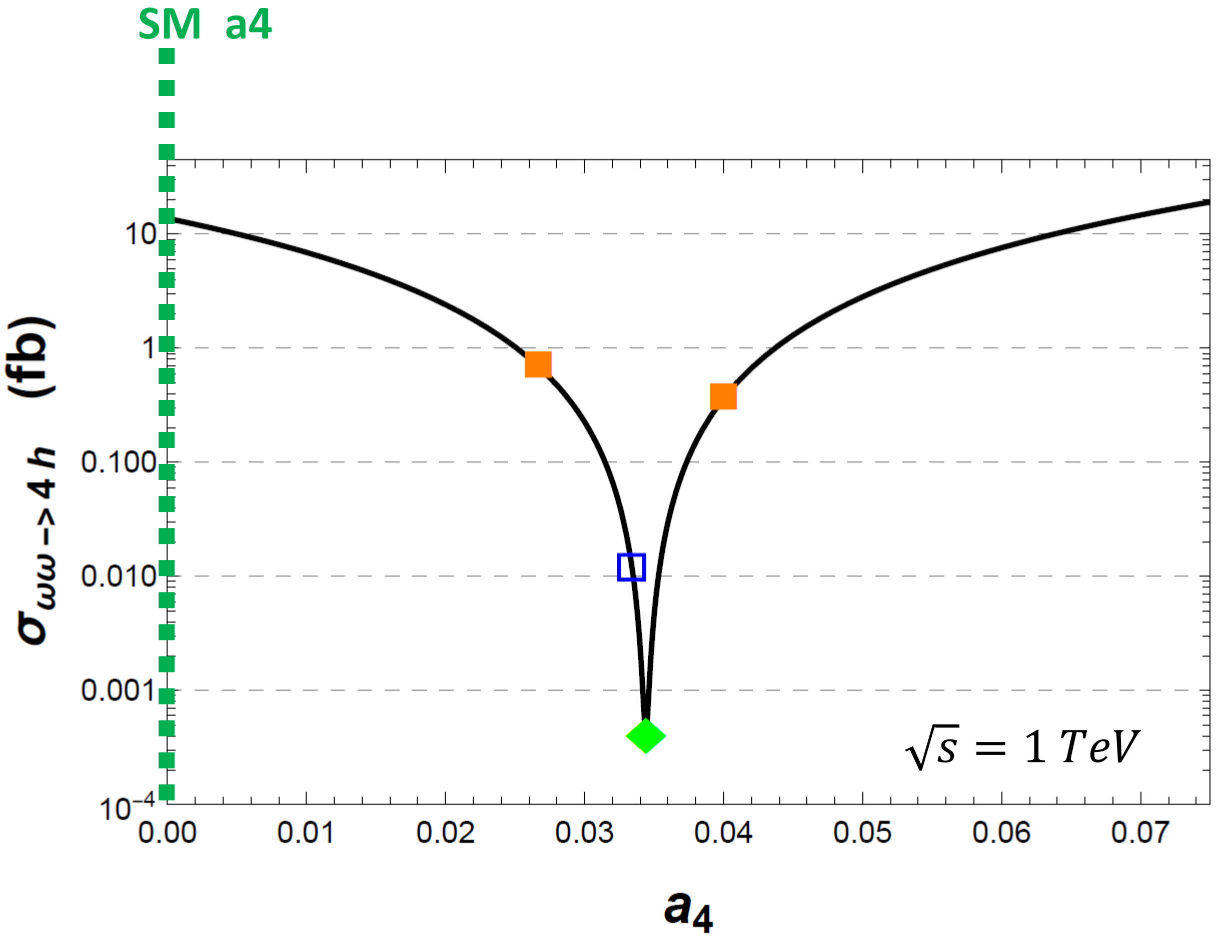

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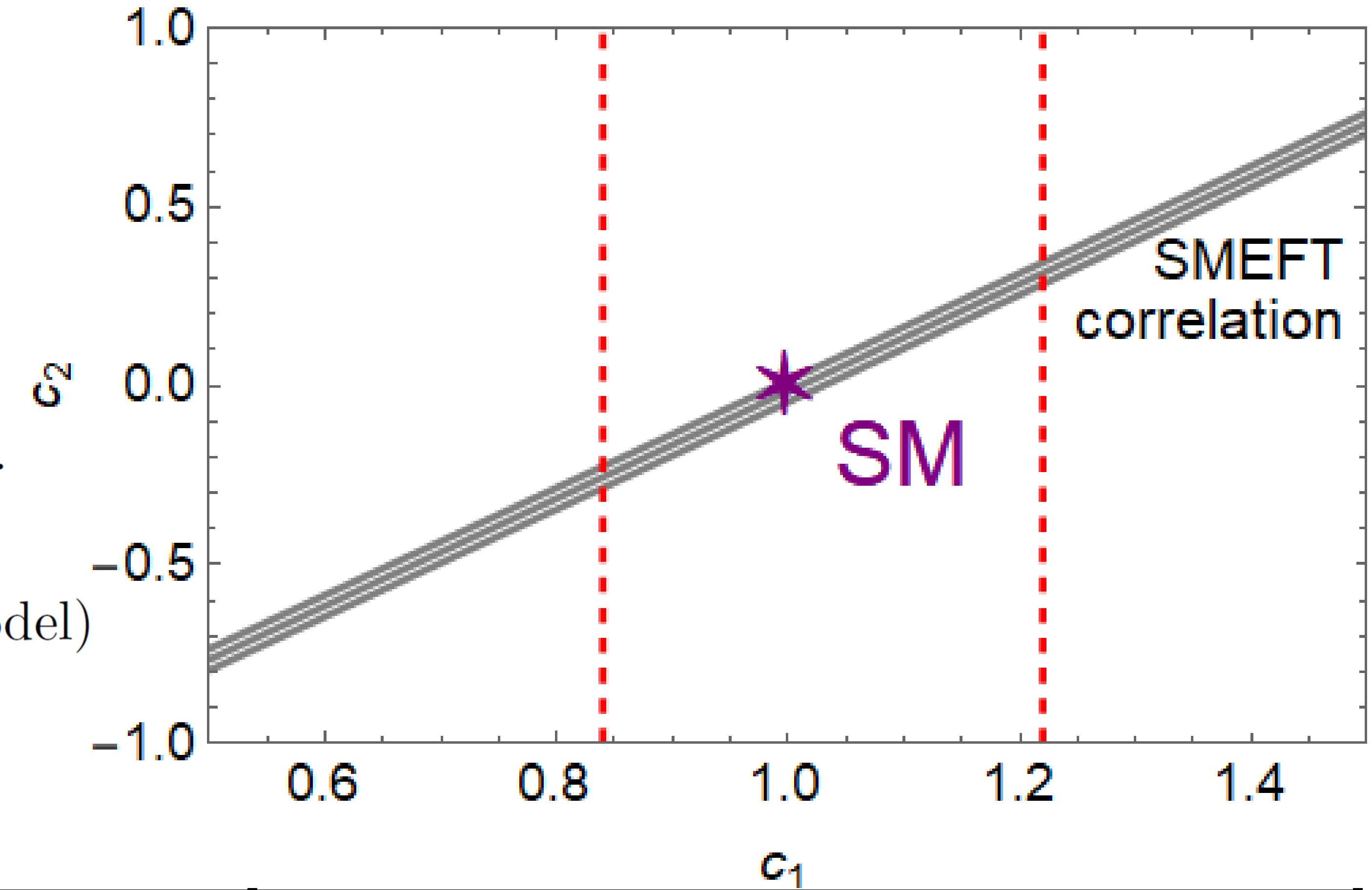
SMEFT theory: other correlations – Yukawa's

$$\mathcal{L}_Y = -\mathcal{G}(h) M_t \bar{t} t \sqrt{1 - \frac{\omega^2}{v^2}} \; + \ldots$$

$$\mathcal{G}(h_{\rm HEFT}) = 1 + c_1 \frac{h_{\rm HEFT}}{v} + c_2 \left(\frac{h_{\rm HEFT}}{v}\right)^2 + \ldots$$

(with $c_1 = 1, \; c_{i\geq 2} = 0$ in the Standard Model)



$$c_2 = 3c_3 = \frac{3}{2}(c_1 - 1) - \frac{1}{4}\Delta a_1$$

SMEFT

$$c_2 = 3c_3 \in [-0.27, 0.35]$$

(*) Gómez-Ambrosio,Llanes-Estrada,Salas-Bernárdez,SC, PRD 106 (2022) 5, 5; Commun.Theor.Phys. 75 (2023) 9, 095202.

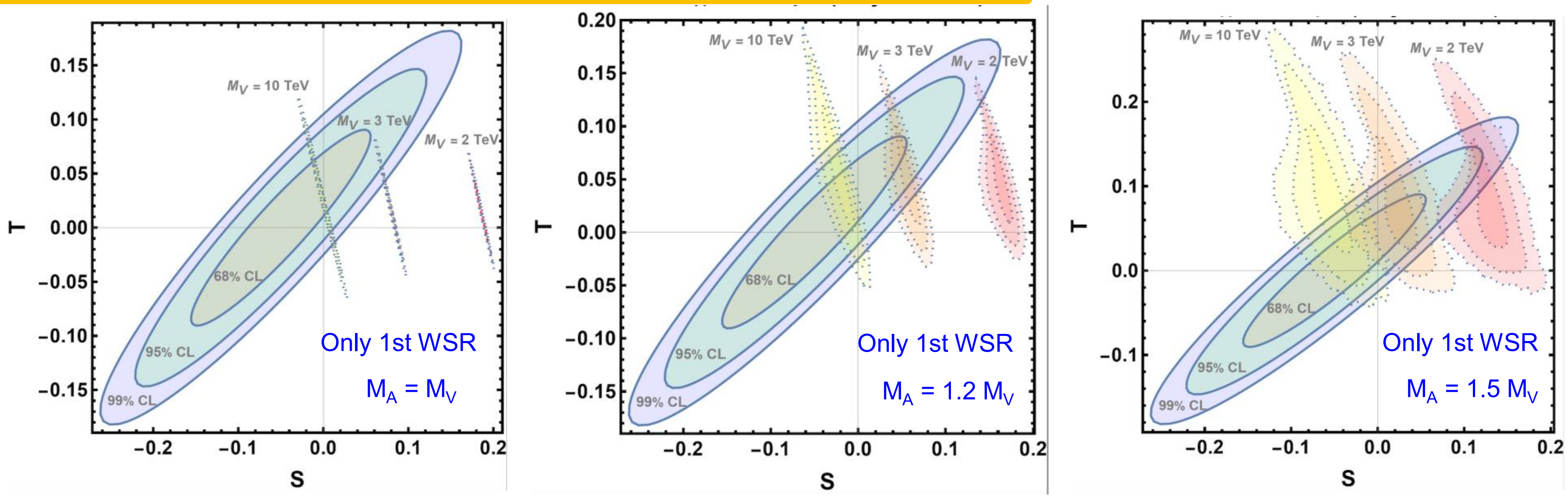
Resonances hidden from LHC?

$S = -0.05 \pm 0.07$ * $T = 0.00 \pm 0.06$*

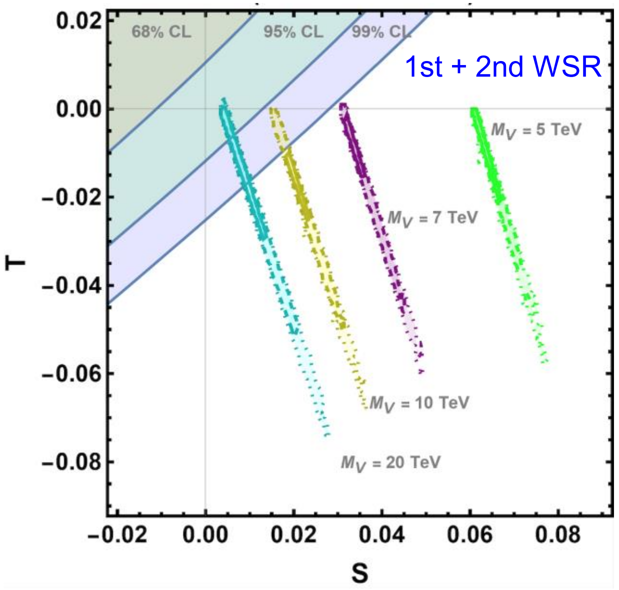
$\kappa_W = 1.023 \pm 0.026$ * $\frac{\widetilde{F}_{V,A}}{F_{V,A}} = 0.00 \pm 0.33$

* PDG '24

Only 1st WSR



1st + 2nd WSR



(*) Pich, Rosell,SC, PRD 112 (2025) 3, 035009

- Study of the oblique (S,T) parameters:

$M_A > M_V \gtrsim 2$ TeV (only the 1st WSR, 95% CL)

$M_A > M_V \gtrsim 10$ TeV (1st and 2nd WSR, 95% CL)

Conclusions

- **SMEFT** $\Longrightarrow$ - Difficult to observe $\boldsymbol{W_L W_L \to 2h}$

  - Difficult to observe a BSM excess

  - Impossible for $\boldsymbol{W_L W_L \to 3h, 4h}$

Far too few events predicted, even in best cases,

even w/ best collider, highest energy, and highest luminosity

($W_L W_L \to 3h, 4h$ hard sub-process cross sections are 3 and 5 orders of magnitude smaller than $W_L W_L \to 2h$)

- Some claim SMEFT is the only *natural* option
  since no BSM state observed in the few-TeV range

Nevertheless: EWPO and LHC VBS do not push this mass bound really that far

- An eventual observation and BSM deviation on:

$\boldsymbol{W_L W_L \to 2h}$ and $\boldsymbol{W_L W_L \to 3h}$ VBS

$\Longrightarrow$ NP is not SMEFT-like

$\Longrightarrow$ HEFT-like pattern expected

BACKUP

- **Compact structure for $W_L W_L \to n \times h$ scattering:**

  [Ruled by very particular combinations, $\hat{\kappa}_{2V} \equiv \hat{a}_2$ for $2h$, $\hat{a}_3$ for $3h$, etc.]

- **Field redefinitions helps us to understand this simplicity**

- **Strong multi-Higgs suppression in SMEFT** wrt to HEFT

  –Even for small O(10%) deviations in SMEFT $a_{1,2,3,4\ldots}$–

  –Much larger XS in pure HEFT–

- ✓ The Lagrangian reads:

$$\Delta\mathcal{L}_{\rm RT} = \frac{v^2}{4}\left(1+\frac{2\kappa_W}{v}h\right)\langle\, u_\mu u^\mu\,\rangle_2$$
$$+\,\langle\, V^1_{3\,\mu\nu}\left(\frac{F_V}{2\sqrt{2}}f_+^{\mu\nu}+\frac{iG_V}{2\sqrt{2}}[u^\mu,u^\nu]+\frac{\widetilde{F}_V}{2\sqrt{2}}f_-^{\mu\nu}+\frac{\widetilde{\lambda}_1^{hV}}{\sqrt{2}}\left[(\partial^\mu h)u^\nu-(\partial^\nu h)u^\mu\right]+C_0^{V_3^1}J_T^{\mu\nu}\right)\rangle_2$$
$$+\,\langle\, A^1_{3\,\mu\nu}\left(\frac{F_A}{2\sqrt{2}}f_-^{\mu\nu}+\frac{\lambda_1^{hA}}{\sqrt{2}}\left[(\partial^\mu h)u^\nu-(\partial^\nu h)u^\mu\right]+\frac{\widetilde{F}_A}{2\sqrt{2}}f_+^{\mu\nu}+\frac{i\widetilde{G}_A}{2\sqrt{2}}[u^\mu,u^\nu]+\widetilde{C}_0^{A_3^1}J_T^{\mu\nu}\right)\rangle_2\,.$$

- ✓ Including resonance masses, we have 12 resonance parameters. This number can be reduced by using short-distance information, but contrary to QCD, we ignore the underlying theory (BSM).

  - ✓ Vanishing form factors at high energies allow us to determine $\left(G_V\,,\widetilde{G}_A\,,\lambda_1^{hA}\,,\widetilde{\lambda}_1^{hV}\right)$ in terms of the remaining parameters:

$$\frac{G_V}{F_A}=-\frac{\widetilde{G}_A}{\widetilde{F}_V}=\frac{\lambda_1^{hA}v}{\kappa_W F_V}=-\frac{\widetilde{\lambda}_1^{hV}v}{\kappa_W\widetilde{F}_A}=\frac{v^2}{F_VF_A-\widetilde{F}_V\widetilde{F}_A}\,.$$

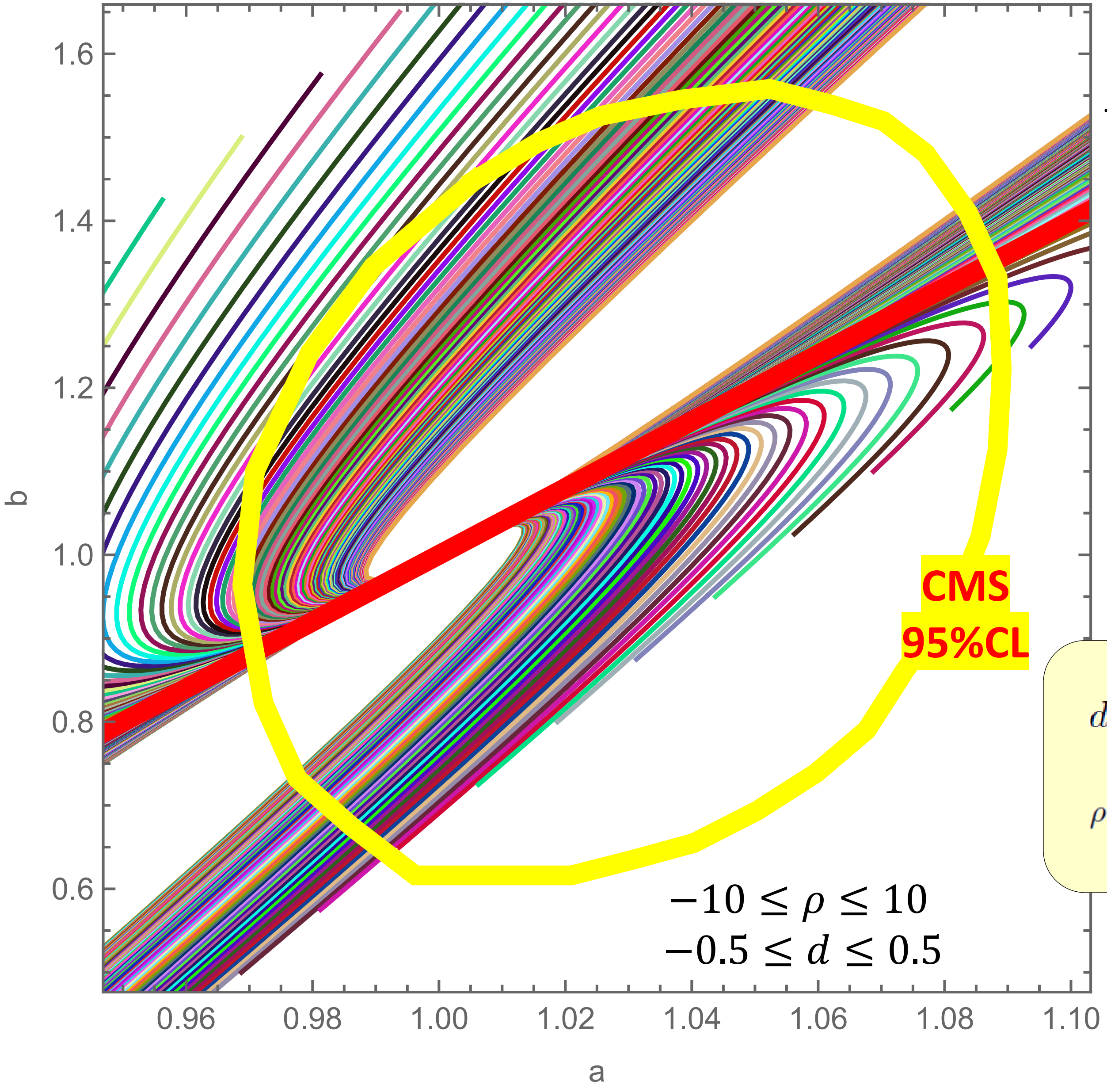
  - ✓ Weinberg sum rules (WSRs) at LO and at NLO.

    - ✓ 1st WSR. Vanishing of the 1/s term of $\Pi_{VV}(s)-\Pi_{AA}(s)$: $\left(F_V^2-\widetilde{F}_V^2\right)-\left(F_A^2-\widetilde{F}_A^2\right)=v^2$
    - ✓ 2nd WSR. Vanishing of the 1/s² term of $\Pi_{VV}(s)-\Pi_{AA}(s)$: $\left(F_V^2-\widetilde{F}_V^2\right)M_V^2-\left(F_A^2-\widetilde{F}_A^2\right)M_A^2=0$
    - ✓ 1st WSR + LHC diboson production imply that contributions from fermionic cuts, terms with , are negligible.

(*) Pich, Rosell,SC, PRD 112 (2025) 3, 035009

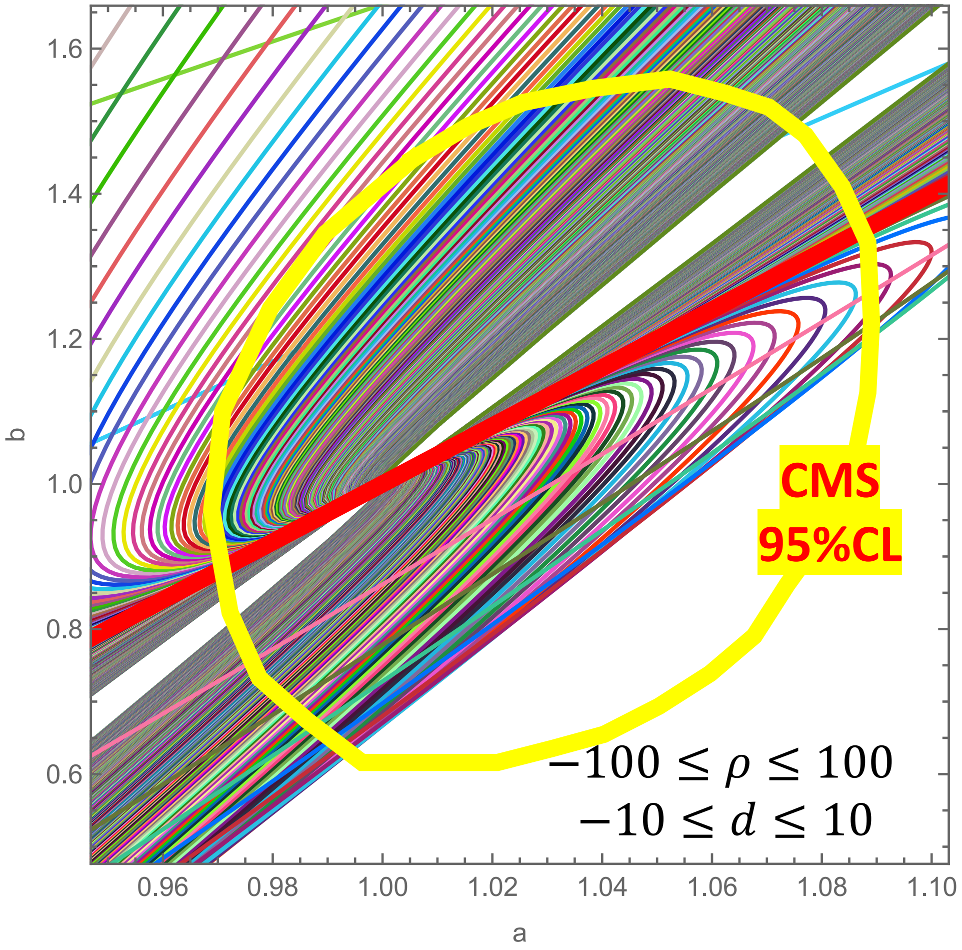
What about D=8 corrections to SMEFT? ➔

- The line turns into a “thick line”/area
- But still can’t go everywhere
- Actually lines (2D surfaces) within $\mathbb{R}^4$ for D=6 ($\mathbb{R}^6$ for D=8)
- Volunteers can play with D=10 corrections



$$d = \frac{2v^2 c^{(6)}_{H\Box}}{\Lambda^2}$$

$$\rho = \frac{c^{(8)}_{H\Box}}{2\left(c^{(6)}_{H\Box}\right)^2}$$



• Chiral counting:

$$\mathcal{L}_{HEFT} = \mathcal{L}_2 + \mathcal{L}_4 + \cdots$$

(See M.J. Herrero's talk for NLO corrections $O(p^4)$ or ask for backup)

•LO Lagrangian, $O(p^2)$:

$$\begin{aligned}\mathcal{L}_2 = & -\frac{1}{2}\langle G_{\mu\nu}G^{\mu\nu}\rangle - \frac{1}{2}\langle W_{\mu\nu}W^{\mu\nu}\rangle - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + \sum_{\psi=q_L,l_L,u_R,d_R,e_R} \bar{\psi} i \not{D} \psi \\ & + \frac{v^2}{4}\langle D_\mu U^\dagger D^\mu U\rangle\, (1+F_U(h)) + \frac{1}{2}\partial_\mu h \partial^\mu h - V(h) \\ & - v\left[\bar{q}_L\left(Y_u + \sum_{n=1}^{\infty} Y_u^{(n)}\left(\frac{h}{v}\right)^n\right) U P_+ q_R + \bar{q}_L\left(Y_d + \sum_{n=1}^{\infty} Y_d^{(n)}\left(\frac{h}{v}\right)^n\right) U P_- q_R \right. \\ & \left. + \bar{l}_L\left(Y_e + \sum_{n=1}^{\infty} Y_e^{(n)}\left(\frac{h}{v}\right)^n\right) U P_- l_R + \text{h.c.}\right] \end{aligned} \qquad \text{(II.2.164)}$$

Handbook of LHC Higgs Cross Sections: 4. Deciphering the Nature of the Higgs Sector - 1610.07922 [hep-ph]

This work's approx. in HEFT: $W_L W_L \to 2h, 3h, 4h \ldots$

- kinematics well over WW threshold: $s \gg m_W^2 \sim m_h^2$

- Mass corrections neglected

- Chiral LO: only $O(\partial^2)$ derivative operators

- Equivalence theorem appr.: $W_L W_L \to n \times h \approx \omega\omega \to n \times h$

[I know: 3h, 4h, etc. looks like science-fiction nowadays]

- Specific $\omega\omega \to n \times h$ stand-alone Mathematica code [link]
- FeynRules + FeynCalc chiral model file @ LO + NLO [link1] [link2]

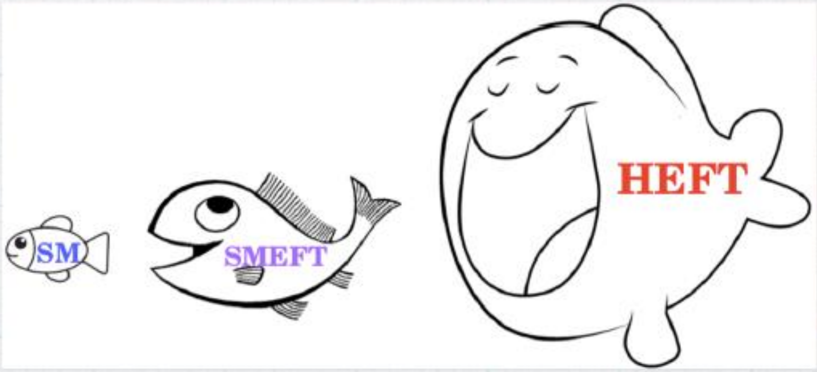
* Delgado,Gómez-Ambrosio,Martínez-Martín,Salas-Bernárdez,SC, JHEP 03 (2024) 037

The Flare Function

* Gómez-Ambrosio, Llanes-Estrada, Salas-Bernárdez, SC, PRD 106 (2022) 5, 5; Commun.Theor.Phys. 75 (2023) 9, 095202
* Domenech, Herrero, Morales, Salas-Bernárdez, 2506.21716 [hep-ph]

- In HEFT: $\mathscr{F}(h)_{HEFT} = 1 + a_1 \frac{h}{v} + a_2 \left(\frac{h}{v}\right)^2 + a_3 \left(\frac{h}{v}\right)^3 + \ldots$
- In the SM: $\mathscr{F}(h)_{SM} = \left(1 + \frac{h}{v}\right)^2$
- In SMEFT?

$$
\begin{aligned}
\mathcal{F}(h_1) &= \left(1 + \frac{h(h_1)}{v}\right)^2 \\
&= 1 + \left(\frac{h_1}{v}\right)\left(2 + 2\frac{c_{H\Box}^{(6)} v^2}{\Lambda^2} + 3\frac{(c_{H\Box}^{(6)})^2 v^4}{\Lambda^4} + 2\frac{c_{H\Box}^{(8)} v^4}{\Lambda^4}\right) + \left(\frac{h_1}{v}\right)^2\left(1 + 4\frac{c_{H\Box}^{(6)} v^2}{\Lambda^2} + 12\frac{(c_{H\Box}^{(6)})^2 v^4}{\Lambda^4} + 6\frac{c_{H\Box}^{(8)} v^4}{\Lambda^4}\right) \\
&+ \left(\frac{h_1}{v}\right)^3\left(8\frac{c_{H\Box}^{(6)} v^2}{3\Lambda^2} + 56\frac{(c_{H\Box}^{(6)})^2 v^4}{3\Lambda^4} + 8\frac{c_{H\Box}^{(8)} v^4}{\Lambda^4}\right) + \left(\frac{h_1}{v}\right)^4\left(2\frac{c_{H\Box}^{(6)} v^2}{3\Lambda^2} + 44\frac{(c_{H\Box}^{(6)})^2 v^4}{3\Lambda^4} + 6\frac{c_{H\Box}^{(8)} v^4}{\Lambda^4}\right) \\
&+ \left(\frac{h_1}{v}\right)^5\left(88\frac{(c_{H\Box}^{(6)})^2 v^4}{15\Lambda^4} + 12\frac{c_{H\Box}^{(8)} v^4}{5\Lambda^4}\right) + \left(\frac{h_1}{v}\right)^6\left(44\frac{(c_{H\Box}^{(6)})^2 v^4}{45\Lambda^4} + 2\frac{c_{H\Box}^{(8)} v^4}{5\Lambda^4}\right) + \mathcal{O}(\Lambda^{-6}).
\end{aligned}
$$



GEOMETRY INTRO

Geometry of the scalar fields (and much more in B. Assi's talk)

SM,

SMEFT,

HEFT... and

geometry:

* R. Alonso, E. E. Jenkins, and A. V. Manohar,
  * "A Geometric Formulation of Higgs Effective Field Theory: Measuring the Curvature of Scalar Field Space," Phys. Lett. B754 (2016) 335–342, arXiv:1511.00724 [hep-ph].
  * "Sigma Models with Negative Curvature," Phys.Lett.B756,358(2016),arXiv:1602.00706 [hep-ph].
  * "Geometry of the Scalar Sector," JHEP 08 (2016) 101, arXiv:1605.03602 [hep-ph]." (Cohen et al., 2021, p. 95)
* T. Cohen, N. Craig, X. Lu, and D. Sutherland:
  * "Is SMEFT Enough?", JHEP 03, 237, arXiv:2008.08597 [hep-ph].
  * "Unitarity Violation and the Geometry of Higgs EFTs", (2021), arXiv:2108.03240 [hep- ph].

we now know that HEFT and SMEFT can be understood geometrically

Old works in 80's:

Boulware,Brown, Annals Phys. 138 (1982) 392

[thanks to M. Knecht for calling out my attention to this]

GEOMETRY INTRO

- Beautiful geometric connection to scalar loop corrections [(*)] provided by the curvature [(x)] of the scalar manifold metric $g_{ij}(\phi) = \begin{bmatrix} F(h)^2 g_{ab}(\varphi) & 0 \\ 0 & 1 \end{bmatrix}$, with $\mathcal{L} = \frac{1}{2} g_{ij} D_\mu \phi^i D^\mu \phi^j$

$$\mathcal{R}_4 = \left(1 - v^2 (F')^2\right) F^2 \quad = \quad \left(1 - \mathcal{K}^2/4\right) \mathcal{F}_C \,,$$

$$\mathcal{R}_2 = \left(1 - v^2 (F')^2\right) - \frac{v^2 F'' F}{(N_\varphi - 1)} \quad = \quad \left(1 - \mathcal{K}^2/4\right) - \frac{\mathcal{F}_C \Omega}{8} \,,$$

$$\mathcal{R}_0 = 2\mathcal{F}_C^{-1} \mathcal{R}_2 - \mathcal{F}_C^{-2} \mathcal{R}_4 \,,$$

$$F = \mathcal{F}_C^{1/2} \quad N_\varphi = 3$$

with Λ^{-2} = the Riemann $\mathbb{R}_{ijmn} \propto \mathscr{R}_{0,2,4} / v^2$ *(loosely speaking, the curvature $\mathbb{R}$)*

- NDA gives you the suppression of individual diagrams ~ $1 / (4\pi v)^2$

 but the <u>full</u> loop suppression is ~ $\mathbb{R}^2 / (4\pi)^2$ and ~ $g^2 \mathbb{R} / (4\pi)^2$

 EFT as an expansion $\mathcal{M} \sim R\, \mathbf{p}^2 + \frac{R^2\, \mathbf{p}^4}{(4\pi)^2} + \frac{R^3\, \mathbf{p}^6}{(4\pi)^4} + \ldots$ in the curvature?

- **SM:** $\mathbb{R}_{ijmn} = 0$ ➔ No $O(p^4)$ renormalization

(*) Guo,Ruiz-Femenia,SC, PRD92 (2015) 074005
(x) Alonso,Jenkins,Manohar, PLB754 (2016) 335; PLB756 (2016) 358; JHEP 1608 (2016) 101

- Depending on the **choice of coordinates** cancellations/suppressions may not be obvious diagram by diagram

  - **Example:** SO(5)/SO(4) MCHM tree LO computations for $\omega\omega \to \omega\omega$

    ** Agashe,Contino,Pomarol '05
    ** Contino et al. '12

HEFT MCHM

$$A(s,t,u)^{\omega\omega\to\omega\omega} = \frac{(1-a^2)\, s}{v^2} = \frac{s}{f^2}\,,$$

HEFT MCHM

$$A(s,t,u)^{\omega\omega\to hh} = \frac{(a^2-b)\, s}{v^2} = \frac{s}{f^2}$$

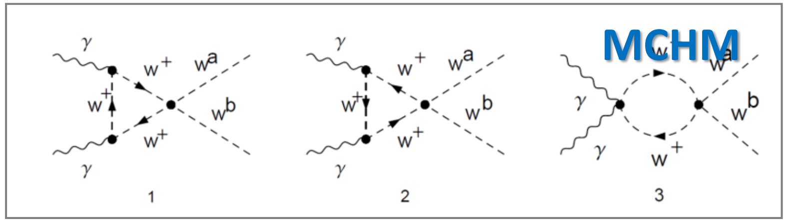
  - **Example:** SO(5)/SO(4) MCHM 1-loop computations for $\gamma\gamma \to \omega\omega$

    ** Agashe,Contino,Pomarol '05
    ** Contino et al. '12

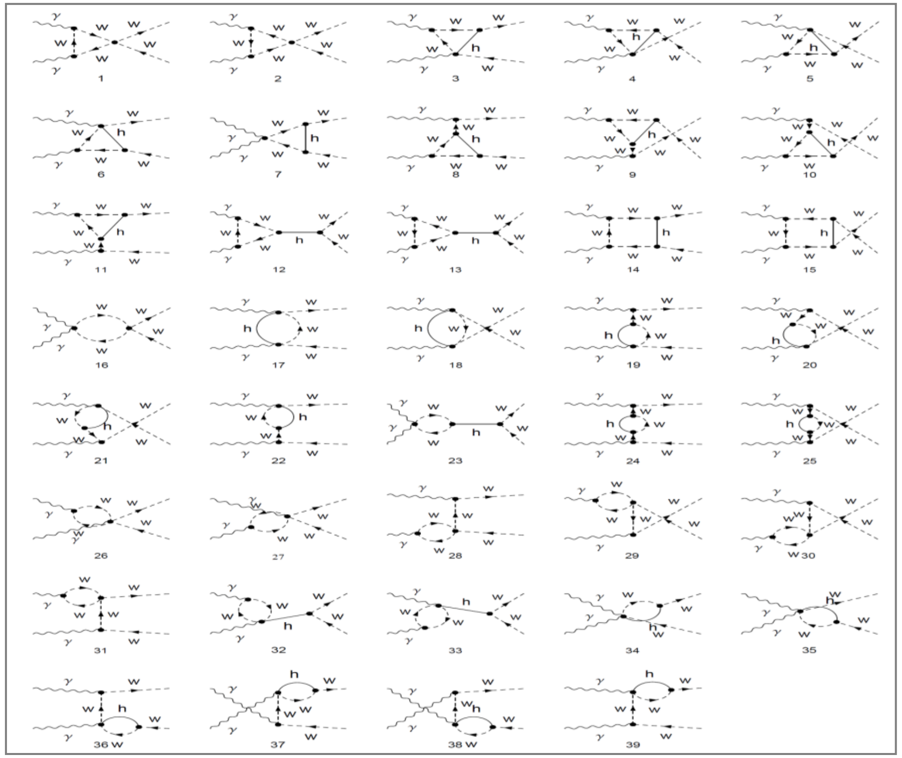
MCHM HEFT

$$A(s,t,u)^{\gamma\gamma\to zz} = -\frac{1}{4\pi^2 f^2} = -\frac{(1-a^2)}{4\pi^2 v^2}$$

$$A(s,t,u)^{\gamma\gamma\to w^+w^-} = -\frac{1}{8\pi^2 f^2} = -\frac{(1-a^2)}{8\pi^2 v^2}$$

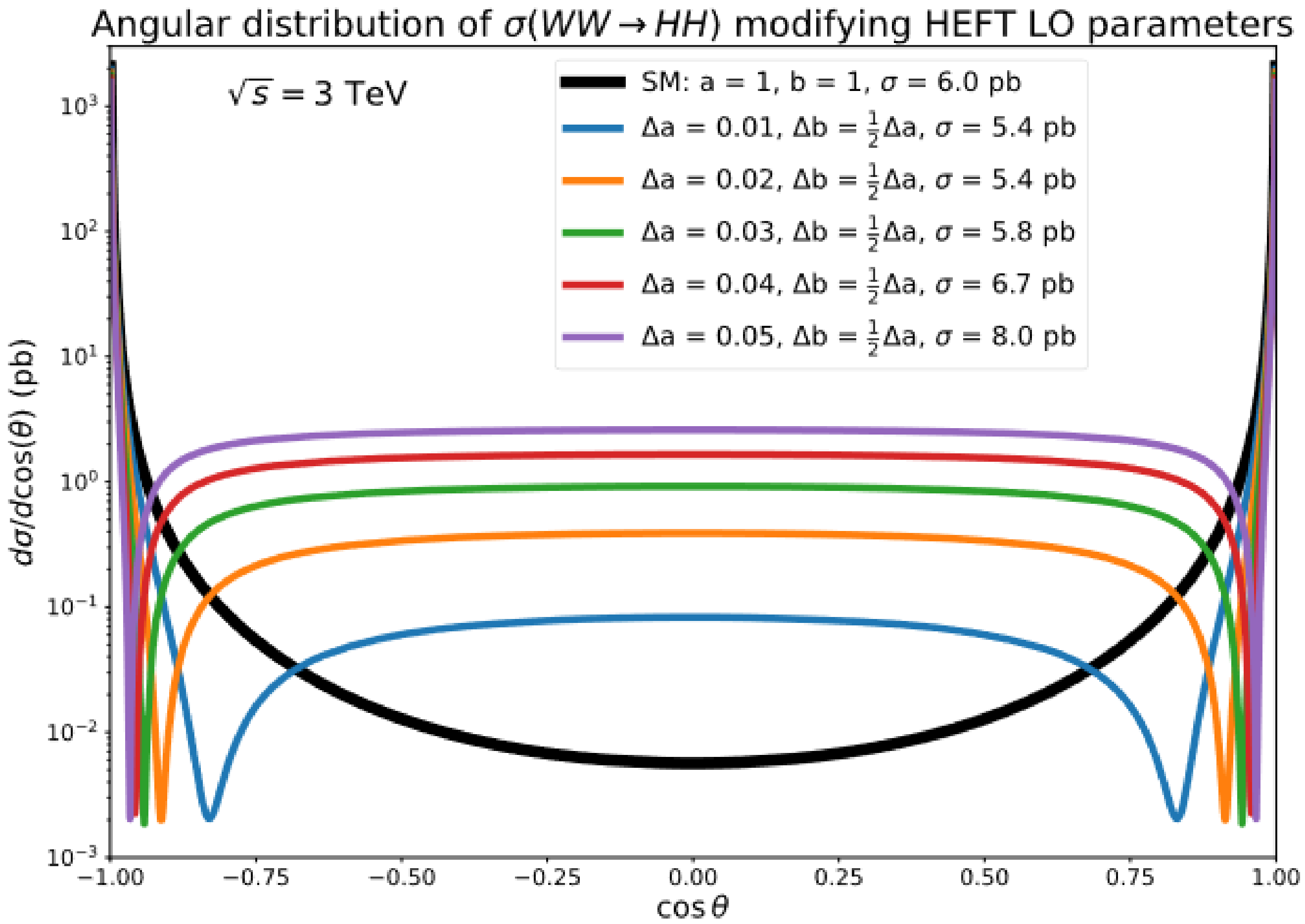


MCHM HEFT

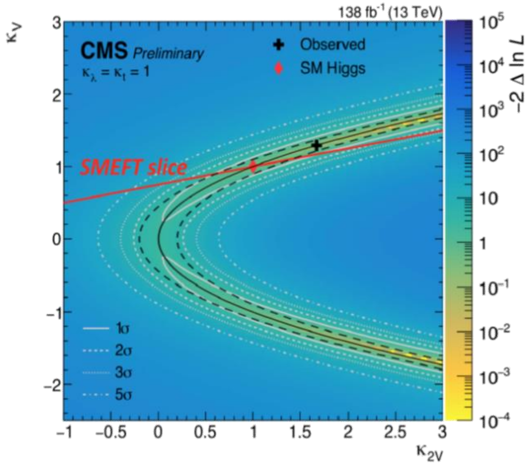


** Delgado,Dobado,Herrero,SC, JHEP 07 (2014) 149

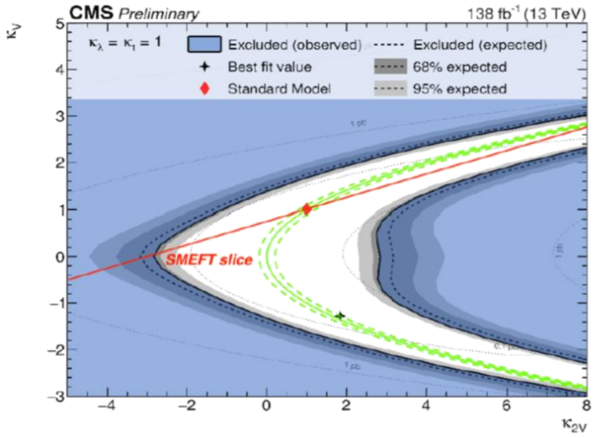
- **IR finite**
- Equiv. Theorem implies a **pure s-wave**
- This HEFT behaviour approximately observed with **real W's** (x) [vs **SM** angular distribution]



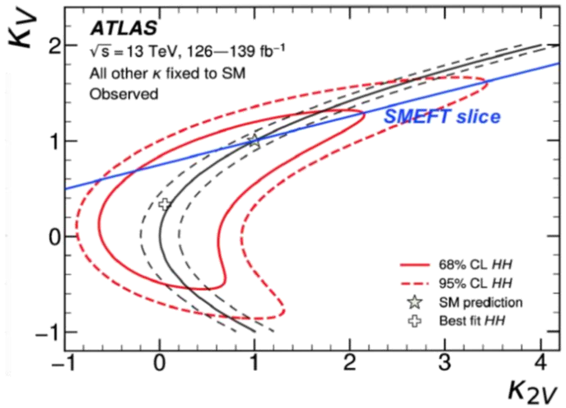
(x) Dávila,Domenech,Herrero,Morales, EPJC 84 (2024) 5, 503.
(x) See also the recent LHC análisis on $\hat{a}_2 = (b - a^2)$: Domenech,García-Mir,Herrero, 2507.20988 [hep-ph]



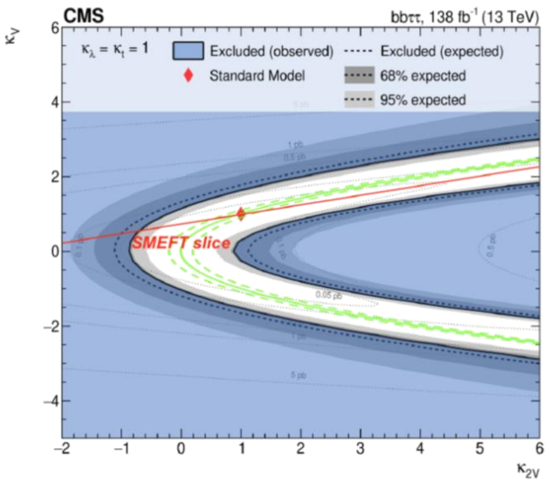
(a)



(b)



(c)



(d)

- Exp. data on hh-production at LHC show an important correlation between (a, b)

[notation: $a = a_1/2 = \kappa_V,\;\; b = a_2 = \kappa_{2V}$]

- NOTE we have superimposed:

  - Parabolles w/ constant $\hat{a}_2 = a_2 - \frac{a_1^2}{4}$

  - D=6 SMEFT prediction $a_2 = 2\,a_1 - 3$

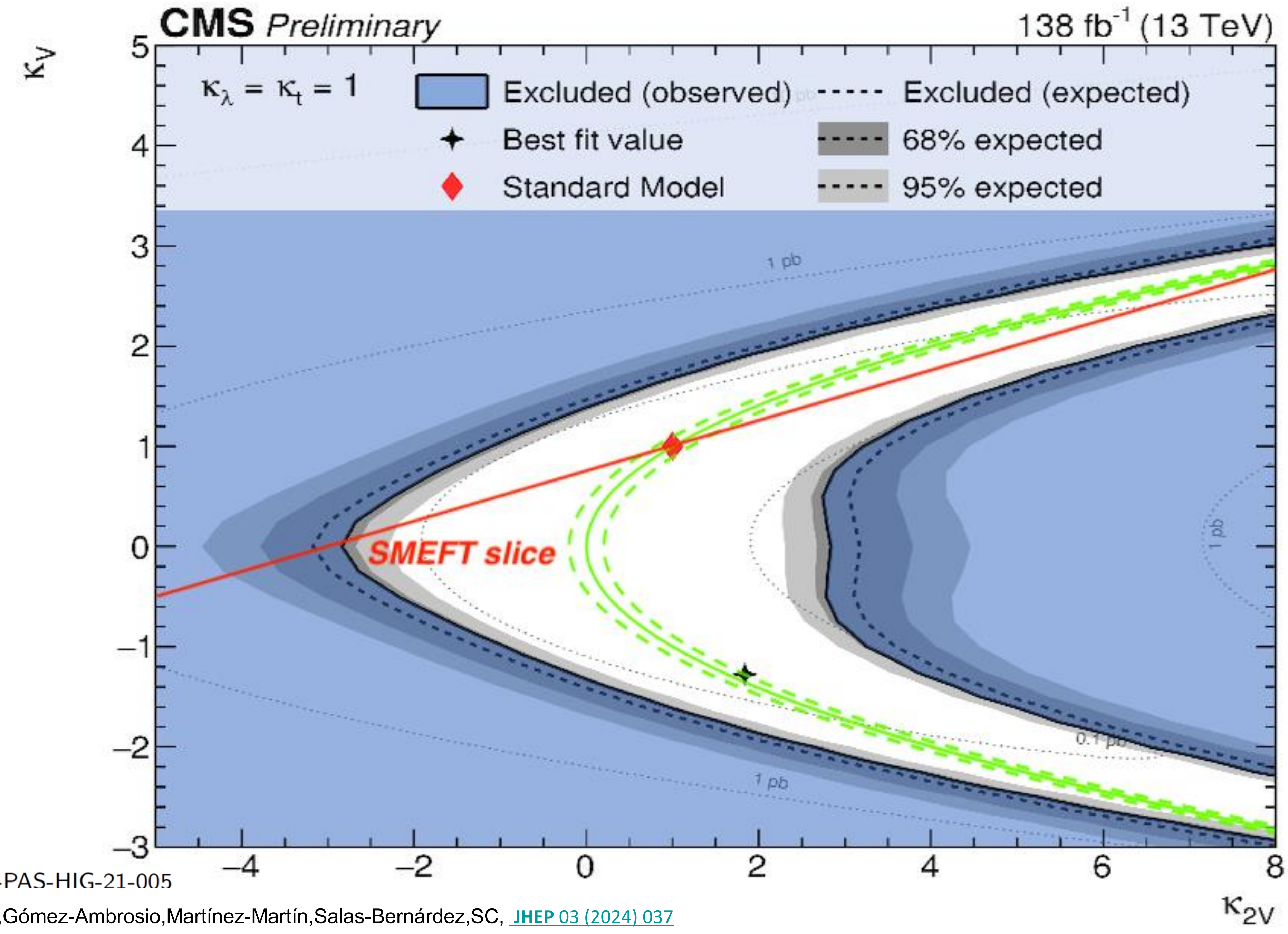
(a) Phys. Rev. Lett. 131 (2023) 041803 [2205.06667]

(b) CMS-PAS-HIG-21-005 **(c)** ATLAS-CONF-2022-050 **(d)** Phys. Lett. B 842 (2023) 137531 [2206.09401]. NOTE: $\kappa_V = a_1/2\,,\; \kappa_{2V} = a_2$.

... + some recente important improvement from HH + H:

CMS PAS HIG-23-006

(b)



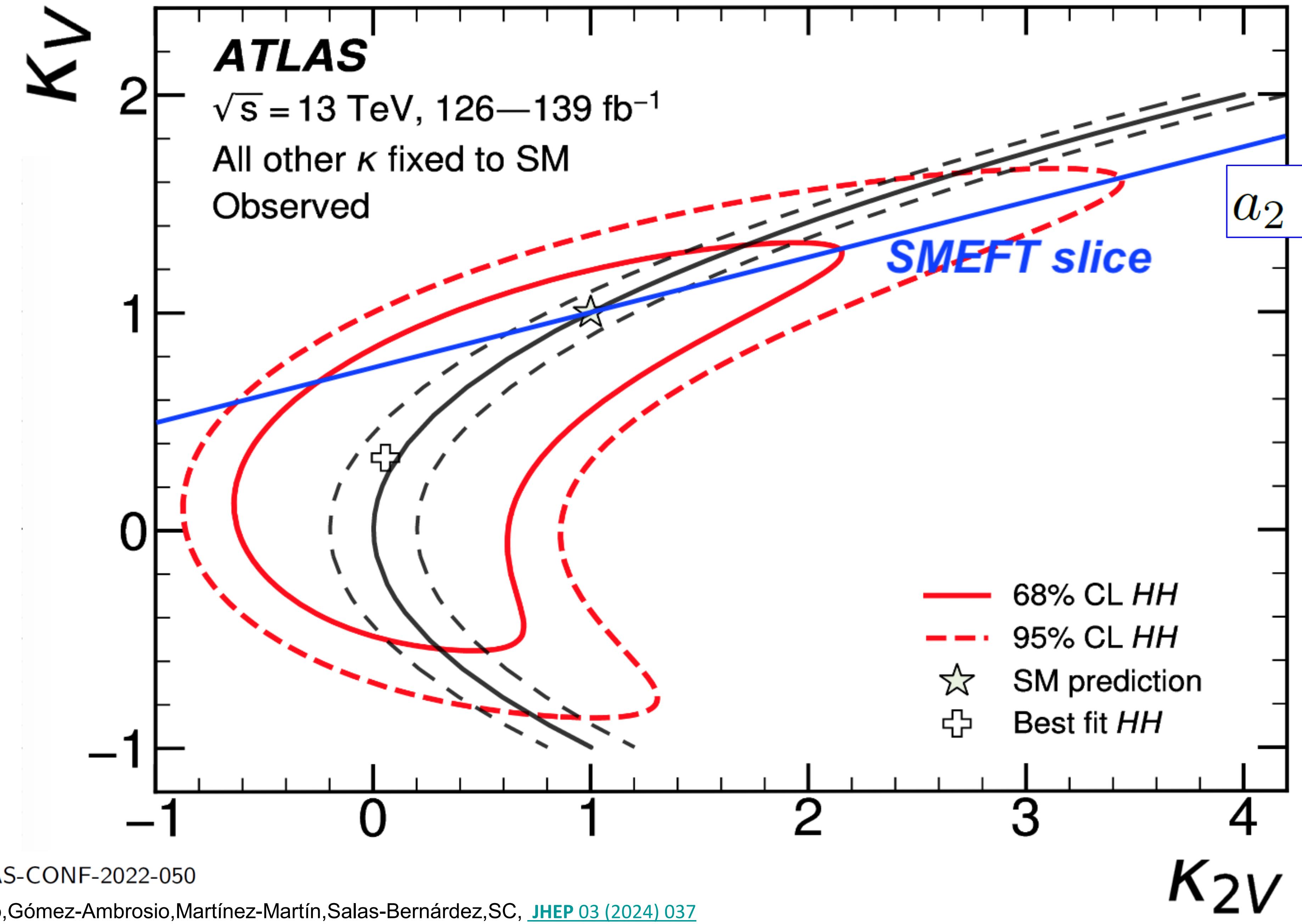
$$a_2 = 2a_1 - 3$$

$$\hat{a}_2 = \pm 0.2$$

(b) CMS-PAS-HIG-21-005

* Delgado,Gómez-Ambrosio,Martínez-Martín,Salas-Bernárdez,SC, JHEP 03 (2024) 037

(c)



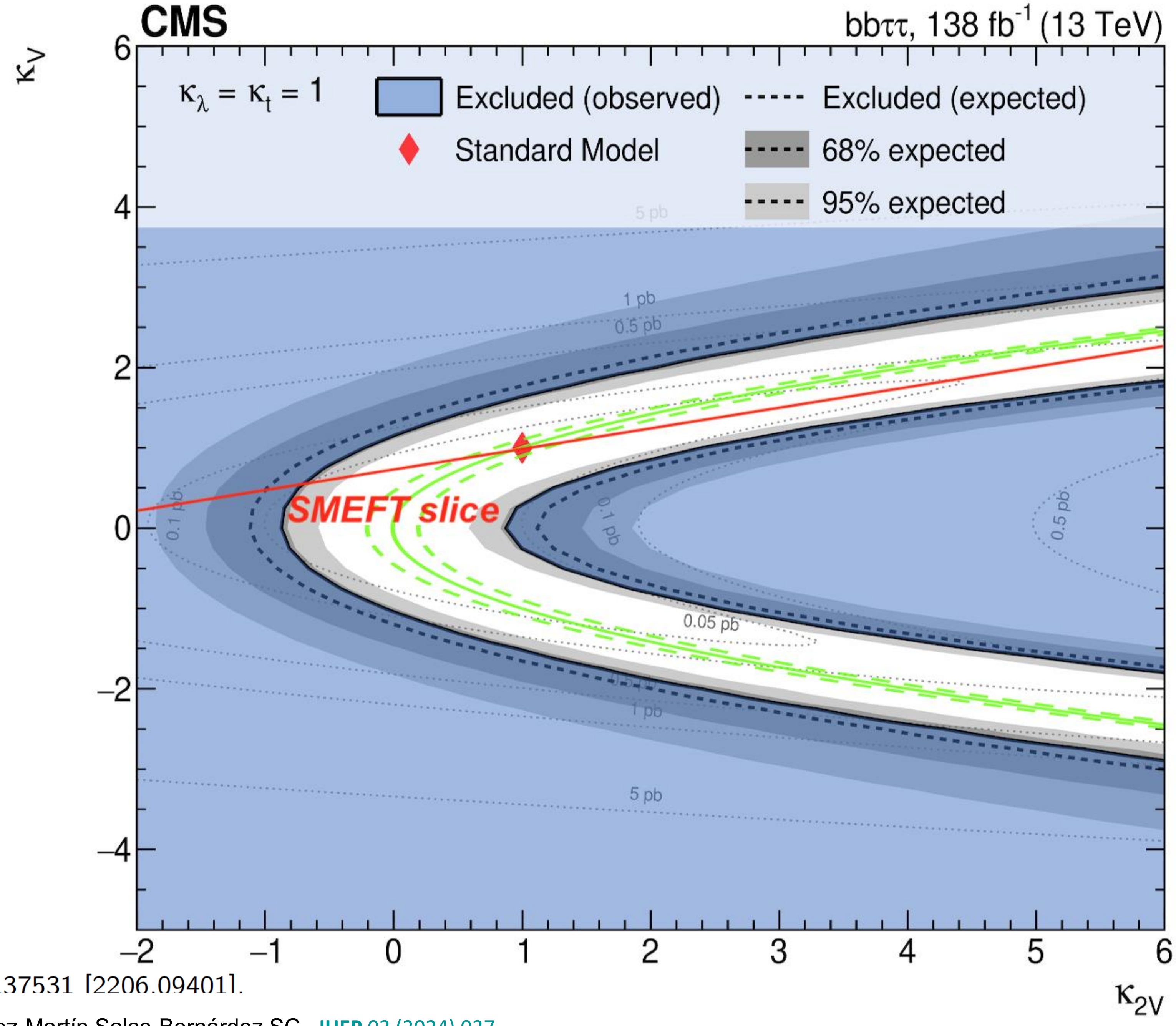
$\boxed{\hat{a}_2 = \pm 0.2}$

$\boxed{a_2 = 2a_1 - 3}$

(c) ATLAS-CONF-2022-050

* Delgado,Gómez-Ambrosio,Martínez-Martín,Salas-Bernárdez,SC, [JHEP 03 (2024) 037]

(d)



$$a_2 = 2a_1 - 3$$

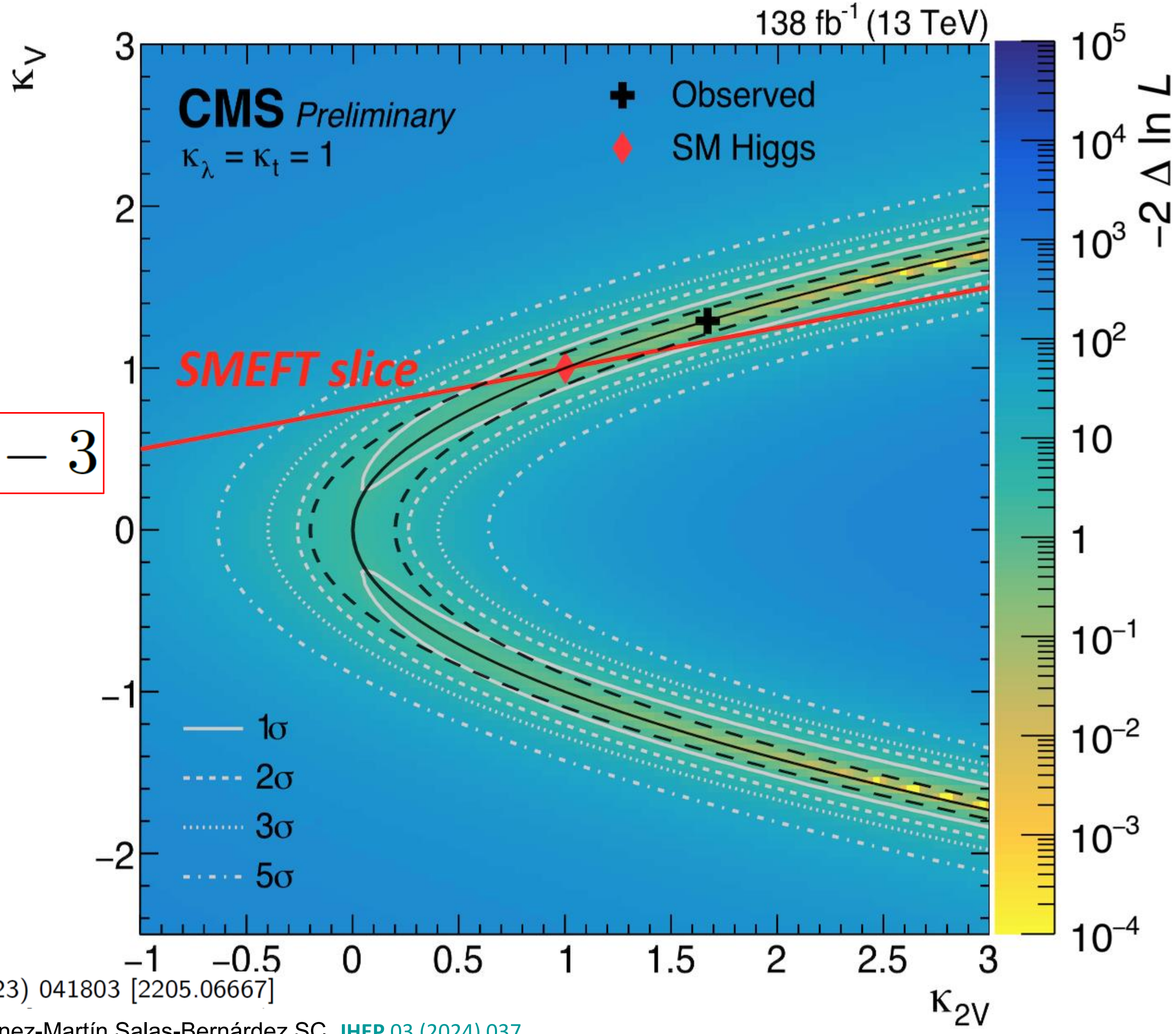
$$\hat{a}_2 = \pm 0.2$$

(d) Phys. Lett. B 842 (2023) 137531 [2206.09401].

* Delgado,Gómez-Ambrosio,Martínez-Martín,Salas-Bernárdez,SC, JHEP 03 (2024) 037

(a)

$$a_2 = 2a_1 - 3$$



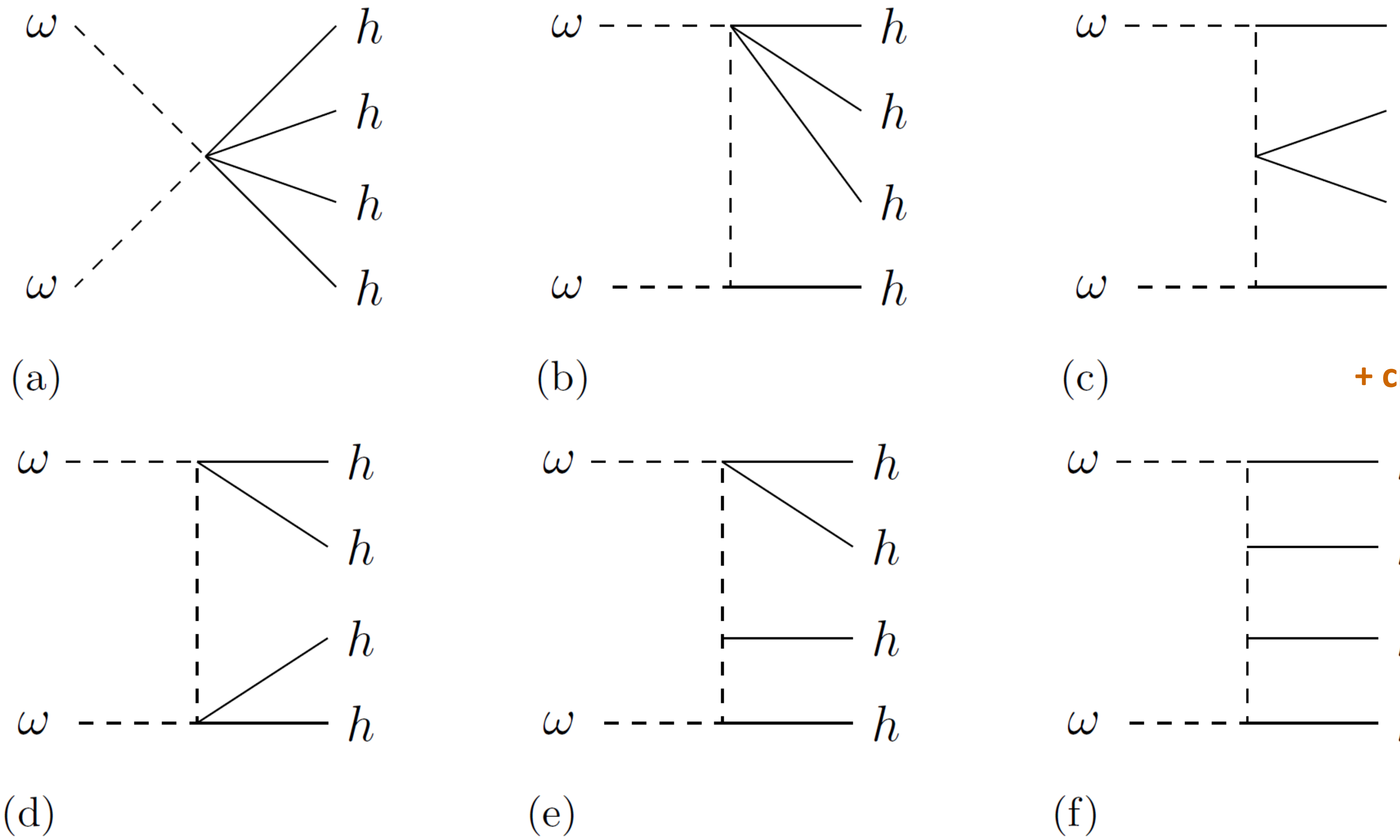
$$\hat{a}_2 = \pm 0.2$$

(a) Phys. Rev. Lett. 131 (2023) 041803 [2205.06667]

* Delgado,Gómez-Ambrosio,Martínez-Martín,Salas-Bernárdez,SC, [JHEP 03 (2024) 037]

$$T_{\omega\omega\to 4h} = -\frac{4s}{v^4}\left(3\hat{a}_4 + \hat{a}_2^2(B-1)\right)$$

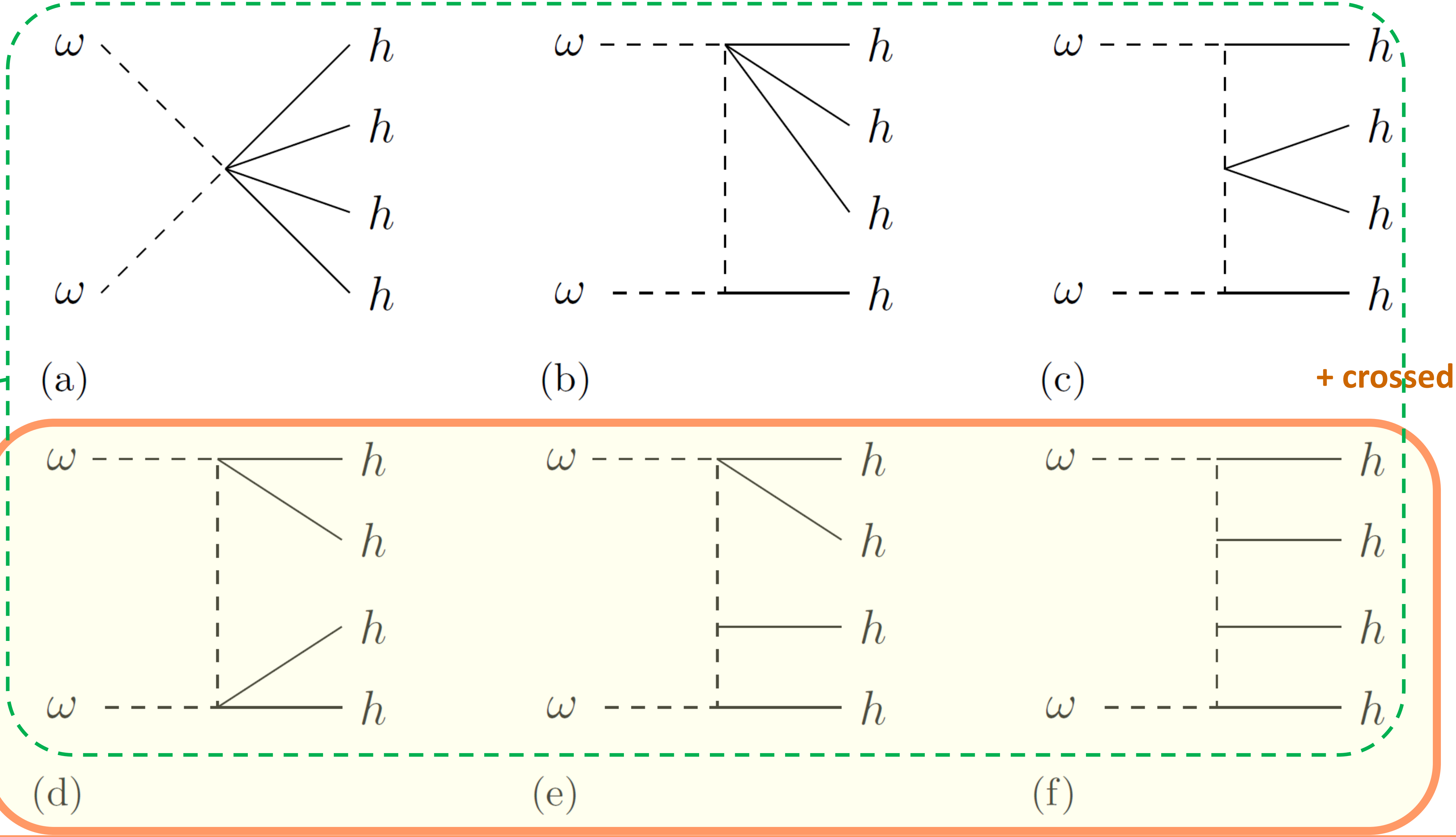
[with permutations: 75 diagrams]



+ crossed

[with permutations: 75 diagrams]

$$T_{\omega\omega\to 4h} = -\frac{4s}{v^4}\left(3\hat{a}_4 + \hat{a}_2^2\,(B-1)\right)$$



Understanding the $T_{\omega\omega\to n\times h}$ structure: field redefinitions

HEFT Lagrangian[1]

[Appelquist et al. - Phys. Rev. D 22 (1980) 200 , Longhitano et al. - Phys. Rev. D 22 (1980) 1166]

$$\mathcal{L}_{\rm HEFT} = \frac{1}{2}\partial_\mu h \partial^\mu h + \frac{1}{2}\mathcal{F}(h)\,\partial_\mu\omega^a\partial^\mu\omega^a \;+ \mathcal{O}(\omega^4)$$

Flare function[2]

[Gómez-Ambrosio, Llanes-Estrada, Salas-Bernárdez and Sanz-Cillero - 2204.01762]

$$\mathcal{F}(h) = 1 + a_1\frac{h}{v} + a_2\left(\frac{h}{v}\right)^2 + a_3\left(\frac{h}{v}\right)^3 + a_4\left(\frac{h}{v}\right)^4 + \mathcal{O}(h^5)$$

$$a \equiv \frac{a_1}{2}\,,\; a_2 \equiv b \quad \text{with} \quad a_{1,\rm SM} = 2\,,\; a_{2,\rm SM} = 1\,,\; a_{3,\rm SM} = 0\,,\; a_{4,\rm SM} = 0$$

- Consider field redefinitions of the form (other coordinates for the fields)

$$\omega^a \to \tilde{\omega}^a(\omega, h) \equiv \omega^a + G_1(h)\,\omega^a$$

$$h \to \tilde{h}(\omega, h) \equiv h + f_1(h)\frac{\omega^2}{v}$$

* Delgado,Gómez-Ambrosio,Martínez-Martín,Salas-Bernárdez,SC, JHEP 03 (2024) 037

** Alessia Nasoni, Master Thesis 2025; Martellini,Nasoni,SC, forthcoming

HEFT lagrangian

$$\mathcal{L}_{\rm HEFT} = \frac{1}{2}\partial_\mu h \partial^\mu h + \frac{1}{2}\mathcal{F}(h)\,\partial_\mu \omega^a \partial^\mu \omega^a \; + \mathcal{O}(\omega^4)$$

Field redefinition

$$\omega^a \to \omega^a + g(h)\,\omega^a\,, \qquad h \to h + \mathcal{N}\,(1+g(h))\,\frac{\omega^a\omega^a}{v}$$

Redefined HEFT Lagrangian for $\boxed{g'(h) = -2\mathcal{N}/[v\,\mathcal{F}(h)]}$

$$\mathcal{L}_{\rm HEFT} = \frac{1}{2}\partial_\mu h \partial^\mu h + \frac{1}{2}\hat{\mathcal{F}}(h)\,\partial_\mu \omega^a \partial^\mu \omega^a + \mathcal{O}(\omega^4)$$

* Delgado,Gómez-Ambrosio,Martínez-Martín,Salas-Bernárdez,SC, JHEP 03 (2024) 037

** Alessia Nasoni, Master Thesis 2025; Martellini,Nasoni,SC, forthcoming

- In order to avoid generating interaction terms proportional to $(\partial h)^2$ or $(\omega\partial\omega)\partial h$ in the LO Lagr. $\mathcal{L}_2$, one needs:

$$f_1(h) = -\tfrac{v}{2}\mathcal{F}(h)G_1'(h)\,(1+G_1(h))$$
$$\mathcal{F}'(h)G_1'(h) = -\mathcal{F}(h)G_1(h);$$

solving for $G_1'(h)$, we find

$$f_1(h) = -\tfrac{v}{2}\tilde{\mathcal{N}}\,(1+G_1(h))$$
$$G_1'(h) = \tilde{\mathcal{N}}\mathcal{F}^{-1}(h);$$

Redefined HEFT lagrangian

$$\mathcal{L}_{\mathrm{HEFT}} = \frac{1}{2}\partial_\mu h \partial^\mu h + \frac{1}{2}\hat{\mathcal{F}}(h)\,\partial_\mu \omega^a \partial^\mu \omega^a + \mathcal{O}(\omega^4)$$

Redefined flare function

$$\hat{\mathcal{F}}(h) = \mathcal{F}(h)\left(1 + g(h)\right)^2$$

* Delgado,Gómez-Ambrosio,Martínez-Martín,Salas-Bernárdez,SC, JHEP 03 (2024) 037

- For a general normalization $\mathcal{N}$ and solution of $g'(h) = -2\mathcal{N}/[v\,\mathcal{F}(h)]$:

$$g(h) = -\frac{2\mathcal{N}}{v}\int_0^h \frac{ds}{\mathcal{F}(s)} = \mathcal{N}\left(-2\frac{h}{v}+2a\frac{h^2}{v^2}+\frac{2}{3}(b-4a^2)\frac{h^3}{v^3}+\frac{1}{2}(a_3-4ab+8a^3)\frac{h^4}{v^4}+\mathcal{O}(h^5)\right)$$

$$\hat{\mathcal{F}}(h) = \mathcal{F}(h)\Big(1+g(h)\Big)^2$$

- However, for the particular normalization $\mathcal{N} = \frac{a_1}{4} = \frac{a}{2}$:

$$g(h) = -a\frac{h}{v} + a^2\frac{h^2}{v^2} + \frac{1}{3}a(b-4a^2)\frac{h^3}{v^3} + \frac{1}{4}a(a_3-4ab+8a^3)\frac{h^4}{v^4} + \mathcal{O}(h^5)$$

$$\boxed{\hat{\mathcal{F}}(h) = 1 + \hat{a}_2\left(\frac{h}{v}\right)^2 + \hat{a}_3\left(\frac{h}{v}\right)^3 + \hat{a}_4\left(\frac{h}{v}\right)^4 + \mathcal{O}(h^5)}$$

Redefined parameters ($\hat{a}_1 = 0$)

$$
\begin{aligned}
\hat{a}_2 &= b - a^2 \\
\hat{a}_3 &= a_3 - \frac{4a}{3}\left(b - a^2\right) \\
\hat{a}_4 &= a_4 - \frac{3}{2} a\, a_3 + \frac{5}{3} a^2 \left(b - a^2\right)
\end{aligned}
$$

Exactly the same effective combinations found

before in the $\omega\omega \to \ n \times h$ scattering amplitudes !!!

* Delgado,Gómez-Ambrosio,Martínez-Martín,Salas-Bernárdez,SC, JHEP 03 (2024) 037

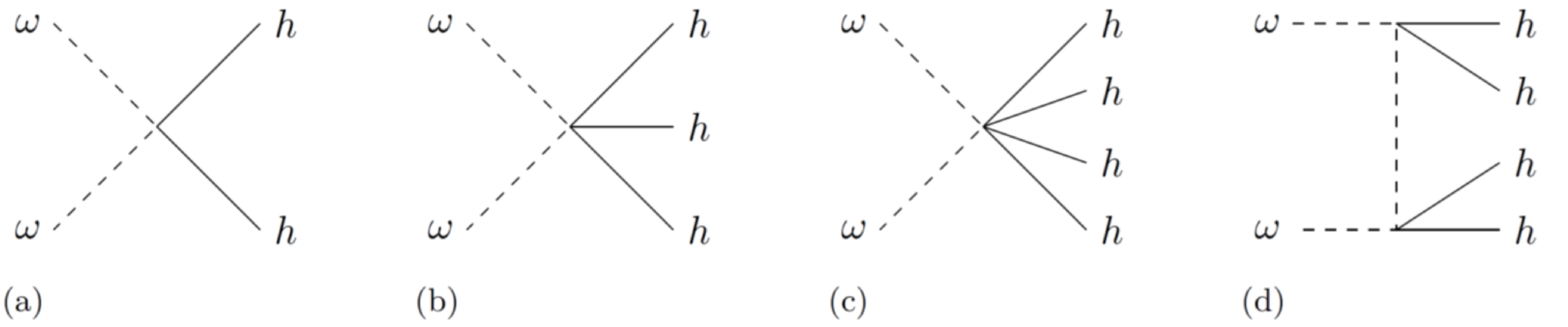


Figure 10. **a)** Only diagram contributing to the process $\omega\omega \to 2h$. **b)** Only diagram contributing to the process $\omega\omega \to 3h$. **c-d)** Only two diagrams contributing to the process $\omega\omega \to 4h$. We have used the simplified Lagrangian (C.6) to generate these amplitudes, so every $\omega\omega h^n$ vertex carries an $\hat{a}_n$ effective coupling. Note that, in addition, one needs to consider all possible permutations for the assignment of the external particles.

* Delgado,Gómez-Ambrosio,Martínez-Martín,Salas-Bernárdez,SC, JHEP 03 (2024) 037

Scanning of the $\omega\omega \to 3h$ cross section predictions for $\sqrt{s} = 1$ TeV:

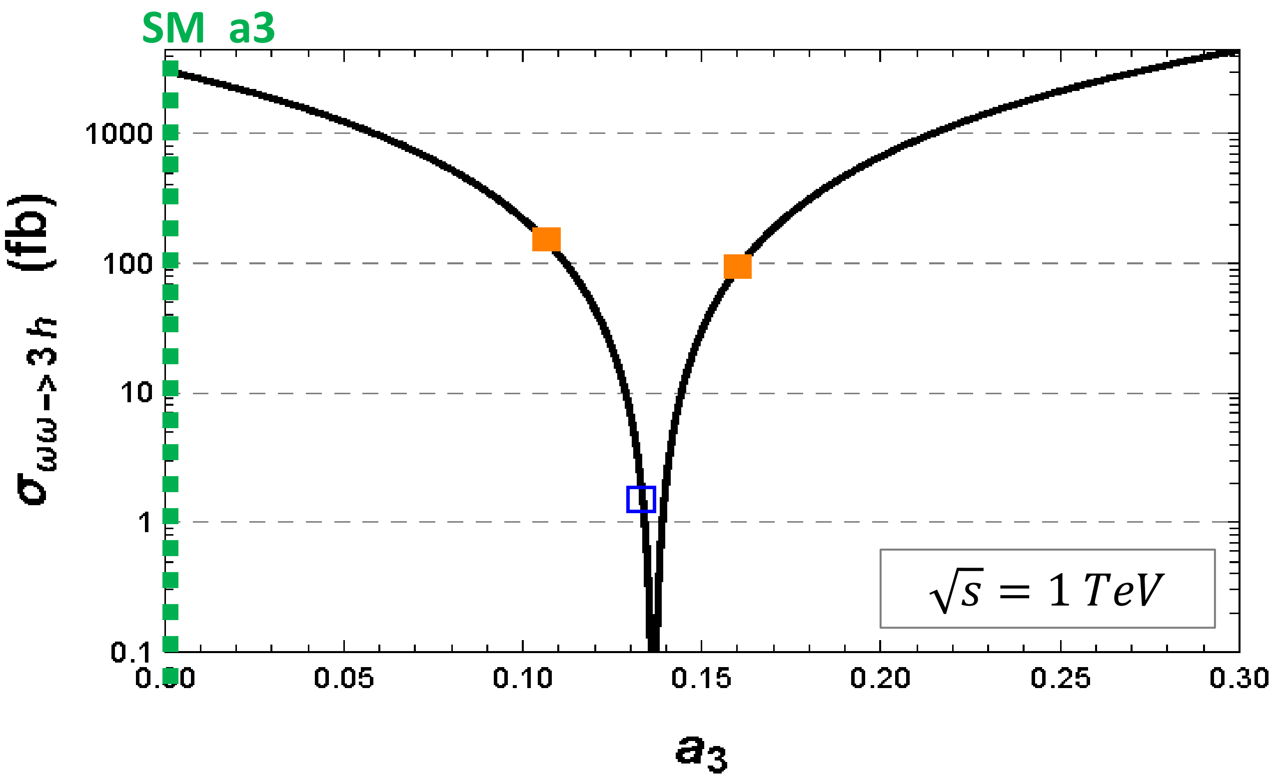
For SMEFT starting @ D=6:
$$\Delta b = 4\Delta a \quad *$$

- Empty Blue square - SMEFT$^{(D=6)}$-BP $\quad (c_{(6)} = 1, \quad v^2/\Lambda^2 \quad = 0.05)$:

$$a = a_1/2 = a^{SMEFT(D=6)} = 1.05, \qquad b = a_2 = a_2^{SMEFT\,(D=6)} = 1.2, \qquad a_3 = a_3^{SMEFT\,(D=6)} = 0.1\hat{3}$$

- Full Orange square - HEFT:

$$a = a_1/2 = a^{SMEFT(D=6)} = 1.05, \qquad b = a_2 = a_2^{SMEFT\,(D=6)} = 1.2, \qquad a_3 = a_3^{SMEFT\,(D=6)} \times (1 \ \pm 20\% \ )$$



$$\sigma^{SMEFT}_{\omega\omega\to3h} \sim \frac{1}{\Lambda^8}$$

- Analogous to previous $a_2 = b = \kappa_{2V}$ scannings for LHC and FCC analyses:

(x) Englert,Naskar,Sutherland, JHEP 11 (2023) 158

* G. Buchalla,Cata,Celis,Krause, NPB 917 (2017) 209-233 (e.g., for Real Singlet Ext),
* Delgado,Gómez-Ambrosio,Martínez-Martín,Salas-Bernárdez,SC, JHEP 03 (2024) 037

Scanning of the $\omega\omega \to 3h$ cross section predictions for $\sqrt{s} = 1$ TeV:

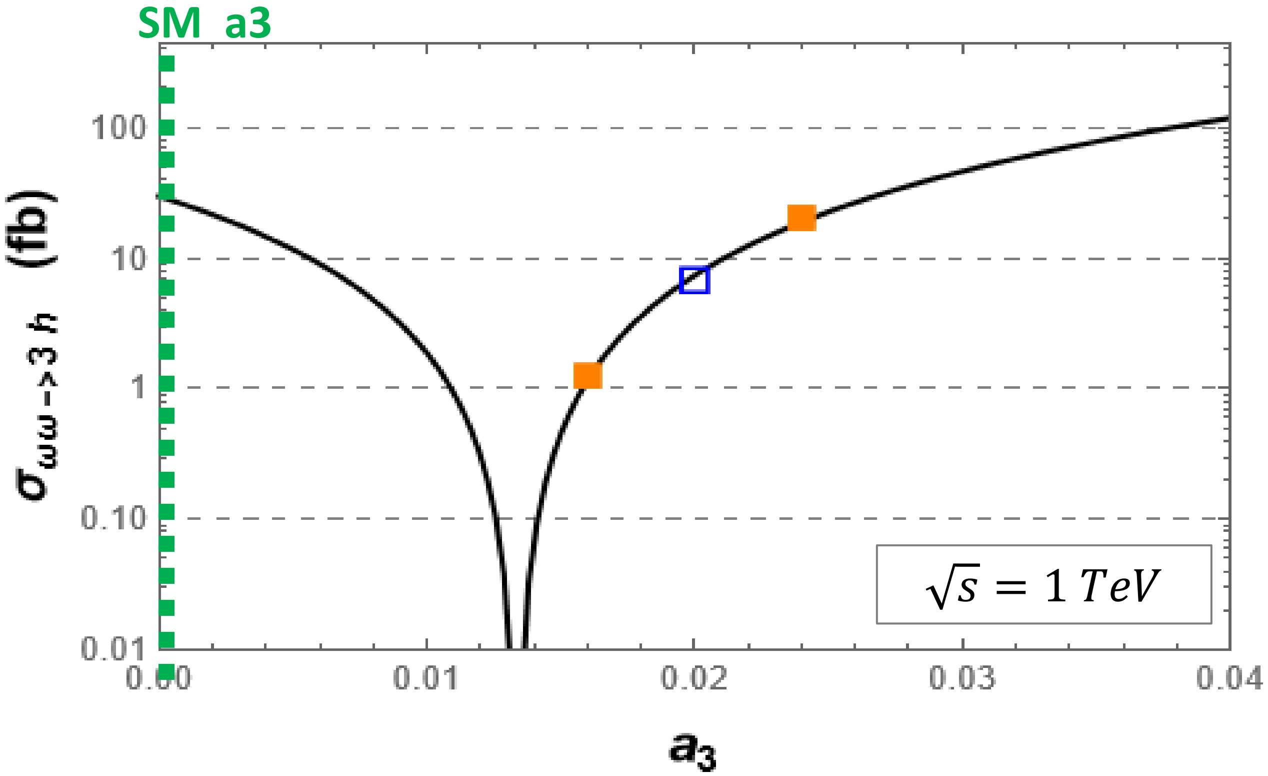
For SMEFT starting @ D=8:
$$\Delta b = 6\Delta a \quad *$$

- Empty Blue square - SMEFT$^{(D=8)}$-BP $\left(c_{(6)} = 0,\ c_{(8)} = 1,\ v^2/\Lambda^2 = 0.05\right)$:

$$a = a_1/2 = a^{SMEFT(D=8)} = 1.05, \qquad b = a_2 = a_2^{SMEFT\,(D=8)} = 1.3, \qquad a_3 = a_3^{SMEFT\,(D=8)} = 0.4$$

- Full Orange square - HEFT:

$$a = a_1/2 = a^{SMEFT(D=8)} = 1.05, \qquad b = a_2 = a_2^{SMEFT\,(D=8)} = 1.3, \qquad a_3 = a_3^{SMEFT\,(D=8)} \times (1 \pm 20\%)$$



$$\sigma^{SMEFT}_{\omega\omega\to 3h} \sim \frac{1}{\Lambda^8}$$

- Analogous to previous $a_2 = b = \kappa_{2V}$ scannings for LHC and FCC analyses:

(x) Englert,Naskar,Sutherland, JHEP 11 (2023) 158

* Dawson,Fontes,Quezada-Calonge,Sanz-Cillero, PRD 108 (2023) 5, 055034 (e.g., for 2HDM),
* Delgado,Gómez-Ambrosio,Martínez-Martín,Salas-Bernárdez,SC, JHEP 03 (2024) 037

- Various public code repositories created:

  - Specific Mathematica stand-alone code for $\omega\omega \rightarrow n \times h$

    https://github.com/alexandresalasb/WWtonHcalculator

  - General FeynRules model file https://github.com/Javomar99/EWET implementing $O(p^2)$ and $O(p^4)$ HEFT Lagrangian

  - New fast+simple Massless Particle Phase-Space Integrator $\mathbb{MaMuPaXS}$ https://github.com/mamupaxs/mamupaxs

* Delgado,Gómez-Ambrosio,Martínez-Martín,Salas-Bernárdez,SC, JHEP 03 (2024) 037; PoS ICHEP2024 (2025) 095 - 2410.19951 [hep-ph]

- We will actually compute the Goldstone-Goldstone scattering,

$$T_{\omega\omega\to n\times h}$$

and extract the corresponding cross section:

$$\sigma_{\omega\omega\to n\times h} = \frac{1}{n!}\frac{1}{2s}\int |T_{\omega\omega\to n\times h}|^2\, d\Pi_n$$

* Delgado,Gómez-Ambrosio,Martínez-Martín,Salas-Bernárdez,SC, JHEP 03 (2024) 037; PoS ICHEP2024 (2025) 095 - 2410.19951 [hep-ph]

$$\omega^+(k_1)\,\omega^-(k_2) \to h(p_1)\,h(p_2)\,h(p_3)\,h(p_4)$$

$$B = f_1 f_2 f_3 f_4 \left(\mathcal{B}_{1234} + \mathcal{B}_{1324} + \mathcal{B}_{1423} + \mathcal{B}_{2314} + \mathcal{B}_{2413} + \mathcal{B}_{3412} \right)$$

$$\mathcal{B}_{ijk\ell} = \frac{z_{ij} z_{k\ell}}{2 f_i f_j z_{ij} - f_i z_i - f_j z_j}$$

where $f_i = qp_i/q^2,\ z_i = 2k_1 p_i/qp_i,\ z_{ij} = z_{ji} = q^2\,(p_i p_j)/[(qp_i)\,(qp_j)]$

$q = k_1 + k_2 = p_1 + p_2 + p_3 + p_4$

* Delgado,Gómez-Ambrosio,Martínez-Martín,Salas-Bernárdez,SC, JHEP 03 (2024) 037; PoS ICHEP2024 (2025) 095 - 2410.19951 [hep-ph]

$$f_i = \|\vec{p}_i\|/\sqrt{s}\ (s = 4\|\vec{k}_1\|^2)$$

(CM)

$$z_i = 2\sin^2(\theta_i/2)$$

$$z_{ij} = 2\sin^2(\theta_{ij}/2)$$

$${}^4B = f_1 f_2 f_3 f_4 \left(\mathcal{B}_{1234} + \mathcal{B}_{1324} + \mathcal{B}_{1423} + \mathcal{B}_{2314} + \mathcal{B}_{2413} + \mathcal{B}_{3412}\right),$$ with each term $\mathcal{B}_{ijk\ell} = \frac{z_{ij} z_{k\ell}}{2 f_i f_j z_{ij} - f_i z_i - f_j z_j}$

only depending on external momenta scalar products of the form $f_i = q p_i/q^2$, $z_i = 2k_1 p_i/q p_i$, $z_{ij} = z_{ji} = q^2 (p_i p_j)/[(q p_i)(q p_j)]$. The total four-momentum is denoted as $q = k_1 + k_2 = p_1 + p_2 + p_3 + p_4$ and the on-shell condition $k^2_{1,2} = p^2_{1,2,3,4} = 0$. Note that here we have rewritten the expression $f_1 f_2 f_3 f_4 \mathcal{B}_{ijk\ell} = (p_i + p_j)^2 (p_k + p_\ell)^2/[4q^2(k_1 - p_i - p_j)^2]$ for on-shell massless external particles in a more convenient form for the phase-space integration. In the center-of-mass (CM) rest-frame these relations define: the three-momentum fractions $f_i = \|\vec{p}_i\|/\sqrt{s}$ $(s = 4\|\vec{k}_1\|^2)$ for each outgoing Higgs boson; the angular functions $z_i = 2\sin^2(\theta_i/2)$ with θ_i being the angle between the i-th Higgs boson and the incoming ω^+ Goldstone boson momenta, $\vec{k}_1$ (that is, $z_1 = 1 - \cos\theta$, $z_2 = 1 + \cos\theta$ as usual in a two-body problem with t and u channels); $z_{ij} = 2\sin^2(\theta_{ij}/2)$, with θ_{ij} being the angle between the i-th and j-th Higgs bosons. We remark that by construction we have factored out the scaling with s so that the kinematic function B only depend on the angular distribution of the Higgses.

* Delgado,Gómez-Ambrosio,Martínez-Martín,Salas-Bernárdez,SC, JHEP 03 (2024) 037; PoS ICHEP2024 (2025) 095 - 2410.19951 [hep-ph]

$$\sigma_{\omega\omega\to 4h} = \frac{8\pi^3}{9s}\left(\frac{s}{16\pi^2 v^2}\right)^4 \left[\left(3\hat{a}_4 - \hat{a}_2^2\right)^2 + 2\left(3\hat{a}_4 - \hat{a}_2^2\right)\hat{a}_2^2\chi_1 + \hat{a}_2^4\chi_2\right]$$

$$\chi_n = \mathcal{V}_4^{-1}\int d\Pi_4\, B^n\,,$$

$$\mathcal{V}_4 = \int d\Pi_4 = s^2\left(24(4\pi)^5\right)^{-1}$$

$$\chi_1 = -0.124984\,(10)$$

$$\chi_2 = 0.0193760\,(16)$$

our phase-space integration code (MaMuPaXS)

* Delgado,Gómez-Ambrosio,Martínez-Martín,Salas-Bernárdez,SC, JHEP 03 (2024) 037; PoS ICHEP2024 (2025) 095 - 2410.19951 [hep-ph]

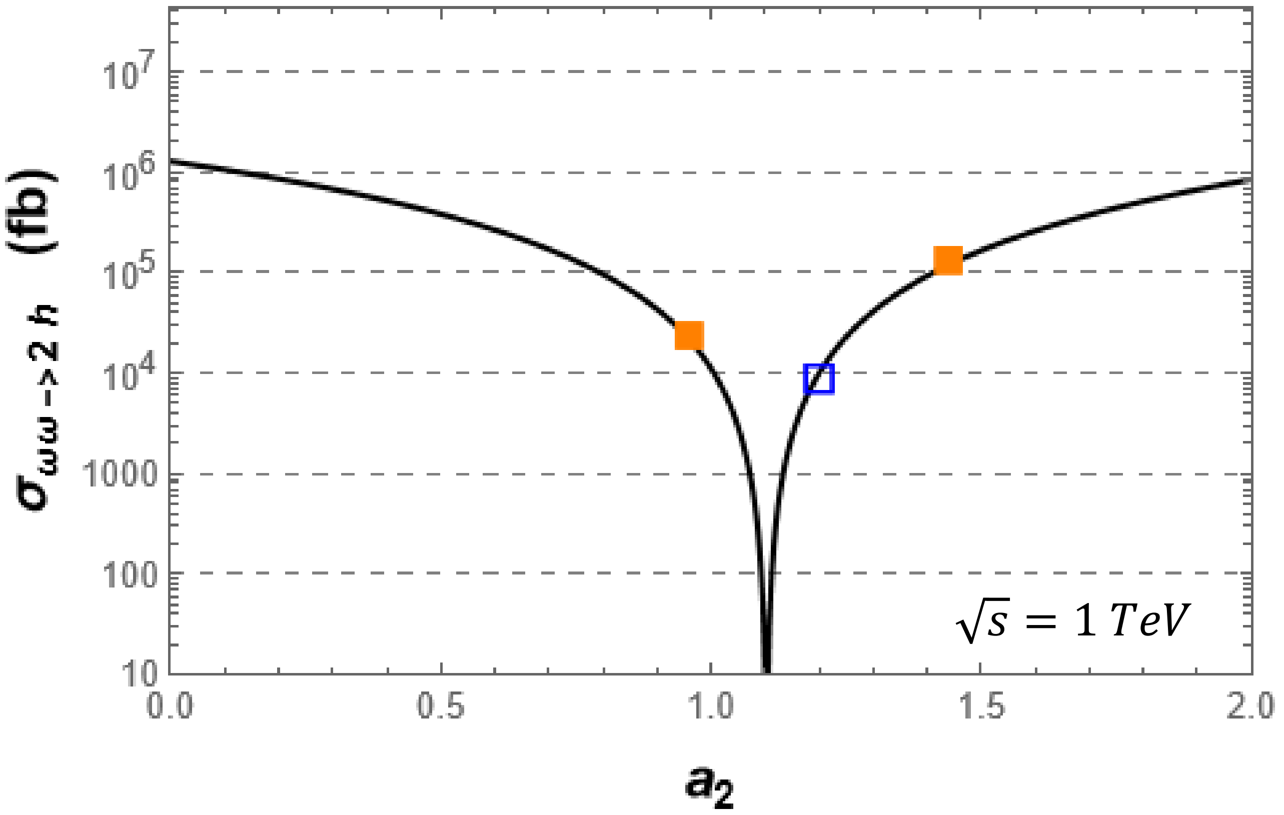
Scanning of the $\omega\omega \to 2h$ cross section predictions for $\sqrt{s} = 1$ TeV:

- Empty blue square - SMEFT$^{(D=6)}$ -BP $(d = 0.1)$:

$$a = a_1/2 = a^{SMEFT(D=6)} = 1.05, \qquad \text{b} = a_2 = a_2^{SMEFT\,(D=6)} = 1.2$$

- Full Orange square - HEFT:

$$a = a_1/2 = a^{SMEFT(D=6)} = 1.05, \qquad \text{b} = a_2 = a_2^{SMEFT\,(D=6)} = 1.2 \times (1 \pm 20\%\ )$$



- Analogous to previous $a_2 = b = \kappa_{2V}$ scannings for LHC and FCC analyses:

(x) Englert,Naskar,Sutherland, JHEP 11 (2023) 158

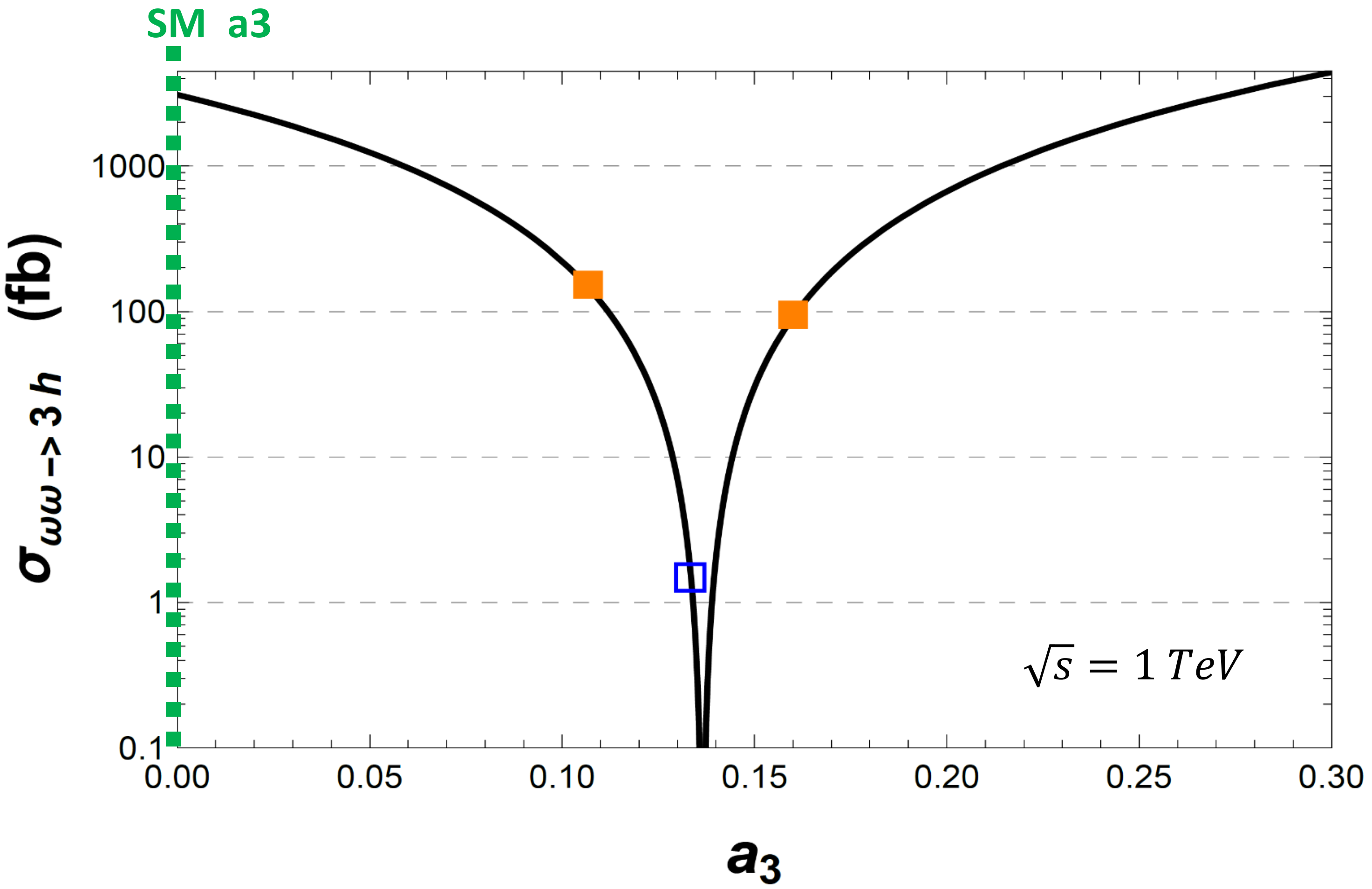


Figure 5. Scan of the $\omega\omega \to 3h$ cross section predictions in terms of a_3 at $\sqrt{s} = 1$ TeV. The inputs $a_1 = a_1^{\text{SMEFT(D=6)}} = 2.1$ and $a_2 = a_2^{\text{SMEFT(D=6)}} = 1.2$ are taken from (4.2), the SMEFT$^{(\text{D=6})}$ BP. We have marked a few especial points: $a_3 = a_3^{\text{SMEFT(D=6)}} = 0.1\widehat{3}$ (empty blue square) and their 20% deviations (full orange squares), $a_3 = 80\% \times a_3^{\text{SMEFT(D=6)}}$ and $a_3 = 120\% \times a_3^{\text{SMEFT(D=6)}}$. We note that, in between, $\sigma_{\omega\omega\to3h}$ vanishes at $a_3 = \frac{2}{3}a_1\left(a_2 - \frac{1}{4}a_1^2\right) = 0.1365$.

* Delgado,Gómez-Ambrosio,Martínez-Martín,Salas-Bernárdez,SC, JHEP 03 (2024) 037

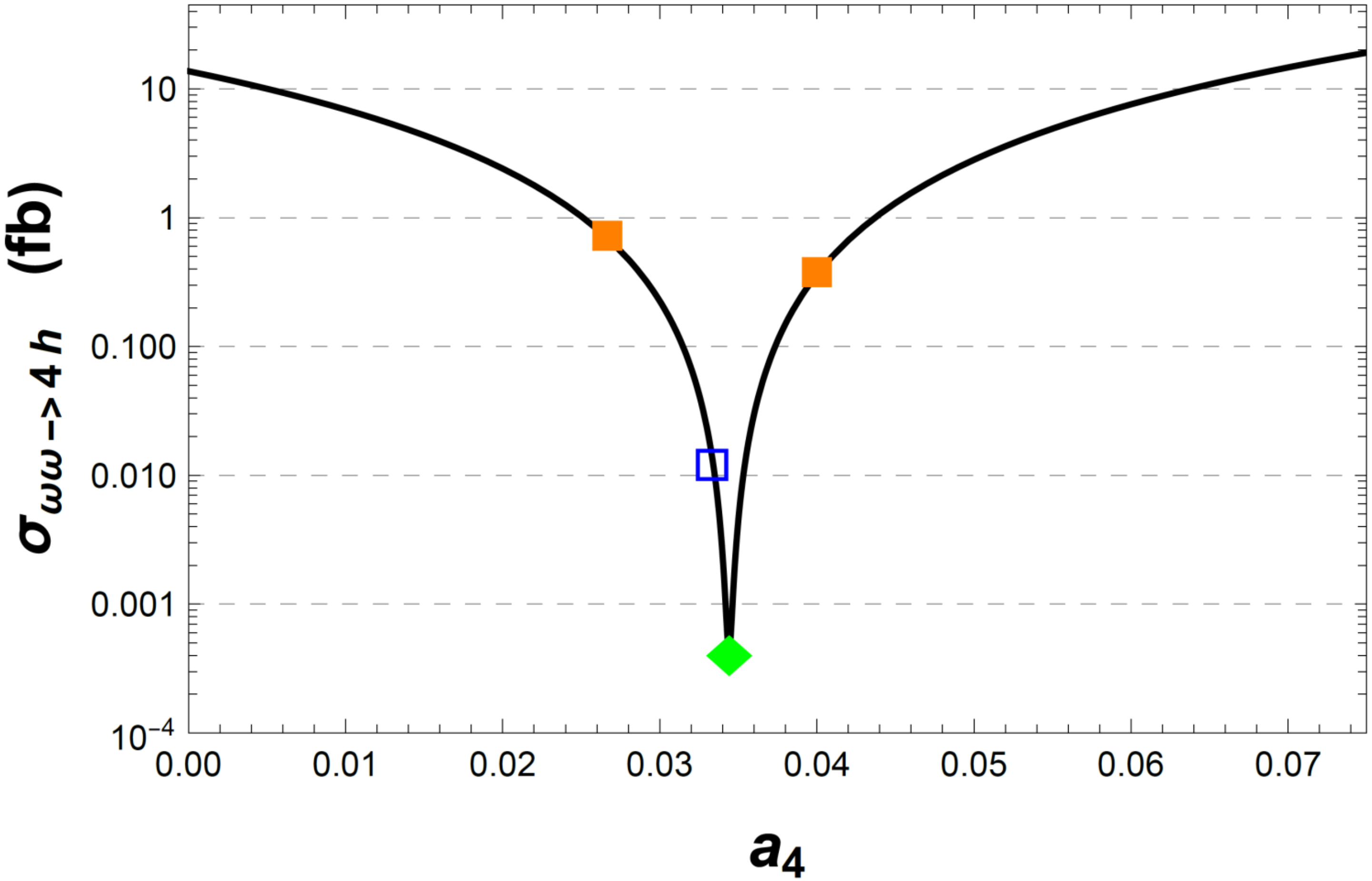


Figure 7. Scanning of the $\omega\omega \to 4h$ cross section predictions in terms of a_4 at $\sqrt{s} = 1$ TeV. The inputs $a_1 = a_1^{\rm SMEFT(D=6)} = 2.1$, $a_2 = a_2^{\rm SMEFT(D=6)} = 1.2$ and $a_3 = a_3^{\rm SMEFT(D=6)} = 0.1\widehat{3}$ are taken from (4.2), the SMEFT$^{\rm (D=6)}$ BP. We have marked a few especial points: $a_4 = a_4^{\rm SMEFT(D=6)} = 0.0\widehat{3}$ (empty blue square) and their 20% deviations (full orange squares), $a_4 = 80\% \times a_4^{\rm SMEFT(D=6)}$ and $a_4 = 120\% \times a_4^{\rm SMEFT(D=6)}$. The cross section's minimum is not zero this time and it is found at $a_4 = \frac{3}{4}a_1 a_3 - \frac{5}{12}a_1^2\hat{a}_2 + \frac{1}{3}\hat{a}_2^2(1-\chi_1) \approx 0.0344$ (filled green diamond).

Correlations accurate at order Λ^{-2}	Correlations accurate at order Λ^{-4}	Λ^{-4} Assuming SMEFT perturbativity
$\Delta a_2 = 2\Delta a_1$		$\lvert\Delta a_2\rvert \leq 5\lvert\Delta a_1\rvert$
$a_3 = \frac{4}{3}\Delta a_1$	$\left(a_3 - \frac{4}{3}\Delta a_1\right) = \frac{8}{3}(\Delta a_2 - 2\Delta a_1) - \frac{1}{3}\left(\Delta a_1\right)^2$	
$a_4 = \frac{1}{3}\Delta a_1$	$\left(a_4 - \frac{1}{3}\Delta a_1\right) = \frac{5}{3}\Delta a_1 - 2\Delta a_2 + \frac{7}{4}a_3 = $ $= \frac{8}{3}(\Delta a_2 - 2\Delta a_1) - \frac{7}{12}\left(\Delta a_1\right)^2$	those for a_3, a_4, a_5, a_6
$a_5 = 0$	$a_5 = \frac{8}{5}\Delta a_1 - \frac{22}{15}\Delta a_2 + a_3 = $ $= \frac{6}{5}(\Delta a_2 - 2\Delta a_1) - \frac{1}{3}\left(\Delta a_1\right)^2$	all the same
$a_6 = 0$ SMEFT	$a_6 = \frac{1}{6}a_5$ SMEFT	SMEFT

$$\Delta a_1 := a_1 - 2 = 2a - 2$$

$$\Delta a_2 := a_2 - 1 = b - 1$$

$$a_1 = \left(2 + 2\frac{c^{(6)}_{H\Box}v^2}{\Lambda^2} + 3\frac{(c^{(6)}_{H\Box})^2 v^4}{\Lambda^4} + 2\frac{c^{(8)}_{H\Box}v^4}{\Lambda^4}\right) \qquad a_2 = \left(1 + 4\frac{c^{(6)}_{H\Box}v^2}{\Lambda^2} + 12\frac{(c^{(6)}_{H\Box})^2 v^4}{\Lambda^4} + 6\frac{c^{(8)}_{H\Box}v^4}{\Lambda^4}\right)$$

(*) Gómez-Ambrosio,Llanes-Estrada,Salas-Bernárdez,SC, PRD 106 (2022) 5, 5; arXiv: 2207.09848 [hep-ph]

Consistent SMEFT range at order Λ^{-2}	Consistent SMEFT range at order Λ^{-4}	Perturbativity of Λ^{-4} SMEFT
$\Delta a_2 \in [-0.12, 0.36]$ $a_3 \in [-0.08, 0.24]$ $a_4 \in [-0.02, 0.06]$ $a_5 = 0$ $a_6 = 0$	ATLAS $a_3 \in [-4.1, 4.0]$ $a_4 \in [-4.2, 3.9]$ $a_5 \in [-1.9, 1.8]$ $a_6 = a_5$	ATLAS $a_3 \in [-3.1, 1.7]$ $a_4 \in [-3.3, 1.5]$ $a_5 \in [-1.5, 0.6]$ $a_6 = a_5$
	CMS $a_3 \in [-3.2, 3.0]$ $a_4 \in [-3.3, 3.0]$ $a_5 \in [-1.5, 1.3]$ $a_6 = a_5$	CMS $a_3 \in [-3.1, 1.7]$ $a_4 \in [-3.3, 1.5]$ $a_5 \in [-1.5, 0.6]$ $a_6 = a_5$

$$|\Delta a_2| \leq 5|\Delta a_1|$$

$a_1/2 = a \in [0.97, 1.09]$ [67]

- ATLAS

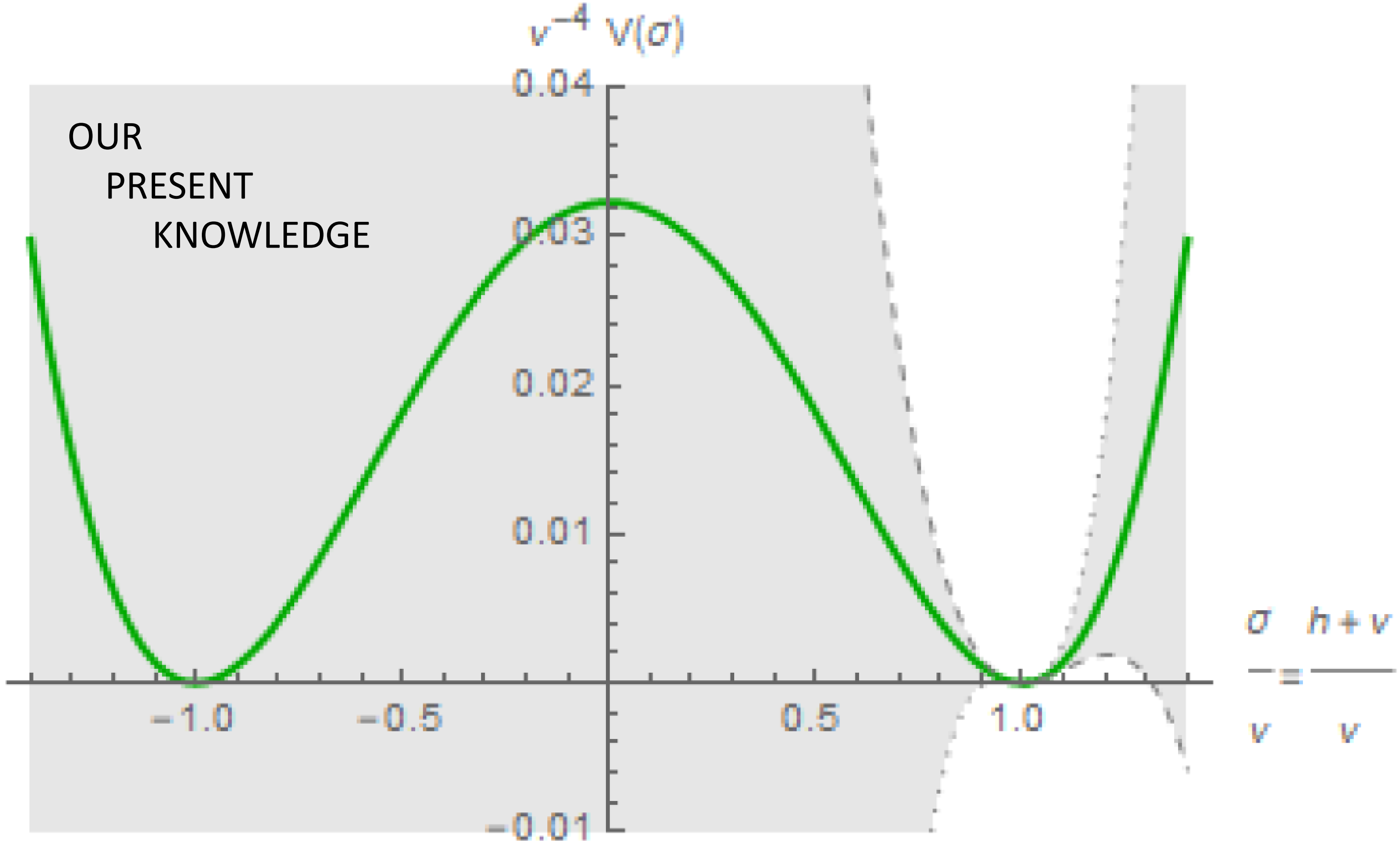
$a_2 = b = \kappa_{2V} \in [-0.43, 2.56]$ [69]

- CMS

$a_2 = b = \kappa_{2V} \in [-0.1, 2.2]$ [68]

(*) Gómez-Ambrosio,Llanes-Estrada,Salas-Bernárdez,SC, 2204.01763 [hep-ph]

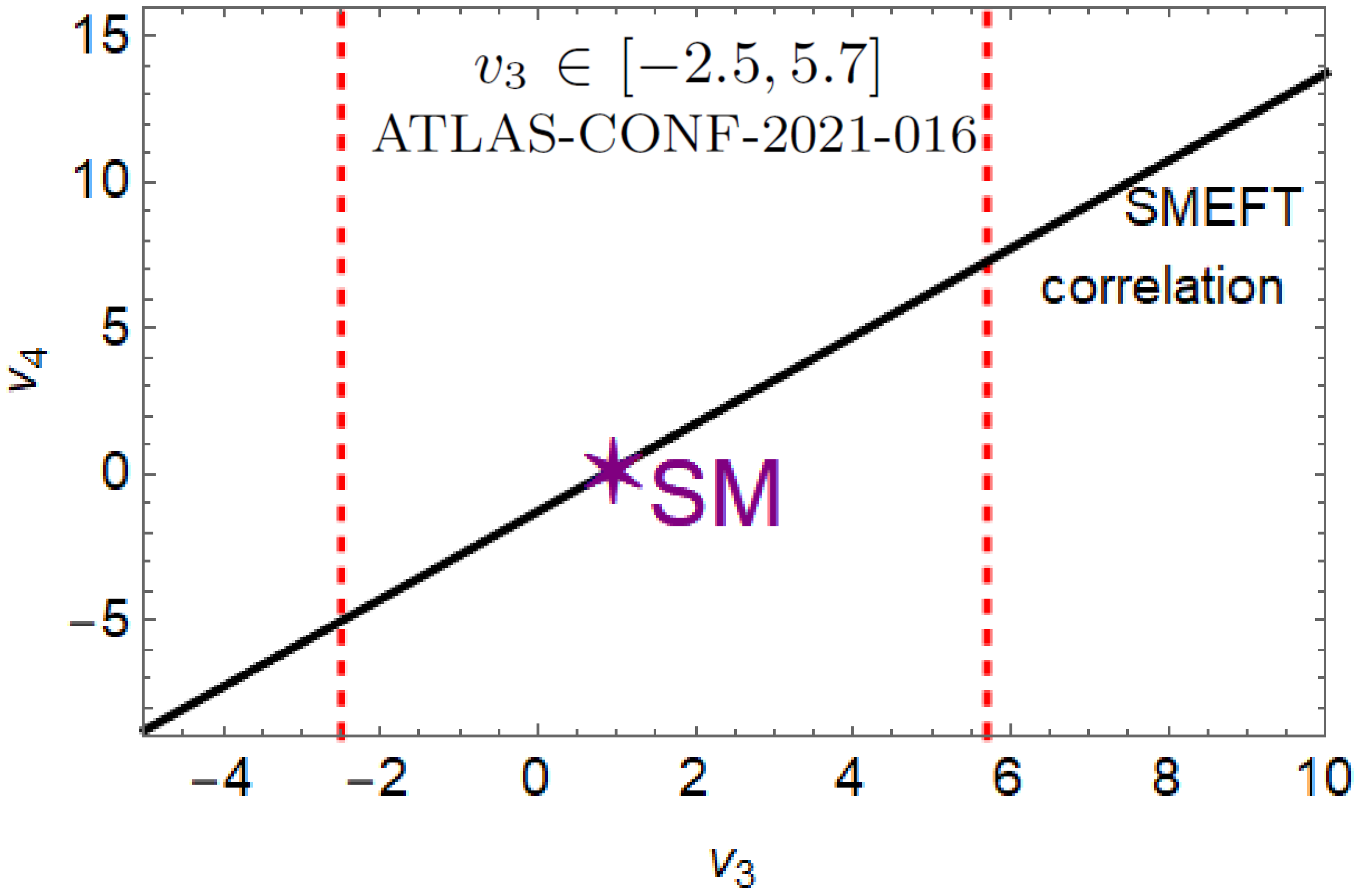
Other correlations: Higgs potential



Other correlations: Higgs potential

$$V_{\rm HEFT} = \frac{m_h^2 v^2}{2}\left[\left(\frac{h_{\rm HEFT}}{v}\right)^2 + v_3\left(\frac{h_{\rm HEFT}}{v}\right)^3 + \right.$$
$$\left. + v_4\left(\frac{h_{\rm HEFT}}{v}\right)^4 + \dots\right],$$

with $v_3 = 1$, $v_4 = 1/4$ and $v_{n\geq 5} = 0$ in the SM



SMEFT	
$\Delta v_4 = \frac{3}{2}\Delta v_3 - \frac{1}{6}\Delta a_1$	$\Delta v_4 \in [-3.8, 8.6]$
$v_5 = 6 v_6 = \frac{3}{4}\Delta v_3 - \frac{1}{8}\Delta a_1$	$v_5 = 6v_6 \in [-1.9, 4.3]$

$a_1/2 \in [0.97, 1.09]$

(*) Gómez-Ambrosio,Llanes-Estrada,Salas-Bernárdez,SC, PRD 106 (2022) 5, 5; arXiv: 2207.09848 [hep-ph]

Correlations in the potential

HEFT correlations from the Custodial preserving SMEFT operators

$$\mathcal{O}_H := (H^\dagger H)^3 \,, \quad \mathcal{O}_{H\Box} := (H^\dagger H)\Box(H^\dagger H) \,.$$

$$v_3 = 1 + \frac{3v^2 c_{H\Box}}{\Lambda^2} + \frac{\mu^2 c_H}{\lambda^2\Lambda^2} \,, \quad v_4 = \frac{1}{4} + \frac{25 v^2 c_{H\Box}}{6\Lambda^2} + \frac{3}{2}\frac{\mu^2 c_H}{\lambda^2\Lambda^2} \,,$$

$$v_5 = \frac{2v^2 c_{H\Box}}{\Lambda^2} + \frac{3}{4}\frac{\mu^2 c_H}{\lambda^2\Lambda^2} \,, \quad v_6 = \frac{v^2 c_{H\Box}}{3\Lambda^2} + \frac{1}{8}\frac{\mu^2 c_H}{\lambda^2\Lambda^2} \,,$$

$$v_{n\geq 7} = 0 \,,$$

with $m_h^2 = -2\mu^2\left(1 + \frac{2c_{H\Box}v^2}{\Lambda^2} + \frac{3}{4}\frac{\mu^2 c_H}{\lambda^2\Lambda^2}\right)$,

$$2\langle |H|^2\rangle = v^2 = -\frac{\mu^2}{\lambda}\left(1 - \frac{3}{4}\frac{\mu^2 c_H}{\lambda^2\Lambda^2}\right).$$

Low-energy EFT (SM + …): representations

- Higgs field representation: SMEFT vs HEFT, a matter of taste? (+)

1) Linear* (SMEFT): in terms of a doublet $\phi = (1+h/v)\, U(\omega^a) \langle\phi\rangle$

$$\mathcal{L}^{\rm L}_{\rm EFT} = (D_\mu\phi)^\dagger D_\mu\phi - \frac{1}{\Lambda^2}(\phi^\dagger\phi)\Box(\phi^\dagger\phi) + \ldots$$
$$= \frac{(v+h)^2}{4}\langle (D_\mu U)^\dagger D_\mu U\rangle + \frac{1}{2}(1+P(h))\,(\partial_\mu h)^2 + \ldots$$

$$\frac{dh^{\rm NL}}{dh^{\rm L}} = \sqrt{1+P(h^{\rm L})}$$

$$h^{\rm NL} = \int_0^{h^{\rm L}} \sqrt{1+P(h)}\, dh$$

? ? ?

$$\frac{v^2}{2}\mathcal{F}_C(h^{\rm NL}) = \frac{(v+h^{\rm L})^2}{2} = \phi^\dagger\phi$$

if there exists an $SU(2)_L \times SU(2)_R$ fixed point $\mathcal{F}_C(h^*)=0$ (x)

$$\mathcal{L}^{\rm NL}_{\rm EFT} = \frac{v^2}{4}\mathcal{F}_c(h)\langle (D_\mu U)^\dagger D_\mu U\rangle + \frac{1}{2}(\partial_\mu h)^2 + \ldots$$

$$\mathcal{F}_C(h) = 1 + \frac{2ah}{v} + \frac{bh^2}{v^2} + \mathcal{O}(h^3)$$

2) Non-linear* (HEFT or EWχL): in terms of 1 singlet h + 3 NGB in $U(\omega^a)$

(+) SC, arXiv:1710.07611 [hep-ph]; PoS EPS-HEP2017 (2017) 460
* Jenkins,Manohar,Trott, JHEP 1310 (2013) 087
* LHCHXSWG Yellow Report [1610.07922]

(x) Transformations:
Giudice,Grojean,Pomarol,Rattazzi, JHEP 0706 (2007) 045
Alonso,Jenkins,Manohar, JHEP 1608 (2016) 101

SMEFT lagrangian

[Gómez-Ambrosio, Llanes-Estrada, Salas-Bernárdez and Sanz-Cillero - 2207.09848]

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \sum_{n=5}^{\infty} \sum_{i} \frac{c_i^{(n)}}{\Lambda^{n-4}} \mathcal{O}_i^{(n)}$$

$\mathcal{O}_{H\Box}$ operator

$$\mathcal{O}_{H\Box}^{(6)} = (H^\dagger H)\Box(H^\dagger H)\,, \quad \mathcal{O}_{H\Box}^{(8)} = (H^\dagger H)^2\Box(H^\dagger H)\,, \quad \partial^2 \equiv \Box$$

SMEFT parameters

$$d = \frac{2v^2 c_{H\Box}^{(6)}}{\Lambda^2} \quad , \quad \rho = \frac{c_{H\Box}^{(8)}}{2(c_{H\Box}^{(6)})^2}$$

SMEFT $\Rightarrow$ HEFT connection

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ (v + h_{\rm SMEFT}) + i\phi_3 \end{pmatrix}$$

Relevant SMEFT at the TeV scale:

$$\mathcal{L}_{\rm SMEFT} = |\partial H|^2 + \frac{c_{H\Box}}{\Lambda^2}(H^\dagger H)\Box(H^\dagger H)$$

To polar-like coordinates:

$$\mathcal{L}_{\rm SMEFT} = \frac{1}{2}\left(1 - 2(v+h)^2\frac{c_{H\Box}}{\Lambda^2}\right)(\partial_\mu h)^2 + \frac{1}{2}(v+h)^2(\partial_\mu \boldsymbol{n}\cdot\partial^\mu \boldsymbol{n})$$

- SMEFT in the *HEFT-form*[(x)] looks like...

$$\mathcal{L}_{\rm SMEFT} = \frac{v^2}{4}\mathcal{F}(h_1)\langle D_\mu U^\dagger D^\mu U\rangle + \frac{1}{2}(\partial_\mu h_1)^2 - V(h) - \frac{c_{H\Box}\left[(v+h_1)^3 \;-\; v^3\right]}{3\Lambda^2}V'(h_1)$$

HEFT [(x)]

$$\mathcal{F}(h_1) = \left(1+\frac{h_1}{v}\right)^2 + \frac{2v^3 c_{H\Box}}{\Lambda^2}\left(1+\frac{h_1}{v}\right)\left(\frac{h_1^3}{3v^3}+\frac{h_1^2}{v^2}+\frac{h_1}{v}\right) + \mathcal{O}\left(\frac{c_{H\Box}^2}{\Lambda^4}\right) =$$
$$= 1 + \left(\frac{h_1}{v}\right)\left(2+2\frac{c_{H\Box}v^2}{\Lambda^2}\right) + \left(\frac{h_1}{v}\right)^2\left(1+4\frac{c_{H\Box}v^2}{\Lambda^2}\right) +$$
$$+ \left(\frac{h_1}{v}\right)^3\left(8\frac{c_{H\Box}v^2}{3\Lambda^2}\right) + \left(\frac{h_1}{v}\right)^4\left(2\frac{c_{H\Box}v^2}{3\Lambda^2}\right),$$

SMEFT correlated coeff.

$$a_1 = 2a = 2\left(1+v^2\frac{c_{H\Box}}{\Lambda^2}\right), \quad a_2 = b = 1+4v^2\frac{c_{H\Box}}{\Lambda^2}, \quad a_3 = \frac{8v^2}{3}\frac{c_{H\Box}}{\Lambda^2}, \quad a_4 = \frac{2v^2}{3}\frac{c_{H\Box}}{\Lambda^2}$$

SMEFT $\Leftarrow$ HEFT connection

From HEFT to SMEFT one has to solve

$$h_{\rm HEFT} = \mathcal{F}^{-1}\left((1 + h_{\rm SMEFT}/v)^2\right)$$

and in order to have an analytic Lagrangian:

$$\mathcal{L}_{\rm SMEFT} = \underbrace{|\partial H|^2}_{=\mathcal{L}_{\rm SM}} + \underbrace{\frac{1}{2}\left[\frac{8|H|^2}{v^2}\left((\mathcal{F}^{-1})'\,(2|H|^2/v^2)\right)^2 - 1\right]\frac{(\partial|H|^2)^2}{2|H|^2}}_{=\Delta\mathcal{L}_{\rm BSM}}$$

Possible non-analyticity

$\Rightarrow$ Provides conditions on the derivatives of the flare function $\mathcal{F}(h)$.

$\Rightarrow$ Correlation of HEFT parameters by assuming an analytic SMEFT.

SMEFT vs HEFT: potential issues

- **Theory:**

HEFT Lagrangian becomes singular in *SMEFT-form*

(coordinates)

- **Phenomenology:**

SMEFT predicts correlations absent in experiment?

(in principle, also absent in HEFT)

Oblique Electroweak Observables: S and T at NLO

✓ Universal oblique corrections via the EW boson self-energies (transverse in the Landau gauge)

$$\mathcal{L}_{\rm v.p.} \doteq -\frac{1}{2} W^3_\mu \Pi^{\mu\nu}_{33}(q^2) W^3_\nu - \frac{1}{2} B_\mu \Pi^{\mu\nu}_{00}(q^2) B_\nu - W^3_\mu \Pi^{\mu\nu}_{30}(q^2) B_\nu - W^+_\mu \Pi^{\mu\nu}_{WW}(q^2) W^-_\nu$$

✓ S parameter*: new physics in the difference between the Z self-energies at $Q^2=M_Z^2$ and $Q^2=0$.

$$e_3 = \frac{g}{g'} \widetilde{\Pi}_{30}(0)\,, \qquad \Pi_{30}(q^2) = q^2 \widetilde{\Pi}_{30}(q^2) + \frac{g^2 \tan\theta_W}{4} v^2\,, \qquad S = \frac{16\pi}{g^2}\left(e_3 - e_3^{\rm SM}\right).$$

✓ T parameter*: custodial symmetry breaking

$$e_1 = \frac{\Pi_{33}(0) - \Pi_{WW}(0)}{M_W^2} \overset{**}{=} \frac{Z^{(+)}}{Z^{(-)}} - 1 \qquad\qquad T = \frac{4\pi}{g'^2 \cos^2\theta_W}\left(e_1 - e_1^{\rm SM}\right)$$

✓ We follow the useful dispersive representation introduced by Peskin and Takeuchi* for S and a dispersion relation for T (checked for the lowest cuts):

$$S = \frac{16\pi}{g^2 \tan\theta_W} \int_0^\infty \frac{{\rm d}t}{t} \left(\rho_S(t) - \rho_S(t)^{\rm SM}\right)$$
$$T = \frac{16\pi}{g'^2 \cos^2\theta_W} \int_0^\infty \frac{{\rm d}t}{t^2} \left(\rho_T(t) - \rho_T(t)^{\rm SM}\right)$$

- ✓ $\rho_S(t)$ and $\rho_T(t)$ are the spectral functions of the W^3B and of the difference of the neutral and charged Goldstone boson self-energies, respectively.
- ✓ They need to be well-behaved at short-distances to get the convergence of the integral.
- ✓ S and T parameters are defined for a reference value for the SM Higgs mass.

* Peskin and Takeuchi '92
** Barbieri et al. '93

✓ We consider only the lightest two-particle absorptive cuts ($\phi\phi, h\phi, \psi\bar{\psi}$) and in general we take as working assumptions $M_A > M_V$ and $\widetilde{F}^2_{V,A} < F^2_{V,A}$.

✓ LO result (T_{LO}=0):

✓ With 1st and 2nd WSR: $S_{\rm LO} = \frac{4\pi v^2}{M_V^2}\left(1+\frac{M_V^2}{M_A^2}\right) \longrightarrow \frac{4\pi v^2}{M_V^2} < S_{\rm LO} < \frac{8\pi v^2}{M_V^2}$

✓ With only the 1st WSR: $S_{\rm LO} > \frac{4\pi v^2}{M_V^2}$

✓ NLO result with 1st and 2nd WSR:

$$
\begin{aligned}
S_{\rm NLO} &= 4\pi v^2\left(\frac{1}{M_V^{r\,2}}+\frac{1}{M_A^{r\,2}}\right) + \Delta S_{\rm NLO}^{\rm P-even} + \Delta S_{\rm NLO}^{\rm P-odd} \\
\Delta S_{\rm NLO}^{\rm P-even} &= \frac{1}{12\pi}\left[(1-\kappa_W^2)\left(\log\frac{M_V^2}{m_h^2}-\frac{11}{6}\right)+\kappa_W^2\left(\frac{M_A^2}{M_V^2}-1\right)\log\frac{M_A^2}{M_V^2}\right] \\
\Delta S_{\rm NLO}^{\rm P-odd} &= \frac{1}{12\pi}\left(\frac{\widetilde{F}_V^2}{F_V^2}+2\kappa_W^2\frac{\widetilde{F}_V\widetilde{F}_A}{F_V F_A}-\kappa_W^2\frac{\widetilde{F}_A^2}{F_A^2}\right)\left(\frac{M_A^2}{M_V^2}-1\right)\log\frac{M_A^2}{M_V^2}+\mathcal{O}\left(\frac{\widetilde{F}_{V,A}^4}{F_{V,A}^4}\right)
\end{aligned}
$$

P-even results correspond to Pich, IR and Sanz-Cillero '13 '14

$$
\begin{aligned}
T_{\rm NLO} &= \Delta T_{\rm NLO}^{\rm P-even} + \Delta T_{\rm NLO}^{\rm P-odd} \\
\Delta T_{\rm NLO}^{\rm P-even} &= \frac{3}{16\pi\cos^2\theta_W}\left[(1-\kappa_W^2)\left(1-\log\frac{M_V^2}{m_h^2}\right)+\kappa_W^2\log\frac{M_A^2}{M_V^2}\right] \\
\Delta T_{\rm NLO}^{\rm P-odd} &= \frac{3}{16\pi\cos^2\theta_W}\left\{2\kappa_W^2\frac{\widetilde{F}_A}{F_A}-2\frac{\widetilde{F}_V}{F_V}+\frac{M_V^2}{M_A^2-M_V^2}\log\frac{M_A^2}{M_V^2}\left(2\frac{\widetilde{F}_V}{F_V}-2\kappa_W^2\frac{M_A^2}{M_V^2}\frac{\widetilde{F}_A}{F_A}\right)\right. \\
&\quad +\frac{M_V^2}{M_A^2-M_V^2}\log\frac{M_A^2}{M_V^2}\left[\left(\kappa_W^2\frac{\widetilde{F}_A^2}{F_A^2}-\frac{\widetilde{F}_V^2}{F_V^2}\right)\left(1+\frac{M_A^2}{M_V^2}\right)+2\frac{\widetilde{F}_V\widetilde{F}_A}{F_V F_A}\left(\kappa_W^2\frac{M_A^2}{M_V^2}-1\right)\right] \\
&\quad \left. +2\left(\frac{\widetilde{F}_V^2}{F_V^2}-\kappa_W^2\frac{\widetilde{F}_A^2}{F_A^2}+(1-\kappa_W^2)\frac{\widetilde{F}_V\widetilde{F}_A}{F_V F_A}\right)\right\}+\mathcal{O}\left(\frac{\widetilde{F}_{V,A}^3}{F_{V,A}^3}\right)
\end{aligned}
$$

Expansion in $\frac{\widetilde{F}_{V,A}}{F_{V,A}}$.

- ✓ We consider only the lightest two-particle absorptive cuts ($\phi\phi, h\phi, \psi\bar{\psi}$) and in general we take as working assumptions $M_A > M_V$ and $\widetilde{F}_{V,A}^2 < F_{V,A}^2$.

- ✓ LO result (T_{LO}=0):
  - ✓ With 1st and 2nd WSR: $S_{\rm LO} = \frac{4\pi v^2}{M_V^2}\left(1+\frac{M_V^2}{M_A^2}\right) \longrightarrow \frac{4\pi v^2}{M_V^2} < S_{\rm LO} < \frac{8\pi v^2}{M_V^2}$
  - ✓ With only the 1st WSR: $S_{\rm LO} > \frac{4\pi v^2}{M_V^2}$

- ✓ NLO result with only the 1st WSR:

$$
\begin{aligned}
S_{\rm NLO} &> \frac{4\pi v^2}{M_V^{r\,2}} + \Delta\widetilde{S}_{\rm NLO}^{\rm P-even} + \Delta\widetilde{S}_{\rm NLO}^{\rm P-odd} \\
\Delta\widetilde{S}_{\rm NLO}^{\rm P-even} &= \frac{1}{12\pi}\left[\left(1-\kappa_W^2\right)\left(\log\frac{M_V^2}{m_h^2}-\frac{11}{6}\right)-\kappa_W^2\left(\log\frac{M_A^2}{M_V^2}-1+\frac{M_A^2}{M_V^2}\right)\right]. \\
\Delta\widetilde{S}_{\rm NLO}^{\rm P-odd} &= \frac{1}{12\pi}\left\{\left(1-\frac{M_A^2}{M_V^2}\right)\left[\frac{\widetilde{F}_V^2}{F_V^2}+\kappa_W^2\frac{\widetilde{F}_A}{F_A}\left(2\frac{\widetilde{F}_V}{F_V}-\frac{\widetilde{F}_A}{F_A}\right)\right]\right. \\
&\quad \left. +\log\frac{M_A^2}{M_V^2}\left(\frac{\widetilde{F}_V^2}{F_V^2}-\kappa_W^2\frac{\widetilde{F}_A^2}{F_A^2}-2\frac{\widetilde{F}_V\widetilde{F}_A}{F_V F_A}\right)\right\}+\mathcal{O}\left(\frac{\widetilde{F}_{V,A}^4}{F_{V,A}^4}\right) \\
T_{\rm NLO} &= \Delta T_{\rm NLO}^{\rm P-even} + \Delta T_{\rm NLO}^{\rm P-odd} \\
\Delta T_{\rm NLO}^{\rm P-even} &= \frac{3}{16\pi\cos^2\theta_W}\left[\left(1-\kappa_W^2\right)\left(1-\log\frac{M_V^2}{m_h^2}\right)+\kappa_W^2\log\frac{M_A^2}{M_V^2}\right] \\
\Delta T_{\rm NLO}^{\rm P-odd} &= \frac{3}{16\pi\cos^2\theta_W}\left\{2\kappa_W^2\frac{\widetilde{F}_A}{F_A}-2\frac{\widetilde{F}_V}{F_V}+\frac{M_V^2}{M_A^2-M_V^2}\log\frac{M_A^2}{M_V^2}\left(2\frac{\widetilde{F}_V}{F_V}-2\kappa_W^2\frac{M_A^2}{M_V^2}\frac{\widetilde{F}_A}{F_A}\right)\right. \\
&\quad +\frac{M_V^2}{M_A^2-M_V^2}\log\frac{M_A^2}{M_V^2}\left[\left(\kappa_W^2\frac{\widetilde{F}_A^2}{F_A^2}-\frac{\widetilde{F}_V^2}{F_V^2}\right)\left(1+\frac{M_A^2}{M_V^2}\right)+2\frac{\widetilde{F}_V\widetilde{F}_A}{F_V F_A}\left(\kappa_W^2\frac{M_A^2}{M_V^2}-1\right)\right] \\
&\quad \left. +2\left(\frac{\widetilde{F}_V^2}{F_V^2}-\kappa_W^2\frac{\widetilde{F}_A^2}{F_A^2}+(1-\kappa_W^2)\frac{\widetilde{F}_V\widetilde{F}_A}{F_V F_A}\right)\right\}+\mathcal{O}\left(\frac{\widetilde{F}_{V,A}^3}{F_{V,A}^3}\right)
\end{aligned}
$$

P-even results correspond to Pich, IR and Sanz-Cillero '13 '14

Expansion in $\frac{\widetilde{F}_{V,A}}{F_{V,A}}$.