Symmetry-restoring finite counterterms of SMEFT four-fermion operator insertions at one-loop

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1. Introduction

- When we do loop calculations in QFTs we often use the Dimensional Regularization procedure.
- In Chiral Theories we need to define the properties of the γ_5 matrix in D-dimensions.
- The only known Mathematically-Consistent way to do so is to use the Breitenlohner-Maison-'t Hooft-Veltman (BMHV) scheme.
- Already at the classical level, this scheme induces Symmetry-Breaking terms in the Lagrangian for which we need to add both Evanescent Divergent and Finite Symmetry-Restoring Counterterms.

2. Renormalization of Gauge Theories

- Quantum Gauge Theories are invariant under the BRST Transformation:

  - Becchi, C.; Rouet, A.; Stora, R. (1974)
  - I.V. Tyutin (1975)

$$\delta_\theta \psi_i(x) = i\theta\omega^\alpha(x)(T_\alpha)_{ij}\psi_j(x)$$

$$\delta_\theta A_\mu^\alpha(x) = \theta\left[\partial_\mu \omega^\alpha(x) + C^\alpha_{\gamma\beta}\omega^\beta(x)A^\gamma_\mu(x)\right]$$

$$\delta_\theta \omega^\alpha(x) = -\frac{1}{2}\theta C^\alpha_{\beta\gamma}\omega^\beta(x)\omega^\gamma(x)$$

$$\delta_\theta \omega^{*\alpha}(x) = -\theta h^\alpha(x)$$

$$\delta_\theta h^\alpha(x) = 0$$

- ❑ ω^α Ghost.
- ❑ $\omega^{*\alpha}$ Antighost.
- ❑ h^α Nakanishi-Lautrup Field.
- ❑ $\epsilon^\alpha(x) = \theta\omega^\alpha(x)$

- The action is:

$$I\left[A_\mu^\alpha, \psi_i, \omega^\alpha, \omega^{*\alpha}, h^\alpha\right] = \underbrace{I_0\left[A_\mu^\alpha, \psi_i\right]}_{\text{Classical Action}} + \underbrace{s\Psi\left[A_\mu^\alpha, \psi_i, \omega^\alpha, \omega^{*\alpha}, h^\alpha\right]}_{\text{Ghost Number}=-1}$$

$$\delta_\theta \chi = \theta(s\chi)$$

$s^2 = 0$ (Nilpotent)

2. Renormalization of Gauge Theories

- BRST Invariance implies that the Quantum Effective Action (QEA) will satisfy the Zinn-Justin Equation:

$s\Gamma \neq 0$ in general

$$(\Gamma, \Gamma) = 0$$

- Zinn-Justin, (1974)

In the Absence of Anomalies

- The QEA is NOT BRST invariant. Loop effects can produce FINITE local effective interactions that break this symmetry. But:

> If the Regularization procedure preserves the BRST Symmetry, then the Divergent Terms of the QEA are BRST Invariant. Thus, they can be Reabsorbed by shifting the Original Parameters of the classical action.

3. BMHV Scheme

- Hooft, G. 't; Veltman, M. (1972)

- The most used Regularization procedure is **Dimensional Regularization**, where we compute the integrals in $D = 4 - 2\epsilon$ dimensions.
- The extension of $\epsilon^{\mu\nu\rho\sigma}$ and γ_5 to D dimensions is non-trivial.

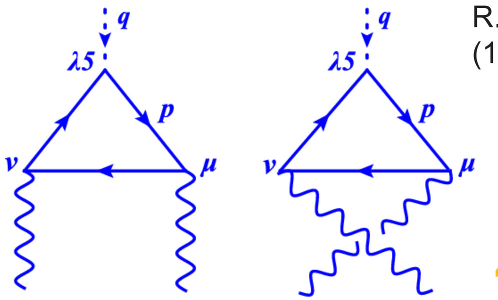
If the Trace is Cylic

If $\{\gamma_\mu, \gamma_5\} = 0$ then $\mathrm{tr}\{\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_5\} = 0$ for $D \neq 2$ or $D \neq 4$.

In $D = 4$ $\mathrm{tr}\{\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_5\} = 4i\epsilon^{\mu\nu\rho\sigma}$.

If $\{\gamma_\mu, \gamma_5\} = 0$ then the scheme is Mathematically Inconsistent

- S. Adler (1969)
- J.S. Bell and R. Jackiw (1969)



3. BMHV Scheme

- Hooft, G. 't; Veltman, M. (1972)
- Breitenlohner, P.; Maison, D. (1977)

- The only known scheme that is mathematically consistent is the BMHV where we split the D -Dimensional space into 4-Dimensional and $(D-4)$-Dimensional subspaces.

$$\bar{\eta}_{\mu\nu} := \begin{cases} \eta_{\mu\nu}, & \mu,\nu \leq 3 \\ 0, & \text{Otherwise} \end{cases} \qquad \hat{\eta}_{\mu\nu} := \eta_{\mu\nu} - \bar{\eta}_{\mu\nu}$$

Projectors: $\bar{p}_\mu := \bar{\eta}_{\mu\nu} p^\nu \qquad \bar{\gamma}_\mu := \bar{\eta}_{\mu\nu} \gamma^\nu \qquad \hat{p}_\mu := \hat{\eta}_{\mu\nu} p^\nu \qquad \hat{\gamma}_\mu := \hat{\eta}_{\mu\nu} \gamma^\nu$

Basic Properties: $\{\gamma_\mu, \gamma_\nu\} = 2\eta_{\mu\nu}\mathbb{I} \qquad \mathrm{tr}\{\mathbb{I}\} = 4$

γ_5 is purely 4-Dimensional

$$\gamma_5 := i\bar{\gamma}^0\bar{\gamma}^1\bar{\gamma}^2\bar{\gamma}^3$$

$$\{\bar{\gamma}_\mu, \gamma_5\} = 0$$
$$[\hat{\gamma}_\mu, \gamma_5] = 0$$

3. BMHV Scheme

- Bélusca-Maïto, H. et al. (2020)

- In this scheme Propagators are purely 4-Dimensional in chiral Theories:

$$\bar{\psi} P_L \gamma_\mu \partial^\mu P_R \psi = \bar{\psi} P_L \bar{\gamma}_\mu \bar{\partial}^\mu P_R \psi \qquad \bar{\psi} P_R \gamma_\mu \partial^\mu P_L \psi = \bar{\psi} P_R \bar{\gamma}_\mu \bar{\partial}^\mu P_L \psi$$

- This leads to Unregularized Amplitudes: $\int \mathrm{d}^D k \, \frac{1}{\bar{k}^2} = \int \mathrm{d}^4 \bar{k} \, \frac{1}{\bar{k}^2} \int \mathrm{d}^{D-4} \hat{k} = \infty \cdot 0$
- We add Ficticious right- or left-handed fields for the left- and right-handed chiral fermions that do not interact $s\psi_f = 0$.

$$\bar{l} \gamma_\mu \partial^\mu l = \bar{l}_L \bar{\gamma}_\mu \bar{\partial}^\mu l_L + \bar{l}_R \bar{\gamma}_\mu \bar{\partial}^\mu l_R + \bar{l}_L \hat{\gamma}_\mu \hat{\partial}^\mu l_R + \bar{l}_R \hat{\gamma}_\mu \hat{\partial}^\mu l_L$$

Evanescent Operator

3. BMHV Scheme

Symmetry Breaking Operator

- The classical action is not BRST invariant due to the propagator:

$$\Delta := s\left(i\,\bar{\psi}_i \gamma_\mu \partial^\mu \psi_i\right) = \hat{\eta}^{\mu\nu} \omega^\alpha \left(t_\alpha^{L,R}\right)_{ij} \bar{\psi}_i \left(\gamma_\mu \vec{\partial}_\nu P_{R,L} + \gamma_\mu \overleftarrow{\partial}_\nu P_{L,R}\right) \psi_j$$

t_α^L →Physical Doublet

t_α^R →Physical Singlet

- It will induce symmetry breaking at the quantum level:

Algebraic Renormalization

$$\mathcal{S}(\Gamma) = (\Gamma, \Gamma) = -2 \lim_{\mathcal{K}(x) \to 0} \int \mathrm{d}^D x \frac{\delta_R \Gamma[X, K, \mathcal{K}]}{\delta \mathcal{K}(x)}$$

Symmetry Breaking at the Quantum Level can be Computed from 1PI Diagrams with Single Insertions of the SB Operator Δ

4. Bonneau Method

$\epsilon \to 0$

- We need to Restore the Symmetry in the limit $D \to 4$. The symmetry breaking contributions that we find in loop caculations with the Evanescent Operator Δ can have three different forms:

1. ϵ Terms: Evanescent operators that vanish when $D \to 4$.
2. $\frac{1}{\epsilon}$ Terms: Absorbed when we eliminate the divergences.
3. $\frac{\epsilon}{\epsilon}$ Terms: They have to be cancelled adding Finite Counterterms.

$$\boxed{\lim_{D\to 4} \mathcal{S}\left(\Gamma + I_{\text{sct}} + I_{\text{fct}}\right) = 0}$$

- I_{sct} Singular Counterterms
- I_{fct} Finite Counterterms

4. Bonneau Method

- We use the Bonneau Method to find the $\frac{\epsilon}{\epsilon}$ terms in the 1PI diagrams with single insertions of the symmetry breaking operator:

$$\Delta := \hat{\eta}^{\mu\nu}\omega^{\alpha}\left(t_{\alpha}^{L,R}\right)_{ij}\bar{\psi}_i\left(\gamma_\mu\vec{\partial}_\nu P_{R,L} + \gamma_\mu\overleftarrow{\partial}_\nu P_{L,R}\right)\psi_j$$

- We have to define the operator $\check{\Delta}$ which is Δ substituting $\hat{\eta}^{\mu\nu}$ by $\check{\eta}^{\mu\nu} := -\hat{\eta}^{\mu\nu}/2\epsilon$. Then, at One Loop, the $\frac{\epsilon}{\epsilon}$ terms are computed using:

$$\lim_{D\to 4} \text{r.s.p.}\left(I_{\varepsilon}^{G^{\check{\Delta}}}\Big|_{\check{\eta}=0}\right)$$

$I_{\epsilon}^{G^{\check{\Delta}}}$ Amplitude of the 1PI diagram with single insertion of $\check{\Delta}$.

- Bonneau, G. (1980)

5. One-Loop SB from Four-Fermion Operators

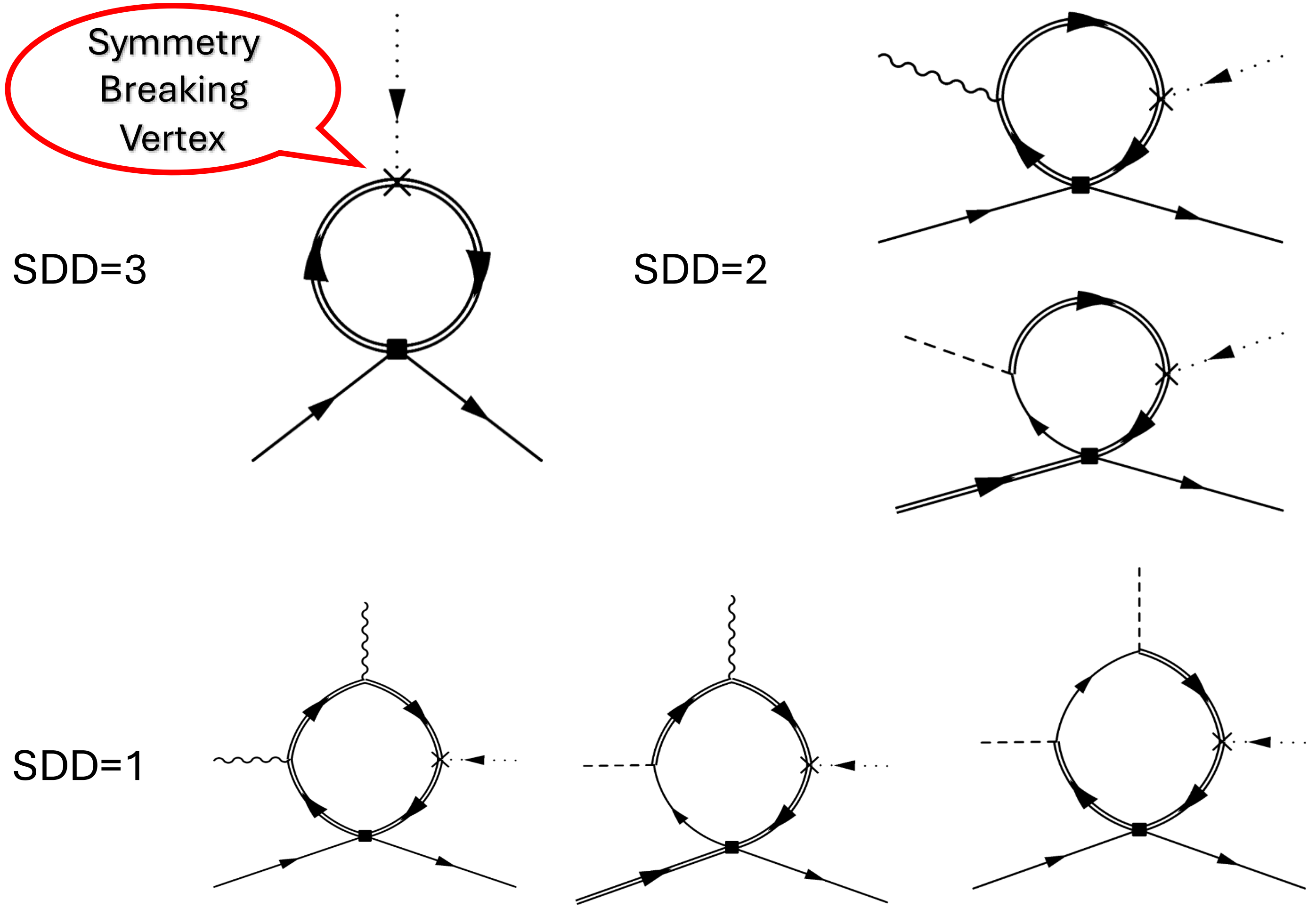
$(\bar{L}L)(\bar{L}L)$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$	
Q_{ll}	$(\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu l_t)$	Q_{ee}	$(\bar{e}_p\gamma_\mu e_r)(\bar{e}_s\gamma^\mu e_t)$	Q_{le}	$(\bar{l}_p\gamma_\mu l_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{uu}	$(\bar{u}_p\gamma_\mu u_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{lu}	$(\bar{l}_p\gamma_\mu l_r)(\bar{u}_s\gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar{q}_p\gamma_\mu\tau^I q_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	Q_{dd}	$(\bar{d}_p\gamma_\mu d_r)(\bar{d}_s\gamma^\mu d_t)$	Q_{ld}	$(\bar{l}_p\gamma_\mu l_r)(\bar{d}_s\gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{eu}	$(\bar{e}_p\gamma_\mu e_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{qe}	$(\bar{q}_p\gamma_\mu q_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar{l}_p\gamma_\mu\tau^I l_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	Q_{ed}	$(\bar{e}_p\gamma_\mu e_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{u}_s\gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar{u}_p\gamma_\mu u_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{u}_s\gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar{u}_p\gamma_\mu T^A u_r)(\bar{d}_s\gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{d}_s\gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{d}_s\gamma^\mu T^A d_t)$

$(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$		B-violating	
Q_{ledq}	$(\bar{l}_p^j e_r)(\bar{d}_s q_t^j)$	Q_{duq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(q_s^{\gamma j})^T C l_t^k\right]$
$Q_{quqd}^{(1)}$	$(\bar{q}_p^j u_r)\varepsilon_{jk}(\bar{q}_s^k d_t)$	Q_{qqu}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r)\varepsilon_{jk}(\bar{q}_s^k T^A d_t)$	Q_{qqq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jn}\varepsilon_{km}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
$Q_{lequ}^{(1)}$	$(\bar{l}_p^j e_r)\varepsilon_{jk}(\bar{q}_s^k u_t)$	Q_{duu}	$\varepsilon^{\alpha\beta\gamma}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{lequ}^{(3)}$	$(\bar{l}_p^j \sigma_{\mu\nu} e_r)\varepsilon_{jk}(\bar{q}_s^k \sigma^{\mu\nu} u_t)$		

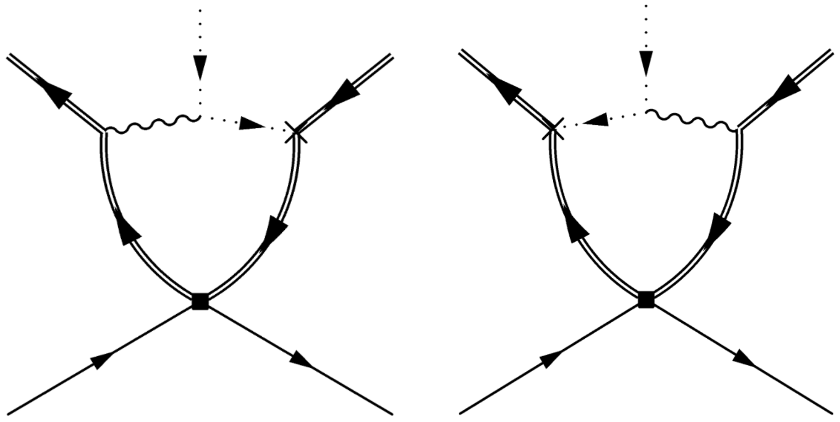
- Grzadkowski, B. et al. (2010)

- The calculation has been done in the Warsaw basis.
- No contributions from evanescent operators at one loop, because of the additional ϵ factor.
- No contributions from B-violating operators at one loop.

5. One-Loop SB from Four-Fermion Operators



There is no contribution coming from SSD=0 diagrams. Some of them are convergent due to the structure of the SB vertex and the other two cancel each other:



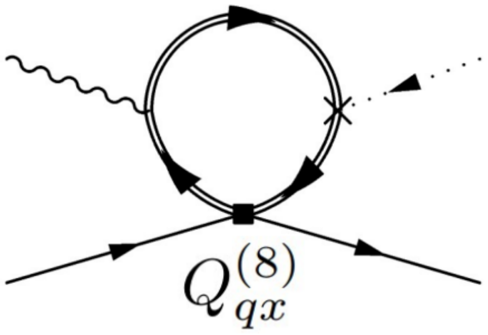
5. One-Loop SB from Four-Fermion Operators

- Up to this point, only the amount of symmetry breaking at one loop has been computed. Now it is necessary to obtain the finite counterterm that cancels that contribution and for that one needs to find the "inverse" of a gauge transformation:

$$\mathcal{S}\,(\Gamma) = \Delta \cdot \Gamma \qquad\qquad \lim_{D\to 4} \mathcal{S}\,(\Gamma + I_{\text{sct}} + I_{\text{fct}}) = 0$$

- Fuentes-Martín, J. et al. (2025)

- This is the most difficult part of the calculation. Main advantage of the spurion method.



$$\propto d_{ABC}\,(\bar{\xi}_R \gamma_\mu T^A \xi_R)\,\partial_\rho \mathbf{G}_\nu^B \partial_\sigma \mathbf{g}^C \epsilon^{\mu\nu\rho\sigma} + \ldots$$

$$s[d_{ABC}\,(\bar{\xi}_R \gamma_\mu T^A \xi_R)\,\partial_\rho \mathbf{G}_\nu^B \mathbf{G}_\sigma^C \epsilon^{\mu\nu\rho\sigma}]$$
$$= d_{ABC}\,(\bar{\xi}_R \gamma_\mu T^A \xi_R)\,\left(\partial_\rho \mathbf{G}_\nu^B \partial_\sigma \mathbf{g}^C + C_{BEF} \mathbf{G}_\sigma^C \mathbf{G}_\nu^E \partial_\rho \mathbf{g}^F\right)\,\epsilon^{\mu\nu\rho\sigma}$$

$$d_{ABC} = 2\,\text{Tr}\{T^A(T^B T^C + T^C T^B)\}$$

6. Conclusions

- The only known Mathematically-Consistent scheme in Dimensional Regularization (BMHV) breaks the gauge symmetry in Chiral Theories.
- In a theory without physical anomalies, the obstructions to the Slavnov-Taylor identities can be eliminated adding the proper set of finite counterterms.
- We have obtained the set of finite counterterms needed to restore the symmetry breaking induced by four-fermion operators in SMEFT at one-loop using the well-known techniques of algebraic renormalization. Results coming soon.

6. Conclusions

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Symmetry Breaking at the Quantum Level

- Symmetry Breaking at Tree-Level:

$$I\left[\chi\right] \to I\left[\chi+\varepsilon F\right] = I\left[\chi\right] + \varepsilon \int \mathrm{d}^D x G\left[x;\chi\right]$$

- Generating Functional:

Antifield

$$Z\left[J,K,\mathcal{K}\right] := e^{iW\left[J,K,\mathcal{K}\right]} = \int \mathcal{D}\chi \exp\left\{ iI\left[\chi\right] + i\int \mathrm{d}^D x F^n\left(x\right) K_n\left(x\right) + i\int \mathrm{d}^D x \chi^n\left(x\right) J_n\left(x\right) + i\int \mathrm{d}^D x G\left(x\right)\mathcal{K}\left(x\right)\right\}$$

Classical Sources

- Classical Field:

$$X^n_{J,K,\mathcal{K}}\left(x\right) := \frac{\delta_R}{\delta J_n\left(x\right)} W\left[J,K,\mathcal{K}\right]$$

- Quantum Effective Action:

$$\Gamma\left[X,K,\mathcal{K}\right] := -\int \mathrm{d}^D x X^n\left(x\right)\left(J_{X,K,\mathcal{K}}\right)_n\left(x\right) + W\left[J_{X,K},K,\mathcal{K}\right].$$

- Antibracket:

$$\left(F,G\right) := \int \left(\frac{\delta_R F\left[\chi,K\right]}{\delta \chi^n\left(x\right)} \frac{\delta_L G\left[\chi,K\right]}{\delta K_n\left(x\right)} - \frac{\delta_R F\left[\chi,K\right]}{\delta K_n\left(x\right)} \frac{\delta_L G\left[\chi,K\right]}{\delta \chi_n\left(x\right)} \right)$$