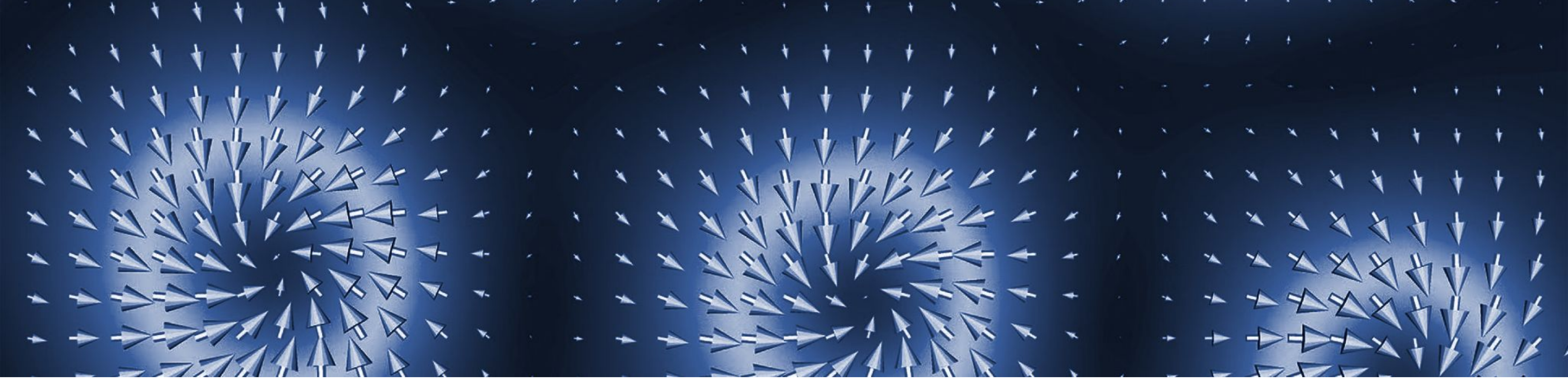

A hot take on electroweak Skyrmions

CPAN 2025 · Valencia, España | November 19th 2025

Luis Gil (Universidad de Granada)

Based on:

M. Chala, J. C. Criado and LG [in prep.]

High Energy Theory

Junta de Andalucía

GOBIERNO DE ESPAÑA

MINISTERIO DE CIENCIA, INNOVACIÓN Y UNIVERSIDADES

Motivation

Skyrme model for chiral perturbation theory

$$\mathcal{L}_{\text{Skyrme}} = \frac{v^2}{4}\,\text{tr}\left(\partial^\mu U^\dagger \partial_\mu U\right) + \frac{1}{32\,e^2}\,\text{tr}\left[U^\dagger\partial_\mu U\,,\,U^\dagger\partial_\nu U\right]^2$$

where $$U(x) = \exp\left(\frac{i\pi_a(x)\sigma_a}{v}\right) \in SU(2)\,, \qquad U^\dagger U = \mathbb{1}_2$$

Skyrme model = approx. to 2-flavors ChPT

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Skyrme model = approx. to 2-flavors ChPT

Goal: Find static configurations of *U* that minimize the energy

Finite energy □ *U(x)* must be continuous and constant at infinity, *U(∞)* = 1

$$U : \mathbb{R}^3 \cup \{\infty\} \cong S^3 \to SU(2) \cong S^3$$

configurations are continuous mappings between 3-spheres!

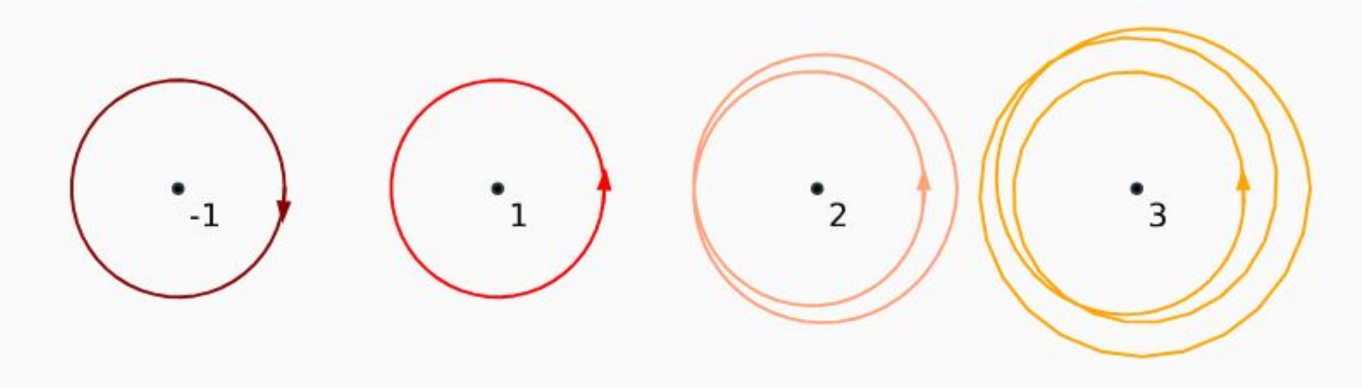
Motivation

Skyrme model for chiral perturbation theory

$U : \mathbb{R}^3 \cup \{\infty\} \cong S^3 \to SU(2) \cong S^3$ continuous mappings between 3-spheres!

These mappings can belong to different **homotopy classes**, which form a group $\pi_3(S^3) \cong \mathbb{Z}$

- **Example:** $S^1 \to S^1$



The **winding number** is:

$$n_U = \frac{1}{24\pi^2}\epsilon_{ijk}\int d^3x \, \langle (U^\dagger \partial_i U)(U^\dagger \partial_j U)(U^\dagger \partial_k U)\rangle$$

It is a **homotopy invariant** (conserved under continuous transformations)

Like time evolution!

Motivation

Skyrme model for chiral perturbation theory

Tony Skyrme

Motivation

Skyrme model for chiral perturbation theory

Tony Skyrme

What if… skyrmions are a low energy description of baryons ($B = n_U$)?

(*) Later confirmed by Adkins, Nappi and Witten

Our setup

Requirements

A theory that contains skyrmions must satisfy:

- **Bosonic fields:** Skyrmions are classical configurations
- **Non-trivial target space:** Need some field that maps to S^3
- **Higher-derivative operators:** Stabilizers for extended configs. (Derrick's theorem)
- **Strong EFT convergence:** Higher-derivative operators cannot become too large

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Naturally realized in the high-temperature limit of QFT!

Our setup

Thermal field theory

- **Generating functional** in thermal field theory (= Euclidean QFT with periodic time):

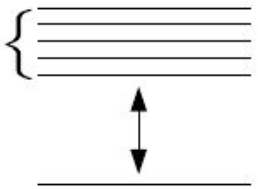
$$\mathcal{Z}_{\rm th} = {\rm Tr}\left(e^{-\beta\mathcal{H}}\right) = \sum_q \langle q\ 0|e^{-\beta\mathcal{H}}|q\ 0\rangle = \mathcal{N} \int_{q(0)=q(-i\beta)} \mathcal{D}q \exp\left(-S_E\right)$$

- Fields decompose in tower of 3D **Matsubara modes** (~ Kaluza-Klein) with thermal masses:

Bosons: $\omega_n = 2\pi n T$

Fermions: $\omega'_n = 2\pi\left(n+\frac{1}{2}\right)T$

$M \sim \pi T$



Bosonic zero mode

3D EFT for bosonic zero modes

Expansion in **m/T**

Our setup

Thermal field theory

$$\mathscr{L} = \frac{v^2}{4}\left\langle \partial_\mu U^\dagger \partial^\mu U \right\rangle + \mathscr{L}_{\text{kin}}^{\psi} - \mathscr{L}_{\text{int}}$$

T >> v

Matching

$$\mathscr{L}_3^{(p^2)} = c_0 T \left\langle \partial_i U^\dagger \partial_i U \right\rangle$$

$$\mathscr{L}_3^{(p^4)} = \frac{c_1}{T}\left\langle \partial_i U^\dagger \partial_i U \right\rangle^2 + \frac{c_2}{T}\left\langle \partial_i U^\dagger \partial_j U \right\rangle \left\langle \partial_i U^\dagger \partial_j U \right\rangle$$

$$\mathscr{L}_3^{(p^6)} = \quad \ldots$$

Bosonic fields (only):

Non-trivial target space:

Higher-derivative operators:

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Bosonic fields (only): ✖ ⭘

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Bosonic fields (only): ✗ ○

Non-trivial target space: ○ ○

Higher-derivative operators:

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Bosonic fields (only):	✗	○
Non-trivial target space:	○	○
Higher-derivative operators:	✗	○

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Thermoskyrmions

- Skyrmions (if they exist) in high-temperature regime are stabilized solely by thermal effects
  □ ***thermoskyrmions***

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- Skyrmions (if they exist) in high-temperature regime are stabilized solely by thermal effects
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- **Properties:**

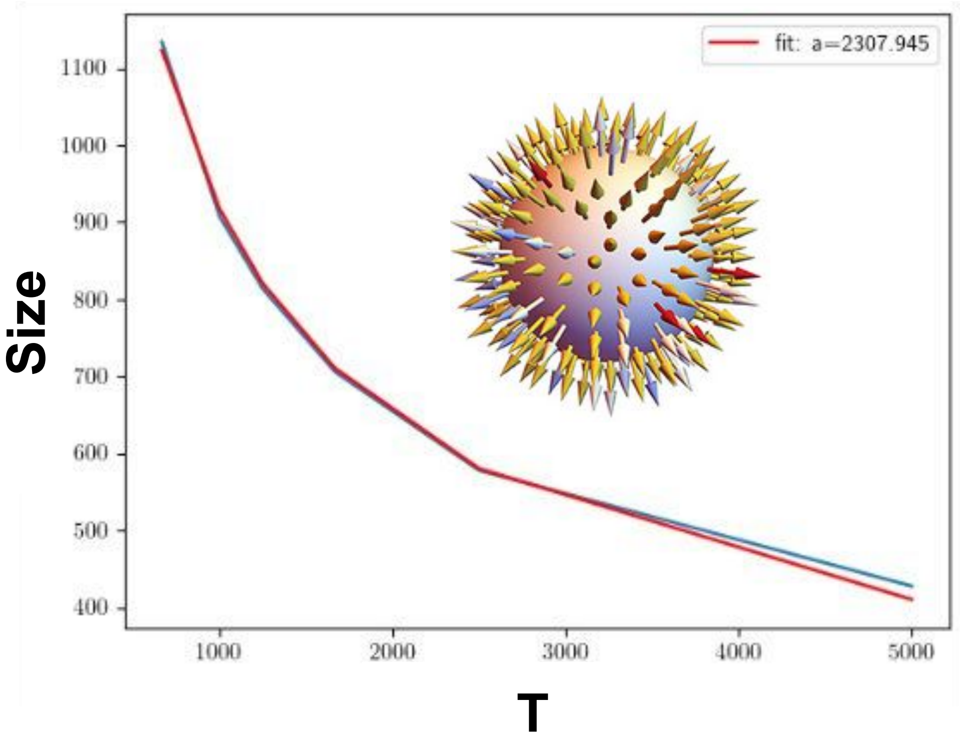
  - **Spherically symmetric**

  - **Stable** against thermal fluctuations
    (minimize the free energy *F*)

  - **Expand** adiabatically as *T* decreases

  - Carry **conserved topological charge**

  - ???



Thank you for your attention!

¡Gracias por vuestra atención!