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$\eta\pi$ phase shift determination from $\eta' \to \eta\pi\pi$ experimental data

David Alejandro Barón Ospina
In collaboration with:
Igor Danilkin
Emilie Passemar

XV CPAN DAYS
2025

Outline

1) Motivation
2) Formalism
3) Dispersive relations
4) Numerical strategy
5) Inputs
6) Perspectives and conclusions

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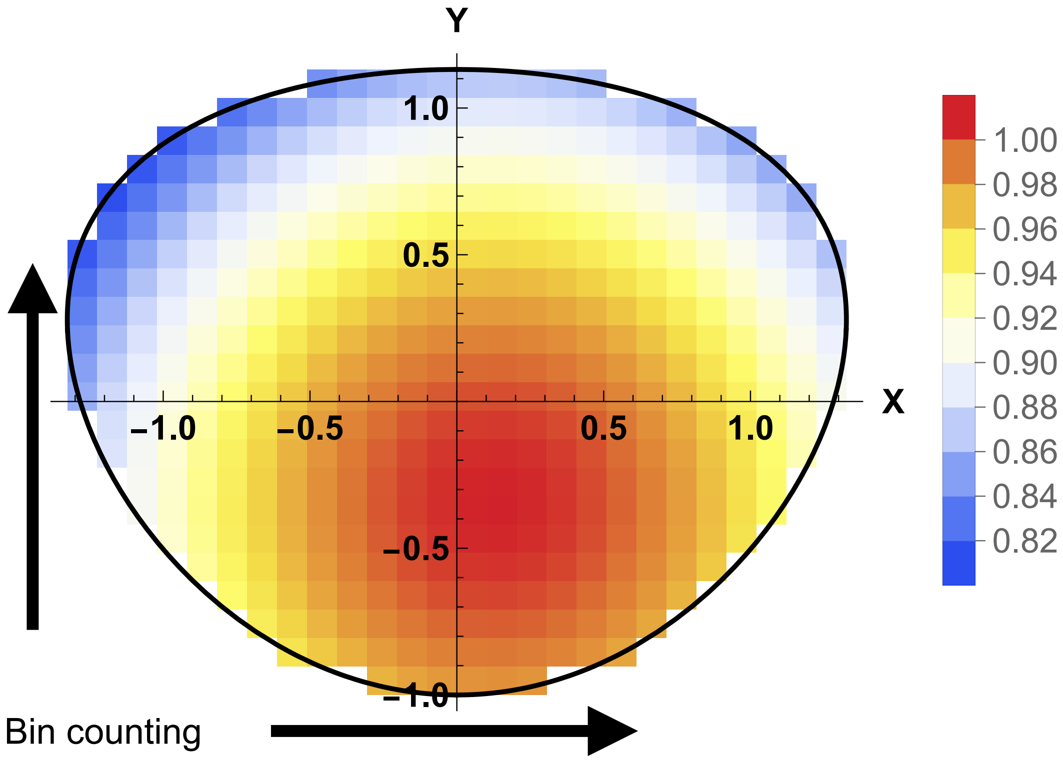

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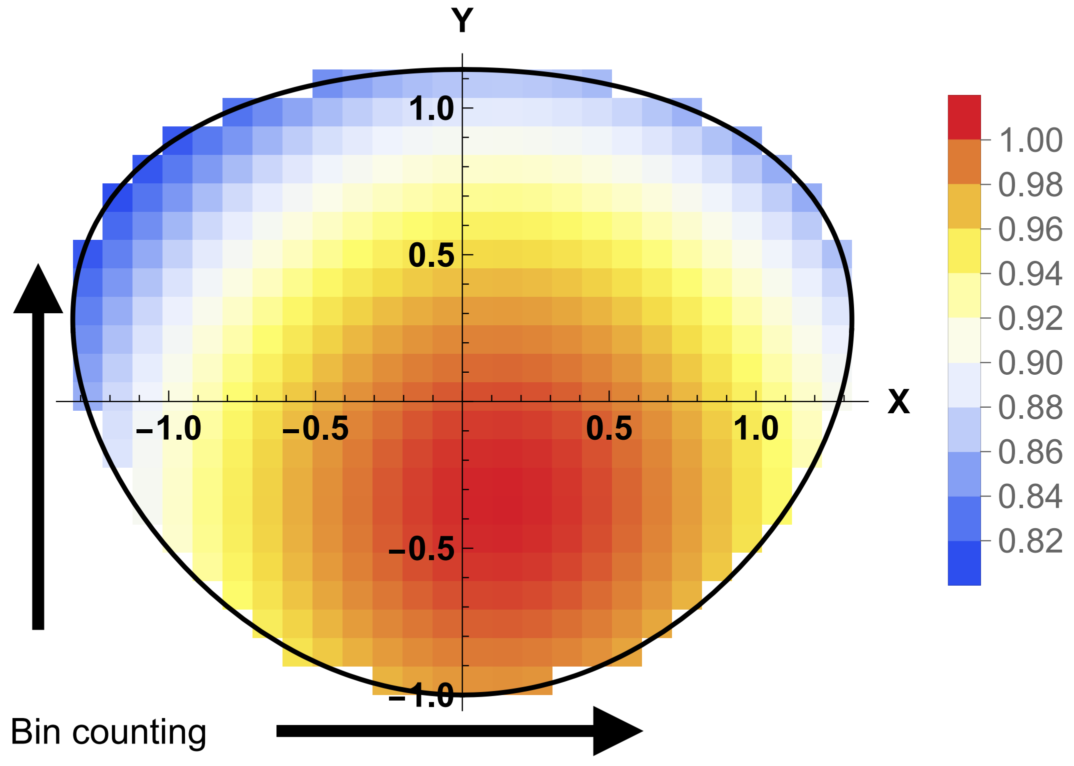

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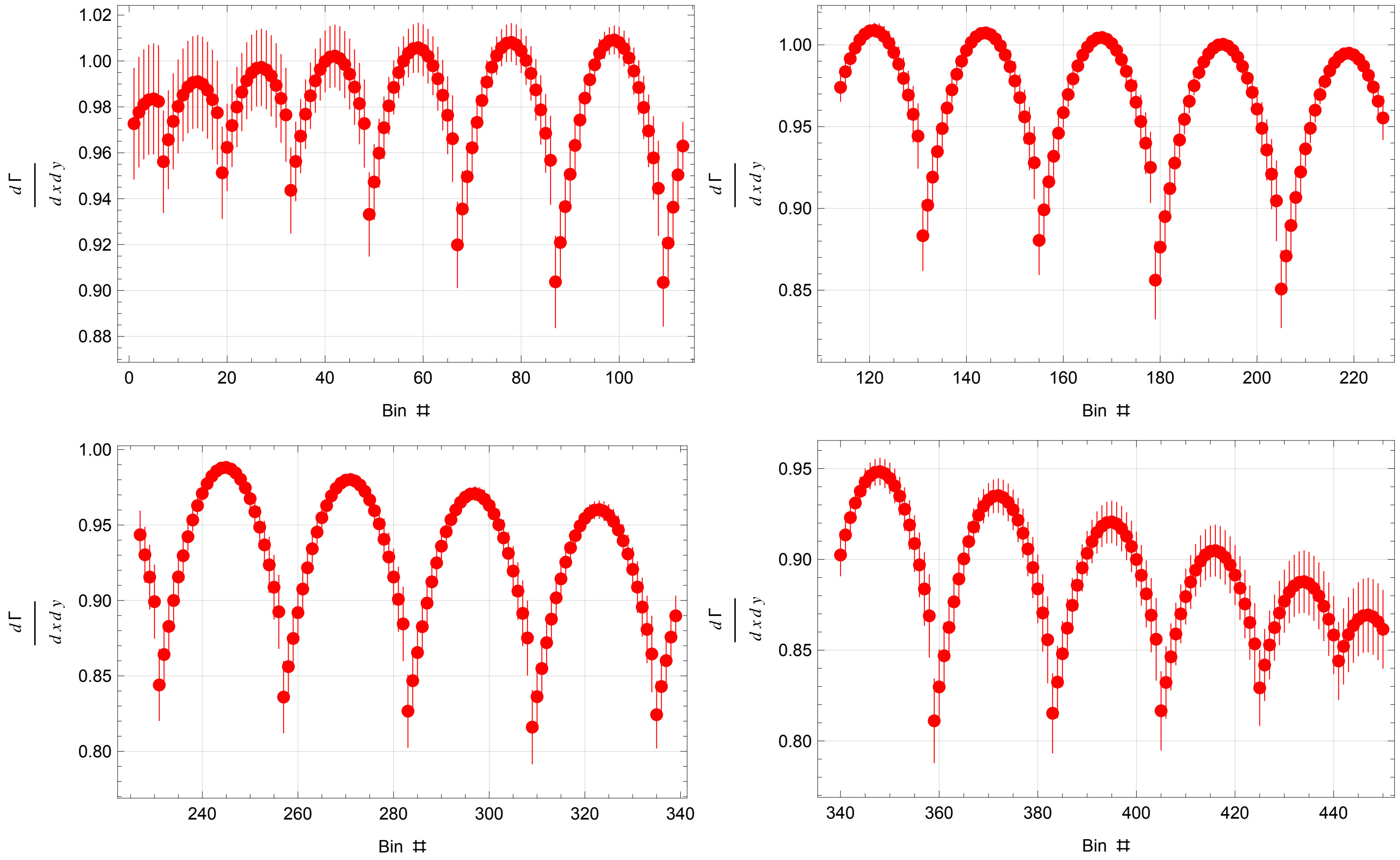

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Tree body final state interactions (FSI)

Incorporated using inelastic inputs in KT

elastic formalism

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The only possibilities are:

$$I_{\pi\pi} = 0, I_{\eta\pi} = 1$$

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$$\eta'\left(P_{\eta'}\right) \rightarrow \eta\left(p_3\right)\pi^i\left(p_1\right)\pi^j\left(p_2\right)$$

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$$+\frac{i}{32\pi^2}\rho_{\pi\eta}(t)T^*_{\eta\pi}(t,\theta_t)\int d\Omega_{t'}\mathscr{M}\left(s',t,u'\right)$$
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$$\rho_{ij}(s) = \frac{\lambda^{1/2}\left(s,m_i^2,m_j^2\right)}{s}$$

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$$\mathrm{Im}\left(\mathscr{T}_I^{ab\,l}(s)\right) = \rho_{ab}(s)\mathscr{T}_I^{ab\,l\,*}(s)\mathscr{T}_I^{ab\,l}(s)\,\theta(s-(m_a+m_b)^2)$$

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$$\mathcal{T}_I^{ab\,l}(s) = \frac{\sin\delta_I^{ab\,l}(s)\; e^{i\delta_I^{ab\,l}(s)}}{\rho_{ab}(s)}$$

$$
\begin{aligned}
\text{Disc}\left(\mathscr{M}(s,t,u)\right) &= i\sin\delta_0^{\pi\pi\ 0}(s)\ e^{-i\delta_0^{\pi\pi\ 0}(s)}\int dz_{s'}\mathscr{M}\left(s,t',u'\right)\\
&+ i\sin\delta_1^{\eta\pi\ 0}(s)\ e^{-i\delta_1^{\eta\pi\ 0}(s)}\int dz_{t'}\mathscr{M}\left(s',t,u'\right)\\
&+ i\sin\delta_1^{\eta\pi\ 0}(s)\ e^{-i\delta_1^{\eta\pi\ 0}(s)}\int dz_{u'}\mathscr{M}\left(s',t',u\right)
\end{aligned}
$$

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Applying the reconstruction theorem

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$$\text{Disc}\left(\mathscr{M}(s,t,u)\right) = i\sin\delta_0^{\pi\pi\;0}(s)\; e^{-i\delta_0^{\pi\pi\;0}(s)}\int dz_{s'}\mathscr{M}\left(s,t',u'\right)$$
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$\Big\downarrow$ Applying the reconstruction theorem

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Applying the reconstruction theorem

$$\text{Disc}(\mathscr{M}_0(s)) = 2i\sin\delta_0^{\pi\pi\ 0}(s)\ e^{-i\delta_0^{\pi\pi\ 0}(s)}\left(\mathscr{M}_0(s) + \frac{1}{2}\int_{-1}^{1} dz_{s'}\left(\mathscr{M}_0^{\pi\eta}(t') + \mathscr{M}_0^{\pi\eta}(u')\right)\right)$$

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Applying the reconstruction theorem

$$\text{Disc}(\mathcal{M}_0(s)) = 2i\sin\delta_0^{\pi\pi\ 0}(s)\ e^{-i\delta_0^{\pi\pi\ 0}(s)}\left(\mathcal{M}_0(s) + \overbrace{\boxed{\frac{1}{2}\int_{-1}^{1}dz_{s'}\left(\mathcal{M}_0^{\pi\eta}(t') + \mathcal{M}_0^{\pi\eta}(u')\right)}}^{\hat{\mathcal{M}}_0(s)}\right)$$

$$\text{Disc}(\mathcal{M}_0^{\pi\eta}(t)) = 2i\sin\delta_1^{\eta\pi\ 0}(t)\ e^{-i\delta_1^{\eta\pi\ 0}(t)}\left(\mathcal{M}_0^{\pi\eta}(t) + \underbrace{\boxed{\int_{-1}^{1}dz_{t'}\left(\mathcal{M}_0(s') + \mathcal{M}_0^{\pi\eta}(u')\right)}}_{\hat{\mathcal{M}}_0^{\eta\pi}(s)}\right)$$

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Outline

1) Motivation
2) Formalism
3) Dispersive relations
4) Numerical strategy
5) Inputs
6) Perspectives and conclusions

Outline

Dispersive relations

Dispersive relations

1) Homogeneous Omnès problem

Dispersive relations

1) Homogeneous Omnès problem

$$\text{disc}\left(f(s)\right) = 2i \lim_{\epsilon\to 0^+} f(s+i\epsilon)\sin\delta(s)\exp[-i\delta(s)]$$

Dispersive relations

1) Homogeneous Omnès problem

$$\text{disc}\left(\boxed{f(s)}\right) = 2i \lim_{\epsilon\to 0^+} f(s+i\epsilon)\sin\delta(s)\exp[-i\delta(s)]$$

$f(s)$ has a cut in the region $s \in [s_0, \infty)$

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Dispersive relations

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$$f(s) = P(s)\Omega(s)$$

Dispersive relations

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Polynomial fixed from the asymptotic behavior of $f(s)$ when $s \to \infty$

Dispersive relations

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Omnès function

Dispersive relations

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Omnès function

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Dispersive relations

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Polynomial fixed from the asymptotic behavior of $f(s)$ when $s \to \infty$

Omnès function

2) Inhomogeneous Omnès problem

$$\mathrm{disc}\, f(s) = 2i \lim_{\epsilon \to 0^+} [f(s+i\epsilon) + \hat{f}(s)]\sin\delta(s)\exp[-i\delta(s)]$$

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$$\text{disc} f(s) = 2i \lim_{\epsilon\to 0^+} [f(s+i\epsilon) + \hat{f}(s)]\sin\delta(s)\exp[-i\delta(s)]$$

$$f(s) = \Omega(s)\left[P_{m-1}(s) + \frac{Q_m(s)}{\pi}\lim_{\epsilon\to 0^+}\int_{s_0}^{\infty} dx \frac{\hat{f}(x)\sin\delta(x)}{|\Omega(x+i\epsilon)|Q_m(x)(x-s)}\right]$$

Dispersive relations

1) Homogeneous Omnès problem

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Omnès function

2) Inhomogeneous Omnès problem

$$\mathrm{disc}\, f(s) = 2i \lim_{\epsilon \to 0^+} [f(s+i\epsilon) + \hat{f}(s)]\sin\delta(s)\exp[-i\delta(s)]$$

$$f(s) = \Omega(s)\left[\boxed{P_{m-1}(s)} + \frac{\boxed{Q_m(s)}}{\pi} \lim_{\epsilon \to 0^+} \int_{s_0}^{\infty} \mathrm{d}x \frac{\hat{f}(x)\sin\delta(x)}{|\Omega(x+i\epsilon)|\boxed{Q_m(x)}(x-s)}\right]$$

Related with the subtractions

Outline

1) Motivation
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Outline

1) Motivation
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Numerical strategy

Numerical strategy

1) Basis functions

Numerical strategy

1) Basis functions

$$\mathscr{M}_0(s) = \Omega_0^{\pi\pi\ 0}(s)\left\{\alpha + \beta\frac{s}{M_{\eta'}^2} + \gamma\frac{s^2}{M_{\eta'}^4} + \frac{s^3}{\pi}\int_{s_0}^{\infty}\frac{\mathrm{d}x}{x^3}\frac{\hat{\mathscr{M}}_0(x)\sin\delta_0^{\pi\pi\ 0}(x)}{\left|\Omega_0^{\pi\pi\ 0}(x)\right|(x-s)}\right\}$$

$$\mathscr{M}_0^{\pi\eta}(t) = \Omega_0^{\pi\eta}(t)\left\{\gamma\frac{t^2}{M_{\eta'}^4} + \frac{t^3}{\pi}\int_{t_0}^{\infty}\frac{\mathrm{d}x}{x^3}\frac{\hat{\mathscr{M}}_0^{\pi\eta}(x)\sin\delta_0^{\pi\eta}(x)}{\left|\Omega_0^{\pi\eta}(x)\right|(x-t)}\right\},$$

Numerical strategy

1) Basis functions

$$\mathscr{M}_0(s) = \Omega_0^{\pi\pi\ 0}(s)\left\{\alpha + \beta\frac{s}{M_{\eta'}^2} + \gamma\frac{s^2}{M_{\eta'}^4} + \frac{s^3}{\pi}\int_{s_0}^{\infty}\frac{\mathrm{d}x}{x^3}\frac{\hat{\mathscr{M}}_0(x)\sin\delta_0^{\pi\pi\ 0}(x)}{\left|\Omega_0^{\pi\pi\ 0}(x)\right|(x-s)}\right\}$$

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System of integral equations solved by iterations

Numerical strategy

1) Basis functions

$$\mathcal{M}_0(s) = \Omega_0^{\pi\pi\,0}(s)\left\{\alpha + \beta\frac{s}{M_{\eta'}^2} + \gamma\frac{s^2}{M_{\eta'}^4} + \frac{s^3}{\pi}\int_{s_0}^{\infty}\frac{\mathrm{d}x}{x^3}\frac{\hat{\mathcal{M}}_0(x)\sin\delta_0^{\pi\pi\,0}(x)}{\left|\Omega_0^{\pi\pi\,0}(x)\right|(x-s)}\right\}$$

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System of integral equations solved by iterations

$$\mathcal{M}(s) = \alpha\mathcal{M}^{\alpha}(s) + \beta\mathcal{M}^{\beta}(s) + \gamma\mathcal{M}^{\gamma}(s) + \lambda\mathcal{M}^{\lambda}(s)$$

Numerical strategy

1) Basis functions

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The initial guess fix the basis function generated

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Numerical strategy

1) Basis functions

$$\mathcal{M}_0(s) = \Omega_0^{\pi\pi\ 0}(s)\left\{\alpha + \beta\frac{s}{M_{\eta'}^2} + \gamma\frac{s^2}{M_{\eta'}^4} + \frac{s^3}{\pi}\int_{s_0}^{\infty}\frac{\mathrm{d}x}{x^3}\frac{\hat{\mathcal{M}}_0(x)\sin\delta_0^{\pi\pi\ 0}(x)}{\left|\Omega_0^{\pi\pi\ 0}(x)\right|(x-s)}\right\}$$

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The initial guess fix the basis function generated

$$\mathcal{M}_0(s) = s\ \Omega_0^{\pi\pi\ 0}(s)$$

$$\mathcal{M}_0^{\eta\pi}(s) = 0$$

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Numerical strategy

1) Basis functions

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$$\mathcal{M}(s) = \alpha\mathcal{M}^{\alpha}(s) + \beta\mathcal{M}^{\beta}(s) + \gamma\mathcal{M}^{\gamma}(s) + \lambda\mathcal{M}^{\lambda}(s)$$

The initial guess fix the basis function generated

$$\mathcal{M}_0(s) = s\ \Omega_0^{\pi\pi\,0}(s)$$
$$\mathcal{M}_0^{\eta\pi}(s) = 0 \longrightarrow$$

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Numerical strategy

1) Basis functions

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$$\mathcal{M}(s) = \alpha\mathcal{M}^{\alpha}(s) + \beta\mathcal{M}^{\beta}(s) + \gamma\mathcal{M}^{\gamma}(s) + \lambda\mathcal{M}^{\lambda}(s)$$

The initial guess fix the basis function generated

$$\mathcal{M}_0(s) = s\,\Omega_0^{\pi\pi\,0}(s) \qquad \longrightarrow \qquad \mathcal{M}_0^{\beta}(s)$$

$$\mathcal{M}_0^{\eta\pi}(s) = 0 \qquad\qquad \mathcal{M}_0^{\eta\pi\,\beta}(s)$$

2) Inhomogeneities

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$\pi\pi$

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$\pi\pi$

$$\hat{\mathcal{M}}_0(s) = \frac{1}{2}\int_{-1}^{1} dz_{s'} \left(\mathcal{M}_0^{\pi\eta}(t') + \mathcal{M}_0^{\pi\eta}(u') \right) = \frac{2}{\kappa_{\pi\pi}(s)} \int_{t_-(s)}^{t_+(s)} dt' \, \mathcal{M}_0^{\pi\eta}(t')$$

2) Inhomogeneities

$\pi\pi$

$$\hat{\mathcal{M}}_0(s) = \frac{1}{2}\int_{-1}^{1} dz_{s'}\left(\mathcal{M}_0^{\pi\eta}(t') + \mathcal{M}_0^{\pi\eta}(u')\right) = \frac{2}{\kappa_{\pi\pi}(s)}\int_{t_-(s)}^{t_+(s)} dt'\,\mathcal{M}_0^{\pi\eta}(t')$$

$$\kappa_{\pi\pi}(s) = \frac{\lambda^{1/2}\left(s, m_\pi^2, m_\pi^2\right)\lambda^{1/2}\left(s, m_{\eta'}^2, m_\eta^2\right)}{s}$$

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2) Inhomogeneities

$\pi\pi$

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$\eta\pi$

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2) Inhomogeneities

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$\eta\pi$

$$\hat{\mathcal{M}}_0^{\eta\pi}(t) = \int_{-1}^{1} dz_{t'}\left(\mathcal{M}_0^{\pi\eta}(u') - \mathcal{M}_0(s')\right) = \frac{1}{\kappa_{\eta\pi}(t)}\left(\int_{u_-(t)}^{u_+(t)} du'\,\mathcal{M}_0^{\pi\eta}(u') - \int_{s_-(t)}^{s_+(t)} ds'\,\mathcal{M}_0(s')\right)$$

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S. Schneider, PhD thesis, (2012).

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S. Schneider, PhD thesis, (2012). First attempt to describe $\eta' \to \eta\pi\pi$, not successful

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T. Isken, PhD thesis, (2021).

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2) Inhomogeneities

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S. Schneider, PhD thesis, (2012). First attempt to describe $\eta' \to \eta\pi\pi$, not successful

T. Isken, PhD thesis, (2021). Inclusion of high energy inelastic effects in the $\pi\pi$ and $\eta\pi$ phase shifts

$$\kappa_{\eta\pi}(s) = \frac{\lambda^{1/2}\left(s, m_\eta^2, m_\pi^2\right) \lambda^{1/2}\left(s, m_{\eta'}^2, m_\pi^2\right)}{s}$$

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2) Inhomogeneities

$\pi\pi$

$$\hat{\mathcal{M}}_0(s) = \frac{1}{2}\int_{-1}^{1} dz_{s'}\left(\mathcal{M}_0^{\pi\eta}(t') + \mathcal{M}_0^{\pi\eta}(u')\right) = \frac{2}{\kappa_{\pi\pi}(s)}\int_{t_-(s)}^{t_+(s)} dt'\,\mathcal{M}_0^{\pi\eta}(t')$$

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S. Schneider, PhD thesis, (2012). First attempt to describe $\eta' \to \eta\pi\pi$, not successful

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2) Inhomogeneities

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D. Winney, IterateKT, github repository. KT code with 3π in the final states

Code crosscheck with Akdag2023 reported basis functions.

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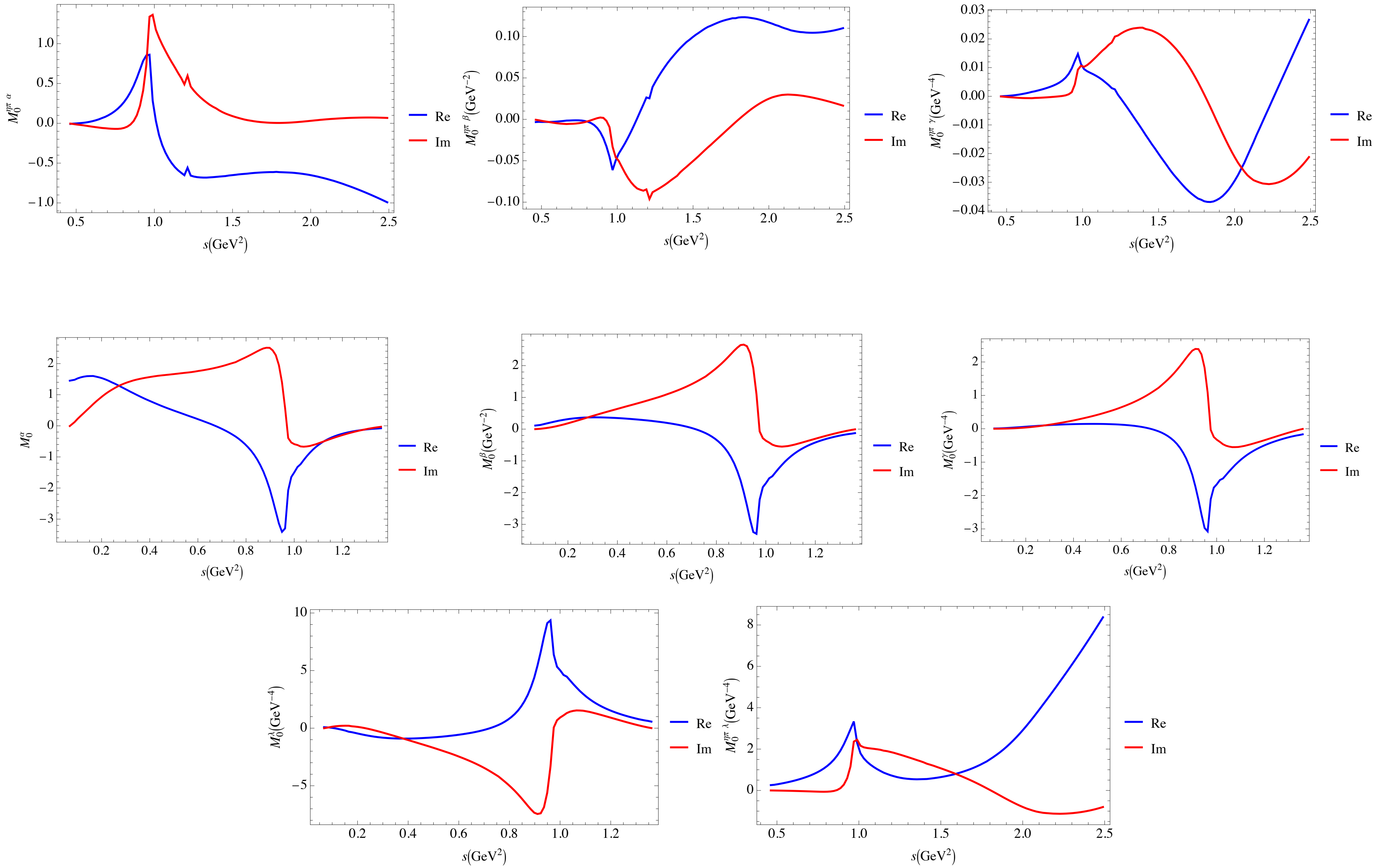

Outline

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1) Motivation
2) Formalism
3) Dispersive relations
4) Numerical strategy
5) Inputs
6) Perspectives and conclusions

Inputs

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$$\text{Im}\left(\mathscr{T}_I^{ab\ 0}(s)\right) = \rho_{ab}(s)\mathscr{T}_I^{ab\ 0\ *}(s)\mathscr{T}_I^{ab\ 0}(s)$$

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Inputs

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$$\mathrm{Im}\left(\mathcal{T}_I^{ab\;0}(s)^{-1}\right) = -\rho_{ab}(s)\theta(s-(m_a+m_b)^2)$$

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Subtraction

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Subtraction

Left-Hand-Cut (LHC)

Inputs

$$\text{Im}\left(\mathcal{T}_I^{ab\,0}(s)\right) = \rho_{ab}(s)\mathcal{T}_I^{ab\,0\,*}(s)\mathcal{T}_I^{ab\,0}(s)$$

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Subtraction

Left-Hand-Cut (LHC)

Right-Hand-Cut (RHC)

Inputs

$$\mathrm{Im}\left(\mathcal{T}_I^{ab\,0}(s)\right) = \rho_{ab}(s)\mathcal{T}_I^{ab\,0\,*}(s)\mathcal{T}_I^{ab\,0}(s)$$

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Analytic in the s complex plane except in the LHC

$$\mathcal{T}_I^{ab\,0}(s)^{-1} = \underbrace{\mathcal{T}_I^{ab\,0}(\tilde{s}_M)^{-1}}_{\text{Subtraction}} + \underbrace{\frac{s-\tilde{s}_M}{\pi}\int_L \frac{ds'}{s'-\tilde{s}_M}\frac{\mathrm{Im}\left(\mathcal{T}_I^{ab\,0}(s')^{-1}\right)}{s'-s}}_{\text{Left-Hand-Cut (LHC)}} + \underbrace{\frac{s-\tilde{s}_M}{\pi}\int_{s0}^{\infty}\frac{ds'}{s'-\tilde{s}_M}\frac{-\rho_{ab}(s')}{s'-s}}_{\text{Right-Hand-Cut (RHC)}}$$

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$$\mathrm{Im}\left(\mathcal{T}_I^{ab\,0}(s)\right) = \rho_{ab}(s)\mathcal{T}_I^{ab\,0\,*}(s)\mathcal{T}_I^{ab\,0}(s)$$

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$$\mathcal{T}_I^{ab\,0}(s)^{-1} \approx \sum_{n=0} C_n\omega^n(s) + \frac{s-\tilde{s}_M}{\pi}\int_{s0}^{\infty}\frac{ds'}{s'-\tilde{s}_M}\frac{-\rho_{ab}(s')}{s'-s}$$

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Left-Hand-Cut (LHC)

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$\omega(s)$ is a conformal mapping that transforms the LHC into the boundary circle $|\omega|=1$ in the ω complex plane. The rest of the s complex plane transforms into the interior of the circle. $\omega(s)$ is real if s belongs to $[s_0,\infty)$.

Inputs

$$\text{Im}\left(\mathcal{T}_I^{ab\,0}(s)\right) = \rho_{ab}(s)\mathcal{T}_I^{ab\,0\,*}(s)\mathcal{T}_I^{ab\,0}(s)$$

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Subtraction Left-Hand-Cut (LHC) Right-Hand-Cut (RHC)

I. Danilkin et al., Phys. Rev. D 107, 074021 (2023).

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Perspectives and conclusions

- We have generated pseudo data that reproduces the experimental fit of BESIII
- We have coded a KT code to compute the basis functions. It converges after 1 or 2 iterations in the majority of the cases. This code have been crosschecked with the basis functions reported in Akdag2023.
- The next step is to fit the pseudo data with the basis functions that generates the dispersive inverse amplitude to fix the conformal mapping parameters.
- The most important challenge is to constrain well enough the dispersive inverse amplitude with χPT inputs.

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