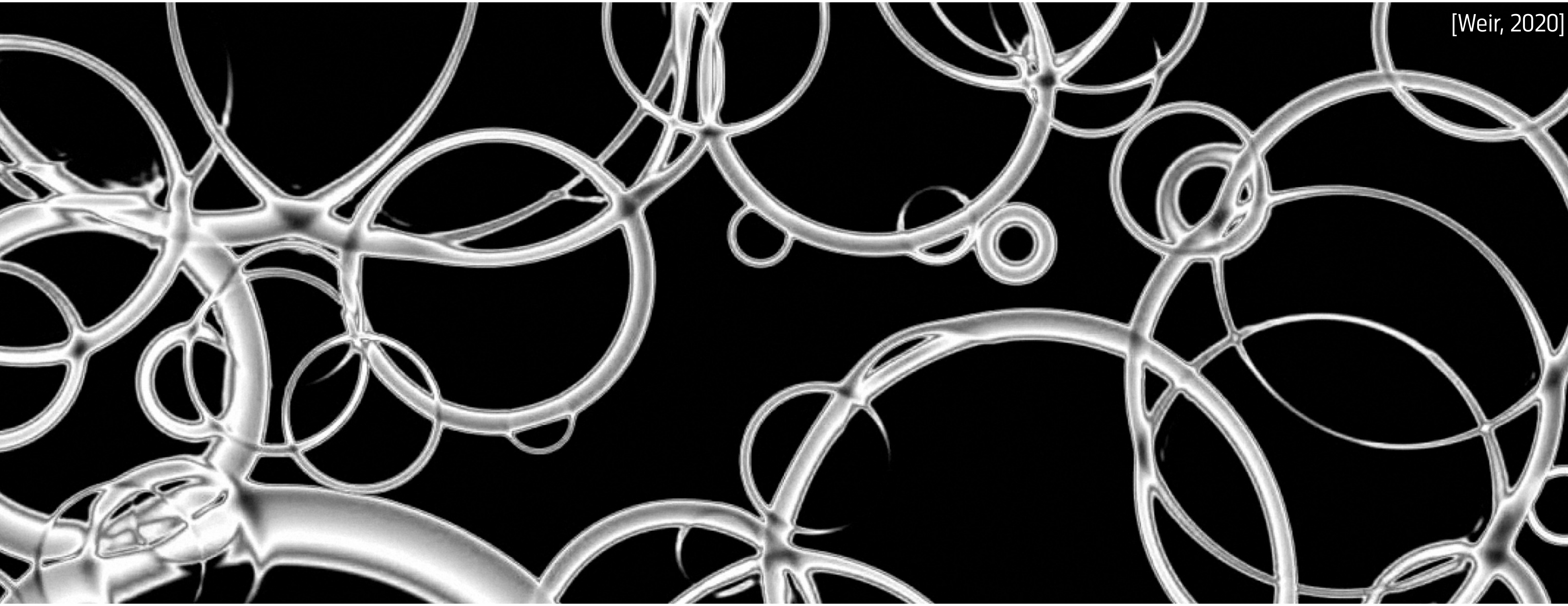
Running Couplings in High-Temperature EFT



[Weir, 2020]

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Based on 2510.26878 (M. Chala, AD, G. Guedes)

High Energy Theory

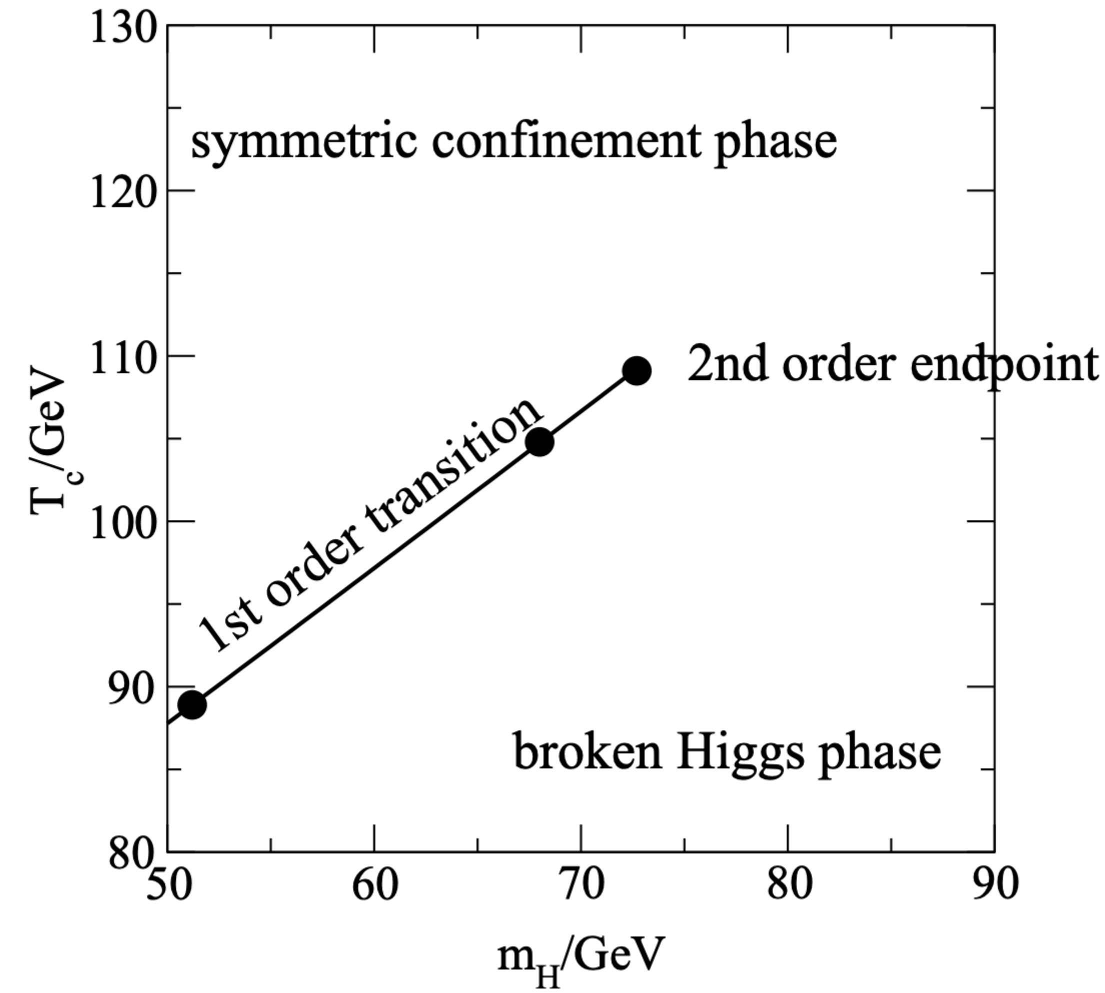
XVII CPAN Days, 19.11.2025, Valencia

Electroweak Phase Transitions (EWPT): Motivation

- Spontaneous EW symmetry breaking — explains masses of elementary particles

If EWPT 1st order:

- Mechanism to satisfy **Sakharov's conditions** for generation of the matter-antimatter asymmetry
- Natural in many **BSM** scenarios
- Perfect candidate for **gravitational wave detection** in close future experiments (LISA, etc)



[Laine, 2000]

Electroweak Phase Transitions: Calculation pipeline

$\mathcal{L}_{\mathrm{4D}}$

EWPT observables

$(T_n, \alpha, v_{\mathrm{wall}}, \Omega_{\mathrm{GW}}, \ldots)$

Electroweak Phase Transitions: Calculation pipeline

$\mathcal{L}_{4\mathrm{D}}$

EWPT observables
$(T_n, \alpha, v_{\mathrm{wall}}, \Omega_{\mathrm{GW}}, \dots)$

- Requires adhoc thermal resummations
  i.e. separation of thermal scales are not taken into account

Electroweak Phase Transitions: Calculation pipeline

$$\begin{tikzcd} \mathcal{L}_{\mathrm{4D}} \arrow[rr, bend left] \arrow[dr, bend right] & & \text{EWPT observables} \ (T_n, \alpha, v_{\mathrm{wall}}, \Omega_{\mathrm{GW}}, \ldots) \\ & \mathcal{L}_{\mathrm{3D\text{-}EFT}} & \end{tikzcd}$$

- <u>Construction of EFT</u>
  i.e. systematic
  accounting of separation
  of scales $T \gg m_h$

Electroweak Phase Transitions: Calculation pipeline

EWPT observables

$$\begin{tikzcd} \mathcal{L}_{\mathrm{4D}} \arrow[rr, bend left] \arrow[dr, bend right] & & (T_n, \alpha, v_{\mathrm{wall}}, \Omega_{\mathrm{GW}}, \dots) \\ & \mathcal{L}_{\mathrm{3D\text{-}EFT}} \arrow[ur, bend right] & \end{tikzcd}$$

- <u>Perturbative methods</u>

  RGEs needed for precision calculations, i.e. resumming logarithms in EFT

- <u>Lattice simulations</u>

  RGEs are crucial in simulations
  i.e. establishing relations between lattice and continuum

Effective Lagrangian

BSM

$\tilde{g}, \Lambda$

Effective Lagrangian

BSM

$\tilde{g}, \Lambda$

$$\Lambda \gtrsim T$$

SMEFT

Effective Lagrangian

BSM

$\tilde{g}, \Lambda$

$\Lambda \gtrsim T$

SMEFT

$T \gg m_h$

See talk of M.C. Fiore

3D-EFT

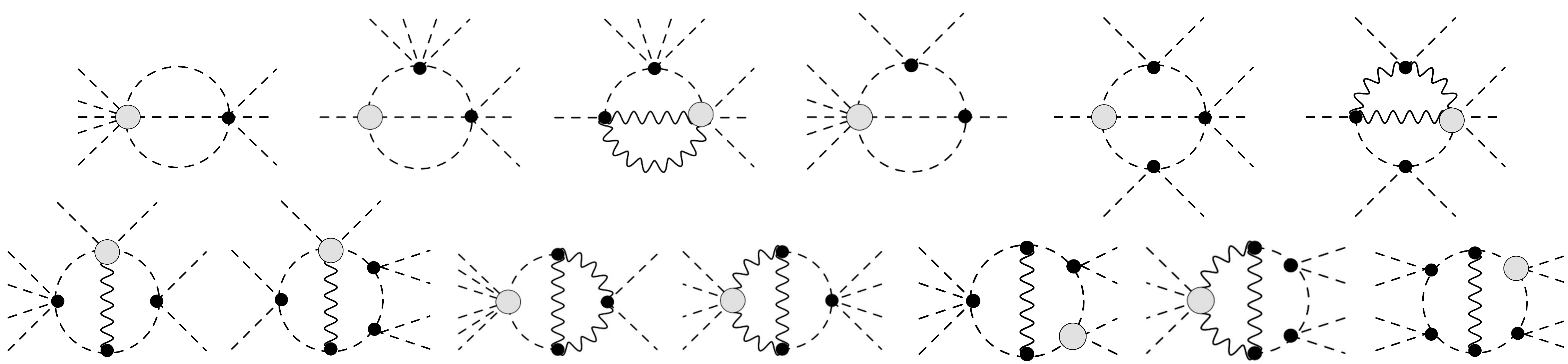
$$\mathcal{L} = \frac{1}{4} W^I_{rs} W^I_{rs} + (D_r\phi)^\dagger (D_r\phi) + m_3^2 |\phi|^2 + \lambda_3 |\phi|^4 + c_{\phi^6} |\phi|^6$$

$$+ c^{(1)}_{\phi^4 D^2} |\phi|^2 \Box |\phi|^2 + c^{(2)}_{\phi^4 D^2} (\phi^\dagger D_r \phi)^\dagger \phi^\dagger D_r \phi + r^{(3)}_{\phi^4 D^2} |\phi|^2 (D_r\phi)^\dagger D_r \phi$$

$$+ c_{\phi^8} |\phi|^8 + c_{\phi^2 W^2} |\phi|^2 W^I_{rs} W^I_{rs} + \dots$$

RGE calculation

- We calculate the leading <u>two-loop</u> contributions to the UV divergences in dimensional regularization of the effective parameters
  (with up to one insertion of the dimension-four operators)



Diagrams for the octic two-loop divergences

Beta functions

- UV-divergences are then used to calculate RGE running according to beta functions

$$\beta_{m_3^2} = \overbrace{-\frac{51}{16}g_3^4 - 9g_3^2\lambda_3 + 12\lambda_3^2}^{\beta^{\mathrm{LO}}_{m_3^2}}$$
$$+ m_3^2\Big[c^{(1)}_{\phi^4 D^2}\left(-14g_3^2 + 32\lambda_3\right)$$
$$+ c^{(2)}_{\phi^4 D^2}\left(3g_3^2 - 8\lambda_3\right) - 20c_{\phi^2 W^2}g_3^2\Big]$$

… (find others in the paper) …

$$\beta_{c_{\phi^8}} = 12c_{\phi^6}\left(296c^{(1)}_{\phi^4 D^2}c_{\phi^6} - 71c^{(2)}_{\phi^4 D^2}c_{\phi^6} + 88c_{\phi^8}\right)$$

Scaling relations for strong first order phase transition

- Requiring the PT to be strong 1st order implies $v \sim T$

$$m_3^2 T \sim \lambda_3 T^2 \sim c_{\phi^6} T^3$$

$$\tilde{g}^2 \sim g\frac{\Lambda}{T}$$

- This implies the relative sizes of the effective couplings

$$m_3^2 \sim g^2 T^2, \lambda_3 \sim g^2 T, c_{\phi^6} \sim g^2$$

$$c_{\phi^4 D^2} \sim g\frac{1}{\Lambda},\ c_{\phi^8} \sim g^3\frac{1}{\Lambda},\ c_{\phi^2 W^2} \sim g^3\frac{1}{\Lambda}$$

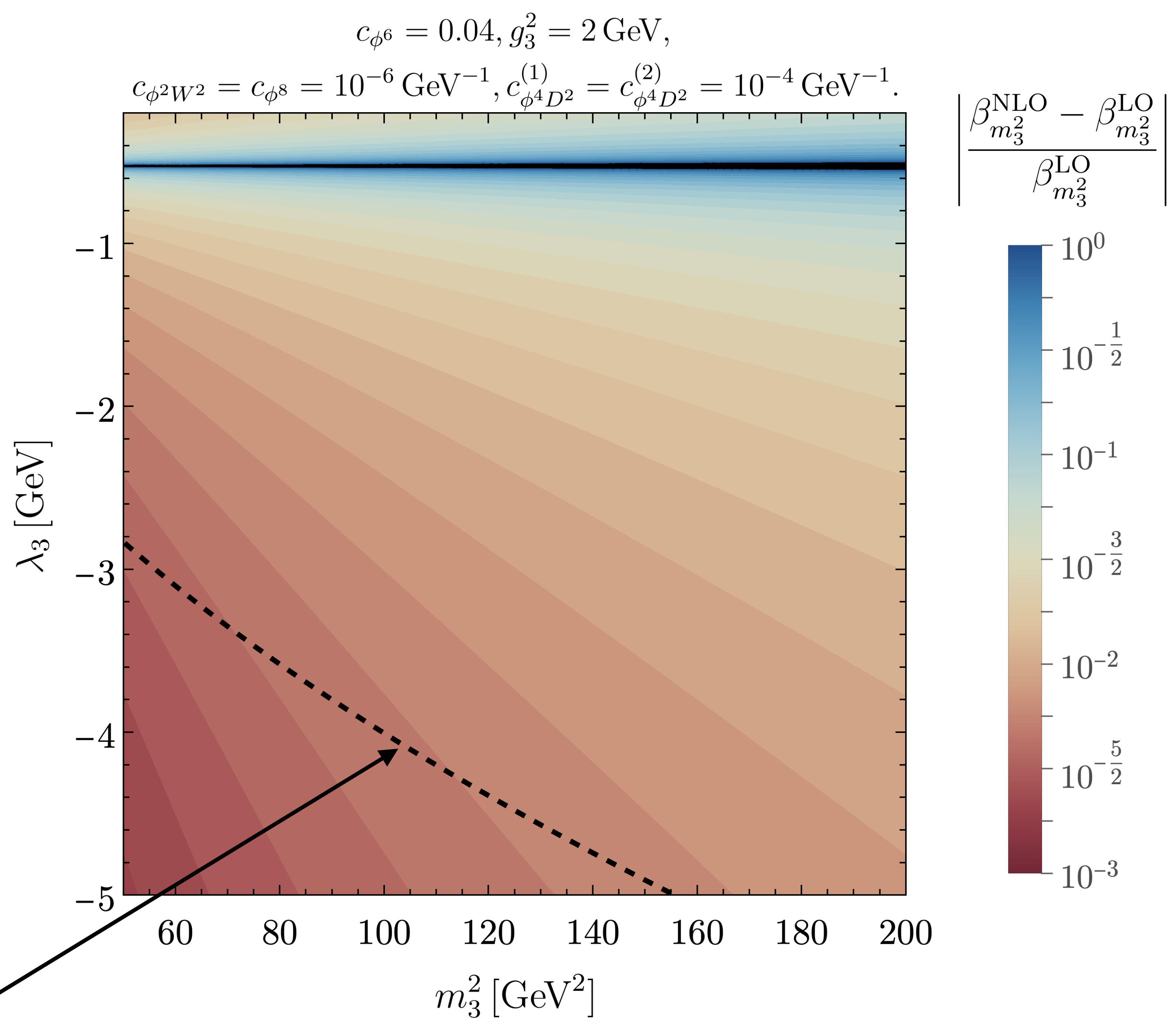
Scaling relations for strong first order phase transition

- Requiring the PT to be strong 1st order implies $v \sim T$

$$
\begin{aligned}
\frac{\partial \log m_3^2}{\partial \log \mu_{3D}} &\sim \overbrace{g^2}^{\text{LO}} + g^3 \frac{T}{\Lambda}\,, \\
\frac{\partial \log \lambda_3}{\partial \log \mu_{3D}} &\sim \frac{\partial \log c_{\phi^6}}{\partial \log \mu_{3D}} \sim \frac{\partial \log c_{\phi^8}}{\partial \log \mu_{3D}} \sim g^2 \\
\frac{\partial \log g_3^2}{\partial \log \mu_{3D}} &\sim g^5 \frac{T}{\Lambda}\,.
\end{aligned}
$$

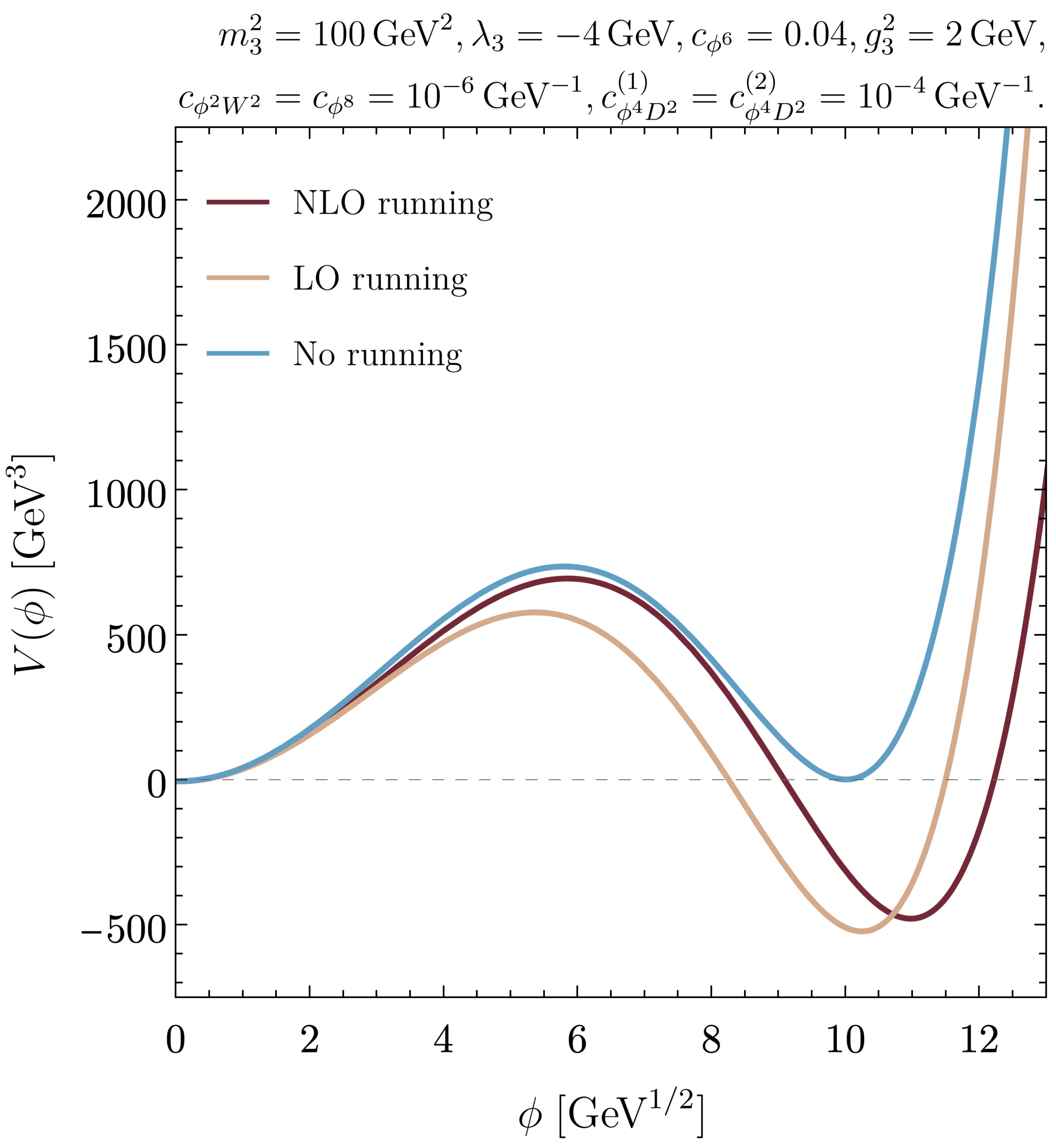
Parametrically of same size as the LO m_3^2-running

Correction to the m_3^2 running

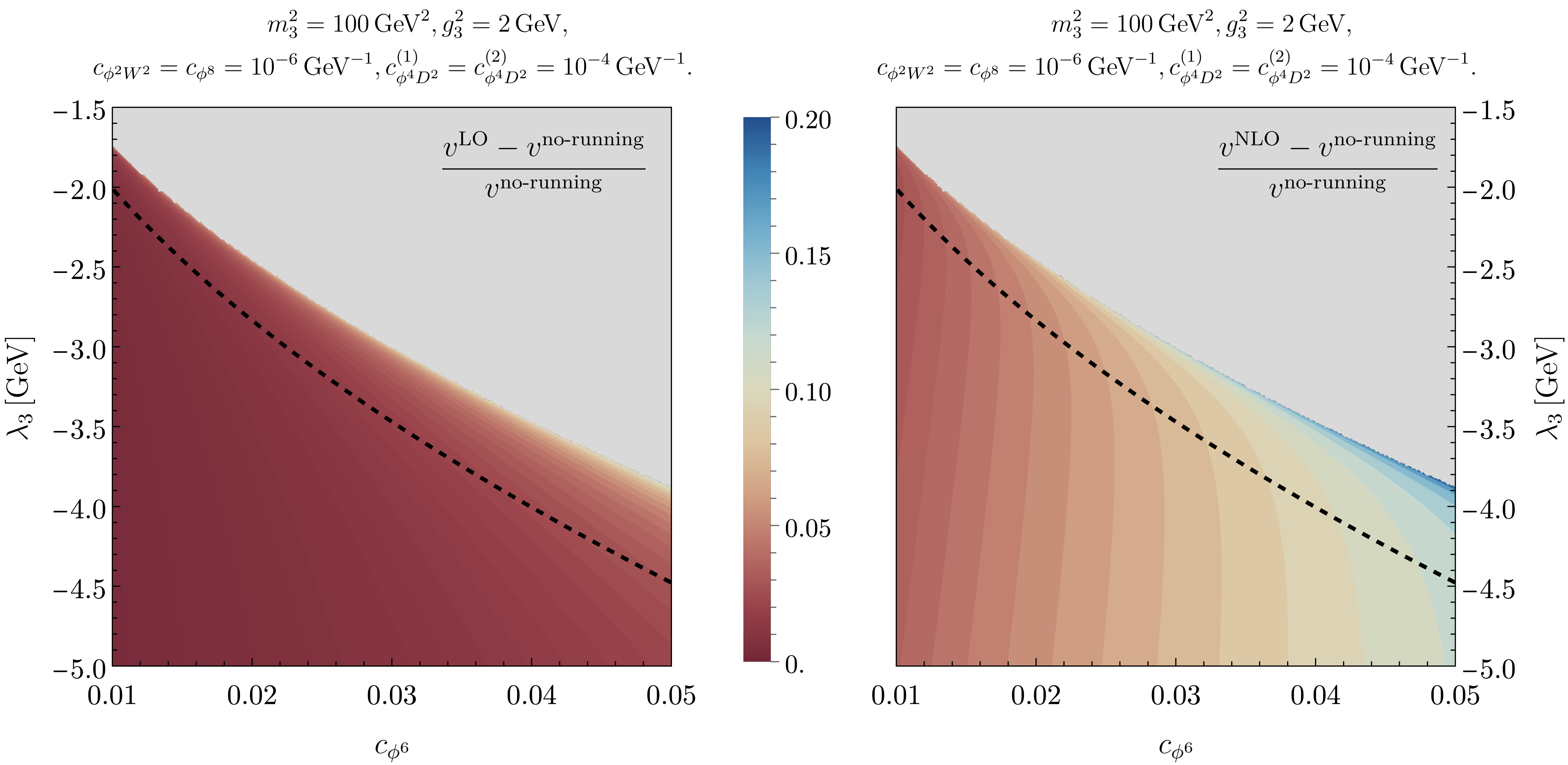


Two local minimas
are of the same depth (i.e. $T = T_c$)

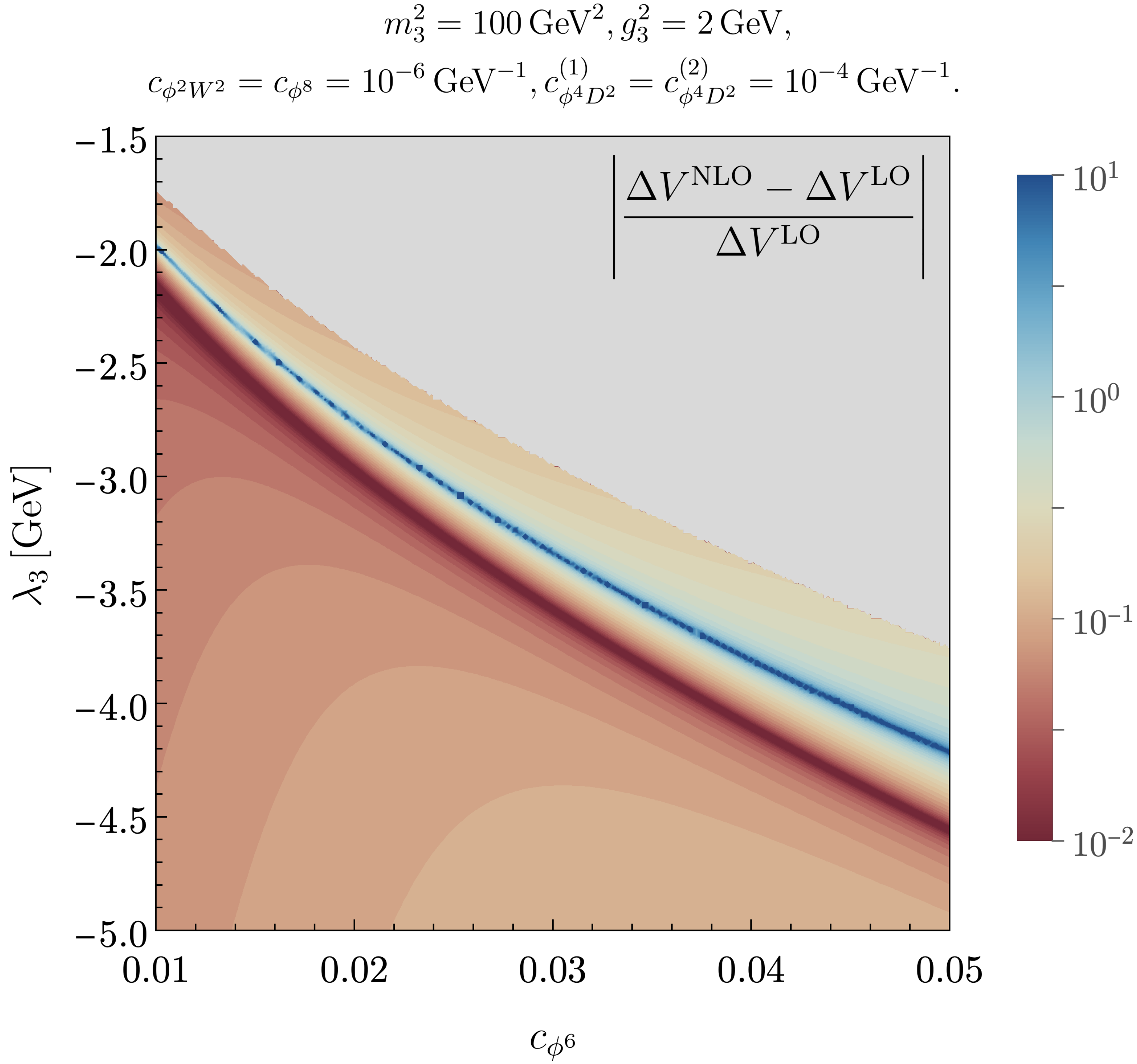
Change in the scalar potential



Shift in the vev value



Shift in the potential value at the minimum



Take-home messages

- EWPT description requires exploitation of separation of thermal scales: EFT techniques
- Precision perturbative calculation require inclusion of RGE running of the effective operators
- Running of higher-dimensional operators (beyond superrenormalizable ones) can be parametrically of the same order as the leading mass parameter running
- Lattice simulations — ultimate tool for study of the EWPT — require the knowledge of RGEs of the effective operators