Universidad de Granada
FTAE
High Energy Theory

New insights into two-loop running in effective field theories

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Two-loop RGEs. Why are they important?

Couplings change with the scale → **Dictated by the RGEs**

Renormalization Group Equations

2-loop RGEs can have a significant impact:
- CP-odd triple-gauge interactions
- Large couplings (top Yukawa) can produce important effects beyond 1-loop

Needed to keep scheme independence in 1-loop matching

After all, we are entering the precision era

Studies in critical phenomena

High-loop calculations needed to extract the spectrum of operators at fixed points

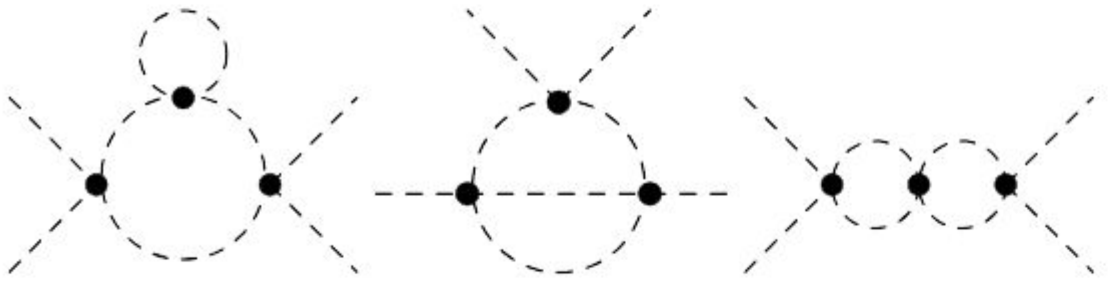
Scalar field theory up to 5 loops

Two-loop RGEs. Computation

RGEs can be obtained directly from the divergences of the theory

Simple scalar theory $\lambda\phi^4$ $\quad \mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \lambda\phi^4$

Relevant 2-loop diagrams:



\+ permutations

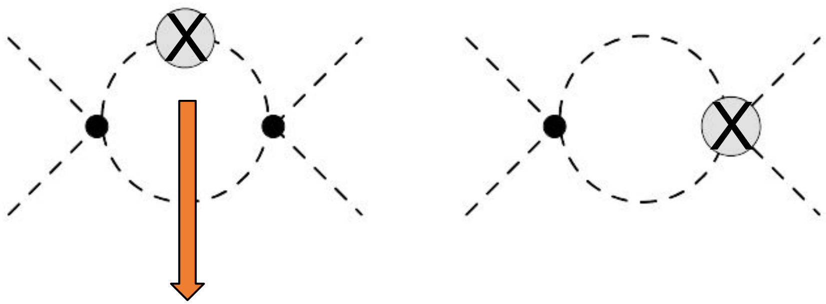
$$\mathcal{A}(\phi\phi\phi\phi) = 64\lambda^3 \int_q \left\{ \frac{108}{q^6 r^2} + \frac{216}{q^2 r^2 (q+r)^4} + \frac{54}{q^2 r^4} \right\}$$

We want UV divergences.
Insertion of an IR mass regulator

Two-loop RGEs. Computation

We forgot about subdivergences!

1-loop diagrams with 1-loop counterterms (CTs):



+ permutations

We need a regulator CT

Need to compute 1-loop CTs and subdivergences

$$\mathcal{A}(\phi\phi\phi\phi) = 64\lambda^3 \int_{qr} \left\{ \frac{108}{(q^2-m^2)^3(r^2-m^2)} + \frac{216}{(q^2-m^2)(r^2-m^2)[(q+r)^2-m^2]^2} + \frac{54}{(q^2-m^2)^2(r^2-m^2)^2} \right\}$$

$$+36 \int_q \left[\frac{16\lambda^2 \delta_{m^2}}{(q^2-m^2)^3} + \frac{4\lambda\delta_\lambda}{(q^2-m^2)^2} \right]$$

Two-loop RGEs. Computation

1. Compute 1-loop counterterms and insert them in 1-loop topologies

2. Regulate IR divergences

   Use an IR mass regulator with $m \to 0$

   Introduce terms such as $\log(m^2)$ which recover the IR divergence in the zero mass limit

   Break gauge invariance for gauge vectors

   Use the R* method

   Algorithmic way to disentangle UV divergences from a Feynman diagram

   Not straightforward to implement

Two-loop RGEs. Computation

New approach using dimensional reduction from 5D to 4D

Dimensional Reduction. Brief Introduction

Typically used in the reduction from 4D to 3D → Thermal Field Theory (at equilibrium)

Focus on 5D to 4D

Spacetime coordinates: $x^M = (x^\mu, y)$

Action (suppose a simple Lagrangian): $S_5 = \int d^4x dy \, \mathcal{L}_5 = \int d^4x dy \left[\frac{1}{2}(\partial_M \phi)^2 - \lambda\phi^4\right]$

Compactify the 5th dimension:

$$\int_{-\infty}^{\infty} dy = \int_0^R dy \qquad \text{and} \qquad \phi_{b/f}(x^\mu, R) = \pm\phi_{b/f}(x^\mu, 0)$$

Use (anti-)periodicity to expand fields in Fourier modes, called Kaluza-Klein modes:

$$\phi(x^\mu, y) = \sum_{n=-\infty}^{\infty} \phi_n(x^\mu) e^{i\omega_n y} \qquad \text{with} \quad \omega_n = \begin{cases} 2n\pi R & \text{for bosons} \\ (2n+1)\pi R & \text{for fermions} \end{cases}$$

Dimensional Reduction. Brief Introduction

$$S_5 = \int d^4x dy \left[\frac{1}{2}(\partial_M \phi)^2 - \lambda\phi^4\right] \to S_4 = \int d^4x \left[\frac{1}{2}\sum_n (\partial_\mu \phi_n)^2 + \frac{1}{2}\sum_n \omega_n^2 \phi_n^2 - \lambda \sum_{m,n,k} \phi_m \phi_n \phi_k \phi_{m+n+k}\right]$$

A single field is decomposed in an infinite tower of fields, each one with mass $\omega_n = 2n\pi R$

If R is big enough we can integrate out every KK mode, except the light zero mode

Compactification of the 5th dimension

$$\underbrace{S_5 \to S_4 \to S_{4\mathrm{EFT}}}_{\text{Dimensional Reduction (DR)}} = \int d^4x \left[\frac{K_4}{2}(\partial_\mu \phi_0)^2 + \frac{1}{2}{m_4}^2 \phi_0^2 - \lambda_4 \phi_0^4\right]$$

Integrating out heavy Matsubara modes

Dimensional Reduction (DR)

2-loop RGEs from Dimensional Reduction

Important Remark

We started with $\lambda\phi^4$ theory in 5D and arrived at $\lambda\phi^4$ theory in 4D

As in every matching procedure, using the Hard Region Expansion:

$$\underbrace{\mathcal{A}_{\mathrm{5D}}}_{\text{finite}} + (\underbrace{\Delta\mathcal{A}_{\mathrm{UV}}}_{\text{UV poles (also considering UV-IR mixing)}} + \underbrace{\Delta\mathcal{A}_{\mathrm{IR}}}_{\text{pure IR poles}}) = \mathcal{A}_{\mathrm{4DEFT}}$$

UV poles correspond to UV poles in the full theory

IR poles correspond to UV poles in the EFT

$\swarrow$

RGEs for the 4DEFT

2-loop RGEs from Dimensional Reduction

If the 5D theory is correctly renormalized (insertion of counterterms):

$$\mathcal{A}_{5\mathrm{D}} + (\cancel{\Delta\mathcal{A}_{\mathrm{UV}}} + \Delta\mathcal{A}_{\mathrm{IR}}) = \mathcal{A}_{4\mathrm{DEFT}}$$

I can read UV divergences for the theory directly from the matching

Renormalization of the 5D theory is very convenient

No 1-loop counterterms for odd-dimensional theory $\rightarrow$ There are no 1-loop topologies with 1-loop CTs insertions

2-loop counterterms have different structure than in 4D

2-loop RGEs from Dimensional Reduction

To illustrate this, imagine that we take $\mathcal{L}_{5D} = \frac{1}{2}(\partial_M \phi)^2 - \lambda\phi^4 + c_{\phi 6}\phi^6$

Mass dimension	In 5D	In 4D
λ	-1	0
$c_{\phi 6}$	-4	-2

If we compute the six-point amplitude, without any kind of renormalization:

$$\mathcal{A}(6\phi)_{\text{div}} = -\frac{1}{\epsilon}\left(\frac{41}{8\pi^4}\lambda^4 - \frac{1701}{16\pi^4}\frac{1}{R^2}\lambda^2 c_{\phi 6} + \frac{783}{2\pi^4}\log\left(R^2\mu^2\right)\lambda^2 c_{\phi 6}\right) - \frac{1}{\epsilon^2}\frac{783}{4\pi^4}\frac{1}{R^2}\lambda^2 c_{\phi 6}$$

How to easily disentangle UV divergences from IR divergences?

- Cannot renormalize $c_{\phi 6}$ in 4D, so this must be a UV divergence. This is just renormalization of the 5D theory.
- Cannot renormalize $c_{\phi 6}$ in 5D, so this must be an IR divergence. This is the renormalization of the 4D theory.

Notice that factors of R are also suggesting the difference…

2-loop RGEs from Dimensional Reduction

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Throw away divergences that do not make sense (dimensionally speaking) in 4D

From the rest you can read the divergences of the 4D theory → 2-loop RGEs

We just performed a matching! No need of renormalizing any theory, treating subdivergences or regulating the IR divergences.

A recap

2-loop RGEs are necessary.

Current precision era require renormalization-group computations beyond one loop. Such requirements also arise in calculations in the context of critical phenomena.

Methods for computing 2-loop RGEs are cumbersome and intricate.

Treatment of subdivergences and IR regulators makes the computation complicated. Other approaches like R* are technically challenging and difficult to implement.

Dimensional Reduction opens a new avenue.

By considering the theory as the IR limit of its 5D analog we can extract the divergences directly from the matching in the DR procedure.

There are some challenges.

Establishing the relation between the 5D and the 4D couplings, not valid for fermions, extra scalars when considering gauge theories.

Thank you for your attention!