From Quarks to TetraQuarks
Heavy-Flavor Fragmentation & Exotic Matter

Francesco Giovanni Celiberto, UAH Madrid



XVI Jornadas CPAN - Valencia

🇪🇸 19/11/2025 🇪🇸

[Phys. Rev. D Letters 111 (2025) 11, L111501 [arXiv:2504.03949]] 🔗

[Phys. Rev. D 112 (2025) 7, 074041 [arXiv:2507.09744]] 🔗

Madrid UAH

talento cm Programa de atracción de talento investigador Comunidad de Madrid

EUGLOH

QCD Hydrogen Atoms

1

¿ Why studying (Heavy) Exotics ?

Precision & Exploration

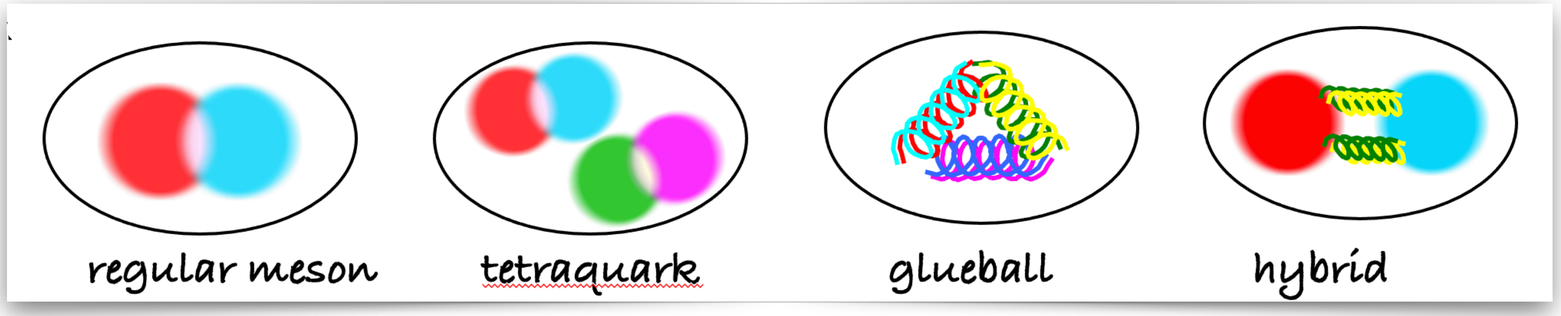
¿ What is the Exotic Matter ?

- Baryons & Mesons → composite states → $|q_1q_2q_3\rangle$ & $|q_1\bar{q}_2\rangle$
- Ordinary Hadrons ← simplest valence-quark colorless states
- QCD color neutrality → TetraQuarks, PentaQuarks, Hybrids allowed
- Exotic Hadrons ← different quantum numbers than ordinary hadrons
- First discovery of an exotic hadron at Belle (🇯🇵) in 2003 → $X(3872)$

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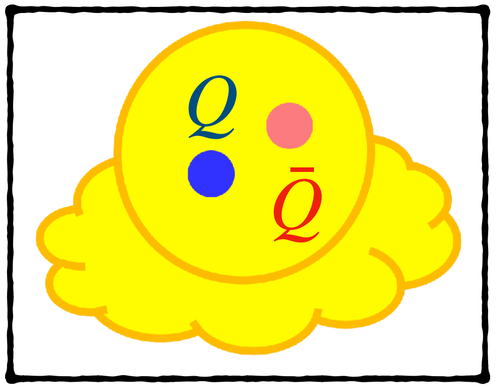
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Hadron Spectroscopy



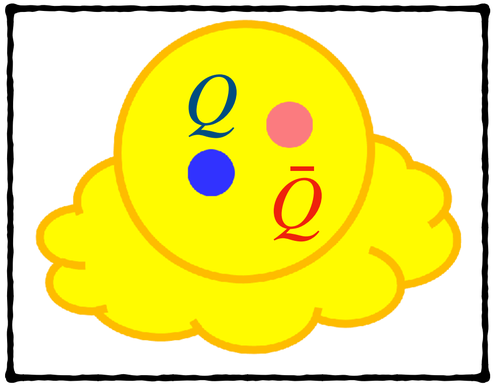
Interpretation of Exotic States

Meson + Continuum

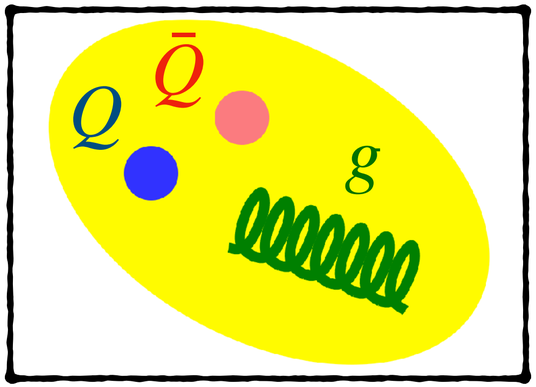


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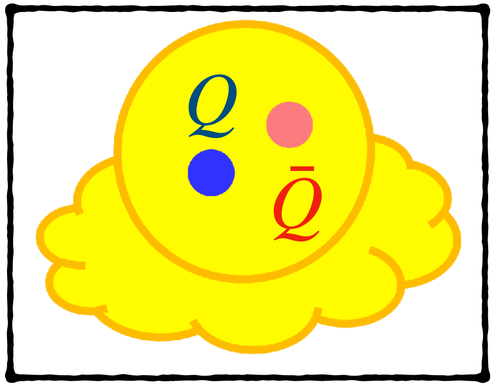


Hybrid

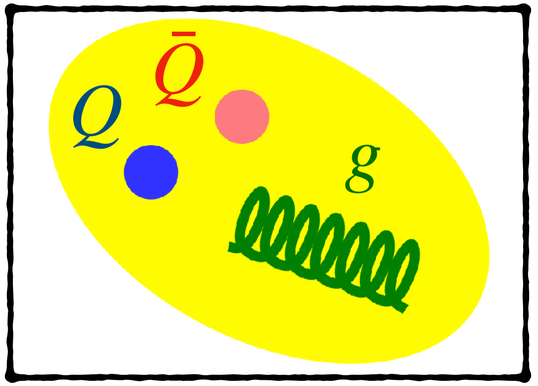


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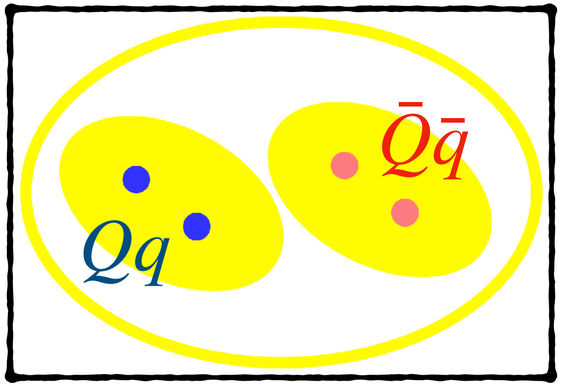
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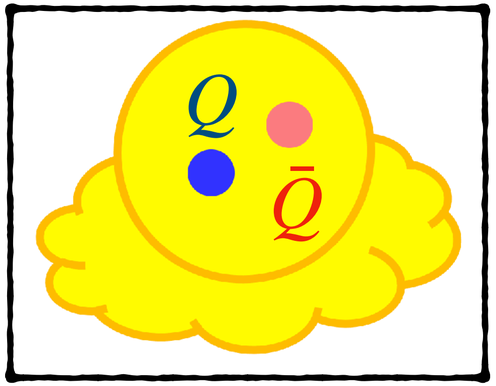


diQuark + AntidiQuark

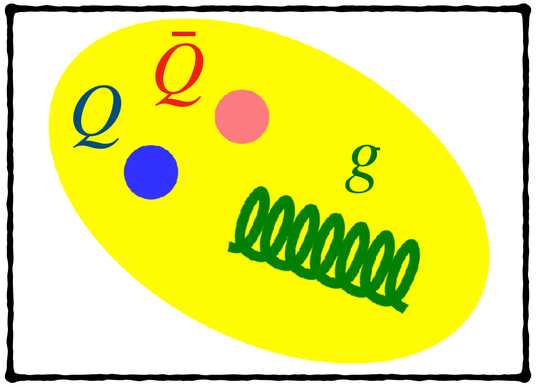


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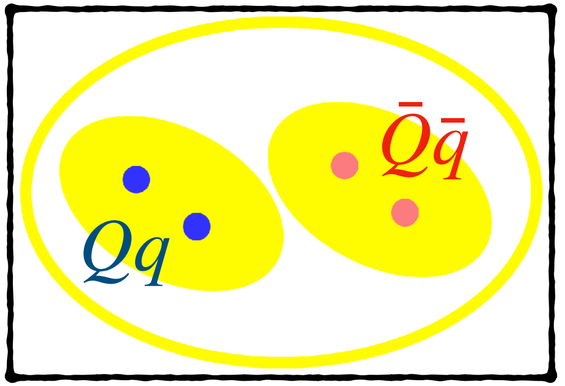
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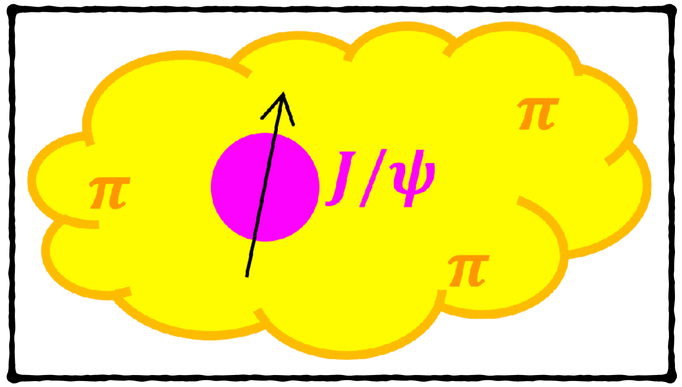


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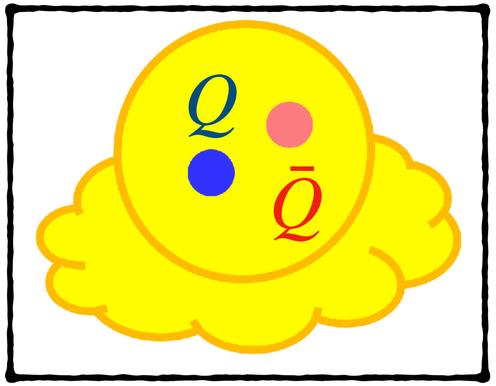




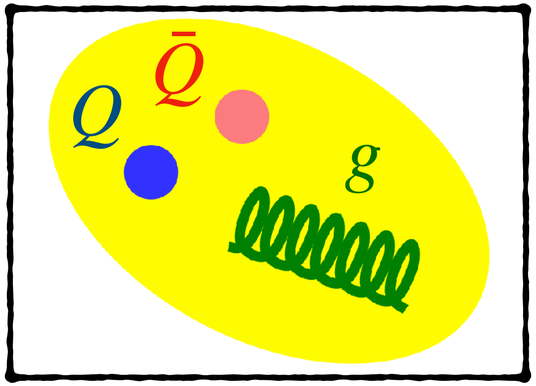
HadroQuarkonium

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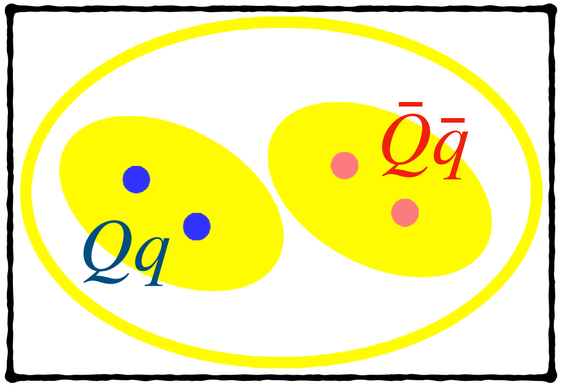
Meson + Continuum

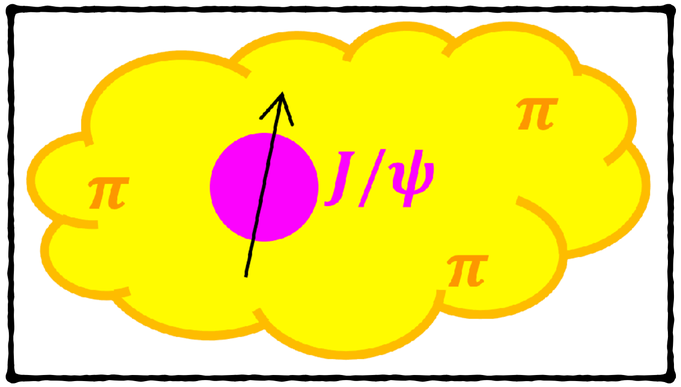


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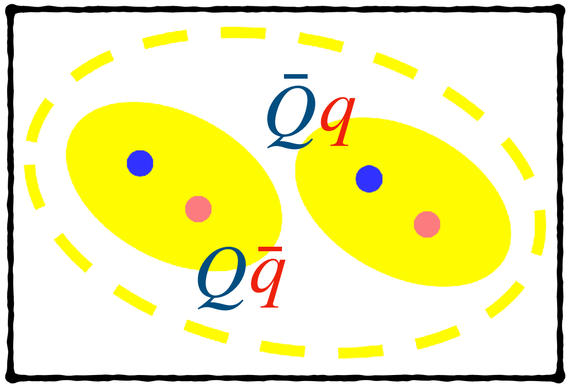


diQuark + AntidiQuark





HadroQuarkonium



Molecule of Mesons

All-Charm TetraQuarks: Experimental Highlights

EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH (CERN)

CERN-EP-2025-118
2025/06/10

CMS-BPH-24-002

Determination of the spin and parity of all-charm tetraquarks

The CMS Collaboration*

arXiv:2506.07944v1 [hep-ex] 9 Jun 2025

Abstract

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Submitted to Nature

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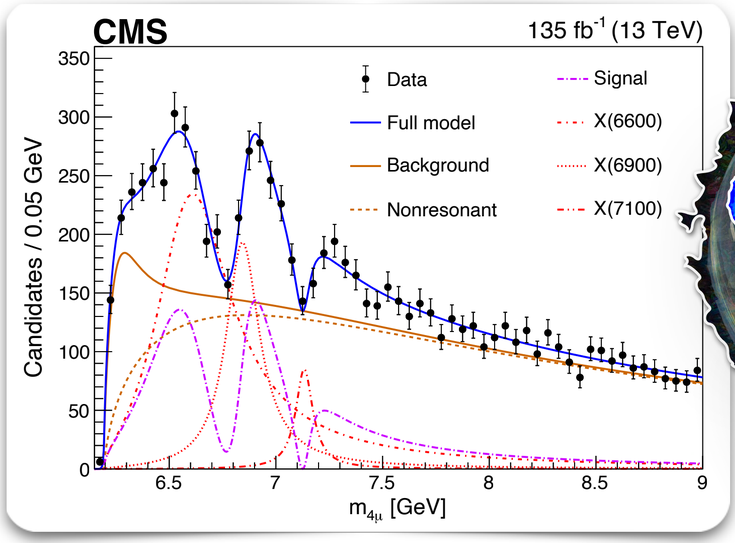
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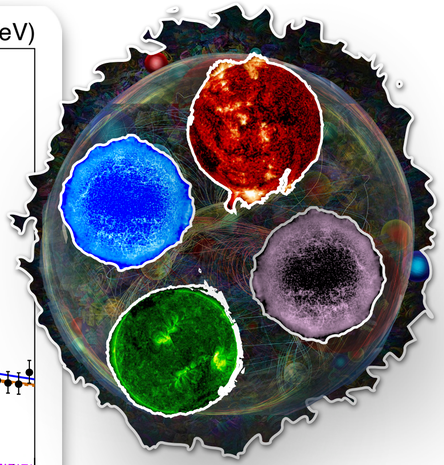
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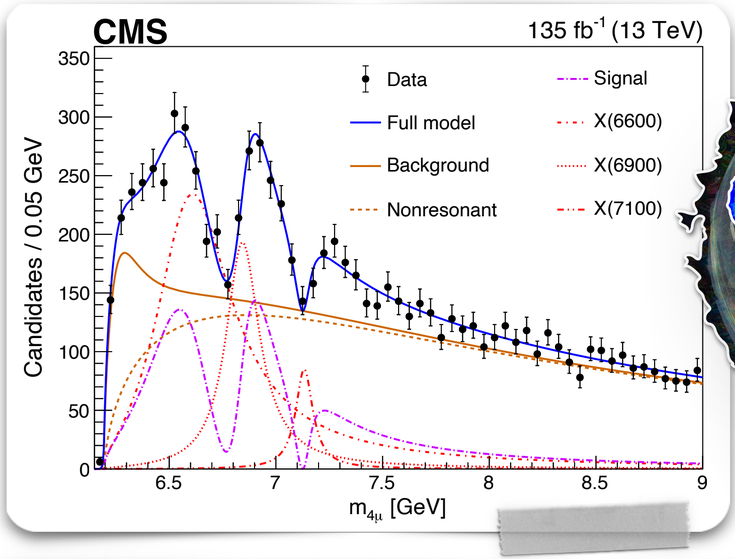
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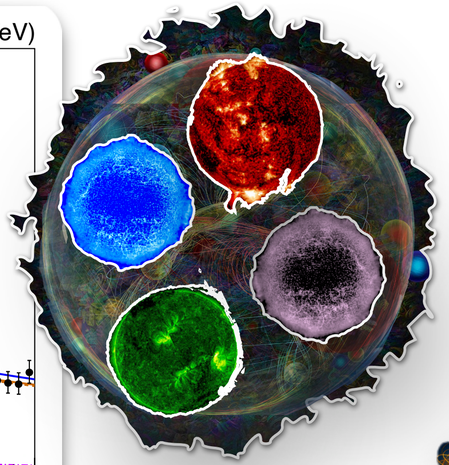
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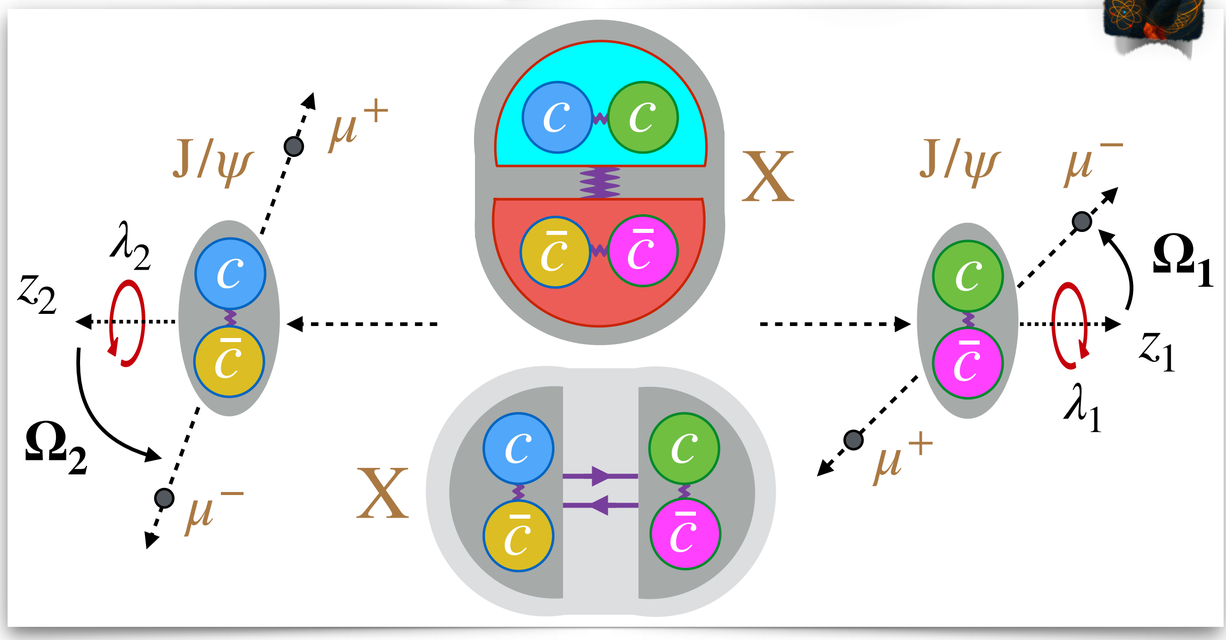
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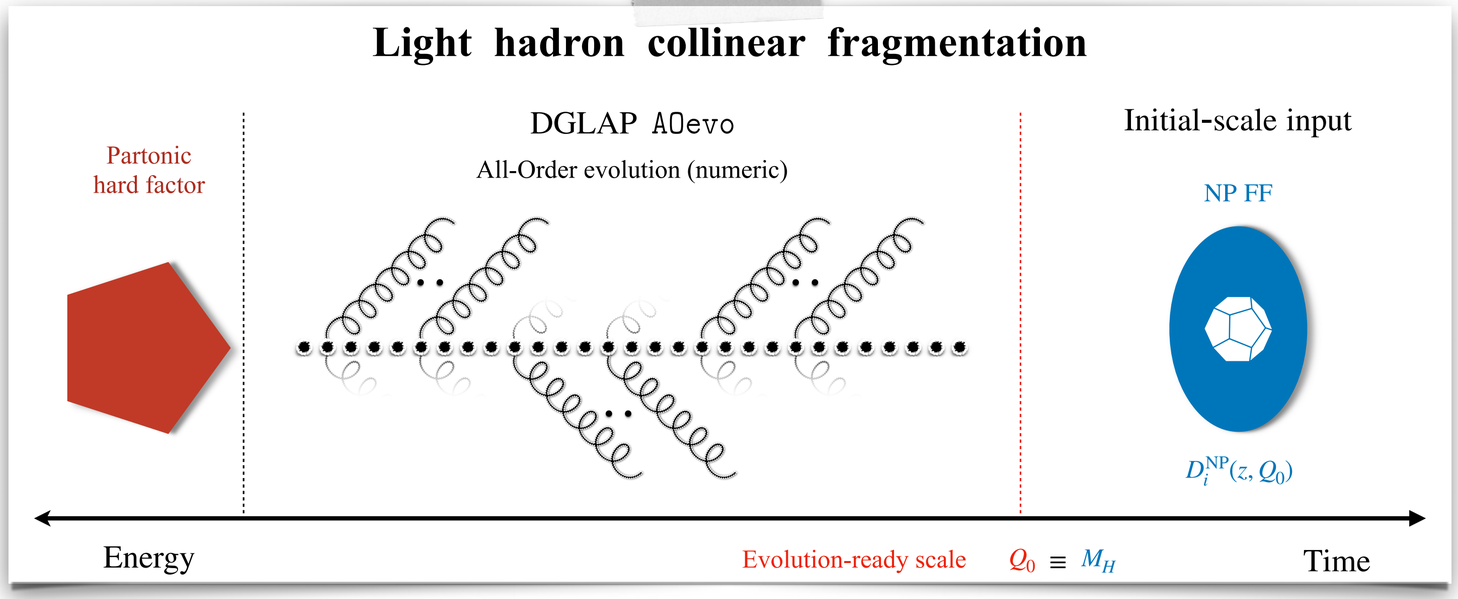




¿ Why Fragmentation is so relevant ?

Fragmentation in QCD (with Heavy Flavors)

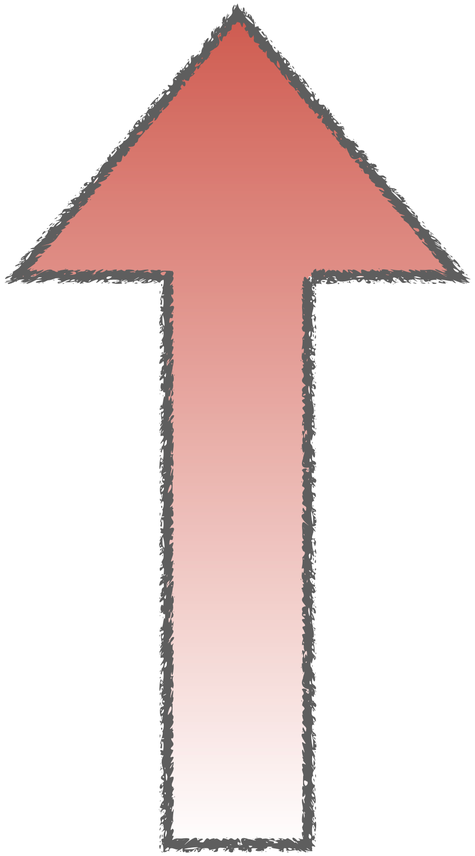
PRECISION CONTROL



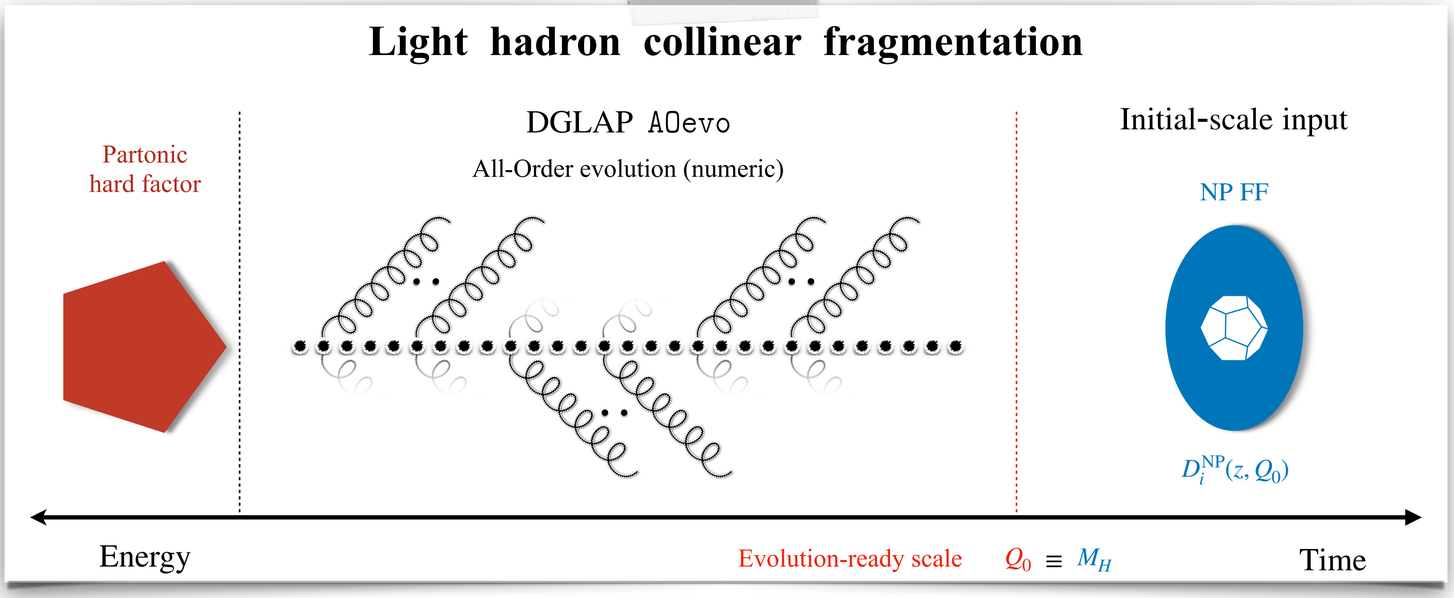
DISCOVERY POTENTIAL

Fragmentation in QCD (with Heavy Flavors)

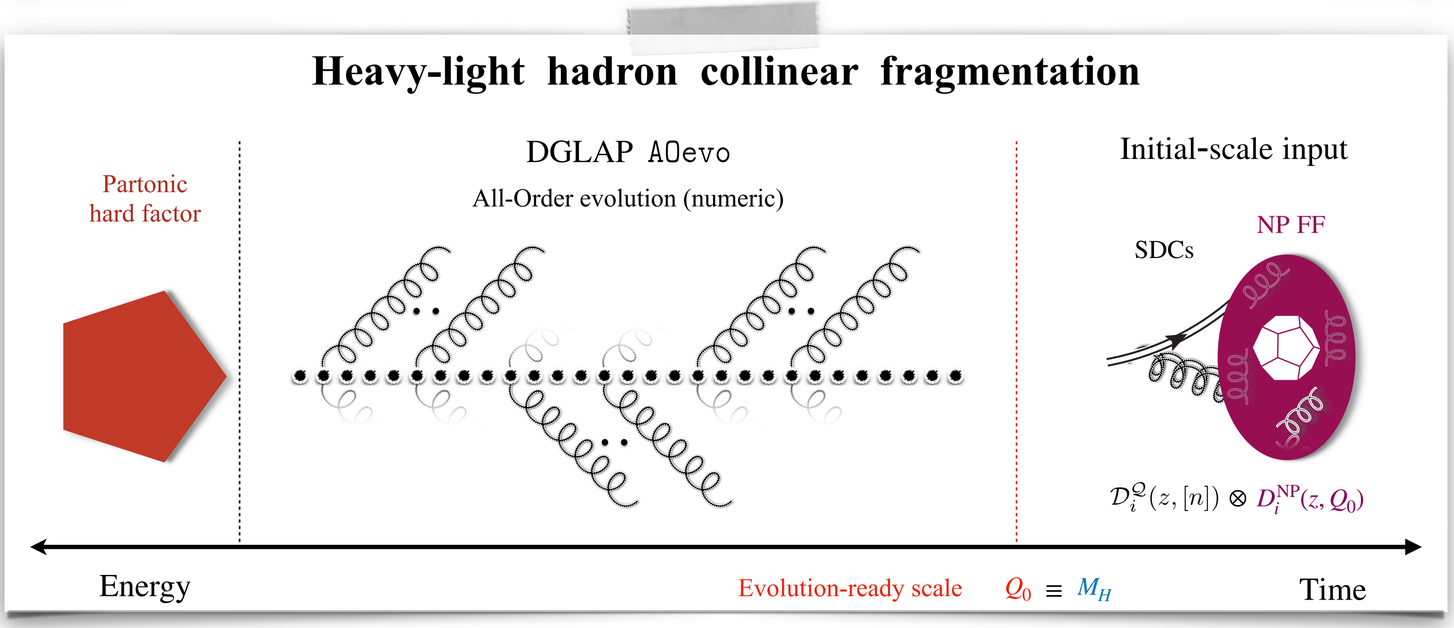
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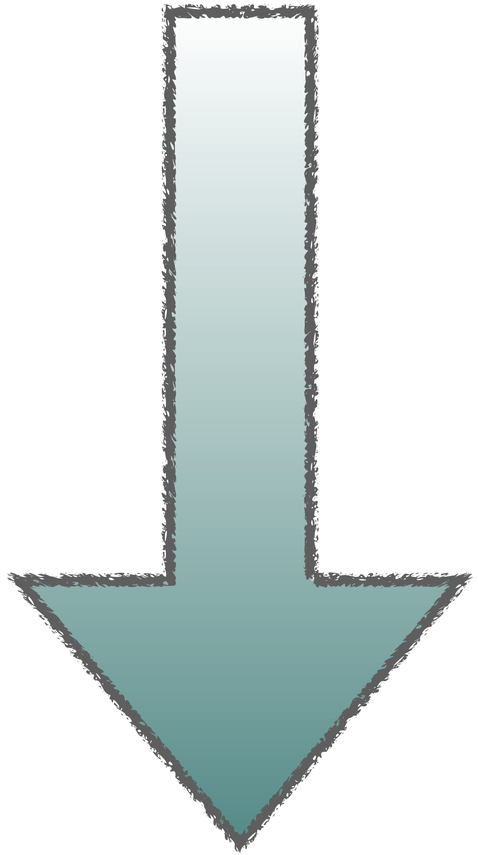


Light hadron collinear fragmentation



Heavy-light hadron collinear fragmentation

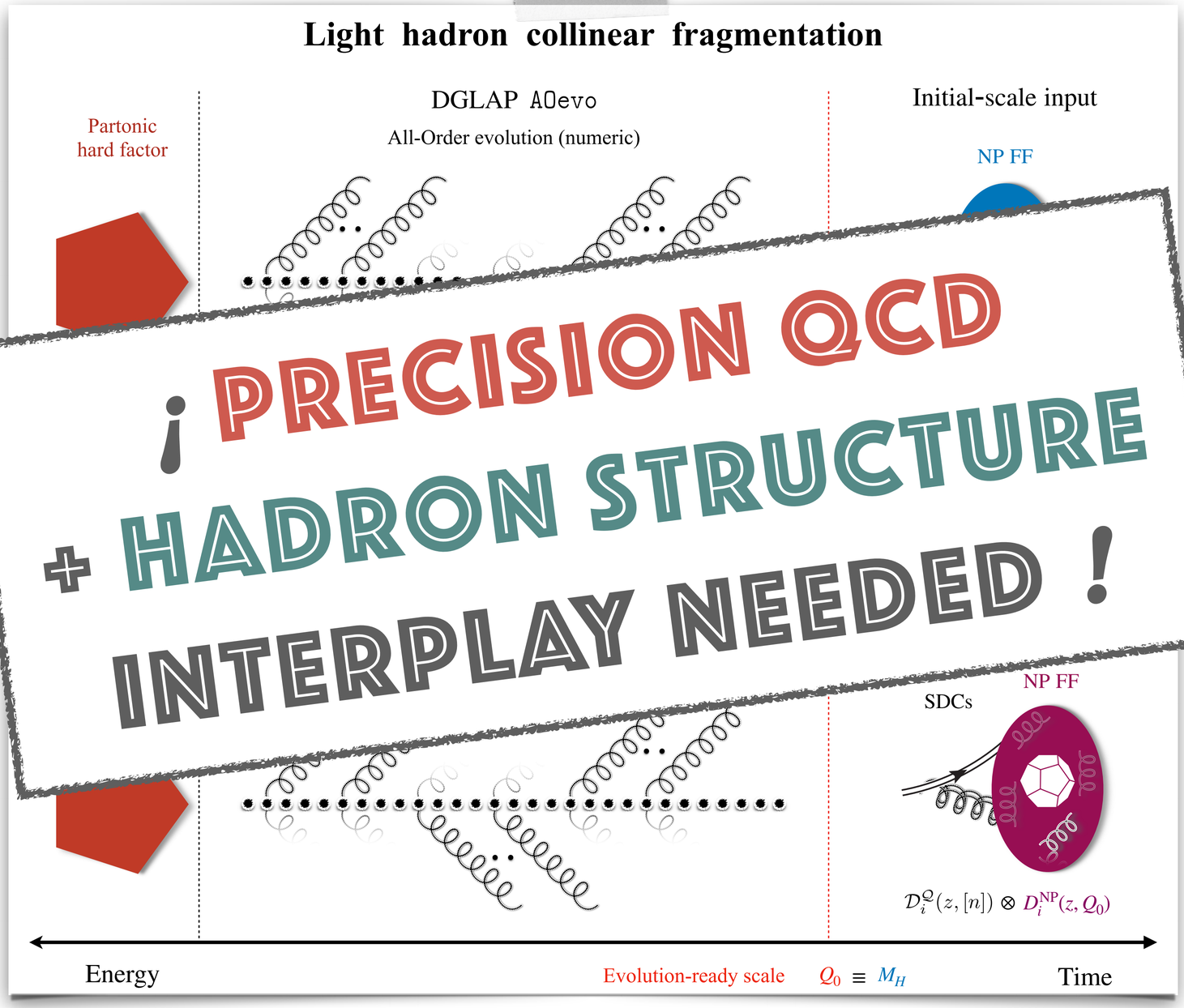




DISCOVERY POTENTIAL

Fragmentation in QCD (with Heavy Flavors)

PRECISION CONTROL



¡ PRECISION QCD + HADRON STRUCTURE INTERPLAY NEEDED !

DISCOVERY POTENTIAL

❷ Heavy-TetraQuark Production via Fragmentation

Heavy-Flavor Non-Relativistic evolution

HADRONIC STRUCTURE

QUARKONIUM THEORY

Heavy-Flavor Non-Relativistic evolution

HADRONIC STRUCTURE

QUARKONIUM THEORY

PRECISION QCD

COLLINEAR FACTORIZATION

Heavy-Flavor Non-Relativistic evolution

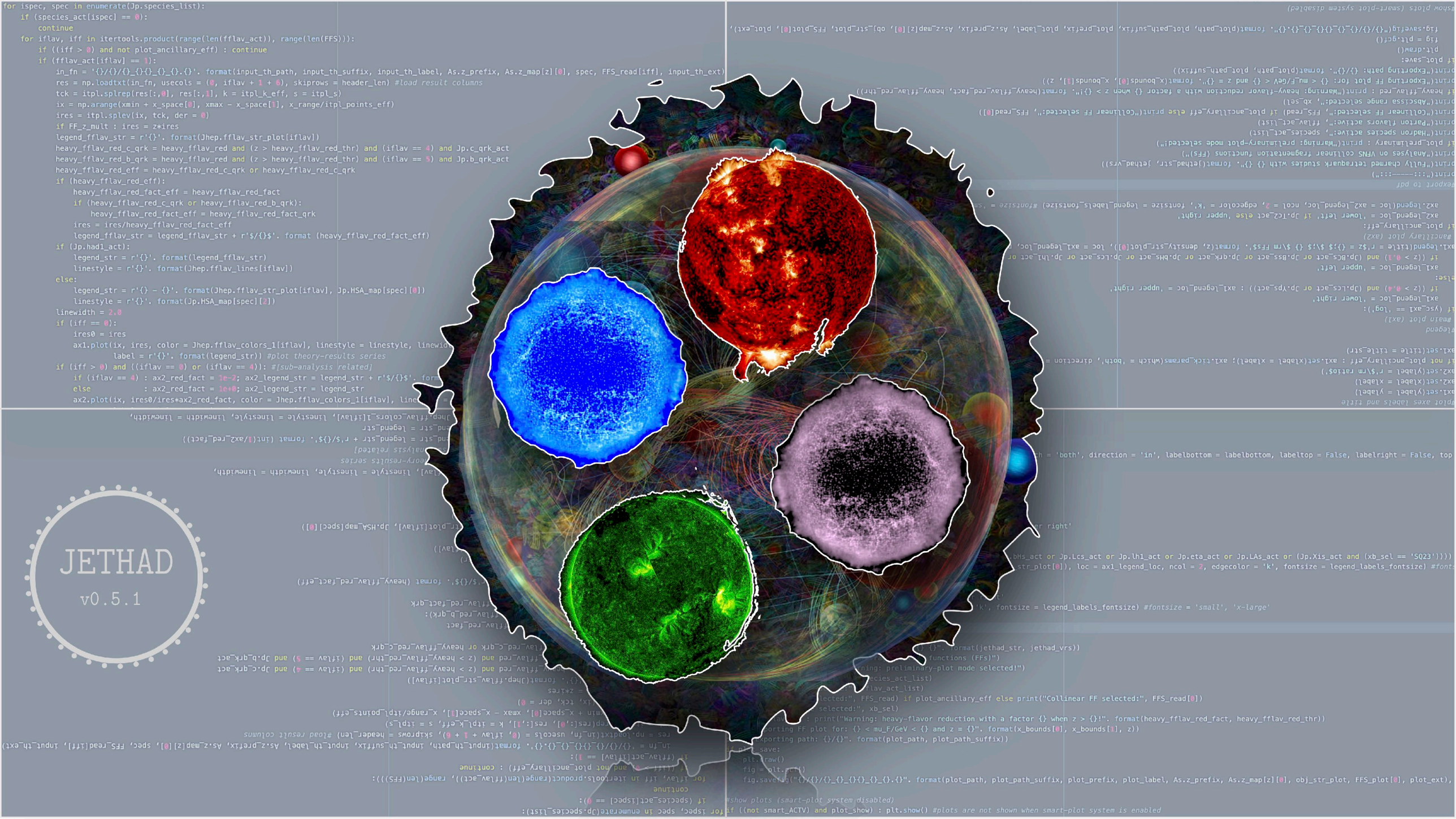
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PRECISION QCD

COLLINEAR FACTORIZATION

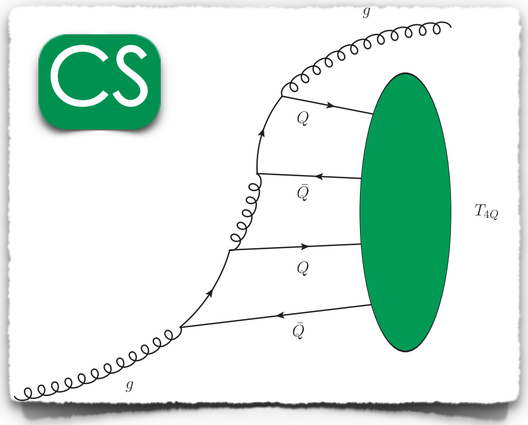
Guiding principle ⇒ ¡ Use the Best of the Two Worlds as much as we can !


JETHAD
v0.5.1

TetraQuark Fragmentation from NRQCD & symJETHAD

LO

Gluon

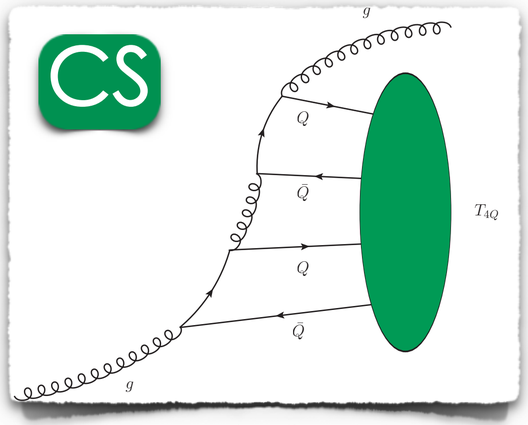


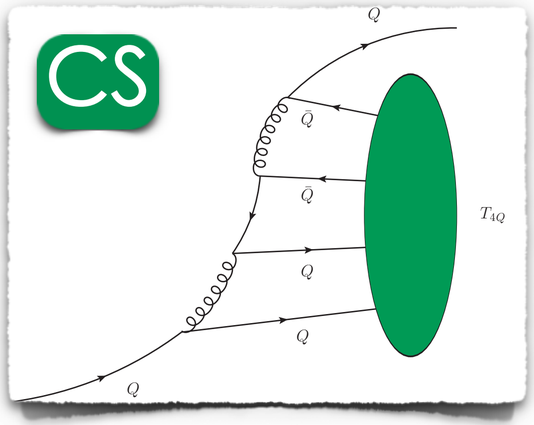
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Constituent heavy quark





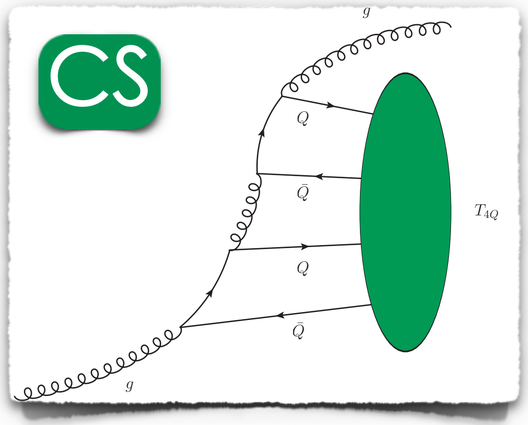
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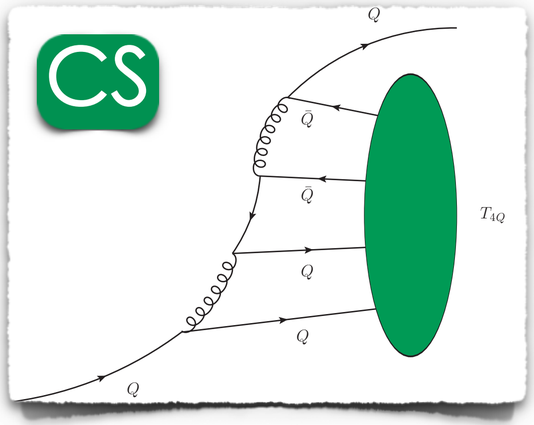
LO

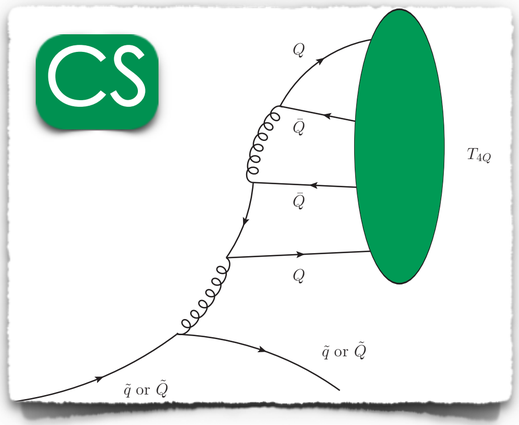
Gluon

Constituent heavy quark

Nonconstituent heavy or light quark







TetraQuark Fragmentation from NRQCD & symJETHAD

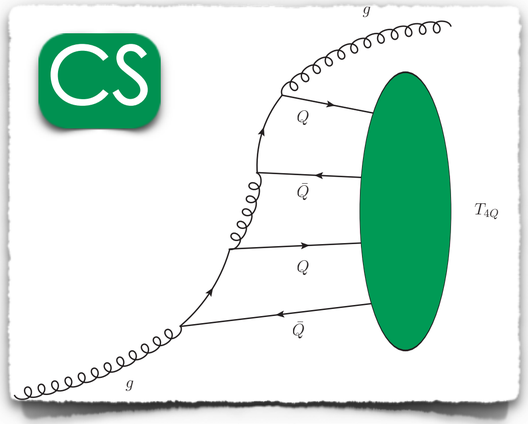
LO

Gluon

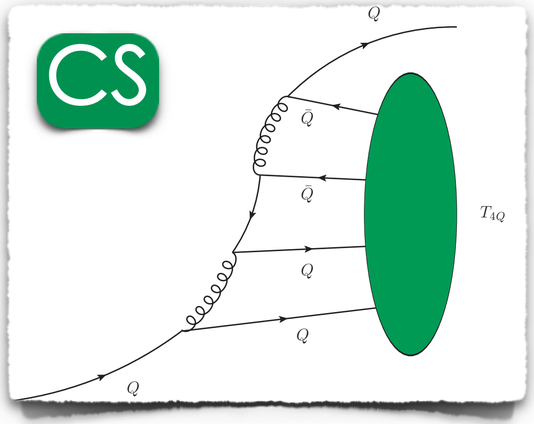
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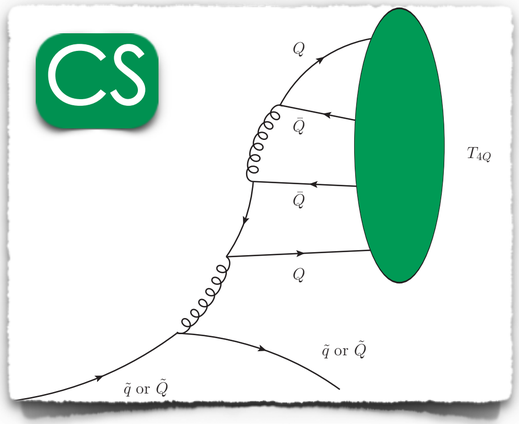
CS



CS



CS



$J^{PC} = 0^{++}$





To be implemented
LO: evolution only

¡!

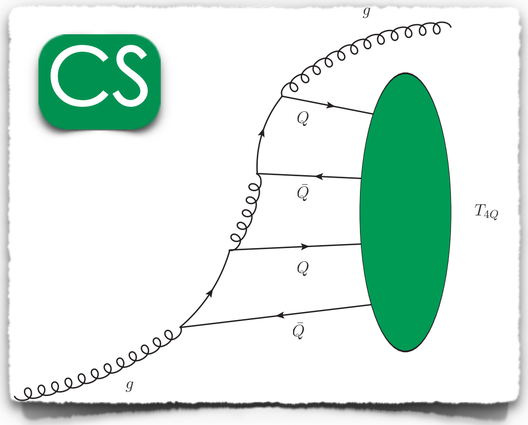
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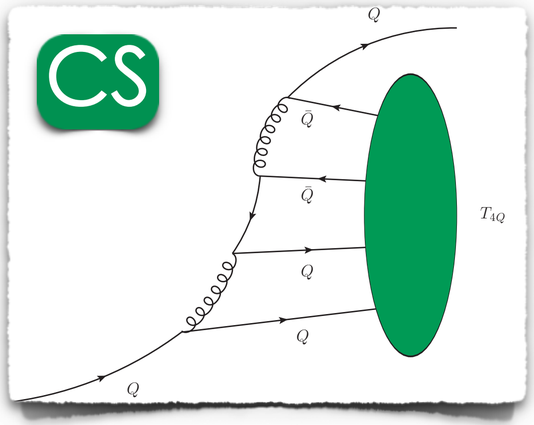
LO

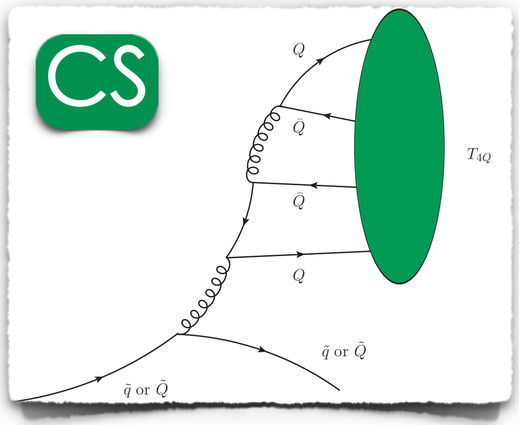
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Nonconstituent heavy or light quark







$J^{PC} = 0^{++}$ ✓ ✓ To be implemented; LO: evolution only ¡!

$J^{PC} = 1^{+-}$ Landau-Yang; LO: evolution only ✗ ✓ C-parity; LO: evolution only ✗

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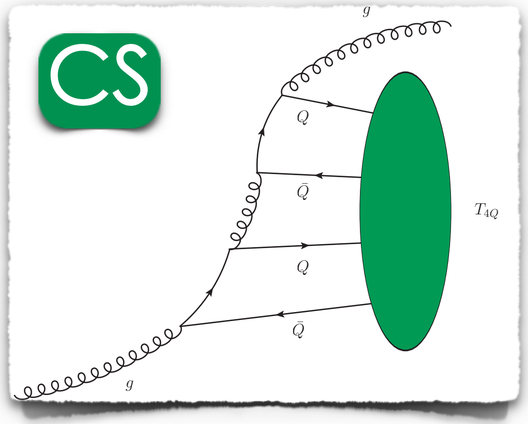
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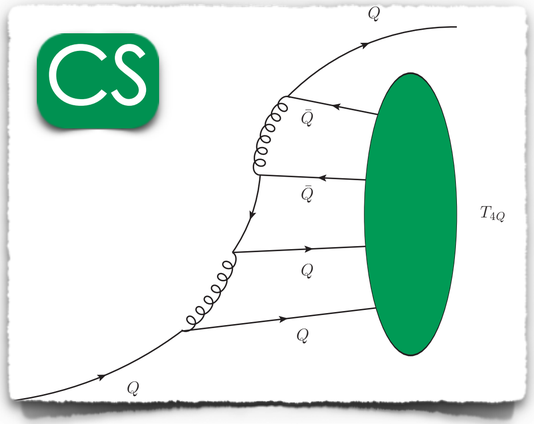
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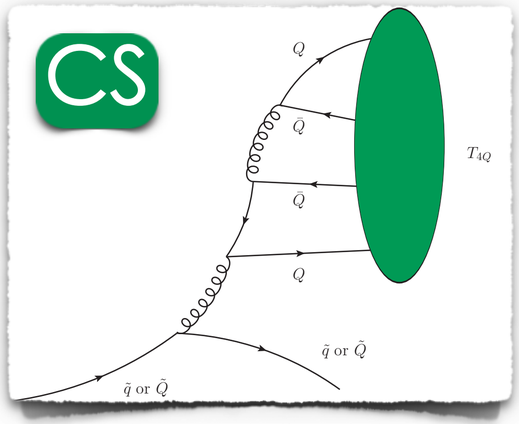
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CS



CS

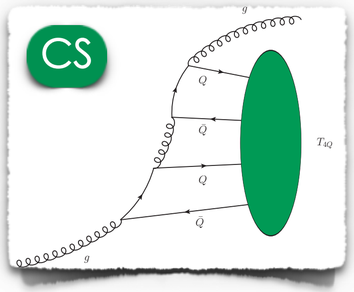


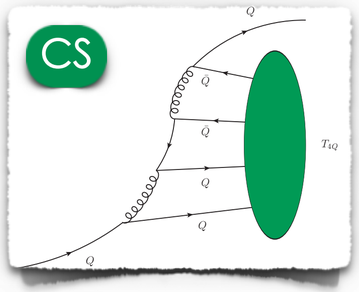
$J^{PC} = 0^{++}$ ✓ ✓ To be implemented LO: evolution only ¡!

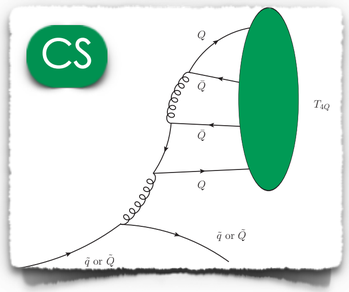
$J^{PC} = 1^{+-}$ Landau-Yang LO: evolution only ✗ ✓ C-parity LO: evolution only ✗

$J^{PC} = 2^{++}$ ✓ ✓ To be implemented LO: evolution only ¡!

HF-NRevo: Initial-scale NRCQD inputs





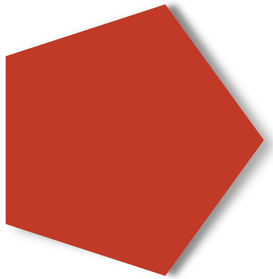


Scalar $[J^{PC} = 0^{++}]$

Tensor $[J^{PC} = 2^{++}]$

HF-NRevo for $T_{4c}(0^{++}, 2^{++})$ collinear fragmentation

Partonic hard factor



DGLAP AOevo

All-Order evolution (numeric)

DGLAP EDevo

Expanded Decoupled evolution

NRQCD

Color-composite LDMEs

SDCs

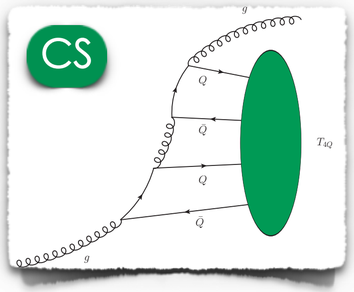
$\mathcal{D}_i^{\mathcal{Q}}(z,[n]) \otimes \langle \mathcal{O}^{\mathcal{Q}}([n]) \rangle$

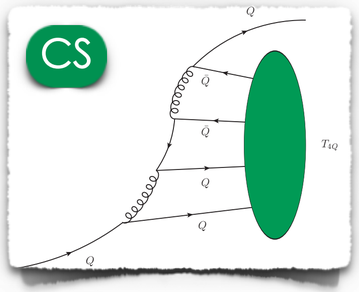
Energy

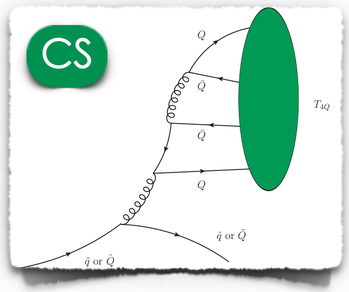
Evolution-ready scale $Q_0 \equiv m_b + 4m_c \quad 5m_c \quad m_{\tilde{q}} + 4m_c$

Time

HF-NRevo: Initial-scale NRCQD inputs





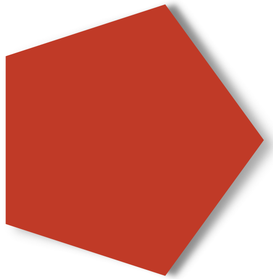


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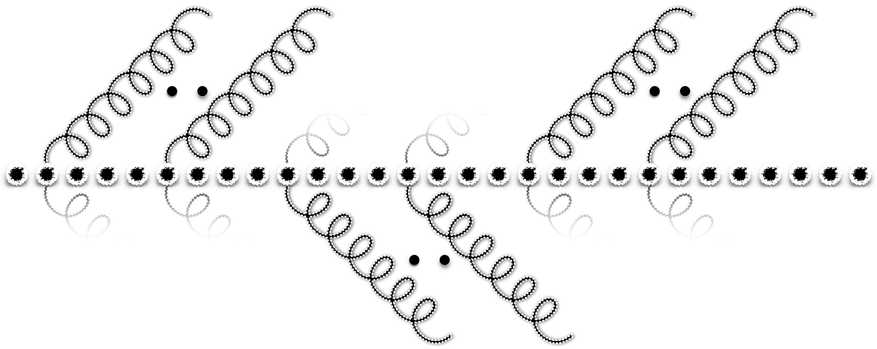
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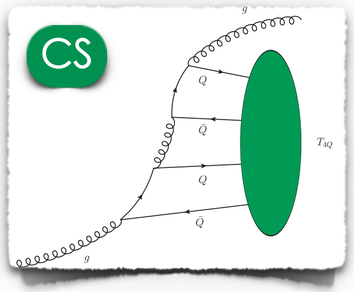
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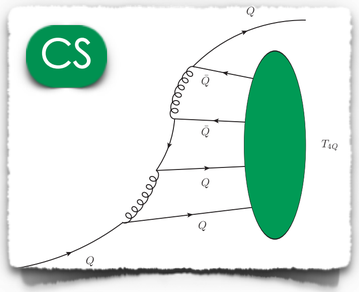
Energy

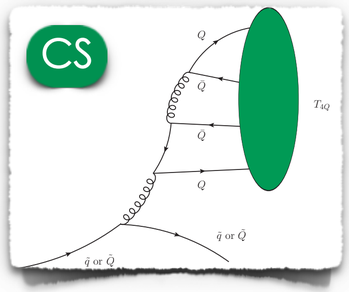
Evolution-ready scale $Q_0 \equiv m_b + 4m_c \quad 5m_c \quad m_{\tilde{q}} + 4m_c$

Time

HF-NRevo: Initial-scale NRCQD inputs





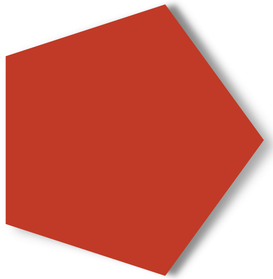


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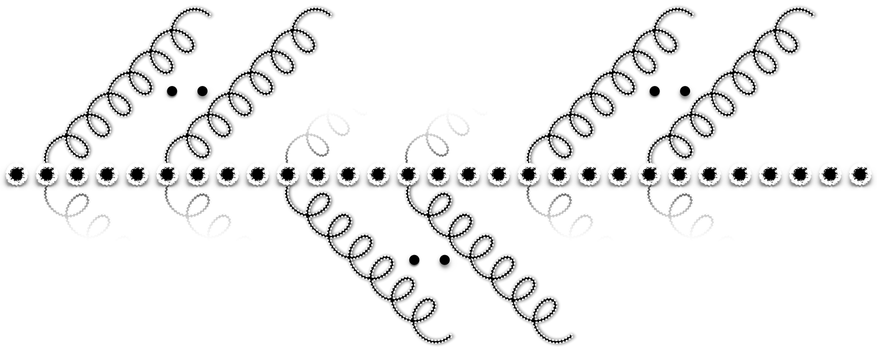
HF-NRevo for $T_{4c}(0^{++}, 2^{++})$ collinear fragmentation

Partonic hard factor



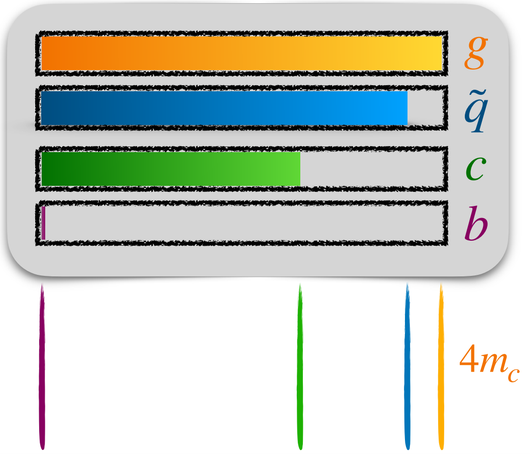
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All-Order evolution (numeric)



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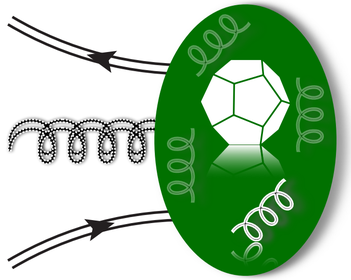
Expanded Decoupled evolution



NRQCD

Color-composite LDMEs

SDCs



$\mathcal{D}_i^{\mathcal{Q}}(z,[n]) \otimes \langle \mathcal{O}^{\mathcal{Q}}([n]) \rangle$

Energy

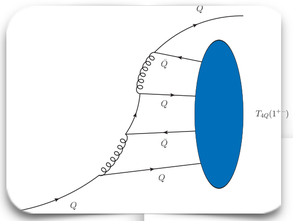
Evolution-ready scale $Q_0 \equiv m_b + 4m_c$ $5m_c$ $m_{\tilde{q}} + 4m_c$

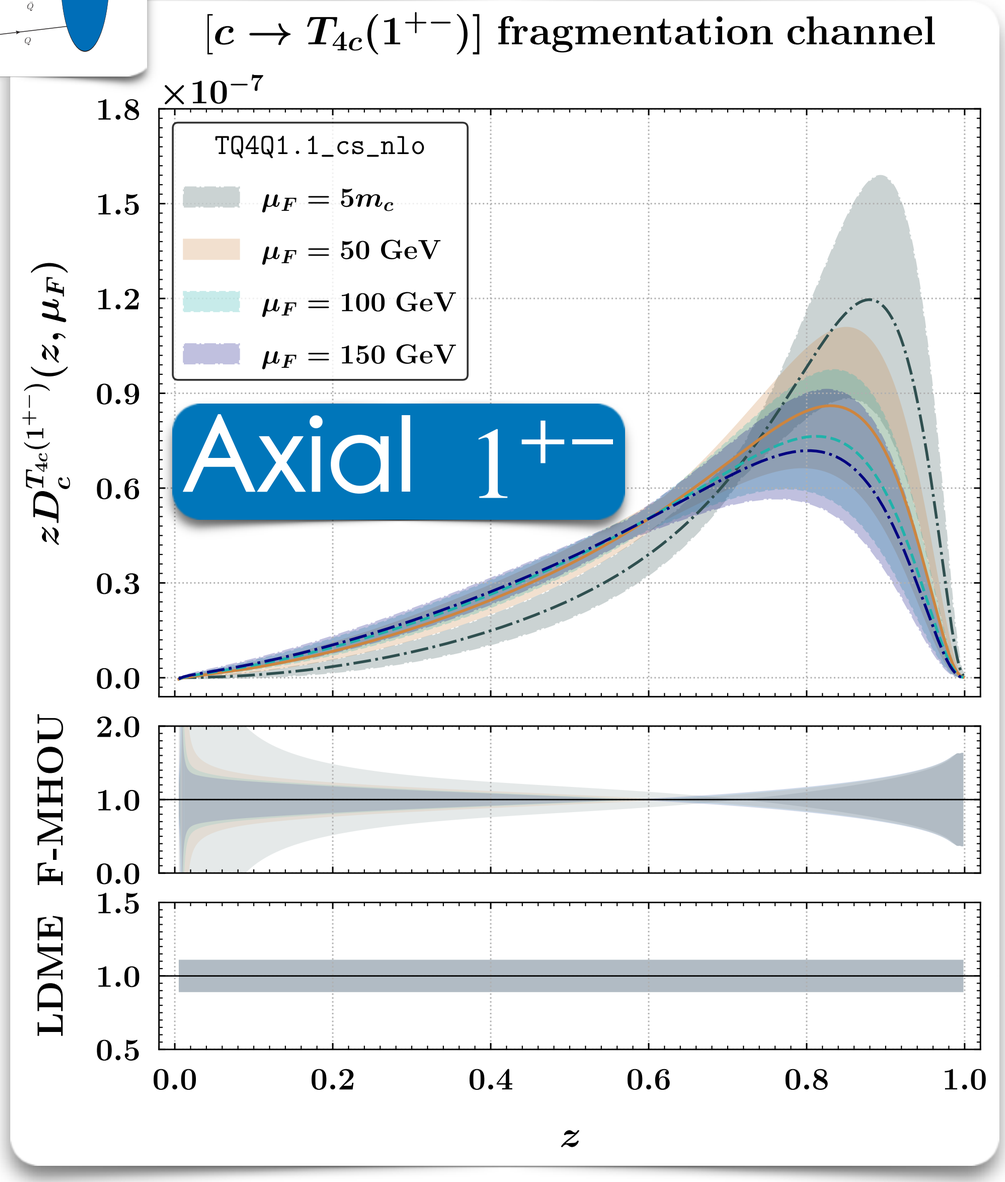
Time

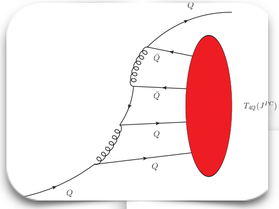
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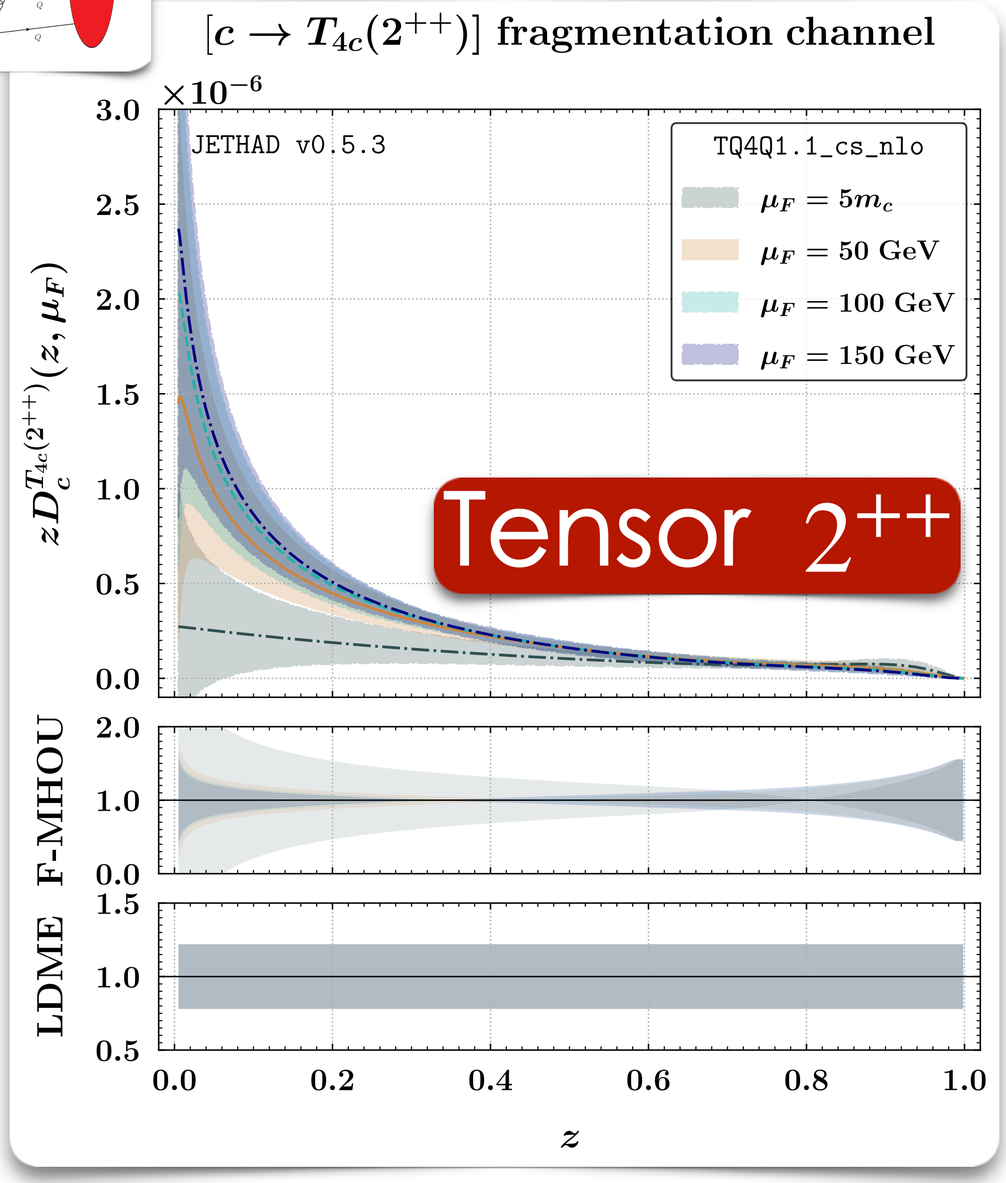
TQ4Q1.1_cs_nlo

TQ4Q1.1: Charm Fragmentation to Charmoniumlike

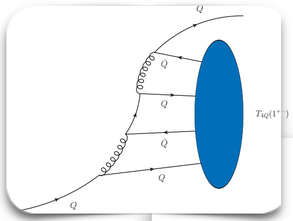


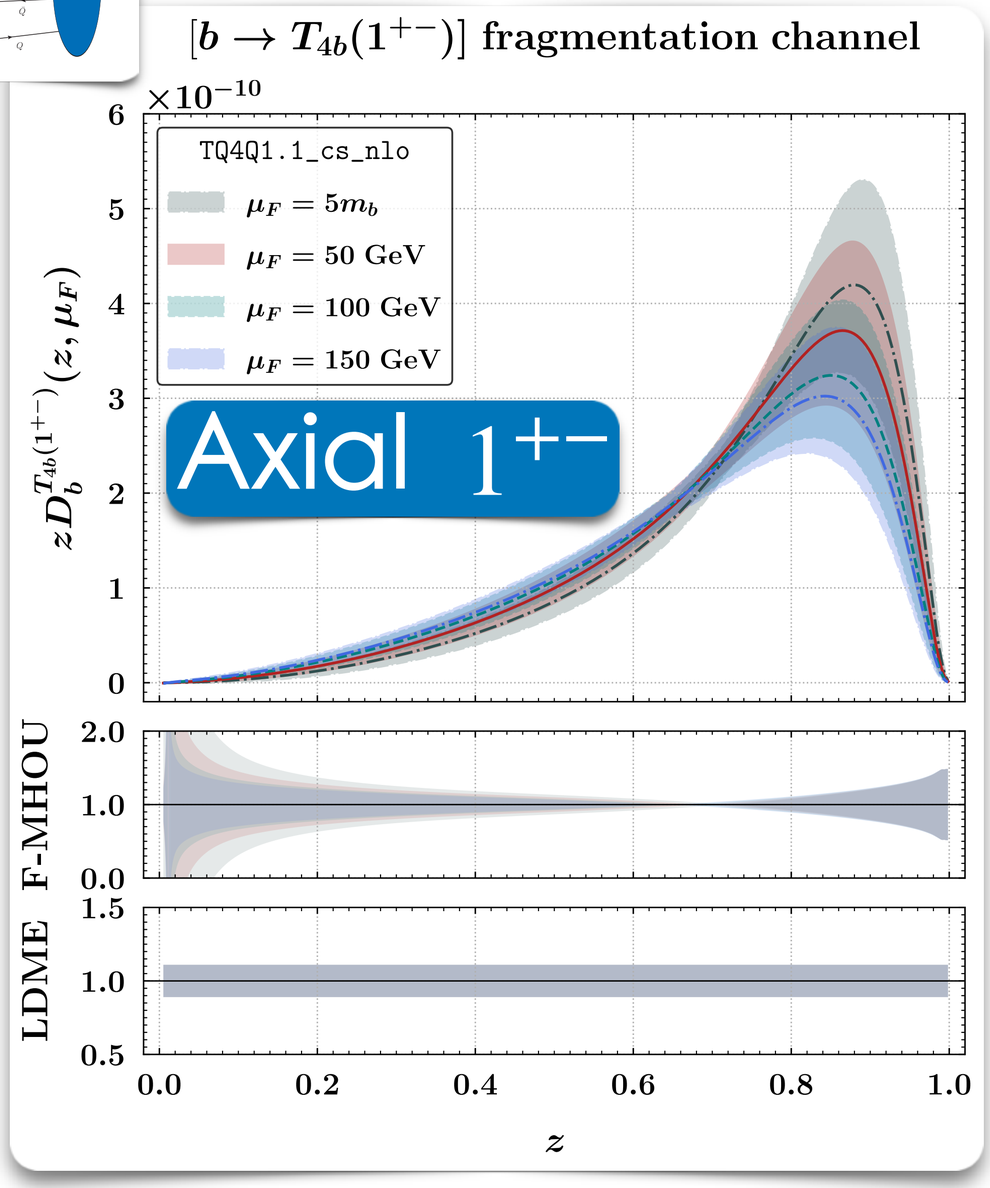


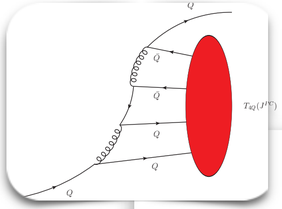


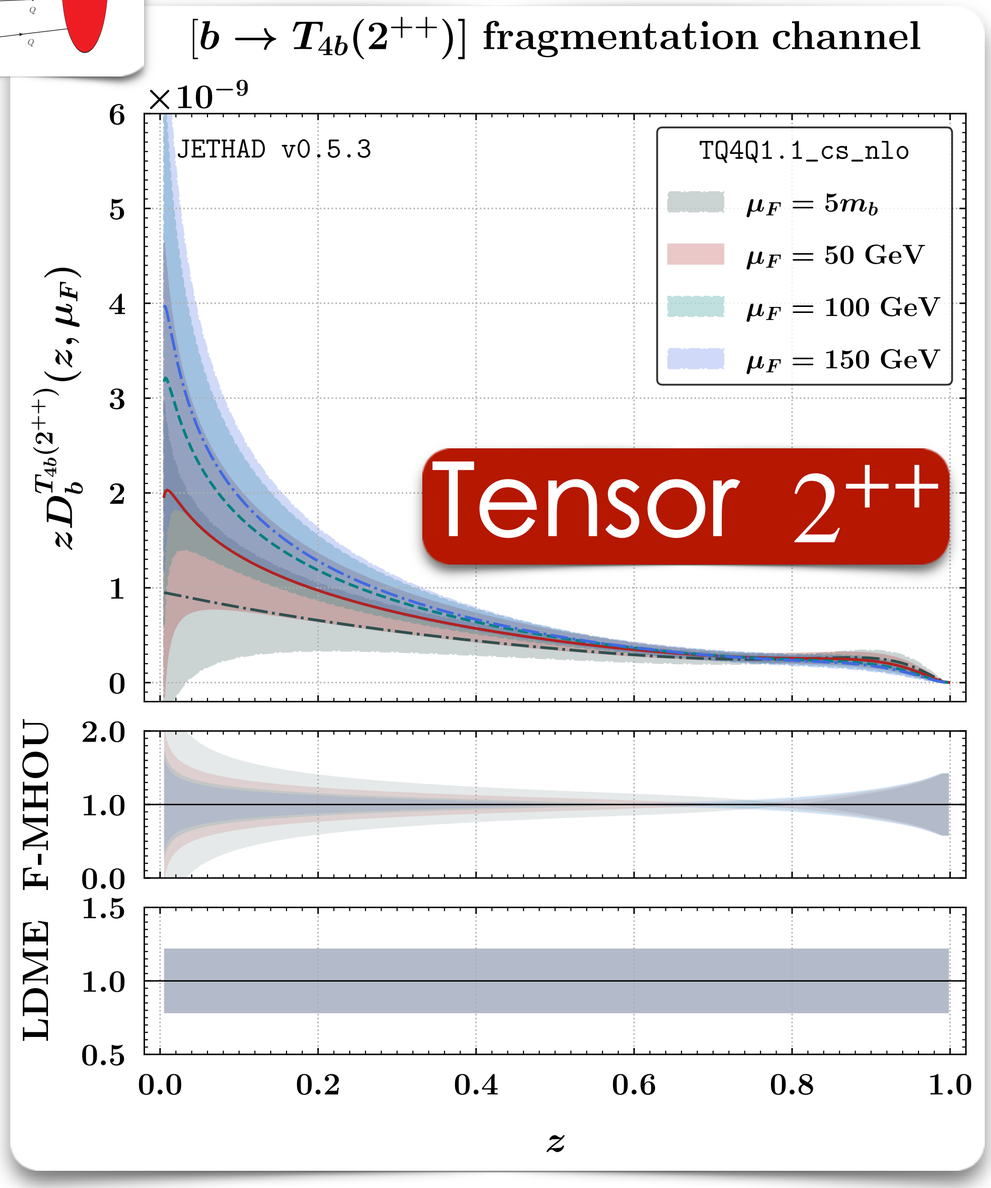


TQ4Q1.1: Bottom Fragmentation to Bottomoniumlike

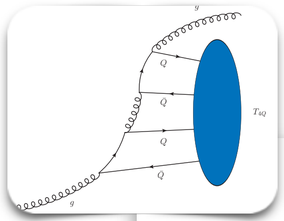


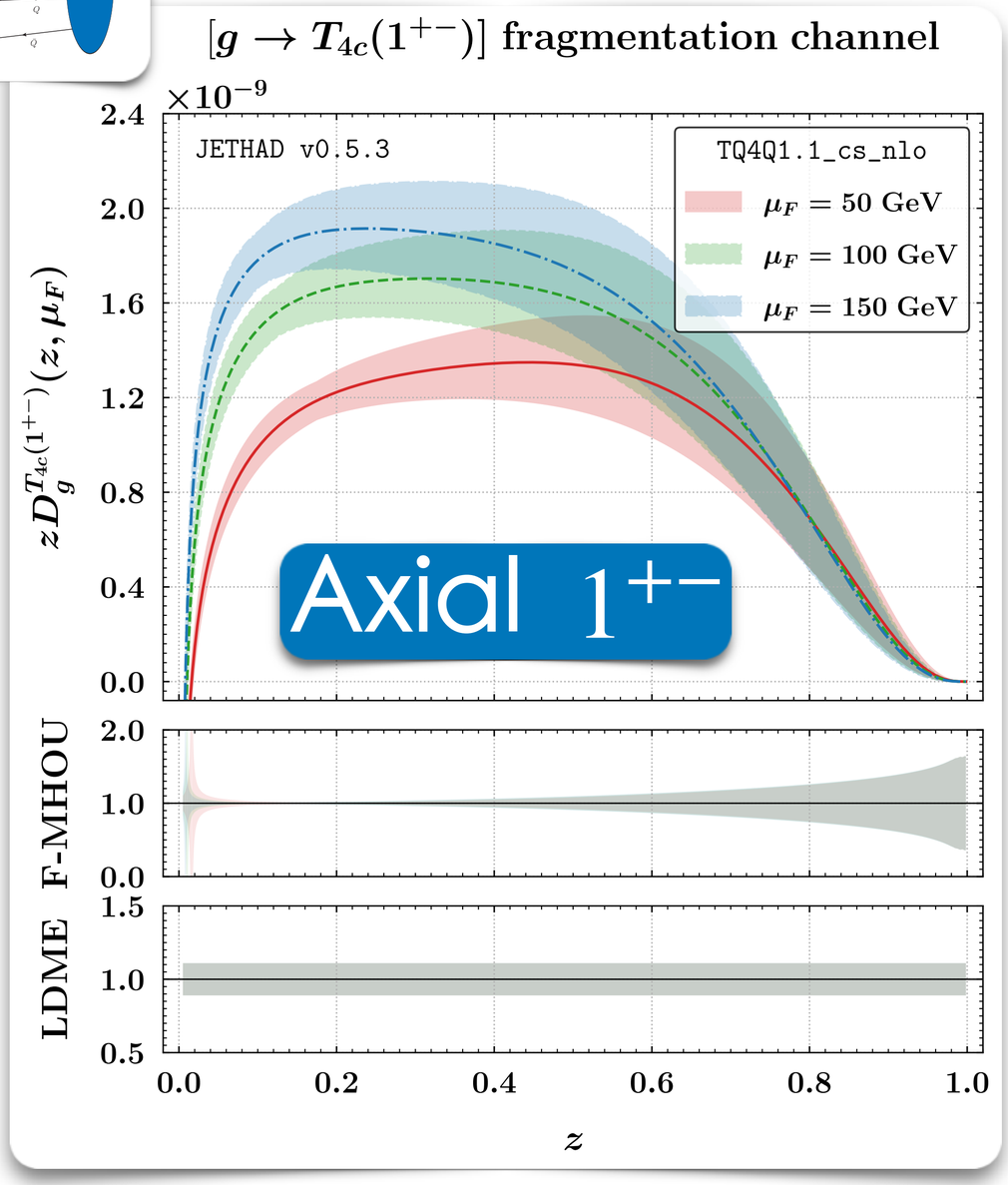


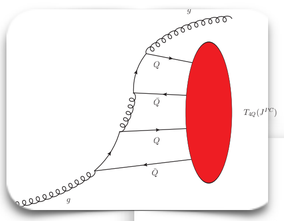


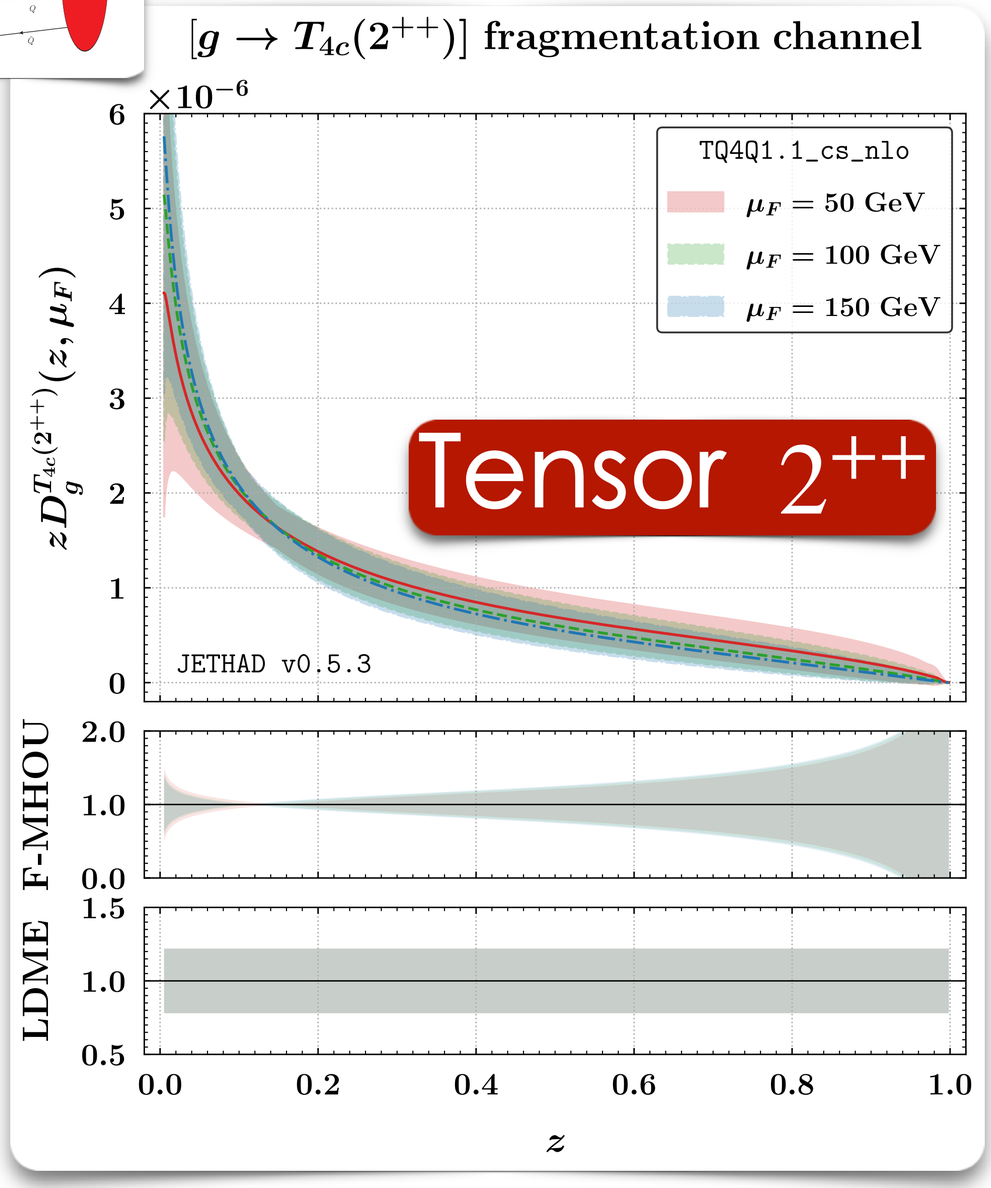


TQ4Q1.1: Gluon Fragmentation to Charmoniumlike

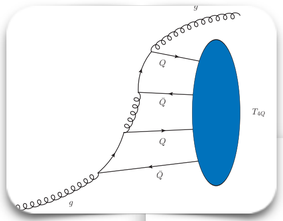


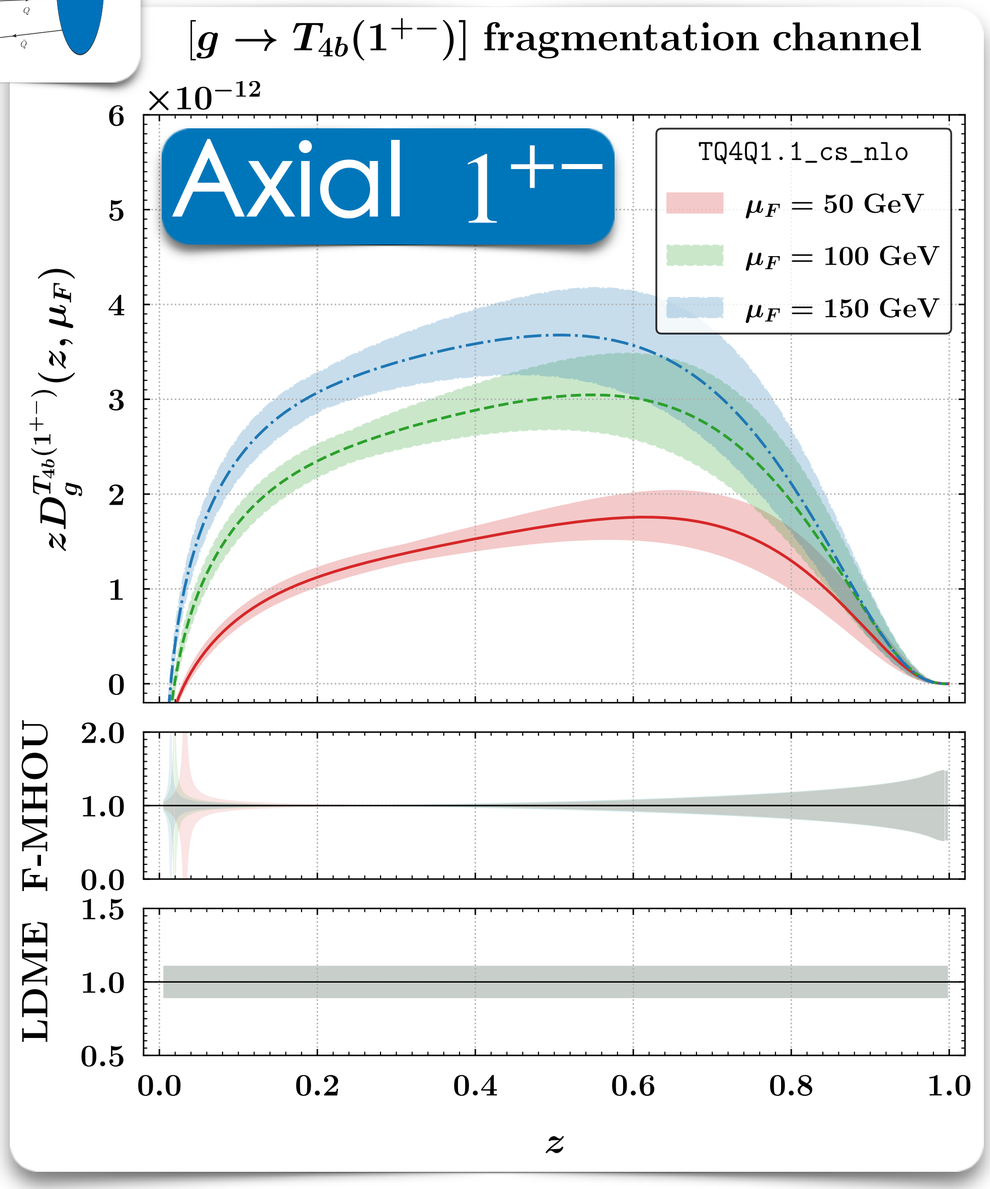


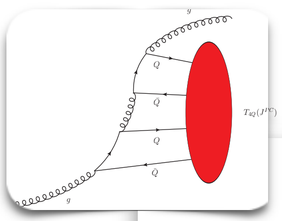


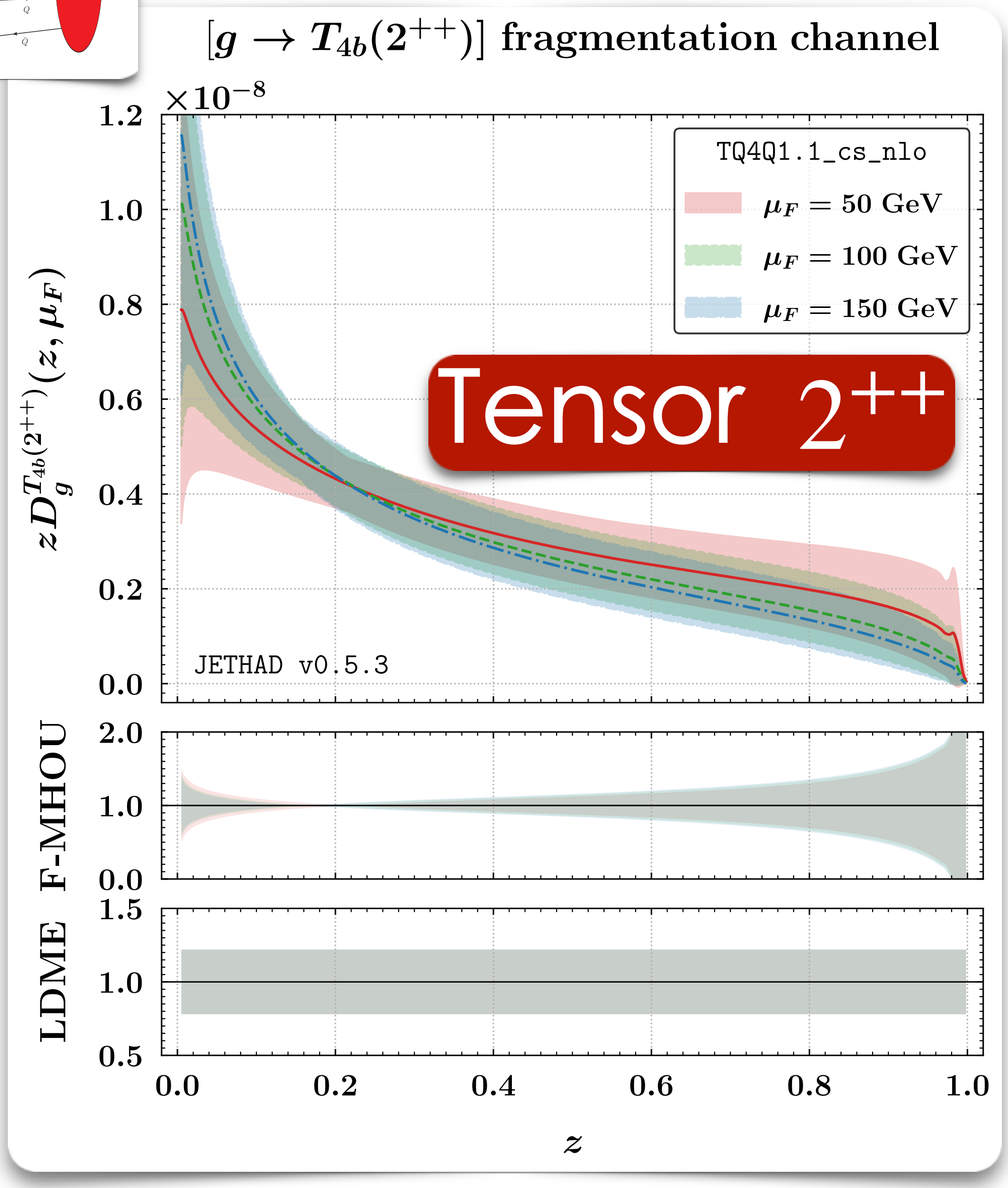


TQ4Q1.1: Gluon Fragmentation to Bottomoniumlike









The TetraQuark + jet Spectrum: Event Yields

HyF fiducial rapidity range: $1.5 < \lvert\Delta Y\rvert < 6.5$			
T_{4Q}	J^{PC}	Events [13 TeV Run 2]	Events [100 TeV FCC]
T_{4c}	0^{++}	489872 ± 83529	6706834 ± 938912
T_{4c}	1^{+-}	3354 ± 471	11424 ± 348
T_{4c}	2^{++}	718363 ± 124716	9606362 ± 1299886
T_{4b}	0^{++}	1750 ± 296	8291 ± 384
T_{4b}	1^{+-}	6 ± 1	22 ± 3
T_{4b}	2^{++}	2371 ± 391	9928 ± 299

TABLE III: Expected event yields for fully heavy tetraquark production at $\sqrt{s} = 13$ TeV and $\sqrt{s} = 100$ TeV, obtained by integrating the NLL-resummed ΔY distributions over the HyF fiducial rapidity range $1.5 < |\Delta Y| < 6.5$ considered in our analysis. The 13 TeV yields assume an integrated luminosity of $\mathcal{L}^{(\mathrm{CMS})} = 138.6\ \mathrm{fb}^{-1}$, corresponding to the total dataset collected by CMS during Run 2 [355–357]. The same luminosity is conservatively adopted for 100 TeV to enable a baseline comparison of energy scaling, independently of specific FCC projections. Uncertainties are propagated from the ΔY distributions and reflect the combined effect of scale variations, FF evolution, and LDME inputs.

Extended rapidity range: $\lvert\Delta Y\rvert < 6.5$			
T_{4Q}	J^{PC}	Events [13 TeV Run 2]	Events [100 TeV FCC]
T_{4c}	0^{++}	1522545 ± 270309	11888862 ± 1801190
T_{4c}	1^{+-}	9367 ± 1323	16976 ± 462
T_{4c}	2^{++}	2270257 ± 417565	17294873 ± 2725859
T_{4b}	0^{++}	4842 ± 822	12128 ± 673
T_{4b}	1^{+-}	16 ± 2	32 ± 4
T_{4b}	2^{++}	6531 ± 1118	14184 ± 435

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Pile-up effects can degrade exp. discrimination

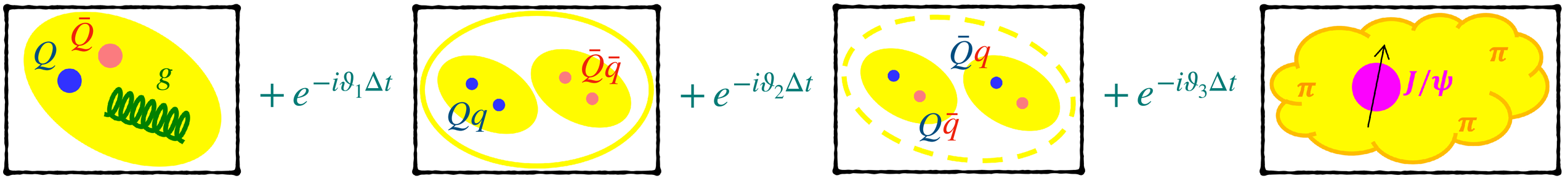
A Multimodal Perspective

Final Remarks

A Portal to Exotic Matter

Exotic Matter Multimodal Fragmentation

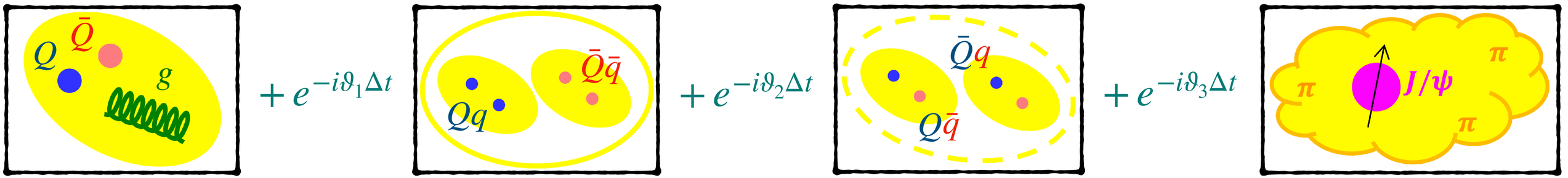
Quantum Superposition



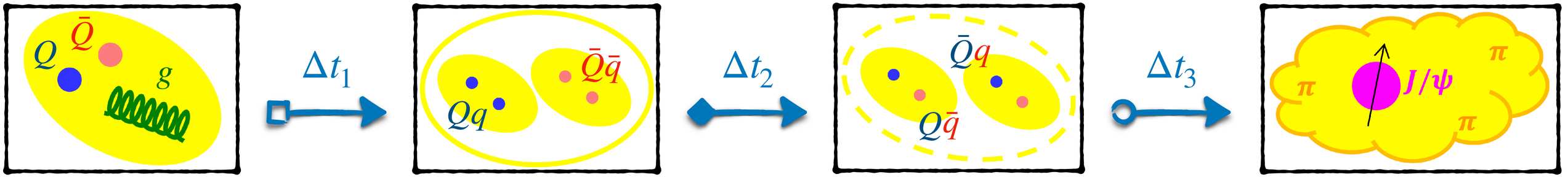
Compact | Diquark | Molecule | Hadro-onium

Exotic Matter Multimodal Fragmentation

Quantum Superposition



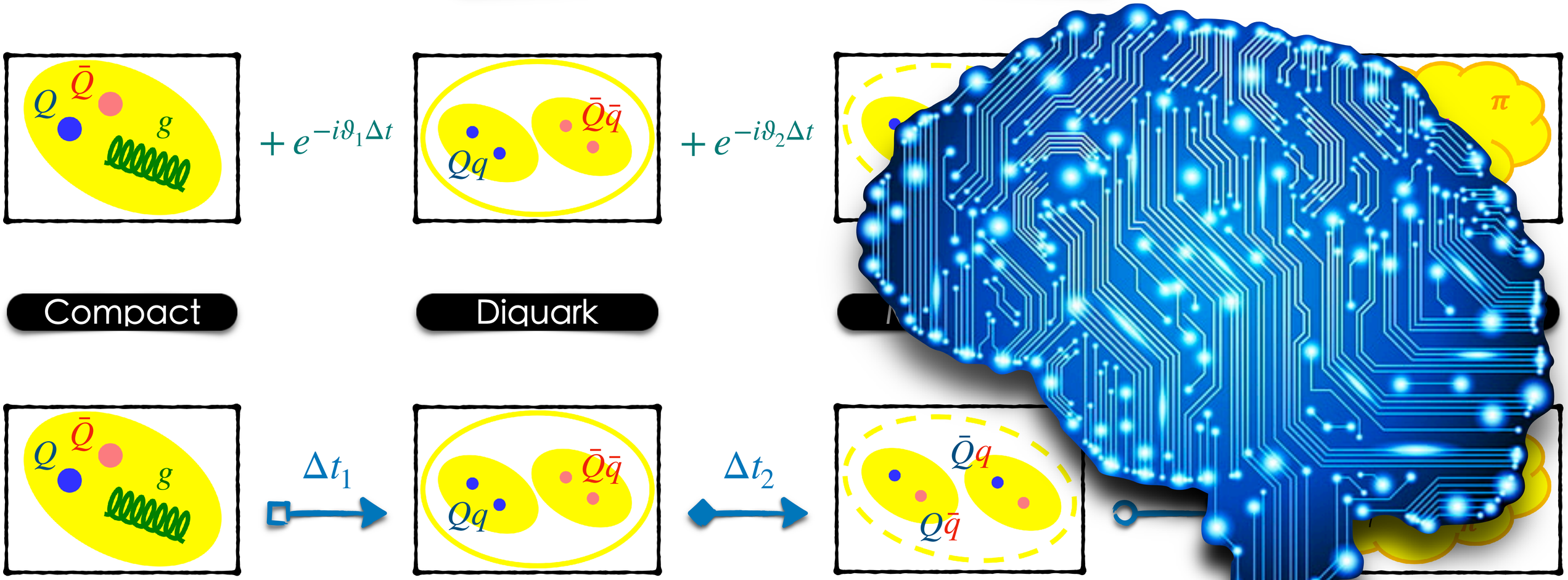
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Energy-Time Evolution

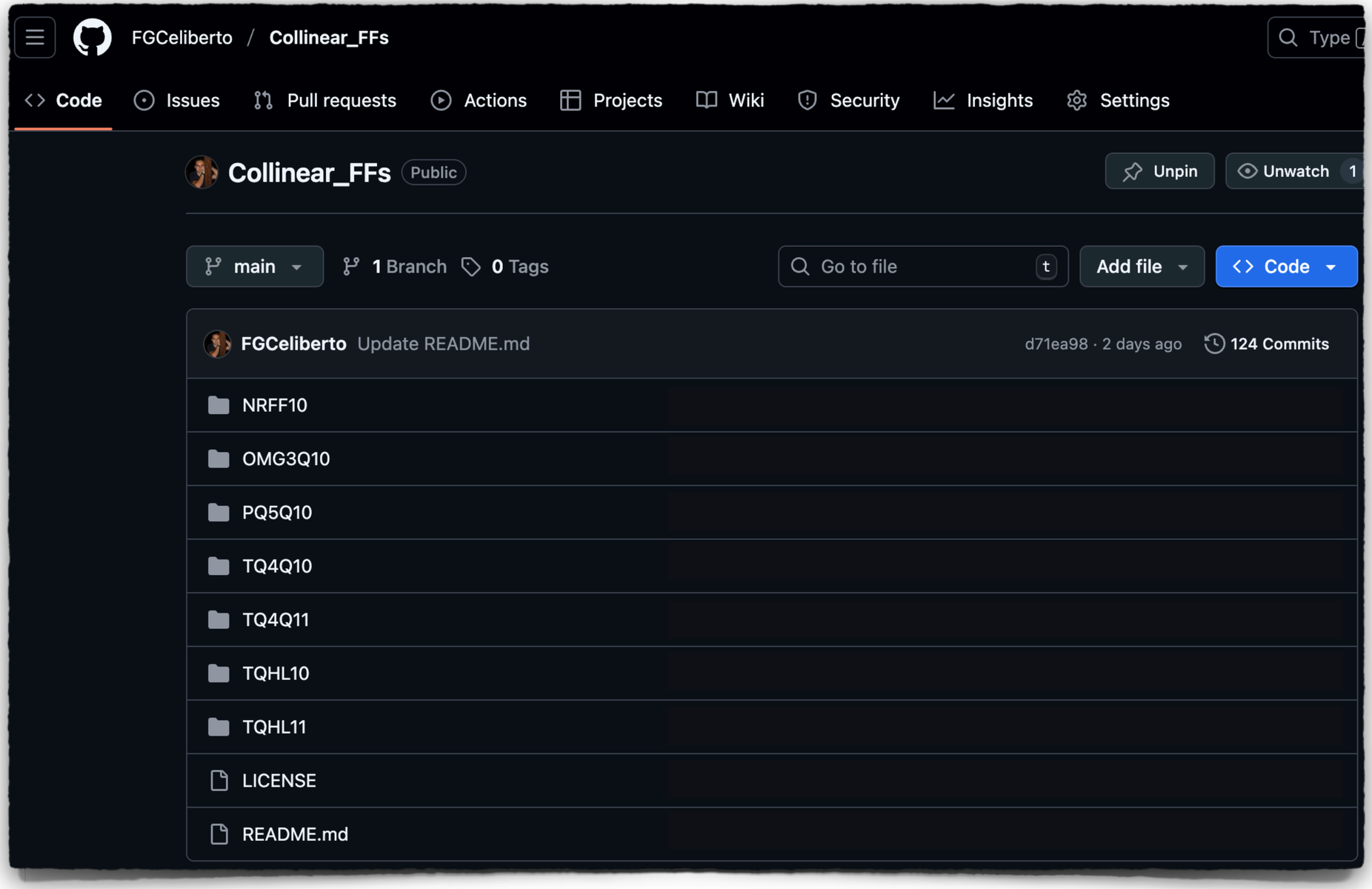
Exotic Matter Multimodal Fragmentation

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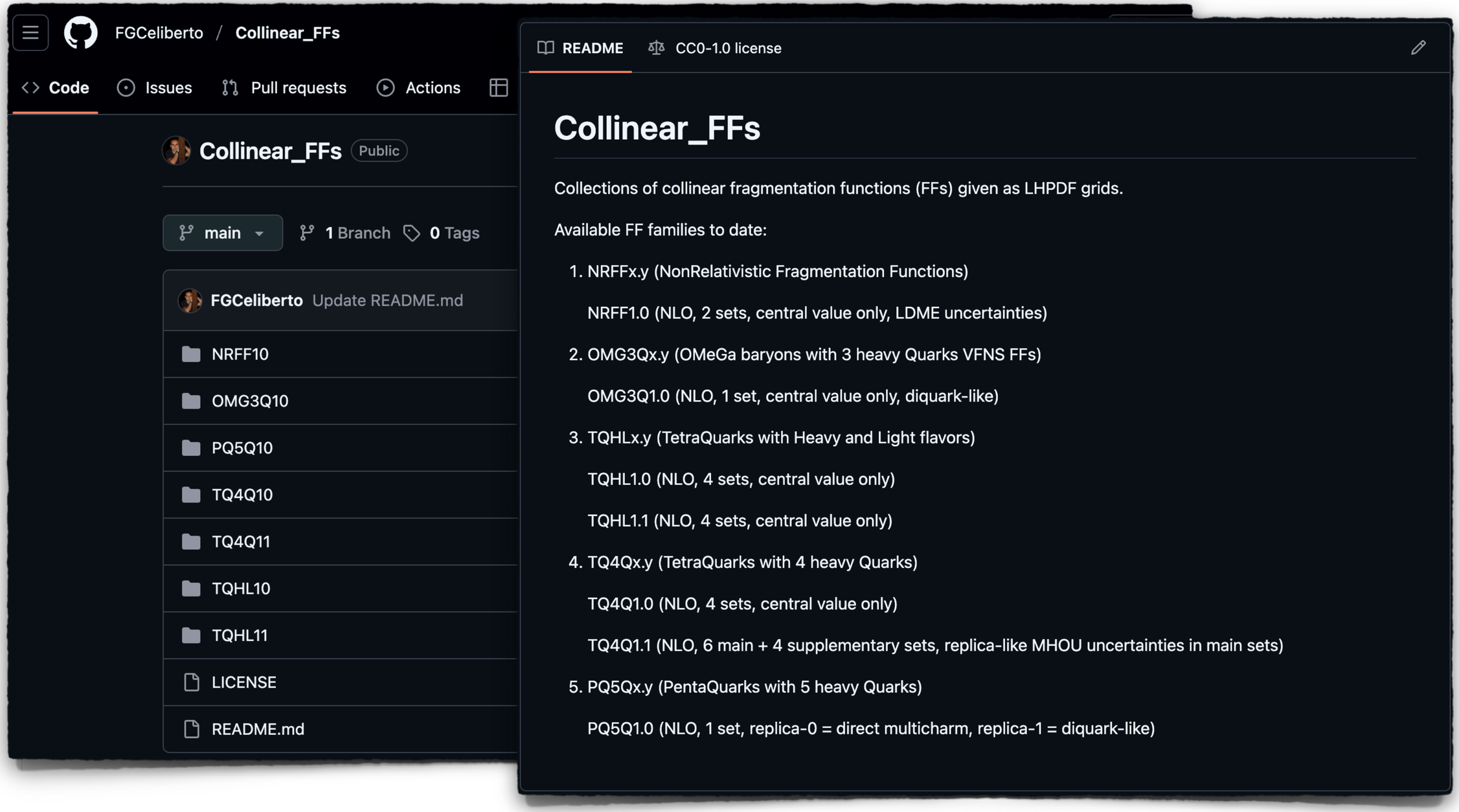


Energy-Time Evolution

Heavy-Flavor Fragmentation from **HF-NRevo**



Heavy-Flavor Fragmentation from **HF-NRevo**



🔗 https://github.com/ FGCeliberto/Collinear_FFs/

Toward **NRFF1.0**, **TQ4Q2.0** and beyond

Toward new directions

Toward **NRFF1.0**, **TQ4Q2.0** and beyond



Quarkonia & Exotics ⇒ precision QCD & production mechanisms

Toward new directions

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HF-NRevo ⇒ basis for explorations of heavy-flavor fragmentation

Toward new directions

Toward **NRFF1.0**, **TQ4Q2.0** and beyond



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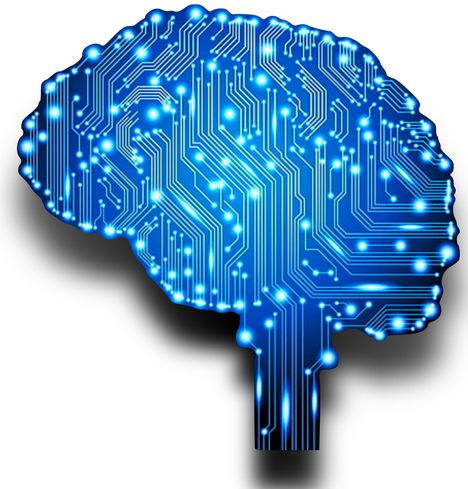


(sym)JETHAD Genisys ⇒ from high-energy to precision QCD

Toward new directions

Toward **NRFF1.0**, **TQ4Q2.0** and beyond

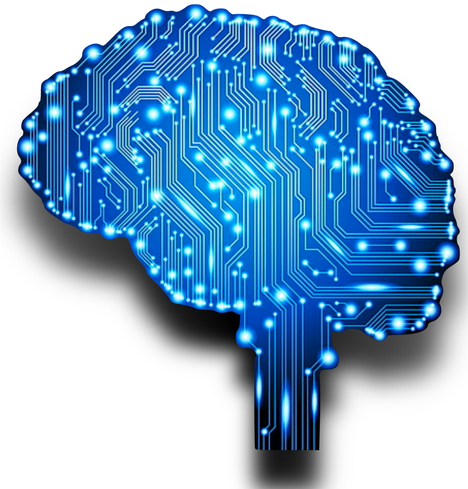
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- **HF-NRevo** ⇒ basis for explorations of heavy-flavor fragmentation
- **(sym)JETHAD Genisys** ⇒ from high-energy to precision QCD
- AI-based extraction of heavy-hadron FFs



Toward new directions

Toward **NRFF1.0**, **TQ4Q2.0** and beyond

- Quarkonia & Exotics ⇒ precision QCD & production mechanisms
- **HF-NRevo** ⇒ basis for explorations of heavy-flavor fragmentation
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¡ Thanks !

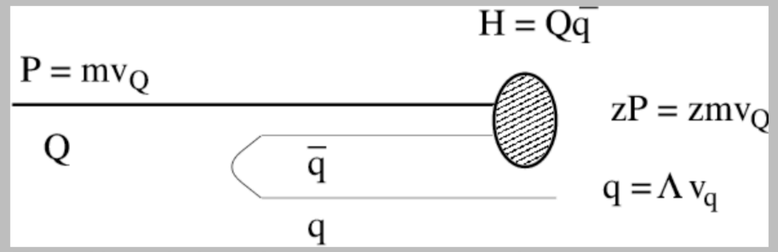
EXTRAS

Highlights on heavy-flavor fragmentation

We observe hadrons, whose momentum is different from that of the parent heavy quark. We usually write this as

$$\text{FF(hadron)} = \text{FF(quark)} \otimes \text{FF(quark}\rightarrow\text{hadron)}$$

What are the characteristics of FF(quark→hadron)?



The heavy quark has momentum $P = mv_Q$, the light quarks have momentum $q = \Lambda v_q$, with Λ a hadronic mass scale

For the binding we need $v_Q \simeq v_q = v$. We have then

$$P = zP + q \qquad\qquad mv = zmv + \Lambda v$$

and therefore

$$\langle z \rangle \simeq 1 - \frac{\Lambda}{m}$$

Hence, FF(quark→hadron) is peaked near $z = 1$ and binding effects scale linearly in the heavy quark mass (Bjorken/Suzuki ($\sim$ 1977))

(NB. more rigorous derivations of this result are available)

Quarkonia as Tools: Precision & Exploration

QUARKONIA

QCD Hydrogen Atoms

Precision determinations of QCD running coupling

$\Upsilon(1S)$ hadronic inclusive decays

[CLEO Collaboration (2006)]
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Forward inclusive reactions

Gluon PDF positivity at low x and low Q^2

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Quarkonia in nuclear matter

Confined matter to QGP transition

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Quarkonia as Tools: Precision & Exploration

Portals to exotic matter and BSM physics

Hadroquarkonium

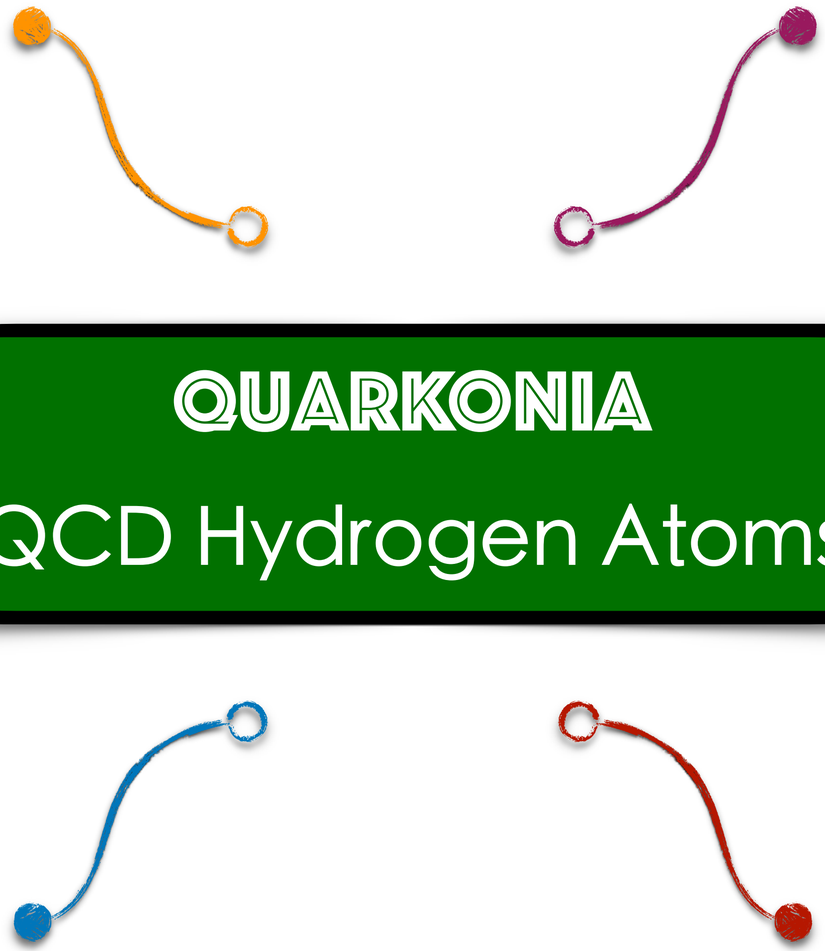
EW boson rare decays

[S. Dubynskiy et al. (2008)]
[F.-K. Guo et al. (2018)]
[J. Ferretti et al. (2020)]
[CMS Collaboration (2019)]

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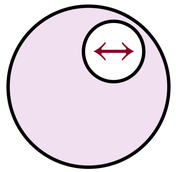
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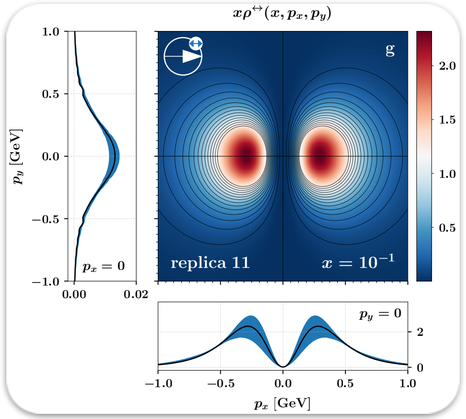
Quarkonium studies: Assets & Challenges

Hadronic structure: Assets



EIC, LHCb, LHCspin

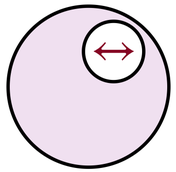
Boer-Mulders



🔗 [A. Bacchetta, F.G. C., M. Radici, P. Taels (EPJC 2020)]

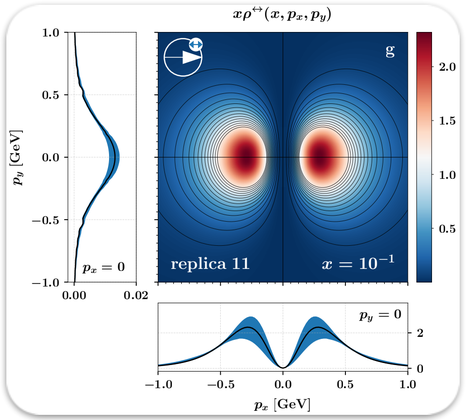
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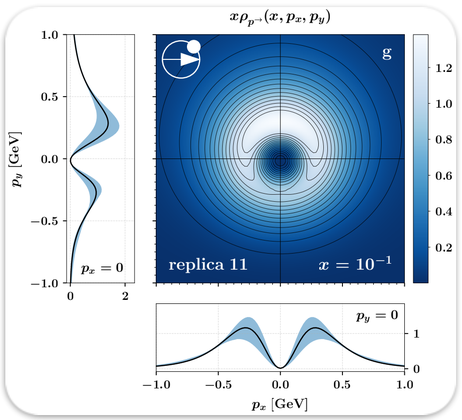
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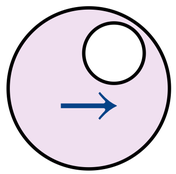
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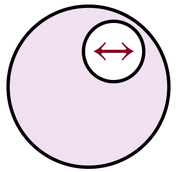


EIC, LHCspin

Sivers

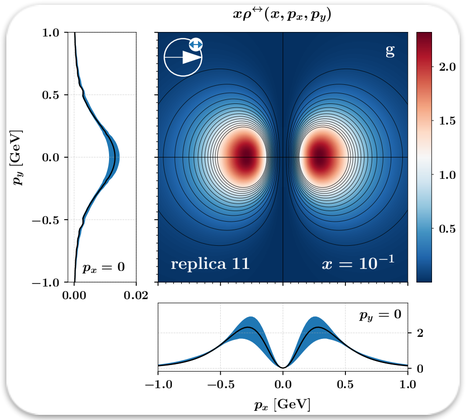
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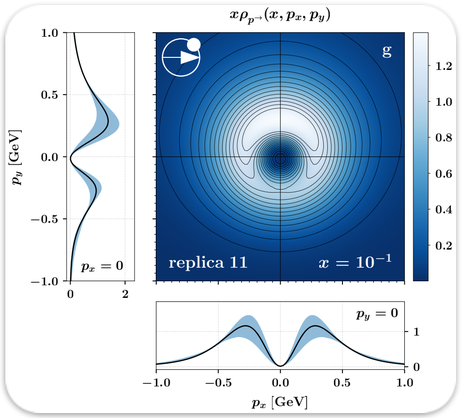


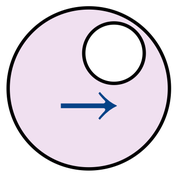
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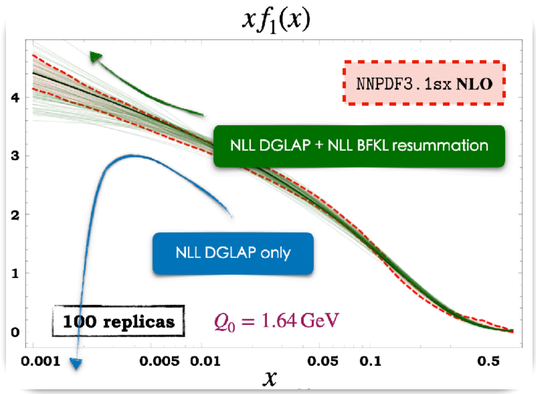


EIC, LHCspin

Sivers

Precision QCD: Challenges

Forward Onia ⇒ Gluon PDF & TMD Positivity

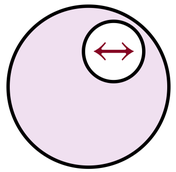


Low x and low Q^2

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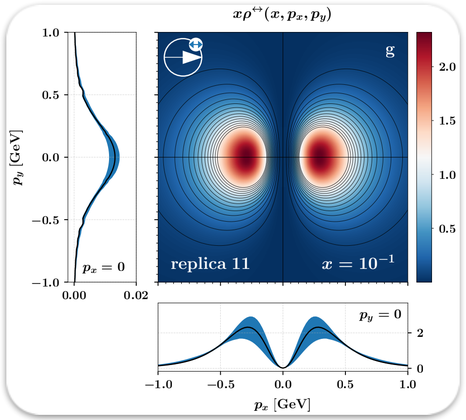
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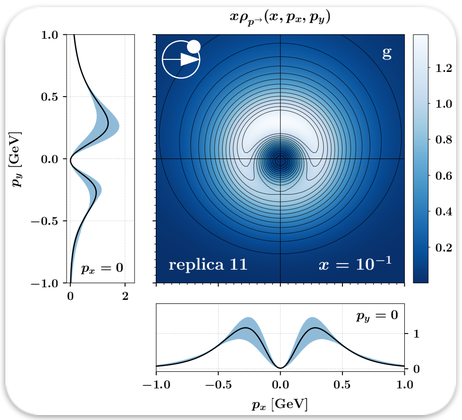


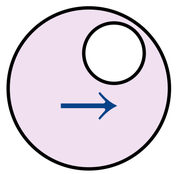
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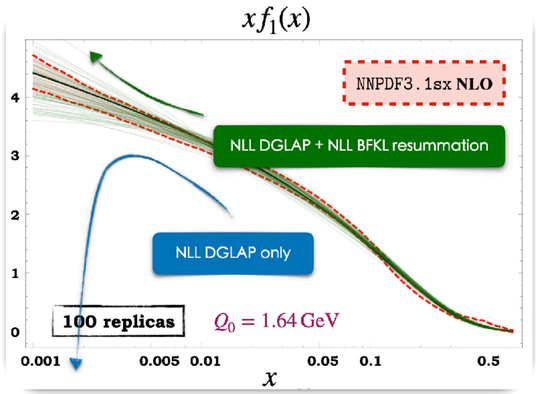


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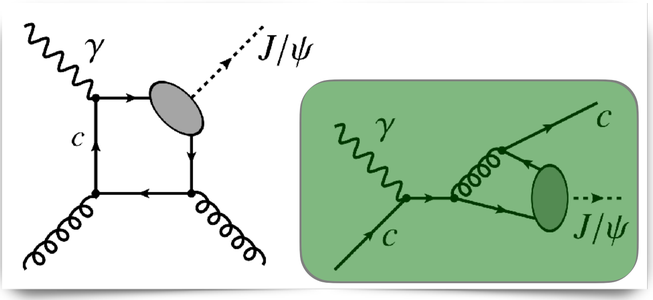


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$\gamma + g \rightarrow J/\psi + c$ ⇒ Valence Intrinsic Charm @EIC

Large x
Moderate to large p_T



(J/ψ @EIC) 🔗 [C. Flore, J.-P. Lansberg, H.-S. Shao, Y. Yedelkina (2020)]
(Intrinsic Charm + valence) studies 🔗🔗 [NNPDF Collaboration (2022, 2023)]

Highlights of Non-Relativistic QCD

NRQCD → effective field theory → nonrelativistic (NR) $Q\bar{Q}$ system

Heavy-quark spinor fields as NR DOFs in the NQRCD Lagrangian

$$\mathcal{L}_{\mathrm{NRQCD}} = \psi^\dagger \left(iD_0 + \frac{\vec{D}^2}{2m_Q} \right) \psi + \chi^\dagger \left(iD_0 - \frac{\vec{D}^2}{2m_Q} \right) \chi + \mathcal{L}_{\mathrm{light}} + \delta\mathcal{L}$$

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[cross section] = [short-distance coeffs.] ⊗ [long-distance matrix elements]

$$d\sigma(H + X) = \sum_n d\hat{\sigma}(Q\overline{Q}\,[n] + X)\,\langle \mathcal{O}^H\,[n] \rangle$$

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$$|\mathcal{Q}\rangle = \mathcal{O}(1)\,|Q\bar{Q}[^3S_1^{(1)}]\rangle + \mathcal{O}(v)\,|Q\bar{Q}[^3P_J^{(8)}g]\rangle + \mathcal{O}(v^2)\,|Q\bar{Q}[^1S_0^{(8)}g]\rangle + \mathcal{O}(v^2)\,|Q\bar{Q}[^3S_1^{(1,8)}gg]\rangle + \mathcal{O}(v^2)\,|Q\bar{Q}[^3D_J^{(1,8)}gg]\rangle + \ldots .$$

HF-NREVO

FOUNDING PILLARS

Heavy-Flavor Non-Relativistic evolution

HF-NRevo

Interpretation

Heavy-Flavor Non-Relativistic evolution

HF-NRevo

Interpretation

Evolution

Heavy-Flavor Non-Relativistic evolution

HF-NRevo

Interpretation

Evolution

Uncertainties

Heavy-Flavor Non-Relativistic evolution

HF-NRevo

Interpretation

- NRQCD ⇒ proxy for initial-scale FF inputs
- A heavy-flavor-scheme based vision
- Low vs large p_T ⇒ FFNS vs VFNS fragmentation

Heavy-Flavor Non-Relativistic evolution

HF-NRevo

Evolution

- Proper DGLAP collinear evolution
- Thresholds at work for all partons
- Two-step strategy $\Rightarrow$ `EDevo` $\otimes$ `AOevo`

Heavy-Flavor Non-Relativistic evolution

HF-NRevo

Uncertainties

- Initial-scale MHOUs ⇒ first systematic analysis
- Idea ⇔ `ThCovM` 🔗🔗 & `MCscales` 🔗 MHOUs
- Future ⇐ extraction of NP LDMEs from data

TOWARD MULTIMODAL FRAGMENTATION

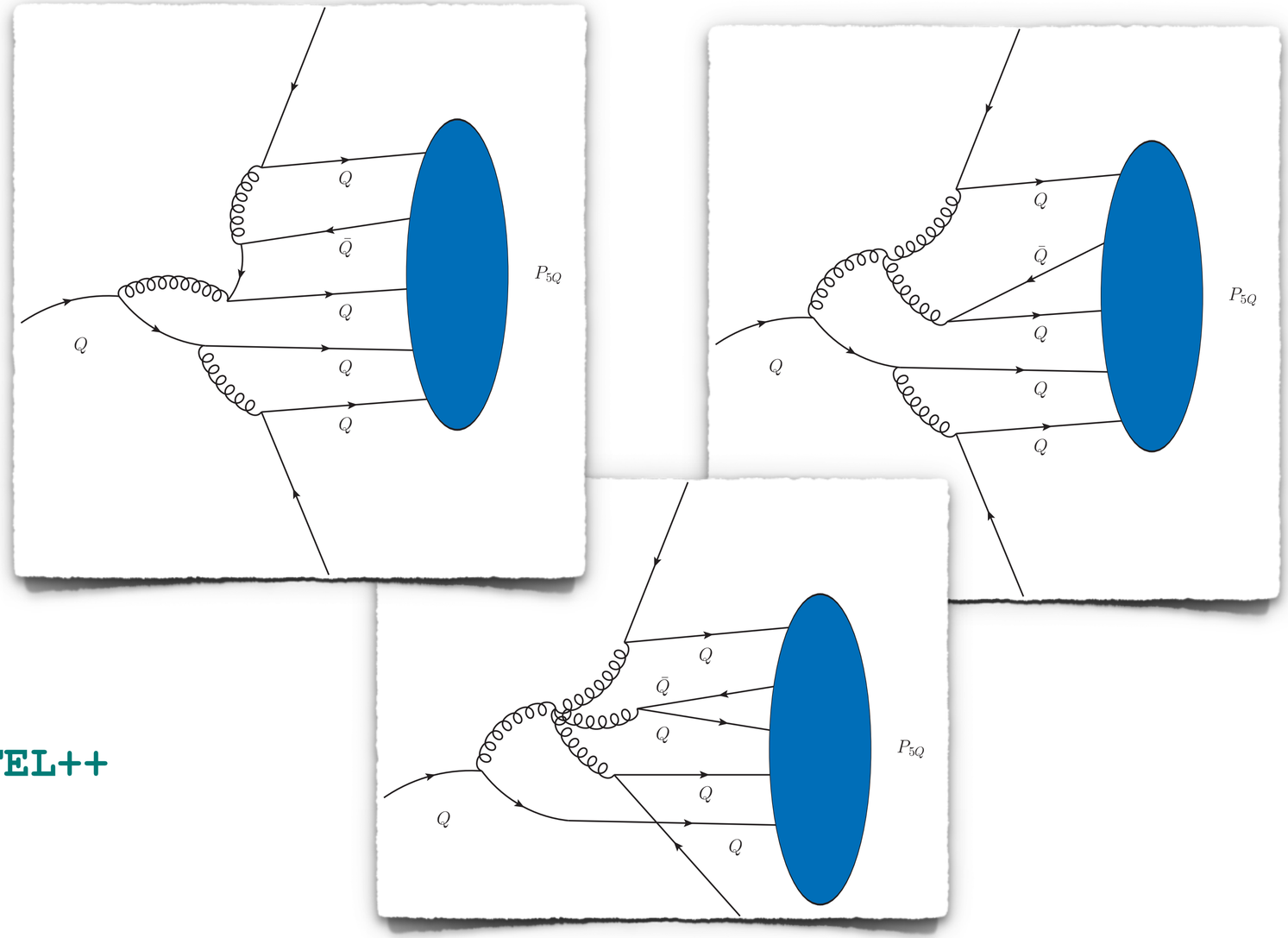
Pentacharm multimodal fragmentation

Mode 1
Direct Multicharm

🔗 [F. G. C., arXiv:2502.11136]

Pentacharm multimodal fragmentation

Mode 1
Direct Multicharm



PQ5Q1.0 FFs: $(Q \to P_{c\bar{c}ccc}) \otimes$ **APFEL++**

4.2 Pentacharm fragmentation

🔗 [F. G. C., arXiv:2502.11136]

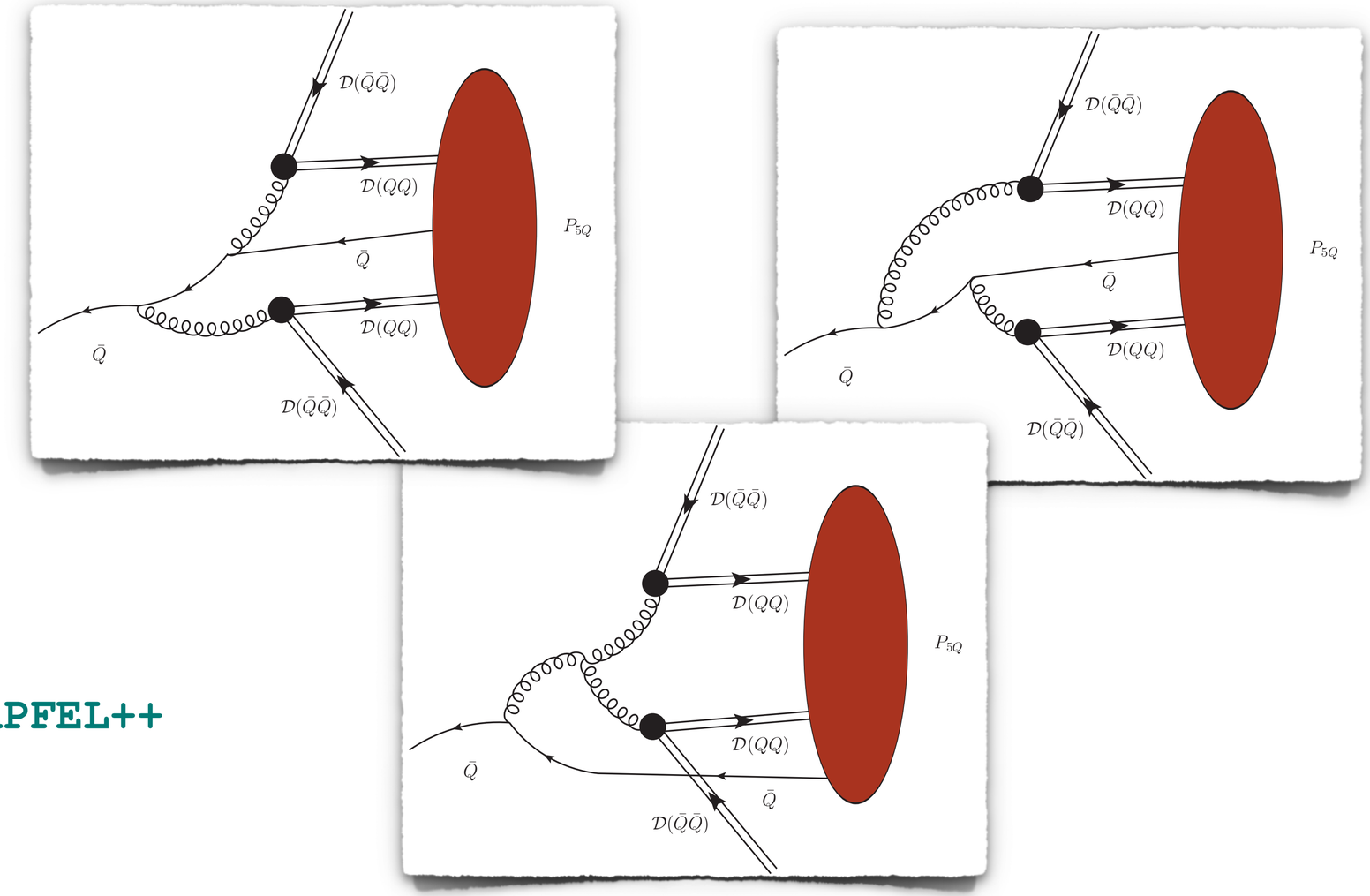
Pentacharm multimodal fragmentation

Mode 2
Scalar Diquark

🔗 [F. G. C., arXiv:2502.11136]

Pentacharm multimodal fragmentation

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Scalar Diquark



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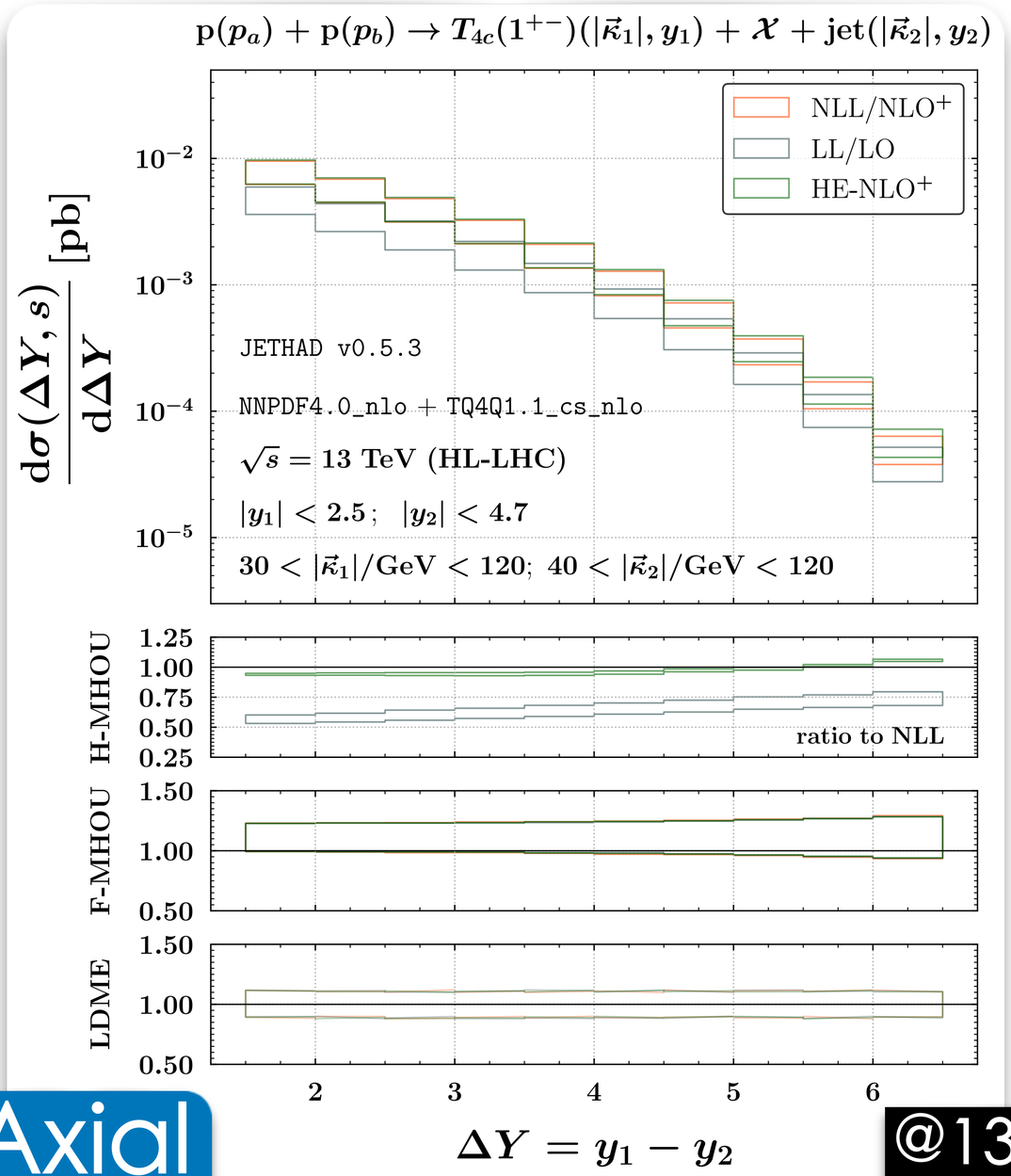
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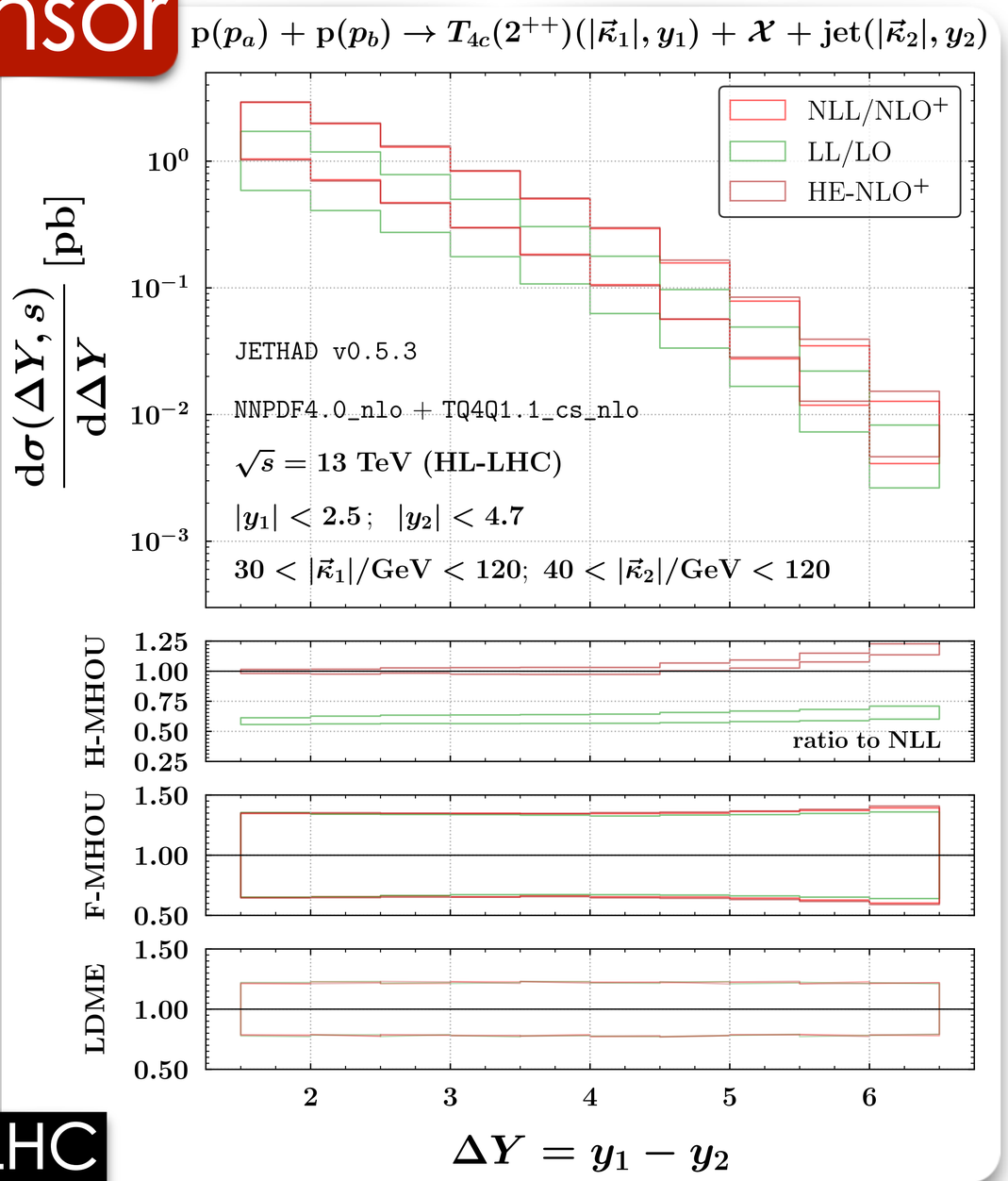
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FORWARD PHYSICS PHENOMENOLOGY

The tetraquark + jet rapidity spectrum from JETHAD

Tensor



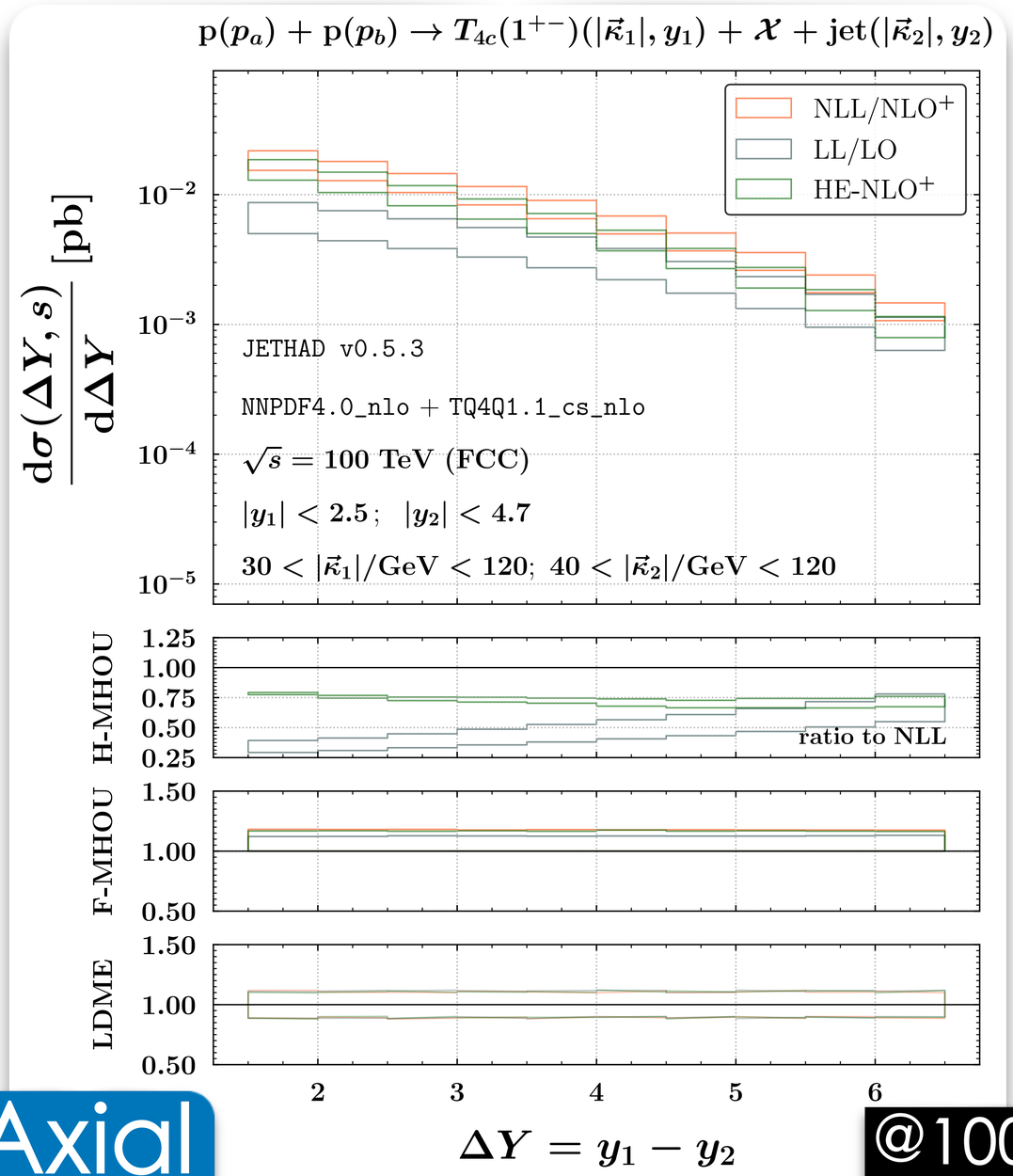


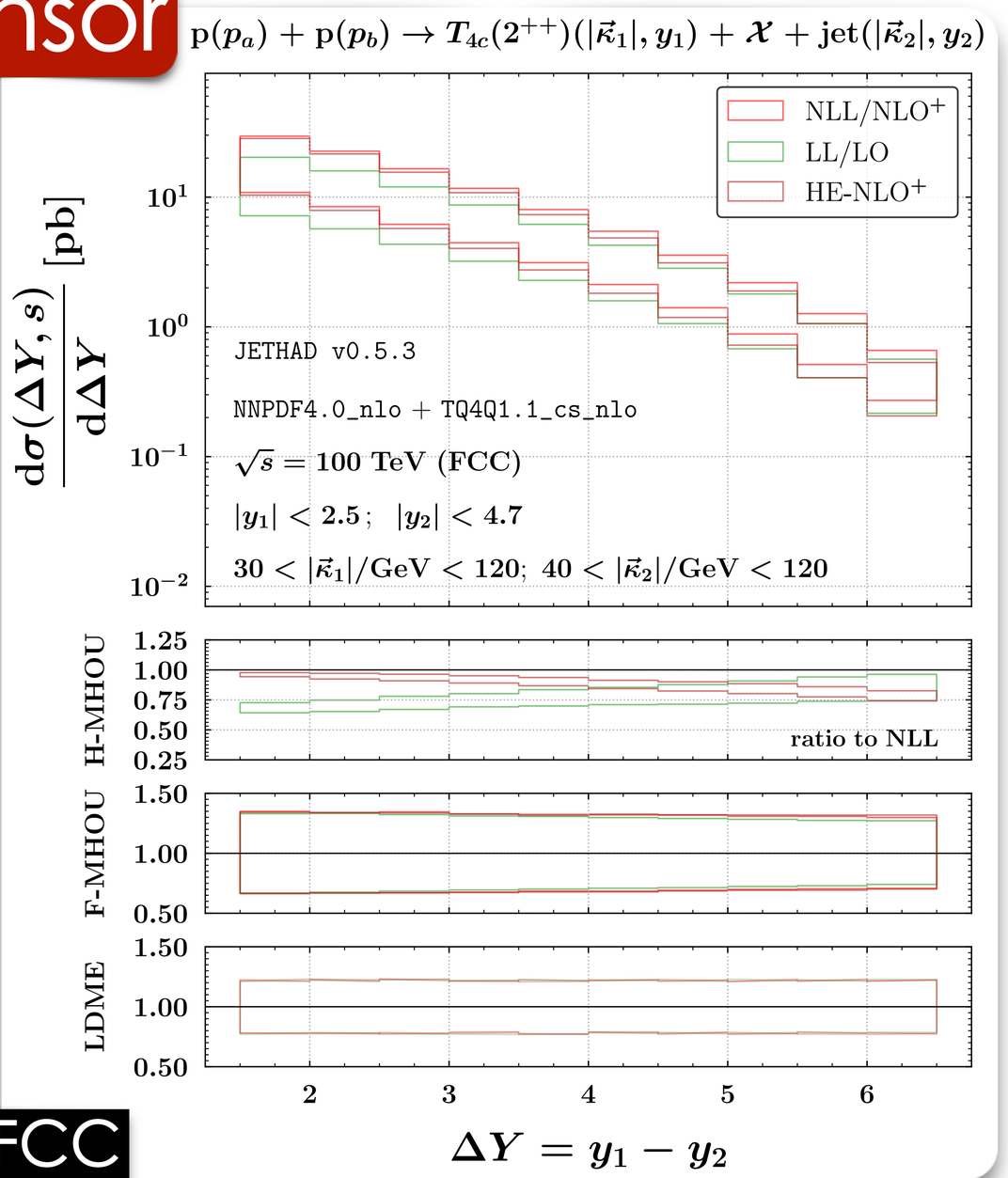
Axial

@13 TeV LHC

The tetraquark + jet rapidity spectrum from JETHAD

Tensor



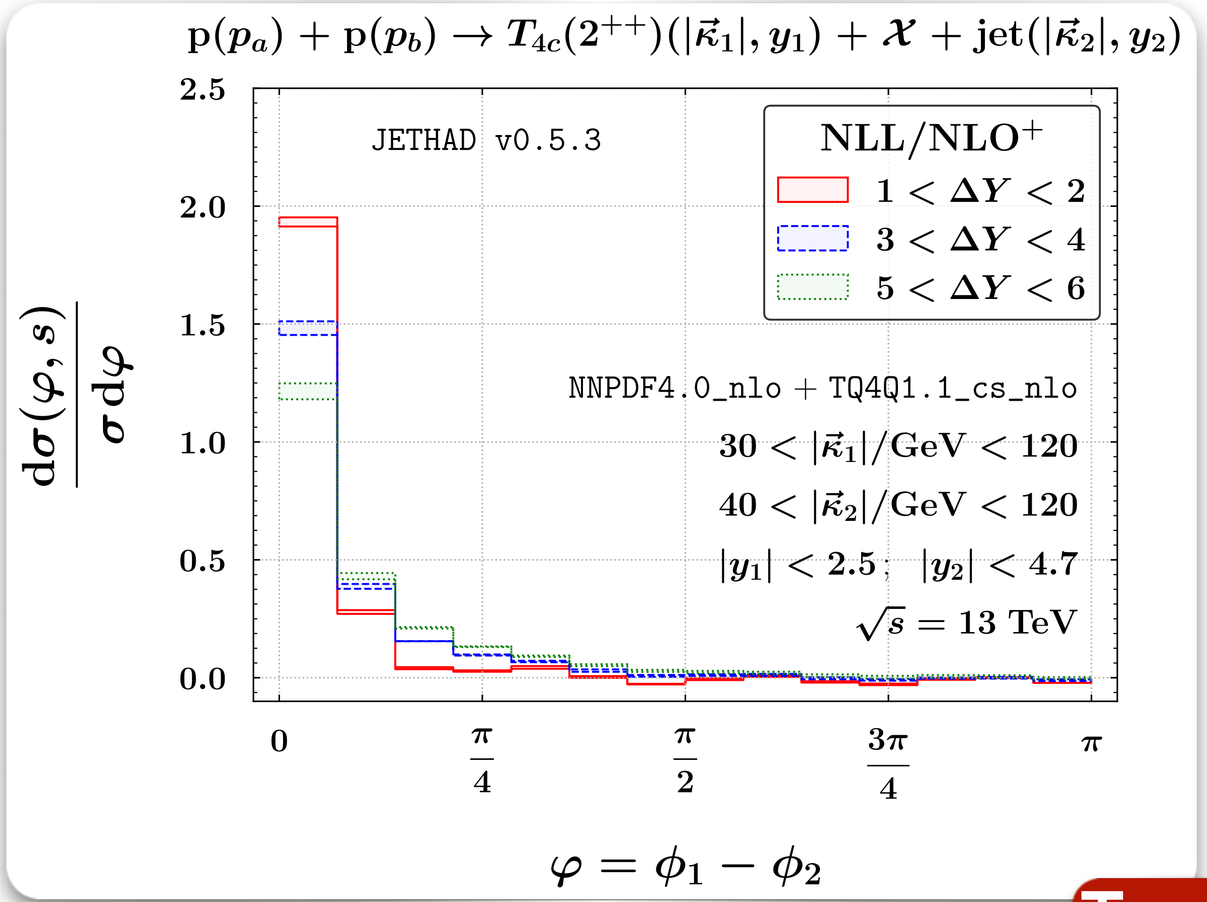


Axial

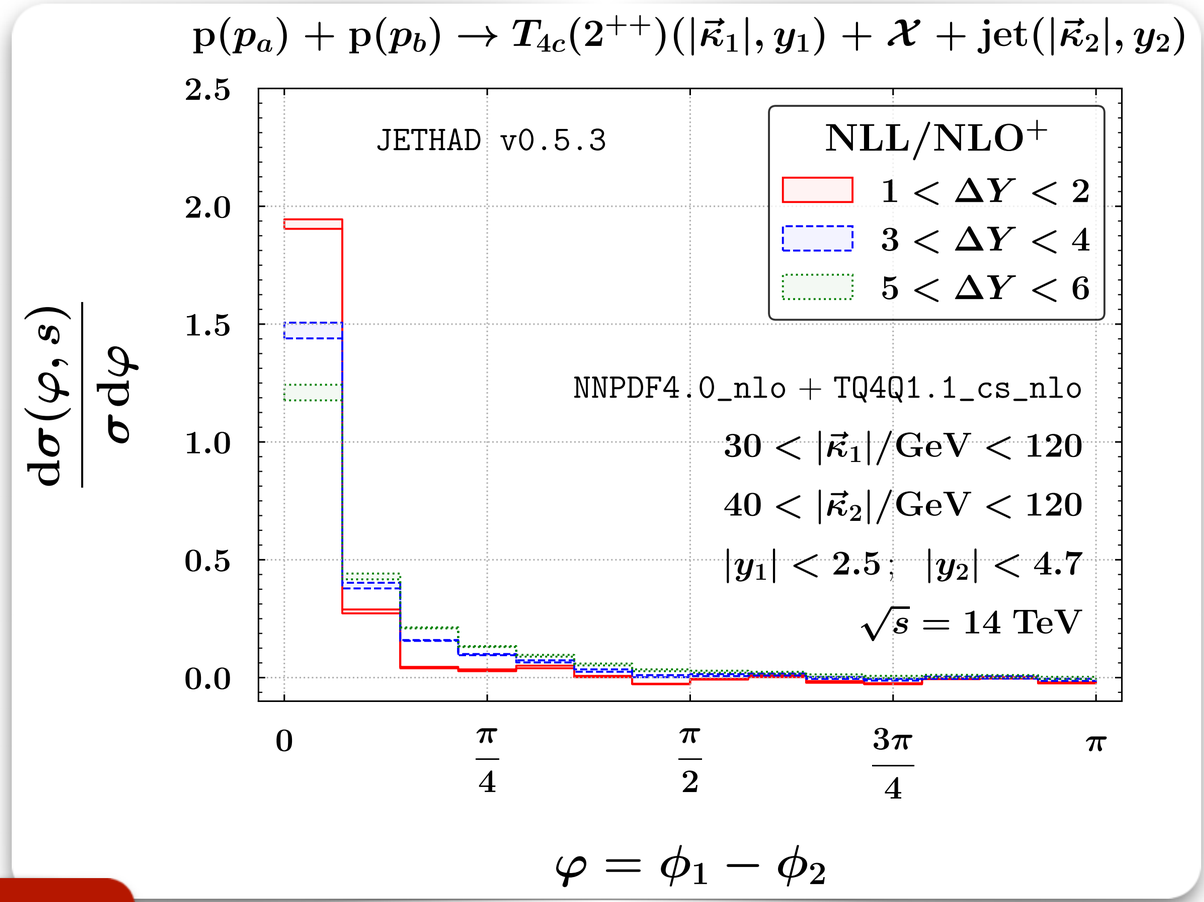
@100 TeV FCC

The tetraquark + jet angular spectrum from JETHAD

@13 TeV LHC



@100 TeV FCC



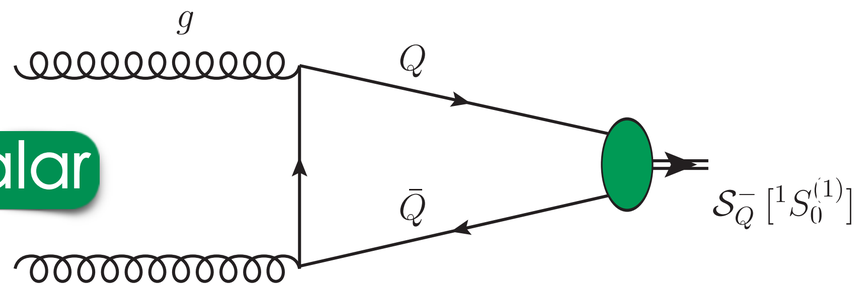
Tensor

QUARKONIUM FRAGMENTATION

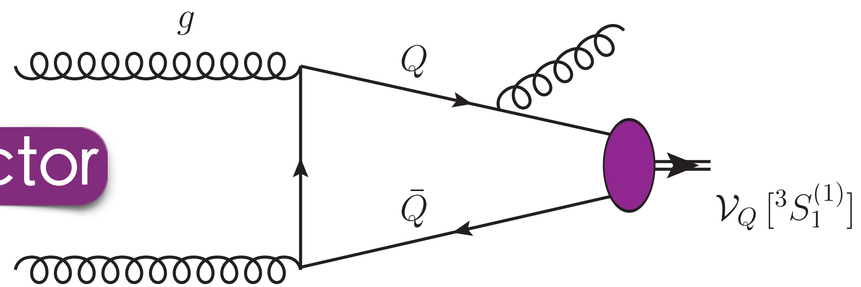
From low energies to high energies

Low p_T $\Rightarrow$ two-parton FFs
Genuine higher-twist FFNS FFs

scalar

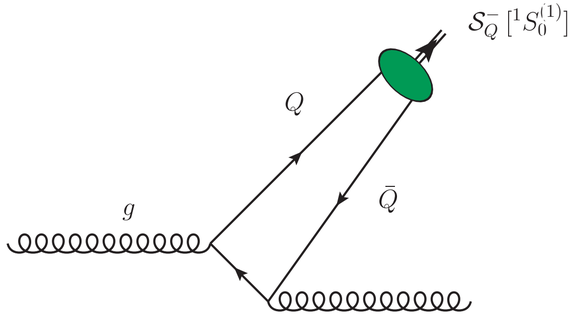


vector

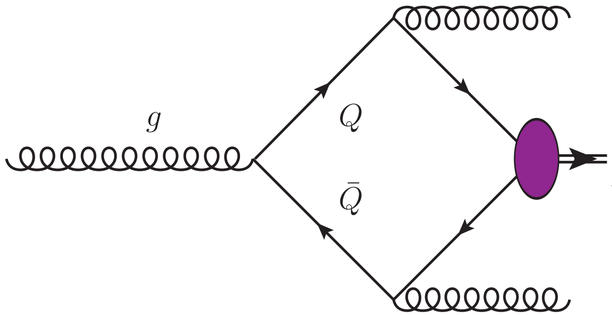


High p_T $\Rightarrow$ single-parton FFs
Twist-2 VFNS FFs + power corrections

scalar



vector

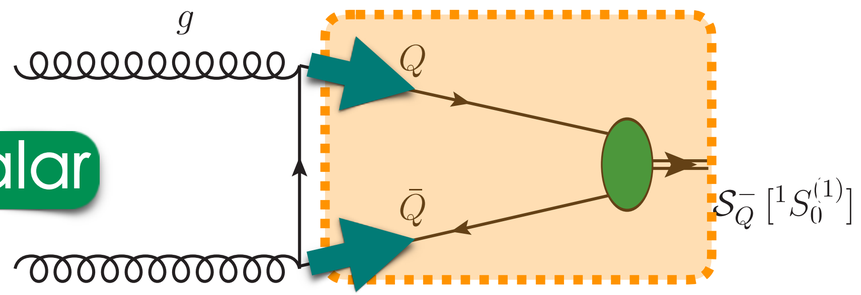


From low energies to high energies

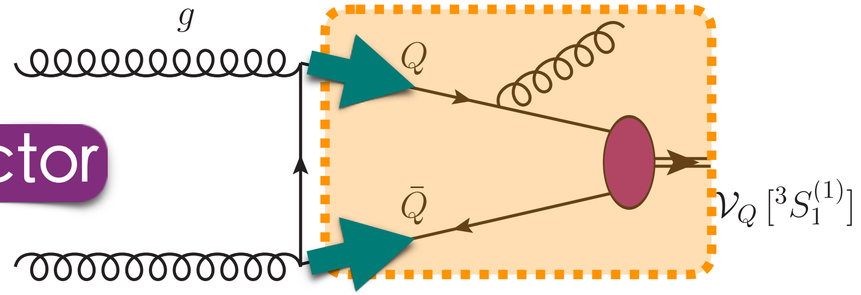
Low p_T $\Rightarrow$ two-parton FFs

Genuine higher-twist FFNS FFs

scalar



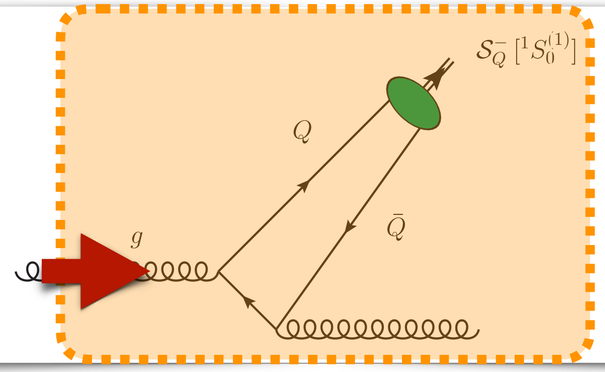
vector



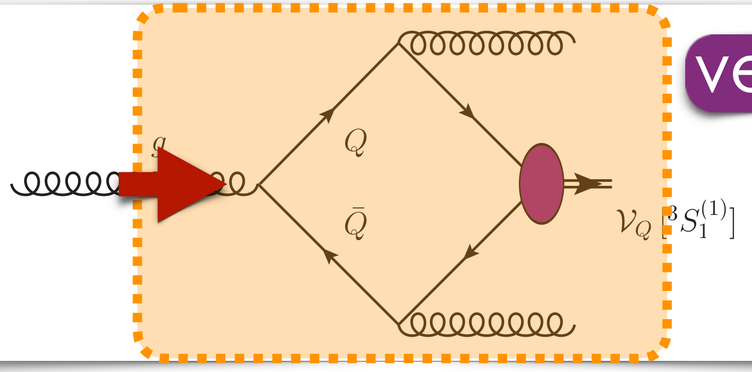
High p_T $\Rightarrow$ single-parton FFs

Twist-2 VFNS FFs + power corrections

scalar



vector



Collinear Fragmentation Correlator (CFC)

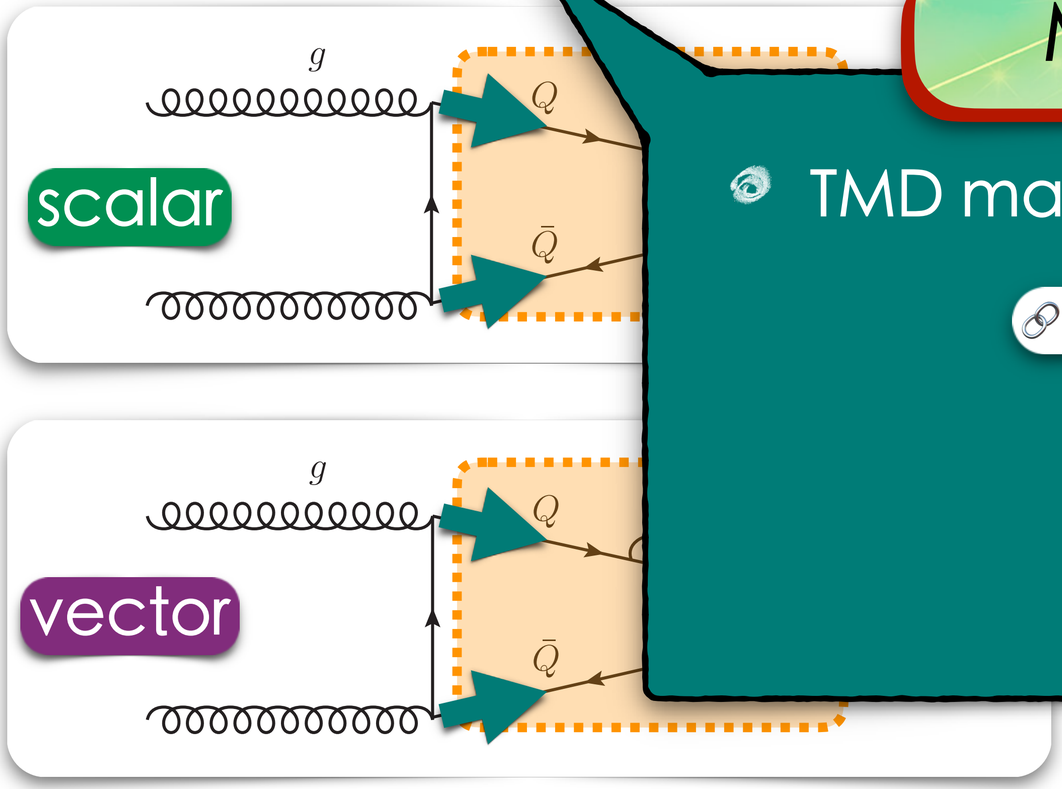
From low energies to high energies

Low p_T $\Rightarrow$ two-parton FFs

Genuine higher-twist FFNS FFs

High p_T $\Rightarrow$ single-parton FFs

Twist-2 VFNS FFs + power corrections



Matching low & high p_T

- TMD matching tails $\Rightarrow$ different singularities

[D. Boer, J. Bor, L. Maxia, C. Pisano, F. Yuan (2023)]

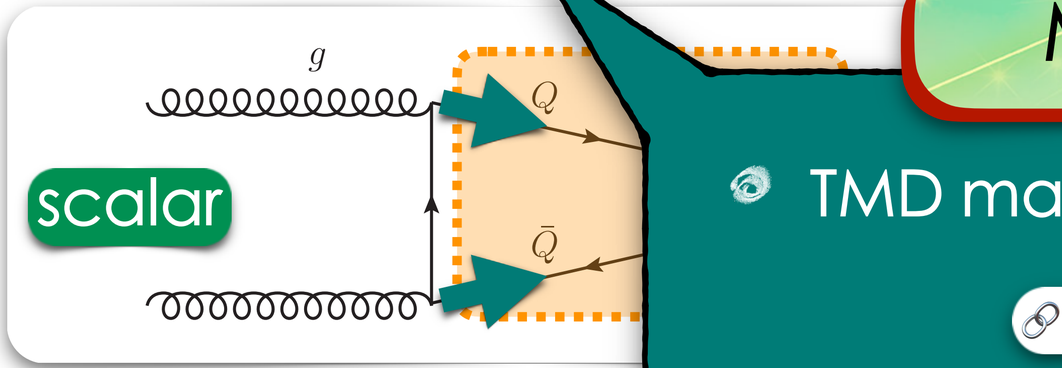
From low energies to high energies

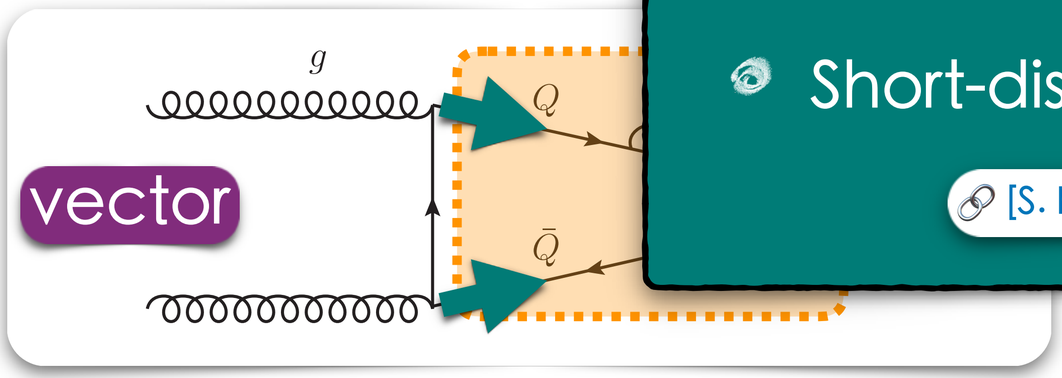
Low p_T $\Rightarrow$ two-parton FFs

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Twist-2 VFNS FFs + power corrections





Matching low & high p_T

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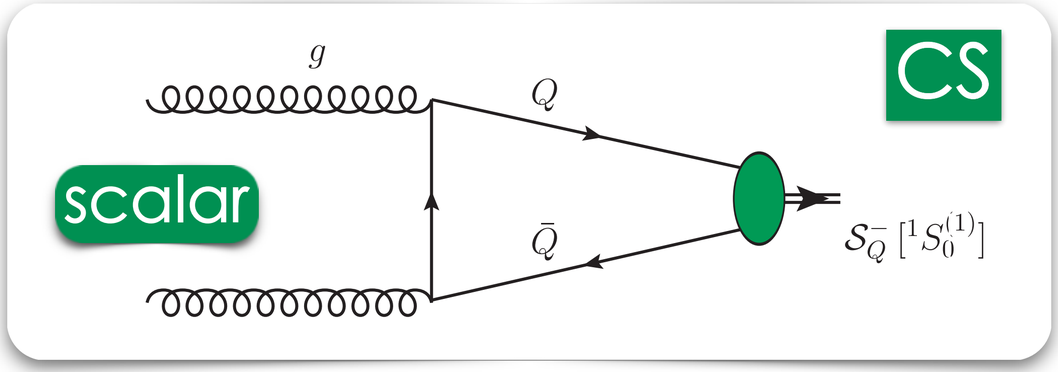
[D. Boer, J. Bor, L. Maxia, C. Pisano, F. Yuan (2023)]

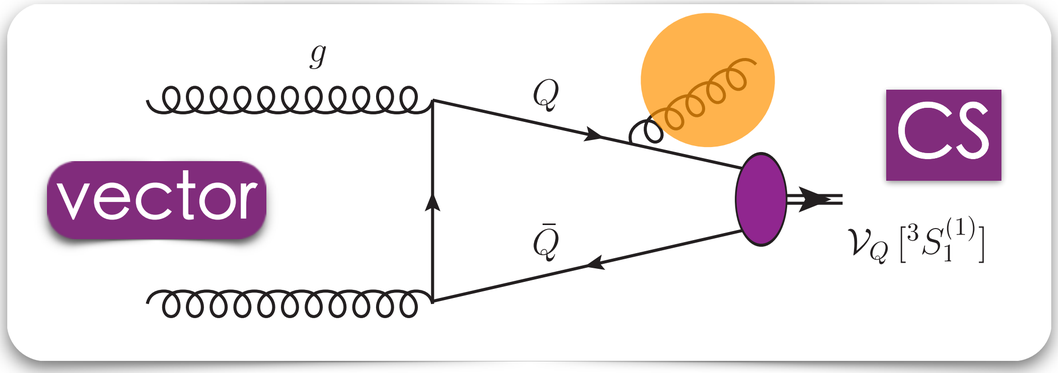
- Short-distance $\Leftrightarrow$ double-parton FFs (SCET)

[S. Fleming, A.K. Leibovich, T. Mehen, I.Z. Rothstein (2012)]

From low energies to high energies

Low energy, low p_T
Short-distance production



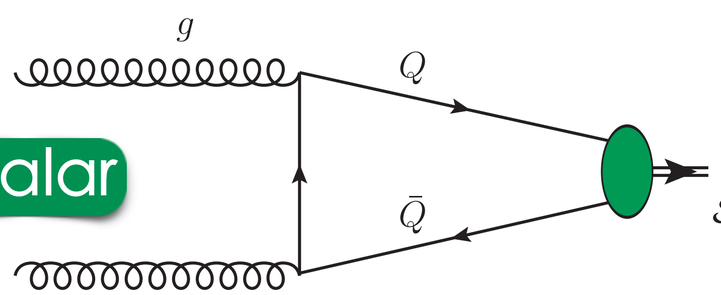


Extra gluon due to Landau-Yang selection rule

From low energies to high energies

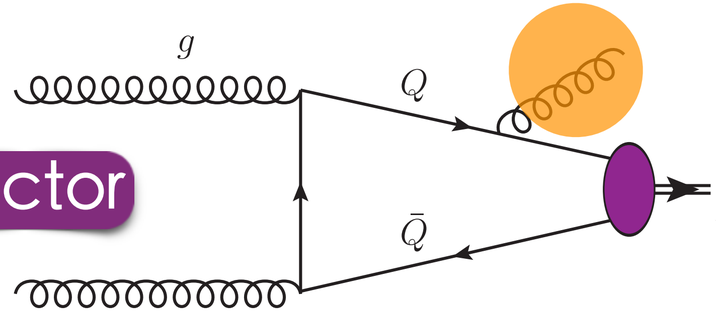
Low energy, low p_T
Short-distance production

scalar



CS

vector

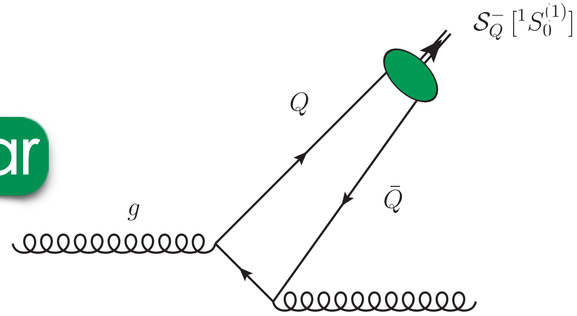


CS

(charmonia)

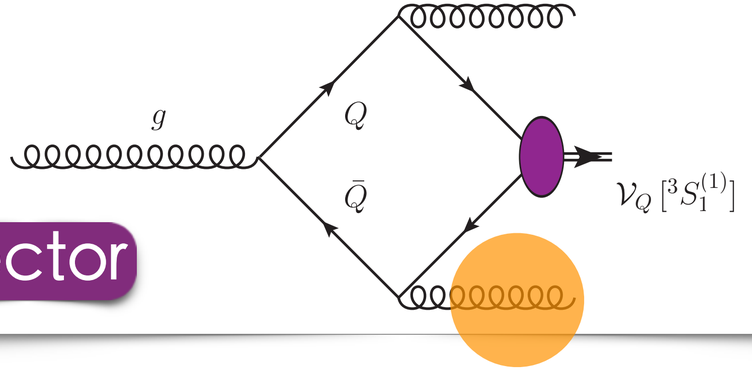
High energy, high p_T
Single-parton fragmentation

scalar



CS

vector



CS

Extra gluon due to Landau-Yang selection rule

HF-NRevo: Initial-scale NRCQD inputs

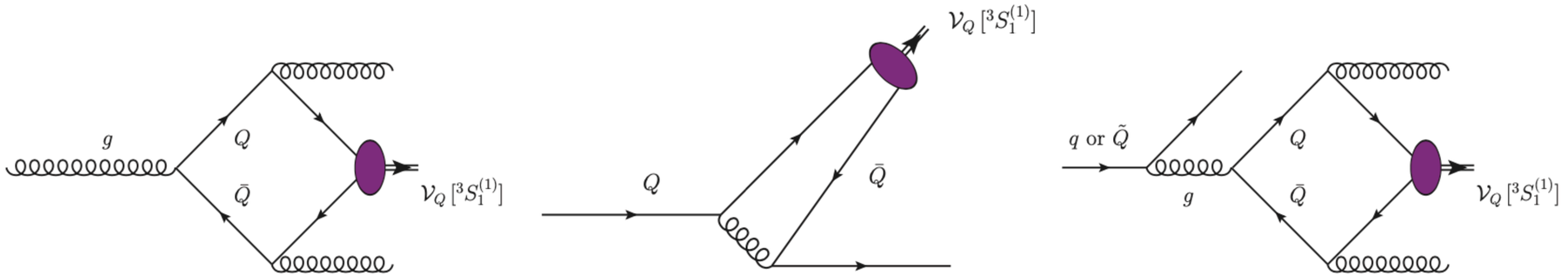


Fig. 1 Left panel: a leading diagram for the fragmentation of a gluon to $\mathcal{V}_Q\,[^3S_1^{(1)}]$ at $\mathcal{O}(\alpha_s^3)$. Central panel: one of the leading diagrams for the fragmentation of a constituent heavy quark (Q) to a S-wave color singlet vector quarkonium $\mathcal{V}_Q\,[^3S_1^{(1)}]$ at $\mathcal{O}(\alpha_s^2)$. Right panel: a leading diagram for the fragmentation of a nonconstituent light (q) or heavy ($\tilde{Q}$) quark to $\mathcal{V}_Q\,[^3S_1^{(1)}]$ at $\mathcal{O}(\alpha_s^4)$. Violet blobs refer to the $\mathcal{V}_Q\,[^3S_1^{(1)}]$ nonperturbative NRQCD LDME.

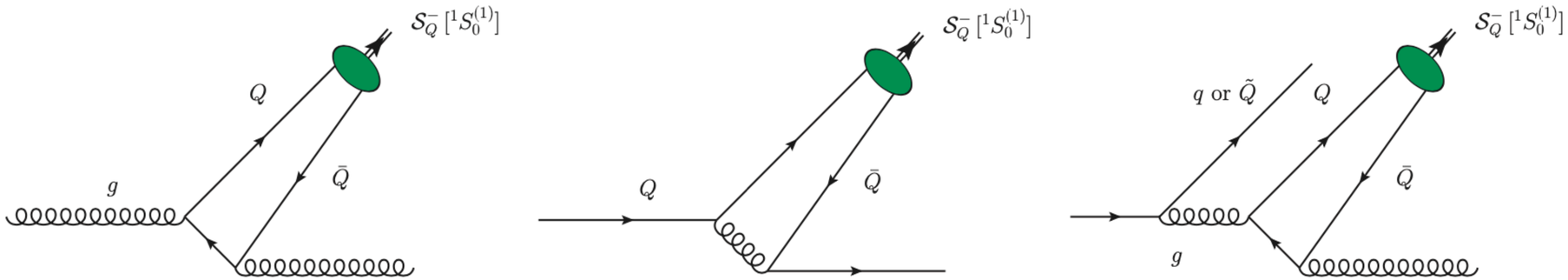
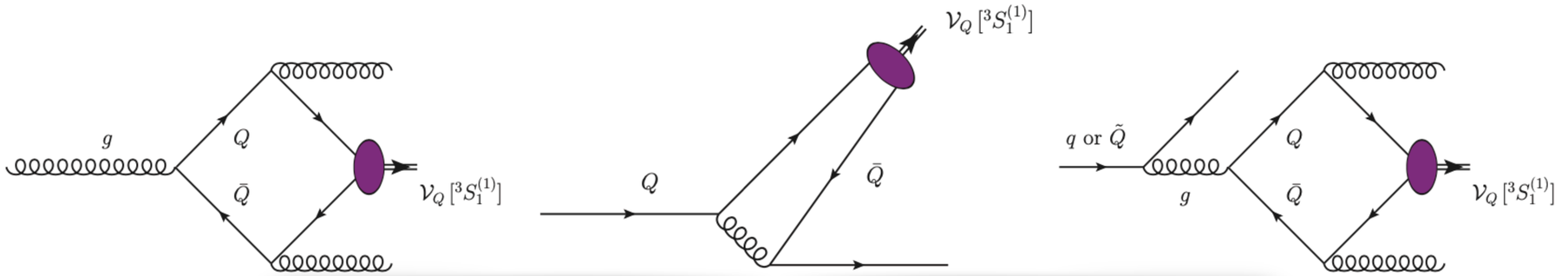


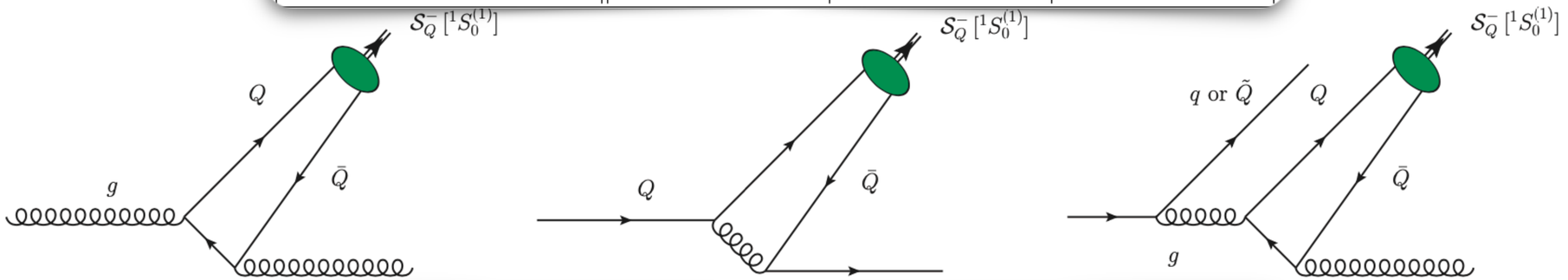
Fig. 2 Left panel: a leading diagram for the fragmentation of a gluon to $\mathcal{S}_Q^-\,[^1S_0^{(1)}]$ at $\mathcal{O}(\alpha_s^2)$. Central panel: one of the leading diagrams for the fragmentation of a constituent heavy quark (Q) to a S-wave color singlet pseudoscalar quarkonium $\mathcal{S}_Q^-\,[^1S_0^{(1)}]$ at $\mathcal{O}(\alpha_s^2)$. Right panel: a leading diagram for the fragmentation of a nonconstituent light (q) or heavy ($\tilde{Q}$) quark to $\mathcal{S}_Q^-\,[^1S_0^{(1)}]$ at $\mathcal{O}(\alpha_s^3)$. Green blobs refer to the $\mathcal{S}_Q^-\,[^1S_0^{(1)}]$ nonperturbative NRQCD LDME.

[F. G. C., in preparation]

HF-NRevo: Initial-scale NRCQD inputs



Quarkonium ($\mathcal{Q}$)	$g \to \mathcal{Q}$	$Q \to \mathcal{Q}$	q or $\tilde{Q} \to \mathcal{Q}$
$\mathcal{Q} \equiv \mathcal{V}_Q\,[^1S_0^{(1)}] \equiv J/\psi, \Upsilon$	–	$\mathcal{O}(\alpha_s^2) \equiv$ LO	–
	$\mathcal{O}(\alpha_s^3) \equiv$ NLO	$\mathcal{O}(\alpha_s^3) \equiv$ NLO	–
	$\mathcal{O}(\alpha_s^4) \equiv$ NNLO	$\mathcal{O}(\alpha_s^4) \equiv$ NNLO	$\mathcal{O}(\alpha_s^4) \equiv$ NNLO
	⋮	⋮	⋮

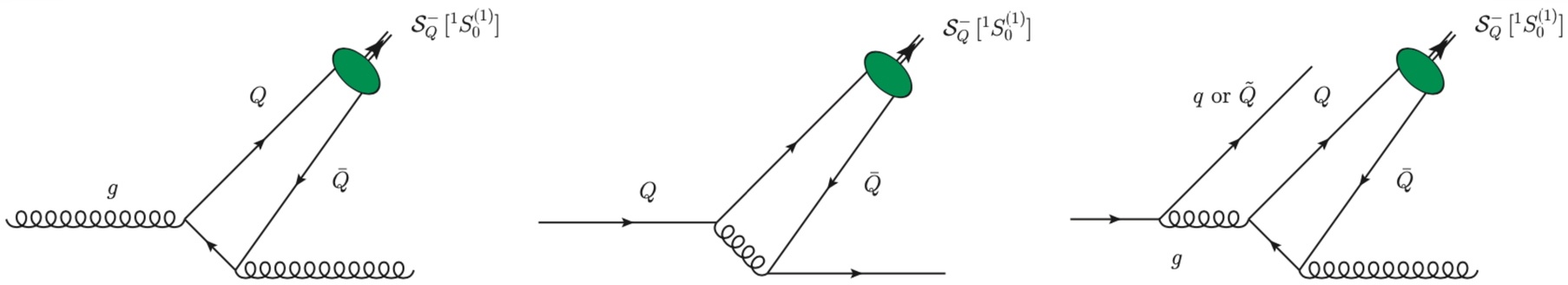


$\mathcal{Q} \equiv \mathcal{S}_Q^-\,[^3S_1^{(1)}] \equiv \eta_c, \eta_b$	$\mathcal{O}(\alpha_s^2) \equiv$ LO	$\mathcal{O}(\alpha_s^2) \equiv$ LO	–
	$\mathcal{O}(\alpha_s^3) \equiv$ NLO	$\mathcal{O}(\alpha_s^3) \equiv$ NLO	$\mathcal{O}(\alpha_s^3) \equiv$ NLO
	$\mathcal{O}(\alpha_s^4) \equiv$ NNLO	$\mathcal{O}(\alpha_s^4) \equiv$ NNLO	$\mathcal{O}(\alpha_s^4) \equiv$ NNLO
	⋮	⋮	⋮

[F. G. C., in preparation]

HF-NRevo: Initial-scale NRCQD inputs

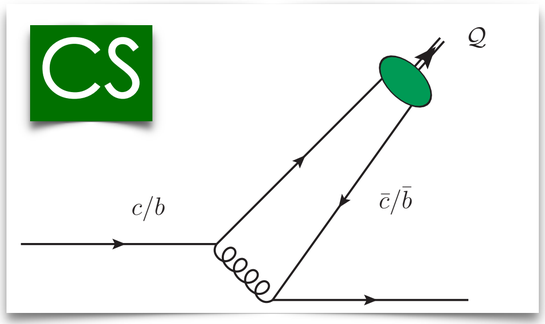
$\mathcal{S}_Q^-\,[^1S_0^{(1)}]$ quarkonium	$\mu_{F,0}(g \to \mathcal{S}_Q^-)$	$\mu_{F,0}(d,u,s \to \mathcal{S}_Q^-)$	$\mu_{F,0}(Q \to \mathcal{S}_Q^-)$	$\mu_{F,0}(\tilde{Q} \to \mathcal{S}_Q^-)$	$Q_0 \equiv \max(\{\mu_{F,0}\})$
η_c	$2m_c$	$m_{d,u,s} + 2m_c$	$3m_c$	$m_b + 2m_c$	$m_b + 2m_c$
η_b	$2m_b$	$m_{d,u,s} + 2m_b$	$3m_b$	$m_c + 2m_b$	$3m_b$



[F. G. C., in preparation]

Vector quarkonium from single-parton fragmentation

¡ *Let us consider* J/ψ *and* Υ *at large* p_T $\rightarrow$ *initial-scale* ***heavy-quark*** + ***gluon*** *from* ***NRQCD*** !



$$D_{\bar{Q}}^{\mathcal{Q}}(z,\mu_F \equiv \mu_0) = D_{\bar{Q}}^{\mathcal{Q},\mathrm{LO}}(z) + \frac{\alpha_s^3(\mu_R)}{m_Q^3}\,|\mathcal{R}_{\mathcal{Q}}(0)|^2\,\Gamma^{\mathcal{Q},\mathrm{NLO}}(z)$$

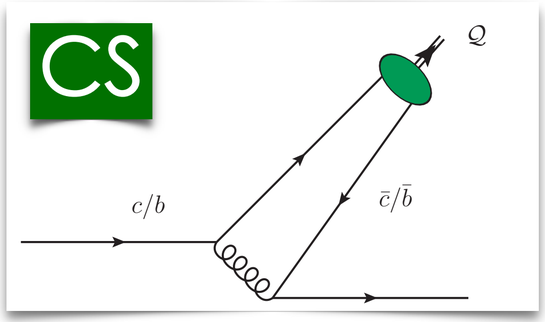
$(\mathcal{Q} \to \bar{\mathcal{Q}}\,\mathcal{Q})$ at $\mu_0 = 3m_{\mathcal{Q}}$

(LO) 🔗 [E. Braaten et al., Phys. Rev. D 48 (1993) 4230-4235]
(NLO) 🔗 [X. Zheng et al., Phys. Rev. D 100 (2019) 1, 014005]

🔗 [My talk at the Higgs Centre, Edinburgh (2023)]
🔗 [F. G. C., M. Fucilla, Eur. Phys. J. C 82 (2022) 10, 929]

Vector quarkonium from single-parton fragmentation

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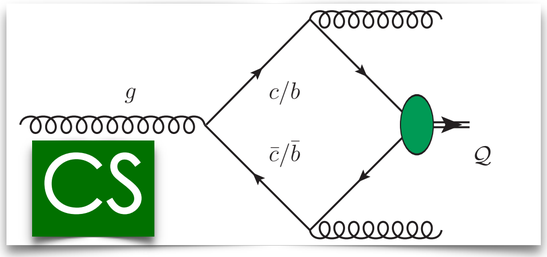
$$D_{\mathcal{Q}}^{\mathcal{Q}}(z,\mu_F \equiv \mu_0) = D_{\mathcal{Q}}^{\mathcal{Q},\mathrm{LO}}(z) + \frac{\alpha_s^3(\mu_R)}{m_Q^3}\,|\mathcal{R}_{\mathcal{Q}}(0)|^2\,\Gamma^{\mathcal{Q},\mathrm{NLO}}(z)$$

$(Q \to \mathcal{Q}\,Q)$ at $\mu_0 = 3m_Q$

(LO) 🔗 [E. Braaten et al., Phys. Rev. D 48 (1993) 4230-4235]

(NLO) 🔗 [X. Zheng et al., Phys. Rev. D 100 (2019) 1, 014005]

+



$$D_g^{\mathcal{Q}}(z,2m_Q) = \frac{5}{36(2\pi)^2}\alpha_s^3(2m_Q)\frac{|\mathcal{R}_{\mathcal{Q}}(0)|^2}{m_{\mathcal{Q}}^3}\int_0^z \mathrm{d}\xi \int_{(\xi+z^2)/2z}^{(1+\xi)/2} \mathrm{d}\tau \frac{1}{(1-\tau)^2(\tau-\xi)^2(\tau^2-\xi)^2}$$

$$\sum_{i=1}^{2} z^i \left[f_i^{(g)}(\xi,\tau) + g_i^{(g)}(\xi,\tau)\frac{1+\xi-2\tau}{2(\tau-\xi)\sqrt{\tau^2-\xi}} \ln\left(\frac{\tau-\xi+\sqrt{\tau^2-\xi}}{\tau-\xi-\sqrt{\tau^2-\xi}}\right)\right],$$

$(g \to \mathcal{Q}\,gg)$ at $\mu_0 = 2m_Q$

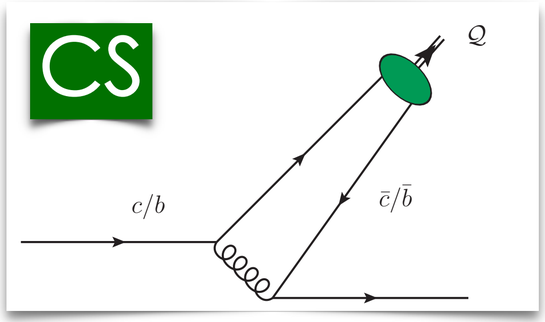
(LO) 🔗 [A. Braaten, T.C Yuan, Phys. Rev. Lett. 71 (1993), 1673]

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🔗 [F. G. C., M. Fucilla, Eur. Phys. J. C 82 (2022) 10, 929]

Vector quarkonium from single-parton fragmentation

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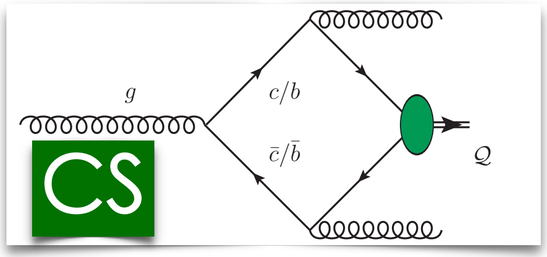


$$D_{\mathcal{Q}}^{\mathcal{Q}}(z,\mu_F \equiv \mu_0) = D_{\mathcal{Q}}^{\mathcal{Q},\mathrm{LO}}(z) + \frac{\alpha_s^3(\mu_R)}{m_Q^3}\,|\mathcal{R}_{\mathcal{Q}}(0)|^2\,\Gamma^{\mathcal{Q},\mathrm{NLO}}(z)$$

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$$D_g^{\mathcal{Q}}(z,2m_Q) = \frac{5}{36(2\pi)^2}\alpha_s^3(2m_Q)\frac{|\mathcal{R}_{\mathcal{Q}}(0)|^2}{m_{\mathcal{Q}}^3}\int_0^z \mathrm{d}\xi \int_{(\xi+z^2)/2z}^{(1+\xi)/2} \mathrm{d}\tau \frac{1}{(1-\tau)^2(\tau-\xi)^2(\tau^2-\xi)^2}$$
$$\sum_{i=1}^{2} z^i \left[f_i^{(g)}(\xi,\tau) + g_i^{(g)}(\xi,\tau)\frac{1+\xi-2\tau}{2(\tau-\xi)\sqrt{\tau^2-\xi}} \ln\left(\frac{\tau-\xi+\sqrt{\tau^2-\xi}}{\tau-\xi-\sqrt{\tau^2-\xi}}\right)\right],$$

$(g \to \mathcal{Q}\,gg)$ at $\mu_0 = 2m_Q$

⊗

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🔗 [F. G. C., M. Fucilla, Eur. Phys. J. C 82 (2022) 10, 929]

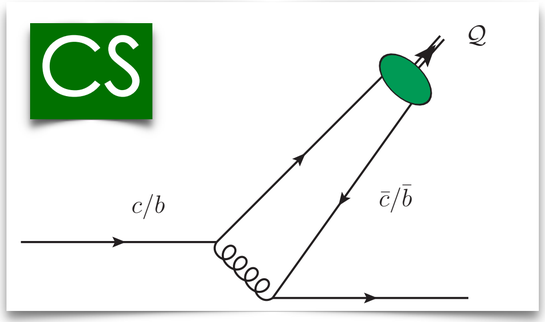
NRFF1.0
quarkonium FFs

=

APFEL++/EKO

Vector quarkonium from single-parton fragmentation

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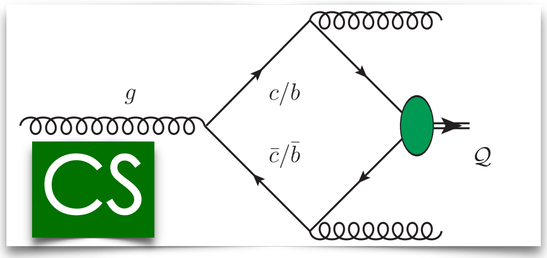


$$D_Q^{\mathcal{Q}}(z,\mu_F \equiv \mu_0) = D_Q^{\mathcal{Q},\mathrm{LO}}(z) + \frac{\alpha_s^3(\mu_R)}{m_Q^3}\,|\mathcal{R}_{\mathcal{Q}}(0)|^2$$

(LO) 🔗 [E. Braaten et al., Phys. Rev. D 48 (1993) 4230-42
(NLO) 🔗 [X. Zheng et al., Phys. Rev. D 100 (2019) 1, 014005]

$(Q \to \mathcal{Q}\,Q)$ at $\mu_0 = 3m_Q$

+



$$D_g^{\mathcal{Q}}(z,2m_Q) = \frac{5}{36(2\pi)^2}\alpha_s^3(2m_Q)\frac{|\mathcal{R}_{\mathcal{Q}}(0)|^2}{m_{\mathcal{Q}}^3}\int_0^z \mathrm{d}\xi \int_{(\xi+z^2)/2z}^{(1+\xi)/2} \mathrm{d}\tau \frac{1}{(1-\tau)^2(z}$$

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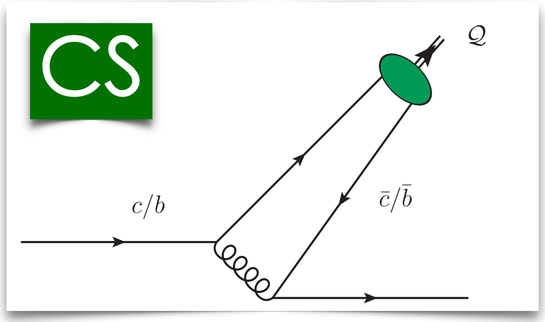
NRFF1.0
quarkonium FFs

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Vector quarkonium from single-parton fragmentation

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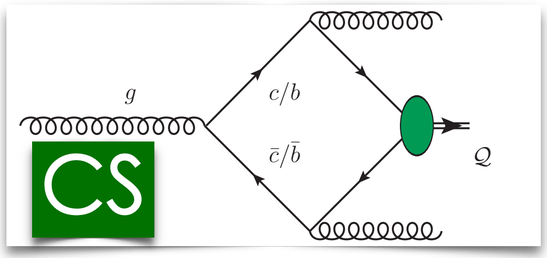


$$D_Q^{\mathcal{Q}}(z,\mu_F\equiv\mu_0)=D_Q^{\mathcal{Q},\mathrm{LO}}(z)+\frac{\alpha_s^3(\mu_R)}{m_Q^3}\,|\mathcal{R}_{\mathcal{Q}}(0)|^2$$

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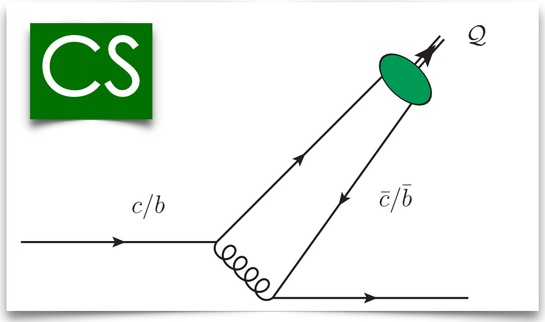
NRFF1.0
quarkonium FFs

=

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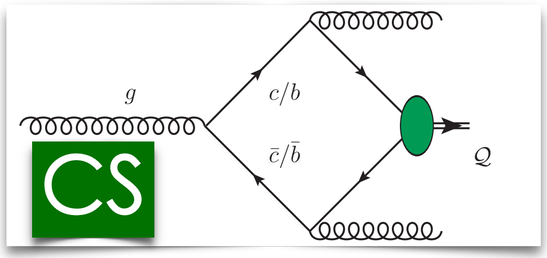
$$D_Q^{\mathcal{Q}}(z,\mu_F \equiv \mu_0) = D_Q^{\mathcal{Q},\mathrm{LO}}(z) + \frac{\alpha_s^3(\mu_R)}{m_Q^3}\,|\mathcal{R}_{\mathcal{Q}}(0)|^2$$

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$(Q \to \mathcal{Q}\,Q)$ at $\mu_0 = 3m_Q$

+



$$D_g^{\mathcal{Q}}(z,2m_Q) = \frac{5}{36(2\pi)^2}\alpha_s^3(2m_Q)\frac{|\mathcal{R}_{\mathcal{Q}}(0)|^2}{m_{\mathcal{Q}}^3}\int_0^z \mathrm{d}\xi \int_{(\xi+z^2)/2z}^{(1+\xi)/2} \mathrm{d}\tau \frac{1}{(1-\tau)^2(z}$$

$$\sum_{i=1}^{2} z^i \left[f_i^{(g)}(\xi,\tau) + g_i^{(g)}(\xi,\tau)\frac{1+\xi-2\tau}{2(\tau-\xi)\sqrt{\tau^2-\xi}} \ln\left(\frac{\tau-\xi+\sqrt{\tau^2-\xi}}{\tau-\xi-\sqrt{\tau^2-\xi}}\right)\right]$$

(LO) 🔗 [A. Braaten, T.C Yuan, Phys. Rev. Lett. 71 (1993), 1673]

$(g \to \mathcal{Q}\,gg)$ at $\mu_0 = 2m_Q$

🔗 [My talk at the Higgs Centre, Edinburgh (2023)]

🔗 [F. G. C., M. Fucilla, Eur. Phys. J. C 82 (2022) 10, 929]

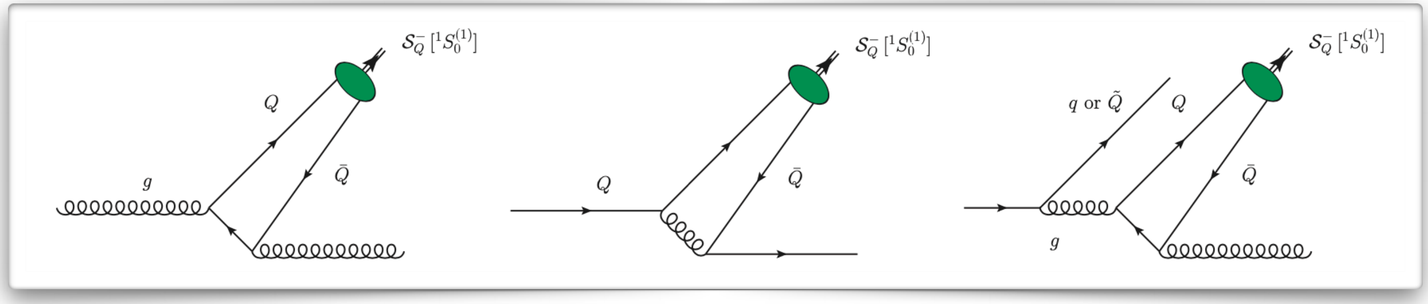
NRFF1.0

quarkonium FFs

=

APFEL++/EKO

HF-NRevo: Initial-scale NRCQD inputs



Scalar η_c

HF-NRevo for $\eta_c[^1S_0^{(1)}]$ collinear fragmentation

Partonic hard factor

DGLAP AOevo

All-Order evolution (numeric)

DGLAP EDevo

Expanded Decoupled evolution

NRQCD

LDMEs

SDCs

g
$\tilde{q}$
c
b
$2m_c$

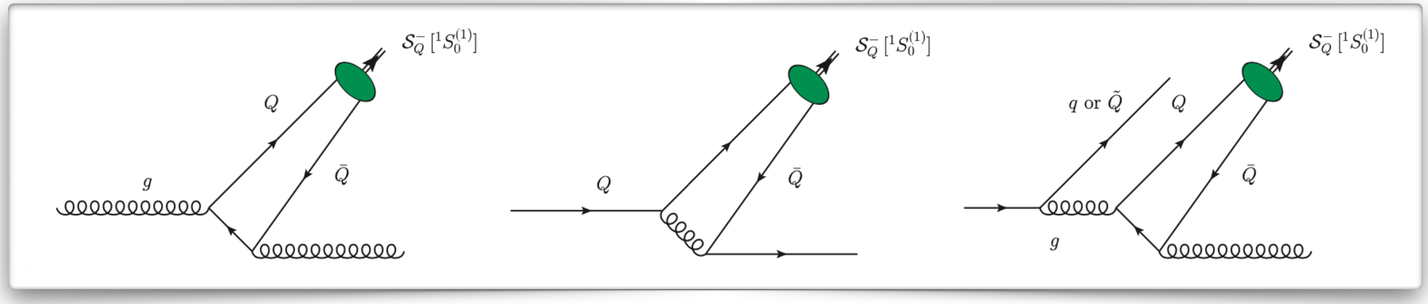
$\mathcal{D}_i^{\mathcal{Q}}(z,[n]) \otimes \langle \mathcal{O}^{\mathcal{Q}}([n]) \rangle$

Energy

Evolution-ready scale $Q_0 \equiv m_b + 2m_c \quad 3m_c \quad m_{\tilde{q}} + 2m_c$

Time

HF-NRevo: Initial-scale NRCQD inputs



Scalar η_c

HF-NRevo for $\eta_c[^1S_0^{(1)}]$ collinear fragmentation

Partonic hard factor

DGLAP AOevo

All-Order evolution (numeric)

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NRQCD

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g
$\tilde{q}$
c
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$2m_c$

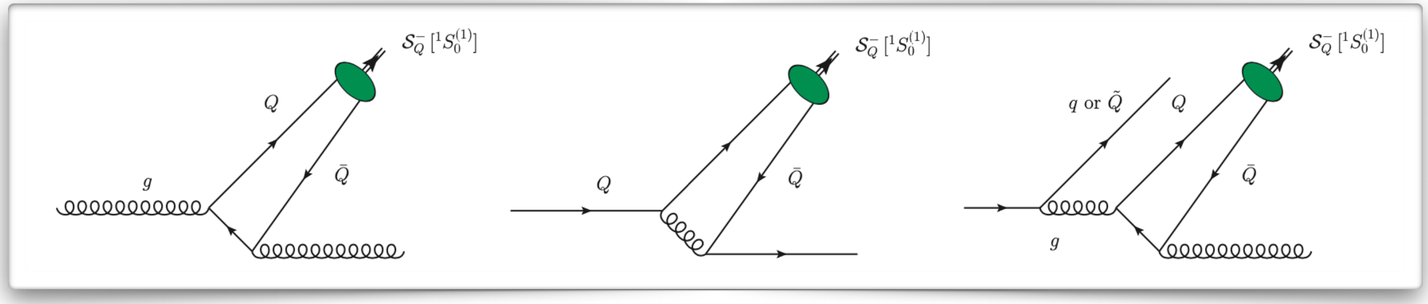
$\mathcal{D}_i^{\mathcal{Q}}(z,[n]) \otimes \langle \mathcal{O}^{\mathcal{Q}}([n]) \rangle$

Energy

Evolution-ready scale $Q_0 \equiv m_b + 2m_c \quad 3m_c \quad m_{\tilde{q}} + 2m_c$

Time

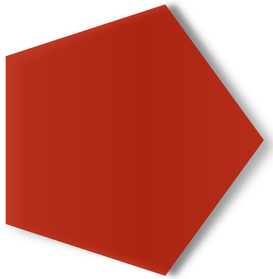
HF-NRevo: Initial-scale NRCQD inputs



Scalar η_c

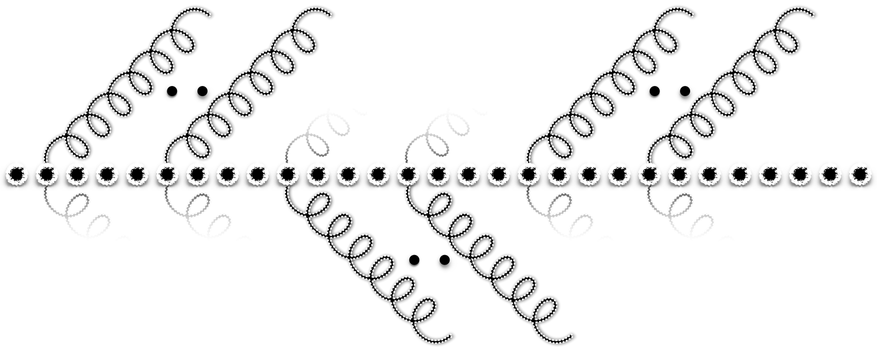
HF-NRevo for $\eta_c[^1S_0^{(1)}]$ collinear fragmentation

Partonic hard factor



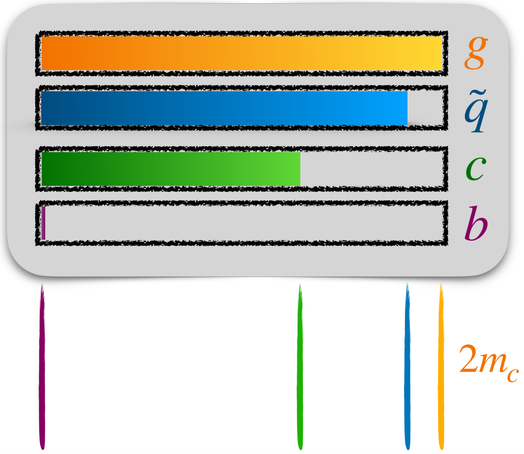
DGLAP AOevo

All-Order evolution (numeric)



DGLAP EDevo

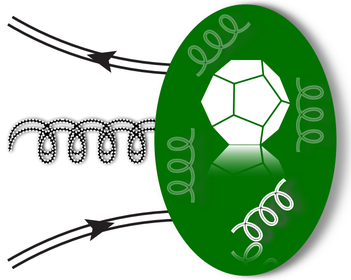
Expanded Decoupled evolution



NRQCD

LDMEs

SDCs



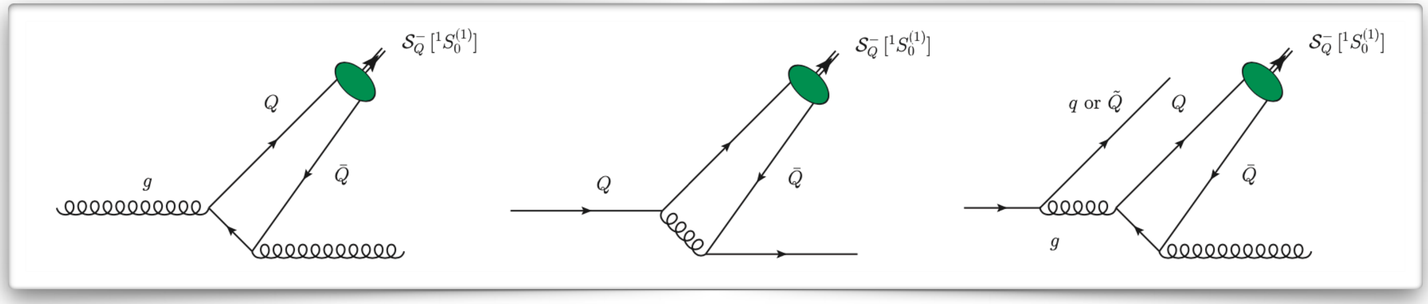
$\mathcal{D}_i^{\mathcal{Q}}(z,[n]) \otimes \langle \mathcal{O}^{\mathcal{Q}}([n]) \rangle$

Energy

Evolution-ready scale $Q_0 \equiv m_b + 2m_c$ $3m_c$ $m_{\tilde{q}} + 2m_c$

Time

HF-NRevo: Initial-scale NRCQD inputs



Scalar η_b

HF-NRevo for $\eta_b[^1S_0^{(1)}]$ collinear fragmentation

Partonic hard factor

DGLAP AOevo

All-Order evolution (numeric)

DGLAP EDevo

Expanded Decoupled evolution

g
$\tilde{q}$
c
b
$2m_b$

NRQCD

SDCs

LDMEs

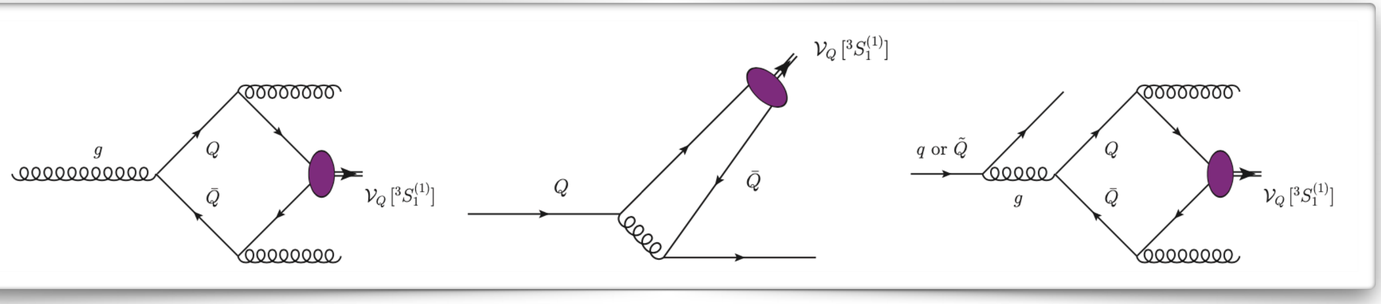
$\mathcal{D}_i^{\mathcal{Q}}(z,[n]) \otimes \langle \mathcal{O}^{\mathcal{Q}}([n]) \rangle$

Energy

Evolution-ready scale $Q_0 \equiv 3m_b$ $m_c + 2m_b$ $m_{\tilde{q}} + 2m_b$

Time

HF-NRevo: Initial-scale NRCQD inputs



Vector J/ψ

HF-NRevo for $J/\psi[^3S_1^{(1)}]$ collinear fragmentation

Partonic hard factor

DGLAP AOevo

All-Order evolution (numeric)

DGLAP EDevo

Expanded Decoupled evolution

g
$\tilde{q}$
c
b

$2m_c$

NRQCD

SDCs

LDMEs

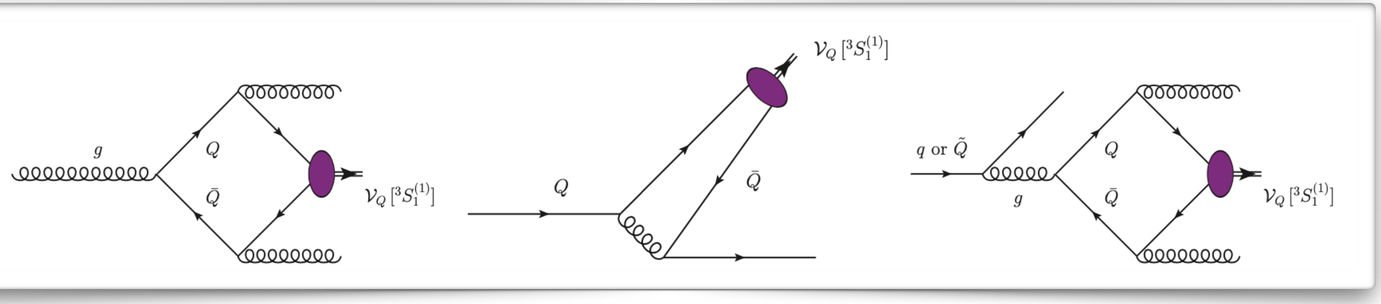
$\mathcal{D}_i^{\mathcal{Q}}(z,[n]) \otimes \langle \mathcal{O}^{\mathcal{Q}}([n]) \rangle$

Energy

Evolution-ready scale $Q_0 \equiv 3m_c$

Time

HF-NRevo: Initial-scale NRCQD inputs



Vector Υ

HF-NRevo for $\Upsilon[^3S_1^{(1)}]$ collinear fragmentation

Partonic hard factor

DGLAP AOevo

All-Order evolution (numeric)

DGLAP EDevo

Expanded Decoupled evolution

g
$\tilde{q}$
c
b
$2m_c$

NRQCD

SDCs

LDMEs

$\mathcal{D}_i^{\mathcal{Q}}(z,[n]) \otimes \langle \mathcal{O}^{\mathcal{Q}}([n]) \rangle$

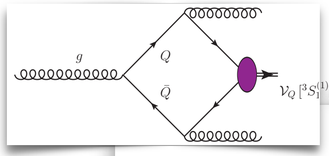
Energy

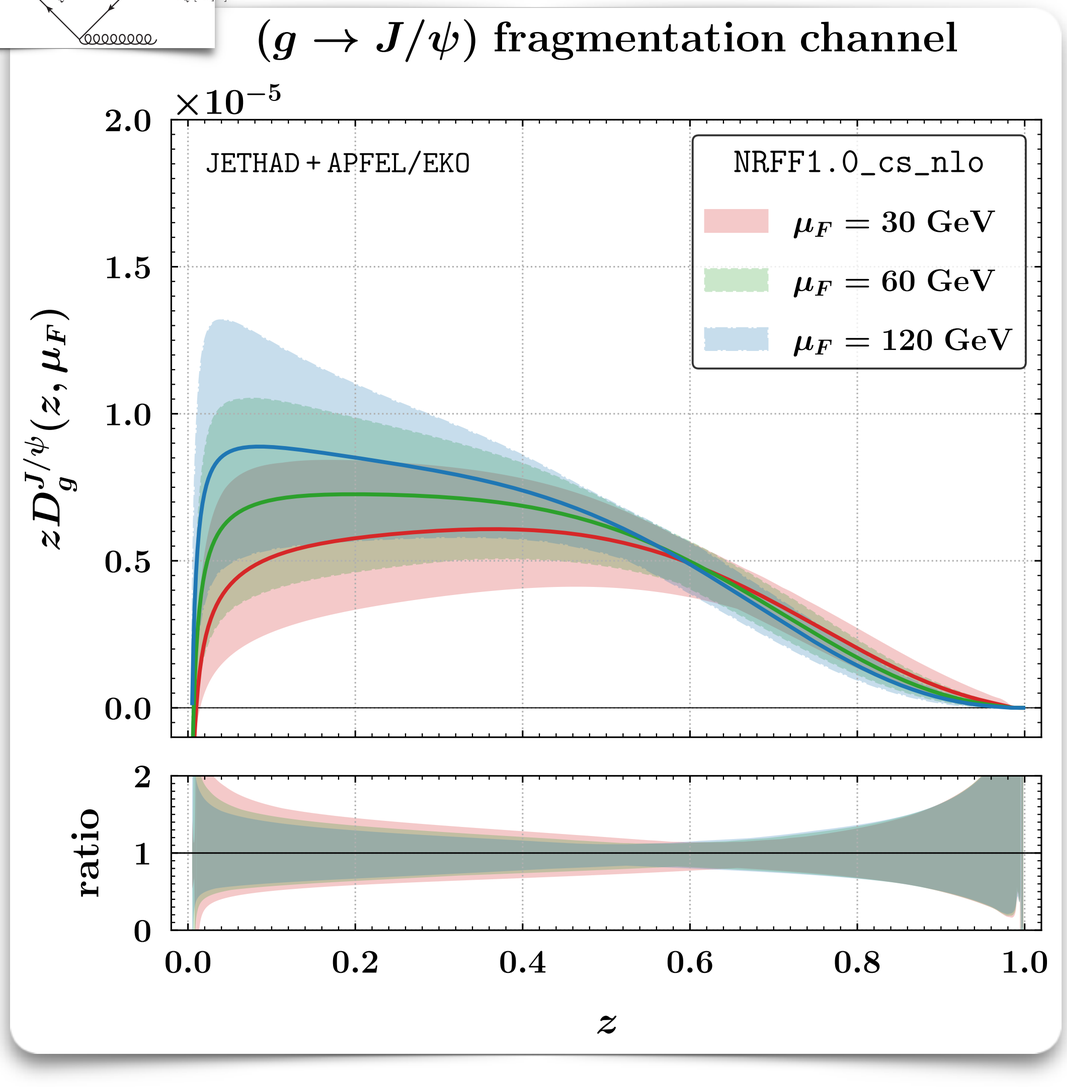
Evolution-ready scale $Q_0 \equiv 3m_b$

Time

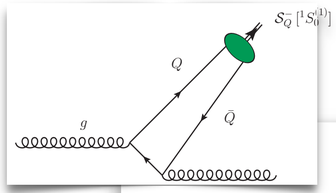
NRFF1.0: Gluon fragmentation to charmonia

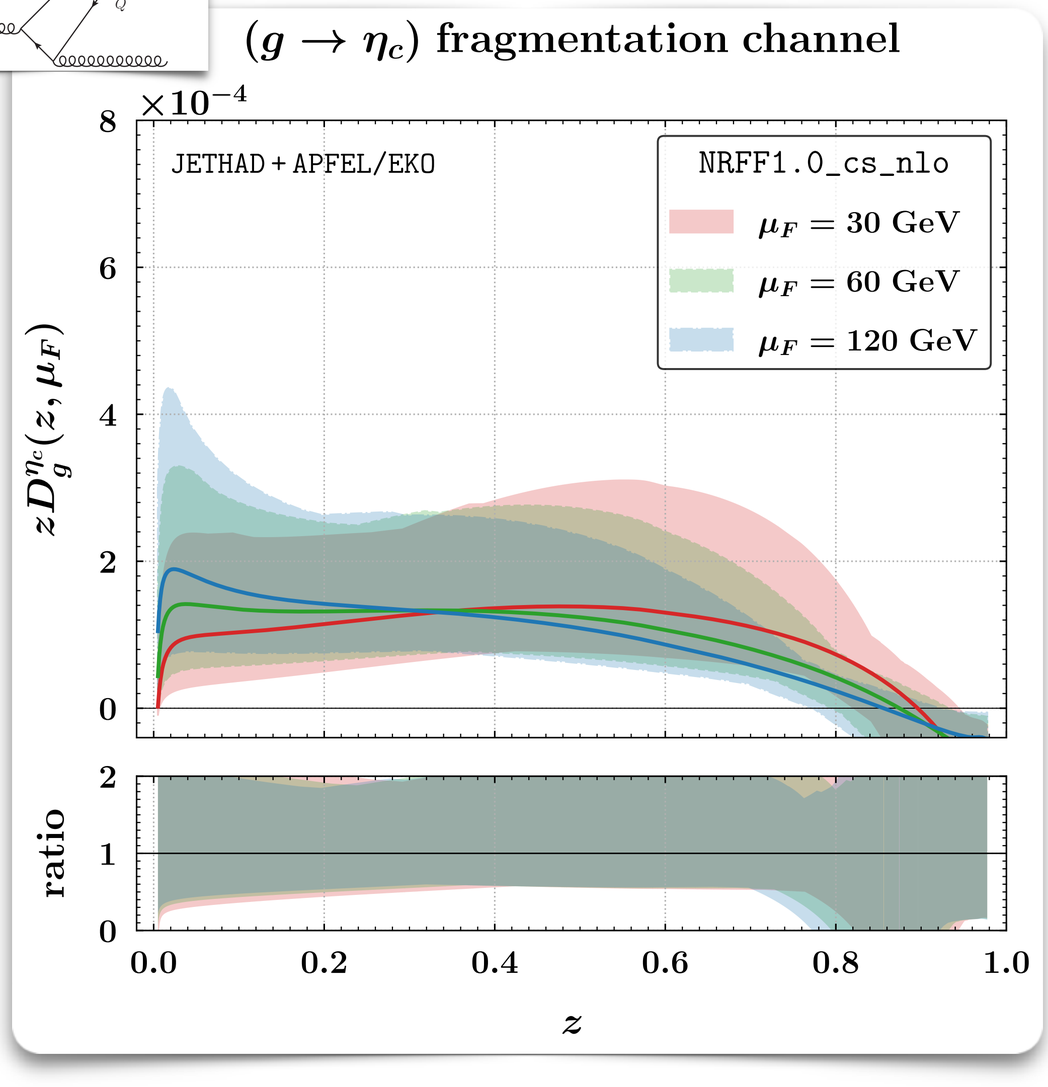
Vector J/ψ





Scalar η_c

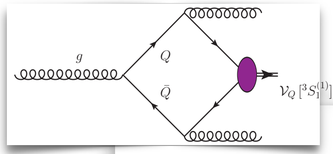


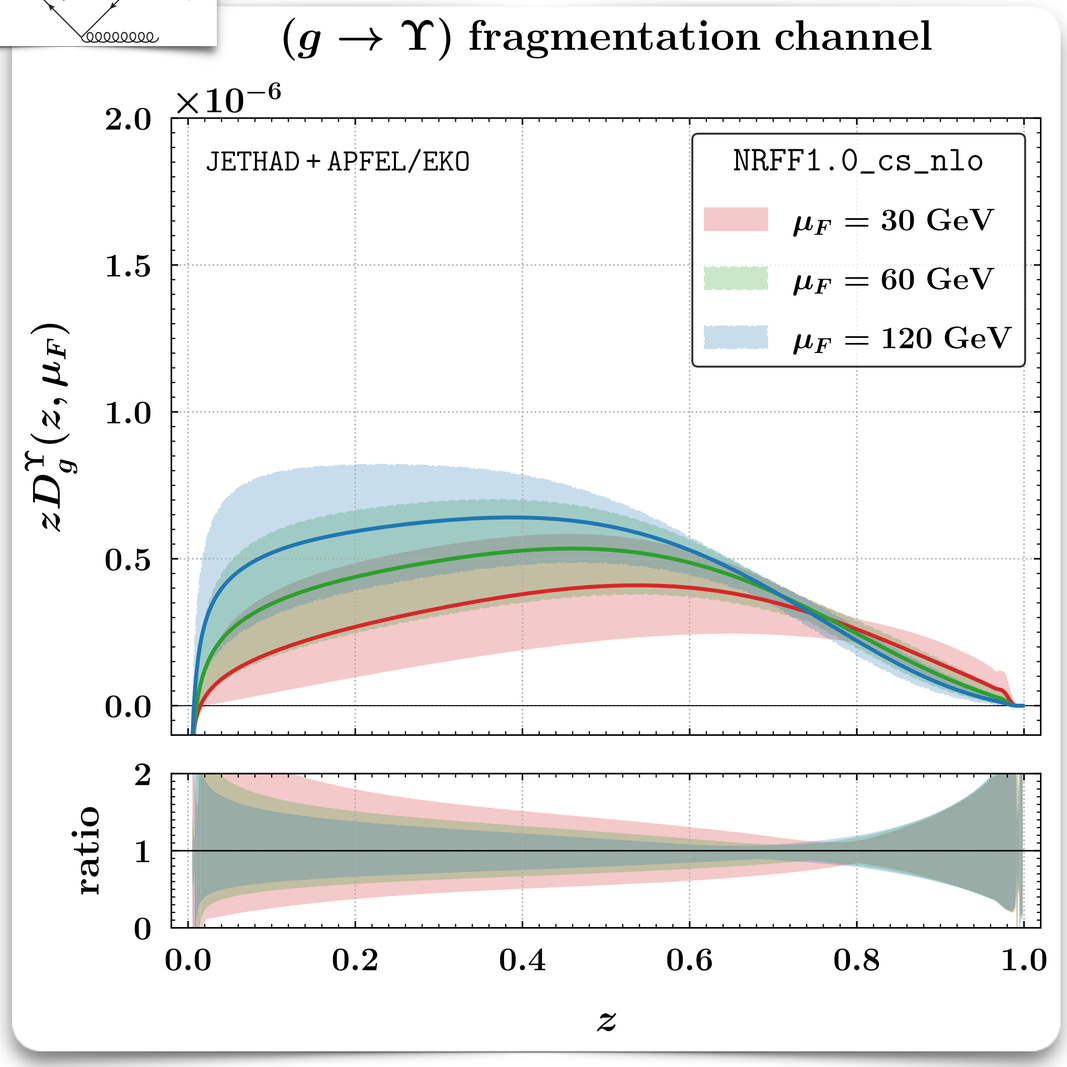


[F. G. C., in preparation]

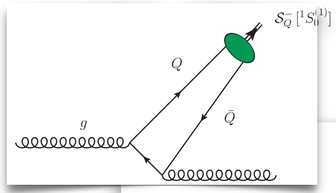
NRFF1.0: Gluon fragmentation to bottomonia

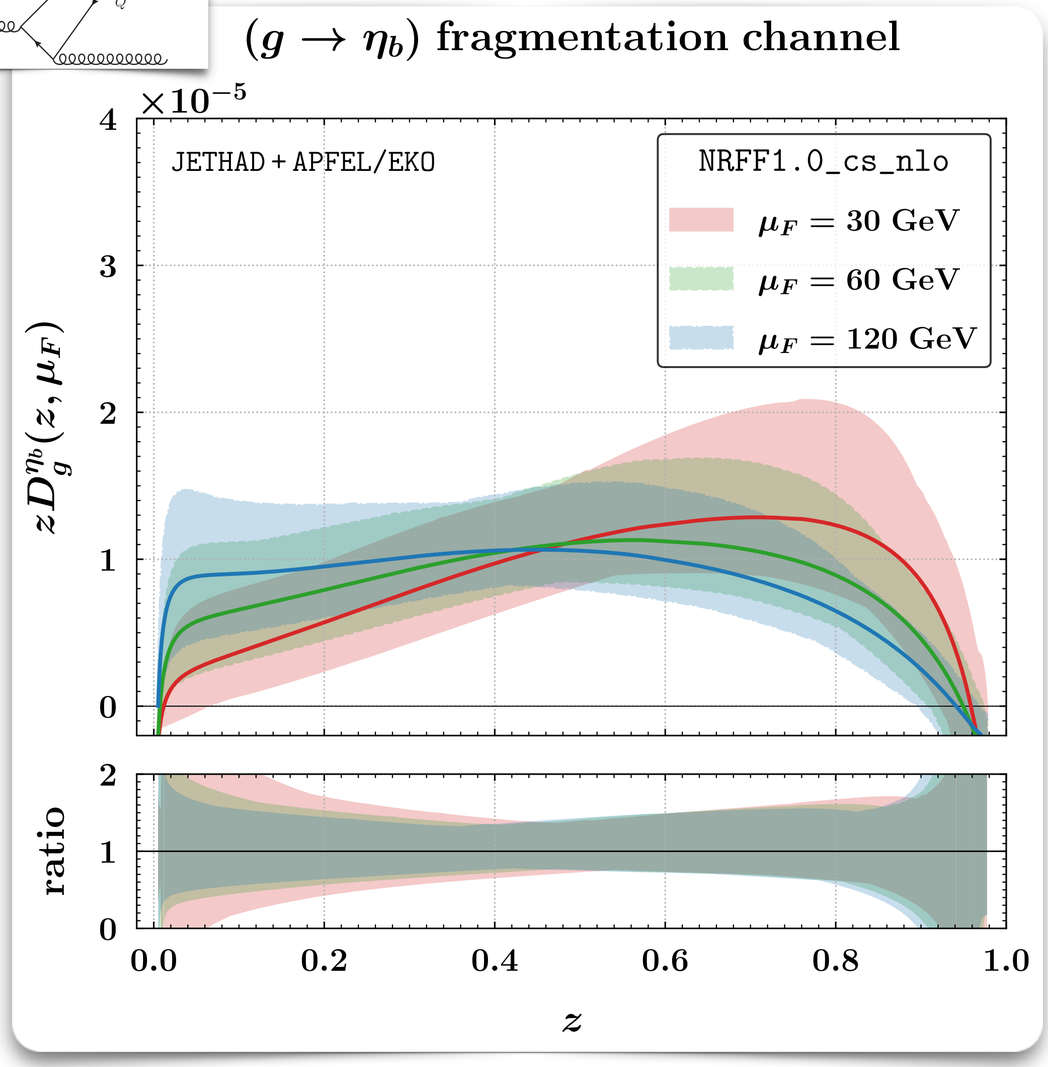
Vector Υ





Scalar η_b

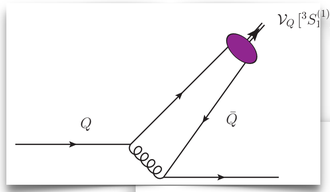


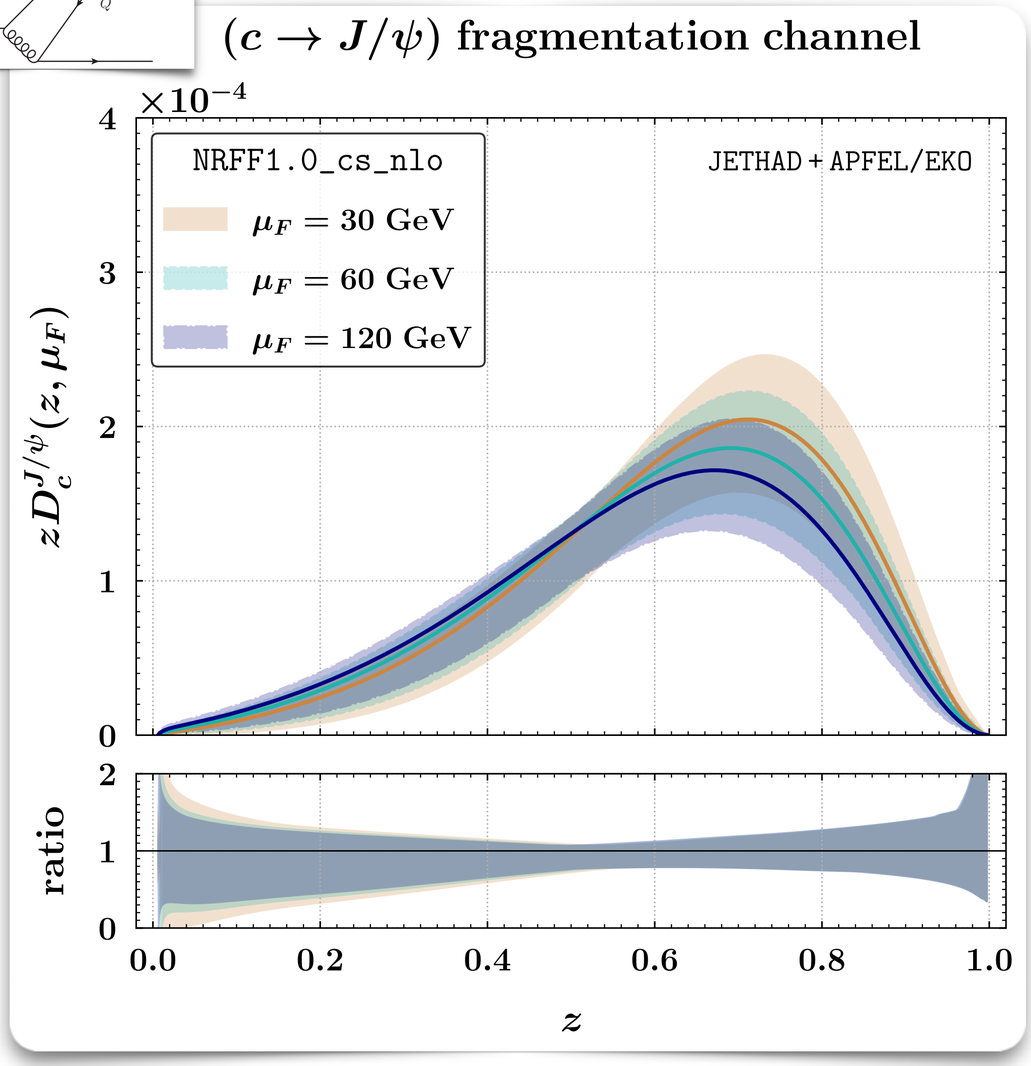


[F. G. C., in preparation]

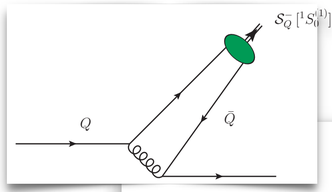
NRFF1.0: Charm fragmentation to charmonia

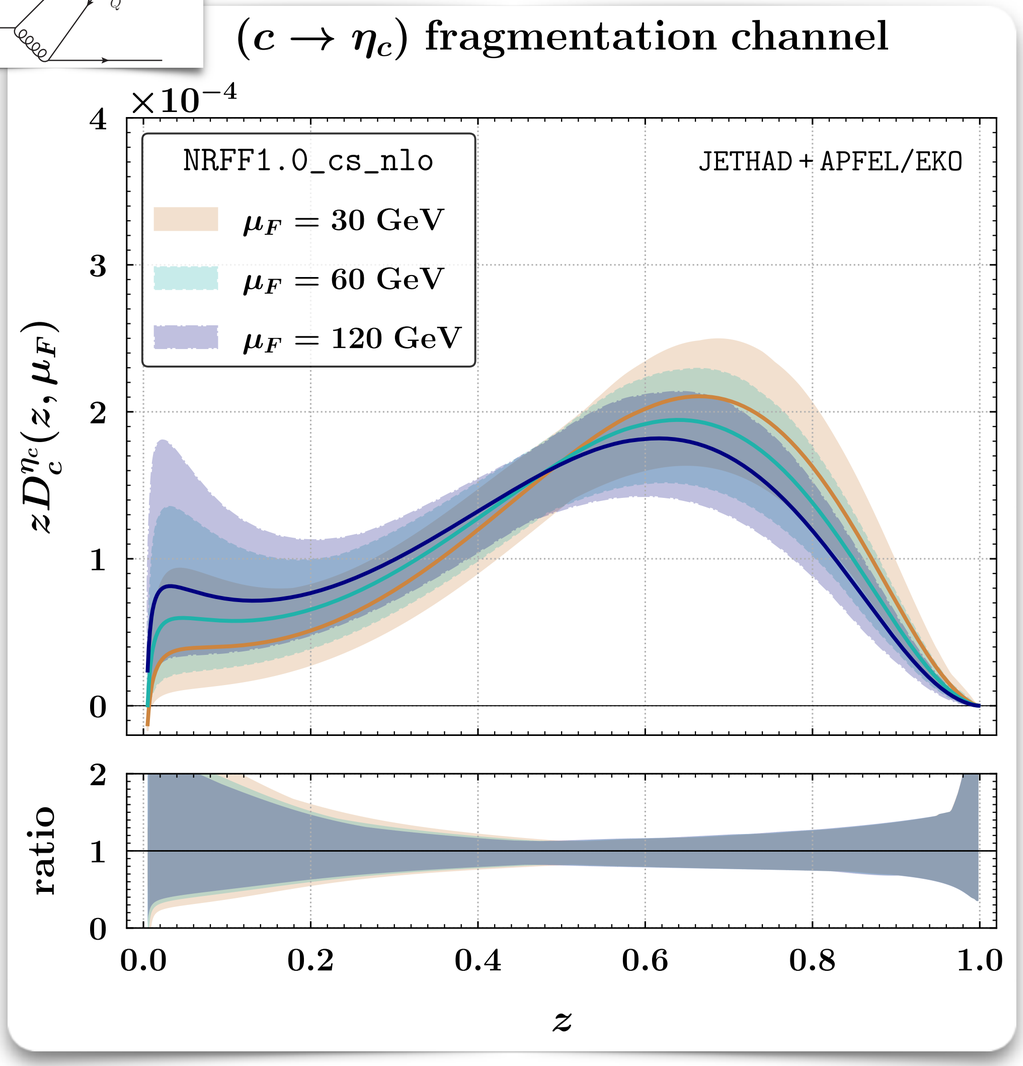
Vector J/ψ





Scalar η_c

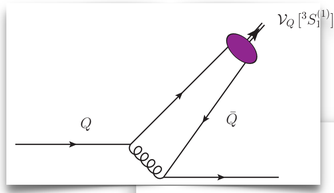


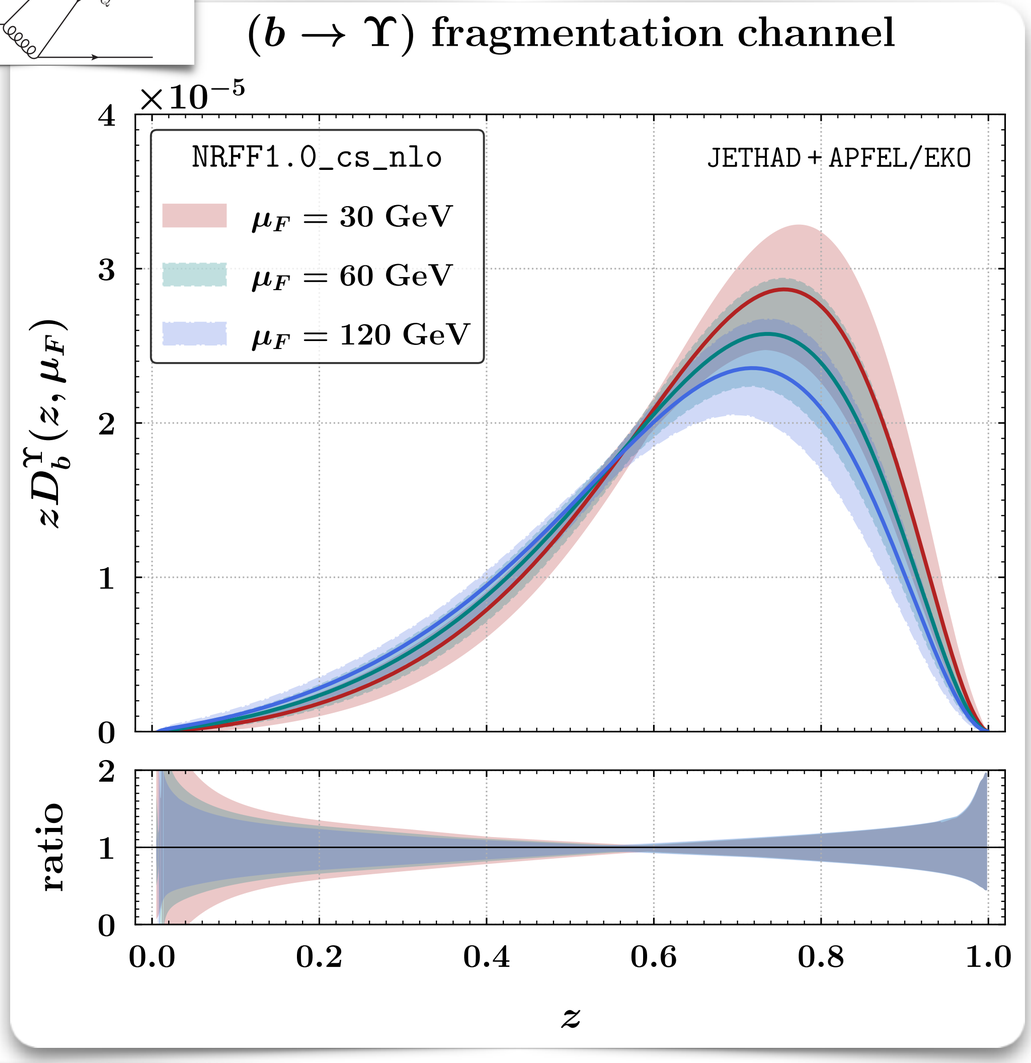


[F. G. C., in preparation]

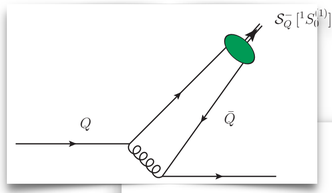
NRFF1.0: Bottom fragmentation to bottomonia

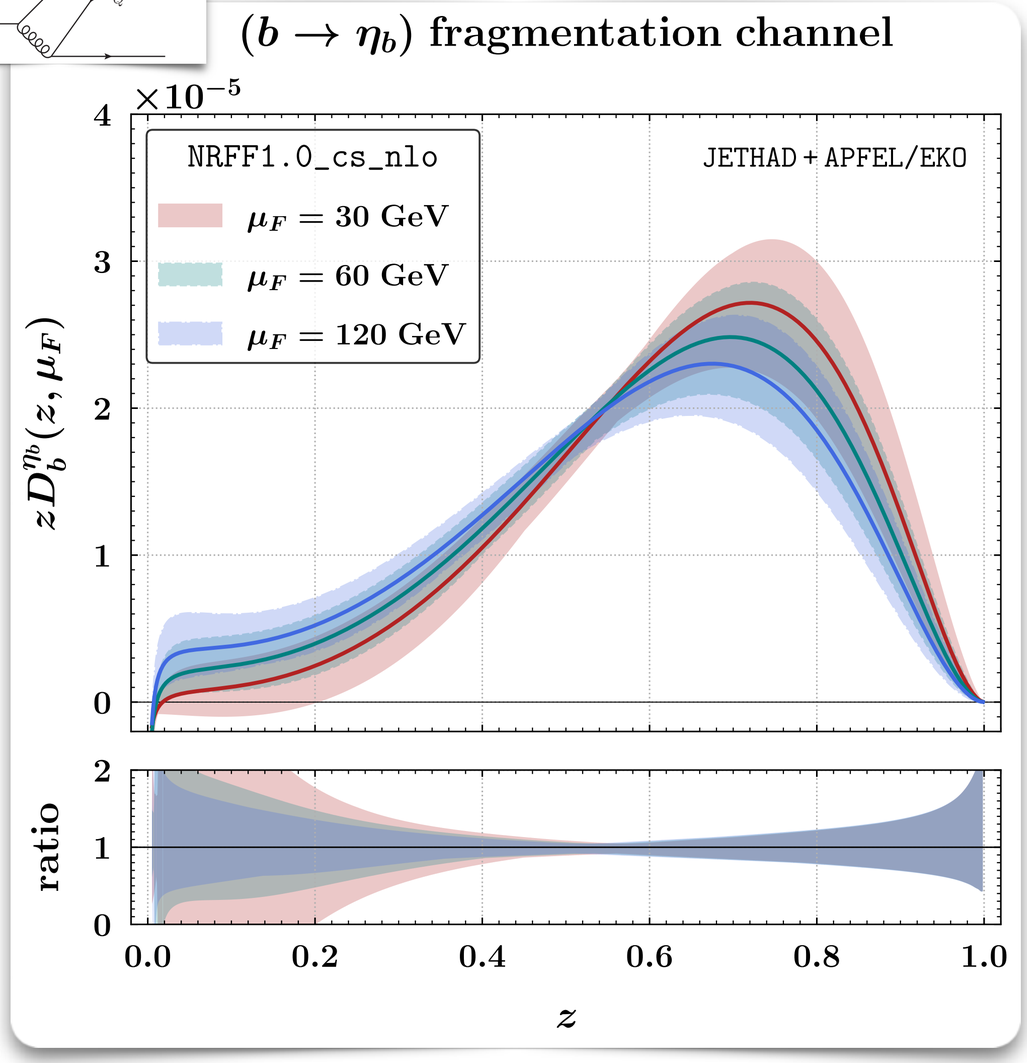
Vector Υ





Scalar η_b





4.1 NRFF1.0 functions

[F. G. C., in preparation]

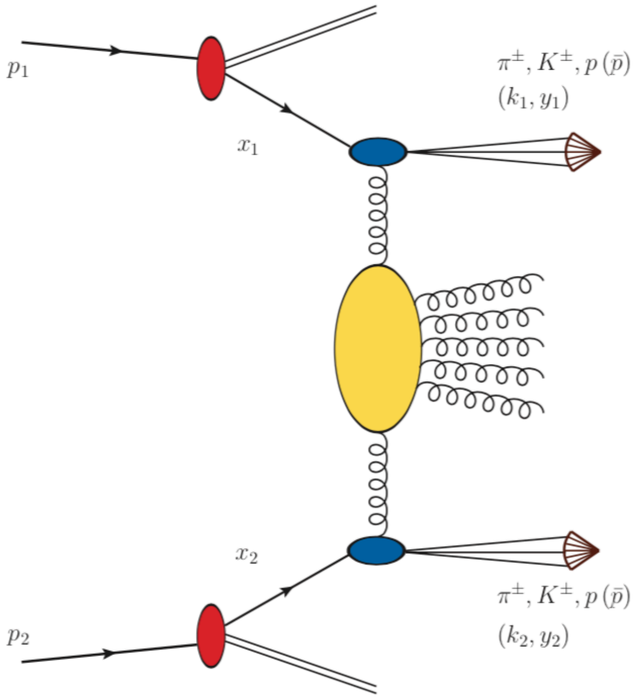
HEAVY-LIGHT HADRONS

From Higgs + jet to bound states

Di-hadron and hadron-jet correlations

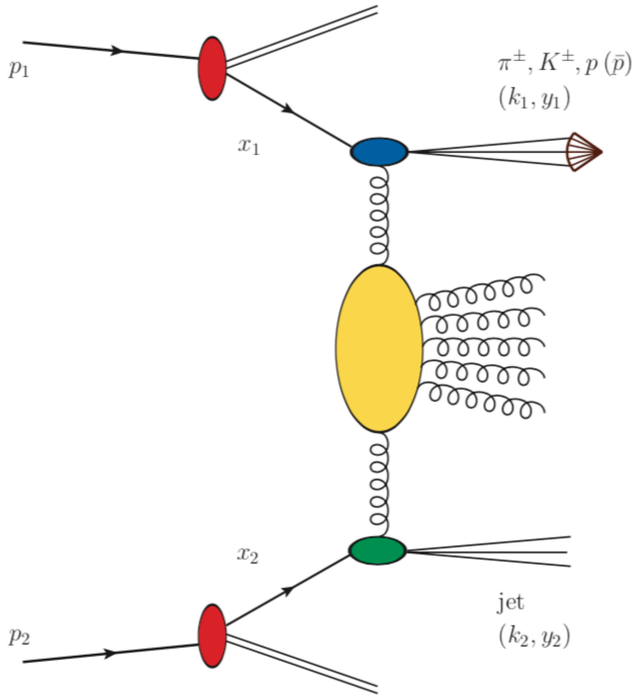
Inclusive di-hadron production

[D.Yu. Ivanov, A. Papa (2012)] (NLO forward-hadron impact factor)
[F.G.C., D.Yu. Ivanov, B. Murdaca, A. Papa (2016, 2017)]



Inclusive hadron-jet production

[A.D. Bolognino, F.G.C., D.Yu. Ivanov, M.M.A. Mohammed, A. Papa (2018)]
[F.G.C. (in preparation)]



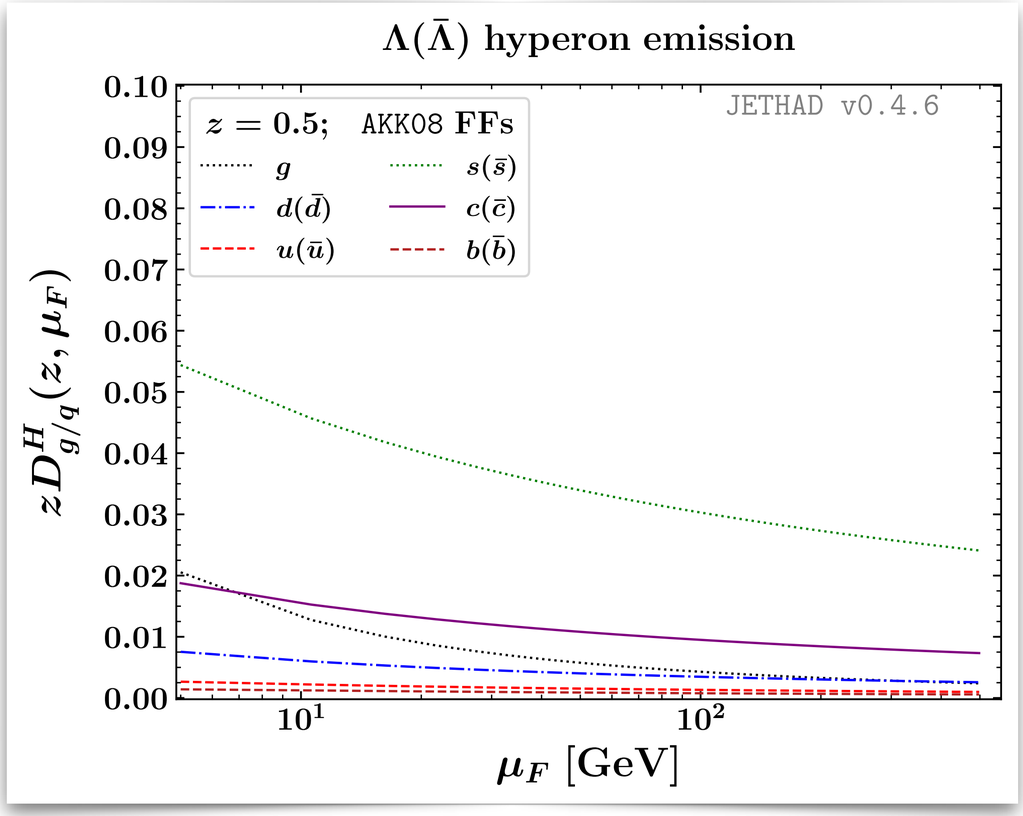
- NLO impact factors known $\Rightarrow$ full NLA BFKL analysis feasible
- PDFs + FFs at work (both), hadrons at smaller rapidities than jets (di-hadron)
- genuine *asymmetric* cuts in transverse momenta (hadron-jet)

🔗 [F. G. C., Eur. Phys. J. C 81 (2021) 8, 691]

Stabilizing effects of heavy-flavor fragmentation

AKK08 VFNS collinear FFs for Λ hyperon: $|uds\rangle$

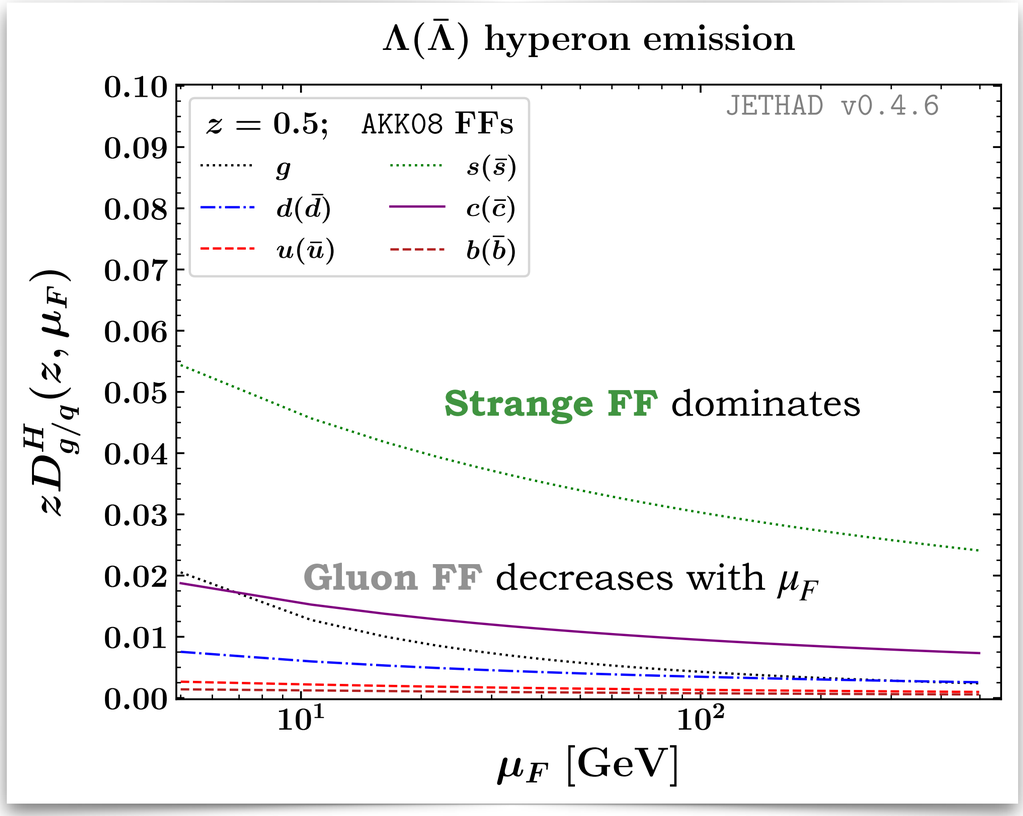
[S. Albino et al., Nucl. Phys. B 803 (2008) 42-104]



Stabilizing effects of heavy-flavor fragmentation

AKK08 VFNS collinear FFs for Λ hyperon: $|uds\rangle$

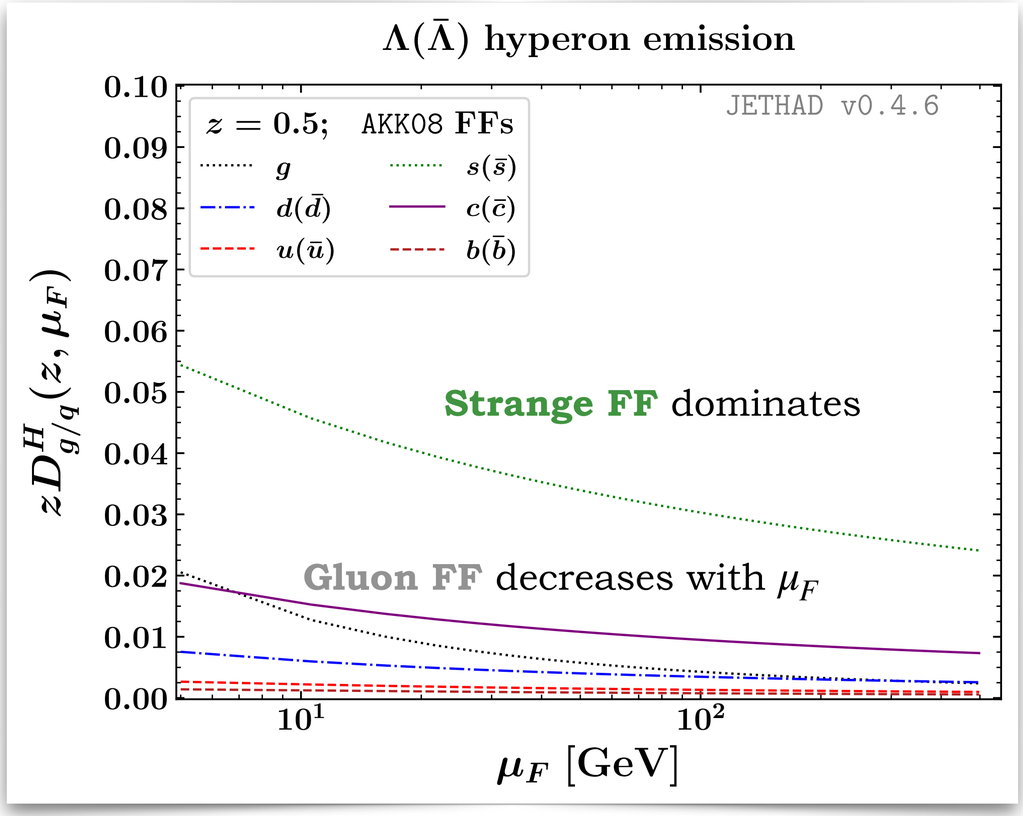
[S. Albino et al., Nucl. Phys. B 803 (2008) 42-104]

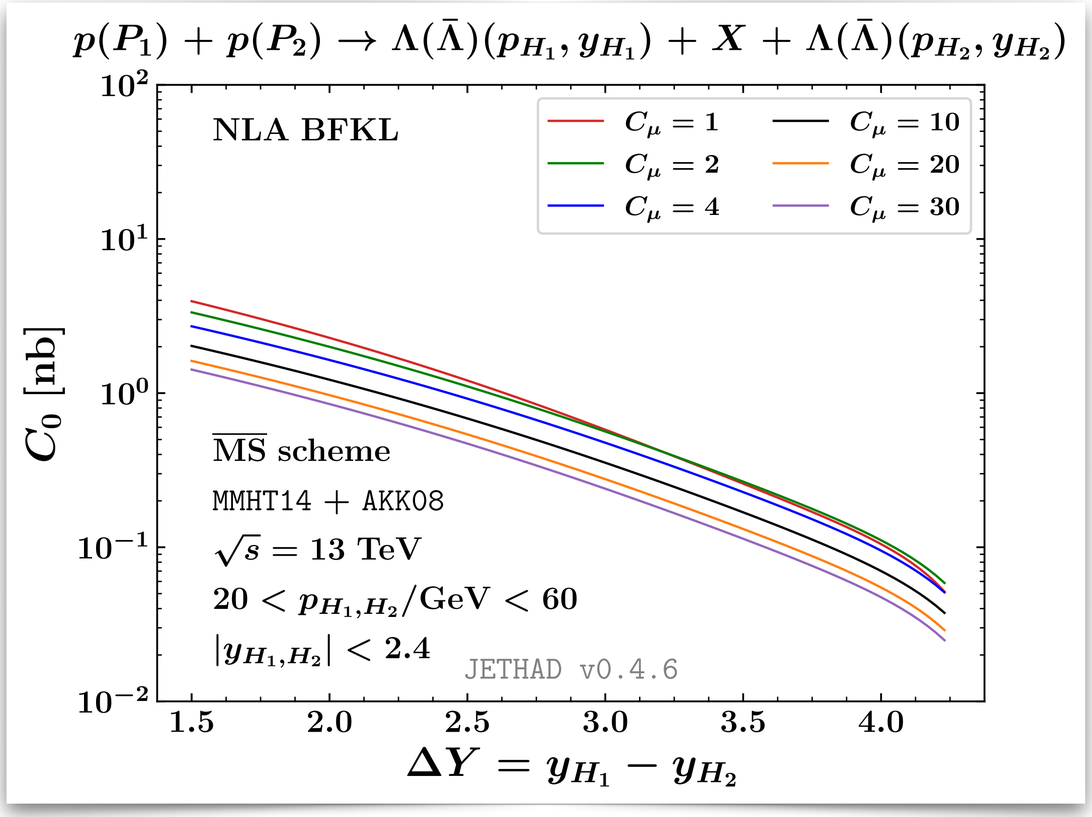


Stabilizing effects of heavy-flavor fragmentation

AKK08 VFNS collinear FFs for Λ hyperon: $|uds\rangle$

[S. Albino et al., Nucl. Phys. B 803 (2008) 42-104]





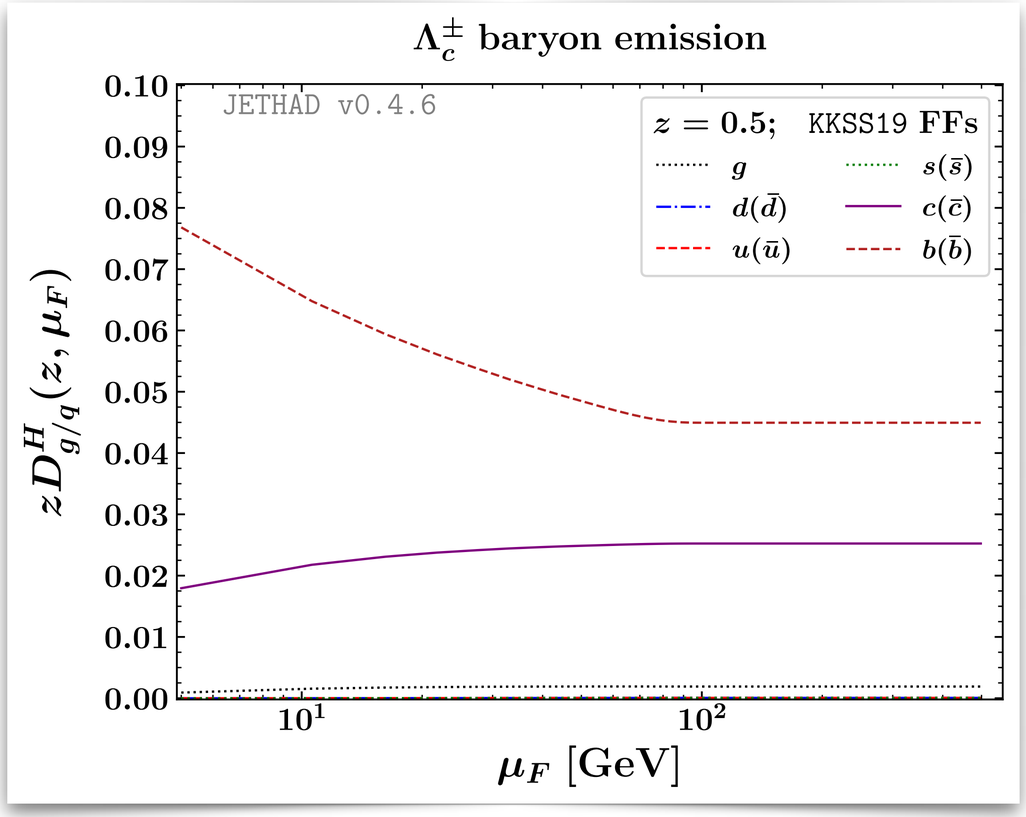
Rapidity distribution sensitive to scale variations

(Λ hyperons) [F. G. C. et al., Phys. Rev. D 102 (2020) 9, 094019]

(cascade Ξ baryons) [F. G. C., Eur. Phys. J. C (in press)]

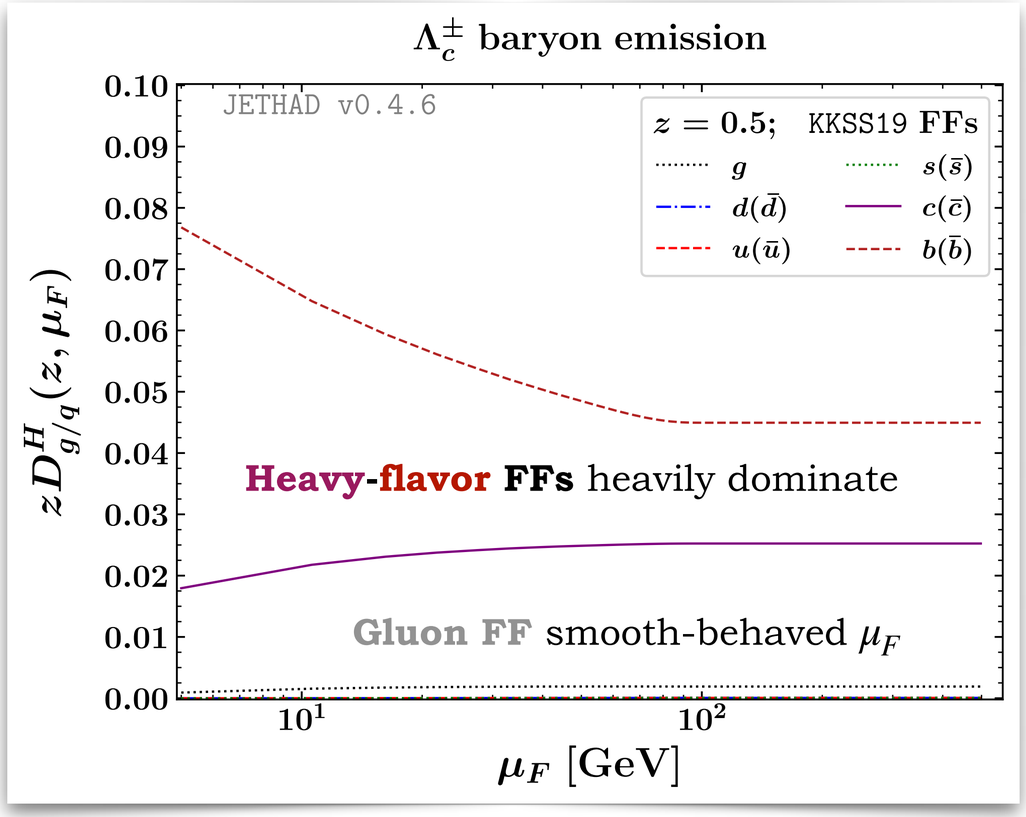
Stabilizing effects of heavy-flavor fragmentation

KKSS19 VFNS collinear FFs for Λ_c baryons: $|udc\rangle$ [B. A. Kniehl et al., Phys. Rev. D 101 (2020) 11, 114021]



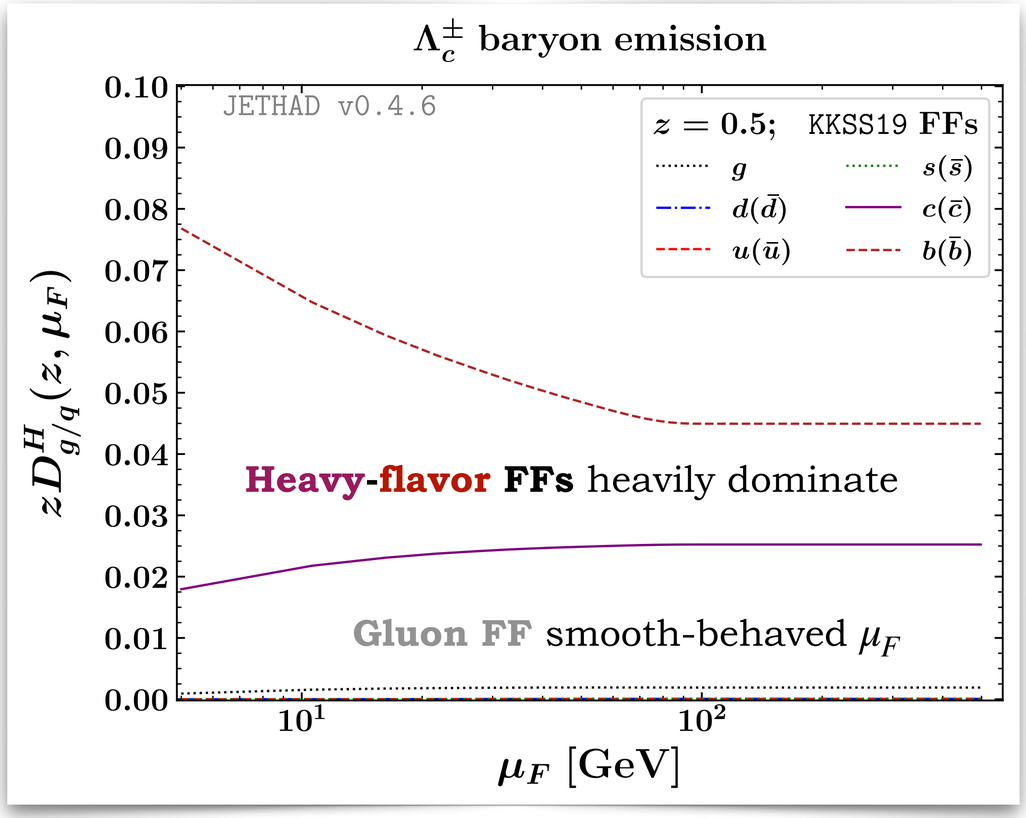
Stabilizing effects of heavy-flavor fragmentation

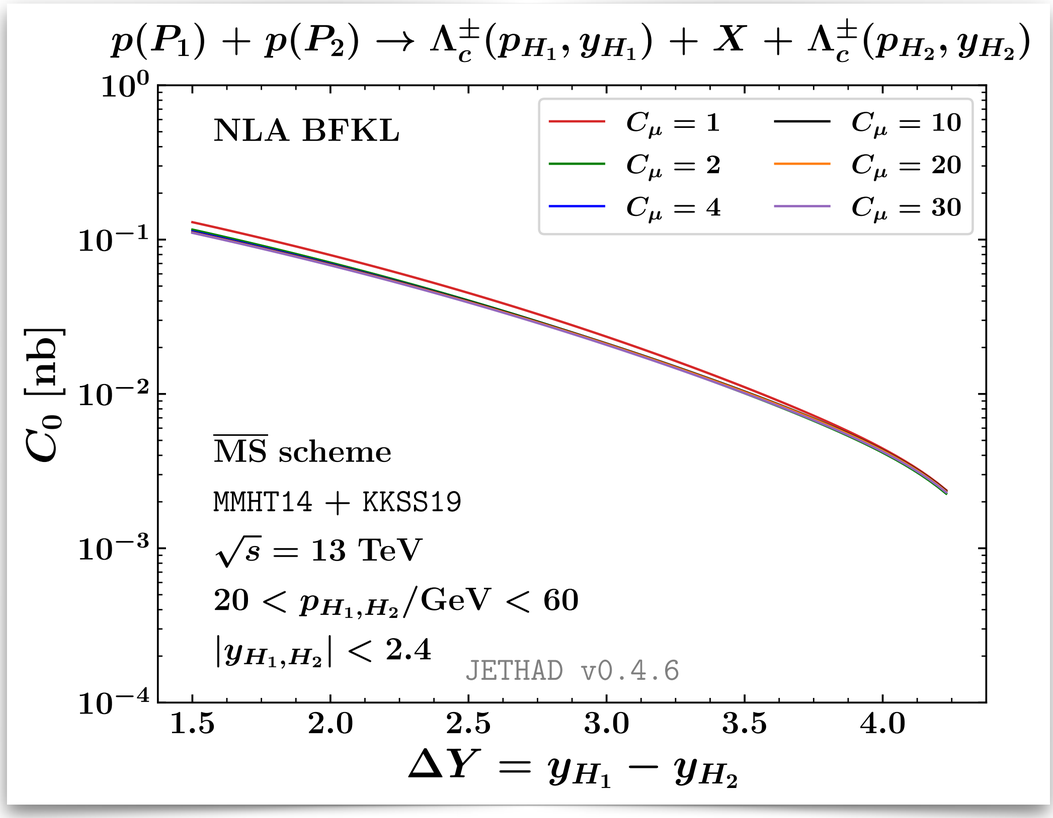
KKSS19 VFNS collinear FFs for Λ_c baryons: $|udc\rangle$ [B. A. Kniehl et al., Phys. Rev. D 101 (2020) 11, 114021]



Stabilizing effects of heavy-flavor fragmentation

KKSS19 VFNS collinear FFs for Λ_c baryons: $|udc\rangle$ [B. A. Kniehl et al., Phys. Rev. D 101 (2020) 11, 114021]





Rapidity distribution stable under scale variations

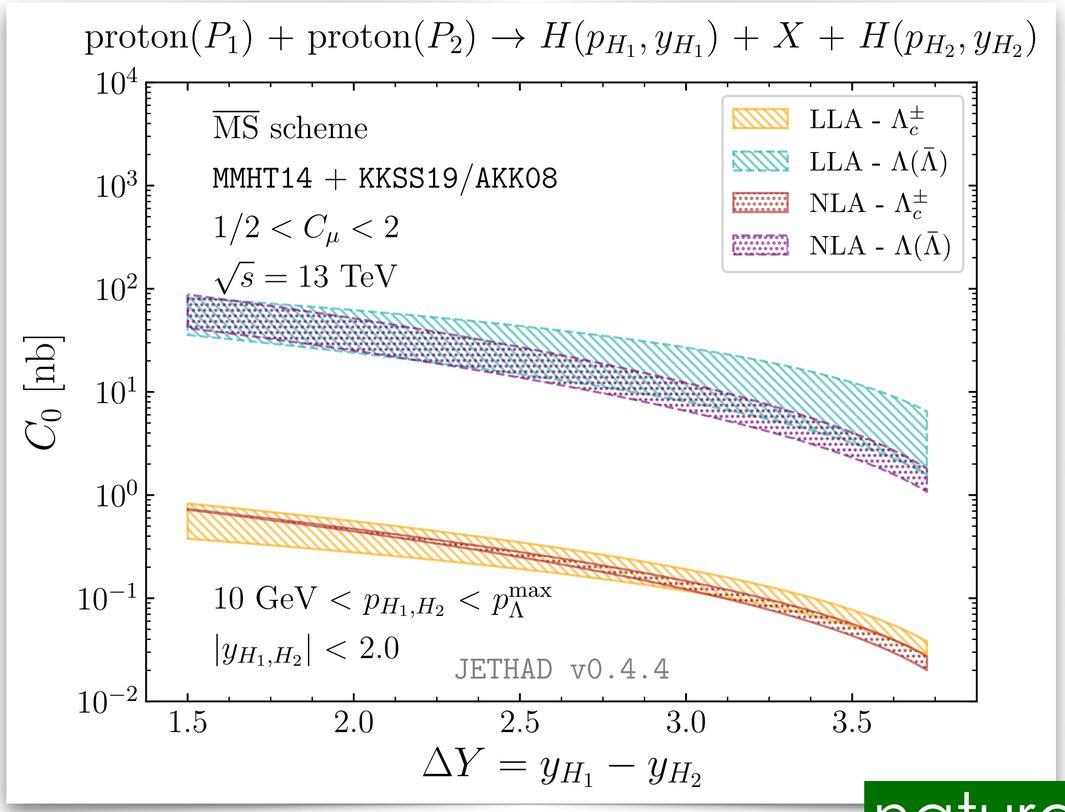
($B_c^{(*)}$ hadrons) [F. G. C., Phys. Lett. B 835 (2022) 137554]

(Λ_c baryons, in this slide) [F. G. C. et al., Eur. Phys. J. C 81 (2021) 8, 780]

(H_b hadrons) [F. G. C. et al., Phys. Rev. D 104 (2021) 11, 114007]

Stability under scale variations & NLL corrections

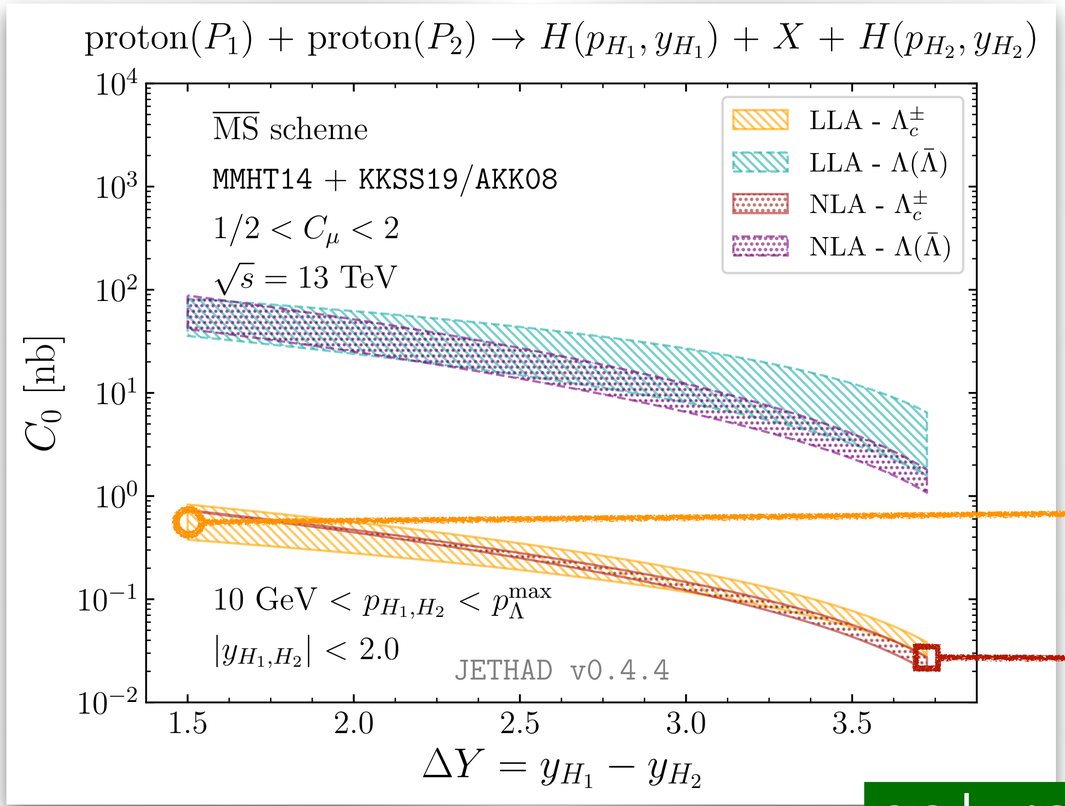
Hybrid factorization @work: Λ_c baryons $|udc\rangle$ versus Λ hyperons $|uds\rangle$

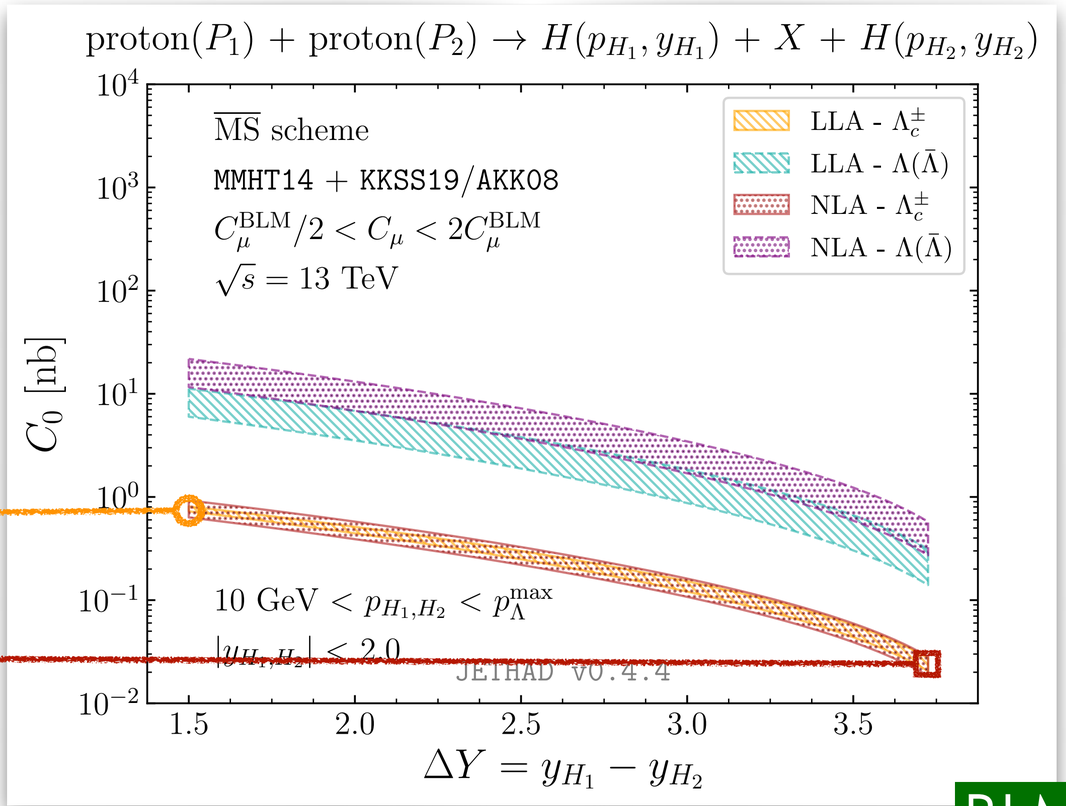


natural

Stability under scale variations & NLL corrections

Hybrid factorization @work: Λ_c baryons $|udc\rangle$ versus Λ hyperons $|uds\rangle$





LL Λ_c

NLL Λ_c

natural

BLM

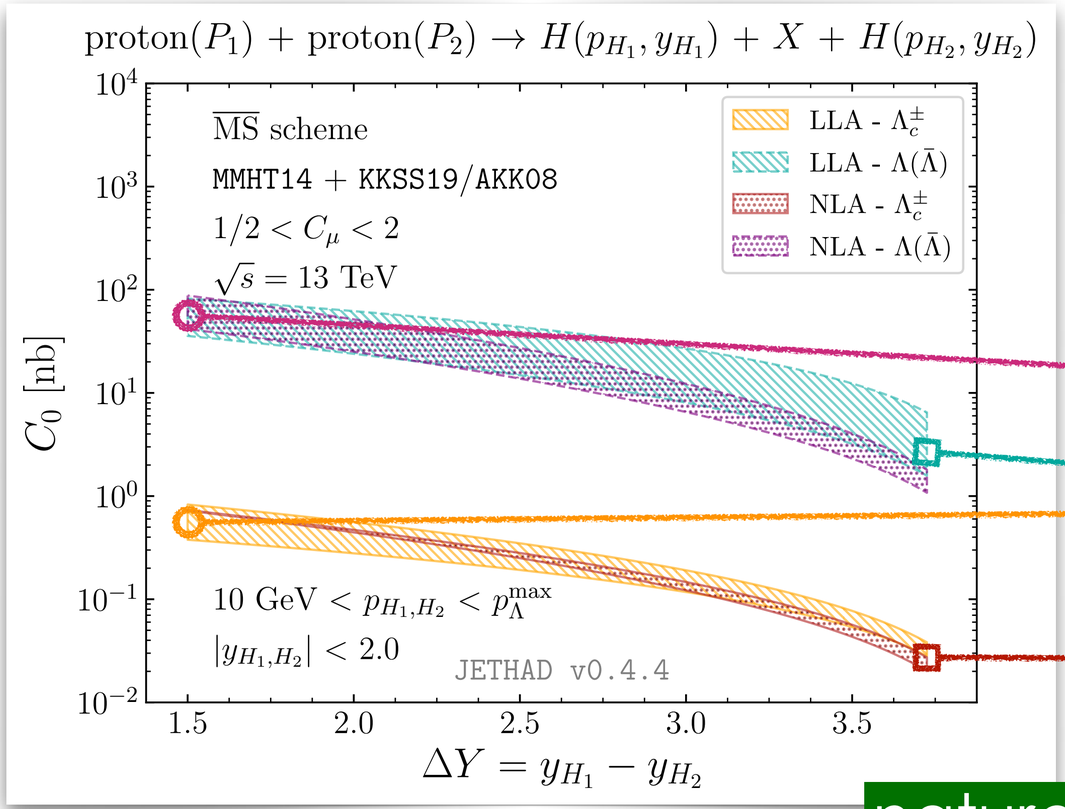
NLL corrections: rapidity distribution stable for Λ_c

(Λ_c baryons, in this slide) [F. G. C. et al., Eur. Phys. J. C 81 (2021) 8, 780]

(H_b hadrons) [F. G. C. et al., Phys. Rev. D 104 (2021) 11, 114007]

Stability under scale variations & NLL corrections

Hybrid factorization @work: Λ_c baryons $|udc\rangle$ versus Λ hyperons $|uds\rangle$

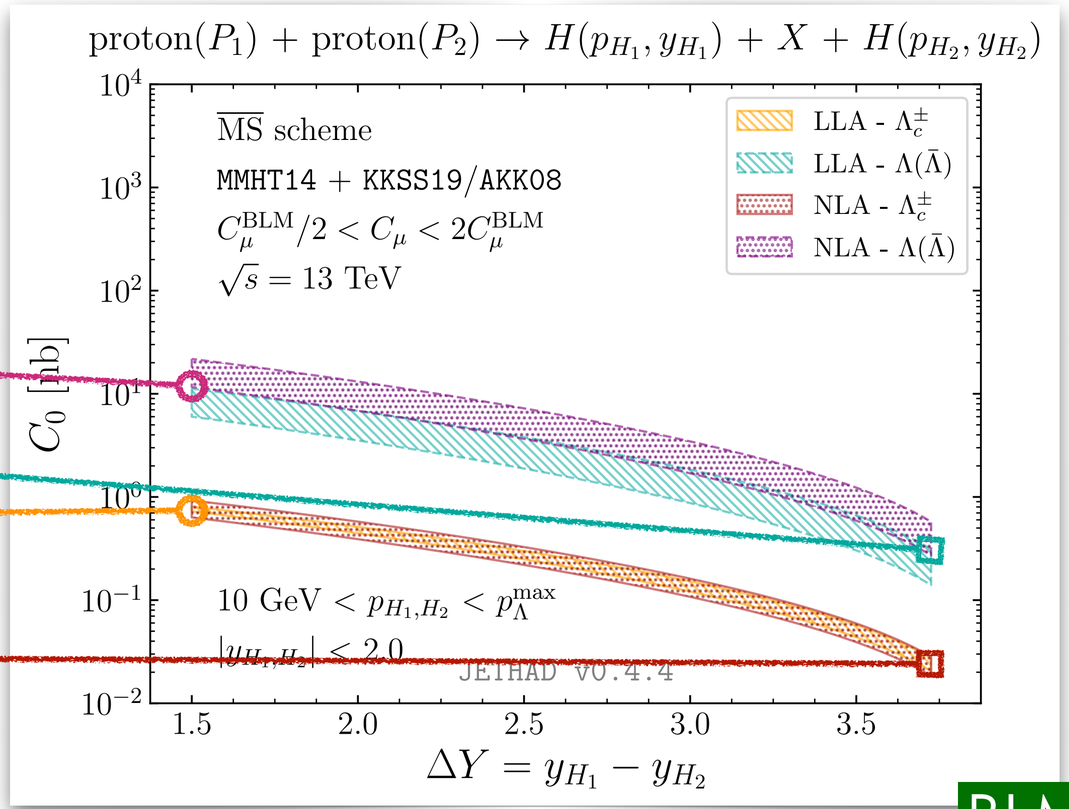


NLL Λ

LL Λ

LL Λ_c

NLL Λ_c



natural

BLM

NLL corrections: rapidity distribution stable for Λ_c, loses $\sim 10^1$ magnitude for Λ

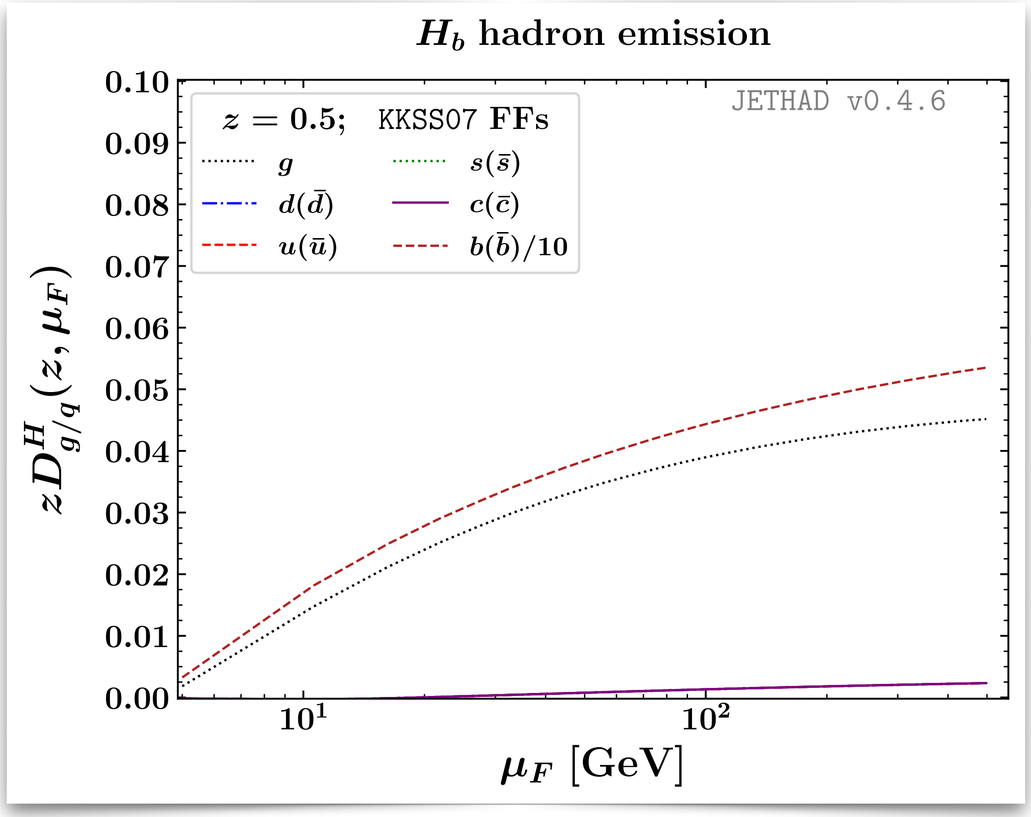
(Λ_c baryons, in this slide) [F. G. C. et al., Eur. Phys. J. C 81 (2021) 8, 780]

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Stabilizing effects of heavy-flavor fragmentation

KKSS07 VFNS collinear FFs for:

$$H_b = B^{\pm}, B^0, B_s^0, \Lambda_b$$



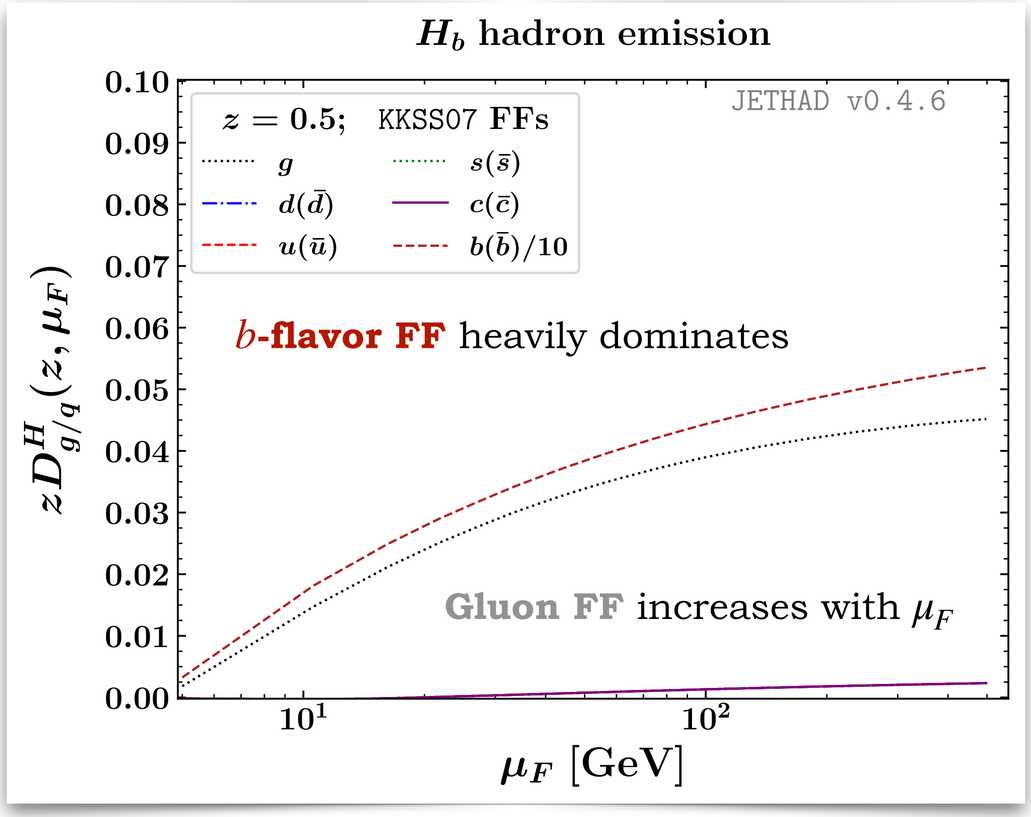
[B. A. Kniehl, H. Spiesberger, Phys. Rev. D 98 (2018) 11, 114010]

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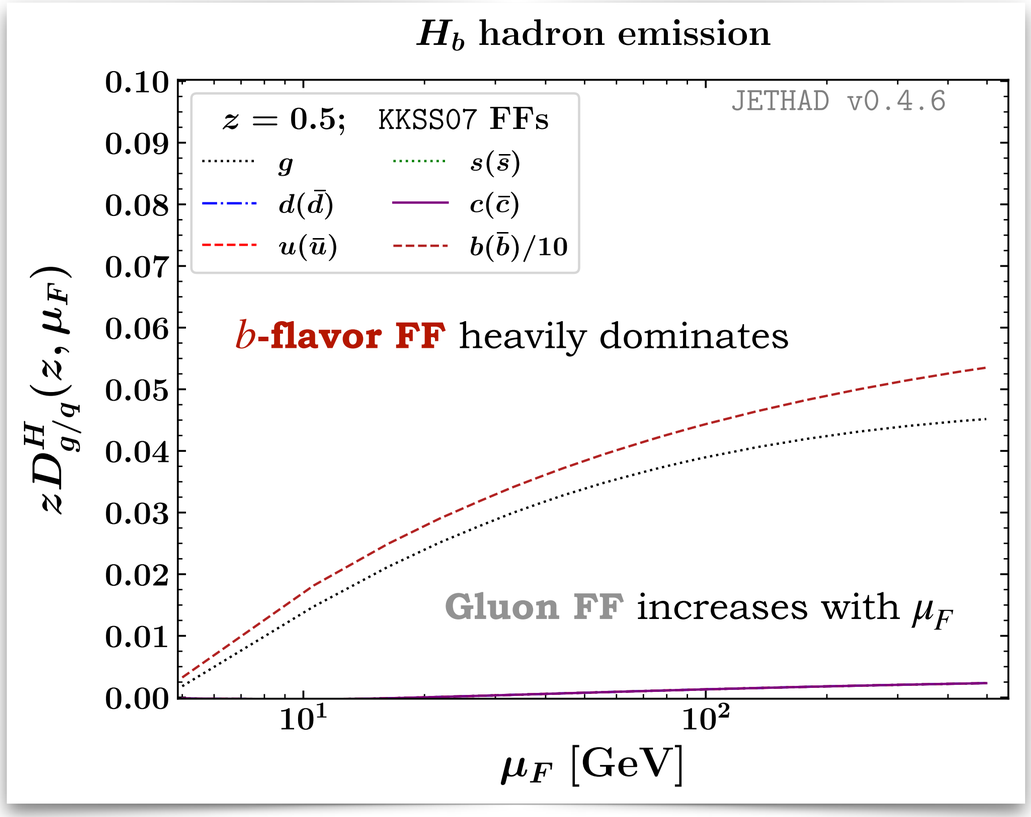
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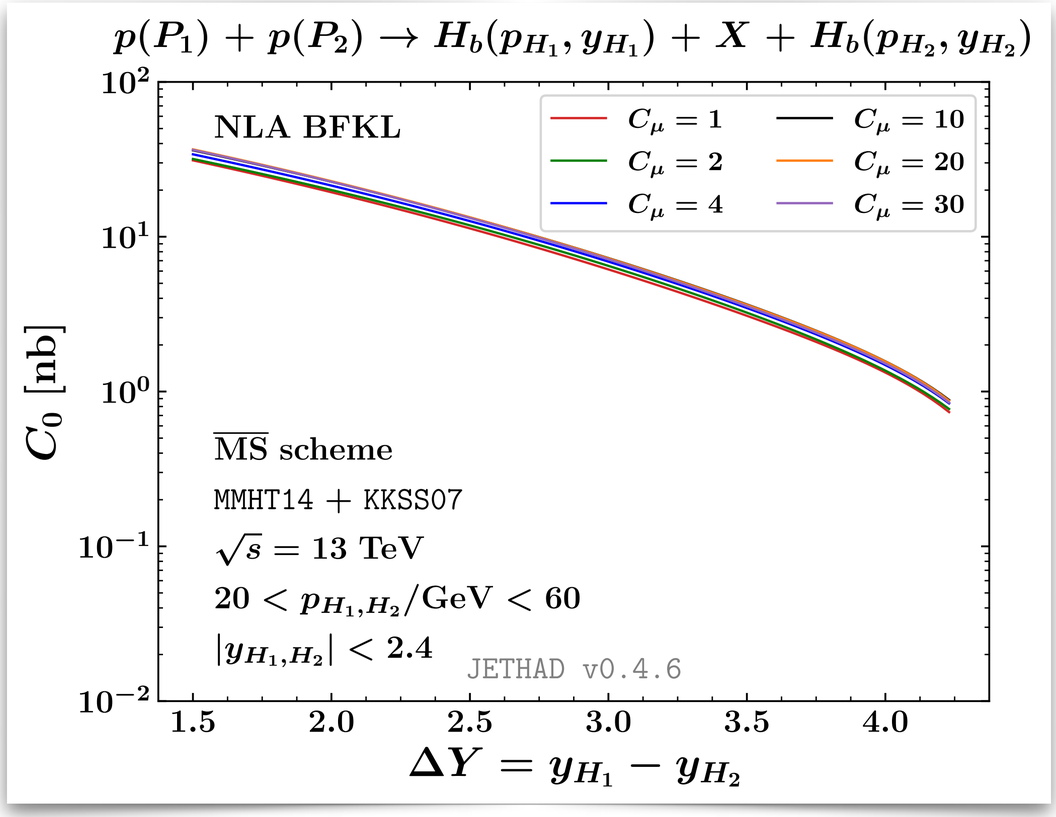
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Rapidity distribution very stable under scale variations

(H_b hadrons, in this slide) [F. G. C. et al., Phys. Rev. D 104 (2021) 11, 114007]

(Λ_c baryons) [F. G. C. et al., Eur. Phys. J. C 81 (2021) 8, 780]

Stabilizing effects of heavy-flavor fragmentation

- Stabilization mechanism encoded in the heavy-flavor gluon FF

- Forward-hadron LO impact factor ⇒ gluon FF enhanced by gluon PDF in collinear convolution

$$c_\Lambda(n,\nu,|\vec{p}|,x) = 2\sqrt{\frac{C_F}{C_A}}(|\vec{p}|^2)^{i\nu-1/2}\int_x^1 \frac{\mathrm{d}z}{z}\left(\frac{z}{x}\right)^{2i\nu-1}\left[\frac{C_A}{C_F}f_g(z)D_g^\Lambda\left(\frac{x}{z}\right)+\sum_{a=q,\bar{q}} f_a(z)D_a^\Lambda\left(\frac{x}{z}\right)\right]$$

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- Forward-hadron NLO impact factor ⇒ a non-diagonal heavy-flavor channel open...

$$\begin{aligned} c_1^{(1)}(n,\nu,|\vec{k}_1|,\alpha_1) \;=\; & 2\sqrt{\frac{C_F}{C_A}}\left(\vec{k}_1^2\right)^{i\nu-\frac{1}{2}}\frac{1}{2\pi}\int_{\alpha_1}^1\frac{dx}{x}\int_{\frac{\alpha_1}{x}}^1\frac{d\zeta}{\zeta}\left(\frac{x\zeta}{\alpha_1}\right)^{2i\nu-1} \\ \times\; & \left[\frac{C_A}{C_F}f_g(x)D_g^h\left(\frac{\alpha_1}{x\zeta}\right)C_{gg}(x,\zeta)+\sum_{a=q,\bar{q}}f_a(x)D_a^h\left(\frac{\alpha_1}{x\zeta}\right)C_{qq}(x,\zeta)\right. \\ +\; & \left. D_g^h\left(\frac{\alpha_1}{x\zeta}\right)\sum_{a=q,\bar{q}}f_a(x)C_{qg}(x,\zeta)+\frac{C_A}{C_F}f_g(x)\sum_{a=q,\bar{q}}D_a^h\left(\frac{\alpha_1}{x\zeta}\right)C_{gq}(x,\zeta)\right] \end{aligned}$$

...but $|C_{gg}| \sim 50 \div 10^4 |C_{gq}|$

(NLO impact factor) 🔗 [D. Yu. Ivanov, A. Papa JHEP 07 (2012) 045]

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Gluon FF rises with energy ⇒ this compensates PDF and BFKL kernel decreasing behavior

(NLO impact factor) 🔗 [D. Yu. Ivanov, A. Papa JHEP 07 (2012) 045]

VECTOR QUARKONIA

Is the natural stability robust?

(1) **KKSS07** and **KKSS19** VFNS collinear FFs share the same extraction technology

¿ *Might natural stability be related to the given FF determination(s)* ?

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(2) **KKSS07** and **KKSS19** VFNS collinear FFs assume no initial-scale gluon, but evolution-driven

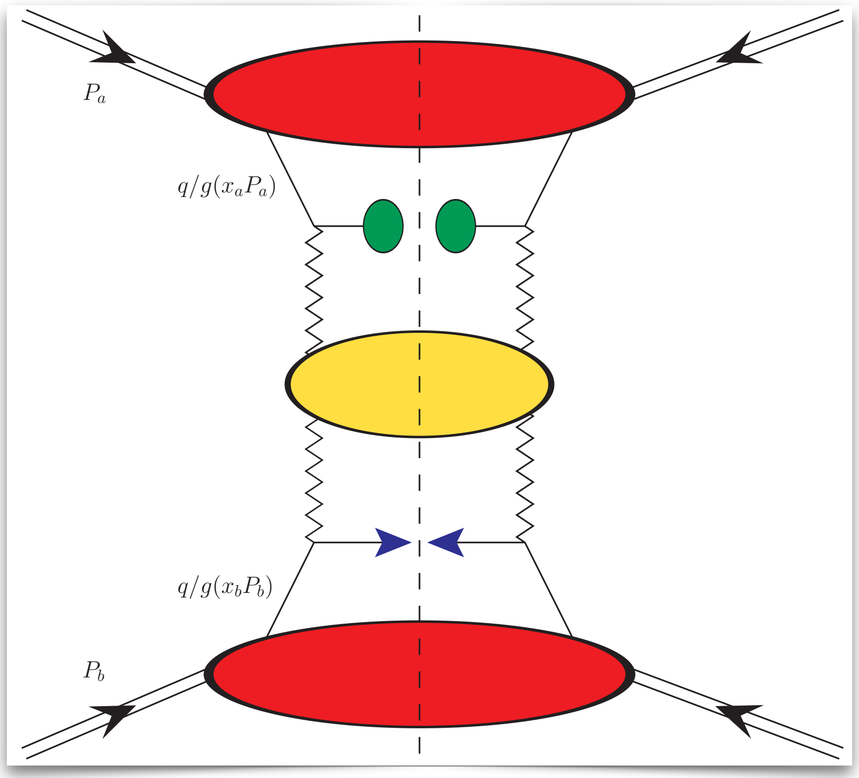
¿ *Might natural stability be artificially generated by this Ansatz* ?

Vector quarkonium from single-parton fragmentation

(1) ¡ *Let us consider* J/ψ *and* Υ *at large* p_T $\rightarrow$ *single-parton* ***fragmentation*** *from* ***NRQCD*** !

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2.1 High-energy resummed cross section

The process under investigation is

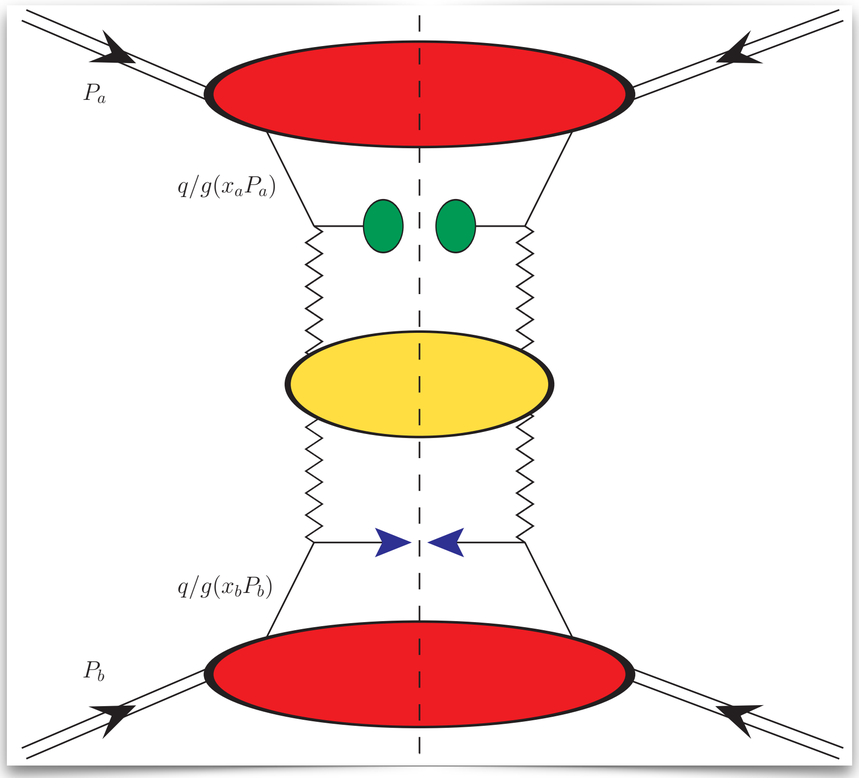
$$p(P_a) + p(P_b) \to \mathcal{Q}(p_{\mathcal{Q}}, y_{\mathcal{Q}}) + X + \text{jet}(p_J, y_J) \ , \tag{1}$$

where $p(P_{a,b})$ stands for an initial proton with momentum $P_{a,b}$, $\mathcal{Q}(p_{\mathcal{Q}}, y_{\mathcal{Q}})$ is a J/ψ or a Υ emitted with momentum $p_{\mathcal{Q}}$ and rapidity $y_{\mathcal{Q}}$, the light jet is tagged with momentum p_J and rapidity y_J, and X denotes all the undetected products of the reaction. High observed transverse momenta, $|\vec{p}_{\mathcal{Q},J}|$, together with a large rapidity separation, $\Delta Y = y_{\mathcal{Q}} - y_J$, are required conditions to get a diffractive semi-hard configuration in the final state. Furthermore the transverse-momentum ranges need to be enough large to ensure the validity of description of the quarkonium production mechanism in terms of single-parton VFNS collinear fragmentation.

🔗 [F. G. C. et al., Eur. Phys. J. C 82 (2022) 10, 929]

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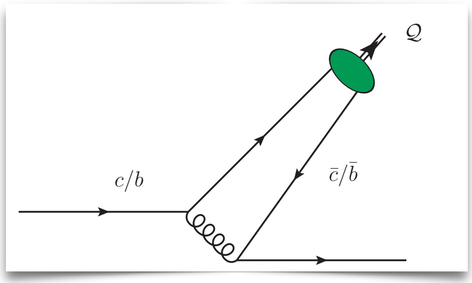
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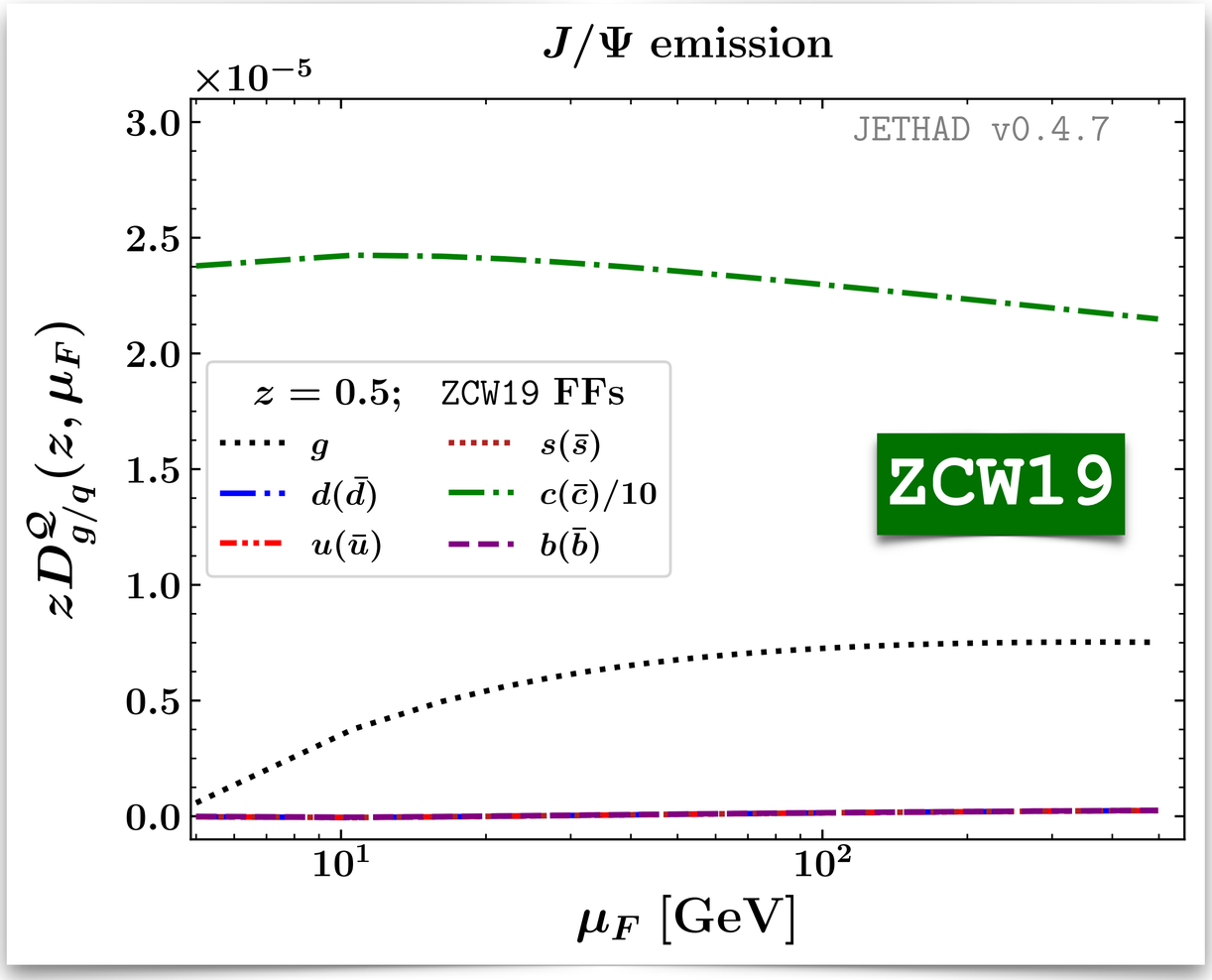
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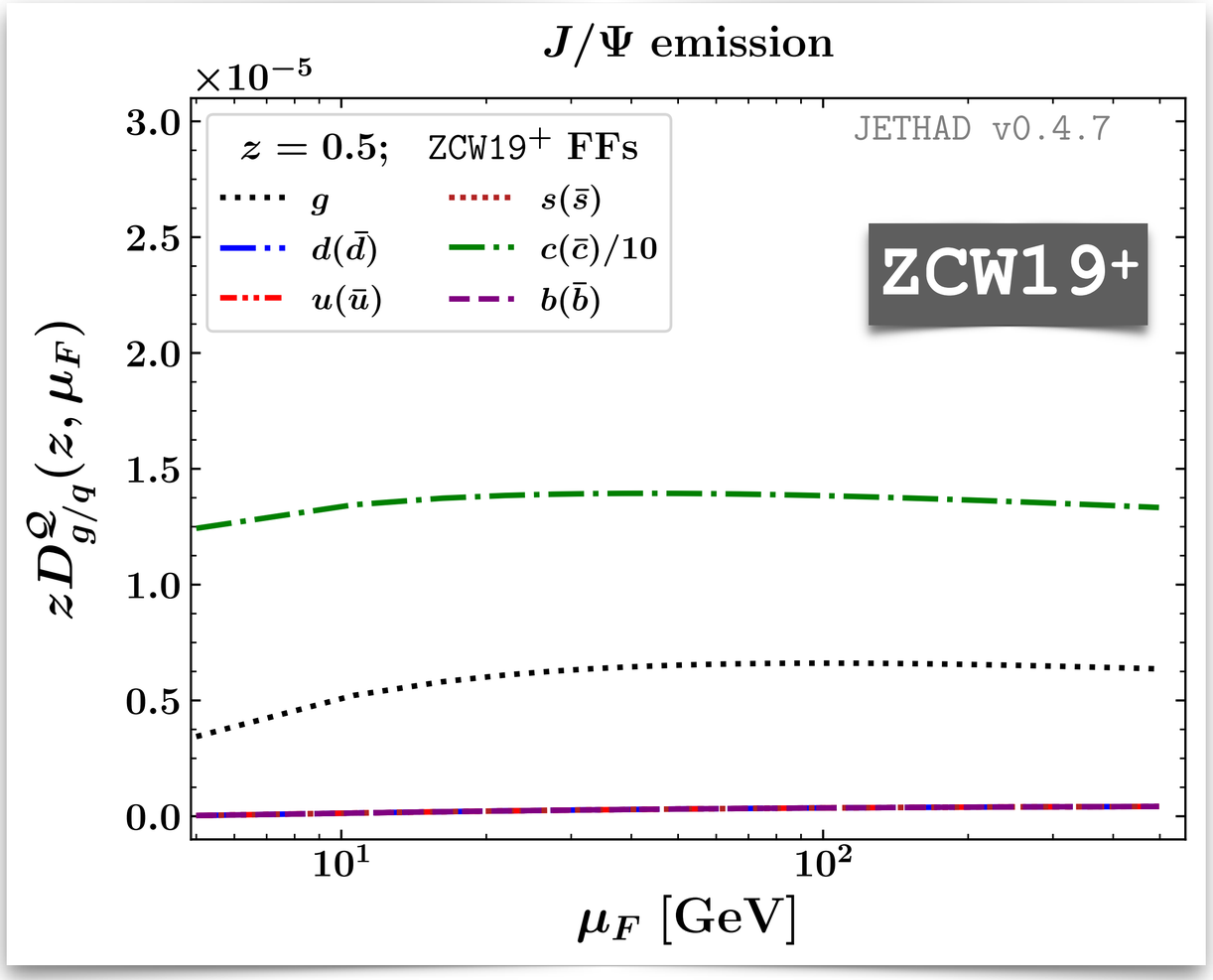


(LO) 🔗 [E. Braaten et al., Phys. Rev. D 48 (1993) 4230-4235]
(NLO) 🔗 [X. Zheng et al., Phys. Rev. D 100 (2019) 1, 014005]

Vector quarkonium + jet at the LHC

J/ψ collinear FFs

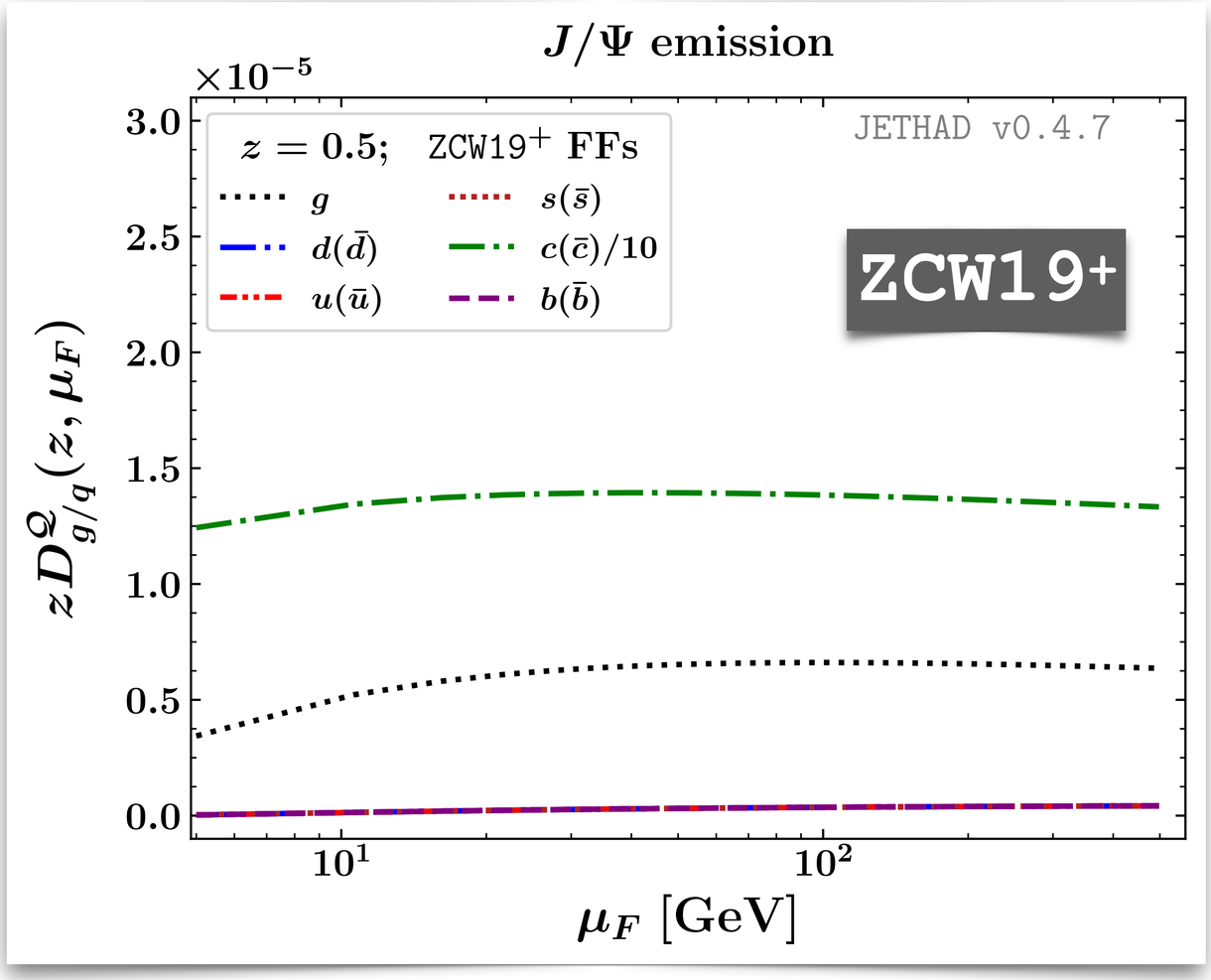




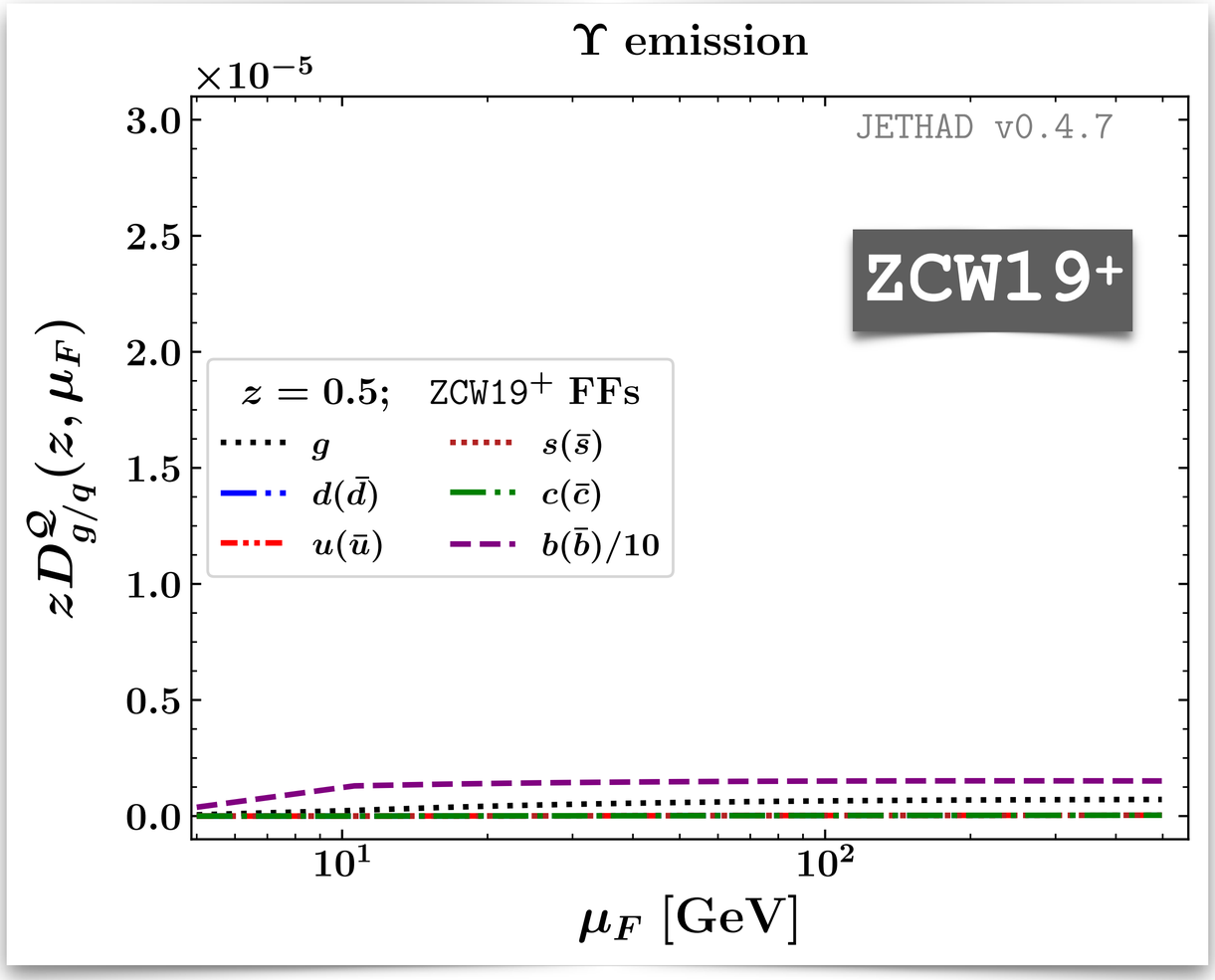
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Υ collinear FFs



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