Decoupling for quark masses

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The cae $n_f \to n_l$

$$\frac{\Lambda^{(n_f)}}{\mu} = \varphi(g^{(n_f)}(\mu)); \qquad \frac{M^{(n_f)}}{m(\mu)} = \eta(g^{(n_f)}(\mu))\,.$$

- Choose relation $\mu = m^{(n_f)}(\mu) = m_\star^{(n_f)}$

$$\frac{\Lambda^{(n_f)}}{M^{(n_f)}} - \frac{\varphi(g_\star^{(n_f)})}{\eta(g_\star^{(n_f)})} = 0$$

  and determine $g_\star^{(n_f)} = g^{(n_f)}(m_\star^{(n_f)})$.
- Decoupling of g with $\xi_g(x) = 1 + \#x^2 + ...$

$$g^{(n_l)}(m_\star^{(n_f)}) = g_\star^{(n_f)} \times \xi_g(g_\star^{(n_f)})$$

- Ratio of RGI (I)

$$\frac{\Lambda^{(n_f)}}{\Lambda^{(n_l)}} = \frac{\varphi^{(n_f)}(g_\star^{(n_f)})}{\varphi^{(n_l)}(g_\star^{(n_f)} \times \xi_g)}$$

- We now use

$$\begin{aligned} \frac{M^{(n_f)}}{m_\star^{(n_f)}} &= \eta^{(n_f)}(g_\star^{(n_f)})\,, \\ \frac{M^{(n_l)}}{m^{(n_l)}(m_\star^{(n_f)})} &= \eta^{(n_l)}(g^{(n_l)}(m_\star^{(n_f)}))\,. \end{aligned}$$

- Therefore using $\xi_m(x) = 1 + \#x^2 + ...$

$$Q_{n_f \to n_l} = \frac{M^{(n_f)}}{M^{(n_l)}} = \frac{\eta^{(n_f)}(g_\star^{(n_f)})}{\eta^{(n_l)}(g^{(n_l)}(m_\star^{(n_f)}))} \times \xi_m(g_\star^{(n_f)})\,.$$

  Analogous to P for Λ decoupling

Quark mass from decoupling

Master decoupling formula

$$\underbrace{\frac{M^{(3)}}{m^{(3)}(\mu_{\rm dec})}}_{\text{Massless } n_f=3} = \underbrace{\frac{M^{(0)}}{m^{(0)}(\mu_{\rm dec})}}_{n_f=0} \times \underbrace{\frac{m^{(0)}(\mu_{\rm dec})}{m^{(3)}(\mu_{\rm dec})} \times \frac{1}{Q_{3\to 0}}}_{PT} + \mathcal{O}\left(\frac{\mu_{\rm dec}^2}{M^2}\right) .$$

What is required?

- Matching condition [2501.06633]

$M/\mu_{\rm dec}$	$\bar{g}^2_{\rm GFT}(\mu_{\rm dec}, M)$	$[\bar{g}^{(0)}_{\rm GF}(\mu_{\rm dec})]^2$	$\rho^{\rm eff}$	$\Lambda^{\rm eff}_{\overline{\rm MS}}$ [MeV]
4	5.258(28)	4.048(18)	0.4843(98)	388.4(10.0)
6	5.347(22)	4.103(15)	0.4523(88)	362.7(9.2)
8	5.479(31)	4.184(19)	0.4390(89)	352.1(9.1)
10	5.669(40)	4.299(24)	0.4382(92)	351.4(9.3)
12	5.780(51)	4.364(31)	0.4319(96)	346.4(9.5)

- LCP for massless computation [DallaBrida:2019wur]

$$\frac{M^{(0)}}{m^{(0)}(\mu_{\rm dec})}$$