THE GAUGE GROUP OF THE STANDARD MODEL

IFIC

Rodrigo Alonso

April 1st

Outline

I. Groups locally & globally

II. Generalised Symmetry

III. The Standard Model Group & Spectrum

VI. Experiment

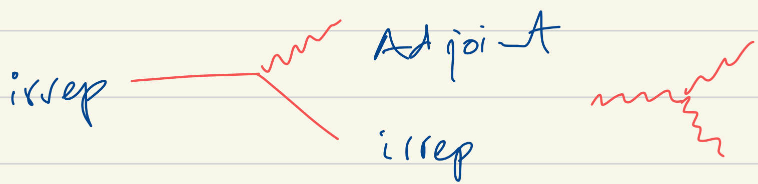
ArXiv:2404.03438
RA, Dimakou & West

What is the Standard Model Group?

I Local vs Global properties of groups.

[Insert appraisal of Gauge Theory here]

Local properties of groups tell us a great deal



$$[T_i, T_j] = i f_{ijk} T^k$$

In particular, given a few inputs, they give the perturbative **S**-matrix, correlators etc

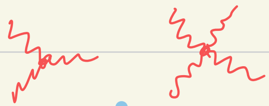
SU(2) versus SO(3)

SU(2): σ_i Pauli Matrices

$$\left[\frac{\sigma_i}{2}, \frac{\sigma_j}{2}\right] = i\varepsilon_{ijk}\frac{\sigma_k}{2}$$

as they act on a doublet ψ

SAME "GLUEDYNAMICS"



SO(3): $T_i = -T_i^T = T_i^\dagger$ 3×3 Matrices

$$[T_i, T_j] = i\varepsilon_{ijk} T_k$$

as they act on 3-vector $\vec{v}$

Would need some non-pert. effect to tell them apart

SU(2) versus SO(3)

SU(2): σ_i Pauli Matrices $\left[\frac{\sigma_i}{2}, \frac{\sigma_j}{2}\right] = i\varepsilon_{ijk}\frac{\sigma_k}{2}$

as they act on a doublet ψ

$$e^{2\pi i \frac{\sigma_3}{2}}\psi = -\psi$$

Rauch et al. '75 Phys. Lett. A

SO(3): $T_i = -T_i^T = T_i^\dagger$ 3 x 3 Matrices

$$[T_i, T_j] = i\varepsilon_{ijk} T_k$$

as they act on 3-vector $\vec{V}$

$$e^{2\pi i T_3}\vec{V} = \vec{V}$$

SU(2) versus SO(3)

σ_i Pauli Matrices $\qquad \left[\frac{\sigma_i}{2}, \frac{\sigma_j}{2}\right] = i\varepsilon_{ijk}\frac{\sigma_k}{2}$

irreps

SU(2):	1	ψ 2	3	4	...
$\sim$					
SO(3):	1	~~2~~	3 $\vec{V}$	~~4~~	

$$\boxed{SO(3) = \frac{SU(2)}{\mathbb{Z}_2}}$$

$\mathbb{Z}_2 = \{1, -1\}$
$\equiv \xi$

$T_i = -T_i^T = T_i^\dagger \quad$ 3×3 Matrices $\quad [T_i, T_j] = i\varepsilon_{ijk} T_k$

$\xi R = R$

in SO(3) reps don't see ξ

Remarks on U(1)

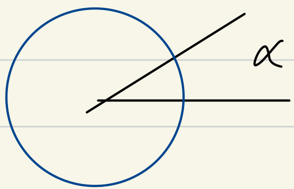
If one declares the group is U(1) $\Rightarrow$ Charge is quantised

$$\psi_Q \to e^{iQ\alpha}\psi_Q \qquad \alpha++ \qquad Q\alpha_1 = 2\pi\mathbb{Z}$$

$$\Rightarrow Q/Q_{min} = \mathbb{Z}$$

$Q^{exp}_{min} = 1$

$\alpha_1 = 2\pi$



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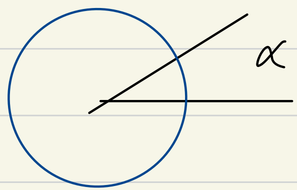
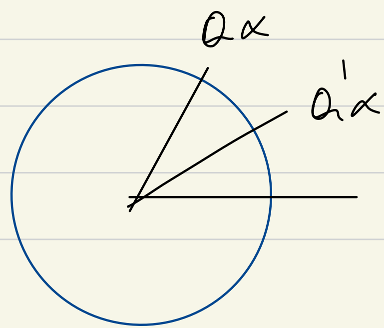


Tomorrow

discover

$Q' = \frac{1}{2} \Rightarrow \alpha_1 = 4\pi$



Again

tends to $\mathbb{R}$

$U(1) = \frac{\mathbb{R}}{\mathbb{Z}}$

Math vs Phy

SU(N)xU(1) versus U(N)

$$\mathbb{1} \to e^{iQ'\theta}\,\mathbb{1}$$

SU(N) x U(1) Unrelated

$$\square \to e^{iQ_F\theta}\, e^{iT_a\theta^a}\,\square$$

$$\square\square \to e^{iQ_S\theta}\, e^{iT_{\square\square}\theta}\,\square\square$$

other than $\frac{Q'}{Q_F}, \frac{Q_S}{Q_F} \in \mathbb{Z}$

U(N) elementary unit:

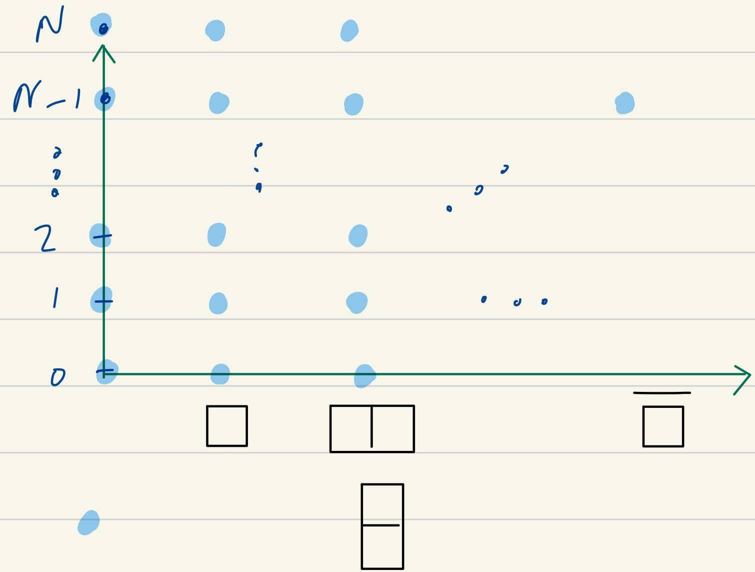
$$\square \to e^{i(\theta Q_F + \theta^a T^a_F)}\,\square$$

$$\square \otimes \square \to e^{2Q_F i\theta + i\theta^a\left(T^a_{\begin{smallmatrix}\square\\\square\end{smallmatrix}} + T^a_{\square\square}\right)}\;\square\square \oplus \begin{matrix}\square\\\square\end{matrix}$$

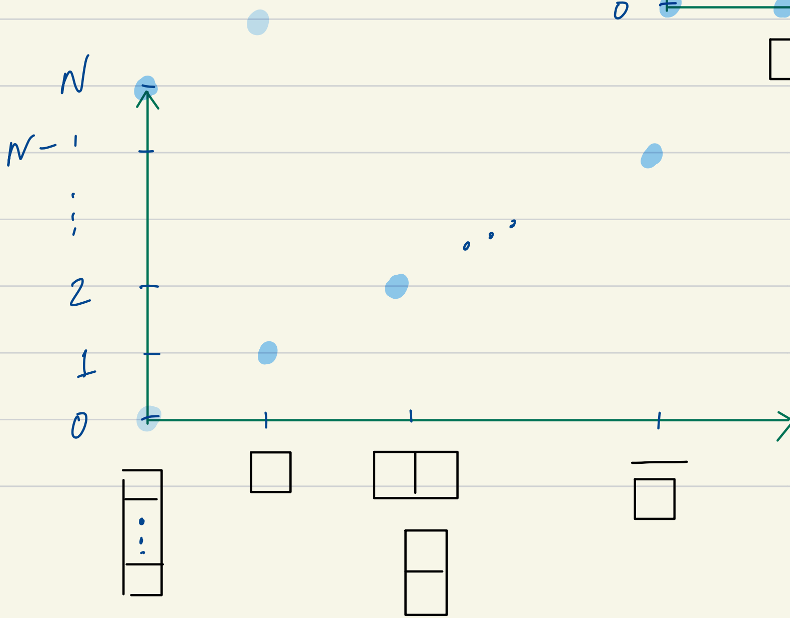
$$\left.\begin{matrix}\square\\\vdots\\\square\end{matrix}\right\}N \to e^{NQ_F i\theta}\begin{matrix}\square\\\vdots\\\square\end{matrix}$$

SU(N)xU(1) versus U(N)

$SU(N) \times U(1)$



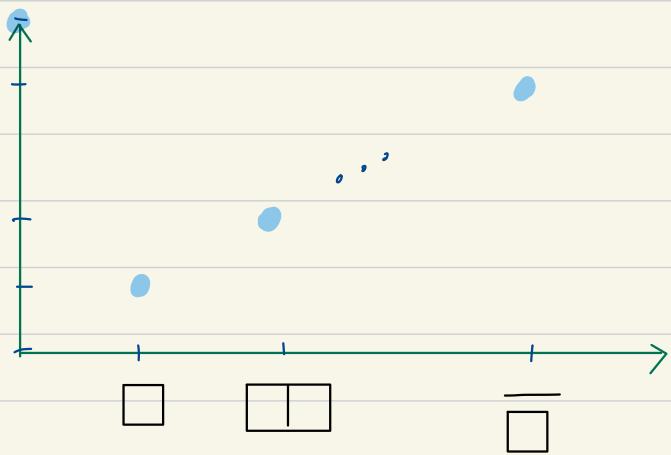
$U(N)$



A way to tell them apart

SU(N)xU(1) versus U(N)

One can again find centre; **U(N)** reps satisfy



$$\mathrm{Exp}\left(\frac{2\pi i n_\psi}{N} - \frac{2\pi i Q}{N Q_F}\right)\psi = \psi$$

$$\equiv \xi \psi$$

ξ spans a $\mathbb{Z}_N$ group

$$\mathbb{Z}_N = \{1, \xi, \xi^2, \cdots \xi^{N-1}\}$$

$$\boxed{U(N) = \frac{SU(N) \times U(1)}{\mathbb{Z}_N}}$$

The centre/zentrum

We can identify all possible isomorphic groups

Given a Lie Algebra, obtain the centre

Z : Centre = subset of elements which commute with everyone else

The "largest" group is the universal cover $\tilde{G}$ while all others are of the form

$$\boxed{G_p = \frac{\tilde{G}}{Z_p}}$$; Z_p subgroup of Z

II Generalised symmetries

Here we could do without them, but they bring a new perspective

Aharony, Seiberg & Tachikawa 2013 D. Tong 2017

- Anomalous symmetries as Non-invertible symmetries
  Choi, Lam & Shao 2022
- Higher form symmetries for

  Monopoles
  Confinement
  Non local-properties
  Axions

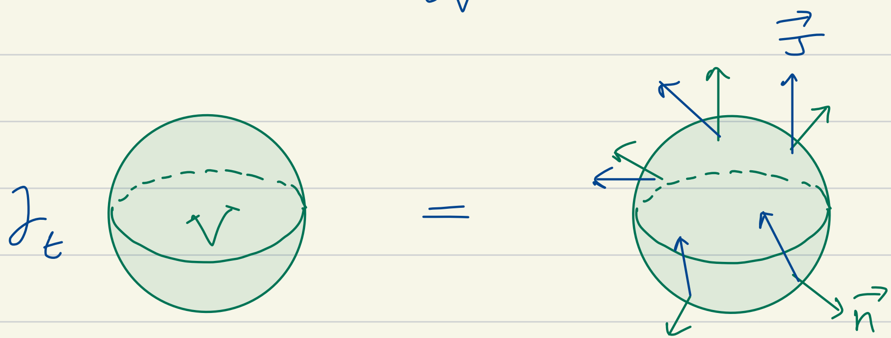
Zero form. Your good old symmetry

CLASSICAL

Divergenceless current

$$\boxed{\partial_\mu J^\mu = 0}$$

$$Q(t) \equiv \int_V d^3x \; J^0(x^\mu)$$



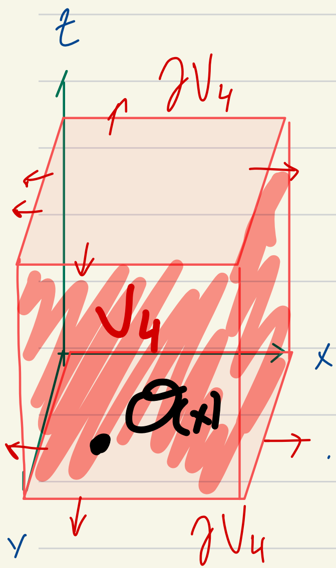
charge given by what escapes at the boundary

$$\partial_0 Q(t) = \int_V d^3x \; \partial_0 J^0 = \int_V d^3x \; \vec{\nabla}\cdot\vec{J} = \int_{\partial V} d\Sigma \; \vec{n}\cdot\vec{J}$$

Zero form. Your good old symmetry

QUANTUM

It acts on Operators ; ie Objects defined at a spacetime-point



$$\mathcal{U}(\alpha, \partial V_4) = \mathrm{Exp}\left(i\alpha \int_{\partial V_4} d^3x \; J^0 \right)$$

$$\langle \mathcal{O}(x) \rangle \equiv \int D\phi \; \mathcal{O}[\phi](x) \, e^{iS[\phi]}$$

$$\delta_\alpha \langle \mathcal{U} \mathcal{O} \mathcal{U}^\dagger \rangle = \langle i\alpha \int_{\partial V_4} d^3y \; J^0(y) \, \mathcal{O}(x) \rangle$$

$$= \langle i\alpha \int_{V_4} d^4y \; \partial_\mu J^\mu(y) \, \mathcal{O}(x) \rangle \underset{\text{Ward Identity}}{=} i\alpha \int d^4y \, \delta^4(x-y) \, q \, \mathcal{O}(x)$$

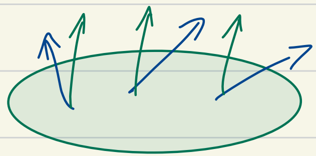
$\Theta(x \text{ in } V_4)$

One form symmetries

CLASSICAL

Now a divergence-less two-form $\boxed{\partial_\mu J^{\mu\nu} = 0}$

$$Fl = \int d\Sigma \, n^i_\Sigma J^{0i} = \int d\Sigma \, n_\mu n^\Sigma_\nu J^{\mu\nu}$$

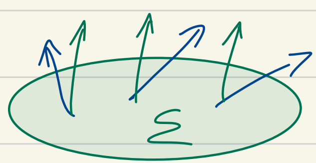


So that

$$\partial_t Fl = \int d\Sigma \, \partial_0 J^{0i} n^i_\Sigma = \int d\Sigma \, n^i_\Sigma \nabla_j J^{ji}$$

$$= \oint dl^i \, \varepsilon_{ijk} J^{jk}$$

∂_t



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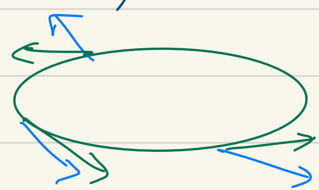


$\partial\Sigma$

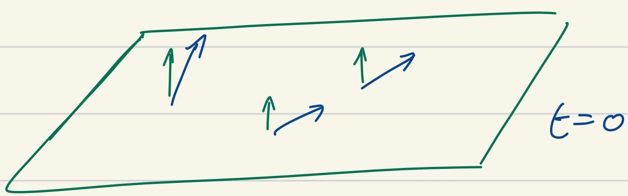
Some familiar example

QED

$\partial_\mu \tilde{F}^{\mu\nu} = 0$ Jacobi's Identity

$\Phi = \int d\vec{\Sigma} \cdot \vec{B}$ Magnetic Flux

If we found a magnetic flux in the universe with boundary large enough to $B(\partial\Sigma) \to 0$ it would always be there



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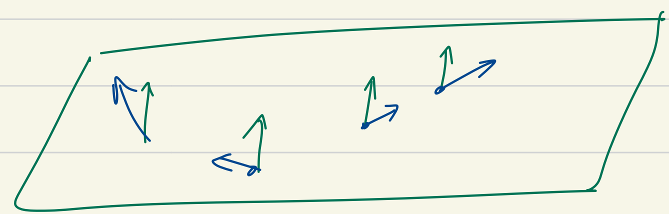
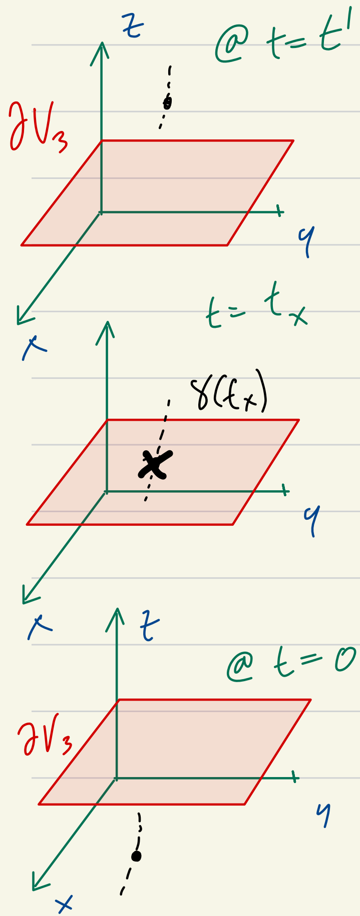


One form symmetries



$$\mathcal{U}(\alpha, \partial V_3) = \mathrm{Exp}\left(i\alpha \int_{\partial V_3} d\Sigma \, n_\mu n^\Sigma_\nu J^{\mu\nu} \right)$$

acts on

Line Operator $W(\gamma)$: γ, $x^\mu(\ell)$ $0 \le \ell \le \ell_0$

$$\delta \langle \mathcal{U} \, W(\gamma) \, \mathcal{U}^\dagger \rangle = i\alpha \left\langle \int_{\partial V_3} d\Sigma \, n^\Sigma_\mu J^{0\mu} \, W(\gamma) \right\rangle$$

$$= i\alpha \left\langle \int_{V_3} \partial_\nu J^{\nu\mu} \, W(\gamma) \right\rangle =$$

$$= i\alpha \left\langle \int d^4y \, \delta(y_3) \, \delta^4(y - x(\ell)) \, \frac{dx^\mu}{d\ell} \, d\ell \, W(\gamma) \right\rangle$$

$$= i\alpha \langle W(\gamma) \rangle \, \text{Linking} \, \# (V_3, \gamma)$$

Notes on one-form symmetries

* Line operators probe the theory non-locally

e.g. $W(\gamma) = T\{ \mathrm{Exp}(i\int_\gamma dx^\mu A_\mu) \}$

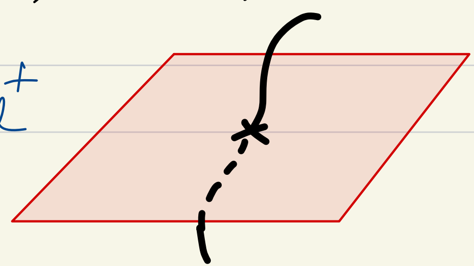
Trip in Gauge Group

* If monopoles exist $\partial \tilde{F} \neq 0$ the symmetry is broken, yet it's still useful

* One forms can also be discrete

Useful in non-abelian Groups

$U\, W(\gamma)\, U^\dagger$



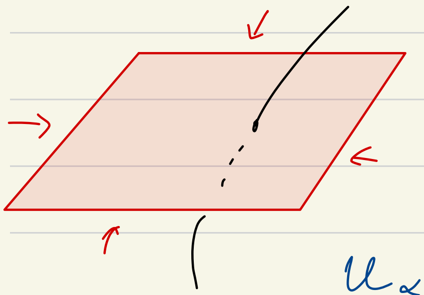
$= \xi\, W(\gamma)$

SU(2) has $\mathbb{Z}_2^{(1)}$ el

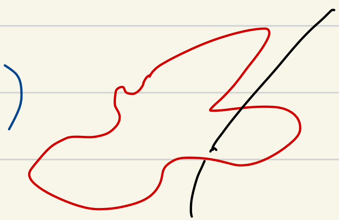
Dirac and t'Hooft

Aharony, Seiberg & Tachikawa

Assume a symmetry operator can be defined in the boundary



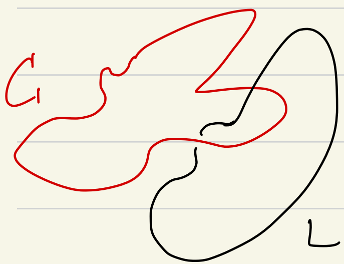
$$\mathcal{U}_\alpha(\Sigma) \to T_g(C_1)$$



$$C = \partial\Sigma$$

$$\mathcal{U}_\alpha(\Sigma)\, W(L)\, \mathcal{U}_\alpha(\Sigma^\dagger) = e^{i\alpha}\, W(L)$$

$$\Downarrow$$



$$T_g(C_1)\, W_q(L) = W_q(L)\, T_g(C_1)\, e^{2\pi i g q} = 1$$

Wilson loop takes a charged particle around so for it to be well defined

t Hooft 1978

Yet for **SO(3)** $g \sim \frac{1}{2}$ $\quad \mathbb{Z}_2^{(1)}$ **mag**

III

The gauge group of the SM

$\tilde{G} \equiv$ \ irreps	q_L	u_R	d_R	l_L	e_R	H
$SU(3)$	3	3	3			
$SU(2)$	2			2		2
$U(1)$	$1/6$	$2/3$	$-1/3$	$-1/2$	-1	$1/2$

Given the spectrum is the a centre ?

$$\mathbb{Z}_3 \times \mathbb{Z}_2 \times U(1) \longrightarrow \mathbb{Z}_6$$

triality

"duality"

Yes: $$\xi \equiv \mathrm{Exp}\left[2\pi i\left(Q_Y + \frac{n_c}{3} + \frac{n_L}{2}\right)\right]$$

The gauge group of the SM

$$\xi^6$$

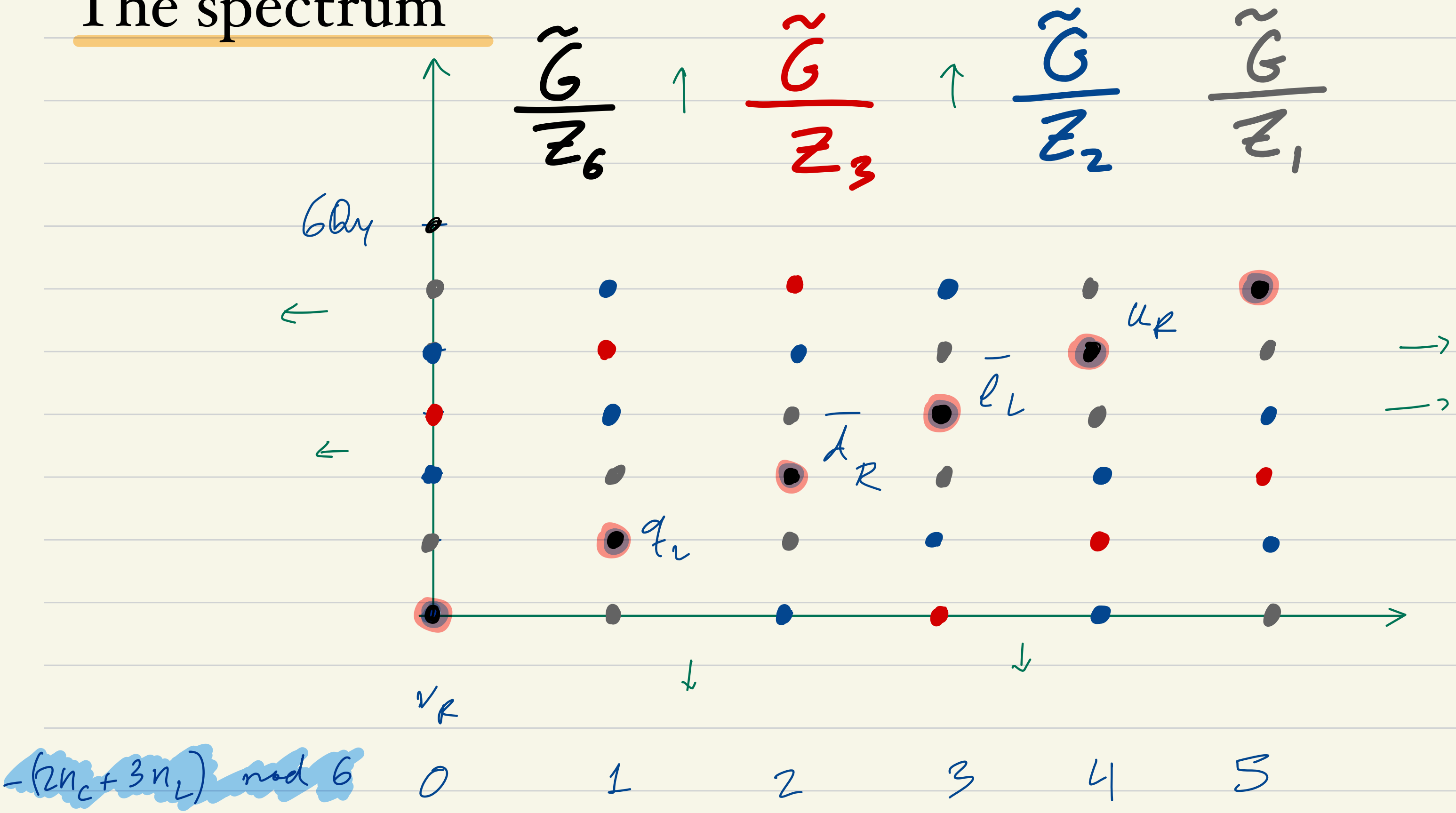
$$\mathbb{Z}_6 = \{ 1, \xi, \xi^2, \xi^3, \xi^4, \xi^5 \}$$

$$\frac{\tilde{G}}{\mathbb{Z}_6} = S[U(3) \times U(2)] \qquad \frac{\tilde{G}}{\mathbb{Z}_3} = U(3) \times SU(2)$$

$$\frac{\tilde{G}}{\mathbb{Z}_2} = SU(3) \times U(2) \qquad \frac{\check{G}}{\mathbb{Z}_1} = \frac{SU(3) \times SU(2) \times U(1)}{\mathbb{Z}_1}$$

Hucks 1991 Tong 2017

The spectrum



The Electric Spectrum

Defining property of Spectrum

$$\xi^{6/p}\,|\text{Any State}\rangle = |\text{Any State}\rangle$$

$$= \left[e^{2\pi i\left(\frac{n_c}{3} + \frac{n_L}{2} + Q_Y\right)}\right]^{6/p} |\text{Any State}\rangle$$

$$= \left[e^{2\pi i\left(\frac{n_c}{3} + \underbrace{T_{3L} + Q_Y}_{Q_{em}}\right)}\right]^{6/p} |\text{Any State}\rangle$$

Condition on Electric Spectrum

The Electric Spectrum

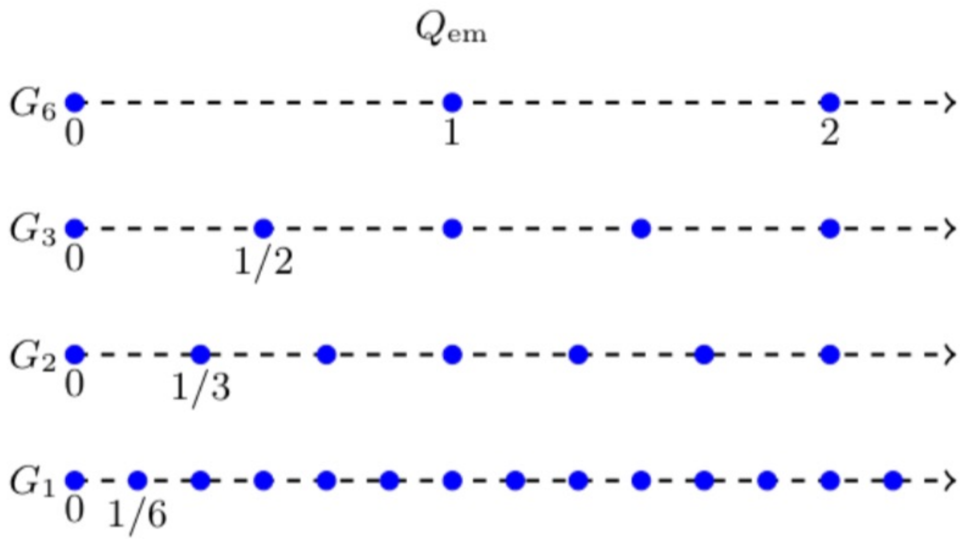
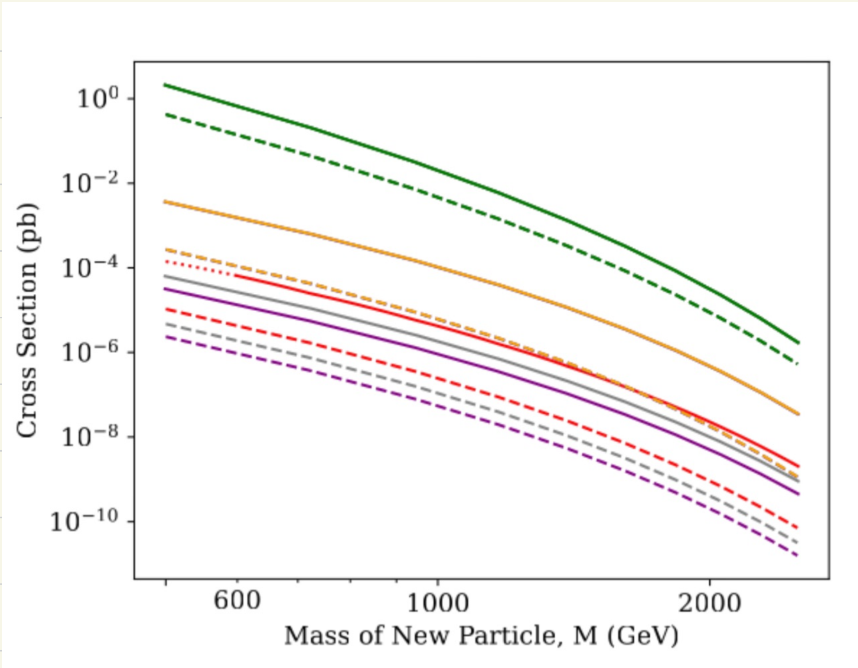


FIG. 4. Electric charge spectrum for hadrons and leptons in the case $k = 0$.

VI The bounds

LHC dixit

$m_{\Xi} > 600\ \text{GeV}$



G_3 : Ξ Λ Ω

$(1,1)_{1/2}\,,\ (1,2)_0\,,\ (3,1)_{1/6}\,,$

G_2 : Σ Δ Θ

$(1,1)_{1/3}\,,\ (1,2)_{1/6}\,,\ (3,1)_0\,,$

G_1 : Φ Λ Θ

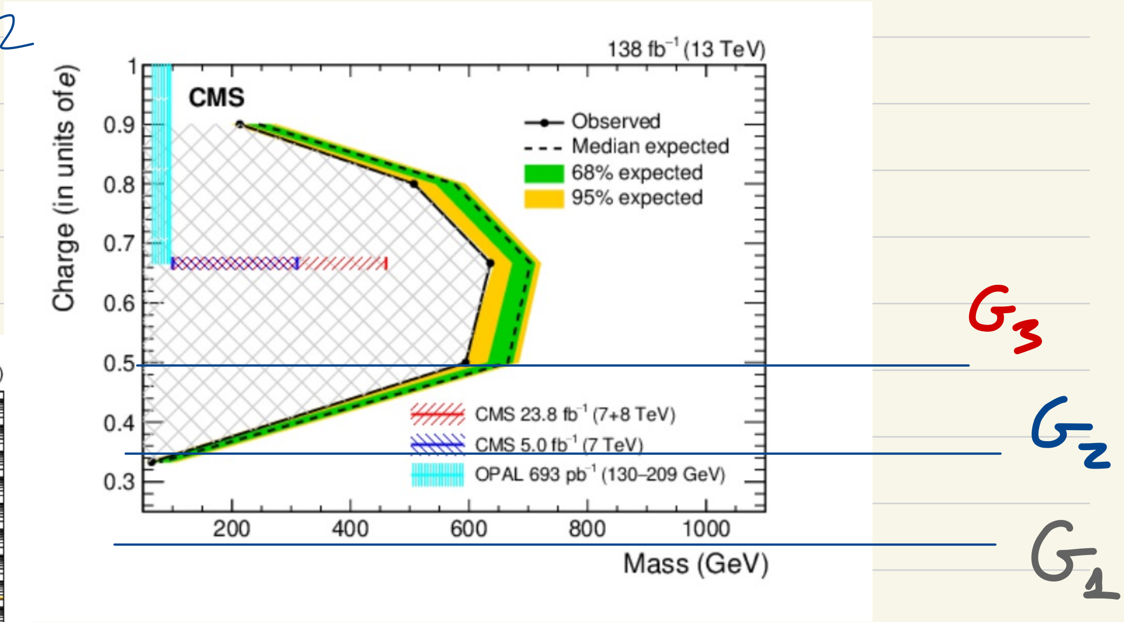
$(1,1)_{1/6}\,,\ (1,2)_0\,,\ (3,1)_0\,.$

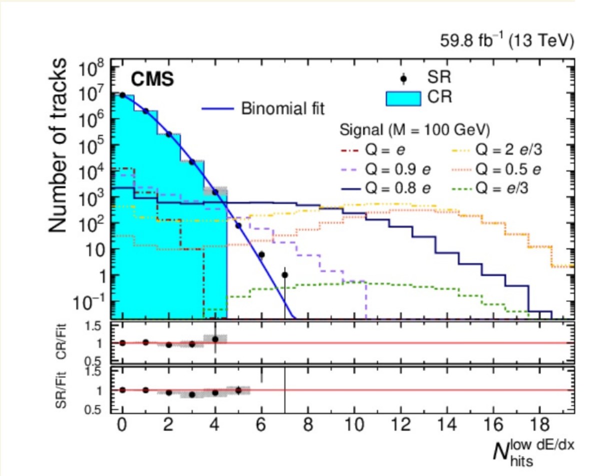
Note these do not decay once produced leaving a charged track in the detector

Searches at the LHC

CMS 2402.09932

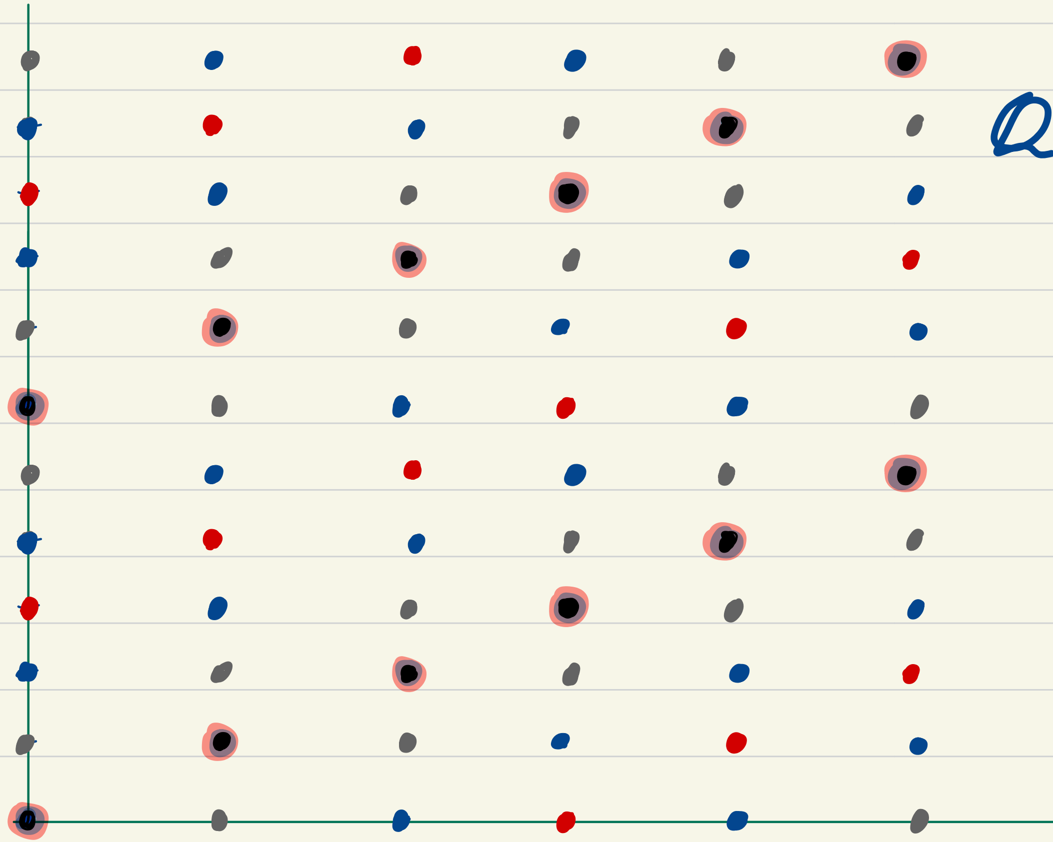
$(1,1)_{Q_4}$





Martin & Koren 2406.17850

The caveat/presumption



Integer

$G_p = 1,2,3,6$

$$Q_{em} = \frac{\mathbb{Z}\; p}{6\,(1+pK)}$$

$K = 0, \pm 1, \pm 2, \ldots$

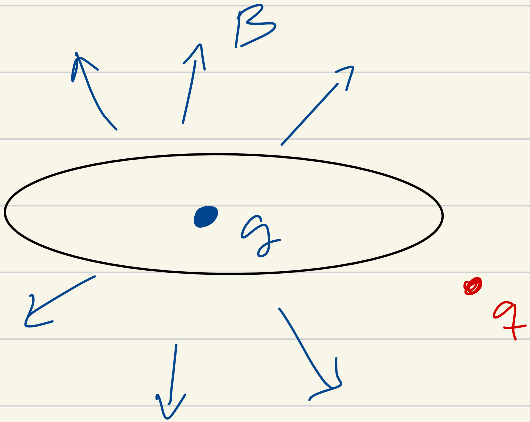
e.g. G_6 $\quad p = 6$

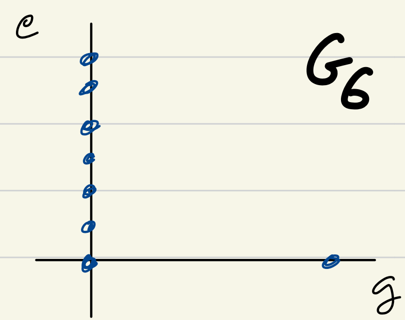
$$\frac{Q^{e^+}}{Q_{min}} = \begin{cases} 1 & K = 0 \\ -5 & K = -1 \\ 7 & K = 1 \end{cases}$$

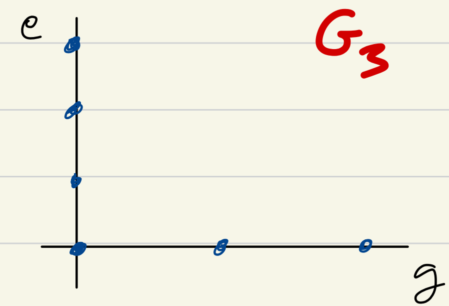
Monopoles

$$e^{iq\int g\, d\phi}\,\psi_q = \psi_q$$

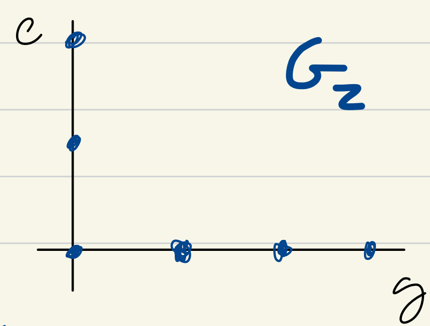
$$2\pi i q g = 2\pi i\, \mathbb{Z}$$

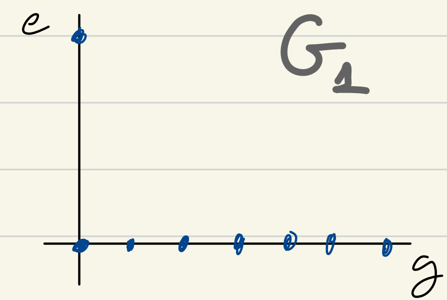






$$(A^+ - A^-)\big|_{\theta=\pi/2} = g\, d\phi$$





The answer to

$$g^{exp}\cdot Q_{min} = 1\ ?$$

Yes Done

No Keep Searching

Exclusive classifier

If $SU(5)$, $\overset{\text{Spin}}{\cancel{SO}}(10)$ then G_6 $K=0$

If $SU(4)\times SU(2)_L \times SU(2)_R$ then G_3 $K=0$

Pati-Salam

If $SU(3)\times SU(3)\times SU(3)$ then G_2 $K=0$

Trinification, Georgi, Glashow & de Rujula

Fifth Workshop on G.U.

Conversely:

If G_3 could have PS but $SU(5)$ ruled out

Summary

The key to the global SM group lies in new fractionally-charged particles

Hadrons $n_c = 0$
& Leptons

Integer $\quad$ $6\,p = 1, 2, 3, 6$

$$Q_{em} = \frac{\mathbb{Z}\ \ p}{6\,(1 + p\,K)}$$

$K = 0, \pm 1, \pm 2, \ldots$

ON THE STANDARD MODEL GAUGE GROUP

RODRIGO ALONSO

CEA Saclay
05/11/2025

DURHAM UNIVERSITY

WHAT I'VE BEEN UP TO