

Flavour Physics in the Standard Model and Beyond

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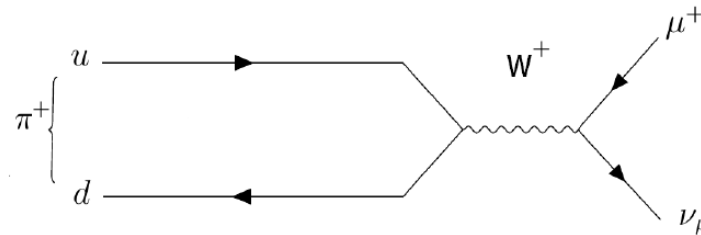
Classification of the processes in the SM

Leptonic Decays

the prototype of these decays is given by

$$\pi^+ \longrightarrow \mu^+ + \nu_\mu \quad (35)$$

which at the fundamental level is given by



$$\Gamma \propto |V_{ij}|^2 f_P^2$$

Other possible leptonic decays are given by

$$K^+ \longrightarrow \mu^+ + \nu_\mu$$

$$D^+ \longrightarrow \mu^+ + \nu_\mu$$

$$B^+ \longrightarrow \tau^+ + \nu_\tau$$

$$\pi^+ \longrightarrow e^+ + \nu_e$$

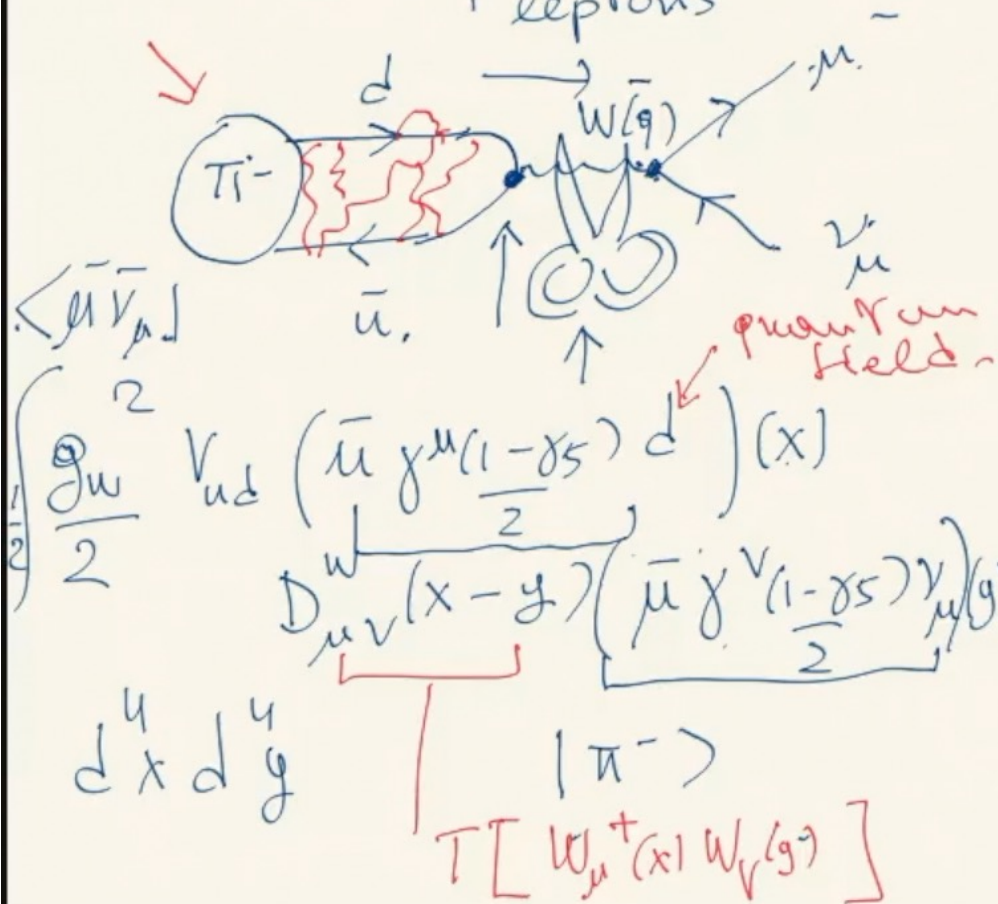
the latter process is suppressed by chirality

*from lattice
calculations*

LEPTONIC DECAYS

$$\pi^- \rightarrow \mu^- \bar{\nu}_\mu$$

↑ leptons



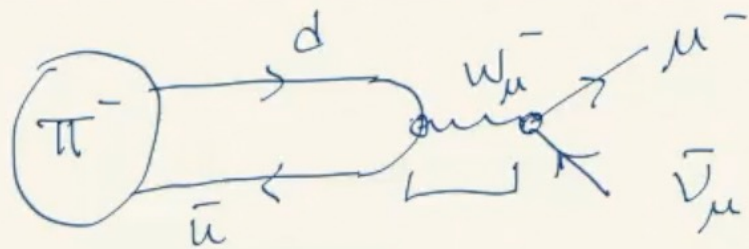
$$\frac{1}{2} \langle \bar{\mu} \bar{\nu}_\mu | \mathcal{L}_I \mathcal{L}_I | \pi^- \rangle$$

$$\mathcal{L}_I = \frac{g_w}{\sqrt{2}} (J_\mu^- W_\mu^+ + J_\mu^+ W_\mu^-)$$

$$J_\mu^- = \left(V_{ud} \bar{u} \gamma^\mu \frac{(1 - \gamma_5)}{2} d \right) +$$

$$V_{cs} \bar{c} \gamma^\mu \frac{(1 - \gamma_5)}{2} s + \dots$$

$$+ \bar{\nu}_\mu \gamma^\mu \frac{(1 - \gamma_5)}{2} \mu + \dots$$



$$\left(\frac{-ig_W}{\sqrt{2}}\right)^2 V_{ud} \bar{u}(p_\mu) \gamma^\mu \frac{(1-\gamma_5)}{2} u(p_{\bar{\nu}_\mu}) \langle 0 | \bar{u} \gamma^\mu \frac{(1-\gamma_5)}{2} d | \pi^- \rangle$$

$$i \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{M_W^2} \right) \frac{1}{-M_W^2 + i\epsilon}$$

$$q = p_\pi \sim 130 \text{ MeV}$$

$$= -i \frac{G_F}{\sqrt{2}} V_{ud} \bar{u}(p_\mu) \gamma^\mu (1-\gamma_5) u(p_{\bar{\nu}_\mu}) \langle 0 | \bar{u} \gamma^\mu (1-\gamma_5) d | \pi^- \rangle$$

$$\begin{aligned} \langle 0 | \bar{u} \gamma^\mu (1-\gamma_5) d | \pi^- \rangle &= \\ &= - \langle 0 | \bar{u} \gamma^\mu \gamma_5 d | \pi^- \rangle = \\ &= -i f_\pi (p_\pi^\mu) \end{aligned}$$

$$A = \frac{G_F}{\sqrt{2}} f_\pi V_{ud} p_\pi^\mu \bar{u}(p_\mu) \gamma^\mu (1-\gamma_5) u(p_{\bar{\nu}_\mu})$$

$$d\Gamma = \frac{1}{2M_\pi} \sum_{\text{final}} \int \frac{d^3 p_\mu}{(2\pi)^3 2E_\mu} \int \frac{d^3 p_{\bar{\nu}_\mu}}{(2\pi)^3 2E_{\bar{\nu}_\mu}} |A|^2 (2\pi)^4 \delta^4(p_\pi - p_\mu - p_{\bar{\nu}_\mu})$$

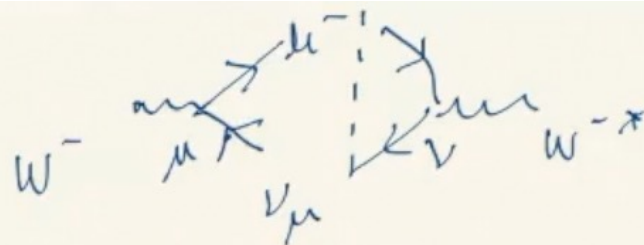
$$|A|^2 = \frac{G_F^2}{2} f_\pi^2 |V_{ud}|^2 p_\pi^\mu p_\pi^\nu \sum_{\text{pol.}} \bar{u}(p_\mu) \gamma^\mu (1-\gamma_5) u(p_{\bar{\nu}}) \bar{u}(p_{\bar{\nu}}) \gamma^\nu (1-\gamma_5) u(p_\mu)$$

$$\begin{aligned} \langle 0 | \bar{u} \gamma^\mu (1 - \gamma_5) d | \pi^- \rangle &= \\ &= \langle 0 | \bar{u} \gamma^\mu \gamma_5 d | \pi^- \rangle = \\ &= i f_\pi (P_\pi^\mu) \end{aligned}$$

$$A = \frac{G_F}{\sqrt{2}} f_\pi V_{ud} P_\pi^\mu \bar{u}(p_\mu) \gamma^\mu (1 - \gamma_5) \nu_\mu$$

$$d\Gamma = \frac{1}{2M_\pi} \sum_{\text{final}} |A|^2 (2\pi)^4 \delta^4(p_\pi - p_\mu - p_\nu) \frac{d^3 p_\mu}{(2\pi)^3 2E_\mu} \frac{d^3 p_\nu}{(2\pi)^3 2E_\nu}$$

$$|A|^2 = \frac{G_F^2}{2} f_\pi^2 |V_{ud}|^2 \underbrace{P_\pi^\mu P_\pi^\nu}_{\text{circled}} \bar{u}(p_\mu) \gamma^\mu (1 - \gamma_5) \nu(p_\nu) \bar{\nu}(p_\nu) \gamma^\nu (1 - \gamma_5) u(p_\mu)$$



$$t_2 (\not{p}_\mu + m_\mu) \gamma^\mu (1 - \gamma_5) \not{p}_\nu \gamma^\nu (1 - \gamma_5)$$

$$= 2 t_2 (\not{p}_\mu + m_\mu) \gamma^\mu \not{p}_\nu \gamma^\nu (1 - \gamma_5)$$

$$= 2 t_2 \not{p}_\mu \gamma^\mu \not{p}_\nu \gamma^\nu (1 - \gamma_5) + 2 m_\mu t_2 \gamma^\mu \not{p}_\nu \gamma^\nu (1 - \gamma_5)$$

$$= 2 \left[\not{p}_\mu^\rho \not{p}_\nu^\sigma + \not{p}_\mu^\sigma \not{p}_\nu^\rho - g^{\rho\sigma} \not{p}_\mu \not{p}_\nu - i \epsilon^{\rho\sigma\alpha\beta} (\not{p}_\mu)_\alpha (\not{p}_\nu)_\beta \right]$$

$$\begin{aligned}
 |A|^2 &= 4G_F^2 |V_{ud}|^2 f_\pi^2 p_\pi^\rho p_\pi^\sigma \\
 &\quad (p_\mu^\rho p_\nu^\sigma + p_\mu^\sigma p_\nu^\rho - g^{\rho\sigma} p_\mu \cdot p_\nu) \\
 &= 4G_F^2 |V_{ud}|^2 f_\pi^2 \{ 2(p_\pi \cdot p_\mu)(p_\pi \cdot p_\nu) \\
 &\quad - m_\pi^2 (p_\mu \cdot p_\nu) \}
 \end{aligned}$$

$$(p_\pi \cdot p_\mu) = (p_\mu + p_\nu) \cdot p_\mu = m_\mu^2 + p_\mu \cdot p_\nu$$

$$(p_\pi \cdot p_\nu) = (p_\mu + p_\nu) \cdot p_\nu = p_\mu \cdot p_\nu$$

$$p_\pi^2 = m_\pi^2 = (p_\mu + p_\nu)^2 = m_\mu^2 + 2p_\mu \cdot p_\nu$$

$$p_\mu \cdot p_\nu = \frac{m_\pi^2 - m_\mu^2}{2}$$

$$d\Gamma = \frac{1}{2m_\pi} G_F^2 |V_{ud}|^2 f_\pi^2$$

$$4m_\mu^2 (m_\pi^2 - m_\mu^2) \circ \textcircled{PS}$$

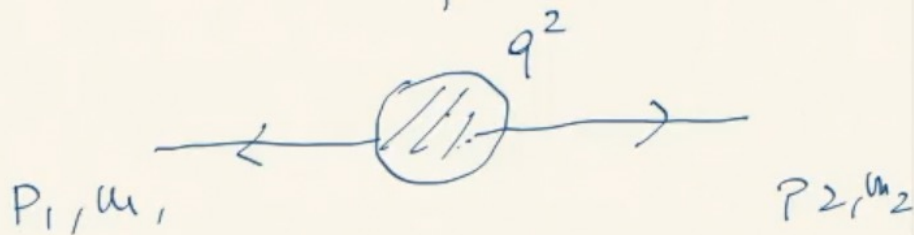
$$\begin{aligned}
 \int PS &= \int (\frac{d^3 p_\mu}{(2\pi)^3 2E_\mu} \frac{d^3 p_\nu}{(2\pi)^3 2E_\nu}) \delta^4(p_\pi - p_\mu - p_\nu) \\
 &= \frac{1}{(2\pi)^2} \pi \left(\frac{|\vec{p}_\mu|}{m_\pi} \right)
 \end{aligned}$$

$$I = \int \delta^4(q - p_1 - p_2) \frac{d^3 p_1}{2E_1} \frac{d^3 p_2}{2E_2}$$

$$= \pi \frac{|\vec{p}_1|}{\sqrt{q^2}}$$

$$|\vec{p}_1| = |\vec{p}_2| \rightarrow$$

$$|\vec{p}_1|^2 = \frac{[(q^2)^2 + (m_1^2)^2 + (m_2^2)^2 - 2q^2 m_1^2 - 2q^2 m_2^2 - 2m_1^2 m_2^2]}{4q^2}$$



$$d\Gamma = \frac{1}{2m_\pi} G_F^2 |V_{ud}|^2 f_\pi^2$$

$$4m_\mu^2 (m_\pi^2 - m_\mu^2) \theta(\text{PS})$$

$$\int \text{PS} = \int (2\pi)^4 \delta^4(p_\pi - p_\mu - p_\nu) \frac{d^3 p_\mu}{(2\pi)^3 2E_\mu} \frac{d^3 p_\nu}{(2\pi)^3 (2E_\nu)}$$

$$= \frac{1}{(2\pi)^2} \pi \left(\frac{|\vec{p}_\mu|}{m_\pi} \right)$$

$$= \frac{1}{(2\pi)^2} \pi \frac{m_\pi^2 - m_\mu^2}{2m_\pi^2}$$

$$= \frac{1}{(2\pi)^2} \frac{\pi}{2} \left(1 - \frac{m_\mu^2}{m_\pi^2} \right)$$

$$\Gamma = \frac{G_F^2 f_\pi^2 m_\mu^2 \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2 m_\pi |V_{ud}|^2}{8\pi}$$

non pert. quality.

$$B^- \rightarrow e^- \bar{\nu}_e$$

$$|V_{ub}|^2$$

$$D^-$$

$$|V_{cd}|^2$$

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{ud}^* \left(1 + \frac{\alpha}{\pi} \log \frac{M_Z}{M_W}\right) (\bar{d} \gamma^\mu (1 - \gamma^5) u) (\bar{\nu}_\ell \gamma_\mu (1 - \gamma^5) \ell),$$

Classification of the processes in the SM

Semi-leptonic Decays

these are the better sources to measure the absolute values of the CKM matrix elements $|V_{ij}|$

$$n \rightarrow p + e^- + \bar{\nu}_e$$

other possible semi-leptonic decays are the following

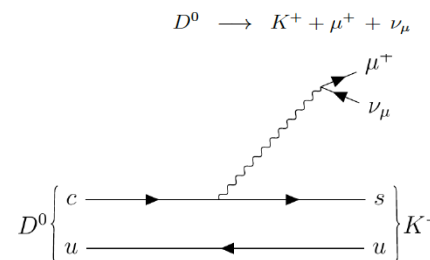
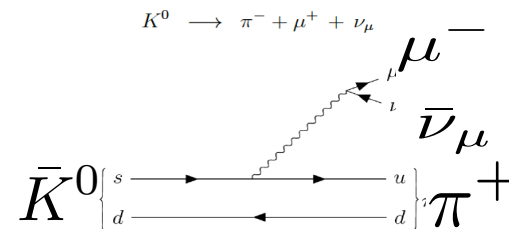
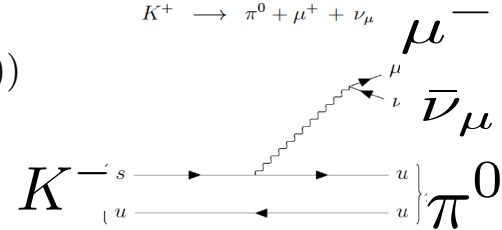
$$\begin{aligned} \pi^+ &\rightarrow \pi^0 + e^+ + \nu_e \\ K^+ &\rightarrow \pi^0 + \mu^+ + \nu_\mu \end{aligned}$$

$$-\frac{G_F}{\sqrt{2}} V_{us} (\bar{u}(x) \gamma_\mu (1 - \gamma_5) s(x)) (\bar{\mu}(x) \gamma_\mu (1 - \gamma_5) \nu_\mu(x))$$

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 |V_{us}|^2}{24\pi^3} |\vec{p}_{\pi^-}|^3 |f^+(q^2)|^2$$

$$\frac{1}{\Gamma} \frac{d\Gamma}{dq^2} \propto |f(q^2)|$$

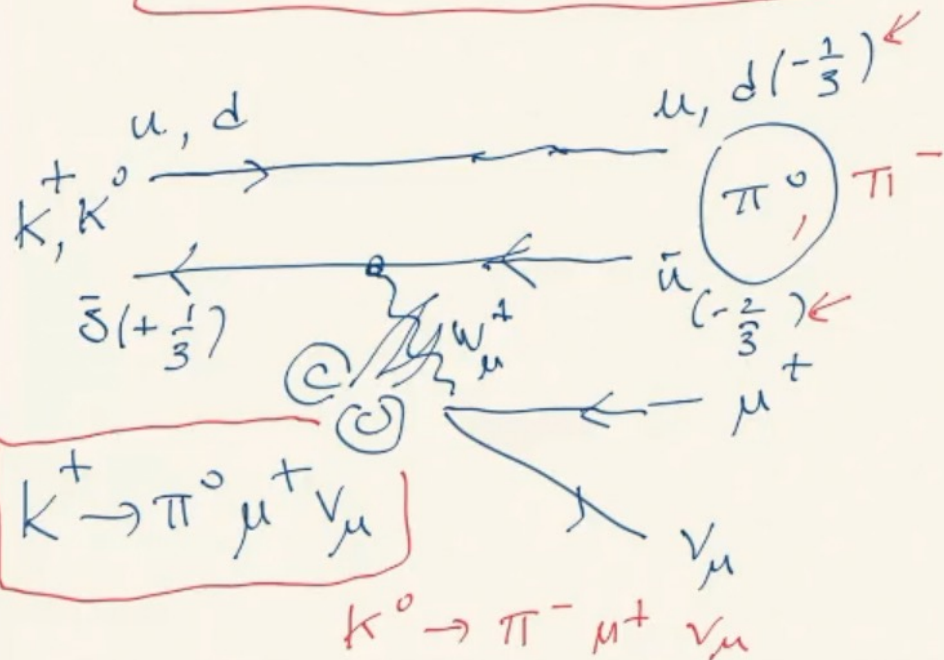
from lattice calculations



$$B^+ \rightarrow \pi^+ \nu_e$$

SEMI LEPTONIC
DECAYS

$$n \rightarrow p + e^- + \bar{\nu}_e$$



$$\langle \pi^0 \mu^+ \nu_\mu | \left(\bar{\nu}_\mu \gamma^\mu (1 - \gamma_5) \mu \right) | K^+ \rangle$$

$$\left(\frac{i G_F}{\sqrt{2}} \right) = \left(- \frac{i G_F}{\sqrt{2}} V_{us} \right)$$

$$\langle \mu^+ \nu_\mu | \bar{\nu}_\mu \gamma^\mu (1 - \gamma_5) \mu | 0 \rangle$$

pseudo scalar $\rightarrow$ $\langle \pi^0 | \bar{s} \gamma^\mu (1 - \gamma_5) u | K^+ \rangle$ pseudo scalar

$$= - \frac{i G_F}{\sqrt{2}} V_{us} \bar{u}(p_\mu) \gamma^\mu (1 - \gamma_5) u(p_\mu)$$

$$\langle \pi^0 | \bar{s} \gamma^\mu u | K^+ \rangle$$

Wigner-Eckart

$$\langle \pi^0 | \bar{S} \gamma^\mu u | K^+ \rangle =$$

$$f^+(q^2) (P_{K^+}^\mu + P_{\pi^0}^\mu) +$$

$$f^-(q^2) (P_{K^+}^\mu - P_{\pi^0}^\mu)$$

$$q = P_{K^+} - P_{\pi^0} = P_{\mu^+} + P_{\nu_\mu}$$

$$f^-(q^2) \neq 0$$

$$f^+(q_{=0}^2) = \text{charge} \\ = \sqrt{2} \left(1 + \frac{(m_S - m_d)^2}{(1 \text{ GeV})^2} \right)$$

$$\langle e^- \nu_e | \bar{e} \gamma_\mu (1 - \gamma_5) \nu_e | 0 \rangle =$$

$$\bar{u}(P_e) \gamma_\mu (1 - \gamma_5) u(P_{\nu_e})$$

$$\langle p | \bar{u} \gamma_\mu (1 - \gamma_5) s | \Lambda^0 \rangle$$

$$\langle p | \bar{u} \gamma_\mu s | \Lambda^0 \rangle =$$

$$\bar{u}^{\sigma_p}(P_p) \left[F_1(q^2) \gamma^\mu + i \frac{\sigma^{\mu\nu} q_\nu F_2(q^2)}{[(m_p + m_{\Lambda^0})/2]} \right]$$

$$+ \left[\frac{q^\mu F_3(q^2)}{(m_p + m_{\Lambda^0})/2} \right] u^{\sigma_{\Lambda^0}}(P_{\Lambda^0})$$

$$q = P_{\Lambda^0} - P_p = P_{\nu_e} + P_{e^-}$$

$$q^\mu \langle \pi(p') | \hat{V}_\mu | \pi(p) \rangle =$$

$$f^+(q^2) (p_\mu + p'_\mu) + f^-(q^2) \underbrace{(p_\mu - p'_\mu)}_{q_\mu}$$

$$= f^+(q^2) (m_\pi^2 - m_\pi^2) + f^-(q^2) q^2 = 0$$

$$f^-(q^2) = 0$$

$$\langle \pi(\vec{p}) | Q(t) | \pi(\vec{p}') \rangle = \int d^3x \langle \pi(\vec{p}') | V_0(\vec{x}, t) | \pi(\vec{p}) \rangle$$

$$\langle \pi(\vec{p}') | Q(0) | \pi(\vec{p}) \rangle = \int d^3x \langle \pi(\vec{p}') | e^{-i\vec{p}' \cdot \vec{x}} V_0(0) e^{i\vec{p} \cdot \vec{x}} | \pi(\vec{p}) \rangle$$

$$= \int d^3x e^{i(\vec{p} - \vec{p}') \cdot \vec{x}} \langle \pi(\vec{p}') | V_0(0) | \pi(\vec{p}) \rangle$$

$$Q_\pi = (2\pi)^3 \delta^3(\vec{p} - \vec{p}') \underbrace{\langle \pi(\vec{p}) | V_0(0) | \pi(\vec{p}) \rangle}_{2E f(q^2=0)}$$

$$(2\pi)^3 2E \delta^3(\vec{p} - \vec{p}')$$

$$\langle B^i J^k B^i \rangle$$

$$J^k |B^i\rangle = 8 \times 8 = 1 + 8 + 8 + 27 + 10 + 10$$

$$[J^k, B^i] = i f^{k i m} R^m \quad R^m \in \mathfrak{g}$$

$$\{J^k, B^i\} = i d^{k i m} J^m \quad J^m \in \mathfrak{g}$$

$$X |m, l, l_z\rangle = \left[r \sin \theta \cos \varphi f_{me}(z) Y_{ll} \right]$$

$$= g(z) \sin \theta (e^{i\varphi} + e^{-i\varphi}) \sin \theta e^{i\varphi}$$

$$\sim g(z) (e^{2i\varphi} + 1) \sin \theta$$

$$(Y_{l, l+1} + Y_{l, l-1}) Y_{ll}$$

$$|1, 1\rangle |1, 1\rangle = |2, 2\rangle$$

$$|1, 0\rangle |1, 1\rangle = a |2, 0\rangle + b |0, 0\rangle$$

$$\rightarrow J_\mu = (\cos \theta_c J_\mu^+ + \sin \theta_c V_\mu^+)$$

$$\langle B^k | J_\mu^+ | B^i \rangle_{i=k} = I$$

SEMI-LEPTONIC HYPERON DECAY

$$B = \begin{pmatrix} \frac{1}{\sqrt{6}} \Lambda^0 + \frac{1}{\sqrt{2}} \Sigma^0 & \Sigma^+ & p \\ \Sigma^- & \frac{1}{\sqrt{6}} \Lambda^0 - \frac{1}{\sqrt{2}} \Sigma^0 & n \\ \bar{\Sigma}^0 & \bar{\Sigma}^+ & -\sqrt{\frac{2}{3}} \Lambda^0 \end{pmatrix} = B^i \frac{\lambda^i}{2}$$

$$\langle B | J^k | B \rangle \sim 8 \times 8 \times 8$$

$$t_2 B^i J^k B^j \sim t_2 J^k [B^j, B^i] + t_2 J^k \{B^j, B^i\}$$

F-couplings D-couplings

EXAMPLES

$$n \rightarrow p e \bar{\nu} \quad [\gamma^4 + (D+F) \gamma^4 \gamma^5] \cos \theta_c$$

$$\Sigma^\pm \rightarrow \Lambda e^\pm \nu \quad \gamma^4 \gamma^5 \sqrt{\frac{2}{3}} D \cos \theta_c$$

$$\Lambda^0 \rightarrow p e \bar{\nu} \quad -\frac{\sqrt{3}}{2} [\gamma^4 + (F + \frac{D}{3}) \gamma^4 \gamma^5] \sin \theta_c$$

SU(3) SYMMETRY ASSUMED

$$F, D \quad F_1 = g_1(0) / \left(1 - \frac{g_1^2}{m_A^2}\right)^2 \quad \text{DIPOLE APPROXIMATION}$$

$|V_{us}|$ not very

sensitive to the

PRECISE VALUE OF m_A

V_{us} from hyperon semileptonic decays

D. Guadagnoli 2005

new data analysis from Cabibbo, Swallow and Winston ('03):

Use of exp. info for decay amplitudes and angular correlations

$$\Gamma_{decay} \simeq |V_{us}|^2 f_1^2(0) \left(1 + 3 \left(\frac{g_1(0)}{f_1(0)} \right)^2 \right) \times \left[\text{known factors, including rad. corrections} \right]$$

✓ Exps. give direct access to the product $|V_{us}| f_1(0)$

- 1) the ratio $g_1(0) / f_1(0)$ is extracted from data,
- 2) SU(3) symmetry is assumed for $f_1(0)$ [as well as for $g_2(0)$].

$$|V_{us}|^{hyp.} = 0.2250 \pm 0.0027$$

$$|V_{us}|^{K13} = 0.2256 \pm 0.0021$$

Decay	g_1/f_1	V_{us}
$\Lambda \rightarrow pe^- \bar{\nu}$	0.718(15)	0.2224 ± 0.0034
$\Sigma^- \rightarrow ne^- \bar{\nu}$	-0.340(17)	0.2282 ± 0.0049
$\Xi^- \rightarrow \Lambda e^- \bar{\nu}$	0.25(5)	0.2367 ± 0.0099
$\Xi^0 \rightarrow \Sigma^+ e^- \bar{\nu}$	$1.32^{+0.22}_{-0.18}$	0.209 ± 0.027
Combined	—	0.2250 ± 0.0027

as the contribution of $\gamma^\mu \gamma_5$ is exactly equal to zero. For the remaining matrix element, as it is a 4-vector under Lorentz transformations, we can only express it through 4-vectors, namely the mesons 4-momentum. Thus, we can write for example that

$$\langle \pi^- | \bar{s} \gamma^\mu u | K^0 \rangle = f^+(q^2)(p_{K^0}^\mu + p_{\pi^-}^\mu) + f^-(q^2)q^\mu$$

where $q^\mu \equiv p_{K^0}^\mu - p_{\pi^-}^\mu$. The quantities $f^+(q^2)$ and $f^-(q^2)$ are called *form factors* and they are functions of q^2 as it is the unique non trivial Lorentz invariant of our problem. Furthermore, they condensate the informations coming from strong interactions, hence we can determine them through non-perturbative techniques, namely numerical simulation on the lattice. Here we put in evidence that in Appendix B we describe the main properties of such framework, namely *Lattice QCD*, also showing an its variant in Appendix B, characterized by the so called *Twisted Mass formalism*.

Another possibility is to describe the hadronic matrix element by introducing a new form factor $f^0(q^2)$, related to the other ones through the relation

$$f^0(q^2) = \frac{q^2}{m_{K^0}^2 - m_{\pi^-}^2} f^-(q^2) + f^+(q^2),$$

so that

$$\langle \pi^- | \bar{s} \gamma^\mu u | K^0 \rangle = f^+(q^2) \left(p_{K^0}^\mu + p_{\pi^-}^\mu - \frac{m_{K^0}^2 - m_{\pi^-}^2}{q^2} q^\mu \right) + f^0(q^2) \frac{m_{K^0}^2 - m_{\pi^-}^2}{q^2} q^\mu.$$

Putting all together, Eq. (1.21) finally becomes

$$\begin{aligned} \frac{d\Gamma}{dq^2} &= \frac{G_F^2 |V_{us}|^2}{24\pi^2} \left(1 - \frac{m_c^2}{q^2} \right)^2 \\ &\times \left[|\vec{p}_{\pi^-}|^3 \left(1 + \frac{m_c^2}{2q^2} \right) f^+(q^2) + m_{K^0}^2 |\vec{p}_{\pi^-}| \left(1 - \frac{m_{\pi^-}^2}{m_{K^0}^2} \right)^2 \frac{3m_c^2}{8q^2} f^0(q^2) \right], \end{aligned} \quad \frac{d\Gamma}{dq^2} = \frac{G_F^2 |V_{us}|^2}{24\pi^3} |\vec{p}_{\pi^-}|^3 |f^+(q^2)|^2$$

where $\vec{p}_{\pi^-}$ is the 3-momentum of the pion, so that

$$|\vec{p}_{\pi^-}| = \left[\left(\frac{m_{K^0}^2 + m_{\pi^-}^2 - q^2}{2m_{K^0}} \right)^2 - m_{\pi^-}^2 \right]^{1/2},$$

Model. To be more specific, we can choose a particular basis in which we can re-express the effective weak Hamiltonian in the Fermi approximation. Referring to the particular quark transition $b \rightarrow c$, as it will be of interest in the future applications present in this Thesis, we can generalize the expression (1.20) valid for a semileptonic transition writing that [37]

$$\mathcal{H}_{eff} = 2\sqrt{2}G_F V_{cb}[(1 + V_L)\mathcal{O}_{V_L} + V_R\mathcal{O}_{V_R} + S_L\mathcal{O}_{S_L} + S_R\mathcal{O}_{S_R} + T_L\mathcal{O}_{T_L}], \quad (1.27)$$

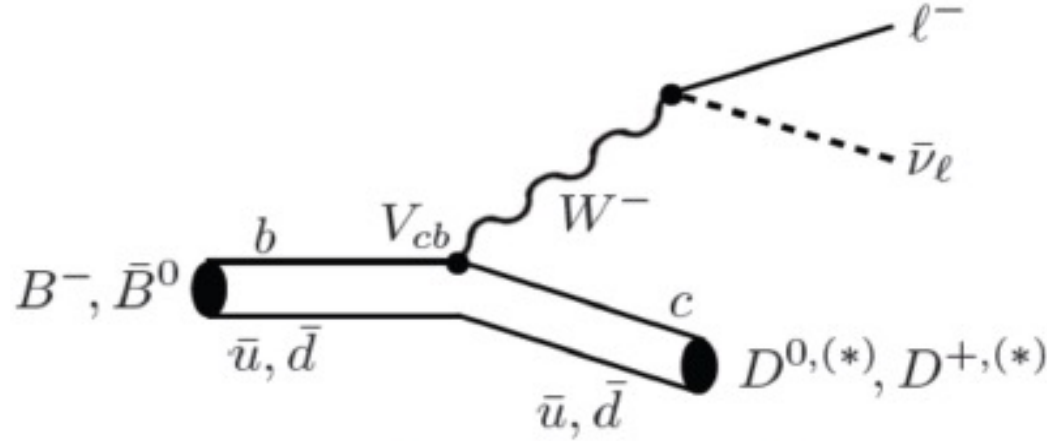


Figure 2.1. Feynman diagram relative to $B^- \rightarrow D^{0(*)}\ell^- \bar{\nu}_\ell$ and $B^0 \rightarrow D^{+(*)}\ell^- \bar{\nu}_\ell$ transitions.

where $V_{L,R}$, $S_{L,R}$ and T_L are the complex Wilson coefficients governing NP contributions while the definitions of NP operators are

$$\begin{aligned} \mathcal{O}_{V_L} &= (\bar{c}\gamma^\mu P_L b)(\bar{\ell}\gamma_\mu P_L \nu_\ell), \\ \mathcal{O}_{V_R} &= (\bar{c}\gamma^\mu P_R b)(\bar{\ell}\gamma_\mu P_L \nu_\ell), \\ \mathcal{O}_{S_L} &= (\bar{c}P_L b)(\bar{\ell}P_L \nu_\ell), \\ \mathcal{O}_{S_R} &= (\bar{c}P_R b)(\bar{\ell}P_L \nu_\ell), \\ \mathcal{O}_{T_L} &= (\bar{c}\sigma^{\mu\nu} P_L b)(\bar{\ell}\sigma_{\mu\nu} P_L \nu_\ell), \end{aligned}$$

D^* , with momentum p , mass m_{D^*} and polarization ϵ , we can define $V(q^2)$ and $A_{0,1,2,3}(q^2)$ form factors, with $q^2 \equiv (p_B - p)^2$, such that ($\epsilon_{0123} = +1$) [66]:

$$\begin{aligned} \langle D^*(p, \epsilon) | \bar{c} \gamma^\mu (1 \mp \gamma_5) b | \bar{B}(p_B) \rangle = & -\frac{2}{m_B + m_{D^*}} \epsilon_{\alpha\beta\gamma}^\mu \epsilon^{*\alpha} p^\beta q^\gamma V(q^2) \\ & \pm i \frac{2m_{D^*}}{q^2} (\epsilon^* \cdot q) q^\mu A_0(q^2) \\ & \mp \frac{i}{m_B - m_{D^*}} [(m_B - m_{D^*}) \epsilon^{*\mu} - (\epsilon^* \cdot q)(p + p_B)^\mu] A_1(q^2) \\ & \mp i \frac{2m_{D^*}}{q^2} (\epsilon^* \cdot q) \left[\frac{q^2}{m_B^2 - m_{D^*}^2} (p + p_B)^\mu - q^\mu \right] A_3(q^2) , \end{aligned} \quad (\text{A.1})$$

$$\bar{B} \rightarrow D^* \ell \nu_\ell$$

together with the identity: $2m_{D^*} A_3(q^2) = [(m_B + m_{D^*}) A_1(q^2) - (m_B - m_{D^*}) A_2(q^2)]$. Note also that $A_3(0) = A_0(0)$, so that eq. (A.1) is finite at the kinematic endpoint. The axial Ward identity yields a non-trivial matrix element for the pseudo-scalar current:

$$\langle D^*(p, \epsilon) | \bar{c} \gamma_5 b | \bar{B}(p_B) \rangle = -i \frac{2m_{D^*} (\epsilon^* \cdot q)}{m_c + m_b} A_0(q^2) , \quad (\text{A.2})$$

while the matrix element for the scalar current must identically vanish. For the sake of completeness, we note that tensor form factors can also be introduced in analogy to eq. (A.1):

$$\begin{aligned} i q_\nu \langle D^*(p, \epsilon) | \bar{c} \sigma^{\mu\nu} (1 \pm \gamma_5) b | \bar{B}(p_B) \rangle = & -\frac{2}{m_B + m_{D^*}} \epsilon_{\alpha\beta\gamma}^\mu \epsilon^{*\alpha} p^\beta q^\gamma T_1(q^2) \\ & \mp \frac{i}{m_B - m_{D^*}} [(m_B - m_{D^*}) \epsilon^{*\mu} - (\epsilon^* \cdot q)(p + p_B)^\mu] T_2(q^2) \\ & \mp \frac{i}{q^2} (\epsilon^* \cdot q) \left[\frac{q^2}{m_B^2 - m_{D^*}^2} (p + p_B)^\mu - q^\mu \right] \\ & \times [(m_B + m_{D^*}) T_1(q^2) - (m_B - m_{D^*}) T_2(q^2)] , \end{aligned} \quad (\text{A.3})$$

together with the algebraic relation: $T_1(0) = T_2(0)$. Large uncertainties in the knowledge of $T_{1,2}(q^2)$ may be presently at hand: so far no tensor contribution has been indeed considered beyond the HQET limit, see for instance [25, 39, 41, 42]. However, within the SM theory, contributions that may involve these form factors do not arise at dimension six

|V_{cb}| extraction from B → D* decay

- **Belle 2018/Belle-II 2023:** $d\Gamma/dx$

Belle Collaboration: PRD '19 [arXiv:1809.03290]

Belle-II Collaboration: PRD '23 [arXiv:2310.01170]

- **Belle 2023:** $(d\Gamma/dx)/\Gamma$

Belle Collaboration, PRD '23 [arXiv:2301.07529]

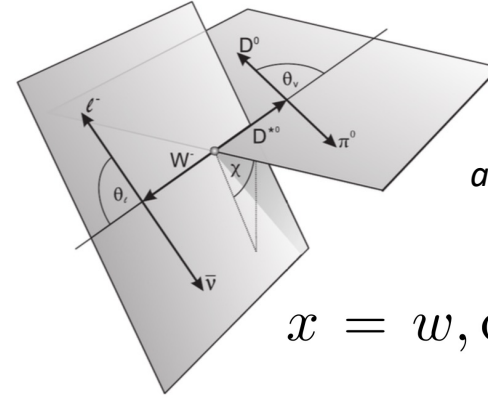


Image taken from
arXiv:1702.01521v2

$$x = w, \cos \theta_l, \cos \theta_v, \chi$$

Theoretically (massless lepton limit) :

$$\left\{ \begin{aligned} \frac{d^4\Gamma}{dw d\cos\theta_v d\cos\theta_\ell d\chi} &= \frac{3}{16\pi} \Gamma_0 \sqrt{w^2 - 1} (1 - 2rw + r^2) \left\{ \underline{H_+^2(w)} \sin^2\theta_v (1 - \cos\theta_\ell)^2 \right. \\ &+ \underline{H_-^2(w)} \sin^2\theta_v (1 + \cos\theta_\ell)^2 + 4 \underline{H_0^2(w)} \cos^2\theta_v \sin^2\theta_\ell \\ &- 2 \underline{H_-(w)} \underline{H_+(w)} \sin^2\theta_v \sin^2\theta_\ell \cos 2\chi \\ &- 2 \underline{H_+(w)} \underline{H_0(w)} \sin 2\theta_v \sin\theta_\ell (1 - \cos\theta_\ell) \cos\chi \\ &\left. + 2 \underline{H_-(w)} \underline{H_0(w)} \sin 2\theta_v \sin\theta_\ell (1 + \cos\theta_\ell) \cos\chi \right\} , \\ \Gamma_0 &\equiv \frac{\eta_{EW}^2 m_B m_{D^*}^2}{(4\pi)^3} G_F^2 |V_{cb}|^2 \end{aligned} \right.$$

$$\begin{aligned} H_+(w) &= f(w) - m_B^2 r \sqrt{w^2 - 1} g(w) , \\ H_-(w) &= f(w) + m_B^2 r \sqrt{w^2 - 1} g(w) , \\ H_0(w) &= \frac{F_1(w)}{m_B \sqrt{1 - 2rw + r^2}} , \end{aligned}$$

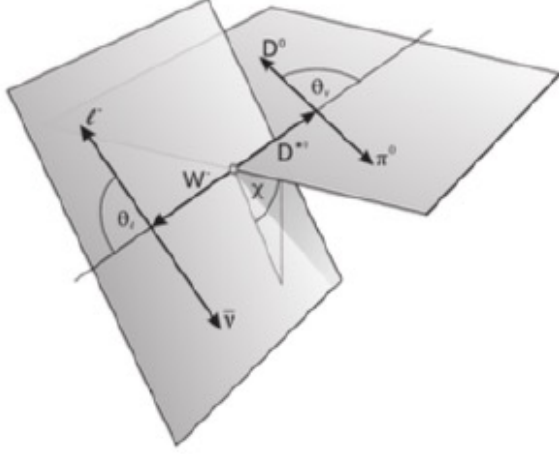


Figure 2.2. Representation of the decay $\bar{B}^0 \rightarrow D^* \bar{\ell} \bar{\nu}$. θ_ℓ is the angle between $\bar{\ell}$ and the direction opposite the $\bar{B}$ meson in the virtual W boson rest frame. θ_D is the angle between the D^* meson and the direction opposite the $\bar{B}$ meson in the D^* rest frame. χ is the angle between the two decay planes spanned by the $W - \bar{\ell}$ and $D^* - \pi^0$ in the $\bar{B}$ meson rest frame.

B.2 $B \rightarrow D^* \ell \nu$

The angular distribution of the differential decay rate of $B \rightarrow D^* \ell \nu$ can be also generally decomposed in terms of Wigner $\mathcal{D}$ -functions:

$$\frac{d^4\Gamma}{dq^2 d\cos\Theta_\ell d\cos\Theta_V d\chi} = \frac{3}{32\pi} \Re \left[G_0^{0,0}(q^2) \Omega_0^{0,0} + G_0^{0,1}(q^2) \Omega_0^{0,1} + G_0^{0,2}(q^2) \Omega_0^{0,2} + G_0^{2,0}(q^2) \Omega_0^{2,0} + G_0^{2,1}(q^2) \Omega_0^{2,1} + G_1^{2,1}(q^2) \Omega_1^{2,1} + G_0^{2,2}(q^2) \Omega_0^{2,2} + G_1^{2,2}(q^2) \Omega_1^{2,2} + G_2^{2,2}(q^2) \Omega_2^{2,2} \right], \quad (\text{B.10})$$

with $\Omega_m^{l_D, l_\ell} \equiv D_{m,0}^{l_D}(\Omega_D') D_{m,0}^{l_\ell}(\Omega_\ell')$. Here Ω_ℓ' and Ω_D' refer to the angular coordinates of the leptonic and hadronic decay components of B in its rest frame respectively as defined in [68]. The additional Wigner $\mathcal{D}$ -functions involved with respect to eq. (B.1) correspond to:

$$D_{1,0}^1 = -\frac{1}{\sqrt{2}} \exp^{-i\phi} \sin\theta, \quad D_{2,0}^2 = \sqrt{\frac{3}{8}} \exp^{-2i\phi} \sin^2\theta, \quad \text{and} \quad D_{1,0}^2 = -\sqrt{\frac{3}{8}} \exp^{-i\phi} \sin 2\theta. \quad (\text{B.11})$$

The functions $G_k^{i,j} = \mathcal{N}_{D^*} \tilde{G}_k^{i,j}$ can be written as:

$$\begin{aligned} \tilde{G}_0^{0,0} &= \frac{4}{9} \left(3E_\ell E_\nu + \frac{\lambda_{\gamma^*}}{4q^2} \right) \left(|H_+^V|^2 + |H_-^V|^2 + |H_0^V|^2 + (V \rightarrow A) \right) \\ &\quad + \frac{4}{3} \left(E_\ell E_\nu + \frac{\lambda_{\gamma^*}}{4q^2} \right) |H^S|^2 + \frac{4}{3} \left(E_\ell E_\nu + \frac{\lambda_{\gamma^*}}{4q^2} \right) |H^P|^2 \\ \tilde{G}_0^{0,1} &= \frac{4\sqrt{\lambda_{\gamma^*}}}{3} \left(\Re [H_+^V \bar{H}_+^A - H_-^V \bar{H}_-^A] - \frac{m_\ell}{\sqrt{q^2}} \Re [H_0^A \bar{H}^P] - \frac{m_\ell}{\sqrt{q^2}} \Re [H_0^V \bar{H}^S] \right), \\ \tilde{G}_0^{0,2} &= -\frac{2\lambda_{\gamma^*}}{9q^2} \left(2|H_0^V|^2 - |H_+^V|^2 - |H_-^V|^2 + (V \rightarrow A) \right) \\ \tilde{G}_0^{2,0} &= -\frac{4}{9} \left(3E_\ell E_\nu + \frac{\lambda_{\gamma^*}}{4q^2} \right) \left(|H_+^V|^2 + |H_-^V|^2 - 2|H_0^V|^2 + (V \rightarrow A) \right) \\ &\quad + \frac{8}{3} \left(E_\ell E_\nu + \frac{\lambda_{\gamma^*}}{4q^2} \right) |H^S|^2 + \frac{8}{3} \left(E_\ell E_\nu + \frac{\lambda_{\gamma^*}}{4q^2} \right) |H^P|^2 \\ \tilde{G}_0^{2,1} &= -\frac{4\sqrt{\lambda_{\gamma^*}}}{3} \left(\Re [H_+^V \bar{H}_+^A - H_-^V \bar{H}_-^A] + 2\frac{m_\ell}{\sqrt{q^2}} \Re [H_0^A \bar{H}^P] + 2\frac{m_\ell}{\sqrt{q^2}} \Re [H_0^V \bar{H}^S] \right), \end{aligned}$$

$$\begin{aligned}
\tilde{G}_0^{2,2} &= -\frac{2}{9} \frac{\lambda_{\gamma^*}}{q^2} \left(4 |H_0^V|^2 + |H_+^V|^2 + |H_-^V|^2 + (V \rightarrow A) \right) , \\
\tilde{G}_1^{2,1} &= \frac{4\sqrt{\lambda_{\gamma^*}}}{\sqrt{3}} \left((H_+^V \bar{H}_0^A + H_+^A \bar{H}_0^V - H_0^V \bar{H}_-^A - H_0^A \bar{H}_-^V) + \frac{m_\ell}{\sqrt{q^2}} (H_+^V \bar{H}^S + H^S \bar{H}_-^V) \right. \\
&\quad \left. + \frac{m_\ell}{\sqrt{q^2}} (H_+^A \bar{H}^P + H^P \bar{H}_-^A) \right) , \\
\tilde{G}_1^{2,2} &= \frac{4}{3} \frac{\lambda_{\gamma^*}}{q^2} (H_+^V \bar{H}_0^V + H_0^V \bar{H}_-^V + (V \rightarrow A)) , \\
\tilde{G}_2^{2,2} &= -\frac{8}{3} \frac{\lambda_{\gamma^*}}{q^2} (H_+^V \bar{H}_-^V m + H_+^A \bar{H}_-^A) . \tag{B.12}
\end{aligned}$$

The hadronic helicity amplitudes of interest H^i , ($i = V, A, S, P$) are given by:

$$\begin{aligned}
H_0^V &= \frac{4im_B m_{D^*}}{\sqrt{q^2}} C_V A_{12} , \\
H_0^A &= \frac{4im_B m_{D^*}}{\sqrt{q^2}} C_A A_{12} , \\
H_\pm^V &= \frac{i}{2(m_B + m_{D^*})} \left(\pm \sqrt{\lambda_B} V - (m_B + m_{D^*})^2 A_1 \right) C_V \\
H_\pm^A &= \frac{i}{2(m_B + m_{D^*})} \left(\pm \sqrt{\lambda_B} V - (m_B + m_{D^*})^2 A_1 \right) C_A , \\
H^P &= \frac{i\sqrt{\lambda_B}}{2} \frac{m_\ell}{q^2} C_A A_0 , \\
H^S &= \frac{i\sqrt{\lambda_B}}{2} \frac{m_\ell}{q^2} C_V A_0 . \tag{B.13}
\end{aligned}$$

with the following definition involved

$$A_{12} = \frac{(m_B + m_{D^*})^2 (m_B^2 - m_{D^*}^2 - q^2) A_1 - \lambda_B A_2}{16m_B m_{D^*}^2 (m_B + m_{D^*})} , \tag{B.14}$$

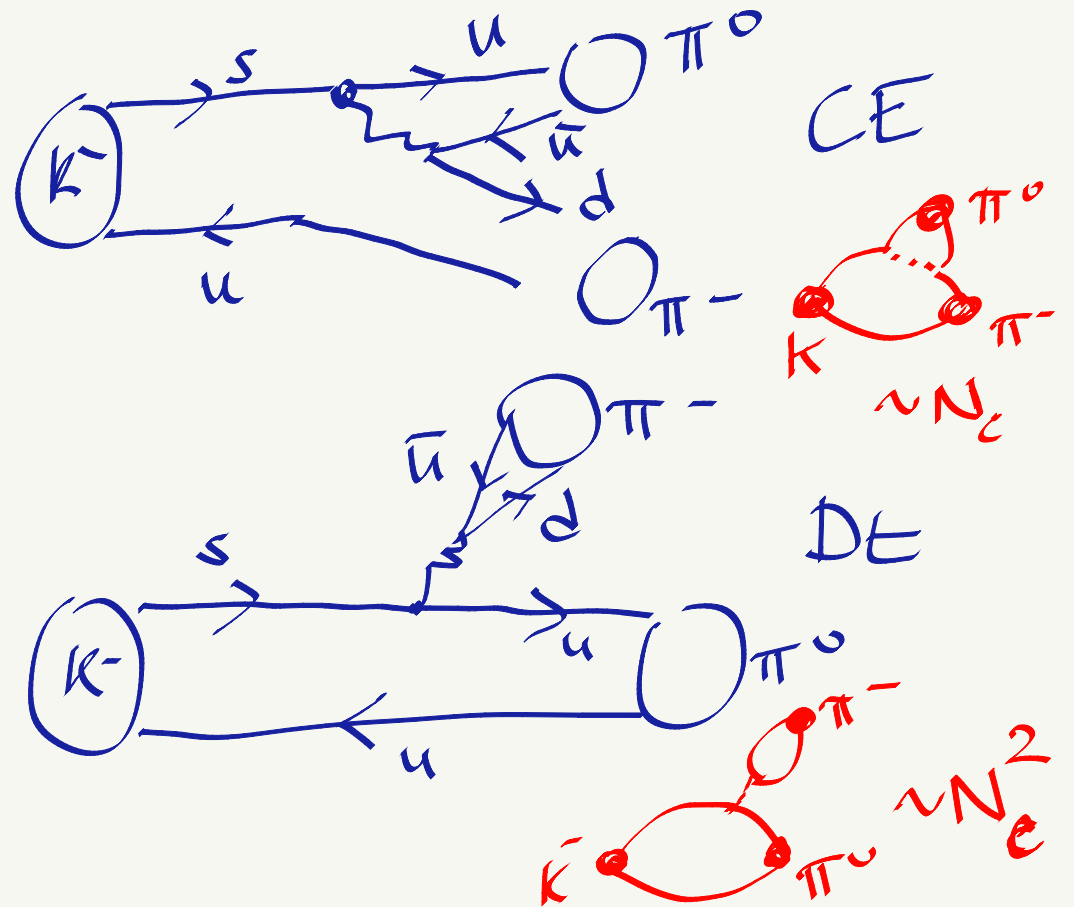
$$\tag{B.15}$$

Non-leptonic Decays

Penguins contractions and all that

$$K^- \rightarrow \pi^- \pi^0$$

$$H_W = -\frac{G_F}{\sqrt{2}} V_{us} V_{ud}^* \bar{u} \gamma^\mu (1-\gamma_5) s \bar{d} \gamma_\mu (1-\gamma_5) u$$

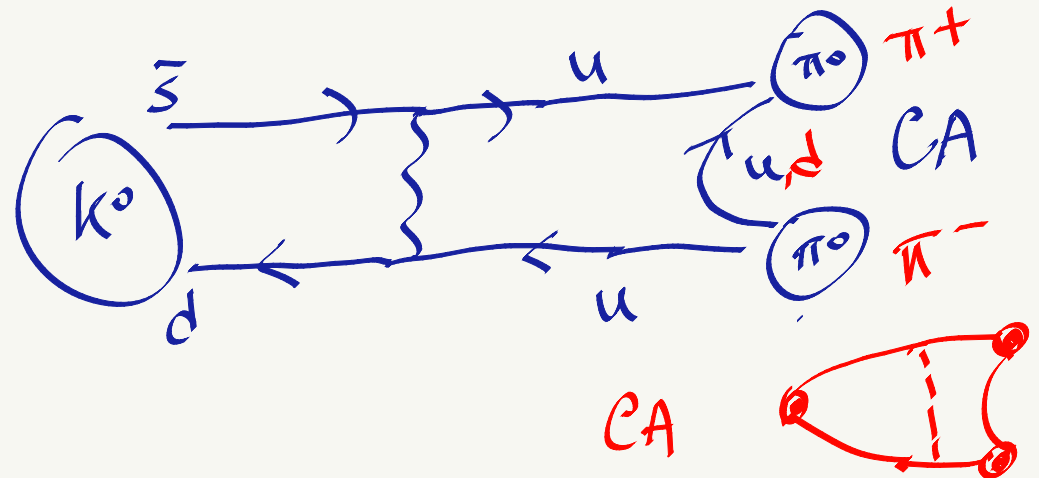
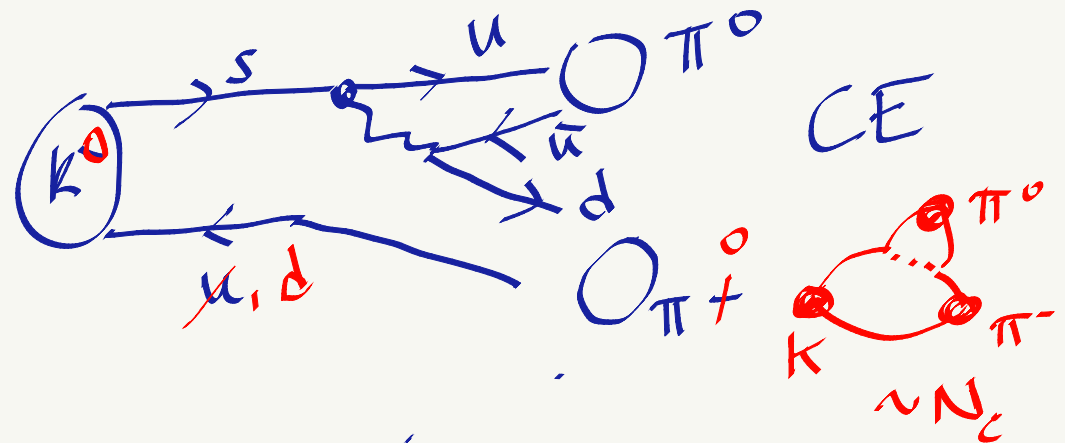


Non-leptonic Decays

Penguins contractions and all that

$$K^0 \rightarrow \pi^+ \pi^-$$

$$H_W = -\frac{G_F}{\sqrt{2}} V_{us} V_{ud}^* \bar{u} \gamma^\mu (1-\gamma_5) s \bar{d} \gamma_\mu (1-\gamma_5) u$$

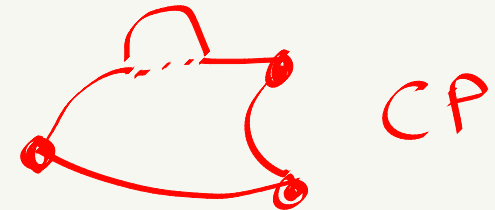
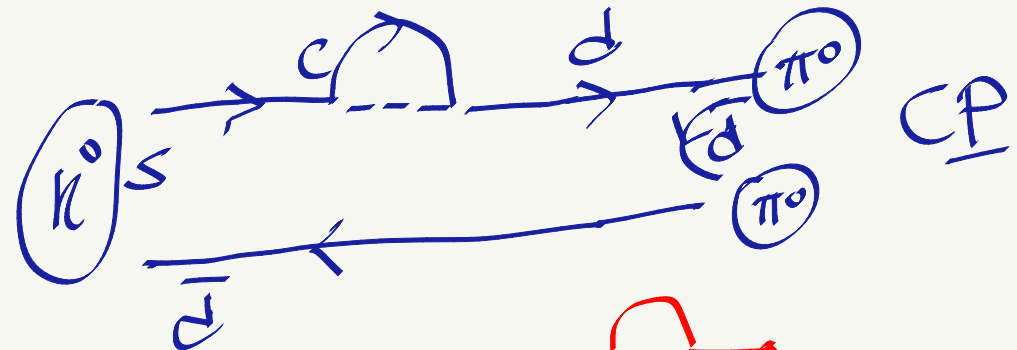


Non-leptonic Decays

Penguins contractions and all that

Penguin diagrams

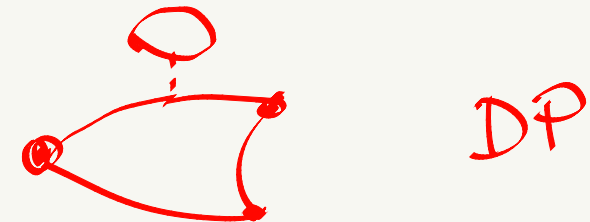
$$H = -\frac{G_F}{\sqrt{2}} V_{cs} V_{cd}^* \bar{c} \gamma_\mu (1 - \gamma_5) s \bar{d} \gamma^\mu (1 - \gamma_5) c$$



other ops

$$\sim V_{cs} V_{cd}^*$$

$$\bar{d} \gamma_\mu (1 - \gamma_5) s \bar{c} \gamma^\mu (1 - \gamma_5) c$$



All Topologies

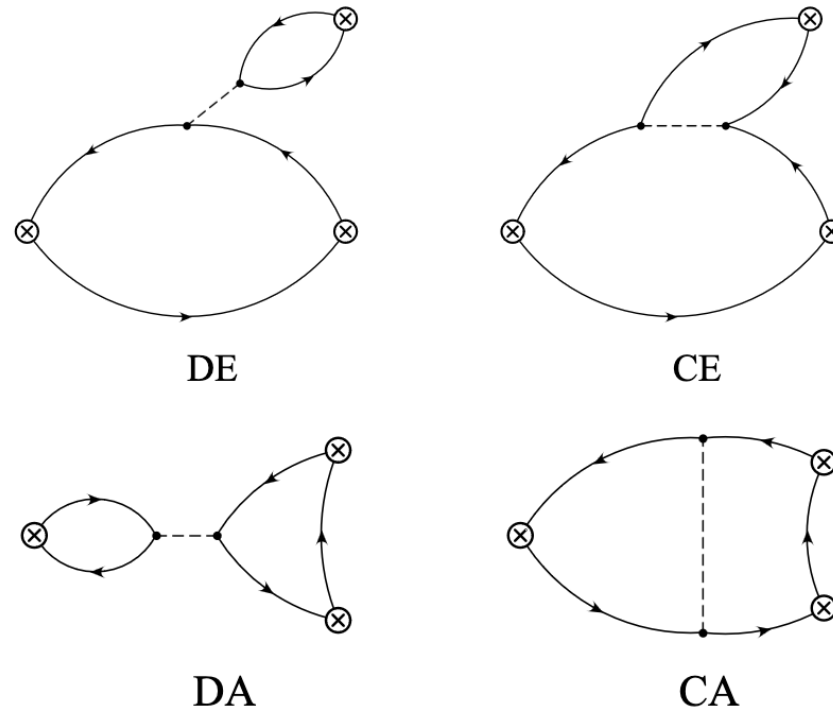
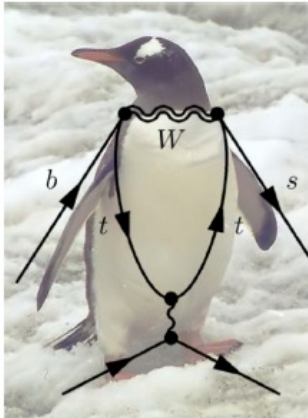
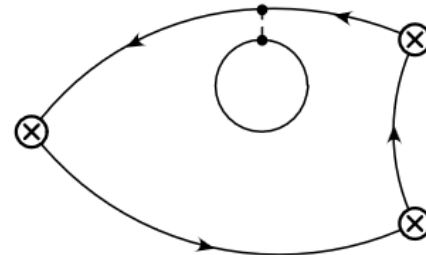


Figure 1: *Non-penguin diagrams. The dashed line represents the four-fermion operator.*

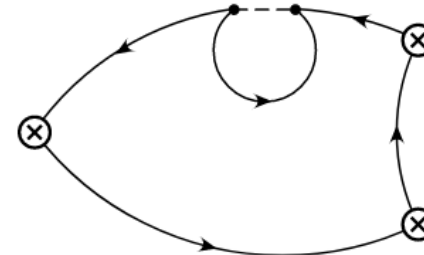
All Topologies



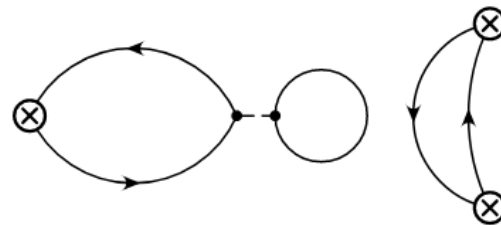
D. Kovalskyi's seminar © CERN (26/7/22)



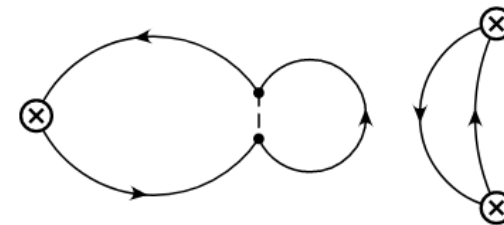
DP



CP



DPA



CPA

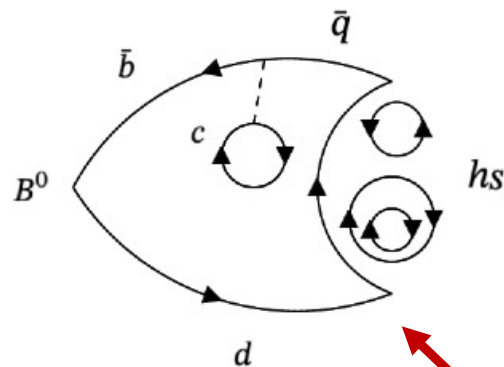


Figure 2: *Penguin diagrams.*

for heavy mesons many particles in the final state

Flavor Changing Neutral Currents

Rare Penguin Radiative Decays

The **main issue** in this Group Meeting will be represented by $b \rightarrow s$ quark transitions: the best example is offered by the **neutral-current semileptonic** $B \rightarrow K^{(*)} l^+ l^-$ transitions!

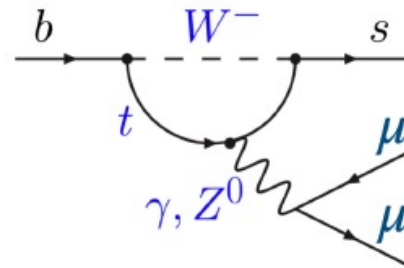
Many interesting properties:

1. **Loop-level processes** (FCNCs are forbidden at tree level in the Standard Model)
2. **CKM-suppressed decays**, where

$$J_{\text{charged}}^\mu = \bar{u}_L^i \boxed{V_{CKM}^{ij}} \gamma^\mu d_L^j + \bar{\nu}_L^i \gamma^\mu \ell_L^i$$



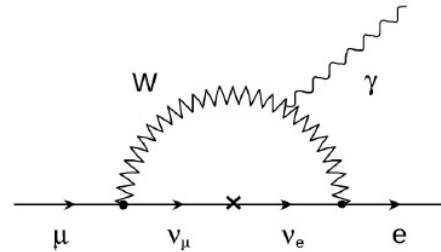
Rare transitions!



$$B^+ \rightarrow K^{(*)+} \gamma$$

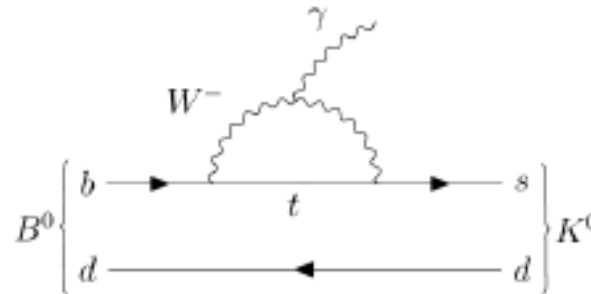
$$B^+ \rightarrow K^{(*)+} \mu^+ \mu^-$$

since different neutrinos have a mass and they can mix,
 $\mu \rightarrow e\gamma$ is a possible decay which satisfies all the symmetry constraints



$$\mathcal{B}(\mu \rightarrow e\gamma) \sim \alpha \frac{m_\nu^4}{m_W^4} \sim 10^{-52}$$

note that the photon is emitted by
the W boson, analogy radiative B decays



$$\bar{B}^0 \quad \bar{K}^0$$

Figura 4: quark process

Radiative Penguins

PENGUINS AND BOXES

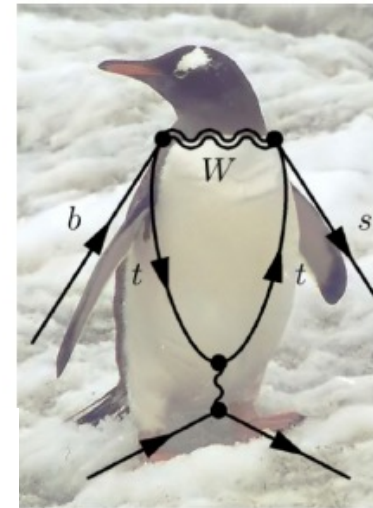
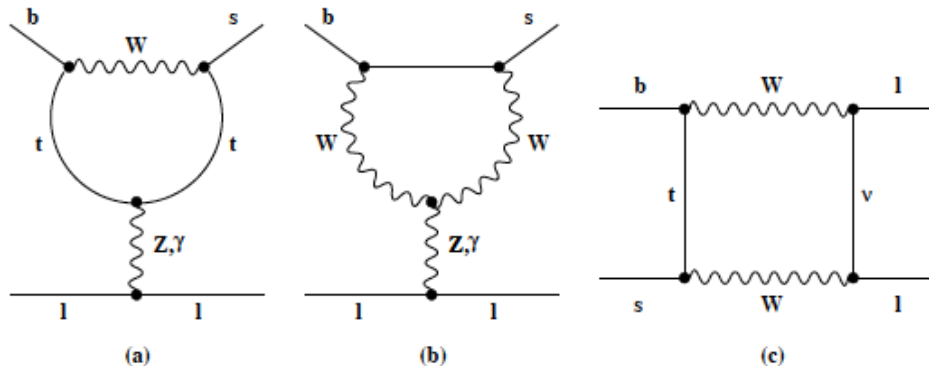
Pure leptonic B_s decays

$$Br(B_s \rightarrow l^+ l^-) = \tau(B_s) \frac{G_F^2}{\pi} \left(\frac{\alpha}{4\pi \sin^2 \Theta_W} \right)^2 \boxed{F_{B_s}^2} m_l^2 m_{B_s} \sqrt{1 - 4 \frac{m_l^2}{m_{B_s}^2}} |V_{tb}^* V_{ts}|^2 Y^2(x_t)$$

G. Buchalla and A.J. Buras, Nucl. Phys. B 400 (1993) 225.

Many interesting properties:

1. **Helicity suppressed**
2. **Non-perturbative hadronic contributions enter via B_s decay constant**



valskeyi's seminar @ CERN (26/7/22)

Lowest order
diagrams
QCD corrections at
NLO or NNLO

BOXES

Mixing of Neutral Mesons

$$K^0 \leftrightarrow \bar{K}^0$$

$$D^0 \leftrightarrow \bar{D}^0$$

$$B^0 \leftrightarrow \bar{B}^0$$

in the case of kaons

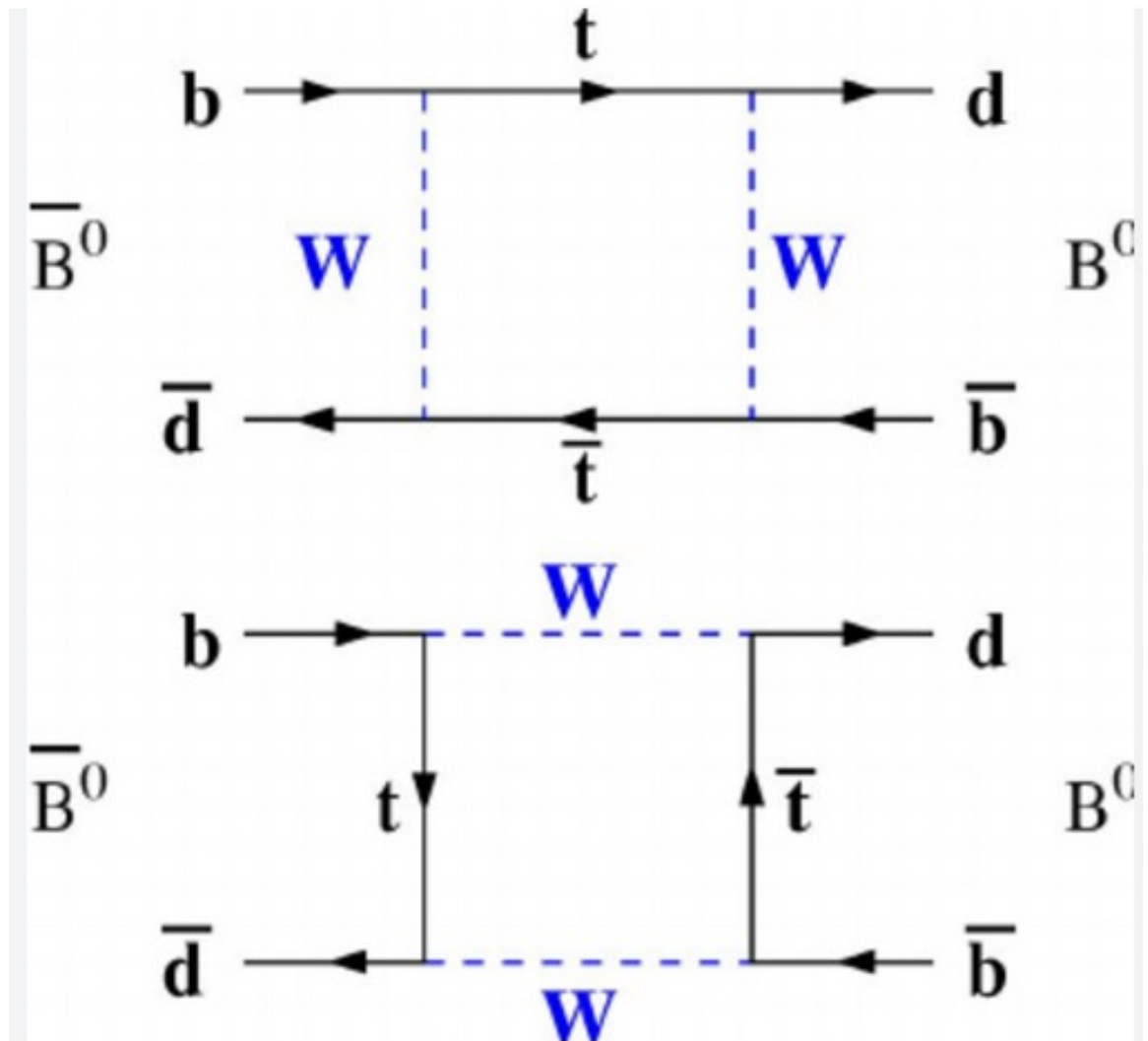
also charm and up quarks

contribute

for D and K meson mixing

there are important long

distance contributions

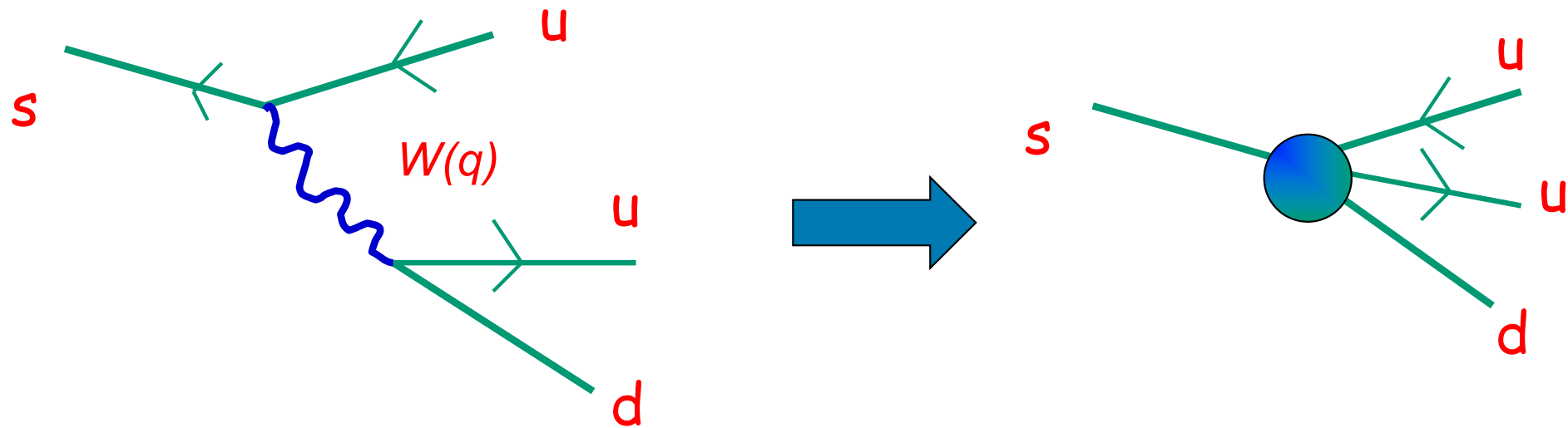


$$\mathcal{H}_{\text{eff}}^{\Delta B=2} = \frac{G_F^2}{16\pi^2} M_W^2 (V_{tb}^* V_{td})^2 \eta_B S_0(x_t) \times$$

$$\times \left[\alpha_s^{(5)}(\mu_b) \right]^{-6/23} \left[1 + \frac{\alpha_s^{(5)}(\mu_b)}{4\pi} J_5 \right] Q^q(\Delta B = 2) + h.c.$$

QCD corrections

The Effective Hamiltonian, Wilson OPE and QCD Corrections



$$q \sim m_K \ll M_W$$

$$\mathcal{H}_{eff} = -\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* (\bar{s} \gamma_\mu (1 - \gamma_5) u) (\bar{u} \gamma^\mu (1 - \gamma_5) d)$$

GENERAL FRAMEWORK: THE OPE

$$A_{FI} (2\pi^4) \delta^4 (p_F - p_I) = \int d^4x d^4y D_{\mu\nu}(x, M_W) \langle F | T[J_\mu(y+x/2) J_\nu^\dagger(y-x/2)] | I \rangle$$


$$\langle F | H^{\Delta S=1} | I \rangle =$$

$$G_F / \sqrt{2} V_{ud} V_{us}^* \frac{\sum_i C_i(\mu) \langle F | Q_i(\mu) | I \rangle}{(M_W)^{di-6}}$$

di= dimension of the operator $Q_i(\mu)$

$C_i(\mu)$ Wilson coefficient: it depends on M_W/μ and $\alpha_W(\mu)$

$Q_i(\mu)$ local operator renormalized at the scale μ

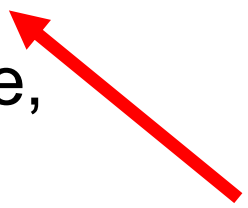
GENERAL FRAMEWORK

$$H^{\Delta S=1} = G_F/\sqrt{2} V_{ud} V_{us}^* \left[(1-\tau) \sum_{i=1,2} z_i (Q_i - Q_i^c) + \tau \sum_{i=1,10} (z_i + y_i) Q_i \right]$$

Where y_i and z_i are short distance coefficients, which are known in perturbation theory at the NLO (Buras et al. + Ciuchini et al.)

$$\tau = -V_{ts}^* V_{td} / V_{us}^* V_{ud}$$

We have to compute $A^{I=0,2}_i = \langle (\pi \pi)_{I=0,2} | Q_i | K \rangle$ with a non perturbative technique (lattice, QCD sum rules, 1/N expansion etc.)



$$A_0 = \sum_i C_i(\mu) \langle (\pi \pi) | Q_i(\mu) | K \rangle_{I=0} (1 - \Omega_{IB})$$

μ = renormalization scale
 μ -dependence cancels if operator
 matrix elements are consistently
 computed

ISOSPIN
BREAKING

$$A_2 = \sum_i C_i(\mu) \langle (\pi \pi) | Q_i(\mu) | K \rangle_{I=2}$$

$\Omega_{IB} = 0.25 \pm 0.08$ (Munich from Buras & Gerard)

0.25 ± 0.15 (Rome Group) 0.16 ± 0.03 (Ecker et al.)

0.10 ± 0.20 Gardner & Valencia, Maltman & Wolf, Cirigliano & al.

$$A^{I=0,2}_i(\mu) = \langle (\pi \pi)_{I=0,2} | Q_i(\mu) | K \rangle$$

$$= Z_{ik}(\mu a) \langle (\pi \pi)_{I=0,2} | Q_k(a) | K \rangle$$

Where $Q_i(a)$ is the bare lattice operator
And a the lattice spacing.

The effective Hamiltonian can then be read as:

$$\langle F | H^{\Delta S=1} | I \rangle = G_F / \sqrt{2} V_{ud} V_{us}^* \sum_i C_i(1/a) \langle F | Q_i(a) | I \rangle$$

In practice the renormalization scale (or $1/a$) are the scales which separate short and long distance dynamics

GENERAL FRAMEWORK

$$\langle H^{\Delta S=1} \rangle = G_F / \sqrt{2} V_{ud} V_{us}^* \dots \sum_i C_i(\mathbf{a}) \langle Q_i(\mathbf{a}) \rangle$$

$$M_W = 100 \text{ GeV}$$

perturbative regime

Effective Theory - quark & gluons

$$a^{-1} = 2\text{-}5 \text{ GeV}$$

Hadronic non-perturbative region

Chiral regime

$$\Lambda_{\text{QCD}}, M_K = 0.2\text{-}0.5 \text{ GeV}$$

100 GeV

perturbative region

Large mass scale: heavy degrees of freedom (m_t , M_W , M_S) are removed and their effect included in the Wilson coefficients

1-2 GeV

non-perturbative region

renormalization scale μ (inverse lattice spacing $1/a$); this is the scale where the quark theory is matched to the effective hadronic theory

Scale of the low energy process
 $\Lambda \sim M_W$

THE SCALE PROBLEM:

Effective theories prefer low scales,
Perturbation Theory prefers large scales

if the scale μ is too low
problems from higher dimensional operators

(Cirigliano, Donoghue, Golowich)

- it is illusory to think that the problem is solved by using dimensional regularization

on the lattice this problem is called
DISCRETIZATION ERRORS

(reduced by using improved actions and/or scales $\mu > 2\text{-}4 \text{ GeV}$)

New local four-fermion operators are generated

$$Q_1 = (\bar{s}_L^A \gamma_\mu u_L^B) (\bar{u}_L^B \gamma_\mu d_L^A) \quad \text{Current-Current}$$

$$Q_2 = (\bar{s}_L^A \gamma_\mu u_L^A) (\bar{u}_L^B \gamma_\mu d_L^B)$$

$$Q_{3,5} = (\bar{s}_R^A \gamma_\mu d_L^A) \sum_q (\bar{q}_{L,R}^B \gamma_\mu q_{L,R}^B) \quad \text{Gluon}$$

$$Q_{4,6} = (\bar{s}_R^A \gamma_\mu d_L^B) \sum_q (\bar{q}_{L,R}^B \gamma_\mu q_{L,R}^A) \quad \text{Penguins}$$

$$Q_{7,9} = 3/2 (\bar{s}_R^A \gamma_\mu d_L^A) \sum_q e_q (\bar{q}_{R,L}^B \gamma_\mu q_{R,L}^B) \quad \text{Electroweak}$$

$$Q_{8,10} = 3/2 (\bar{s}_R^A \gamma_\mu d_L^B) \sum_q e_q (\bar{q}_{R,L}^B \gamma_\mu q_{R,L}^A) \quad \text{Penguins}$$

+ Chromomagnetic and electromagnetic operators

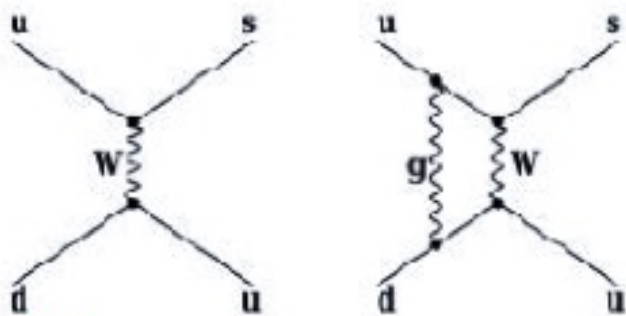
$$\mathcal{A}(K \rightarrow \pi\pi) = \sum_i C_W^i(\mu) \langle \pi\pi | O_i(\mu) | K \rangle$$

Weak Hamiltonian for $K \rightarrow \pi\pi$

Weak Hamiltonian is given by local four-quark operator *Courtesy by Xu Feng*

$$\mathcal{H}^{\Delta S=1} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \left\{ \sum_{i=1}^{10} [z_i(\mu) + \tau y_i(\mu)] Q_i \right\}, \quad \tau = -\frac{V_{td} V_{ts}^*}{V_{ud} V_{us}^*}$$

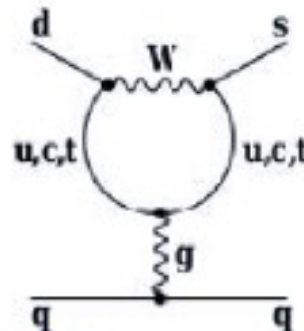
- $\tau = -\frac{V_{td} V_{ts}^*}{V_{ud} V_{us}^*} = 1.543 + 0.635i$
- $z_i(\mu)$ and $y_i(\mu)$ are perturbative Wilson coefficients
- Q_i are local four-quark operator



Current-current operator

Q_1, Q_2

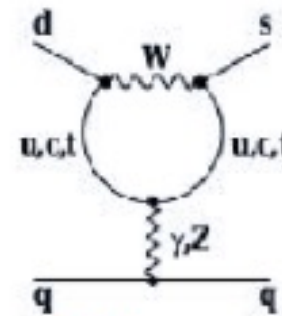
dominate $\text{Re}[A_0], \text{Re}[A_2]$



QCD penguin

$Q_3 - Q_6$

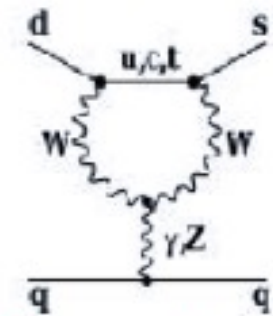
Q_6 dominate $\text{Im}[A_0]$



Electro-weak penguin

$Q_7 - Q_{10}$

Q_7, Q_8 dominate $\text{Im}[A_2]$



~~CP~~ Violation in the Neutral Kaon System

Expanding in several “small” quantities

$$\eta^{00} = \frac{\langle \pi^0 \pi^0 | H_W | K_L \rangle}{\langle \pi^0 \pi^0 | H_W | K_S \rangle} \sim \varepsilon - 2 \varepsilon'$$

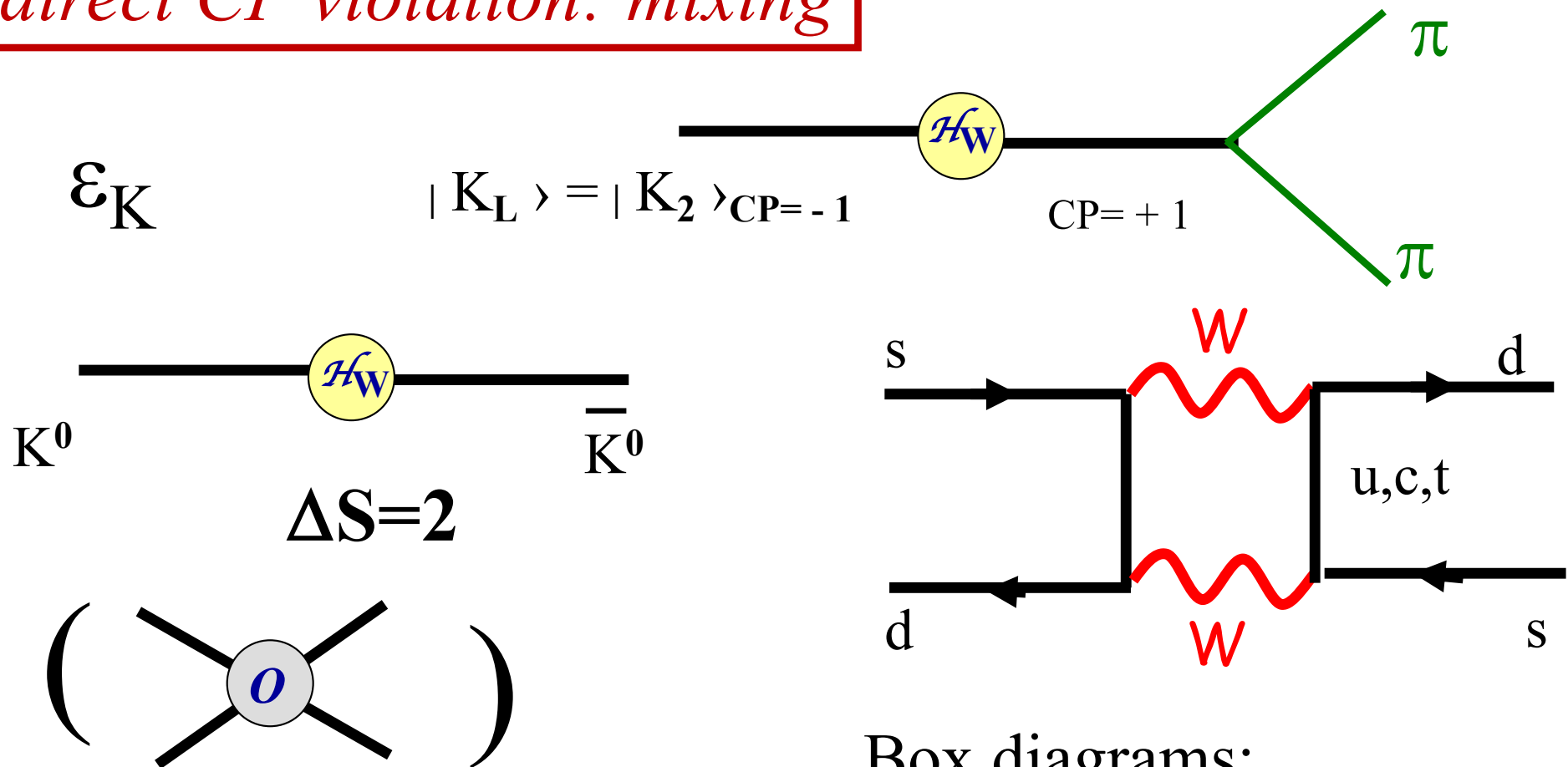
$$\eta^{+-} = \frac{\langle \pi^+ \pi^- | H_W | K_L \rangle}{\langle \pi^+ \pi^- | H_W | K_S \rangle} \sim \varepsilon + \varepsilon'$$

Conventionally:

$$| K_S \rangle = | K_1 \rangle_{CP=+1} + \varepsilon | K_2 \rangle_{CP=-1}$$

$$| K_L \rangle = | K_2 \rangle_{CP=-1} + \varepsilon | K_1 \rangle_{CP=+1}$$

Indirect CP violation: mixing



Complex $\Delta S=2$ effective coupling

Box diagrams:

They are also responsible
for $B^0 - \bar{B}^0$ mixing

$$\Delta m_{d,s}$$

$$| \varepsilon_K | \sim C_\varepsilon A^2 \lambda^6 \sigma \sin \delta$$

$$\frac{\{F(x_c, x_t) + F(x_t)[A^2 \lambda^4 (1 - \sigma \cos \delta)] - F(x_c)\}}{B_K}$$

$$\eta = \sigma \sin \delta \quad \rho = \sigma \cos \delta$$

*Inami-Lin
Functions + QCD
Corrections (NLO)*

$$C_\varepsilon = \frac{G_F^2 M_W^2 M_K f_K^2}{6 \sqrt{2} \pi^2 \Delta \mathcal{M}_K}$$

$$\langle \bar{K}^0 | (\bar{s} \gamma_\mu (1 - \gamma_5) d)^2 | K^0 \rangle = 8/3 f_K^2 M_K^2 B_K$$

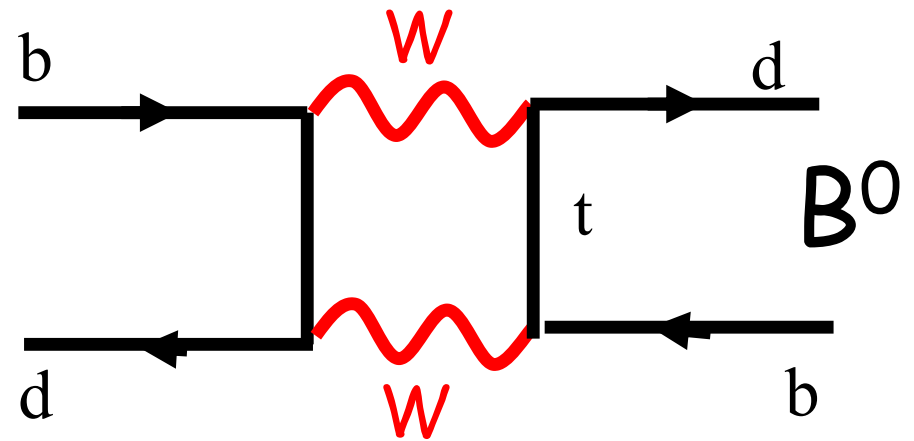
$B^0 - \bar{B}^0$ mixing

$$H = \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix}$$

$$\mathcal{H}_{eff}^{\Delta B=2} = \text{diagram with a circle labeled } O \text{ and four external lines}$$

$\bar{B}^0$

$\Delta B=2$ Transitions



$$\propto (\bar{d} \gamma_\mu (1 - \gamma_5) b)^2$$

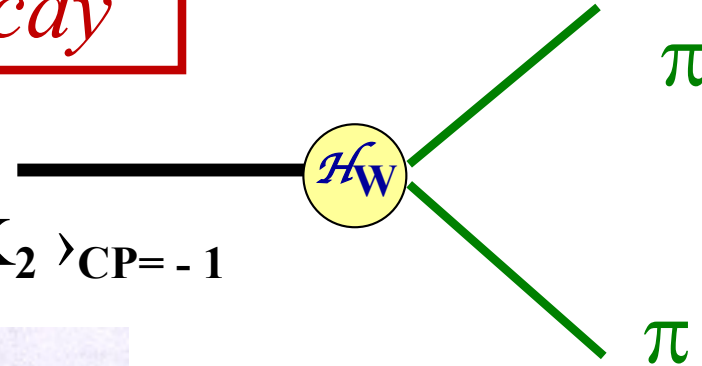
CKM

Hadronic
matrix
element

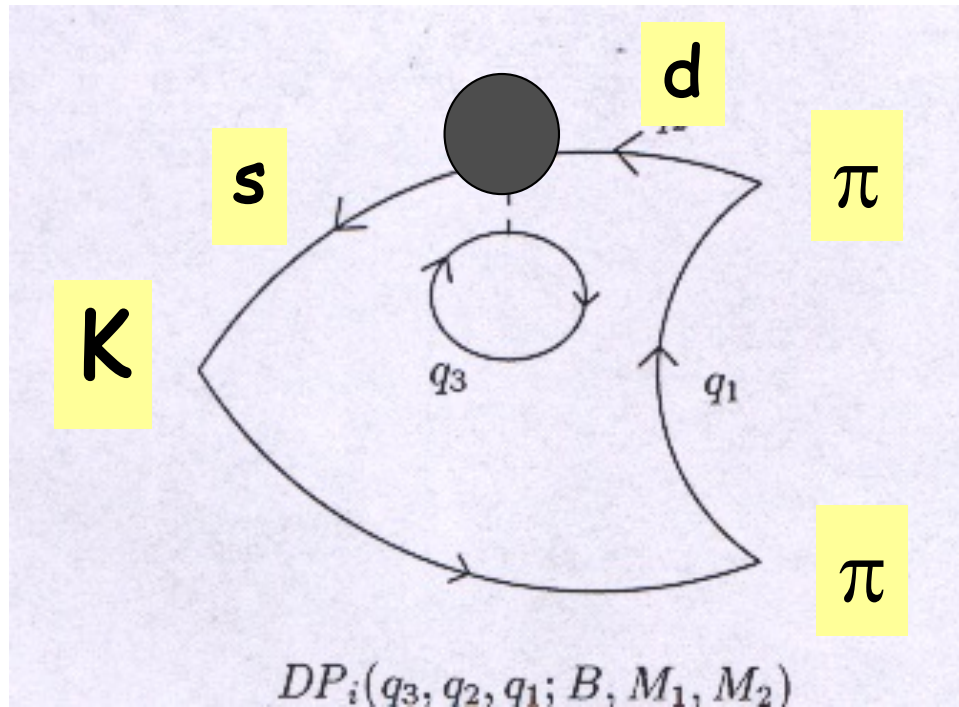
$$\Delta m_{d,s} = \frac{G_F^2 M_W^2}{16 \pi^2} A^2 \lambda^6 F_{tt} \left(\frac{m_t^2}{M_W^2} \right) \langle O \rangle$$

Direct CP violation: decay

$$|K_L\rangle = |K_2\rangle_{CP=-1}$$



$$CP = +1$$



Complex $\Delta S=1$ effective coupling

$$L^{\cancel{CP}} = L^{\Delta F=0} + L^{\Delta F=1} + L^{\Delta F=2}$$

$$\Delta F=0 \quad d_e < 1.5 \cdot 10^{-27} \text{ e cm} \quad d_N < 6.3 \cdot 10^{-26} \text{ e cm}$$

$$\Delta F=1 \quad \varepsilon' / \varepsilon$$

+ B decays (see later)

$$\Delta F=2 \quad \varepsilon \quad \text{and} \quad B \rightarrow J/\psi K_s$$

The extraordinary progress of the experimental measurements requires accurate theoretical predictions

Precision flavor physics requires the control of hadronic effects for which lattice QCD simulations are essential

$$Q^{EXP} = V_{CKM} \langle F | \hat{O} | I \rangle$$

SM

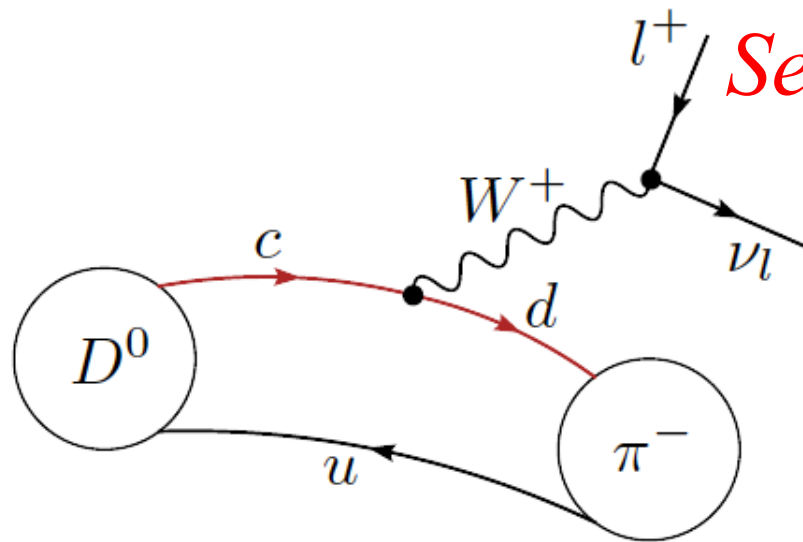
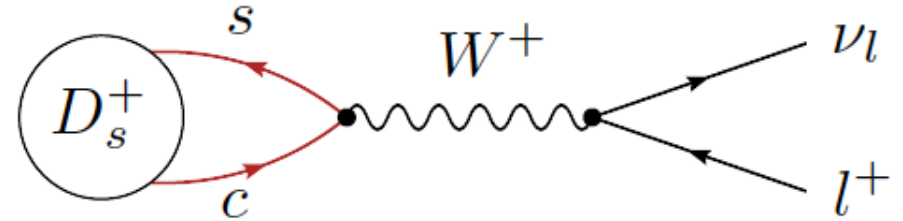
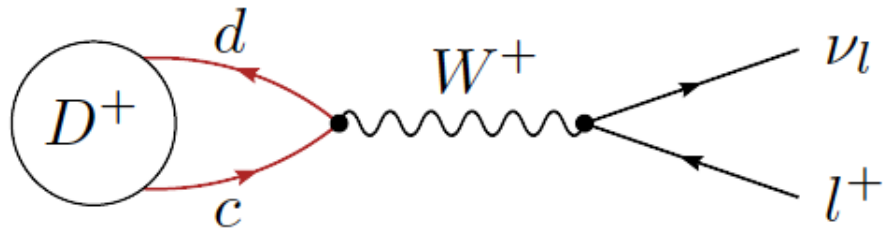
$$Q^{EXP} = \sum_i C_{SM}^i(M_W, m_t, \alpha_s) \langle F | \hat{O}_i | I \rangle + \sum_{i'} C_{Beyond}^{i'}(\tilde{m}_\beta, \alpha_s) \langle F | \hat{O}_{i'} | I \rangle$$

BSM

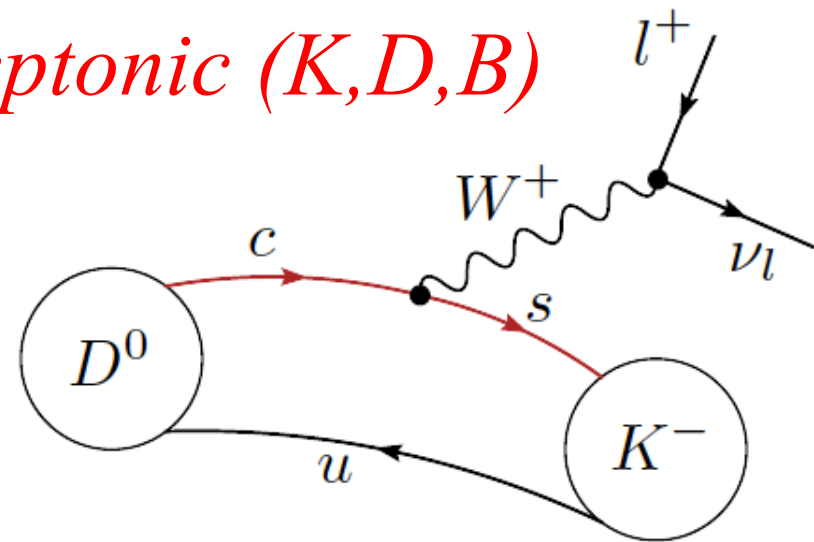
*What can be computed and
what cannot be computed*



Leptonic (π, K, D, B)

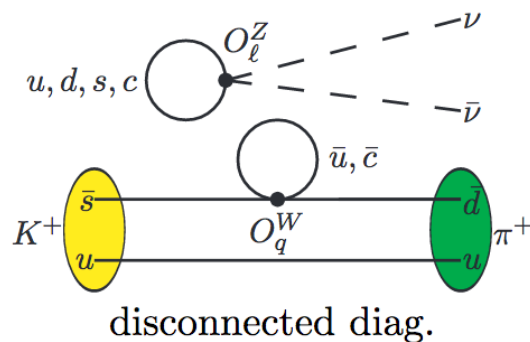
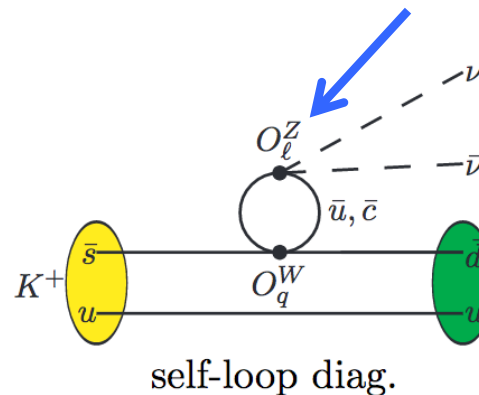
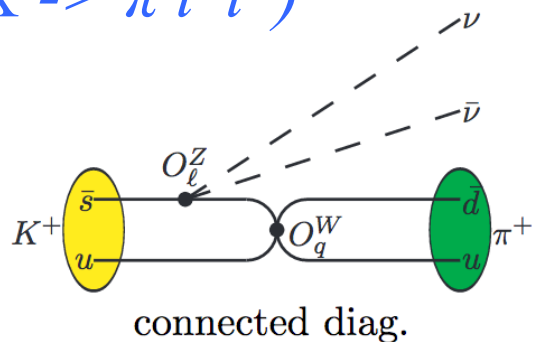


Semileptonic (K, D, B)



(some) Radiative and Rare
(also $K \rightarrow \pi l^+ l^-$)

long distance effects

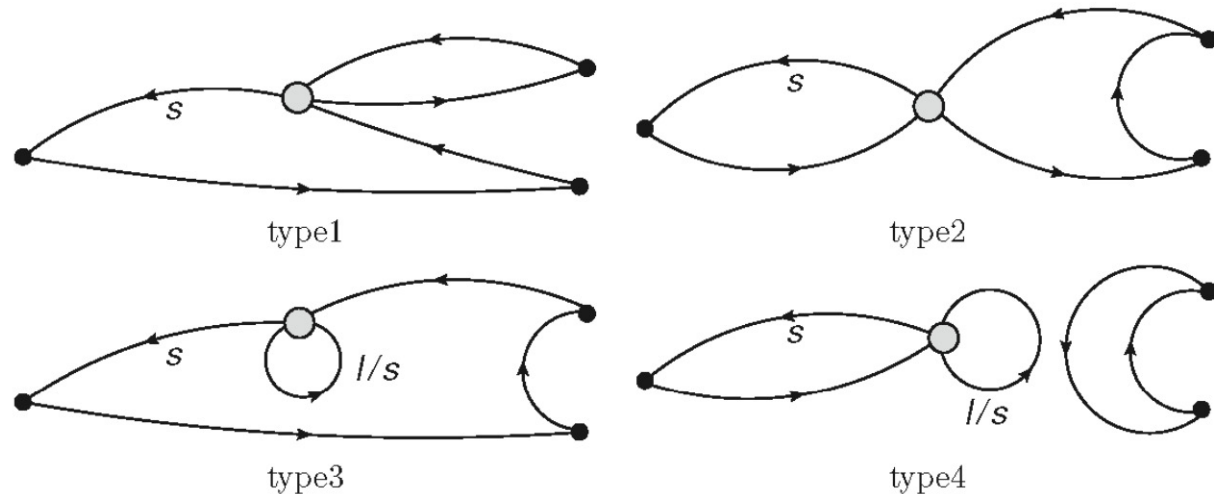


Non-leptonic

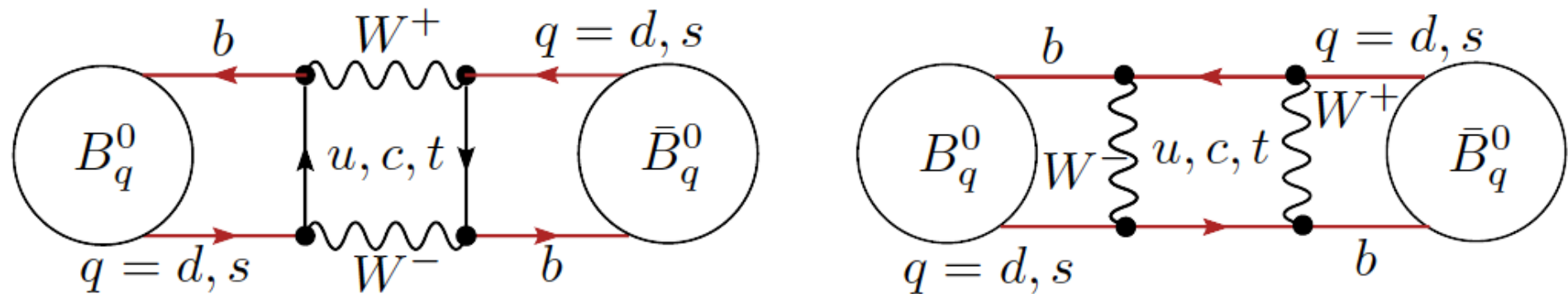
but only below the inelastic threshold

(may be also 3 body decays) now changing

$B \rightarrow \pi\pi, K\pi$, etc. No May be!



Neutral meson mixing (local)



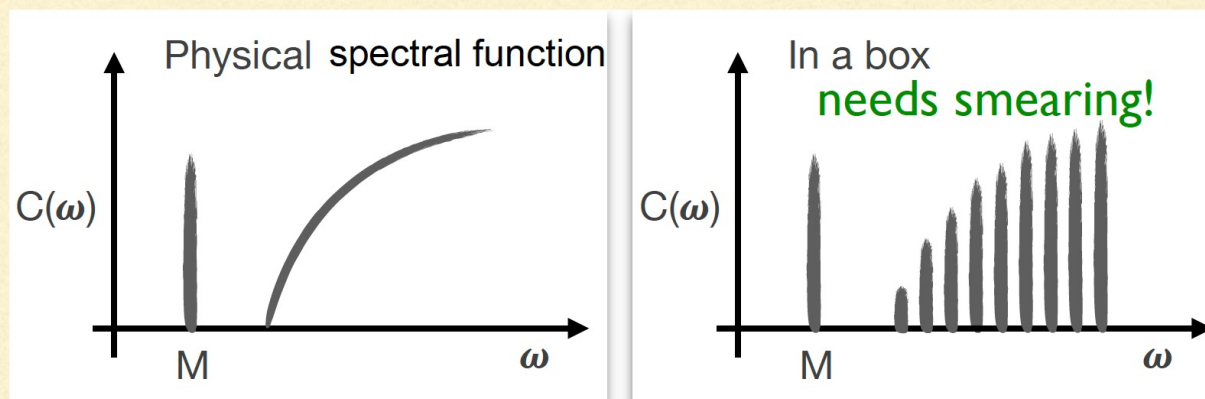
+ some long distance contributions to K and D neutral meson mixing + short distance contributions to $B \rightarrow K^{()} l^+ l^-$*

INCLUSIVE DECAYS ON THE LATTICE

Inclusive processes impractical to treat directly on the lattice. Vacuum current correlators computed in euclidean space-time are related to $e^+e^- \rightarrow$ hadrons or τ decay via analyticity. In our case the correlators have to be computed in the B meson, but analytic continuation more complicated: two cuts, decay occurs only on a portion of the physical cut.

While the lattice calculation of the spectral density of hadronic correlators is an **ill-posed problem**, the spectral density is accessible after smearing

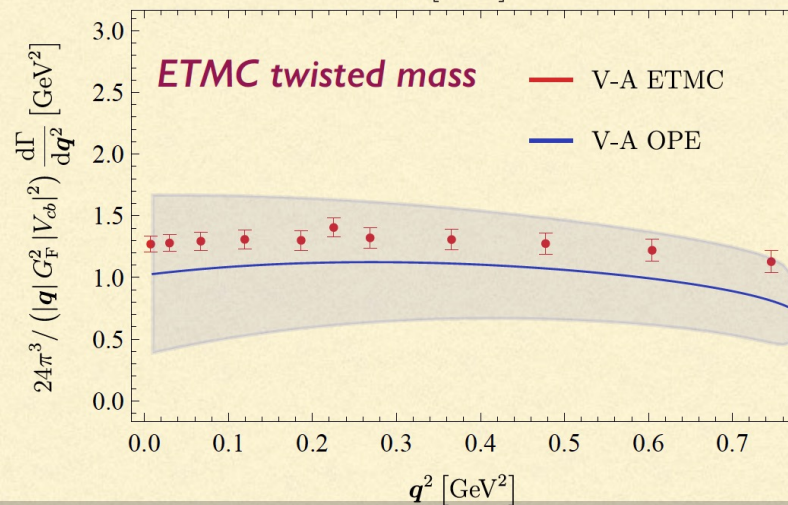
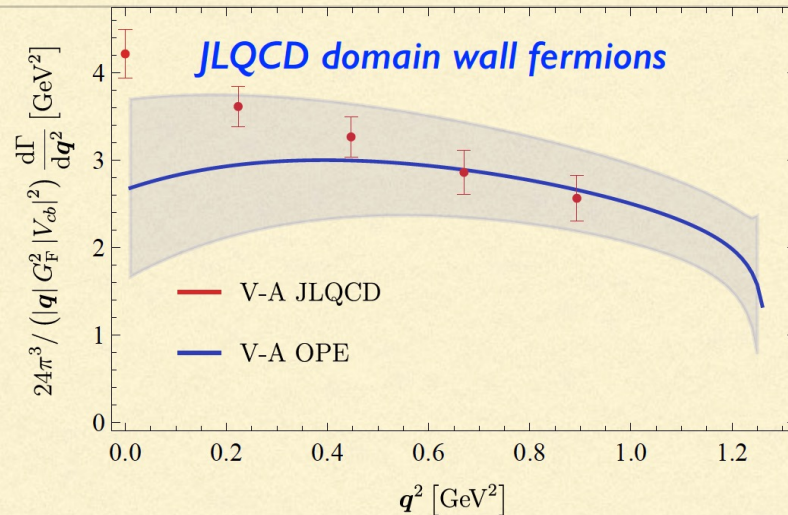
Hansen, Meyer, Robaina, Hansen, Lupo, Tantalò, Bailas, Hashimoto, Ishikawa



W. Jay @Snowmass
workshop

courtesy of P. Gambino

LATTICE vs OPE



m_b^{kin} (JLQCD)	2.70 ± 0.04
$\overline{m}_c(2 \text{ GeV})$ (JLQCD)	1.10 ± 0.02
m_b^{kin} (ETMC)	2.39 ± 0.08
$\overline{m}_c(2 \text{ GeV})$ (ETMC)	1.19 ± 0.04
μ_π^2	0.57 ± 0.15
ρ_D^3	0.22 ± 0.06
$\mu_G^2(m_b)$	0.37 ± 0.10
ρ_{LS}^3	-0.13 ± 0.10
$\alpha_s^{(4)}(2 \text{ GeV})$	0.301 ± 0.006

OPE inputs from fits to exp data (physical m_b), HQE of meson masses on lattice

1704.06105, J.Phys.Conf.Ser. 1137 (2019) 1, 012005

We include $O(1/m_b^3)$ and $O(\alpha_s)$ terms

Hard scale $\sqrt{m_c^2 + q^2} \sim 1 - 1.5 \text{ GeV}$

We do not expect OPE to work at high $|q|$

Twisted boundary conditions allow for any value of $\vec{q}^2$

Smaller statistical uncertainties

Evaluating hadronic amplitudes on the lattice through the spectral representation

Giuseppe Gagliardi, INFN Sezione di Roma Tre

R. Frezzotti, V. Lubicz, G. Martinelli, F. Mazzetti, C.T. Sachrajda,
F. Sanfilippo, S. Simula, N. Tantalo

ETMC meeting, 8-10 February 2023, Bern.

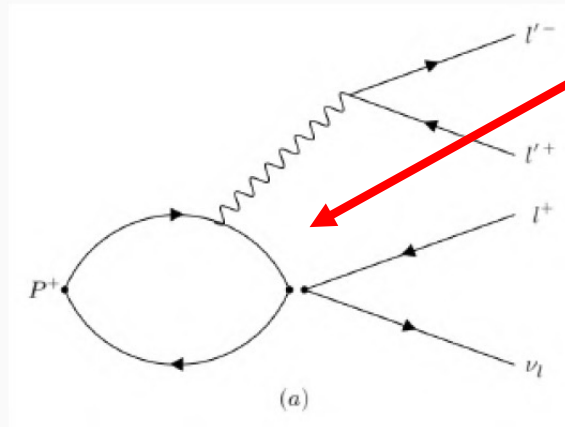


Gagliardi – Pisa

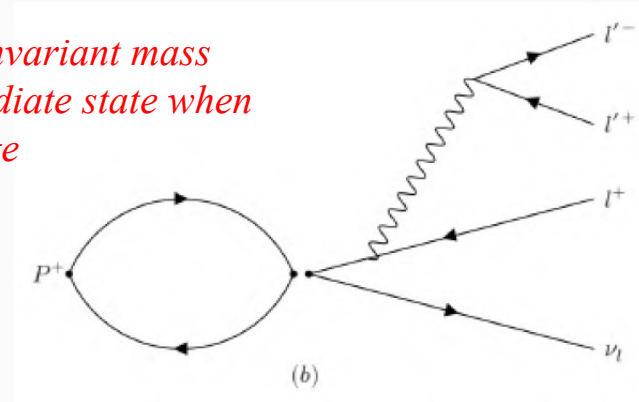
More at the IFIC Colloquium

Radiative decays (new paper also disconnected diagrams)

The $P^+ \equiv D\gamma^5 U \rightarrow l'^+ l'^- l^+ \nu_l$ decays



Large invariant mass intermediate state when q^2 large



- Diagram (b) is perturbative, only QCD input is **decay constant** f_P .
- Diagram (a) is **non-perturbative**. Virtual photon γ^* emitted from either a U -type or a D -type quark line. For $P^+ = D_s^+$: $U = c$, $D = s$.

Non-perturbative QCD contribution encoded in the **hadronic tensor**

$$H_W^{\mu\nu}(k, p) = \int d^4x e^{ik \cdot x} \langle 0 | T[J_{\text{em}}^\mu(x) J_W^\nu(0)] | P(p) \rangle, \quad W = V, A$$

- $k = (E_\gamma, \mathbf{k})$ is photon 4-momentum, p is P -meson 3-momentum.
- We **neglect** SU(3)-vanishing quark-line disconnected diagrams.

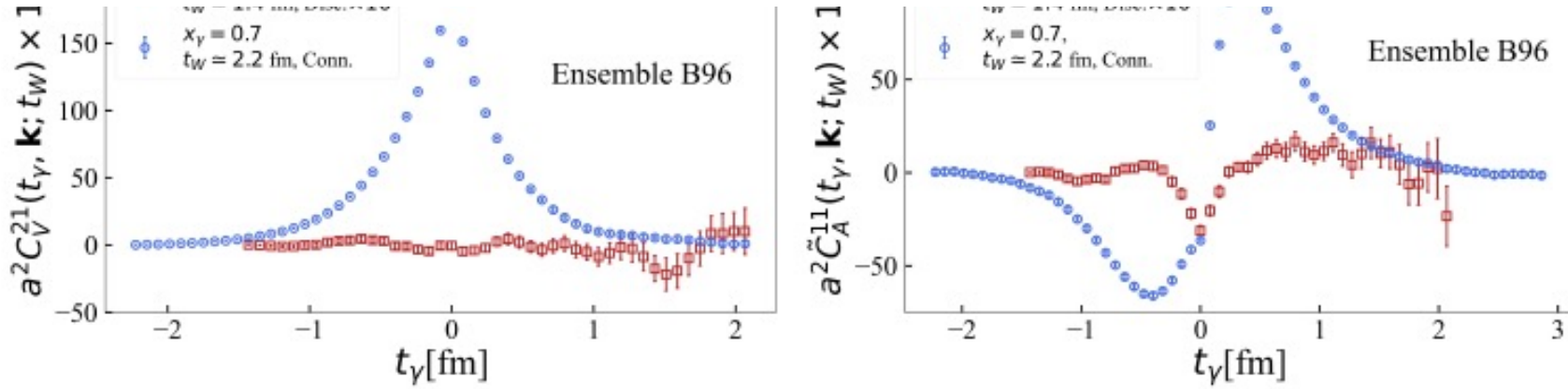
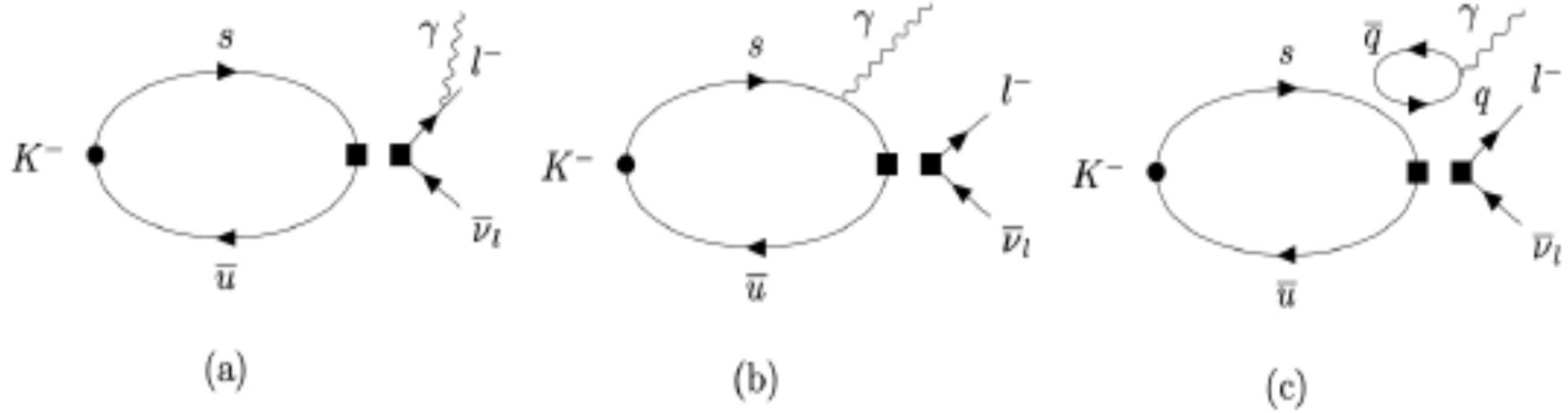


FIG. 4. Vector (left panel) and axial correlators (right panel) as a function of t_γ . In each of the two panel, we show both the quark-connected (blue circles) and the quark-disconnected (red squares) contributions. The quark-connected contributions shown in the figure correspond to photon emission by the up anti-quark. The results correspond to our determination on the 396 ensemble, with $x_\gamma \simeq 0.7$. The quark-disconnected contributions have been multiplied by a factor of 10 for visualization purposes.

The disconnected contribution is very small and negligible within the current uncertainties of the quark-connected contribution

$\sin 2\beta$ is measured directly from $B \rightarrow J/\psi K_s$ decays at Babar & Belle

$$A_{J/\psi K_s} = \frac{\Gamma(B_d^0 \rightarrow J/\psi K_s, t) - \bar{\Gamma}(B_d^0 \rightarrow J/\psi K_s, t)}{\Gamma(B_d^0 \rightarrow J/\psi K_s, t) + \bar{\Gamma}(B_d^0 \rightarrow J/\psi K_s, t)}$$

$$A_{J/\psi K_s} = \sin 2\beta \sin (\Delta m_d t)$$

DIFFERENT LEVELS OF THEORETICAL UNCERTAINTIES (STRONG INTERACTIONS)

- 1) First class quantities, with reduced or negligible uncertainties

$$A_{CP}(B \rightarrow J/\psi K_s) \quad \gamma \text{ from } B \rightarrow DK$$

$$K^0 \rightarrow \pi^0 \nu \bar{\nu}$$

- 2) Second class quantities, with theoretical errors of O(10%) or less that can be reliably estimated

$$\varepsilon_K, \Delta M_{d,s}$$

$$\Gamma(B \rightarrow c, u), K^+ \rightarrow \pi^+ \nu \bar{\nu}$$

- 3) Third class quantities, for which theoretical predictions are model dependent (BBNS, charming, etc.)

In case of discrepancies we cannot tell whether is new physics or
we must blame the model

$$B \rightarrow K \pi \quad B \rightarrow \pi^0 \pi^0$$

$$B \rightarrow \phi K_s$$

Flavour Physics

1963: *Cabibbo Angle*

1964: *CP violation in K decays **

1970 *GIM Mechanism*

1973: *CP Violation needs at least three quark families (CKM) **

1975: *discovery of the tau lepton – 3rd lepton family **

1977: *discovery of the b quark - 3rd quark family **

2003/4: *CP violation in B meson decays*

** Nobel Prize*



- ▶ the tiny branching ratio of the decay $K_L \rightarrow \mu^+ \mu^-$ led to the prediction of the charm quark to suppress FCNCs

(Glashow, Iliopoulos, Maiani 1970)

!!



- ▶ the measurement of the frequency of kaon anti-kaon oscillations allowed a successful prediction of the charm quark mass

(Gaillard, Lee 1974)

(direct discovery of the charm quark in 1974 at SLAC and BNL)

- ▶ the observation of CP violation in kaon anti-kaon oscillations led to the prediction of the 3rd generation of quarks

(Kobayashi, Maskawa 1973)

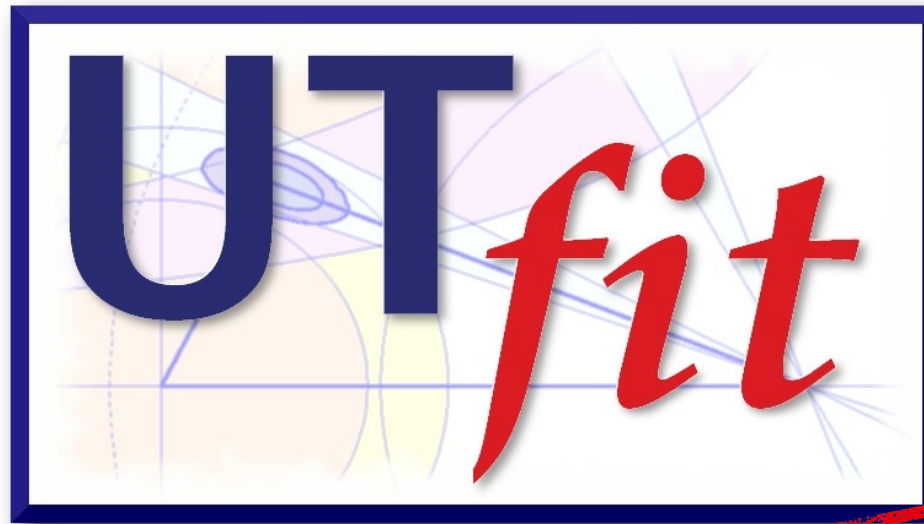
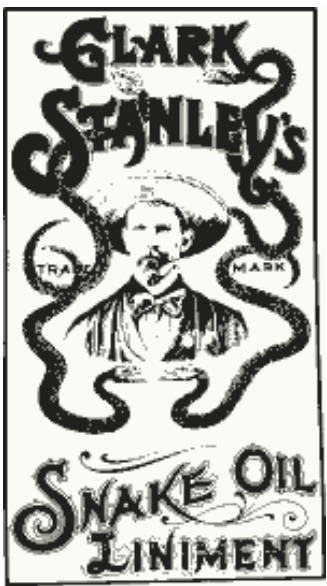
- ▶ the measurement of the frequency of $B - \bar{B}$ oscillations allowed to predict the large top quark mass

(various authors in the late 80's)

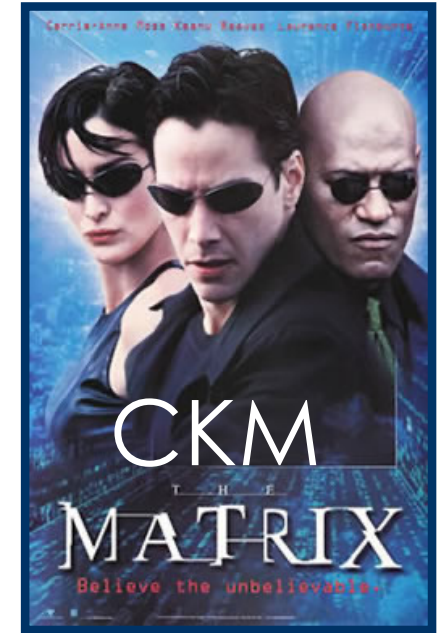


(direct discovery of the bottom quark in 1977 at Fermilab)

(direct discovery of the top quark in 1995 at Fermilab)



www.utfit.org



*M. Bona, M. Ciuchini, D. Derkach, R. Di Palma, F. Ferrari,
V. Lubicz, G. Martinelli, M. Pierini, L. Silvestrini, S. Simula,
A. Stocchi, C. Tarantino, V. Vagnoni,
M. Valli, and L. Vittorio*

The Unitarity Triangle Analysis

- Flavor-changing processes and CP violation in the SM ruled by 4 parameters in the 3x3 CKM (unitary) matrix

$$\bar{V}_{\text{CKM}} = \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

- $A, \lambda, \bar{\rho}$ and $\bar{\eta}$

$$\bar{\rho} = \rho(1 - \lambda^2/2 + \dots) \quad \bar{\eta} = \eta(1 - \lambda^2/2 + \dots)$$

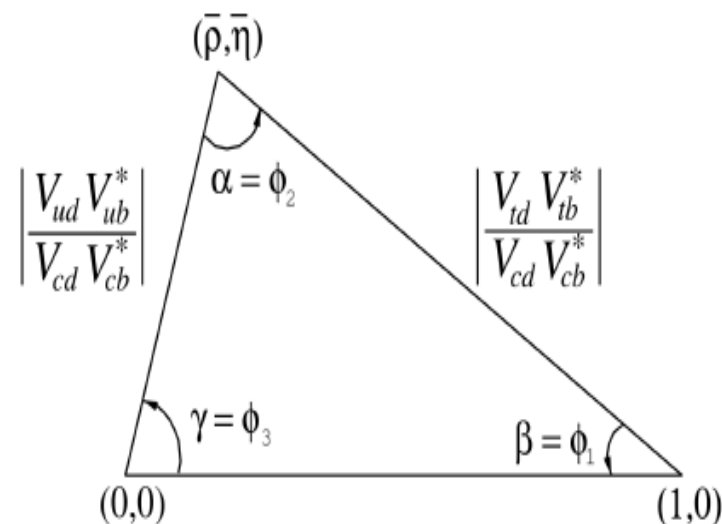
- Small value sin of Cabibbo angle (λ) makes the CKM matrix close to diagonal

- Unitarity implies relations between elements, that can be represented as a triangle in a plane

- By determining the CKM matrix

$$\begin{aligned} \sin \theta_{12} &= \lambda \\ \sin \theta_{23} &= A \lambda^2 \\ \sin \theta_{13} &= A \lambda^3(\rho - i\eta) \end{aligned}$$

$$\delta_{13} = \gamma = \phi_3$$

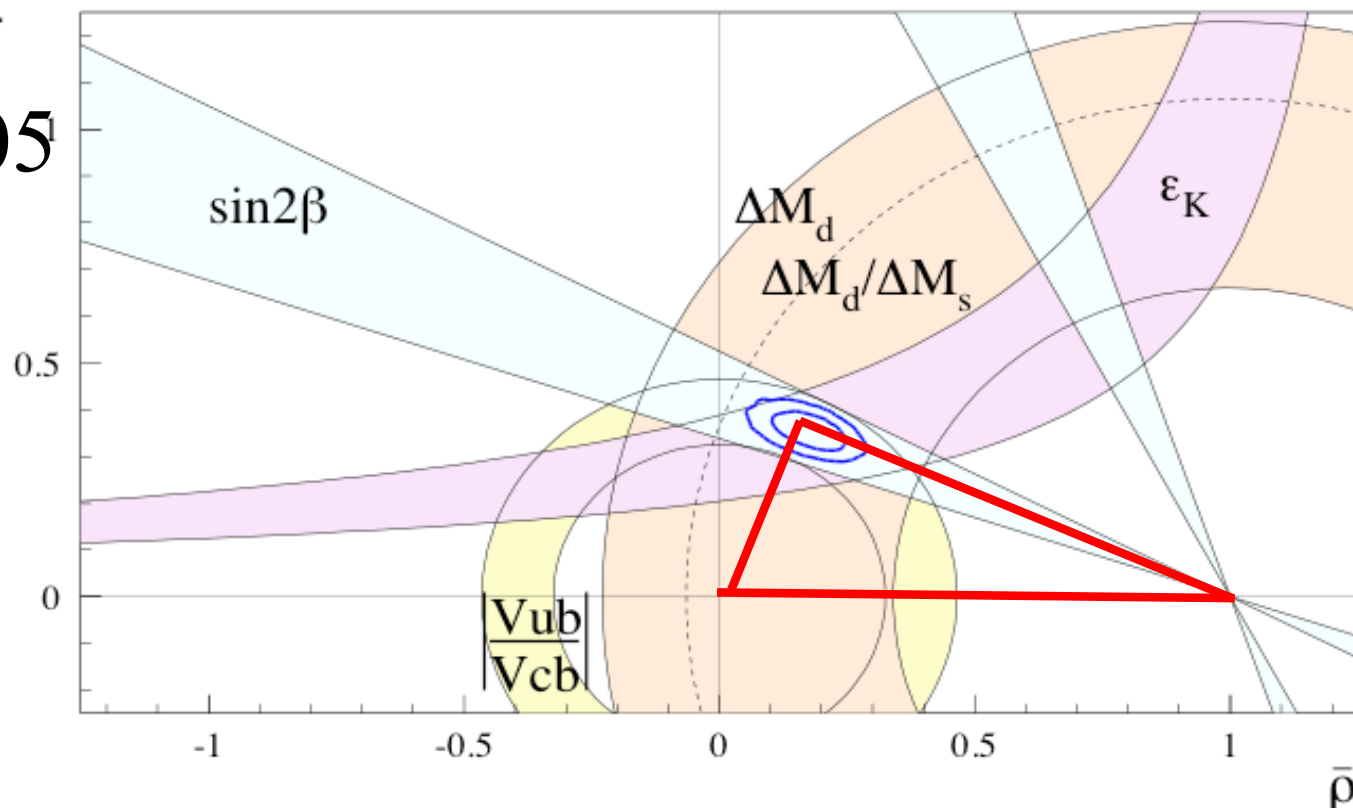


Unitary Triangle SM

2005

semileptonic decays

Experimental cor



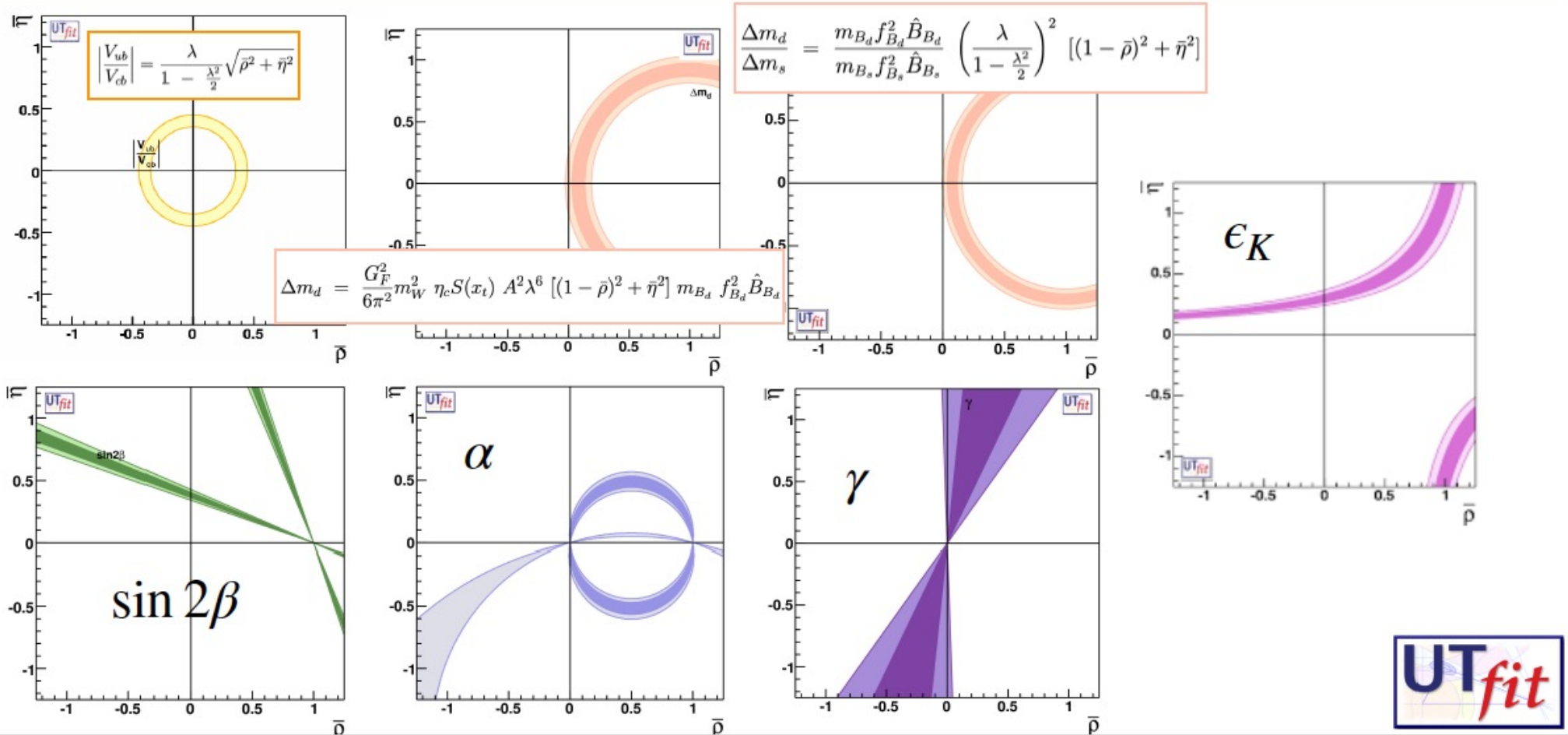
Meas.	$V_{CKM} \times \text{other}$	$(\bar{\rho}, \bar{\eta})$
$\frac{b \rightarrow u}{b \rightarrow c}$	$ V_{ub}/V_{cb} ^2$	$\bar{\rho}^2 + \bar{\eta}^2$
Δm_d	$ V_{td} ^2 f_{B_d}^2 B_{B_d}$	$(1 - \bar{\rho})^2 + \bar{\eta}^2$
$\frac{\Delta m_d}{\Delta m_s}$	$\left \frac{V_{td}}{V_{ts}} \right ^2 \xi^2$	$(1 - \bar{\rho})^2 + \bar{\eta}^2$
ϵ_K	$f(A, \bar{\eta}, \bar{\rho}, B_K)$	$\propto \bar{\eta}(1 - \bar{\rho})$
$A(J/\psi K^0)$	$\sin 2\beta$	$\frac{2\bar{\eta}(1 - \bar{\rho})}{\sqrt{\bar{\eta}^2 + (1 - \bar{\rho})^2}}$

$B_{d,s}^0 - \bar{B}_{d,s}^0$ mixing

$K^0 - \bar{K}^0$ mixing

B_d

UT constraints



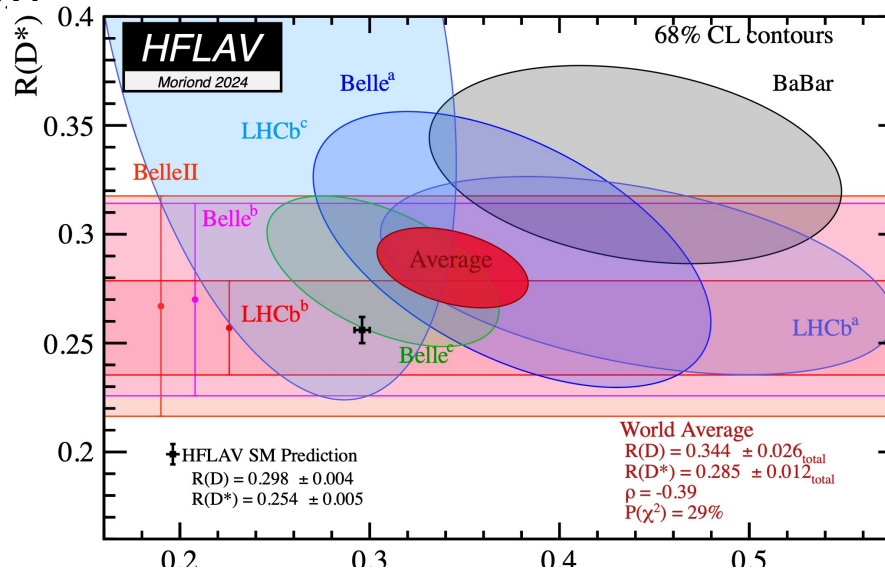
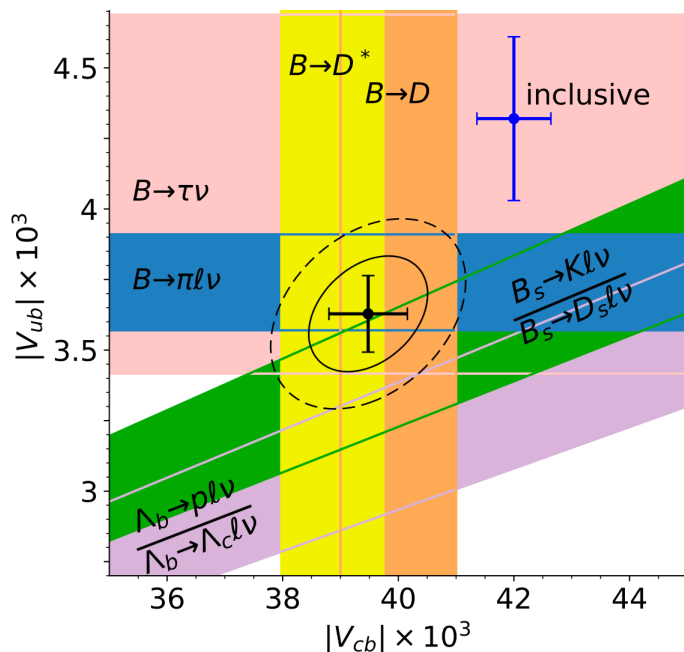
redundancy is the big strength of the UT analysis
 one can remove a subset of inputs and still determine the CKM
 one can exclude $\eta=0$ using only CP conserving processes

Tension(s) in $b \rightarrow c$ decays ? Charged Currents & Tree level

1. $|V_{cb}|$ (and $|V_{ub}|$) puzzle

2. Lepton Flavor Universality Violation

FLAG Review 2021 [EPJC '22 (2111.09849)]



HFLAV Collaboration, PRD '23 [arXiv:2206.07501] (updated plot)

An important CKM unitarity test is the Unitarity Triangle (UT) formed by

$$1 + \frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} + \frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*} = 0$$

V_{cb} plays an important role in UT

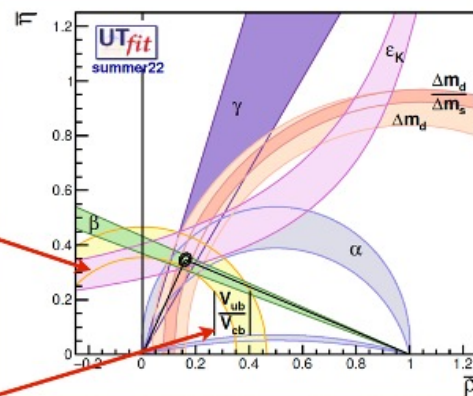
$$\varepsilon_K \approx x|V_{cb}|^4 + \dots$$

and in the prediction of FCNC:

$$\propto |V_{tb}V_{ts}|^2 \simeq |V_{cb}|^2 [1 + O(\lambda^2)]$$

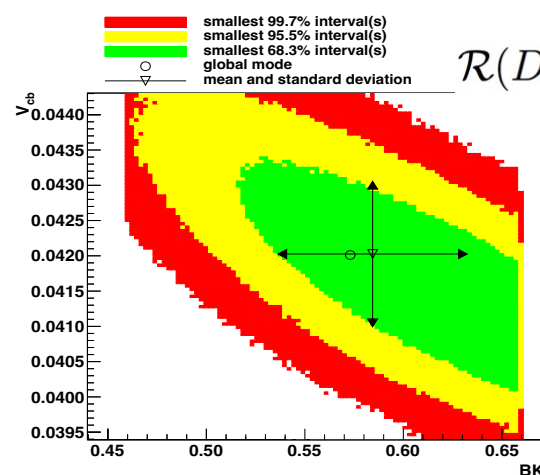
where it often dominates the theoretical uncertainty.

V_{ub}/V_{cb} constrains directly the UT



$$\mathcal{R}(D) = \frac{\mathcal{B}(B \rightarrow D\tau\nu_\tau)}{\mathcal{B}(B \rightarrow D\ell\nu_\ell)},$$

$$\mathcal{R}(D^*) = \frac{\mathcal{B}(B \rightarrow D^*\tau\nu_\tau)}{\mathcal{B}(B \rightarrow D^*\ell\nu_\ell)}$$



Our ability to determine precisely V_{cb} is crucial for indirect NP searches

$$\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) \propto |V_{cb}|^2$$

The tension strongly depends on the method used in the theoretical analysis

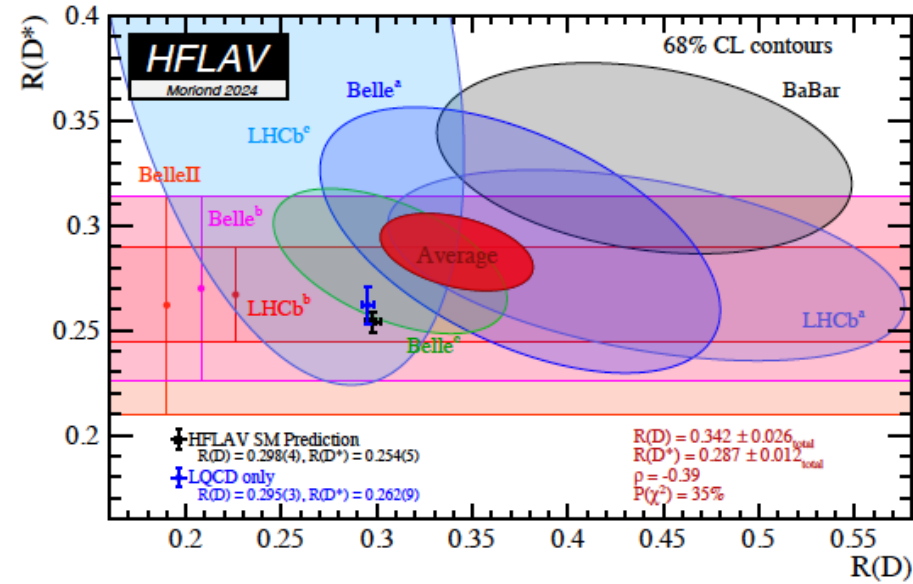
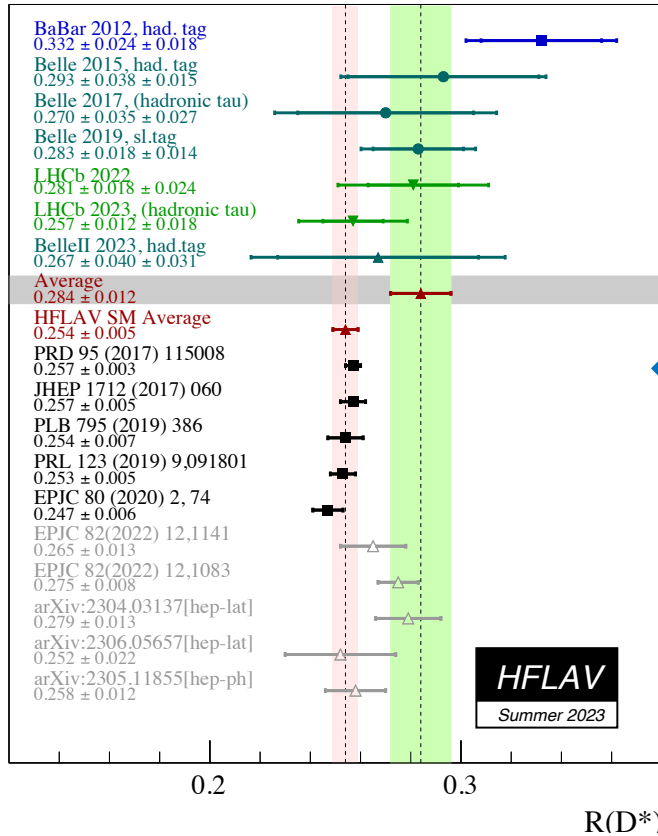
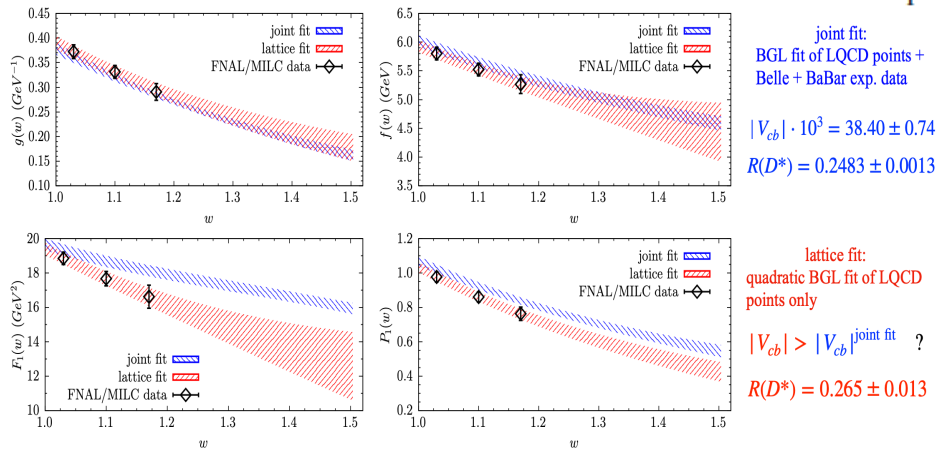


Figure 9. Measurements of $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$ listed in Table 3 and their two-dimensional average. Contours correspond to 68% CL for both the bands and the ellipses. The black and blue points with error bars are two recent SM predictions for $\mathcal{R}(D^*)$ and $\mathcal{R}(D)$. The SM prediction reported is based on the results summarized in Table 1. This prediction and the experimental average deviate from each other by about 3.3σ . The SM prediction based only on LQCD calculations is also reported, where $\mathcal{R}(D)$ is taken from FLAG [25], while $\mathcal{R}(D^*)$ is taken from Ref. [28]. The deviation from the experimental average and this prediction is about 2.5σ . The measurements are listed in Table 3.



simultaneous fit of the lattice points and experimental data to determine the shape of the FFs and to extract $|V_{cb}|$

*** slope differences between exp's and theory $\rightarrow$ bias on $|V_{cb}|^{\text{joint fit}} ?$ ***

*Klaver S & Rotondo M,
doi 10.3390/sym16080964*

$$\mathcal{R}(D^*) = \frac{\mathcal{B}(B \rightarrow D^* \tau \nu_\tau)}{\mathcal{B}(B \rightarrow D^* \ell \nu_\ell)}$$

$$\mathcal{R}(D) = \frac{\mathcal{B}(B \rightarrow D \tau \nu_\tau)}{\mathcal{B}(B \rightarrow D \ell \nu_\ell)},$$

V_{cb} and V_{ub}

Latest inputs from arXiv:2310.03680

$$|V_{cb}| \text{ (excl)} = (40.13 \pm 0.55) 10^{-3}$$

$$|V_{cb}| \text{ (incl)} = (41.97 \pm 0.48) 10^{-3}$$

from arXiv:2310.20324

from arXiv:2202.10285

$$|V_{ub}| \text{ (excl)} = (3.57 \pm 0.23) 10^{-3}$$

$$|V_{ub}| \text{ (incl)} = (4.13 \pm 0.26) 10^{-3}$$

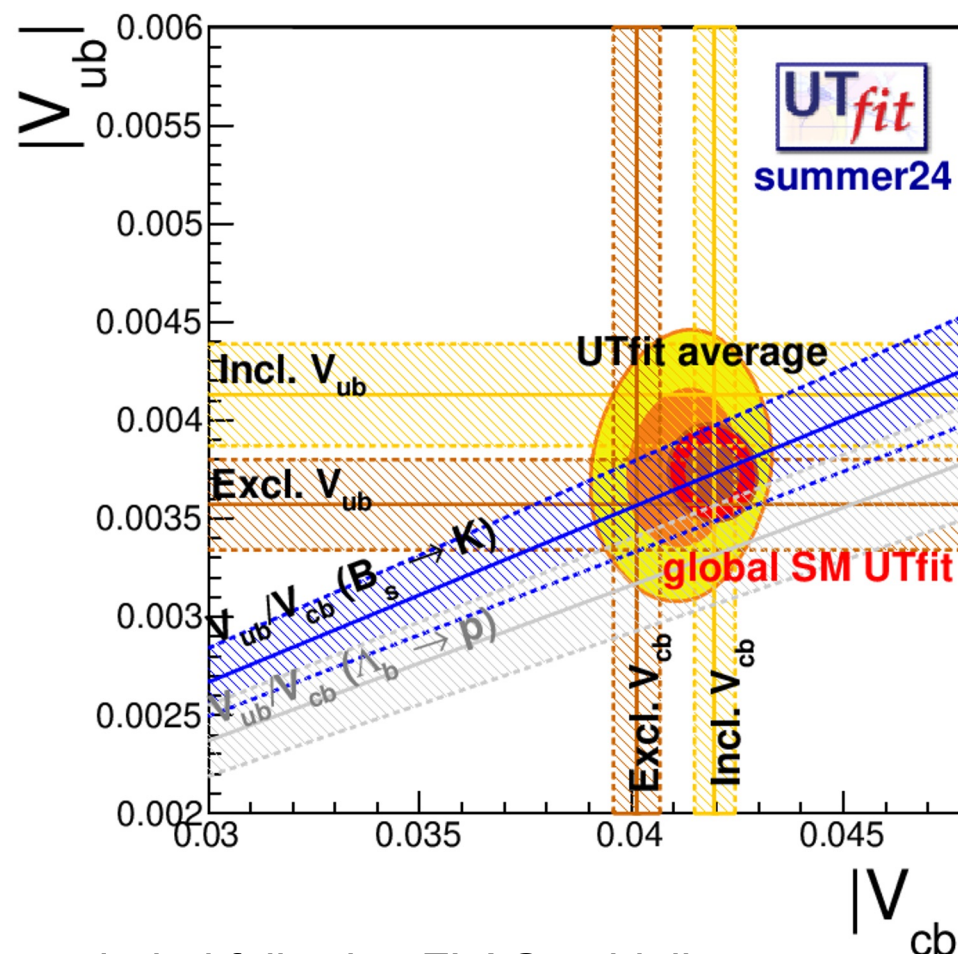
PDG 2024

from arXiv:2310.03680

$$|V_{ub} / V_{cb}| = (8.7 \pm 0.9) 10^{-2}$$

$$|V_{ub} / V_{cb}| \text{ (LHCb)} = (7.9 \pm 0.6) 10^{-2}$$

2

 Λ_b , excluded following FLAG guidelines

$$|V_{cb}| \text{ (incl)} = (42.00 \pm 0.47) 10^{-3}$$

M. Fael et al. Eur.Phys.J.ST 233 (2024) 2, 325-346

V_{cb} and V_{ub}

Inputs to the global fit
from 2D à la D'Agostini averages

$$|V_{cb}|_{\text{UTfit}} = (41.20 \pm 0.74) 10^{-3}$$

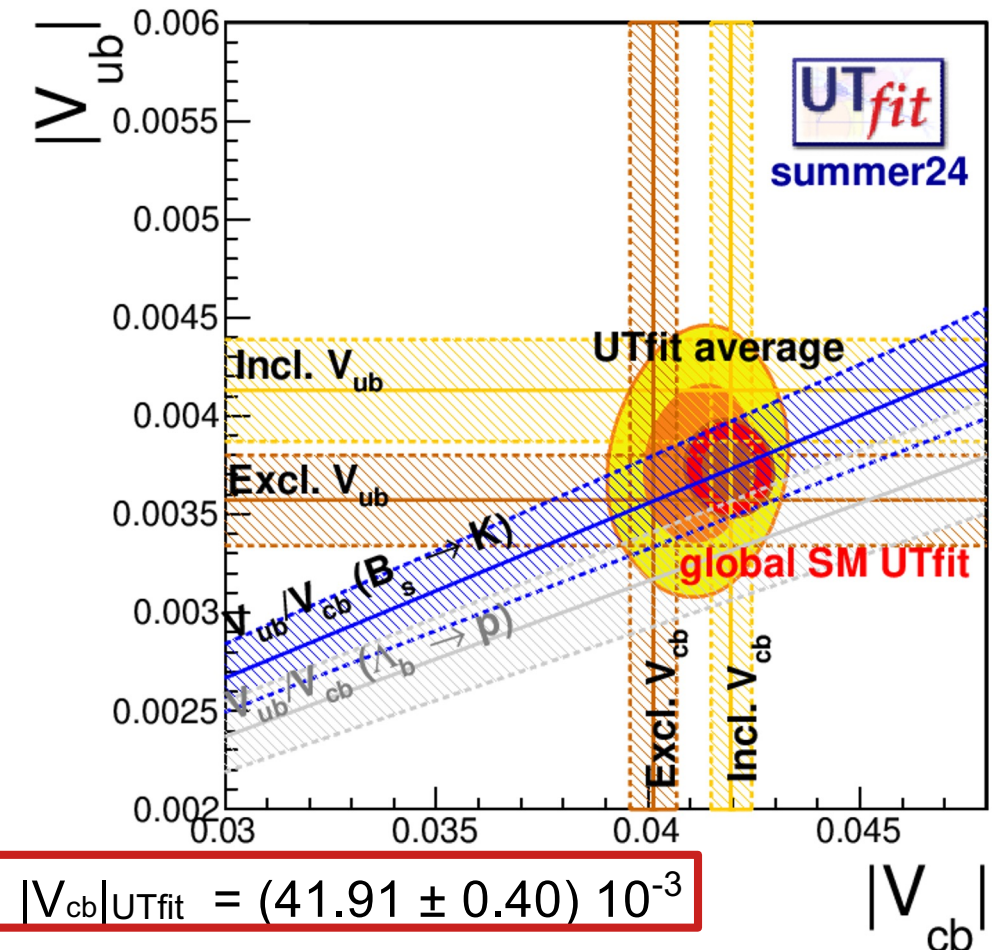
$$|V_{ub}|_{\text{UTfit}} = (3.84 \pm 0.35) 10^{-3}$$

UTfit predictions:

$$|V_{cb}|_{\text{UTfit}} = (42.19 \pm 0.48) 10^{-3}$$

$$|V_{ub}|_{\text{UTfit}} = (3.72 \pm 0.10) 10^{-3}$$

UTfit full fit



$$|V_{cb}|_{\text{UTfit}} = (41.91 \pm 0.40) 10^{-3}$$

$$|V_{ub}|_{\text{UTfit}} = (3.73 \pm 0.09) 10^{-3}$$

New Analyses (G.M., S.Simula, L.Vittorio 2310.03680)

NEW EXCLUSIVE $V_{cb} = (39.92 \pm 0.64) 10^{-3}$ from B $\rightarrow$ D* from bin by bin analysis

$$|V_{cb}| (incl) = (41.97 \pm 0.48) 10^{-3}$$

2.6 σ difference

Finauri & Gambino 2310.20324

$$|V_{cb}| (incl) = (41.69 \pm 0.63) 10^{-3}$$

2.0 σ difference

F. Bernlochner et al. 2205.10274

$$\text{NEW } V_{ub}/V_{cb} = (8.7 \pm 0.9) 10^{-2}$$

FLAG UNDERESTIMATES OF THE UNCERTAINTY

The larger error reduces the correlation between V_{ub} and V_{cb}

	$ V_{cb} \cdot 10^3$			
experiment	FNAL/MILC	HPQCD	JLQCD	Average
Belle '18 [19]	39.64 (74)	39.11 (81)	39.92 (74)	39.58 (98)
$\chi^2/(d.o.f.)$	3.71	1.14	0.04	0.26
Belle '23 [13]	40.87 (115)	41.03 (125)	41.38 (134)	41.11 (138)
$\chi^2/(d.o.f.)$	1.80	0.11	0.31	0.03
BelleII '23 [14]	39.35 (77)	39.98 (102)	40.20 (85)	39.79 (94)
$\chi^2/(d.o.f.)$	0.63	0.09	0.42	0.29

$$|V_{cb}| = (39.92 \pm 0.64) \cdot 10^{-3}$$

$$|V_{cb}| \cdot 10^3 = 39.87 \pm 0.55$$

$$\text{Ufit Prediction } V_{cb} = (42.19 \pm 0.48) 10^{-3}$$

$$V_{ub} = (3.72 \pm 0.10) 10^{-3}$$

see also

e-Print: [2409.10492](#)

↙ [2410.17974](#)

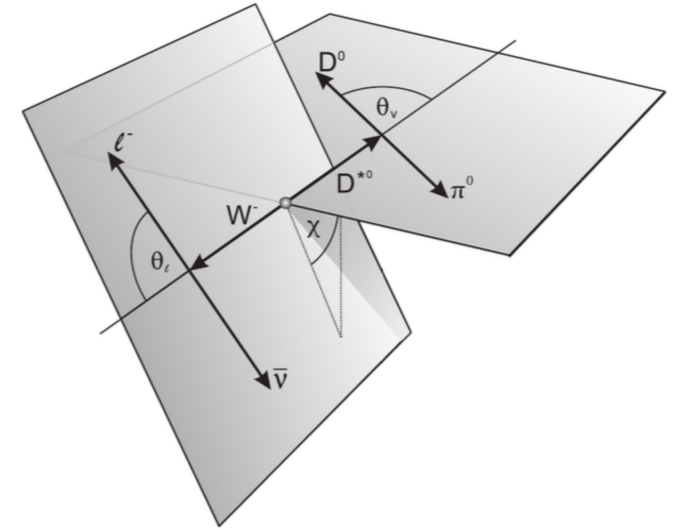
$$\text{from the total width } V_{cb} = (39.7 \pm 0.8) 10^{-3}$$

$A_{FB,l}$ is the forward-backward asymmetry with respect to

$$z = \cos \theta_\ell \quad A_{FB,\ell} = \frac{\int_{-1}^0 dz \frac{d\Gamma}{dz} - \int_0^1 dz \frac{d\Gamma}{dz}}{\Gamma}$$

$P_\tau(D^*)$ is the τ polarization

$F_{\ell,\tau}$ is the fraction of longitudinal polarization of the D^* with a lepton/tau in the final state



$$\mathcal{R}(D^*) = \frac{\mathcal{B}(B \rightarrow D^* \tau \nu_\tau)}{\mathcal{B}(B \rightarrow D^* \ell \nu_\ell)} \quad \mathcal{R}(D) = \frac{\mathcal{B}(B \rightarrow D \tau \nu_\tau)}{\mathcal{B}(B \rightarrow D \ell \nu_\ell)},$$

R(D*) and the polarization observables

As done for semileptonic $B \rightarrow D$ decays, we can immediately compute the theor. expectation values of several quantities:

G. Martinelli, S. Simula, LV, EPJC '24 [arXiv:2310.03680]

Lattice FFs	$R(D^*)$	$P_\tau(D^*)$	$F_{L,\tau}$	$F_{L,\ell}$	$A_{FB,\ell}$	F_L^τ in two q^2 bins	
						low- q^2 bin	high- q^2 bin
FNAL/MILC [15]	0.275(8)	-0.529(7)	0.418(9)	0.450(19)	0.261(14)	0.486(15)	0.381(5)
HPQCD [16]	0.266(12)	-0.543(18)	0.399(23)	0.435(42)	0.265(30)	0.459(38)	0.367(14)
JLQCD [17]	0.247(8)	-0.509(11)	0.448(16)	0.516(29)	0.220(21)	0.534(25)	0.398(10)
Average [15]-[17] (PDG scale factor)	0.262(9) (1.8)	-0.525(7) (1.3)	0.422(10) (1.4)	0.465(22) (1.5)	0.251(13) (1.2)	0.495(17) (1.4)	0.383(6) (1.4)
Combined [15]-[17]	0.259(5)	-0.521(6)	0.425(7)	0.473(14)	0.252(10)	0.498(12)	0.384(4)
Experimental value	0.284(12) [36]	$-0.38 \pm 0.51^{+0.21}_{-0.16}$ [38]	0.49(8) [39, 40]	0.520(6) [13, 14]	0.232(10) [13, 14]	0.51(7)(3) [40]	0.35(8)(2) [40]

less than 2σ difference

2.4 σ tension

measurements in the τ -sector

Speculations of New Physics coupled to light lepton generations?

M. Jung and D. Straub, JHEP '19 [1801.01112]
M. Fedele et al, PRD '23 [2305.15457]
P. Colangelo et al, PRD '24 [2401.12304]

[13, 14]: Belle23 and BelleII23 [38]: Belle Coll., PRL '17 [1612.00529]

[36]: HFLAV Coll, PRD '23 [2206.07501]

[39]: Belle Coll., arXiv:1903.03102

[40]: LHCb Coll., arXiv:2311.05224

Prediction	$P_\tau(D^*)$	$F_L^{D^*}$	
FNAL/MILC, A. Bazavov et al. [9]	-0.529 ± 0.007	0.418 ± 0.009	
HPQCD, J. Harrison, C. Davies [10]	-0.547 ± 0.019	0.395 ± 0.024	
JLQCD, Y. Aoki et al. [11]	-0.509 ± 0.011	0.448 ± 0.016	
G. Martinelli, S. Simula, L. Vittorio [28]	-0.521 ± 0.006	0.425 ± 0.007	2023
M. Bordone, A. Juttner [29]	$-0.522 \pm 0.006^{+13}_{-22}$	$0.424 \pm 0.007^{+21}_{-25}$	2024

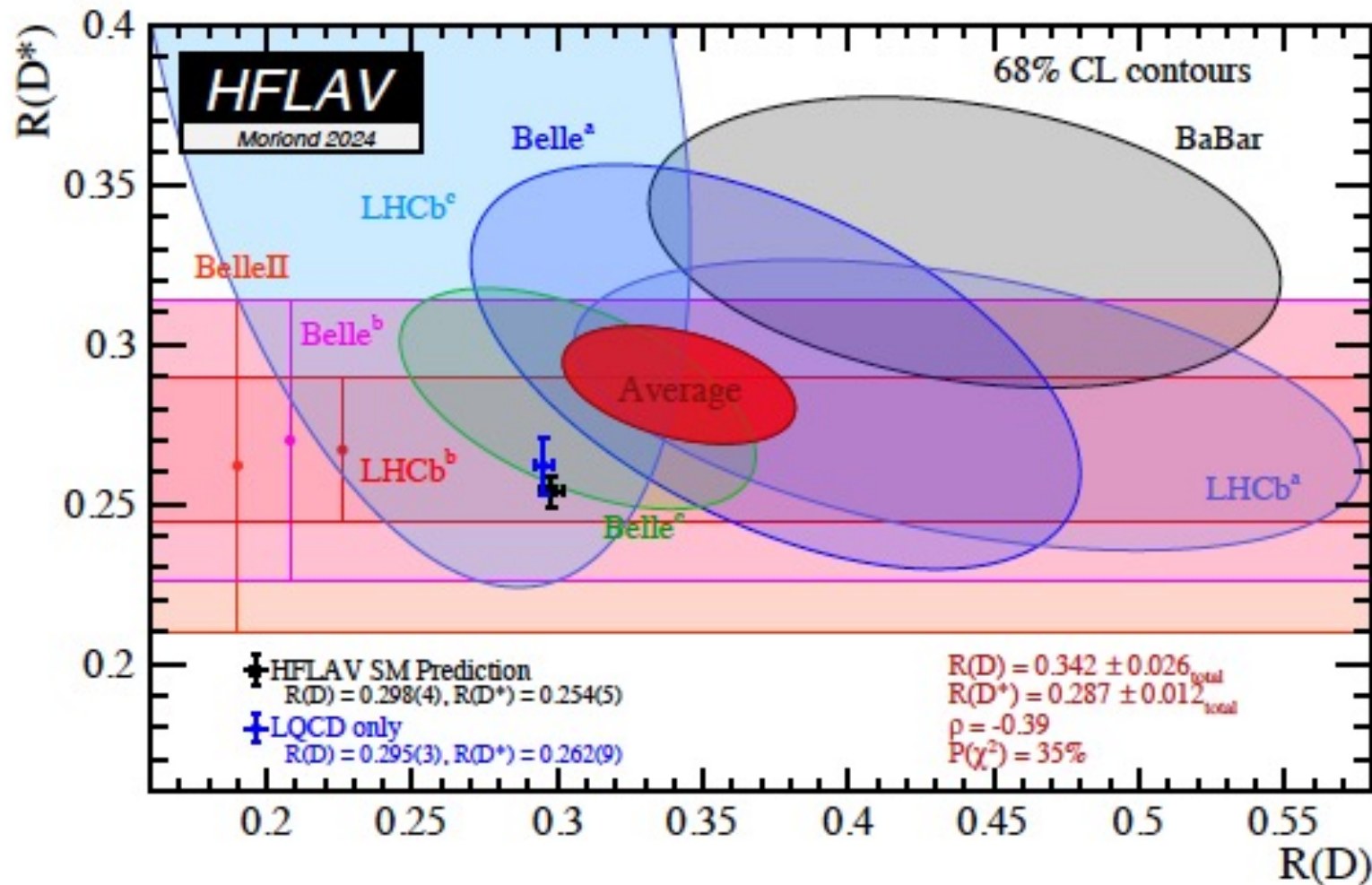
Prediction	$\mathcal{R}(D^*)$	$\mathcal{R}(D)$
D. Bigi, P. Gambino [7]		0.299 ± 0.003
P. Gambino, M. Jung, S. Schacht [17]	$0.254^{+0.007}_{-0.006}$	
M. Bordone, M. Jung, D. van Dyk [18]	0.247 ± 0.006	0.298 ± 0.003
F. Bernlochner, et al. [19]	0.257 ± 0.003	0.299 ± 0.003
S. Jaiswal, S. Nandi, S.K. Patra [20]	0.257 ± 0.005	0.299 ± 0.004
BABAR Coll. [21]	0.253 ± 0.005	
G. Martinelli, S. Simula, L. Vittorio [16]		0.296 ± 0.008
Arithmetic average (HFLAV) [22]	0.254 ± 0.005	0.298 ± 0.004
D. Bigi, P. Gambino, S. Schacht [23]	0.260 ± 0.008	
M. Bordone, M. Jung, D. van Dyk [18]	0.250 ± 0.003	0.297 ± 0.003
F. Bernlochner, et al. [24]	0.249 ± 0.003	0.288 ± 0.004
FLAG Coll. [25]		0.2951 ± 0.0031
G. Martinelli, S. Simula, L. Vittorio [26]	0.275 ± 0.008	
I. Ray, S. Nandi [27]	0.258 ± 0.012	
FNAL/MILC, A. Bazavov et al. [9]	0.265 ± 0.013	
HPQCD, J. Harrison, C. Davies [10]	0.273 ± 0.015	
JLQCD, Y. Aoki et al. [11]	0.252 ± 0.022	
G. Martinelli, S. Simula, L. Vittorio [28]	0.262 ± 0.009	
M. Bordone, A. Juttner [29]	$0.262 \pm 0.005 \pm 0.012$	

Flag 2024 $\mathcal{R}(D) 0.294(5)$
 $\mathcal{R}(D^*)=0.258(5)_{2+1}$; $0.275(15)_{2+1+1}$
 $\rightarrow 259(5)$

2023
2024

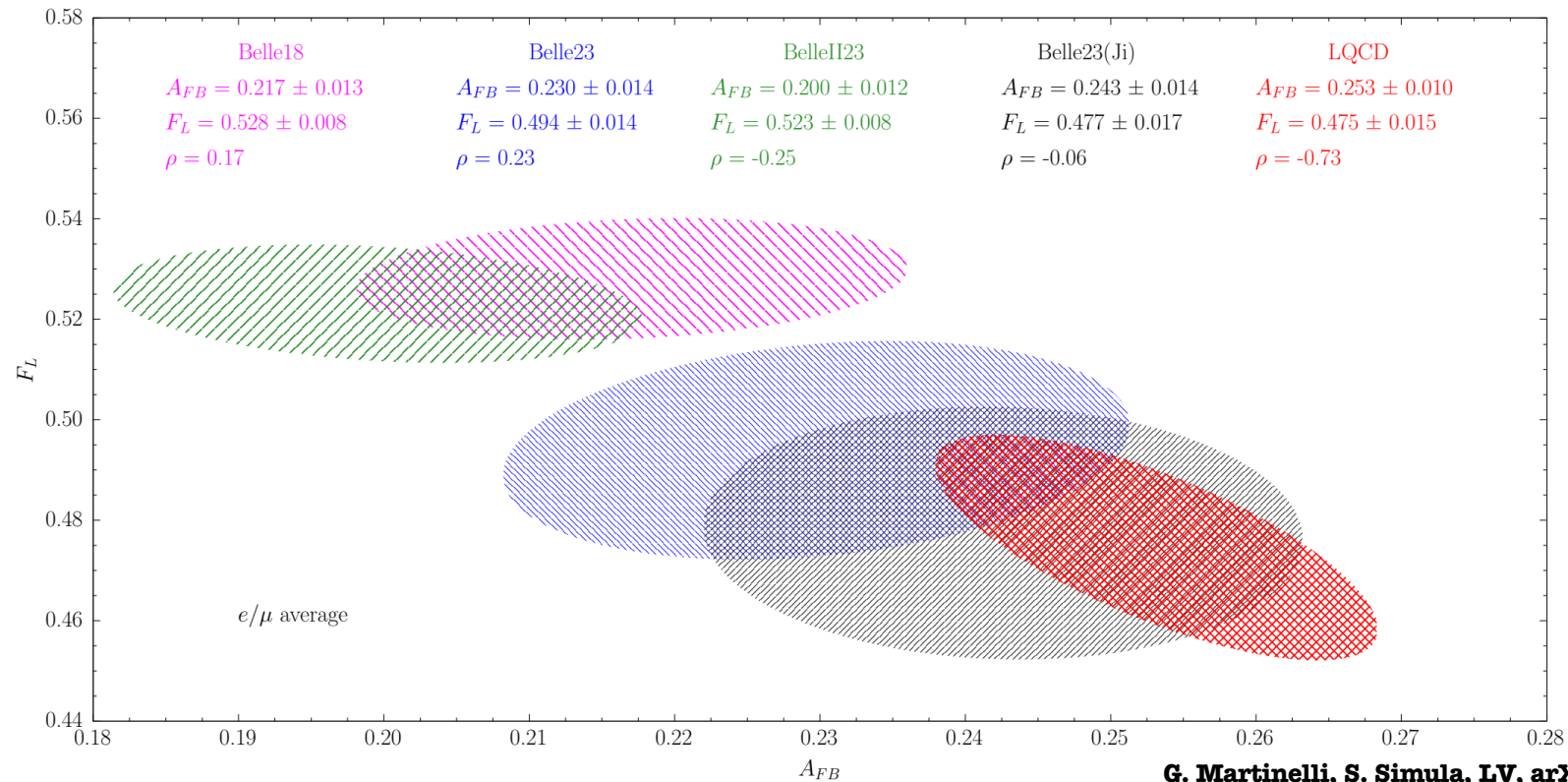
From the nice review S
 Klaver and M Rotondo
 doi
 10.3390/sym16080964

Experiment	$\mathcal{R}(D^*)$	$\mathcal{R}(D)$	ρ
BaBar [42,43]	$0.332 \pm 0.024_{\text{stat}} \pm 0.018_{\text{syst}}$	$0.440 \pm 0.058_{\text{stat}} \pm 0.042_{\text{syst}}$	-0.27
Belle ^a [44]	$0.293 \pm 0.038_{\text{stat}} \pm 0.015_{\text{syst}}$	$0.375 \pm 0.064_{\text{stat}} \pm 0.026_{\text{syst}}$	-0.49
Belle ^b [49]	$0.270 \pm 0.035_{\text{stat}}^{+0.028}_{-0.025_{\text{syst}}}$		
Belle ^c [48]	$0.283 \pm 0.018_{\text{stat}} \pm 0.014_{\text{syst}}$	$0.307 \pm 0.037_{\text{stat}} \pm 0.016_{\text{syst}}$	-0.51
LHCb ^a [53]	$0.281 \pm 0.018_{\text{stat}} \pm 0.024_{\text{syst}}$	$0.441 \pm 0.060_{\text{stat}} \pm 0.066_{\text{syst}}$	-0.43
LHCb ^b [56,57,64]	$0.267 \pm 0.012_{\text{stat}} \pm 0.019_{\text{syst}}$		
LHCb ^c [54]	$0.402 \pm 0.081_{\text{stat}} \pm 0.085_{\text{syst}}$	$0.249 \pm 0.043_{\text{stat}} \pm 0.047_{\text{syst}}$	-0.39
Belle II [45]	$0.262^{+0.041}_{-0.039_{\text{stat}}}^{+0.035}_{-0.032_{\text{syst}}}$		
Average	0.287 ± 0.012	0.342 ± 0.026	-0.39



contours correspond to 68% CL, SM predictions fitting together data and FFs give a deviation of 3.3 sigma (HFLAV) , LQCD only 2.5 sigma $R(D)$ stable in different calculations.

Polarization observables A_{FB} vs F_L



Relevant differences exist also among different experimental (and not only lattice) datasets, this is an impediment to look for new physics

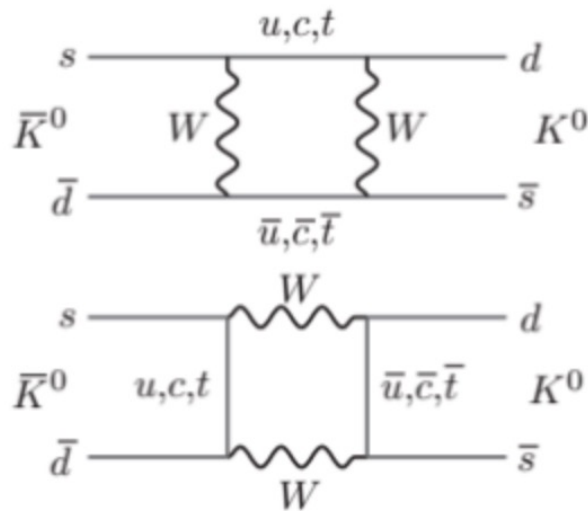
Power corrections to the CP-violation parameter ε_K

M. Ciuchini^(a), E. Franco^(b), V. Lubicz^(c,a), $\varepsilon_K^{exp} = 2.228 \pm 0.011) \cdot 10^{-3}$
 G. Martinelli^(d,b), L. Silvestrini^(b), C. Tarantino^(c,a)

2021: an estimate from the $1/mc$ expansion of the effective Hamiltonian + UTfit

$$\varepsilon_K = 2.00(15) \times 10^{-3}$$

Computing the long-distance contributions to ε_K



Ziyuan Bai
 Columbia University, USA
bzyhty@gmail.com

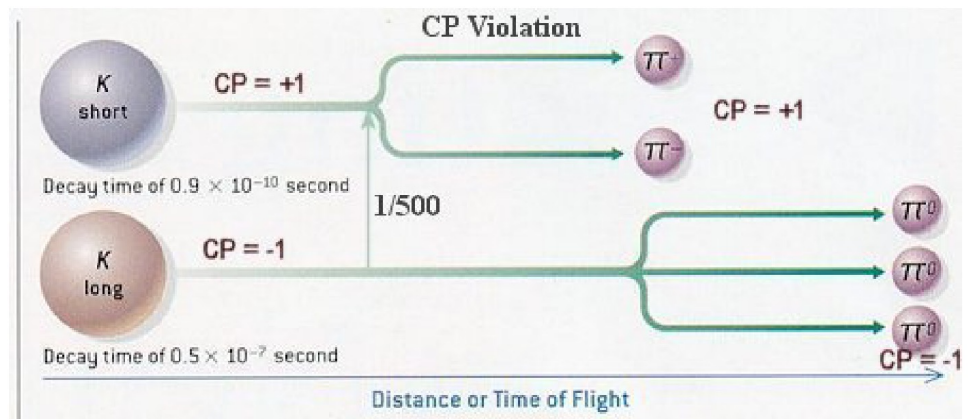
Norman Christ^{*†}
 Columbia University, USA
 E-mail: nhc@phys.columbia.edu

RBC and UKQCD Collaborations

*2015: a real exploratory calculation
 no physical masses, no
 extrapolation to the continuum
 etc.*

$$|\varepsilon| = \underbrace{(1.806(41))}_{tt} + \underbrace{0.891(11)}_{ut_{SD}} + \underbrace{0.209(6)}_{ut_{LD}} + \underbrace{0.112(13)}_{\text{Im}(A_0)} \times 10^{-3} = 3.019(45) \times 10^{-3}$$

Final result for ϵ'



- Combining our new result for $\text{Im}(A_0)$ and our 2015 result for $\text{Im}(A_2)$, and again using expt. for the real parts, we find

$$\text{Re} \left(\frac{\epsilon'}{\epsilon} \right) = \text{Re} \left\{ \frac{i\omega e^{i(\delta_2 - \delta_0)}}{\sqrt{2}\epsilon} \left[\frac{\text{Im}A_2}{\text{Re}A_2} - \frac{\text{Im}A_0}{\text{Re}A_0} \right] \right\}$$

$$= 0.00217(26)(62)(50)$$

IB + EM

stat sys

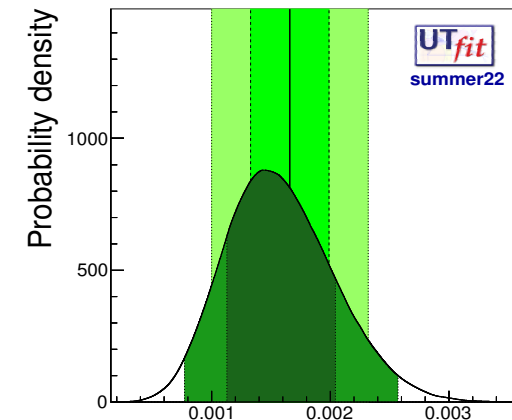
A second group should do this calculation!!

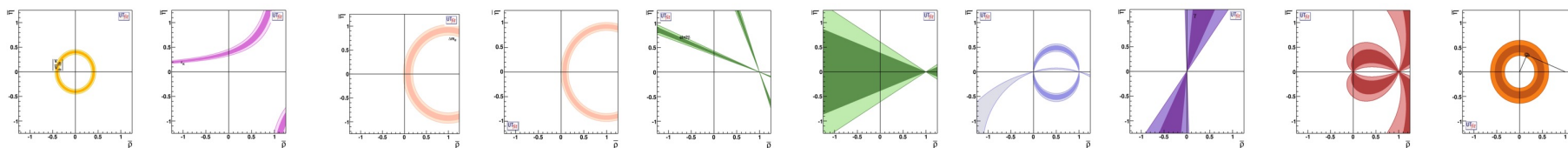
Consistent with experimental result:

$$\text{Re}(\epsilon'/\epsilon)_{\text{expt}} = 0.00166(23)$$

RBC/UKQCD: $\epsilon'/\epsilon = 16.7 \times 10^{-4}$

UKQCD: $\epsilon'/\epsilon = 15.2(4.7) \times 10^{-4}$

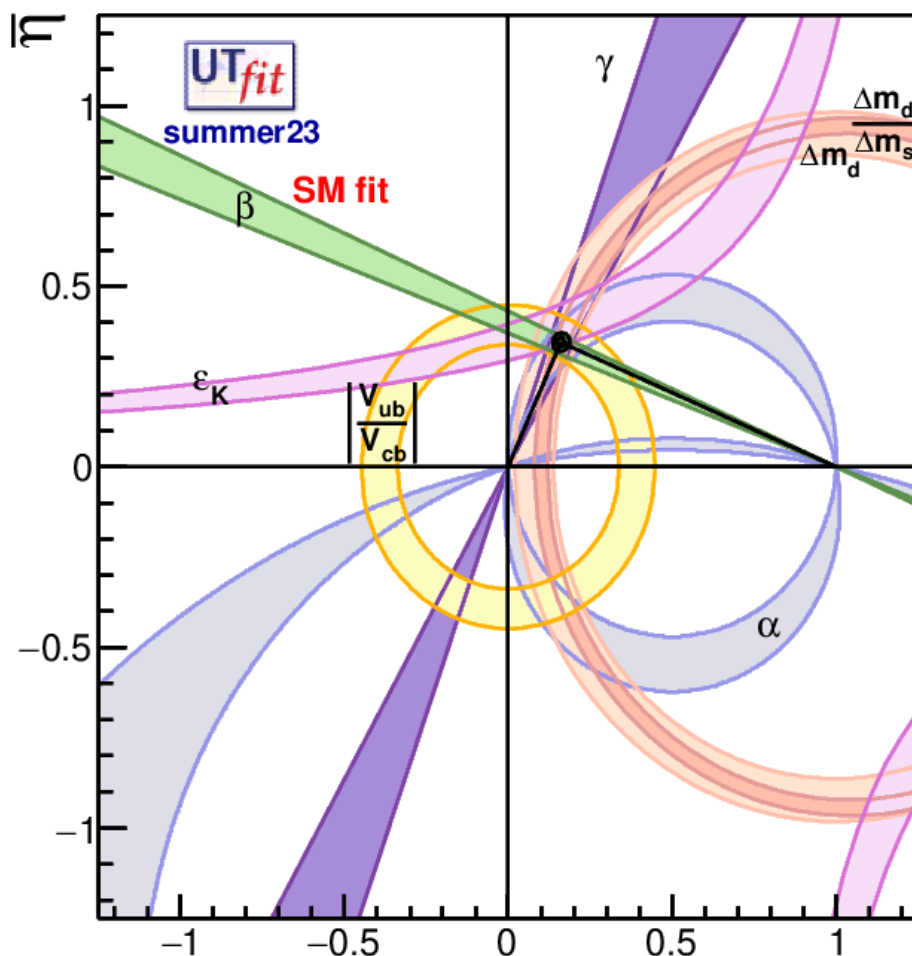




September 24

$$\bar{\rho} = 0.158 \pm 0.009 \quad \bar{\eta} = 0.352 \pm 0.011$$

In the hadronic sector, the SM CKM pattern represents the principal part of the flavor structure and of CP violation

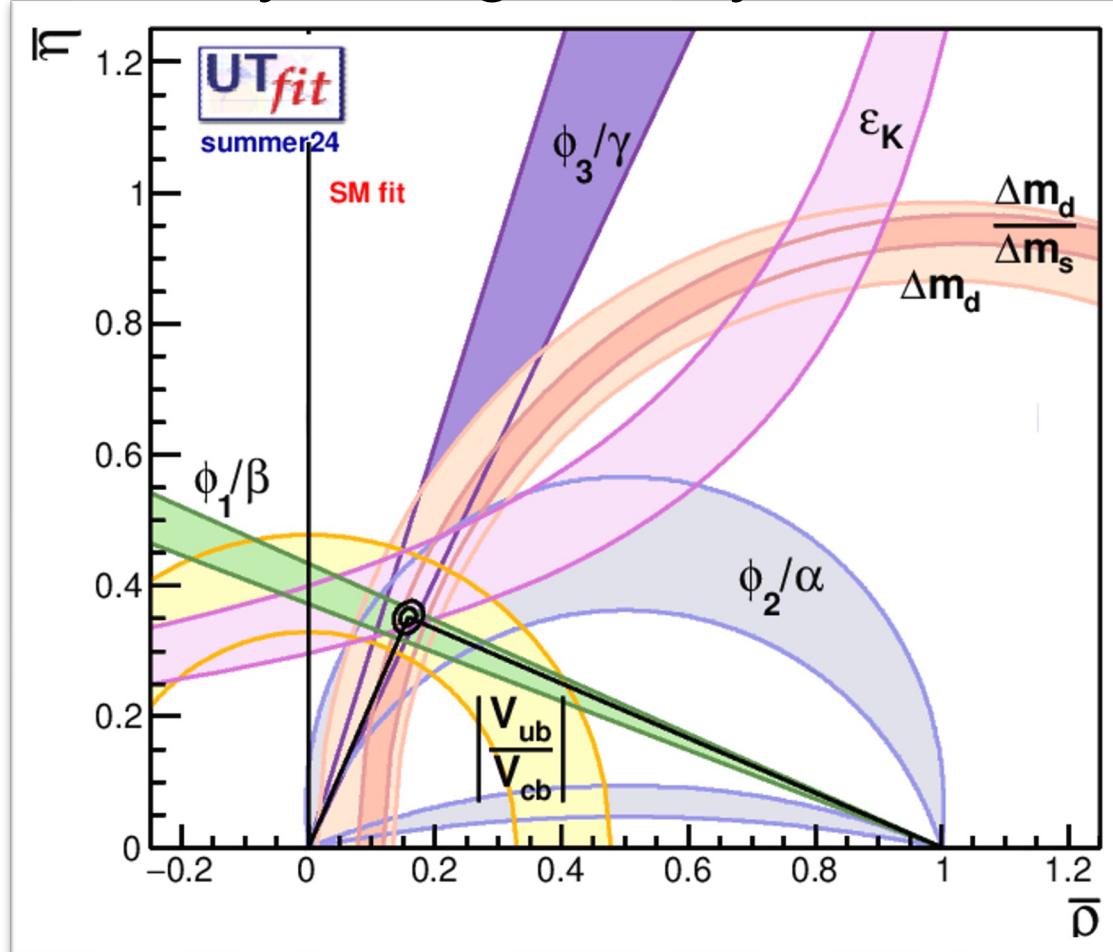


$$\begin{aligned} \alpha &= (91.4 \pm 1.4)^\circ \\ \sin 2\beta &= 0.763 \pm 0.030 \\ \gamma &= (65.6 \pm 1.4)^\circ \\ A &= 0.826 \pm 0.011 \\ \lambda &= 0.2250 \pm 0.0007 \end{aligned}$$

Consistence on an over constrained fit of the CKM parameters

CKM matrix is the dominant source of flavour mixing and CP violation

Unitarity Triangle analysis in the SM:



levels @
95% Prob

$$\rho = 0.158 \pm 0.009$$

$$\eta = 0.352 \pm 0.010$$

$$\lambda = 0.2250 \pm 0.0007$$

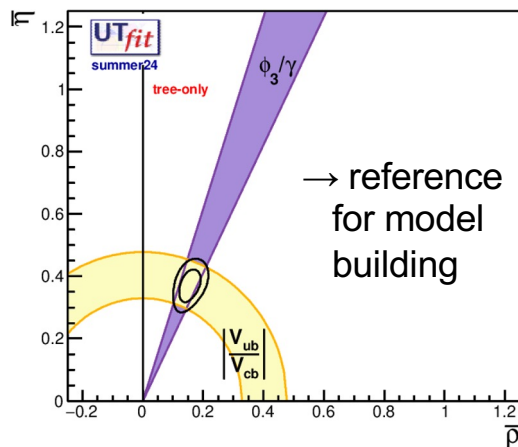
$$A = 0.826 \pm 0.009$$

Some interesting configurations

“Tree only”

$$\rho = \pm 0.156 \pm 0.024$$

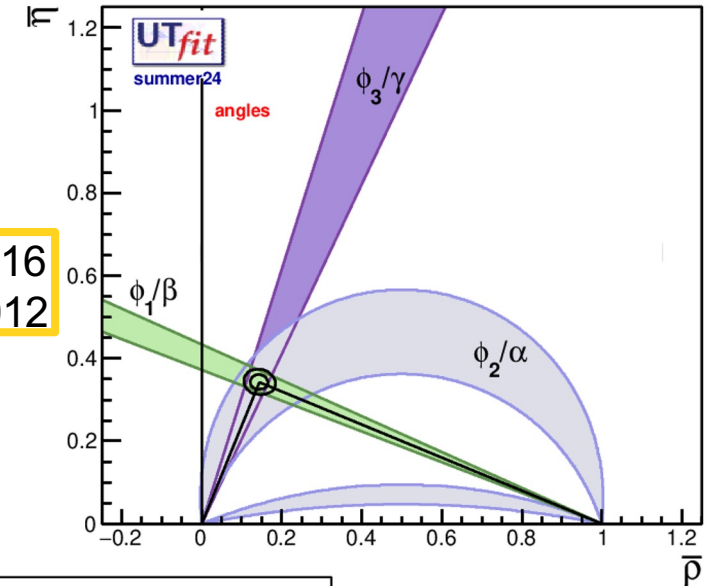
$$\eta = \pm 0.372 \pm 0.035$$



Angles only

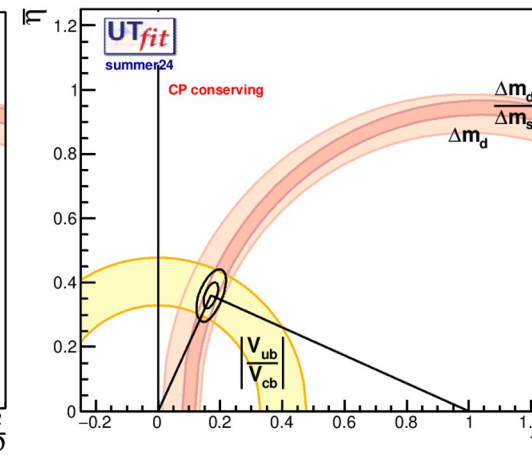
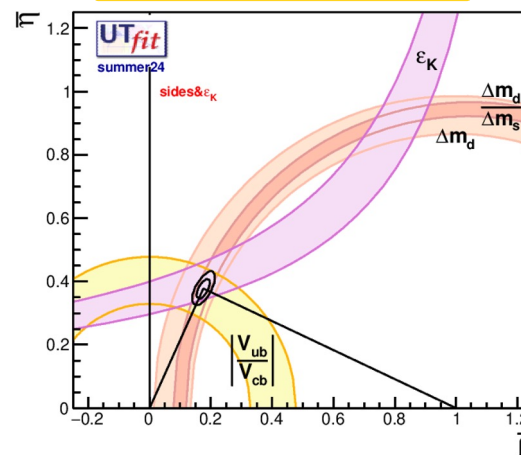
$$\rho = 0.144 \pm 0.016$$

$$\eta = 0.343 \pm 0.012$$

Sides and ϵ_K

$$\rho = 0.176 \pm 0.015$$

$$\eta = 0.377 \pm 0.022$$



CP conserving constraints

$$\rho = 0.170 \pm 0.017$$

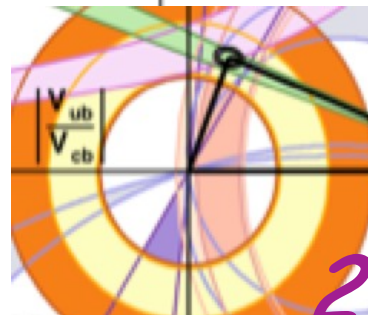
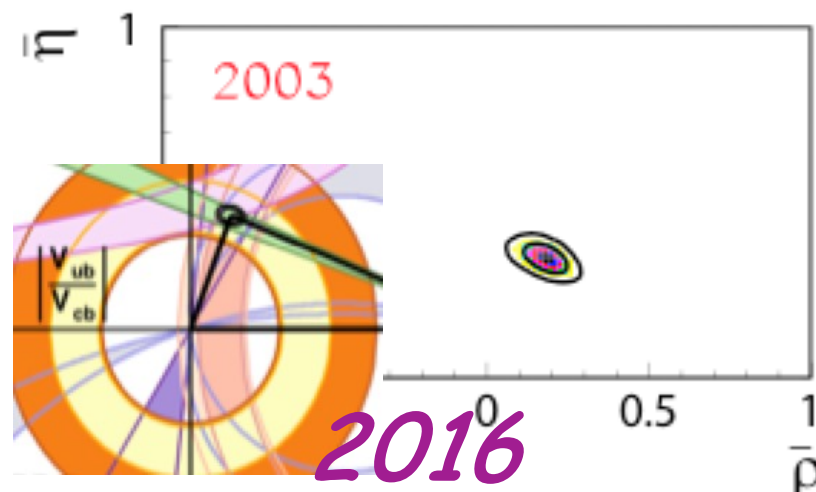
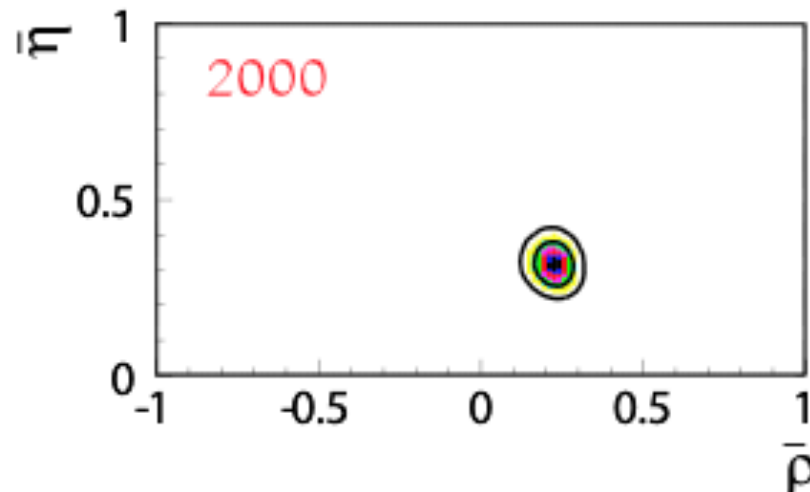
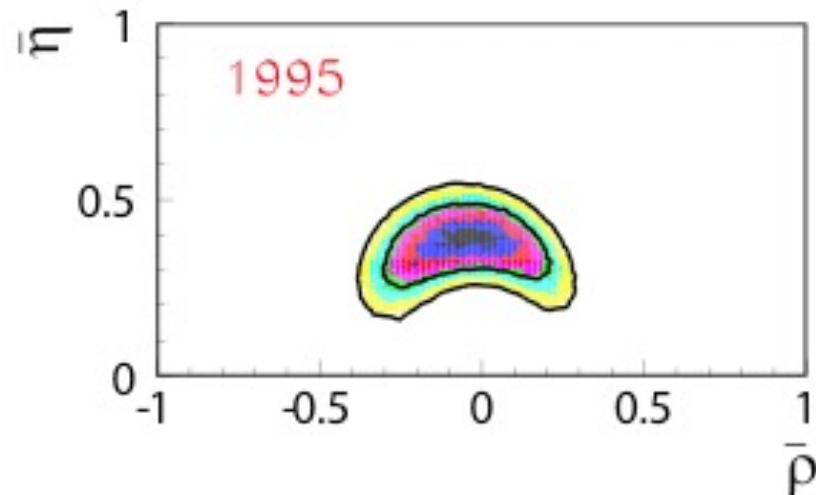
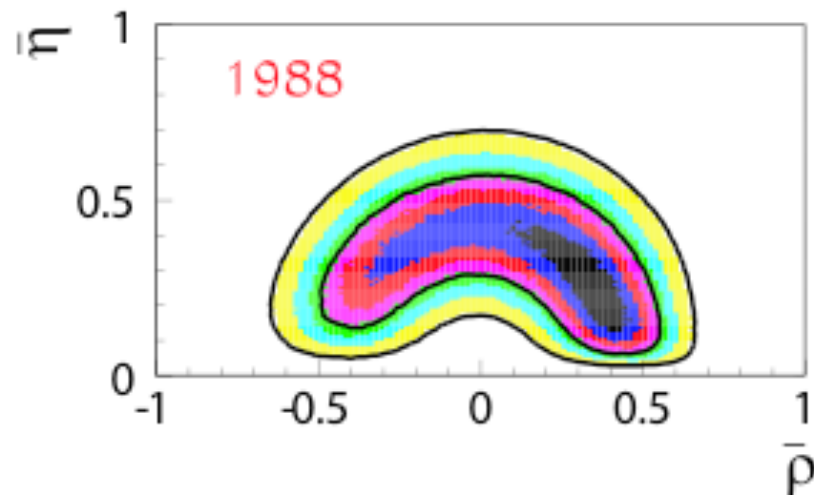
$$\eta = 0.361 \pm 0.035$$

Result summary

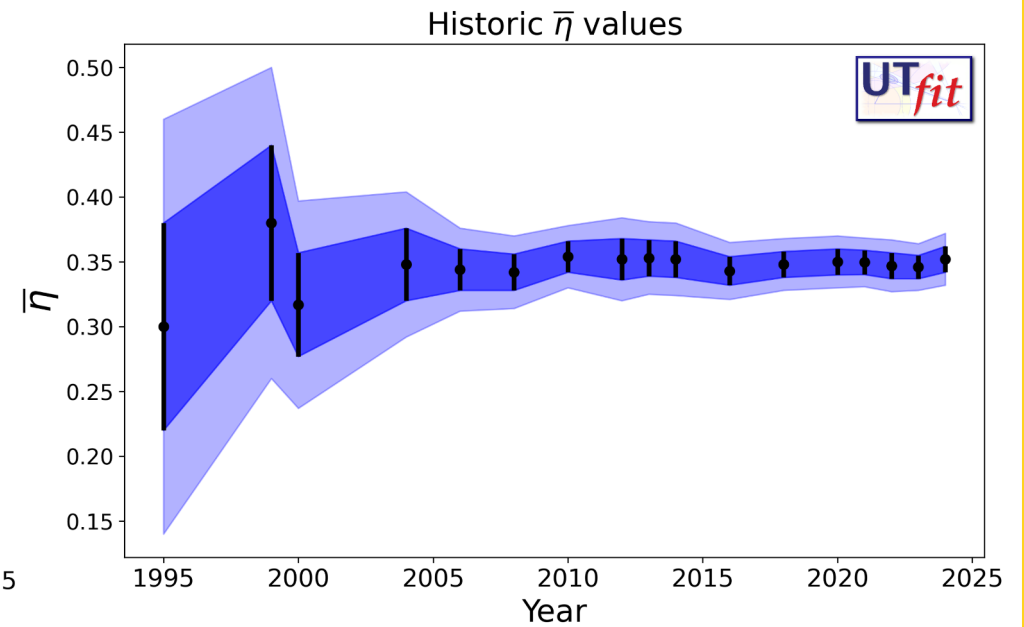
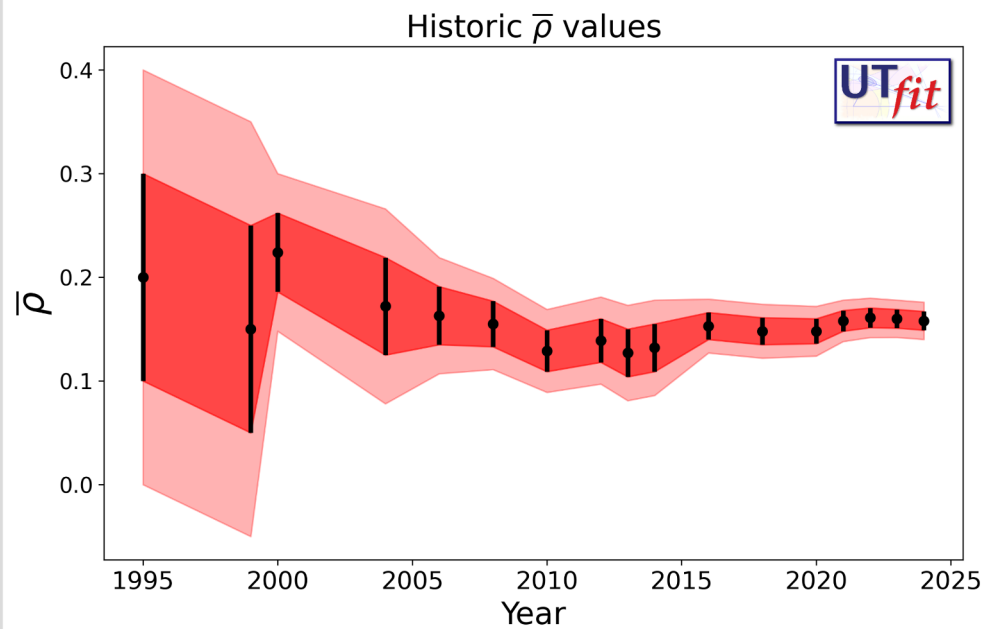
Observables	Measurement	Prediction	Pull ($\# \sigma$)
$\sin 2\beta$	0.692 ± 0.019	0.763 ± 0.030	~ 2
γ	67.2 ± 2.9	65.6 ± 1.4	< 1
α	95 ± 8	91.4 ± 1.4	< 1
$ V_{cb} \cdot 10^3$	41.20 ± 0.74	42.19 ± 0.48	~ 1.1
$ V_{cb} \cdot 10^3$ (excl)	40.13 ± 0.55		~ 2.8
$ V_{cb} \cdot 10^3$ (incl)	41.97 ± 0.48		< 1
$ V_{ub} \cdot 10^3$	3.84 ± 0.35	3.72 ± 0.10	< 1
$ V_{ub} \cdot 10^3$ (excl)	3.57 ± 0.23	—	< 1
$ V_{ub} \cdot 10^3$ (incl)	4.13 ± 0.26	—	~ 1.4
$\text{BR}(B \rightarrow \tau \nu)[10^4]$	1.09 ± 0.24	0.88 ± 0.05	< 1

PROGRESS SINCE 1988

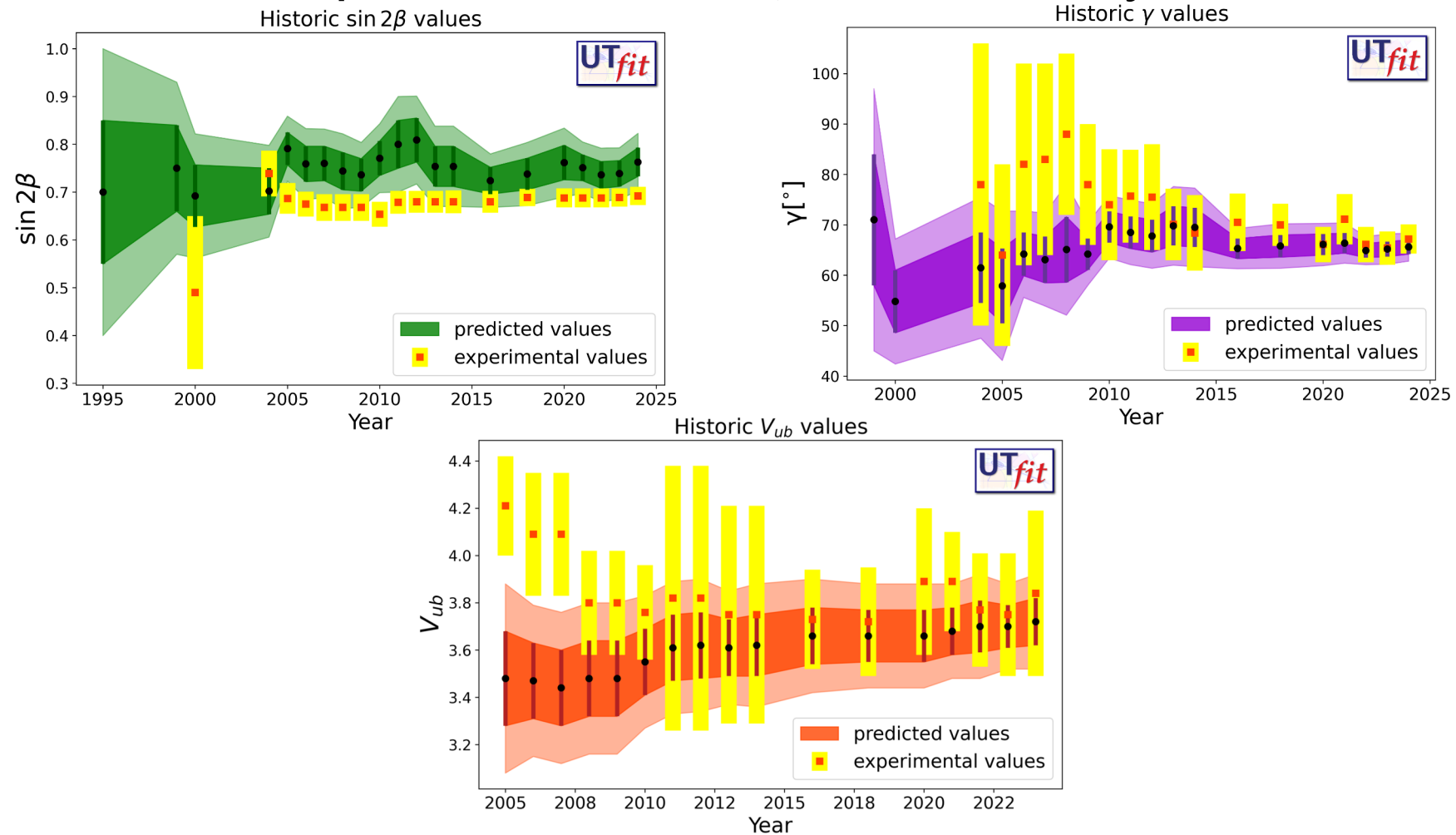
Experimental progress so impressive that we can fit the hadronic matrix elements (in the SM)



UTfit results across the years:



UTfit and experimental results across the years:

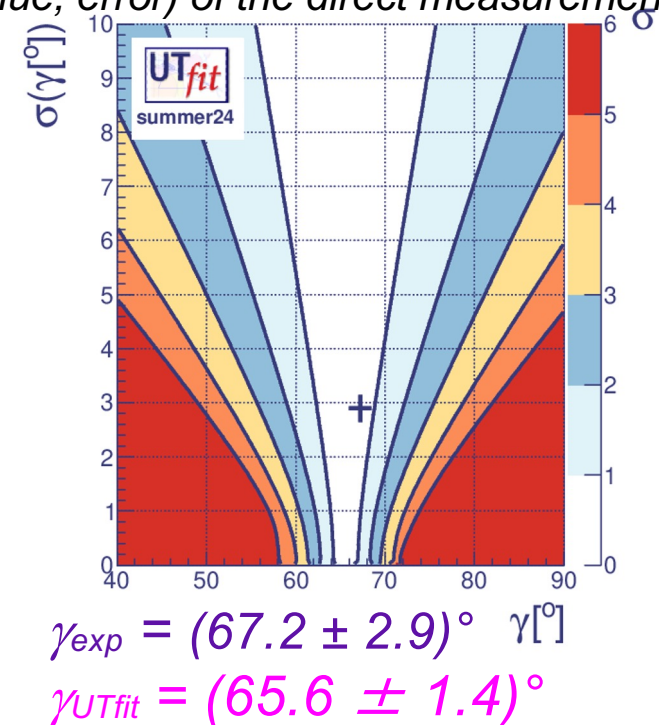
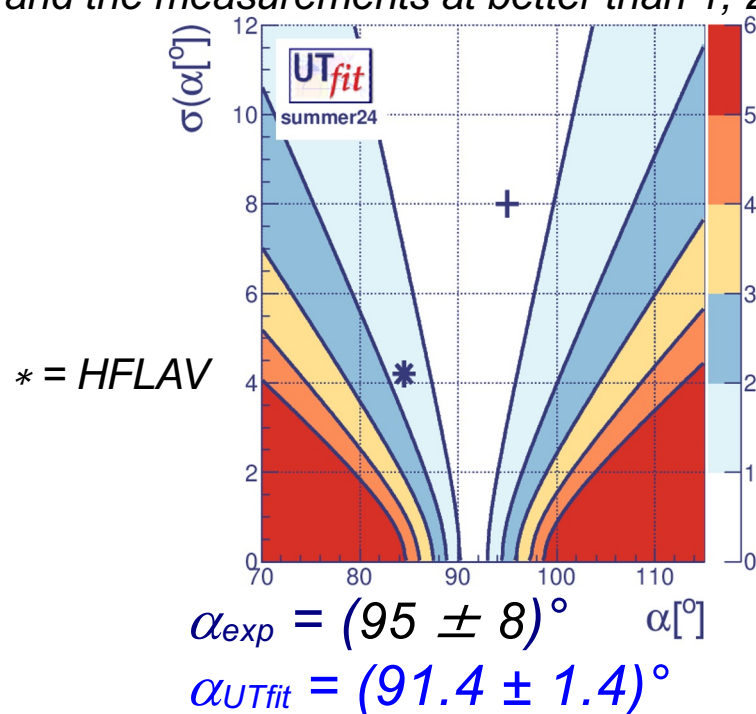


Compatibility plots

A way to “measure” the agreement of a single measurement with the indirect determination from the fit using all the other inputs: test for the SM description of the flavour physics

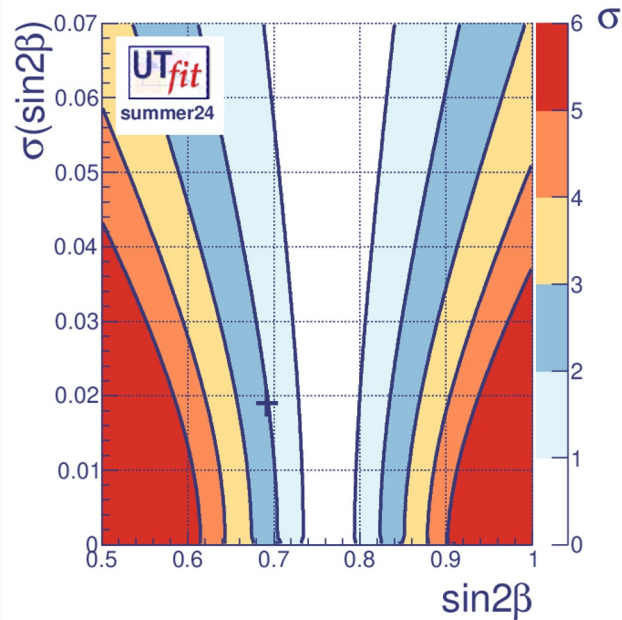
Colour code: agreement between the predicted values and the measurements at better than 1, 2, ... $n\sigma$

The cross has the coordinates (x,y)=(central value, error) of the direct measurement



$$\gamma = (68.4 \pm 2.6)^\circ \quad \text{F. Betti et al. } e\text{-Print: } \underline{2409.06449} \text{ [hep-ph]}$$

Checking the usual tensions..

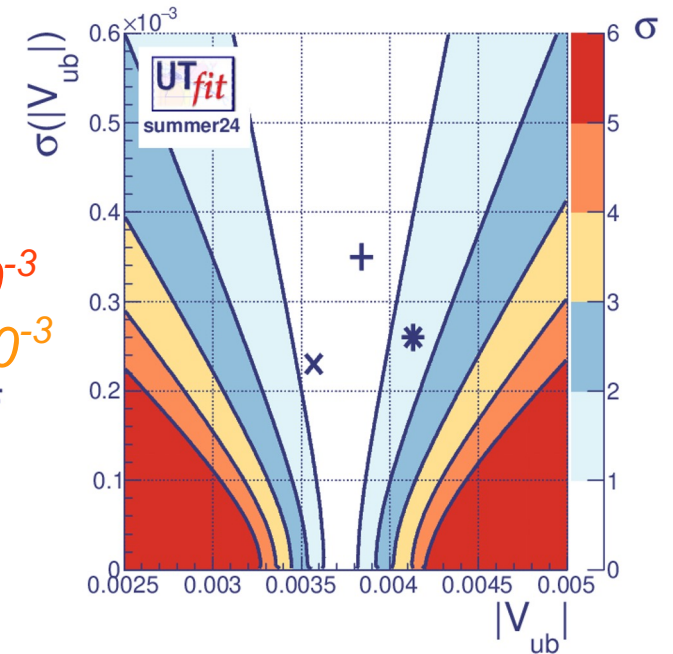
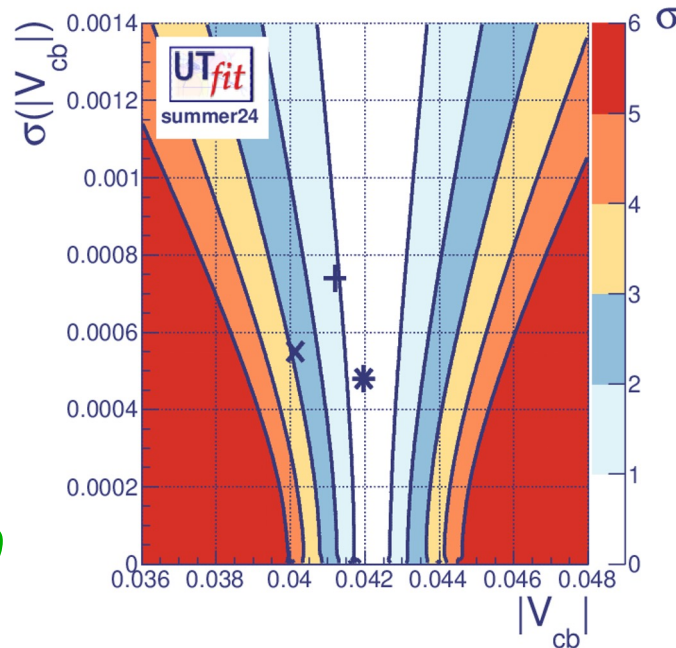


$$\sin 2\beta_{\text{exp}} = 0.692 \pm 0.019$$

$$\sin 2\beta_{\text{UTfit}} = 0.763 \pm 0.030$$

$$V_{cb_{\text{exp}}} = (41.20 \pm 0.74) \cdot 10^{-3}$$

$$V_{cb_{\text{UTfit}}} = (42.19 \pm 0.48) \cdot 10^{-3}$$



$$V_{ub_{\text{exp}}} = (3.84 \pm 0.35) \cdot 10^{-3}$$

$$V_{ub_{\text{UTfit}}} = (3.72 \pm 0.10) \cdot 10^{-3}$$

x = exclusive

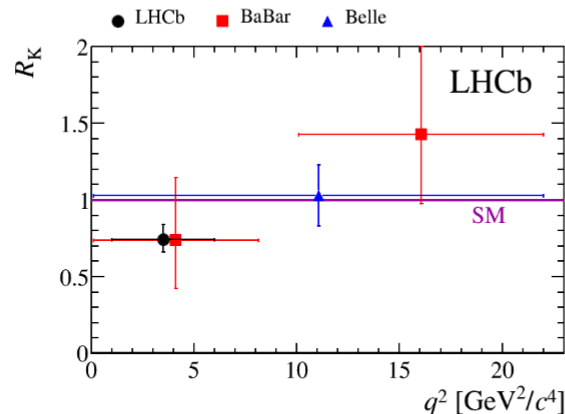
* = inclusive

An instructive story: Tension(s) in $b \rightarrow s$ decays ? Neutral Currents & Loop level

Reminder:

$$R_K = B(B^+ \rightarrow K^+ \mu^+ \mu^-) / B(B^+ \rightarrow K^+ e^+ e^-)$$

- Test of lepton universality : $R_K \sim 1$ in SM, with negligible theoretical uncertainties



LHCb, PRL 113 151601

Belle, PRL 103 171801

BaBar, PRD 86 032012

$$R_K(1 < q^2 < 6 \text{ GeV}^2) = 0.745^{+0.090}_{-0.074}(\text{stat}) \pm 0.036(\text{syst})$$

- Compatible with SM at 2.6σ
- Experimentally challenging
 - lower trigger efficiency for electrons, resolution deteriorated by bremsstrahlung
- Other modes suitable for same test:
 $B^0 \rightarrow K^{*0} l^+ l^-$, $B_s \rightarrow \phi l^+ l^-$, $\Lambda_B \rightarrow \Lambda l^+ l^-$

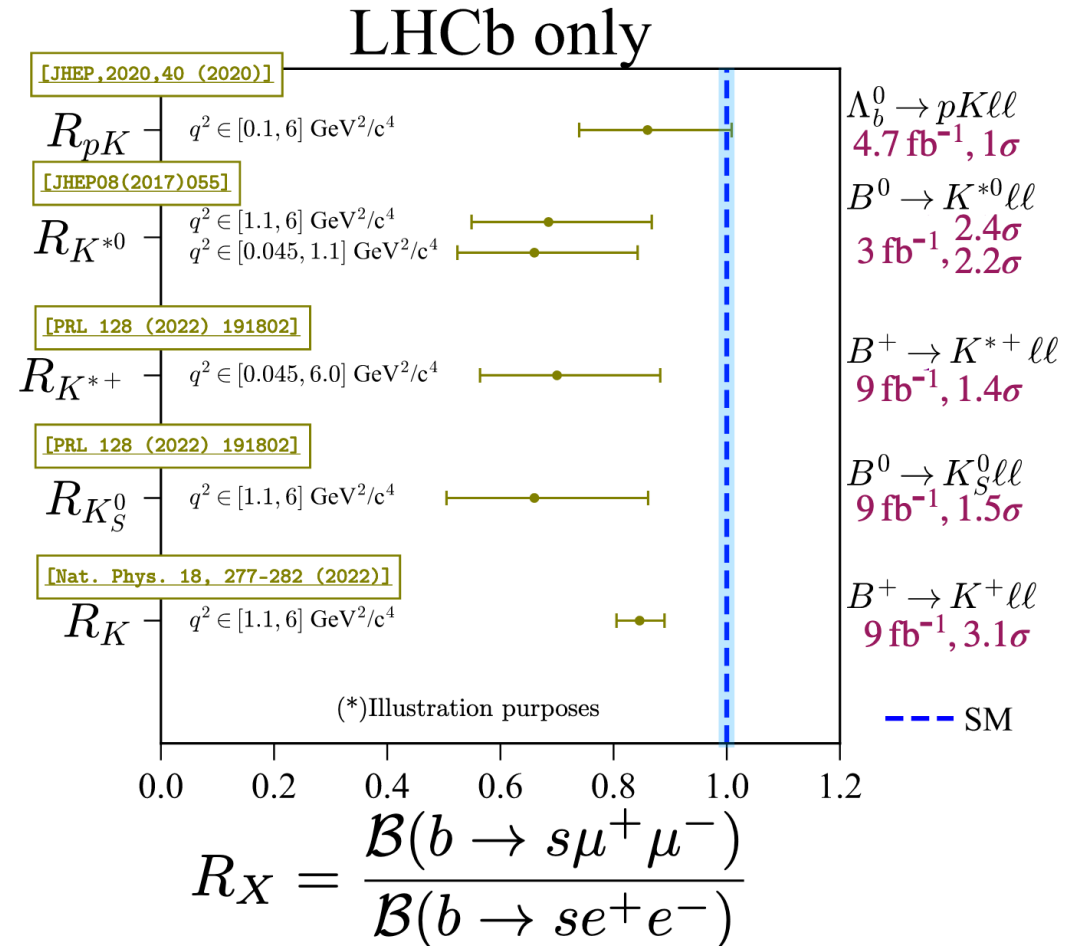
Old slide

Excitement

Analysis

Lepton Flavour Universality (LFU) tests in $b \rightarrow s\ell^+\ell^-$

- ◆ Coherent pattern of tension to SM in LFU test with $b \rightarrow s\ell^+\ell^-$ transition:
- ◆ R_X ratio extremely well predicted in SM
 - Cancellation of hadronic uncertainties at 10^{-4}
 - $\mathcal{O}(1\%)$ QED correction [Eur.Phys.J.C 76 (2016) 8]
 - Statistically limited
- ◆ Any departure from unity is a clear sign of New Physics



(*) Measurements from Belle not shown (larger statistical uncertainties)

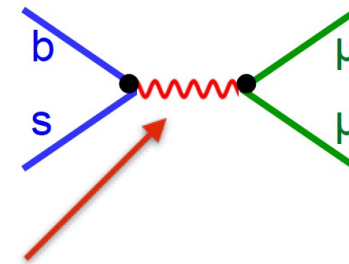
A EFT description

$b \rightarrow s$

A relatively sizable New Physics effect...

~30% of the Standard Model contribution (arising at one loop)

...hinting towards a relatively low New Physics scale:



generic tree $\frac{1}{\Lambda_{\text{NP}}^2} (\bar{s} \gamma_\nu P_L b) (\bar{\mu} \gamma^\nu \mu)$

$$\Lambda_{\text{NP}} \simeq 35 \text{ TeV} \times (C_9^{\text{NP}})^{-1/2}$$

MFV tree $\frac{1}{\Lambda_{\text{NP}}^2} V_{tb} V_{ts}^* (\bar{s} \gamma_\nu P_L b) (\bar{\mu} \gamma^\nu \mu)$

$$\Lambda_{\text{NP}} \simeq 7 \text{ TeV} \times (C_9^{\text{NP}})^{-1/2}$$

generic loop $\frac{1}{\Lambda_{\text{NP}}^2} \frac{1}{16\pi^2} (\bar{s} \gamma_\nu P_L b) (\bar{\mu} \gamma^\nu \mu)$

$$\Lambda_{\text{NP}} \simeq 3 \text{ TeV} \times (C_9^{\text{NP}})^{-1/2}$$

MFV loop $\frac{1}{\Lambda_{\text{NP}}^2} \frac{1}{16\pi^2} V_{tb} V_{ts}^* (\bar{s} \gamma_\nu P_L b) (\bar{\mu} \gamma^\nu \mu)$

$$\Lambda_{\text{NP}} \simeq 0.6 \text{ TeV} \times (C_9^{\text{NP}})^{-1/2}$$

Harakiri!



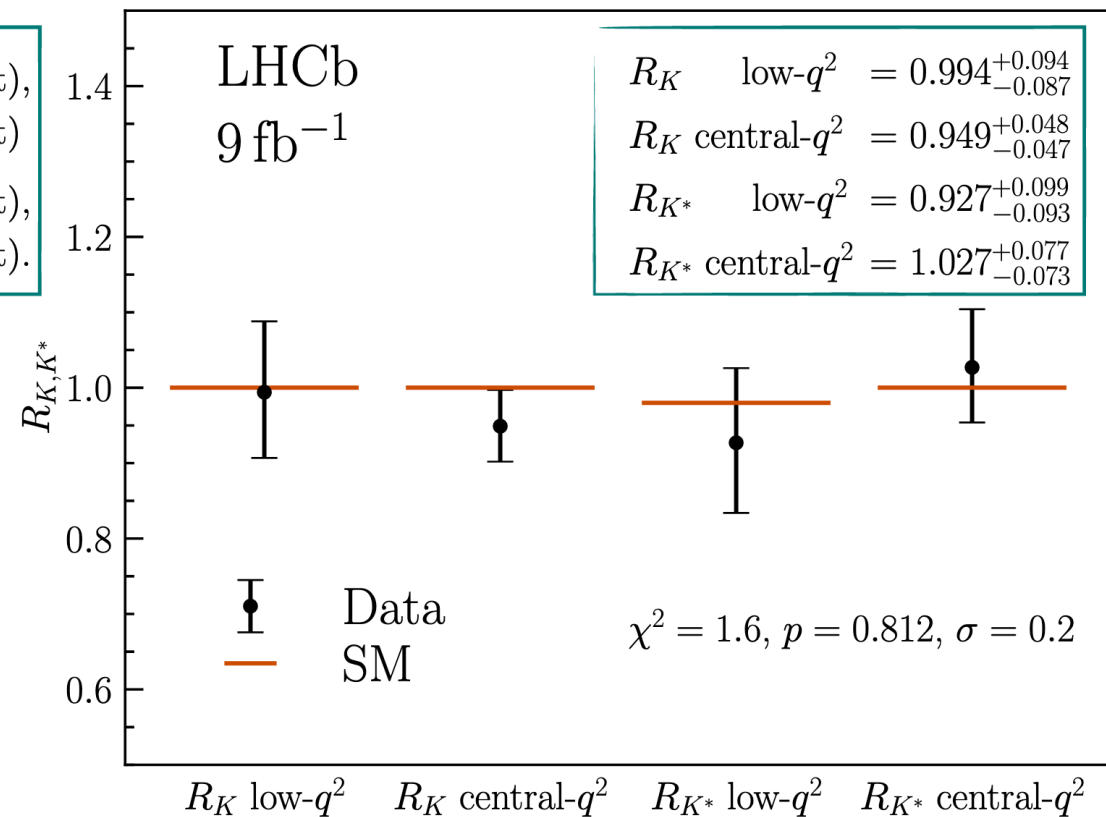
Analysis: results

Results

$$\begin{aligned} \text{low-}q^2 \begin{cases} R_K &= 0.994^{+0.090}_{-0.082} \text{ (stat)}^{+0.027}_{-0.029} \text{ (syst)}, \\ R_{K^*} &= 0.927^{+0.093}_{-0.087} \text{ (stat)}^{+0.034}_{-0.033} \text{ (syst)} \end{cases} \\ \text{central-}q^2 \begin{cases} R_K &= 0.949^{+0.042}_{-0.041} \text{ (stat)}^{+0.023}_{-0.023} \text{ (syst)}, \\ R_{K^*} &= 1.027^{+0.072}_{-0.068} \text{ (stat)}^{+0.027}_{-0.027} \text{ (syst)}. \end{cases} \end{aligned}$$

◆ Most precise and accurate LFU test in $b \rightarrow s\ell\ell$ transition

◆ Compatible with SM with a simple χ^2 test on 4 measurement at 0.2σ

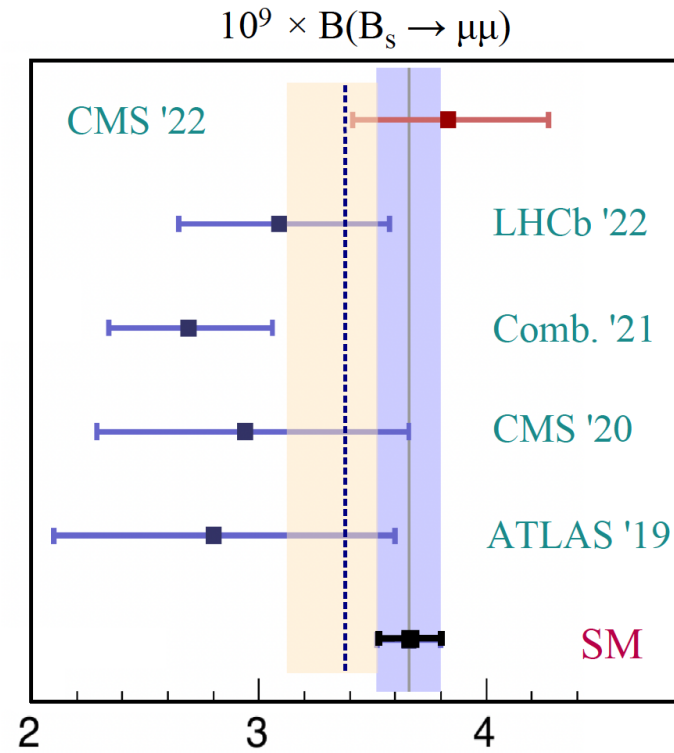
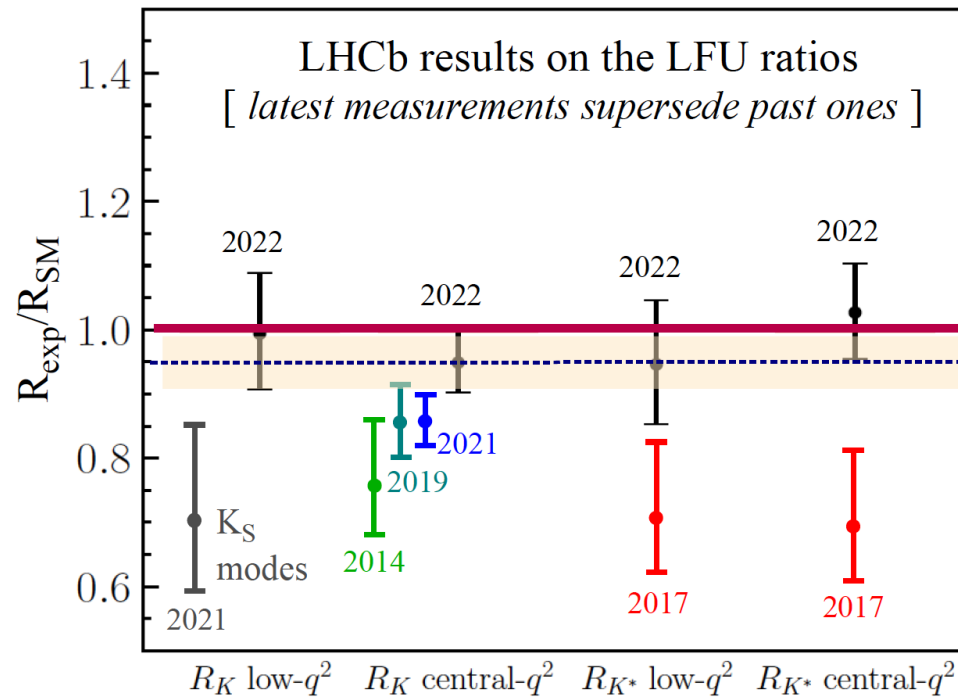


Hints of non-universality in B-physics

III. ~~LFU~~ anomaly in NC & BR($B_s \rightarrow \mu\mu$)

- Clean SM predictions
(LFU ratios + no long-distance in $B_s \rightarrow \mu\mu$)
- ~~Highest significance till summer 2022~~

$$\begin{aligned} \text{low-}q^2 \begin{cases} R_K &= 0.994^{+0.090}_{-0.082} \text{ (stat)}^{+0.027}_{-0.029} \text{ (syst)}, \\ R_{K^*} &= 0.927^{+0.093}_{-0.087} \text{ (stat)}^{+0.034}_{-0.033} \text{ (syst)} \end{cases} \\ \text{central-}q^2 \begin{cases} R_K &= 0.949^{+0.042}_{-0.041} \text{ (stat)}^{+0.023}_{-0.023} \text{ (syst)}, \\ R_{K^*} &= 1.027^{+0.072}_{-0.068} \text{ (stat)}^{+0.027}_{-0.027} \text{ (syst)}. \end{cases} \end{aligned}$$



$$\begin{aligned} \text{BR}(B_s \rightarrow \mu\mu)_{\text{exp}} &= (3.41 \pm 0.29) \times 10^{-9} & 9\% \\ \text{BR}(B_s \rightarrow \mu\mu)_{\text{SM}} &= (3.47 \pm 0.14) \times 10^{-9} & 4\% \end{aligned}$$

still

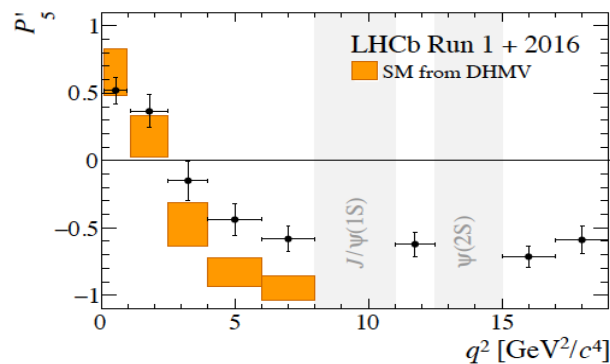
B anomalies in the post- R_K era

Nazila Mahmoudi

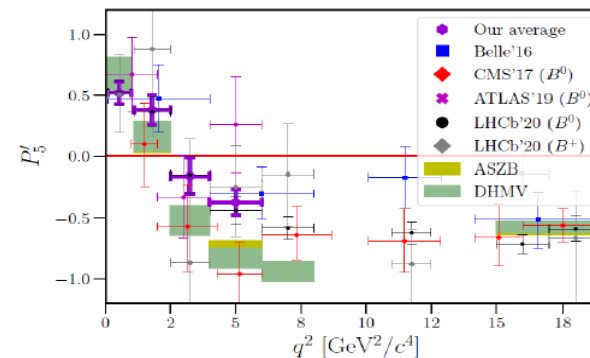
Corfu 2023

Tension in the angular observables - 2020 updates

$P'_5(B^0 \rightarrow K^{*0} \mu^+ \mu^-)$: 2020 LHCb update with 4.7 fb^{-1} : $\sim 2.9\sigma$ local tension

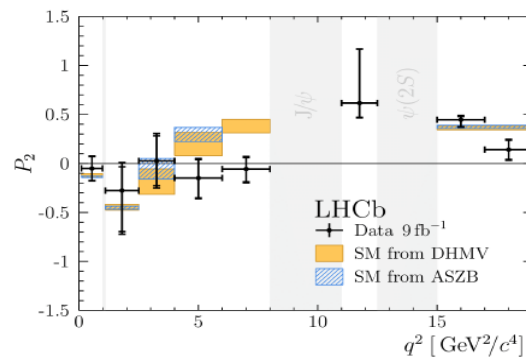


Phys. Rev. Lett. 125, 011802 (2020)

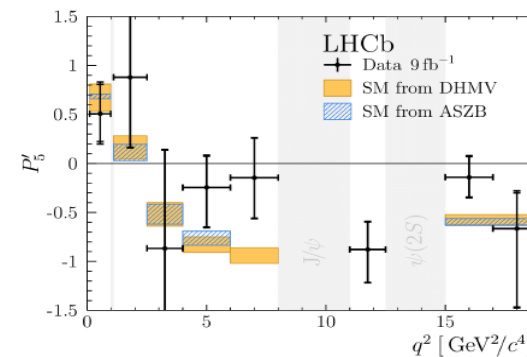


ATLAS-CONF-2017-023; CMS-PAS-BPH-11-008

First measurement of $B^+ \rightarrow K^{*+} \mu^+ \mu^-$ angular observables using the full Run 1 and Run 2 dataset (9 fb^{-1}):



Phys. Rev. Lett. 126, 161802 (2021)



*anomalies hunters never surrender
desperately seeking Susan NP*

*theoretical estimates of
unknown amplitudes*

The results confirm the global tension with respect to the SM!

Comparison of one-operator NP fits:

All observables 2022 ($\chi^2_{\text{SM}} = 253.3$)			
	b.f. value	χ^2_{min}	Pull _{SM}
δC_9	-0.95 ± 0.13	215.8	6.1σ
δC_9^e	0.82 ± 0.19	232.4	4.6σ
δC_9^μ	-0.92 ± 0.11	195.2	7.6σ
δC_{10}	0.08 ± 0.16	253.2	0.5σ
δC_{10}^e	-0.77 ± 0.18	230.6	4.8σ
δC_{10}^μ	0.43 ± 0.12	238.9	3.8σ
δC_{LL}^e	0.42 ± 0.10	231.4	4.7σ
δC_{LL}^μ	-0.43 ± 0.07	213.6	6.3σ

All observables 2023 ($\chi^2_{\text{SM}} = 231.3$)			
	b.f. value	χ^2_{min}	Pull _{SM}
δC_9	-0.96 ± 0.13	230.7	6.3σ
δC_9^e	0.21 ± 0.16	269.2	1.3σ
δC_9^μ	-0.69 ± 0.12	240.4	5.5σ
δC_{10}	0.15 ± 0.15	270.0	1.0σ
δC_{10}^e	-0.18 ± 0.14	269.3	1.3σ
δC_{10}^μ	0.16 ± 0.10	268.3	1.6σ
δC_{LL}	-0.54 ± 0.12	249.1	4.7σ
δC_{LL}^e	0.10 ± 0.08	269.2	1.3σ
δC_{LL}^μ	-0.23 ± 0.06	257.4	3.7σ

$\delta C_{\text{LL}}^\ell$ basis corresponds to $\delta C_9^\ell = -\delta C_{10}^\ell$.

But ... really a reliable estimate of uncertainties is missing and theory must be improved otherwise we will continue to generate anomalies out of our ignorance

Known unknowns in $B \rightarrow K^* \mu \mu$

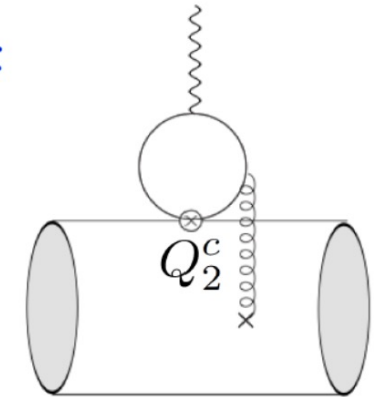
$$H_V^\lambda = \frac{4iG_F m_B}{\sqrt{2}} \frac{e^2}{16\pi^2} \lambda_t \left\{ C_9^{\text{eff}} \tilde{V}_{L\lambda} + \frac{m_B^2}{q^2} \left[\frac{2m_b}{m_B} C_7^{\text{eff}} \tilde{T}_{L\lambda} - 16\pi^2 h_\lambda \right] \right\}$$

$$h_\lambda(q^2) = \frac{\epsilon_\mu^*(\lambda)}{m_B^2} \int d^4x e^{iqx} \langle \bar{K}^* | T \{ j_{\text{em}}^\mu(x) \mathcal{H}_{\text{eff}}^{\text{had}}(0) \} | \bar{B} \rangle$$

Non-factorizable power-suppressed contributions of 4-quark operators to the matrix element

- dominated by

$$\begin{aligned} Q_1^c &= (\bar{s}_L \gamma_\mu T^a c_L) (\bar{c}_L \gamma^\mu T^a b_L), \\ Q_2^c &= (\bar{s}_L \gamma_\mu c_L) (\bar{c}_L \gamma^\mu b_L), \end{aligned}$$



the charm pair can be close to the resonant region

See IFIC
Colloquium

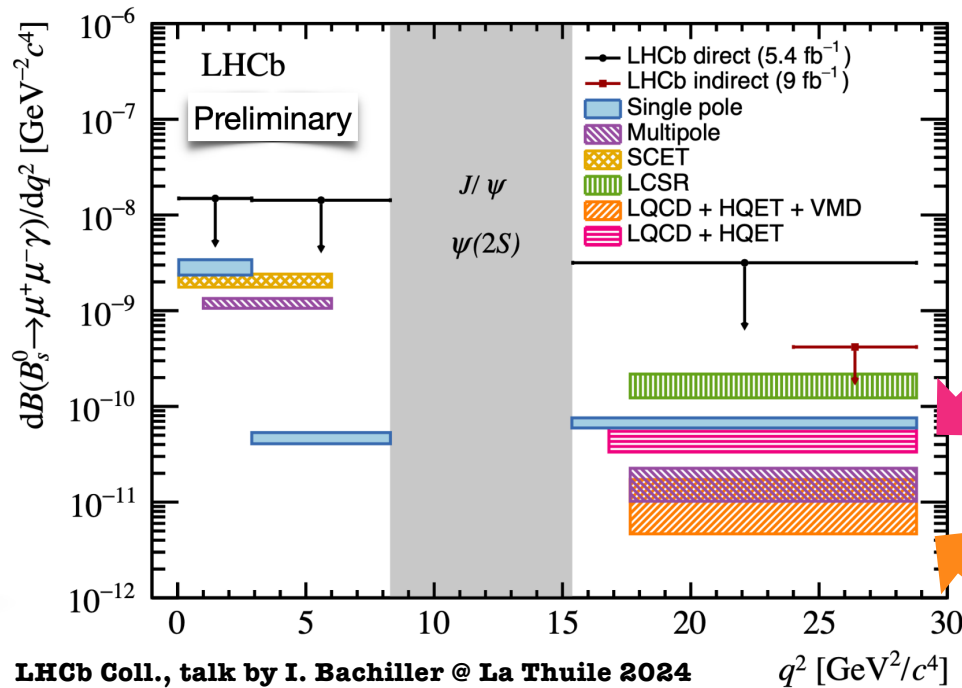
Do we know how to compute them?

In general, no!

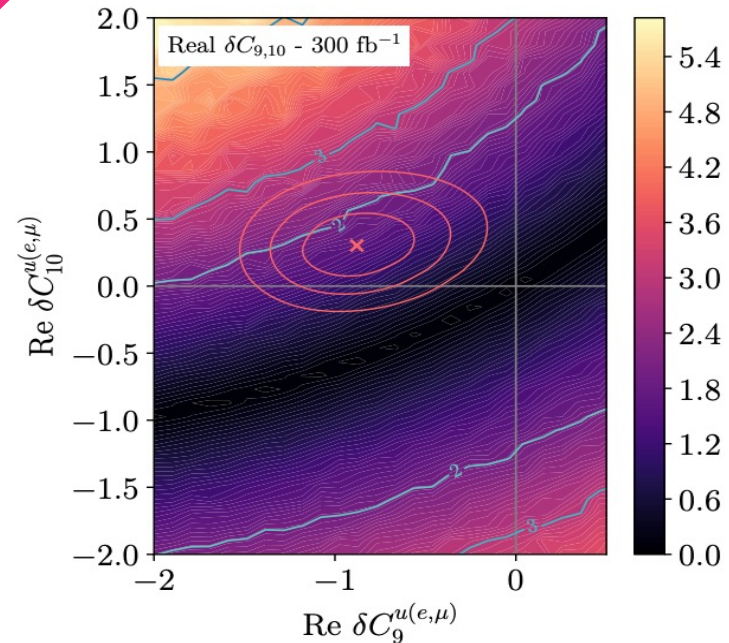
Courtesy by L. Silvestrini

Look for complementary $b \rightarrow s$ transitions

$B_s \rightarrow \mu\mu\gamma$ @ high- q^2 : in this range the observables depend on the same short distance effects as those present in $B \rightarrow K(*) l^+ l^-$ but long distance contributions are expected to be rather small



Theoretical progresses:
 First lattice calculation by the
 Rome-Southampton Collaboration
 G. Gagliardi et al. (2402.03262)



Guadagnoli, Normand, Simula, Vittorio,
 JHEP '23 [2308.00034]

Look for complementary $b \rightarrow s$ transitions

$B \rightarrow K^{(*)}\nu\nu$: short distance contributions dominate

Main uncertainties

1. $|V_{cb}|$
2. Form factors

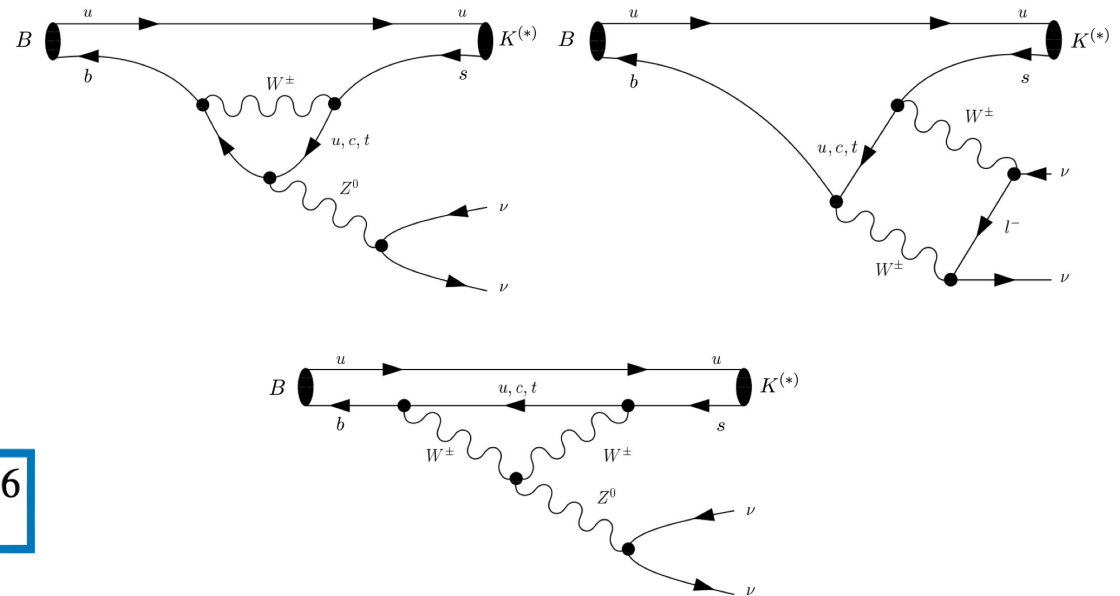
SM prediction *

$$\mathcal{B}(B^\pm \rightarrow K^\pm \nu \bar{\nu}) = (4.44 \pm 0.30) \times 10^{-6}$$

Jernej F. Kamenik(Frascati) Christopher Smith(Karlsruhe U., TTP)
Phys.Lett.B 680 (2009) 471-475 [0908.1174](#) [hep-ph]

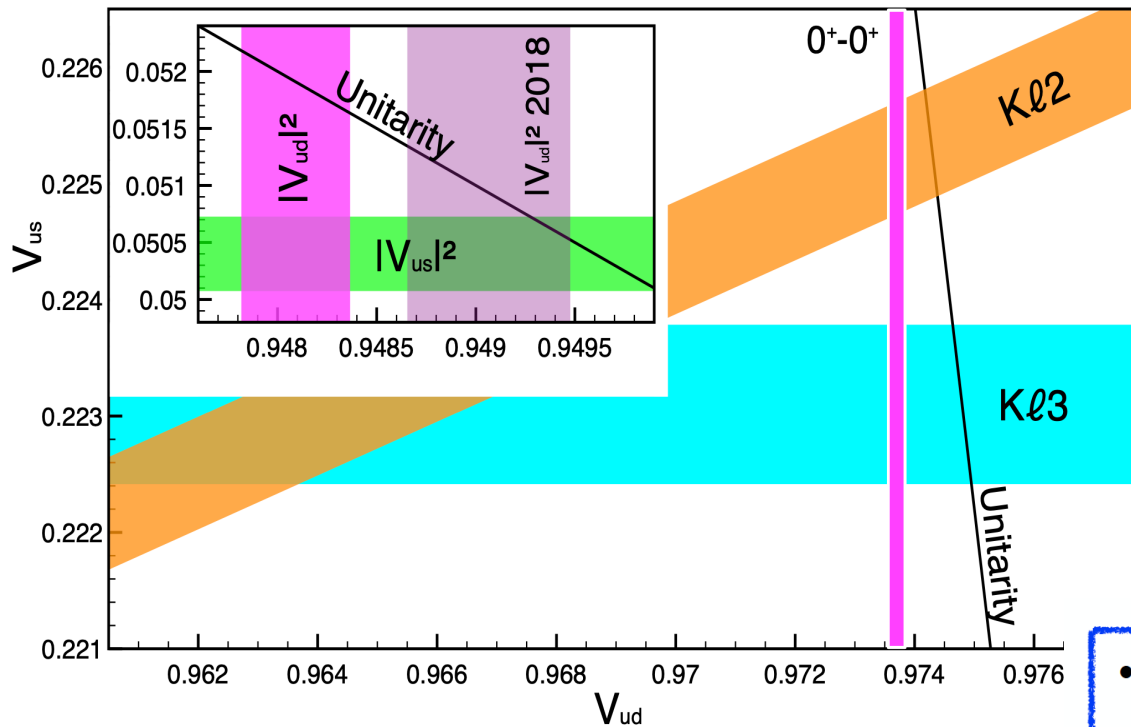
D. Becirevic, G. Piazza & O. Sumensari, EPJC '23 [arXiv:2301.06990]
Phys.Lett.B 848 (2024) 138411 [2309.02246](#) *

$$\mathcal{B}(B^+ \rightarrow K^+ \nu \bar{\nu}) \Big|_{\text{Belle-II}} = (2.4 \pm 0.7) \times 10^{-5}$$



Tension in $B \rightarrow K\nu\nu$:
 2.8σ

Tensions with the unitarity of the first CKM row ?



M. Gorshteyn, talk @ CKM23 conference

Both theory and experiments demands a closer scrutiny of systematic errors

For the enthusiasts

- Until ~2018, bands *did* intersect in the same region on the unitarity circle ($< 2\sigma$)
- Main changes since then:
 - V_{us} from Kl3 decreased ($\langle V \rangle$ increased with smaller uncertainty, 2+1+1 lattice QCD)
 - V_{ud} decreased (radiative corrections in nuclear & neutron increased with smaller uncertainty, dispersive)

$$|V_{ud}|^2 + |V_{us}|^2 + |\cancel{V_{ub}}|^2 = 0.9985(6)_{V_{ud}}(4)_{V_{us}}$$

$\sim 0.95 \quad \sim 0.05 \quad \sim 10^{-5}$

Determination of $|V_{ud}|$ and $|V_{us}|$

$|V_{ud}|$

M. Gorshteyn, talk @ CKM23 conference

- Nuclear decays 0^+-0^+ (es.: $^{14}\text{O} \rightarrow ^{14}\text{N}$)

$$|V_{ud}^{0^+-0^+}| = 0.97370 (1)_{\text{exp, nucl}} (3)_{\text{NS}} (1)_{\text{RC}} [3]_{\text{total}}$$

- Neutron β decay

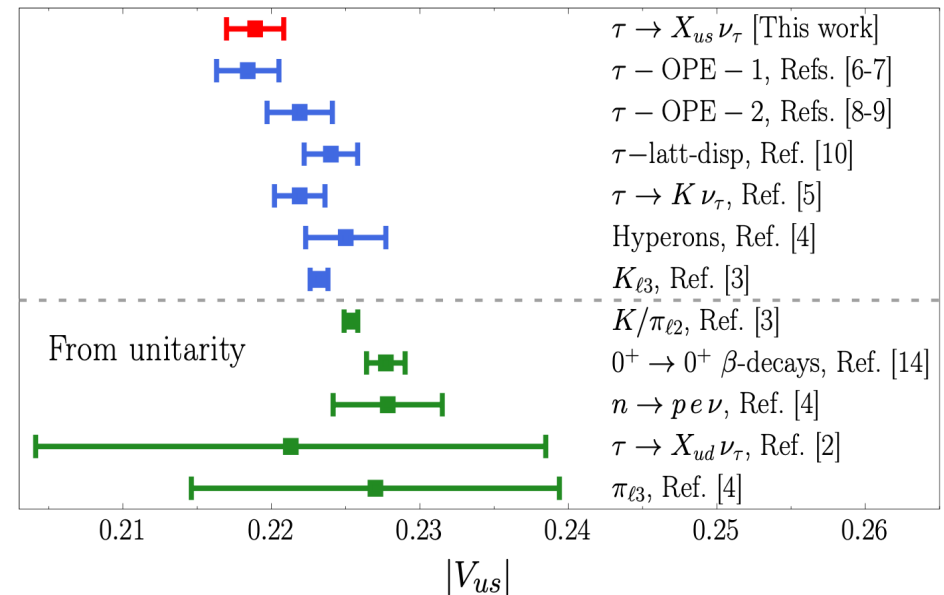
$$|V_{ud}^{\text{free n}}| = 0.9733 (2)_{\tau_n} (3)_{g_A} (1)_{\text{RC}} [4]_{\text{total}}$$

- $\pi^+ \rightarrow \pi^0 e^+ \nu$:

$$|V_{ud}^{\pi\ell^3}| = 0.9739 (27)_{\text{exp}} (1)_{\text{RC}}$$

improvable in the future with PIONEER (2203.01981)

$|V_{us}|$

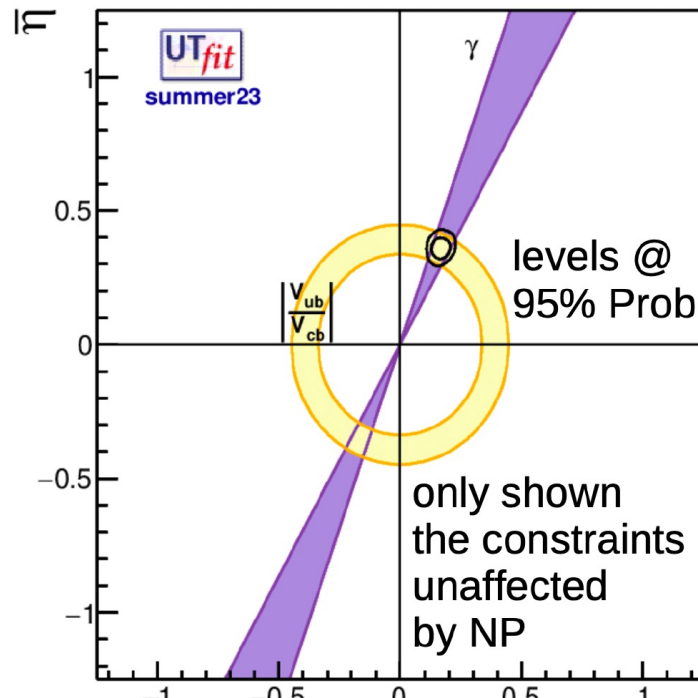


ETMC Collaboration [arXiv:2403.05404]



.... beyond
the Standard Model

Results of BSM analysis: CKM parameters

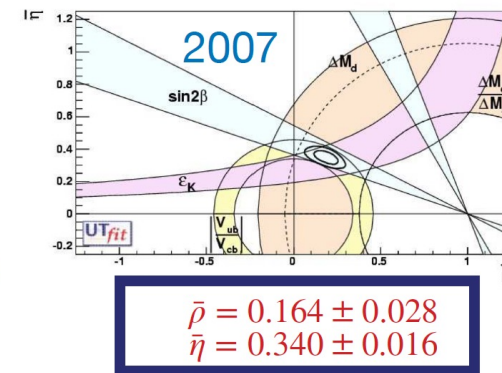


CKM parameters from BSM analysis

$$\bar{\rho} = 0.167 \pm 0.025$$

$$\bar{\eta} = 0.361 \pm 0.027$$

CKM
parameters
known (even in
presence of NP
effects) with
similar precision
of pre-LHC SM
analysis
2004



1. The CKM phase is different from zero
2. The CKM phase is the dominant source of CP violation at low energy
3. No evidence for corrections to CKM
4. NP contributions to observed FCNC at most comparable (smaller) than the CKM ones
5. NP contributions very small in $s \rightarrow d$, $c \rightarrow u$, $b \rightarrow d$, $b \rightarrow s$

$$Q^{EXP} = V_{CKM} \langle F | \hat{O} | I \rangle$$

Constrains on NP from UTfit

$$Q^{EXP} = \sum_i C_{SM}^i(M_W, m_t, \alpha_s) \langle F | \hat{O}_i | I \rangle + \sum_{i'} C_{Beyond}^{i'}(\tilde{m}_\beta, \alpha_s) \langle F | \hat{O}_{i'} | I \rangle$$

UT generalization Beyond the Standard Model

- fit simultaneously for the CKM and the NP parameters (generalized UT analysis)
- parameterize BSM effects in $\Delta F = 2$ Hamiltonian in model-independent
- use all available experimental information
- find out NP contributions to $\Delta F=2$ transitions

$$A_q = C_{B_q} e^{2i\phi_{B_q}} A_q^{SM} e^{2i\phi_q^{SM}} = \left(1 + \frac{A_q^{NP}}{A_q^{SM}} e^{2i(\phi_q^{NP} - \phi_q^{SM})} \right) A_q^{SM} e^{2i\phi_q^{SM}}$$

$$\begin{aligned} \Delta m_{q/K} &= C_{B_q/\Delta m_K} (\Delta m_{q/K})^{SM} \\ A_{CP}^{B_d \rightarrow J/\psi K_s} &= \sin 2(\beta + \phi_{B_d}) \\ A_{SL}^q &= \text{Im}(\Gamma_{12}^q / A_q) \\ \varepsilon_K &= C_\varepsilon \varepsilon_K^{SM} \\ A_{CP}^{B_s \rightarrow J/\psi \phi} &\sim \sin 2(-\beta_s + \phi_{B_s}) \\ \Delta \Gamma^q / \Delta m_q &= \text{Re}(\Gamma_{12}^q / A_q) \end{aligned}$$

New local four-fermion operators are generated

$$Q_1 = (\bar{b}_L^A \gamma_\mu d_L^A) (\bar{b}_L^B \gamma_\mu d_L^B) \quad \text{SM}$$

$$Q_2 = (\bar{b}_R^A d_L^A) (\bar{b}_R^B d_L^B)$$

$$Q_3 = (\bar{b}_R^A d_L^B) (\bar{b}_R^B d_L^A)$$

$$Q_4 = (\bar{b}_R^A d_L^A) (\bar{b}_L^B d_R^B)$$

$$Q_5 = (\bar{b}_R^A d_L^B) (\bar{b}_L^B d_R^A)$$

+ those obtained by $L \leftrightarrow R$

Similarly for the s quark e.g.

$$(\bar{s}_R^A d_L^A) (s_R^B d_L^B)$$

$$\langle \bar{K}^0 | O_1(\mu) | K^0 \rangle = \frac{8}{3} M_K^2 f_K^2 B_1(\mu) ,$$

$$\langle \bar{K}^0 | O_2(\mu) | K^0 \rangle = -\frac{5}{3} \left(\frac{M_K}{m_s(\mu) + m_d(\mu)} \right)^2 M_K^2 f_K^2 B_2(\mu) ,$$

$$\langle \bar{K}^0 | O_3(\mu) | K^0 \rangle = \frac{1}{3} \left(\frac{M_K}{m_s(\mu) + m_d(\mu)} \right)^2 M_K^2 f_K^2 B_3(\mu) ,$$

$$\langle \bar{K}^0 | O_4(\mu) | K^0 \rangle = 2 \left(\frac{M_K}{m_s(\mu) + m_d(\mu)} \right)^2 M_K^2 f_K^2 B_4(\mu) ,$$

$$\langle \bar{K}^0 | O_5(\mu) | K^0 \rangle = \frac{2}{3} \left(\frac{M_K}{m_s(\mu) + m_d(\mu)} \right)^2 M_K^2 f_K^2 B_5(\mu) ,$$

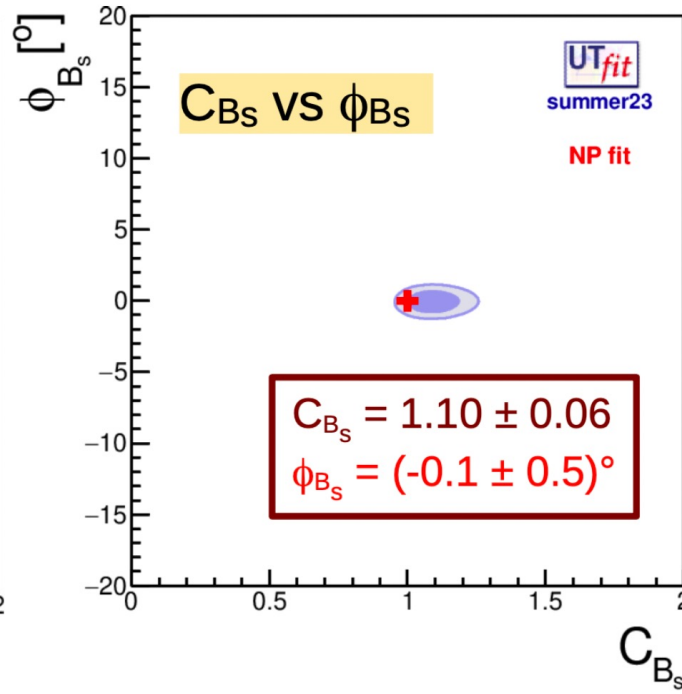
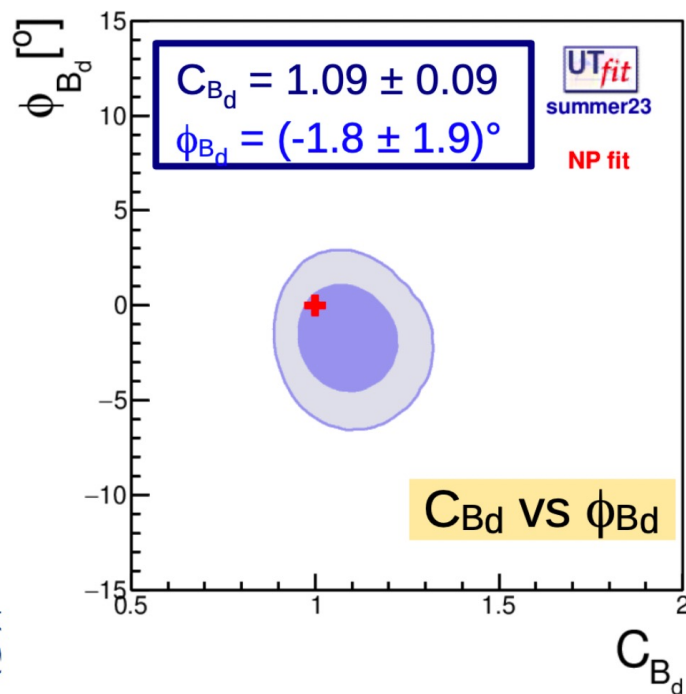
Results of BSM analysis: New Physics parameters

$$A_q = C_{B_q} e^{2i\phi_{B_q}} A_q^{SM} e^{2i\phi_q^{SM}}$$

K system

$$C_{e_K} = 1.09 \pm 0.10$$

dark: 68%
light: 95%
SM: red cross

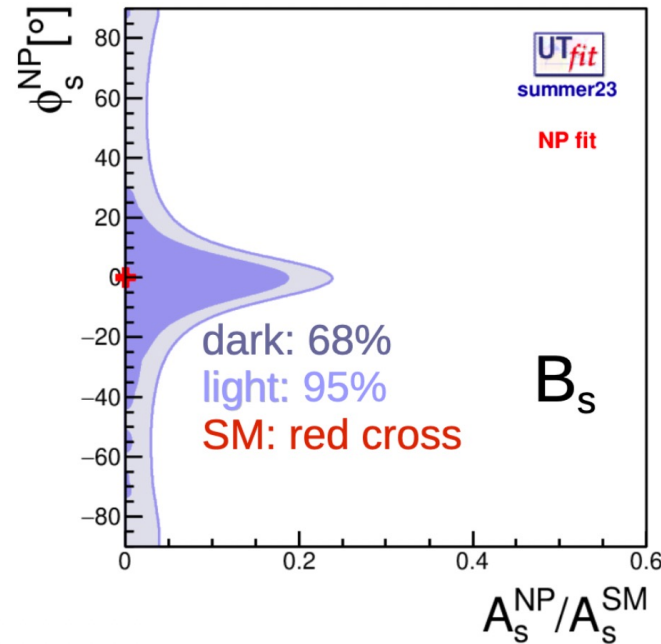
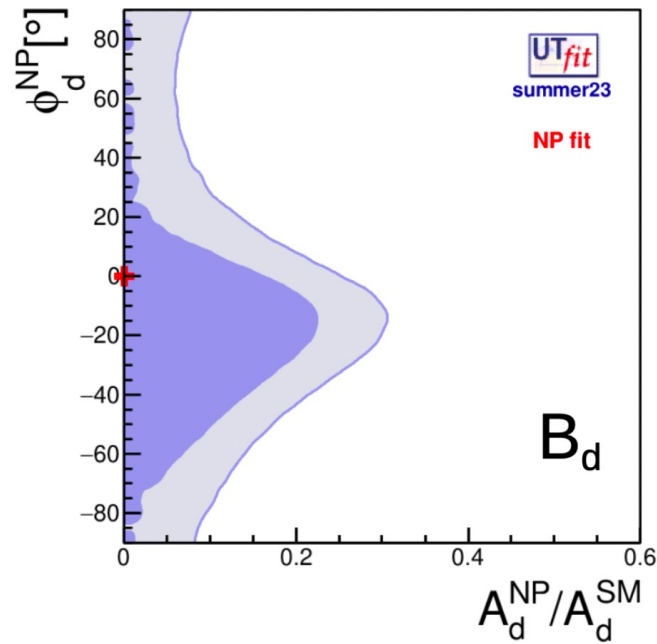


Results of BSM analysis: New Physics parameters

$$A_q = \left(1 + \frac{A_q^{NP}}{A_q^{SM}} e^{2i(\phi_q^{NP} - \phi_q^{SM})} \right) A_q^{SM} e^{2i\phi_q^{SM}}$$

The ratio of NP/SM amplitudes is:
 < 25% @68% prob. (35% @95%) in B_d mixing
 < 25% @68% prob. (30% @95%) in B_s mixing

dark: 68%
 light: 95%
 SM: red cross



Testing the new-physics scale

M. Bona *et al.* (UTfit)
JHEP 0803:049,2008
arXiv:0707.0636

R
G
E

At the high scale

new physics enters according to its specific features

At the low scale

use OPE to write the most general effective Hamiltonian. the operators have different chiralities than the SM

NP effects are in the Wilson Coefficients C

$$C_i(\Lambda) = F_i \frac{L_i}{\Lambda^2}$$

$$\mathcal{H}_{\text{eff}}^{\Delta B=2} = \sum_{i=1}^5 C_i Q_i^{bq} + \sum_{i=1}^3 \tilde{C}_i \tilde{Q}_i^{bq}$$

$$Q_1^{q_i q_j} = \bar{q}_{jL}^\alpha \gamma_\mu q_{iL}^\alpha \bar{q}_{jL}^\beta \gamma^\mu q_{iL}^\beta,$$

$$Q_2^{q_i q_j} = \bar{q}_{jR}^\alpha q_{iL}^\alpha \bar{q}_{jR}^\beta q_{iL}^\beta,$$

$$Q_3^{q_i q_j} = \bar{q}_{jR}^\alpha q_{iL}^\beta \bar{q}_{jR}^\beta q_{iL}^\alpha,$$

$$Q_4^{q_i q_j} = \bar{q}_{jR}^\alpha q_{iL}^\alpha \bar{q}_{jL}^\beta q_{iR}^\beta,$$

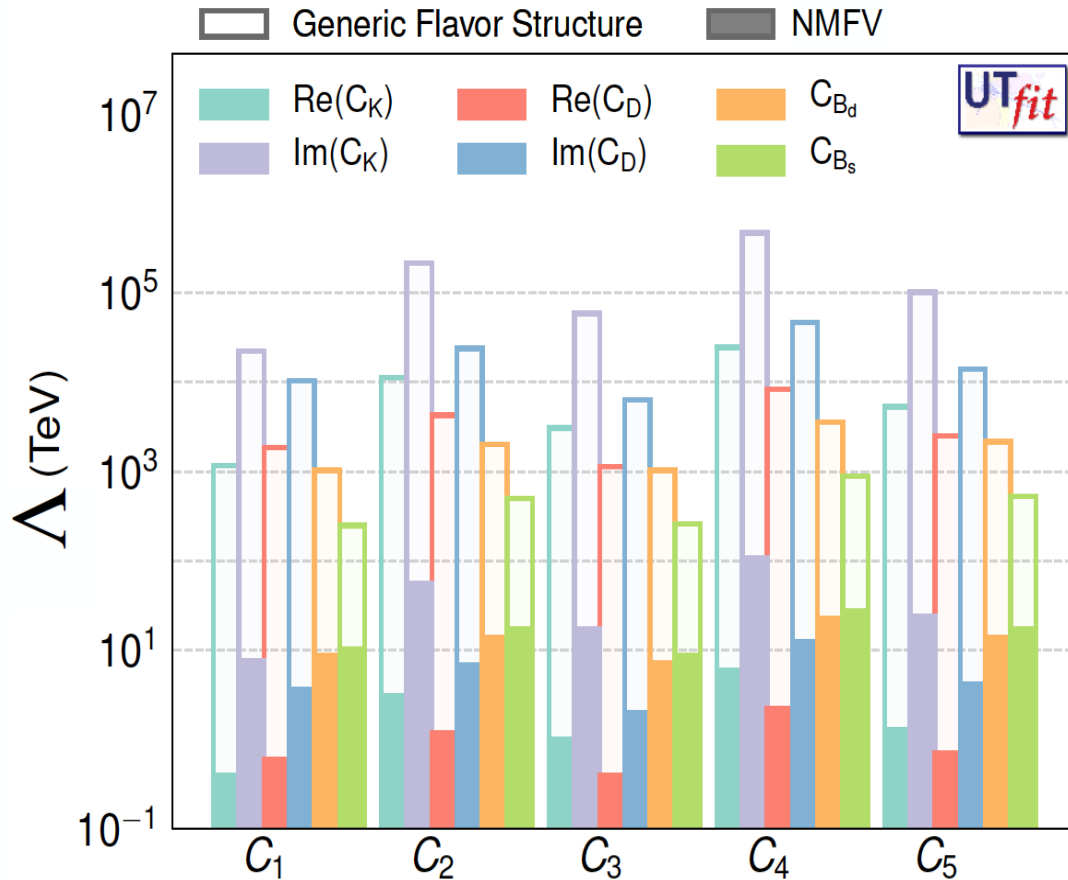
$$Q_5^{q_i q_j} = \bar{q}_{jR}^\alpha q_{iL}^\beta \bar{q}_{jL}^\beta q_{iR}^\alpha.$$

F_i : function of the NP flavour couplings

L_i : loop factor (in NP models with no tree-level FCNC)

Λ : NP scale (typical mass of new particles mediating $\Delta F=2$ processes)

Results of BSM analysis: probing New Physics Scale



- $\alpha \sim \alpha_w$ in case of loop coupling
- through weak interactions*

$$\Lambda > 1.3 \cdot 10^4 \text{ TeV}$$

$$\mathcal{H}_{\text{eff}}^{\Delta B=2} = \sum_{i=1}^5 C_i Q_i^{bq} + \sum_{i=1}^3 \tilde{C}_i \tilde{Q}_i^{bq}$$

$$Q_1^{q_i q_j} = \bar{q}_{jL}^\alpha \gamma_\mu q_{iL}^\alpha \bar{q}_{jL}^\beta \gamma^\mu q_{iL}^\beta,$$

$$Q_2^{q_i q_j} = \bar{q}_{jR}^\alpha q_{iL}^\alpha \bar{q}_{jR}^\beta q_{iL}^\beta,$$

$$Q_3^{q_i q_j} = \bar{q}_{jR}^\alpha q_{iL}^\beta \bar{q}_{jR}^\beta q_{iL}^\alpha,$$

$$Q_4^{q_i q_j} = \bar{q}_{jR}^\alpha q_{iL}^\alpha \bar{q}_{jL}^\beta q_{iR}^\beta,$$

$$Q_5^{q_i q_j} = \bar{q}_{jR}^\alpha q_{iL}^\beta \bar{q}_{jL}^\beta q_{iR}^\alpha.$$

$$C_i(\Lambda) = F_i \frac{L_i}{\Lambda^2}$$

- **Generic:** $C(\Lambda) = \alpha/\Lambda^2$ $F_i \sim 1$, arbitrary phase
- **NMFV:** $C(\Lambda) = \alpha \times |F_{\text{SM}}|/\Lambda^2$ $F_i \sim |F_{\text{SM}}|$, arbitrary phase

- $\alpha \sim \alpha_w$ in case of loop coupling
- through weak interactions*

$$\Lambda > 2.7 \text{ TeV}$$

- 1) NP must explain the strong hierarchy of the Fermion couplings/masses
- 2) If the scale of NP it is not too high it must suppresses FCNC processes at an acceptable level

$$Y_t \sim 1 \quad Y_c \sim 10^{-2} \quad Y_u \sim 10^{-5}$$

$$Y_b \sim 10^{-2} \quad Y_s \sim 10^{-3} \quad Y_d \sim 10^{-5}$$

$$Y_\tau \sim 10^{-2} \quad Y_\mu \sim 10^{-3} \quad Y_e \sim 10^{-6}$$

$$|V_{us}| \sim 0.2 \quad |V_{cb}| \sim 0.04 \quad |V_{ub}| \sim 0.004 \quad \delta \sim 1$$

$$0.1 \sim g', \quad g, \quad g_s, \quad \lambda \sim 1.$$

***FUTURE, BSM:** It is difficult to make predictions,
especially about the future*



4th generation



extended Higgs
sectors



extended
technicolor



left-right
symmetry



leptoquarks



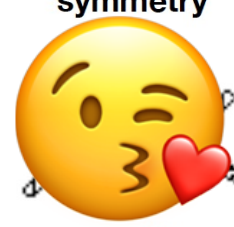
universal extra
dimensions



large extra
dimensions



warped extra
dimensions



gauge-Higgs
unification



Higgsless
models



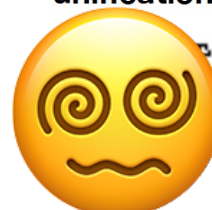
MSSM



CMSSM



NMSSM



νMSSM



SUSY GUTs



unparticles



Little Higgs



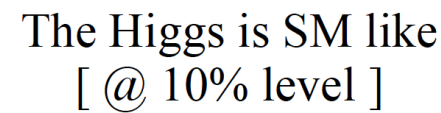
hidden valleys



not yet thought of ...

3rd family
NP

Universal NP



If these ideas corrects, new non-standard effects should emerge soon both at low and at high energies ($\rightarrow$ very interesting opportunities for run-3...).

absence says more than presence

FRANK HERBERT

(Dune)

THANKS FOR YOUR ATTENTION

