

Flavour Physics in the Standard Model and Beyond

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Some General Consideration:

Discrete, Continuum, Global o Local

Discrete Symmetries C P T:

- 1) Charge Conjugation
- 2) Parity
- 3) Time reversal

All violated at the quantum level

CPT conserved in a local relativistic
quantum field theory (vacuum)

C, CP and CPT and their violation are related to the foundations of modern physics (Relativistic quantum mechanics, Locality, Matter-Antimatter properties, Cosmology etc.)

Although in the Standard Model (SM) all ingredients are present, new sources of $\cancel{CP}$ beyond the SM are necessary to explain quantitatively the BAU

Almost all New Physics Theories generate new sources of $\cancel{CP}$

$$S = \bar{\psi}_1 \psi_2 \quad \text{Scalar}$$

$$P = \bar{\psi}_1 \gamma_5 \psi_2 \quad \text{pseudoscalar}$$

$$V_\mu = (V_0, \vec{V}) \quad V^\mu = (V_0, -\vec{V}) \quad \text{vector}$$

$$\hookrightarrow \bar{\psi}_1 \gamma_\mu \psi_2$$

$$A_\mu = \bar{\psi}_1 \gamma_\mu \gamma_5 \psi_2 \quad \text{axial vector}$$

~~$$T_{\mu\nu} = \bar{\psi}_1 \sigma_{\mu\nu} \psi_2 \quad \text{tensor}$$~~

$$\text{Parity} \quad V_0 \rightarrow V_0 \quad \vec{V} \rightarrow -\vec{V}$$

$$\hat{P}^\dagger \psi(\vec{x}, t) \hat{P} = P_{\alpha\beta} \psi_\beta(-\vec{x}, t)$$

$$\hat{P}^2 = \hat{I} \quad \hat{P}^\dagger = \hat{P} \quad \hookrightarrow P = \gamma^0$$

$$\begin{aligned} S &\rightarrow S & P &\rightarrow -P \\ V_\mu &\rightarrow V^\mu & A_\mu &\rightarrow -A^\mu \end{aligned}$$

Parity

$$\tilde{V}_i = \bar{\psi}_1 \sigma_{0i} \psi_2 \rightarrow -\tilde{V}_i \quad \tilde{A}_i = \epsilon_{ijk} \bar{\psi}_1 \sigma^{jk} \psi_2 \rightarrow \tilde{A}_i \quad i, j, k = 1, 2, 3$$

$$\psi = a_{e^-}^\dagger e^{-ip \cdot x} u_L(p) + (b_{e^+}^\dagger)^\dagger e^{ip \cdot x} v_L(p)$$

↑
annihilate an electron

↑
create a positron

$$\psi_c = b_{e^+}^\dagger e^{-ip \cdot x} u_L(p) + (a_{e^-}^\dagger)^\dagger e^{ip \cdot x} v_L(p)$$

$$e^{i\alpha} (\psi_c)_\alpha = (\hat{C}^\dagger \psi)_\alpha = C_{\alpha\beta} \psi_\beta^\dagger$$

$$\hat{C}^2 = \hat{I} \quad \hat{C}^\dagger \hat{C} = 1 \quad \hat{C} = \hat{C}^\dagger = \hat{C}^{-1}$$

$$C \gamma^\mu C = -(\gamma^\mu)^*$$

$$C = \pm i \gamma^2$$

$$S = \bar{\psi}_1 \psi_2 \rightarrow \bar{\psi}_2 \psi_1$$

$$P = \bar{\psi}_1 \gamma_5 \psi_2 \rightarrow \bar{\psi}_2 \gamma_5 \psi_1$$

$$V_\mu = \bar{\psi}_1 \gamma_\mu \psi_2 \rightarrow -\bar{\psi}_2 \gamma_\mu \psi_1$$

$$A_\mu = \bar{\psi}_1 \gamma_\mu \gamma_5 \psi_2 \rightarrow \bar{\psi}_2 \gamma_\mu \gamma_5 \psi_1$$

$$\bar{\psi}_1 \sigma_{\mu\nu} \psi_2 \rightarrow -\bar{\psi}_2 \sigma_{\mu\nu} \psi_1$$

$$C^\dagger A_\mu C = -A_\mu$$

photon

$$C^\dagger W_\mu^\pm C = -W_\mu^\mp$$

charged
vector boson

$$e A_\mu \bar{\psi} \gamma^\mu \psi \rightarrow e (-A_\mu) (-\bar{\psi} \gamma^\mu \psi)$$

invariant

Charge
Conjugation

$$g W_\mu^+ \bar{d} \gamma^\mu (1 - \gamma_5) u$$

P

$$g (W^+)_\mu \bar{d} (\gamma_\mu + \gamma_\mu \gamma_5) u =$$

$$g W_\mu^+ \bar{d} \gamma^\mu (1 + \gamma_5) u$$

C

$$g (-W_\mu^-) [-\bar{u} \gamma^\mu d - \bar{u} \gamma^\mu \gamma_5 d]$$

$$= g W_\mu^- \bar{u} \gamma^\mu (1 + \gamma_5) d$$

4 Inversione temporale T

La meccanica classica è invariante sotto inversione temporale: l'equazione di Newton resta immutata se effettuo la trasformazione $t \longrightarrow -t$.

$$m \frac{dx^2}{dt^2} = V(x)$$

In meccanica quantistica le cose sono più complicate. Riflettiamo anzitutto su cosa è l'inversione temporale. L'evoluzione temporale trasforma certe quantità fisiche $\vec{r}, \vec{P}, E$ dal tempo 0 al tempo t :

$$\vec{r}(0) \vec{P}(0) E(0) \longrightarrow \vec{r}(t) \vec{P}(t) E(t)$$

Definiamo una traslazione in avanti nel tempo:

$$\vec{r}(-T) \vec{P}(-T) E(-T) \longrightarrow \vec{r}(0) \vec{P}(0) E(0)$$

Questo è un sistema che evolve dal passato al futuro. Il sistema ottenuto dalla operazione di inversione temporale evolve dal futuro al passato, pertanto:

$$\vec{r}(0) \vec{P}(0) E(0) \longrightarrow \vec{r}(-T) \vec{P}(-T) E(-T)$$

Per concretezza si considera il processo $\sigma + \sigma \longrightarrow \pi + \pi$. Il processo inverso temporalmente è $\pi + \pi \longrightarrow \sigma + \sigma$. Se il sistema presenta una simmetria sotto inversione temporale i due processi hanno la stessa ampiezza. Se la simmetria è rotta, i due processi hanno una diversa ampiezza.

4.1 Digressione: antiunitarietà

Sappiamo che l'evoluzione temporale per una funzione d'onda è regolata da:

$$\psi(\mathbf{x}, t) = e^{-iEt} \psi(\mathbf{x}, 0)$$

Applicando la trasformazione $t \longrightarrow -t$ si ottiene:

$$\psi \sim e^{iEt} \psi(\mathbf{x}, 0)$$

Questa espressione è insoddisfacente perché manca il segno $-$ all'esponente. Si osserva però che la ψ^* verifica una relazione soddisfacente:

$$\psi^* \sim e^{-iEt} \psi^*(\mathbf{x}, 0)$$

Consideriamo un generico operatore di evoluzione temporale $\mathbf{U}$ e due stati $|A\rangle$ e $|B\rangle$. Gli stati evoluti temporali saranno:

$$\begin{aligned} |A'\rangle &= \mathbf{U} |A\rangle \\ |B'\rangle &= \mathbf{U} |B\rangle \end{aligned}$$

Generalmente, per conservare le ampiezze di probabilità, si richiede che:

$$\langle B' | A' \rangle = \langle B | \mathbf{U}^\dagger \mathbf{U} | A \rangle = \langle B | A \rangle$$

ottenendo $\mathbf{U}^\dagger = \mathbf{U}$. Nel caso dell'operatore inversione temporale non possiamo procedere in questa maniera. Richiediamo allora che valga:

$$\langle B' | A' \rangle = \langle B | A \rangle^* \quad (12)$$

Un operatore che verifica (12) è detto antiunitario.

Studiamo come si comporta un operatore antiunitario quanto è applicato ad una combinazione lineare di stati. Sia $\langle \psi_j | \psi_i \rangle = \delta_{ij}$ una base ortonormale. Abbiamo:

$$|\phi_i\rangle = \mathbf{U} |\psi_i\rangle = \sum_j |\psi_j\rangle \langle \psi_j | \mathbf{U} |\psi_i\rangle = \sum_j U_{ji} |\psi_j\rangle$$

dove si è usata la decomposizione dell'identità.

Consideriamo uno stato $|A\rangle$, decomposto in base $|\psi_i\rangle$

$$|A\rangle = \sum_i \langle \psi_i | A \rangle |\psi_i\rangle = \sum_i a_i |\psi_i\rangle$$

Avremo:

$$|A'\rangle = \mathbf{U} |A\rangle = \mathbf{U} \sum_i a_i |\psi_i\rangle$$

D'altro canto:

$$|A'\rangle = \sum_i \langle \psi'_i | A' \rangle |\psi'_i\rangle = \sum_i \langle \psi_i | A \rangle^* |\psi'_i\rangle = \sum_i a_i^* \mathbf{U} |\psi_i\rangle$$

Confrontando si conclude che un operatore agisce su una combinazione lineare di stati restituendo la combinazione lineare delle immagini degli stati, effettuata con i **coefficienti complessi coniugati**.

4.2 Determinazione dell'operatore $\mathbf{T}$

Vogliamo determinare una forma per l'operatore di inversione temporale $\mathbf{T}$. Definiamo l'operatore nel modo seguente:

$$(\psi_T)_\alpha = (\mathbf{T}^+ \psi(\mathbf{x}, t) \mathbf{T})_\alpha = \tau_{\alpha\beta} \psi_\beta^*(\mathbf{x}, -t)$$

Quindi:

$$\begin{cases} \psi_T = \mathbf{T} \psi^* \\ \psi_T^+ = (\psi^*)^+ \mathbf{T}^+ \end{cases}$$

Se effettuo due volte l'operazione di inversione temporale, il sistema rimane invariato, pertanto $\mathbf{T}^2 = \mathbf{I}$.

Vediamo cosa accade alla equazione di Dirac.

$$\begin{aligned} (i\cancel{D} - m)\psi &= 0 \\ (i\gamma^0\partial_0 + i\bar{\gamma} \cdot \bar{\partial} - m)\psi &= 0 \end{aligned}$$

Coniugando l'equazione e moltiplicando per $\mathbf{T}^2$ si ottiene:

$$(-i(\gamma^0)^* \partial_0 - i\bar{\gamma}^* \cdot \bar{\partial} - m) \mathbf{T} \mathbf{T} \psi^* = 0$$

Riconoscendo ψ_T e moltiplicando per $\mathbf{T}^+$ da sinistra si ottiene:

$$\mathbf{T}^+ (-i(\gamma^0)^* \partial_0 - i\bar{\gamma}^* \cdot \bar{\partial} - m) \mathbf{T} \psi_T = 0$$

Desidero che per la ψ_T valga l'equazione di Dirac seguente:

$$(-i\gamma^0\partial_0 + i\bar{\gamma} \cdot \bar{\partial} - m)\psi_T = 0$$

Scelgo:

$$\mathbf{T} = i\gamma^1\gamma^3$$

Si può verificare che questa scelta soddisfa i requisiti richiesti e $\mathbf{T}^+ = \mathbf{T}$, ovvero l'operatore di inversione temporale è hermitiano.

Let $q(\mathbf{x}, t)$ be the Dirac field operator that describes a quark of flavor $q = u, \dots, t$, $q^\dagger(\mathbf{x}, t)$ denotes its Hermitean adjoint, and $\bar{q} = q^\dagger \gamma^0$. The baryon number operator (36) is

$$\hat{B} = \frac{1}{3} \sum_q \int d^3x : q^\dagger(\mathbf{x}, t) q(\mathbf{x}, t) :, \quad (109)$$

and the colons denote normal ordering. Let C, P denote the unitary and T the anti-unitary operator which implement the charge conjugation, parity, and time reversal transformations, respectively, in the space of states. Their action on the quark fields is, adopting standard phase conventions,

$$Pq(\mathbf{x}, t)P^{-1} = \gamma^0 q(-\mathbf{x}, t), \quad (110)$$

$$Pq^\dagger(\mathbf{x}, t)P^{-1} = q^\dagger(-\mathbf{x}, t)\gamma^0, \quad (111)$$

$$Cq(\mathbf{x}, t)C^{-1} = i\gamma^2 q^\dagger(\mathbf{x}, t), \quad (112)$$

$$Cq^\dagger(\mathbf{x}, t)C^{-1} = iq(\mathbf{x}, t)\gamma^2, \quad (113)$$

$$Tq(\mathbf{x}, t)T^{-1} = -i q(\mathbf{x}, -t)\gamma_5\gamma^0\gamma^2, \quad (114)$$

$$Tq^\dagger(\mathbf{x}, t)T^{-1} = -i\gamma^2\gamma^0\gamma_5 q^\dagger(\mathbf{x}, -t), \quad (115)$$

where γ^0 , γ^2 , and $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ denote Dirac matrices. Then

$$P : q^\dagger(\mathbf{x}, t) q(\mathbf{x}, t) : P^{-1} = : q^\dagger(-\mathbf{x}, t) q(-\mathbf{x}, t) :, \quad (116)$$

$$C : q^\dagger(\mathbf{x}, t) q(\mathbf{x}, t) : C^{-1} = : q(\mathbf{x}, t) q^\dagger(\mathbf{x}, t) : = - : q^\dagger(\mathbf{x}, t) q(\mathbf{x}, t) :, \quad (117)$$

$$T : q^\dagger(\mathbf{x}, t) q(\mathbf{x}, t) : T^{-1} = : q^\dagger(\mathbf{x}, -t) q(\mathbf{x}, -t) : . \quad (118)$$

With these relations we immediately obtain:

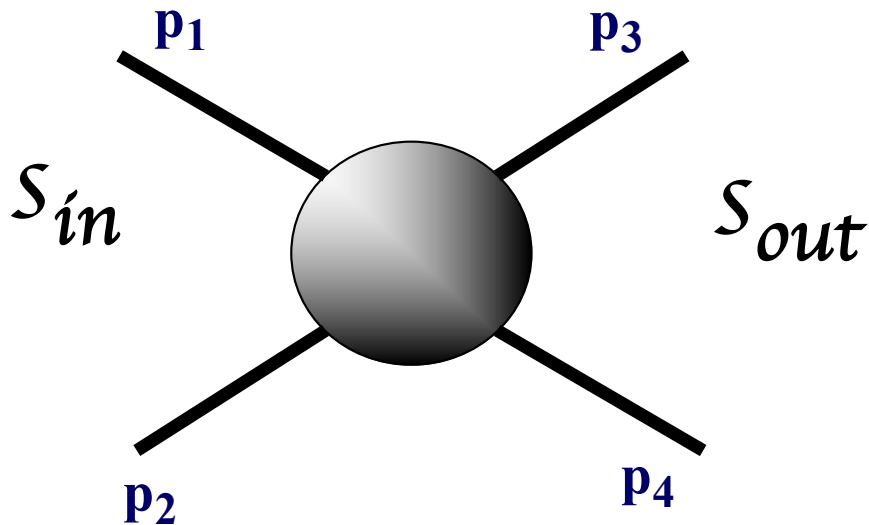
$$P\hat{B}P^{-1} = \hat{B}, \quad (119)$$

$$C\hat{B}C^{-1} = -\hat{B}. \quad (120)$$

Consequences of a Symmetry

$$[S, H] = 0 \rightarrow |E, \mathbf{p}, s\rangle$$

We may find states which are simultaneously eigenstates of $\underline{S}$ and of the Energy



$$CP |K_1^0\rangle = + |K_1^0\rangle$$

$$CP |K_2^0\rangle = - |K_2^0\rangle$$

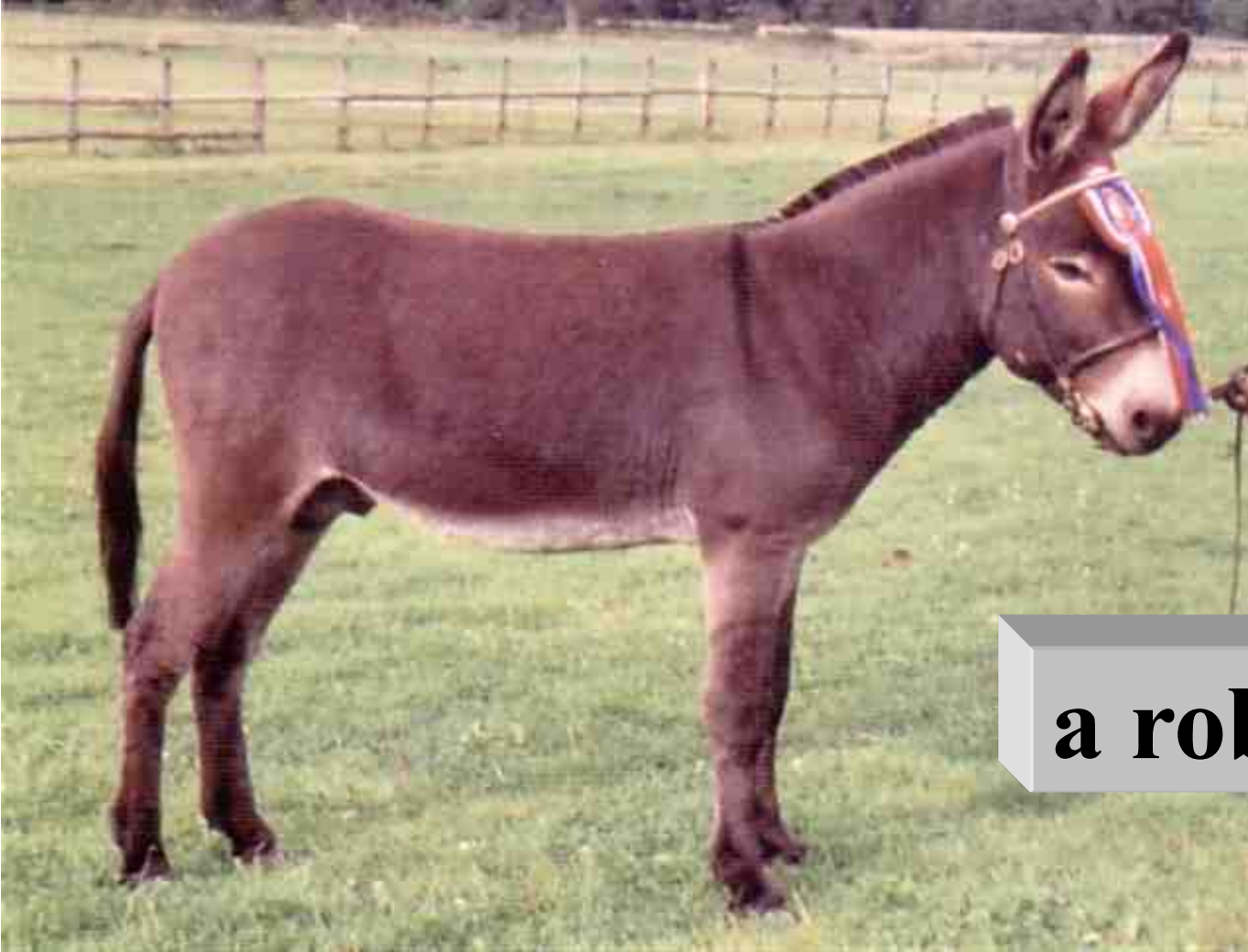
$$\langle \pi\pi | K_1^0 \rangle \neq 0$$

$$\langle \pi\pi | K_2^0 \rangle = 0$$

$$|K_{S,L}^0\rangle = \alpha |K_1^0\rangle + \beta |K_2^0\rangle$$

if CP is conserved
either $\alpha=0$ or $\beta=0$

the Standard Model



a robust animal

the Standard Model and beyond

*Vacuum
Energy*

Hierarchy

*Vacuum
Stability*

$$\mathcal{L} = \Lambda^4 + \Lambda^2 H^2 + \lambda H^4 +$$
$$(D_\mu H)^2 + \bar{\psi} \not{D} \psi + F_{\mu\nu}^2 + F_{\mu\nu} \tilde{F}_{\mu\nu} \quad \text{Strong } \cancel{CP}$$
$$Y H \bar{\psi} \psi + \frac{1}{\Lambda} (\bar{L} H)^2 + \frac{1}{\Lambda^2} \sum_i C_i O_i + \dots$$

*Flavor
puzzle*

*Neutrino
Masses*

New Physics
*Possible breaking of
accidental
symmetries*

*Fundamental and Effective
Theories*

Local Gauge Symmetry $SU(3)_c \otimes SU(2)_W \otimes U(1)_Y$

The SM Lagrangian
(since 1973 in its full content)

$$\mathcal{L}_{\sim SM} = -\frac{1}{4}F_{\mu\nu}^a F^{a\mu\nu} + i\bar{\psi} \not{D}\psi \quad (\sim 1975-2000)$$

$$+ |D_\mu h|^2 - V(h) \quad (\sim 1990-2012\text{-now})$$

$$+ \psi_i \lambda_{ij} \psi_j h + h.c. \quad (\sim 2000\text{- now})$$

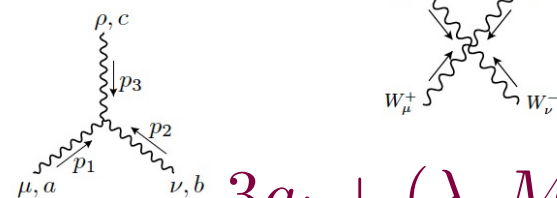
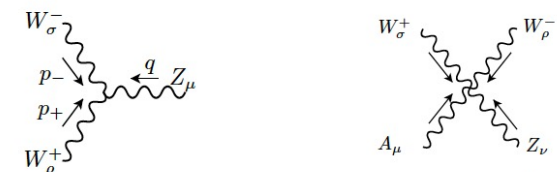
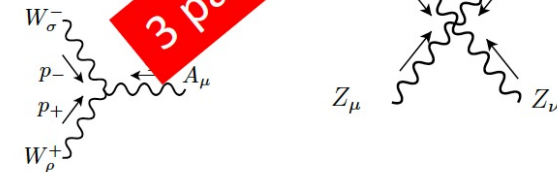
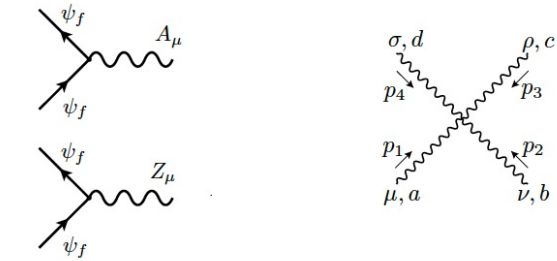
In () the approximate dates of the experimental confirmation
of the various lines (at different levels)

The synthetic nature of the SM exhibited

The Standard Model

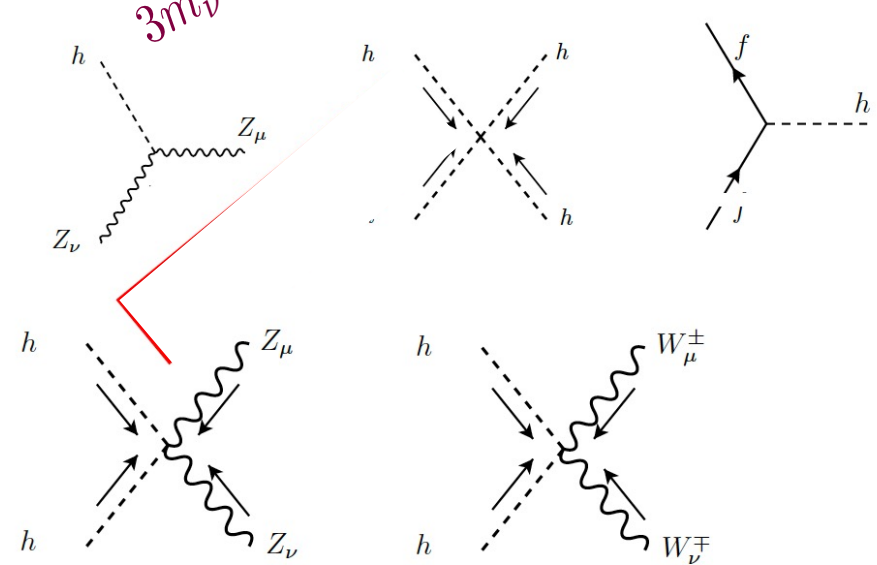
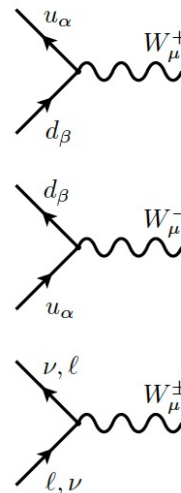
$$SU(3) \times SU(2) \times U(1)_Y$$

Higgs + gauge principle



3 parameters

$$3m_\nu + (2+1)\delta_{PMNS} + 3\theta_{PMNS} = 9$$



from elegance to caos !!

If we are looking for the suspect that could be hiding some secret obviously the higgs is the one!

$$3g_i + (\lambda, M_H) + 6m_q + 3m_\ell + \delta + 3\theta_{CKM} + \theta_{QCD} = 19$$

The Weirddness of the Standard Model

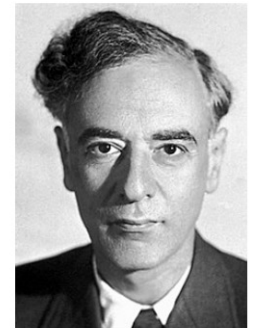
- Three families

“who ordered that ?” I. Rabi



- Fundamental breaking of Parity

“space cannot be asymmetric!” L. Landau



- Activity: $3m_\nu + (2+1)\delta_{PMNS} + 3\theta_{PMNS} = 9$
 3 gauge couplings+ 16 higgs couplings (+ 7 higgs-neutrino) !
 + the coupling θ of strong CP violation

“has too many arbitrary features for [its] predictions to be taken very seriously” S. Weinberg '67

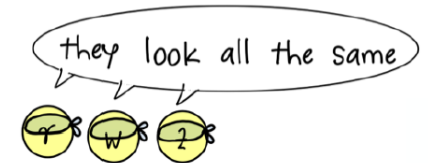


$$3g_i + (\lambda, M_H) + 6m_q + 3m_\ell + \delta + 3\theta_{CKM} + \theta_{QCD} = 19$$

Hints of NP structure: Flavor symmetries of the SM

- Standard Model (SM) gauge sector is *flavor blind*!

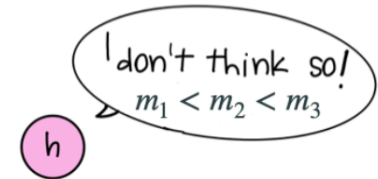
$$\mathcal{G}_F(\text{SM}) = U(3)^5 \equiv U(3)_q \times U(3)_u \times U(3)_d \times U(3)_\ell \times U(3)_e$$



Turn on Yukawas



$$Y_{ij} \bar{\Psi}_L^i H \Psi_R^j$$



$$\mathcal{G}_F(\text{SM}) = U(1)_B \times U(1)_L$$

the role of accidental symmetries

courtesy of B.A. Stefanek

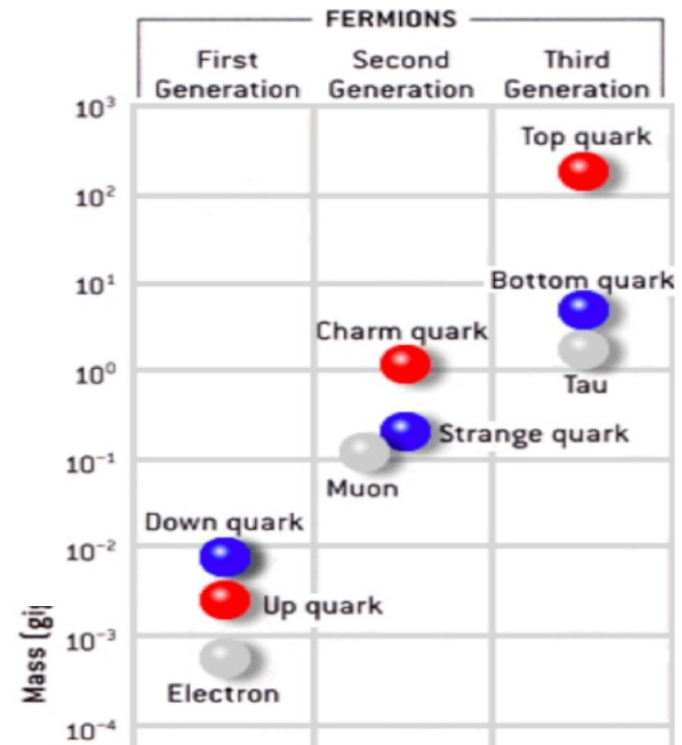
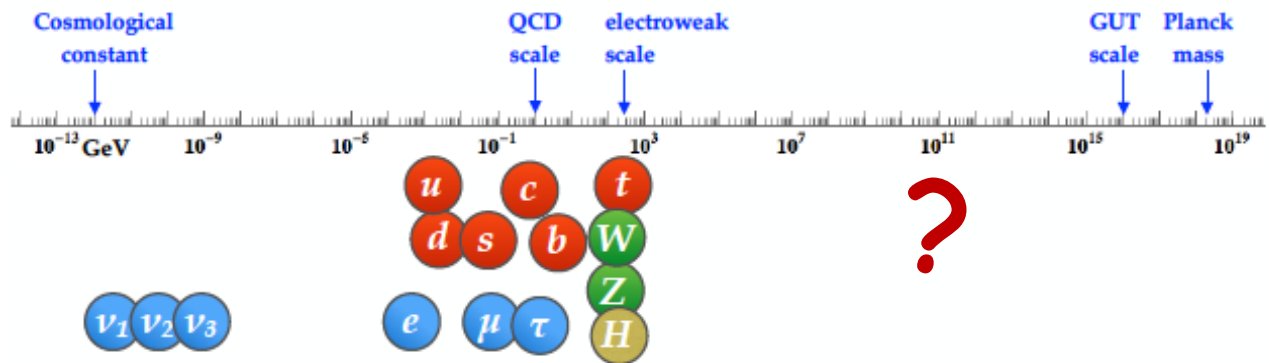


Illustration from a G. Isidori talk

$m_\nu \leq 1\text{ eV}$

Quark
Masses from
Lattice QCD

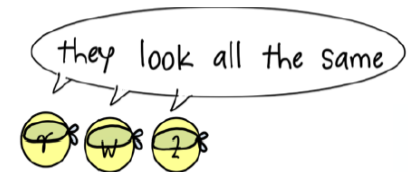
Input	Lattice/Exp
$m_u^{\overline{\text{MS}}}(2\text{ GeV})$	2.20(9) MeV
$m_d^{\overline{\text{MS}}}(2\text{ GeV})$	4.69(2) MeV
$m_s^{\overline{\text{MS}}}(2\text{ GeV})$	93.14(58) MeV
$m_c^{\overline{\text{MS}}}(3\text{ GeV})$	993(4) MeV
$m_c^{\overline{\text{MS}}}(m_c^{\overline{\text{MS}}})$	1277(5) MeV
$m_b^{\overline{\text{MS}}}(m_b^{\overline{\text{MS}}})$	4196(19) MeV
$m_t^{\overline{\text{MS}}}(m_t^{\overline{\text{MS}}})$ (GeV) to be updated	163.44(43)

Table 3 Full lattice inputs. The values of the different quantities have been c taking the weighted average of the $N_f = 2 + 1$ and $N_f = 2 + 1 + 1$ FLAG num

Hints of NP structure: Flavor symmetries of the SM

- Standard Model (SM) gauge sector is **flavor blind and CP conserving**

$$\mathcal{G}_F(\text{SM}) = U(3)^5 \equiv U(3)_q \times U(3)_u \times U(3)_d \times U(3)_\ell \times U(3)_e$$

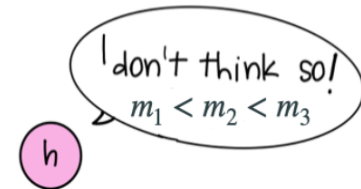


The Higgs introduces the only known non-gauge couplings

Turn on Yukawas



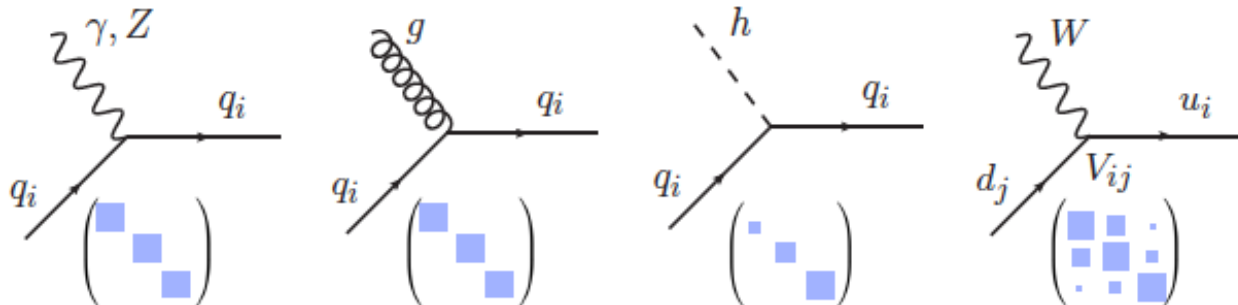
$$Y_{ij} \bar{\Psi}_L^i H \Psi_R^j$$



$$\mathcal{G}_F(\text{SM}) = U(1)_B \times U(1)_L$$

Higgs couplings are not flavor blind

courtesy of B.A. Stefanek



$$\mathcal{L}_{int} = \overset{\text{electromagnetic}}{-e A^\mu J_\mu^{em}} - \overset{\text{neutral currents}}{\frac{g_W}{2 \cos \theta_W} Z^\mu J_\mu^Z} - \overset{\text{charged currents}}{\frac{g_W}{2\sqrt{2}} [W^\mu (J^W)_\mu^\dagger + h.c.]}$$

$$J_\mu^Z = 2J_\mu^3 - 2 \sin^2 \theta_W J_\mu^{em}$$

$$W_\mu (J_W^\dagger)^\mu + h.c. = W_\mu^- \bar{u} \gamma^\mu (1 - \gamma_5) d + W_\mu^+ \bar{d} \gamma^\mu (1 - \gamma_5) u$$

*All these currents play a
fundamental role
in flavor physics (exp or theo)*

for Hermiticity $g_1 = g_2$

$$g_1 = (g_2)^*$$

$$\mathcal{L} = g_1 W_\mu^- \bar{u} \gamma^\mu (1 - \gamma_5) d + g_2 W_\mu^+ \bar{d} \gamma^\mu (1 - \gamma_5) u \quad g_1 \neq g_2$$

$$[W_\mu^- \bar{u} \gamma^\mu (1 - \gamma_5) d]^\dagger = W_\mu^+ d^\dagger [(\gamma^\mu)^\dagger (\gamma^0)^\dagger - (\gamma_5)^\dagger (\gamma^\mu)^\dagger (\gamma^0)^\dagger] u = W_\mu^+ d^\dagger [\gamma^0 (\gamma^0 (\gamma^\mu)^\dagger \gamma^0) - \gamma_5 \gamma^0 (\gamma^0 (\gamma^\mu)^\dagger \gamma^0)] u$$

$$(\gamma^0 (\gamma^\mu)^\dagger \gamma^0) = \gamma^\mu \quad = W_\mu^+ \bar{d} [\gamma^\mu] - \gamma^\mu \gamma_5 u$$

$$\mathcal{L} = g_w \left[W_\mu^- \bar{u} \gamma^\mu (1 - \gamma_5) d + W_\mu^+ \bar{d} \gamma^\mu (1 - \gamma_5) u \right]$$

hermiticity

$$\mathcal{P}^\dagger \mathcal{L} \mathcal{P} = g_w \left[W_\mu^- \bar{u} \gamma^\mu (1 + \gamma_5) d + W_\mu^+ \bar{d} \gamma^\mu (1 + \gamma_5) u \right]$$

parity violation

$$\mathcal{C}^\dagger \mathcal{L} \mathcal{C} = g_w \left[W_\mu^+ \bar{d} \gamma^\mu (1 + \gamma_5) u + W_\mu^- \bar{u} \gamma^\mu (1 + \gamma_5) d \right]$$

charge conj. violation

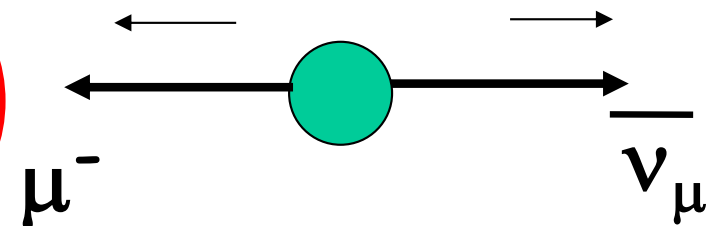
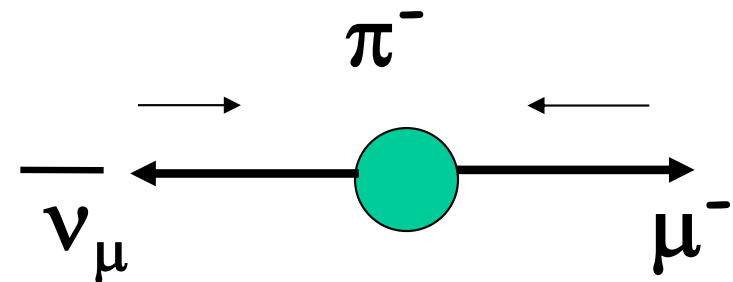
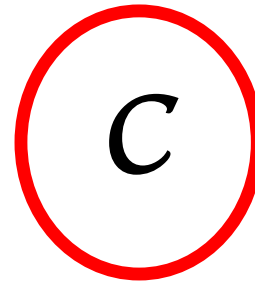
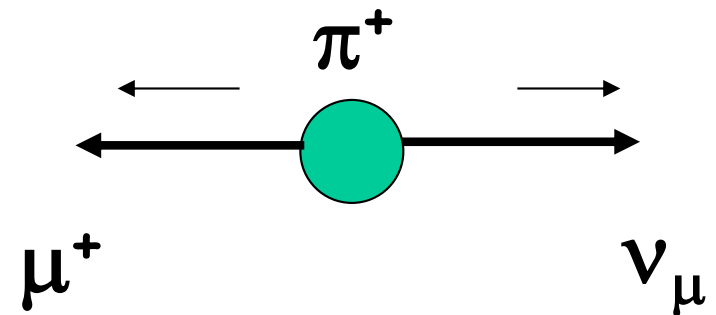
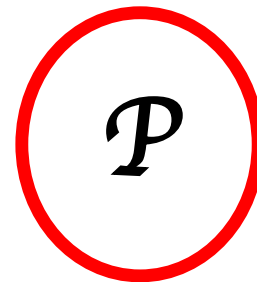
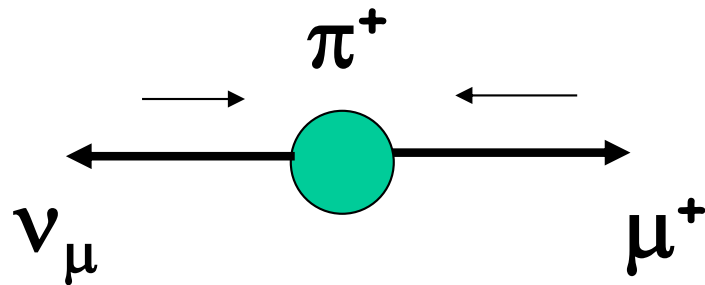
$(\mathcal{CP})^\dagger \mathcal{L} (\mathcal{CP}) = \mathcal{L}$

 CP conserved

Relativistic
Quantum
Mechanics



Antimatter
CPT Theorem



CP Violation was
discovered about
37 years ago in
 $K^0 - \bar{K}^0$ mixing
(weak
interactions)

The usual mantra
*reasons to go
beyond the SM(s):*



“Experimental” evidence

1. *Neutrino Masses*
2. *Dark Matter and Dark Energy*
3. *Matter-Antimatter Asymmetry*

“Theoretical” evidence

1. *SM instability (hierarchy, naturalness)*
2. *Flavour Physics (families, Yukawa couplings, CP violation for both quarks and leptons)*
3. *Unification of forces and quantization of gravity*
4. *Equality of the electric charges of p and e*



Observational

Questions from (problems for) the SM

Structural

0. Which rationale for matter quantum numbers?

$$\text{E.g.: } |Q_n - Q_p - Q_e| < 10^{-21}e$$

1. Phenomena unaccounted for

neutrino masses

matter-antimatter asymmetry

Dark matter

inflation?

2. Why $\theta \lesssim 10^{-10}$?

$$\theta G_{\mu\nu} \tilde{G}^{\mu\nu}$$

Axions? A discrete space-time symmetry?

3. $\mathcal{O}_i : d(\mathcal{O}_i) \leq 4$ only?

neutrino masses

Are the protons forever?

Gravity

What about individual L_i conservations?

To be discussed later

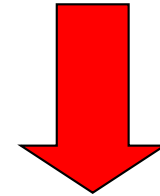
4. Lack of calculability

$\Rightarrow$ the hierarchy problem
the flavour puzzle

$\Leftarrow$ none of the 15 masses
predicted in the SM

**Weak Couplings,
Quark Masses
and
CP Violation in
the Standard Model**

In the Standard Model the quark mass matrix, from which the CKM Matrix and $\mathcal{CP}$ originate, is determined by the Yukawa Lagrangian which couples fermions and Higgs



$$L_{quarks} = L^{kinetic} + L^{weak\ int} + L^{yukawa}$$

CP invariant

$\mathcal{CP}$ and symmetry breaking are closely related !

QUARK FAMILIES

1)

$$q_L \equiv \begin{pmatrix} u_L \\ d_L \end{pmatrix}$$

$$\begin{aligned} U_R &= u_R \\ D_R &= d_R \end{aligned}$$

2)

$$q_L \equiv \begin{pmatrix} c_L \\ s_L \end{pmatrix}$$

$$\begin{aligned} U_R &= c_R \\ D_R &= s_R \end{aligned}$$

3)

$$q_L \equiv \begin{pmatrix} t_L \\ b_L \end{pmatrix}$$

$$\begin{aligned} U_R &= t_R \\ D_R &= b_R \end{aligned}$$

Only experiments can fix the gauge group and the particles quantum numbers

$$Y=1/6$$

$$Y_{U,D}=Q_{U,D}=2/3,-1/3$$

QUARK MASSES ARE GENERATED BY DYNAMICAL SYMMETRY BREAKING

$$H = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \quad H^C = i\tau_2 H^*$$

$$\phi^+ \rightarrow 0 \quad \phi^0 \rightarrow \frac{V}{\sqrt{2}}$$

Charge +2/3

$$\mathcal{L}_{\text{Yukawa}} \equiv \sum_{i,k=1,N} \left[Y_{i,k} (q_L^i H^C) U_R^k + X_{i,k} (q_L^i H) D_R^k + \text{h.c.} \right]$$

Charge -1/3

$$\sum_{i,k=1,N} \left[m_{i,k}^u (\bar{u}_L^i u_R^k) + m_{i,k}^d (\bar{d}_L^i d_R^k) + \text{h.c.} \right]$$

Elementary Particles

Quarks	u	c	t	γ
	d	s	b	g
Leptons	ν_e	ν_μ	ν_τ	Z
	e	μ	τ	W

Force Carriers

Three Generations of Matter

$$\sum_{i,k=1,N} \left[m^u_{i,k} (\bar{u}^i_L u^k_R) + m^d_{i,k} (\bar{d}^i_L d^k_R) + \text{h.c.} \right]$$

It is easy to show the a necessary and sufficient condition for CP invariance is

$$m^{u,d}_{i,k} = \text{real}$$

- 1) there is no compelling symmetry for $m^{u,d}_{i,k}$ to be real
- 2) in field theory, all that may happen will happen [see below]
- 3) symmetries and accidental symmetries
e.g. separate conservation of lepton and baryon numbers (it follows from gauge symmetry and renormalizability)

Diagonalization of the Mass Matrix

The mass matrix (M) is not even Hermitean;
Up to singular cases, however, it can always be diagonalized
by 2 unitary transformations

$$u_L^i \rightarrow U_L^{ik} u_L^k \qquad u_R^i \rightarrow U_R^{ik} u_R^k$$

$$M' = U_L^\dagger M U_R \qquad (M')^\dagger = U_R^\dagger (M)^\dagger U_L$$

$$\mathcal{L}^{\text{mass}} \equiv m_{\text{up}} (\bar{u}_L u_R + \bar{u}_R u_L) + m_{\text{ch}} (\bar{c}_L c_R + \bar{c}_R c_L) + \\ m_{\text{top}} (\bar{t}_L t_R + \bar{t}_R t_L)$$

$$\mathcal{L}_{\text{CC}}^{\text{weak int}} = \frac{g_W}{2\sqrt{2}} (J^-_\mu W^+_\mu + J^+_\mu W^-_\mu)$$



The Cabibbo-Kobayashi-Maskawa Matrix

$$J_{\mu}^{+} W_{\mu}^{-} = (\bar{u} \gamma_{\mu} (1 - \gamma_5) d + \dots) W_{\mu}^{-} =$$

$$\bar{u}_L \gamma_{\mu} d_L W_{\mu}^{-} \rightarrow \bar{u}_L (U_L^u)^{\dagger} \gamma_{\mu} (U_L^d) d_L W_{\mu}^{-} =$$

$$\bar{u}_L V^{\text{CKM}} \gamma_{\mu} d_L W_{\mu}^{-}$$

where $V^{\text{CKM}} = (U_L^u)^{\dagger} (U_L^d)$ is a unitary matrix

With $N(N-1)/2$ Euler angles and $[N^2 - N(N-1)/2]$

phases; not all these phases are physical

neutral currents remain diagonal in flavor (no FCNC at tree level)

$$u_L \rightarrow e^{i\phi_u} u_L \quad d_L \rightarrow e^{i\phi_d} d_L \quad \text{etc.}$$

Thus finally we have $N(N-1)/2$ angles and
 $(N-1)(N-2)/2$ phases

Diagonalization of the Mass Matrix

Up to singular cases, the mass matrix can always be diagonalized by 2 unitary transformations

$$u_L^i \rightarrow U_L^{ik} u_L^k \qquad u_R^i \rightarrow U_R^{ik} u_R^k$$

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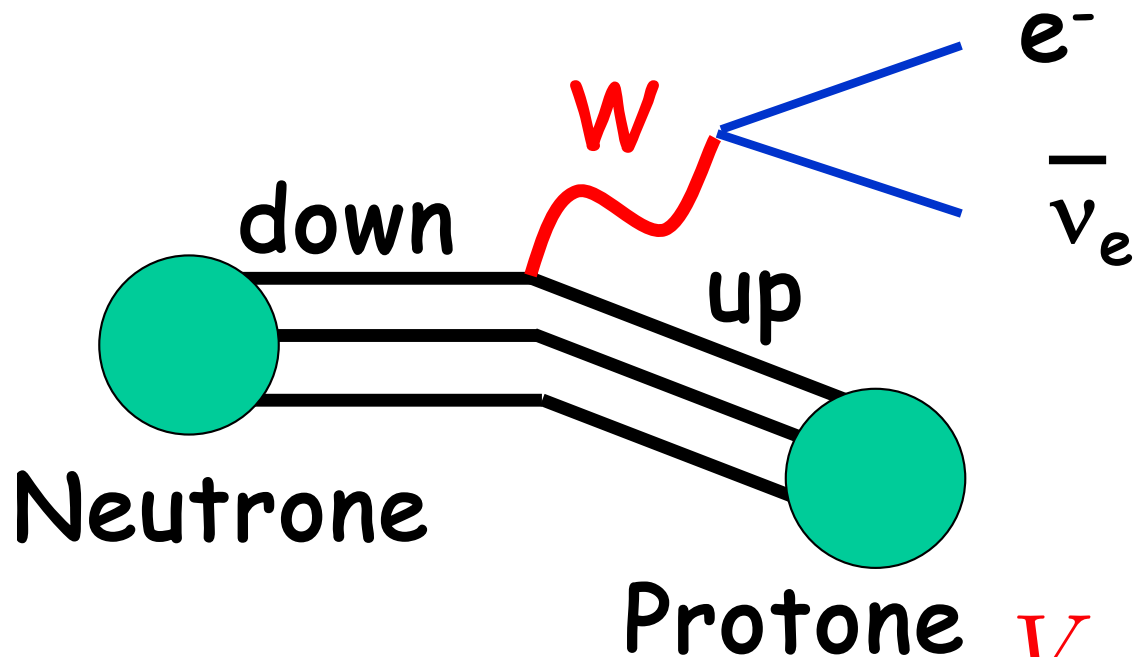
$$L_{CC}^{\text{weak int}} = \frac{g_W}{\sqrt{2}} (J_\mu^- W_\mu^+ + h.c.)$$

$$\rightarrow \frac{g_W}{\sqrt{2}} (\bar{u}_L \mathbf{V}^{CKM} \gamma_\mu d_L W_\mu^+ + \dots)$$

Weak Interactions

β -Decays

$$n \rightarrow p + e^- + \bar{\nu}_e$$

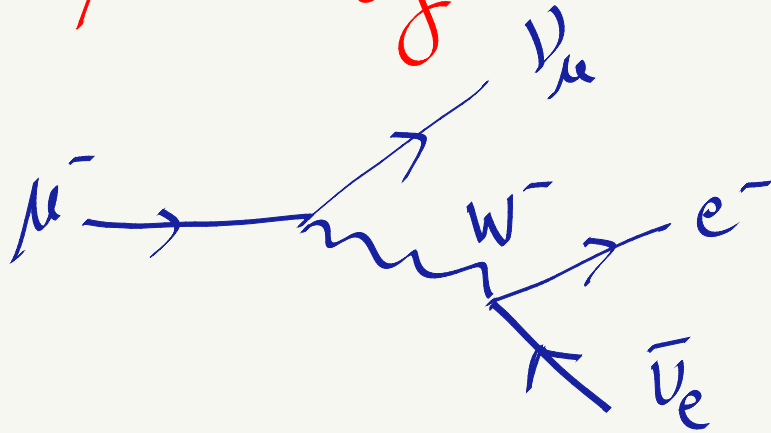


$$\mathcal{H}_{eff} = -\frac{G_F}{\sqrt{2}} V_{ud} [\bar{u}\gamma_\mu(1 - \gamma_5)d][\bar{e}\gamma^\mu(1 - \gamma_5)\nu_e]$$

$$i \frac{-g_{\mu\nu} + q_\mu q_\nu / M_W^2}{q^2 - M_W^2 + i\varepsilon} \sim i \frac{g_{\mu\nu}}{M_W^2} \quad q^2 \ll M_W^2 \quad \frac{G_F}{\sqrt{2}} = \frac{g_W^2}{8M_W^2}$$

μ -decay

[E13]



$$H_{\text{eff}} = -\frac{G_F}{\sqrt{2}} \bar{\nu}_\mu \gamma^\mu (1 - \gamma_5) \mu \times \bar{e} \gamma_\mu (1 - \gamma_5) \nu_e$$

$$\frac{d\Gamma}{dx} = \frac{G_F^2}{96\pi^3} \sqrt{x^2 - 4\beta^2} \times (3 - 2x)$$

$x = 2E_e/m_\mu$ $\beta = m_e/m_\mu$

$$2\beta \leq x \leq 1 + \beta^2$$

$\beta = 0$

$$\Gamma = \frac{G_F^2 m_\mu^5}{192\pi^3}$$

Very recently, in an important theoretical development, van Ritbergen and Stuart completed the evaluation of $\mathcal{O}(\alpha^2)$ corrections to τ_μ in the local V-A theory, in the limit $m_e^2/m_\mu^2 \rightarrow 0$ [7,8]. Their final answer can be expressed succinctly as

$$C(m_\mu) = \frac{\alpha(m_\mu)}{\pi} c_1 + \left(\frac{\alpha(m_\mu)}{\pi} \right)^2 c_2, \quad (1)$$

$$c_1 = \frac{1}{2} \left(\frac{25}{4} - \pi^2 \right); \quad c_2 = 6.700. \quad (2)$$

In Eq. (1), $\alpha(m_\mu)c_1/\pi$ is the one-loop result [1] and $\alpha(m_\mu)$ is a running coupling defined by

$$\alpha(m_\mu) = \frac{\alpha}{1 - \left(\frac{2\alpha}{3\pi} + \frac{\alpha^2}{2\pi^2} \right) \ln \frac{m_\mu}{m_e}}. \quad (3)$$

$$\Gamma = \frac{G_F^2 m_\mu^5}{192\pi^3} [1 + C(m_\mu)] f(\rho^2) + \mathcal{O}\left(\frac{m_\mu^2}{M_W^2}\right)$$

$$f(z) = 1 - 8z - 12z^2 \log z + 8z^3 - z^4$$

The total rate grows like m^5 , that is the reason the top quark cannot form bound states

$N(N-1)/2$ angles and $(N-1)(N-2)/2$ phases

$N=3$ 3 angles + 1 phase KM
the phase generates complex couplings i.e. CP
violation;

6 masses + 3 angles + 1 phase = 10 parameters

V_{ud}	V_{us}	V_{ub}
V_{cd}	V_{cs}	V_{cb}
V_{td}	V_{ts}	V_{tb}

$$N^2 - \frac{N(N-1)}{2} - (2N-1) = \frac{(N-1)(N-2)}{2}$$

$$= \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{bmatrix}$$

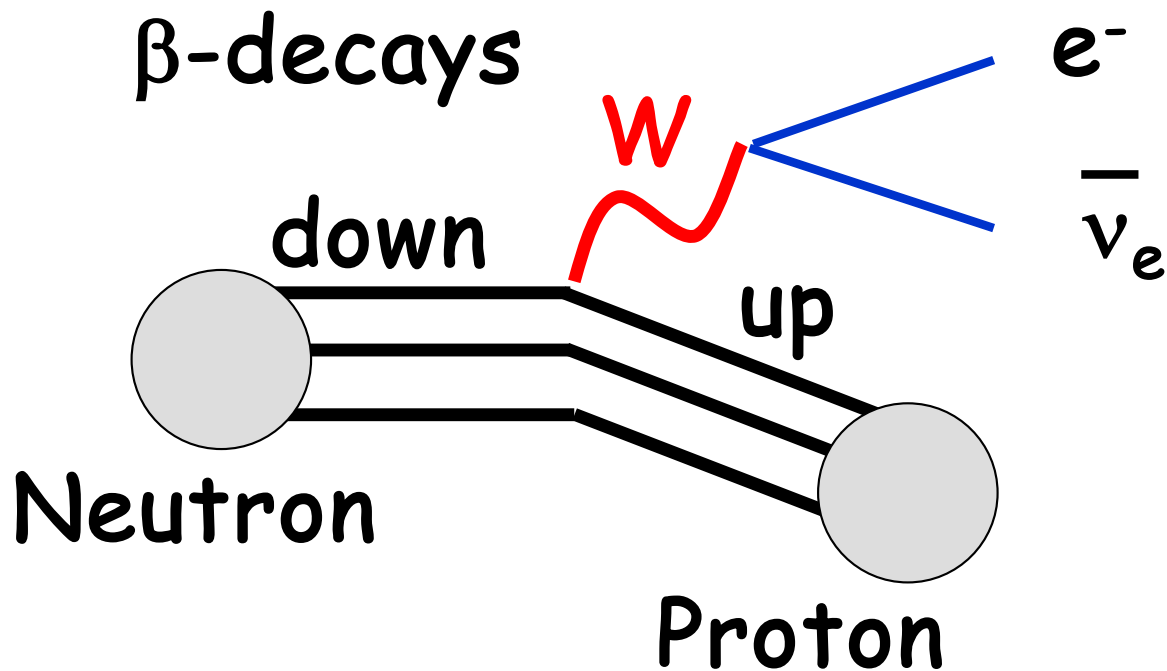
**NO Flavour Changing Neutral Currents (FCNC)
at Tree Level**
**(FCNC processes are good candidates for observing
NEW PHYSICS)**

**CP Violation is natural with three quark
generations (Kobayashi-Maskawa)**

**With three generations all CP
phenomena are related to the same
unique parameter (δ)**

Quark masses & Generation Mixing

V_{ud}	V_{us}	V_{ub}
V_{cd}	V_{cs}	V_{cb}
V_{td}	V_{ts}	V_{tb}



$$\begin{aligned}
 |V_{ud}| &= 0.9735(8) \\
 |V_{us}| &= 0.2196(23) \\
 |V_{cd}| &= 0.224(16) \\
 |V_{cs}| &= 0.970(9)(70) \\
 |V_{cb}| &= 0.0406(8) \\
 |V_{ub}| &= 0.00409(25) \\
 |V_{tb}| &= 0.99(29) \\
 |V_{td}| &= (0.999)
 \end{aligned}$$

updated values next slide

CKM December 2022

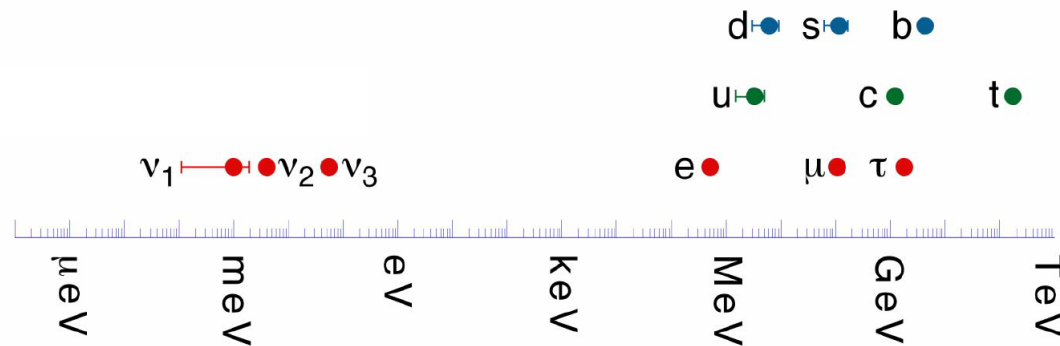
$$V_{\text{CKM}} = \begin{pmatrix} 0.97431(19) & 0.22517(81) & 0.003715(93) e^{-i(65.1(1.3))^{\circ}} \\ -0.22503(83) e^{+i(0.0351(1))^{\circ}} & 0.97345(20) e^{-i(0.00187(5))^{\circ}} & 0.0420(5) \\ 0.00859(11) e^{-i(22.4(7))^{\circ}} & -0.04128(46) e^{+i(1.05(3))^{\circ}} & 0.999111(20) \end{pmatrix}$$

Rend.Lincei Sci.Fis.Nat. 34 (2023) 37-57 e-Print: [2212.03894 \[hep-ph\]](#)

Questions

how to extend the SM in order to accommodate neutrino masses?

why neutrino masses are so small, compared with the charged fermion masses?



why lepton mixing angles are so different from those of the quark sector?

$$U_{PMNS} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} + \text{corrections}$$

$$V_{CKM} \approx \begin{pmatrix} 1 & O(\lambda) & O(\lambda^4 \div \lambda^3) \\ O(\lambda) & 1 & O(\lambda^2) \\ O(\lambda^4 \div \lambda^3) & O(\lambda^2) & 1 \end{pmatrix}$$

$\lambda \approx 0.22$

courtesy of ???

Textures

There is a clear correlation between mixings and masses

$$m_u \sim 3 \text{ MeV} \quad m_c \sim 1200 \text{ MeV} \quad m_t \sim 170 \text{ GeV}$$

$$m_d \sim 7 \text{ MeV} \quad m_s \sim 110 \text{ MeV} \quad m_b \sim 4.3 \text{ GeV}$$

Horizontal $U(2)$: $\psi_L \quad \psi_L^c$

$$L_{\text{higgs}} = \frac{Y_H}{M} \left[(\psi_L^a)(\psi_L^b)^c S^{ab} + (\psi_L^a)(\psi_L^b)^c A^{ab} \right]$$

Symmetric
tensor

Antisymmetric
tensor

$$M^d = M \begin{pmatrix} 0 & -\sqrt{x} \\ \sqrt{x} & 1+x \end{pmatrix}$$

$$\sin \theta_c \sim \sqrt{m_d / m_s}$$

R.Gatto '70

$$\text{diag}(M) = M \begin{pmatrix} x & \\ & 1 \end{pmatrix} \quad x = m_d / m_s$$

$$V_1 = \begin{pmatrix} 1 \\ \sqrt{x} \end{pmatrix} \quad \lambda_1 = M x$$

$$V_2 = \begin{pmatrix} -\sqrt{x} \\ 1 \end{pmatrix} \quad \lambda_2 = M$$

**Masses &
Mixings
(including the
CP phases)
are related !!**

$$\begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{bmatrix}$$

$$c_{ij} = \cos \theta_{ij} \quad s_{ij} = \sin \theta_{ij} \quad c_{ij} \geq 0 \quad s_{ij} \geq 0$$

$$0 \leq \delta \leq 2\pi \quad |s_{12}| \sim \sin \theta_c$$

$$\text{for small angles} \quad |s_{ij}| \sim |V_{ij}|$$

The Wolfenstein Parametrization

$1 - 1/2 \lambda^2$	λ	$A \lambda^3(\rho - i \eta)$
$-\lambda$	$1 - 1/2 \lambda^2$	$A \lambda^2$
$A \lambda^3 \times$ $(1 - \rho - i \eta)$	$-A \lambda^2$	1

$\mathbf{V}_{ub}$

$+ O(\lambda^4)$

$\mathbf{V}_{td}$

$$\lambda \sim 0.2 \quad A \sim 0.8$$

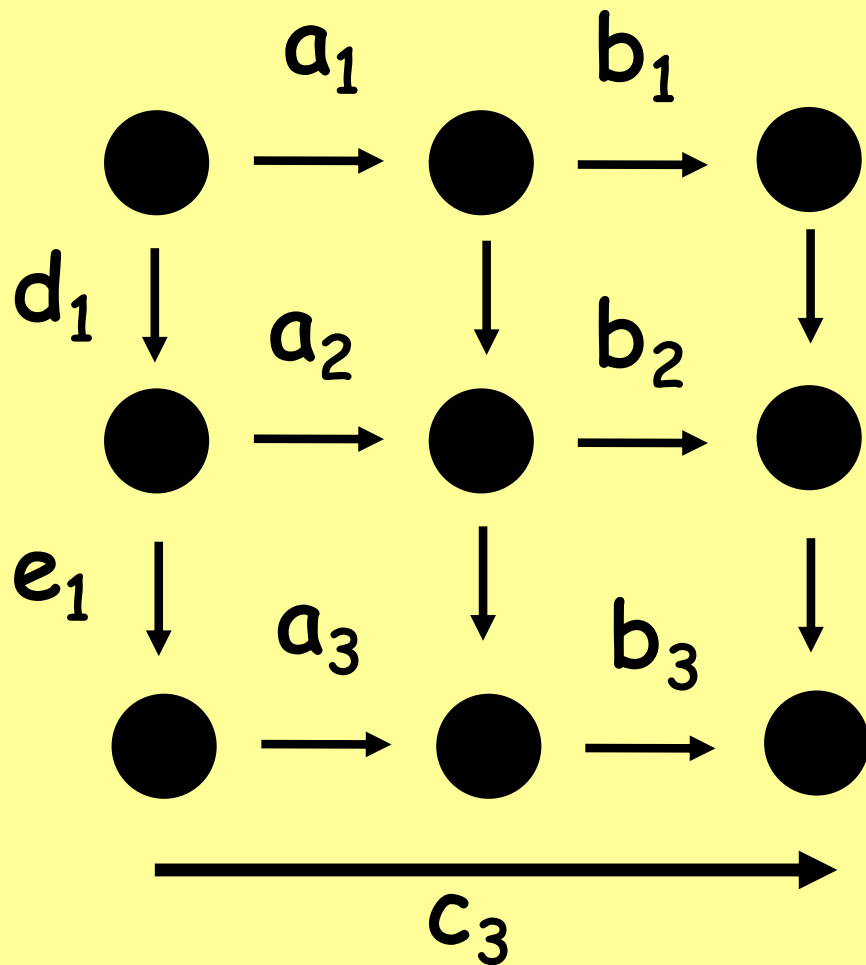
$$\eta \sim 0.2 \quad \rho \sim 0.3$$

$$\sin \theta_{12} = \lambda$$

$$\sin \theta_{23} = A \lambda^2$$

$$\sin \theta_{13} = A \lambda^3(\rho - i \eta)$$

The Bjorken-Jarlskog Unitarity Triangle



$|V_{ij}|$ is invariant under phase rotations

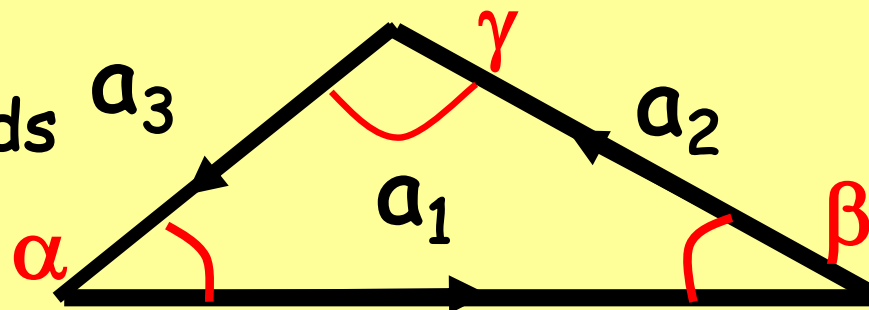
$$a_1 = V_{11} V_{12}^* = V_{ud} V_{us}^*$$

$$a_2 = V_{21} V_{22}^* \quad a_3 = V_{31} V_{32}^*$$

$$a_1 + a_2 + a_3 = 0$$

($b_1 + b_2 + b_3 = 0$ etc.)

Only the orientation depends on the phase convention



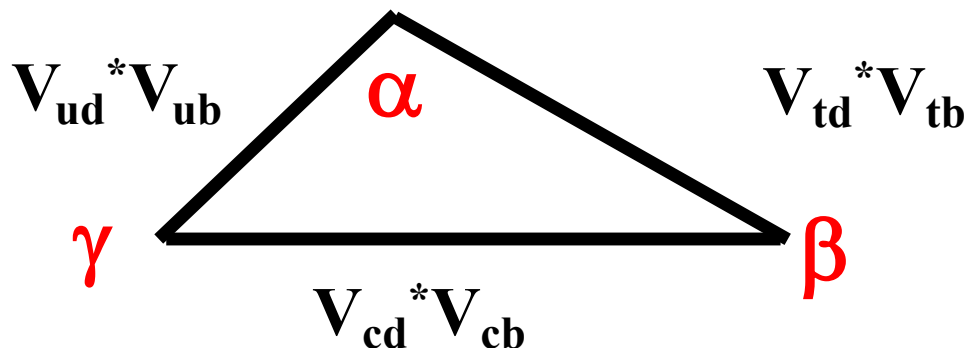
Physical quantities correspond to invariants under phase reparametrization i.e. $|a_1|, |a_2|, \dots, |e_3|$ and the area of the Unitary Triangles

$$J = \text{Im} (a_1 a_2^*) = |a_1 a_2| \sin \beta$$

a precise knowledge of the moduli (angles) would fix J

$$\cancel{\mathcal{CP}} \propto J$$

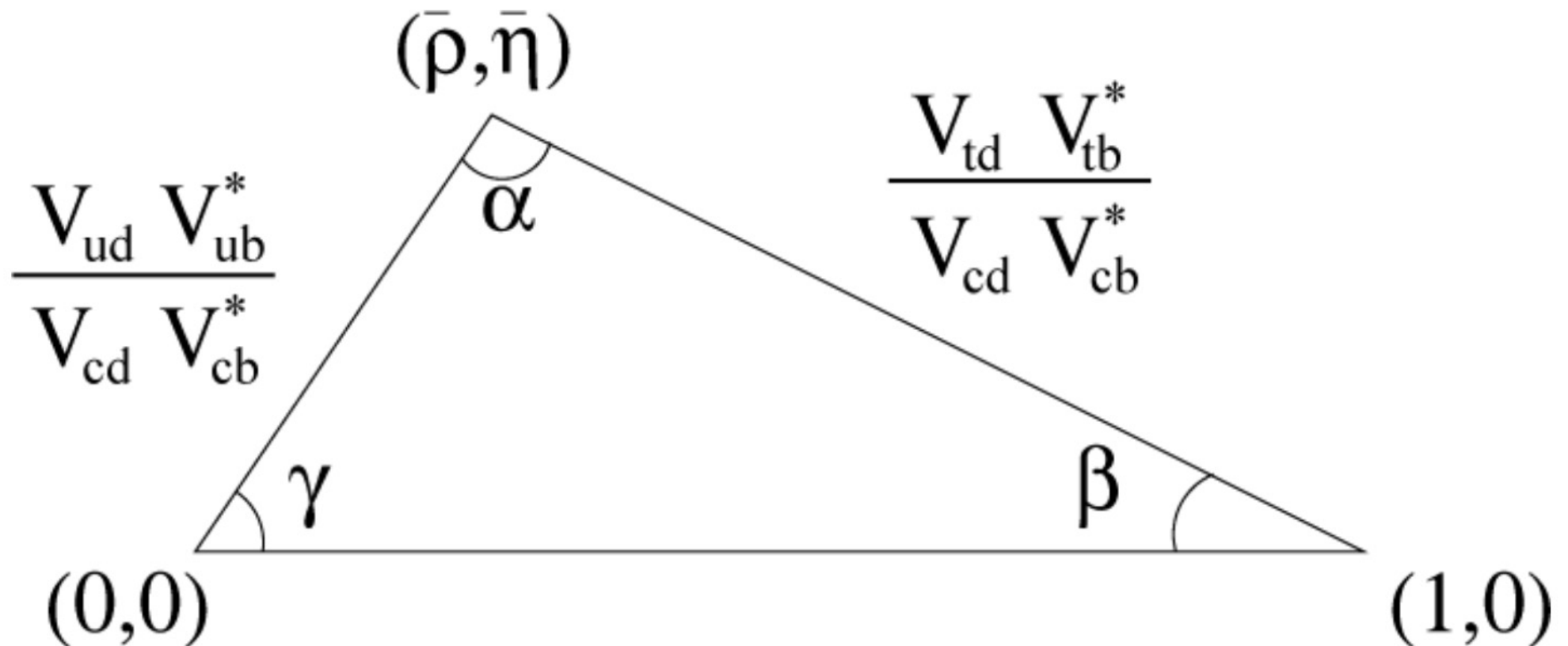
$$V_{ud}^* V_{ub} + V_{cd}^* V_{cb} + V_{td}^* V_{tb} = 0$$



$$\gamma = \delta_{CKM}$$



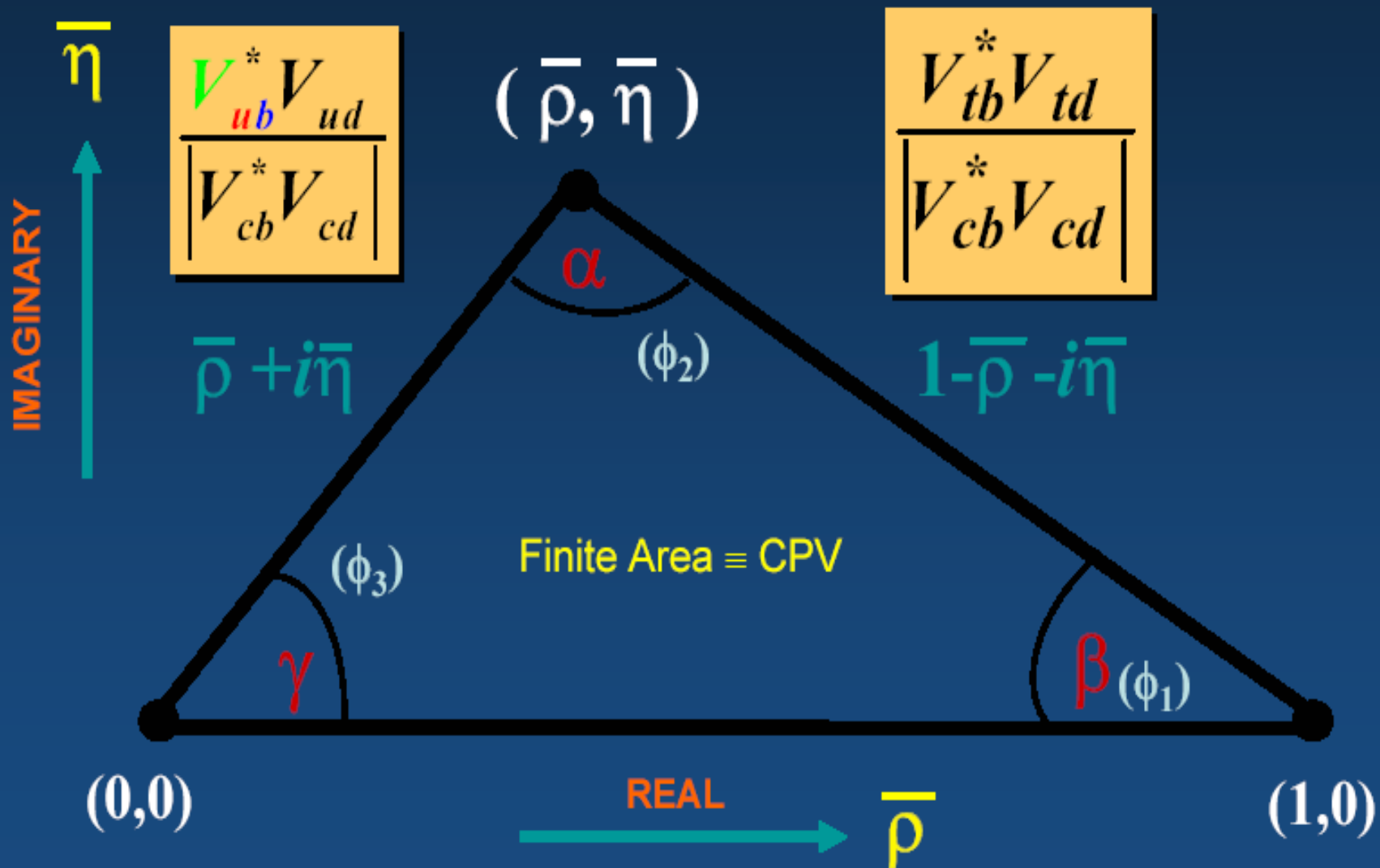
$$\bar{\rho} = \rho (1 - \lambda^2/2) \quad \bar{\eta} = \eta (1 - \lambda^2/2)$$



The Standard Triangle of the Standard Model

Unitarity:

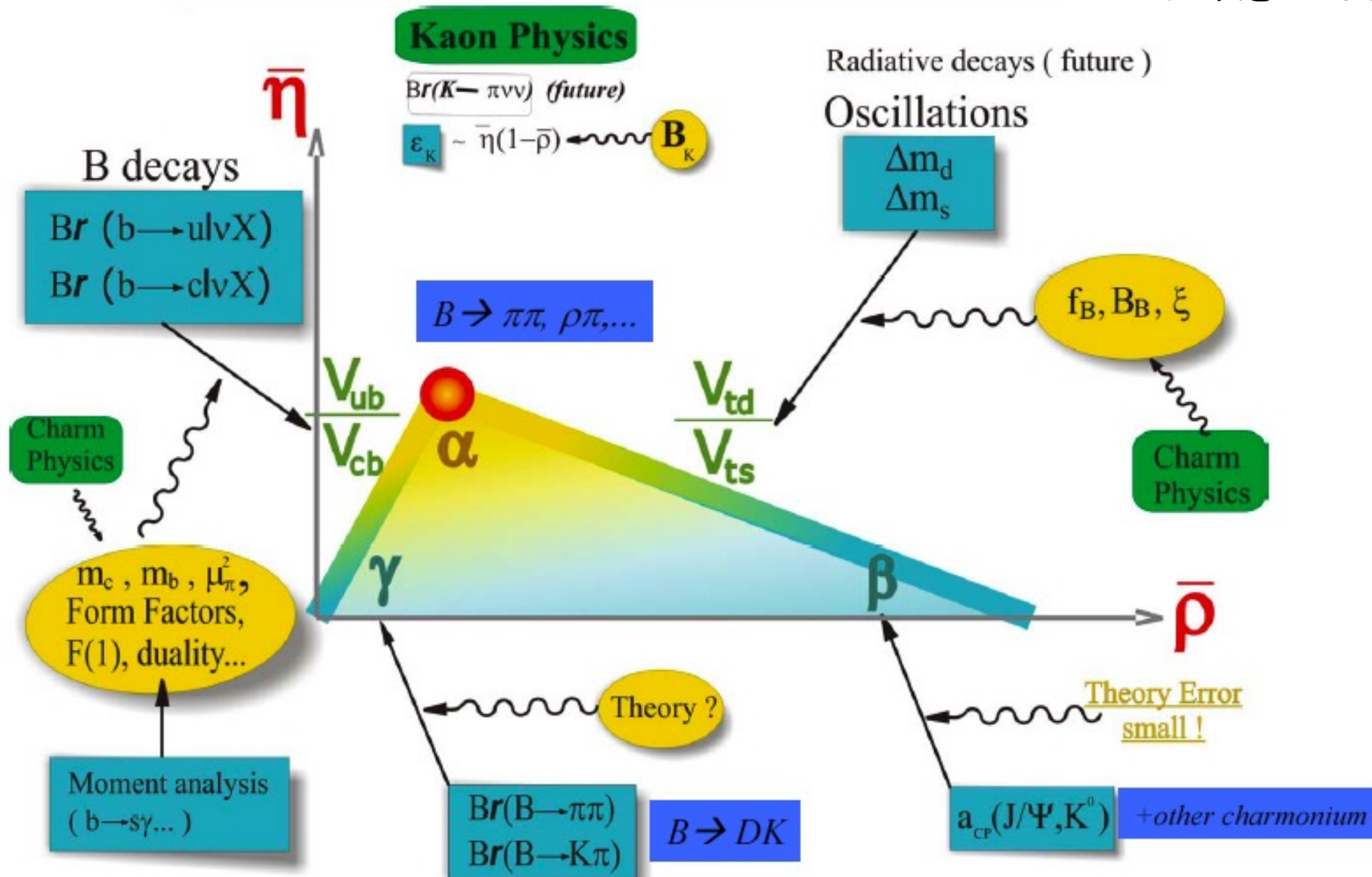
$$V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0$$



Visualization of the unitarity of the CKM matrix

Unitarity Triangle in the $(\bar{\rho}-\bar{\eta})$ plane

From
A. Stocchi
ICHEP 2002





$$\mathcal{L}_\theta = \theta \tilde{G}^{\mu\nu a} G_{\mu\nu}^a$$

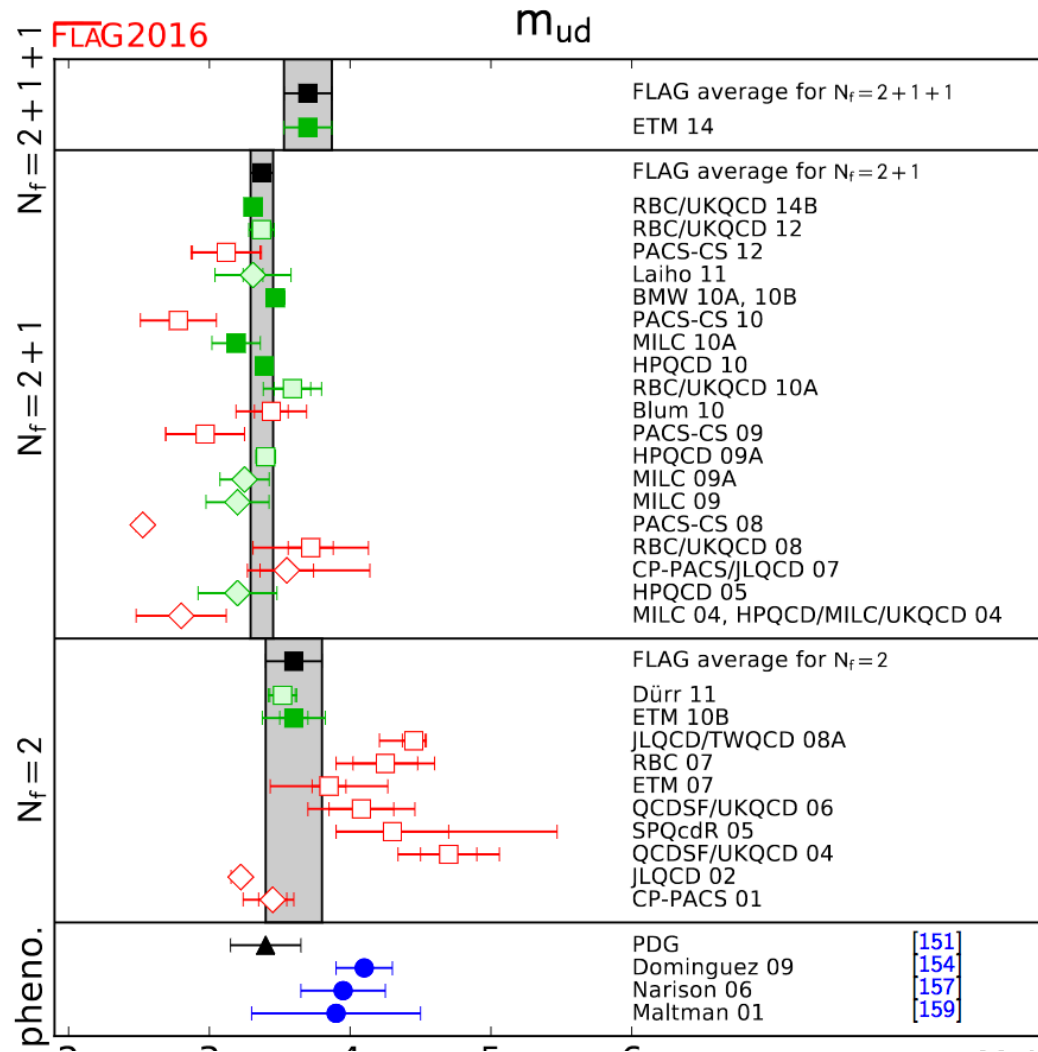
$$\tilde{G}_{\mu\nu}^a = \varepsilon_{\mu\nu\rho\sigma} G_{\rho\sigma}^a$$

$$\mathcal{L}_\theta \sim \theta \vec{E}^a \cdot \vec{B}^a$$

This term violates CP and gives a contribution to the electric dipole moment of the neutron

$$e_n < 3 \cdot 10^{-26} \text{ e cm}$$

$$\theta < 10^{-10} \quad \text{which is quite unnatural !!}$$

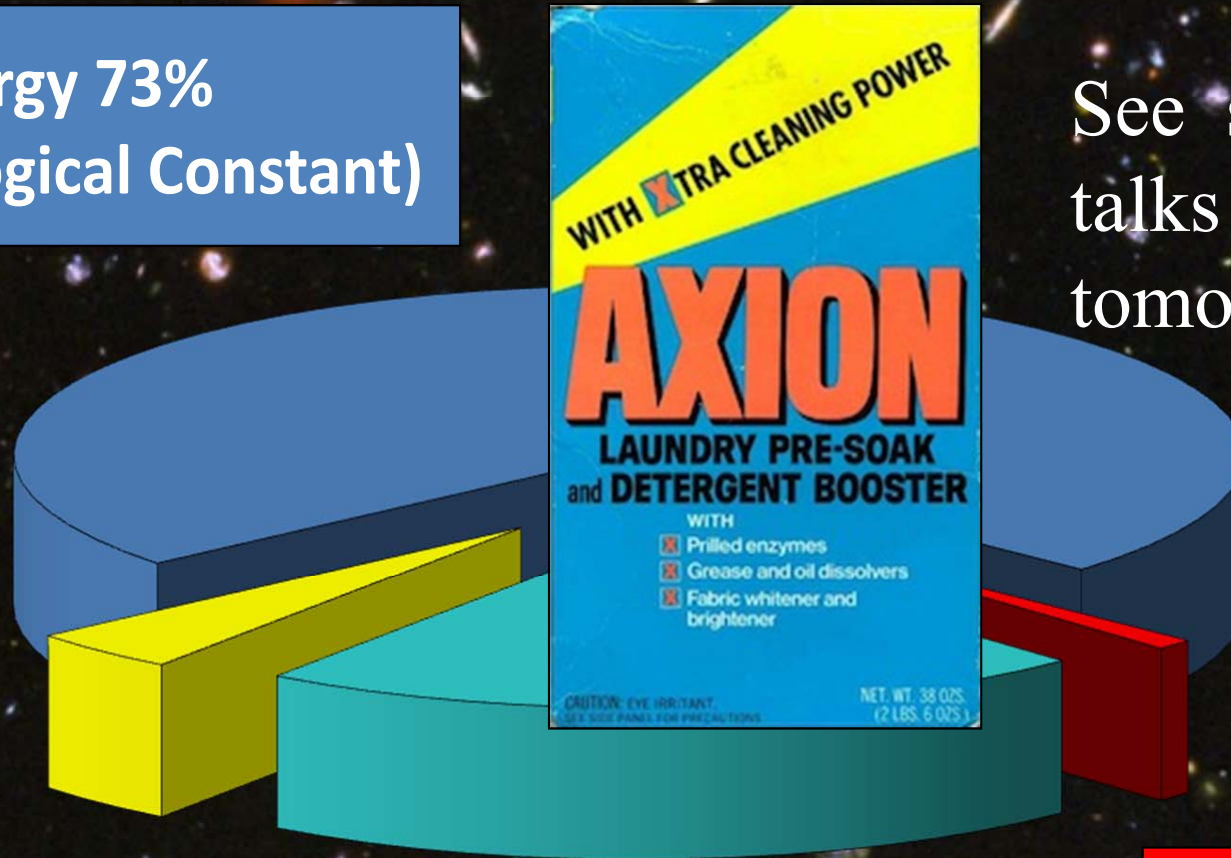


N_f	m_u	m_d	m_u/m_d	R	Q
2+1+1	2.36(24)	5.03(26)	0.470(56)	35.6(5.1)	22.2 (1.6)
2+1	2.16(9)(7)	4.68(14)(7)	0.46(2)(2)	35.0(1.9)(1.8)	22.5(6)(6)
2	2.40(23)	4.80(23)	0.50(4)	40.7(3.7)(2.2)	24.3(1.4)(0.6)

Raffelt

See several
talks on axions
tomorrow

Dark Energy 73%
(Cosmological Constant)

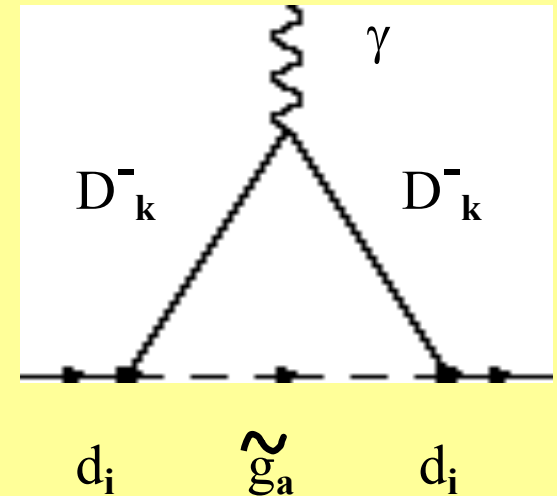
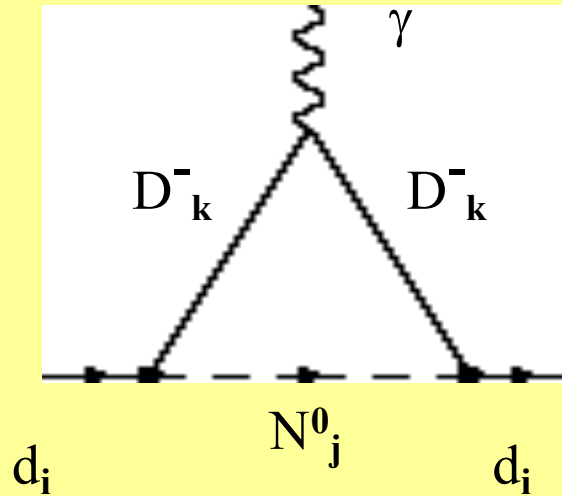
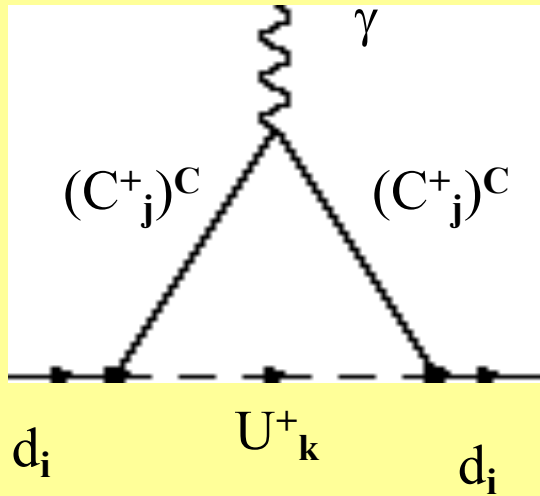


Ordinary Matter 4%
**(of this only about
10% luminous)**

**Dark Matter
23%**

**Neutrinos
0.1–2%**

Neutron electric dipole moment in SuperSymmetry



$$\begin{aligned} \mathcal{L}^{\Delta F=0} = & -i/2 C_e \bar{\Psi} \sigma_{\mu\nu} \gamma_5 \Psi F^{\mu\nu} \\ & -i/2 C_C \bar{\psi} \sigma_{\mu\nu} \gamma_5 t^a \psi G^{\mu\nu a} \\ & -1/6 C_g f_{abc} G^a_{\mu\rho} G^{b\rho}_{\nu} G^c_{\lambda\sigma} \epsilon^{\mu\nu\lambda\sigma} \end{aligned}$$

C_e, C_g can be computed perturbatively

Why Flavor Physics is so important:

*It is sensitive to NP scales $\Lambda_{NP} \gg E_{collider}$
since FCNC are suppressed in the SM by loops and
small $|V_{ij}|$ (Cabibbo suppression)*

SM Flavor puzzle:

*Why flavor parameters are so small and hierarchical?
(and different from the neutrino sector)*

NP Flavor puzzle:

*If NP is at the TeV scale, why FCNC
effects are so small that they have not be detected yet?*

WHY RARE DECAYS ?

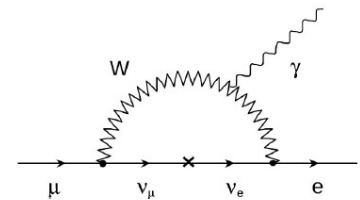
Rare decays are a manifestation of broken (accidental) symmetries e.g. of physics beyond the Standard Model

Proton decay

$\mu \rightarrow e + \gamma$

$\nu_i \rightarrow \nu_k$ found !

baryon and lepton number conservation
lepton flavor number



$$\mathcal{B}(\mu \rightarrow e \gamma) \sim \alpha \frac{m_\nu^4}{m_W^4} \sim 10^{-52}$$

Rare decays allowed in the SM

$q_i \rightarrow q_k + \nu \bar{\nu}$

$q_i \rightarrow q_k + l^+ l^-$

$q_i \rightarrow q_k + \gamma$

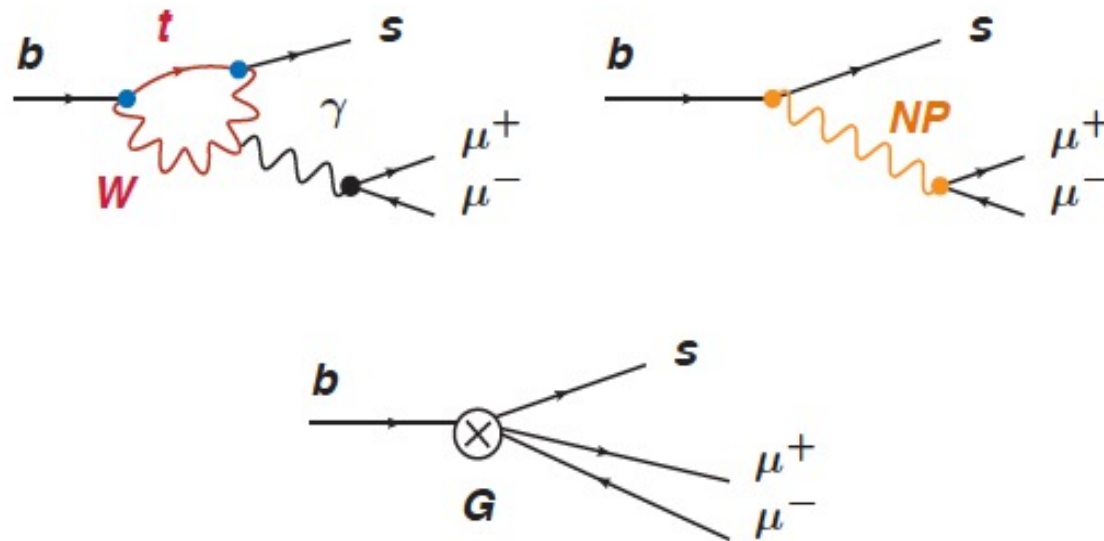
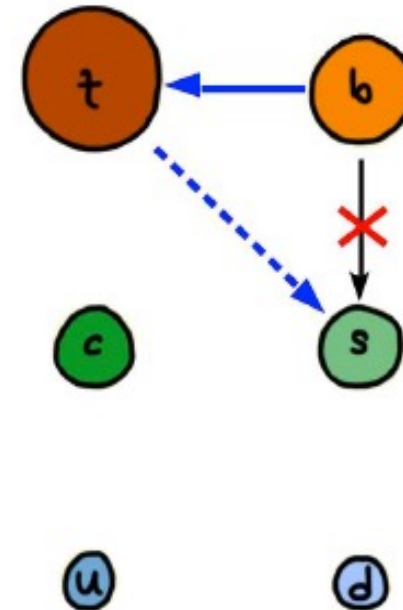
these decays occur only via loops and are suppressed by CKM because of GIM

**THUS THEY ARE SENSITIVE TO
NEW PHYSICS**

Flavor Changing Neutral Currents in the SM

In the SM, flavor changing neutral currents (FCNCs) are absent at the tree level

FCNCs can arise at the **loop level**
they are suppressed by **loop factors**
and small **CKM elements**

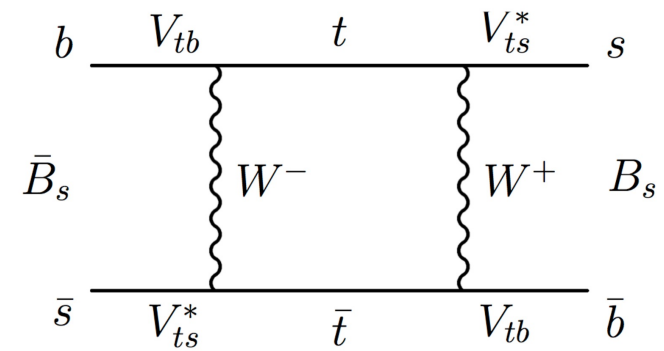


$$G \sim \frac{1}{16\pi^2} \frac{g^4}{m_W^2} \frac{m_t^2}{m_W^2} V_{tb} V_{ts}^* + \frac{C_{NP}}{\Lambda_{NP}^2}$$

→ measuring low energy flavor observables gives information
on new physics flavor couplings and the new physics mass scale

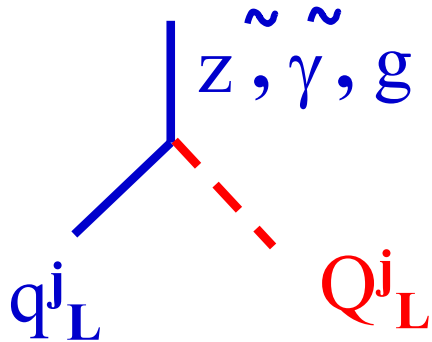
B⁰ - $\bar{B}^0$ mixing

Standard Model CKM



$$\Delta m_{B_s} = \frac{G_F^2 M_W^2}{16\pi^2} A^2 \lambda^6 F_{tt} \left(\frac{m_t^2}{M_W^2} \right) \langle B_s | (\bar{s} \gamma_\mu (1 - \gamma_5) b)^2 | \bar{B}_s \rangle$$

Hadronic matrix element

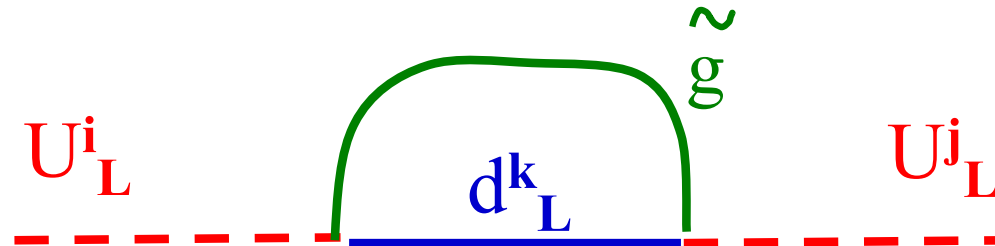


In general the mixing mass matrix of the SQuarks (SMM) is not diagonal in flavour space analogously to the quark case

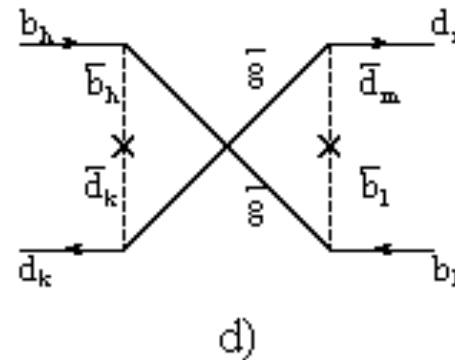
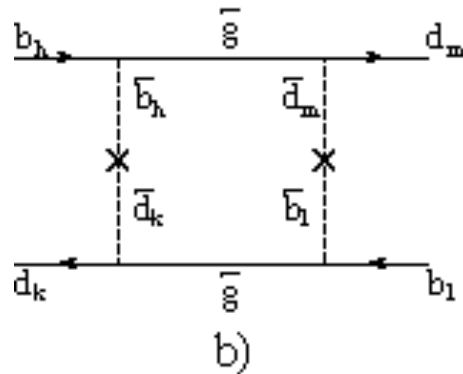
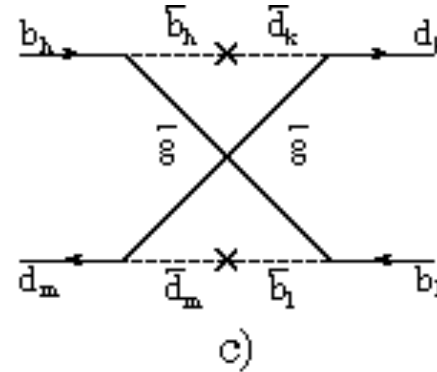
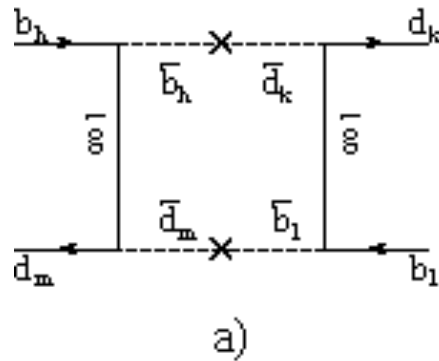
We may either
Diagonalize the SMM

or Rotate by the same
Matrices the SUSY partners of
the u- and d- like quarks

$$(Q_L^j)' = U_L^{ij} Q_L^j$$

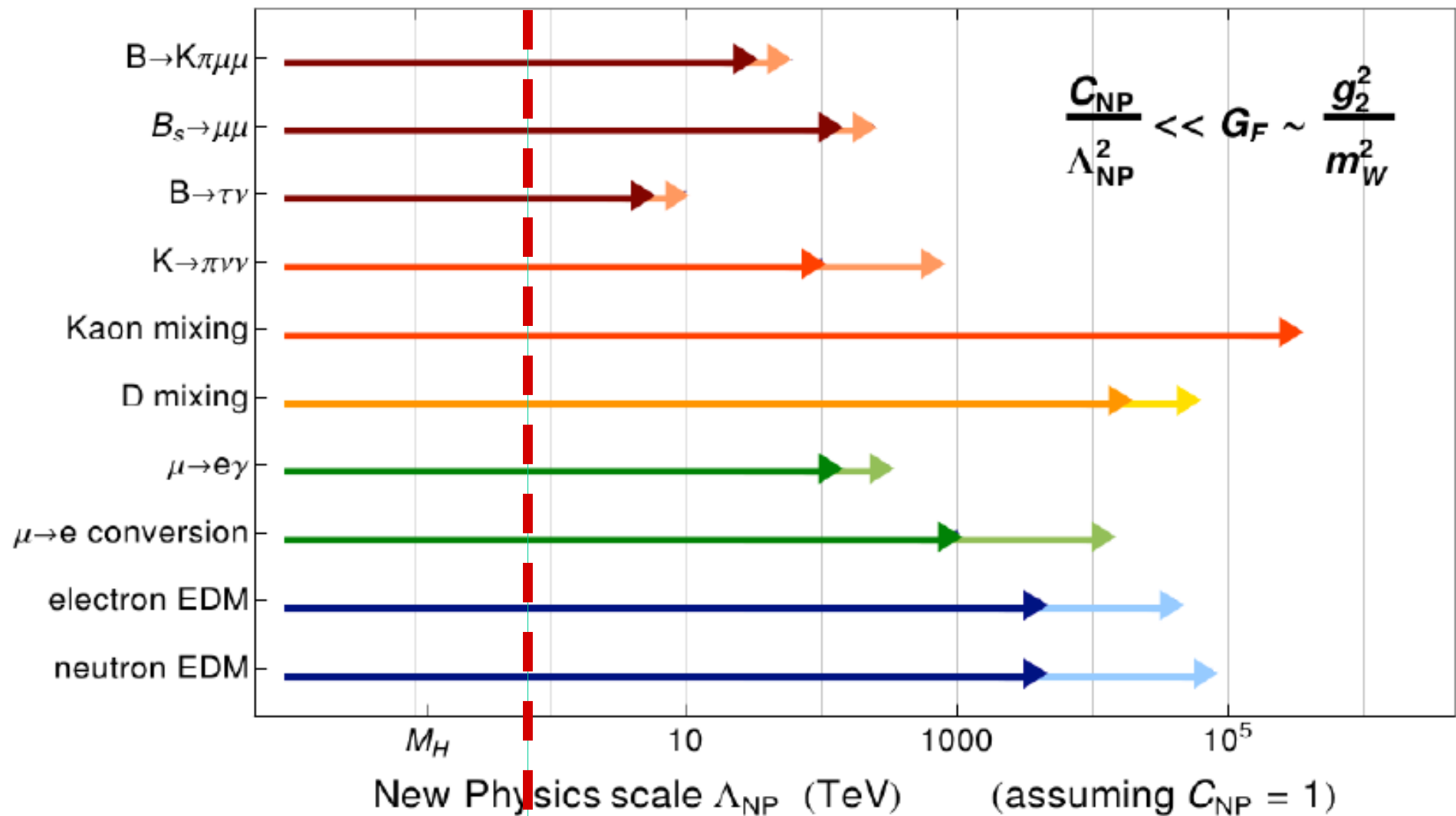


In the latter case the Squark Mass Matrix is not diagonal



$$(m^2_Q)_{ij} = m^2_{average} \mathbf{1}_{ij} + \Delta m_{ij}^2 \quad \delta_{ij} = \Delta m_{ij}^2 / m^2_{average}$$

Sensitivity to New Physics from Flavor I



Approximate LHC direct reach

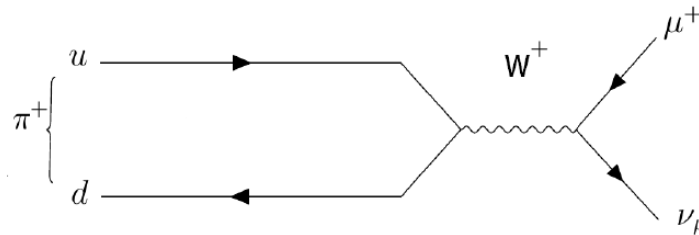
Classification of the processes in the SM

Leptonic Decays

the prototype of these decays is given by

$$\pi^+ \longrightarrow \mu^+ + \nu_\mu \quad (35)$$

which at the fundamental level is given by



$$\Gamma \propto |V_{ij}|^2 f_P^2$$

Other possible leptonic decays are given by

$$K^+ \longrightarrow \mu^+ + \nu_\mu$$

$$D^+ \longrightarrow \mu^+ + \nu_\mu$$

$$B^+ \longrightarrow \tau^+ + \nu_\tau$$

$$\pi^+ \longrightarrow e^+ + \nu_e$$

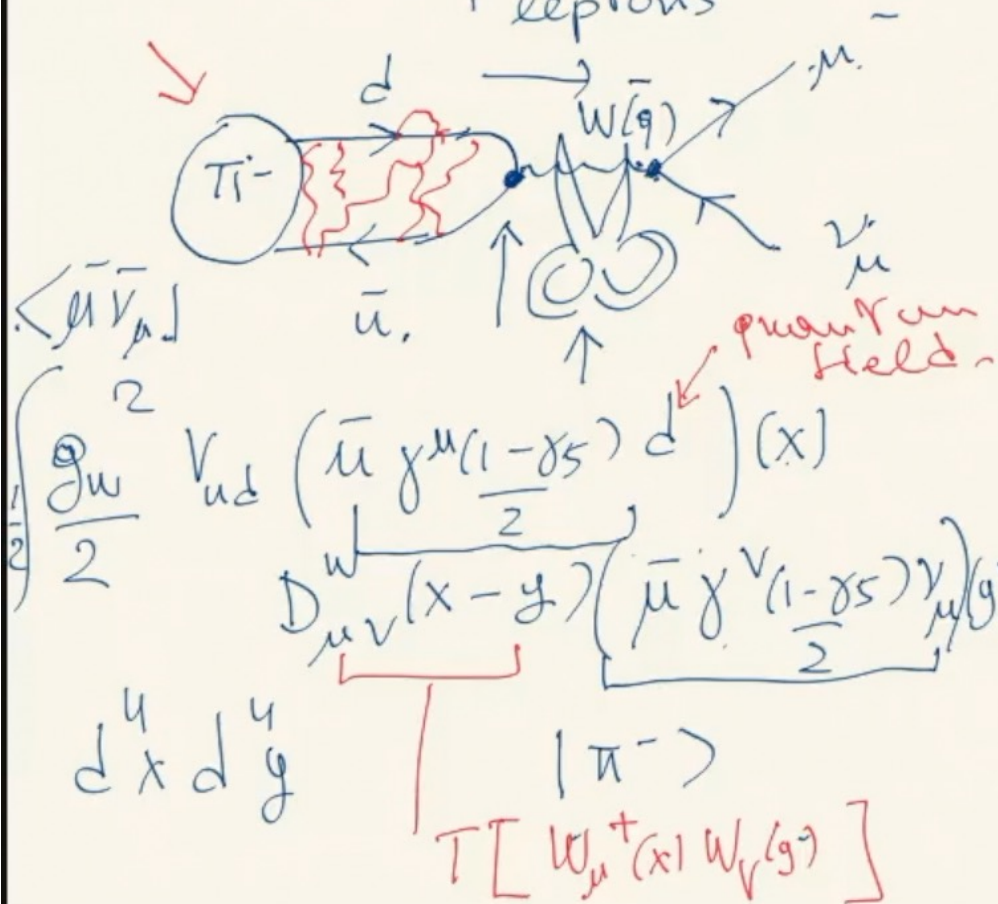
the latter process is suppressed by chirality

*from lattice
calculations*

LEPTONIC DECAYS

$$\pi^- \rightarrow \mu^- \bar{\nu}_\mu$$

↑ leptons



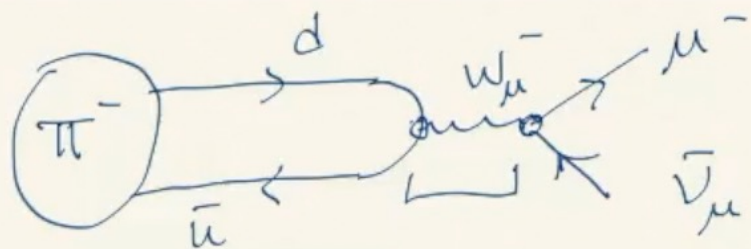
$$\frac{1}{2} \langle \bar{\mu} \bar{\nu}_\mu | \mathcal{L}_I \mathcal{L}_I | \pi^- \rangle$$

$$\mathcal{L}_I = \frac{g_w}{\sqrt{2}} (J_\mu^- W_\mu^+ + J_\mu^+ W_\mu^-)$$

$$J_\mu^- = \bar{u} \gamma^\mu (1-\gamma_5) d + \bar{e} \gamma^\mu (1-\gamma_5) \nu_e + \dots$$

$$J_\mu^+ = \bar{d} \gamma^\mu (1-\gamma_5) u + \bar{\nu}_e \gamma^\mu (1-\gamma_5) e + \dots$$

$$+ \bar{\nu}_\mu \gamma^\mu (1-\gamma_5) \mu + \dots$$



$$\left(\frac{-ig_W}{\sqrt{2}}\right)^2 V_{ud} \bar{u}(p_\mu) \gamma^\mu \frac{(1-\gamma_5)}{2} u(p_{\bar{\nu}_\mu}) \langle 0 | \bar{u} \gamma^\mu \frac{(1-\gamma_5)}{2} d | \pi^- \rangle$$

$$i \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{M_W^2} \right) \frac{1}{-M_W^2 + i\epsilon}$$

$$q = p_\pi \sim 130 \text{ MeV}$$

$$= -i \frac{G_F}{\sqrt{2}} V_{ud} \bar{u}(p_\mu) \gamma^\mu (1-\gamma_5) u(p_{\bar{\nu}_\mu}) \langle 0 | \bar{u} \gamma^\mu (1-\gamma_5) d | \pi^- \rangle$$

$$\begin{aligned} \langle 0 | \bar{u} \gamma^\mu (1-\gamma_5) d | \pi^- \rangle &= \\ - \langle 0 | \bar{u} \gamma^\mu \gamma_5 d | \pi^- \rangle &= \\ - i f_\pi (p_\pi^\mu) \end{aligned}$$

$$A = \frac{G_F}{\sqrt{2}} f_\pi V_{ud} p_\pi^\mu \bar{u}(p_\mu) \gamma^\mu (1-\gamma_5) u(p_{\bar{\nu}_\mu})$$

$$d\Gamma = \frac{1}{2M_\pi} \sum_{\text{final states}} |A|^2 (2\pi)^4 \delta^4(p_\pi - p_\mu - p_{\bar{\nu}_\mu}) \frac{d^3 p_\mu}{(2\pi)^3 2E_\mu} \frac{d^3 p_{\bar{\nu}_\mu}}{(2\pi)^3 2E_{\bar{\nu}_\mu}}$$

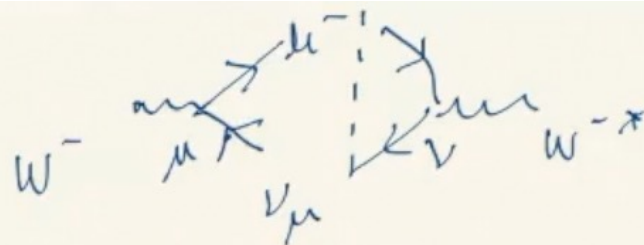
$$|A|^2 = \frac{G_F^2}{2} f_\pi^2 |V_{ud}|^2 p_\pi^\mu p_\pi^\nu \sum_{\text{pol.}} \bar{u}(p_\mu) \gamma^\mu (1-\gamma_5) u(p_{\bar{\nu}}) \bar{u}(p_{\bar{\nu}}) \gamma^\nu (1-\gamma_5) u(p_\mu)$$

$$\begin{aligned} \langle 0 | \bar{u} \gamma^\mu (1 - \gamma_5) d | \pi^- \rangle &= \\ &= \langle 0 | \bar{u} \gamma^\mu \gamma_5 d | \pi^- \rangle = \\ &= i f_\pi (P_\pi^\mu) \end{aligned}$$

$$A = \frac{G_F}{\sqrt{2}} f_\pi V_{ud} P_\pi^\mu \bar{u}(p_\mu) \gamma^\mu (1 - \gamma_5) \nu_\mu$$

$$d\Gamma = \frac{1}{2M_\pi} \sum_{\text{final}} |A|^2 (2\pi)^4 \delta^4(p_\pi - p_\mu - p_\nu) \frac{d^3 p_\mu}{(2\pi)^3 2E_\mu} \frac{d^3 p_\nu}{(2\pi)^3 2E_\nu}$$

$$|A|^2 = \frac{G_F^2}{2} f_\pi^2 |V_{ud}|^2 \underbrace{P_\pi^\mu P_\pi^\nu}_{\text{circled}} \bar{u}(p_\mu) \gamma_\mu (1 - \gamma_5) \nu(p_\nu) \bar{\nu}(p_\nu) \gamma_\nu (1 - \gamma_5) u(p_\mu)$$



$$t_2 (\not{p}_\mu + m_\mu) \gamma^\mu (1 - \gamma_5) \not{p}_\nu \gamma^\nu (1 - \gamma_5)$$

$$= 2 t_2 (\not{p}_\mu + m_\mu) \gamma^\mu \not{p}_\nu \gamma^\nu (1 - \gamma_5)$$

$$= 2 t_2 \not{p}_\mu \gamma^\mu \not{p}_\nu \gamma^\nu (1 - \gamma_5) + 2 m_\mu t_2 \gamma^\mu \not{p}_\nu \gamma^\nu (1 - \gamma_5)$$

$$= 2 \left[\not{p}_\mu^\rho \not{p}_\nu^\sigma + \not{p}_\mu^\sigma \not{p}_\nu^\rho - g^{\rho\sigma} \not{p}_\mu \not{p}_\nu - i \epsilon^{\rho\sigma\alpha\beta} (\not{p}_\mu)_\alpha (\not{p}_\nu)_\beta \right]$$

$$\begin{aligned}
 |A|^2 &= 4G_F^2 |V_{ud}|^2 f_\pi^2 p_\pi^\rho p_\pi^\sigma \\
 &\quad (p_\mu^\rho p_\nu^\sigma + p_\mu^\sigma p_\nu^\rho - g^{\rho\sigma} p_\mu \cdot p_\nu) \\
 &= 4G_F^2 |V_{ud}|^2 f_\pi^2 \{ 2(p_\pi \cdot p_\mu)(p_\pi \cdot p_\nu) \\
 &\quad - m_\pi^2 (p_\mu \cdot p_\nu) \}
 \end{aligned}$$

$$(p_\pi \cdot p_\mu) = (p_\mu + p_\nu) \cdot p_\mu = m_\mu^2 + p_\mu \cdot p_\nu$$

$$(p_\pi \cdot p_\nu) = (p_\mu + p_\nu) \cdot p_\nu = p_\mu \cdot p_\nu$$

$$p_\pi^2 = m_\pi^2 = (p_\mu + p_\nu)^2 = m_\mu^2 + 2p_\mu \cdot p_\nu$$

$$p_\mu \cdot p_\nu = \frac{m_\pi^2 - m_\mu^2}{2}$$

$$d\Gamma = \frac{1}{2m_\pi} G_F^2 |V_{ud}|^2 f_\pi^2$$

$$4m_\mu^2 (m_\pi^2 - m_\mu^2) \circ \textcircled{PS}$$

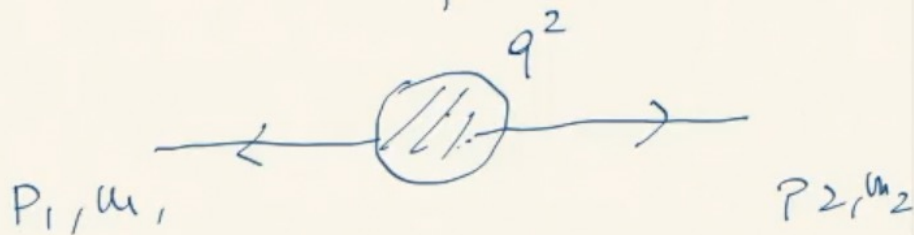
$$\begin{aligned}
 \int PS &= \int (\frac{d^3 p_\mu}{(2\pi)^3 2E_\mu} \frac{d^3 p_\nu}{(2\pi)^3 2E_\nu}) \delta^4(p_\pi - p_\mu - p_\nu) \\
 &= \frac{1}{(2\pi)^2} \pi \left(\frac{|\vec{p}_\mu|}{m_\pi} \right)
 \end{aligned}$$

$$I = \int \delta^4(q - p_1 - p_2) \frac{d^3 p_1}{2E_1} \frac{d^3 p_2}{2E_2}$$

$$= \pi \frac{|\vec{p}_1|}{\sqrt{q^2}}$$

$$|\vec{p}_1| = |\vec{p}_2| \rightarrow$$

$$|\vec{p}_1|^2 = \frac{[(q^2)^2 + (m_1^2)^2 + (m_2^2)^2 - 2q^2 m_1^2 - 2q^2 m_2^2 - 2m_1^2 m_2^2]}{4q^2}$$



$$d\Gamma = \frac{1}{2m_\pi} G_F^2 |V_{ud}|^2 f_\pi^2$$

$$4m_\mu^2 (m_\pi^2 - m_\mu^2) \theta(\text{PS})$$

$$\int \text{PS} = \int (2\pi)^4 \delta^4(p_\pi - p_\mu - p_\nu) \frac{d^3 p_\mu}{(2\pi)^3 2E_\mu} \frac{d^3 p_\nu}{(2\pi)^3 (2E_\nu)}$$

$$= \frac{1}{(2\pi)^2} \pi \left(\frac{|\vec{p}_\mu|}{m_\pi} \right)$$

$$= \frac{1}{(2\pi)^2} \pi \frac{m_\pi^2 - m_\mu^2}{2m_\pi^2}$$

$$= \frac{1}{(2\pi)^2} \frac{\pi}{2} \left(1 - \frac{m_\mu^2}{m_\pi^2} \right)$$

$$\Gamma = \frac{G_F^2 f_\pi^2 m_\mu^2 \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2 m_\pi |V_{ud}|^2}{8\pi}$$

non pert. quality.

$$B^- \rightarrow e^- \bar{\nu}_e$$

$$|V_{ub}|^2$$

$$D^-$$

$$|V_{cd}|^2$$

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{ud}^* \left(1 + \frac{\alpha}{\pi} \log \frac{M_Z}{M_W}\right) (\bar{d} \gamma^\mu (1 - \gamma^5) u) (\bar{\nu}_\ell \gamma_\mu (1 - \gamma^5) \ell),$$

Classification of the processes in the SM

Semi-leptonic Decays

these are the better sources to measure the absolute values of the CKM matrix elements $|V_{ij}|$

$$n \rightarrow p + e^- + \bar{\nu}_e$$

other possible semi-leptonic decays are the following

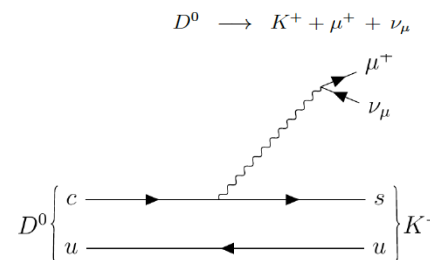
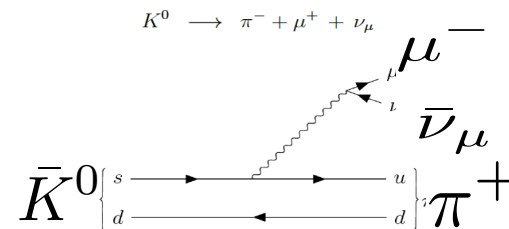
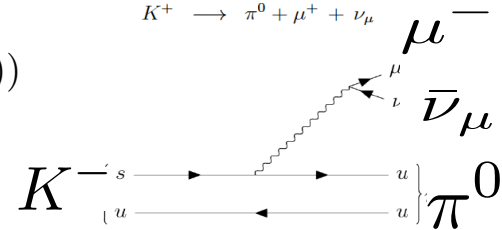
$$\begin{aligned} \pi^+ &\rightarrow \pi^0 + e^+ + \nu_e \\ K^+ &\rightarrow \pi^0 + \mu^+ + \nu_\mu \end{aligned}$$

$$-\frac{G_F}{\sqrt{2}} V_{us} (\bar{u}(x) \gamma_\mu (1 - \gamma_5) s(x)) (\bar{\mu}(x) \gamma_\mu (1 - \gamma_5) \nu_\mu(x))$$

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 |V_{us}|^2}{24\pi^3} |\vec{p}_{\pi^-}|^3 |f^+(q^2)|^2$$

$$\frac{1}{\Gamma} \frac{d\Gamma}{dq^2} \propto |f(q^2)|$$

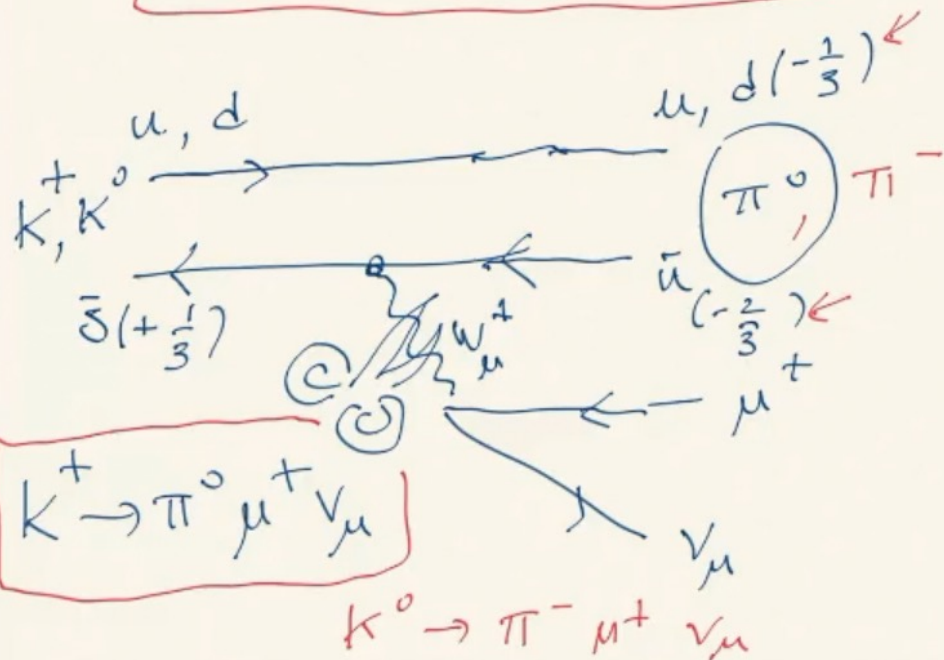
from lattice calculations



$$B^+ \rightarrow \pi^+ \nu_e$$

SEMI LEPTONIC
DECAYS

$$n \rightarrow p + e^- + \bar{\nu}_e$$



$$\langle \pi^0 \mu^+ \nu_\mu | \left(\bar{\nu}_\mu \gamma^\mu (1 - \gamma_5) \mu \right) | K^+ \rangle$$

$$\left(\frac{-i G_F}{\sqrt{2}} \right) = \left(-i G_F V_{us} \right)$$

$$\langle \mu^+ \nu_\mu | \bar{\nu}_\mu \gamma^\mu (1 - \gamma_5) \mu | 0 \rangle$$

$$= -i \frac{G_F}{\sqrt{2}} V_{us} \bar{u}(p_\mu) \gamma^\mu (1 - \gamma_5) u(p_{\mu^+}) \langle \pi^0 | \bar{s} \gamma^\mu u | K^+ \rangle$$

Wigner-Eckart

$$\langle \pi^0 | \bar{S} \gamma^\mu u | K^+ \rangle =$$

$$f^+(q^2) (P_{K^+}^\mu + P_{\pi^0}^\mu) +$$

$$f^-(q^2) (P_{K^+}^\mu - P_{\pi^0}^\mu)$$

$$q = P_{K^+} - P_{\pi^0} = P_{\mu^+} + P_{\nu_\mu}$$

$$f^-(q^2) \neq 0$$

$$f^+(q_{=0}^2) = \text{charge} \\ = \sqrt{2} \left(1 + \frac{(m_s - m_d)^2}{(1 \text{ GeV})^2} \right)$$

$$\langle e^- \nu_e | \bar{e} \gamma_\mu (1 - \gamma_5) \nu_e | 0 \rangle =$$

$$\bar{u}(P_e) \gamma_\mu (1 - \gamma_5) u(P_{\nu_e})$$

$$\langle p | \bar{u} \gamma_\mu (1 - \gamma_5) s | \Lambda^0 \rangle$$

$$\langle p | \bar{u} \gamma_\mu s | \Lambda^0 \rangle =$$

$$\bar{u}^{\sigma_p}(P_p) \left[F_1(q^2) \gamma^\mu + i \frac{\sigma^{\mu\nu} q_\nu F_2(q^2)}{[(m_p + m_{\Lambda^0})/2]} \right]$$

$$+ \left[\frac{q^\mu F_3(q^2)}{(m_p + m_{\Lambda^0})/2} \right] u^{\sigma_{\Lambda^0}}(P_{\Lambda^0})$$

$$q = P_{\Lambda^0} - P_p = P_{\nu_e} + P_{e^-}$$

$$q^\mu \langle \pi(p') | \hat{V}_\mu | \pi(p) \rangle =$$

$$f^+(q^2) (p_\mu + p'_\mu) + f^-(q^2) \underbrace{(p_\mu - p'_\mu)}_{q_\mu}$$

$$= f^+(q^2) (m_\pi^2 - m_\pi^2) + f^-(q^2) q^2 = 0$$

$$f^-(q^2) = 0$$

$$\langle \pi(\vec{p}) | Q(t) | \pi(\vec{p}') \rangle = \int d^3x \langle \pi(\vec{p}') | V_0(\vec{x}, t) | \pi(\vec{p}) \rangle$$

$$\langle \pi(\vec{p}') | Q(0) | \pi(\vec{p}) \rangle = \int d^3x \langle \pi(\vec{p}') | e^{-i\vec{p}' \cdot \vec{x}} V_0(0) e^{i\vec{p} \cdot \vec{x}} | \pi(\vec{p}) \rangle$$

$$= \int d^3x e^{i(\vec{p} - \vec{p}') \cdot \vec{x}} \langle \pi(\vec{p}') | V_0(0) | \pi(\vec{p}) \rangle$$

$$Q_\pi = (2\pi)^3 \delta^3(\vec{p} - \vec{p}') \underbrace{\langle \pi(\vec{p}) | V_0(0) | \pi(\vec{p}) \rangle}_{2E f(q^2=0)}$$

$$(2\pi)^3 2E \delta^3(\vec{p} - \vec{p}')$$

$$\langle B^i J^k B^i \rangle$$

$$J^k |B^i\rangle = 8 \times 8 = 1 + 8 + 8 + 27 + 10 + 10$$

$$[J^k, B^i] = i f^{k i m} R^m \quad R^m \in \mathfrak{g}$$

$$\{J^k, B^i\} = i d^{k i m} J^m \quad J^m \in \mathfrak{g}$$

$$X |m, l, l_z\rangle = \left[r \sin \theta \cos \varphi f_{me}(z) Y_{ll} \right]$$

$$= g(z) \sin \theta (e^{i\varphi} + e^{-i\varphi}) \sin \theta e^{i\varphi}$$

$$\sim g(z) (e^{2i\varphi} + 1) \sin \theta$$

$$(Y_{l, l+1} + Y_{l, l-1}) Y_{ll}$$

$$|1, 1\rangle |1, 1\rangle = |2, 2\rangle$$

$$|1, 0\rangle |1, 1\rangle = a |2, 0\rangle + b |0, 0\rangle$$

$$\rightarrow J_\mu = (\cos \theta_c J_\mu^+ + \sin \theta_c V_\mu^+)$$

$$\langle B^k | J_\mu^+ | B^i \rangle_{i=k} = I$$

SEMI-LEPTONIC HYPERON DECAY

$$B = \begin{pmatrix} \frac{1}{\sqrt{6}} \Lambda^0 + \frac{1}{\sqrt{2}} \Sigma^0 & \Sigma^+ & p \\ \Sigma^- & \frac{1}{\sqrt{6}} \Lambda^0 - \frac{1}{\sqrt{2}} \Sigma^0 & n \\ \bar{\Sigma}^0 & \bar{\Sigma}^+ & -\sqrt{\frac{2}{3}} \Lambda^0 \end{pmatrix} = B^i \frac{\lambda^i}{2}$$

$$\langle B | J^k | B \rangle \sim 8 \times 8 \times 8$$

$$t_2 B^i J^k B^j \sim t_2 J^k [B^j, B^i] + t_2 J^k \{B^j, B^i\}$$

F-couplings D-couplings

EXAMPLES

$$n \rightarrow p e \bar{\nu} \quad [\gamma^4 + (D+F) \gamma^4 \gamma^5] \cos \theta_c$$

$$\Sigma^\pm \rightarrow \Lambda e^\pm \nu \quad \gamma^4 \gamma^5 \sqrt{\frac{2}{3}} D \cos \theta_c$$

$$\Lambda^0 \rightarrow p e \bar{\nu} \quad -\frac{\sqrt{3}}{2} [\gamma^4 + (F + \frac{D}{3}) \gamma^4 \gamma^5] \sin \theta_c$$

SU(3) SYMMETRY ASSUMED

$$F, D \quad F_1 = g_1(0) / \left(1 - \frac{g_1^2}{m_A^2}\right)^2 \quad \text{DIPOLE APPROXIMATION}$$

$|V_{us}|$ not very

sensitive to the

PRECISE VALUE OF m_A

V_{us} from hyperon semileptonic decays

D. Guadagnoli 2005

new data analysis from Cabibbo, Swallow and Winston ('03):

Use of exp. info for decay amplitudes and angular correlations

$$\Gamma_{decay} \simeq |V_{us}|^2 f_1^2(0) \left(1 + 3 \left(\frac{g_1(0)}{f_1(0)} \right)^2 \right) \times \left[\text{known factors, including rad. corrections} \right]$$

✓ Exps. give direct access to the product $|V_{us}| f_1(0)$

- 1) the ratio $g_1(0) / f_1(0)$ is extracted from data,
- 2) SU(3) symmetry is assumed for $f_1(0)$ [as well as for $g_2(0)$].

$$|V_{us}|^{hyp.} = 0.2250 \pm 0.0027$$

$$|V_{us}|^{K13} = 0.2256 \pm 0.0021$$

Decay	g_1/f_1	V_{us}
$\Lambda \rightarrow pe^- \bar{\nu}$	0.718(15)	0.2224 ± 0.0034
$\Sigma^- \rightarrow ne^- \bar{\nu}$	-0.340(17)	0.2282 ± 0.0049
$\Xi^- \rightarrow \Lambda e^- \bar{\nu}$	0.25(5)	0.2367 ± 0.0099
$\Xi^0 \rightarrow \Sigma^+ e^- \bar{\nu}$	$1.32^{+0.22}_{-0.18}$	0.209 ± 0.027
Combined	—	0.2250 ± 0.0027

as the contribution of $\gamma^\mu \gamma_5$ is exactly equal to zero. For the remaining matrix element, as it is a 4-vector under Lorentz transformations, we can only express it through 4-vectors, namely the mesons 4-momentum. Thus, we can write for example that

$$\langle \pi^- | \bar{s} \gamma^\mu u | K^0 \rangle = f^+(q^2)(p_{K^0}^\mu + p_{\pi^-}^\mu) + f^-(q^2)q^\mu$$

where $q^\mu \equiv p_{K^0}^\mu - p_{\pi^-}^\mu$. The quantities $f^+(q^2)$ and $f^-(q^2)$ are called *form factors* and they are functions of q^2 as it is the unique non trivial Lorentz invariant of our problem. Furthermore, they condensate the informations coming from strong interactions, hence we can determine them through non-perturbative techniques, namely numerical simulation on the lattice. Here we put in evidence that in Appendix B we describe the main properties of such framework, namely *Lattice QCD*, also showing an its variant in Appendix B, characterized by the so called *Twisted Mass formalism*.

Another possibility is to describe the hadronic matrix element by introducing a new form factor $f^0(q^2)$, related to the other ones through the relation

$$f^0(q^2) = \frac{q^2}{m_{K^0}^2 - m_{\pi^-}^2} f^-(q^2) + f^+(q^2),$$

so that

$$\langle \pi^- | \bar{s} \gamma^\mu u | K^0 \rangle = f^+(q^2) \left(p_{K^0}^\mu + p_{\pi^-}^\mu - \frac{m_{K^0}^2 - m_{\pi^-}^2}{q^2} q^\mu \right) + f^0(q^2) \frac{m_{K^0}^2 - m_{\pi^-}^2}{q^2} q^\mu.$$

Putting all together, Eq. (1.21) finally becomes

$$\begin{aligned} \frac{d\Gamma}{dq^2} &= \frac{G_F^2 |V_{us}|^2}{24\pi^2} \left(1 - \frac{m_c^2}{q^2} \right)^2 \\ &\times \left[|\vec{p}_{\pi^-}|^3 \left(1 + \frac{m_c^2}{2q^2} \right) f^+(q^2) + m_{K^0}^2 |\vec{p}_{\pi^-}| \left(1 - \frac{m_{\pi^-}^2}{m_{K^0}^2} \right)^2 \frac{3m_c^2}{8q^2} f^0(q^2) \right], \end{aligned} \quad \frac{d\Gamma}{dq^2} = \frac{G_F^2 |V_{us}|^2}{24\pi^3} |\vec{p}_{\pi^-}|^3 |f^+(q^2)|^2$$

where $\vec{p}_{\pi^-}$ is the 3-momentum of the pion, so that

$$|\vec{p}_{\pi^-}| = \left[\left(\frac{m_{K^0}^2 + m_{\pi^-}^2 - q^2}{2m_{K^0}} \right)^2 - m_{\pi^-}^2 \right]^{1/2},$$

Model. To be more specific, we can choose a particular basis in which we can re-express the effective weak Hamiltonian in the Fermi approximation. Referring to the particular quark transition $b \rightarrow c$, as it will be of interest in the future applications present in this Thesis, we can generalize the expression (1.20) valid for a semileptonic transition writing that [37]

$$\mathcal{H}_{eff} = 2\sqrt{2}G_F V_{cb}[(1 + V_L)\mathcal{O}_{V_L} + V_R\mathcal{O}_{V_R} + S_L\mathcal{O}_{S_L} + S_R\mathcal{O}_{S_R} + T_L\mathcal{O}_{T_L}], \quad (1.27)$$

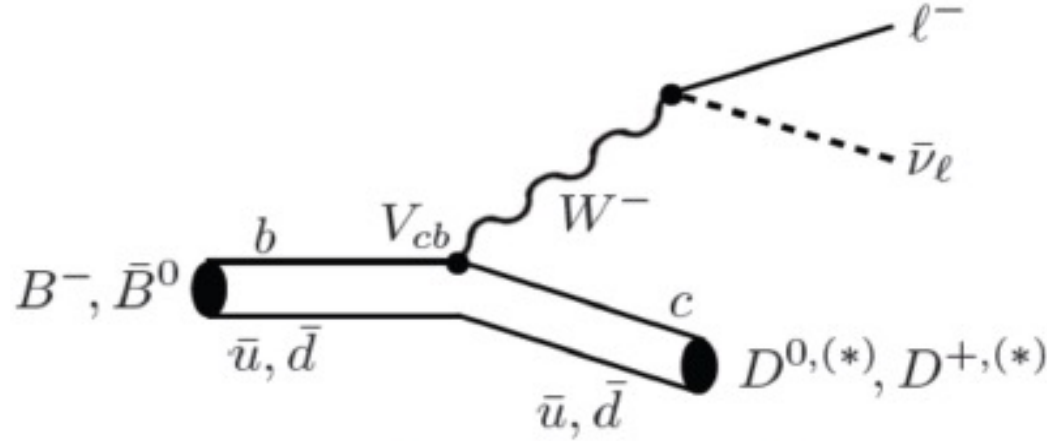


Figure 2.1. Feynman diagram relative to $B^- \rightarrow D^{0(*)}\ell^-\bar{\nu}_\ell$ and $B^0 \rightarrow D^{+(*)}\ell^-\bar{\nu}_\ell$ transitions.

where $V_{L,R}$, $S_{L,R}$ and T_L are the complex Wilson coefficients governing NP contributions while the definitions of NP operators are

$$\begin{aligned} \mathcal{O}_{V_L} &= (\bar{c}\gamma^\mu P_L b)(\bar{\ell}\gamma_\mu P_L \nu_\ell), \\ \mathcal{O}_{V_R} &= (\bar{c}\gamma^\mu P_R b)(\bar{\ell}\gamma_\mu P_L \nu_\ell), \\ \mathcal{O}_{S_L} &= (\bar{c}P_L b)(\bar{\ell}P_L \nu_\ell), \\ \mathcal{O}_{S_R} &= (\bar{c}P_R b)(\bar{\ell}P_L \nu_\ell), \\ \mathcal{O}_{T_L} &= (\bar{c}\sigma^{\mu\nu} P_L b)(\bar{\ell}\sigma_{\mu\nu} P_L \nu_\ell), \end{aligned}$$

D^* , with momentum p , mass m_{D^*} and polarization ϵ , we can define $V(q^2)$ and $A_{0,1,2,3}(q^2)$ form factors, with $q^2 \equiv (p_B - p)^2$, such that ($\epsilon_{0123} = +1$) [66]:

$$\begin{aligned} \langle D^*(p, \epsilon) | \bar{c} \gamma^\mu (1 \mp \gamma_5) b | \bar{B}(p_B) \rangle = & -\frac{2}{m_B + m_{D^*}} \epsilon_{\alpha\beta\gamma}^\mu \epsilon^{*\alpha} p^\beta q^\gamma V(q^2) \\ & \pm i \frac{2m_{D^*}}{q^2} (\epsilon^* \cdot q) q^\mu A_0(q^2) \\ & \mp \frac{i}{m_B - m_{D^*}} [(m_B - m_{D^*}) \epsilon^{*\mu} - (\epsilon^* \cdot q)(p + p_B)^\mu] A_1(q^2) \\ & \mp i \frac{2m_{D^*}}{q^2} (\epsilon^* \cdot q) \left[\frac{q^2}{m_B^2 - m_{D^*}^2} (p + p_B)^\mu - q^\mu \right] A_3(q^2) , \end{aligned} \quad (\text{A.1})$$

$$\bar{B} \rightarrow D^* \ell \nu_\ell$$

together with the identity: $2m_{D^*} A_3(q^2) = [(m_B + m_{D^*}) A_1(q^2) - (m_B - m_{D^*}) A_2(q^2)]$. Note also that $A_3(0) = A_0(0)$, so that eq. (A.1) is finite at the kinematic endpoint. The axial Ward identity yields a non-trivial matrix element for the pseudo-scalar current:

$$\langle D^*(p, \epsilon) | \bar{c} \gamma_5 b | \bar{B}(p_B) \rangle = -i \frac{2m_{D^*} (\epsilon^* \cdot q)}{m_c + m_b} A_0(q^2) , \quad (\text{A.2})$$

while the matrix element for the scalar current must identically vanish. For the sake of completeness, we note that tensor form factors can also be introduced in analogy to eq. (A.1):

$$\begin{aligned} i q_\nu \langle D^*(p, \epsilon) | \bar{c} \sigma^{\mu\nu} (1 \pm \gamma_5) b | \bar{B}(p_B) \rangle = & -\frac{2}{m_B + m_{D^*}} \epsilon_{\alpha\beta\gamma}^\mu \epsilon^{*\alpha} p^\beta q^\gamma T_1(q^2) \\ & \mp \frac{i}{m_B - m_{D^*}} [(m_B - m_{D^*}) \epsilon^{*\mu} - (\epsilon^* \cdot q)(p + p_B)^\mu] T_2(q^2) \\ & \mp \frac{i}{q^2} (\epsilon^* \cdot q) \left[\frac{q^2}{m_B^2 - m_{D^*}^2} (p + p_B)^\mu - q^\mu \right] \\ & \times [(m_B + m_{D^*}) T_1(q^2) - (m_B - m_{D^*}) T_2(q^2)] , \end{aligned} \quad (\text{A.3})$$

together with the algebraic relation: $T_1(0) = T_2(0)$. Large uncertainties in the knowledge of $T_{1,2}(q^2)$ may be presently at hand: so far no tensor contribution has been indeed considered beyond the HQET limit, see for instance [25, 39, 41, 42]. However, within the SM theory, contributions that may involve these form factors do not arise at dimension six

|V_{cb}| extraction from B → D* decay

- **Belle 2018/Belle-II 2023:** $d\Gamma/dx$

Belle Collaboration: PRD '19 [arXiv:1809.03290]

Belle-II Collaboration: PRD '23 [arXiv:2310.01170]

- **Belle 2023:** $(d\Gamma/dx)/\Gamma$

Belle Collaboration, PRD '23 [arXiv:2301.07529]

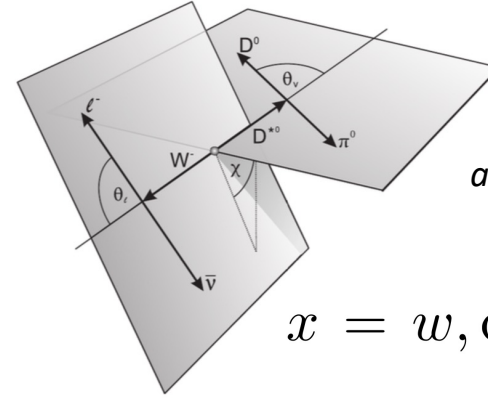


Image taken from
arXiv:1702.01521v2

$$x = w, \cos \theta_l, \cos \theta_v, \chi$$

Theoretically (massless lepton limit) :

$$\left\{ \begin{aligned} \frac{d^4\Gamma}{dw d\cos\theta_v d\cos\theta_\ell d\chi} &= \frac{3}{16\pi} \Gamma_0 \sqrt{w^2 - 1} (1 - 2rw + r^2) \left\{ \underline{H_+^2(w)} \sin^2\theta_v (1 - \cos\theta_\ell)^2 \right. \\ &+ \underline{H_-^2(w)} \sin^2\theta_v (1 + \cos\theta_\ell)^2 + 4 \underline{H_0^2(w)} \cos^2\theta_v \sin^2\theta_\ell \\ &- 2 \underline{H_-(w)} \underline{H_+(w)} \sin^2\theta_v \sin^2\theta_\ell \cos 2\chi \\ &- 2 \underline{H_+(w)} \underline{H_0(w)} \sin 2\theta_v \sin\theta_\ell (1 - \cos\theta_\ell) \cos\chi \\ &\left. + 2 \underline{H_-(w)} \underline{H_0(w)} \sin 2\theta_v \sin\theta_\ell (1 + \cos\theta_\ell) \cos\chi \right\} , \\ \Gamma_0 &\equiv \frac{\eta_{EW}^2 m_B m_{D^*}^2}{(4\pi)^3} G_F^2 |V_{cb}|^2 \end{aligned} \right.$$

$$\begin{aligned} H_+(w) &= f(w) - m_B^2 r \sqrt{w^2 - 1} g(w) , \\ H_-(w) &= f(w) + m_B^2 r \sqrt{w^2 - 1} g(w) , \\ H_0(w) &= \frac{F_1(w)}{m_B \sqrt{1 - 2rw + r^2}} , \end{aligned}$$

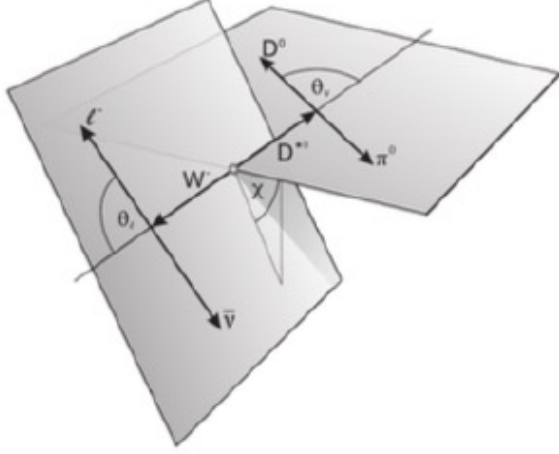


Figure 2.2. Representation of the decay $\bar{B}^0 \rightarrow D^* \bar{\ell} \bar{\nu}$. θ_ℓ is the angle between $\bar{\ell}$ and the direction opposite the $\bar{B}$ meson in the virtual W boson rest frame. θ_D is the angle between the D meson and the direction opposite the B meson in the D^* rest frame. χ is the angle between the two decay planes spanned by the $W - \ell$ and $D^* - \bar{\nu}$ in the B meson rest frame.

B.2 $B \rightarrow D^* \ell \nu$

The angular distribution of the differential decay rate of $B \rightarrow D^* \ell \nu$ can be also generally decomposed in terms of Wigner $\mathcal{D}$ -functions:

$$\frac{d^4\Gamma}{dq^2 d\cos\Theta_\ell d\cos\Theta_V d\chi} = \frac{3}{32\pi} \Re \left[G_0^{0,0}(q^2) \Omega_0^{0,0} + G_0^{0,1}(q^2) \Omega_0^{0,1} + G_0^{0,2}(q^2) \Omega_0^{0,2} + G_0^{2,0}(q^2) \Omega_0^{2,0} + G_0^{2,1}(q^2) \Omega_0^{2,1} + G_1^{2,1}(q^2) \Omega_1^{2,1} + G_0^{2,2}(q^2) \Omega_0^{2,2} + G_1^{2,2}(q^2) \Omega_1^{2,2} + G_2^{2,2}(q^2) \Omega_2^{2,2} \right], \quad (\text{B.10})$$

with $\Omega_m^{l_D, l_\ell} \equiv D_{m,0}^{l_D}(\Omega_D') D_{m,0}^{l_\ell}(\Omega_\ell')$. Here Ω_ℓ' and Ω_D' refer to the angular coordinates of the leptonic and hadronic decay components of B in its rest frame respectively as defined in [68]. The additional Wigner $\mathcal{D}$ -functions involved with respect to eq. (B.1) correspond to:

$$D_{1,0}^1 = -\frac{1}{\sqrt{2}} \exp^{-i\phi} \sin\theta, \quad D_{2,0}^2 = \sqrt{\frac{3}{8}} \exp^{-2i\phi} \sin^2\theta, \quad \text{and} \quad D_{1,0}^2 = -\sqrt{\frac{3}{8}} \exp^{-i\phi} \sin 2\theta. \quad (\text{B.11})$$

The functions $G_k^{i,j} = \mathcal{N}_{D^*} \tilde{G}_k^{i,j}$ can be written as:

$$\begin{aligned} \tilde{G}_0^{0,0} &= \frac{4}{9} \left(3E_\ell E_\nu + \frac{\lambda_{\gamma^*}}{4q^2} \right) \left(|H_+^V|^2 + |H_-^V|^2 + |H_0^V|^2 + (V \rightarrow A) \right) \\ &\quad + \frac{4}{3} \left(E_\ell E_\nu + \frac{\lambda_{\gamma^*}}{4q^2} \right) |H^S|^2 + \frac{4}{3} \left(E_\ell E_\nu + \frac{\lambda_{\gamma^*}}{4q^2} \right) |H^P|^2 \\ \tilde{G}_0^{0,1} &= \frac{4\sqrt{\lambda_{\gamma^*}}}{3} \left(\Re [H_+^V \bar{H}_+^A - H_-^V \bar{H}_-^A] - \frac{m_\ell}{\sqrt{q^2}} \Re [H_0^A \bar{H}^P] - \frac{m_\ell}{\sqrt{q^2}} \Re [H_0^V \bar{H}^S] \right), \\ \tilde{G}_0^{0,2} &= -\frac{2\lambda_{\gamma^*}}{9q^2} \left(2|H_0^V|^2 - |H_+^V|^2 - |H_-^V|^2 + (V \rightarrow A) \right) \\ \tilde{G}_0^{2,0} &= -\frac{4}{9} \left(3E_\ell E_\nu + \frac{\lambda_{\gamma^*}}{4q^2} \right) \left(|H_+^V|^2 + |H_-^V|^2 - 2|H_0^V|^2 + (V \rightarrow A) \right) \\ &\quad + \frac{8}{3} \left(E_\ell E_\nu + \frac{\lambda_{\gamma^*}}{4q^2} \right) |H^S|^2 + \frac{8}{3} \left(E_\ell E_\nu + \frac{\lambda_{\gamma^*}}{4q^2} \right) |H^P|^2 \\ \tilde{G}_0^{2,1} &= -\frac{4\sqrt{\lambda_{\gamma^*}}}{3} \left(\Re [H_+^V \bar{H}_+^A - H_-^V \bar{H}_-^A] + 2\frac{m_\ell}{\sqrt{q^2}} \Re [H_0^A \bar{H}^P] + 2\frac{m_\ell}{\sqrt{q^2}} \Re [H_0^V \bar{H}^S] \right), \end{aligned}$$

$$\begin{aligned}
\tilde{G}_0^{2,2} &= -\frac{2}{9} \frac{\lambda_{\gamma^*}}{q^2} \left(4 |H_0^V|^2 + |H_+^V|^2 + |H_-^V|^2 + (V \rightarrow A) \right) , \\
\tilde{G}_1^{2,1} &= \frac{4\sqrt{\lambda_{\gamma^*}}}{\sqrt{3}} \left((H_+^V \bar{H}_0^A + H_+^A \bar{H}_0^V - H_0^V \bar{H}_-^A - H_0^A \bar{H}_-^V) + \frac{m_\ell}{\sqrt{q^2}} (H_+^V \bar{H}^S + H^S \bar{H}_-^V) \right. \\
&\quad \left. + \frac{m_\ell}{\sqrt{q^2}} (H_+^A \bar{H}^P + H^P \bar{H}_-^A) \right) , \\
\tilde{G}_1^{2,2} &= \frac{4}{3} \frac{\lambda_{\gamma^*}}{q^2} (H_+^V \bar{H}_0^V + H_0^V \bar{H}_-^V + (V \rightarrow A)) , \\
\tilde{G}_2^{2,2} &= -\frac{8}{3} \frac{\lambda_{\gamma^*}}{q^2} (H_+^V \bar{H}_-^V m + H_+^A \bar{H}_-^A) . \tag{B.12}
\end{aligned}$$

The hadronic helicity amplitudes of interest H^i , ($i = V, A, S, P$) are given by:

$$\begin{aligned}
H_0^V &= \frac{4im_B m_{D^*}}{\sqrt{q^2}} C_V A_{12} , \\
H_0^A &= \frac{4im_B m_{D^*}}{\sqrt{q^2}} C_A A_{12} , \\
H_\pm^V &= \frac{i}{2(m_B + m_{D^*})} \left(\pm \sqrt{\lambda_B} V - (m_B + m_{D^*})^2 A_1 \right) C_V \\
H_\pm^A &= \frac{i}{2(m_B + m_{D^*})} \left(\pm \sqrt{\lambda_B} V - (m_B + m_{D^*})^2 A_1 \right) C_A , \\
H^P &= \frac{i\sqrt{\lambda_B}}{2} \frac{m_\ell}{q^2} C_A A_0 , \\
H^S &= \frac{i\sqrt{\lambda_B}}{2} \frac{m_\ell}{q^2} C_V A_0 . \tag{B.13}
\end{aligned}$$

with the following definition involved

$$A_{12} = \frac{(m_B + m_{D^*})^2 (m_B^2 - m_{D^*}^2 - q^2) A_1 - \lambda_B A_2}{16m_B m_{D^*}^2 (m_B + m_{D^*})} , \tag{B.14}$$

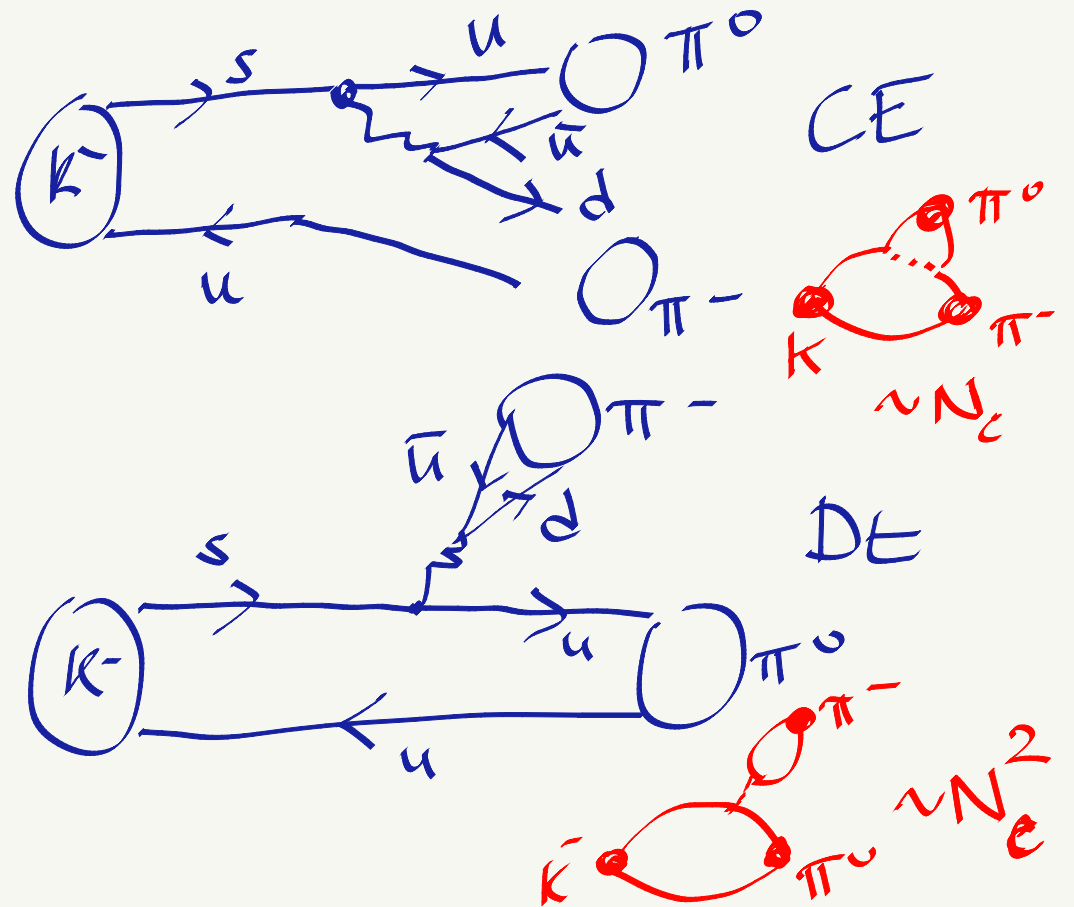
$$\tag{B.15}$$

Non-leptonic Decays

Penguins contractions and all that

$$K^- \rightarrow \pi^- \pi^0$$

$$H_W = -\frac{G_F}{\sqrt{2}} V_{us} V_{ud}^* \bar{u} \gamma^\mu (1-\gamma_5) s \bar{d} \gamma_\mu (1-\gamma_5) u$$

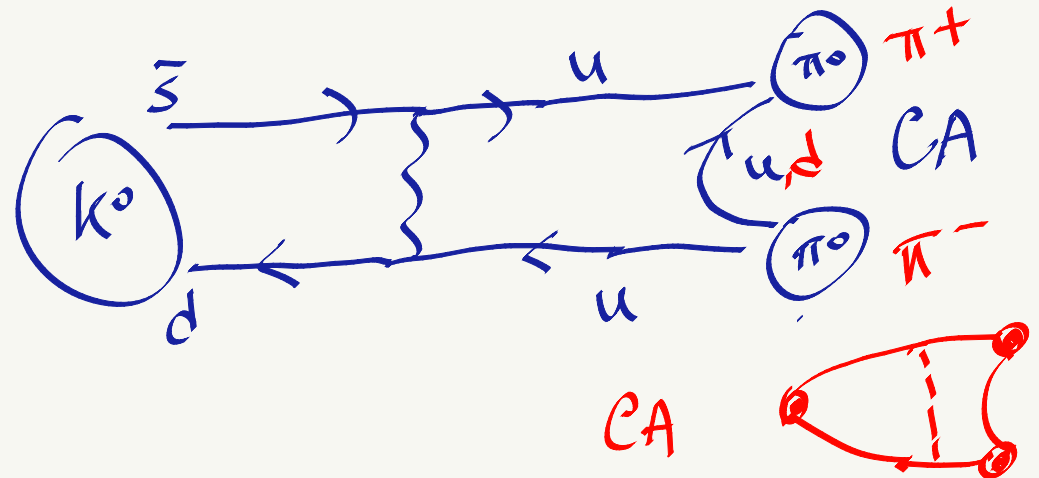
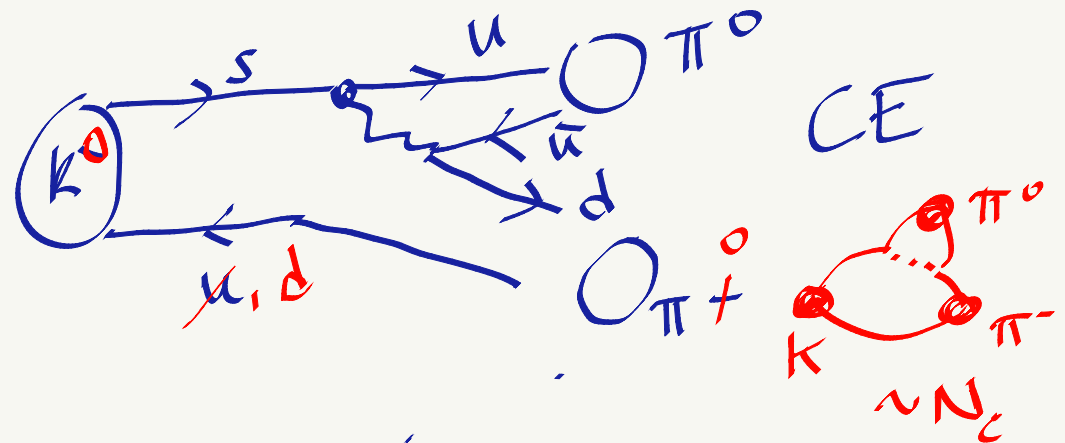


Non-leptonic Decays

Penguins contractions and all that

$$K^0 \rightarrow \pi^+ \pi^-$$

$$H_W = -\frac{G_F}{\sqrt{2}} V_{us} V_{ud}^* \bar{u} \gamma^\mu (1-\gamma_5) s \bar{d} \gamma_\mu (1-\gamma_5) u$$

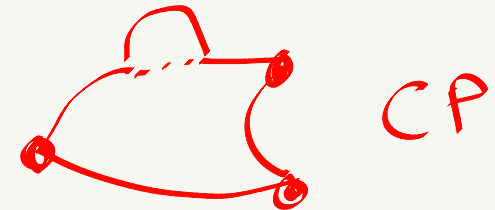
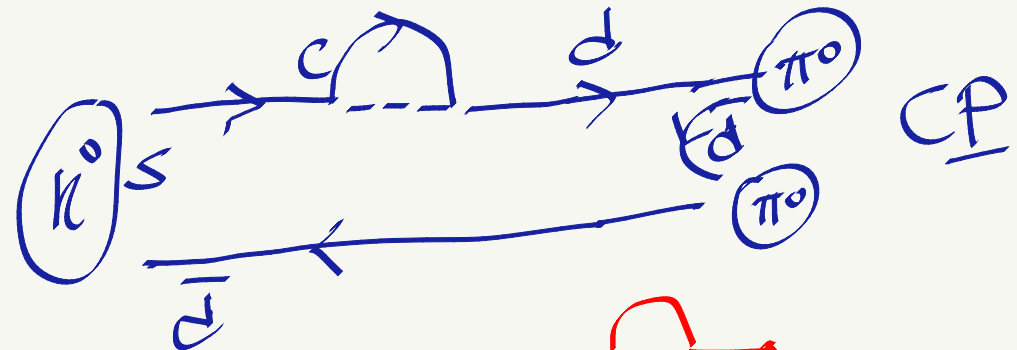


Non-leptonic Decays

Penguins contractions and all that

Penguin diagrams

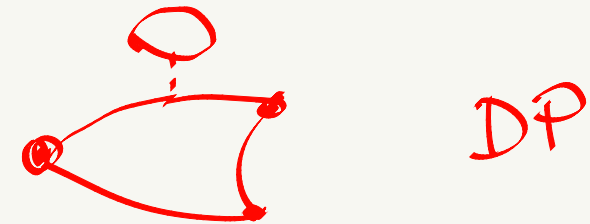
$$H = -\frac{G_F}{\sqrt{2}} V_{cs} V_{cd}^* \bar{c} \gamma_\mu (1 - \gamma_5) s \bar{d} \gamma^\mu (1 - \gamma_5) c$$



other ops

$$\sim V_{cs} V_{cd}^*$$

$$\bar{d} \gamma_\mu (1 - \gamma_5) s \bar{c} \gamma^\mu (1 - \gamma_5) c$$



All Topologies

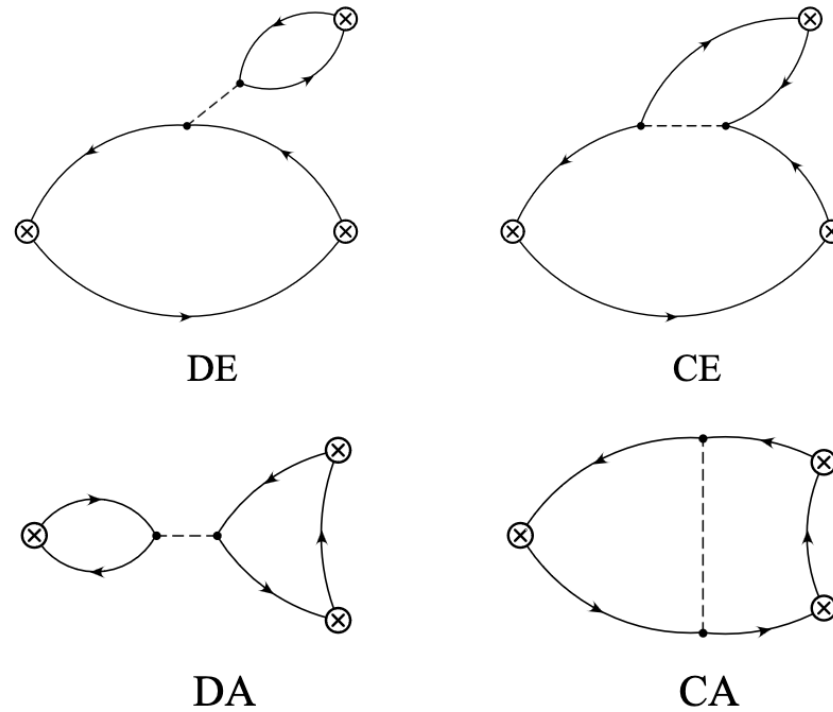
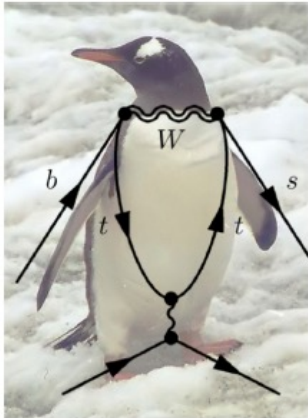
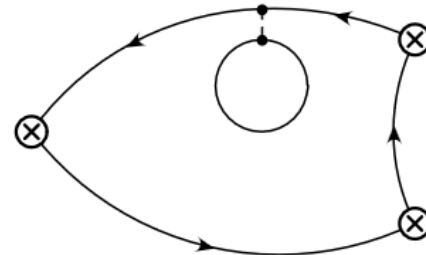


Figure 1: *Non-penguin diagrams. The dashed line represents the four-fermion operator.*

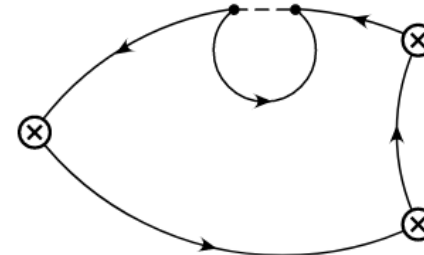
All Topologies



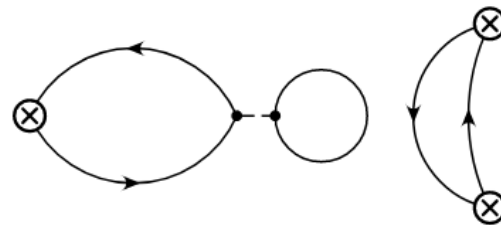
D. Kovalskyi's seminar © CERN (26/7/22)



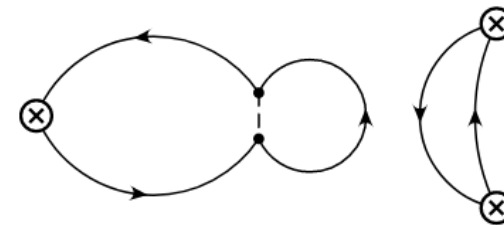
DP



CP



DPA



CPA

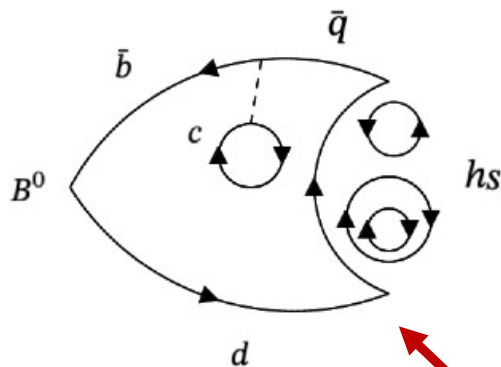


Figure 2: *Penguin diagrams.*

for heavy mesons many particles in the final state

Flavor Changing Neutral Currents

Rare Penguin Radiative Decays

The **main issue** in this Group Meeting will be represented by $b \rightarrow s$ quark transitions: the best example is offered by the **neutral-current semileptonic** $B \rightarrow K^{(*)} l^+ l^-$ transitions!

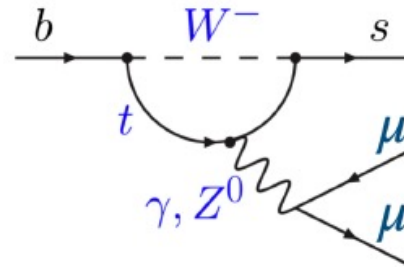
Many interesting properties:

1. **Loop-level processes** (FCNCs are forbidden at tree level in the Standard Model)
2. **CKM-suppressed decays**, where

$$J_{\text{charged}}^\mu = \bar{u}_L^i \boxed{V_{CKM}^{ij}} \gamma^\mu d_L^j + \bar{\nu}_L^i \gamma^\mu \ell_L^i$$



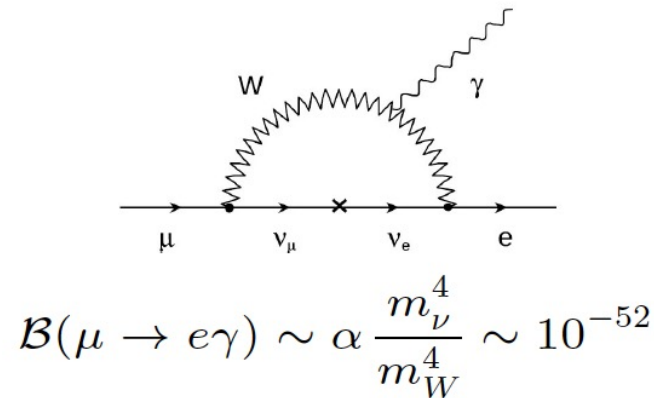
Rare transitions!



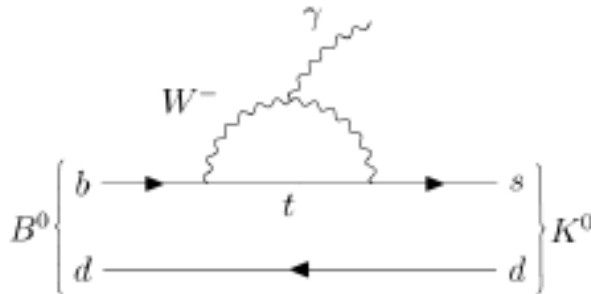
$$B^+ \rightarrow K^{(*)+} \gamma$$

$$B^+ \rightarrow K^{(*)+} \mu^+ \mu^-$$

since different neutrinos have a mass and they can mix,
 $\mu \rightarrow e\gamma$ is a possible decay which satisfies all the symmetry constraints



note that the photon is emitted by
the W boson, analogy radiative B decays



$\bar{B}^0$ $\bar{K}^0$

Figura 4: quark process

Radiative Penguins

PENGUINS AND BOXES

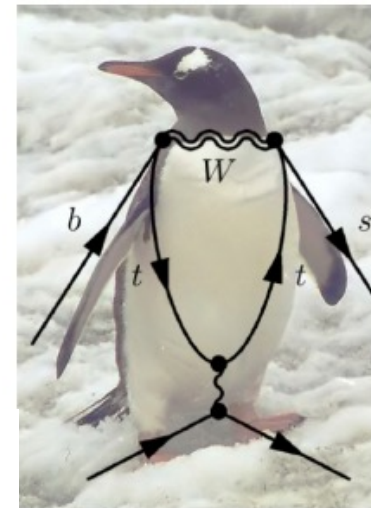
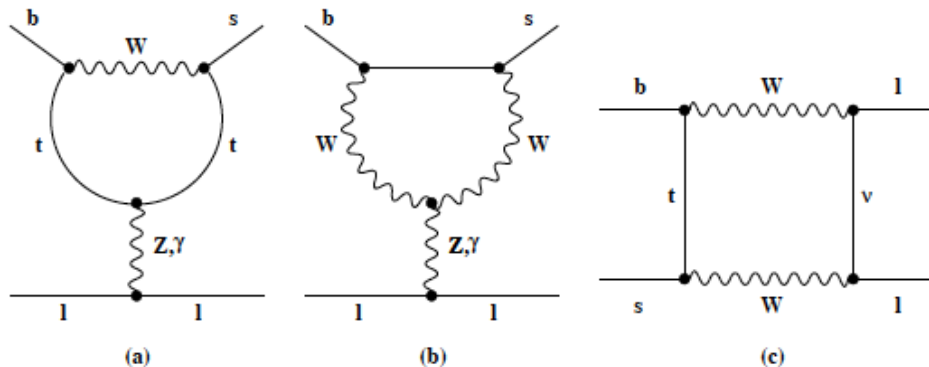
Pure leptonic B_s decays

$$Br(B_s \rightarrow l^+ l^-) = \tau(B_s) \frac{G_F^2}{\pi} \left(\frac{\alpha}{4\pi \sin^2 \Theta_W} \right)^2 \boxed{F_{B_s}^2} m_l^2 m_{B_s} \sqrt{1 - 4 \frac{m_l^2}{m_{B_s}^2}} |V_{tb}^* V_{ts}|^2 Y^2(x_t)$$

G. Buchalla and A.J. Buras, Nucl. Phys. B 400 (1993) 225.

Many interesting properties:

1. **Helicity suppressed**
2. **Non-perturbative hadronic contributions enter via B_s decay constant**



valskeyi's seminar @ CERN (26/7/22)

Lowest order
diagrams
QCD corrections at
NLO or NNLO

BOXES

Mixing of Neutral Mesons

$$K^0 \leftrightarrow \bar{K}^0$$

$$D^0 \leftrightarrow \bar{D}^0$$

$$B^0 \leftrightarrow \bar{B}^0$$

in the case of kaons

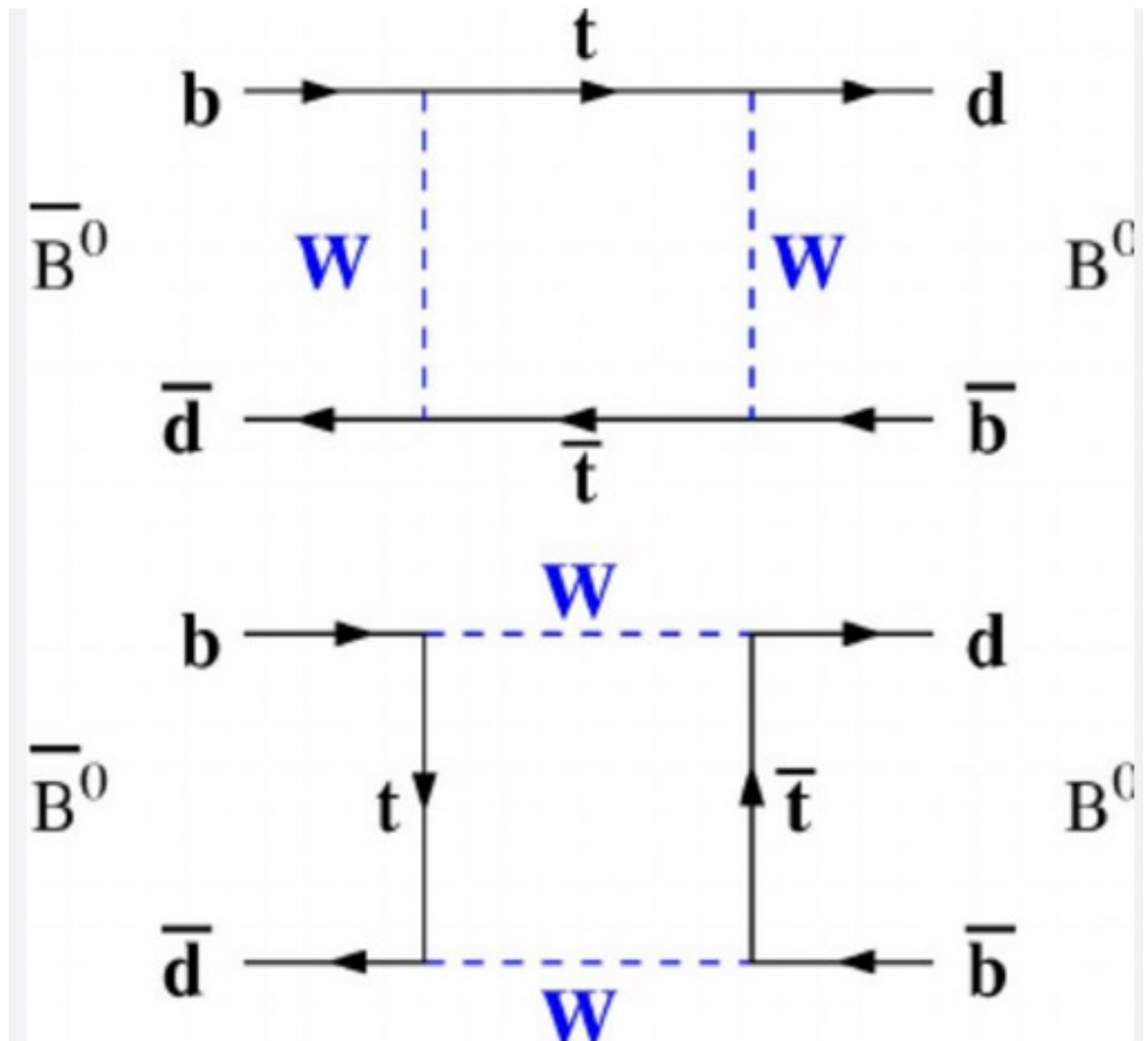
also charm and up quarks

contribute

for D and K meson mixing

there are important long

distance contributions



$$\mathcal{H}_{\text{eff}}^{\Delta B=2} = \frac{G_F^2}{16\pi^2} M_W^2 (V_{tb}^* V_{td})^2 \eta_B S_0(x_t) \times$$

$$\times \left[\alpha_s^{(5)}(\mu_b) \right]^{-6/23} \left[1 + \frac{\alpha_s^{(5)}(\mu_b)}{4\pi} J_5 \right] Q^q(\Delta B = 2) + h.c.$$

QCD corrections