中央研究院
ACADEMIA SINICA

Atmospheric lepton fluxes

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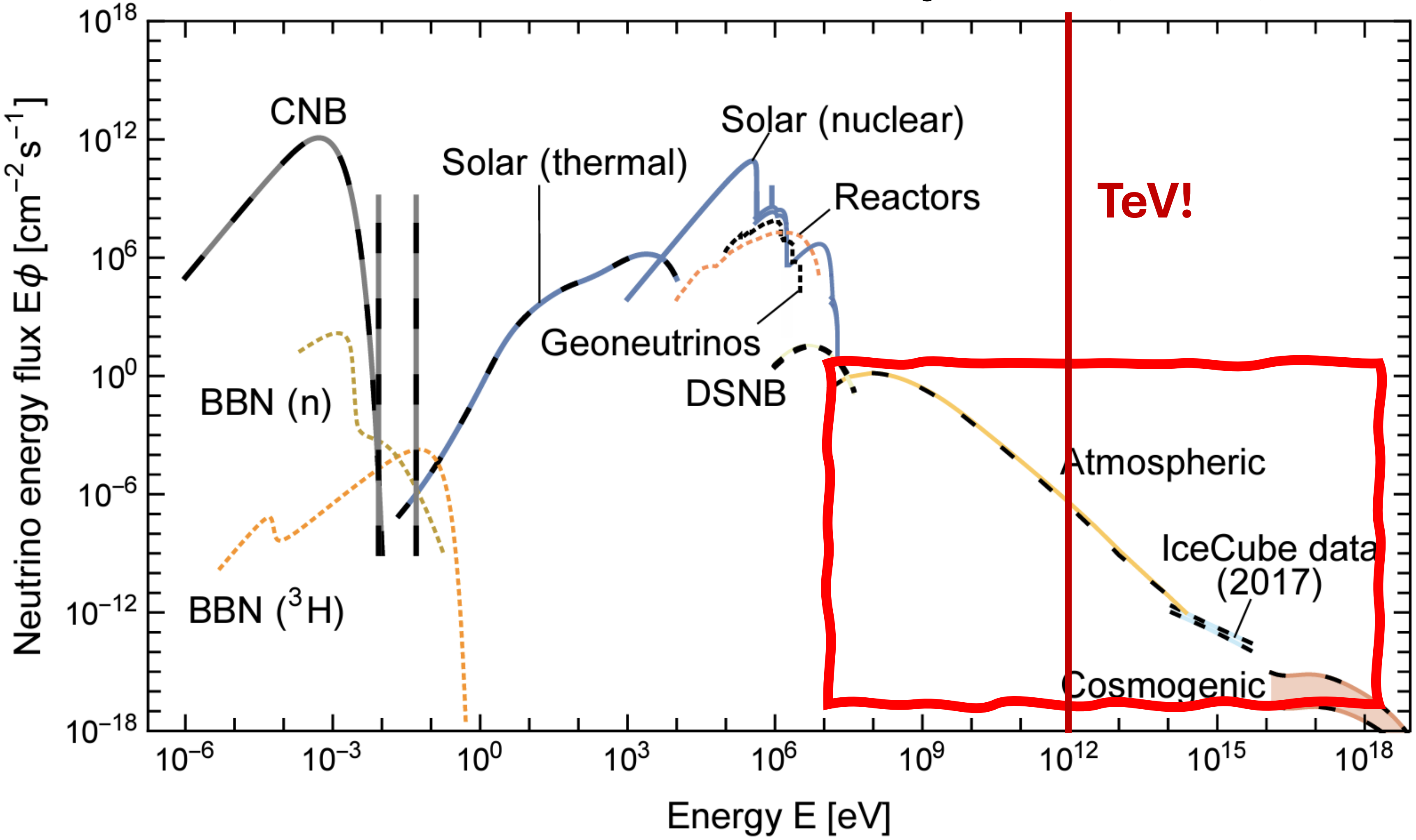
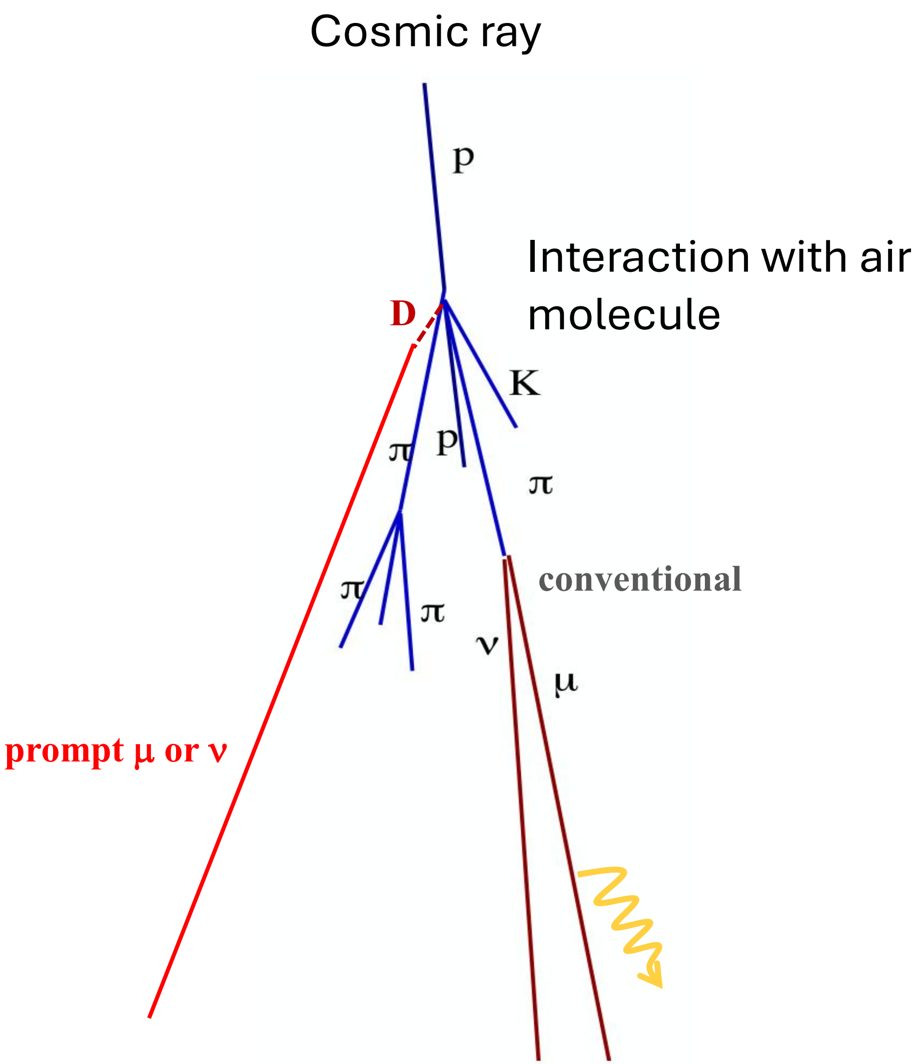
04 / 02 / 2025

Atmospheric neutrinos

Vitagliano, Tamborra, Raffelt 2019, 1910.11878



Origin of atmospheric leptons



conventional

$$p, A + \text{air} \rightarrow \pi^{\pm}, \pi^{0}, \text{K}^{\pm}, \text{K}^{0}_{\text{S,L}}$$

muons and muon neutrinos

$$\pi^{\pm}, K^{\pm} \rightarrow \mu^{\pm}\nu_{\mu}(\bar{\nu}_{\mu})$$

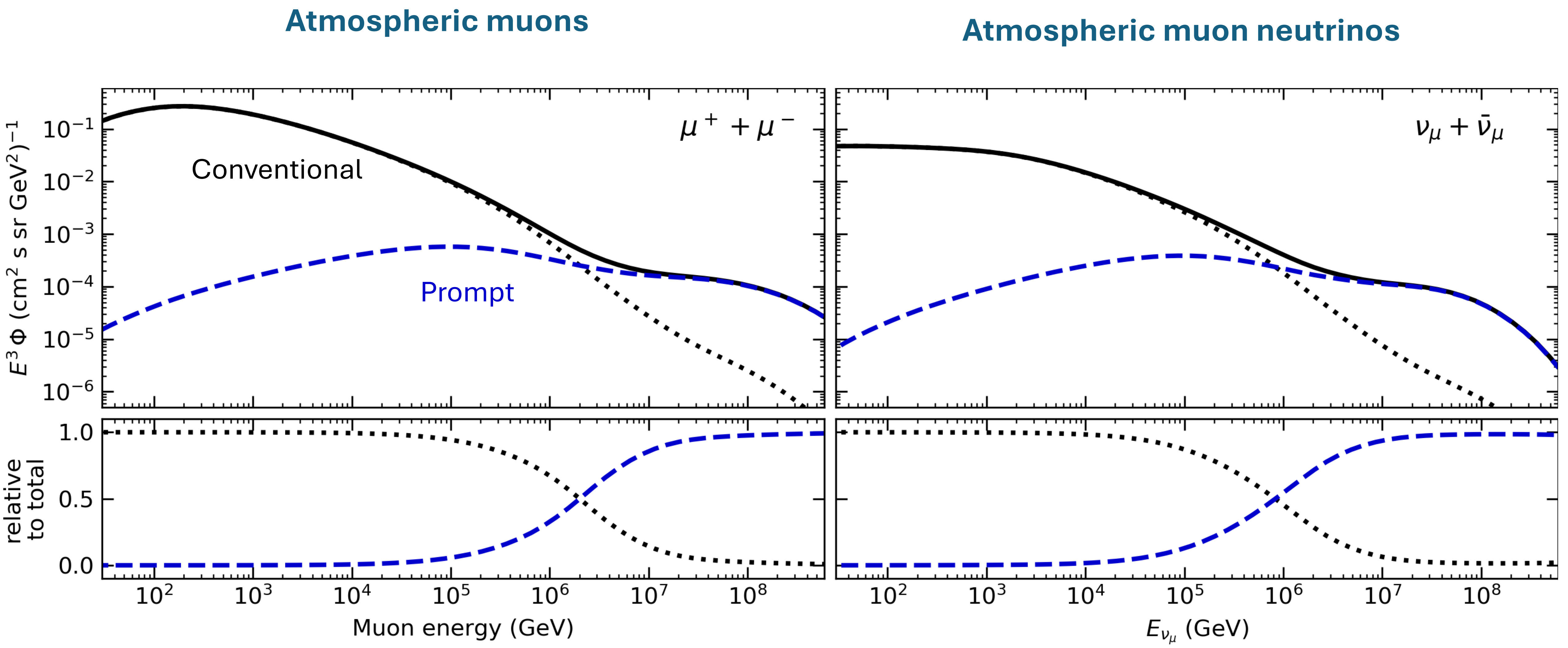
electron neutrinos

$$K^{\pm}, K^{0}_{L} \rightarrow [\pi^{\pm}, \pi^{0}]e^{\pm}\nu_{e}(\bar{\nu}_{e})$$

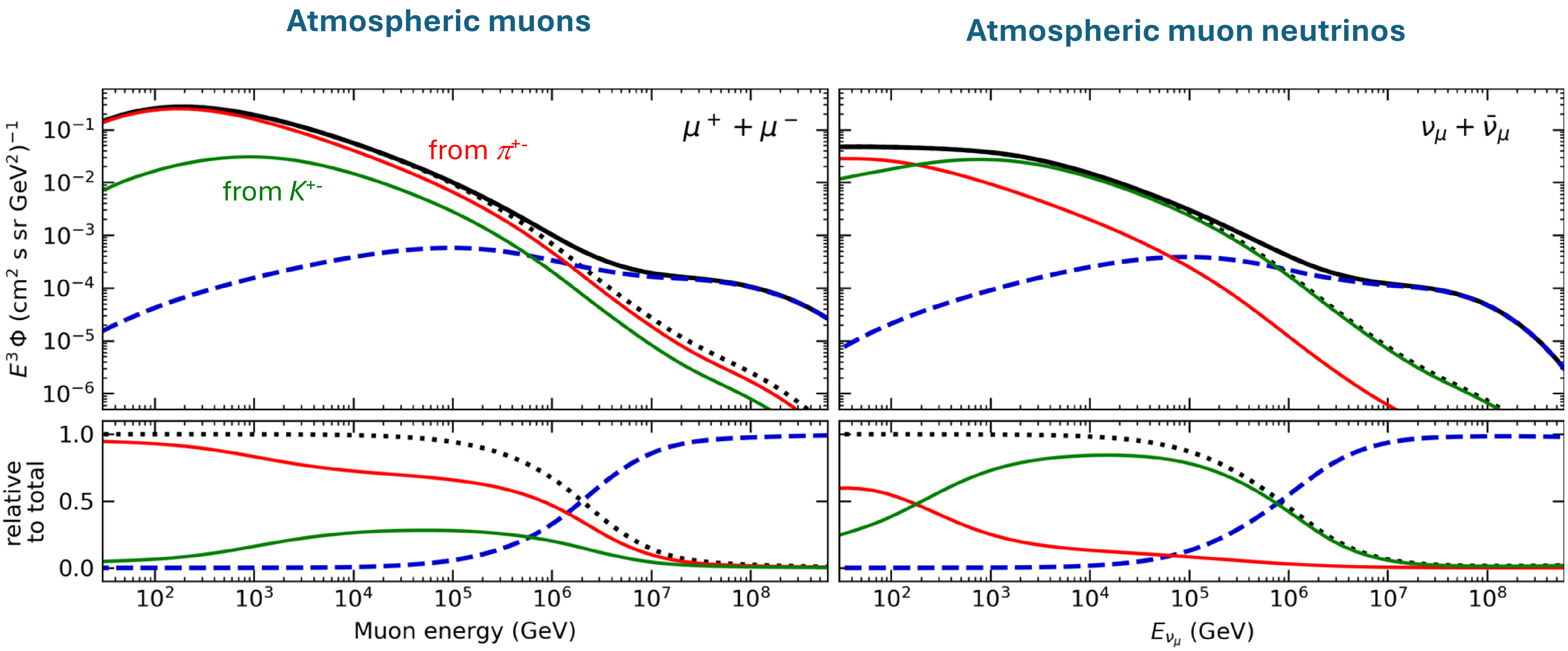
prompt

$$p, A + \text{air} \rightarrow \text{D}, \Lambda_{\text{C}} \rightarrow \nu_{\mu}, \nu_{\text{e}}, \mu$$

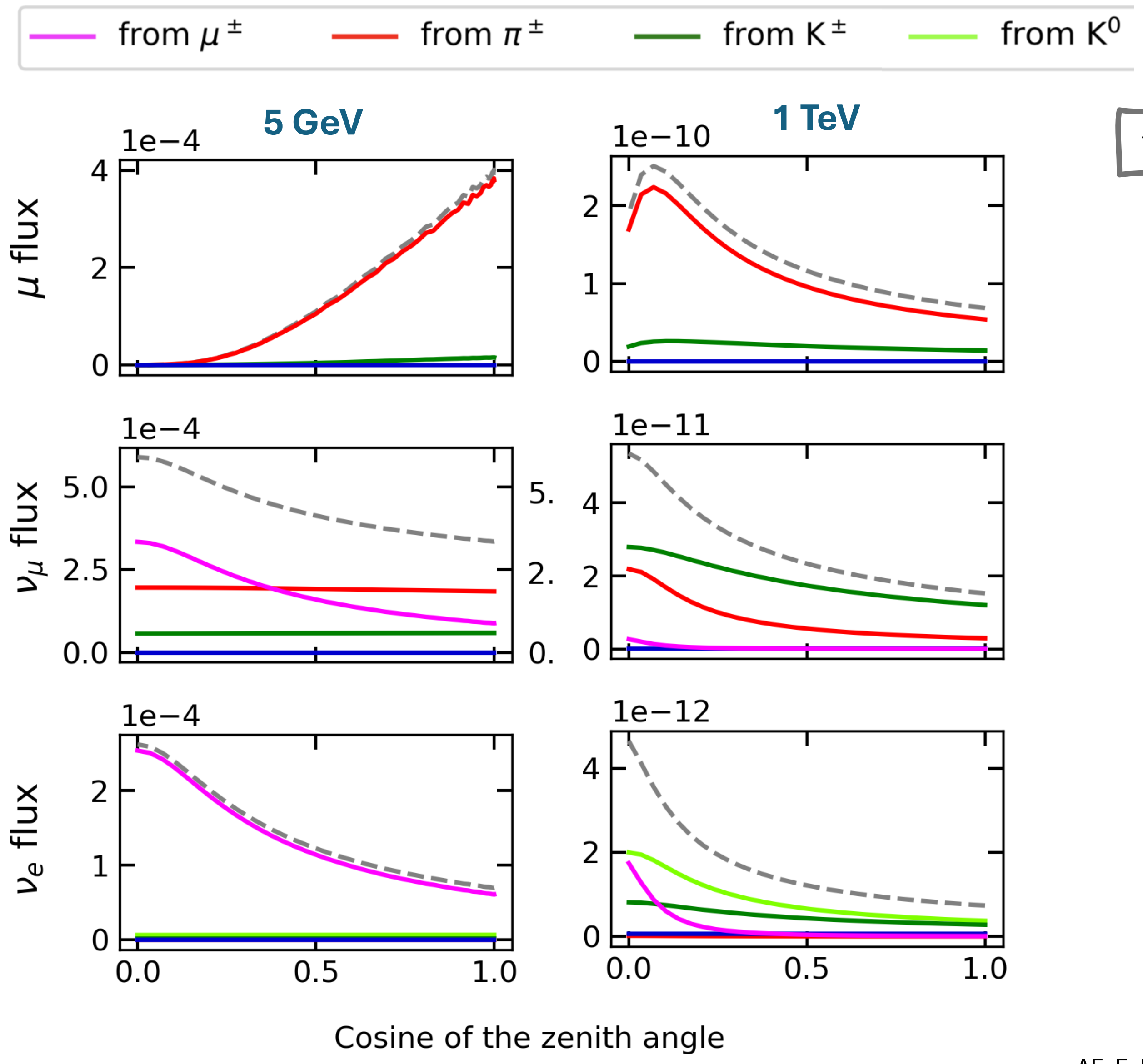
Smooth power-law like spectra, defined by CR spectrum...

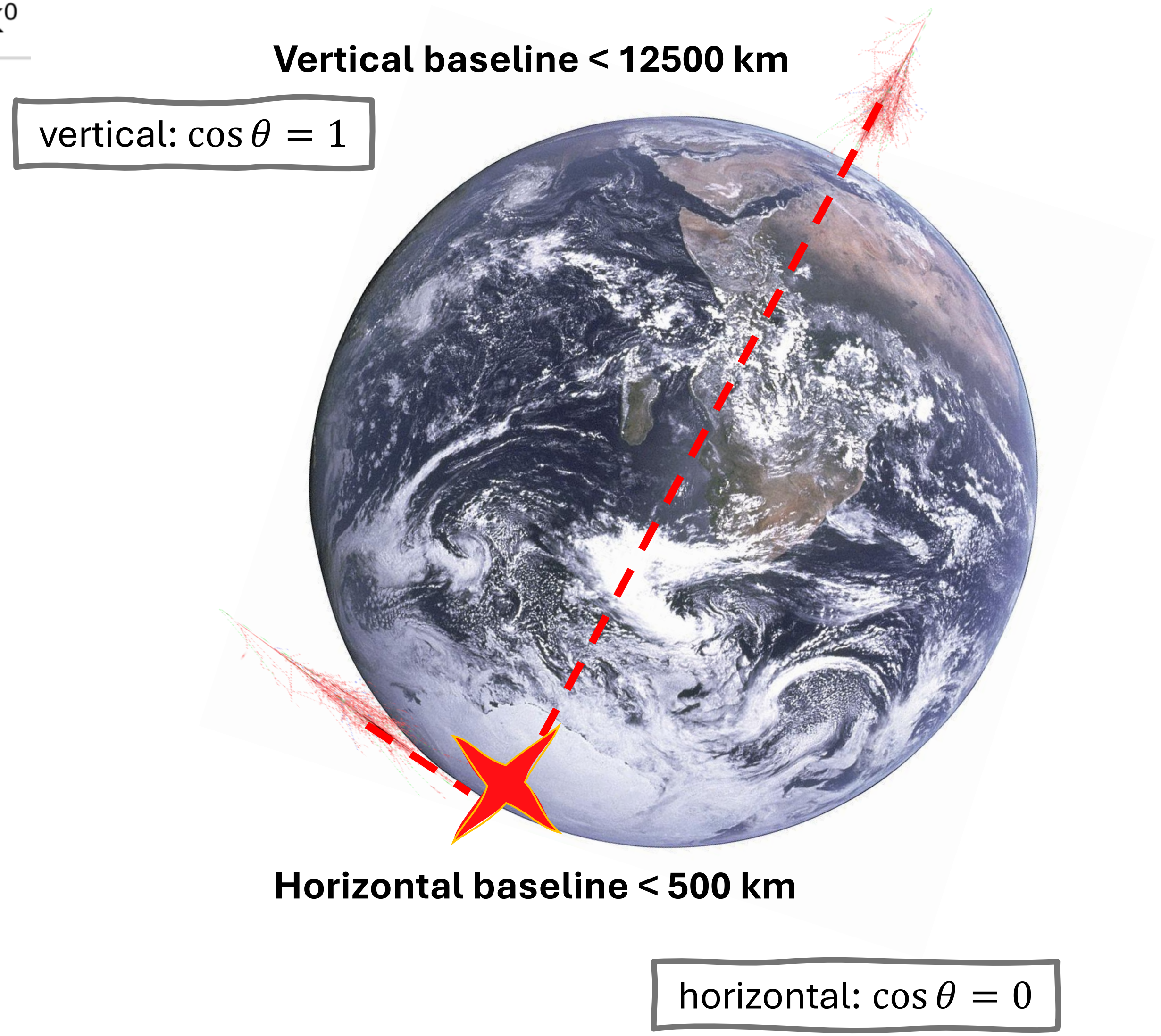


...and the interplay of parent hadrons



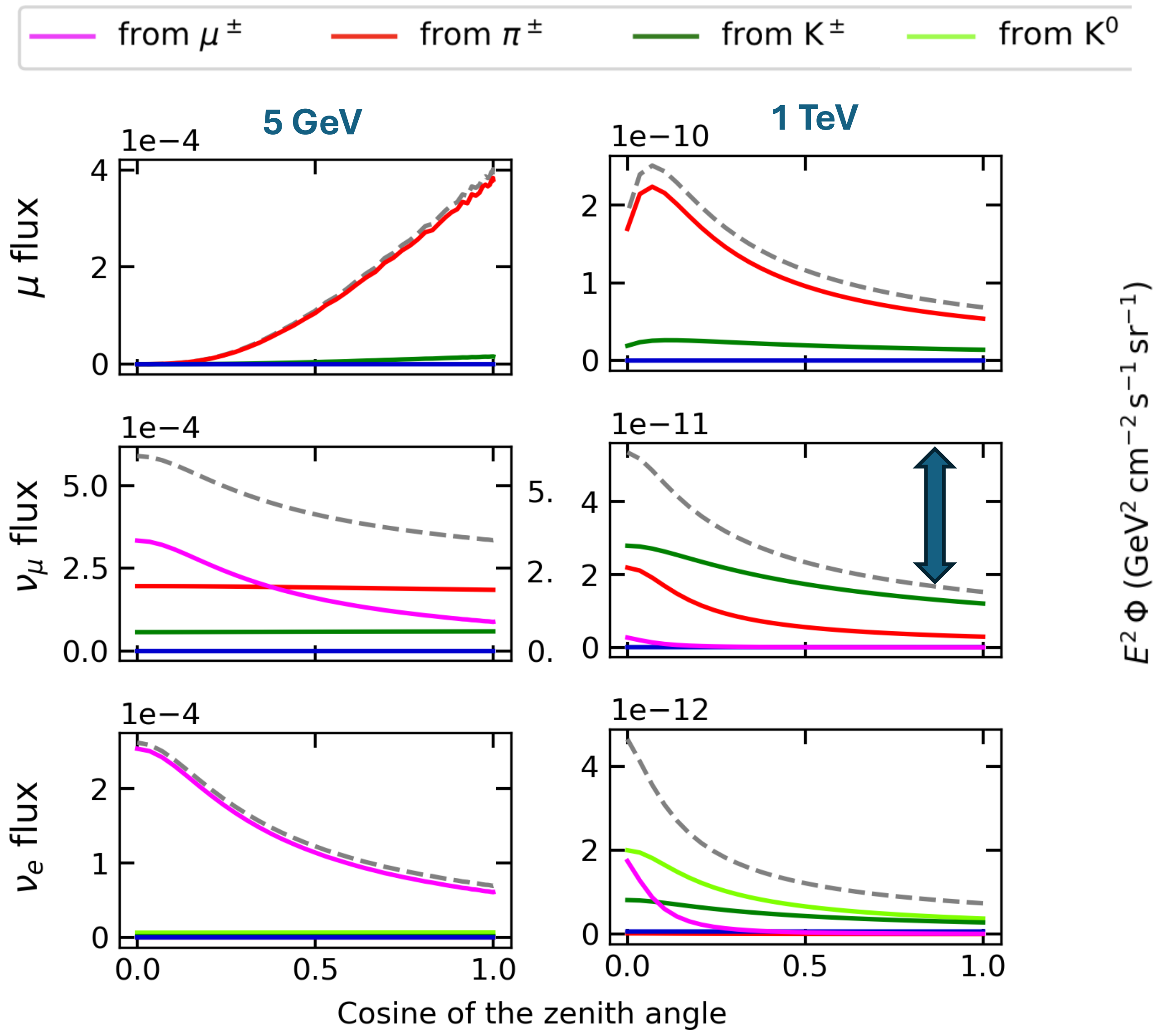
Zenith distribution

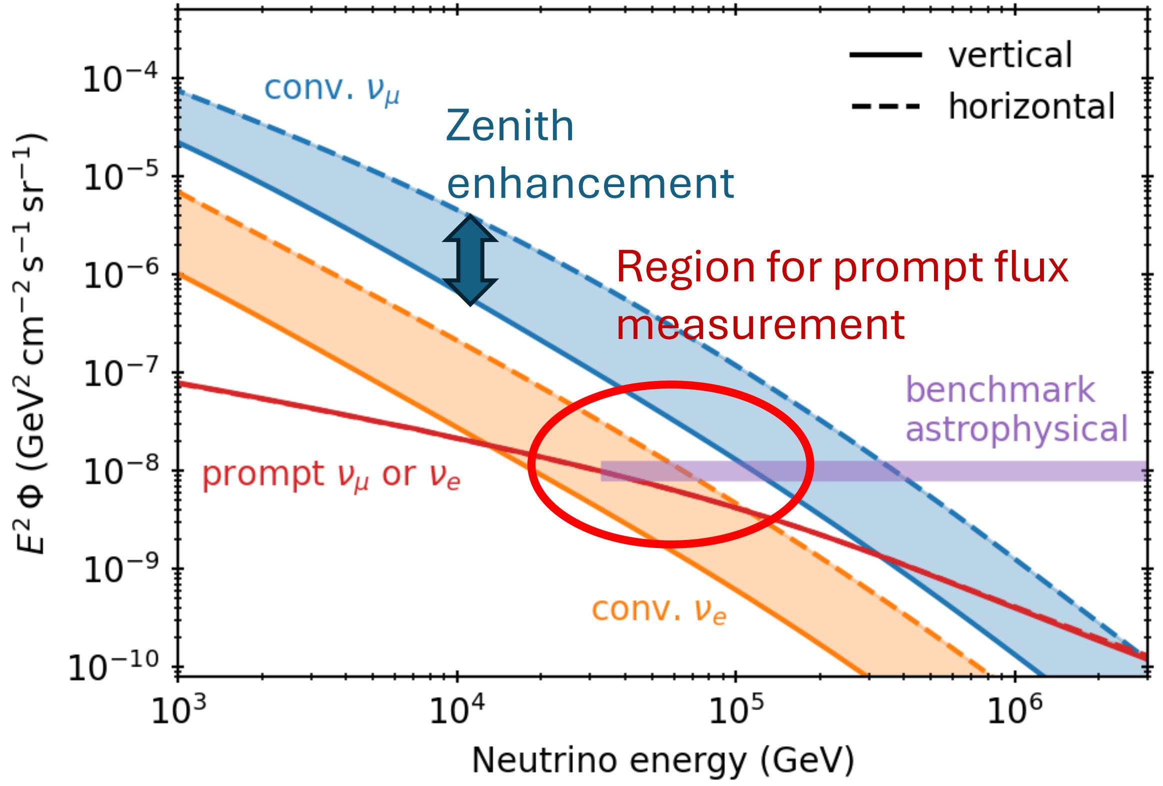




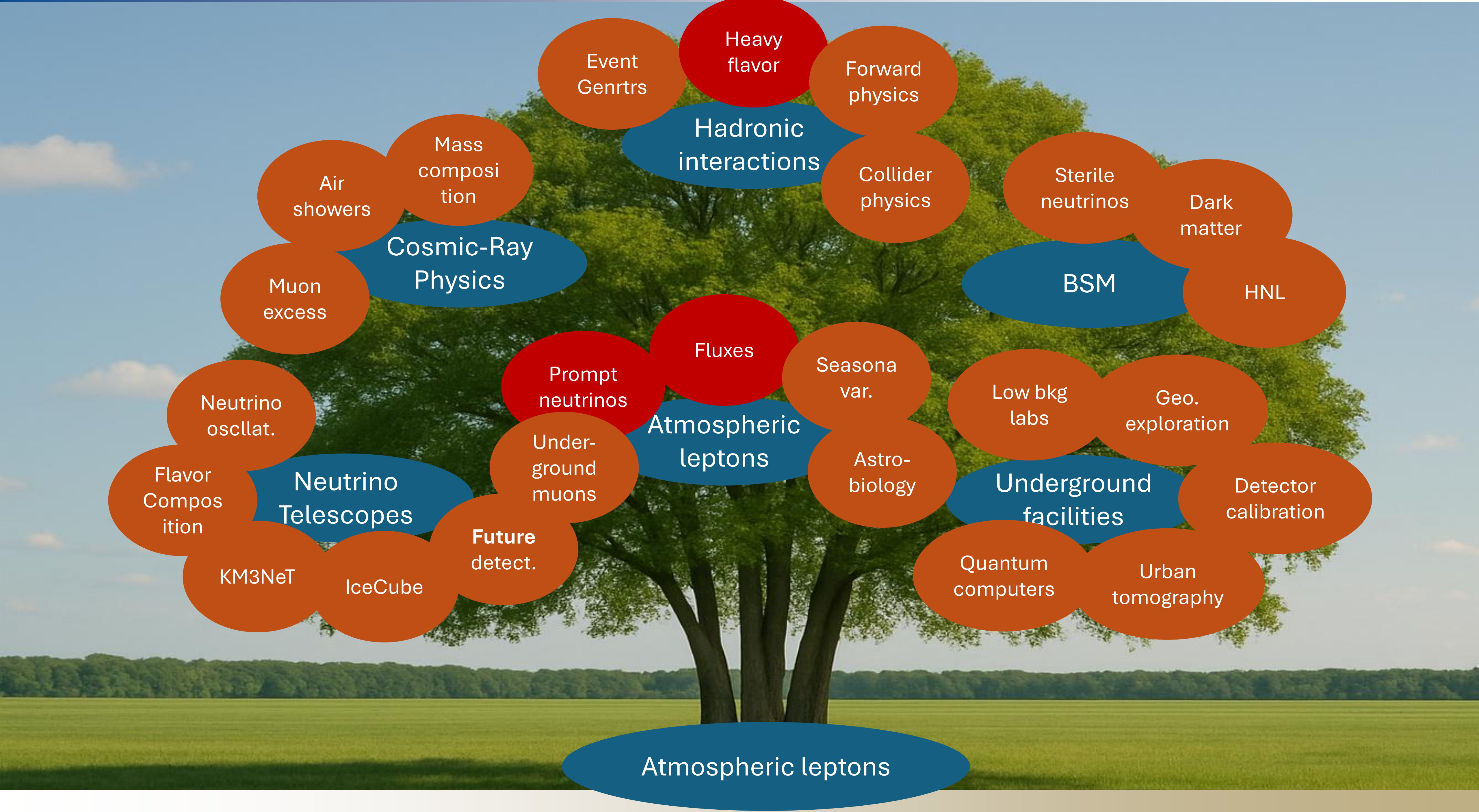
AF, F. Riehn, R. Engel, T.K. Gaisser, T. Stanev, PRD 100 2019

Zenith distribution



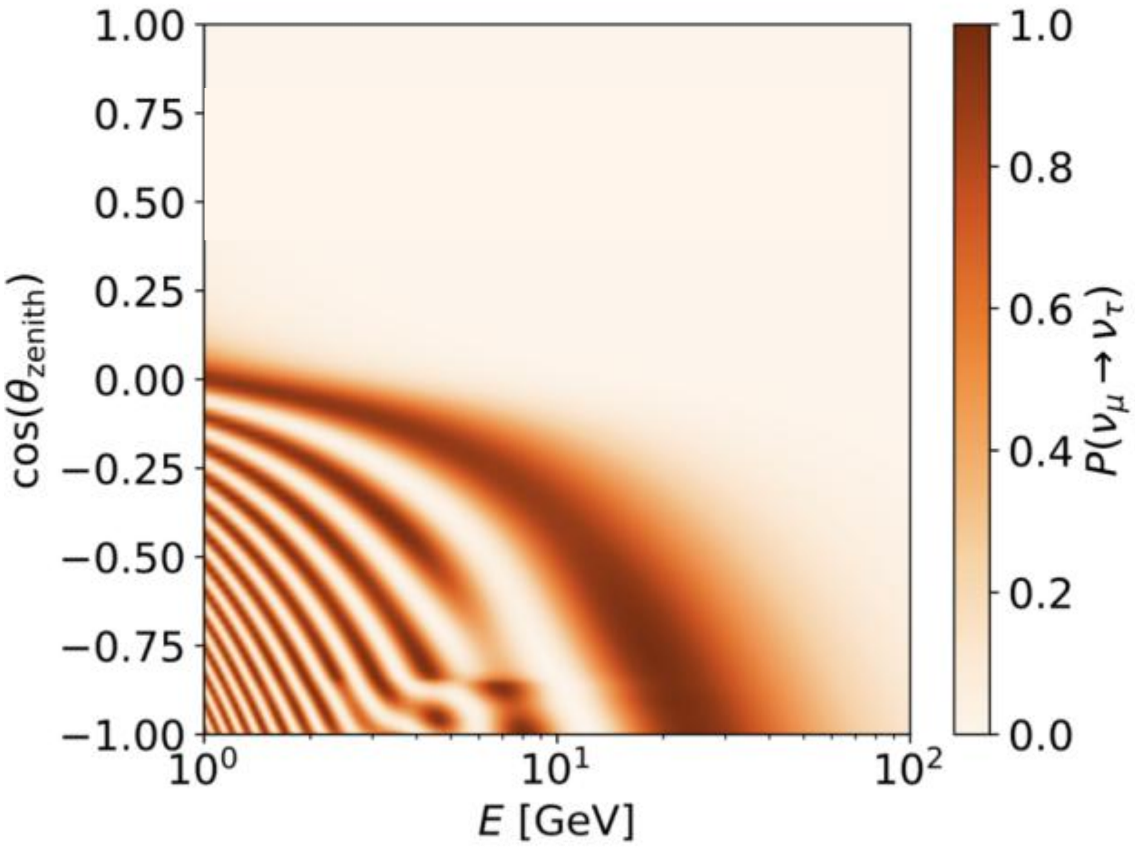


AF, F. Riehn, R. Engel, T.K. Gaisser, T. Stanev, PRD 100 2019


Event Genrtrs
Heavy flavor
Forward physics
Hadronic interactions
Collider physics
Mass composi tion
Air showers
Cosmic-Ray Physics
Muon excess
Sterile neutrinos
Dark matter
BSM
HNL
Fluxes
Prompt neutrinos
Seasona var.
Atmospheric leptons
Under-ground muons
Astro-biology
Low bkg labs
Geo. exploration
Underground facilities
Detector calibration
Quantum computers
Urban tomography
Neutrino oscllat.
Flavor Compos ition
Neutrino Telescopes
KM3NeT
IceCube
Future detect.
Atmospheric leptons

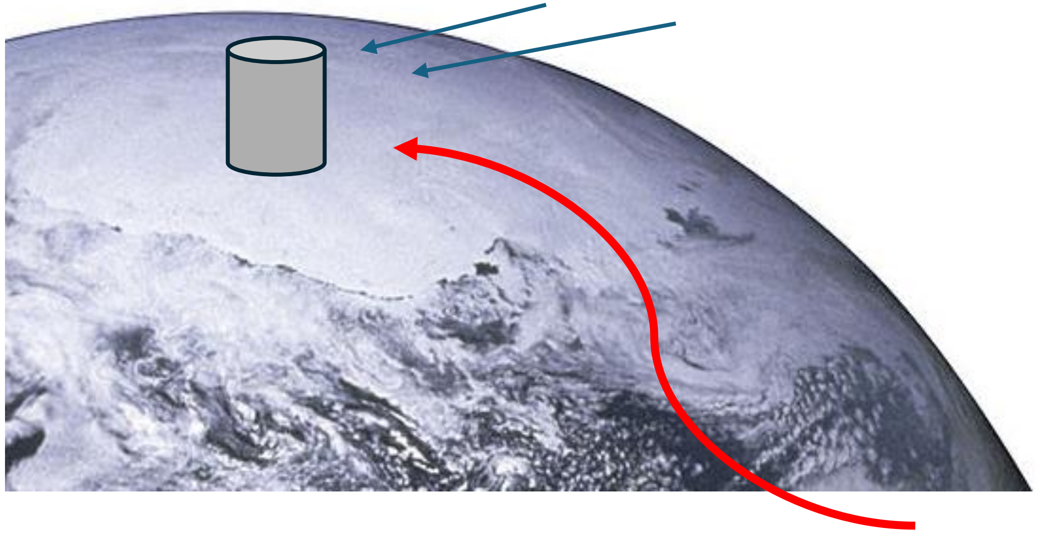
Measurement principle with high-energy atmospheric ν flux

Example: Imprint of neutrino oscillations in **energy X zenith space**



T. Stuttard, IceCube

Unperturbed flux <u>model</u>
without oscillation impact



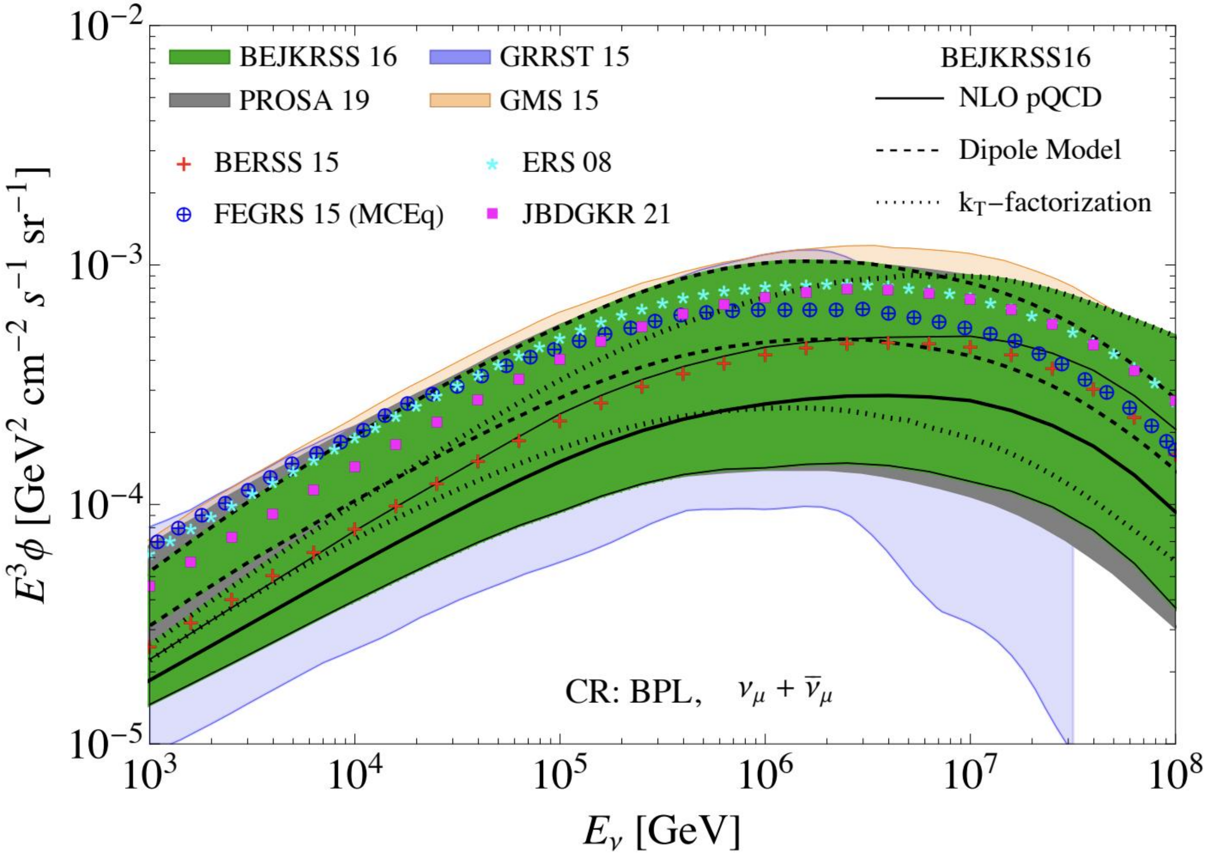
Distorted flux hypothesis with:

- Oscillations → neutrino properties, standard + sterile
- Expected absorption effects → neutrino cross sections**, Earth tomography (Alex Wen NU311)**
- **<u>Prompt neutrinos</u>** → (forward) charm production cross section, **intrinsic charm (G. Sigl NU229)**

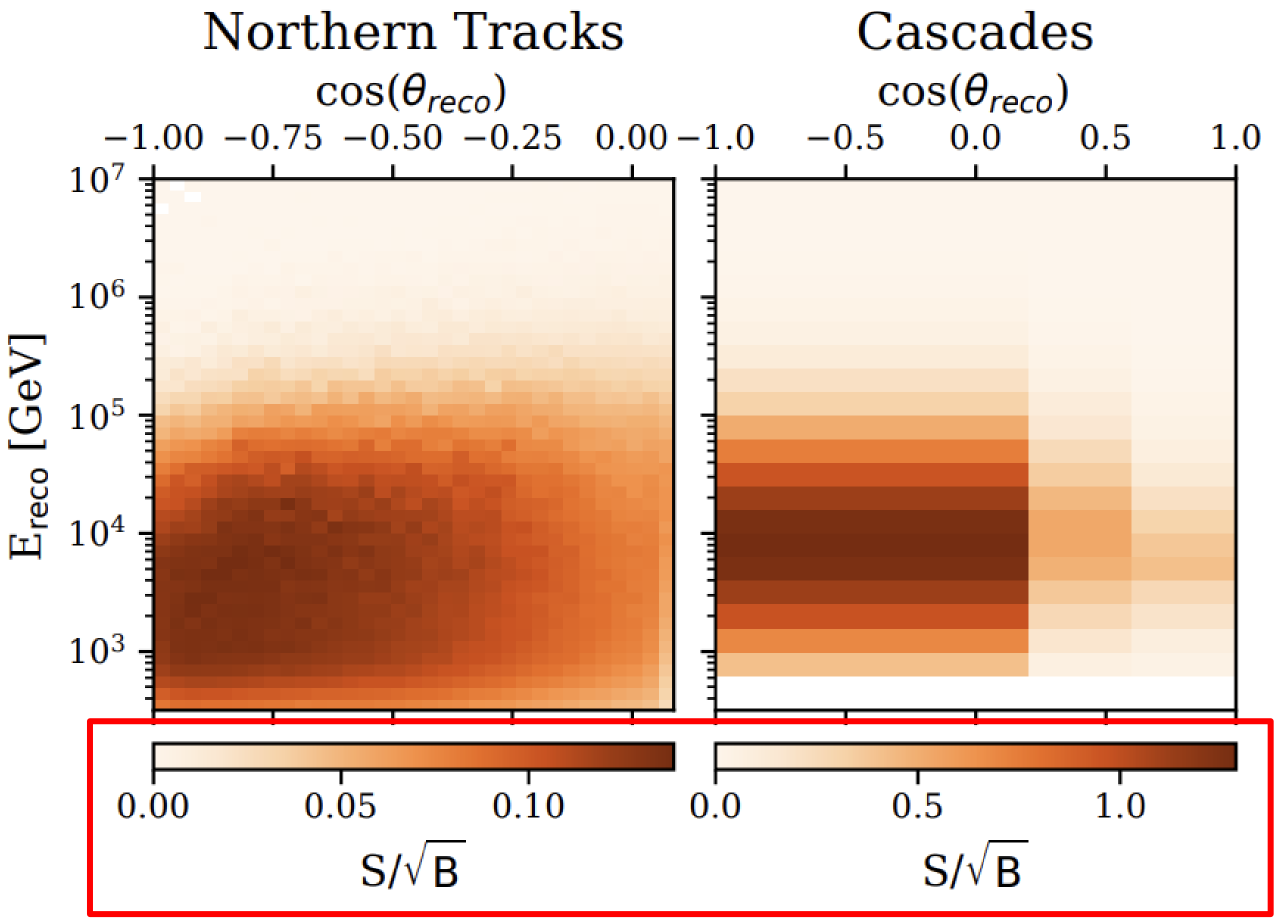
Undiscovered prompt neutrinos

- Term coined in the late 70s – early 80s by Volkova and Gaisser & Halzen
- Phase space for atmospheric charm is not covered by collider detectors (too forward) → interesting for particle physicists
- Large uncertainties from pQCD (factorization and renormalization scale)
- pQCD might be incomplete (intrinsic charm)
- The fragmentation ($c \rightarrow D$) function is not well known for forward charm and high energy
- Expected similar rate of ν_μ and ν_e but not μ because of additional decays of mesons

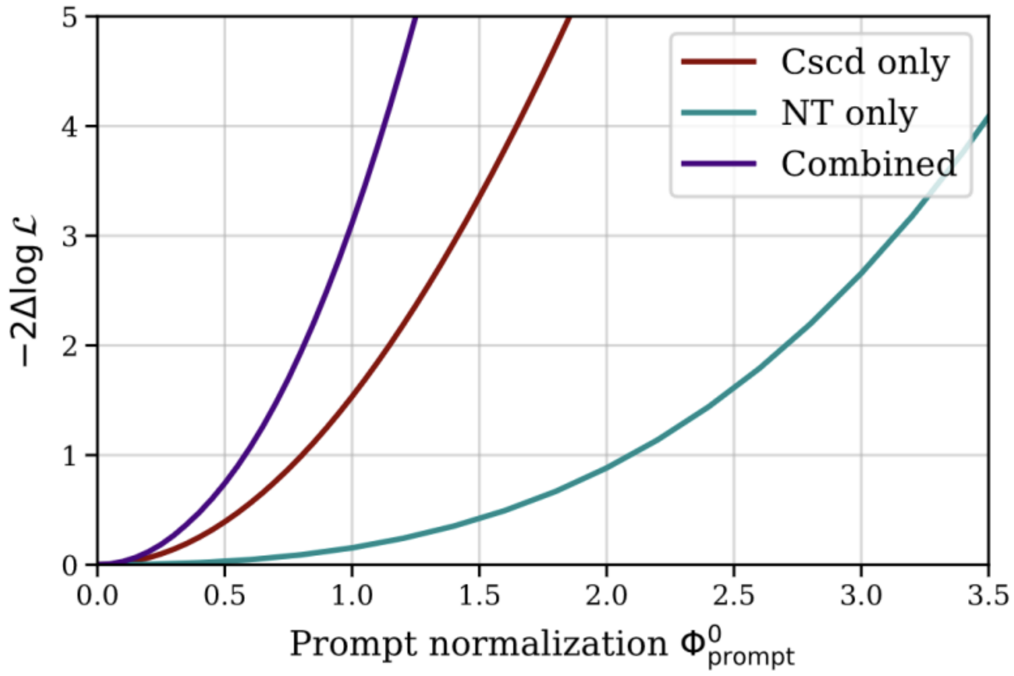
Forward Physics Facility Snowmass arXiv: 2203.05090



Measurement of prompt neutrinos by IceCube



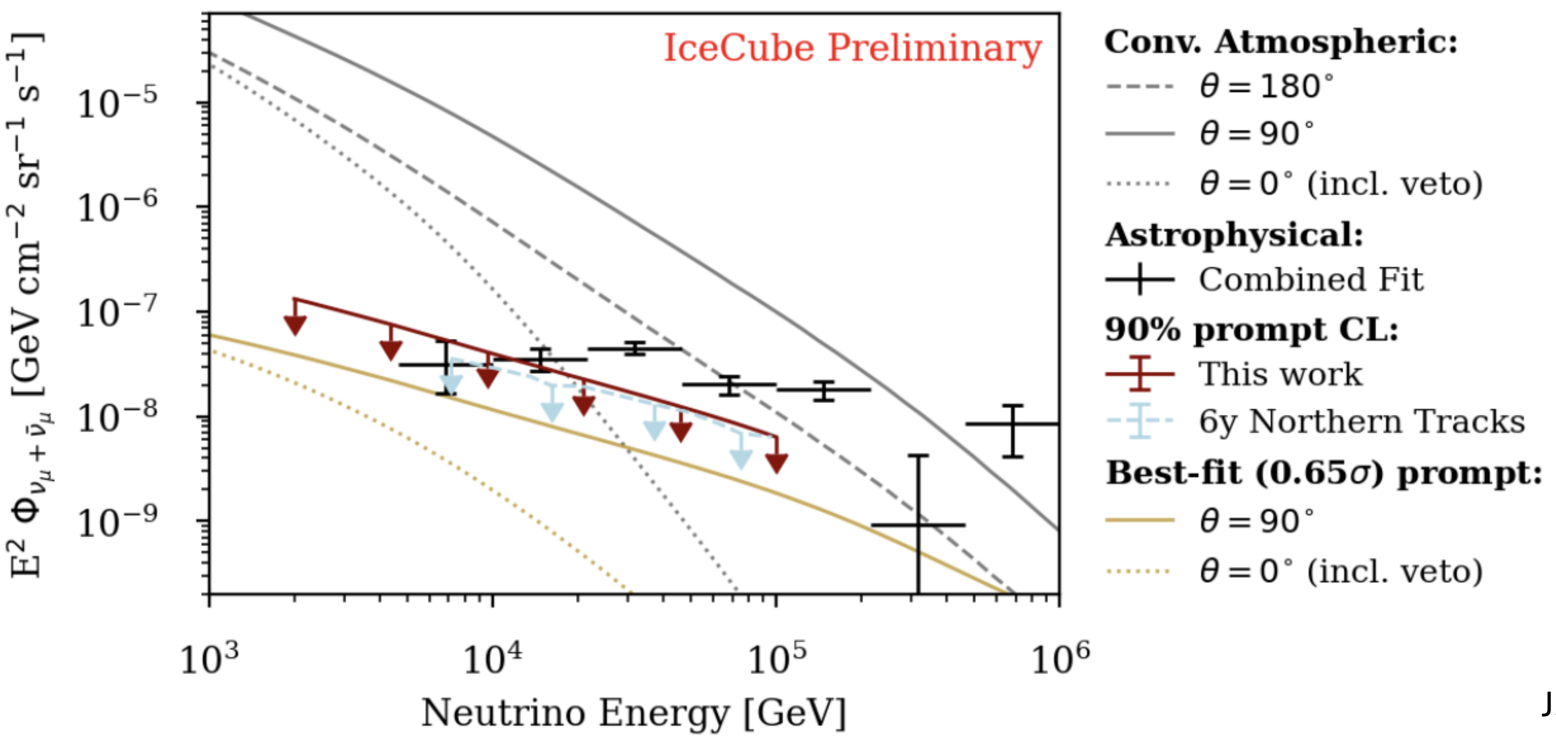
Measurement combines tracks and cascades using IceCube's "GlobalFit" Framework.



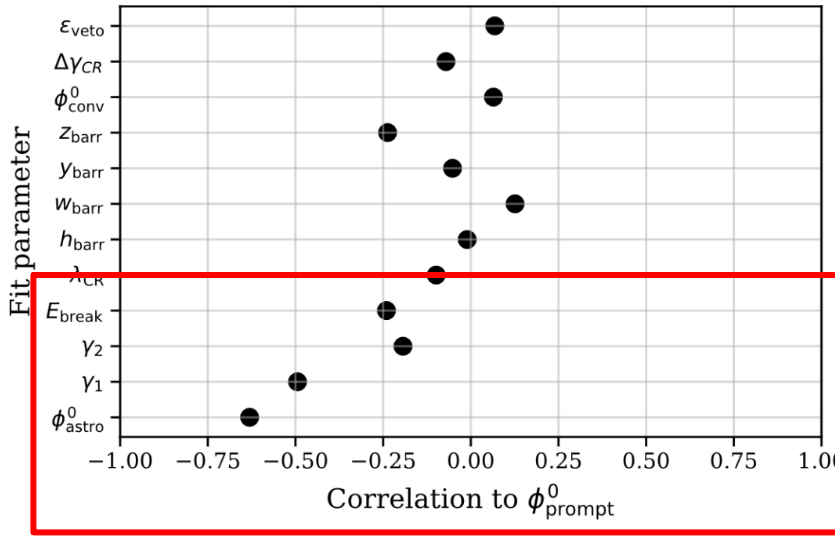
J. Böttcher for IceCube at ICRC2023 & PhD thesis RWTH

Measurement of prompt neutrinos by IceCube

- Non-zero prompt normalization best fit @ >1 σ
- Value compatible with "pQCD" predictions (SIBYLL 2.3c)
- Some degeneracy with diffuse astrophysical flux
- **Atm. flux model dependence**



Correlations with astrophysical neutrino flux model

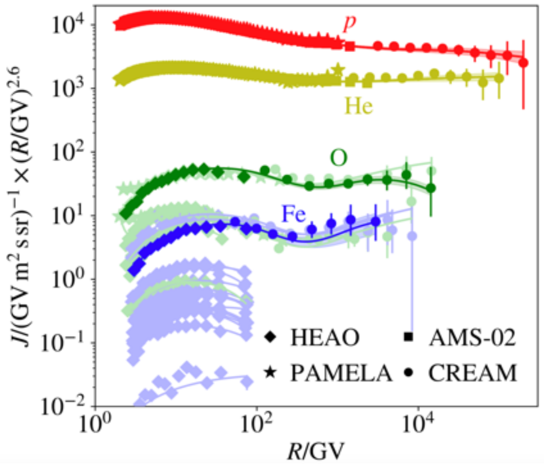


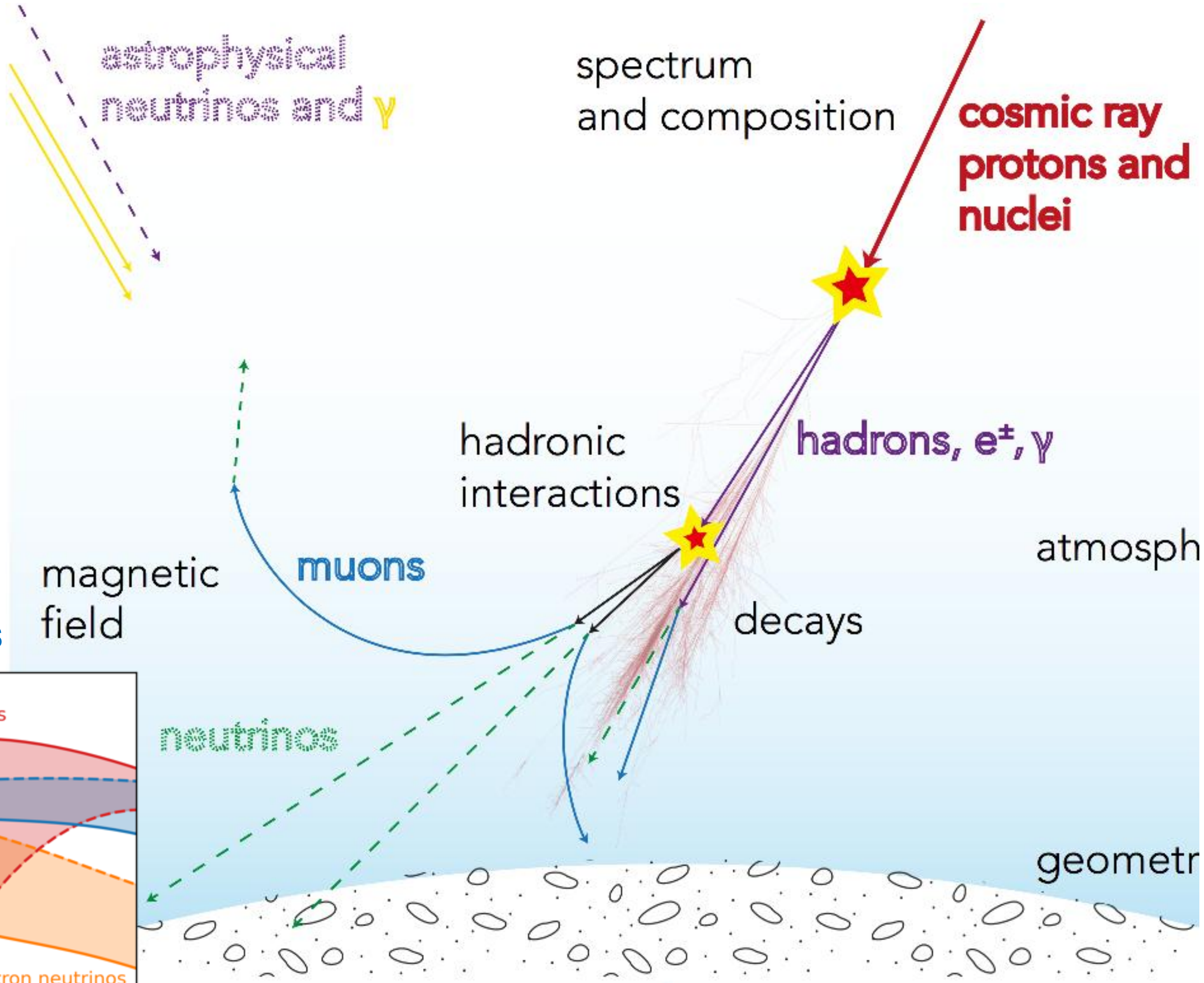
Degeneracy with atm. flux model

Model	H4a-GST	DAEMONFlux
H4a	1.00	0.84
GST	0.98	0.10
GSF	1.11	0.87
Honda	0.70	0.97
DAEMONFlux	0.73	1.00

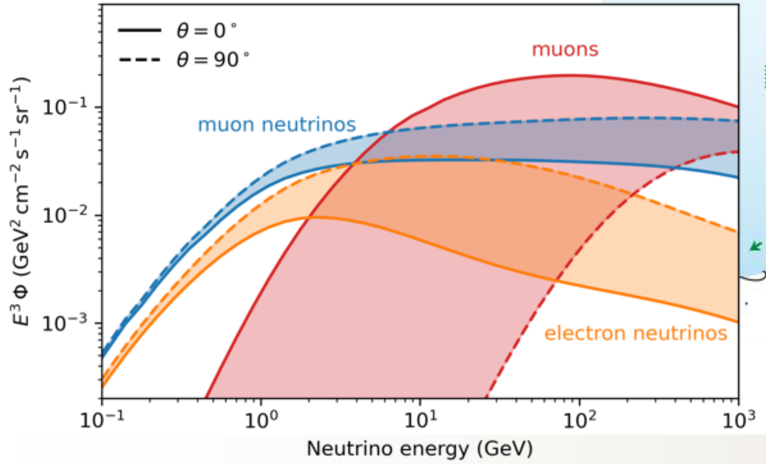
J. Böttcher for IceCube at ICRC2023 & PhD thesis RWTH

Flux modeling





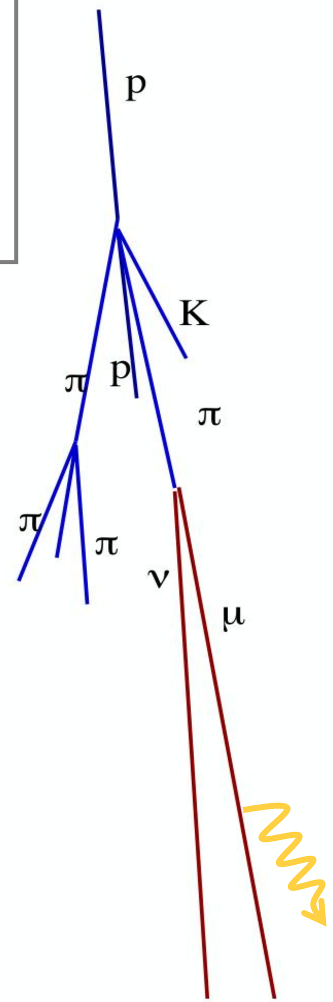
Atmospheric neutrinos



Flux calculation methods

1D particle cascade Monte Carlo:

- CORSIKA 7: AF, Becker Tjus, Desiati, PRD86 114024 (2012)
- High-energy part of HKKMS and Bartol calculations
  M. Honda et al., PRD **92**, 023004 (2015), Barr et al. PRD 70, 023006 (2004)
- FLUKA: G. Battistoni et al. Astroparticle Physics **12**, 315 (1999)



Approximate semi-analytical solutions of cascade equations:

Gaisser, Engel, Resconi book (2016) or e.g.,

$$\Phi_\ell(E) = \frac{\phi_\mathrm{N}(E)}{1 - Z_\mathrm{NN}} \sum_{\substack{h=\pi,\\ \mathrm{K},\mathrm{K}^0_\mathrm{L},\ldots}} \frac{Z_{\mathrm{N}h,\gamma} Z_{h\to\ell,\gamma}}{1 + B_h E \cos\theta / \varepsilon_h}$$

$\phi_N(E)$: cosmic ray flux

Z_{Nh}: particle production yields

B_h and $Z_{h\to l}$: kinematic factors

Matrix Cascade Equations (MCEq)

AF, F. Riehn, R. Engel, T.K. Gaisser, T. Stanev, PRD 100 2019

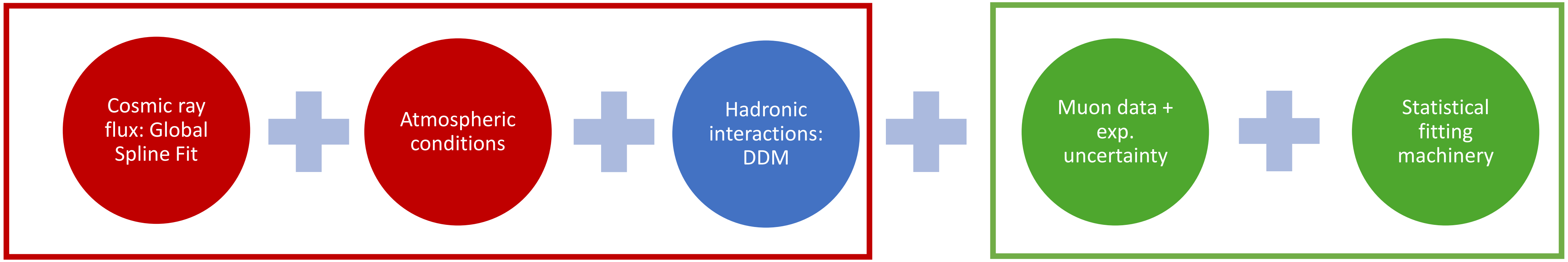
- Iterative solution of coupled cascade equations
- Very fast and accurate
- Open source https://github.com/afedynitch/MCEc
- Now also in 2D (energy-angle)

$$\frac{\mathrm{d}}{\mathrm{d}X}\vec{\Phi} = -\vec{\nabla}_E(\mathrm{diag}(\vec{\mu})\vec{\Phi}) + (-\mathbf{1} + \mathbf{C})\mathbf{\Lambda}_\mathrm{int}\vec{\Phi} + \frac{1}{\rho(X)}(-\mathbf{1} + \mathbf{D})\mathbf{\Lambda}_\mathrm{dec}\vec{\Phi}$$

`daemonflux` approach to modeling atmospheric fluxes

- The open-source code **MCEq** solves the equations accurately, but **flux predictions depend on the arbitrary choice of input models**
- Difficult to quantify **theoretical error**
- *Data-driven input models* parameterize external **data and uncertainty, MCEq propagates it to the flux predictions**

”Flexible” flux model with uncertainty priors from data

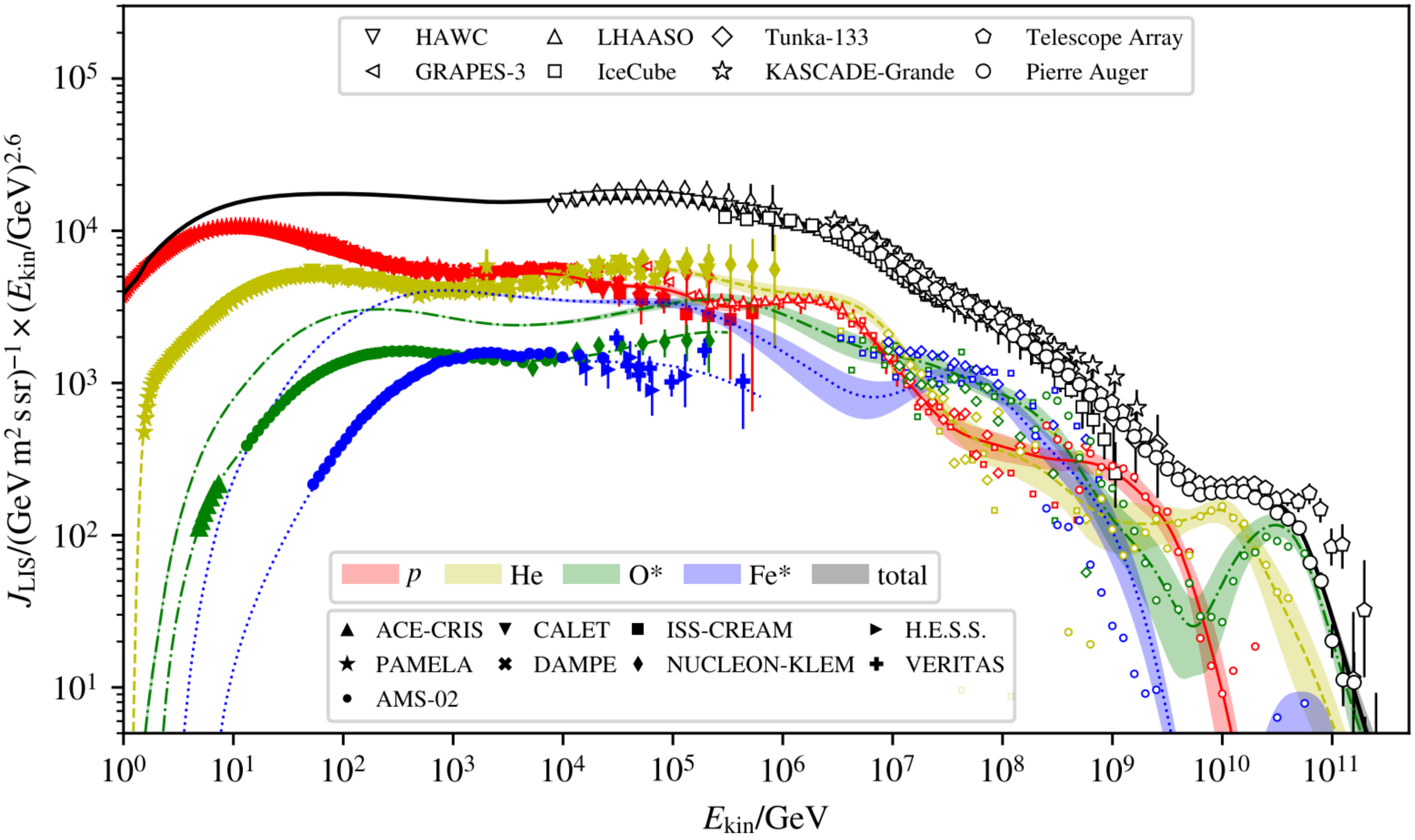


Cross-calibration with atmospheric muons

Extensive reference list in backup

Global Spline Fit Cosmic Ray Model

2025



Aim:

- Agnostic flux parameterization (minimal assumptions)
- Stat. and syst. uncertainties of data sets
- Parameterization with uncertainties and correlations

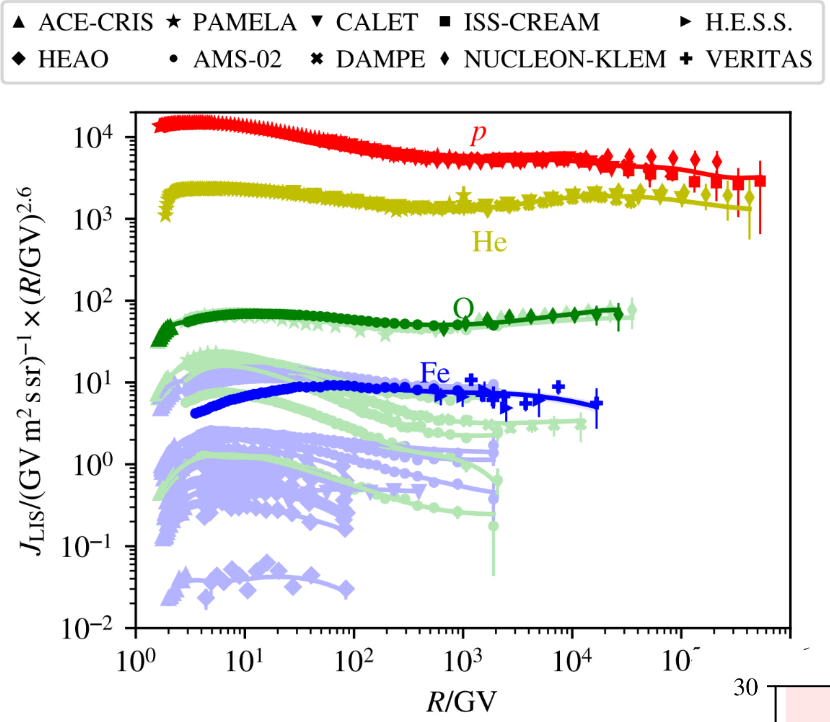
News since 2017:

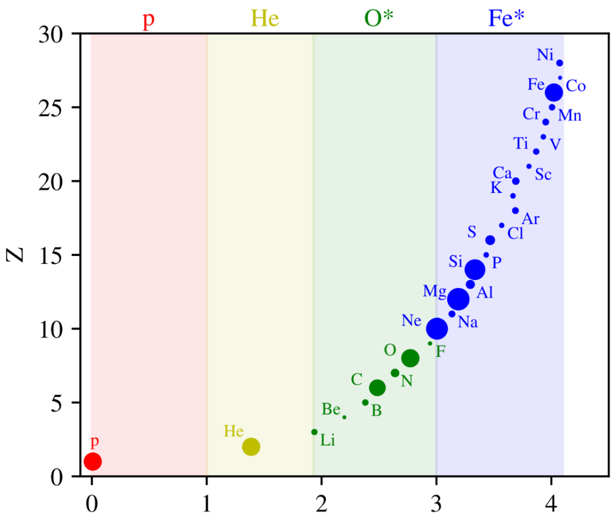
- Proton spectrum known from **data from GeV to 10 EeV**
- **Up to < ~200 TeV relevant features of the CR flux are almost perfectly known**

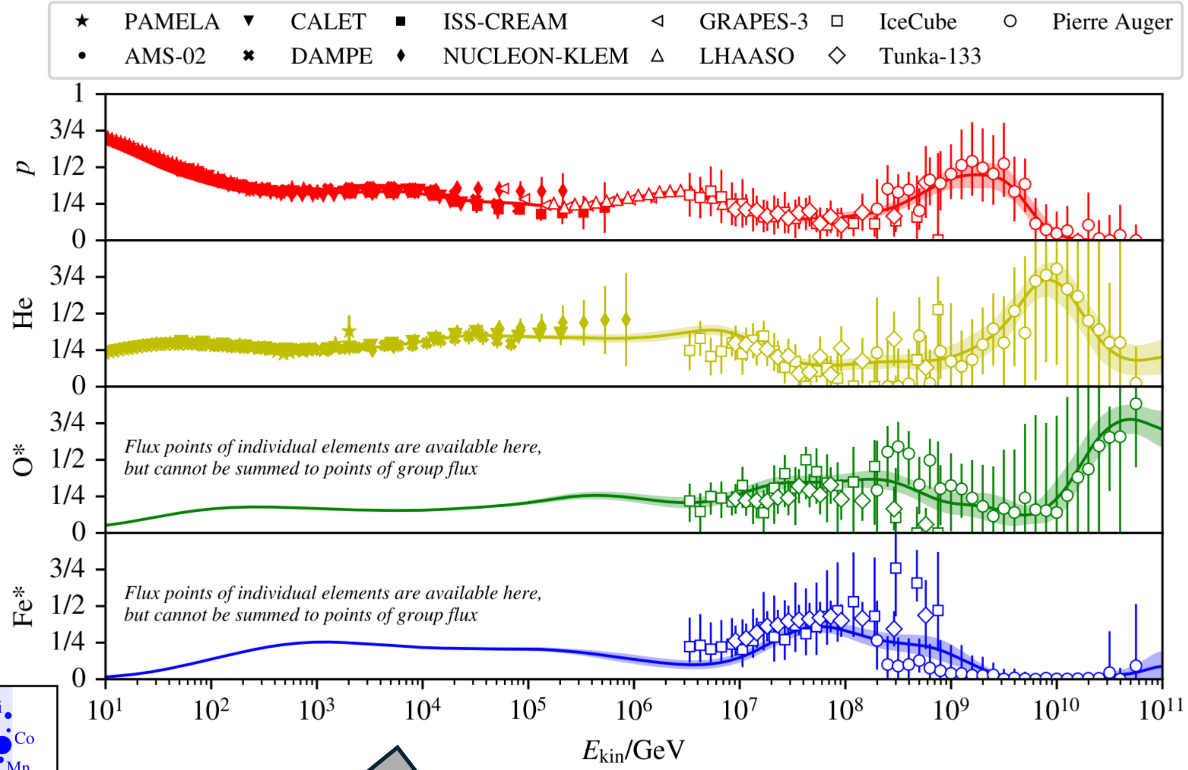
Dembinski, AF, Gaisser, ICRC 2017 & H. Dembinski 2019 & **Fujisue**, AF, Engel UHECR2024 & Dembinski, AF, Engel, **Fujisue** ICRC2025

Connects direct and indirect CR measurements

2025

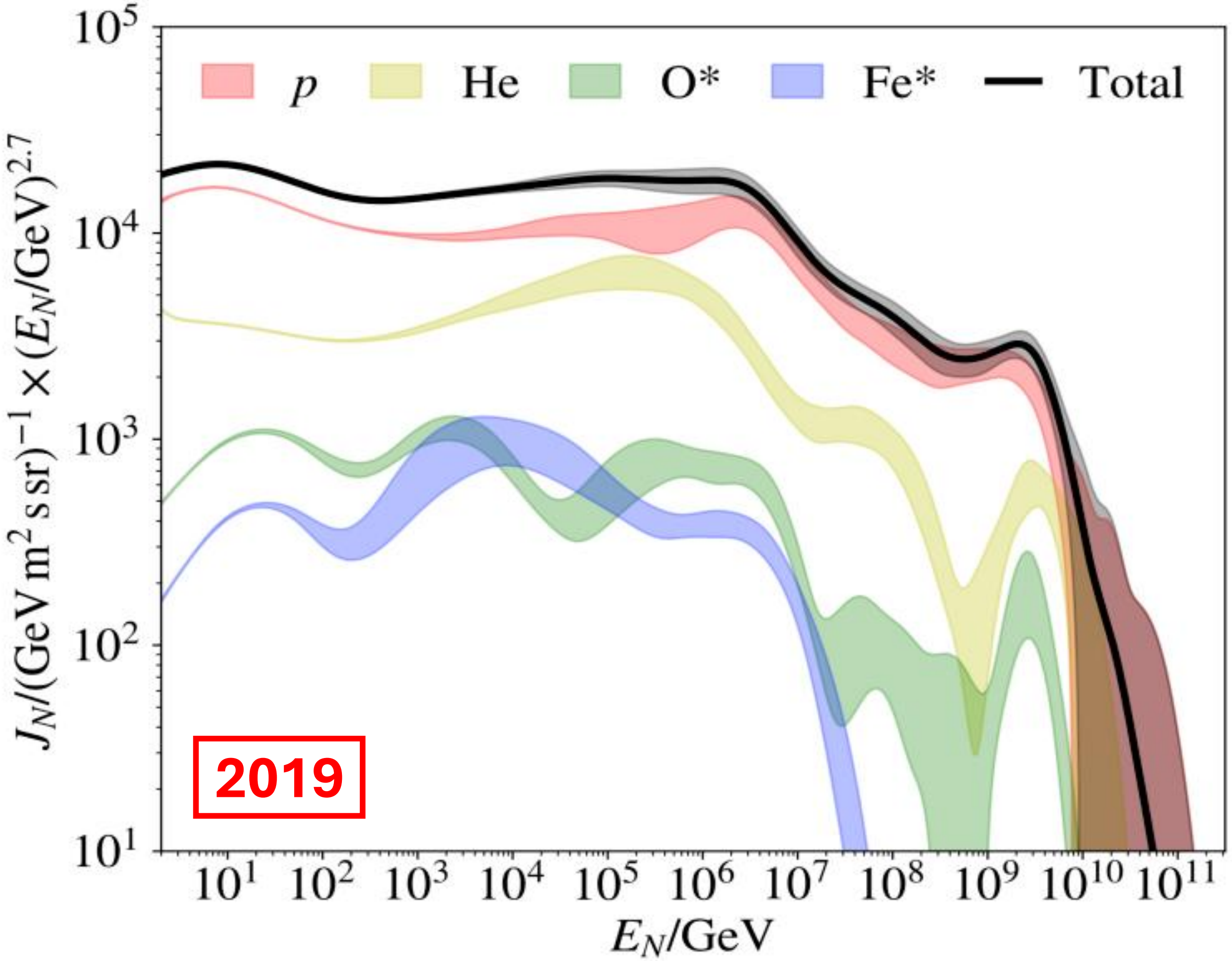


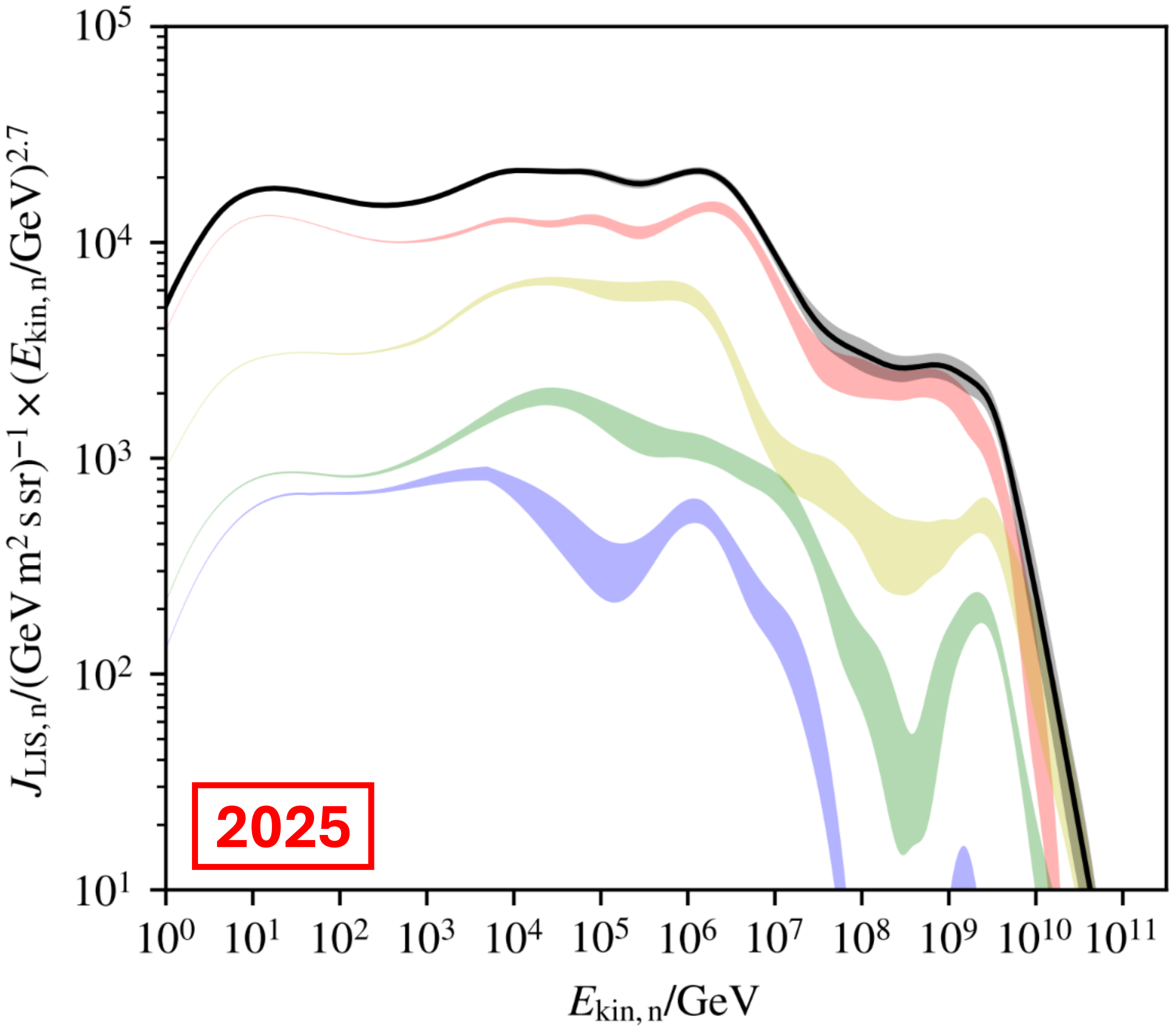




- Define and fit 4 mass groups globally
- Element fluxes have constant ratios within a mass group outside of the data range

Nucleon fluxes (MCEq & daemonflux input)

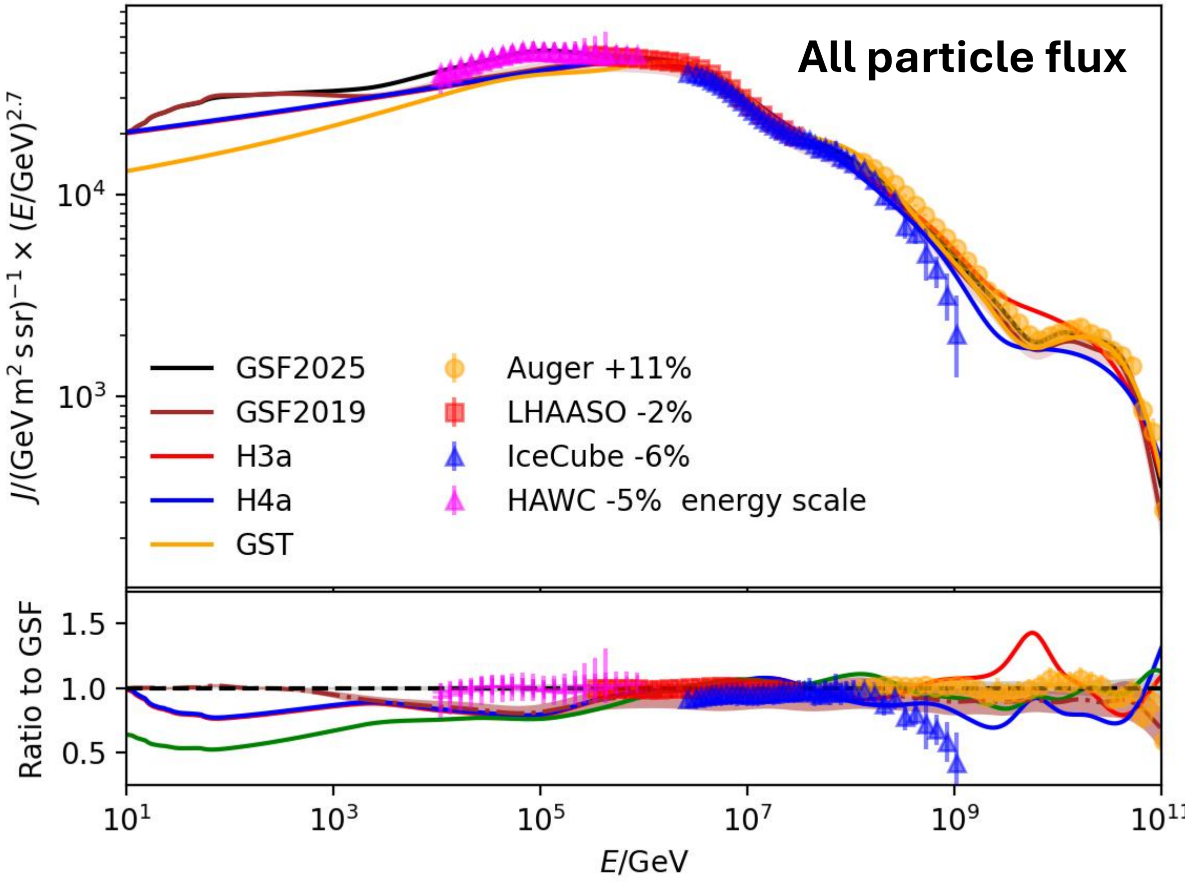


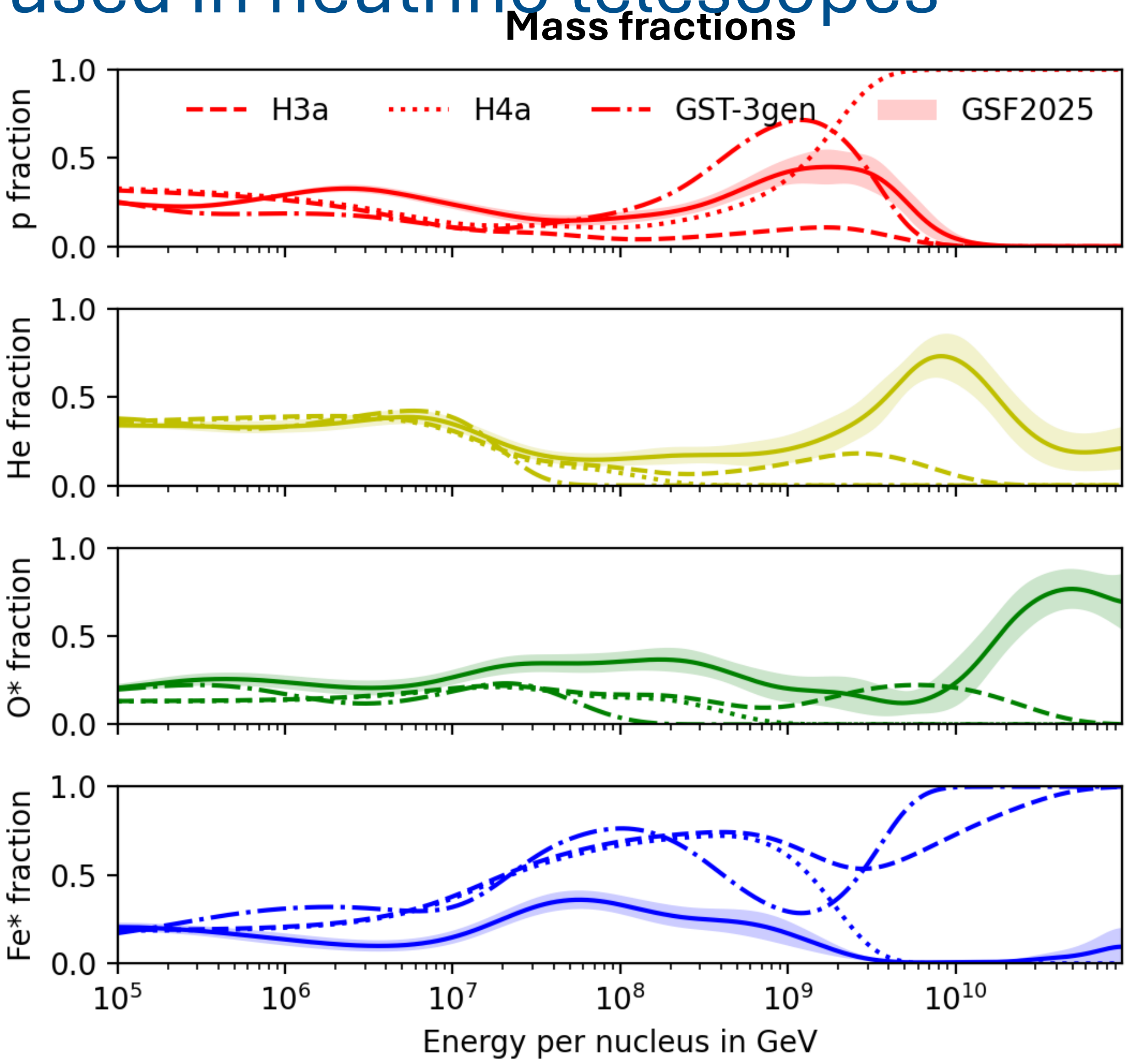


- Dominated by proton spectrum, which is known from data!
- **Several new breaks** in recent data

For daemonflux: reduce parameter space to 6 with PCA on covariance

Comparison with other models used in neutrino telescopes

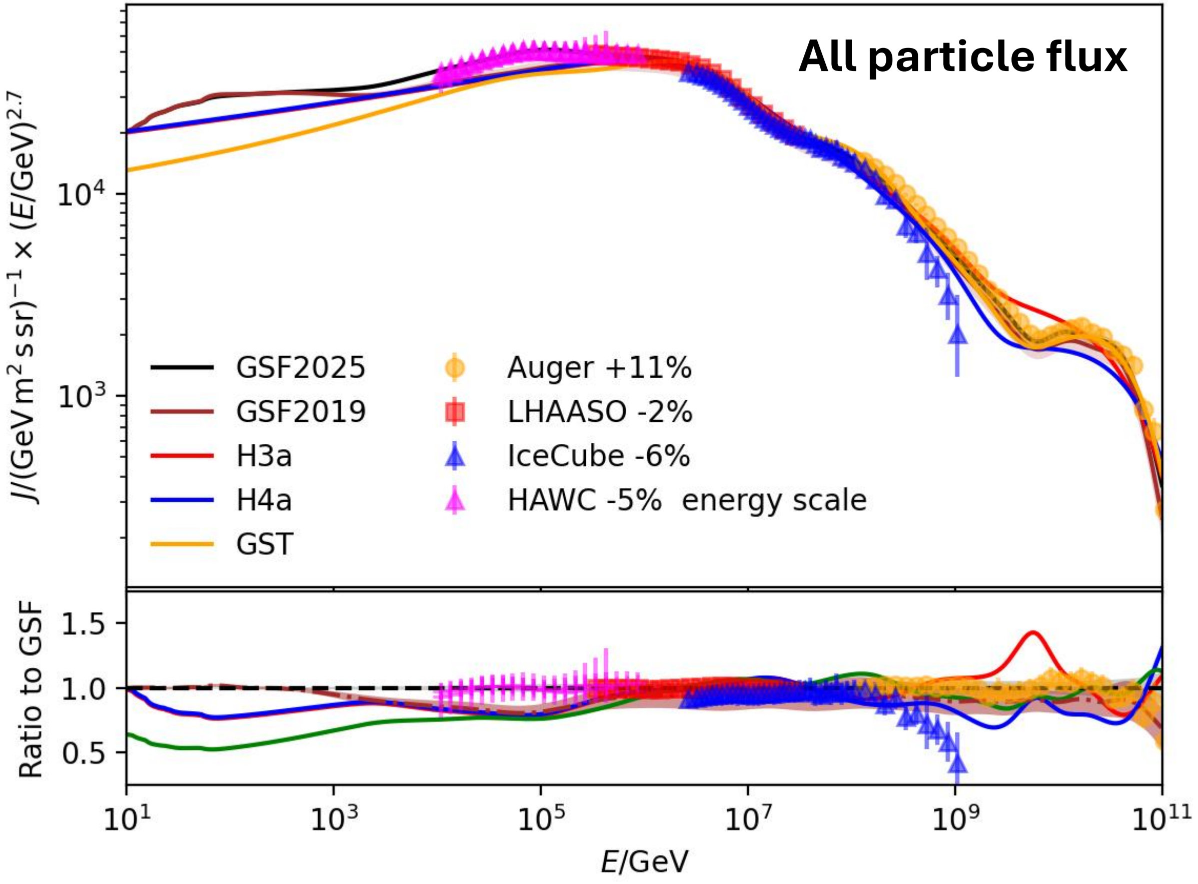


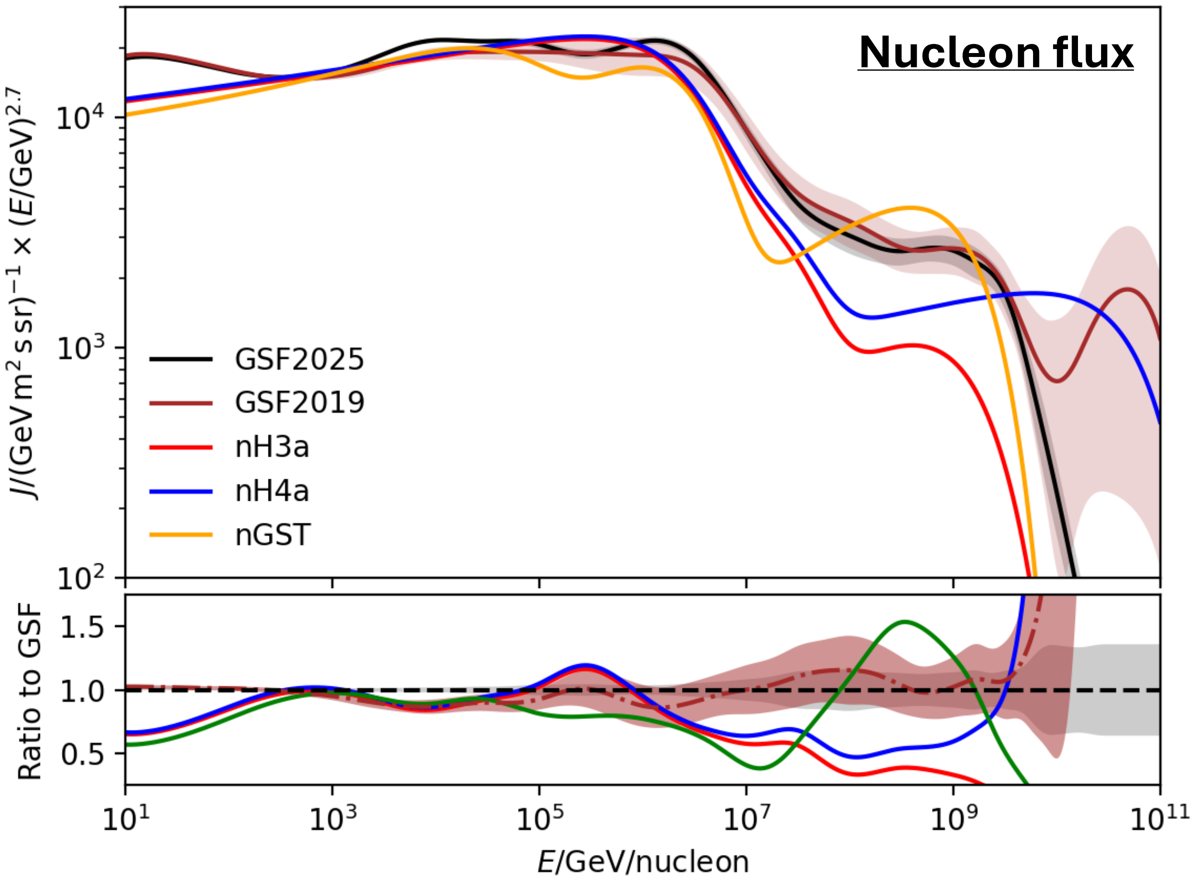


- **Bracketing doesn’t work** at most of the relevant energy range
- CR observations reveal new features and more precise data

GST: T. Gaisser, T. Stanev and S. Tilav, Front. Phys. (Beijing) **8** 748-758 (2013), H3/4a: T.K. Gaisser, ApJ35, (2012)

Comparison with other models used in neutrino telescopes





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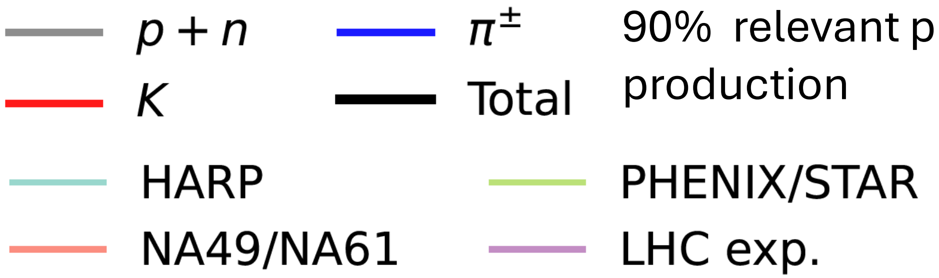
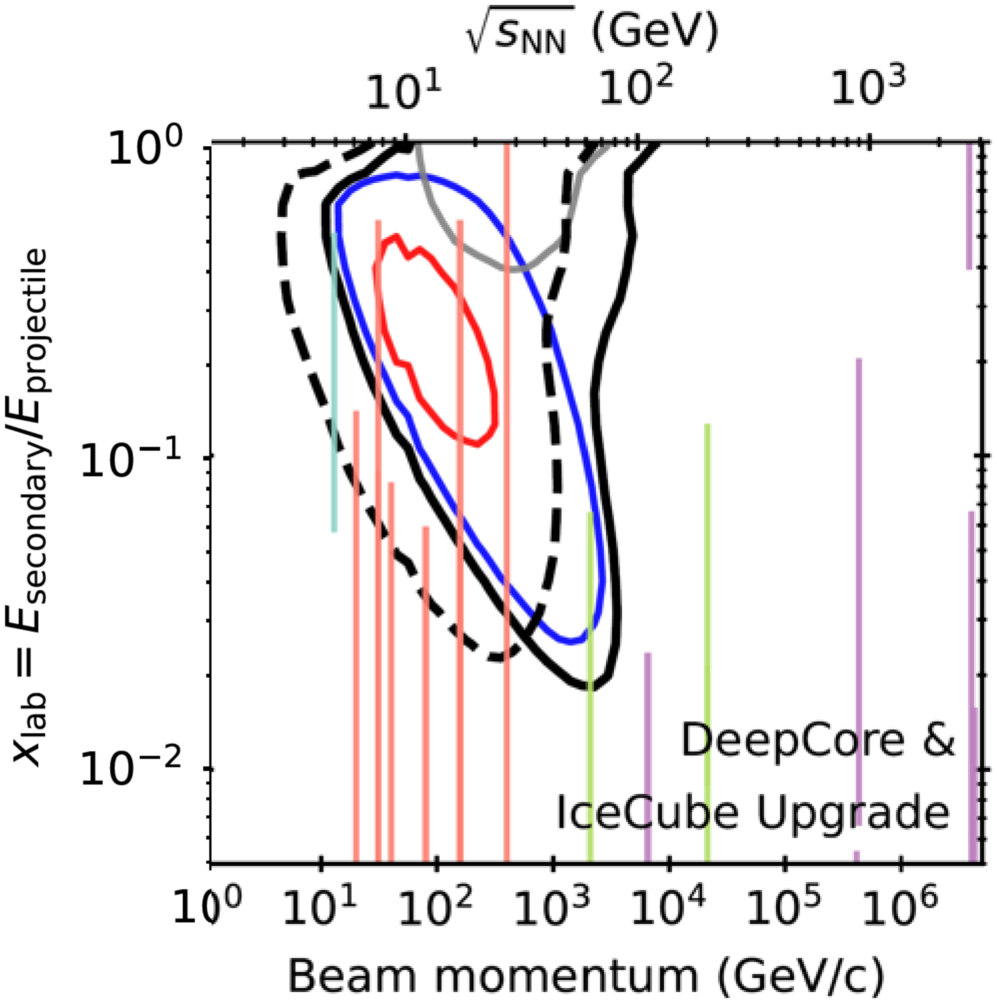
- CR observations reveal new features and more precise data

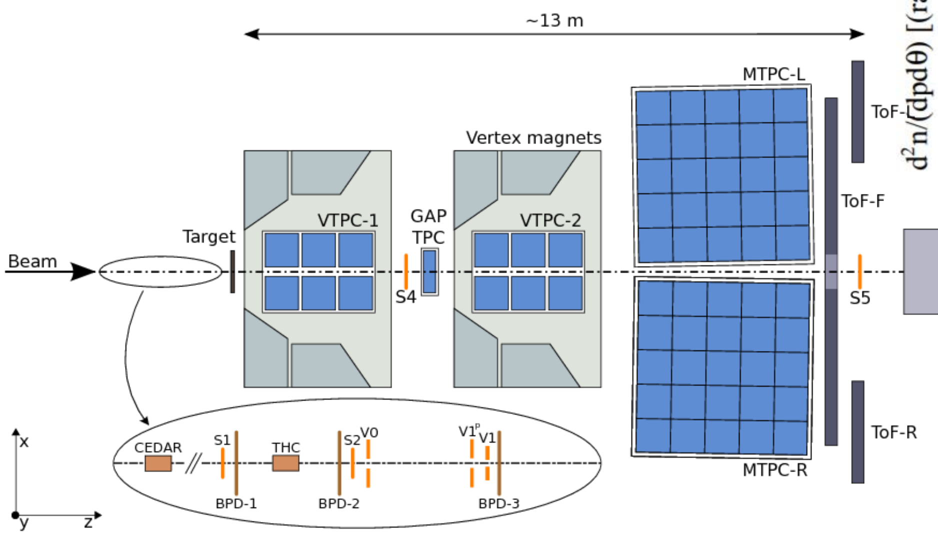
Data-driven hadronic model (DDM)

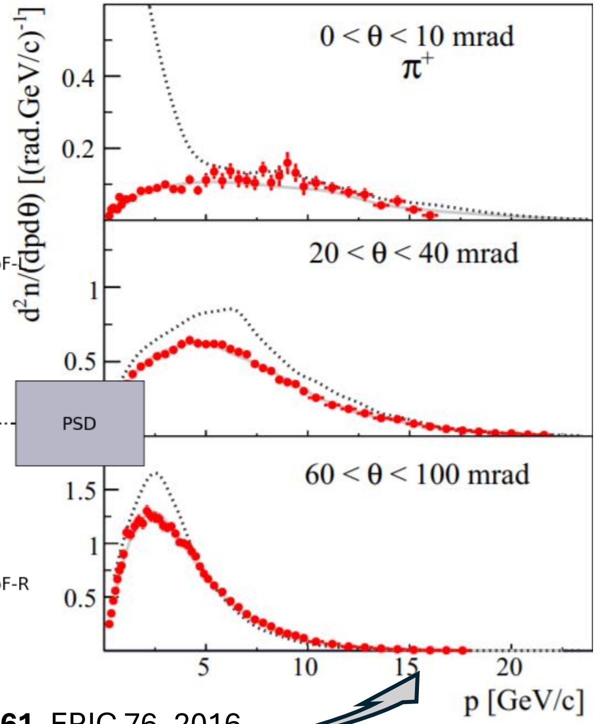


90% relevant phase space neutrino production

Accelerator data taken at fixed beam energies





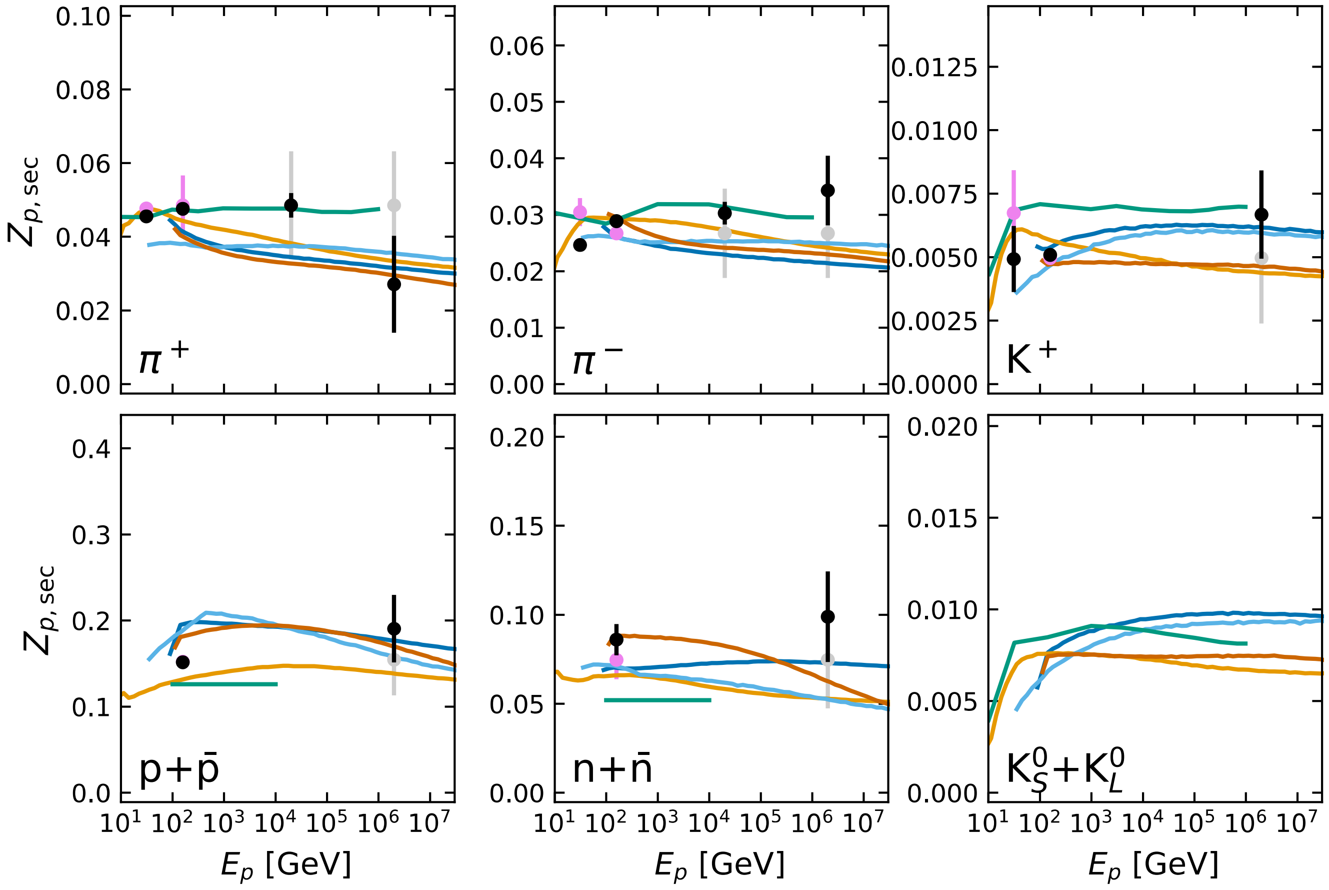


Proton on carbon, 31 GeV, **NA61**, EPJC 76, 2016

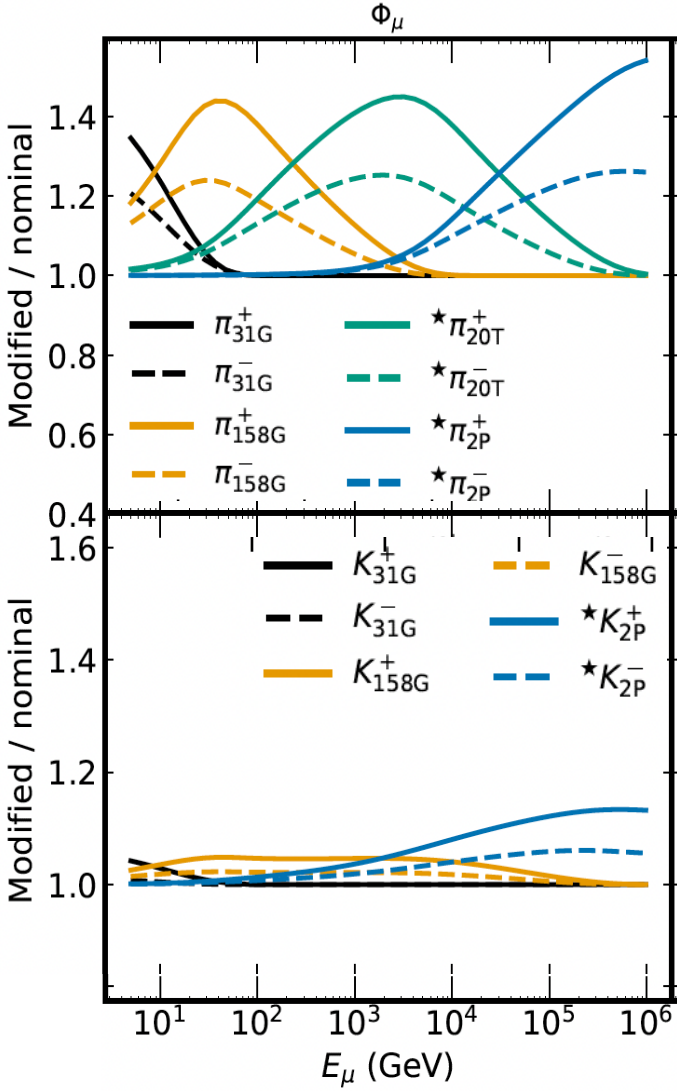
AF & M. Huber, PRD 106, 2022

Inter-/extrapolation of hadronic yields across energies

J. P. Yanez & AF, PRD 107, 2023



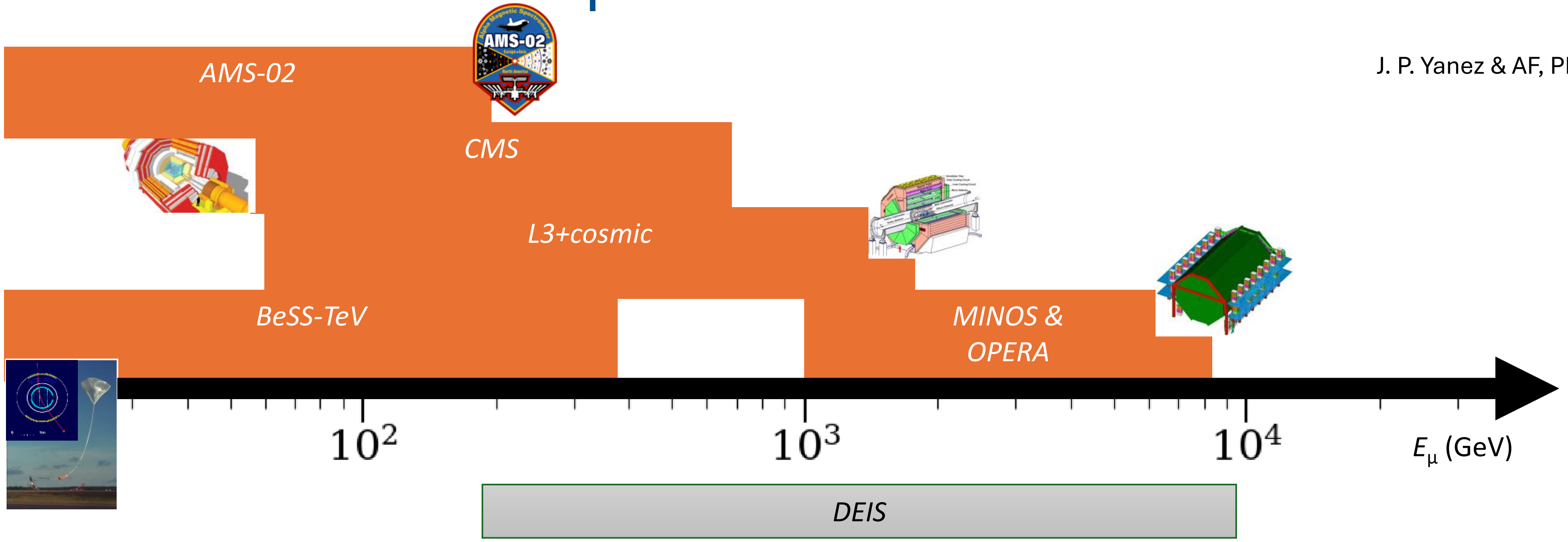
Set of gradients



- Interpolate parameterizations of fixed-target data taken at fixed energies
- Add additional degrees of freedom with loose priors when extrapolating

Calibrate CR and hadronic parameters on muons

J. P. Yanez & AF, PRD 107, 2023



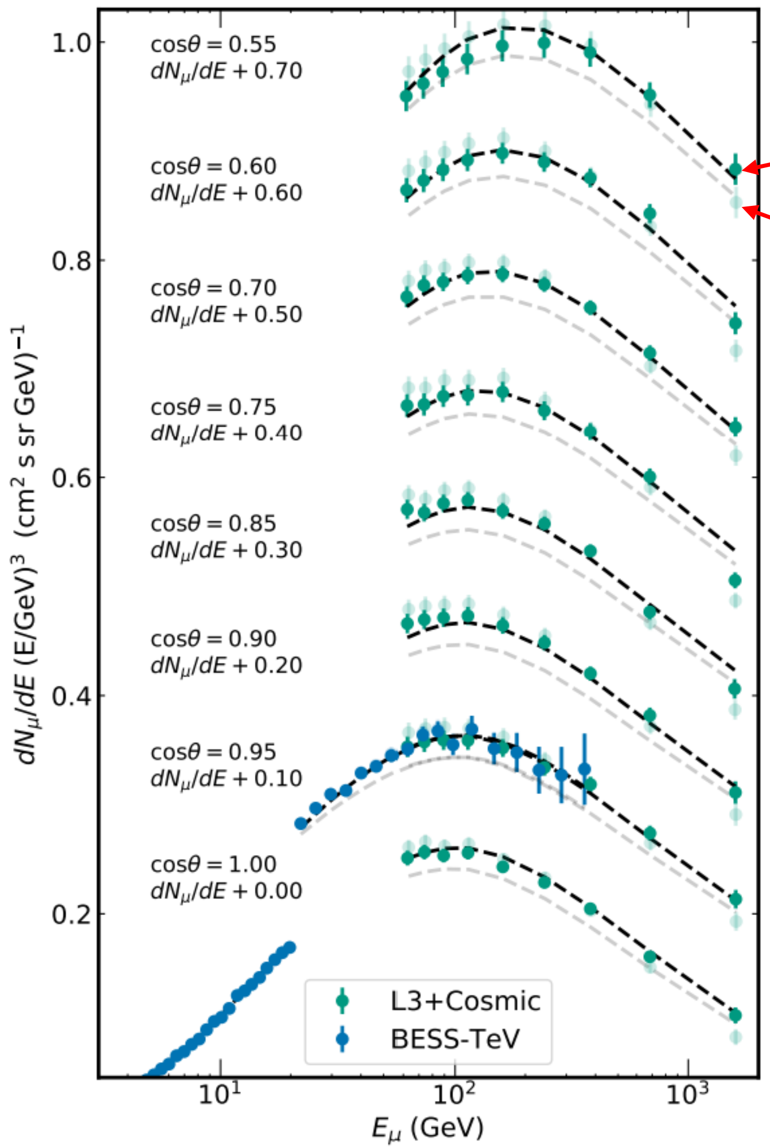
Experiments disclosing systematic uncertainties. Most provide correction functions for the data.

Experiment	Energy (GeV)	Measurements	Unit	Systematics	Location	Altitude	Zenith range
BESS-TeV [44]	0.6-400	Φ_μ	p_μ	C	36.2°N, 140.1°W	30 m	0-25.8°
CMS [45]	5-1000	R_{μ^+/μ^-}	p_μ	Q	46.31°N, 6.071°E	420 m	$p\cos\theta_z$
L3+C [46]	20-3000	Φ_μ,R_{μ^+/μ^-}	p_μ	C	46.25°N, 6.02°E	450 m	0-58°
DEIS [47]	5-10000	Φ_μ	p_μ	Q	32.11°N, 34.80°E	5 m	78.1-90°
MUTRON [48]	80-10000	R_{μ^+/μ^-}	p_μ	Q	35.67°N, 139.70°E	5 m	87-90°
MINOS [49]	1000-7000	R_{μ^+/μ^-}	E_μ	C	47.82°N, 92.24°W	5 m	unfolded
OPERA [50]	891-7079	R_{μ^+/μ^-}	E_μ	Q	42.42°N, 13.51°E	5 m	$E\cos\theta^*$

Muon fluxes and cross-calibrated data

J. P. Yanez & AF, PRD 107, 2023

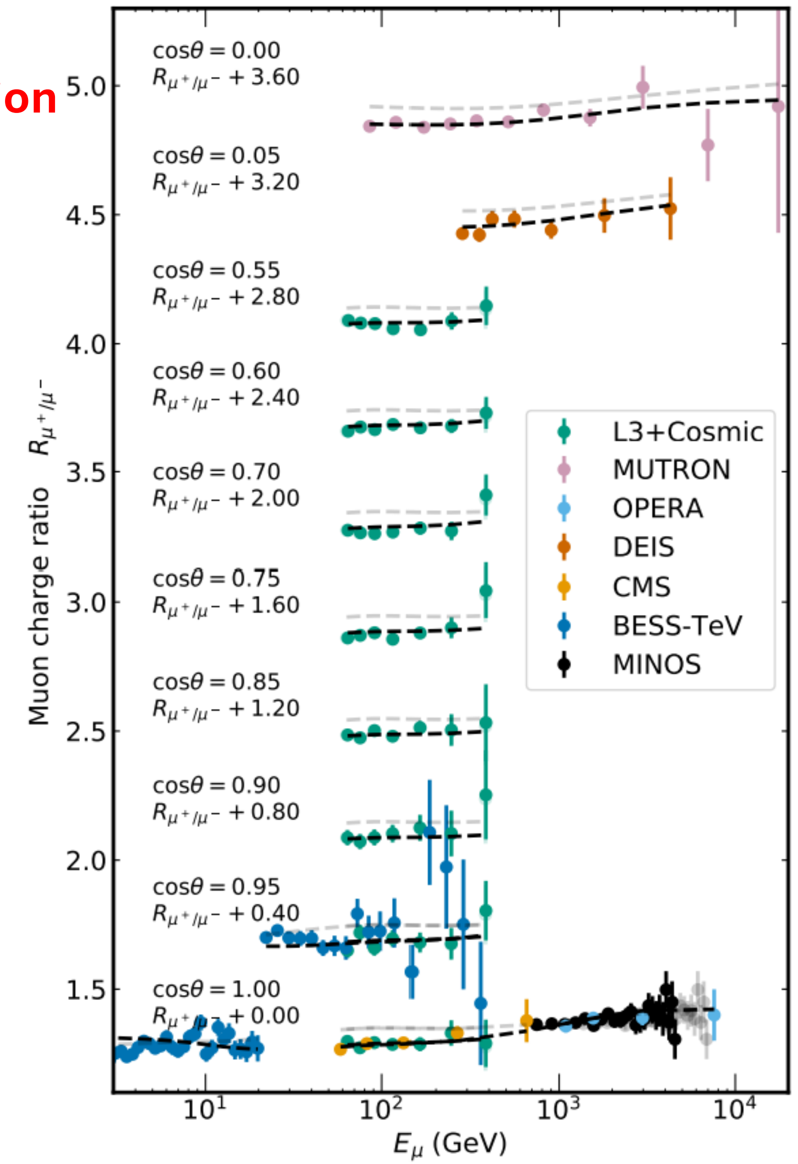
Muon flux



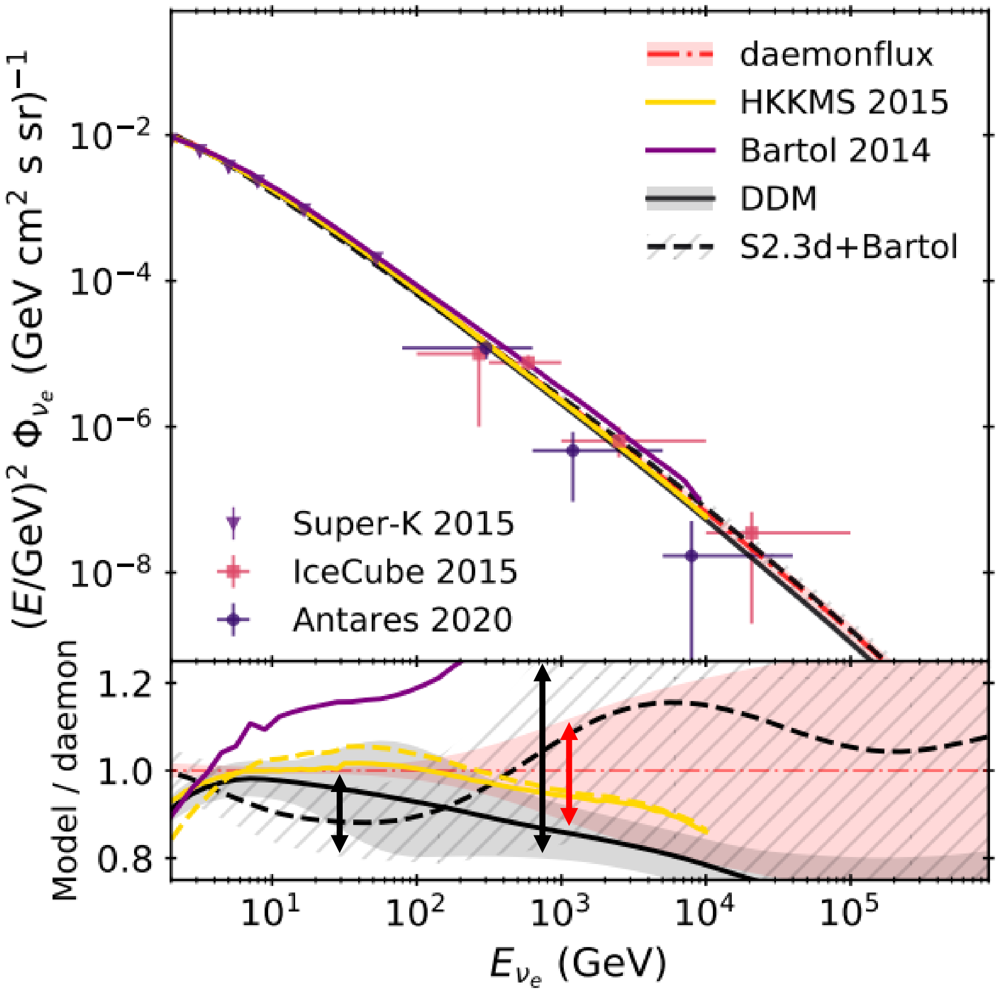
Data w/ syst. correction

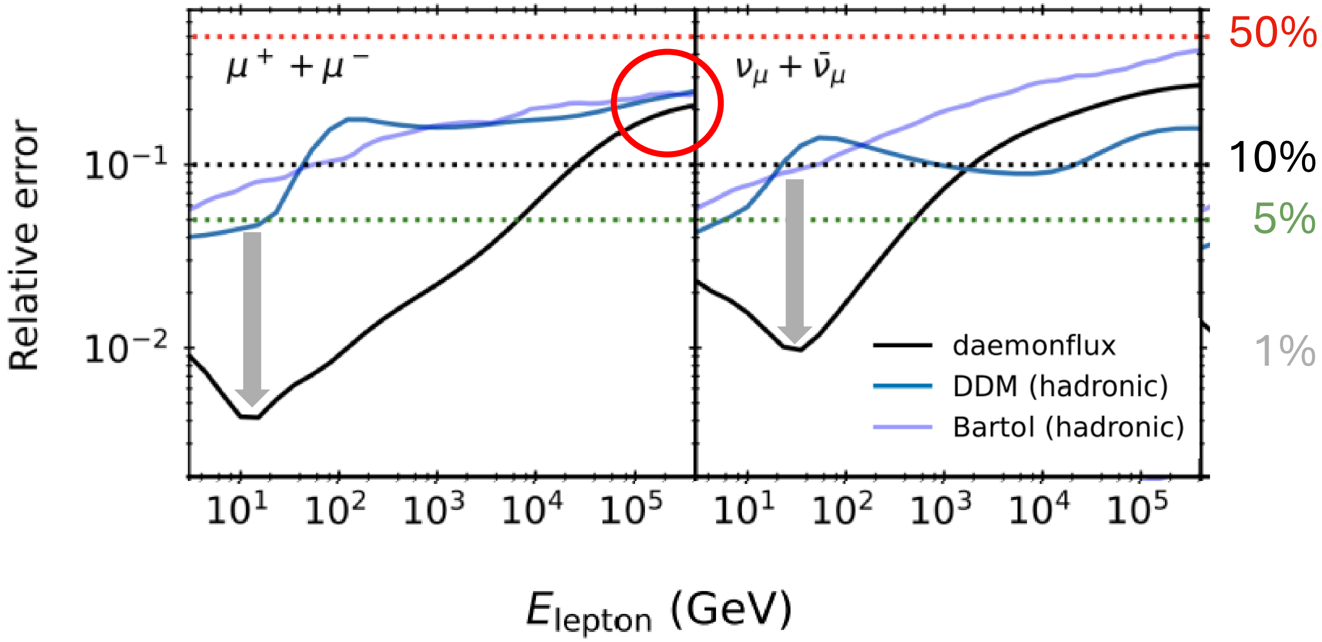
Data w/o syst. correction

Muon charge ratio



Resulting neutrino fluxes and uncertainty



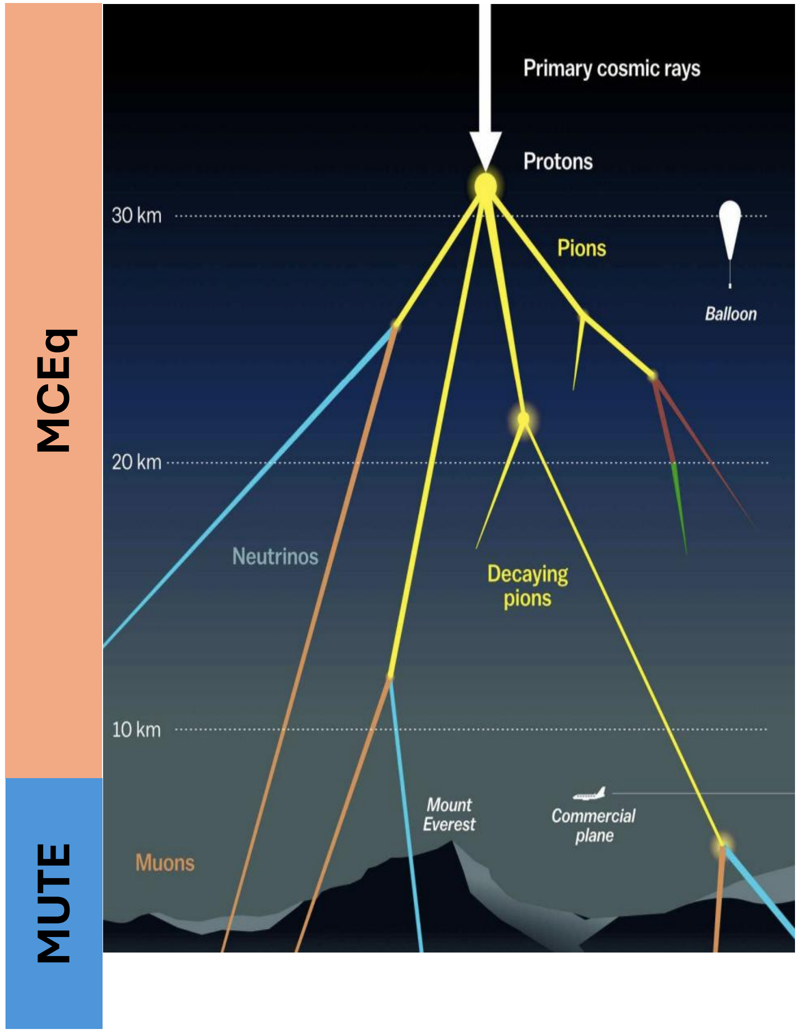


- Uncertainties in daemonflux are driven by uncertainties in the muon and CR data → high precision < 1 TeV
- Extrapolation uncertainty comparable to previous models

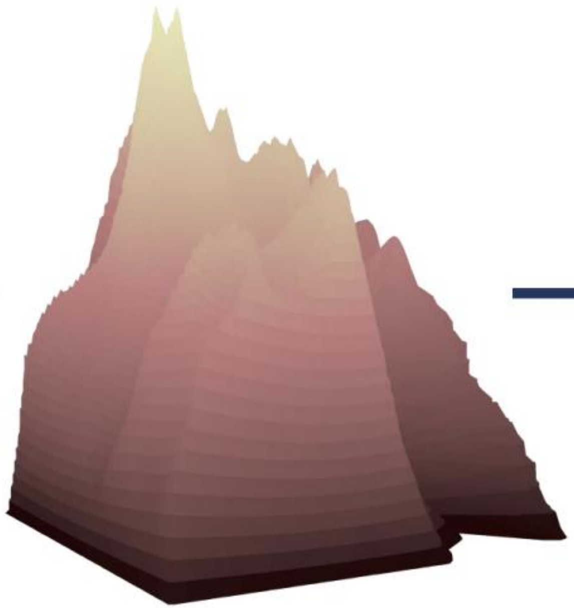
MUTE

Talk by W. Woodley GWA / CRA / CPP / 13

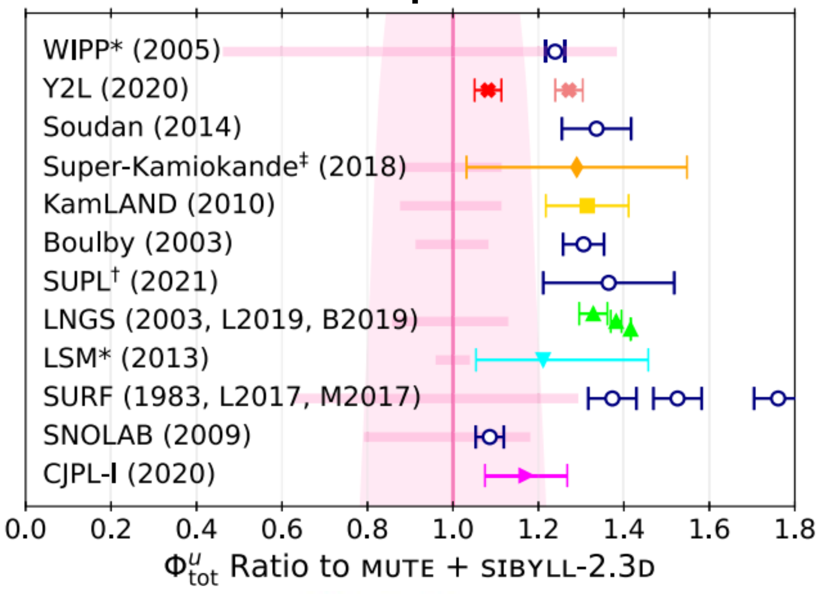
- The MUTE code is using MCEq and PROPOSAL
- Predicts muon spectra underground
- Underground measurements reflect higher energy muons at the surface



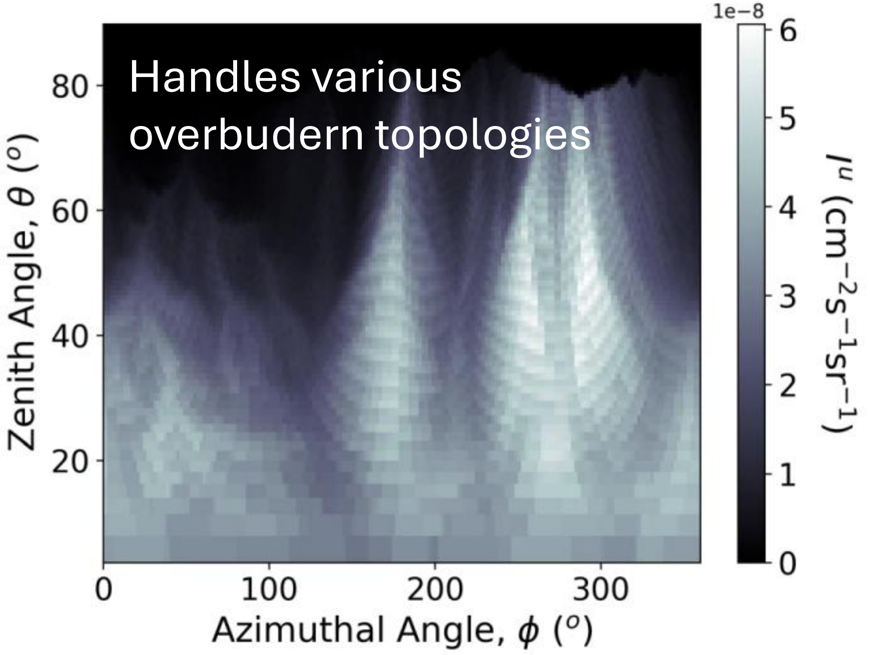
$X(\theta, \phi)$



MCEq + SIBYLL 2.3d + GSF



$I^u(\theta, \phi)$



*AF, **W. Woodley**, M.-C. Piro, ApJ 928 27 (2022)*

***W. Woodley**, AF, M.-C. Piro., PRD 110, (2024)*

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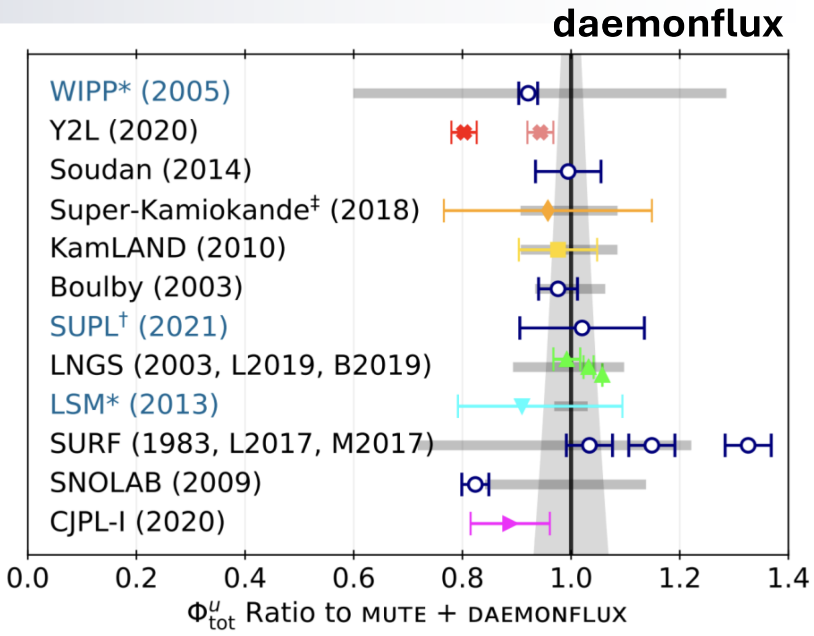

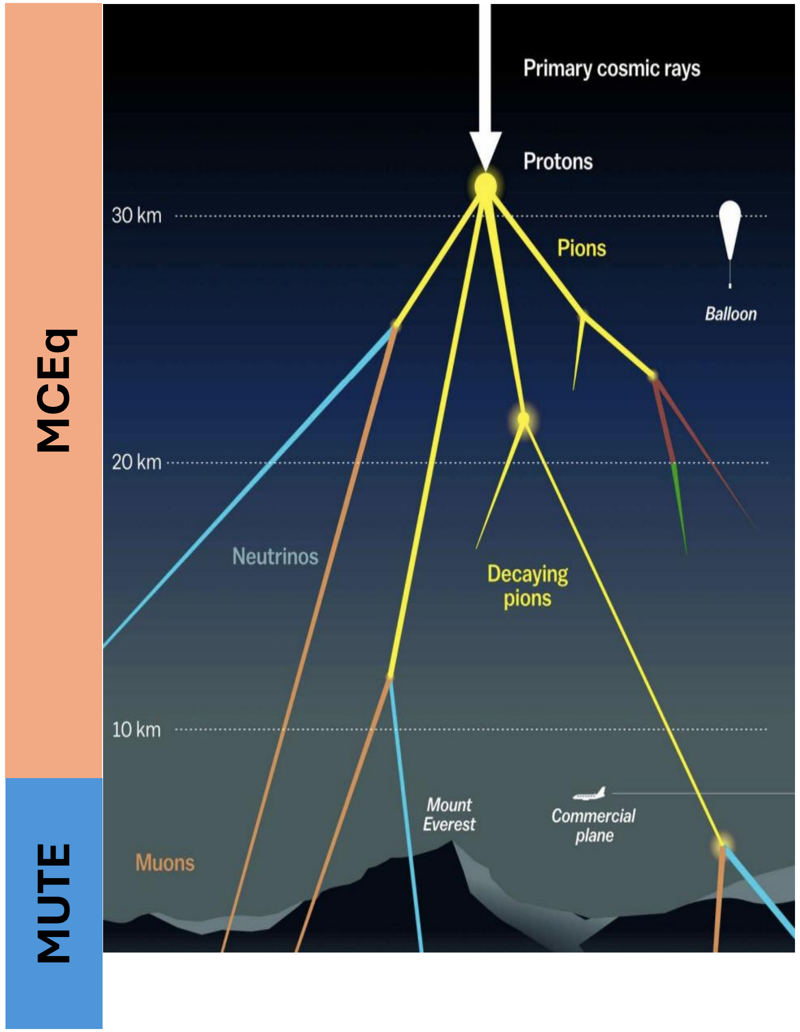

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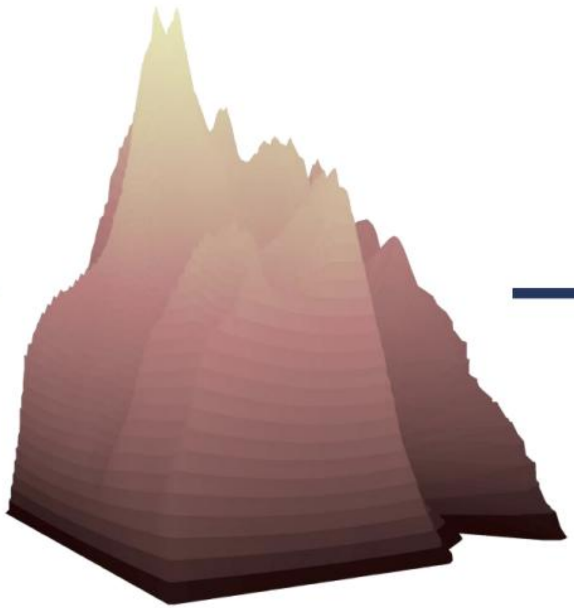

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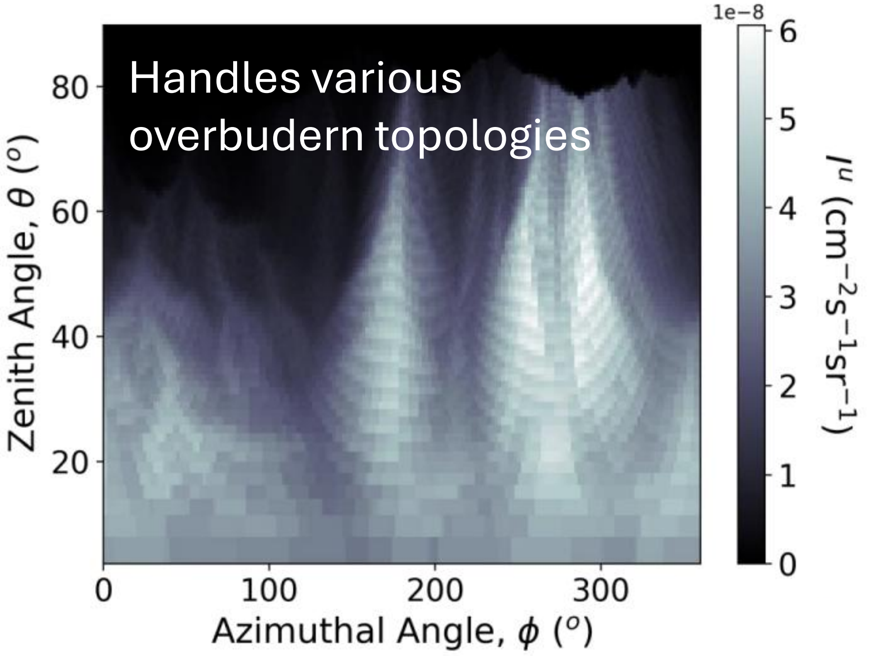
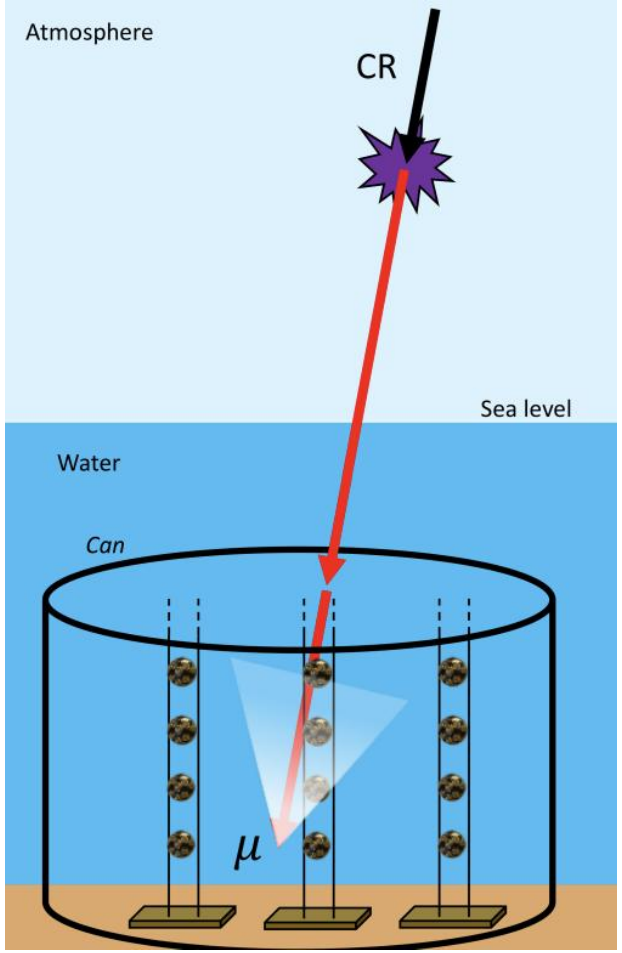


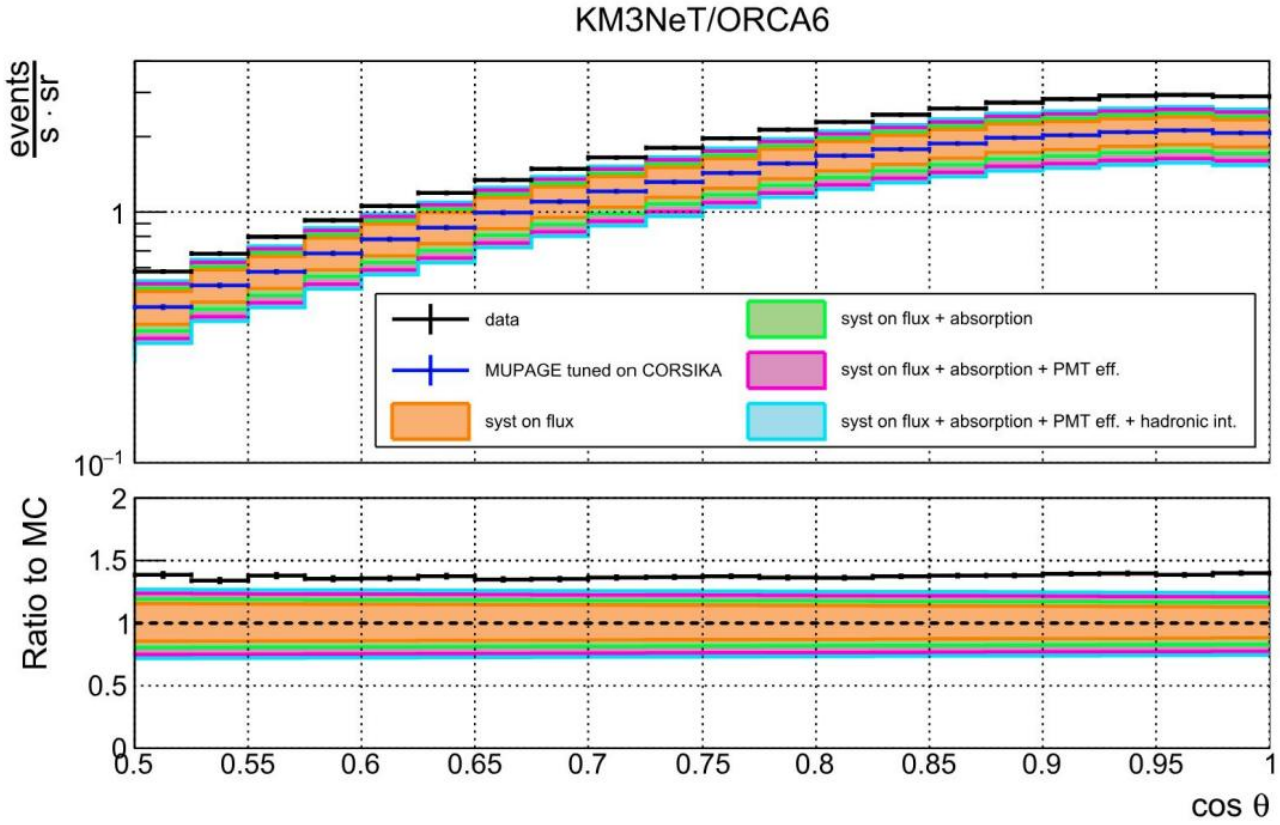
*AF, **W. Woodley**, M.-C. Piro, ApJ 928 27 (2022)*

***W. Woodley**, AF, M.-C. Piro., PRD 110, (2024)*

Progress on muon measurements @ KM3NeT



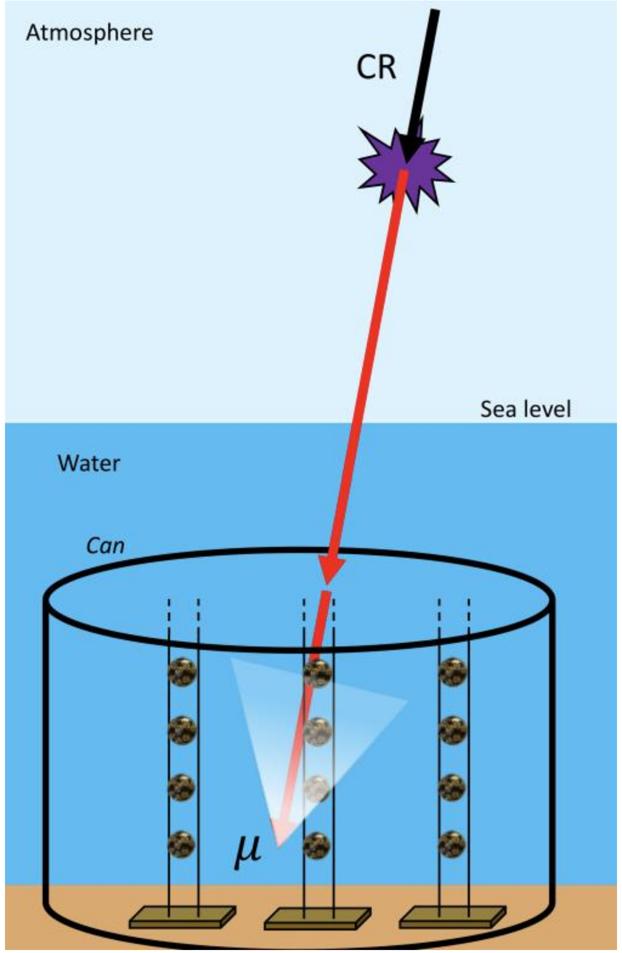
Muon bundle rate in KM3NeT ORCA compared to Sibyll2.3d + GSF2017



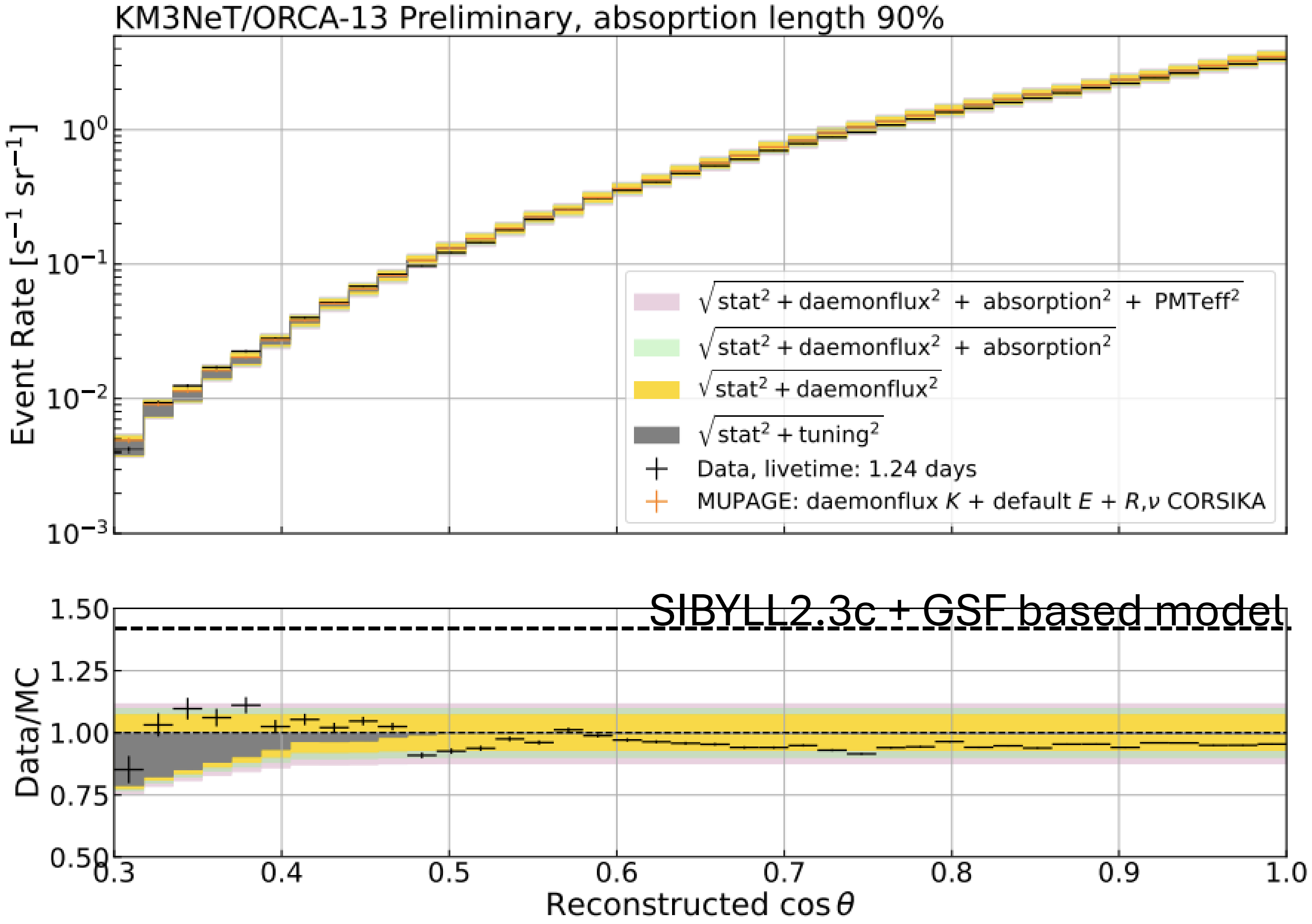
KM3NeT, EPJC 84 (2024)

- Comparing MUPAGE MC tuned to CORSIKA with SIBYLL2.3D and GSF
- Observe large x1.4 disagreement in Data/MC

Progress on muon measurements @ KM3NeT



Venugopal Ellajosyula's talk @ Cosmic Rays / 306



- Significantly better agreement with their data
- Flux model precision is used to study water properties

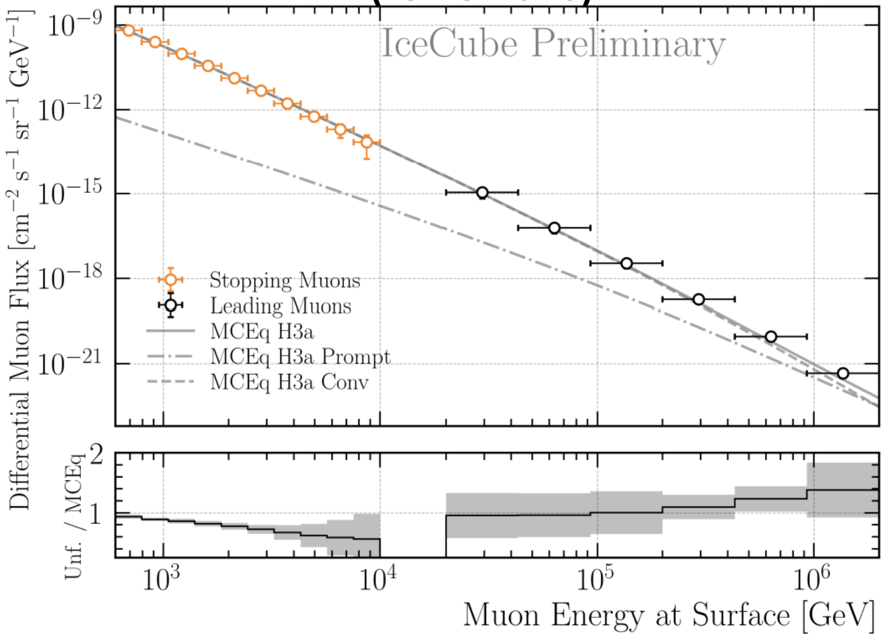
Roadmap for MCEq-based models

1. **New hadronic models**: SIBYLL 2.3e, QGSJET-III, EPOS-LHC-R (this year)
2. Release of **GSF 2025:** including covariance matrix (early 2026)
3. **Daemonflux:** update with GSF2025 and underground muon data (2026)
4. **Full zenith/azimuth atmosphere:** found performance boost x100000 for parallel calculations, model should remain lightweight (late 2026)
5. **Low-energy focus**: fluxes < 5 GeV, geomagnetic cutoff + 3D (2027+)

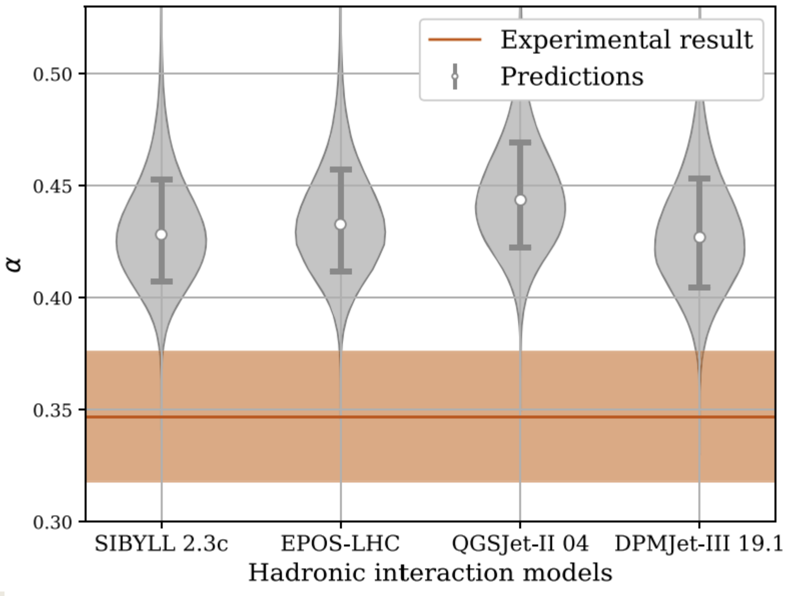
Final remarks

- Made progress in atmospheric flux modeling and measurements. Model precision unlocks new types of tests and measurements (see e.g., Venu's talk)
- Is MCEq + SIBYLL/EPOS/QGSJET + X a bad model combination? → No, served well over the years within the systematic uncertainties of the models and experiments
- Do we observe a **muon deficit** in atmospheric leptons similar to the muon puzzle in UHECR? → maybe
- Early adopters (IceCube Sterile Neutrino Search PRL133 2024, thanks Alfonso and MEOWS team) successfully analysed data with daemonflux, and **more rigorous tests by Neutrino Telescopes have not yet been finalized**

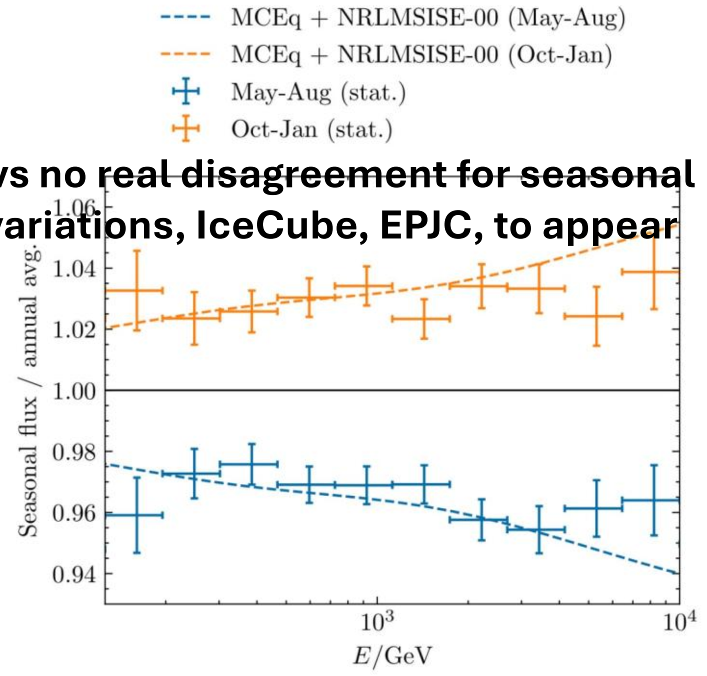
Muon flux unfolding by P. Gutjahr (ICRC 2025)



Seasonal variations: mild disagreement with model predictions. IceCube, EPJC83, 2023



..vs no real disagreement for seasonal variations, IceCube, EPJC, to appear

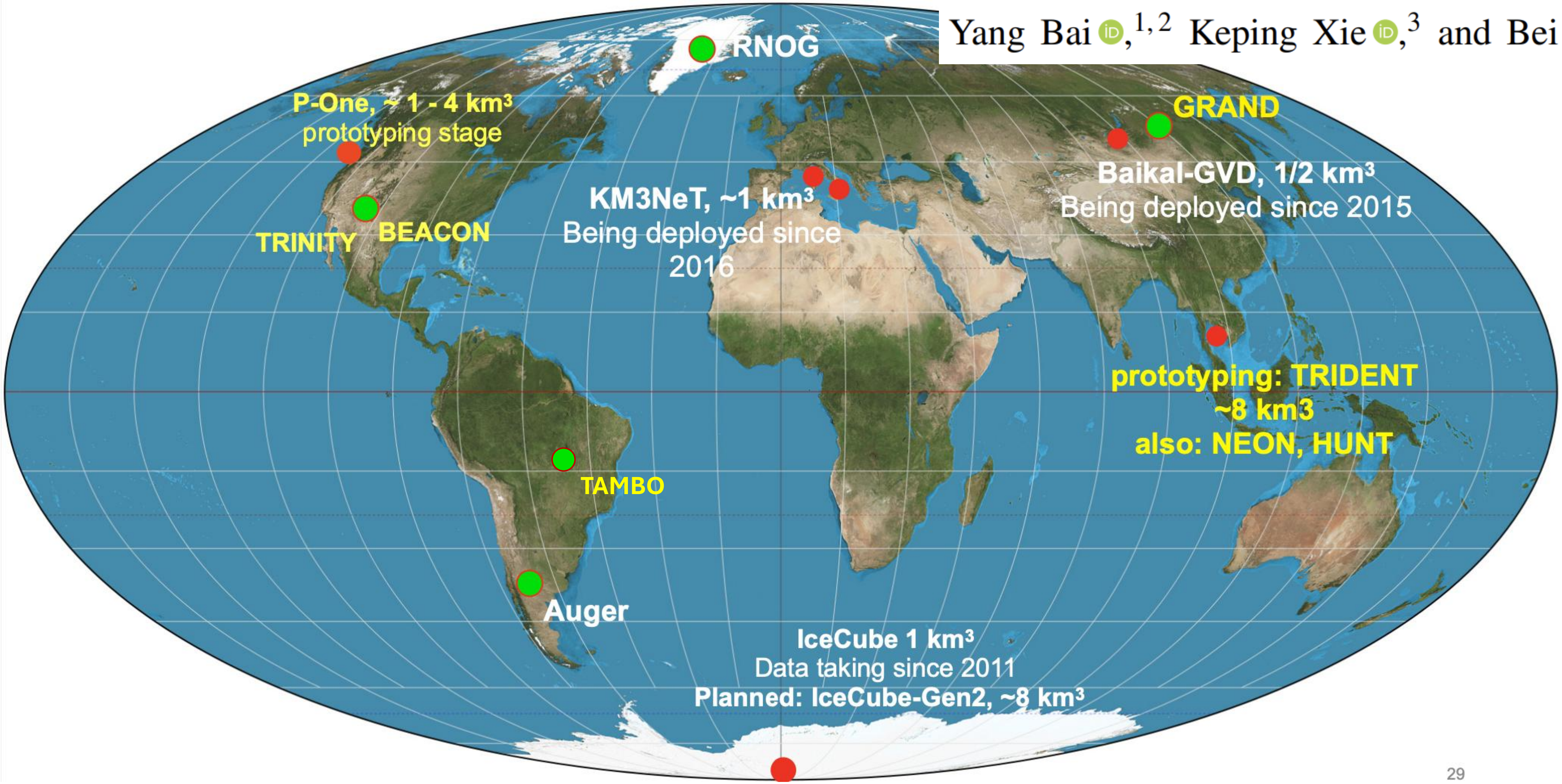


arXiv:2510.13948

Future

Large Neutrino "Collider"

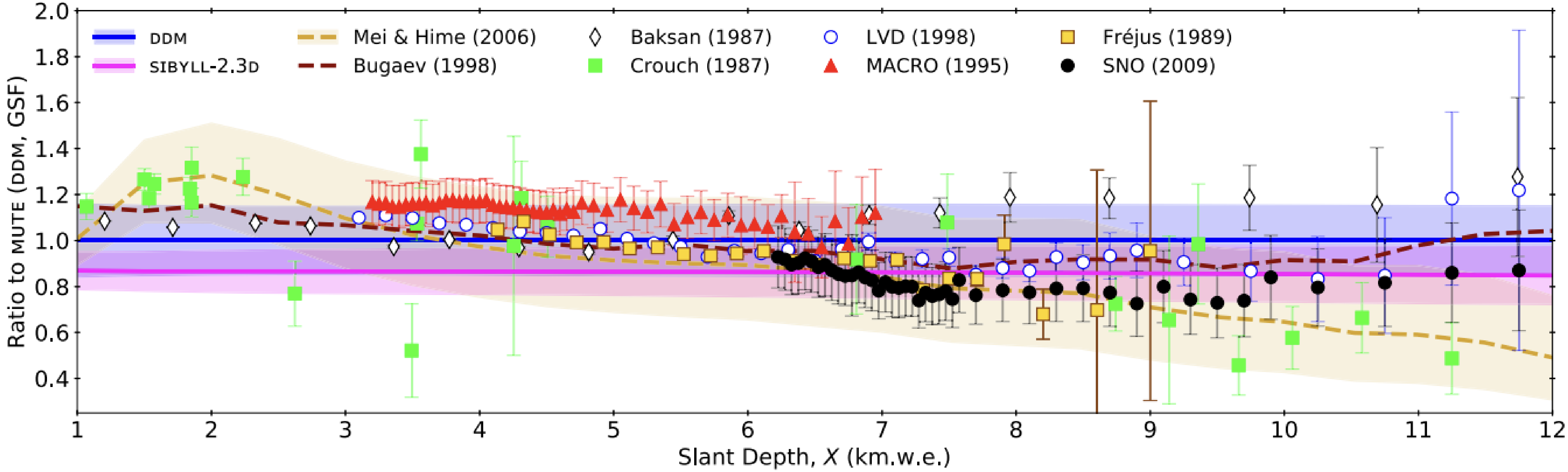
Yang Bai[1,2] Keping Xie[3] and Bei Zhou[4,5]



Vertical equivalent underground fluxes

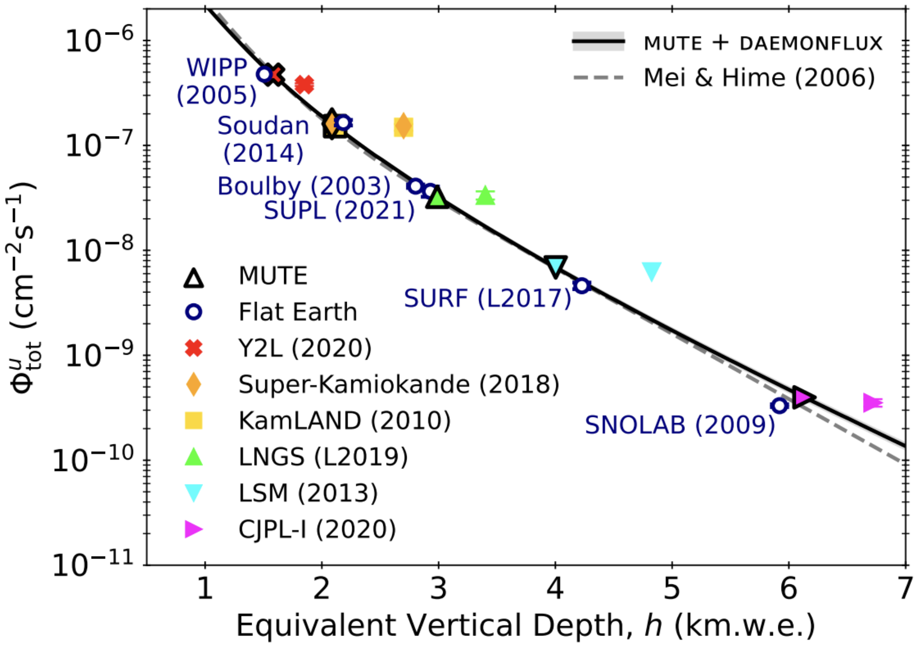
https://github.com/wjwoodley/mute

AF, W. Woodley, M.-C. Piro, ApJ 928 27 (2022)



Data was found to be more constraining
than the theoretical uncertainties (bands).

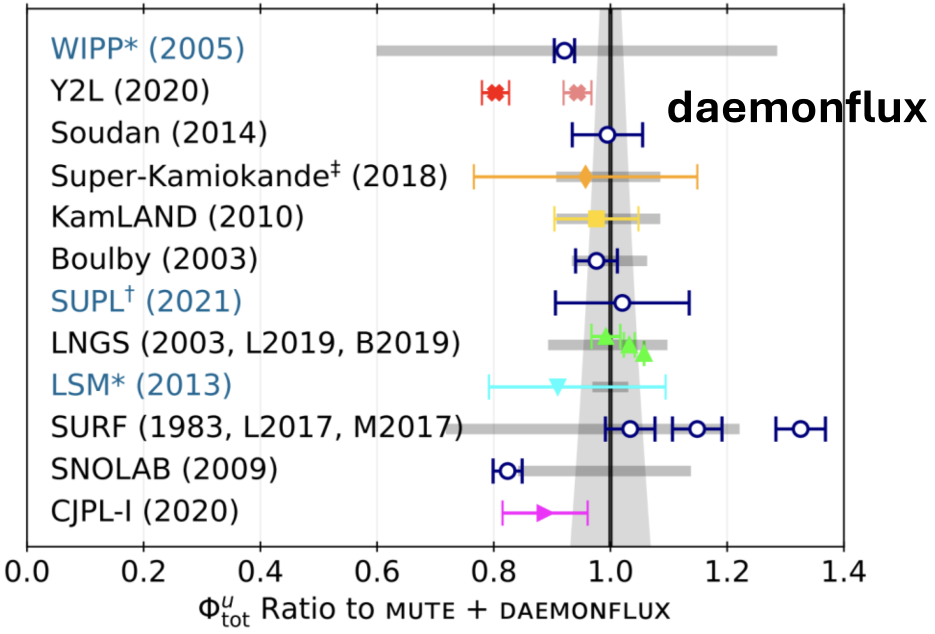
Studied problem in high detail

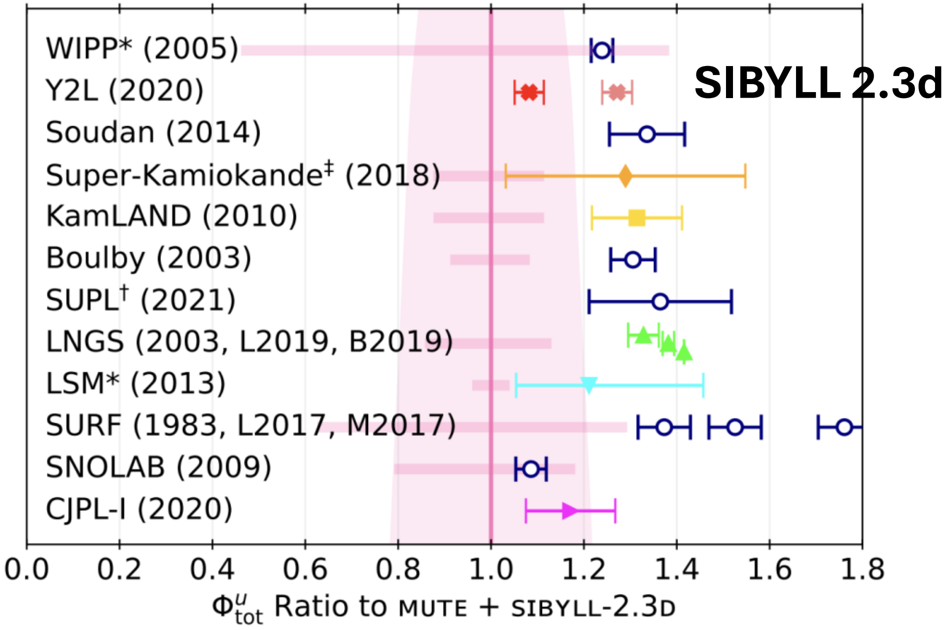


- Modeled topography above labs
- Modeled chemical rock composition → critical
- Elaborated a reduced list of experiments with systematic uncertainties, necessary conditions published, consistent errors and measurements
- All preparations made for next-gen "daemonflux" fit

***W. Woodley**, AF, M.-C. Piro., PRD 110, (2024)*

Relative difference to predicted total muon rate





Transport and cascading of particles

Equations for fluxes of particles of type *h* in the atmosphere:

Depth along CR trajectory *l:*

$$X(h_0) = \int_0^{h_0} \mathrm{d}\ell \; \rho_{\mathrm{air}}(\ell)$$

$$\frac{\mathrm{d}\Phi_h(E,X)}{\mathrm{d}X} =$$

- absorption by **interactions**

- absorption by **decays**

Depend on density or *X*

- ionization and radiation **losses**

+ particle production in **hadronic interactions**

+ particle production through decays

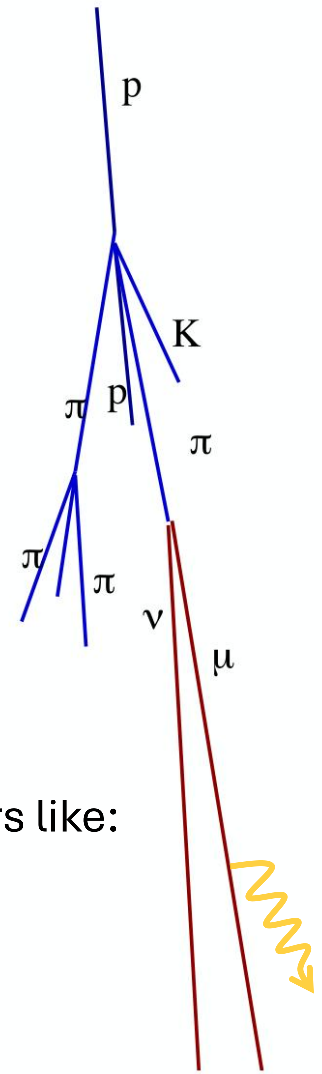
Coupling between particle types

!

Initial condition is the flux of cosmic ray nucleons at *X=0.*

Event generators like:

- SIBYLL
- DPMJET
- Pythia
- EPOS
- QGSJet



Transport and cascading of particles

Equations for fluxes of particles of type *h* in the atmosphere:

$$\frac{\mathrm{d}\Phi_h(E,X)}{\mathrm{d}X} = -\frac{\Phi_h(E,X)}{\lambda_{\mathrm{int},h}(E)} - \frac{\Phi_h(E,X)}{\lambda_{\mathrm{dec},h}(E,X)} - \frac{\partial}{\partial E}(\mu(E)\Phi_h(E,X)) + \sum_k \int_E^\infty \mathrm{d}E_k \, \frac{\mathrm{d}N_{k(E_k)\to h(E)}}{\mathrm{d}E} \frac{\Phi_k(E_k,X)}{\lambda_{\mathrm{int},k}(E_k)} + \sum_k \int_E^\infty \mathrm{d}E_k \, \frac{\mathrm{d}N^{\mathrm{dec}}_{k(E_k)\to h(E)}}{\mathrm{d}E} \frac{\Phi_k(E_k,X)}{\lambda_{\mathrm{dec},k}(E_k,X)}$$

Depend on density or *X*

Coupling between particle types

Depth along CR trajectory *l:*

$$X(h_0) = \int_0^{h_0} \mathrm{d}\ell \; \rho_{\mathrm{air}}(\ell)$$

Initial condition is the flux of cosmic ray nucleons at *X=0*.

!

Event generators like:

- SIBYLL
- DPMJET
- Pythia
- EPOS
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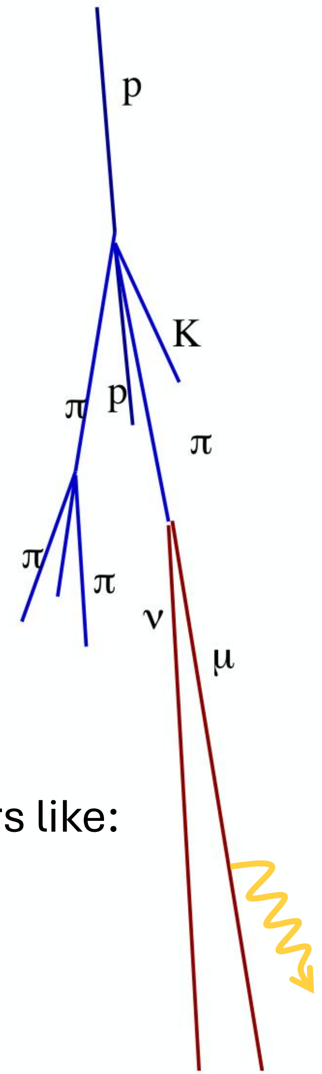


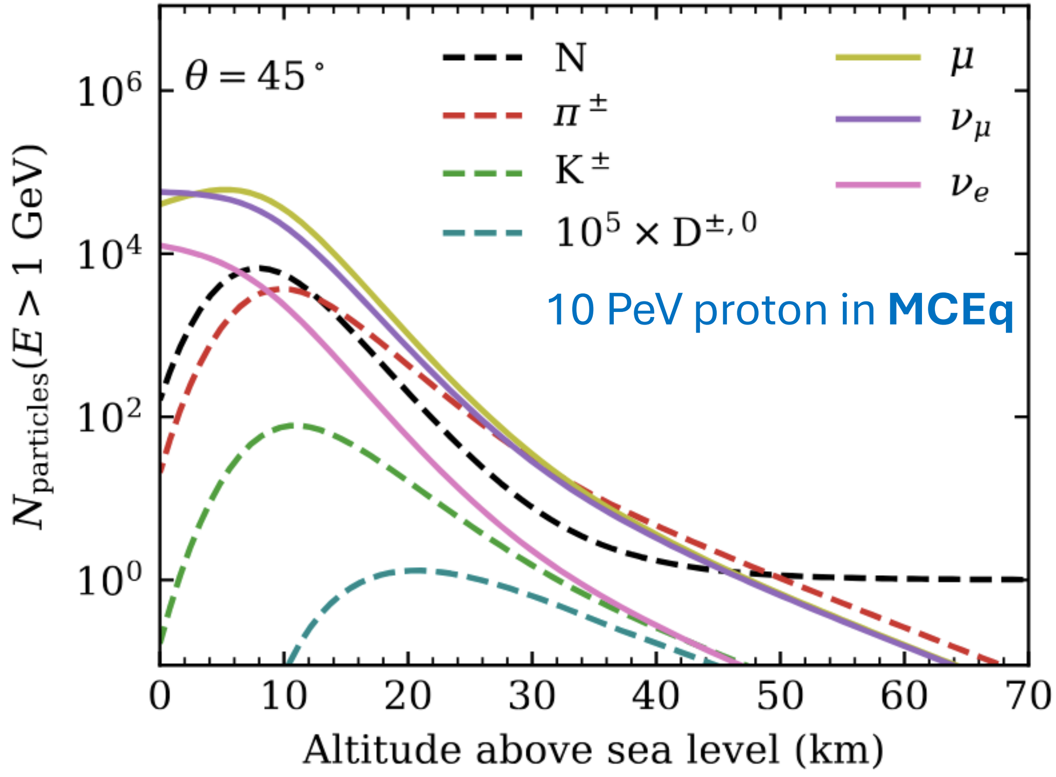
Transport and cascading of particles

Equations for fluxes of particles of type *h* in the atmosphere:

$$\frac{\mathrm{d}\Phi_h(E,X)}{\mathrm{d}X} = -\frac{\Phi_h(E,X)}{\lambda_{\mathrm{int},h}(E)} - \frac{\Phi_h(E,X)}{\lambda_{\mathrm{dec},h}(E,X)} - \frac{\partial}{\partial E}(\mu(E)\Phi_h(E,X)) + \sum_k \int_E^\infty \mathrm{d}E_k \frac{\mathrm{d}N_{k(E_k)\to h(E)}}{\mathrm{d}E} \frac{\Phi_k(E_k,X)}{\lambda_{\mathrm{int},k}(E_k)} + \sum_k \int_E^\infty \mathrm{d}E_k \frac{\mathrm{d}N^{\mathrm{dec}}_{k(E_k)\to h(E)}}{\mathrm{d}E} \frac{\Phi_k(E_k,X)}{\lambda_{\mathrm{dec},k}(E_k,X)}$$

Depend on density or *X*

Coupling between particle types



MCEq: Matrix Cascade Equations

1. Express integrals via midpoint rule as matrix-vector multiplication
2. Arrange all particles in a large, **sparse** matrix (like a state-space model in control theory)
3. Study stability and eigenvalues, deal with stiffness

$$\frac{\mathrm{d}\Phi^h_{E_i}}{\mathrm{d}X} = -\frac{\Phi^h_{E_i}}{\lambda^h_{\mathrm{int},E_i}} - \frac{\Phi^h_{E_i}}{\lambda^h_{\mathrm{dec},E_i}(X)} - \vec{\nabla}_i(\mu^h_{E_i}\Phi^h_{E_i}) + \sum_{E_k \geq E_i}^{E_N}\sum_{\ell}\frac{c_{\ell(E_k)\to h(E_i)}}{\lambda^\ell_{\mathrm{int},E_k}}\Phi^\ell_{E_k} + \sum_{E_k \geq E_i}^{E_N}\sum_{\ell}\frac{\mathrm{d}_{\ell(E_k)\to h(E_i)}}{\lambda^\ell_{\mathrm{dec},E_k}(X)}\Phi^\ell_{E_k}$$

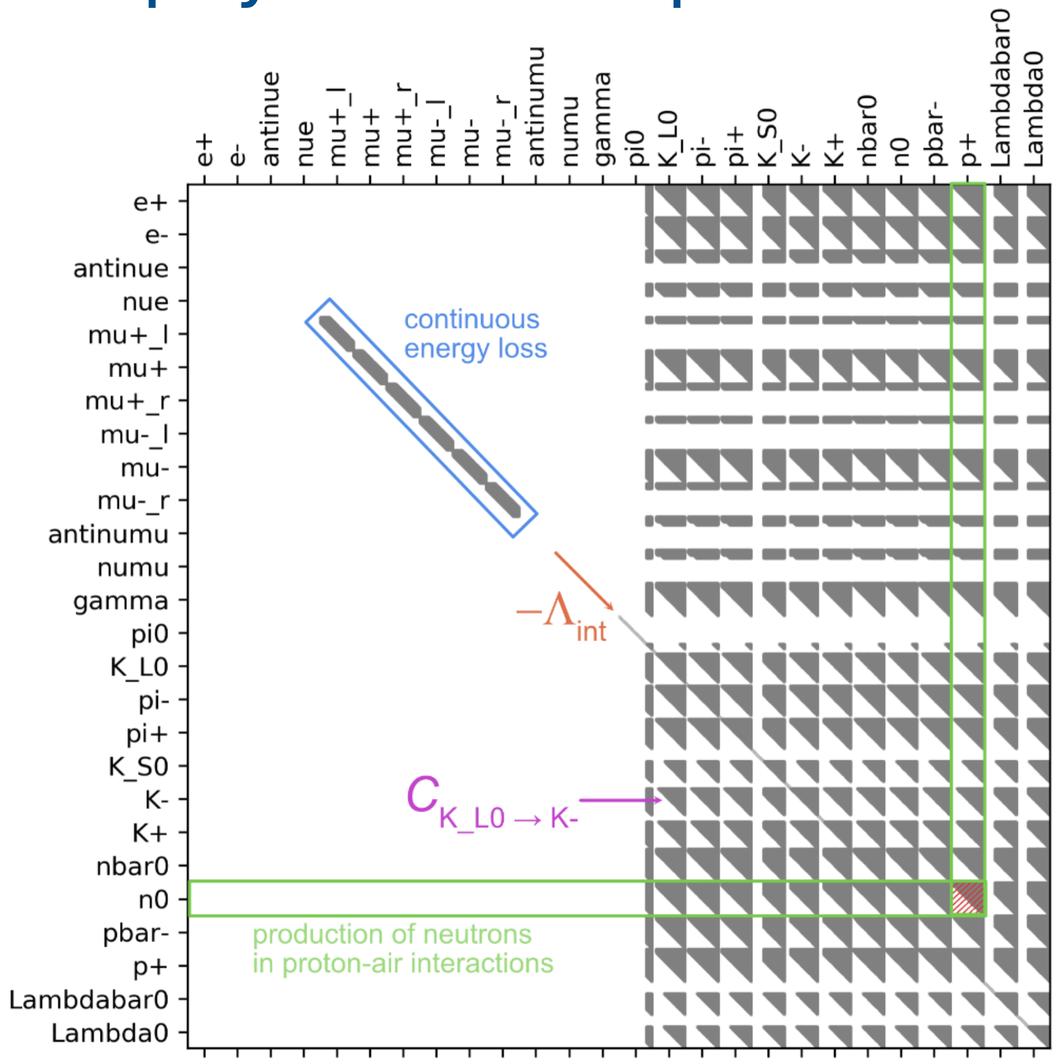
Depend on density or *X*

Coupling between particle types

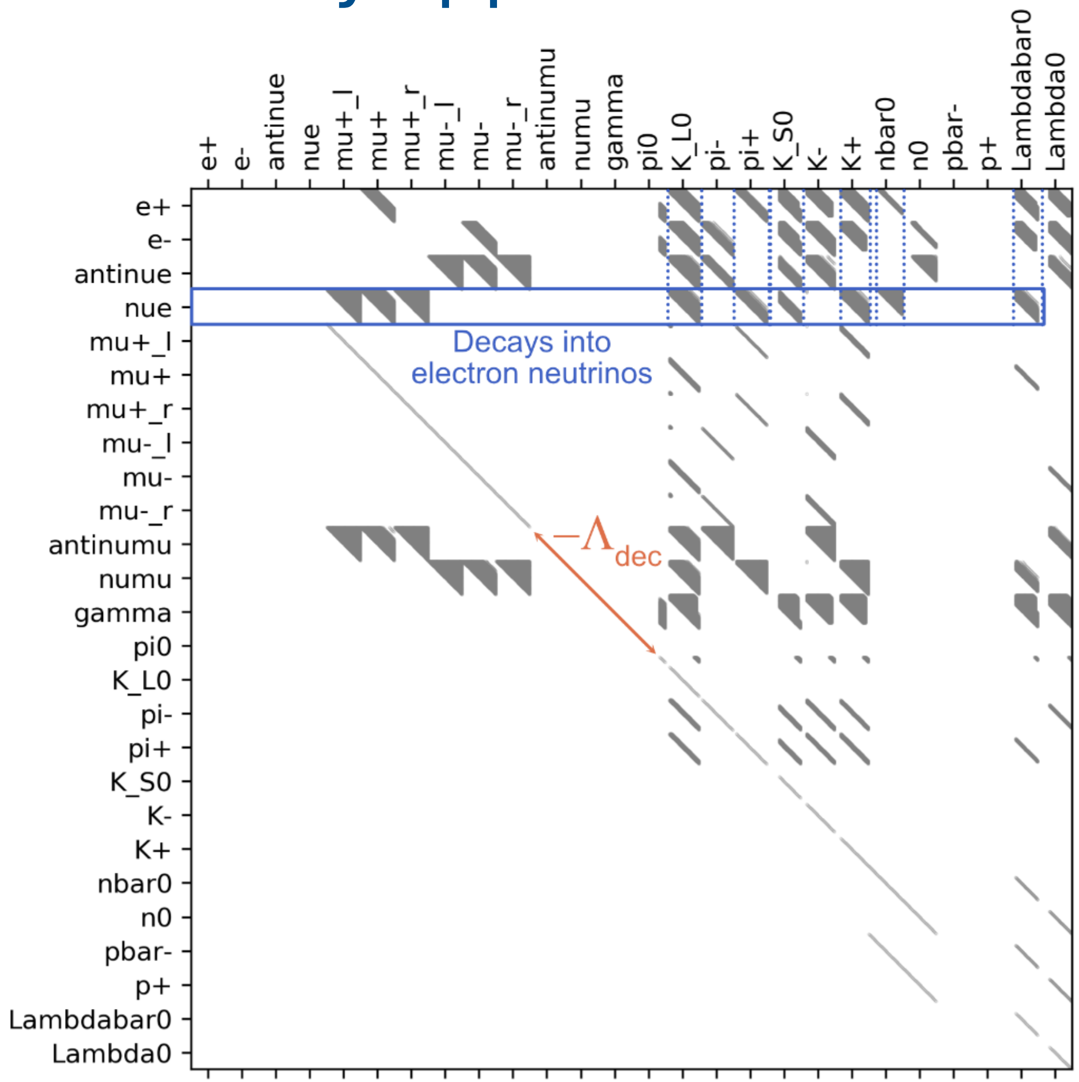
Rewrite as a simple matrix equation, implement using BLAS and solve iteratively

$$\frac{\mathrm{d}}{\mathrm{d}X}\vec{\Phi} = -\vec{\nabla}_E(\mathrm{diag}(\vec{\mu})\vec{\Phi}) + (-\mathbf{1} + \mathbf{C})\mathbf{\Lambda}_{\mathrm{int}}\vec{\Phi} + \frac{1}{\rho(X)}(-\mathbf{1} + \mathbf{D})\mathbf{\Lambda}_{\mathrm{dec}}\vec{\Phi}$$

The physics of the problem is immediately apparent

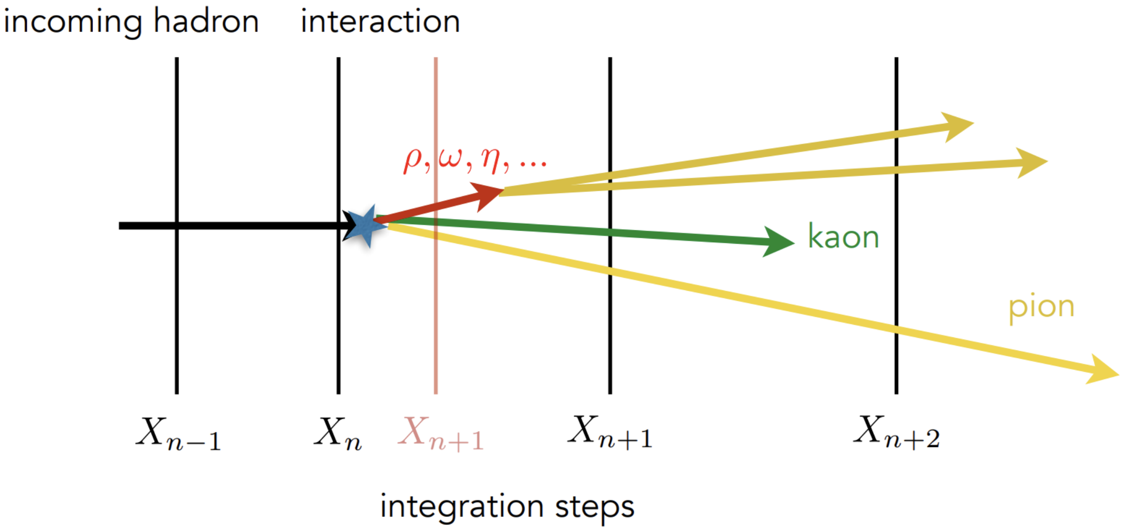


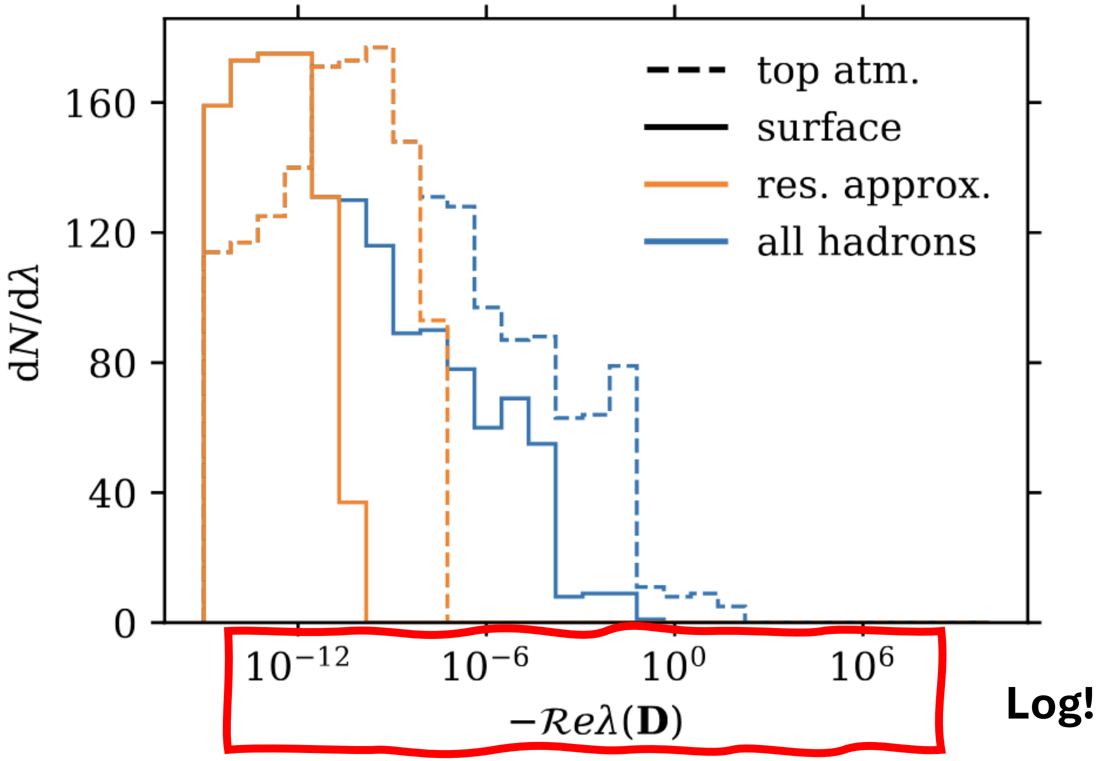
Interaction matrix *C*



Decay matrix *D*

Eigenvalue analysis → Eliminate stiffness → Resonance approx.





- Large negative eigenvalues (from decay of short-lived particles) → Solution attenuates too quickly
  → Instability and oscillations
- → Introduce **the Resonance Approximation** (semi-analytical extension)

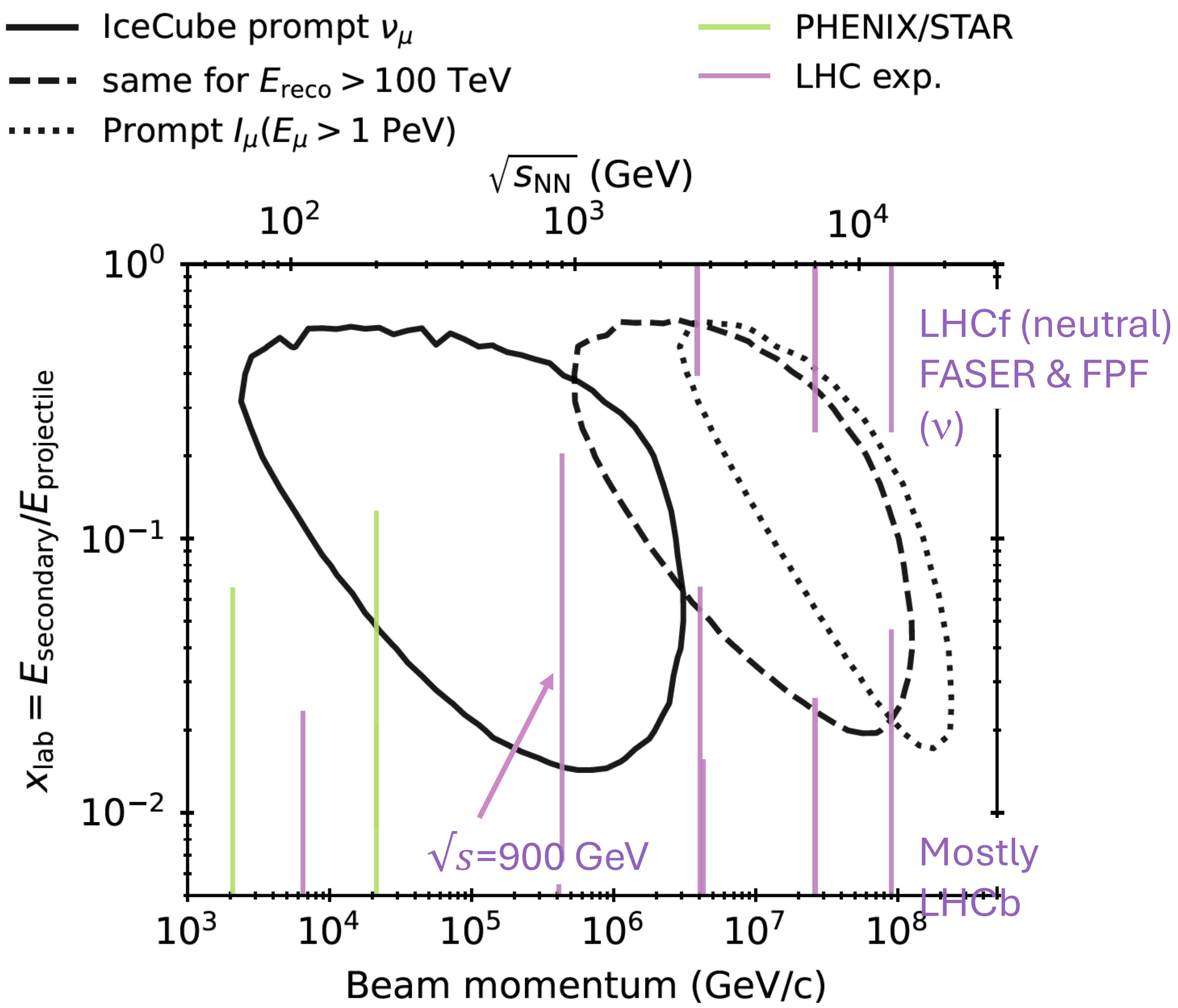
$$\vec{\Phi}^{\omega} = \left(\Phi^{\omega}_{E_0} \cdots \Phi^{\omega}_{E_i} \;\middle|\; \Phi^{\omega}_{E_{i+1}} \cdots \Phi^{\omega}_{E_N} \right)^T$$

$\lambda_{dec} < t_{mix}\lambda_{int}$	$\lambda_{dec} \geq t_{mix}\lambda_{int}$
$\Phi^{\omega}_{E_0} \cdots \Phi^{\omega}_{E_i}$ $\equiv 0$	$\Phi^{\omega}_{E_{i+1}} \cdots \Phi^{\omega}_{E_N}$
treat as resonance	transport as particle

Eigenvalues from the **diagonalization** of decay and interaction matrix

$$\vec{\Phi} = \sum_{i=1}^{n} c_i e^{\lambda_i^* X} \vec{\Psi}_i,$$

Charm production cross section inaccessible to present-day colliders

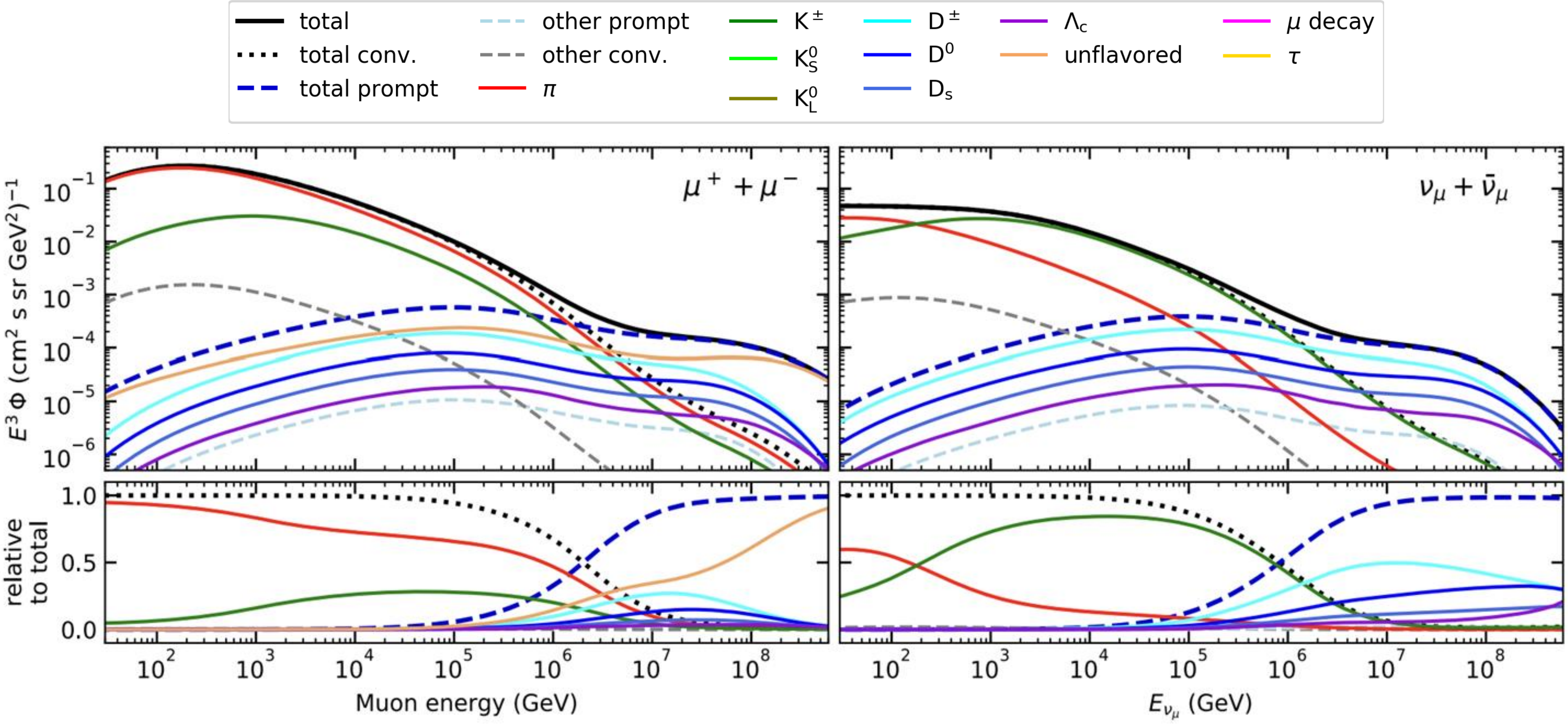


- Each line represents a collider running at fixed $\sqrt{s}$
- Gap in x between LHC coverage is due to the beam pipe
- Detectors need particle ID capability & sufficient luminosity
- Indirect constraints from new forward detectors like FASER and the proposed FPF (see 2203.05090)
- New insights expected from proton-oxygen collisions in Run3

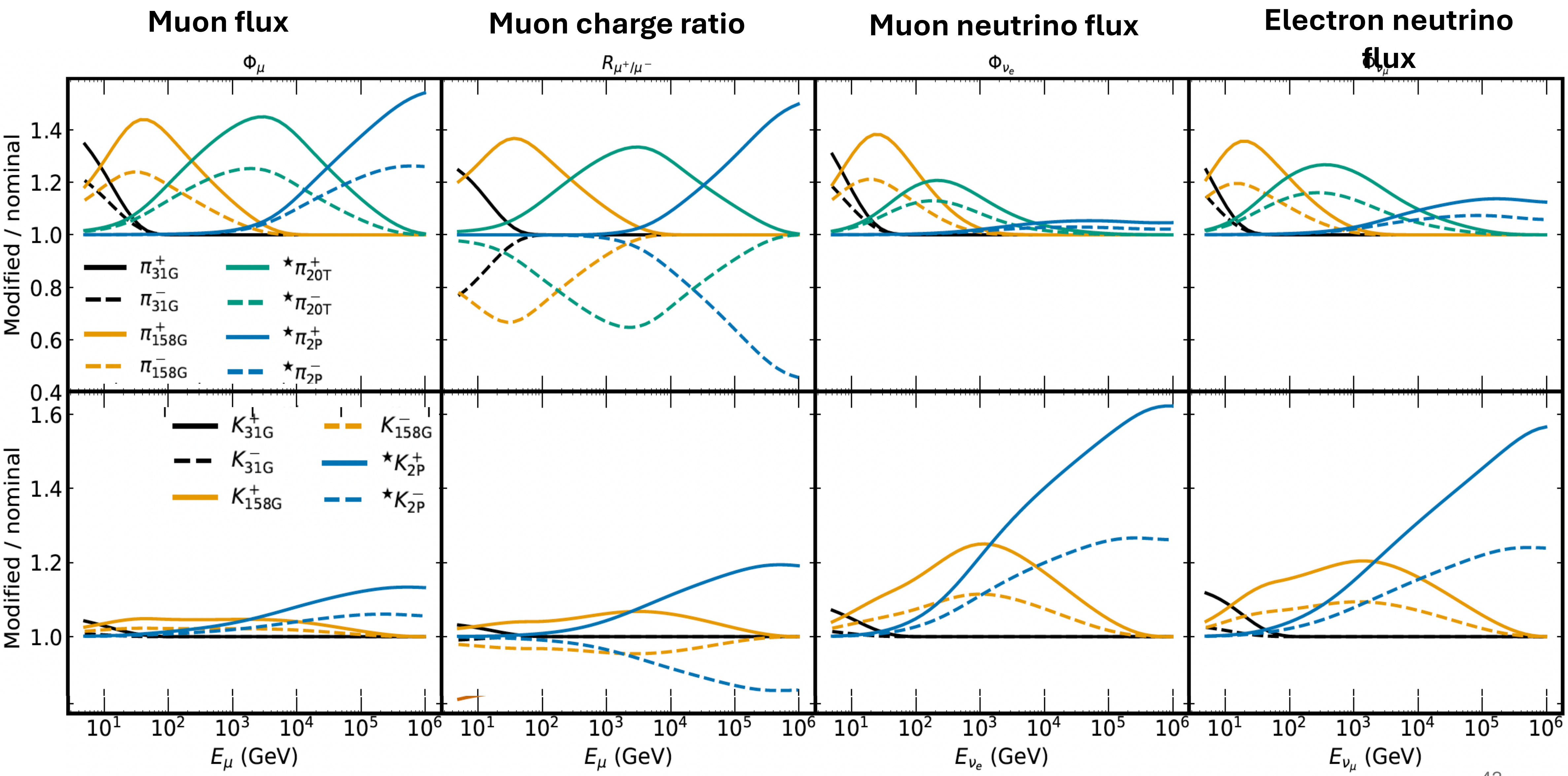
AF & M. Huber, arXiv:2205.14766

Neutrino contours: 90% events in IceCube

New level of detail: resolve the hadronic origin of atm. leptons

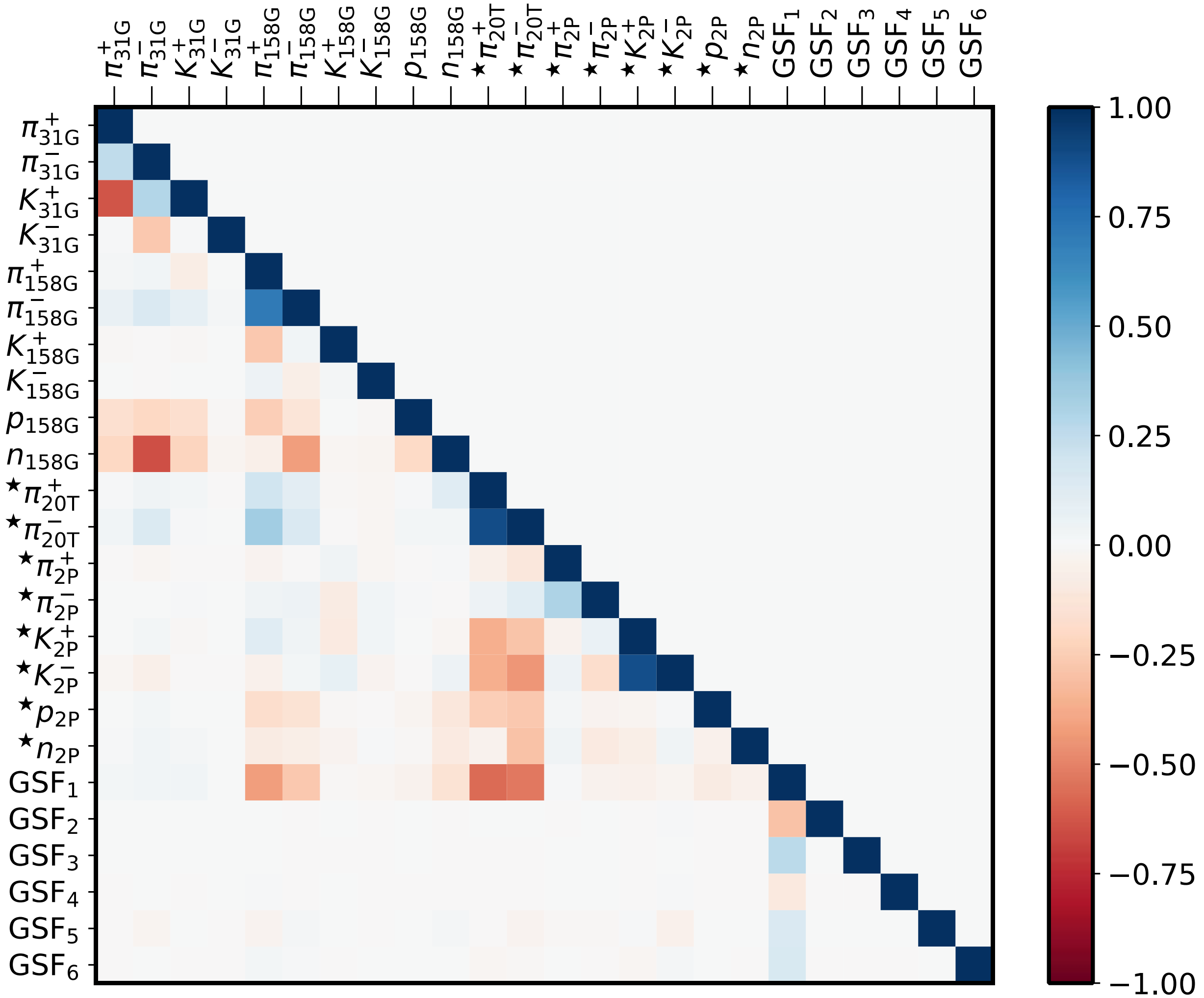


Gradients defined by parameters of DDM and GSF



J. P. Yanez & AF, arXiv:2303.00022

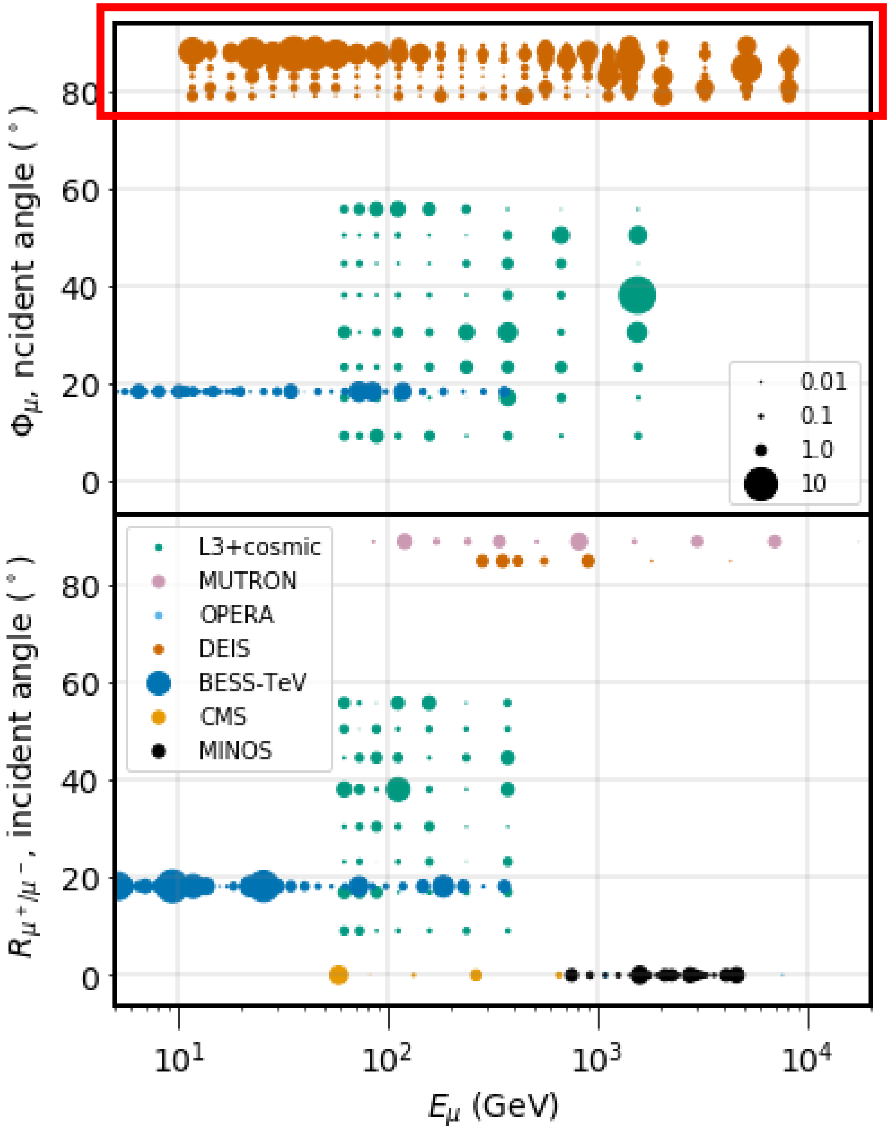
Fit quality



Physics parameter part of the correlation matrix:
Total 34 parameters: 18 hadrons + 6 GSF + 10 experimental

Contribution to Chi2

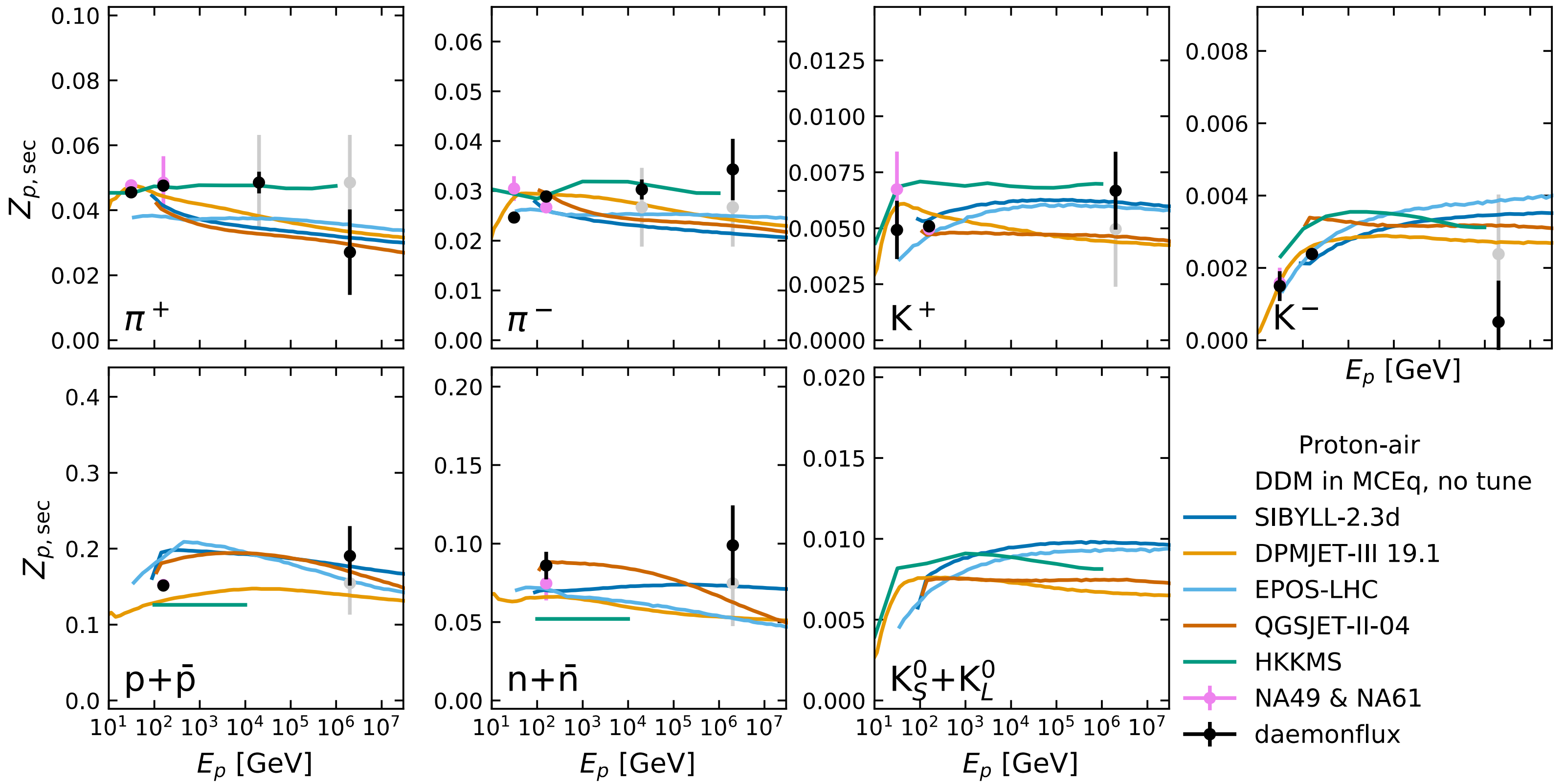
Not used!



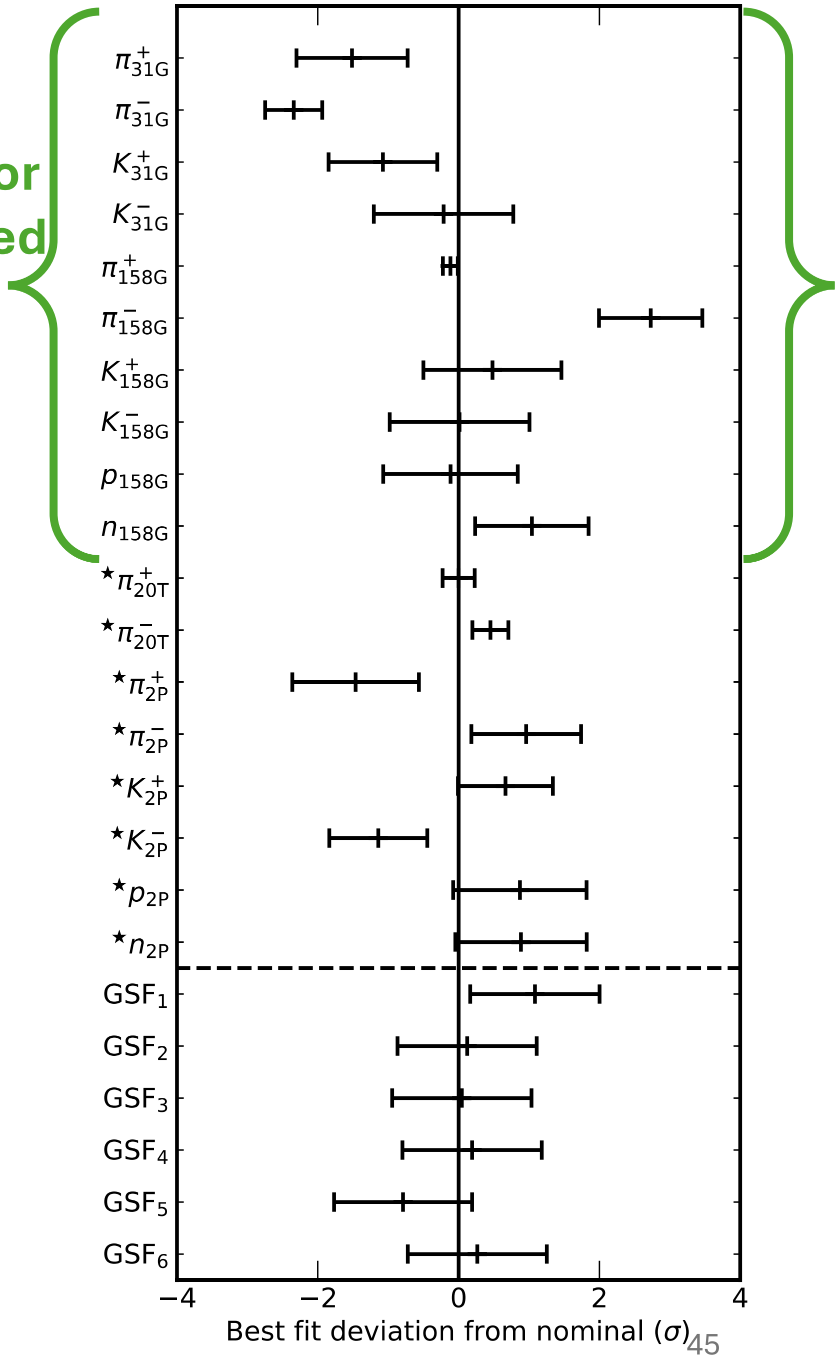
Chi^2 199/ 217 dof (approximate)
P-value = 81%

J. P. Yanez & AF, arXiv:2303.00022

Fitted parameter values



Accelerator constrained



J. P. Yanez & AF, arXiv:2303.00022

Global fit to recent, well documented CR measurements

Experiment	Detector type
ACE-CRIS [5, 6]	satellite
HEAO [7]	satellite
PAMELA [8, 9]	satellite
AMS-02 [10–13]	satellite
CALET [14–19]	satellite
DAMPE [20–23]	satellite
ISS-CREAM [24]	satellite
NUCLEON-KLEM [25]	satellite
GRAPES-3 [26]	surface array
H.E.S.S. [27]	Cherenkov telescope
VERITAS [28]	Cherenkov telescope
HAWC [29, 30]	surface array
LHAASO [31, 32]	optical + surface
IceCube [33, 34]	surface array
Tunka [35, 36]	optical
KASCADE-Grande [37]	surface array
TA [38, 39]	optical + surface
Auger [40–45]	optical + surface

New or updated data set

Hadronic interaction models:
average used, sys. uncertainty from envelope

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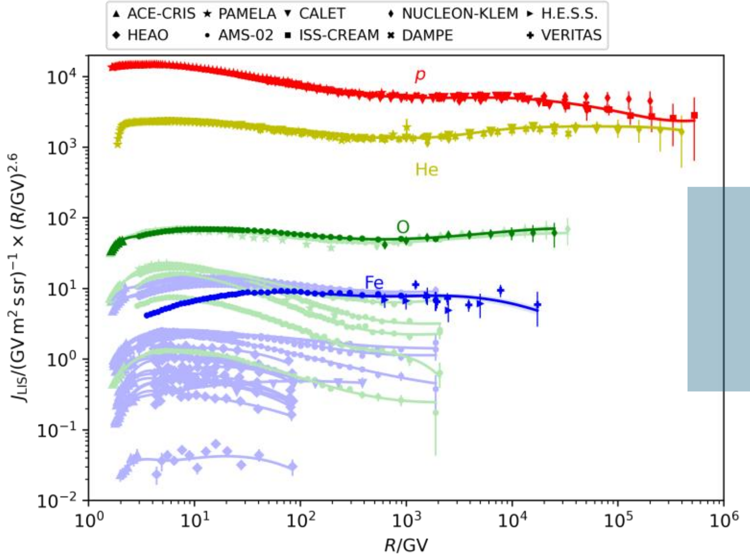
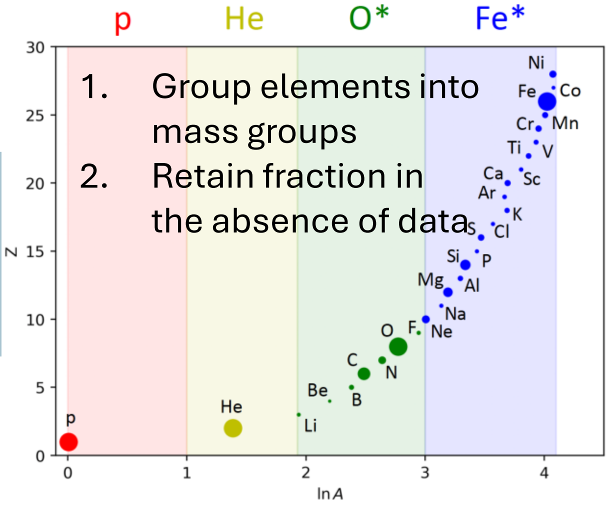
The Global Spline Fit (GSF) – bridging two experimental worlds

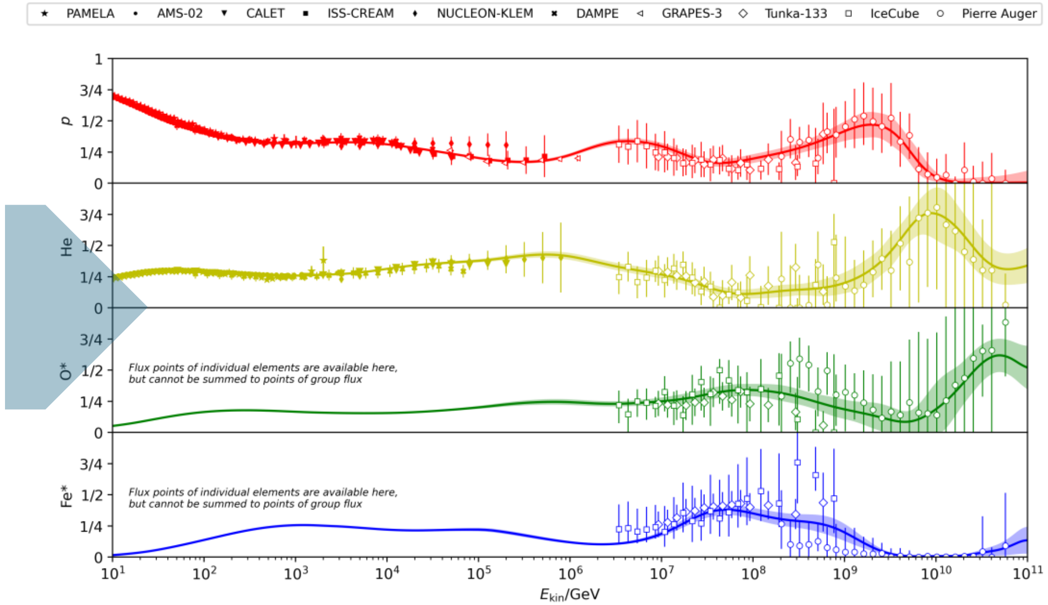
Direct experiments measure elements

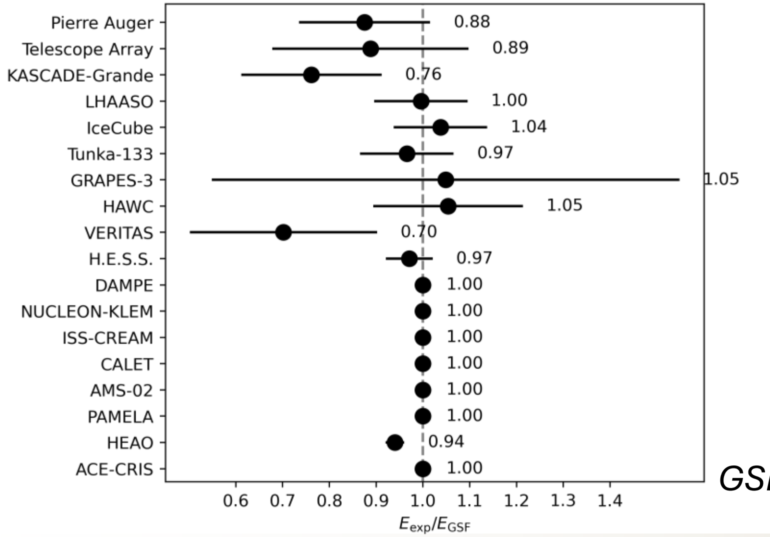




1. Group elements into mass groups
2. Retain fraction in the absence of data

Indirect experiments measure mass groups



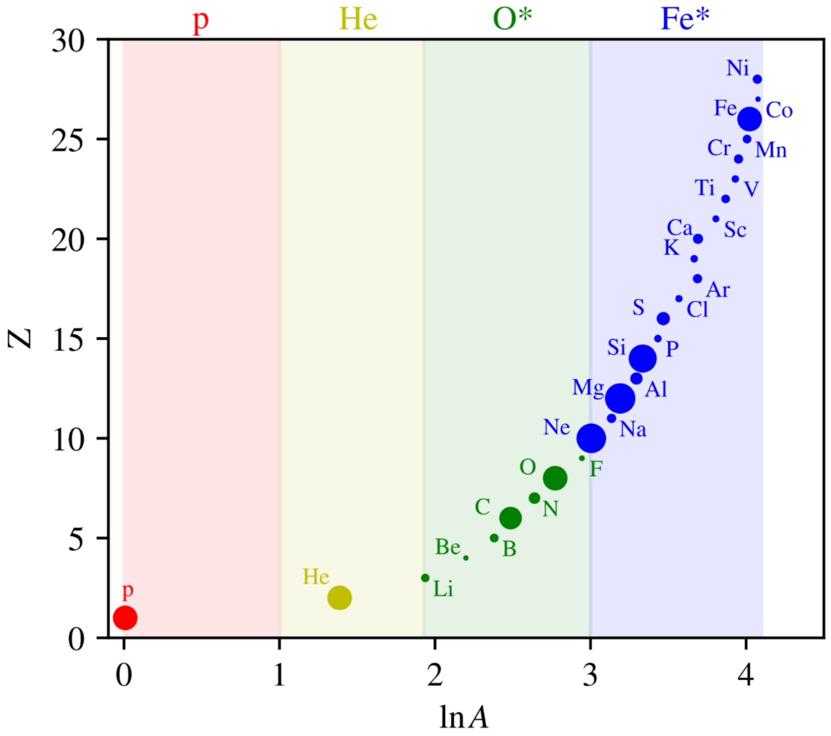


- **Motivation**: represent observations incl. uncertainties
- The project started in 2016 and was implemented by Hans Dembinski (based on his Auger-era codes)
- ICRC2017 proc. has 159 citations by now
- Kozo Fujisue continues the work on GSF since 2024

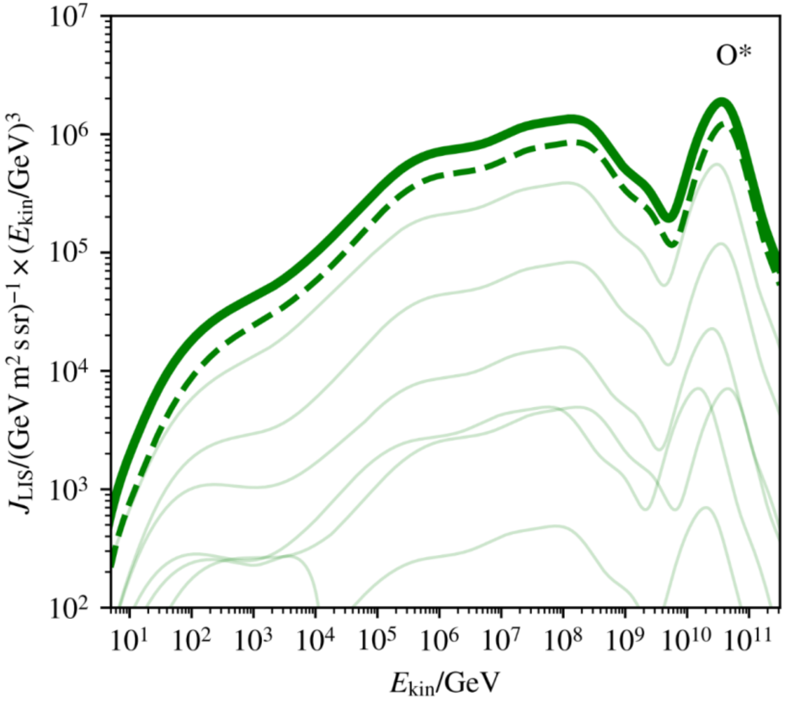
GSF2024: Fujisue, Dembinski, AF, Engel, UHECR 2024

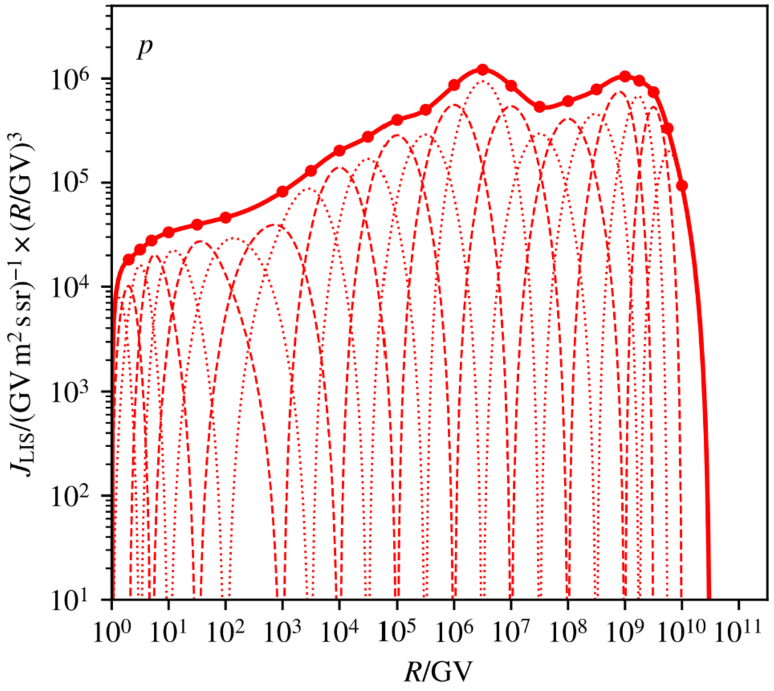
Fit 4 mass groups

2025



Oxygen mass group O*



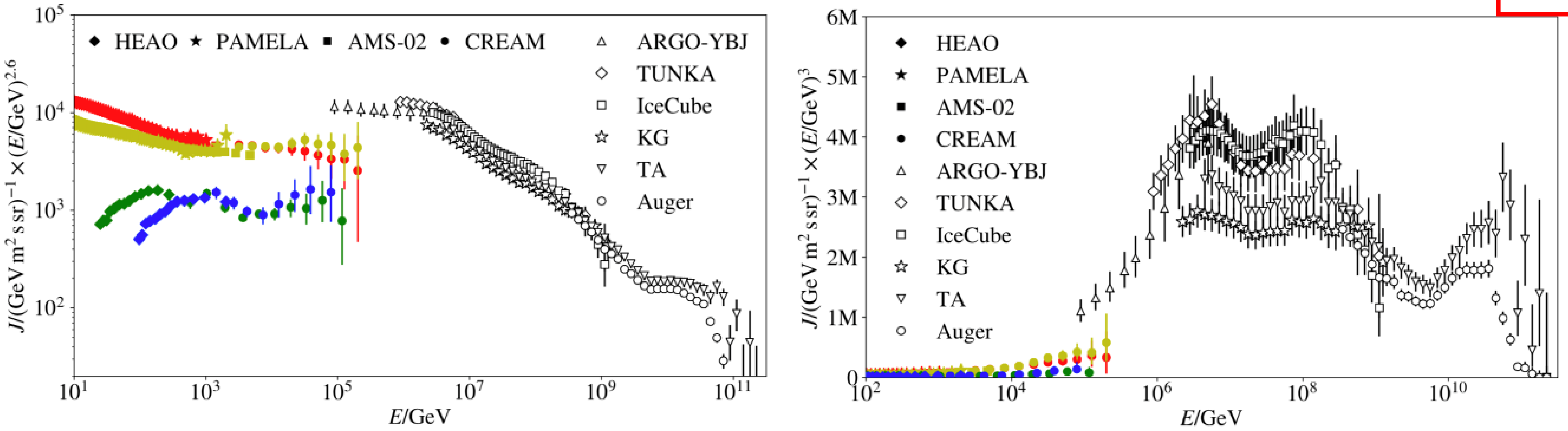


- Uses B-splines to fit **four** mass groups from GeV to 100 EeV
- Interpolates direct satellite/balloon element data at low energies
- Fits mass groups to indirect experimental data
- Takes into account systematics → exclusively uses experiments with systematics

Energy scale systematics

Original data

2017



- The determination of **energy scale in air-shower experiments is uncertain**
- This is caused by inconsistencies of **hadronic interaction models and reconstruction methods**
- **Fit** each experiment's energy scale **using native uncertainty estimate as penalty/prior**
- Sum **remaining systematic** uncertainty in quadrature with stats

$$\tilde{J}(\tilde{E}) = J(E)\frac{\mathrm{d}E}{\mathrm{d}\tilde{E}} = J\left(\frac{\tilde{E}}{1+z_E}\right)\frac{1}{1+z_E}$$

Flux distortion caused by energy-scale offset z_E

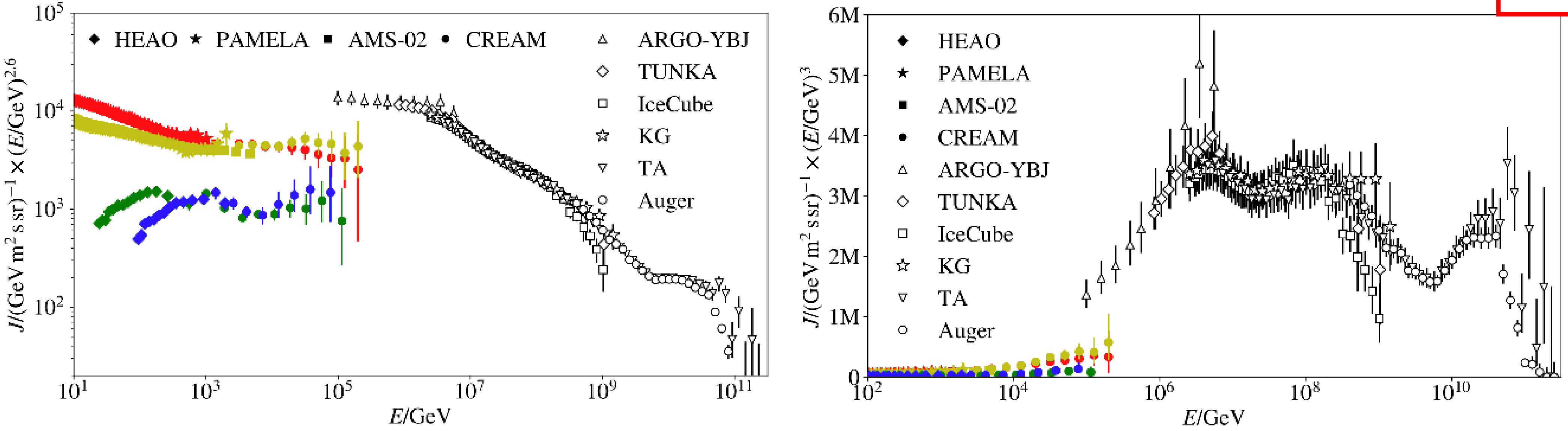
$$S = \sum_i z_i^2 + \sum_j \left(\frac{z_{Ej}}{(\sigma[E]/E)_j}\right)^2$$

Flux residuals Energy-scale offset residuals

Combined fit to all-particle, mass group flux and energy scale

Adjusted data

2017



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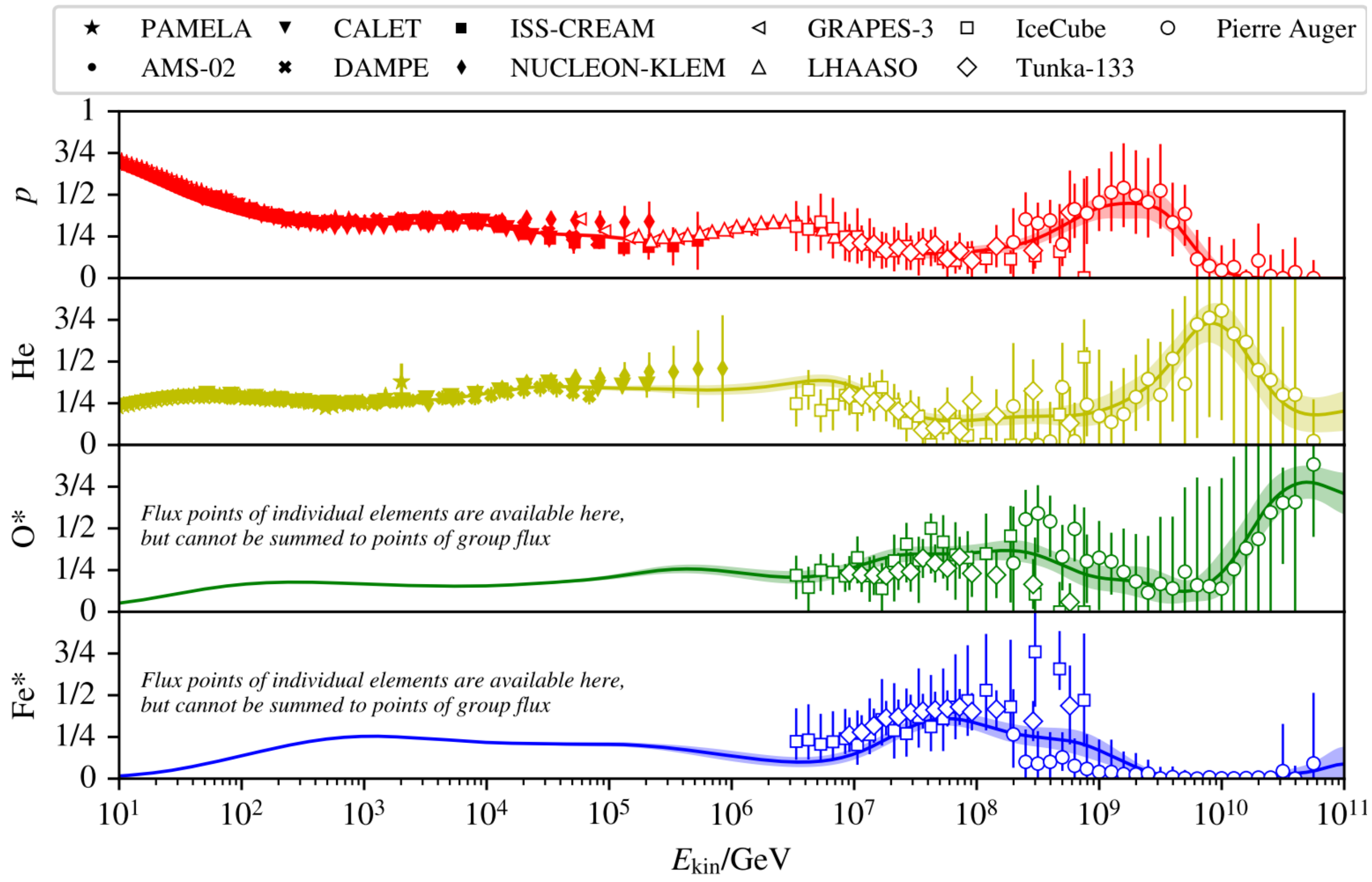
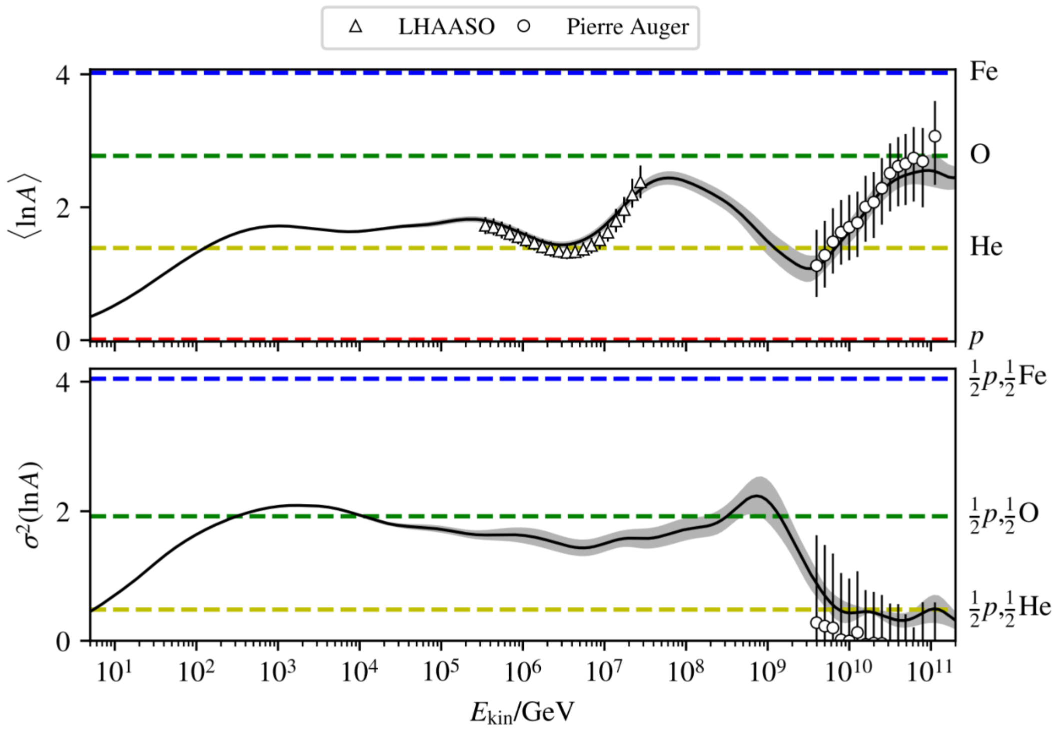
Flux residuals Energy-scale offset residuals

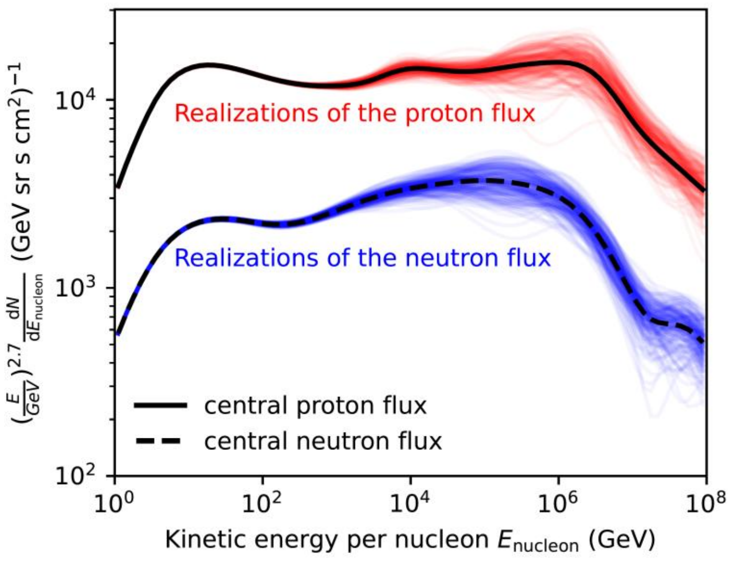
GSF 2025: mass composition

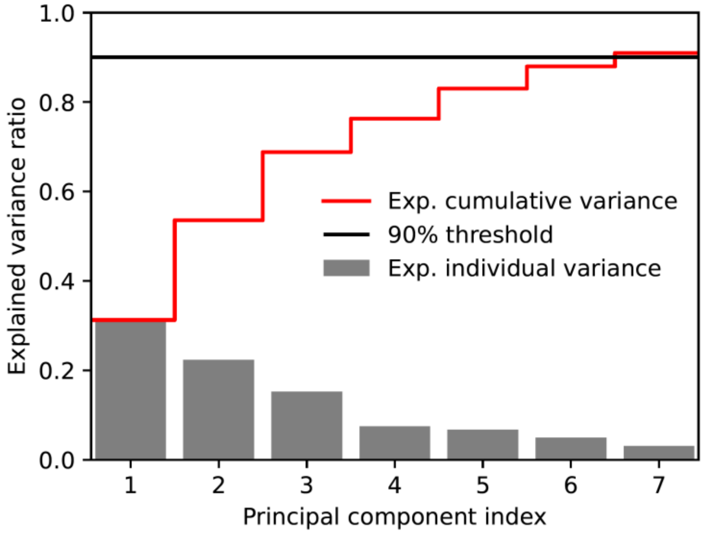


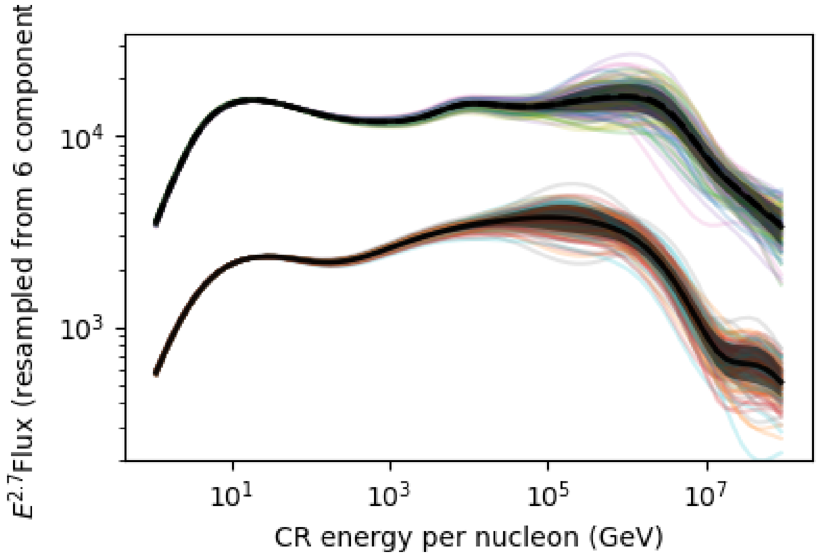


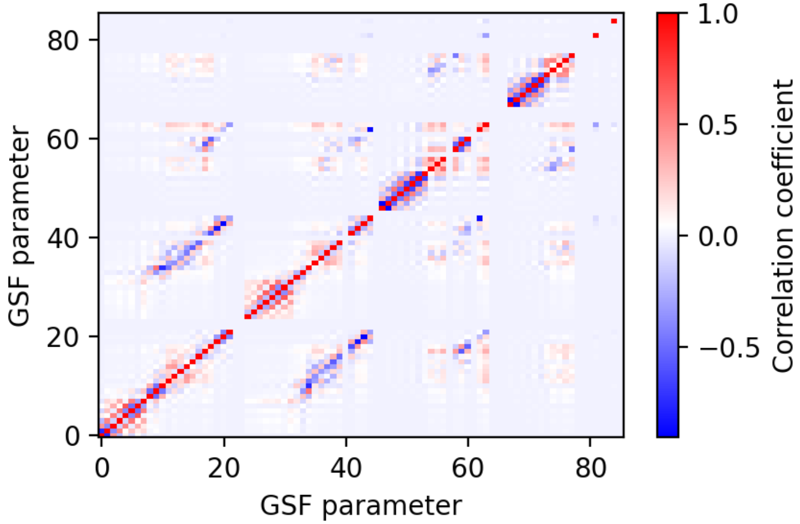
- lnA information is now fully used, LHAASO, Auger SD
- Tests showed that just lnA moments are insufficient to fit 4 components

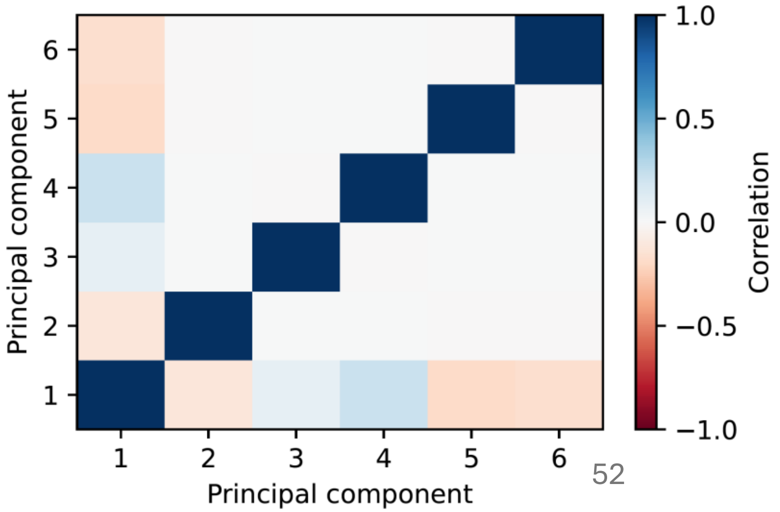
Dimensionality reduction to ~6 parameters





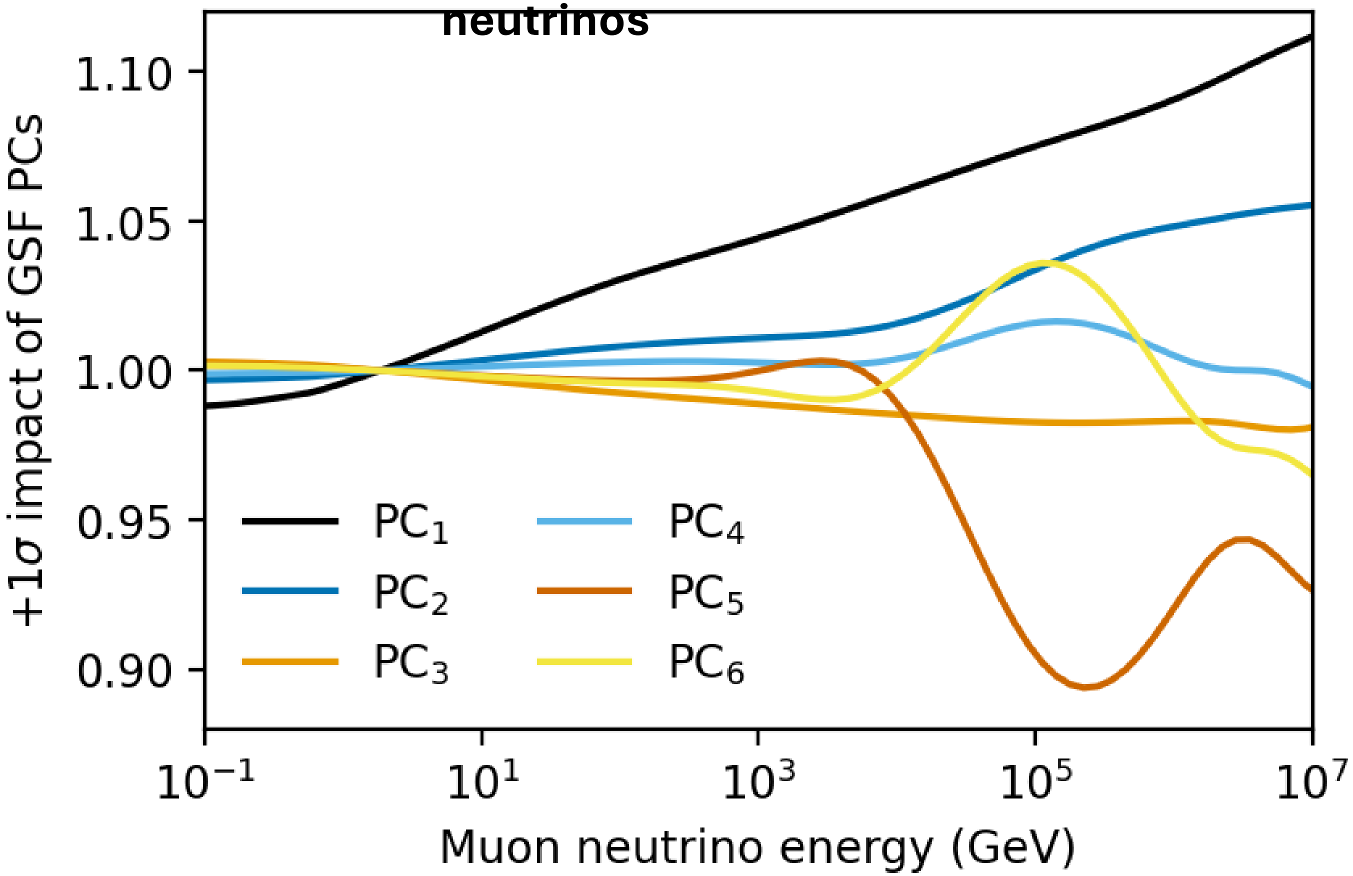






6 Principal components of CR nucleon fluxes

zenith-averaged muon neutrinos



- Component 1 is a "global" spectral index correction
- Sum of components can reproduce 90% allowed shapes from the 1-sigma range of GSF

→ 6 simple, data-motivated nuisance parameters for systematics calculations

→ **GSF2025 may need fewer than 6 parameters**