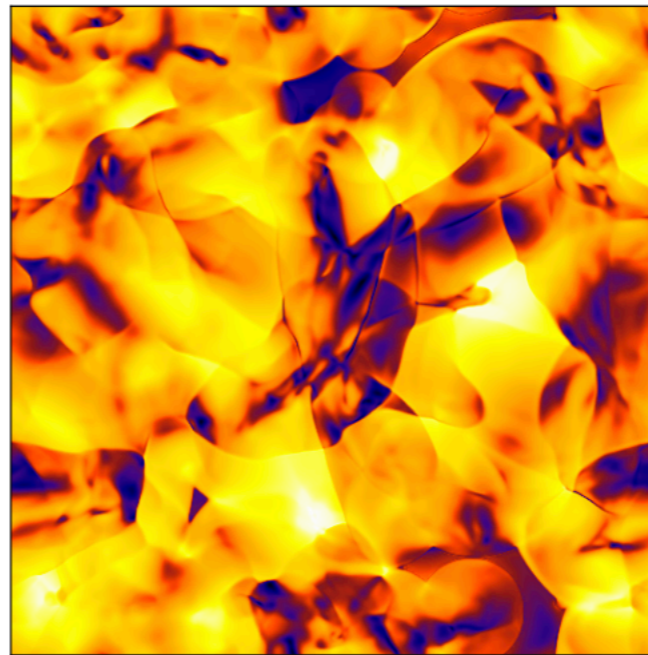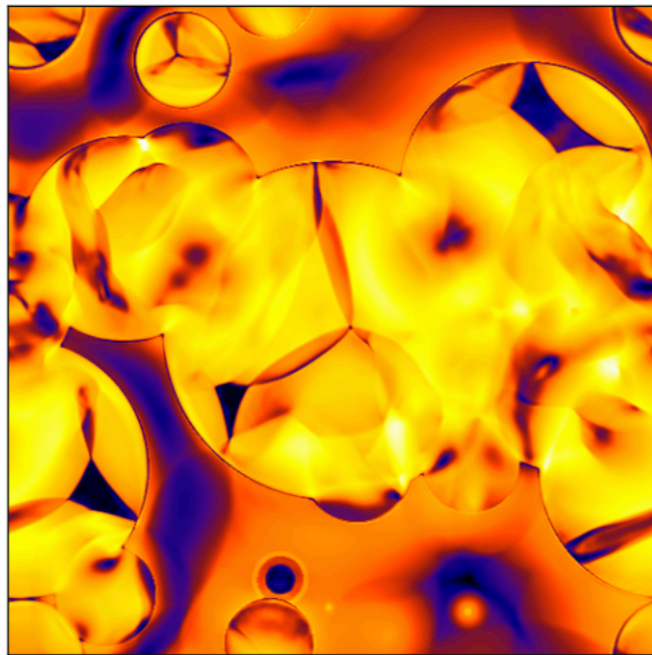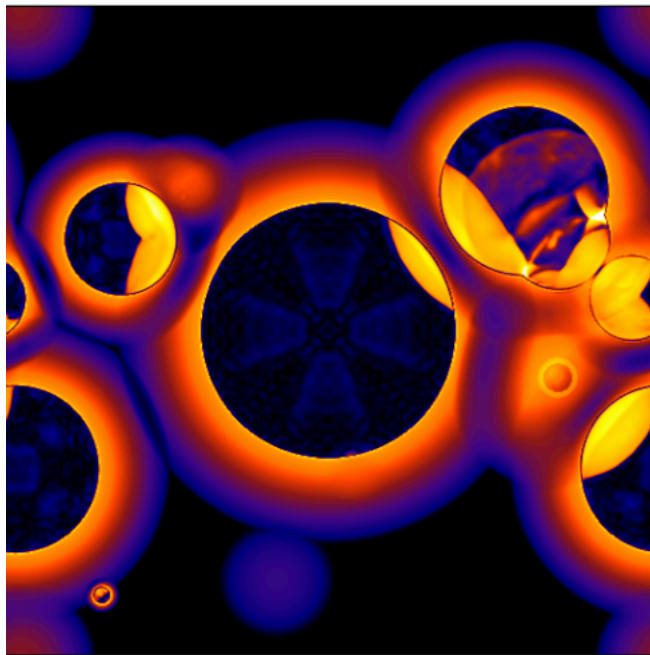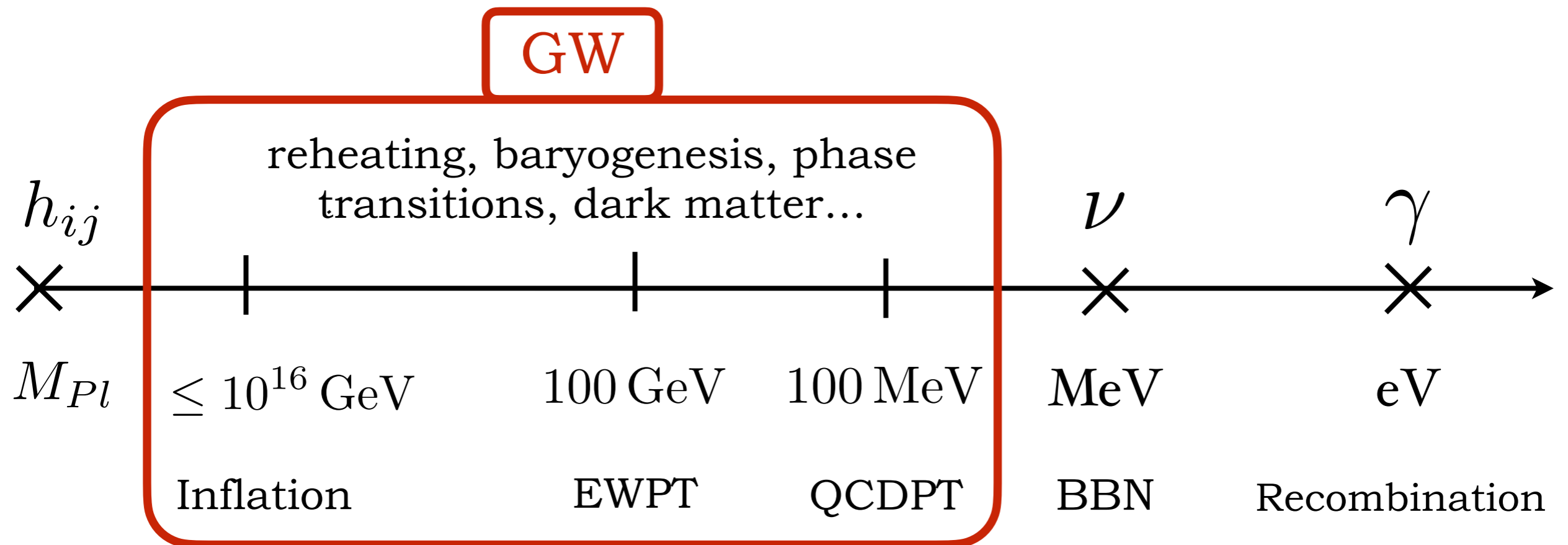# Stochastic gravitational waves backgrounds

Chiara Caprini
(CERN & University of Geneva)
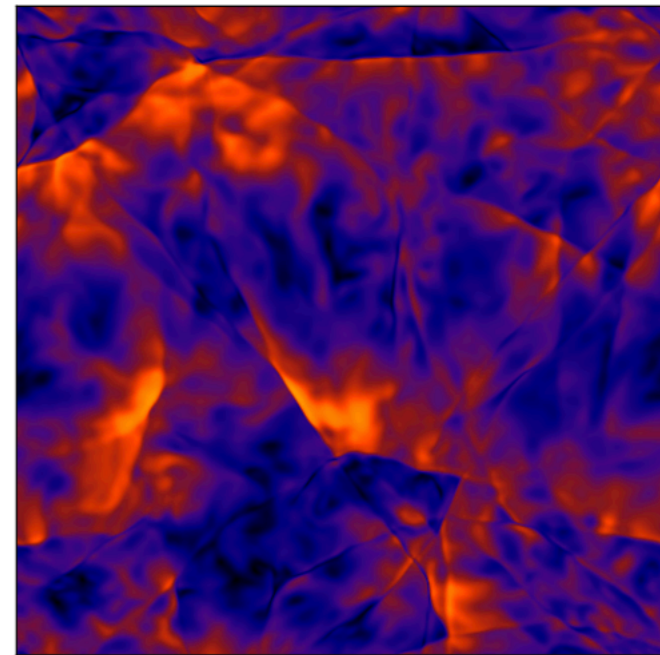
# GWs from the early universe: motivation

- GW emission processes in the early universe form a fossil radiation, whose detection would bring *direct information from very early stages of the universe evolution,* to which we have no access through em radiation

- amazing discovery potential, linked to high energy physics. Which energy scales can one access?

GW

reheating, baryogenesis, phase transitions, dark matter…

$h_{ij}$

$\nu$

$\gamma$

$M_{Pl}$  $\leq 10^{16}\,\text{GeV}$  $100\,\text{GeV}$  $100\,\text{MeV}$  MeV  eV

Inflation  EWPT  QCDPT  BBN  Recombination
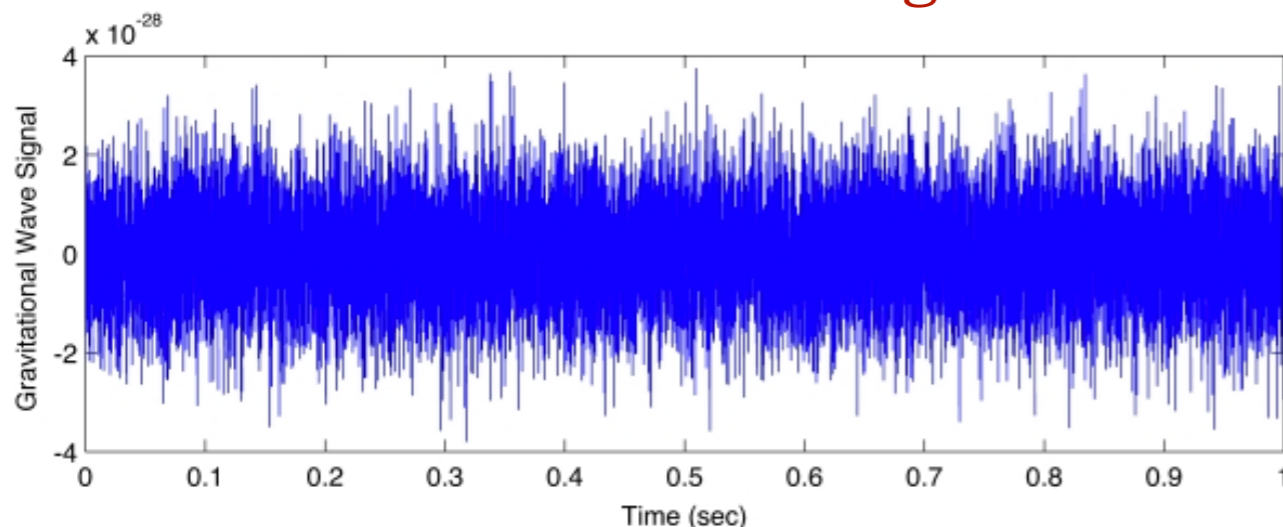
# GWs from the early universe: motivation

Suppose the GW generating process has a *characteristic time and length scale*

$$\ell_* \leq \tau_* \leq H_*^{-1} \quad \blacktriangleright \quad \text{GW frequency} \quad f_* \sim \frac{1}{\ell_*} \geq H_*$$

causal horizon at
generation time

Consequence: GW signals from the primordial universe have *too small correlation scale with respect to the detector resolution* -> only the statistical properties of the signal can be accessed, $h_{ij}(\mathbf{x},t)$ must be treated as a *random variable*

Stochastic GW background



Angular size of the
full sky: 360°

$\ell_{H_0}^2$

$\ell^2(T_*)$

Angular size of a
correlated region for
$T_* \sim 100$ GeV: $10^{-12°}$

# GWs from the early universe: motivation

Suppose the GW generating process has a *characteristic time and length scale*

$$\ell_* \leq \tau_* \leq H_*^{-1} \quad \blacktriangleright \quad \text{GW frequency} \quad f_* \sim \frac{1}{\ell_*} \geq H_*$$
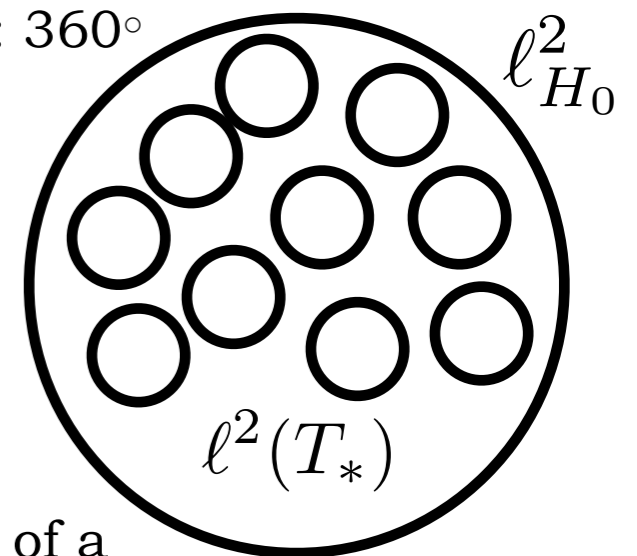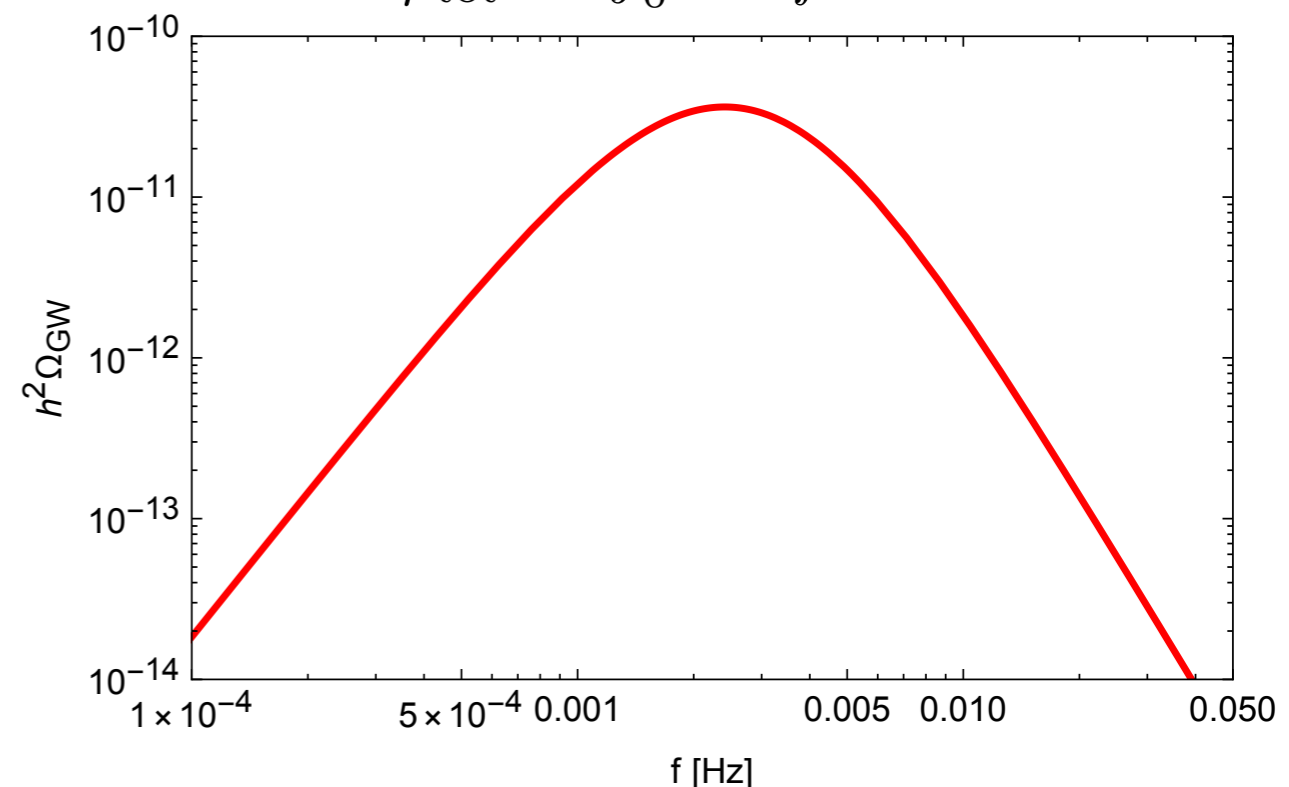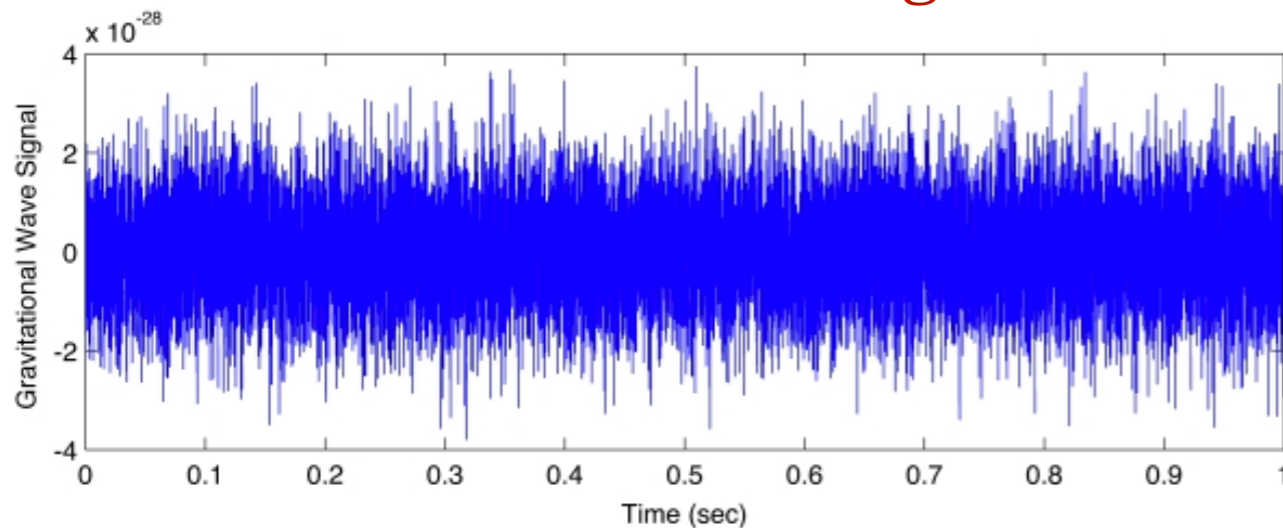
causal horizon at
generation time

Consequence: GW signals from the primordial universe have *too small correlation scale with respect to the detector resolution* -> only the statistical properties of the signal can be accessed -> GW energy power spectrum

$$\frac{\rho_{\text{GW}}}{\rho_{\text{tot}}} = \int_0^{+\infty} \frac{df}{f}\, \Omega_{\text{GW}}(f)$$

Stochastic GW background

# GWs from the early universe: motivation

Suppose the GW generating process has a *characteristic time and length scale*

$$\ell_* \leq \tau_* \leq H_*^{-1} \qquad \blacktriangleright \qquad \text{GW frequency} \quad f_* \sim \frac{1}{\ell_*} \geq H_*$$

GW frequency redshifted to today:
(source operating during the radiation dominated era)

$$f = f_* \frac{a_*}{a_0} = \frac{1.65 \times 10^{-5}}{\ell_* H_*} \left( \frac{g(T_*)}{100} \right)^{1/6} \frac{T_*}{100 \text{GeV}} \text{ Hz}$$



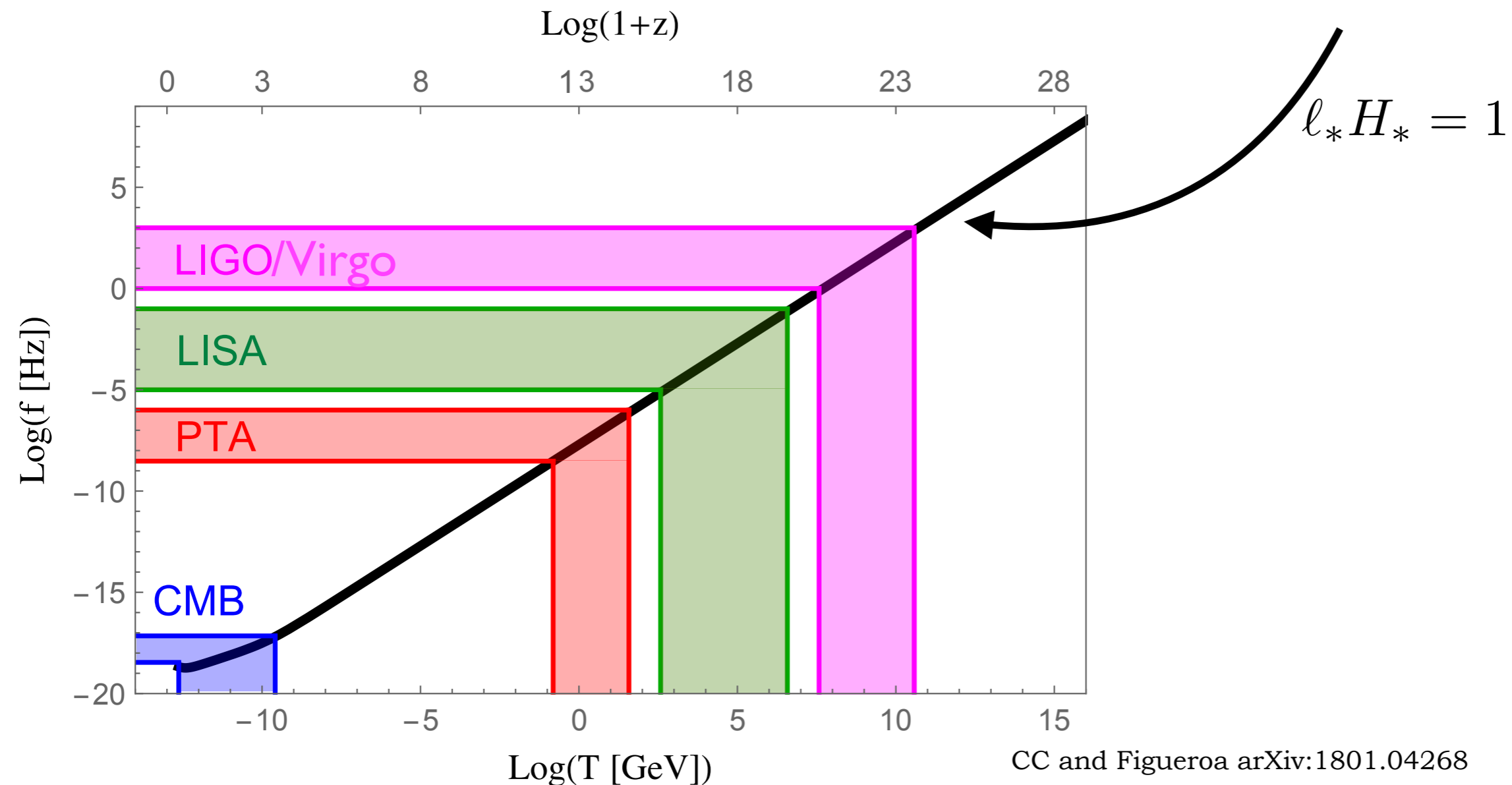CC and Figueroa arXiv:1801.04268

# GWs from the early universe: motivation

Suppose the GW generating process has a *characteristic time and length scale*

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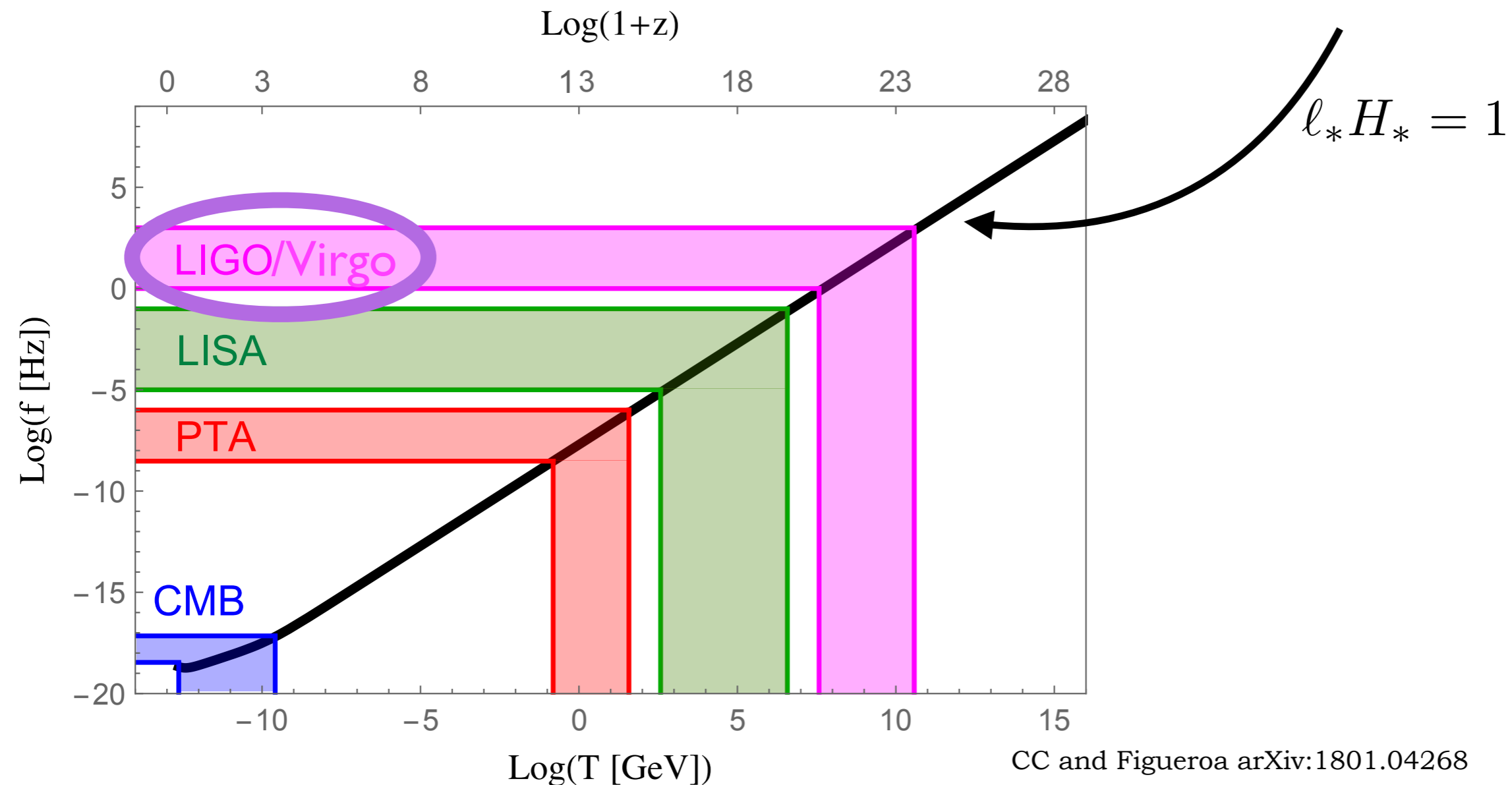(source operating during the radiation dominated era)

$$f = f_* \frac{a_*}{a_0} = \frac{1.65 \times 10^{-5}}{\ell_* H_*} \left( \frac{g(T_*)}{100} \right)^{1/6} \frac{T_*}{100 \text{GeV}} \text{Hz}$$

$\ell_* H_* = 1$



CC and Figueroa arXiv:1801.04268

# GWs from the early universe: motivation

## LIGO Virgo Kagra: operating ground based interferometers

### Timeline:

- Most probably no cosmological SGWB detection, masked by astrophysical foreground detection expected for ~2030

- **~2035** 3rd generation projects: Einstein Telescope and Cosmic Explorer, factor 20 improvement in sensitivity
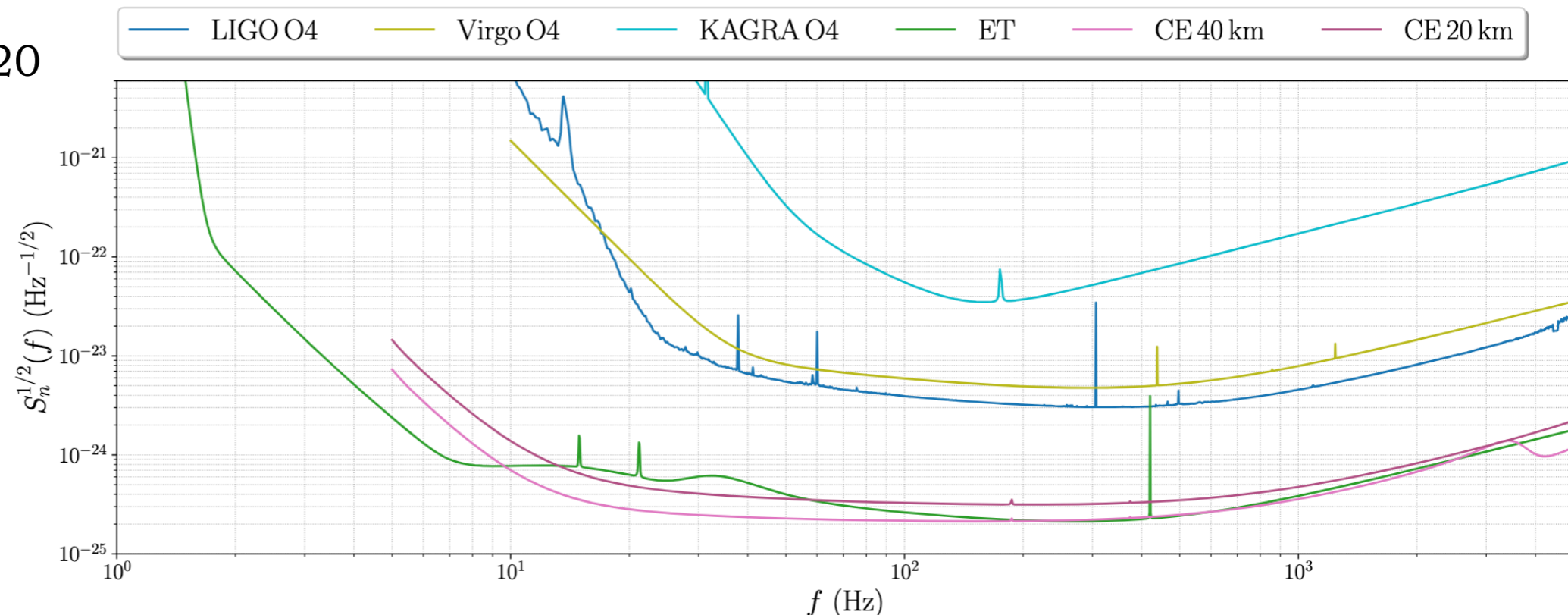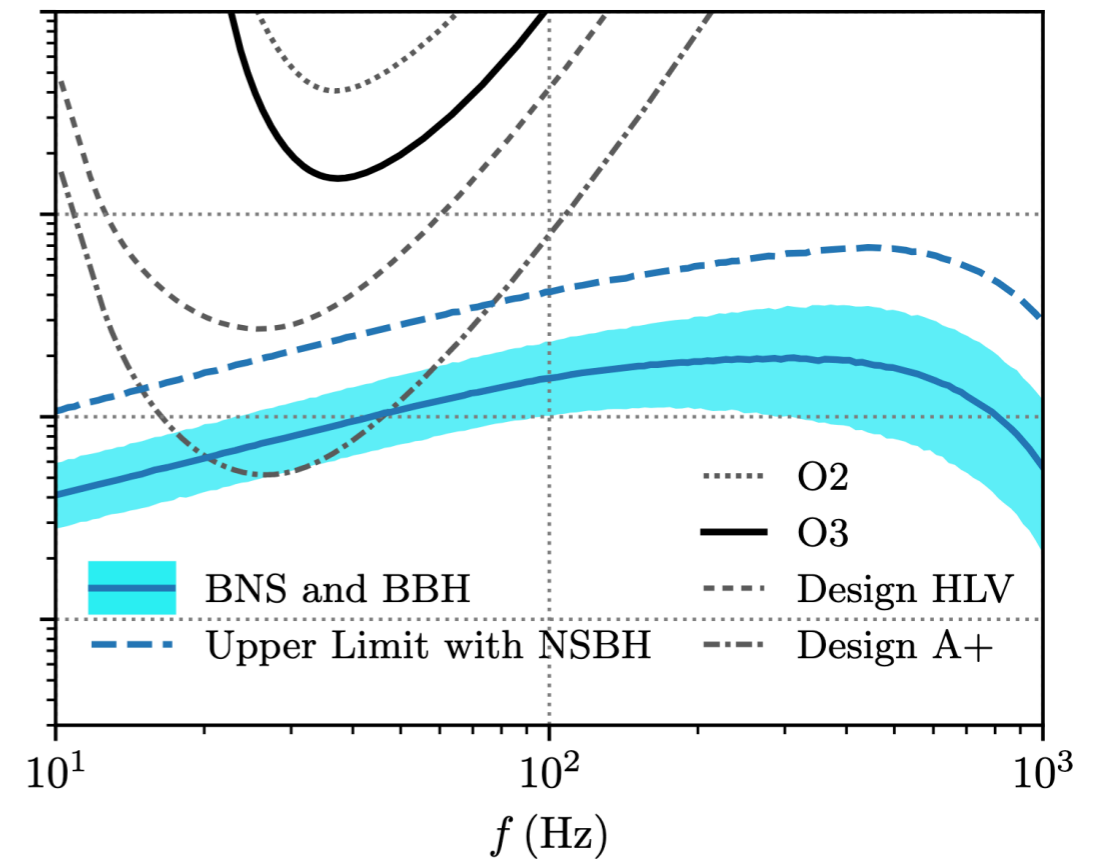
Iacovelli et al, arXiv:2207.02771

# GWs from the early universe: motivation

**LIGO Virgo Kagra: operating ground based interferometers**

$$1\,\text{Hz} < f < 1000\,\text{Hz} \quad \blacktriangleright \quad 10^{6}\,\text{GeV} \lesssim T_{*} \lesssim 10^{10}\,\text{GeV}$$
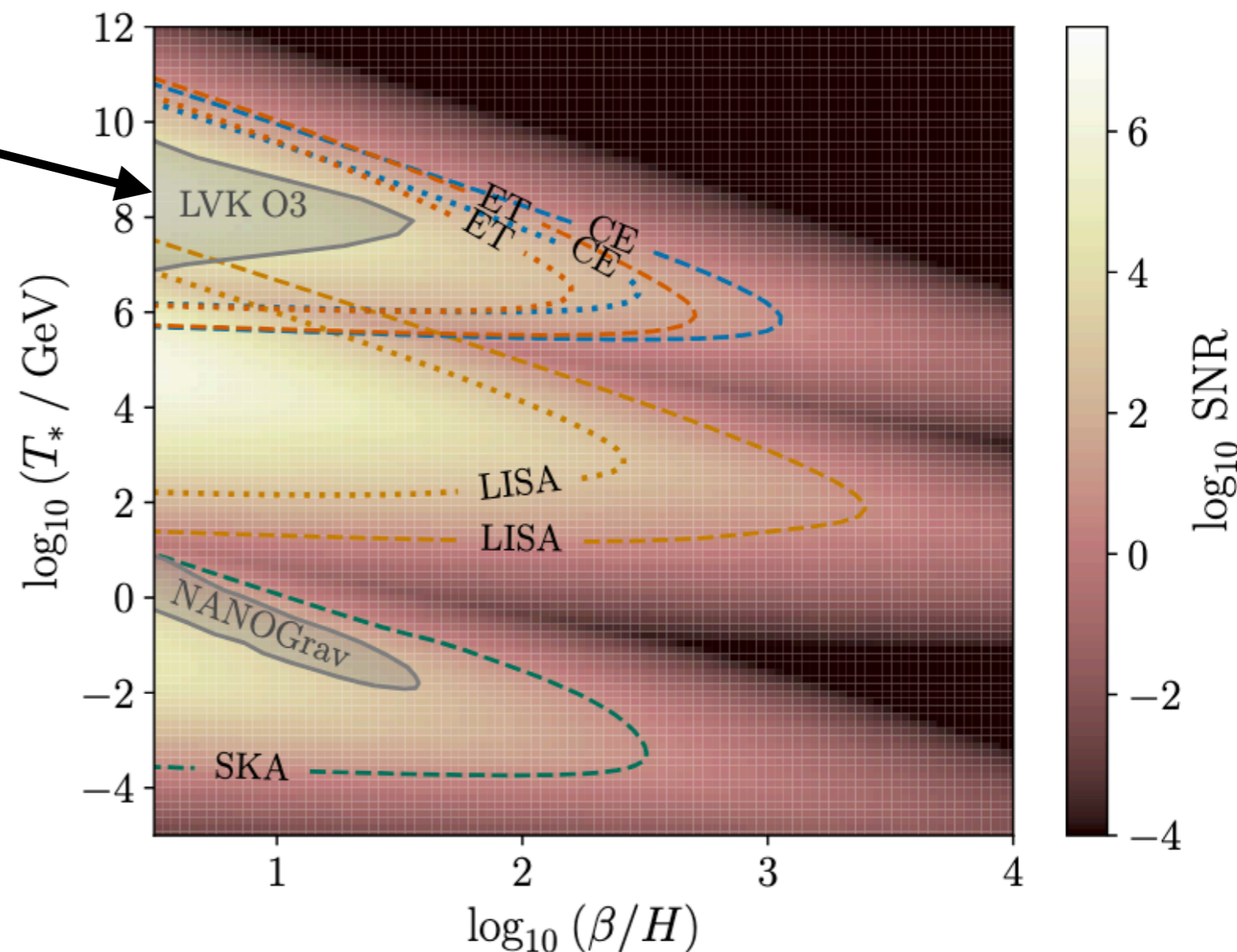
CC et al, ArXiv:2406.02359

LVK constraints from non-detection

Badger et al, arXiv:2209.14707

**Peccei-Quinn phase transition**

$$T_{\text{PQ}} \sim F_{a}$$

$$10^{7-8}\,\text{GeV} \lesssim F_{a} \lesssim 10^{10-11}\,\text{GeV}$$
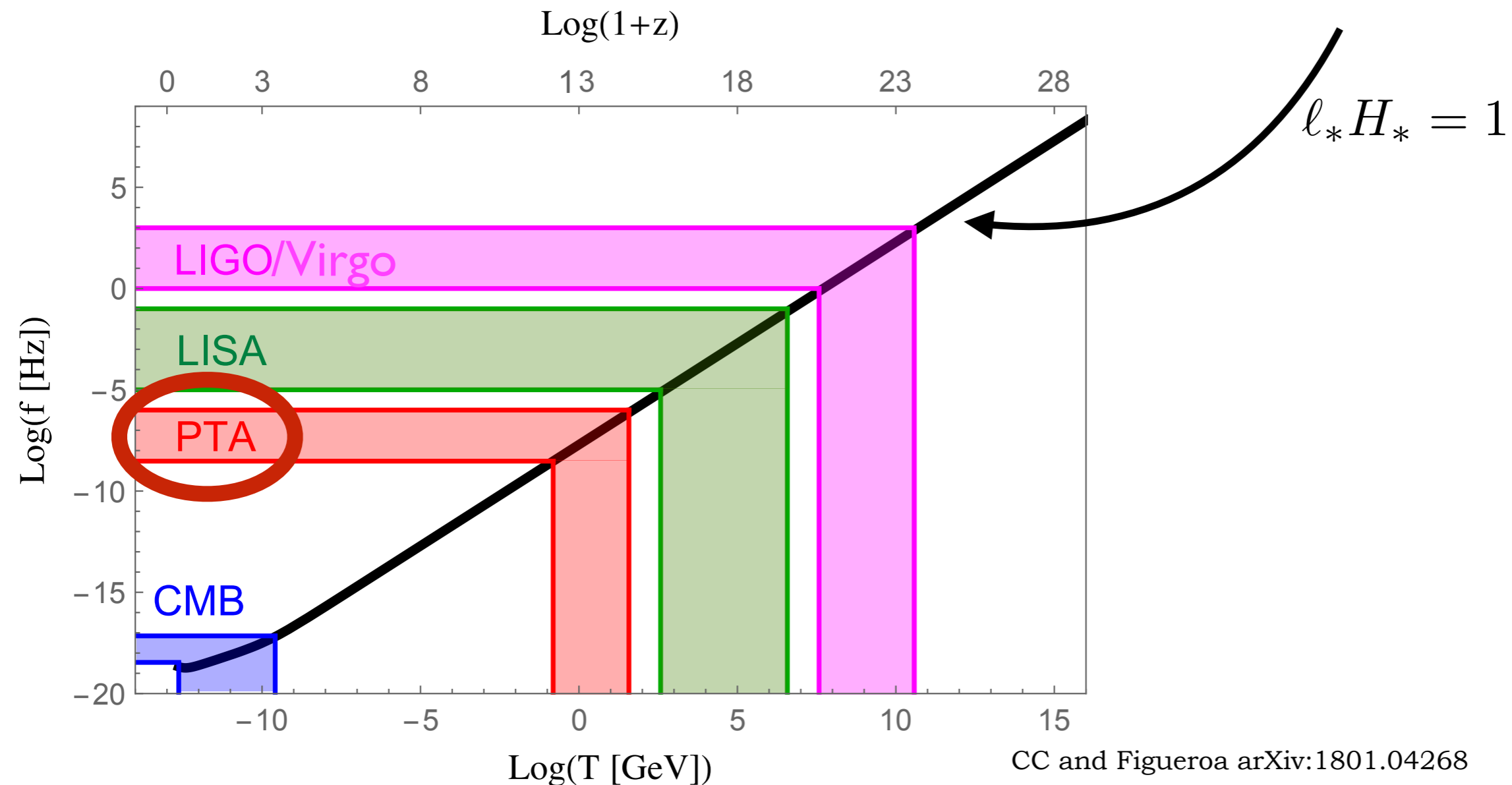


Parameter to which the signal amplitude is *inversely* proportional

# GWs from the early universe: motivation

Suppose the GW generating process has a *characteristic time and length scale*

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$\ell_* H_* = 1$

CC and Figueroa arXiv:1801.04268

# GWs from the early universe: motivation

## Pulsar Timing Arrays: first SGWB detection
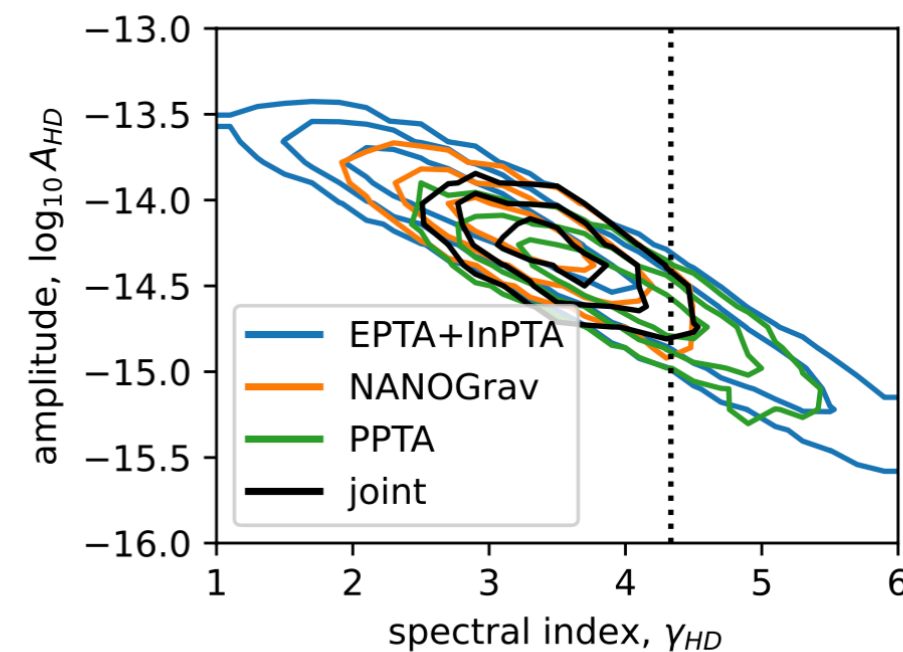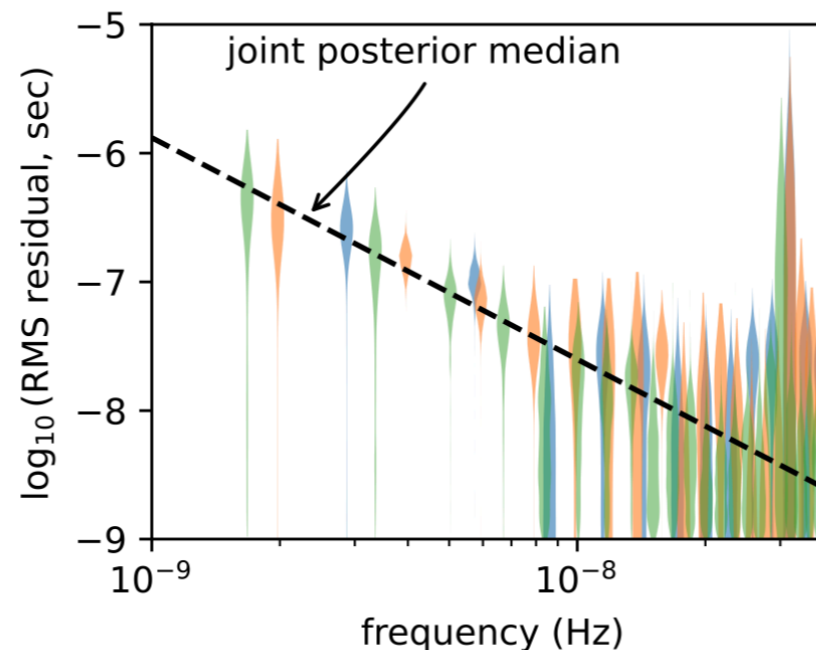
**Timeline:**

- **2024**: announcement of evidence for the signal and for the expected response of pulsars to GWs
- 2026: IPTA paper should improve the evidence
- ~2030: SKA

The SGWB from super-massive black hole binaries at the centre of galaxies is the best candidate source for this signal
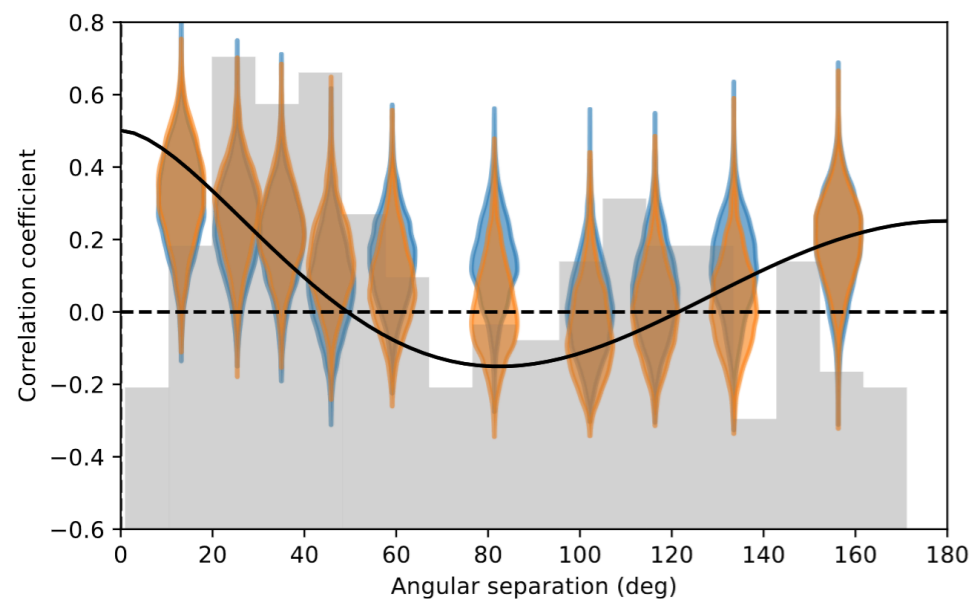


J. Antoniadis et al, arXiv:2306.16214

IPTA Collaboration, arXiv:2309.00693

# GWs from the early universe: motivation

## Pulsar Timing Arrays: first SGWB detection

$$10^{-9}\,\mathrm{Hz} < f < 10^{-7}\,\mathrm{Hz} \quad \blacktriangleright \quad 1\,\mathrm{MeV} \lesssim T_* \lesssim 1\,\mathrm{GeV}$$

CC et al, ArXiv:2406.02359

PTAs offer the possibility
to probe the
**QCD energy scale**

Parameter space
region that could
explain the
measurement

Afzal et al arXiv:2306.16219
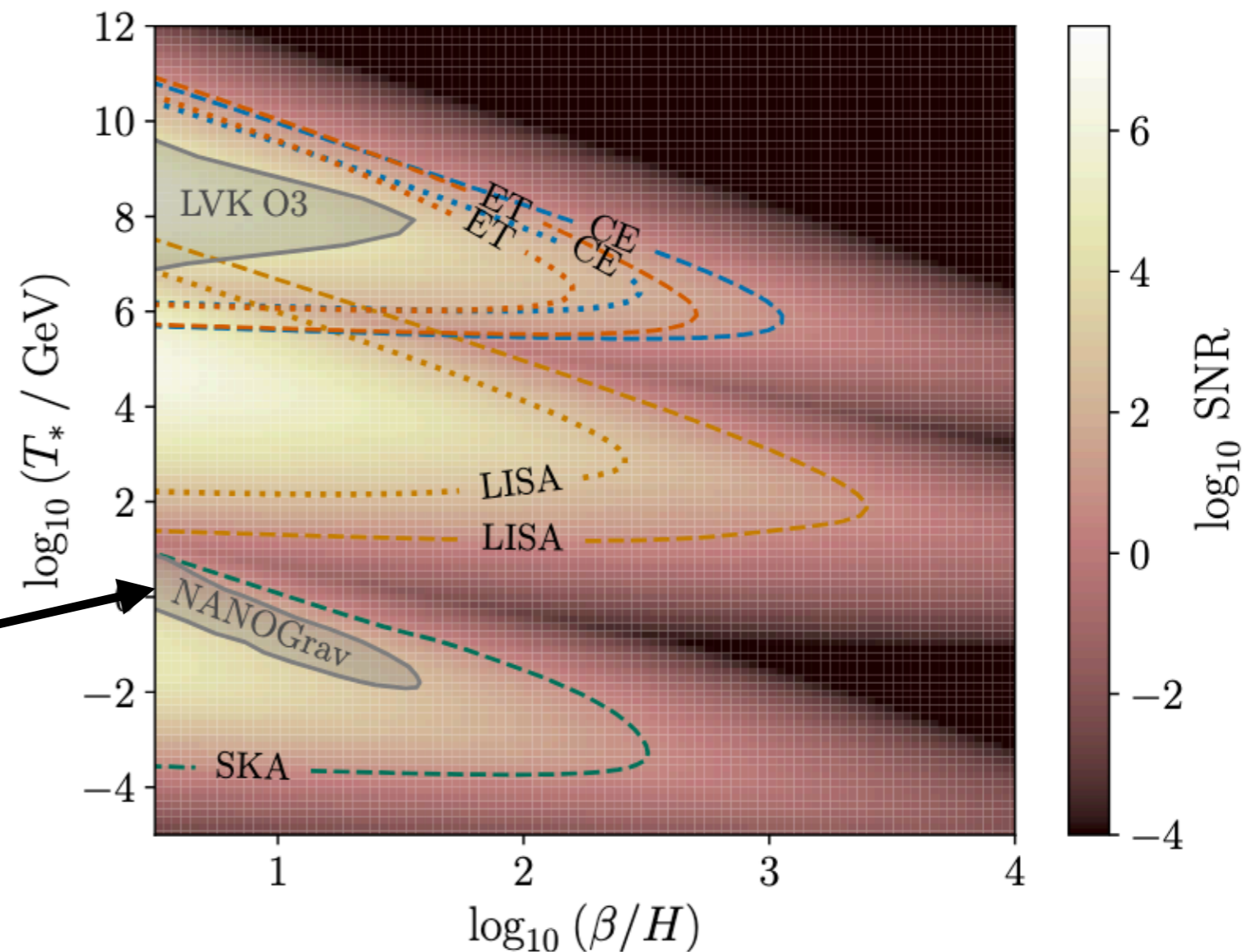


Parameter to which the signal amplitude is *inversely* proportional

# GWs from the early universe: motivation

Suppose the GW generating process has a *characteristic time and length scale*

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GW frequency redshifted to today:
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$$f = f_* \frac{a_*}{a_0} = \frac{1.65 \times 10^{-5}}{\ell_* H_*} \left( \frac{g(T_*)}{100} \right)^{1/6} \frac{T_*}{100\,\text{GeV}} \,\text{Hz}$$



CC and Figueroa arXiv:1801.04268

# GWs from the early universe: motivation

## Laser interferometer space antenna

**Timeline:**
- 2024 mission adopted by ESA, 30% NASA, science Red Book
- ~10 years: mission construction
- **Middle 2030s**: launch (Ariane 6) + 1.5 years to get to orbit, 6-12 months for commissioning
- nominal mission duration ~4 years, up to 10 years



LISA Red Book arXiv:2402.07571
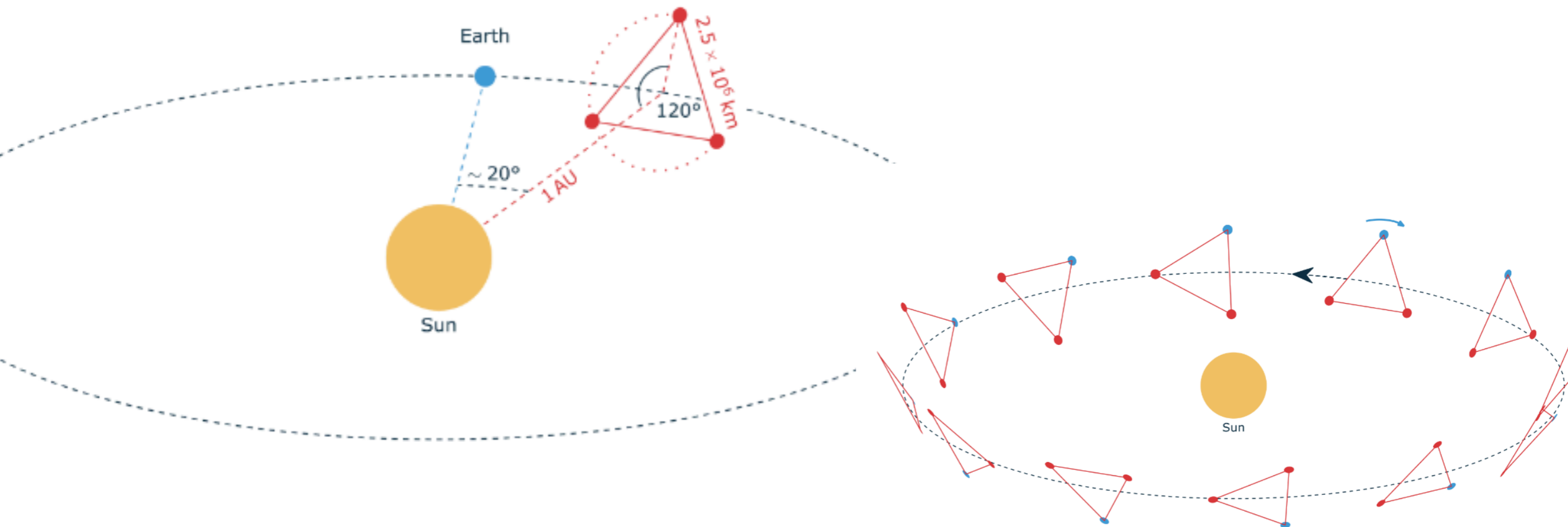
# GWs from the early universe: motivation

## Laser interferometer space antenna

$$10^{-5}\,\mathrm{Hz} < f < 0.1\,\mathrm{Hz} \quad \longrightarrow \quad 10\,\mathrm{GeV} \lesssim T_* \lesssim 10^5\,\mathrm{GeV}$$

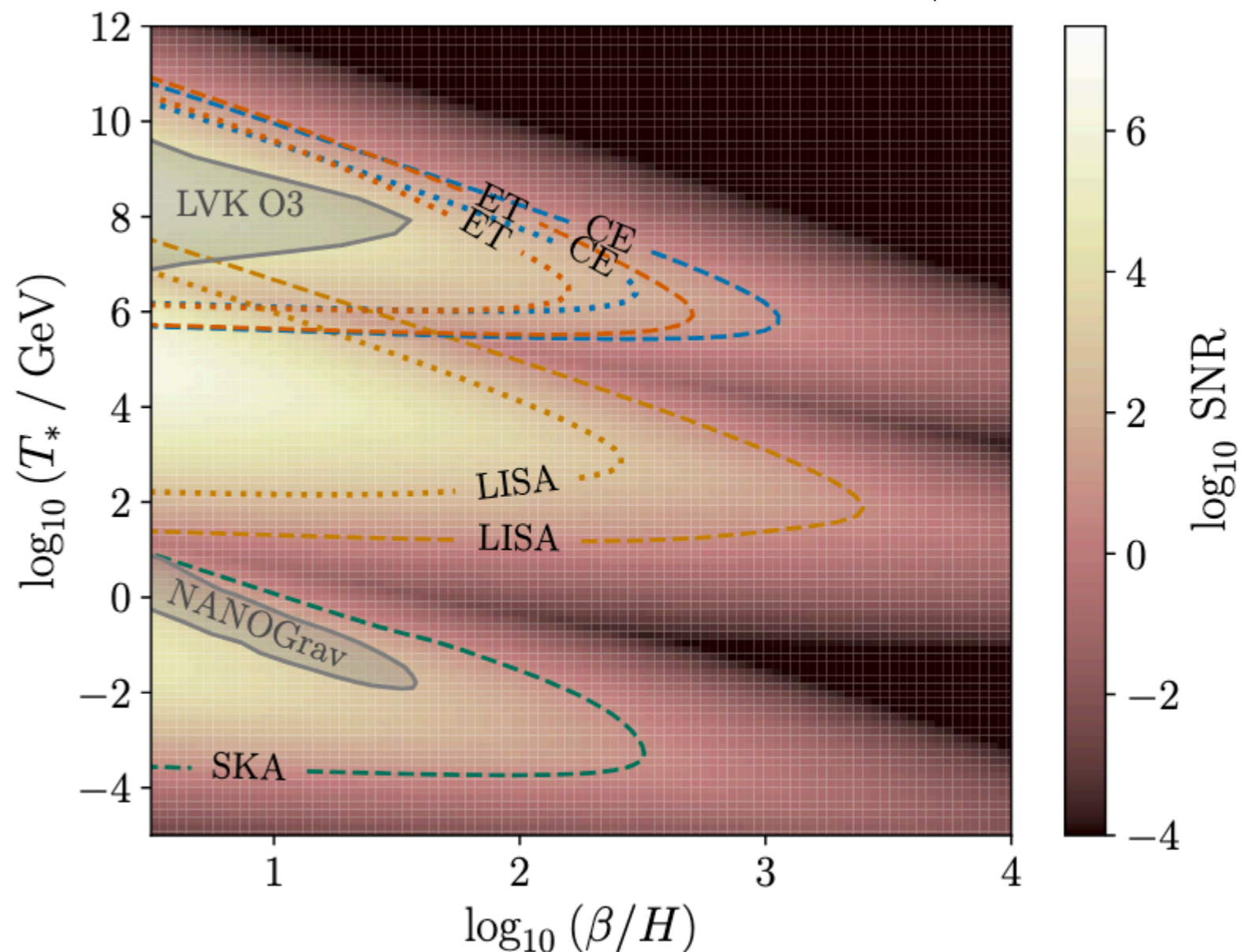LISA offers the possibility to probe the **EW energy scale and beyond**

CC et al, ArXiv:2406.02359



Parameter to which the signal amplitude is *inversely* proportional

# GW sources in the early universe

$$ds^2 = -dt^2 + a^2(t)[(\delta_{ij} + h_{ij})dx^i dx^j] \qquad \bar{G}_{\mu\nu} + \delta G_{\mu\nu} = 8\pi G \left(\bar{T}_{\mu\nu} + \delta T_{\mu\nu}\right)$$

$$\ddot{h}_{ij} + 3H\dot{h}_{ij} + k^2 h_{ij} = \boxed{16\pi G\, \Pi_{ij}^{TT}}$$

GW SOURCE:
tensor anisotropic stress

Scaling of the signal
with the source
characteristics

$$\Omega_{\mathrm{GW}}^* = \frac{\rho_{\mathrm{GW}}^*}{\rho_{\mathrm{tot}}^*} \sim (H_*\ell_*)^2 \left(\frac{\Pi}{\rho_{\mathrm{tot}}^*}\right)^2$$

setting $k \sim \dfrac{1}{\ell_*}$

$$\Omega_{\mathrm{GW}}^{\mathrm{today}} \sim 10^{-5}\Omega_{\mathrm{GW}}^* \gtrsim 10^{-11} \qquad \Longrightarrow \qquad (H_*\ell_*)\left(\frac{\Pi}{\rho_{\mathrm{tot}}^*}\right) \gtrsim 10^{-3}$$
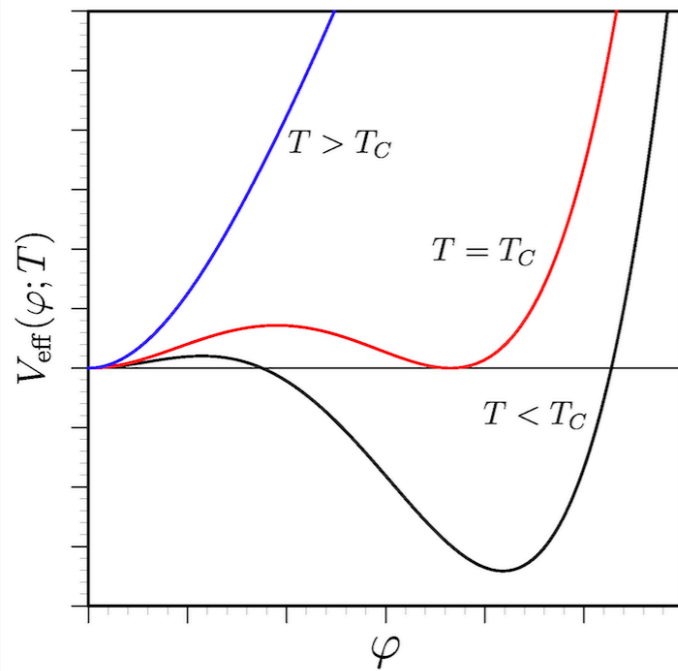
order of the
sensitivity of LISA

High anisotropic stresses
needed for a detectable signal

# GW sources in the early universe

Which processes in the early universe can produce high anisotropic stresses?

**Typically phase transitions, if they are first order...**



- Bubble collision (scalar field gradients)  $\Pi_{ij}^{TT} \sim [\partial_i \phi \partial_j \phi]^{TT}$

- Bulk fluid motion  $\Pi_{ij}^{TT} \sim [\gamma^2 (\rho + p) v_i v_j]^{TT}$

- Electromagnetic fields  $\Pi_{ij}^{TT} \sim [-E_i E_j - B_i B_j]^{TT}$



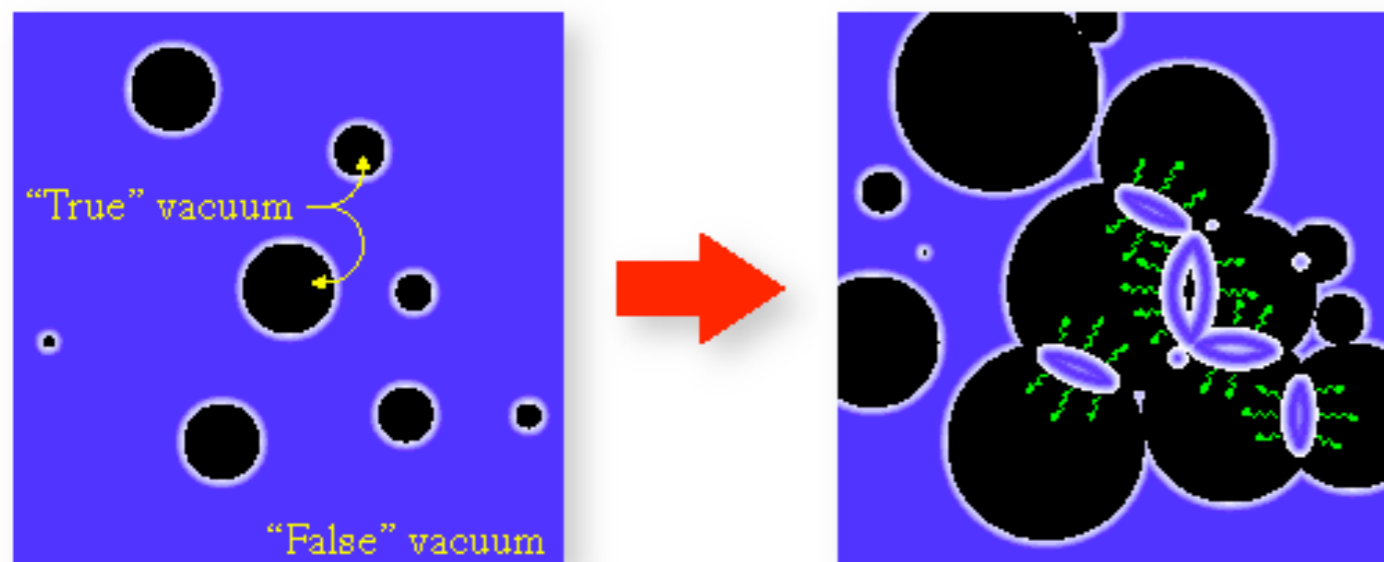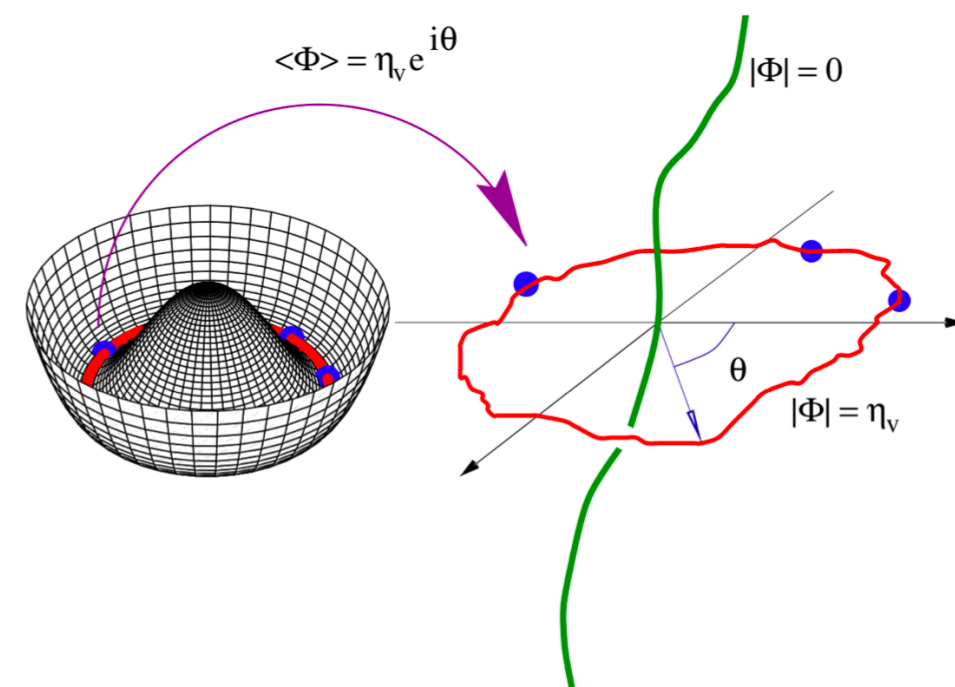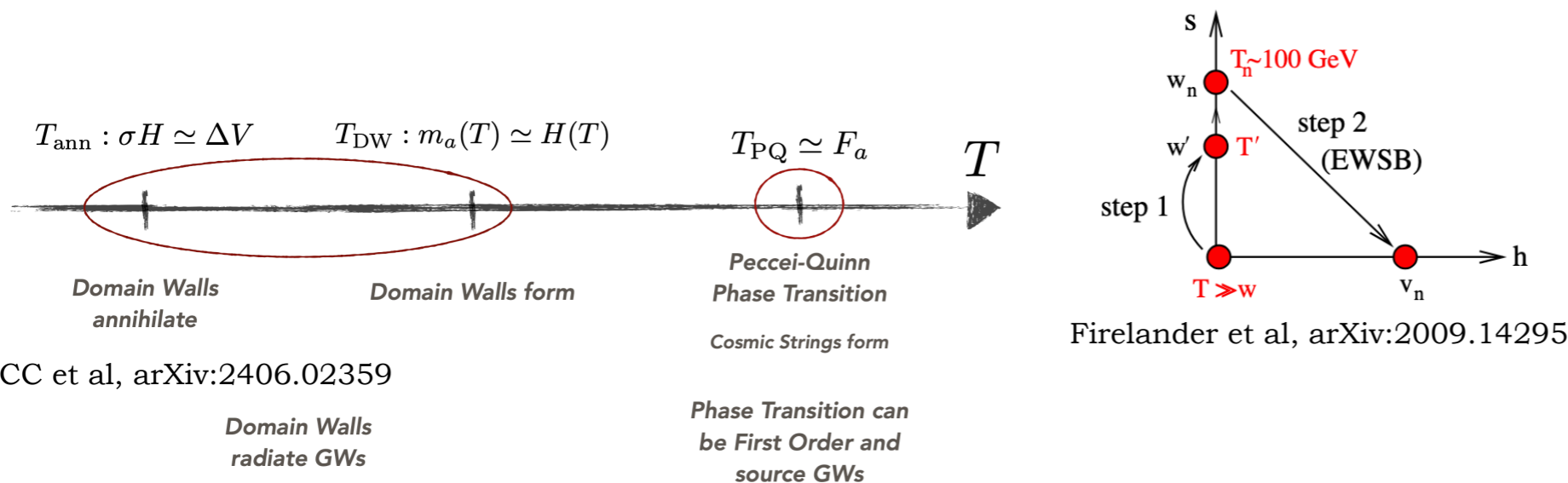**... or if they produce topological defects**

# GW sources in the early universe

Which processes in the early universe can produce high anisotropic stresses?

**Typically phase transitions, if they are first order...**

- **EW sector extensions:** SM + light scalars (SM+singlet, SUSY, 2HDM, composite Higgs...)
- **Effective approaches:** heavy new physic represented by higher dimensional operators
- **Conformal models:** e.g. conformal symmetry breaking with dilaton
- **New symmetries:** extend the SM with e.g. $U(1)_{B-L}$
- **Hidden sectors:** provide also dark matter candidates, PT can be as strong as one wants
- **QCDPT:** might become first order if the lepton asymmetry in the universe is large
- **Peccei Quinn** can be first order depending on the realisation

$T_{\text{ann}} : \sigma H \simeq \Delta V$     $T_{\text{DW}} : m_a(T) \simeq H(T)$     $T_{\text{PQ}} \simeq F_a$     $T$

Domain Walls annihilate

Domain Walls form

Peccei-Quinn Phase Transition

Cosmic Strings form

CC et al, arXiv:2406.02359

Domain Walls radiate GWs

Phase Transition can be First Order and source GWs

Firelander et al, arXiv:2009.14295

**... or if they produce topological defects**

- **Axion or axion-like** models
- Spontaneous breaking of higher symmetry patterns possibly connected to **GUT**

**Many models that can lead to potentially detectable GW signals: can we distinguish them?**

# GW from a first order PT

$$\Omega^*_{\mathrm{GW}} \sim (H_* \ell_*)^2 \left( \frac{\Pi}{\rho^*_{\mathrm{tot}}} \right)^2$$

- **the characteristic scale of the source:**

Size of the bubbles at collision
(towards the end of the PT)
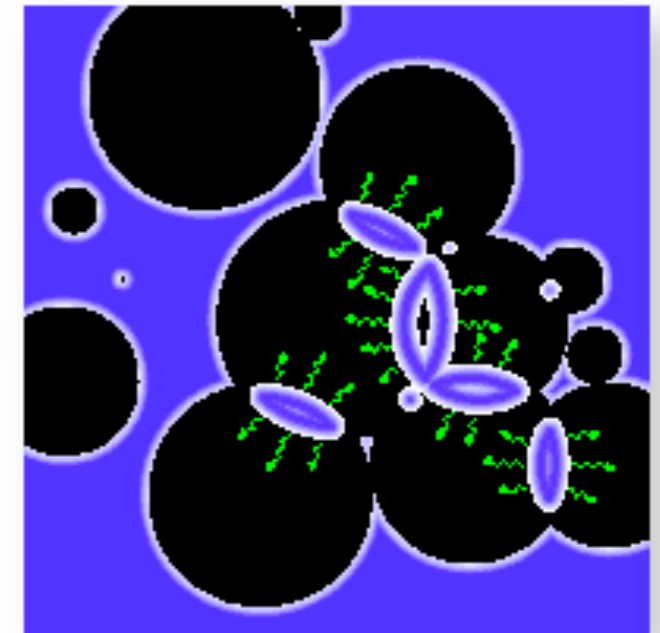
$$\ell_* \sim R_* \sim \frac{v_w}{\beta}$$



- Transition rate
parameter
("duration" of the PT):

$$\beta = \left. \frac{d}{dt} S(t) \right|_{t_*}$$

from    $\Gamma(t) = \Gamma(t_*) \exp[(\beta(t - t_*)]$

Probability of tunnelling per unit volume and time

$$\Gamma(t) \simeq \max \left\{ \frac{1}{R_0^4} \left( \frac{S_4(\varphi)}{2\pi} \right)^2 \exp[-S_4(\varphi)], T^4 \left( \frac{S_3(\varphi, T)}{2\pi T} \right)^{3/2} \exp\left[ -\frac{S_3(\varphi, T)}{T} \right] \right\}$$

# GW from a first order PT

$$\Omega^*_{\mathrm{GW}} \sim (H_* \ell_*)^2 \left( \frac{\Pi}{\rho^*_{\mathrm{tot}}} \right)^2$$

- **the characteristic scale of the source:**

Size of the bubbles at collision (towards the end of the PT)

$$\ell_* \sim R_* \sim \frac{v_w}{\beta}$$



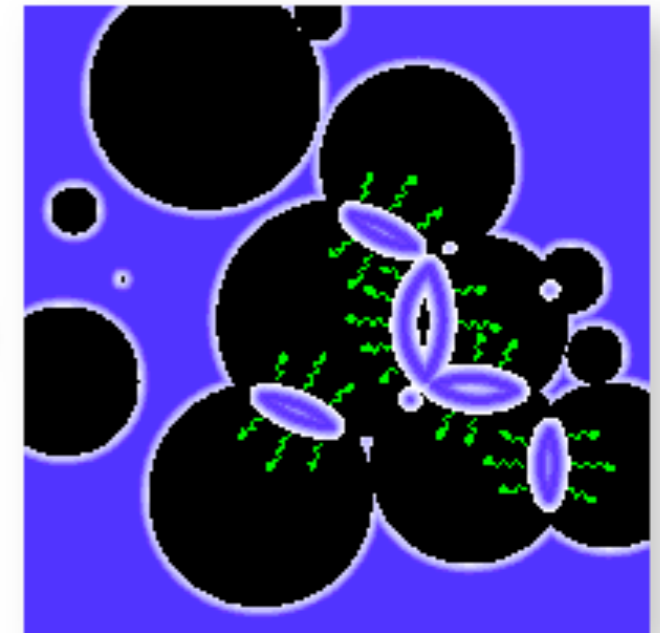- Transition rate parameter ("duration" of the PT):

$$\beta = \frac{d}{dt} S(t) \Big|_{t_*}$$

- bubble wall velocity $v_w$

Difficult to estimate!

Thermal PT: terminal wall velocity (steady state bubble) given by the balance among the driving force (pressure difference) and the friction force (interaction of the wall with particles in the surrounding plasma)

Often used is a phenomenological description introducing a friction parameter (hopefully covering several particle theory models)

See e.g. Huber and Sopena arXiv:1302.1044

# GW from a first order PT

$$\Omega^*_{\mathrm{GW}} \sim (H_* \ell_*)^2 \left( \frac{\Pi}{\rho^*_{\mathrm{tot}}} \right)^2$$

- **the characteristic scale of the source:**

  Size of the bubbles at collision
  (towards the end of the PT)
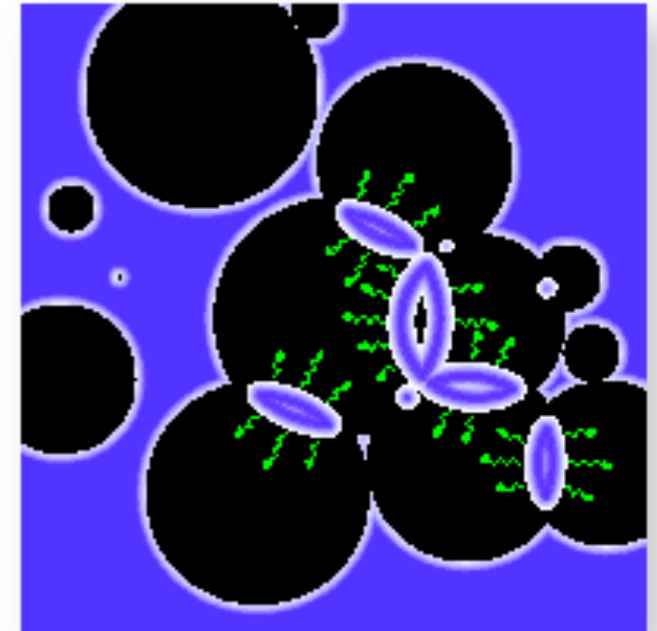
  $$\ell_* \sim R_* \sim \frac{v_w}{\beta}$$



- Transition rate parameter ("duration" of the PT):

  $$\beta = \frac{d}{dt} S(t) \Big|_{t_*}$$

- bubble wall velocity $v_w$

- redshift to get the characteristic frequency of the GW signal today: temperature scale of the PT

$$f = f_* \frac{a_*}{a_0} = \frac{1.65 \times 10^{-5}}{\ell_* H_*} \left( \frac{g(T_*)}{100} \right)^{1/6} \frac{T_*}{100\,\mathrm{GeV}} \, \mathrm{Hz}$$
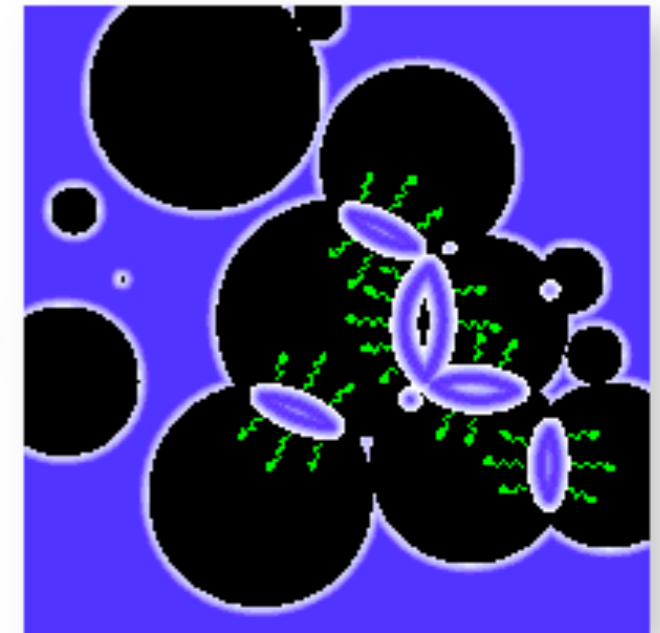
# GW from a first order PT

$$\Omega^*_{\mathrm{GW}} \sim (H_* \ell_*)^2 \left( \frac{\Pi}{\rho^*_{\mathrm{tot}}} \right)^2$$

- **the anisotropic stress energy fraction:**

1. The colliding bubble walls source anisotropic stresses

    -> gradient energy in the scalar field $\quad K = \dfrac{\rho_\phi}{\rho_{\mathrm{tot}}}$



2. The coupling with the surrounding fluid sets it into motion

    bulk fluid motion sources anisotropies stress via
    - *Sound waves* (compressional mode, linear)
    - *Turbulence* (vortical mode, non-linear, possibly MHD) $\quad \tau_{\mathrm{nl}} \sim \dfrac{\ell_*}{v_{\mathrm{rms}}} \leq H_*^{-1}$

    -> kinetic energy in the bulk fluid motion $\quad K = \dfrac{\rho_v}{\rho_{\mathrm{tot}}}$

Which process dominates depends on the $\alpha_{\mathrm{bag}} = \dfrac{V_0(\varphi_f)}{\frac{\pi^2}{30} g_f T_f^4}$
strength of the PT

# GW from a first order PT

$$\Omega^*_{\mathrm{GW}} \sim (H_* \ell_*)^2 \left( \frac{\Pi}{\rho^*_{\mathrm{tot}}} \right)^2$$

- **Strong PT $\alpha \gtrsim \mathcal{O}(1)$**

   Supercooling, short inflationary phase, bubbles possibly accelerating to the speed of light: GW sources are *bubble collisions and/or collisions of surrounding thin fluid shells*

   $$K = \frac{\rho_\phi}{\rho_{\mathrm{tot}}}$$

- **Weak PT $\alpha \lesssim \mathcal{O}(10^{-2})$**

   Potential energy subdominant, GW production from fluid motion surrounding the bubbles, small velocity/enthalpy perturbations: *sound waves*

   $$K = \frac{\rho_v}{\rho_{\mathrm{tot}}}$$

- **Intermediate PT $\alpha \sim \mathcal{O}(10^{-1}) - \mathcal{O}(1)$**

   GW production from fluid motion but velocity/enthalpy perturbations can be high: non-linear compressional and vortical turbulence

Which process dominates depends on the strength of the PT $\quad \alpha_{\mathrm{bag}} = \dfrac{V_0(\varphi_f)}{\frac{\pi^2}{30} g_f T_f^4}$

# GW from a first order PT

$$\Omega_{\mathrm{GW}}^*(f) = \tilde{\Omega}\,(\ell_* H_*)^2 \left(\frac{\Pi}{\rho_{\mathrm{tot}}^*}\right)^2 S(f)$$

How much kinetic energy is in anisotropic stresses?

What is the spectral shape of the GW signal as a function of frequency?

- **numerical simulations** are necessary because of non-linear dynamics and/or complicated fluid shells profiles and/or intrinsic randomness of the process

  - link the PT strength to the actual energy available in the GW source
  - understand which source dominates are how they are connected
  - predict the signal amplitude and spectral shape

- several codes exist that tackle the problem with different characteristics: with/without scalar field dynamics, relativistic motion, expansion of the universe, magnetic field...

  M. Hindmarsh et al, arXiv:1304.2433 and following, A. Roper Pol et al, arXiv:1903.08585 and following, R. Jinno et al, arXiv:2209.04369 and following

- each source produces GW signals with different features (peak, slopes...) that are still work in progress
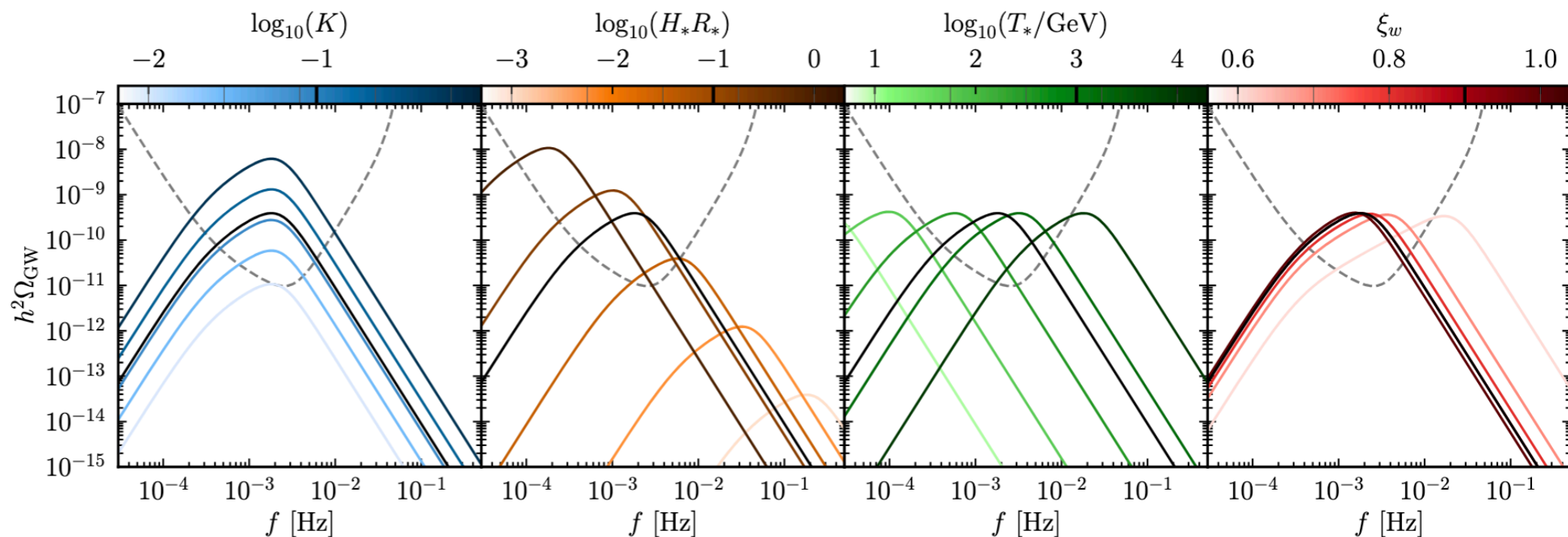
# GW from a first order PT

To summarise, the following "thermodynamic" parameters enter in the GW signal:

$$T_*, \ \alpha, \ \frac{\beta}{H_*}$$ ⟶ Determined by the effective potential

$$v_w, K$$ ⟶ Determined by the bubble expansion dynamics and interaction, and by the fluid dynamics (sound speed fixed)

If the PT is strong and non-linearities in the bulk fluid develop: fraction $\varepsilon = \dfrac{K_{\text{turb}}}{K}$ of kinetic energy in turbulent motions
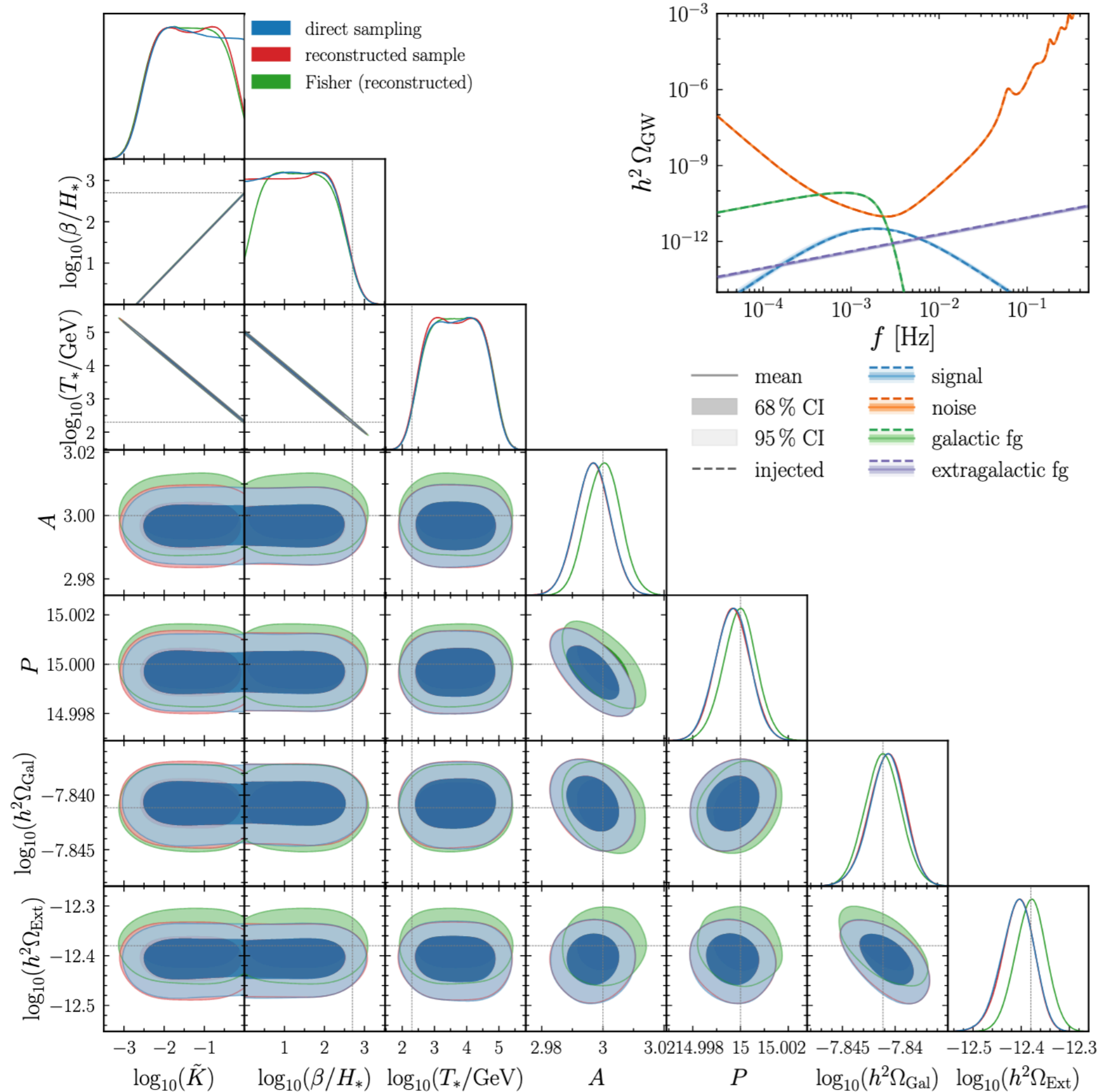
**Most of these parameters are known (at least in principle) given a PT model + numerical simulations of the fluid dynamics**

However, there are degeneracies to extract them from the GW detection



LISA CosWG, arXiv:2403.03723

(b) sound waves (black: $K = 0.1$, $H_* R_* = 0.1$, $\xi_w = 0.9$, $T_* = 1\,\text{TeV}$)
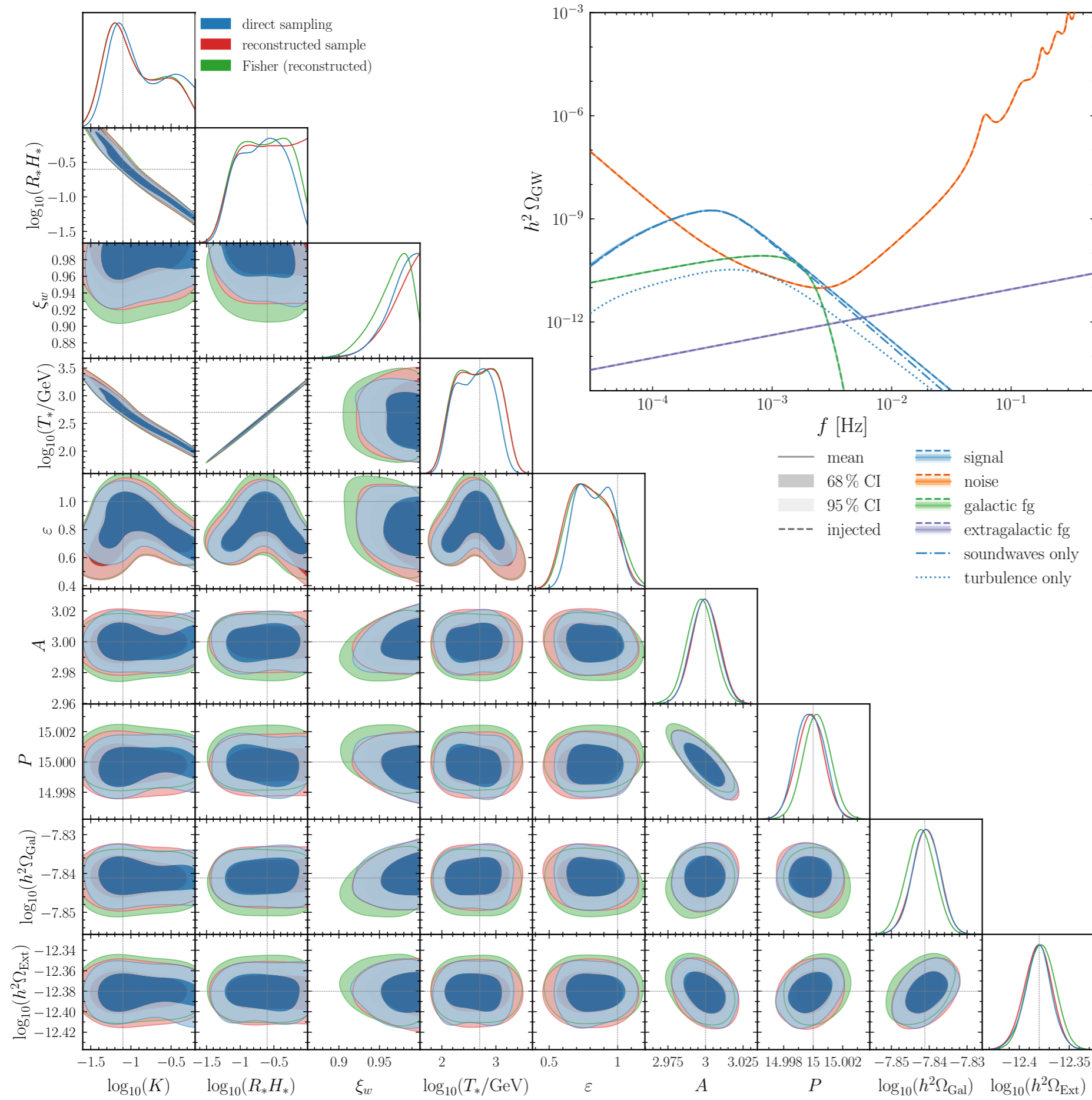
# Examples of detectable signal from the EWPT



**Template-based** reconstruction of the *thermodynamic parameters* of the first order PT for **bubble collisions**

accounting for **foregrounds** and **assuming a two-parameters noise model**

LISA CosWG, arXiv:2403.03723

# Examples of detectable signal from the EWPT



**Template-based** reconstruction of the *thermodynamic parameters* of the first order PT for **sound waves + turbulence**

accounting for **foregrounds** and **assuming a two-parameters noise model**

LISA CosWG, arXiv:2403.03723
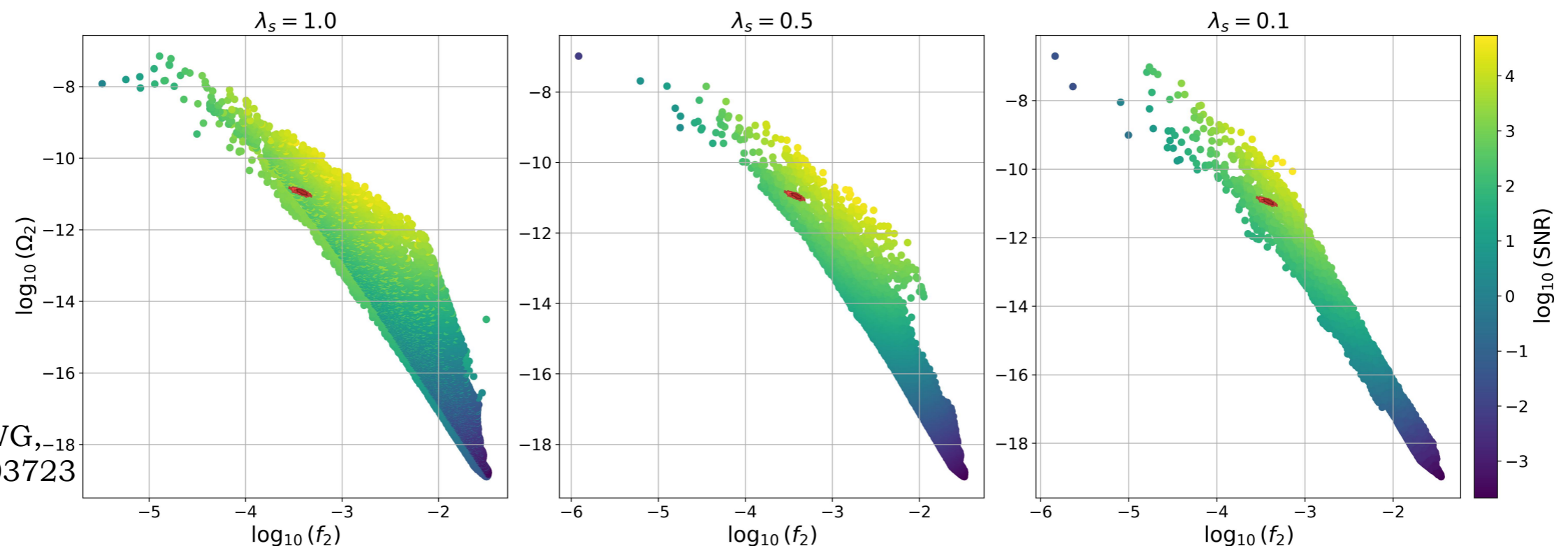
# Examples of detectable signal from the EWPT

Is it possible to infer the underlying PT model from the GW measurement?
No, too many degeneracies

but it is possible to constrain the model once you assume it:
the thermodynamic parameters reconstructed from the SGWB measurement can be
mapped to those of the particle physics model underlying the PT,
possibly synergising with current and future particle physics experiments

**Example: Z₂ singlet extension of the Standard Model**

**Step 1.** Predict the GW signal from the model parameters
Peak amplitude and frequency of the GW signal + SNR in LISA for three fixed quartic couplings
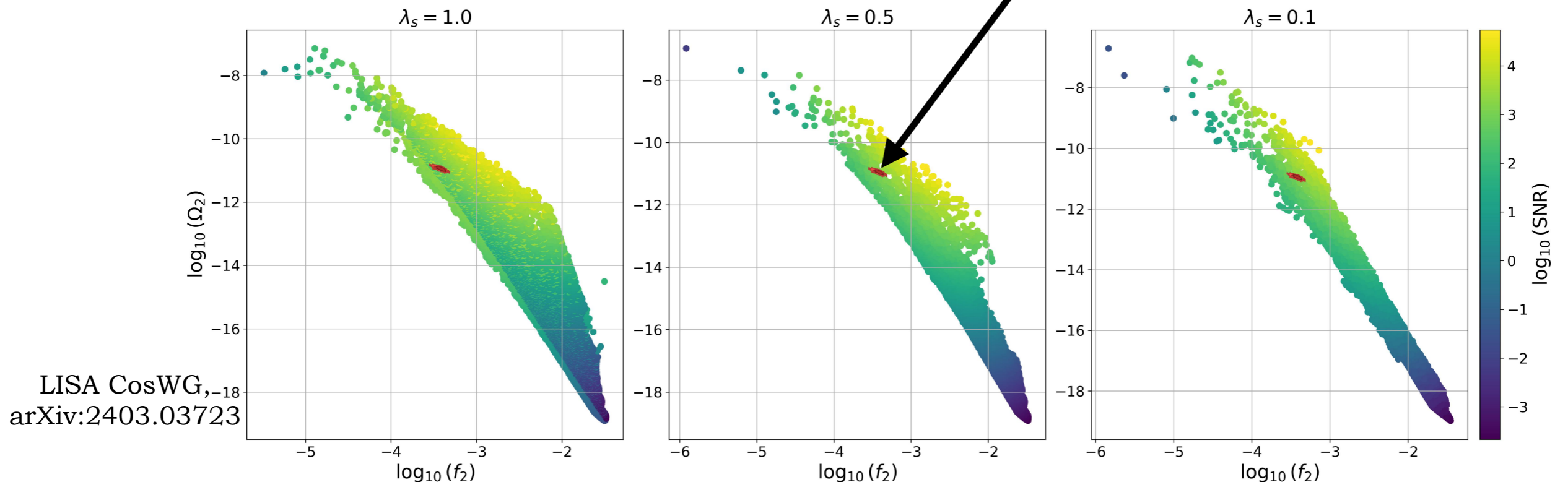


LISA CosWG, arXiv:2403.03723

# Examples of detectable signal from the EWPT



**Step 2.** LISA measures a signal compatible with bubble collisions from the first order PT: the thermodynamic parameters are reconstructed from the LISA measurement
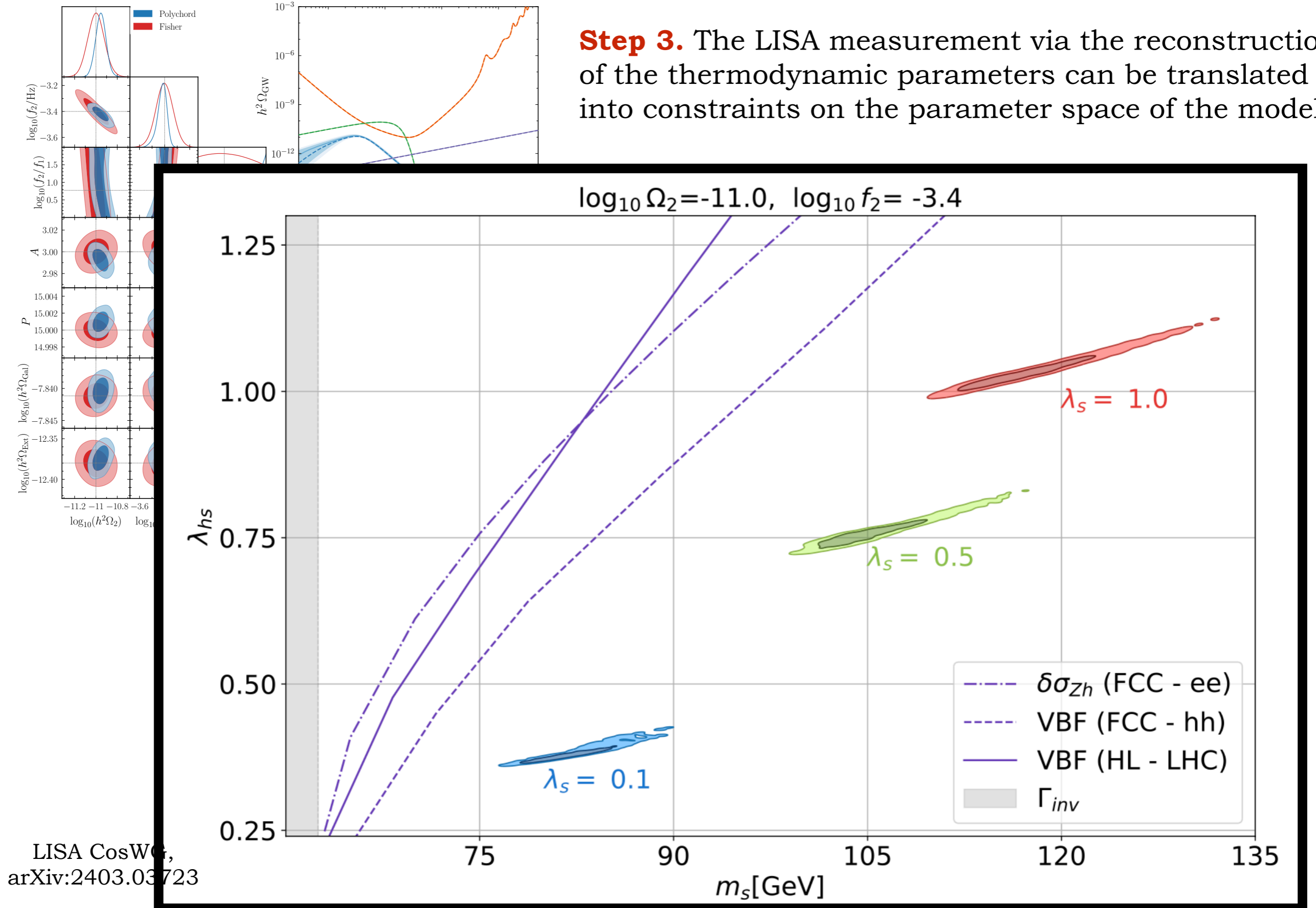
LISA measurement
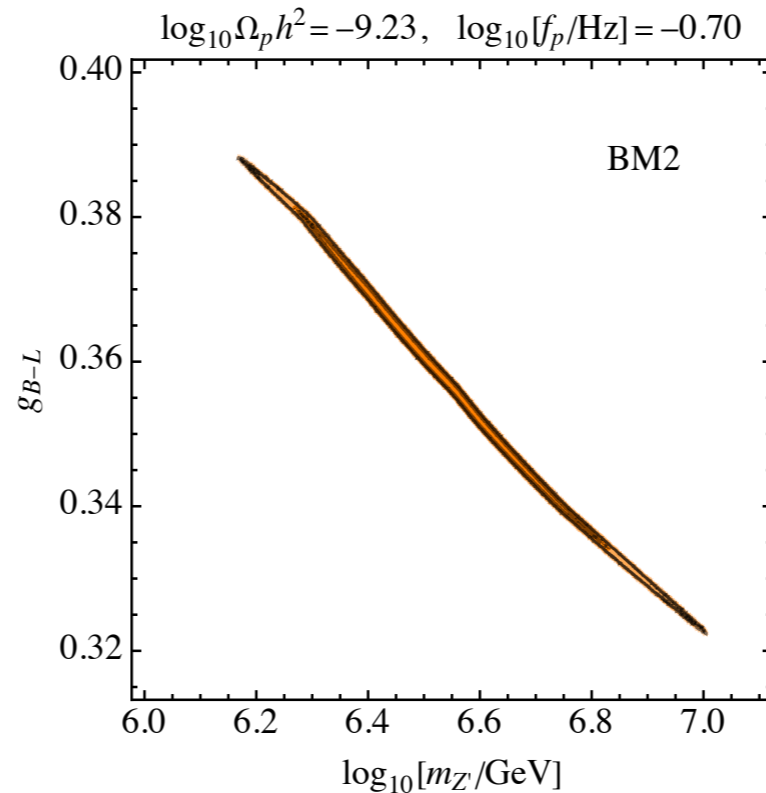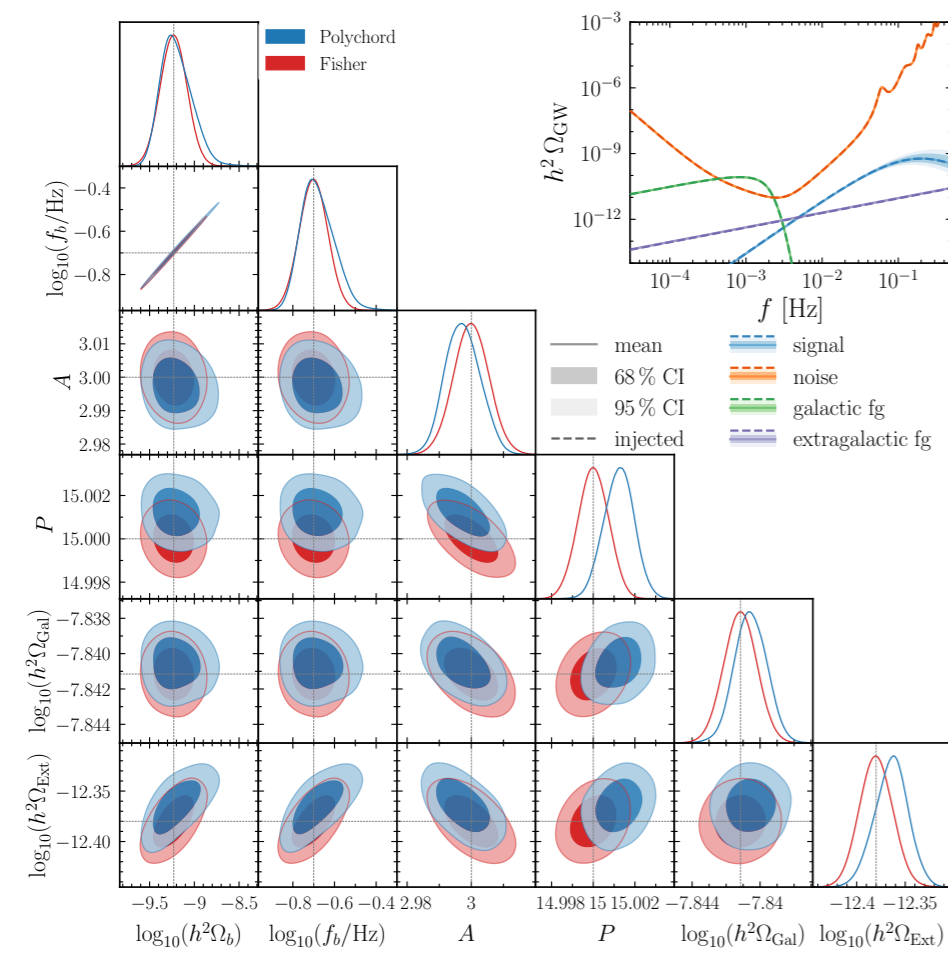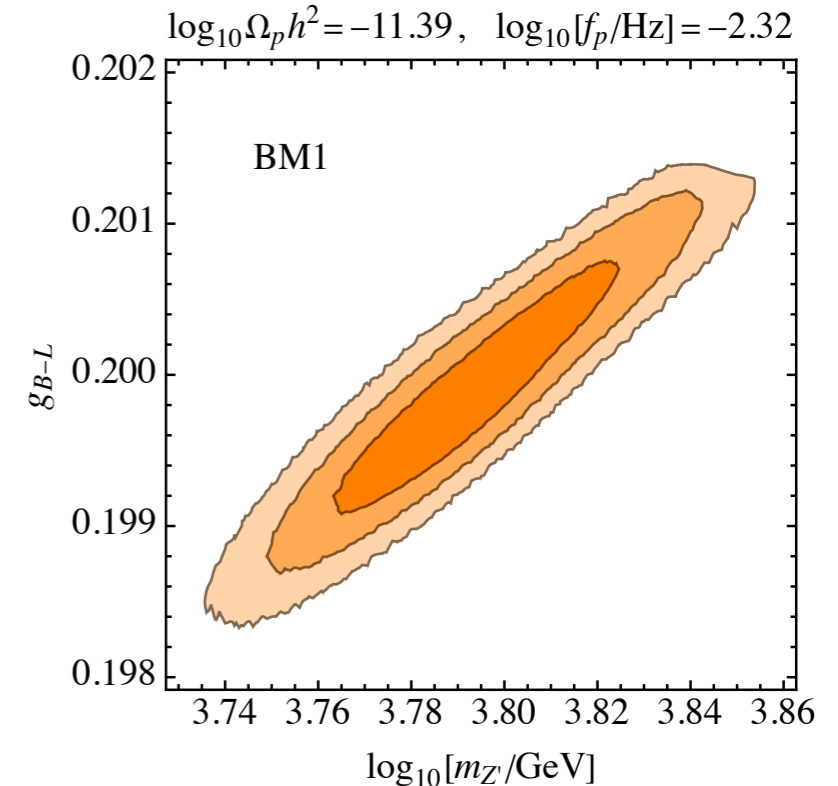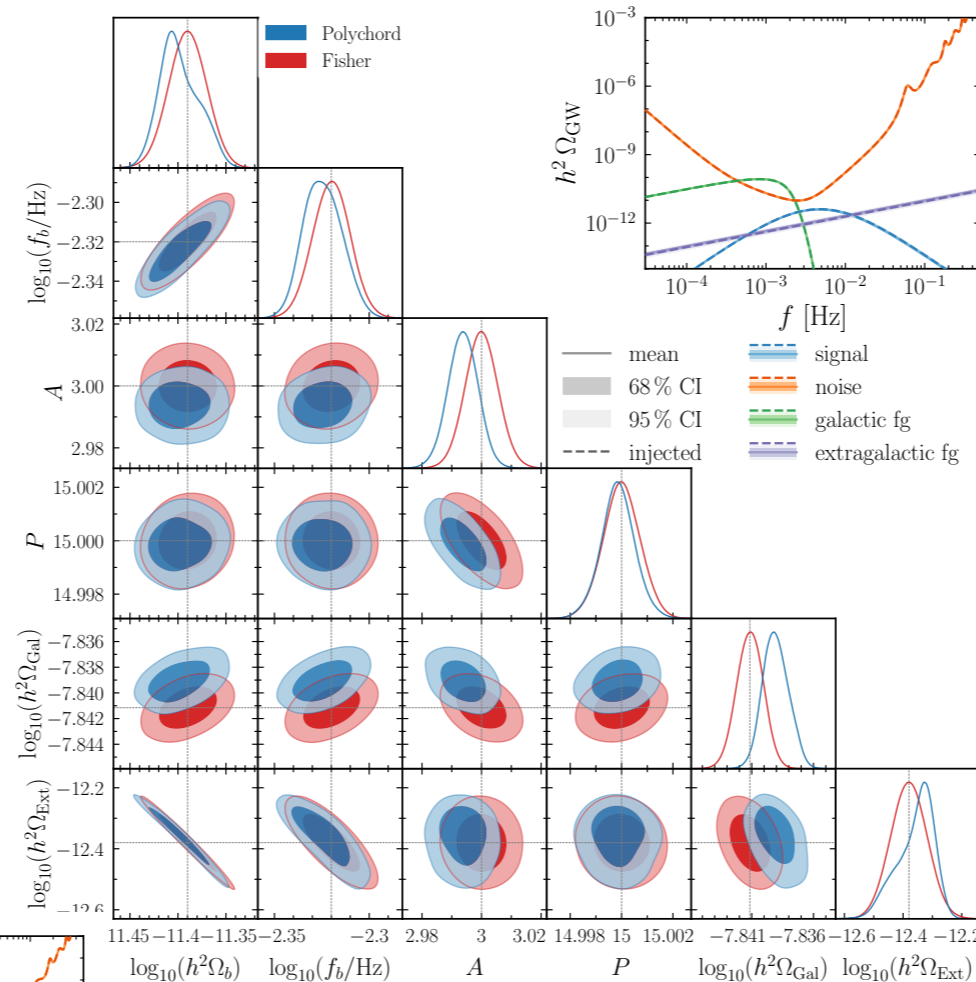
LISA CosWG, arXiv:2403.03723
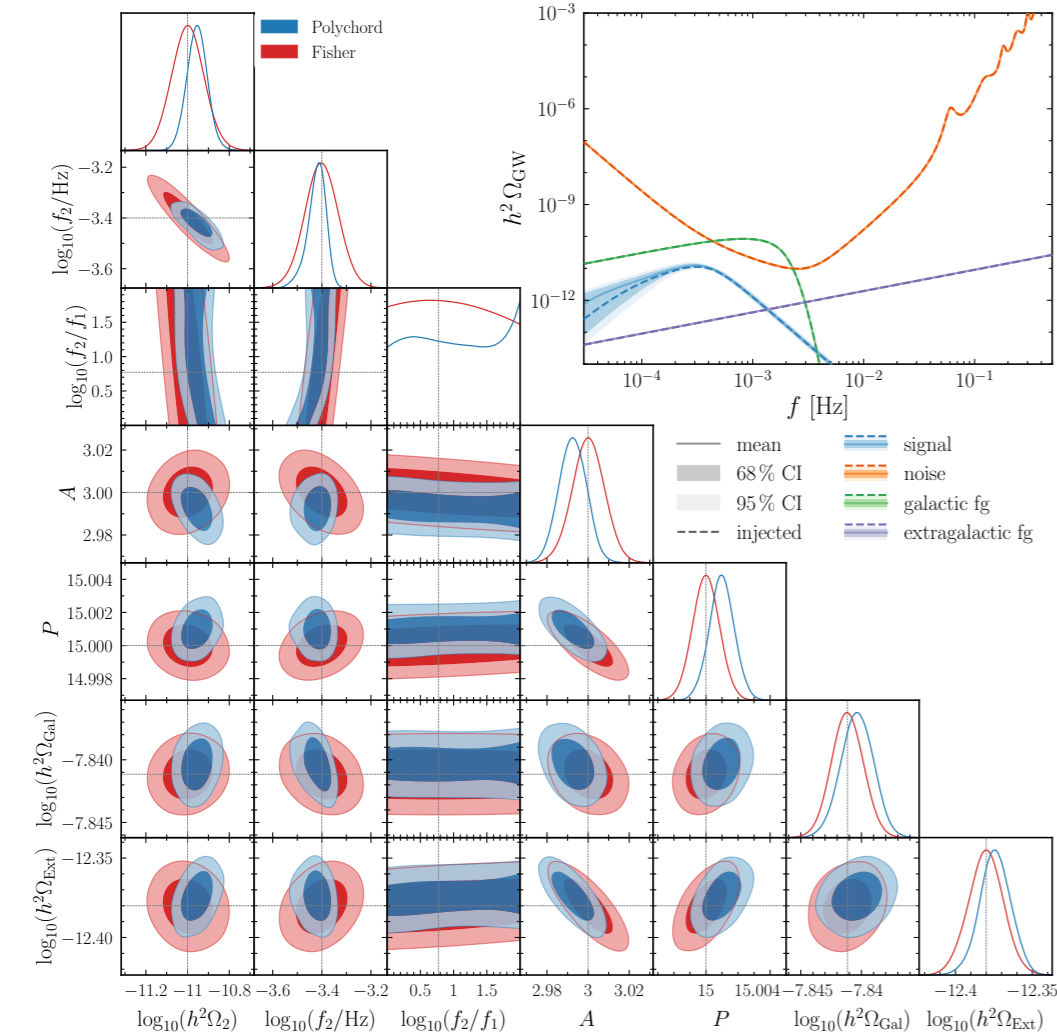
# Examples of detectable signal from the EWPT



**Step 3.** The LISA measurement via the reconstruction of the thermodynamic parameters can be translated into constraints on the parameter space of the model

$\log_{10} \Omega_2 = -11.0, \quad \log_{10} f_2 = -3.4$

$\lambda_s = 1.0$

$\lambda_s = 0.5$

$\lambda_s = 0.1$

Legend:
- $\delta\sigma_{Zh}$ (FCC - ee)
- VBF (FCC - hh)
- VBF (HL - LHC)
- $\Gamma_{inv}$

Axis labels: $\lambda_{hs}$, $m_s$[GeV]

LISA CosWG, arXiv:2403.03723

# Examples of detectable signal from the EWPT



**Same procedure, another example: Standard Model extended with U(1)$_{B-L}$**

$\log_{10}\Omega_p h^2 = -11.39, \quad \log_{10}[f_p/\text{Hz}] = -2.32$

BM1

$\log_{10}\Omega_p h^2 = -9.23, \quad \log_{10}[f_p/\text{Hz}] = -0.70$

BM2

Two different measurements at LISA lead to different constraints on the model parameter space

LISA CosWG, arXiv:2403.03723

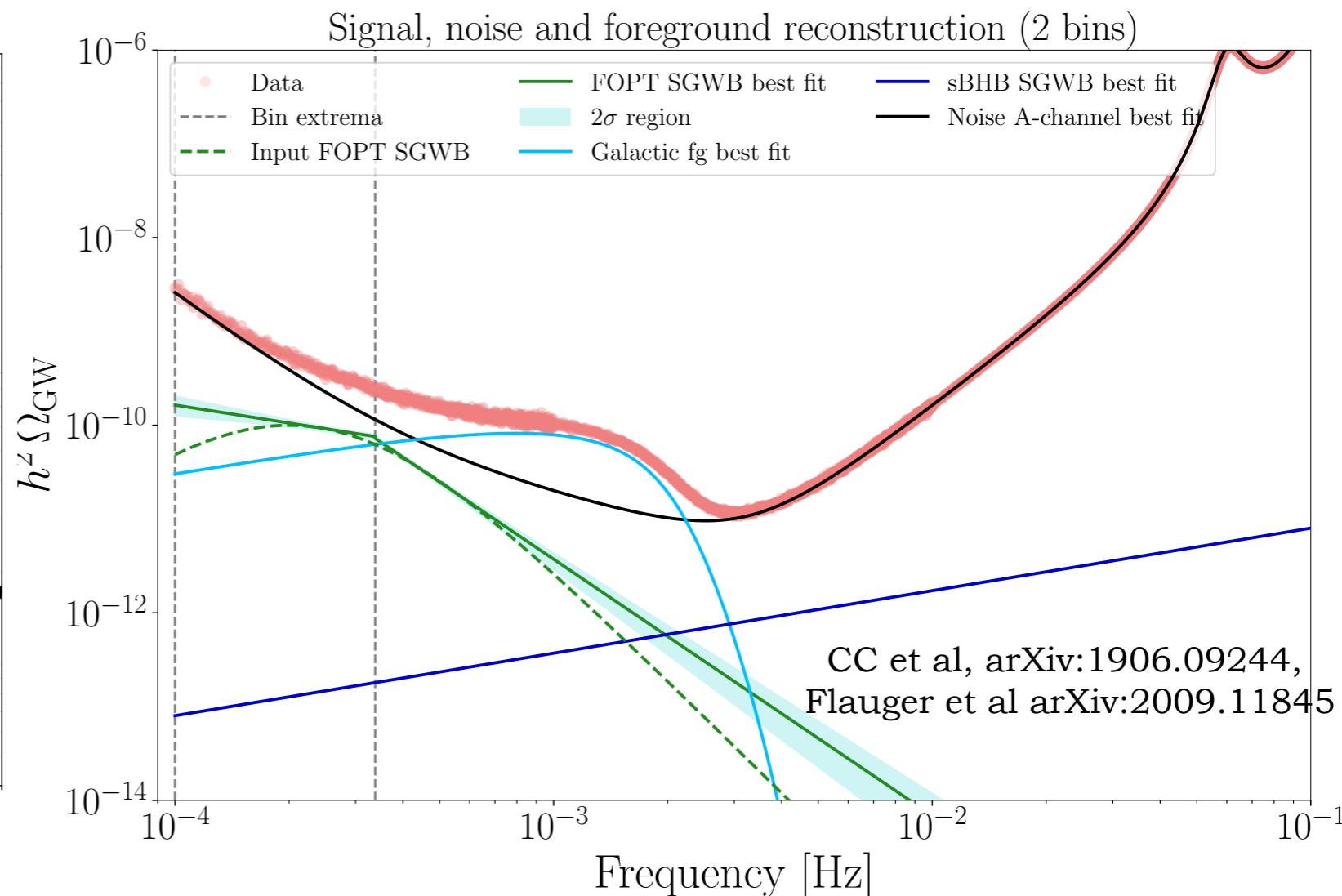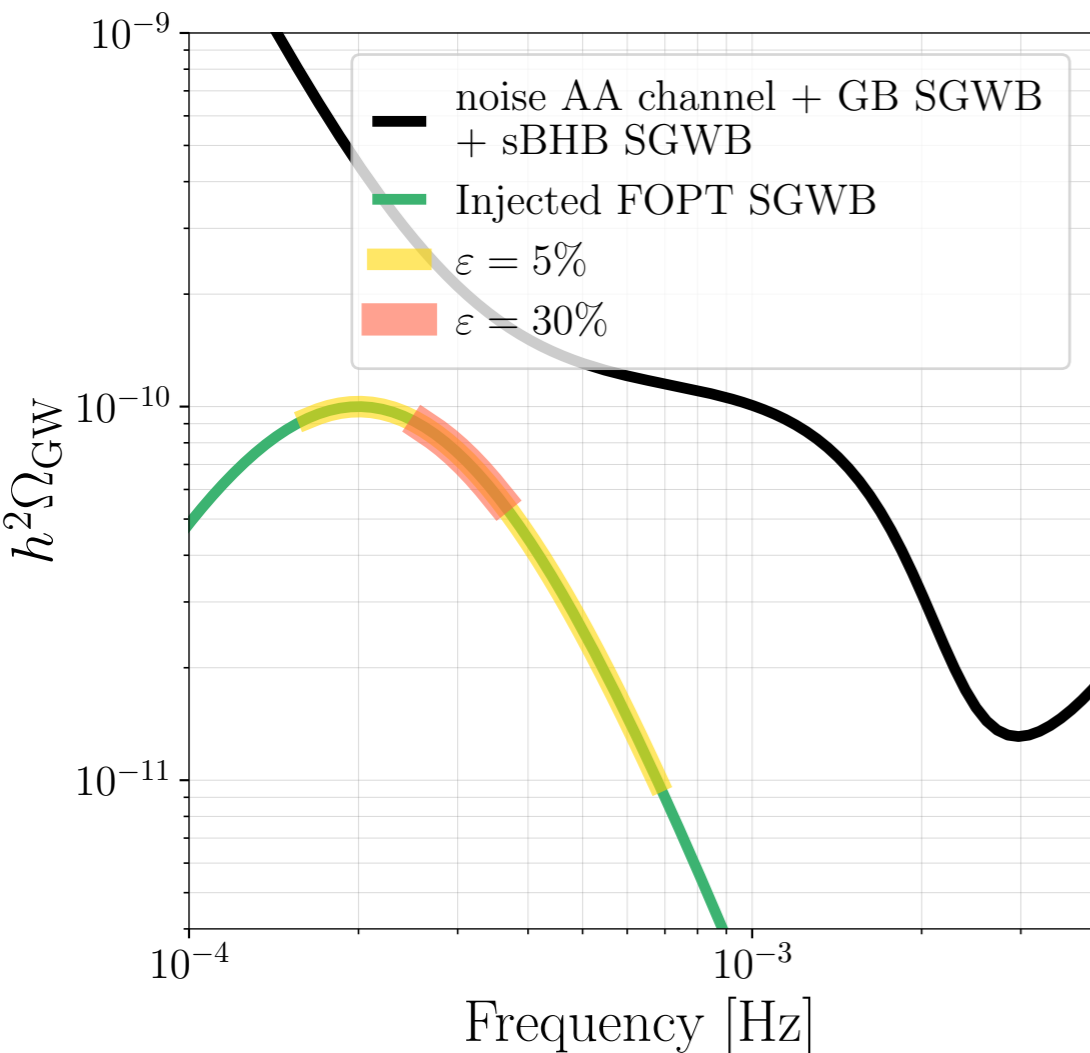# Detection challenges

Is it possible to reconstruct the GW signal spectral shape, to **identify** that the source is a first order PT, *provided we have a model for the LISA instrument noise*?

LISA Red Book arXiv:2402.07571

Signal from a singlet extension with $m_s = 0.94\,\mathrm{GeV},\ \lambda_s = 1,\ \lambda_{hs} = 0.92$

N. Karnesis, arXiv:1906.09027



CC et al, arXiv:1906.09244,
Flauger et al arXiv:2009.11845

# An example of possible detection at European Pulsar Timing Arrays

PTAs are sensitive to energy scales around the **QCD scale** and the signal is compatible with GWs generated by MHD turbulence at the QCD scale
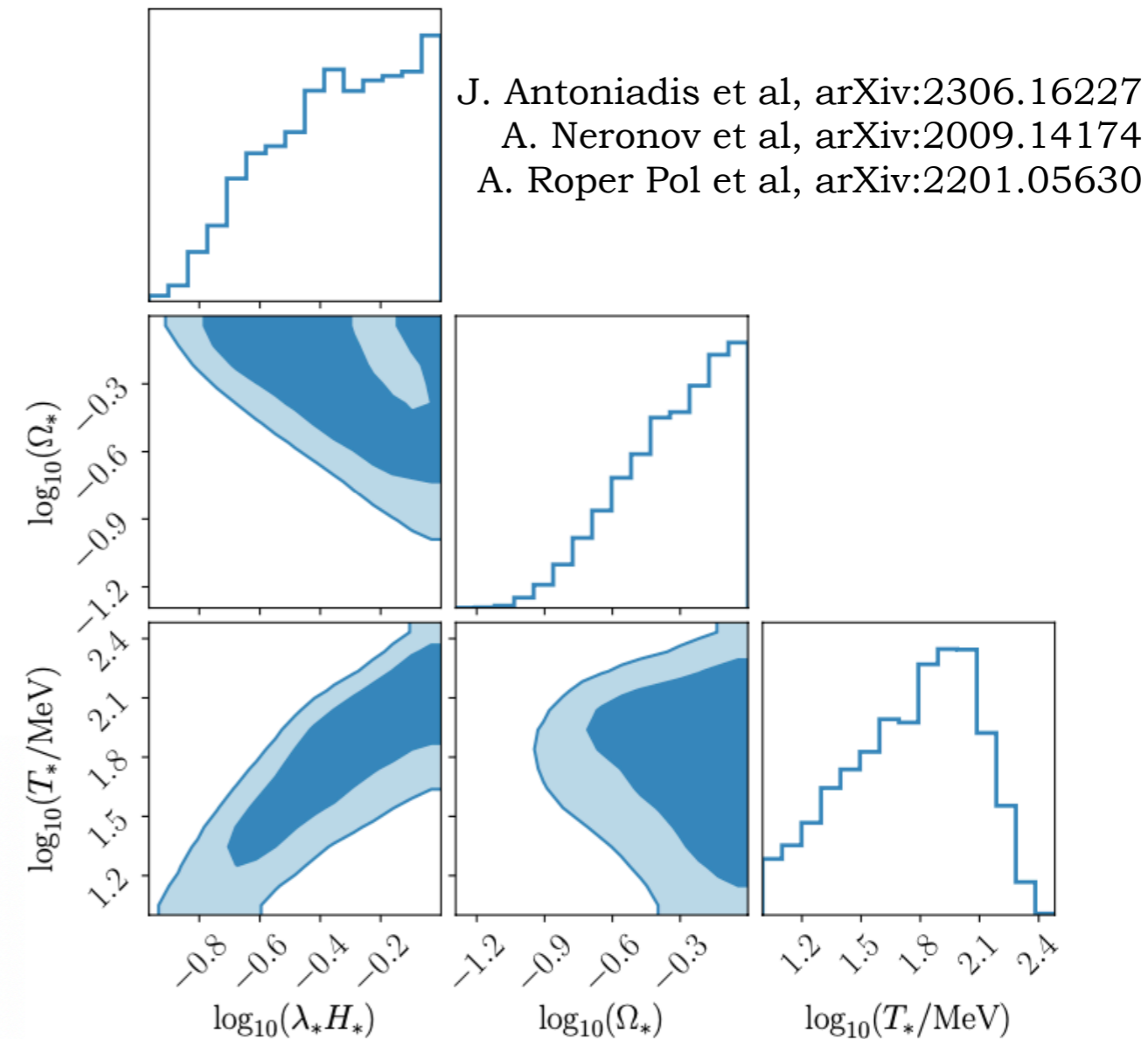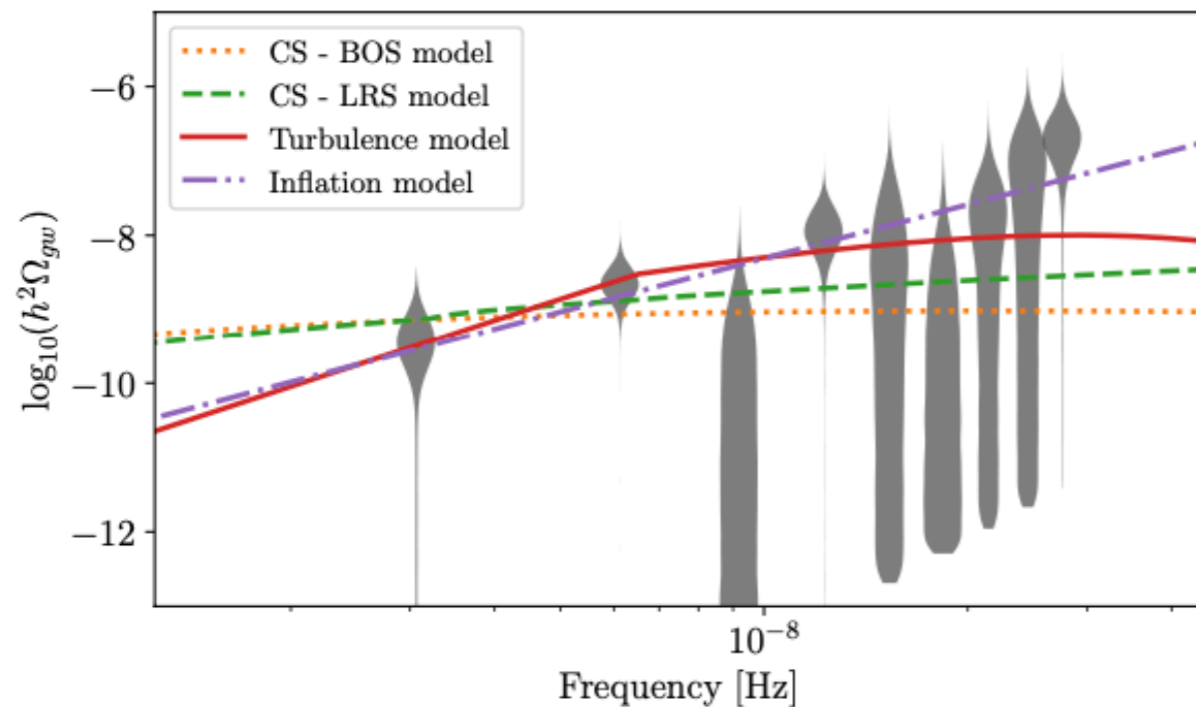


J. Antoniadis et al, arXiv:2306.16227
A. Neronov et al, arXiv:2009.14174
A. Roper Pol et al, arXiv:2201.05630

## Can we assess the signal origin?



Afzal et al arXiv:2306.16219
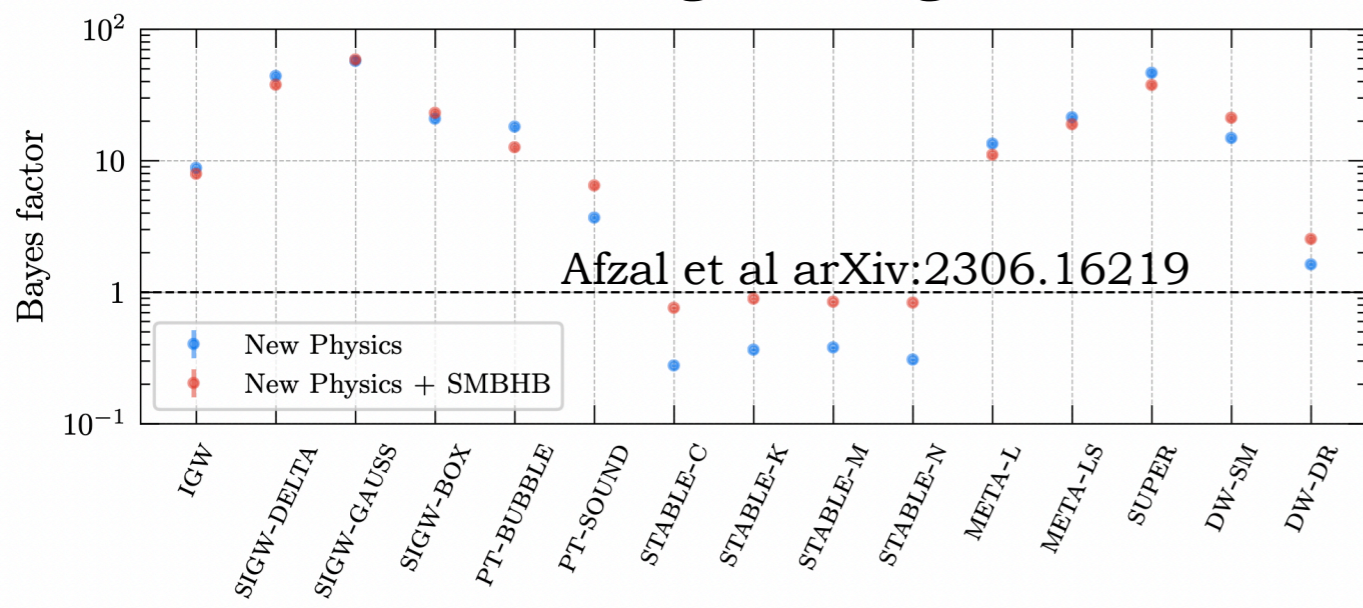


A first order QCDPT?
D. Schwarz and Stuke, arXiv:0906.3434
M. Middeldorf-Wygas et al, arXiv:2009.00036

# To summarise

- Stochastic GW backgrounds from the early universe form a fossil radiation which has the potential to provide information on high energy physics

- Present and future GW detectors have **frequency ranges serendipitously adapted** to probe interesting energy scales in the universe, at which e.g. phase transitions are expected to occur

- GW production from a phase transition has a very rich phenomenology, but **the signals are difficult to predict**: complex, non-linear physics + freedom in model building + uncertainty on the early universe conditions

- For first order phase transitions, the GW spectral shape posses in general less features than the parameters determining the signal: there are **degeneracies**. Furthermore, there are degeneracies between the thermodynamic parameters and the underlying particle physics model

- A (non-)detection can be exploited to **constrain given models,** but not to infer which model produced the signal

- In reality, things are even more complicated: LISA is the only observatory with no associated smoking gun of a SGWB detection, so will we be able to **distinguish a SGWB from the instrumental noise**?