Generic isocurvature fluctuations of scalar spectators

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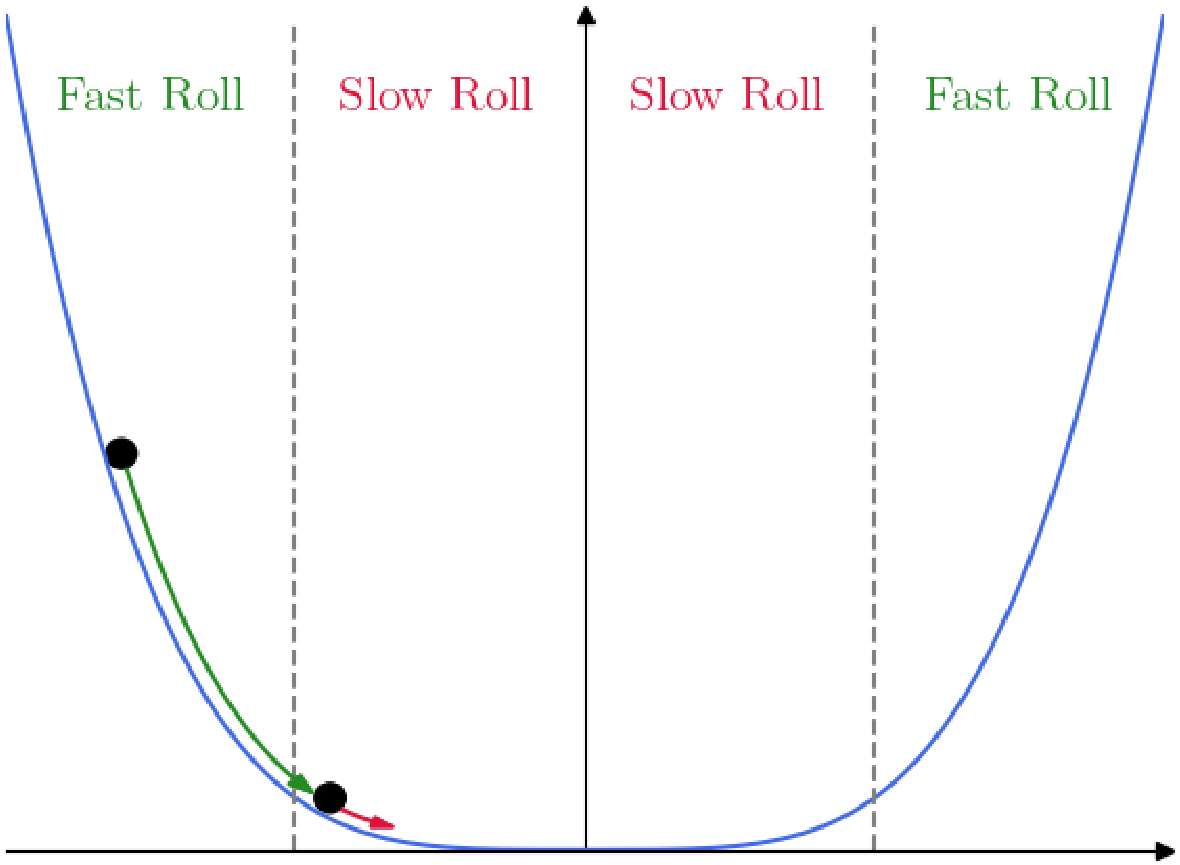
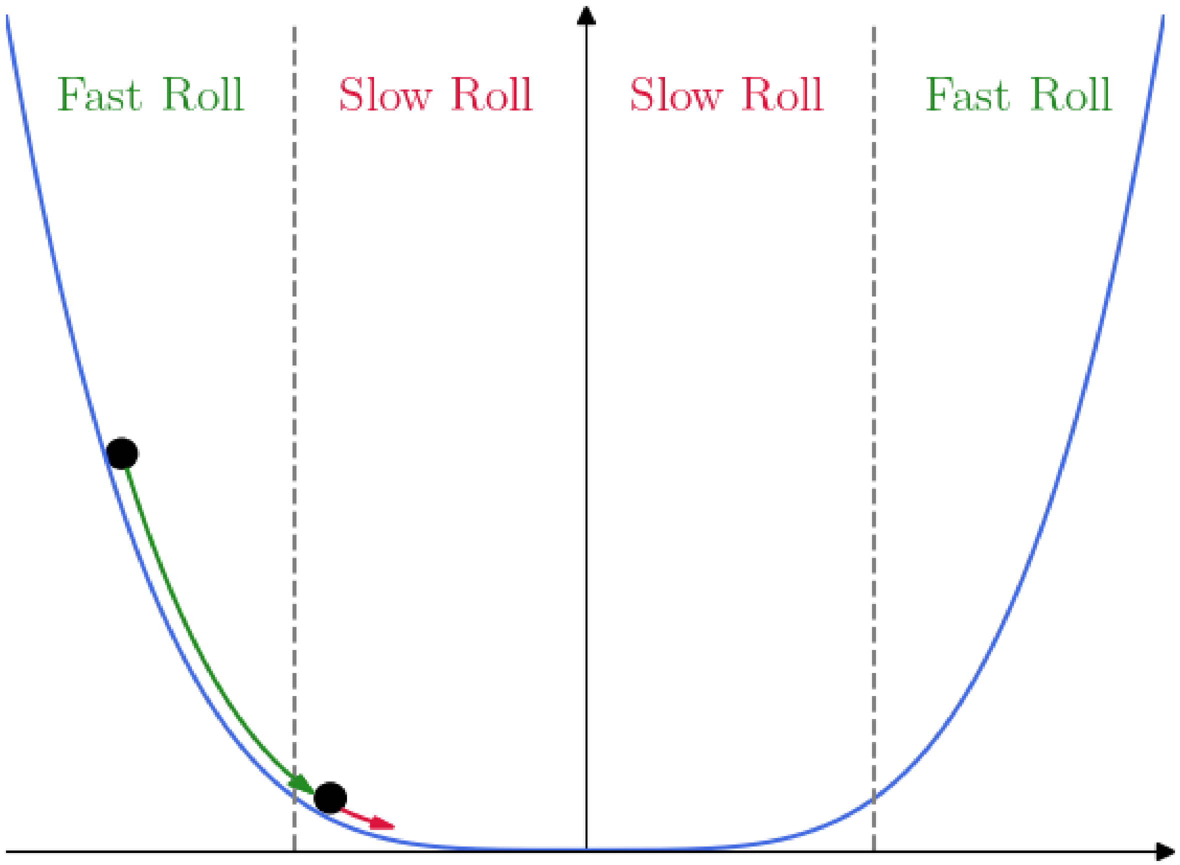
Introduction

- Scalar fields acquire isocurvature fluctuations during inflation
  - Light field ($m \lesssim H_I$) → isocurvature at all scales → constrained by CMB
  - Heavy field ($m \gtrsim H_I$) → no isocurvature
- Need $m \sim H_I$ for blue-tilted spectrum
  - Interesting signatures at small scales, e.g., GWs, non-Gaussianities
- Can model build, e.g., effective mass m_{eff} during inflation
- If scalar has nontrivial potential and inflation not too long, dynamics can generically produce blue tilt!
- Dynamics yield predictive relic abundance → target in DM parameter space

Schematic picture



Schematic picture



Slow-roll condition:

$$V'' < 3H_I^2$$

m_{eff}^2

Outline

I. Inflationary dynamics

II. Perturbations

III. Relic abundance

Inflationary dynamics

- Suppose scalar dominated by condensate $\phi(x,t) = \phi_0(t) + \delta\phi(x,t)$
- Slow-roll parameter $\alpha \equiv V''(\phi_0)/3H_I^2$
- Condensate satisfies

$$\ddot{\phi}_0 + 3H_I\dot{\phi}_0 + V'(\phi_0) = 0$$

Inflationary dynamics (fast roll)

- Suppose scalar dominated by condensate $\phi(x,t) = \phi_0(t) + \delta\phi(x,t)$
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$$\ddot{\phi}_0 + 3H_I\dot{\phi}_0 + V'(\phi_0) = 0$$

- If initially $\alpha \gg 1$, scalar oscillates and redshifts

$$\rho \sim \exp(-3(1+w)N) \implies \alpha \sim \exp(-\#N)$$

Inflationary dynamics (slow roll)

- Suppose scalar dominated by condensate $\phi(x,t) = \phi_0(t) + \delta\phi(x,t)$
- Slow-roll parameter $\alpha \equiv V''(\phi_0)/3H_I^2$
- Condensate satisfies

$$\ddot{\phi}_0 + 3H_I\dot{\phi}_0 + V'(\phi_0) = 0$$

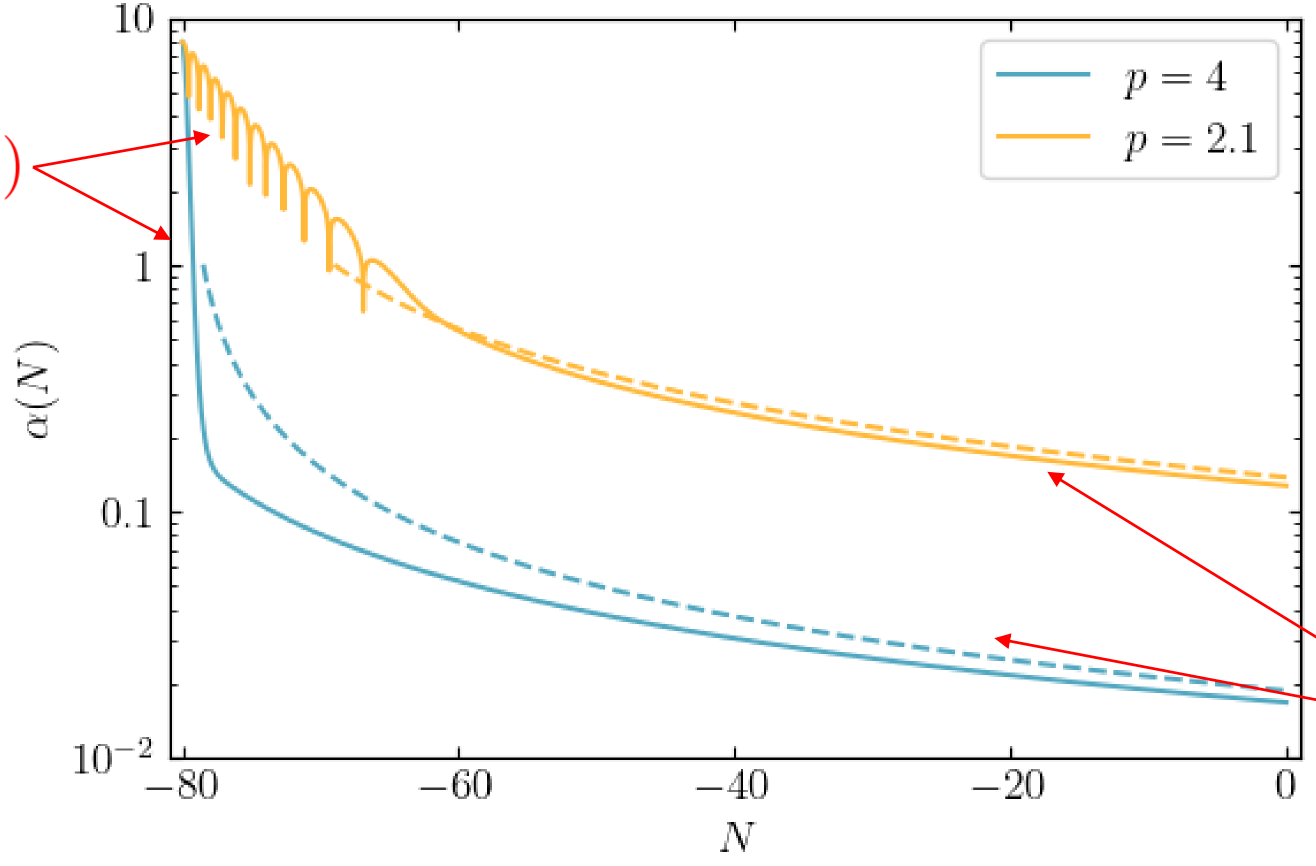
- Slow-roll solution

$$\dot{\phi}_0 = -\frac{V'(\phi_0)}{3H_I}$$

- Valid when $|\ddot{\phi}_0| < H_I|\dot{\phi}_0| \implies \alpha < 1$
- Slow roll leads to $\alpha \sim 1/N$

Evolution of α

$\alpha \sim \exp(-\#N)$



$V(\phi) \sim \phi^p$

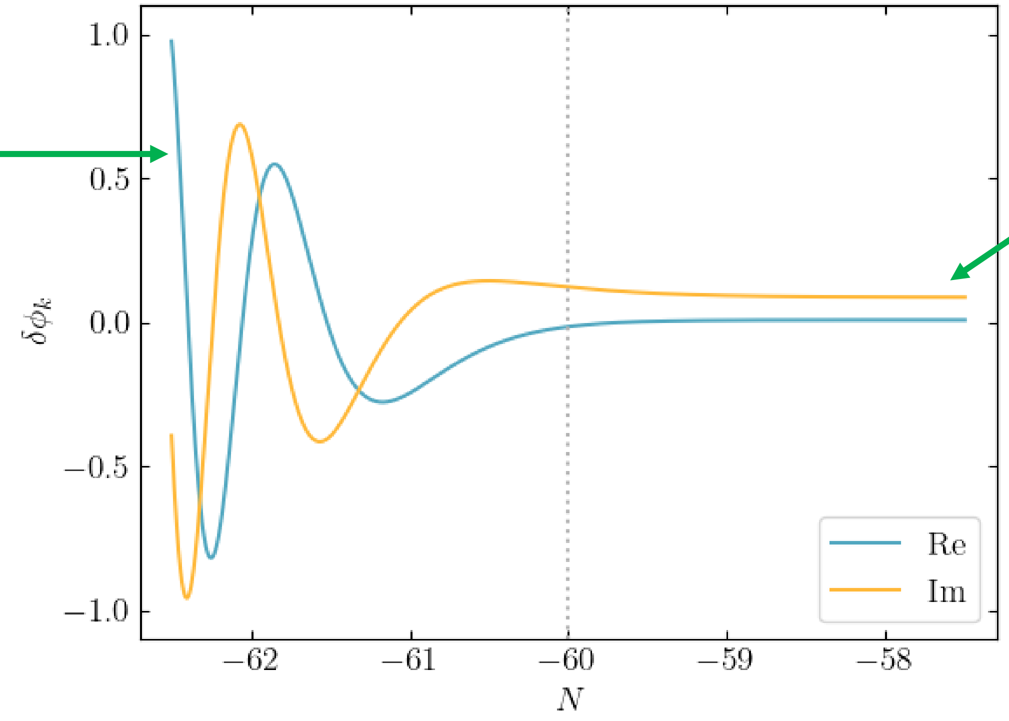
$\alpha \sim 1/N$

Coefficient generically $\mathcal{O}(1)$, but can be large

Perturbations (no potential)

$$\delta\ddot{\phi}_k + 3H_I\delta\dot{\phi}_k + \frac{k^2}{a^2}\delta\phi_k = 0$$

Oscillate and decay inside horizon (Bunch-Davies)

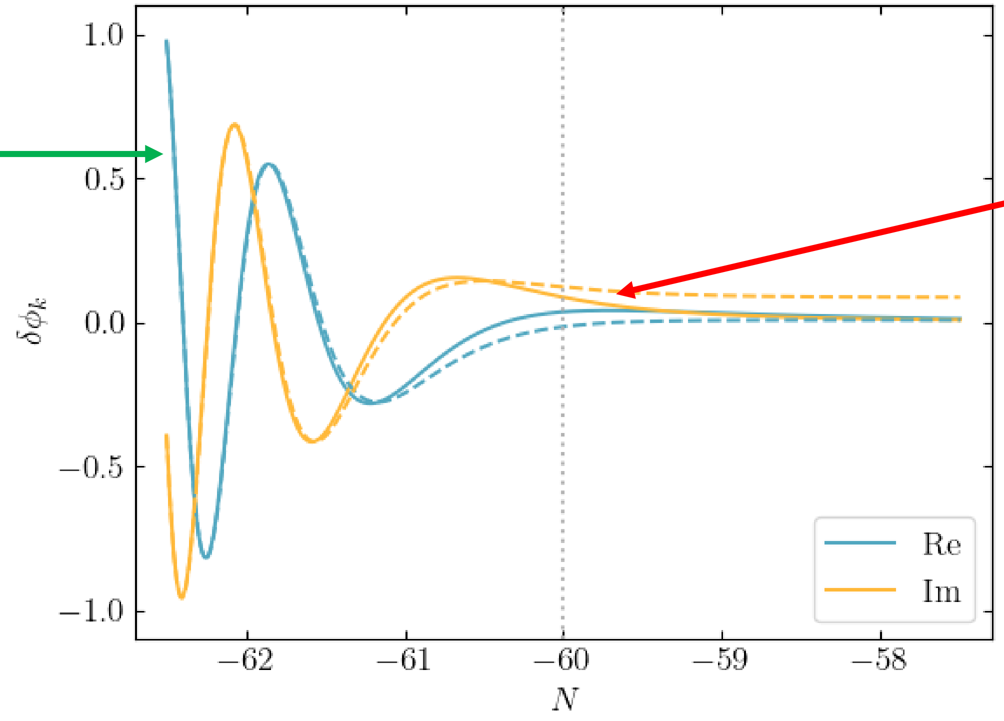


Constant outside horizon

Perturbations (with potential)

$$\delta\ddot{\phi}_k + 3H_I\delta\dot{\phi}_k + \left(\frac{k^2}{a^2} + V''(\phi_0)\right)\delta\phi_k = 0$$

Oscillate and decay inside horizon (Bunch-Davies)



Decays outside horizon

Spectral tilt

- Potential causes decay $\delta\phi_k \sim \exp(-\alpha N)$ outside horizon:

$$P_\delta(k)|_{\mathrm{rh}} \sim \exp\left(-2\int_{N_*}^{N_{\mathrm{rh}}} \alpha(N)dN\right),$$

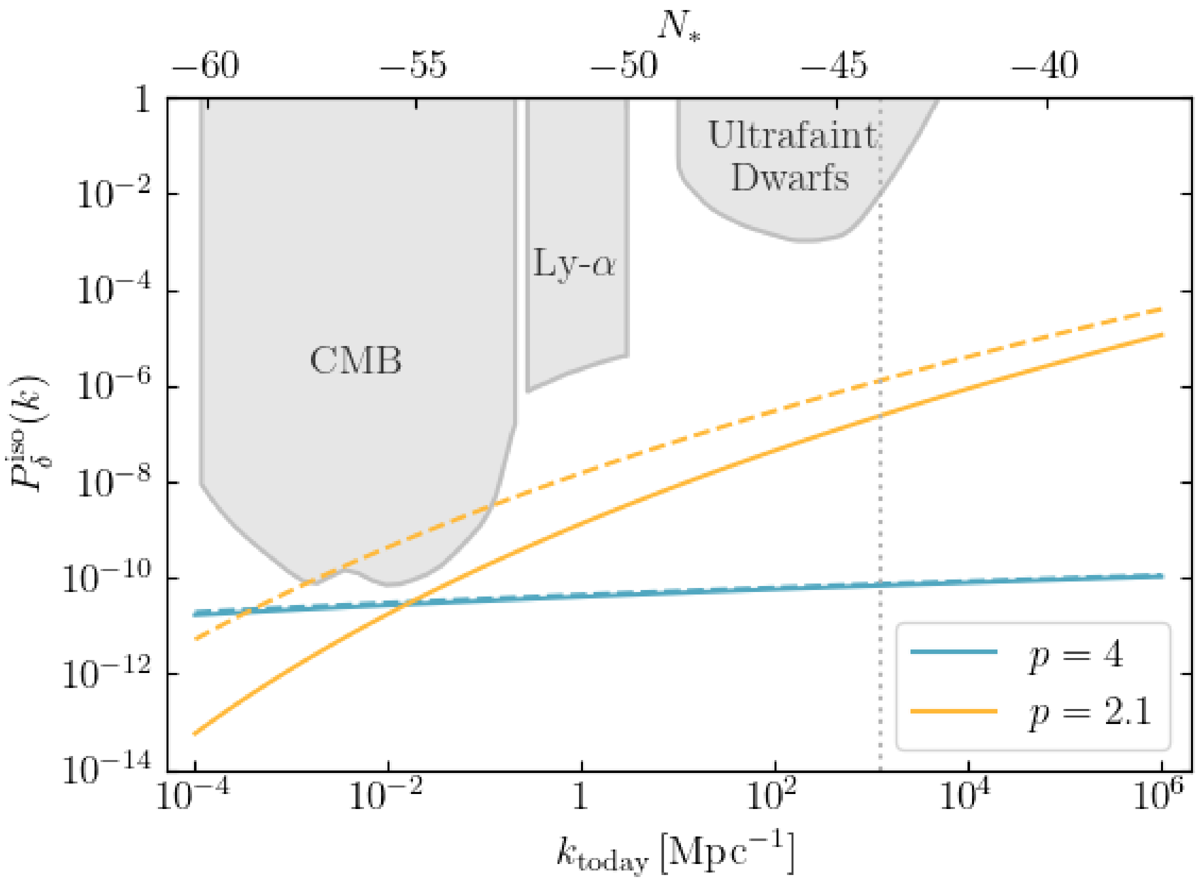
where N_* denotes horizon crossing

- Longer wavelength modes spend longer outside horizon → blue tilt!

$$\frac{d\log P_\delta}{d\log k} = 2\alpha(N_*)$$

Isocurvature power spectrum

Larger α, larger tilt!



$$V(\phi) \sim \phi^p$$

$$H_I = 10^{12}\,\mathrm{GeV}$$

$$N_{\rm tot} = 80$$

Relic abundance

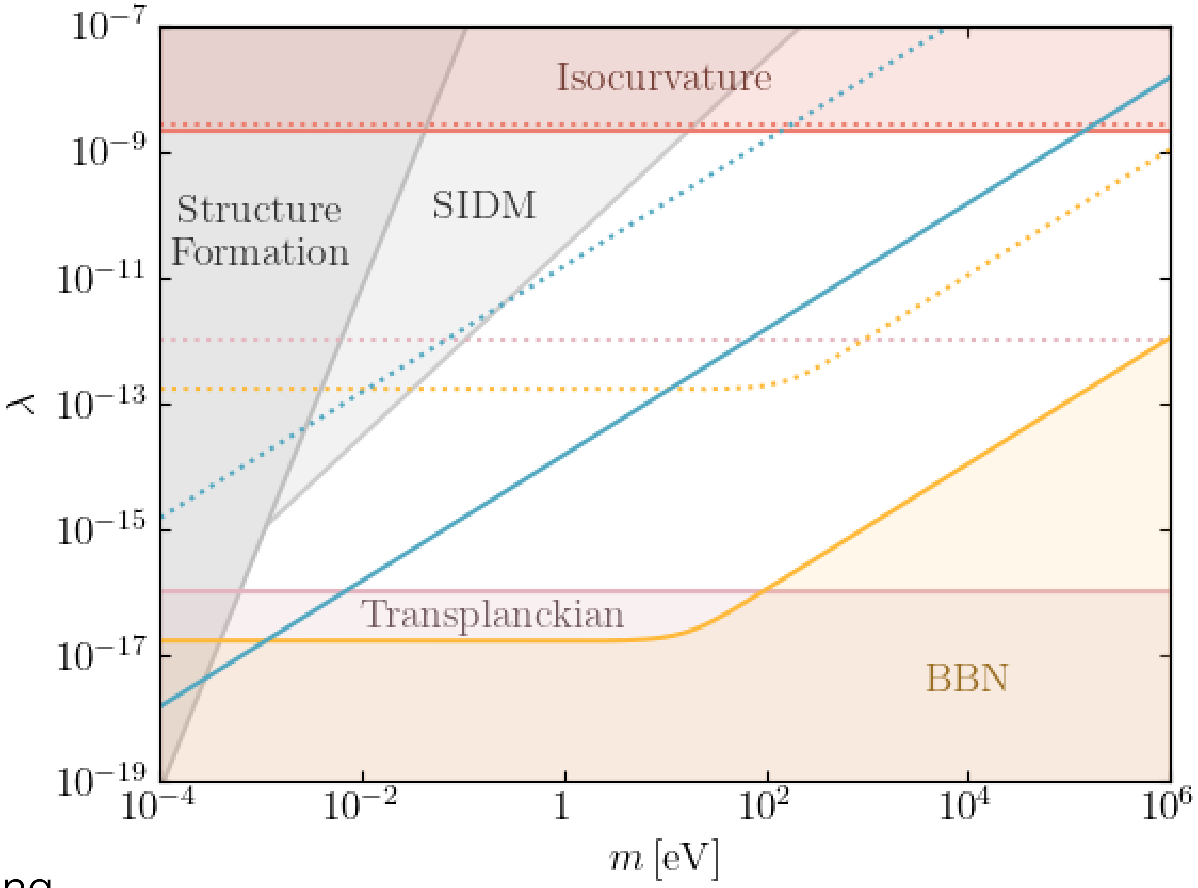
- If ϕ long-lived, energy density redshifts in phases:

$$V(\phi) = \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4 \implies \begin{cases} 1.\ \rho \propto a^0 & \text{[frozen]} \\ 2.\ \rho \propto a^{-4} & \text{[quartic]} \\ 3.\ \rho \propto a^{-3} & \text{[matter]} \end{cases}$$

[quartic] ← potential dependent

- Final abundance independent of initial conditions (for fixed inflationary parameters)

Quartic dark matter



$$H_I = 10^{10}\,\text{GeV} \quad \text{(solid)}$$
$$= 10^{12}\,\text{GeV} \quad \text{(dashed)}$$

$$N_{\text{tot}} = 80$$

*assumes instantaneous reheating

Conclusion

- If scalar has potential and inflation not too long, generic blue-tilted isocurvature!
- Scalar spends most of inflation near slow-roll boundary, where $m_{\rm eff} \sim H_I$
- Enhanced isocurvature on small scales → GWs or non-Gaussianities
- If scalar long-lived, relic abundance is predictable
- Quartic scalar can be all of DM with well-defined parameter space target!

Backup Slides

Slow-roll solution

- During slow roll, condensate EOM can be written as

$$\frac{d\alpha}{dN} = -\kappa\alpha^2, \qquad \kappa \equiv \frac{V'''(\phi_0)V'(\phi_0)}{V''(\phi_0)^2} \left[= \frac{p-2}{p-1} \text{ for } V(\phi) \sim \phi^p\right]$$

- Solved by

$$\alpha(N) = \frac{1}{\kappa(N - N_{\rm sr}) + 1}, \qquad \alpha(N_{\rm sr}) \equiv 1$$

Necessary conditions

$$V''(\phi_{0,i}) > 3H_I^2 \qquad \text{(begin in fast-roll regime)}$$

$$V''(\phi) < 3H_I^2 \text{ for some } \phi \qquad \text{(slow-roll regime exists)}$$

$$V'''(\phi)V'(\phi) > 0 \qquad \text{(roll toward slow-roll regime)}$$

$$V(\phi_{0,i}) \ll 3H_I^2 M_{\rm pl}^2 \qquad \text{(spectator subdominant)}$$

Perturbation solution

- In terms of $f_k = a\delta\phi_k$ and $\tau = -(aH_I)^{-1}$, perturbation EOM becomes

$$\partial_\tau^2 f_k + \left(k^2 - \frac{\partial_\tau^2 a}{a} + a^2 V''(\phi_0)\right) f_k = 0$$

- For $|k\tau| \gg 1$, harmonic oscillator of frequency k
- Bunch-Davies initial conditions

$$f_k(\tau \to -\infty) = \frac{1}{\sqrt{2k}} e^{-ik\tau}$$

- For $|k\tau| \ll 1$,

$$\partial_\tau^2 f_k + \frac{3\alpha - 2}{\tau^2} f_k = 0 \implies f_k \sim (-\tau)^{\alpha-1} \implies \delta\phi_k \sim \exp(-\alpha N)$$

Power spectrum

- Spectrum of field fluctuations (at reheating)

$$P_{\delta\phi}(k)|_{\rm rh} \equiv \frac{k^3}{2\pi^2}|\delta\phi_k|^2 \approx \left(\frac{H_I}{2\pi}\right)^2 \cdot \exp\left(-2\int_{N_*}^{N_{\rm rh}} \alpha(N)dN\right)$$

- Related to density perturbations by

$$\delta_k \equiv \frac{\delta\rho_k}{\rho} \approx \frac{V'(\phi_0)}{V(\phi_0)}\delta\phi_k$$

- Spectrum of density perturbations (at reheating)

$$P_\delta(k)|_{\rm rh} \approx \frac{1}{12\pi^2\alpha_{\rm rh}}\left[\frac{V'^2V''}{V^2}\right]_{\rm rh} \exp\left(-2\int_{N_*}^{N_{\rm rh}} \alpha(N)dN\right)$$

Late-time evolution

- "Separate universes": each patch undergoes same dynamics as zero mode
- Relic abundance calculation gives

$$\rho_f \propto \rho_{\rm rh}^{3/8} H_I^{3/2}$$

- Final perturbations consist of isocurvature + adiabatic

$$\delta_f = \frac{3}{8}\delta_{\rm rh} - \frac{3}{2}\Phi$$

- Applies for long-wavelength modes

$$\int dt \frac{k}{\sqrt{3}a} \lesssim 1 \implies k_{\rm today} \lesssim 1200\,{\rm Mpc}^{-1} \cdot \left(\frac{m}{10\,{\rm eV}}\right)$$