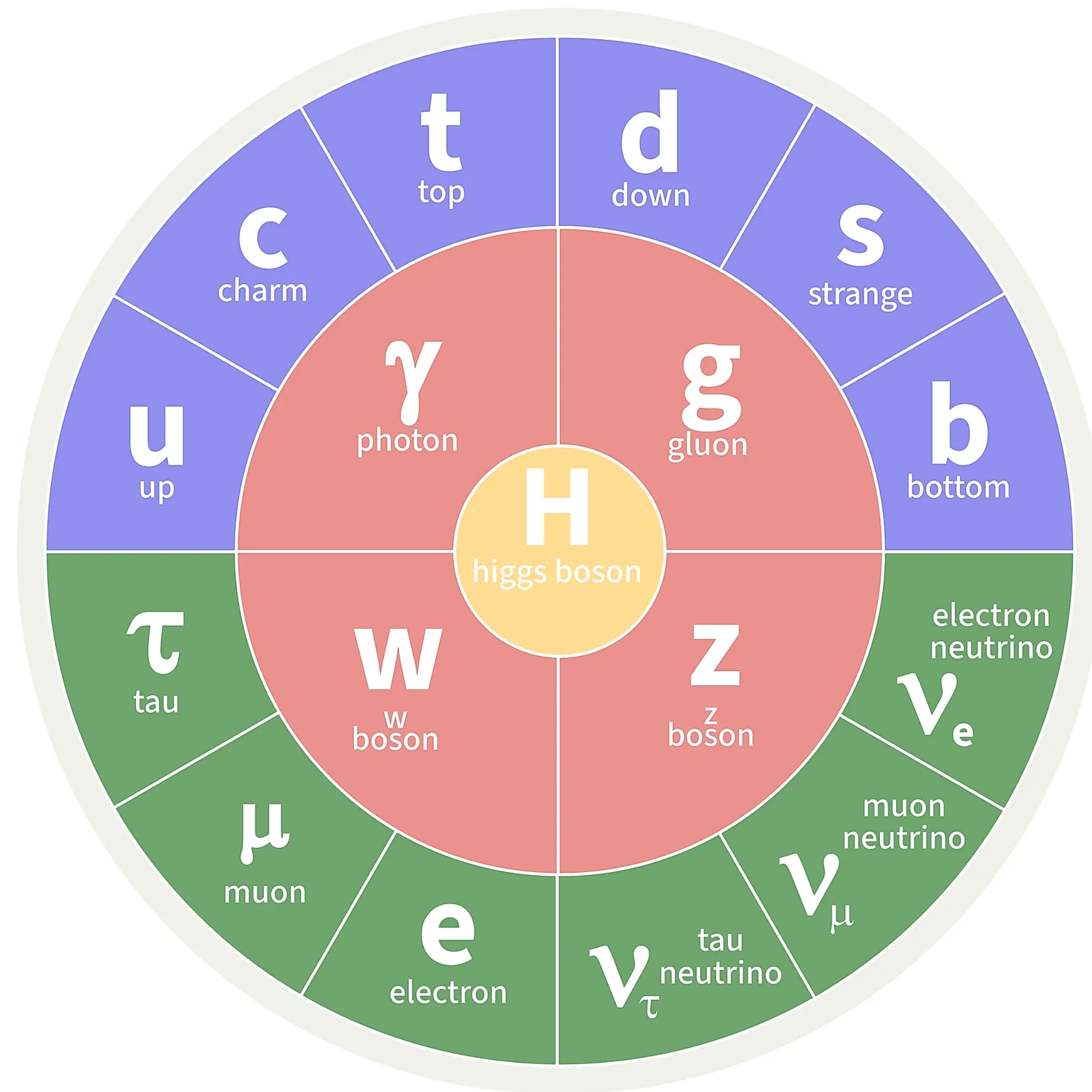


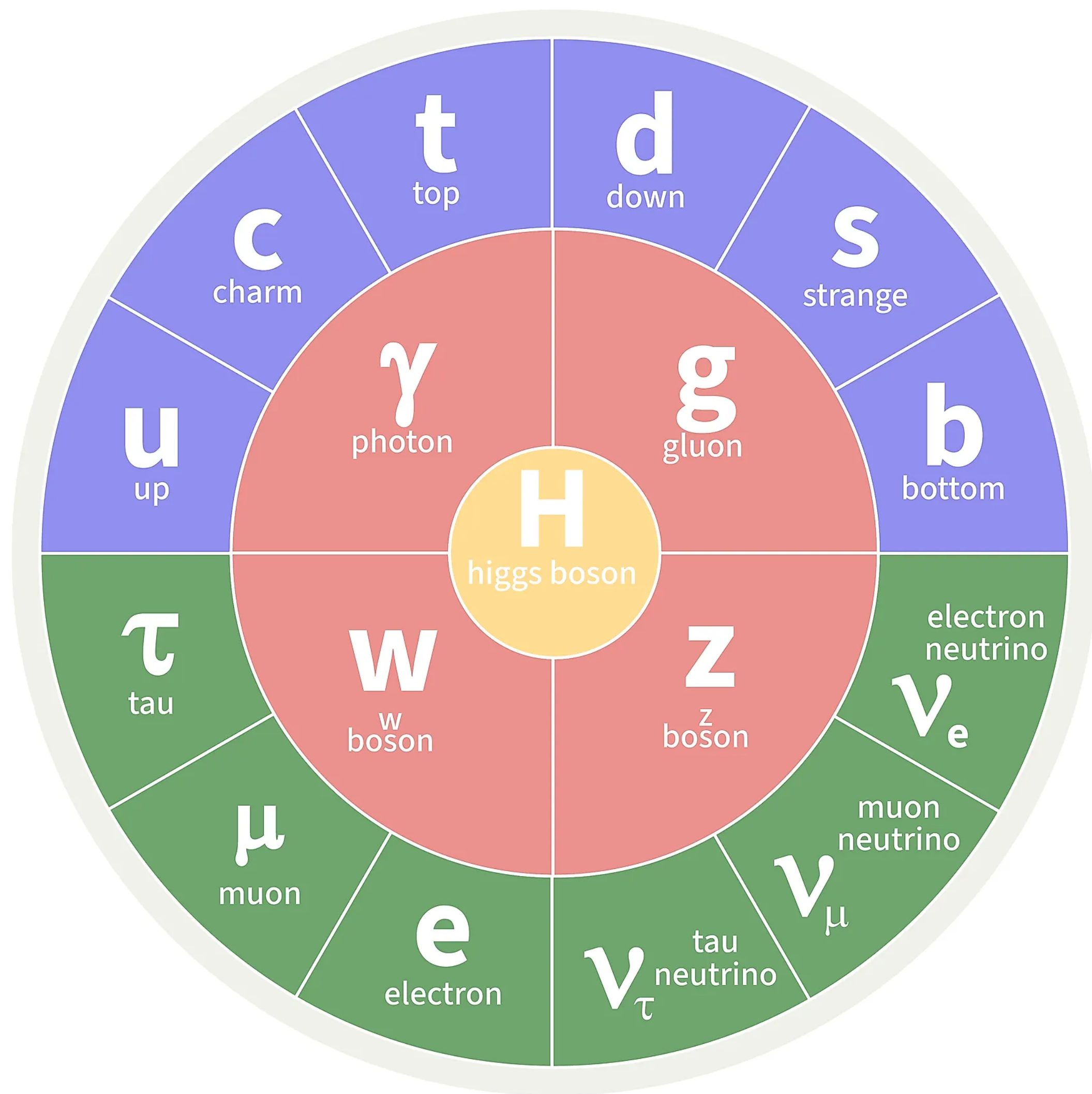
# Baryogenesis in an Axiverse

David Cyncynates, ICTP Trieste

Based on [2511.XXXXX] and [26XX.XXXXX] with Pouya Asadi & Stefania Gori







**Gauge hierarchy**

**Cosmological constant**

**Flavor**

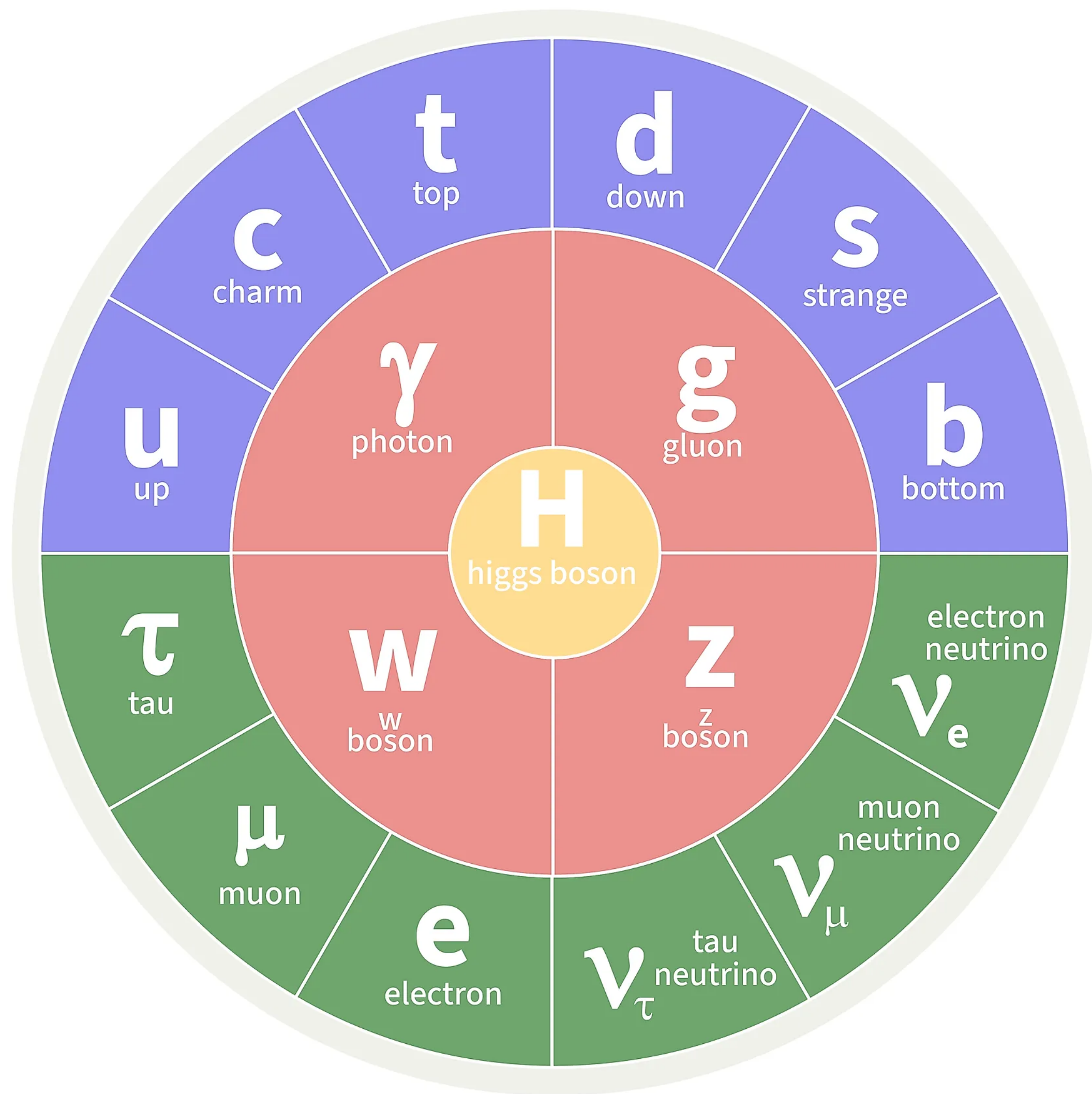
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**Dark Matter**

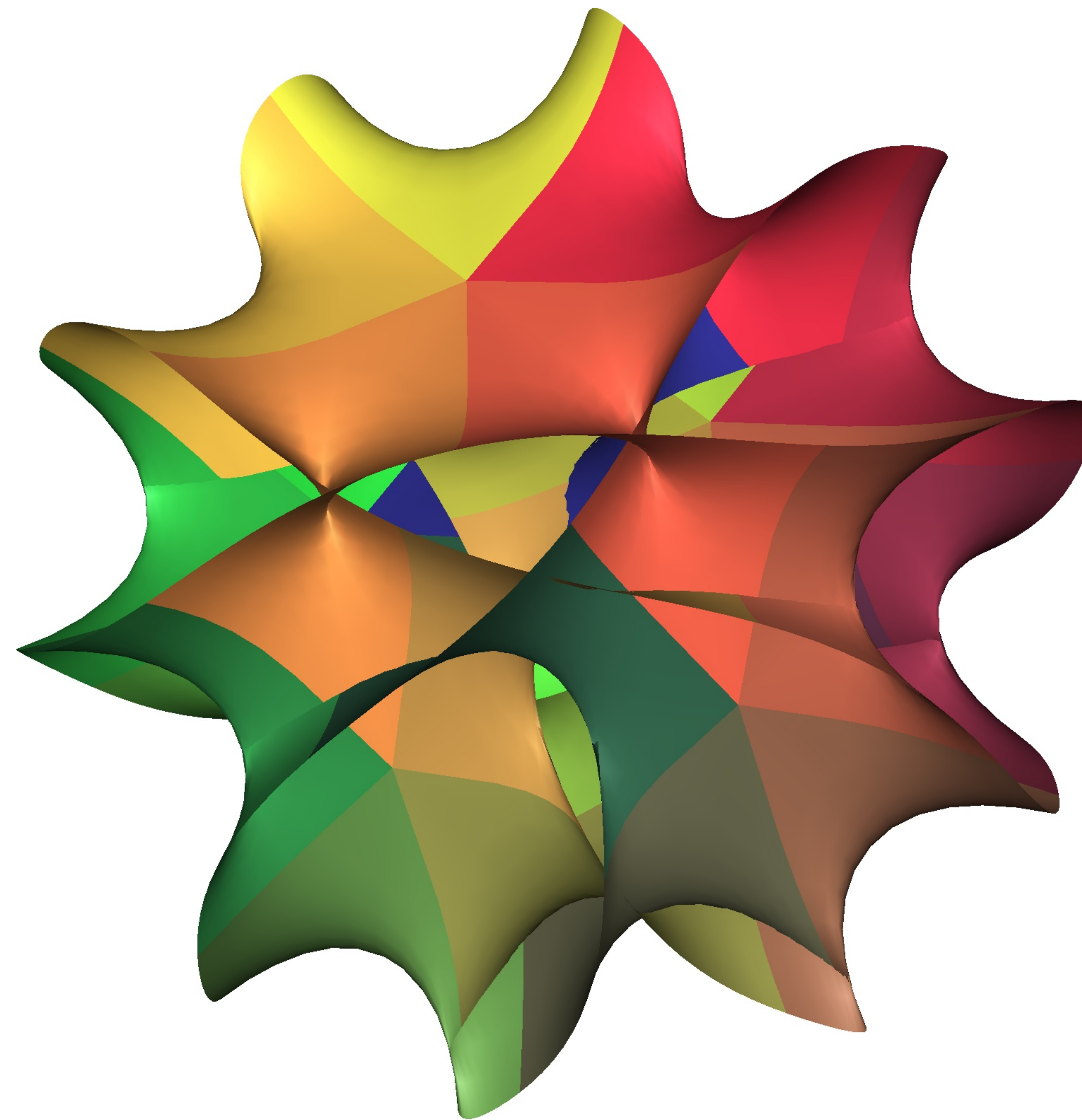
**Baryon asymmetry**

**Quantum gravity**

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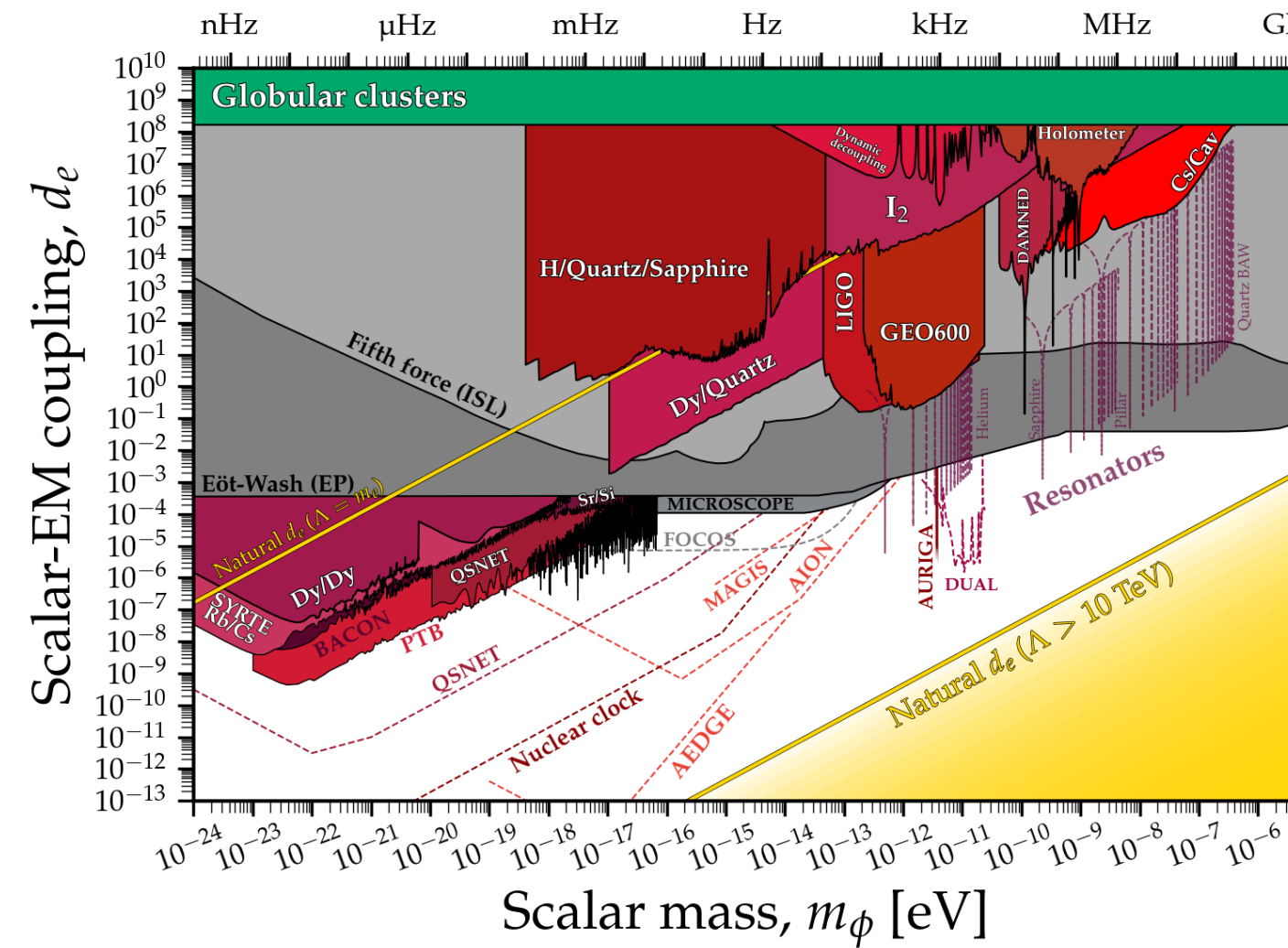
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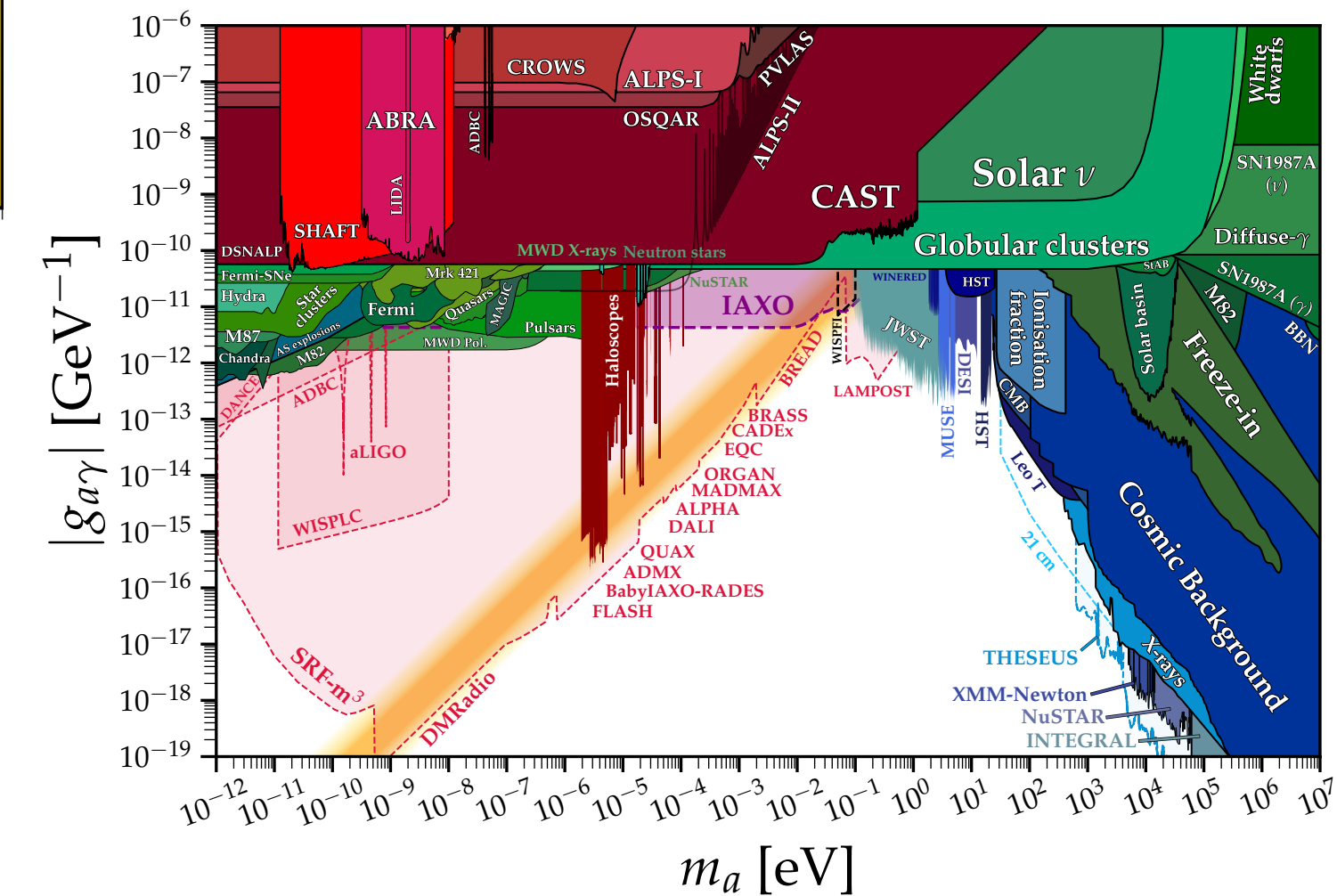
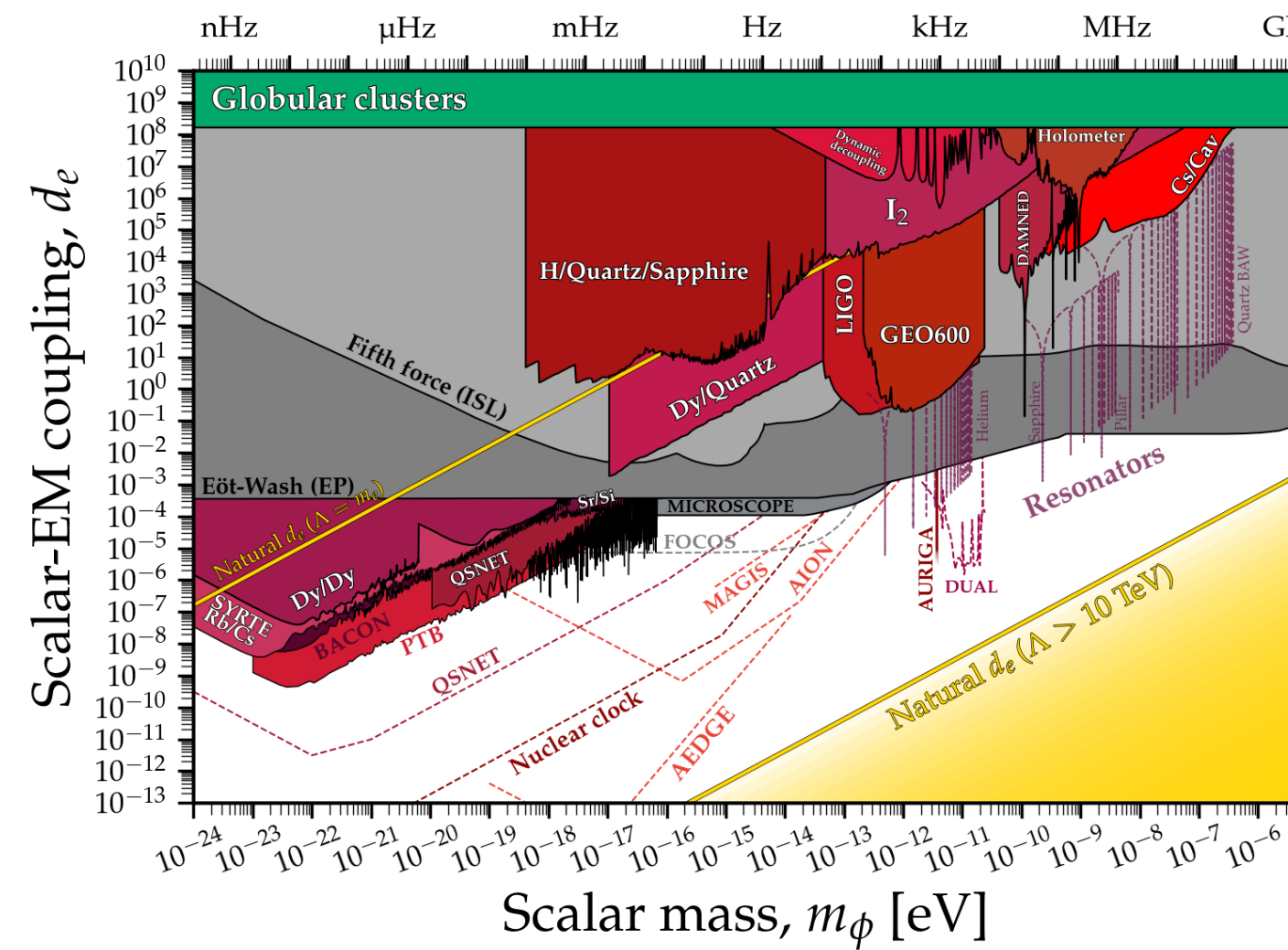
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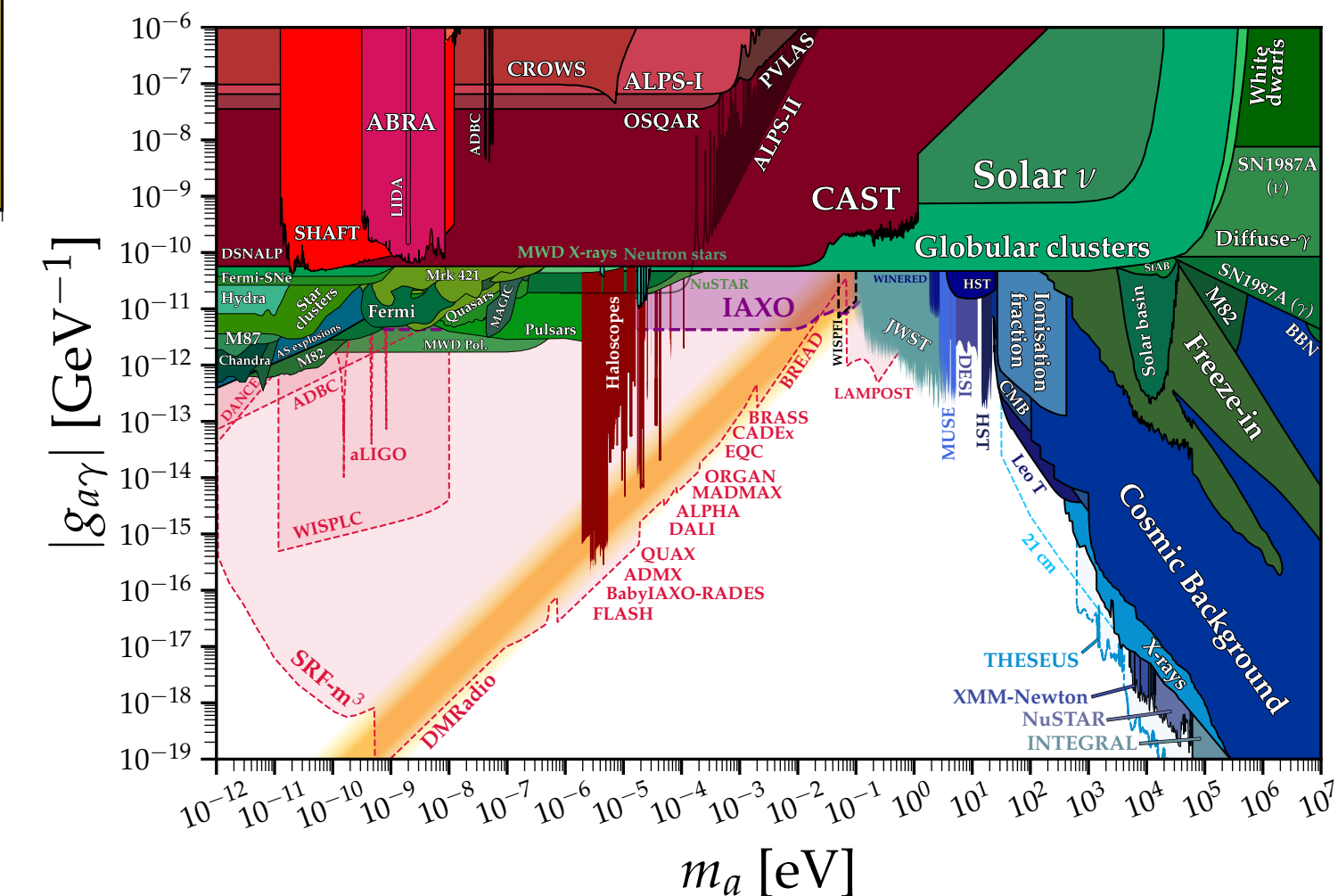
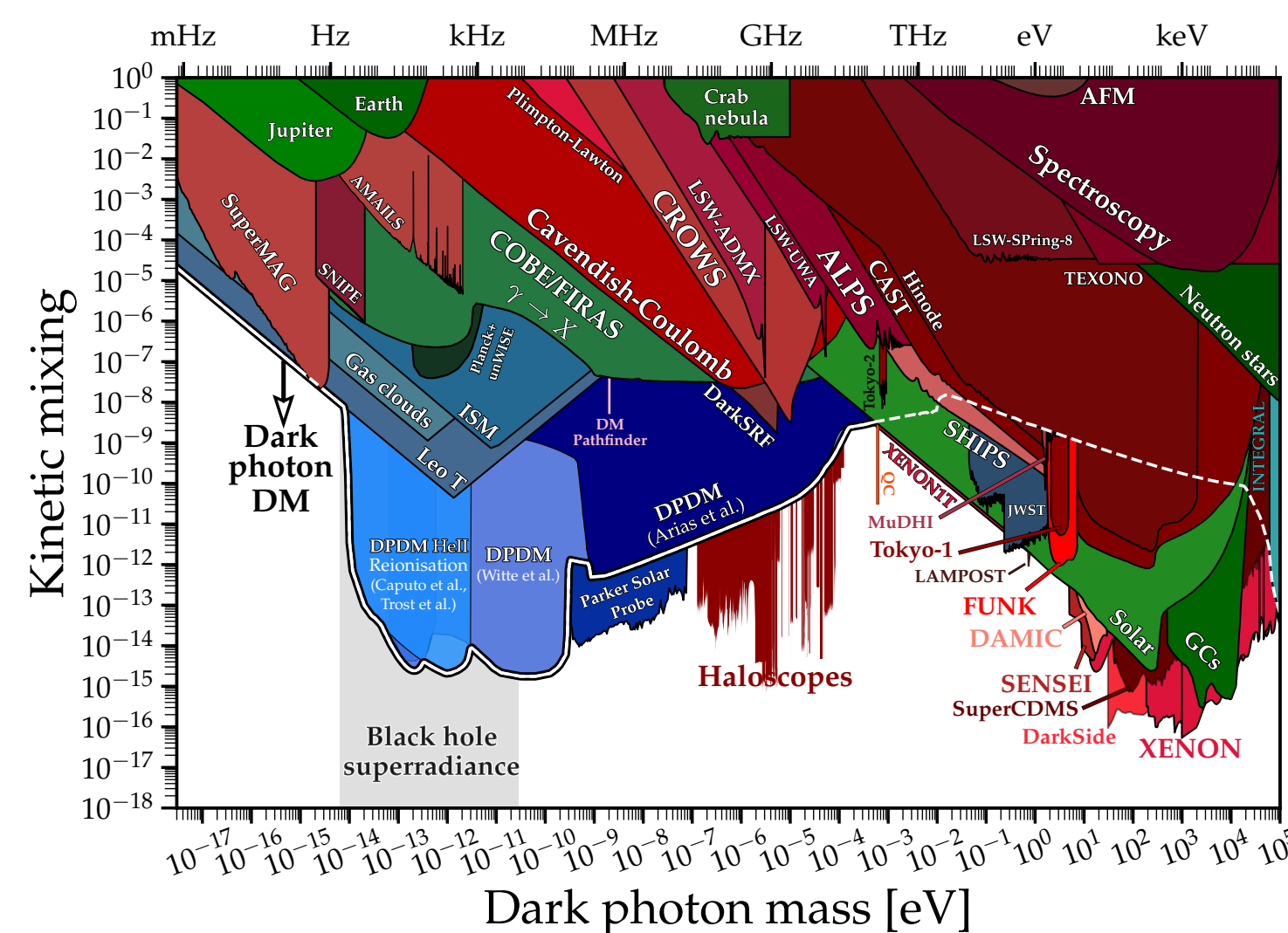
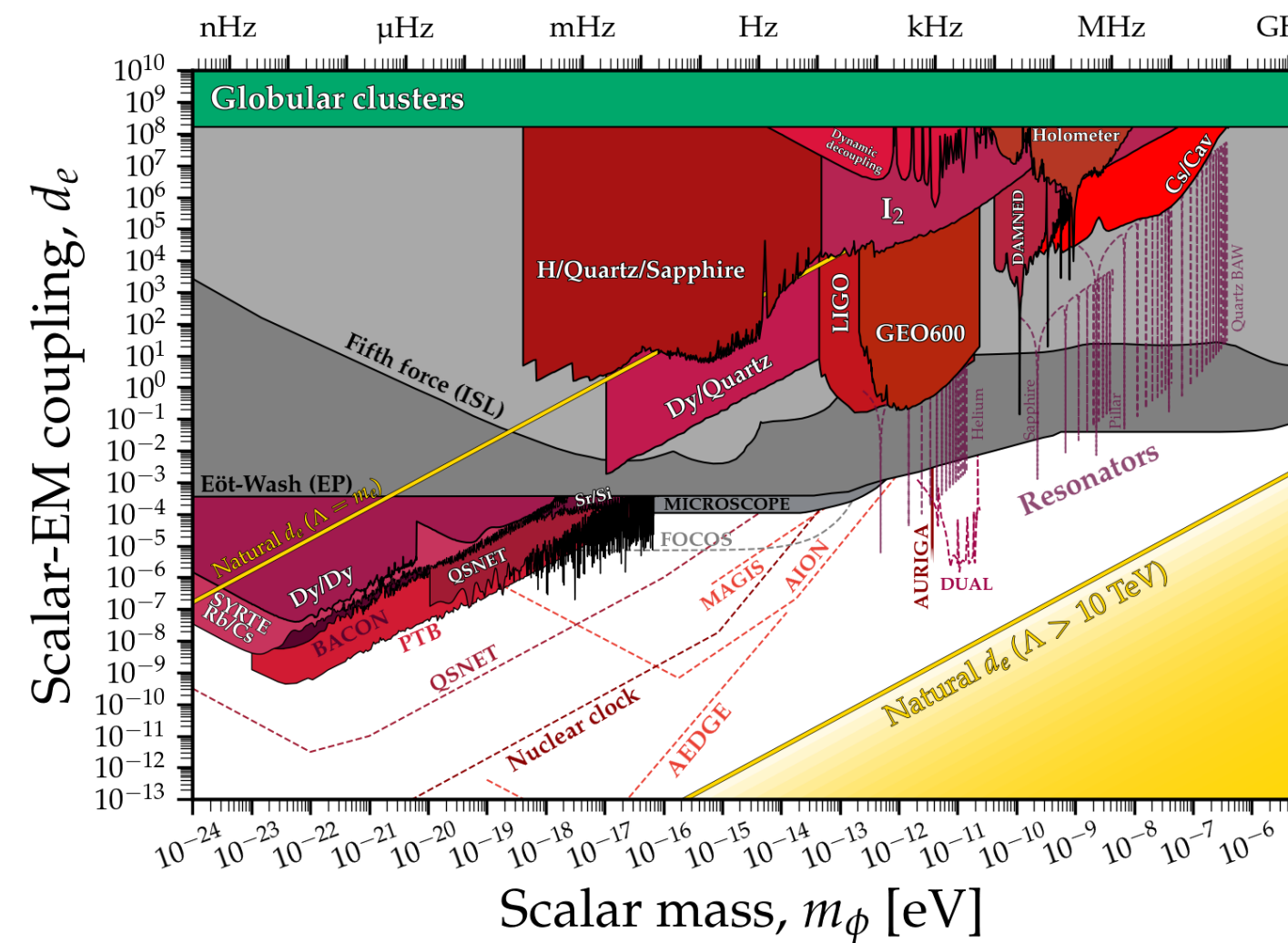
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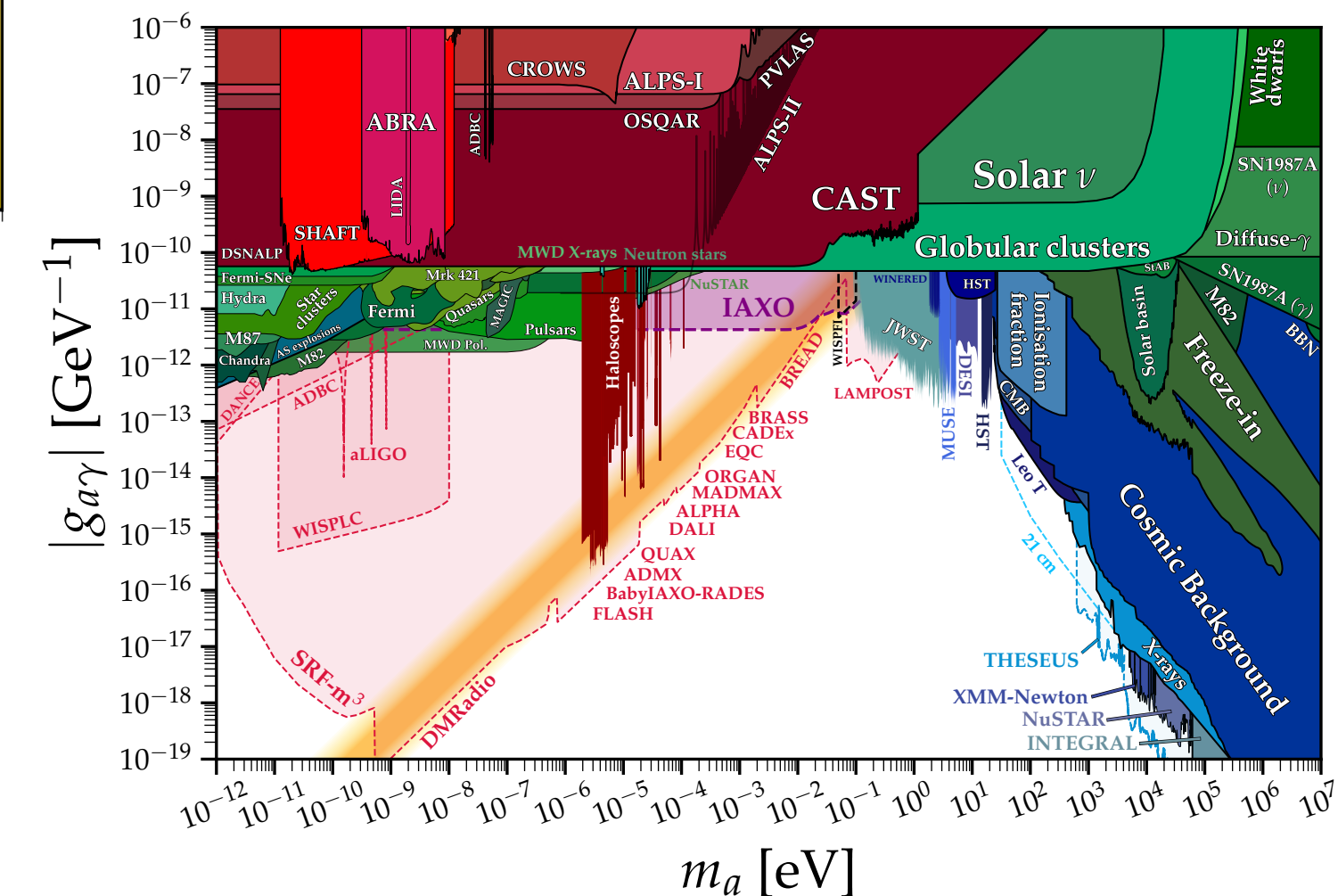
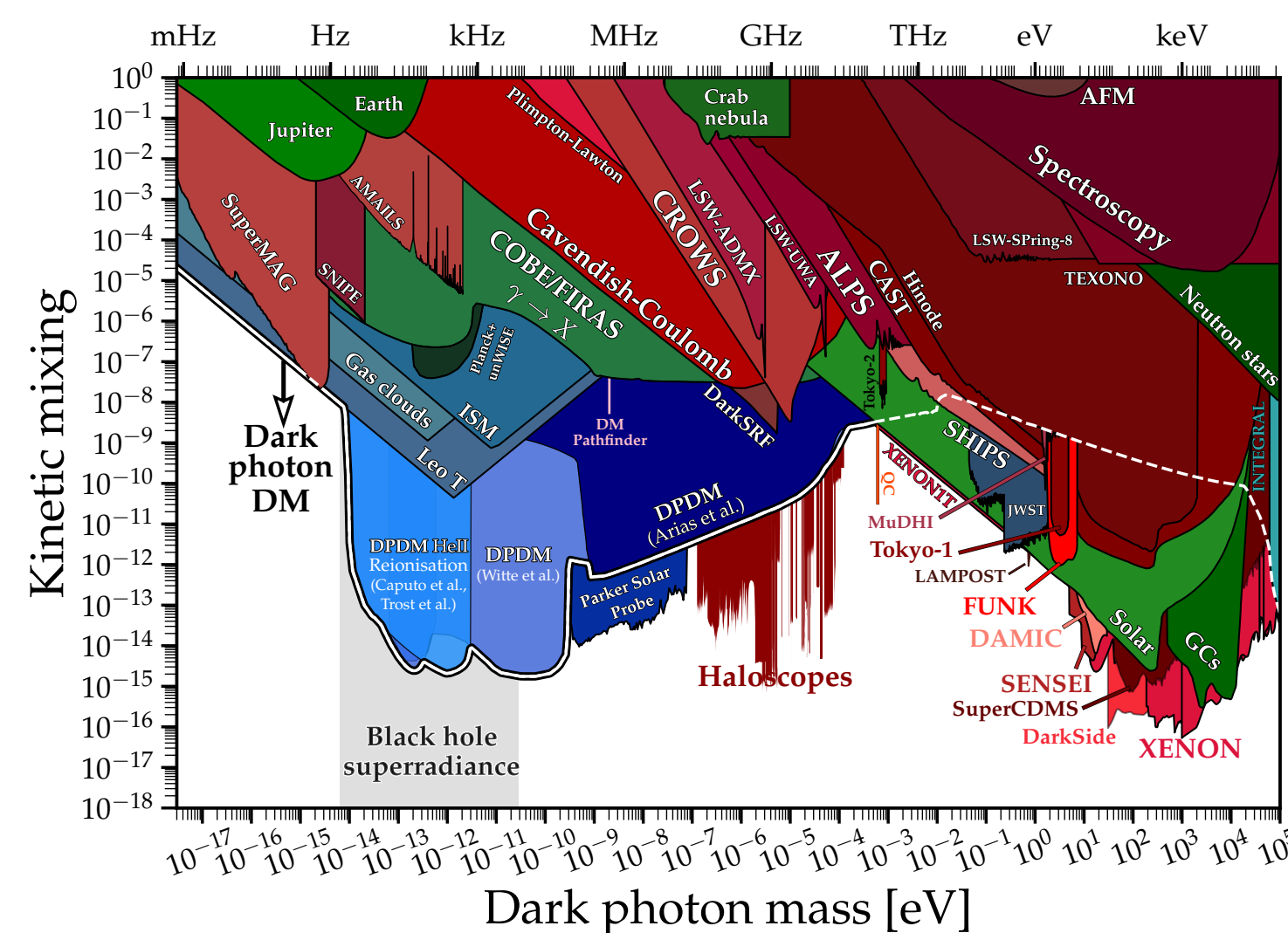
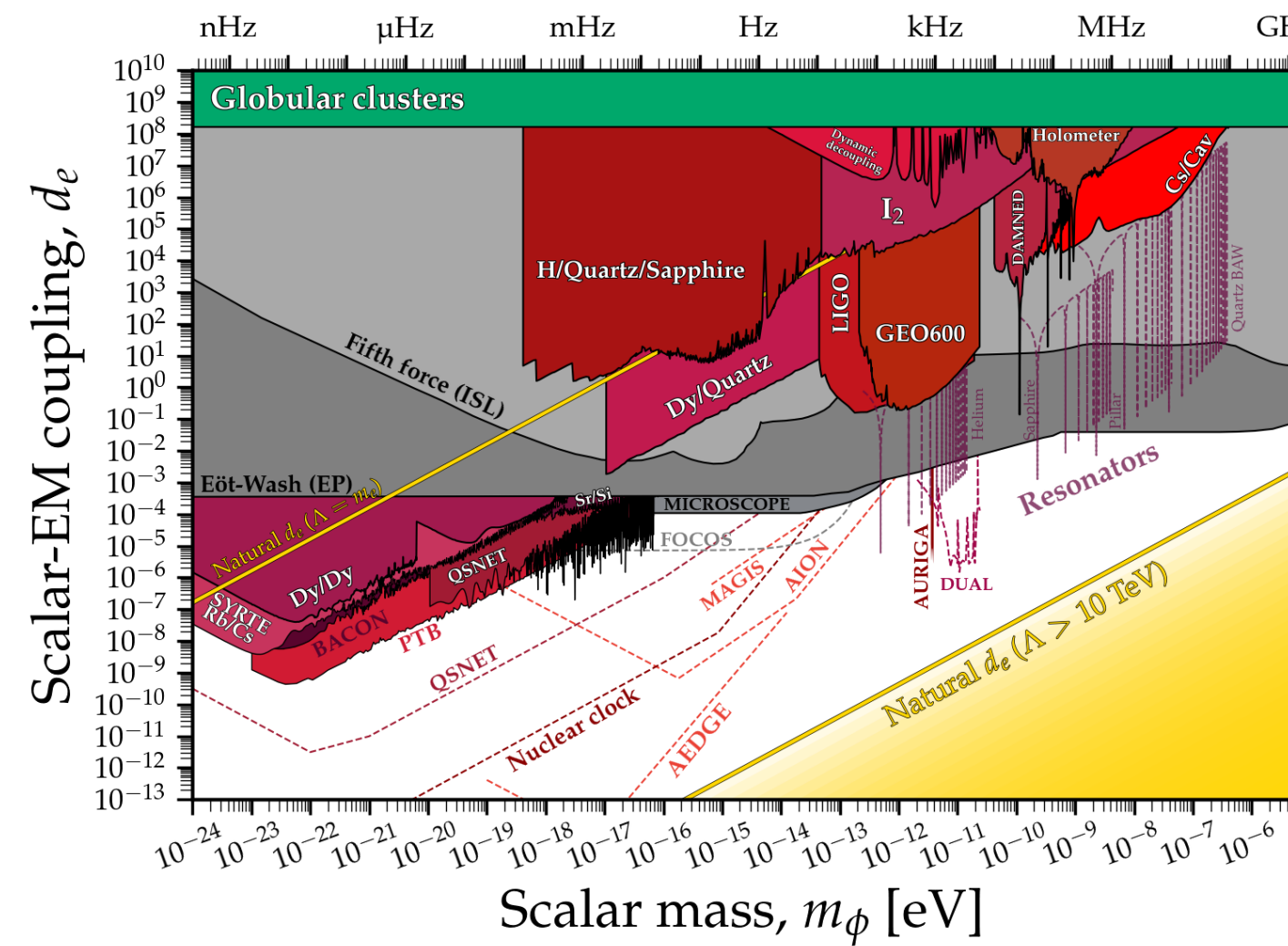


ICTP Trieste

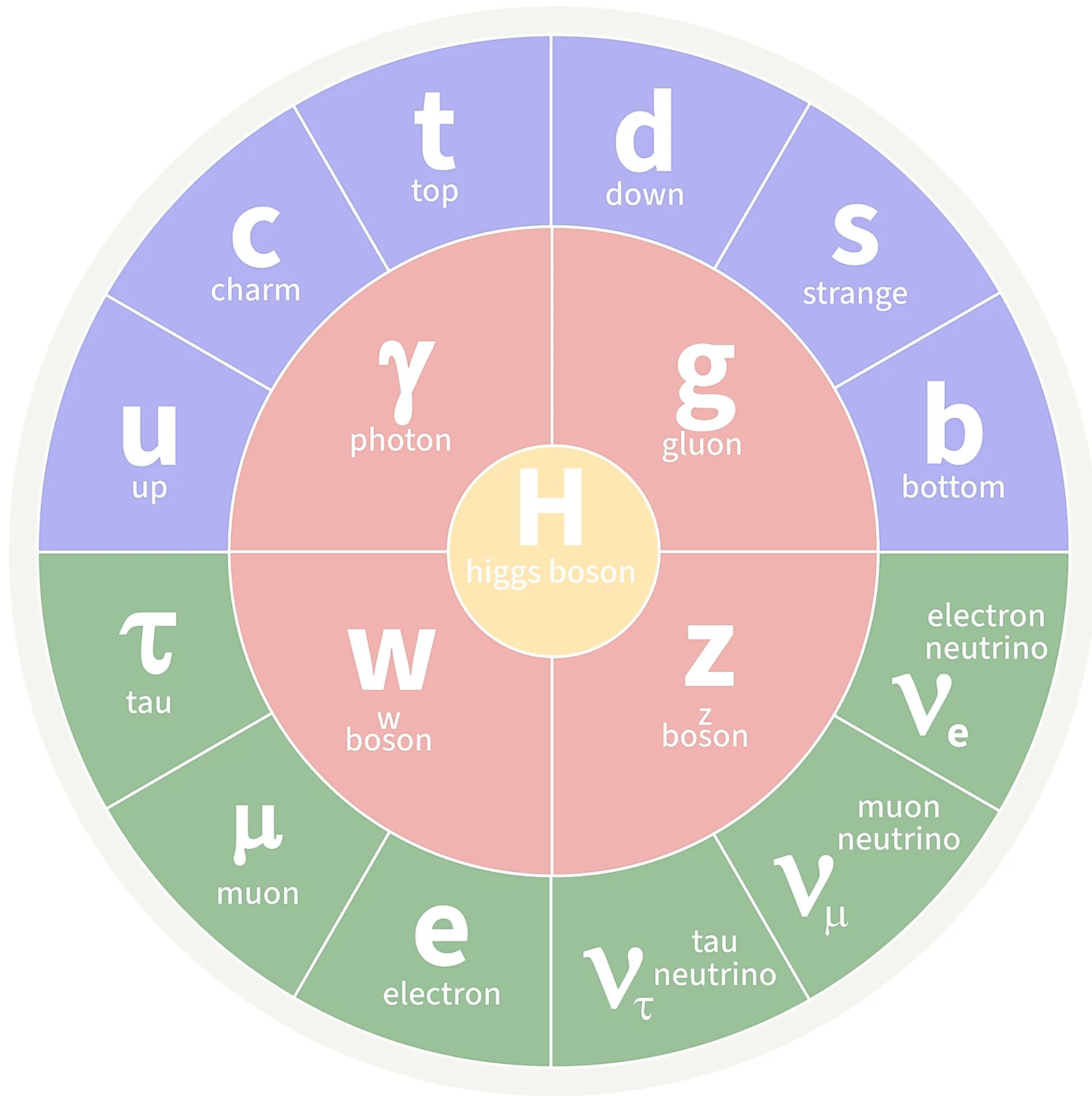
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ICTP Trieste



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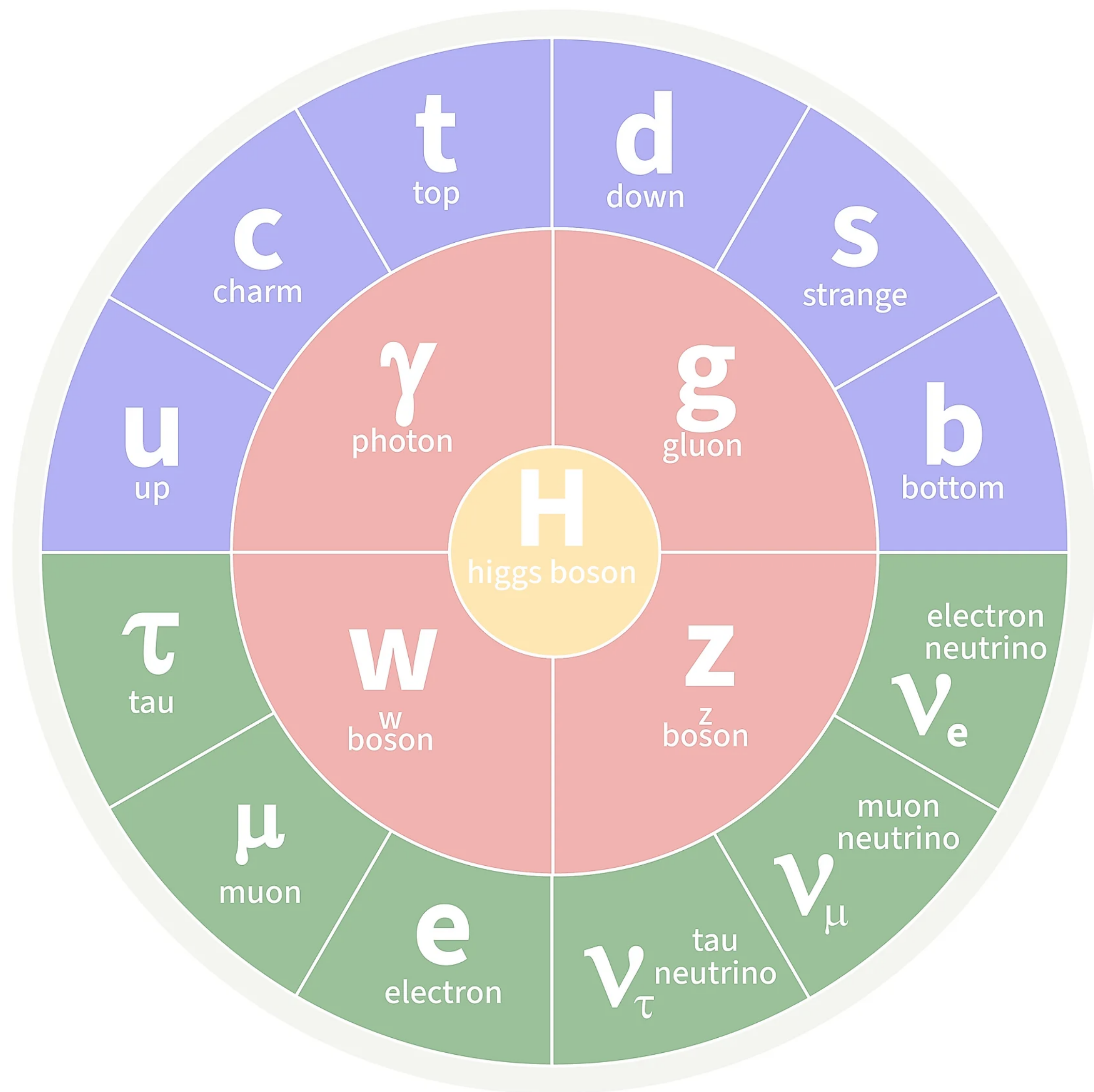
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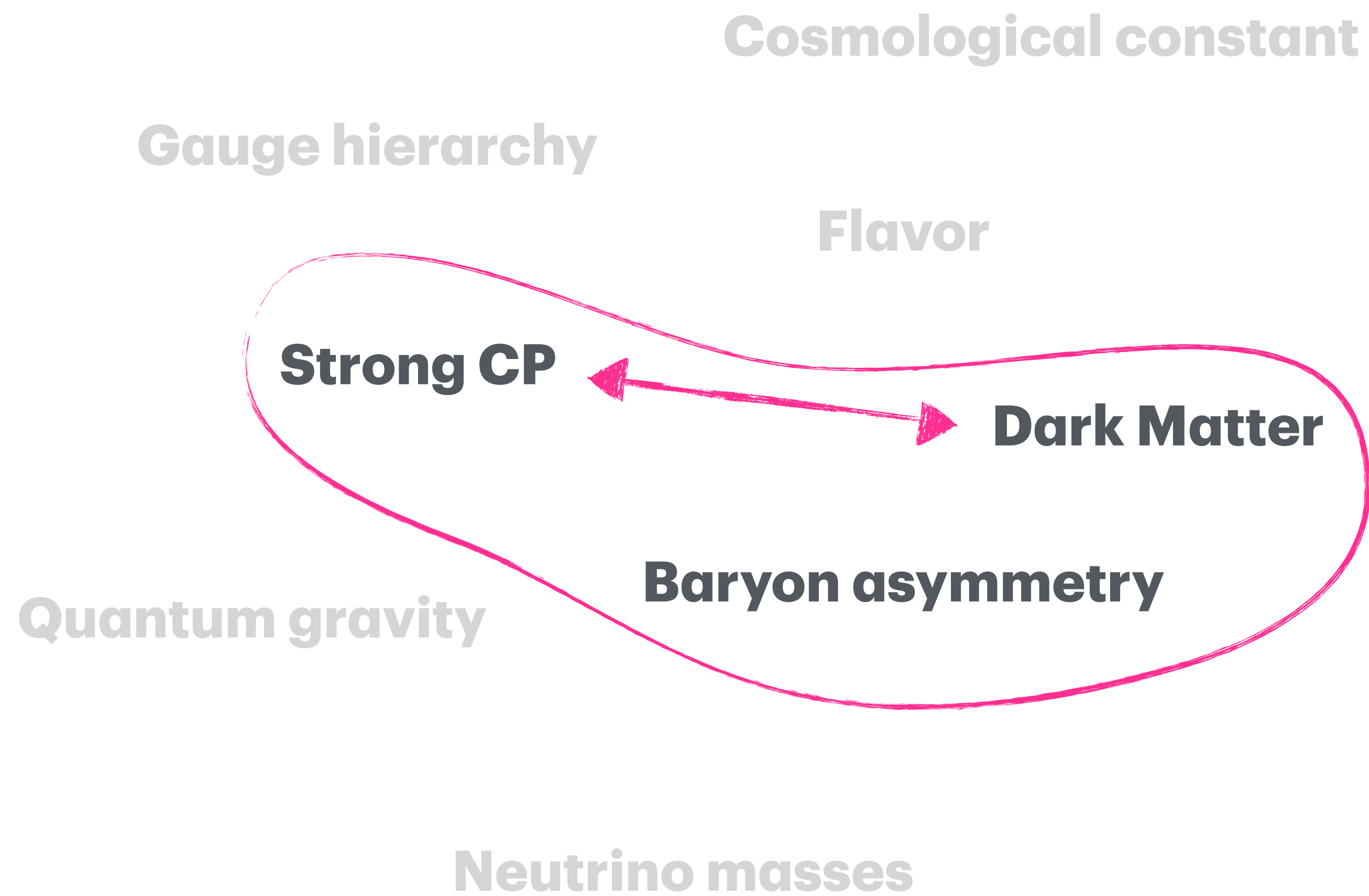
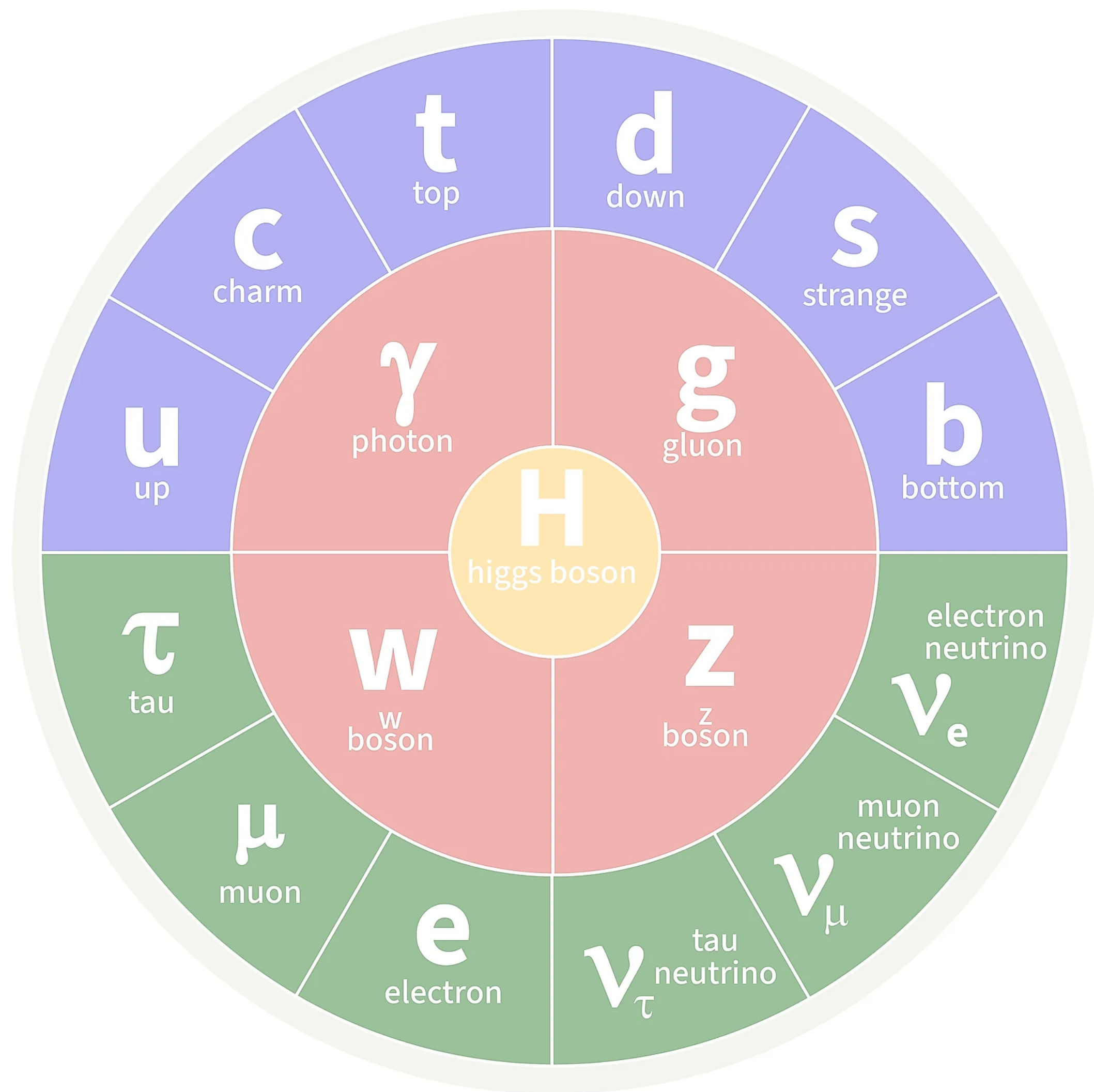
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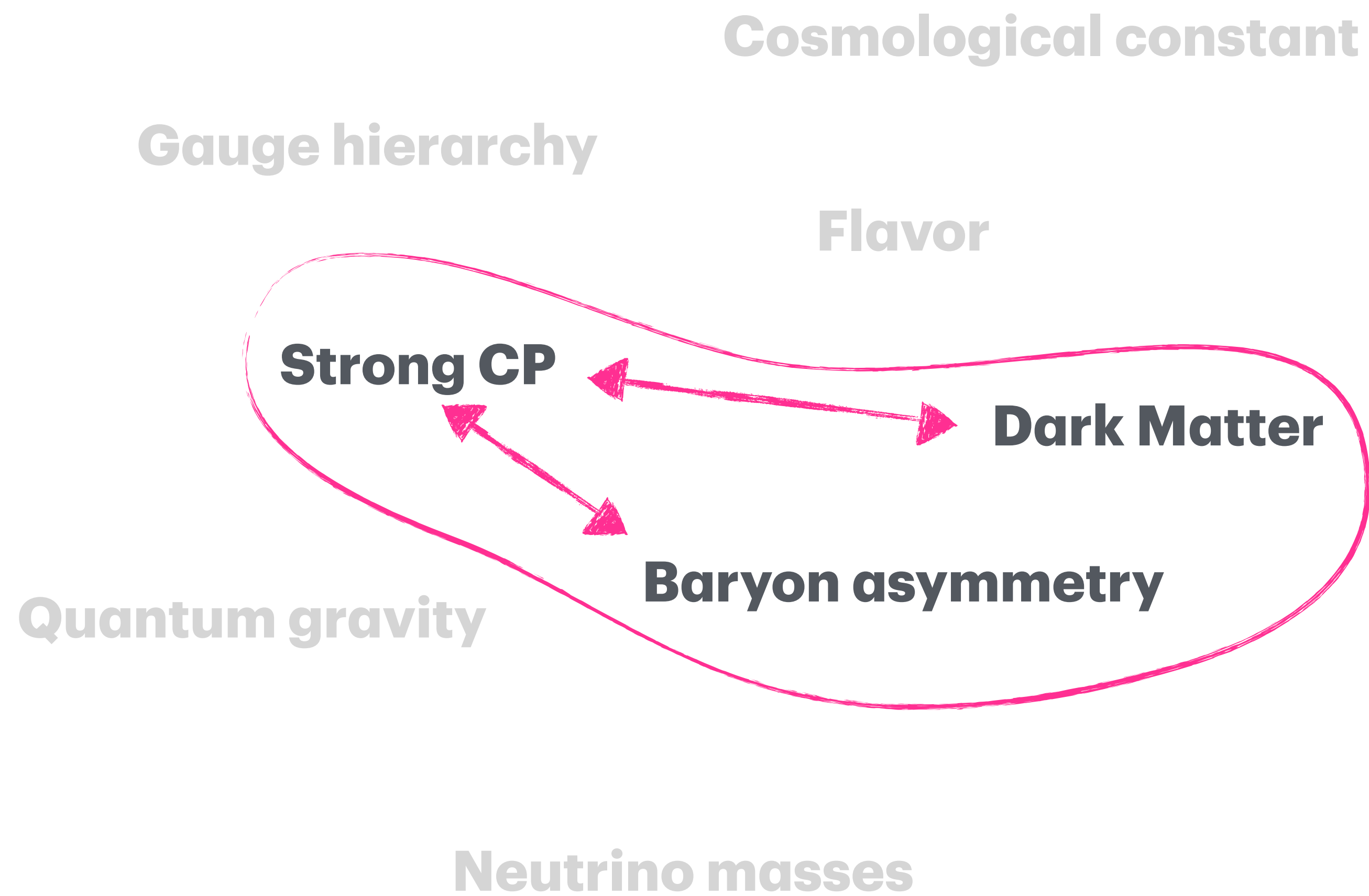
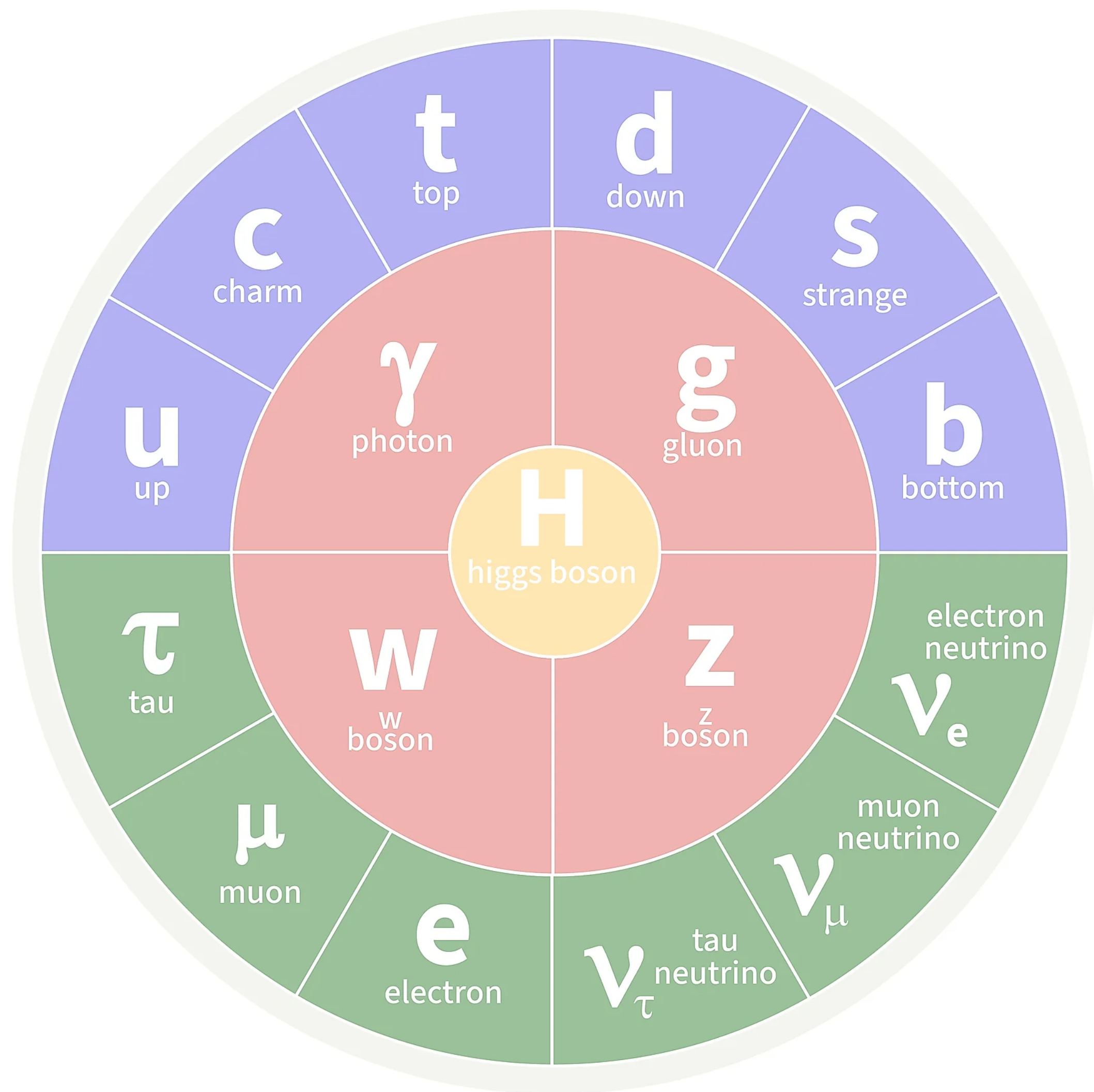
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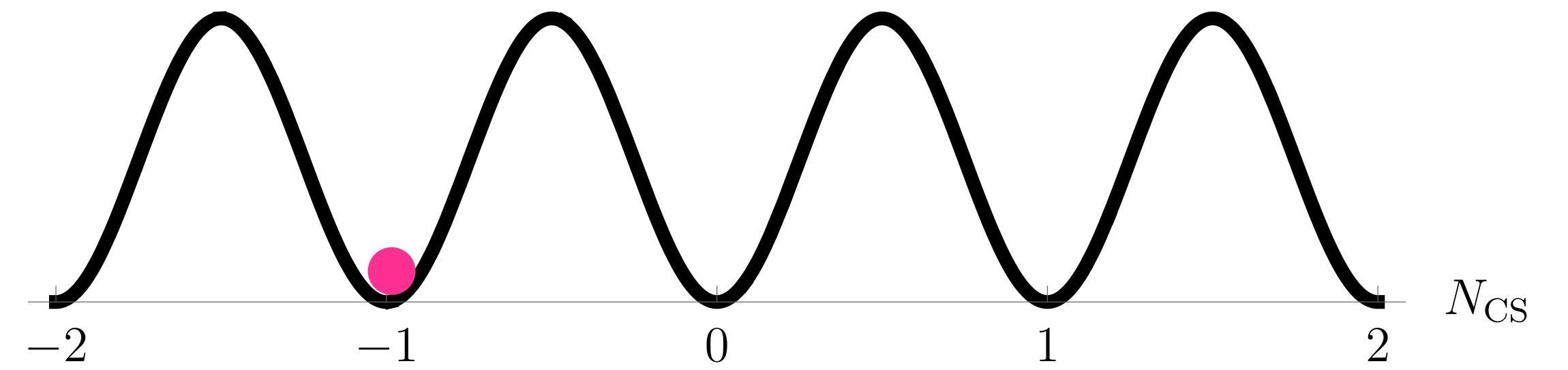
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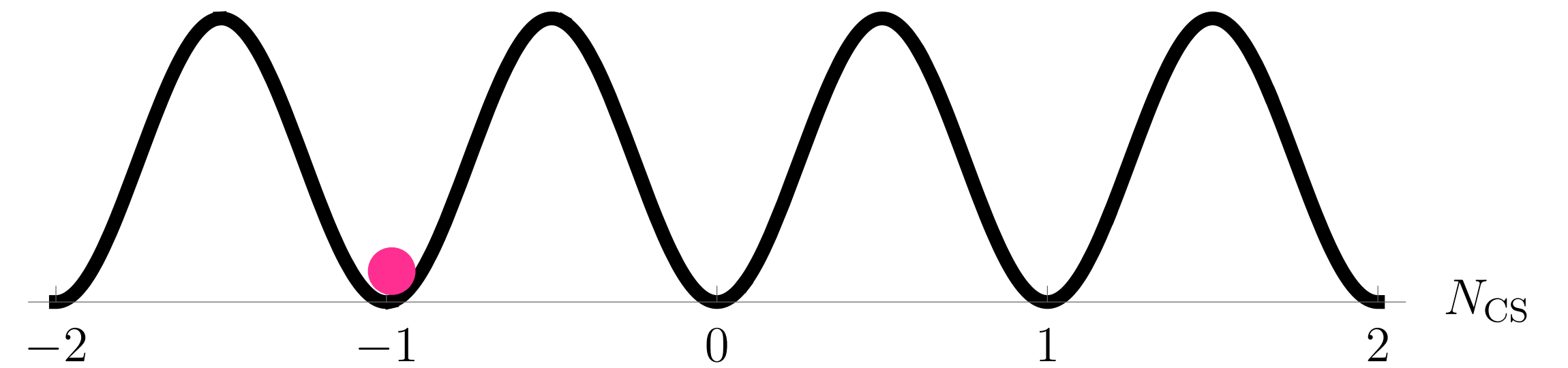


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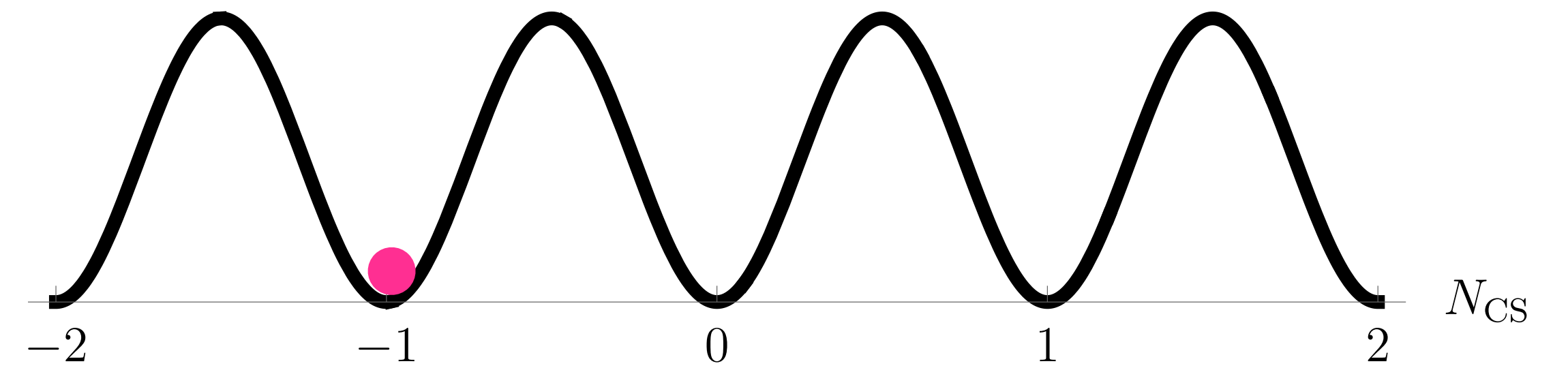
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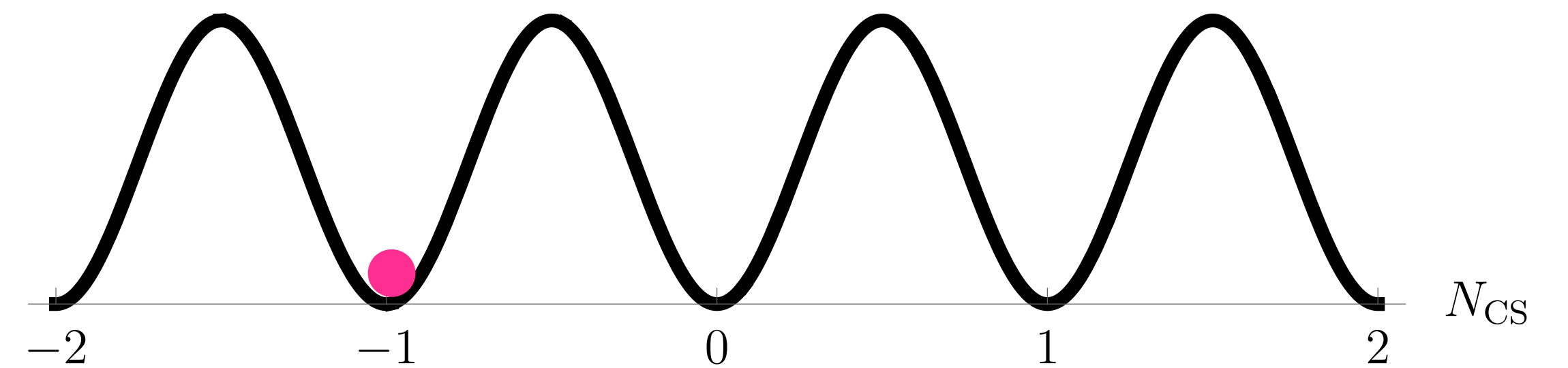
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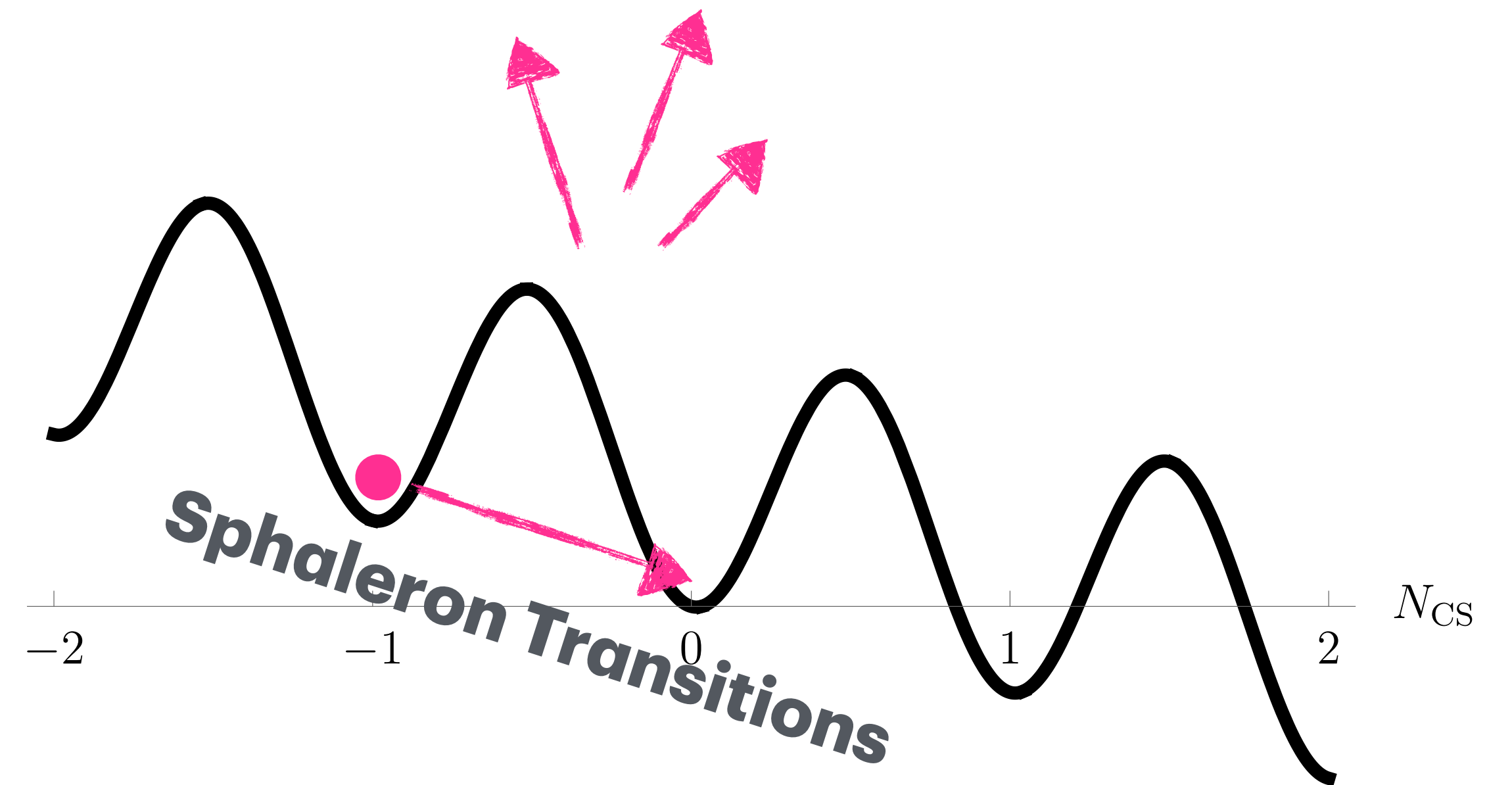
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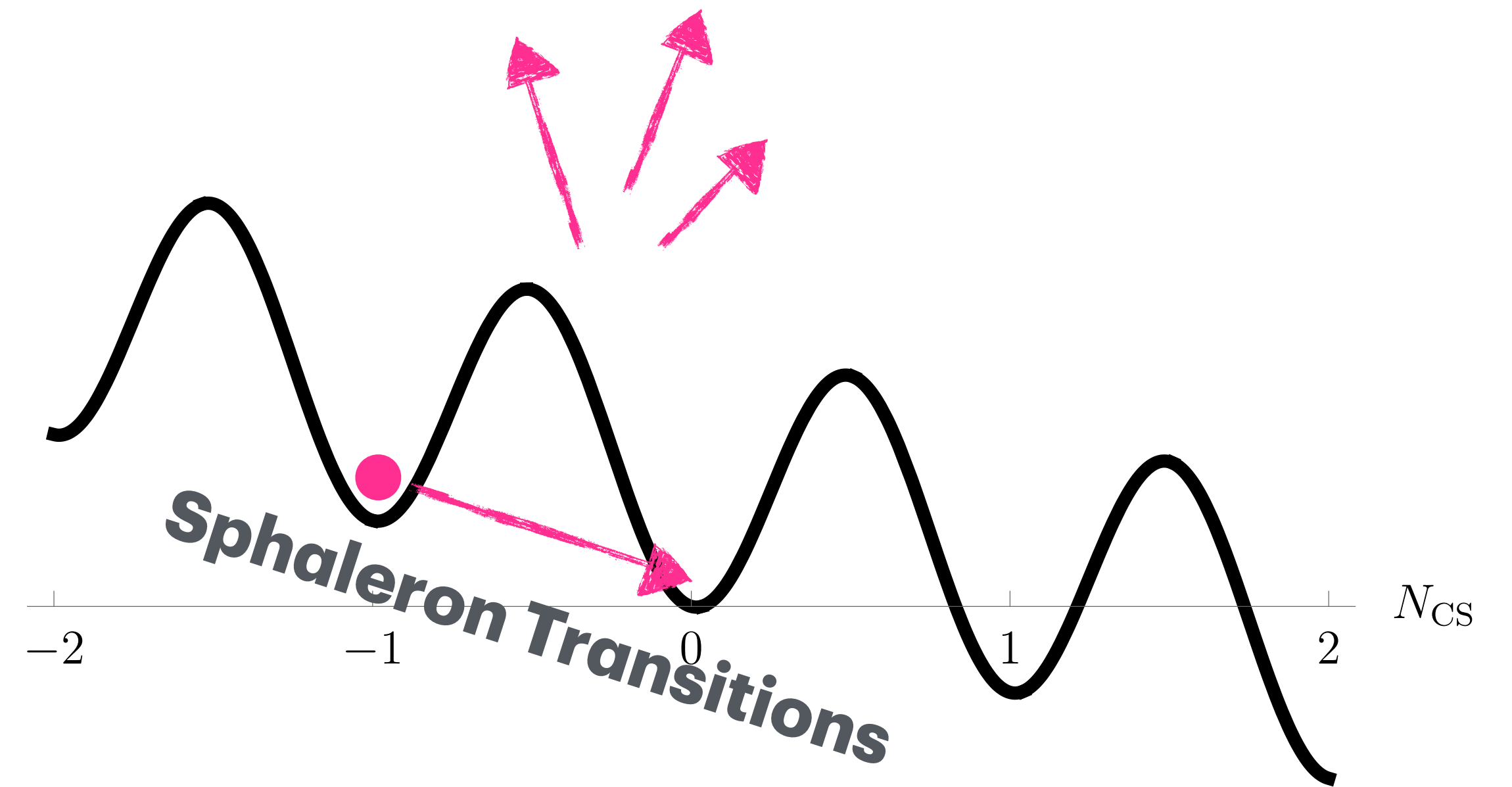
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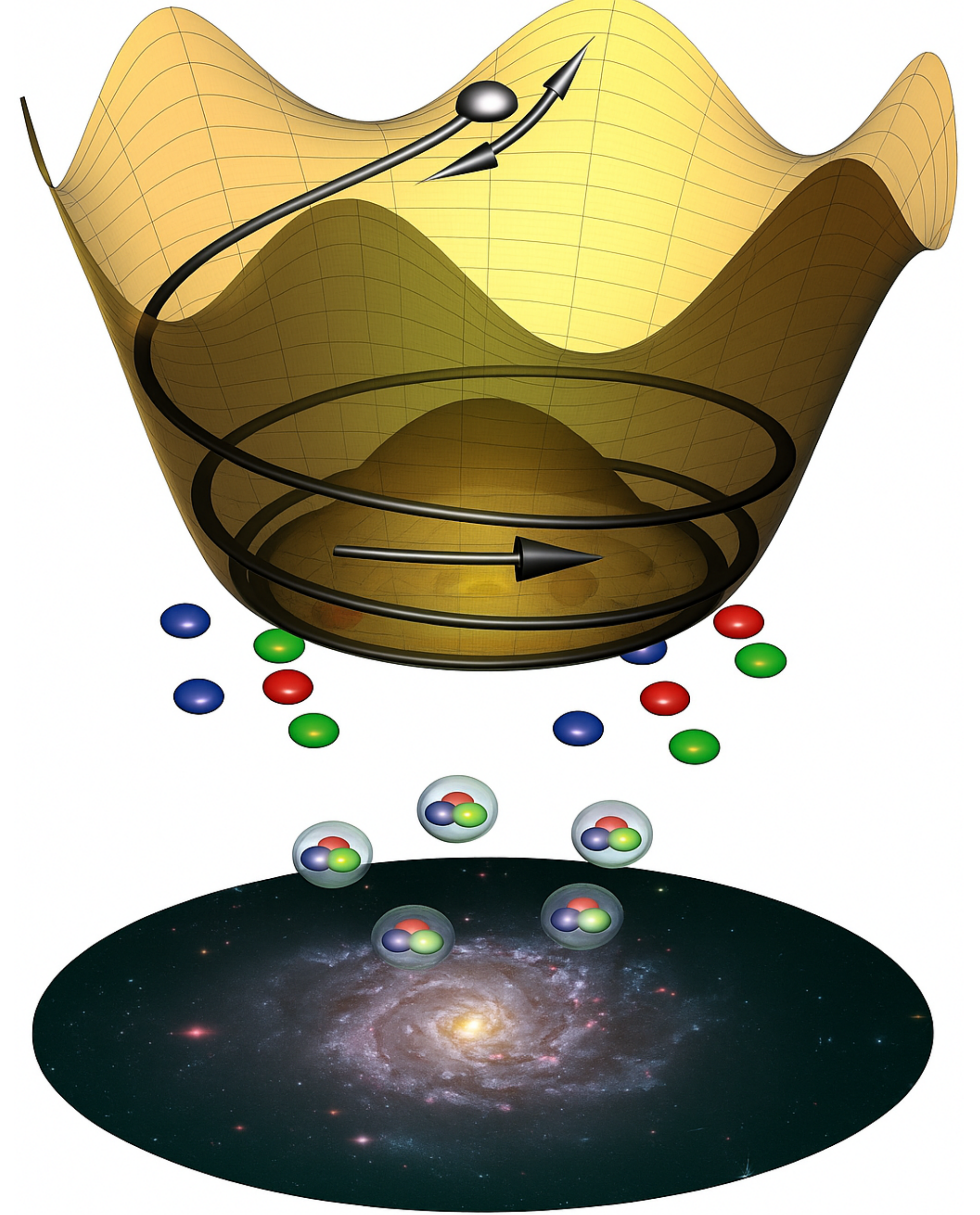
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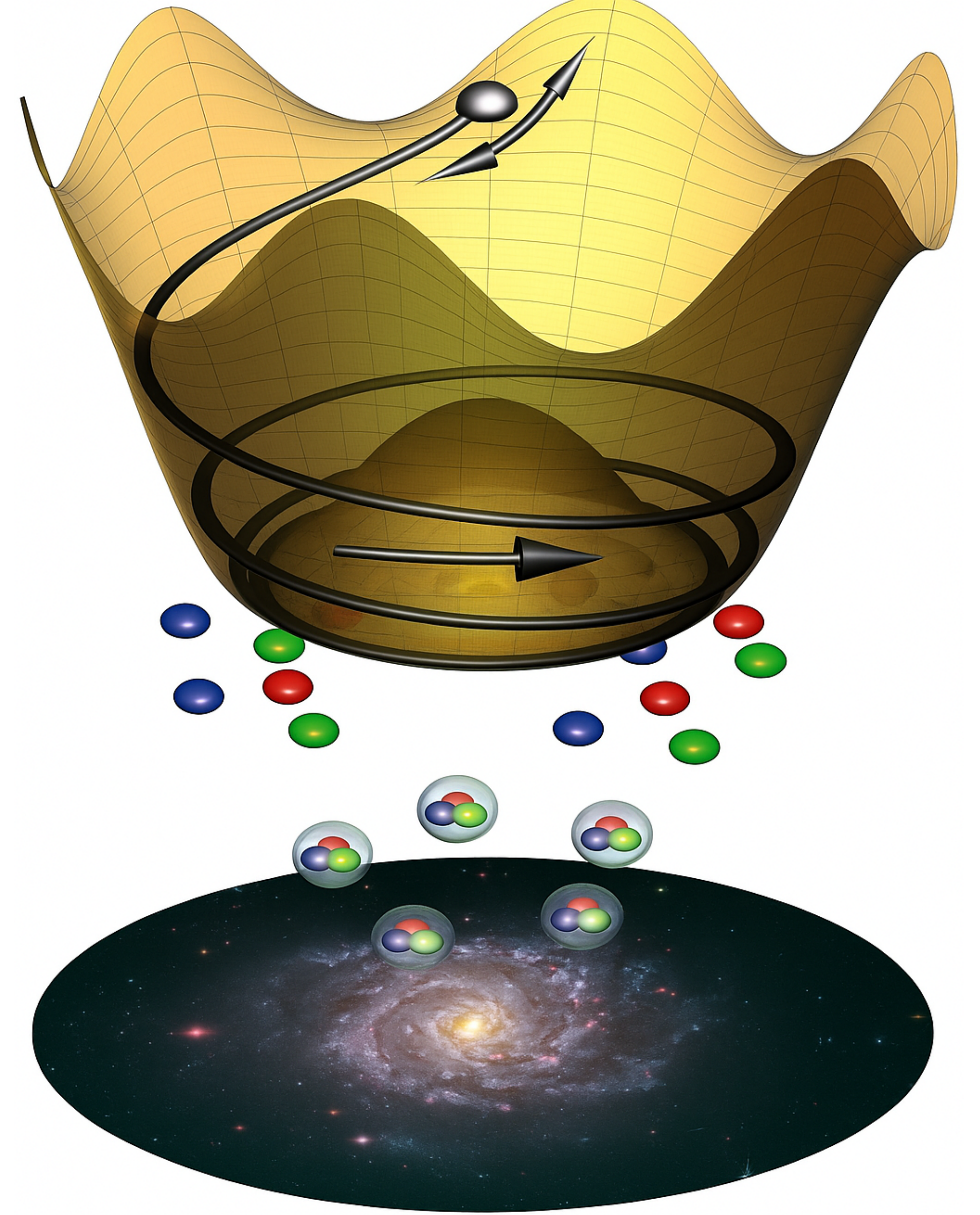
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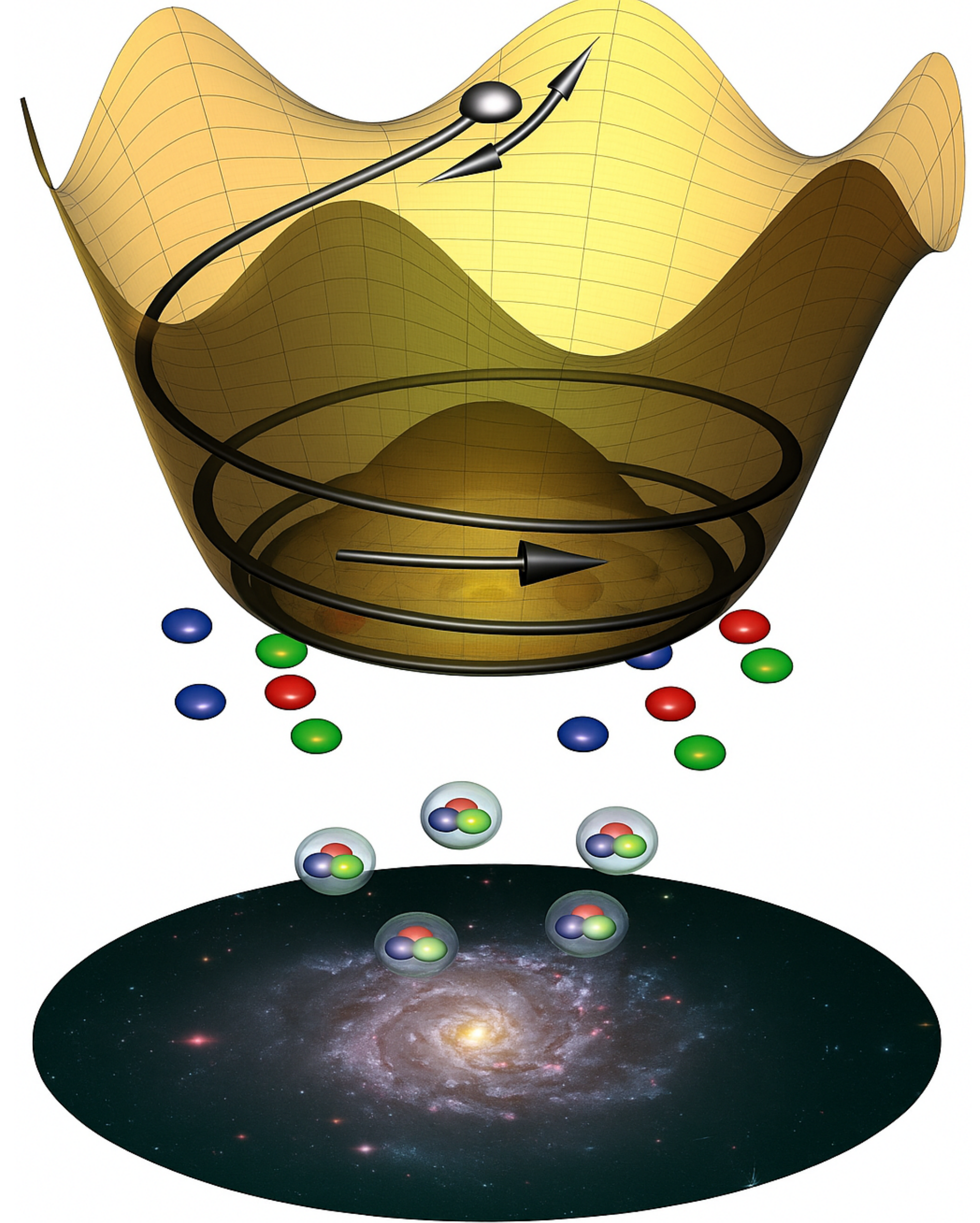
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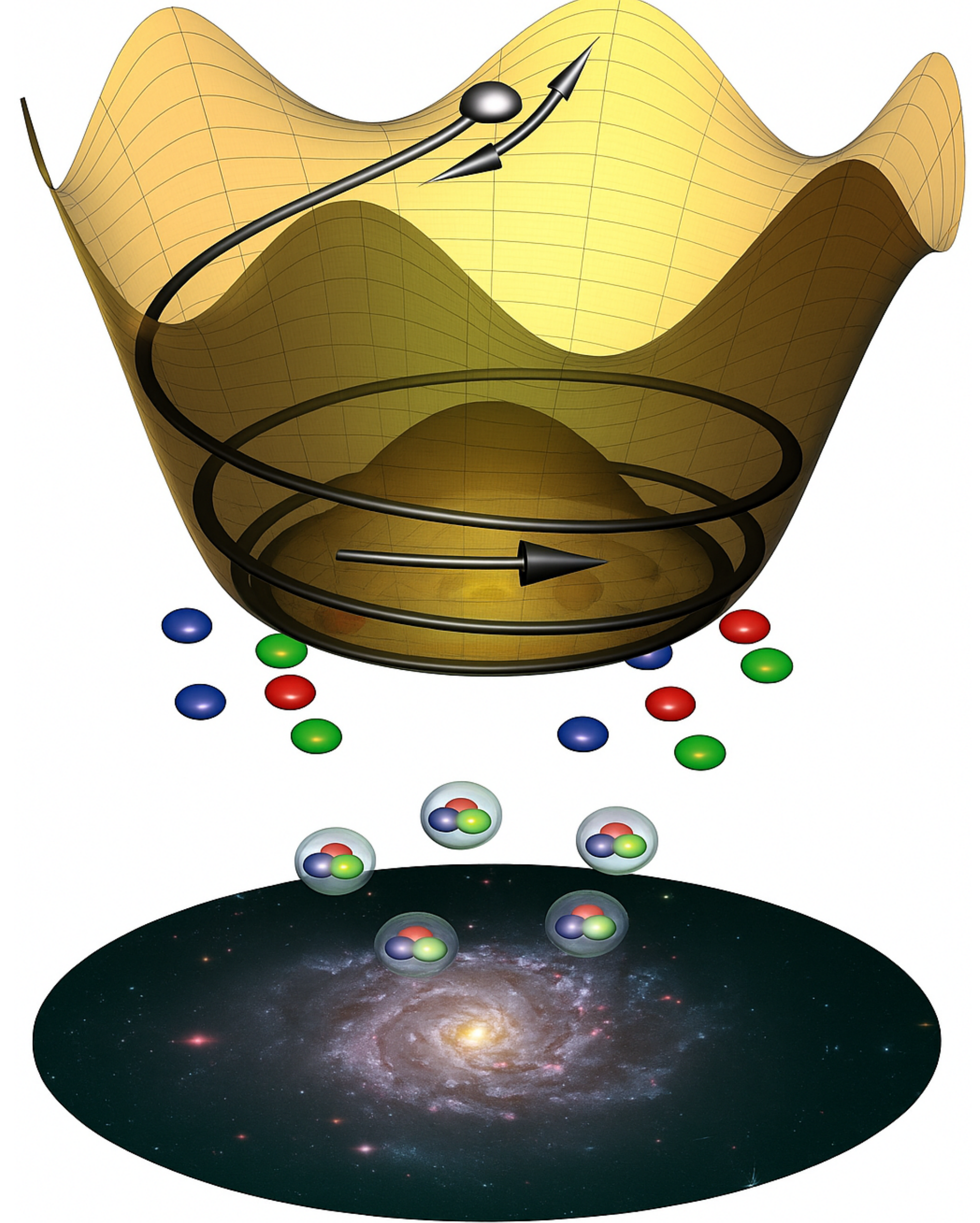
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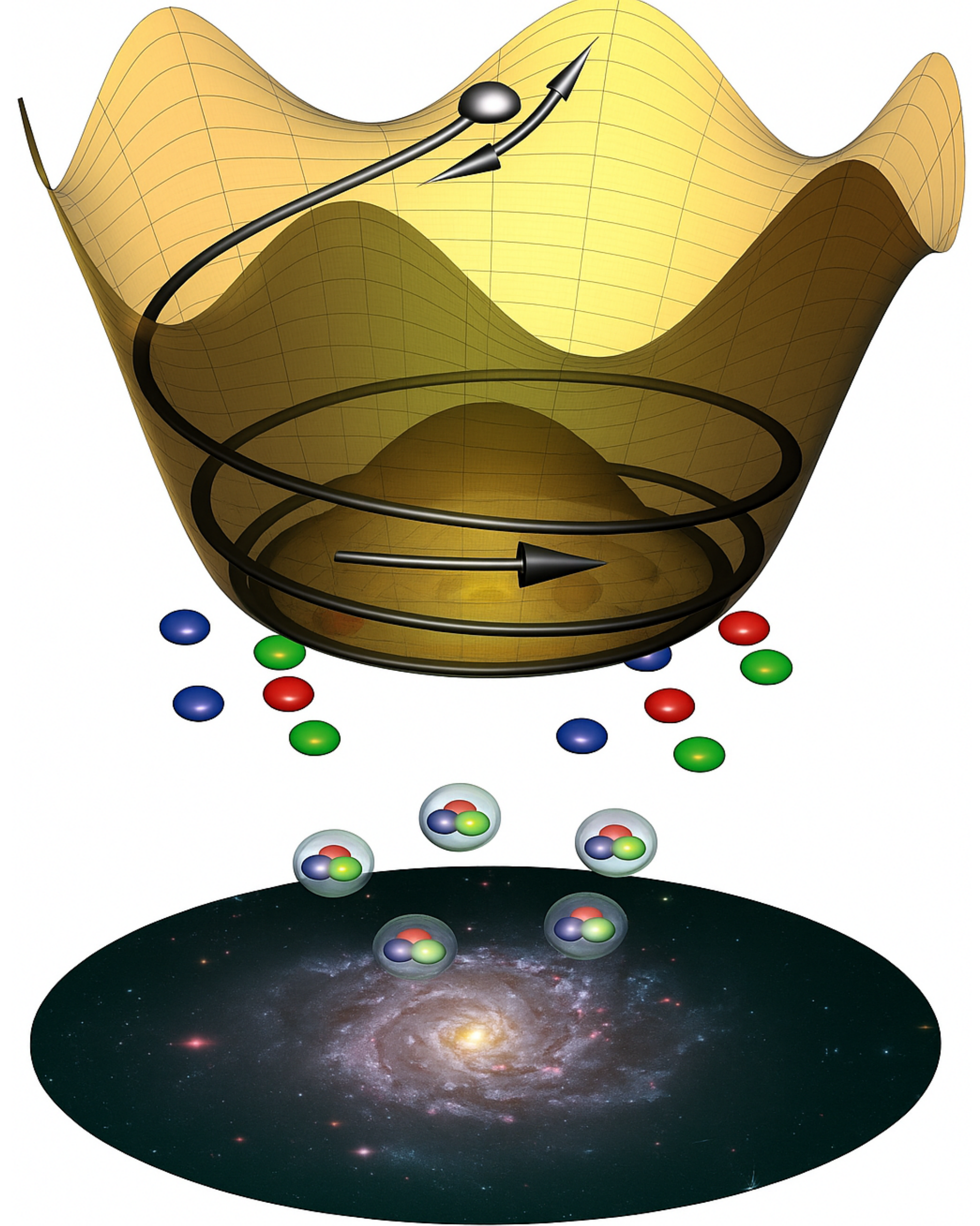
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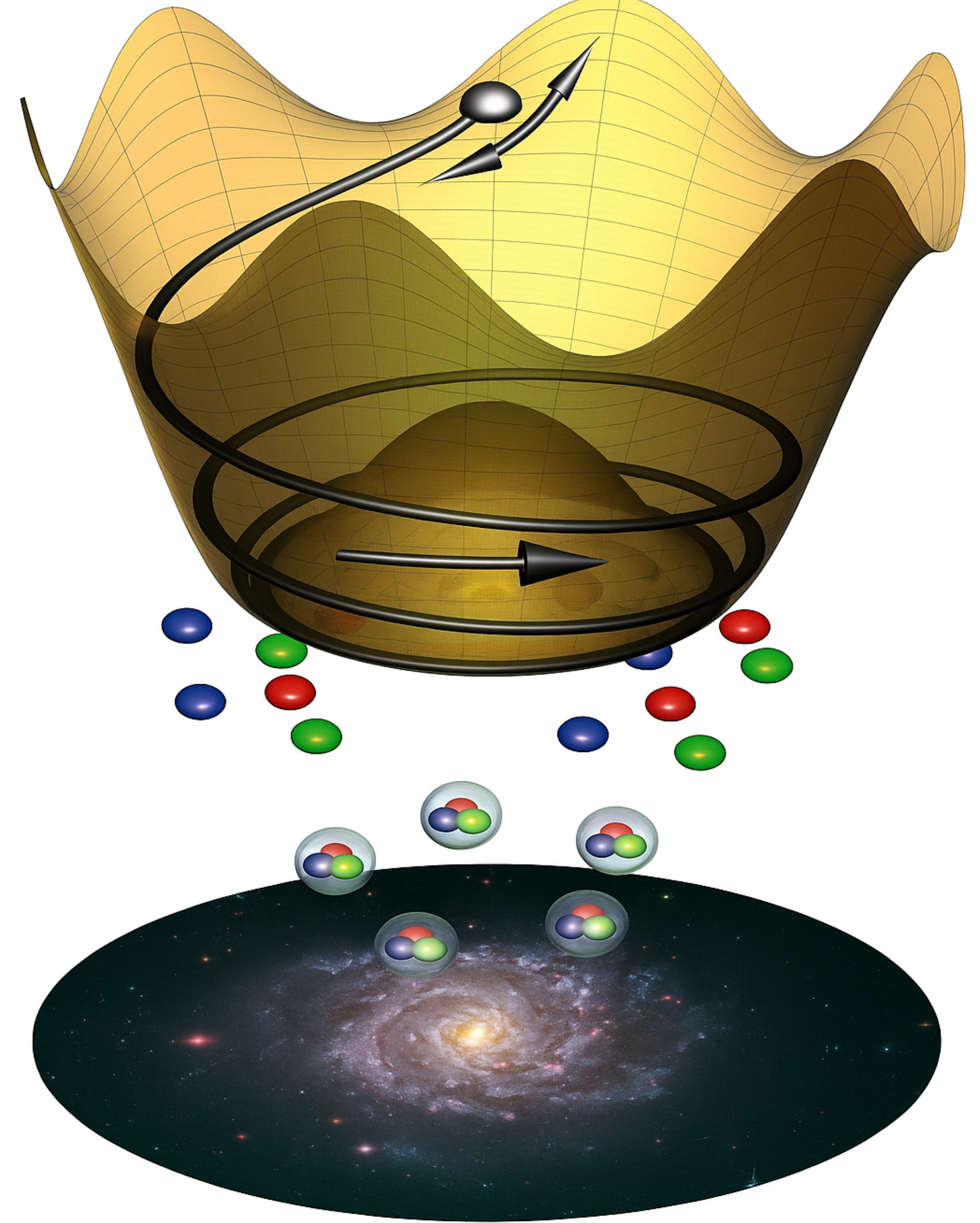
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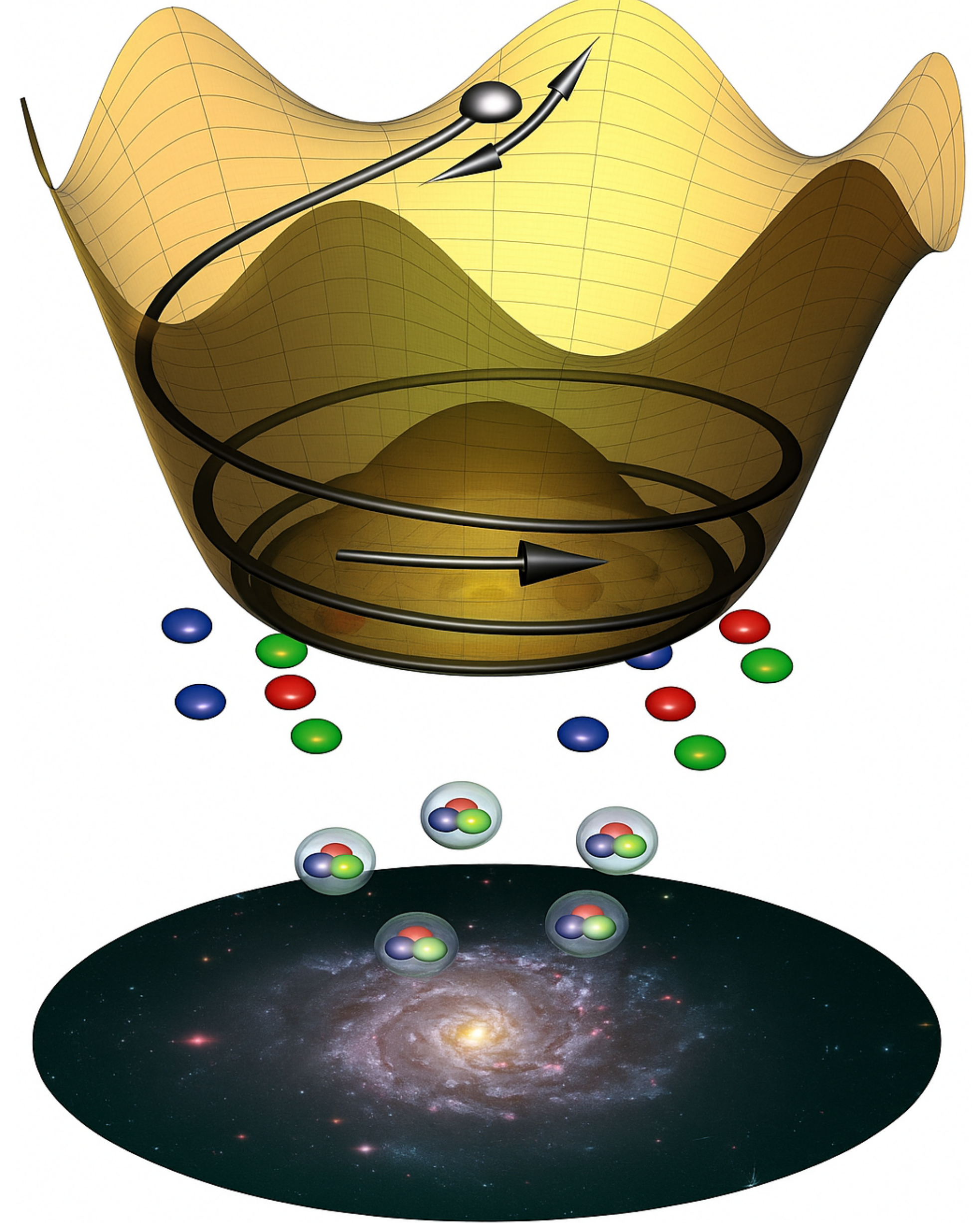
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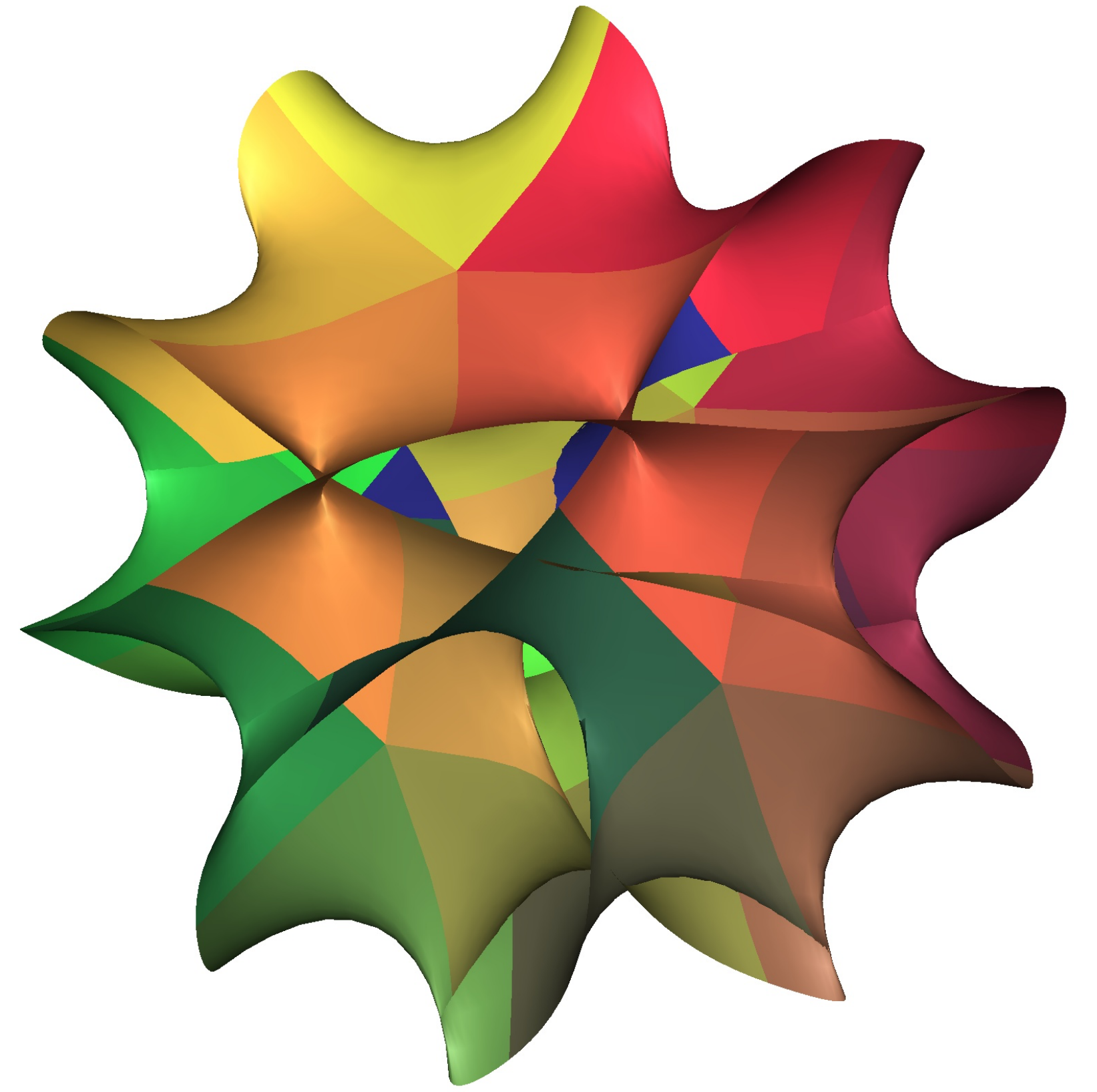
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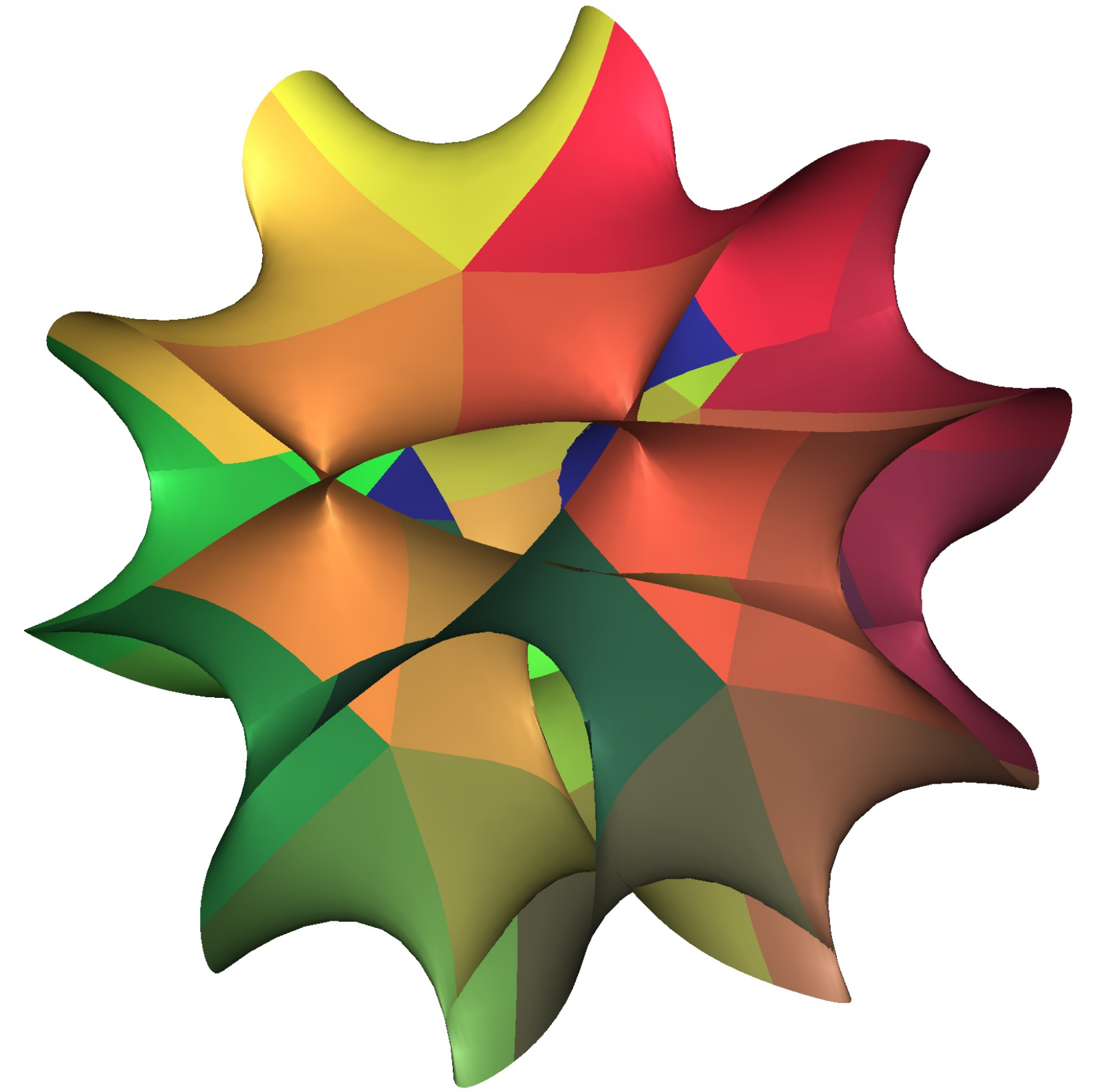
- Both resolved naturally if the QCD axion is one of many in the String Axiverse

# The string axiverse



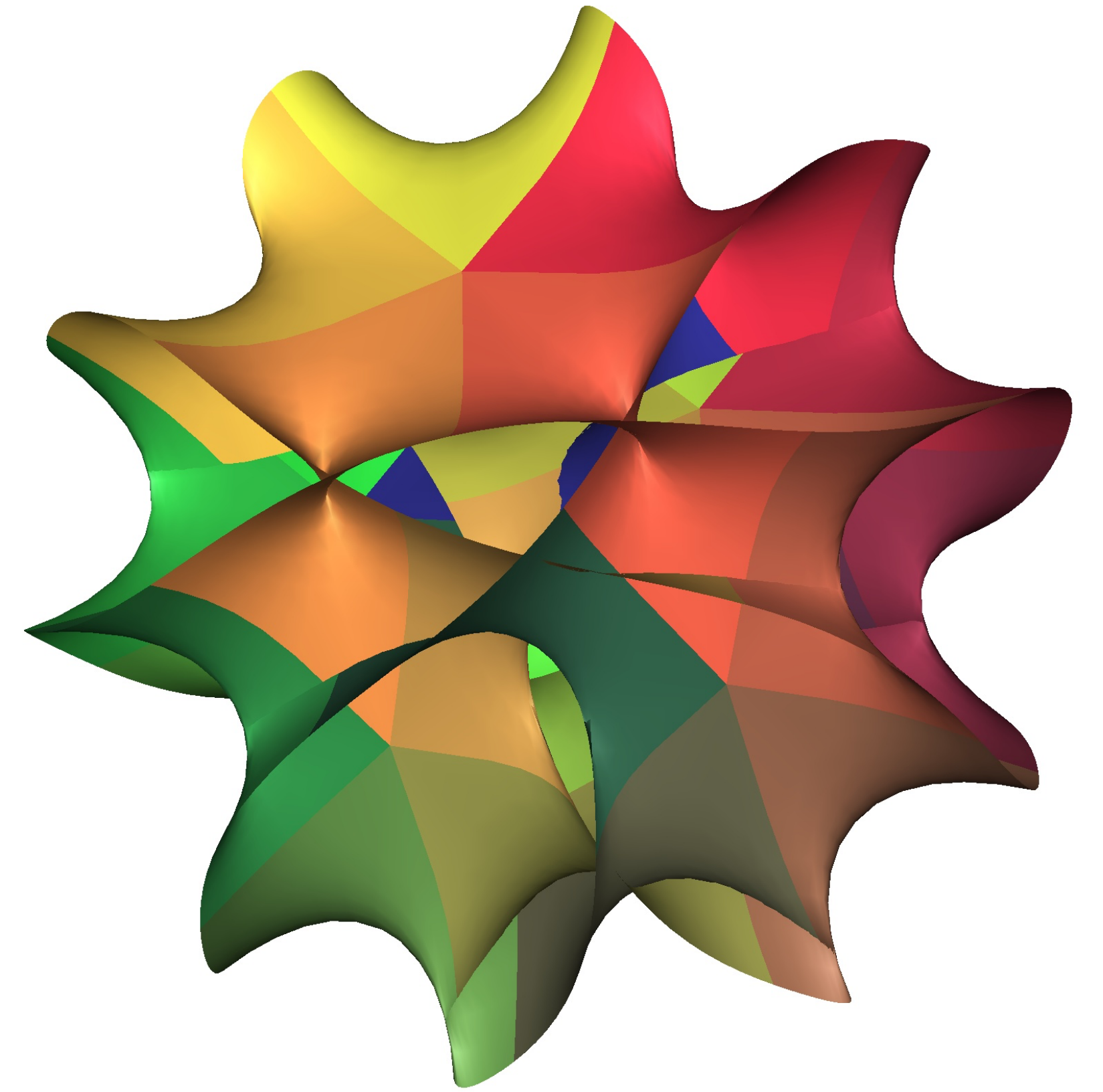
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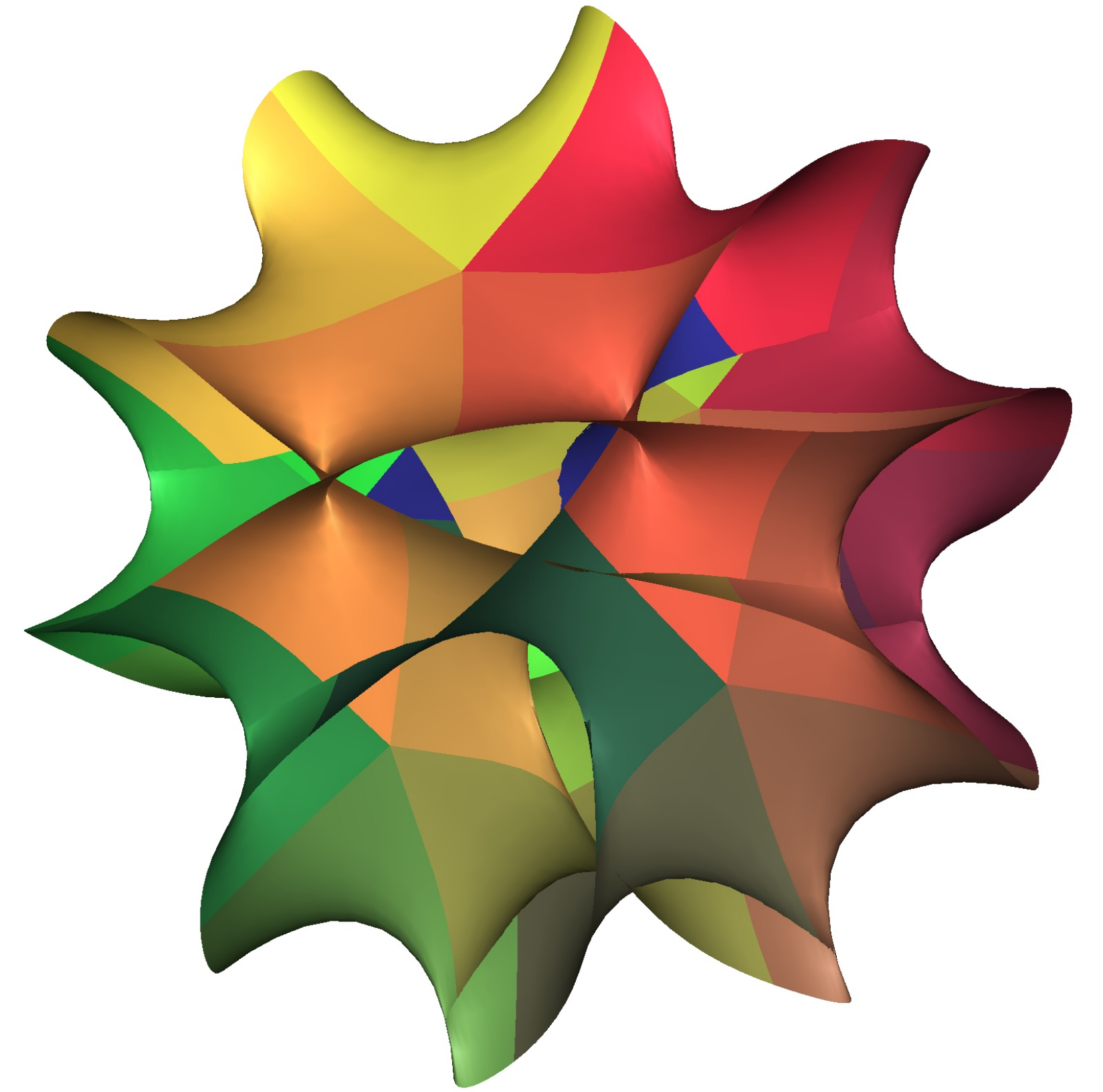
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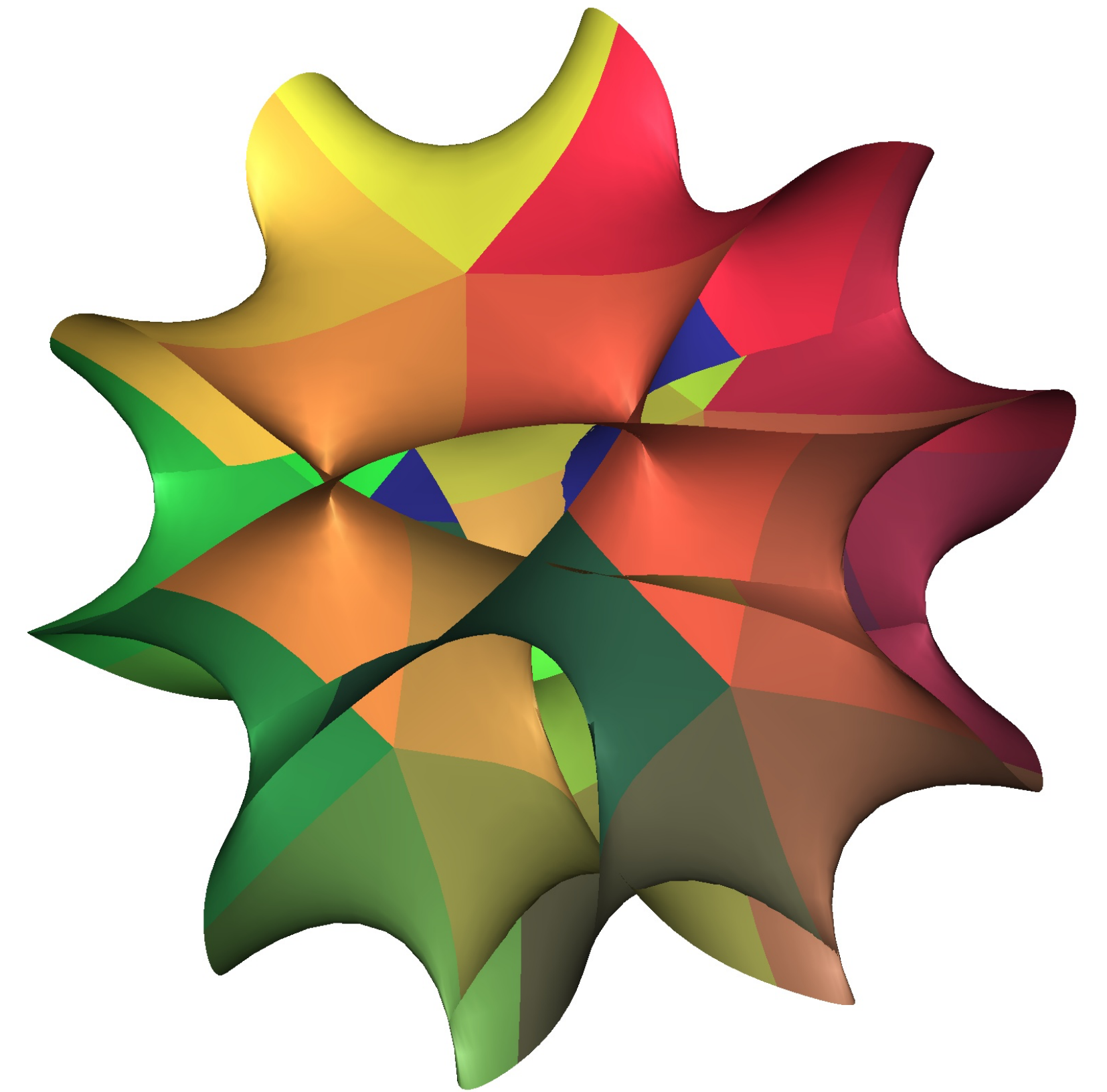
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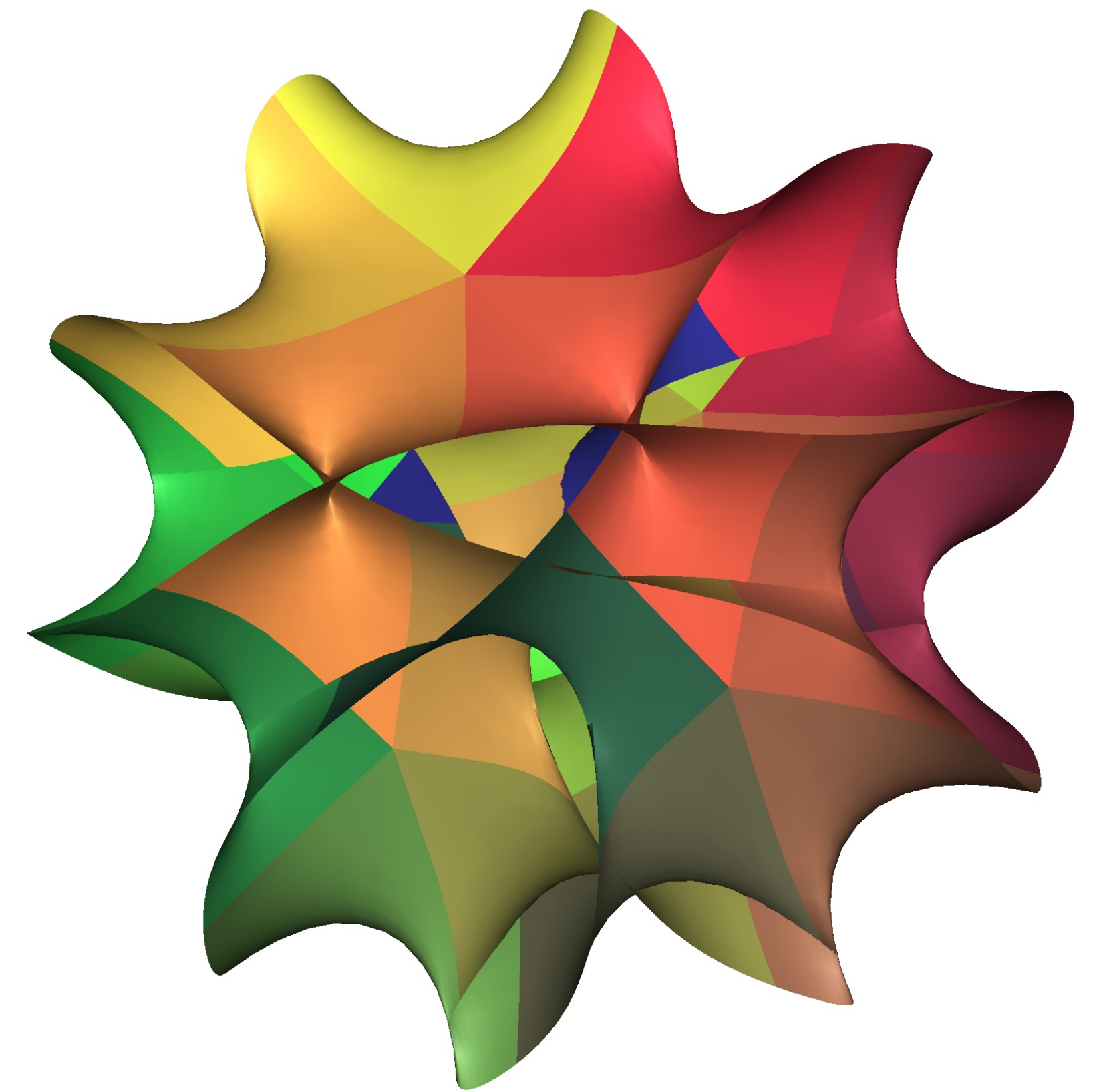
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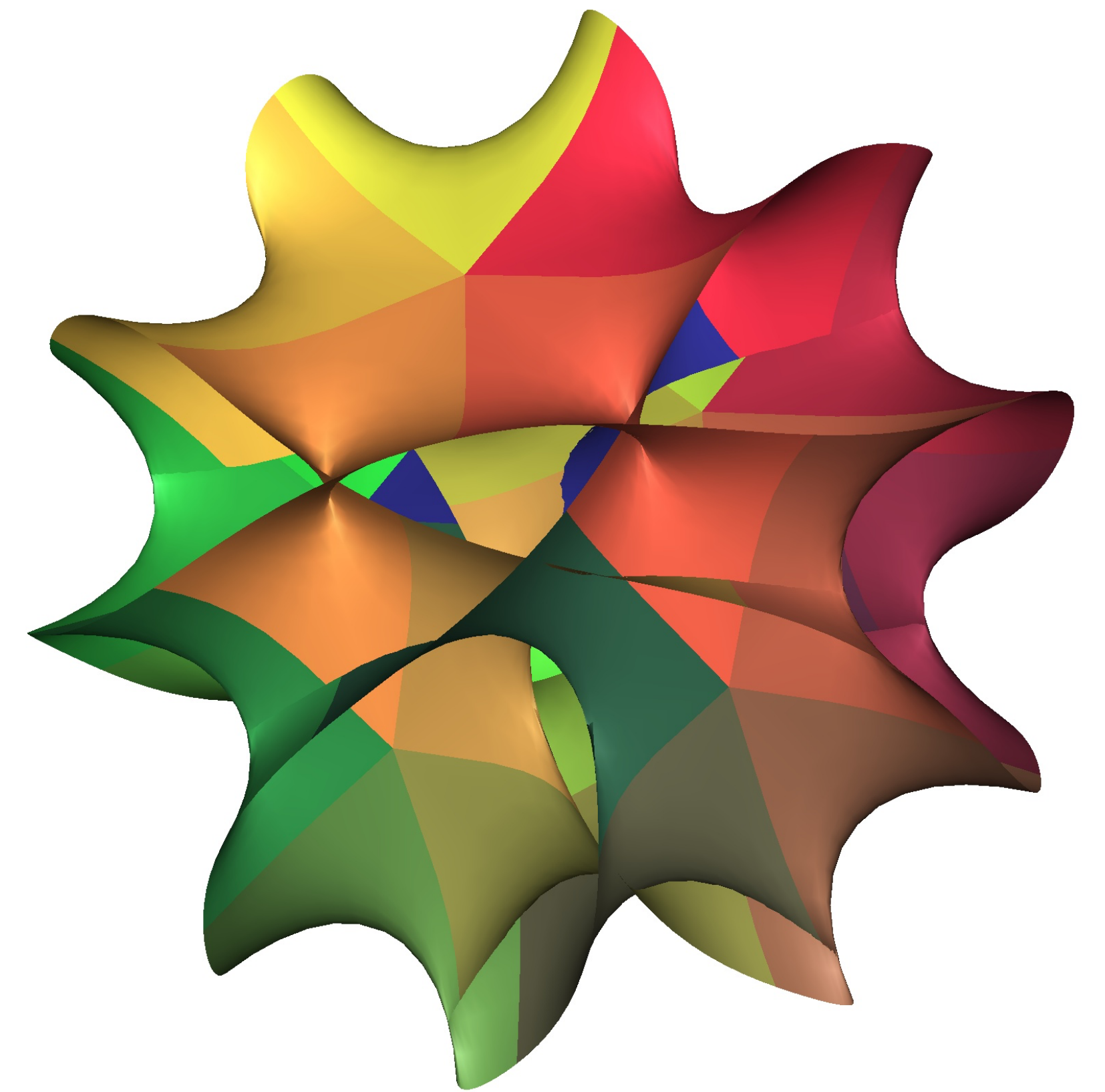
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Most-general two-axion Lagrangian that solves strong CP


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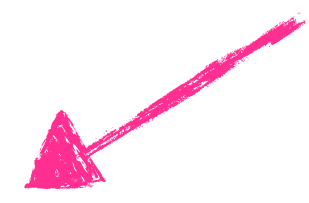
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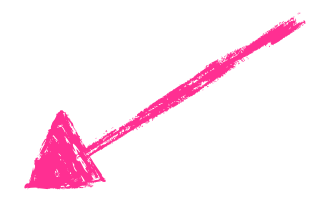
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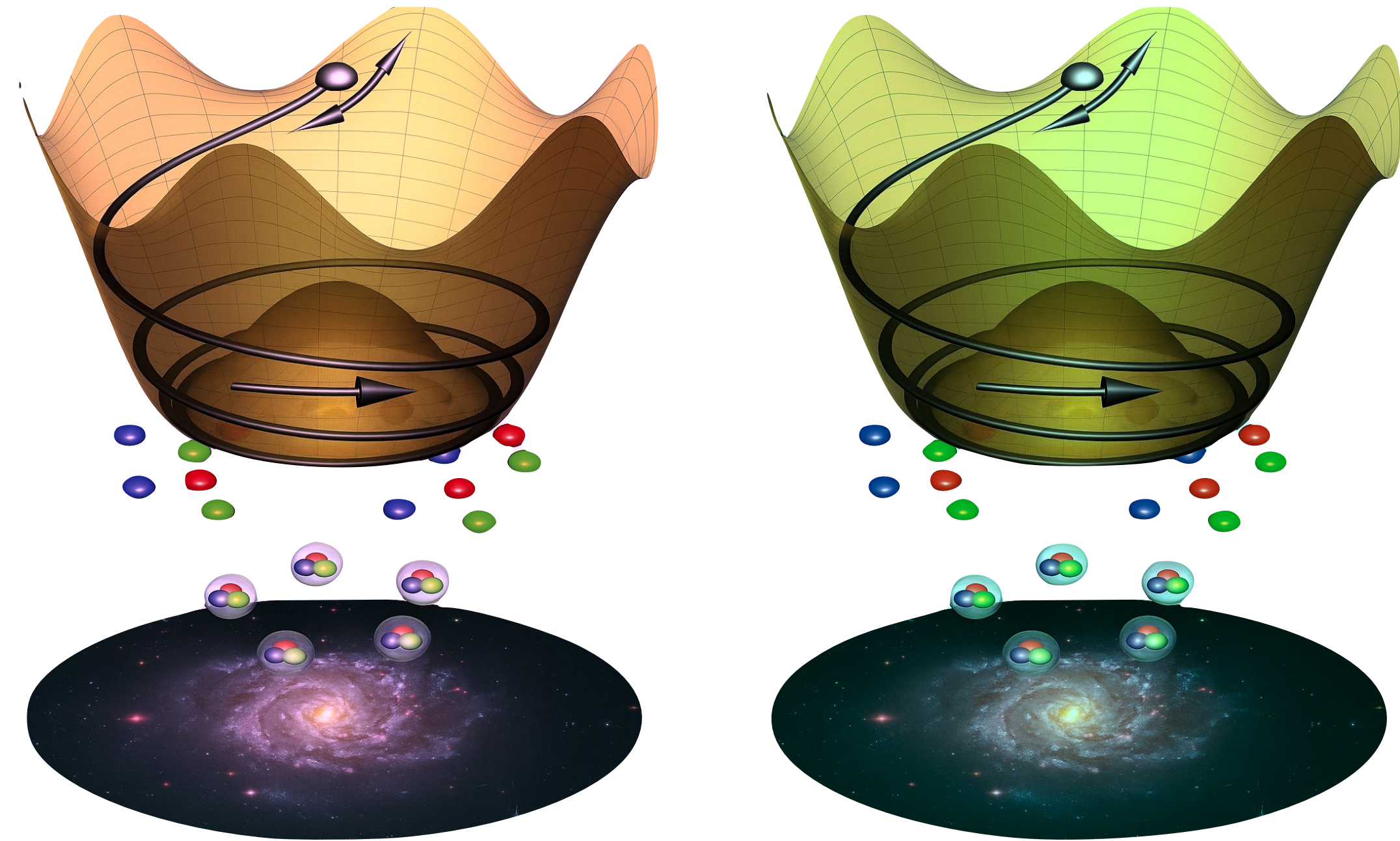
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# A two-axion example

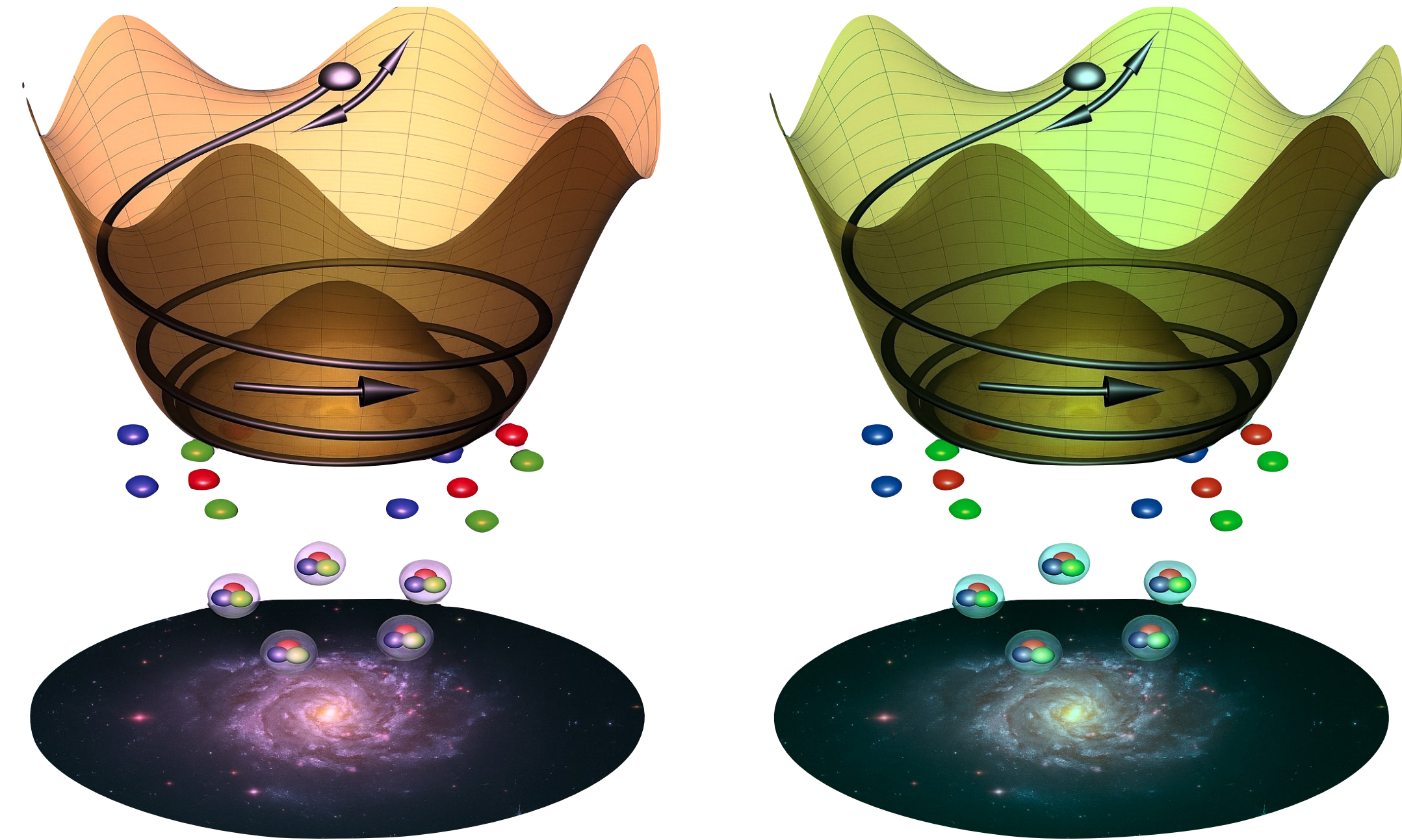
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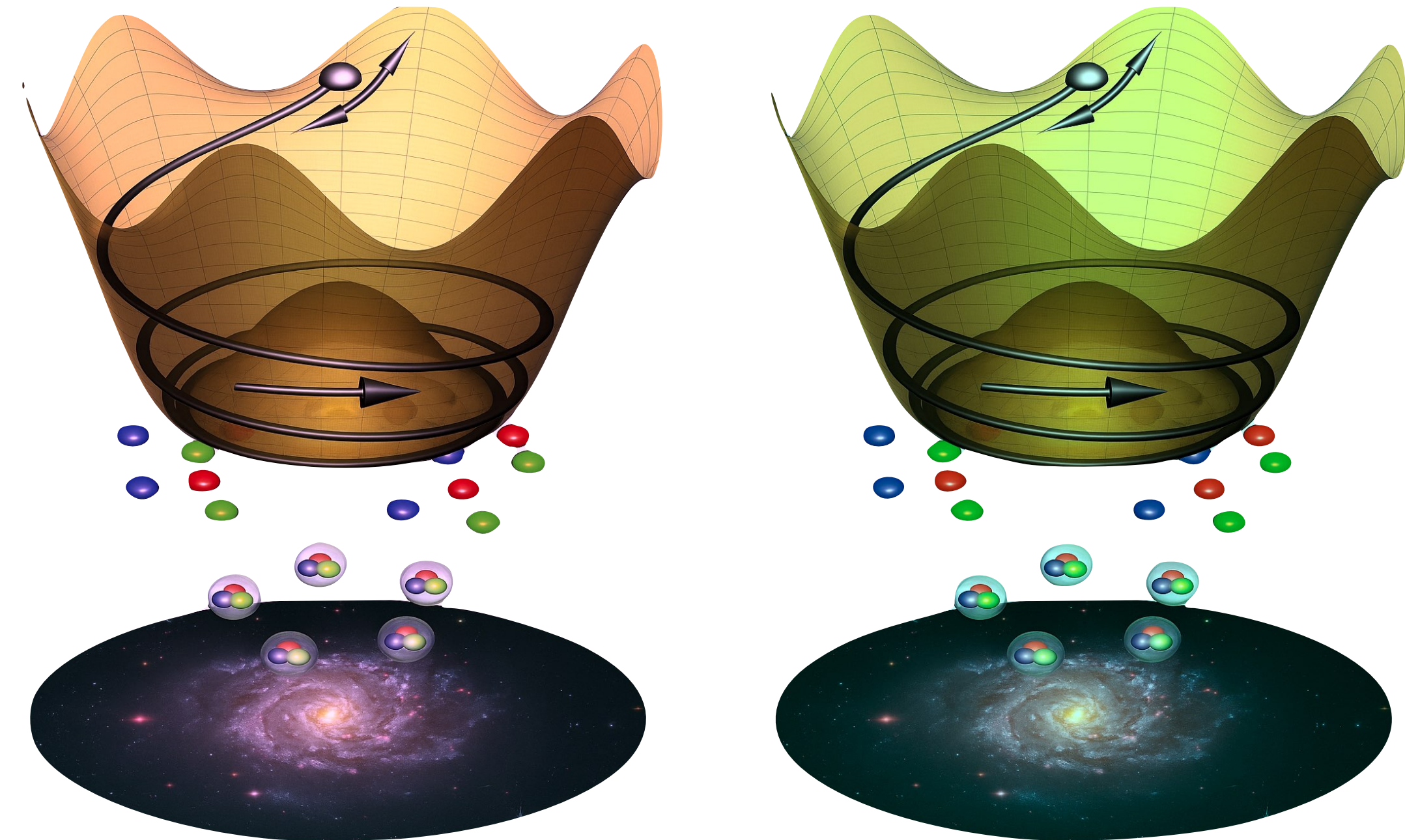


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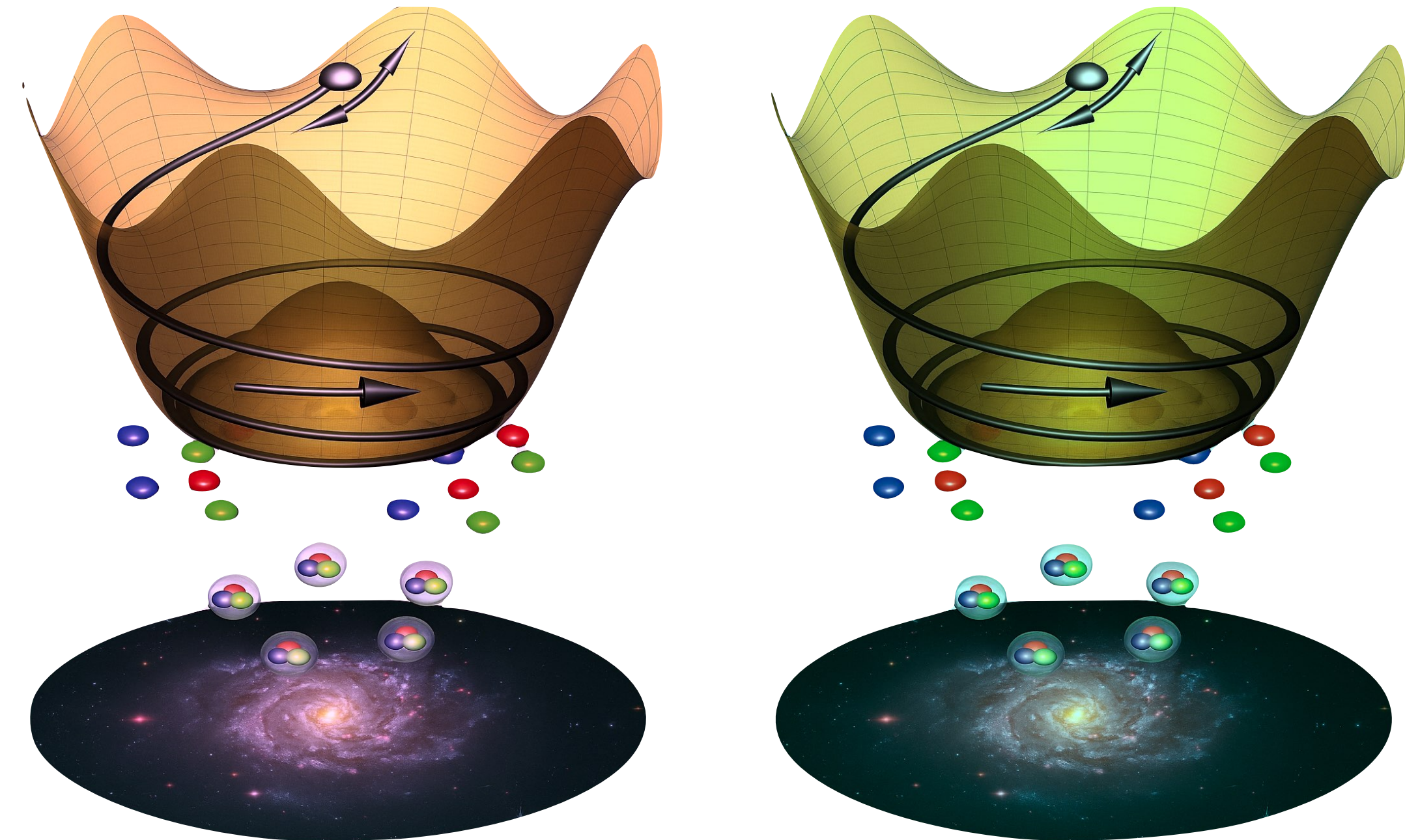
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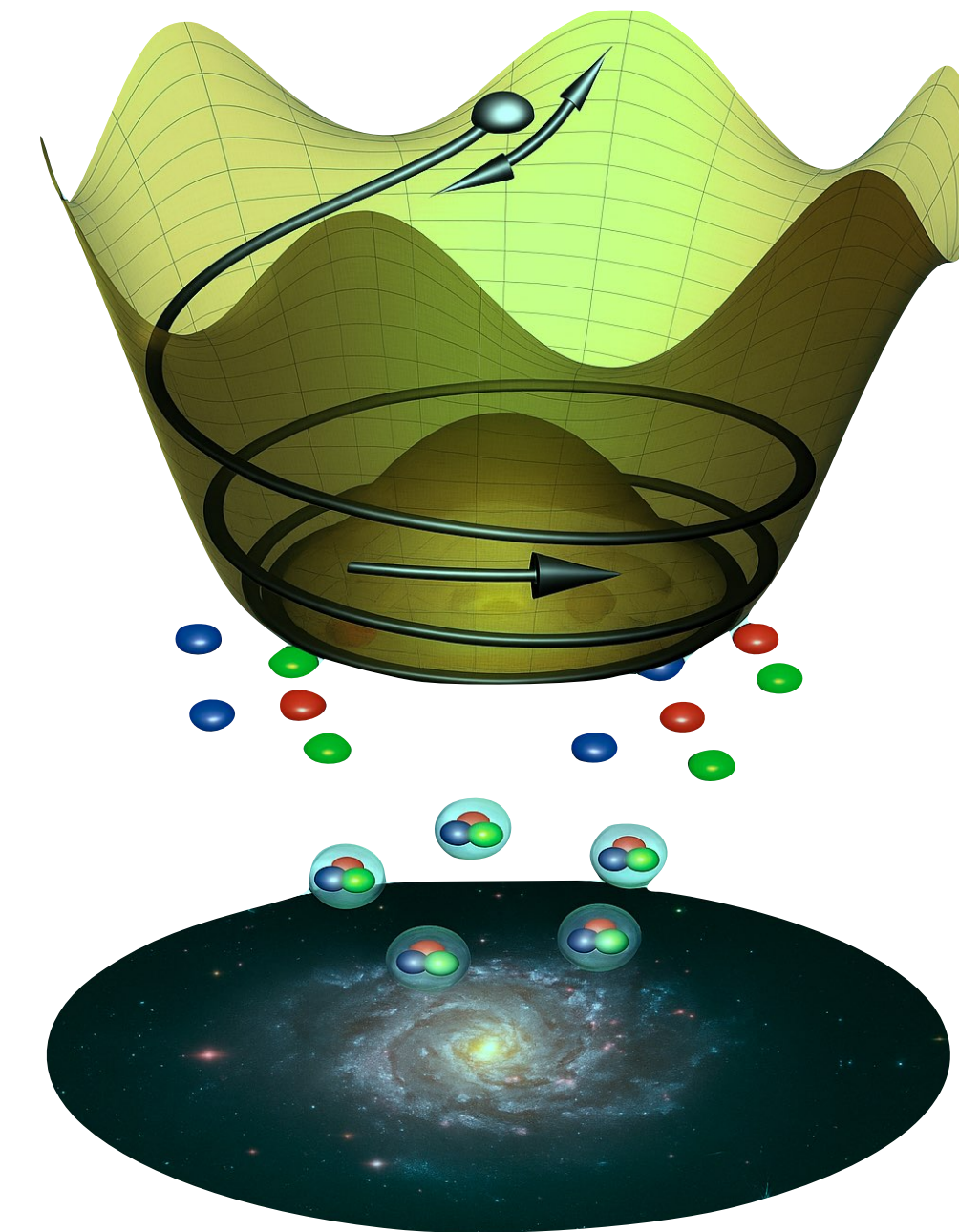
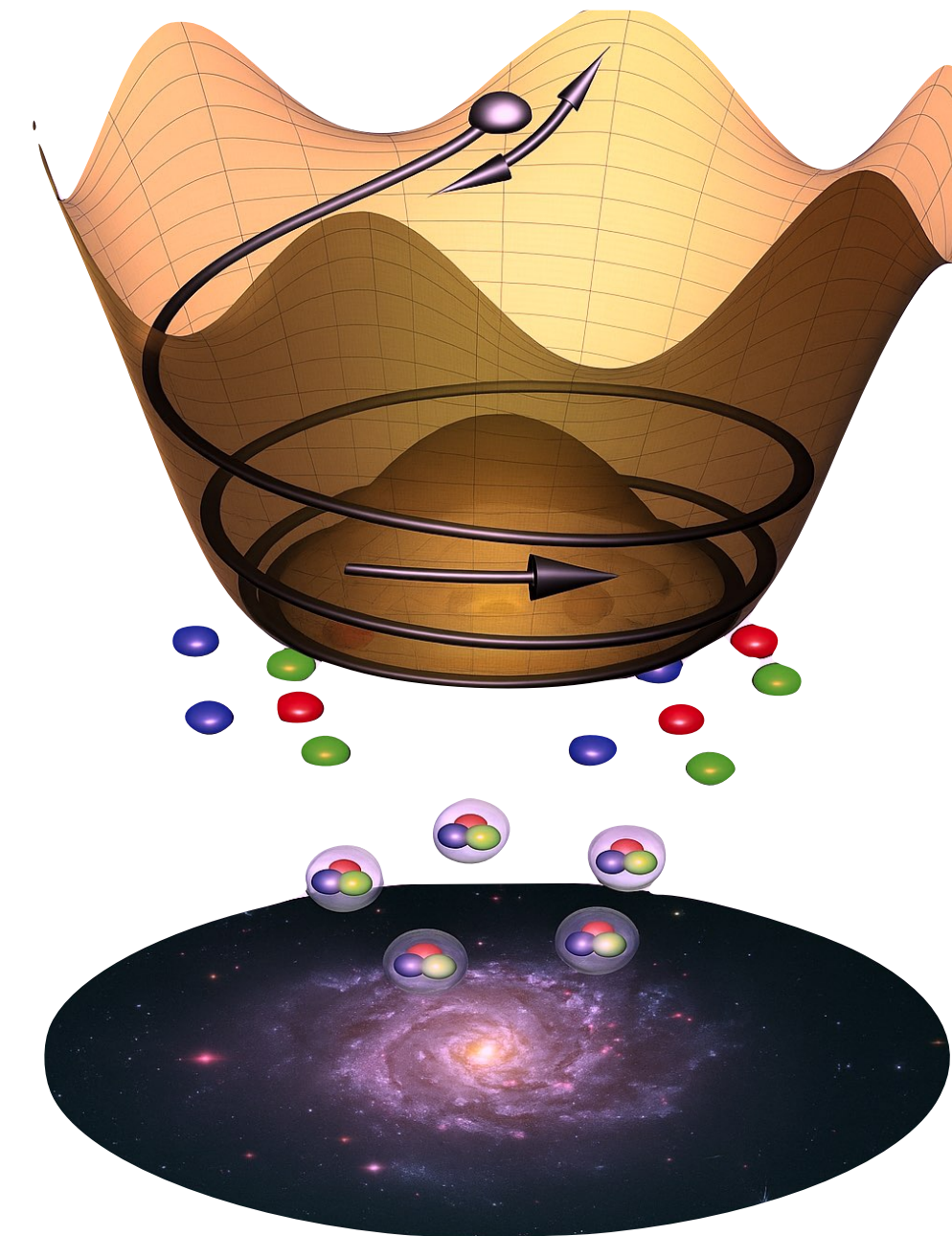
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# A two-axion example

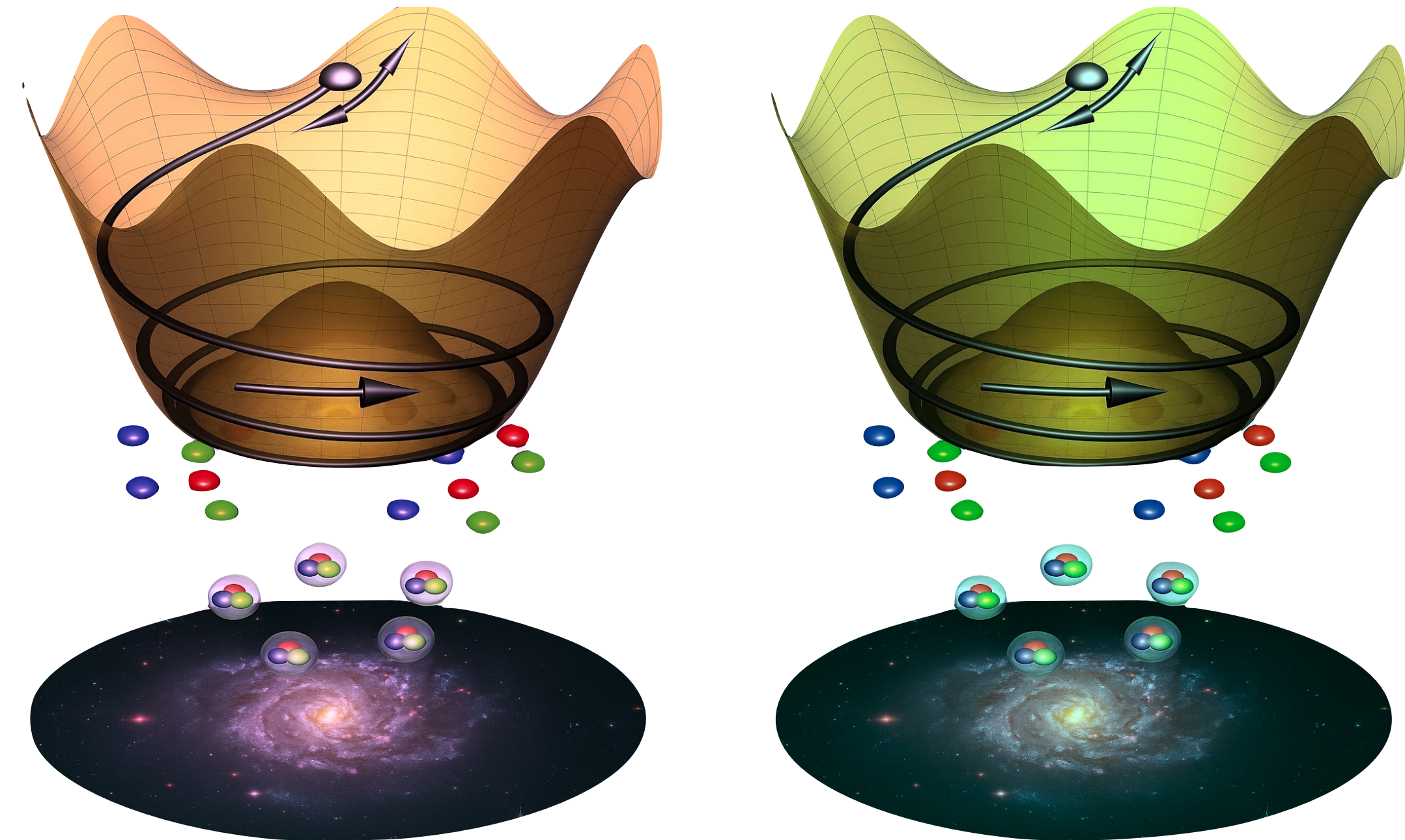
$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \frac{1}{2}f_1^2(\partial\theta_1)^2 + \frac{1}{2}f_2^2(\partial\theta_2)^2 + \frac{\alpha_c}{8\pi}(C_1\theta_1 + C_2\theta_2)\tilde{G}_c G_c + \frac{\alpha_d}{8\pi}\theta_2\tilde{G}_d G_d$$

- **Quality & IC:**

- Axions are protected by gauge symmetry — no quality problem.

~~$\Phi^n / \Lambda^{n-4} \ll \pi \cdot c.$~~

- PQ breaking provided radion dynamics (e.g. in Goldberger Wise mechanism)



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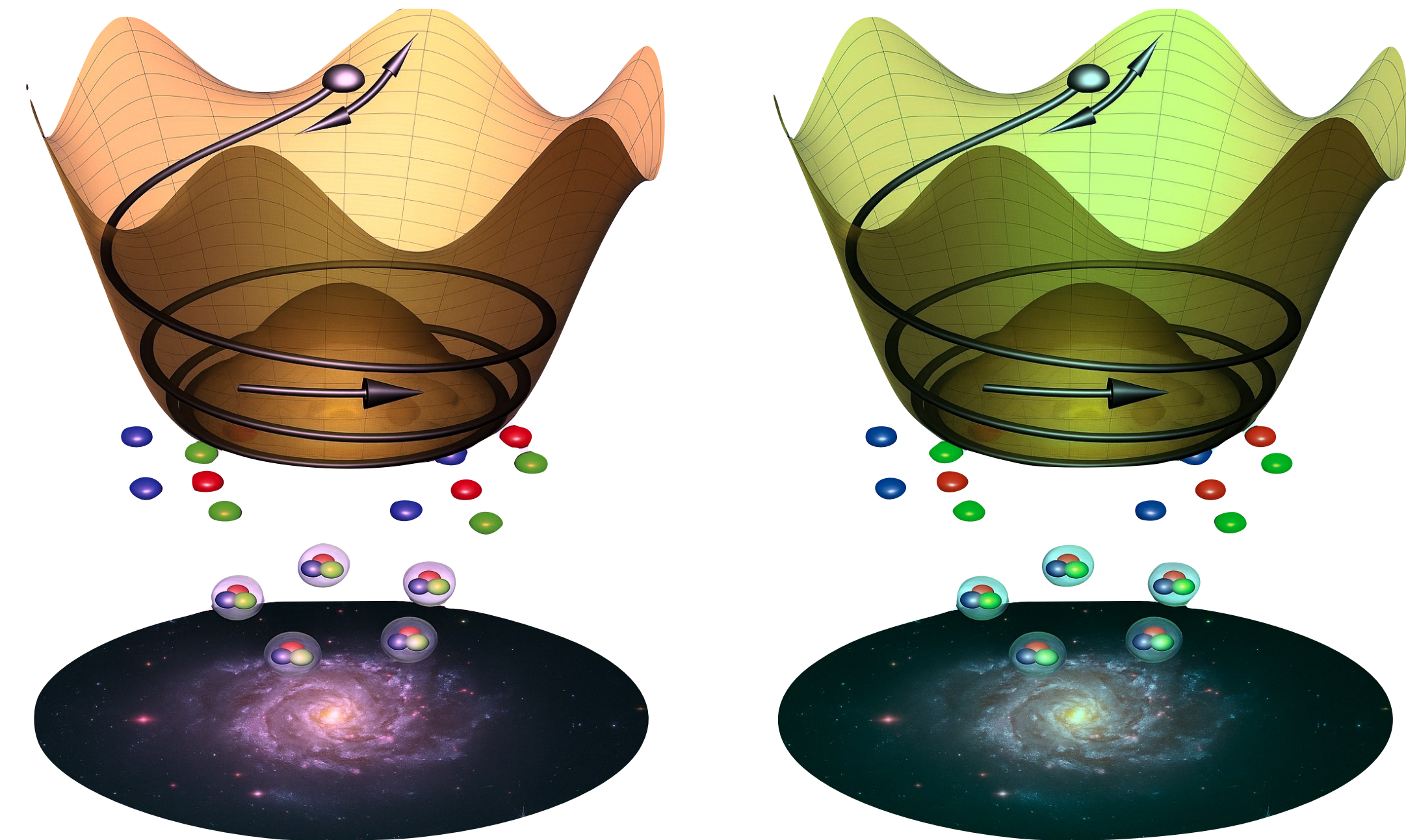
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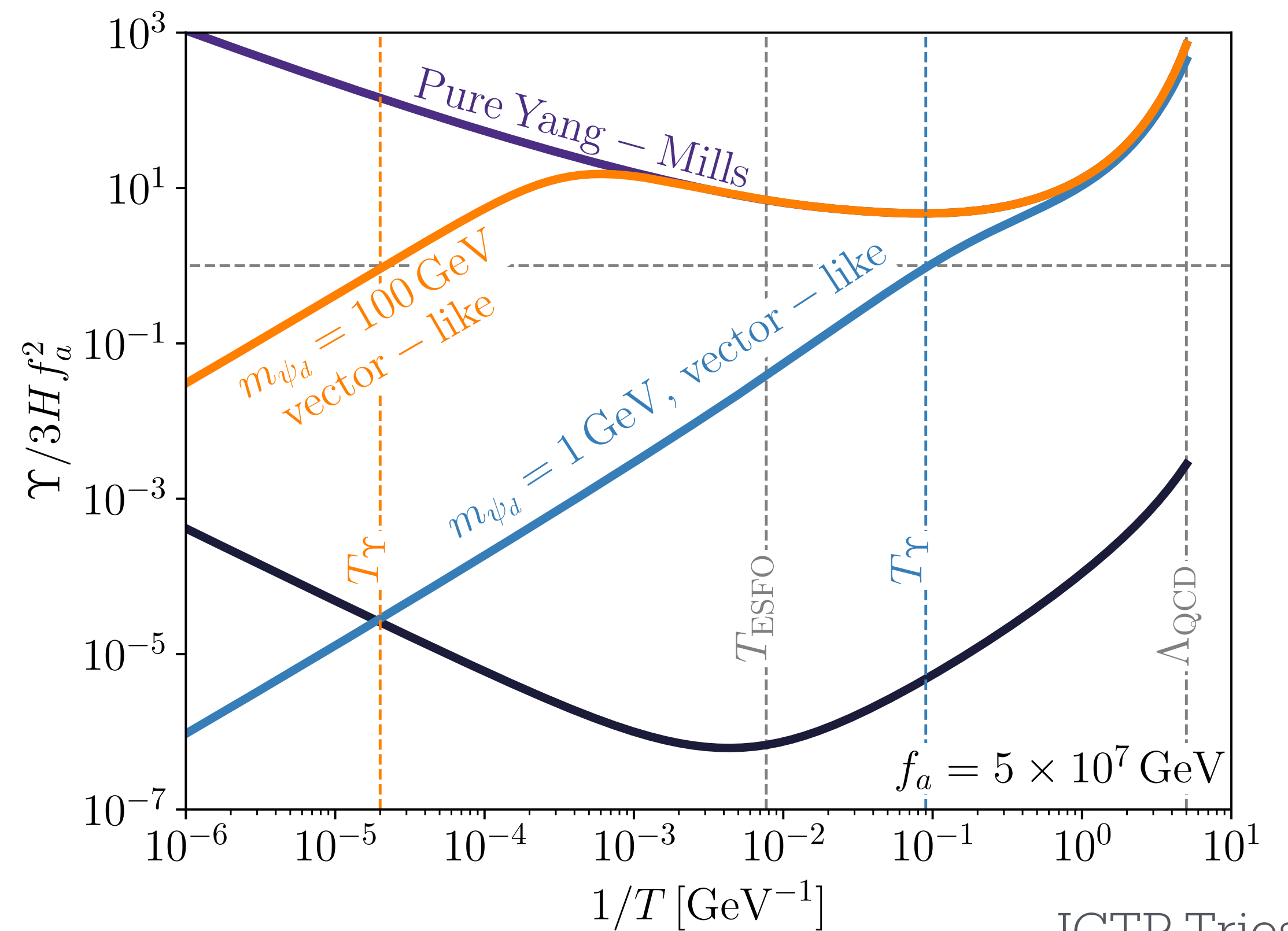
$$V_{PQ} = e^{-ML} \cos(n_1\theta_1 + n_2\theta_2)$$

$$V(L) \sim M^4(e^{\alpha ML} + e^{-\beta ML})$$



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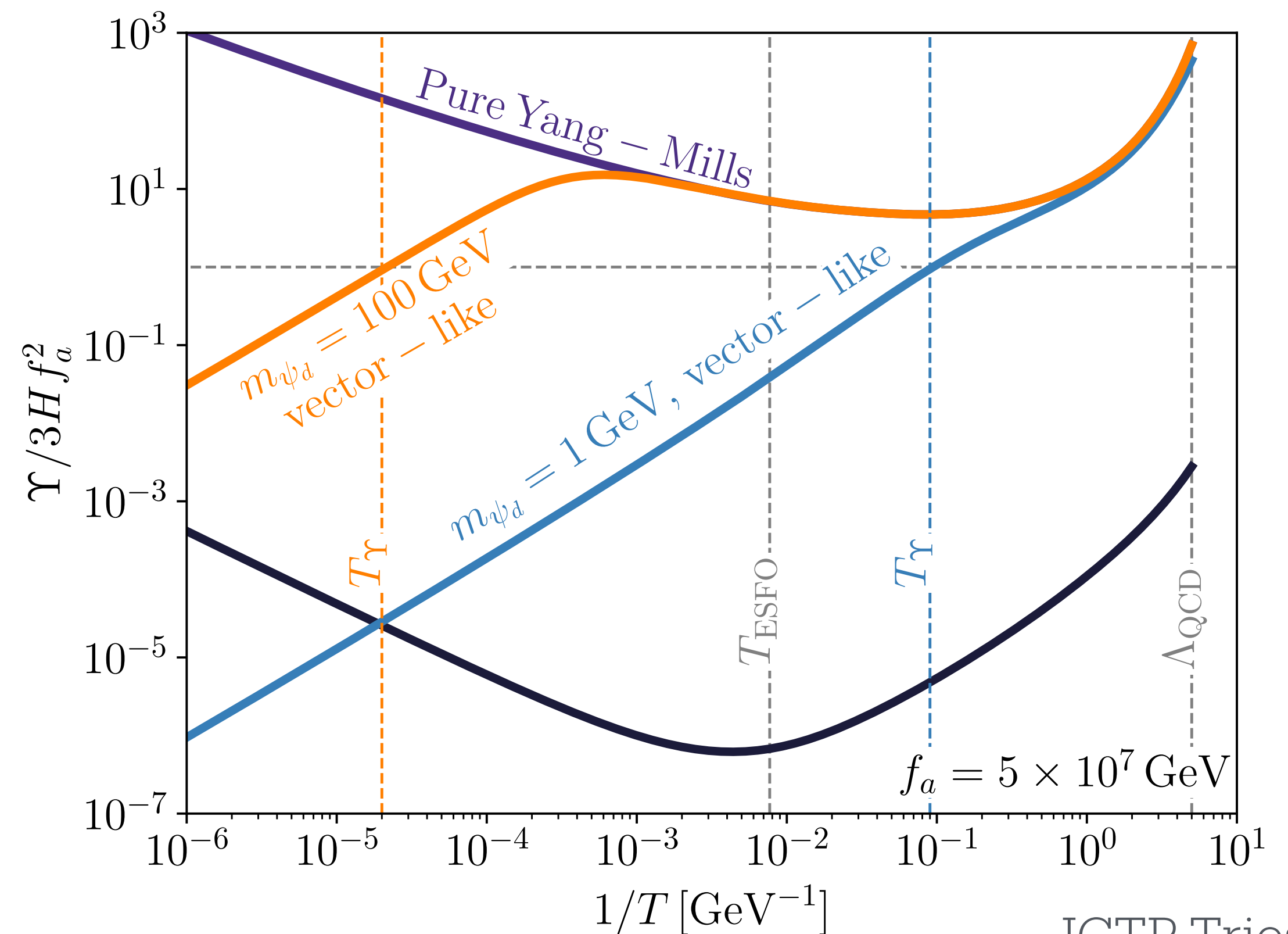
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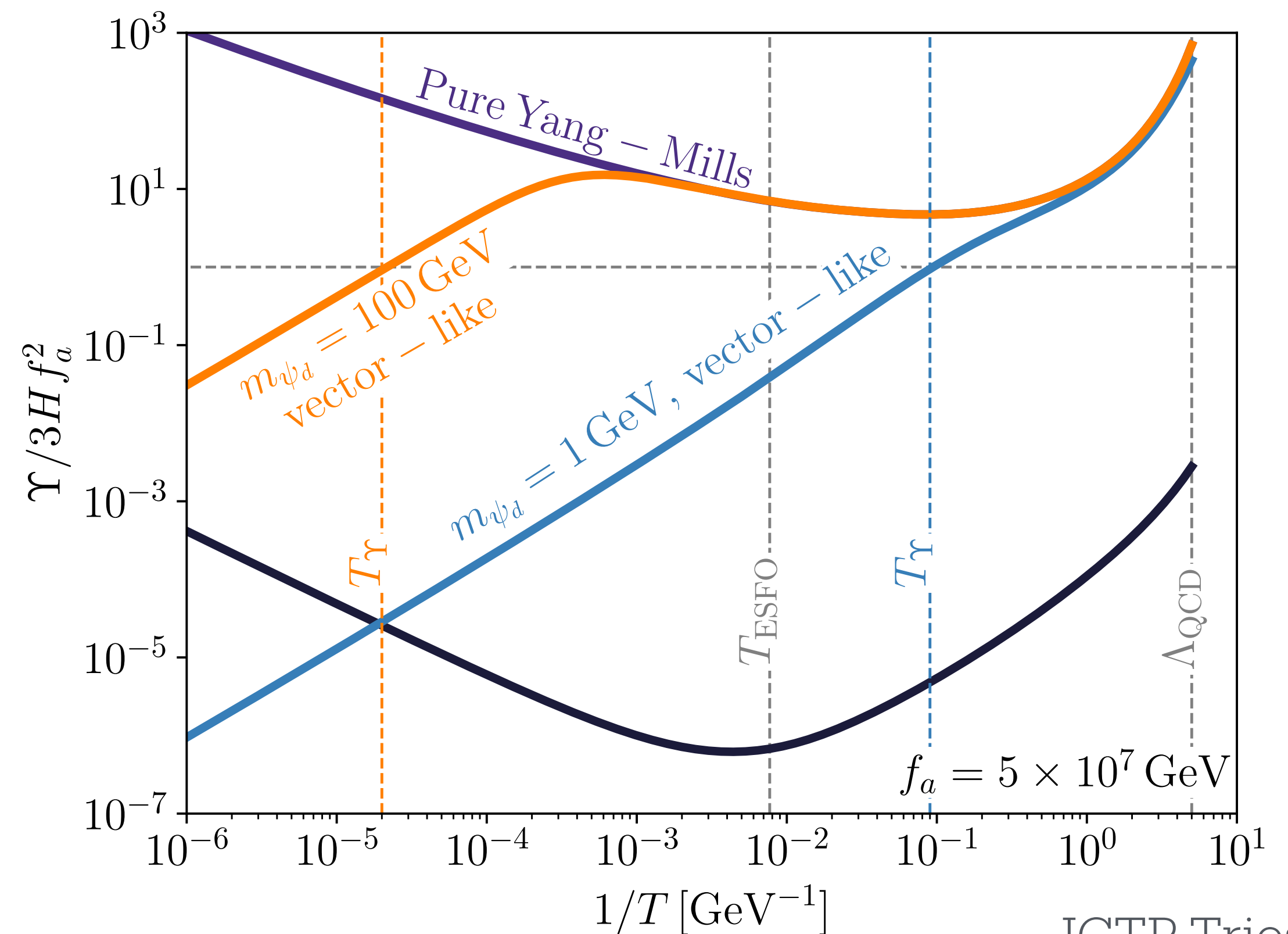
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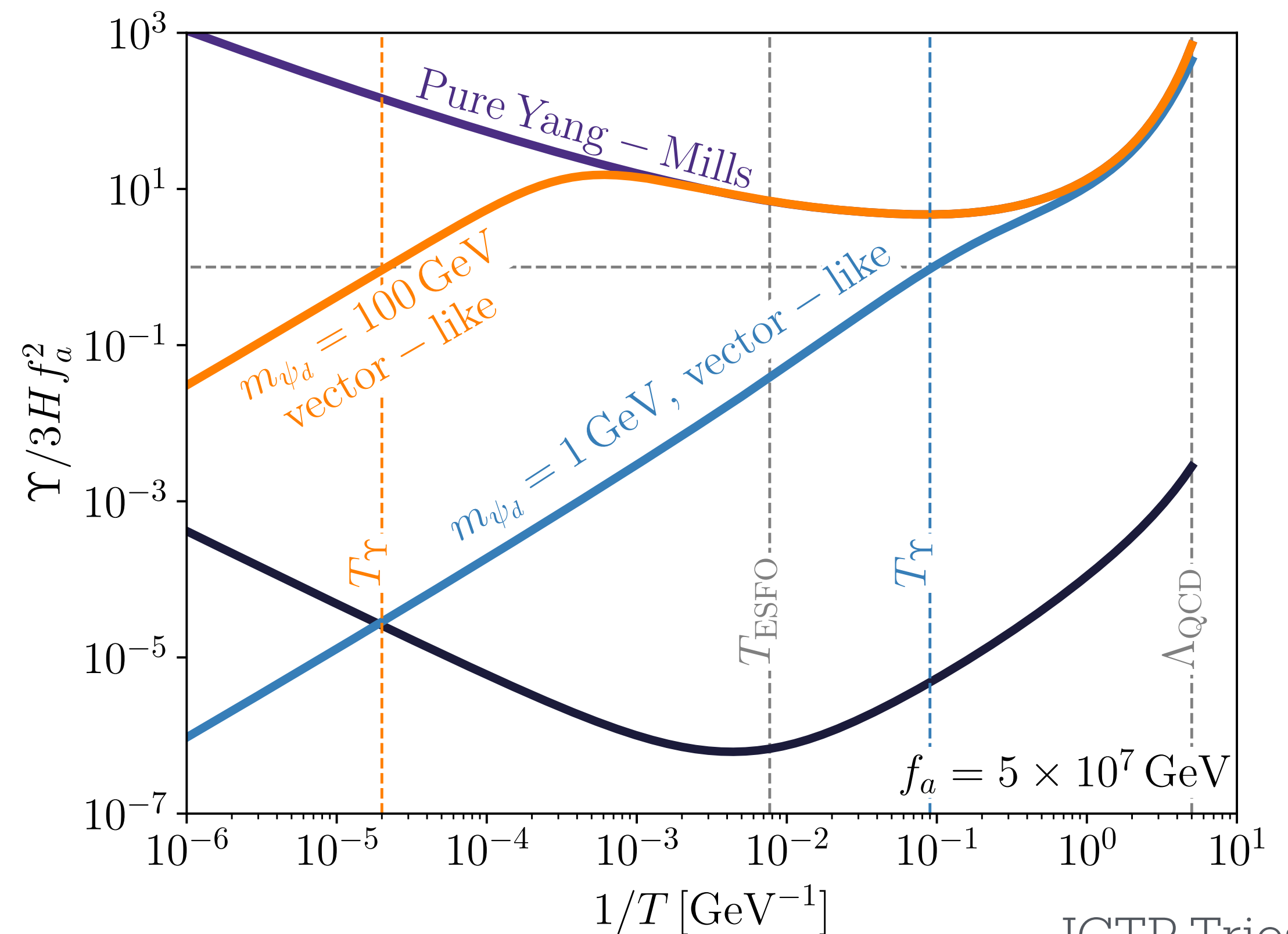
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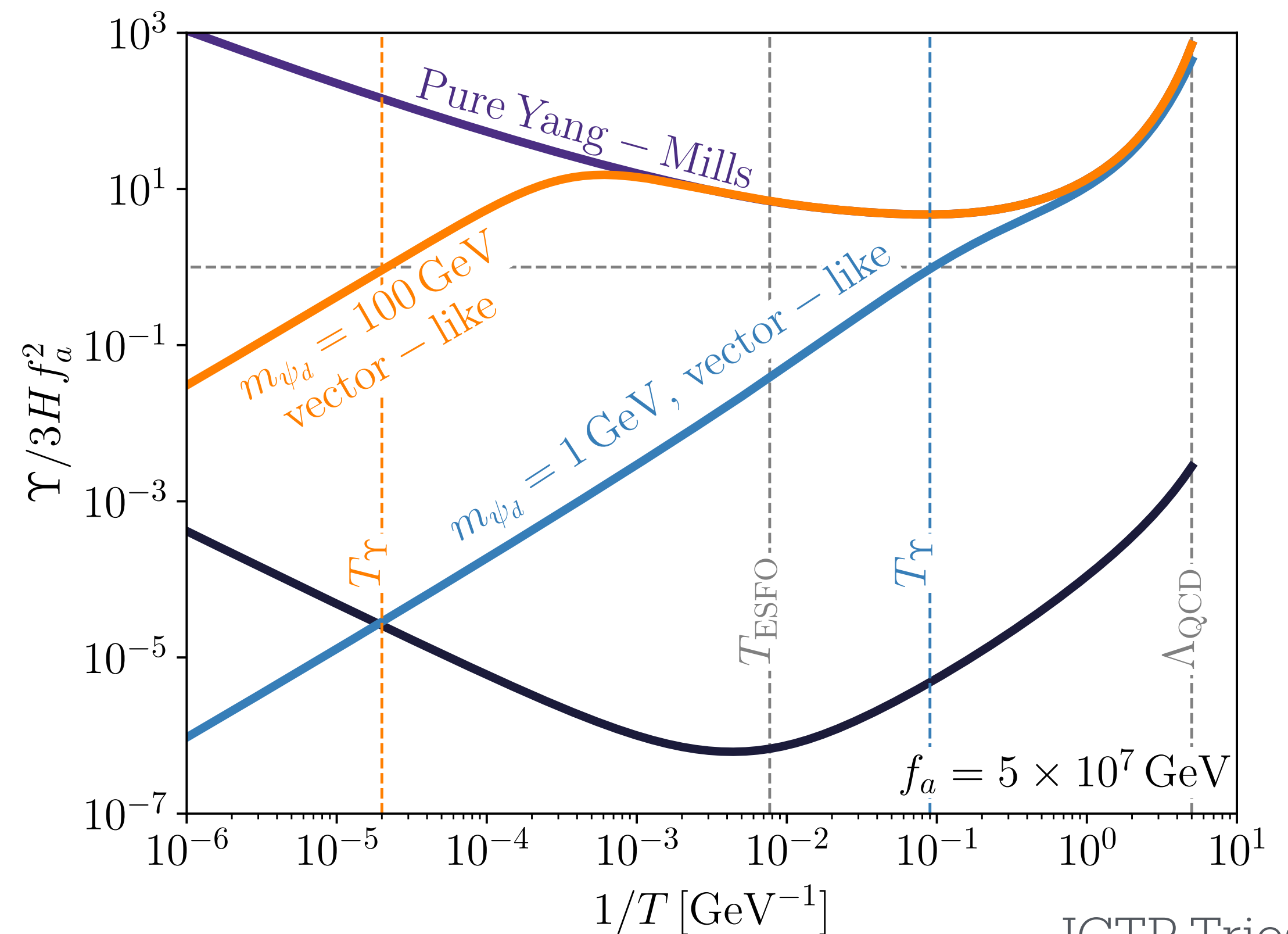
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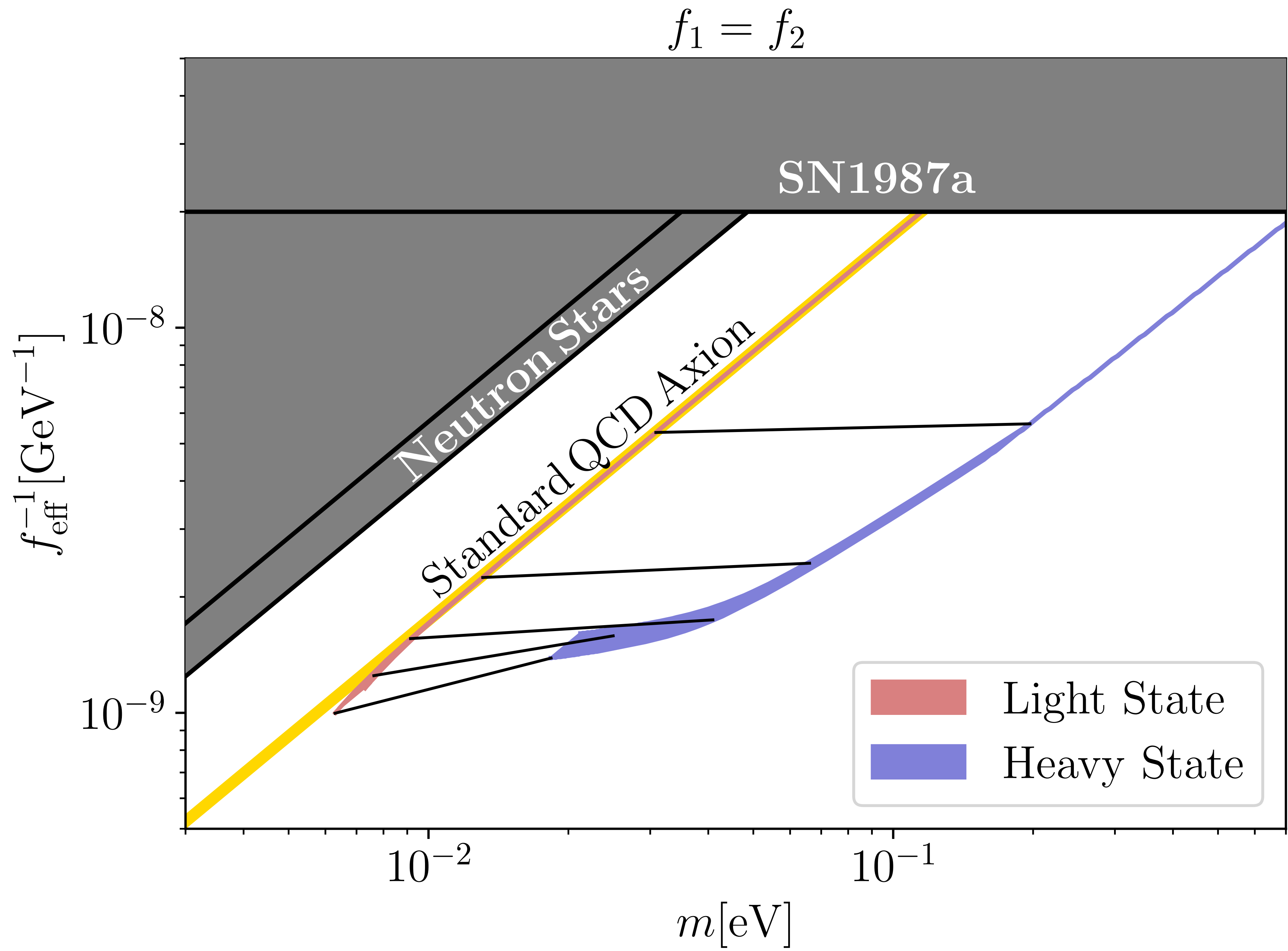


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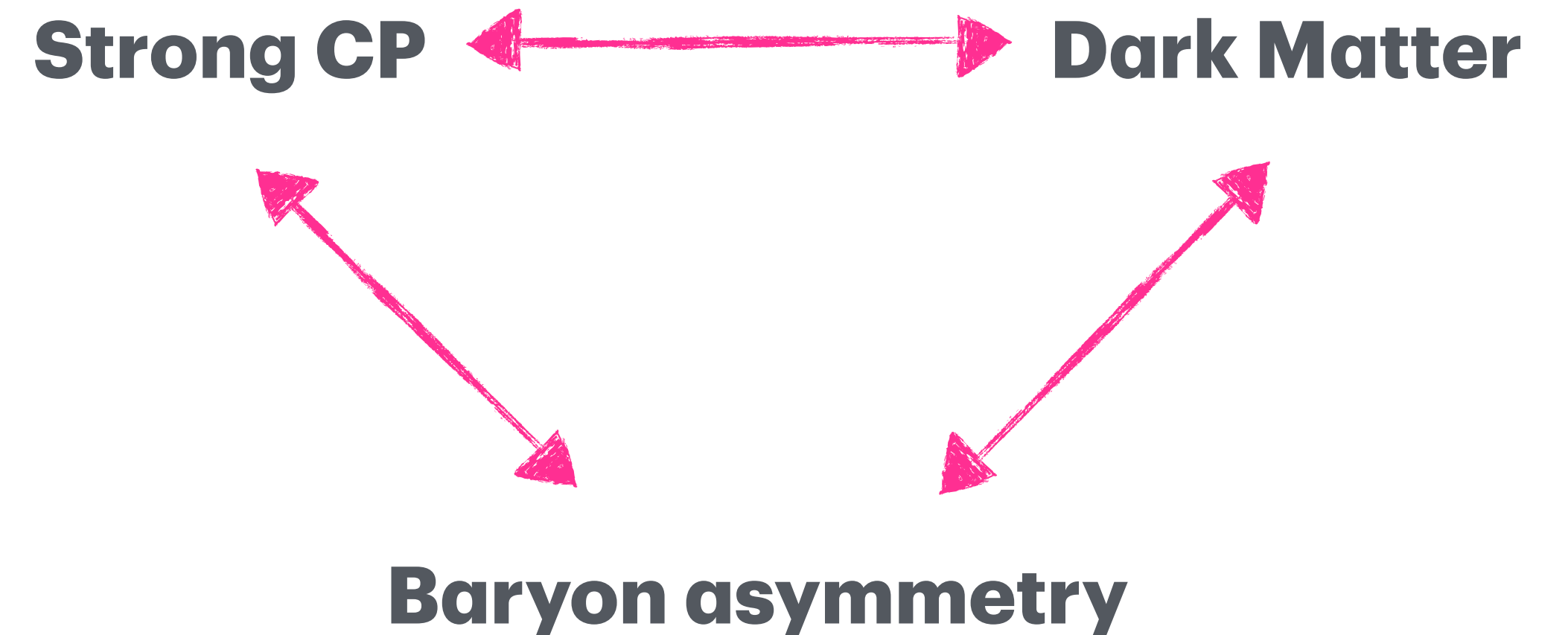
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→ strong sphaleron damping of  $\dot{\theta}_2$
- Mixing with  $\theta_1$ 
  - Avoid overclosure





# Phenomenology

- **Haloscope:** EV-scale QCD-like axion and heavier partner
- **Collider:** new weak-charged fermions with  $m_f \in [0.25, 4.0] \text{ TeV}$
- **Astrophysics:** Axion minihalos
- **Cosmology:** Dark glueball decay signatures from BBN

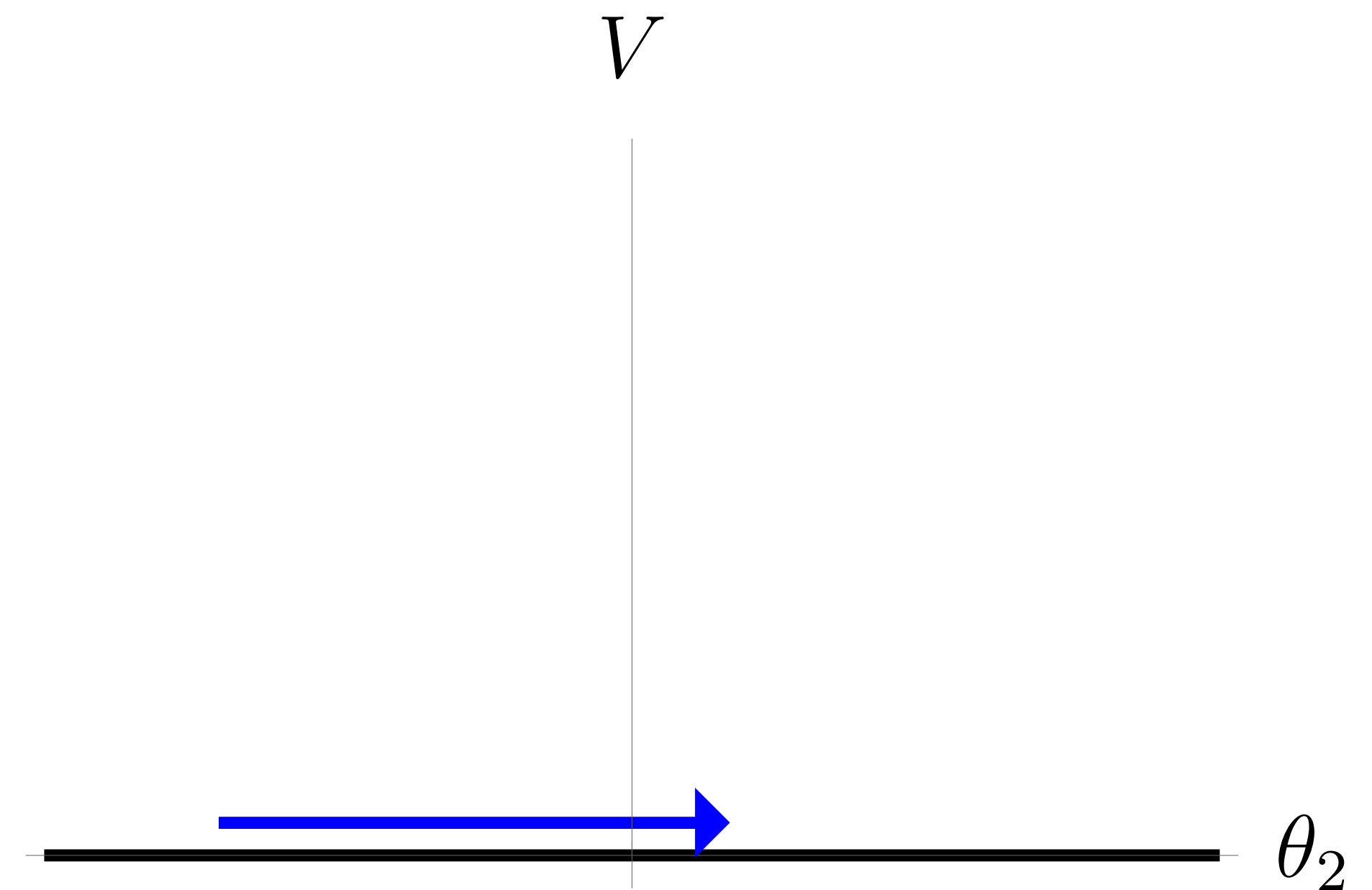
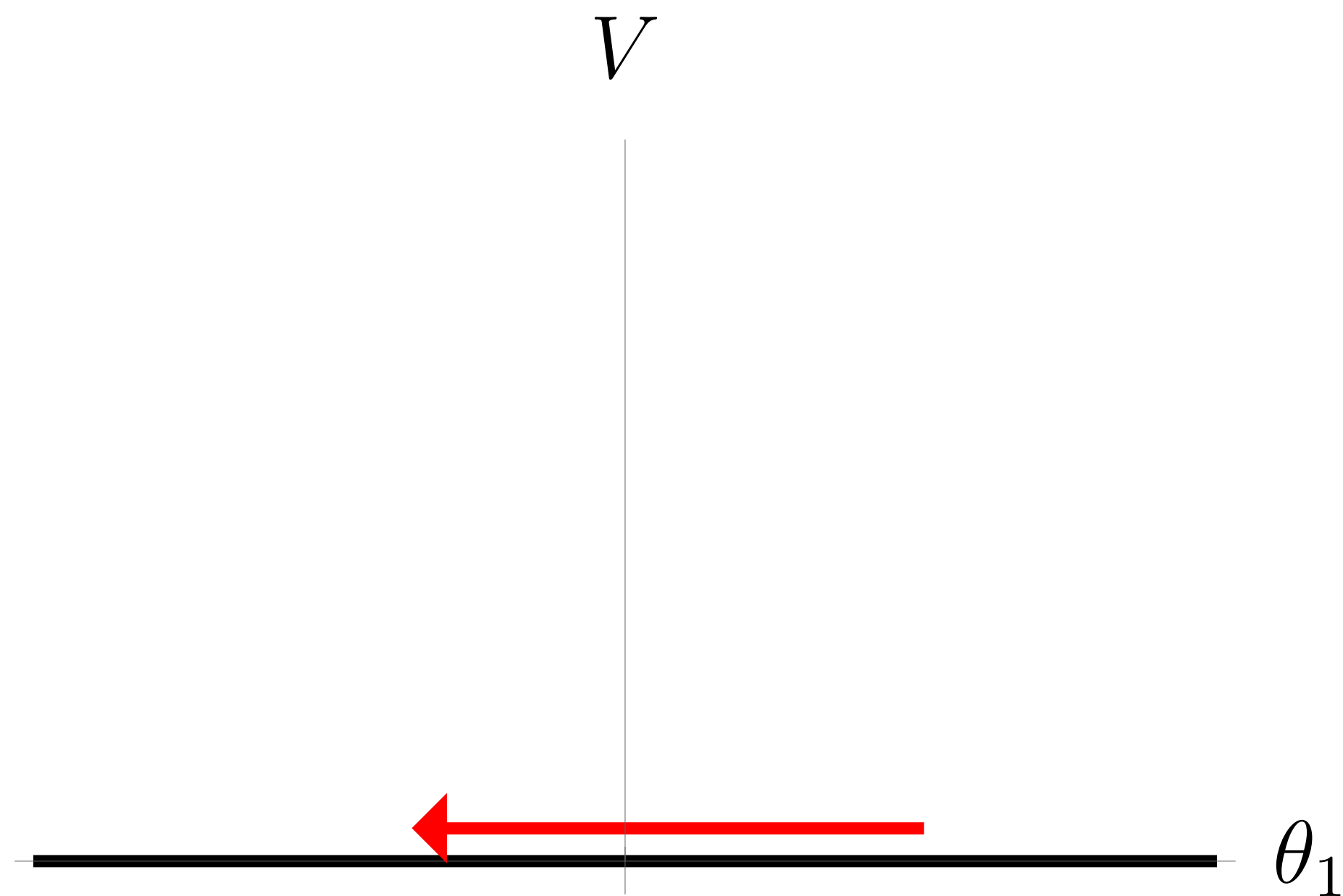


Thank you

# Dynamics

$$0 = \ddot{\theta}_1 + 3H\dot{\theta}_1 + \frac{\chi_c}{f_1^2} \sin(\theta_1 + \theta_2)$$

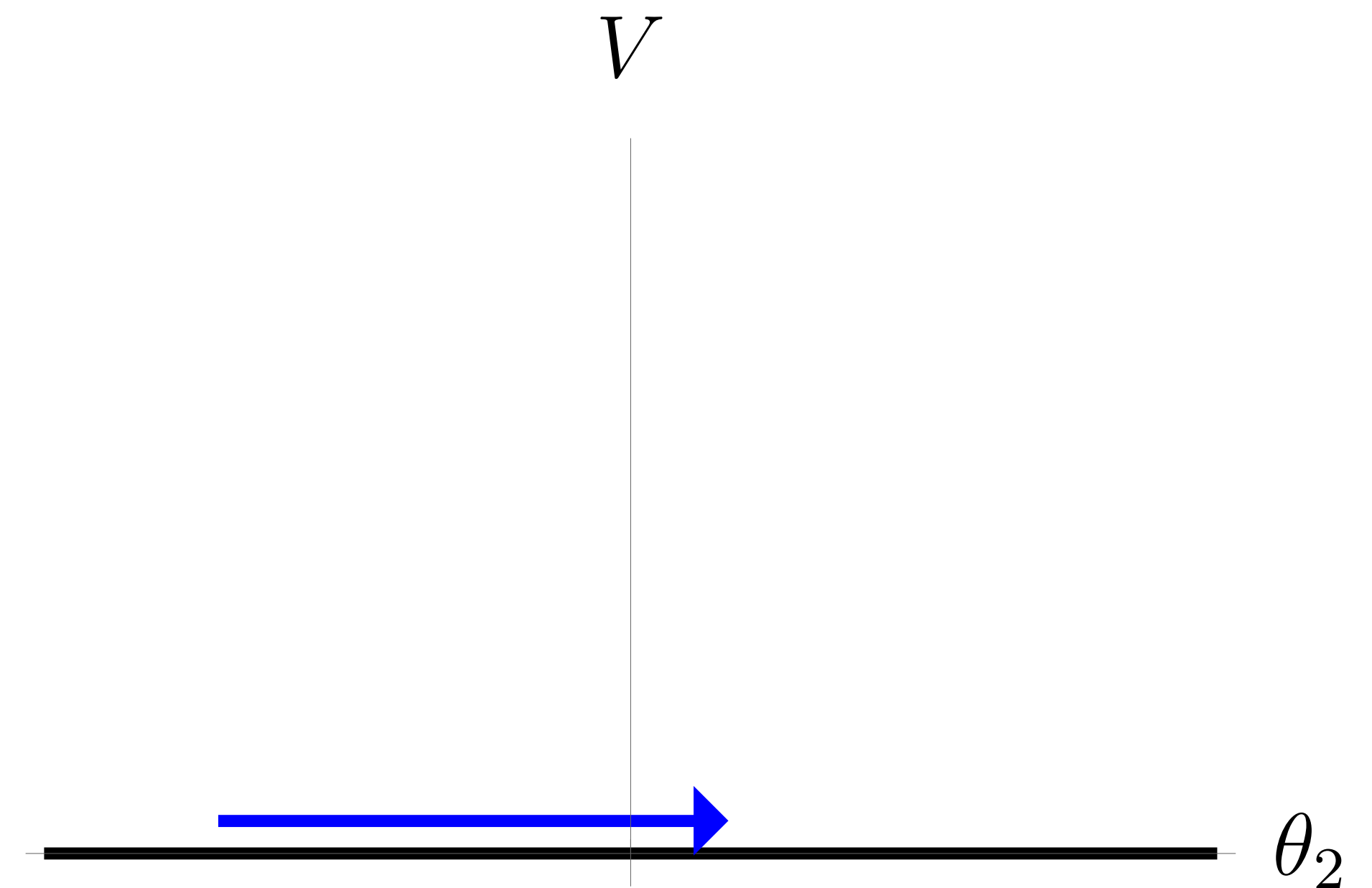
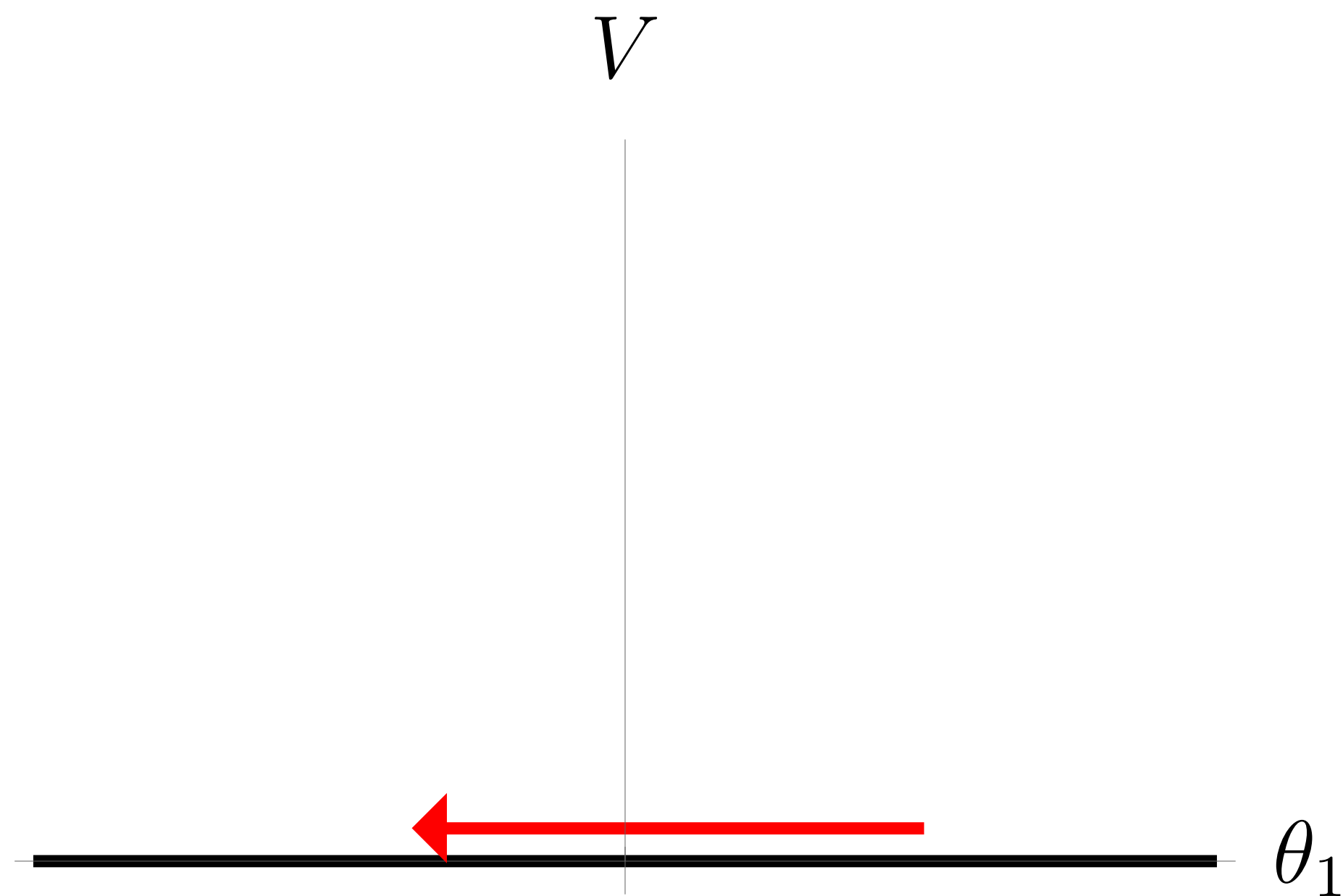
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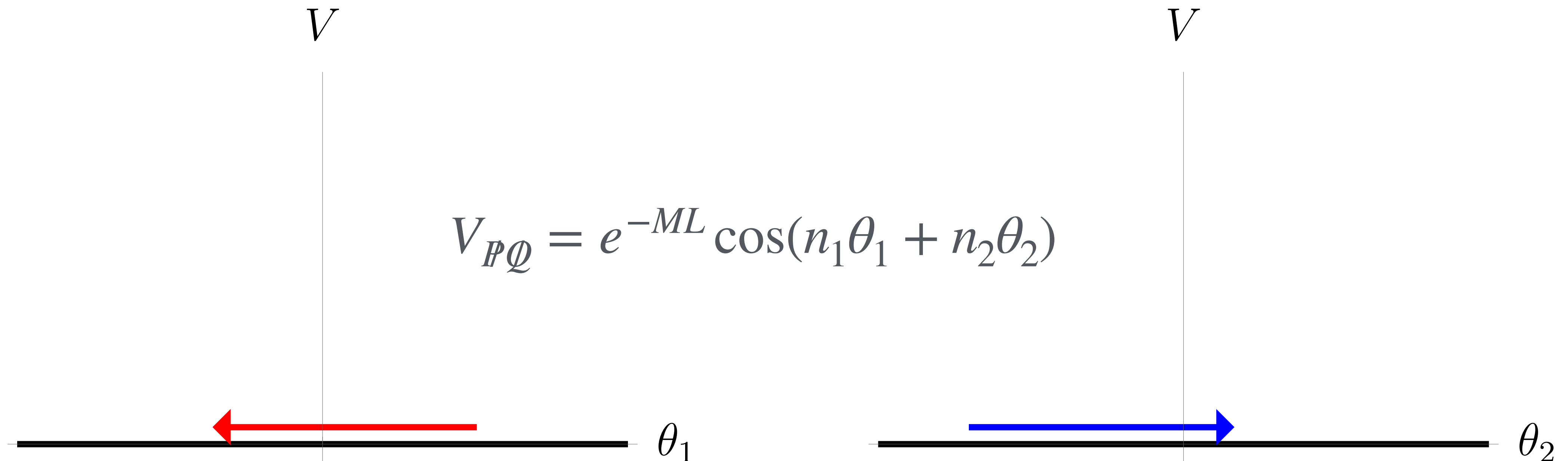
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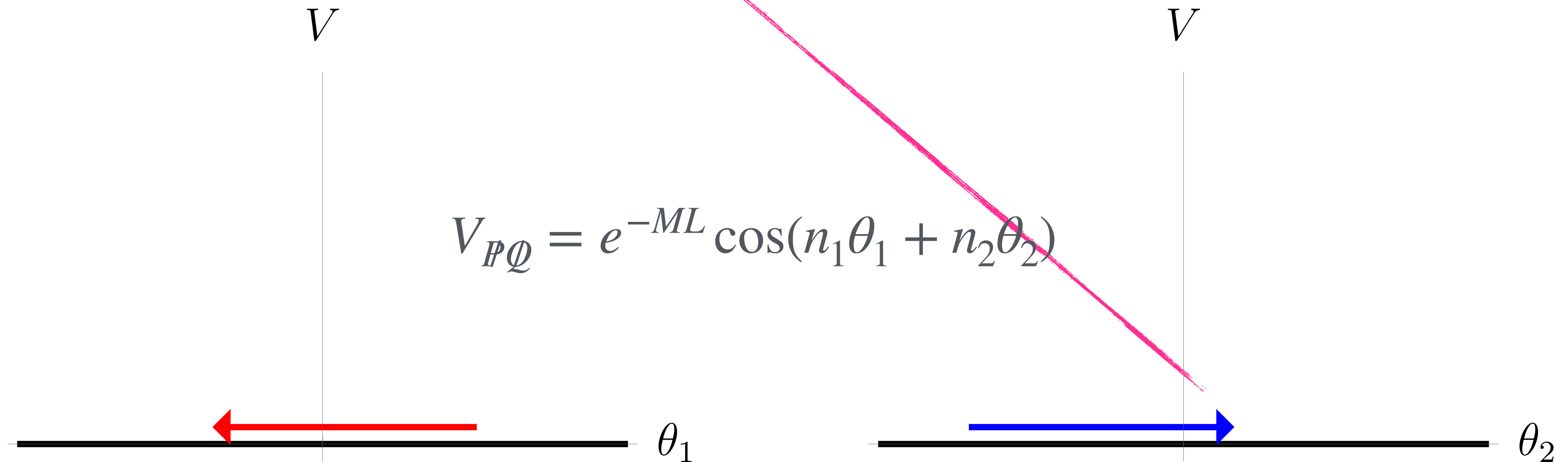


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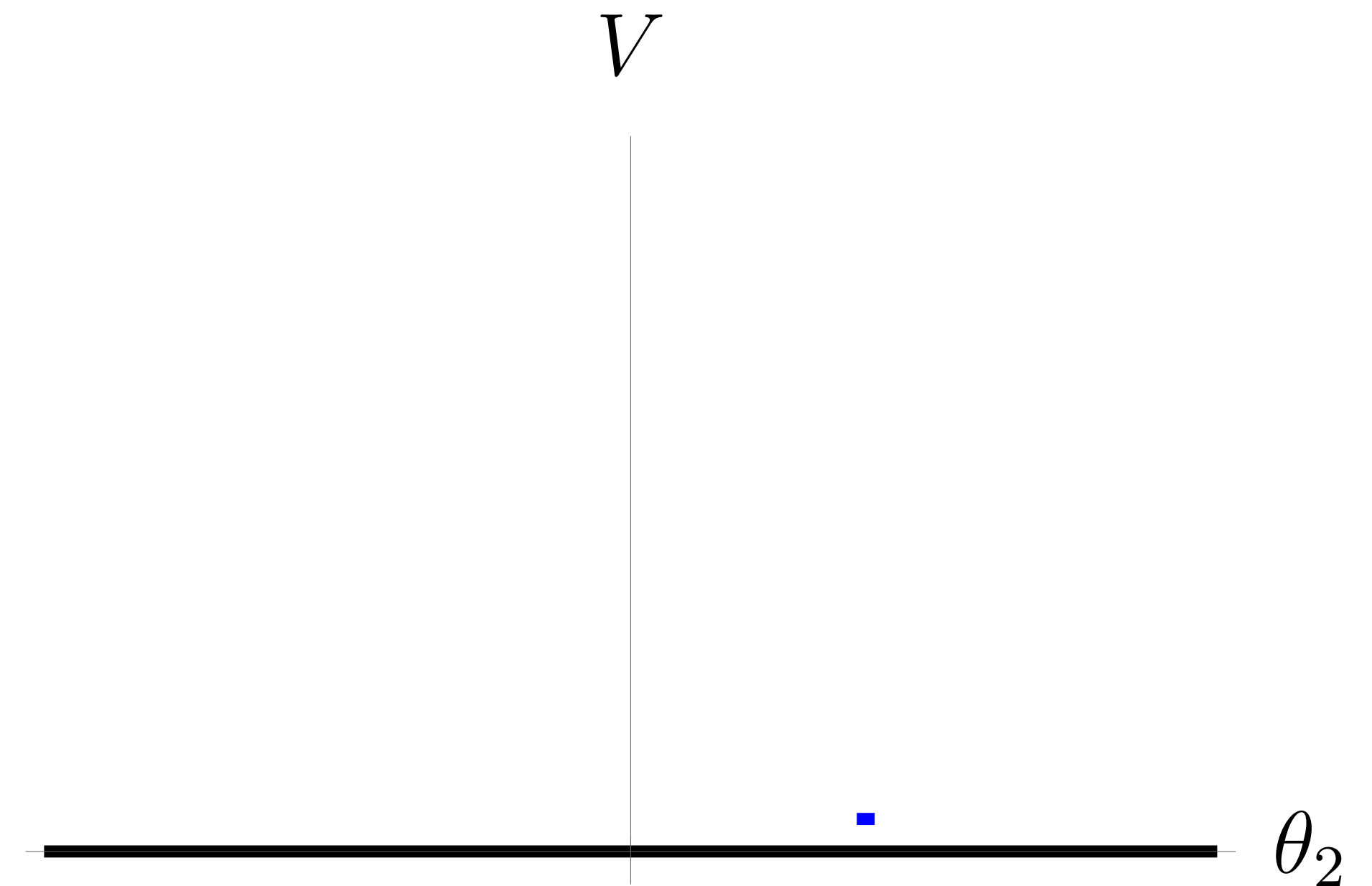
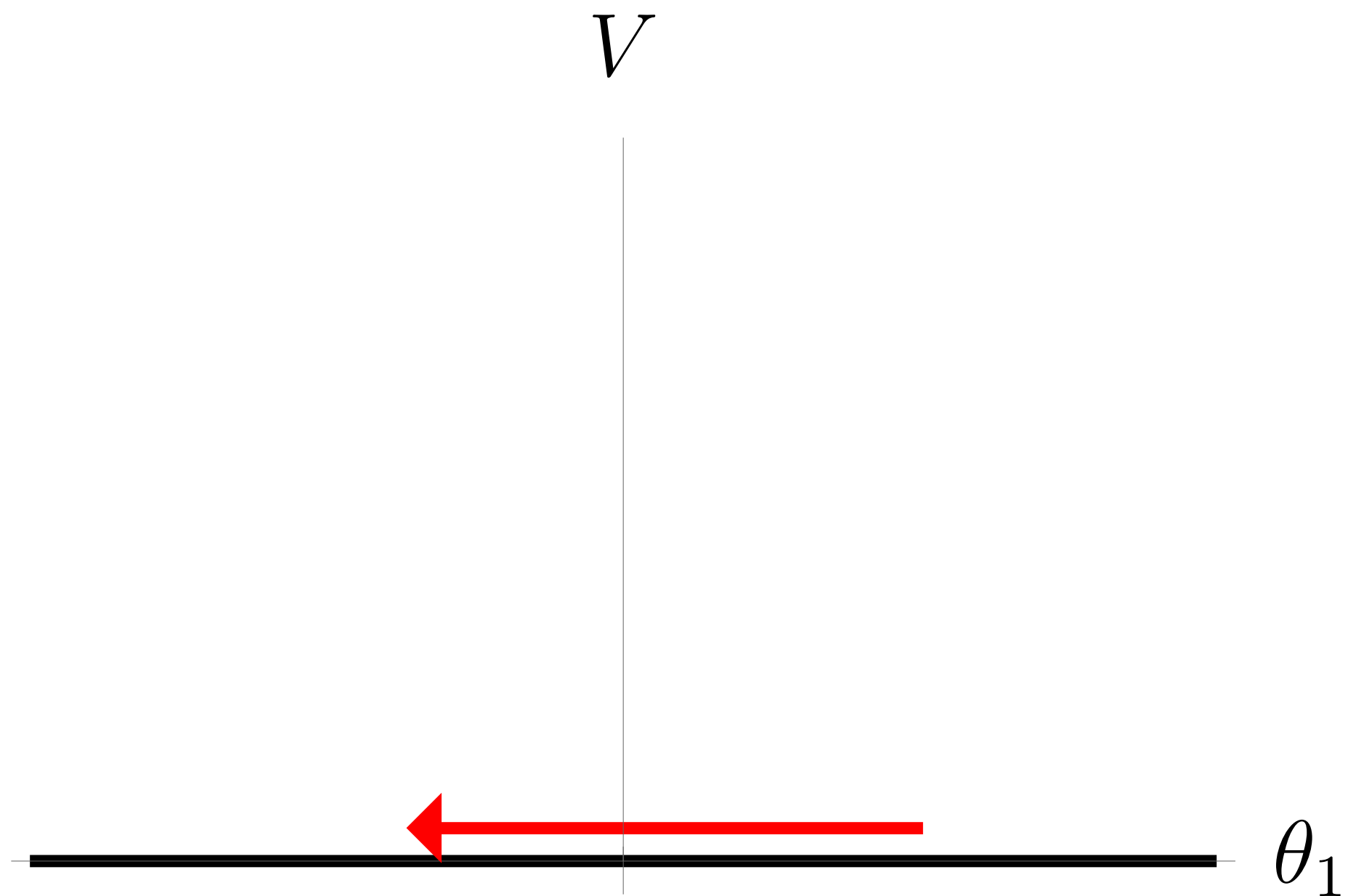
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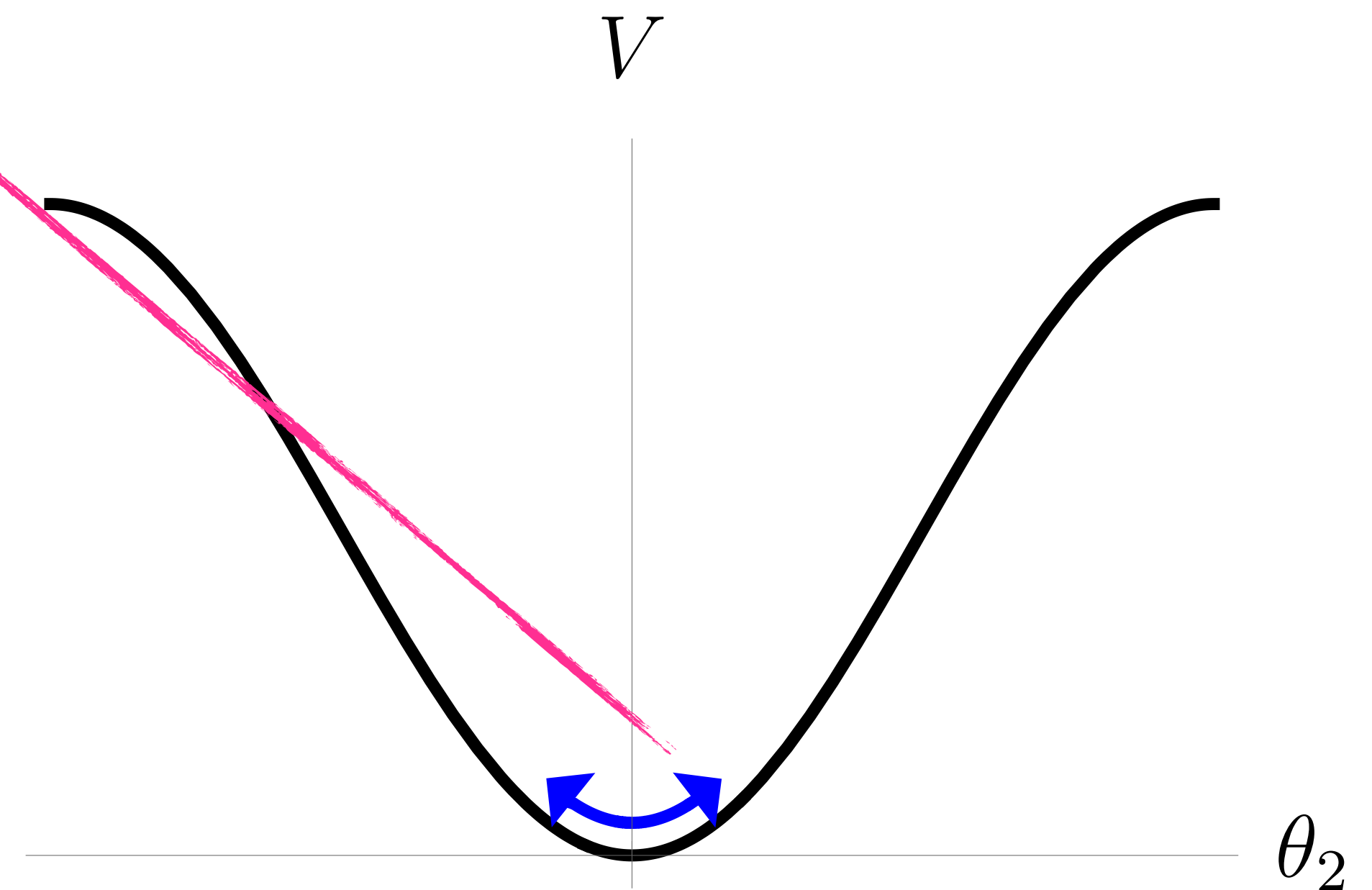
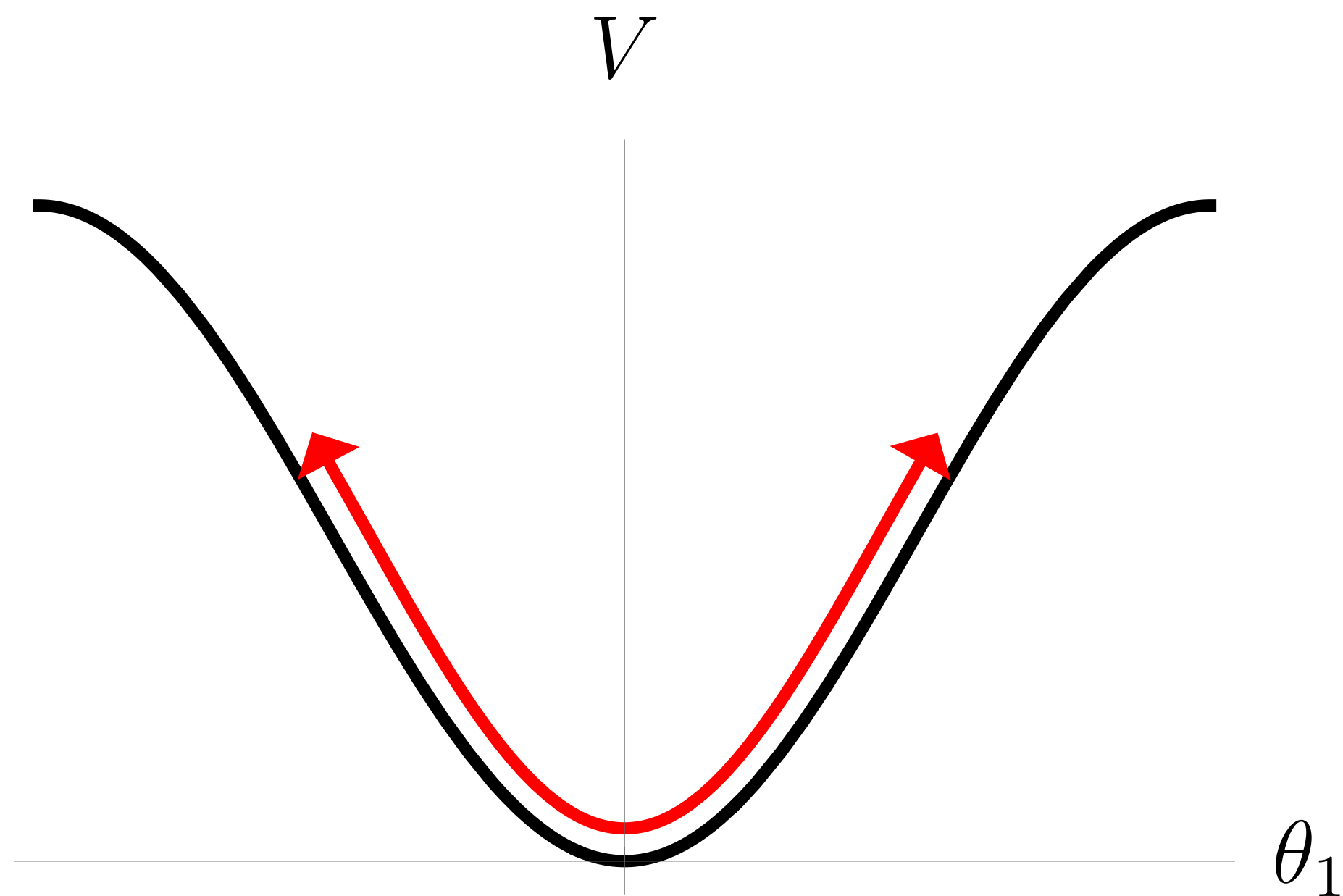
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