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Exploring ultra-high energy neutrino experiments through the lens of the transport equation

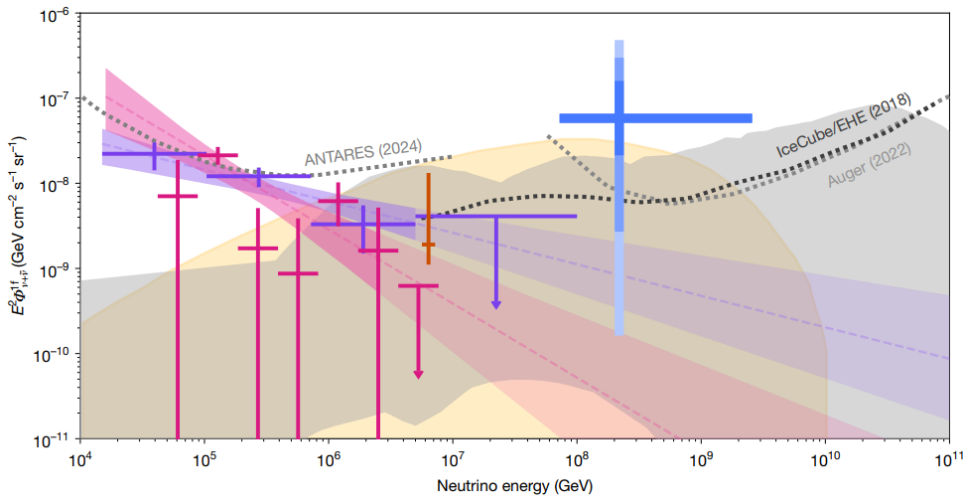
Based on arXiv:2507.10665

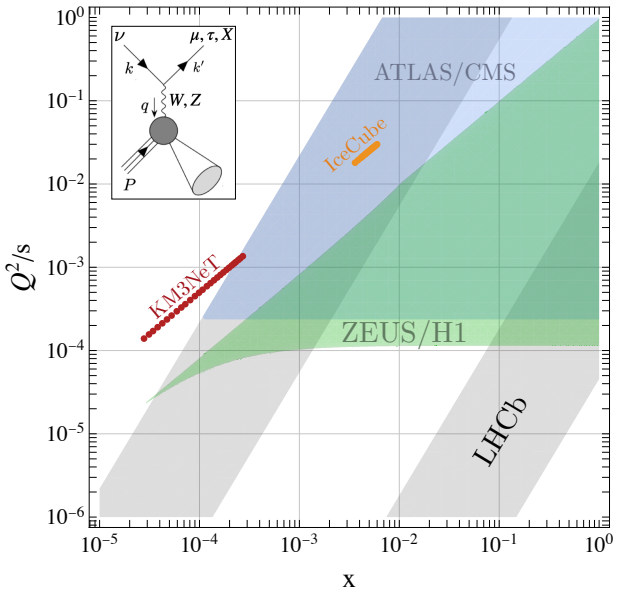
with Diego Redigolo, Michele Tammaro, and Andrea Tesi

Outline of the talk

Neutrino Telescopes are huge "fixed" target experiments

- Measure flux of neutrinos (or other particles!)
- Infer properties of neutrino, Earth, ...





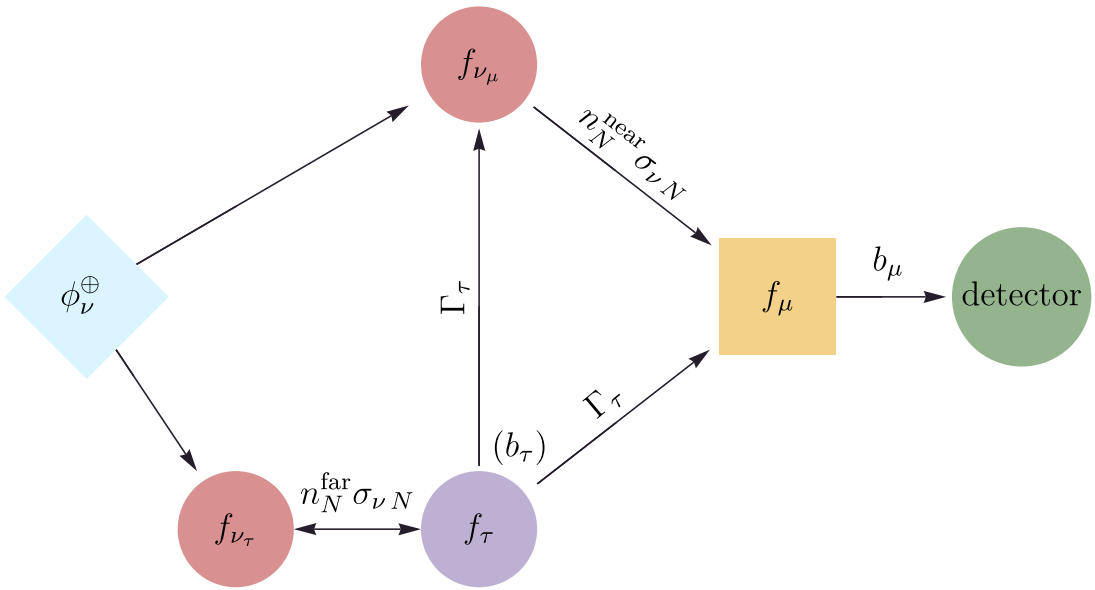
Muon Telescopes?

- Computation of rate of muon tracks from first principles

$$\langle N_\mu \rangle = \int_\nu \int \frac{d^3 p_\mu}{(2\pi)^3} \mathbf{f}_\mu \left(t, \vec{x}, \vec{p}\right) \longrightarrow \text{Boltzmann Equation}$$

Beacom, et al., 02
Dutta, et al., 02
Dutta, et al., 05
Gonzalez-Garcia, et al., 09
Gaisser, et al., 16

- Only unknown →



- Solve neutrino-tau system once and for all, feed into equation for muons

Boltzmann Equation

$$\frac{\partial f_\mu}{\partial t} + \dot{\vec{x}} \cdot \frac{\partial f_\mu}{\partial \vec{x}} + \dot{\vec{p}} \cdot \frac{\partial f_\mu}{\partial \vec{p}} = K\left(t, \vec{x}, \vec{p}\right)$$

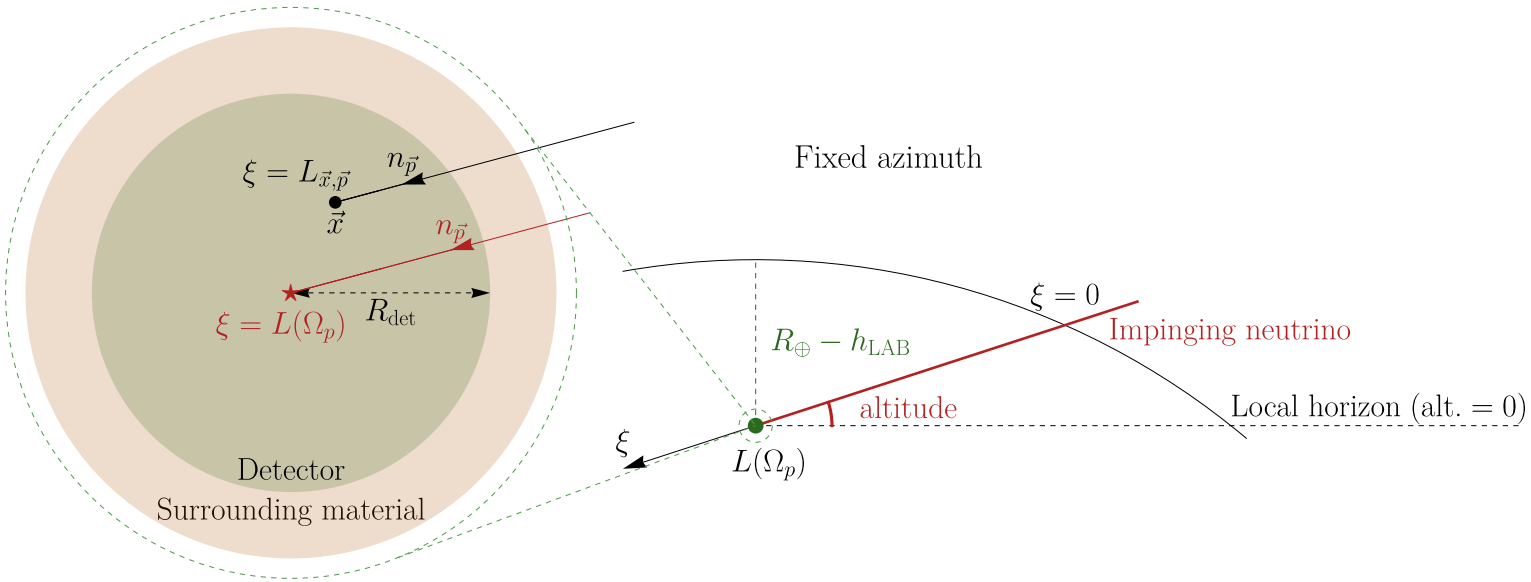
- Kernel encodes upscatterings $\supset \frac{\mathrm{d}\sigma}{\mathrm{d}^3 p_\mu}$, decays $\supset \frac{\mathrm{d}\Gamma}{\mathrm{d}^3 p_\mu}$
- QED processes are fast, and they contribute to $\dot{\vec{p}}$

$$-\frac{\mathrm{d}E_\ell}{\mathrm{d}t} = -v\frac{\mathrm{d}E_\ell}{\mathrm{d}x} \approx -b_\ell v E_\ell$$

- Muon decay and ν_μ repopulation is slow, we neglect it

Under these approximations, equation is linear

Line-of-sight



- Find trajectories along which the equation becomes ordinary
- $f_\mu\left(t, \vec{x}, \vec{p}\right) = \int_0^L \mathrm{d}\xi K(t(\xi), \vec{x}(\xi), \vec{p}(\xi))$
- Neutrino energy - conversion point relation:

$$E_\mu^{\rm meas} \sim E_\mu^{\rm prod} e^{-b_\mu \xi} \sim E_\nu e^{-b_\mu \xi}$$

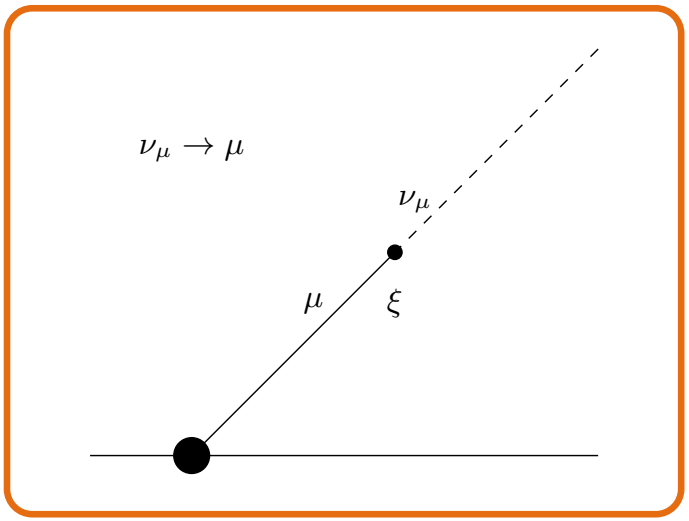
- Integral ranges over $R_{\rm eff} \sim 1/b_\mu \gtrsim R_{\rm det}$

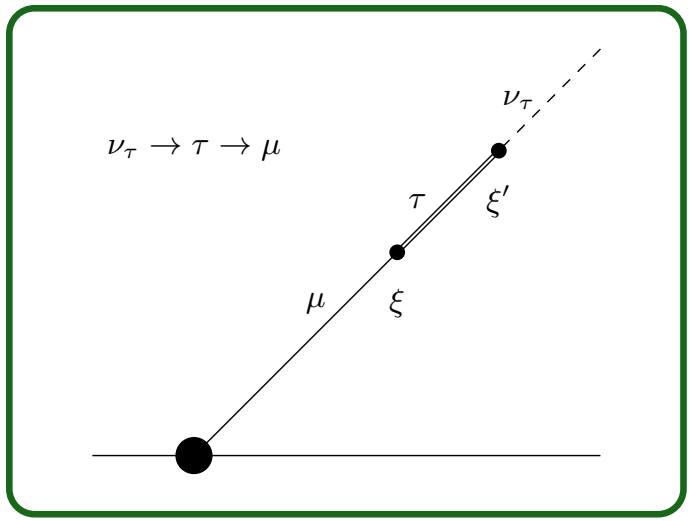
Expected rate of events

$$\left\langle \frac{\mathrm{d}N_\mu}{\mathrm{d}t\mathrm{d}E_\mu\mathrm{d}\Omega_\mu} \right\rangle = \frac{\text{number of muons}}{\text{time} \times \text{muon energy} \times \text{solid angle}} =$$

$$4\pi A_T E_\mu^2 \left[\int^L \mathrm{d}\xi \int \mathrm{d}E_\nu \,\mathrm{Att.}\, n_N(\xi) \frac{\mathrm{d}\sigma}{\mathrm{d}^3 p_\mu(\xi)} \phi_\nu^\oplus \left(E_\nu\right) \right.$$

$$\left. + \int^L \mathrm{d}\xi \int^\xi \mathrm{d}\xi' \int dE_\nu \,\mathrm{d}^3 p_\tau \,\mathrm{Att.}\, n_N(\xi') \frac{\mathrm{d}\sigma}{\mathrm{d}^3 p_\tau(\xi')} \frac{\mathrm{d}\Gamma}{\mathrm{d}^3 p_\mu(\xi)} D_\tau\left(\xi, \xi'\right) \phi_\nu^\oplus \left(E_\nu\right) \right]$$





Case study (and prompt): KM3-230213A

- Assessment of tension with IceCube
- Tension quantified in terms of

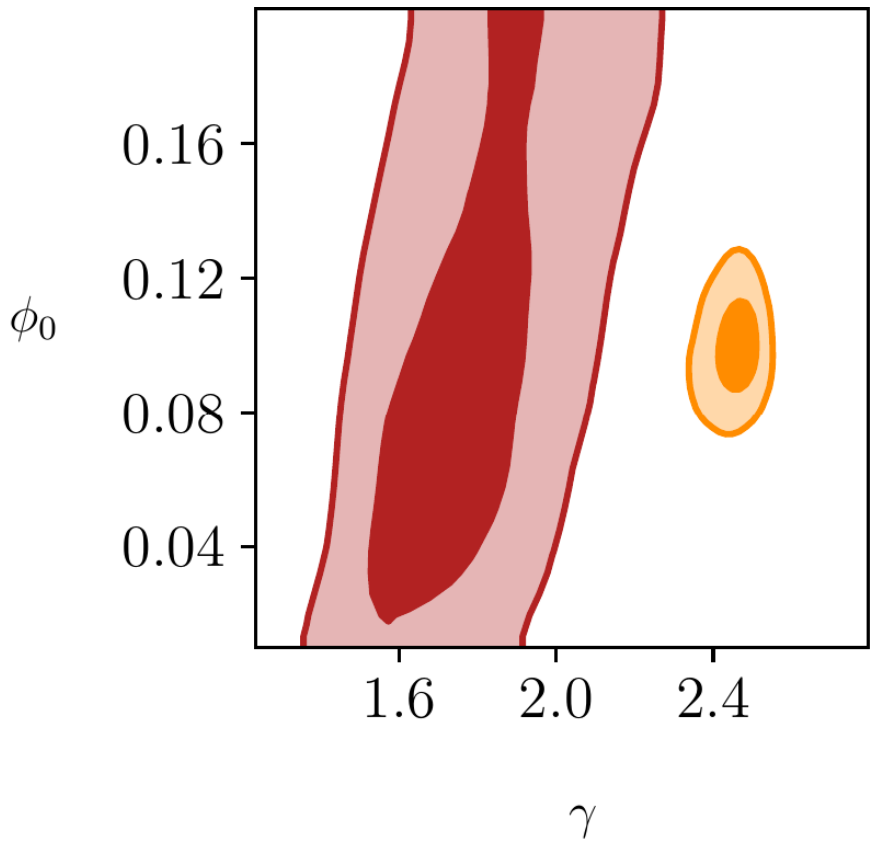
$$\Delta = \left. \frac{\#_{\mathrm{IC}}}{\#_{\mathrm{KM3NeT}}} \right|_{\text{KM3-230213A bin}}$$

1. Diffuse flux, both power law and energy localized
2. Point source, which may be transient

Diffuse power-law

$$\phi_\nu^{\oplus} = \phi_0^* \left(\frac{E_\nu}{E_*}\right)^{-\gamma} \longrightarrow \Delta \sim 70 \longrightarrow 3.1\,\sigma \text{ tension}$$

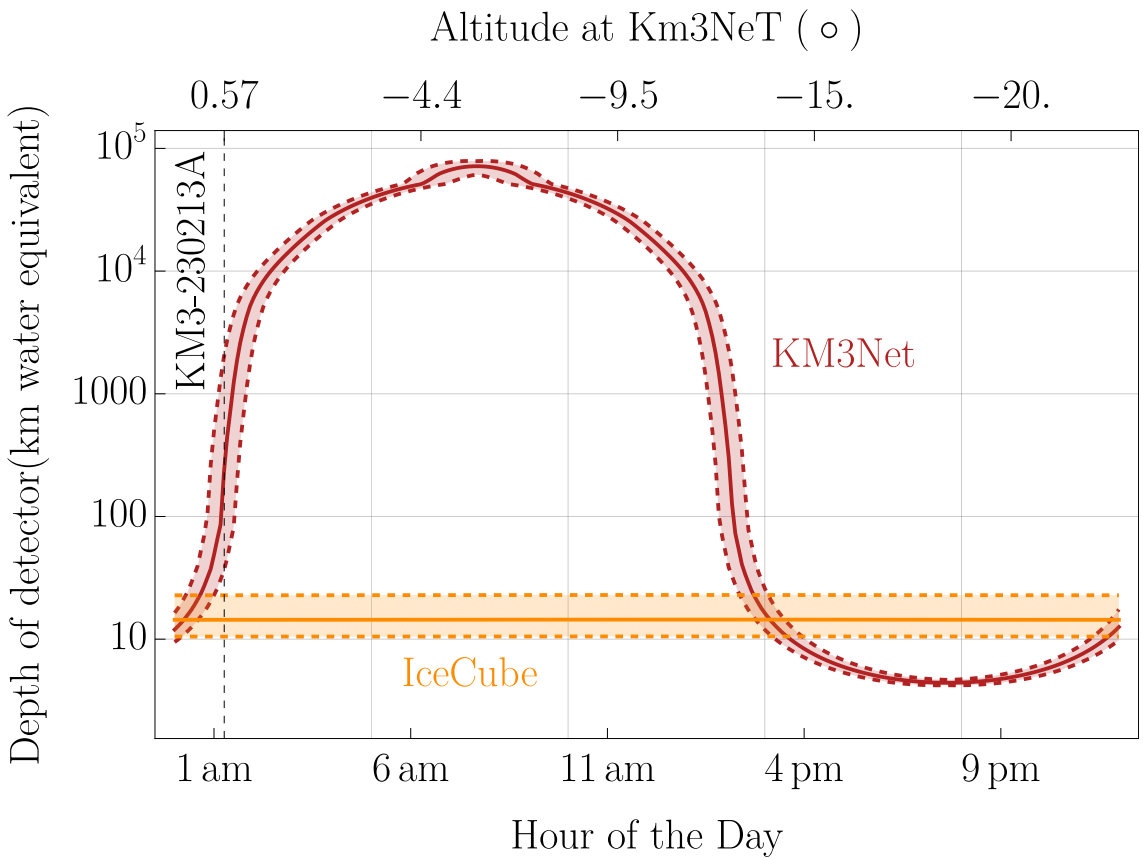
compatible w/ Li, et al., 25



- Egg: IceCube through-going
- Veg. bacon: KM3NeT

Point source

$$\phi_\nu^{\oplus}(\Omega_\nu) \propto \delta^{(2)}\left(\Omega_\nu - \Omega_{\rm PS}(t)\right)$$



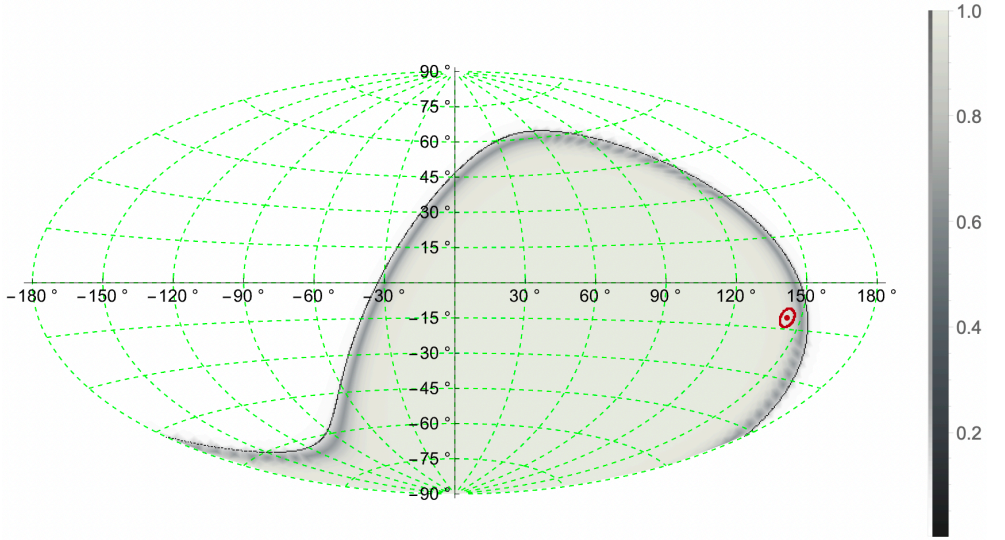
$$\Delta_{\rm PS} \approx \frac{[TA]_{\rm IC}}{[TA]_{\rm KM3NeT}} \cdot \frac{\langle \text{Att.} \rangle_{\rm IC}}{\langle \text{Att.} \rangle_{\rm KM3NeT}} \cdot \frac{R_{\rm eff}^{\rm IC}}{R_{\rm eff}^{\rm KM3NeT}} \approx 140$$

Larger tension than diffuse

unlike Li, et al., 25

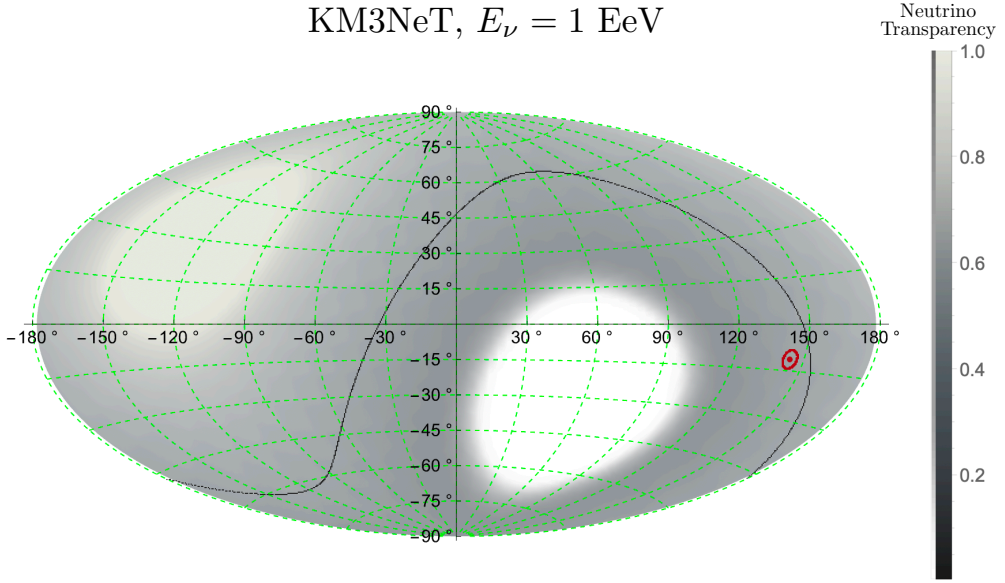
Neutrino transparency

IceCube, $E_\nu = 1$ EeV



$$\langle e^{-L/L_\nu} \rangle_{\rm IC} \approx 0.99$$

KM3NeT, $E_\nu = 1$ EeV



$$\langle e^{-L/L_\nu} \rangle_{\rm KM} \approx 0.48$$

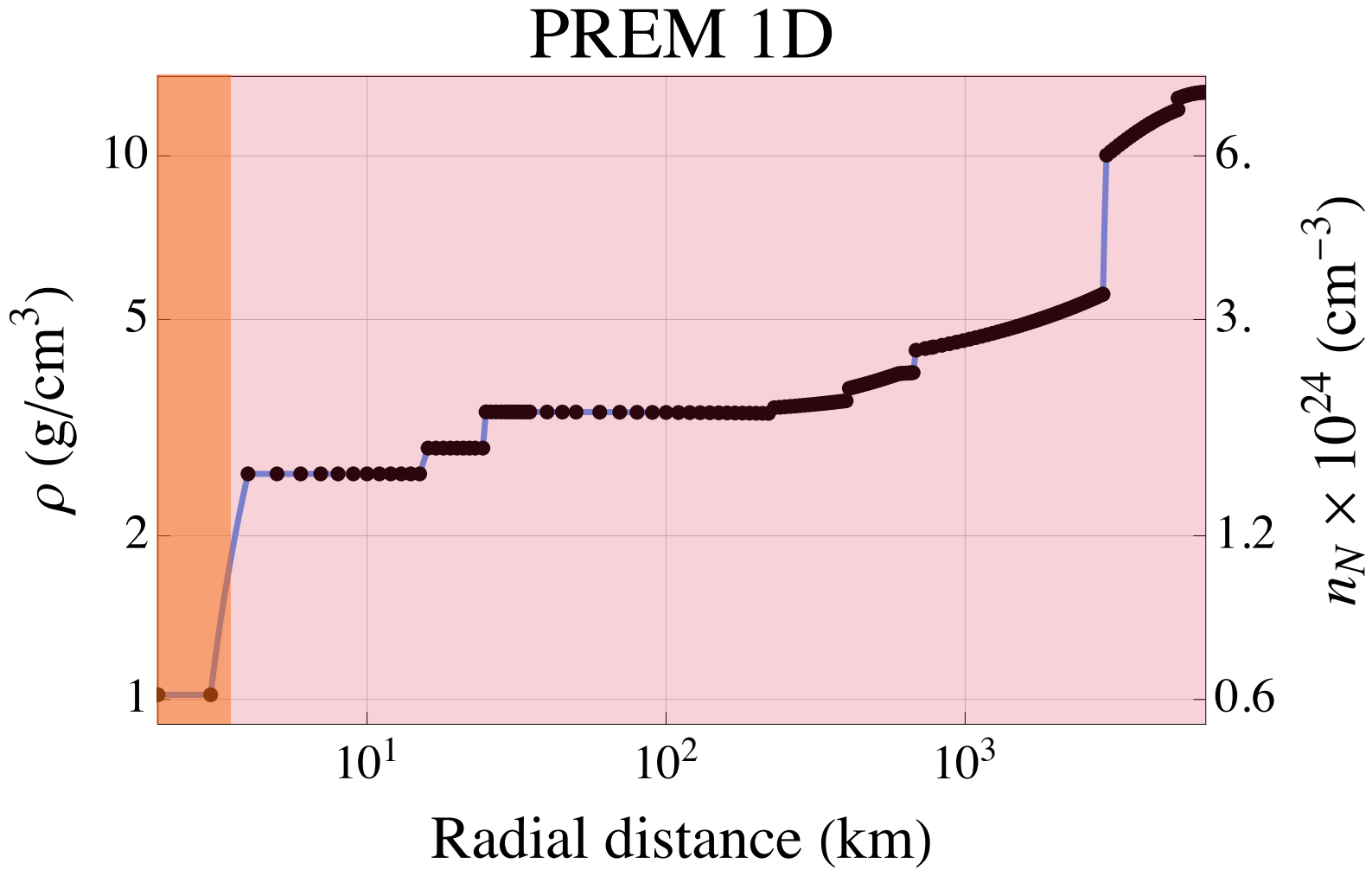
Summary of tension with IC

Model	$\Delta_{\mathrm{IC/KM}}$	B **factor**	$\Delta\sigma$
Diffuse (power-law)	70	21	3.1
PS	140	≈ 35	3.8
PS (transient)	12	≈ 2.2	1.6
Diffuse (energy localized)	70	≈ 3.4	2.4

Our approach

- First-principle computation
- Flux at the crust and ν physics (nuisance) parameter directly linked to experimental rate, not hard-coded in MC
- Can perform fit directly from measured quantity (muon energies)
- Simple system of equations, can be implemented in a code

Thank you for the attention!

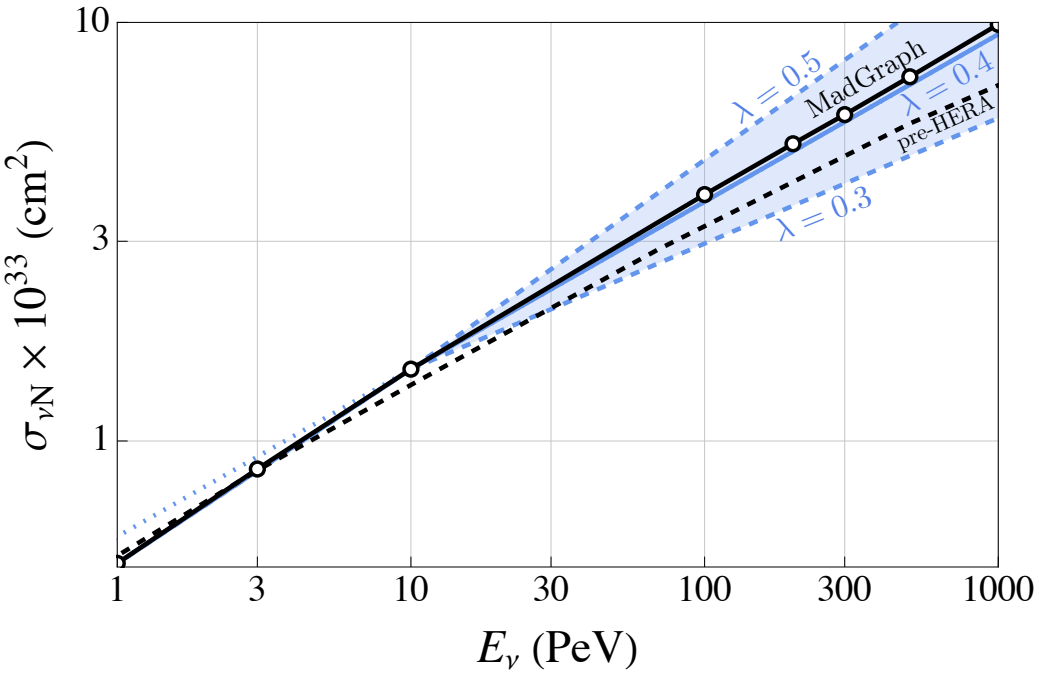

PREM 1D
10
5
2
1
ρ (g/cm^3)
10^1
10^2
10^3
Radial distance (km)
6.
3.
1.2
0.6
$n_N \times 10^{24}$ (cm^{-3})

Neutrino cross-section

- In the limit of small x ($E_\nu > 10\,\mathrm{PeV}$), neutrino DIS cross section scales as

$$\frac{\mathrm{d}^2\sigma_{\nu N}}{\mathrm{d}x\mathrm{d}y} \approx \frac{G_F^2 M_W^2}{\pi x^\lambda}\left[\frac{(1+(1-y)^2)\tilde{s}}{(1+xy\tilde{s})^2}\right]$$

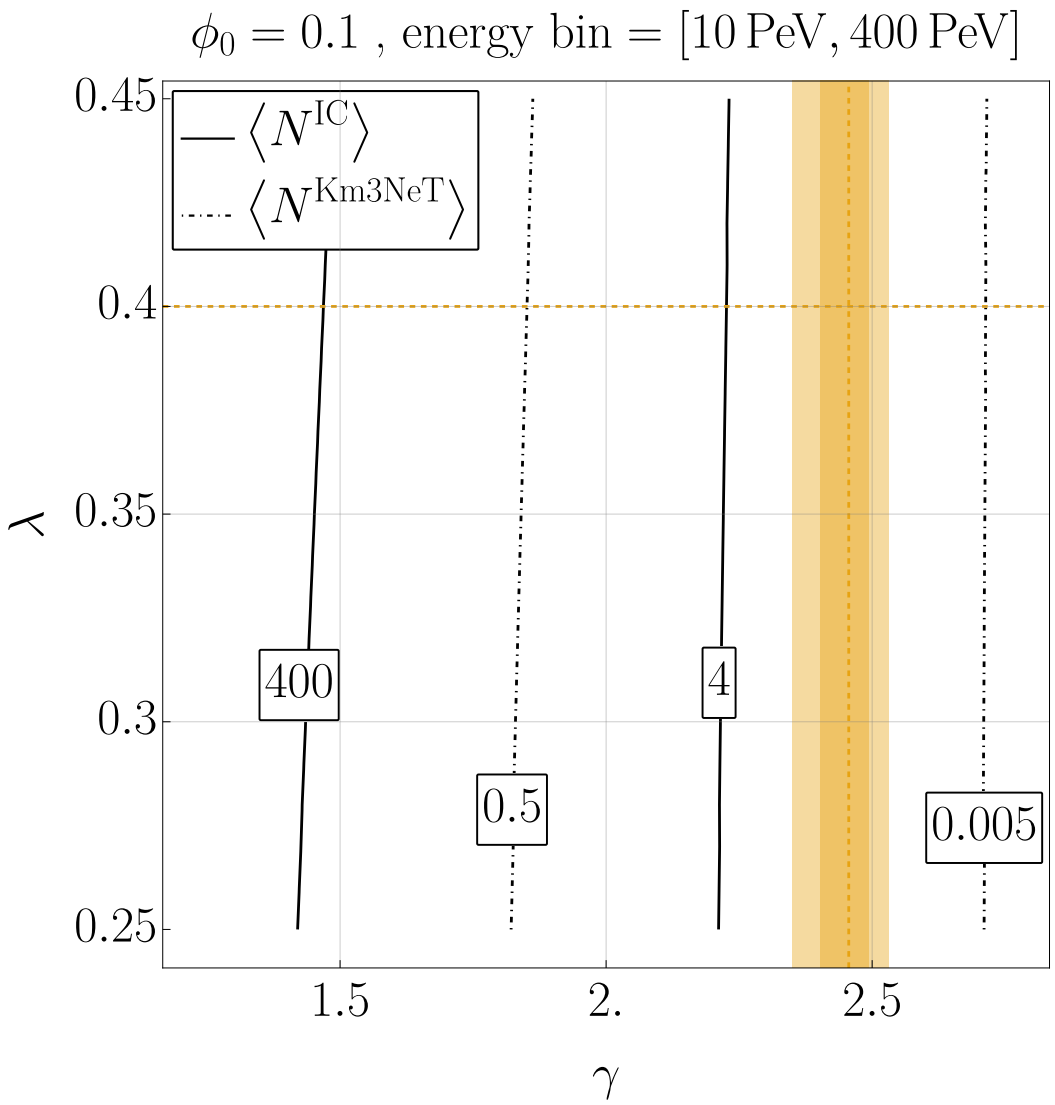
$$\sigma_{\nu\,N}(E_\nu) 1.48 \times 10^{-33}\ \mathrm{cm}^2 \times \left(\frac{E_\nu}{10\ \mathrm{PeV}}\right)^\lambda$$



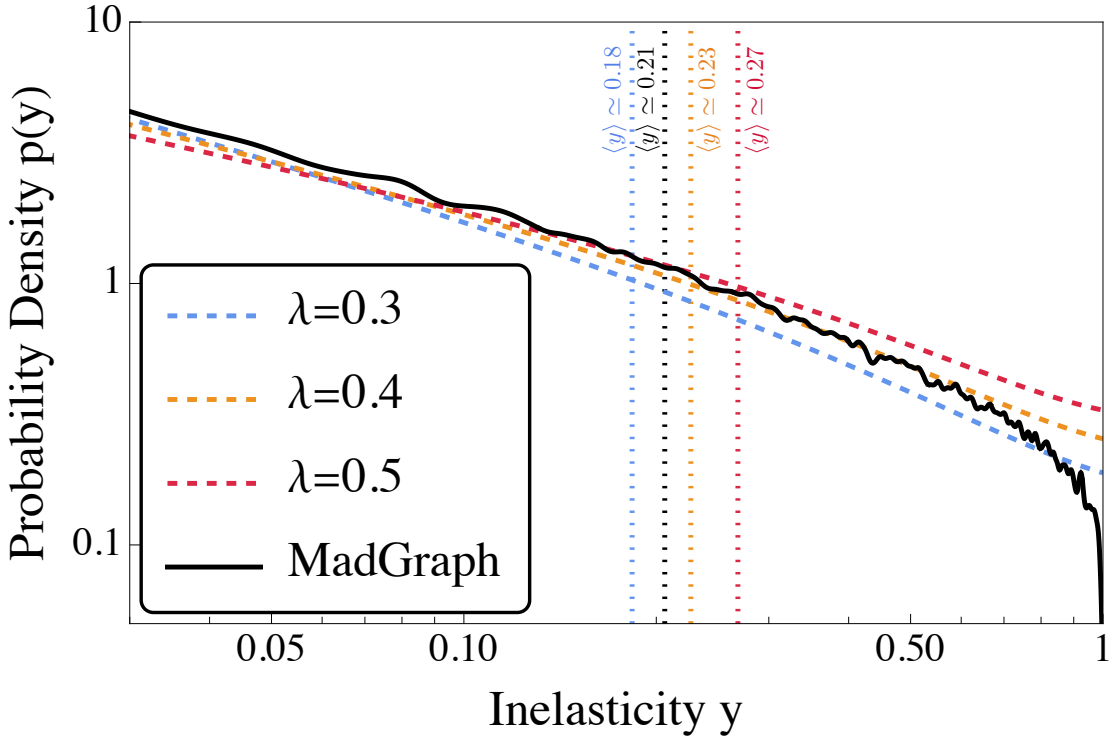
LHAPDF6

- Eventually it can scale at most as $\log^2(E_\nu)$ Froissart, 1961

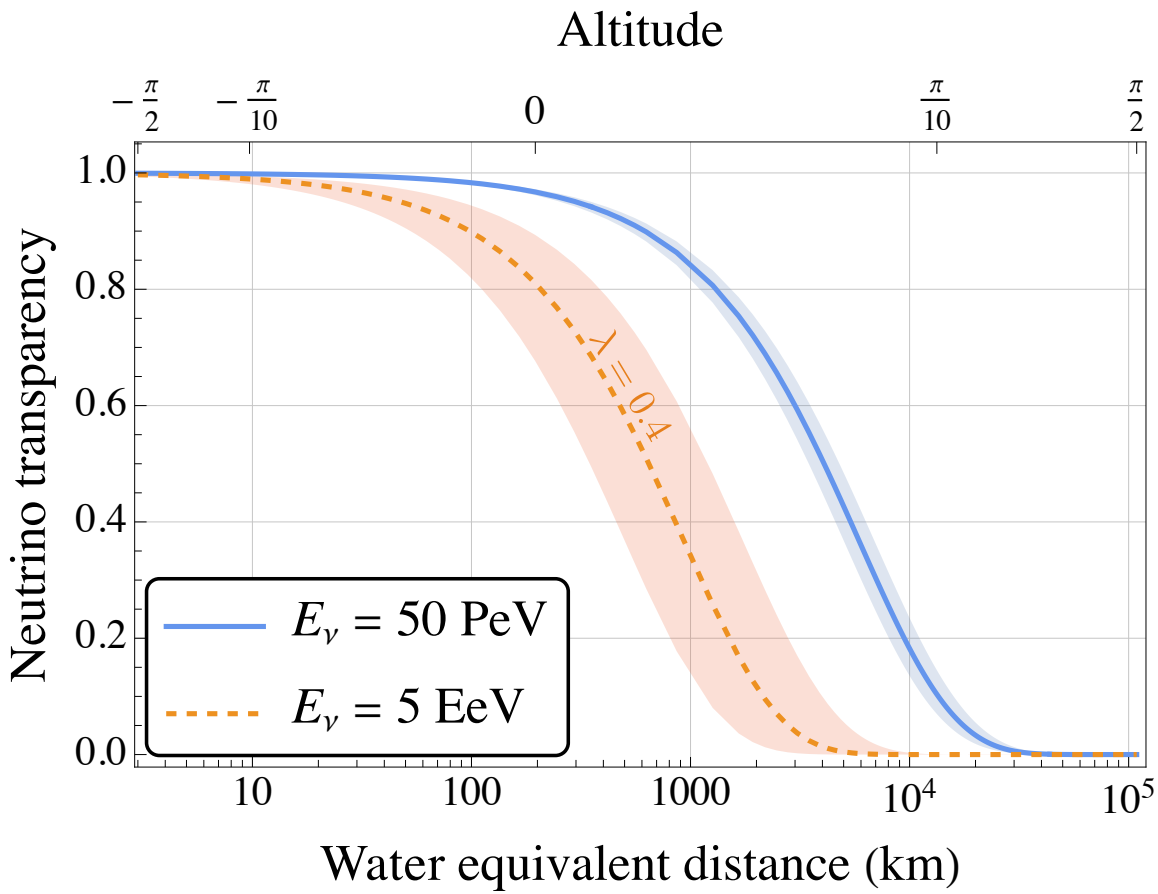
- Expected number of events in UHE bins largely independent on λ



$$\frac{1}{\sigma_{\nu N}}\frac{\mathrm{d}\sigma_{\nu N}}{\mathrm{d}^3 p_\mu} = \delta^{(2)}(\Omega_\nu - \Omega_p)\frac{(1-y)}{E_\mu^3}\mathcal{P}(y)$$



$$D(\xi, E_\nu) = \exp\left(-\int_0^\xi \frac{d\zeta}{L_\nu(\zeta, E_\nu)}\right) = \exp\left(-\frac{\xi_{\rm we}}{L_\nu^{\rm water}(E_\nu)}\right),$$



$$\begin{cases} \dfrac{\partial f_{\nu_\mu}}{\partial t} + \dot{\vec{x}} \cdot \dfrac{\partial f_{\nu_\mu}}{\partial \vec{x}} + \dot{\vec{p}}_{\nu_\mu} \cdot \dfrac{\partial f_{\nu_\mu}}{\partial \vec{p}_{\nu_\mu}} = C_{\text{flavor-ind.}}(t, \vec{x}, \vec{p}_{\nu_\mu}) + K^{\tau}_{\text{decay}}(t, \vec{x}, \vec{p}_{\nu_\mu}) \\ \dfrac{\partial f_{\nu_\tau}}{\partial t} + \dot{\vec{x}} \cdot \dfrac{\partial f_{\nu_\tau}}{\partial \vec{x}} + \dot{\vec{p}}_{\nu_\tau} \cdot \dfrac{\partial f_{\nu_\tau}}{\partial \vec{p}_{\nu_\tau}} = C_{\text{flavor-ind.}}(t, \vec{x}, \vec{p}_{\nu_\tau}) + K^{\tau}_{\text{decay}}(t, \vec{x}, \vec{p}_{\nu_\tau}) \\ \dfrac{\partial f_{\tau}}{\partial t} + \dot{\vec{x}} \cdot \dfrac{\partial f_{\tau}}{\partial \vec{x}} + \dot{\vec{p}}_{\tau} \cdot \dfrac{\partial f_{\tau}}{\partial \vec{p}_{\tau}} = K^{\nu_\tau}_{\text{scattering}}(t, \vec{x}, \vec{p}_{\tau}) - \Gamma_\tau(\vec{p}_\tau) f_\tau \end{cases},$$

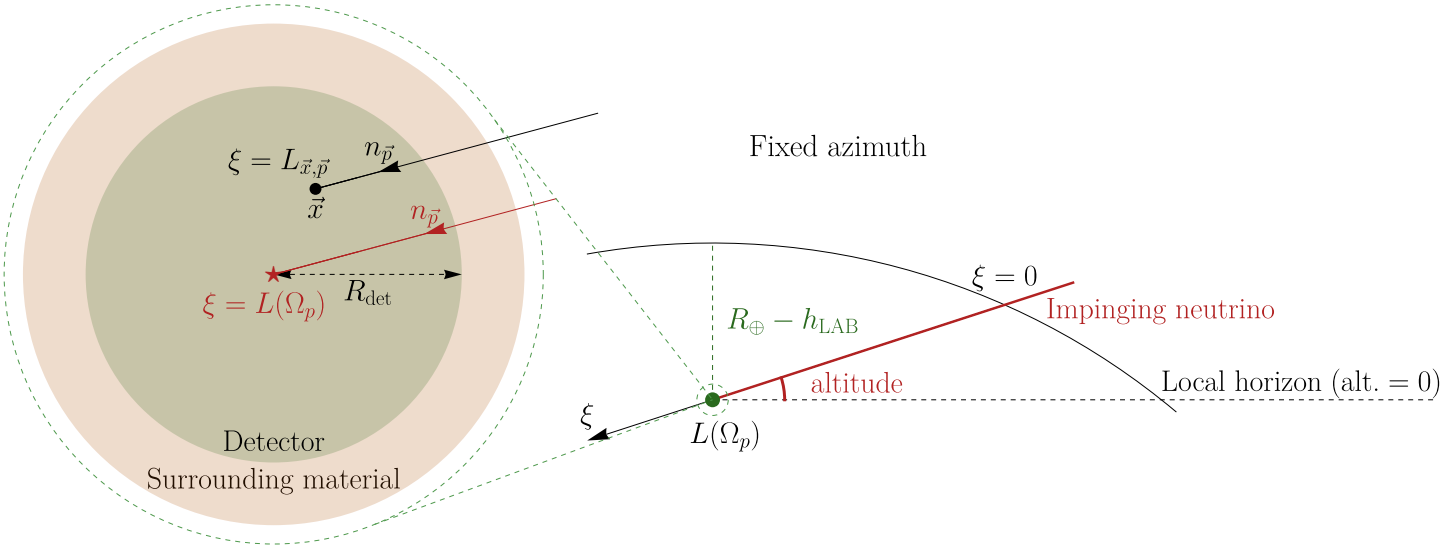
- Neutrino attenuation Beacom, et al., 02

$$\frac{\partial \phi_\nu}{\partial \xi} = -\frac{1}{\lambda(E_\nu)} \left[\phi_\nu(\xi, E_\nu) - \int_0^1 \frac{dy}{1-y} \mathcal{P}(y) \frac{\sigma_{\text{NC}}}{\sigma_{\text{tot}}} \phi_\nu(\xi, \frac{E_\nu}{1-y}) \right]$$

- Taus are more complex Dutta, et al., 05

$$f_\tau \sim \int d\xi' \int dE_\nu \text{Att.} \phi^{\oplus}_\nu(E_\nu) n^{\text{far}}_N E^2_\tau \frac{\mathrm{d}\sigma}{\mathrm{d}^3 p_\tau(\xi')} \exp\left[(m_\tau/E_\tau) \,/\, (d_\tau b_\tau) \left(1 - e^{-b_\tau \xi'}\right) \right]$$

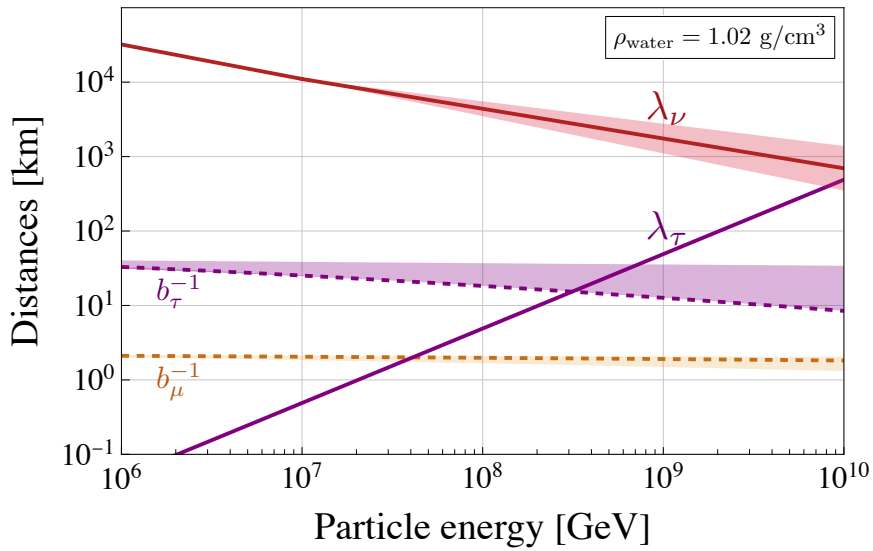
$$\begin{cases} \mathrm{d}t/\mathrm{d}\xi = /v \\ \mathrm{d}\vec{x}/\mathrm{d}\xi = \hat{n}_p \\ \mathrm{d}\vec{p}/\mathrm{d}\xi = -b_\mu(\xi)\vec{p} \end{cases} \Longrightarrow \begin{cases} t(\xi) = t + (\xi - L_{x,p})/v \\ \vec{x}(\xi) = \vec{x} - (L_{x,p} - \xi)\hat{n}_p \\ \vec{p}(\xi) = p\,\hat{n}_p\, \exp(\int_{\xi}^{L_{x,p}} \mathrm{d}\xi' b(\xi')) \end{cases}$$



$$-\left\langle \frac{\mathrm{d}E_\ell}{\mathrm{d}x} \right\rangle = a_\ell(E_\ell) + b_\ell(E_\ell)E_\ell \approx b_\ell E_\ell$$ PDG

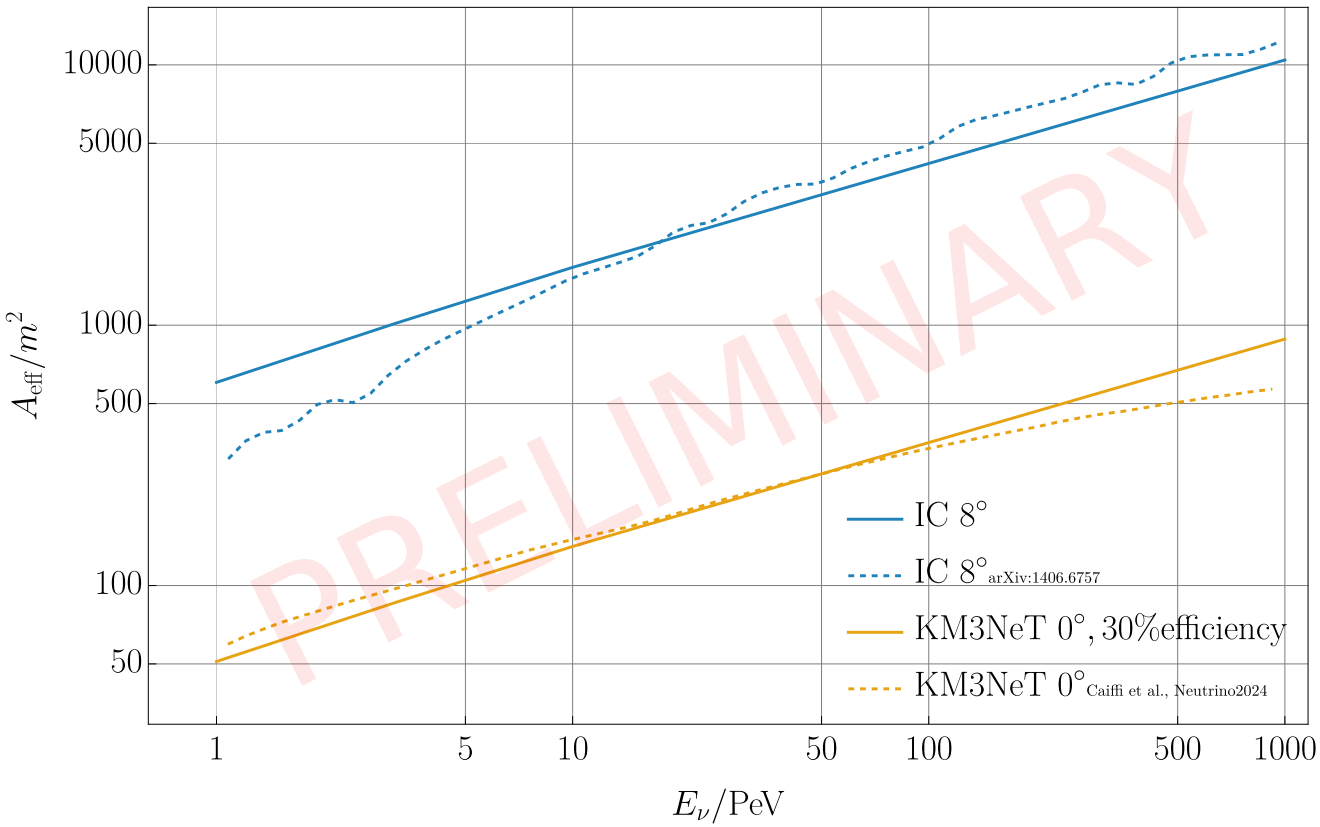
Very different for the three leptons

- $b_\mu^{-1} \sim$ few km $\rightarrow$ tracks, bangs
- $b_e^{-1} \sim \left(\frac{m_e}{m_\mu}\right)^2 b_\mu^{-1} \approx 0 \rightarrow$ no tracks, only bangs
- $b_\tau^{-1} \sim$ tens of km, with large uncertainties due to DIS
  - But decays in shorter lenghts (unless very high energy) $\rightarrow$ muon tracks, double bangs



Effective Area

$$A_{\rm eff}(E_\nu, \Omega_\nu) = A_T \int_0^{L(\Omega_{\vec{p}})} \mathrm{d}\xi \int_{\Delta_{E_\mu} \otimes \Delta_{\Omega_{\vec{p}}}} \mathrm{d}E_\mu \mathrm{d}\Omega_p \underbrace{\epsilon\,(E_\mu, \Omega_{\vec{p}})}_{???} \overbrace{\mathcal{T}(E_\mu, \Omega_{\vec{p}} | E_\nu, \Omega_\nu; \xi)}^{4\pi E_\mu^2 \mathrm{Att.}\, \frac{d\sigma(p(\xi))}{d^3 p(\xi)} n_N(\xi)}$$



IC: Aartsen, et al., 14

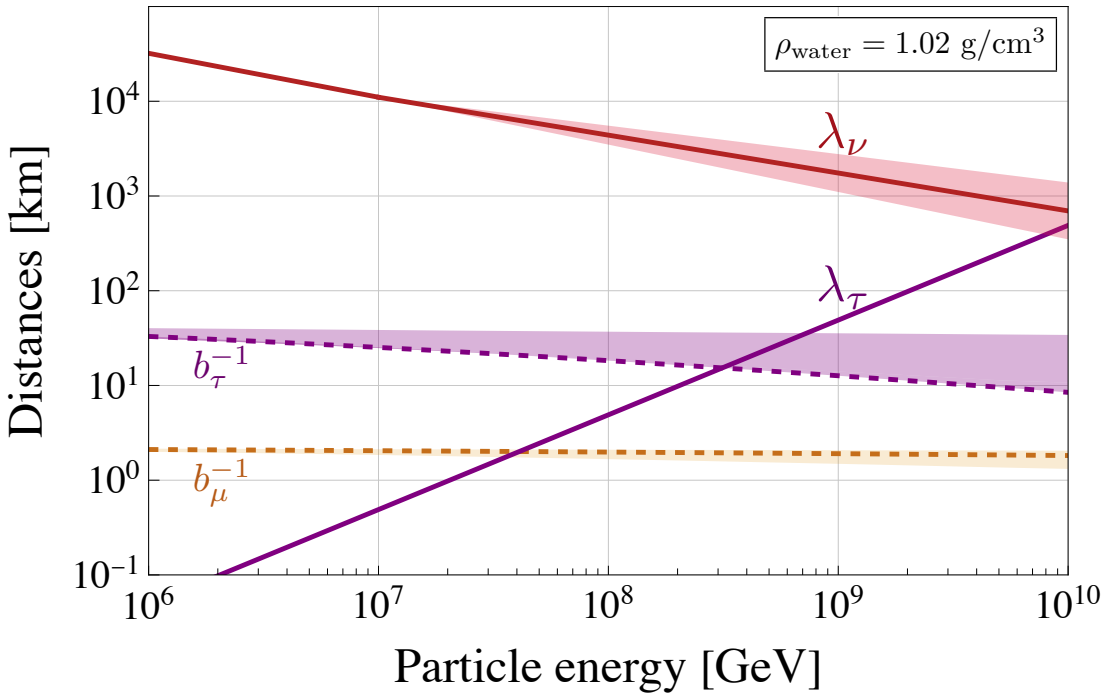
KM3NeT: Caiffi, et al., 25

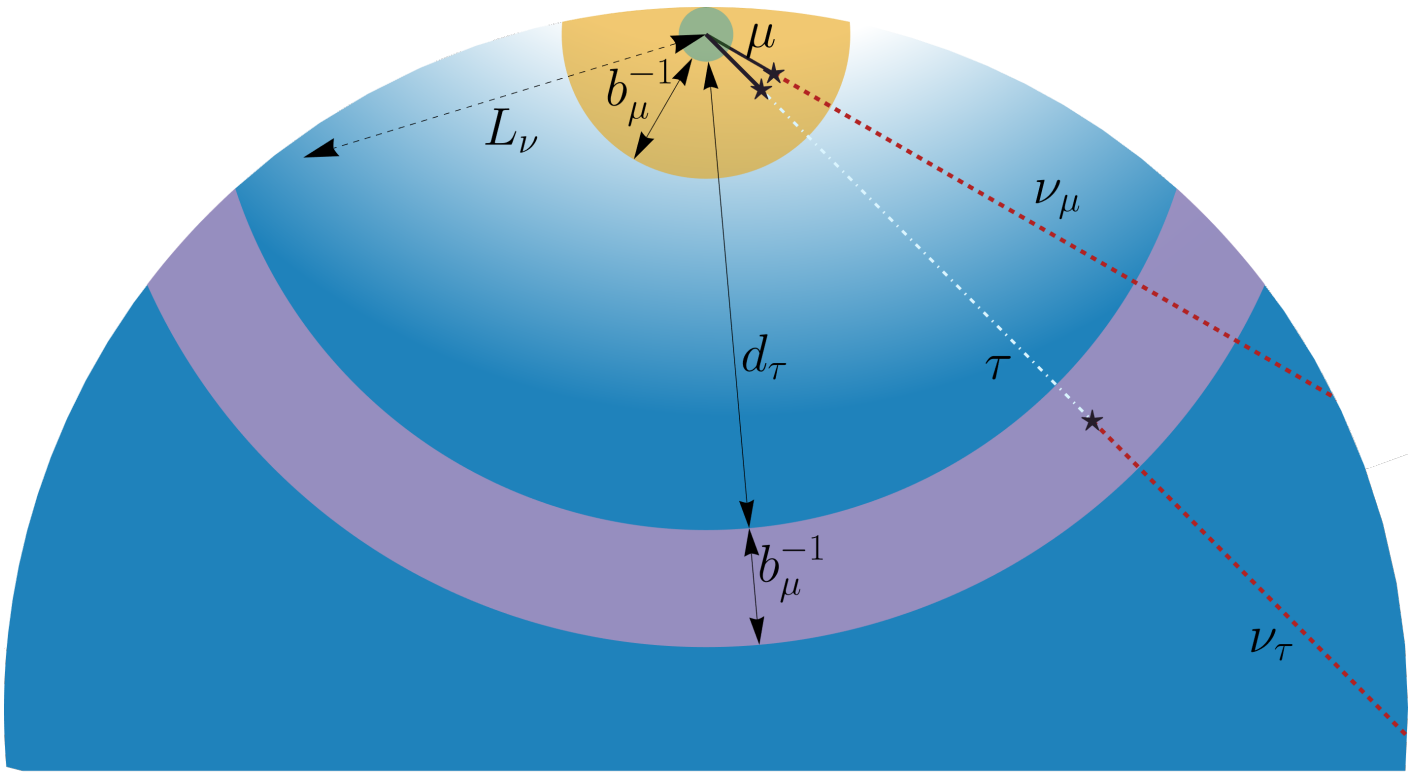
τ-induced events

- Neglecting ξ' dependence of attenuation, as $b_\tau \to 0$ $(p_\tau(\xi) \approx p_\tau)$

$$f_\tau \propto \left(1 - e^{\xi/d_\tau}\right) \to \left(1 - e^{-L/d_\tau}\right)$$

- Naïvely, at $E_\tau \gtrsim 1\,\mathrm{EeV}$, yield from PS is
  - unsuppressed at KM3NeT
  - supressed as $L_{\mathrm{IC}}/d_\tau \lesssim 0.2$
- $\min\{b_\tau^{-1}, d_\tau\}$ sets the rules of the game, never drastically larger than L_{IC}




μ
b_μ^{-1}
L_ν
ν_μ
d_τ
τ
b_μ^{-1}
ν_τ

$$B_{1\to 2}(D) \equiv \frac{\int P(\hat{\theta}_1|D)d\hat{\theta}_1}{\int P(\hat{\theta}_2|D)d\hat{\theta}_1} = \frac{\int \mathcal{L}_D(\hat{\theta}_1)\pi_1(\hat{\theta}_1)d\hat{\theta}_1}{\int \mathcal{L}_D(\hat{\theta}_2)\pi_2(\hat{\theta}_2)d\hat{\theta}_2}\,,$$

- $B_{1\to 2} \gg 1 \to$ model 1 favored by data wrt model 2

$$B_{D_1/D_2} = \frac{\int \mathcal{L}_{D_2}(\hat{\theta}|\mathrm{M})\pi(\hat{\theta})d\hat{\theta}}{\int \mathcal{L}_{D_2}(\hat{\theta}|\mathrm{M})\pi_{\mathrm{ref_{D1}}}(\hat{\theta})d\hat{\theta}}$$

- $\pi_{\mathrm{ref_{D1}}}(\hat{\theta})$ yields a good fit of D_1 for model M
- If $\pi(\hat{\theta})$ such that $B_{D_1/D_2} \gg 1 \to$ tension between datasets D_1 and D_2

Remark ($B \sim \chi^2$ with d d.o.f)

$$\Delta\sigma = \sqrt{2}\,\mathrm{erfc}^{-1}\left[Q\left(\frac{d}{2}; 2\ln B\right)\right],$$

where $Q(a,z) \equiv \Gamma(a,z)/\Gamma(a,0)$, and $\Gamma(a,z) = \int_z^\infty dt\, t^{a-1}e^{-t}$.

$$\mathcal{L}_{\mathrm{IC},0}(\hat{N}) = e^{-\Delta \times \hat{N}} \quad \text{and} \quad \mathcal{L}_{\mathrm{KM3},1} = \hat{N} e^{-\hat{N}} \, .$$