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Constraints on Neutrino Secret Interactions from Multi-messenger neutrinos scattering on CνB

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Neutrino Self interactions - motivation

- νSI often arise naturally in well-motivated BSM models, including
  - DM–ν interaction models, spontaneous breaking of lepton number symmetry, Majoron models, etc
  - Neutrino mass generation
- Neutrinos Are the Perfect Portal to New Physics
  - sensitive probes of hidden sectors
- Cosmological Relevance
  - May help resolve cosmological tensions
    - Hubble tension - disagreement between late- and early-time measurements
    - Matter power spectrum
  - Production of dark matter in the early Universe
- Not Yet Excluded by Data

CMB - Cosmic microwave background
LSS - large-scale structures
BBN - Big Bang nucleosynthesis

Neutrino Self interactions

$$\mathcal{L} = g_{ij}\overline{\nu}_i\hat{\phi}\nu_j \ , \ \hat{\phi} = \{\phi, \gamma_\mu\phi^\mu, \gamma_5\gamma_\mu\phi^\mu, \gamma_5\phi\} \qquad \nu\nu \to \phi \to \nu\nu$$

$$\mathcal{L} = g_{\alpha\beta}\overline{\nu}_\alpha\Gamma_\mu\nu_\beta\phi^\mu, \ \nu_\alpha = \sum_{i=1}^{4} U_{\alpha i}\nu_i \qquad \nu\nu_s \to \phi \to \nu\nu_s$$

$$\mathcal{L} = g_\nu^{\alpha\beta}\overline{\nu}_\alpha\Gamma_\mu\nu_\beta\phi^\mu + g_\chi\overline{\chi}\Gamma_\mu\chi\phi^\mu \qquad \nu\overline{\nu} \to \phi \to \chi\overline{\chi}$$

Coupling
- ➢ Active + Active
- ➢ Active + Sterile
- ➢ Active + DM

Coupling to neutrinos
- ➢ Flavor universal
- ➢ Flavor non-universal

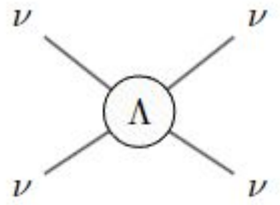
Free parameters
- ➢ Mediator mass
- ➢ Coupling strength

Neutrino nature
- ➢ Dirac Neutrino
- ➢ Maiorana Neutrino

Type of mediator
- ➢ massive scalar
- ➢ pseudoscalar
- ➢ vector
- ➢ axial-vector field

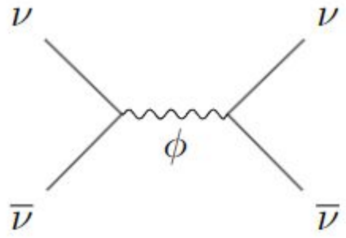
Neutrino Self interactions

In the scenario of minimal coupling, neutrinos may couple to a massive **scalar**, pseudoscalar, **vector**, or axial-vector field .



$$\mathcal{L}_{\text{v}} = g_{ij}\overline{\nu}_i\gamma_\mu\nu_j\phi^\mu \ , \ (i, j = e, \mu, \tau)$$

$$\mathcal{L}_{\text{s}} = g_{ij}\overline{\nu}_i\phi\nu_j$$

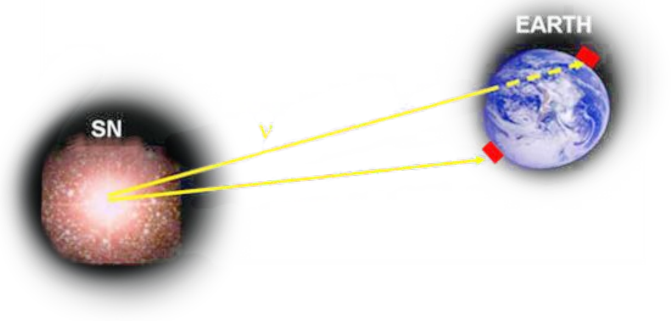


Framework: Flavor universal coupling.
Universal neutrino masses.
Mediator coupling to other particles is effectively negligible.

The mediator mass M and coupling strength g, are free parameters.

How nSI can be measured and constrained?

High energy astrophysical neutrinos Scattering on Cosmic Neutrino Background (CνB)



High energy neutrinos (HEν) must travel tremendous distances from the source to the detector on Earth. And if we assume existence of νSI, instead of free-streaming, these HEν may scatter on the abundant CνB, which consists of relic neutrinos with very low effective temperature. Thus such a scattering will ensure a **visible energy loss**, and **will remove the HE neutrino from the initial flux**.

★ Average CνB density

$n_\nu = 112\ cm^{-3}$ *per flavour*
$n_\nu \sim 340\ cm^{-3}$ *total*

★ Effective CνB temperature

$T^{eff}_{CnB} \simeq 1.7\ 10^{-4}$ eV

Optical Depth

Idea: If we observe a neutrino from a distant source, it means the space between the source and us is not opaque to neutrinos.

To ensure that, we impose the transparency condition:

$$\tau \leq 1$$

$\tau = 1$ corresponds to one interaction length on average along the line of sight.

For nearby sources (small redshift) at distance D, this condition translates into

$$\tau \equiv D/\lambda \leq 1$$

The detection of neutrinos from a HEν source requires that t**he mean free path of neutrinos through the CνB is comparable to or greater than the distance to the source**. So we set this condition to the mean free path to experience at least one interaction

$$v_{Moller} = \frac{|\mathbf{v}_X - \mathbf{v}_\nu|}{|\mathbf{v}_X|}$$

Mean free path

Our next step is to calculate the mean free path, which is the inverse of the interaction rate:

$$\lambda^{-1} = \int \frac{d\mathbf{p}_X}{(2\pi)^3} f(\mathbf{p}_X)\, v_{Moller}\, \sigma(s)$$

- *interaction rate*

$$\sigma \sim g^4$$

This results in limits to the neutrino self interaction coupling:

$$g \leq \left(\frac{\lambda|_{g\to 1}}{D} \right)^{1/4}$$

X - stands for background

Two background regimes

Non-relativistic:

$$\lambda_{NR}^{-1} = n_X\, \sigma(s) \qquad n_X = \frac{1}{4\pi^2}\int_0^\infty dE_X E_X^2 f(E_X)$$

$$\frac{|p_X|}{E_X} \to 0, \quad s \to m_X^2 + 2Em_X$$

Ultra relativistic: ★

$$\lambda_{UR}^{-1} = \frac{\sqrt{2}}{4\pi^2}\int_0^\infty dE_X E_X^2\, f(E_X) \int dz \sqrt{1-z}\, \sigma(z, E_X)$$

$$\frac{|p_X|}{E_X} \to 1, \quad s \to 2EE_X(1-z),$$

Mass regimes

Heavy massive mediator limit

$$\sigma(s) = g^4 \frac{as}{M^4}$$

Full mass dependence ★

+

Resonance

Massless mediator limit

$$\sigma(s) = g^4 \frac{a}{s}$$

Sources of UHE

Source	SN1987A	NGC 1068	PKS 0735+178	TXS 0506+056	KM3-230213A
Mean energy	10 MeV	10 TeV	171 TeV	290 TeV	220 PeV
Distance D	55 kpc	13.4 Mpc	1.9 Gpc	1.3 Gpc	-
Redshift z	0.00045	0.00379	0.424	0.336	-

Supernova 1987

Neutrino events by LSD, BUST, IMB and Kamiokande II.

NGC 1068:

Active Galactic Nuclei. Neutrino events were detected by IceCube.

TXS 0506+056.

Blazar. Neutrino events were detected by IceCube and Baikal-GVD

PKS 0735+178

Blazar. Neutrino events were detected by IceCube, Baikal-GVD, BUST, and KM3NeT.

Blazar: $E_B \in (TeV - PeV)$

The origin of high-energy astrophysical neutrinos: new results and prospects - Sergey Troitsky
Neutrino Echoes from Multimessenger Transient Sources - K. Murase Ian M. Shoemaker

RESULTS

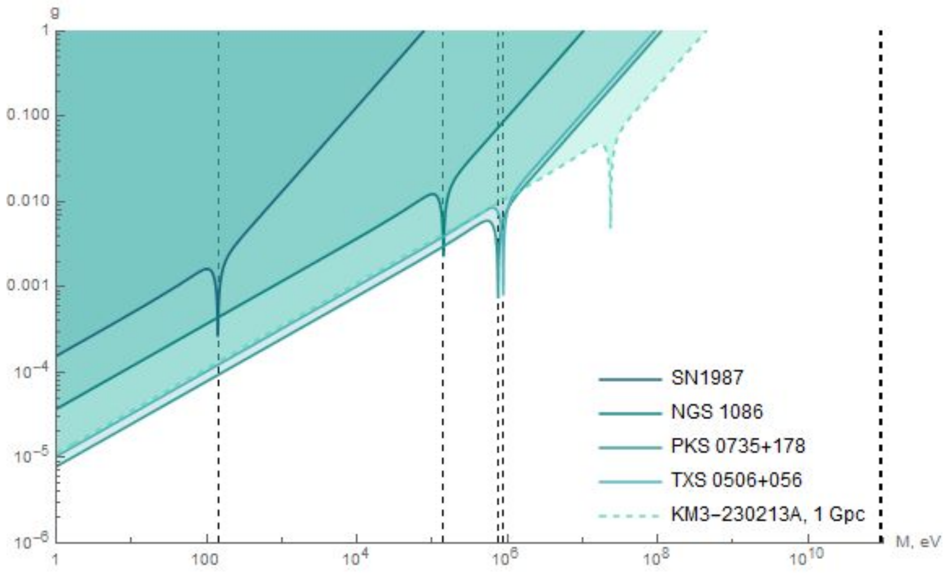
FLAVOR UNIVERSAL COUPLING CONSTANT

Exclusion plots for the coupling constant g from the vector mediator mass M

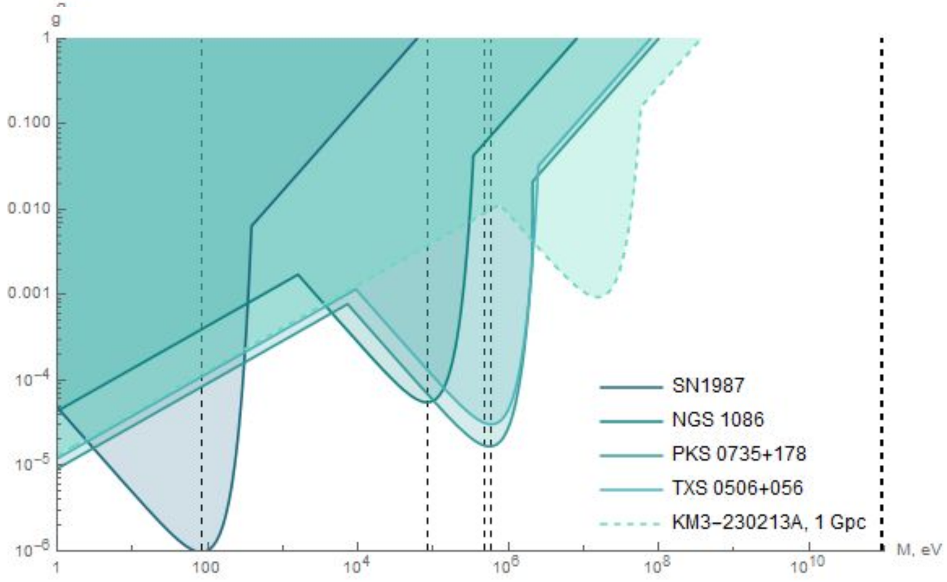
$$g \le \left(\frac{\lambda|_{g\to 1}}{D} \right)^{1/4}$$

The regions above the curves are the regions of exclusion.

NR CVB regime



UR CVB regime



Toy neutrino mass: $m = 10^{-3}$ eV

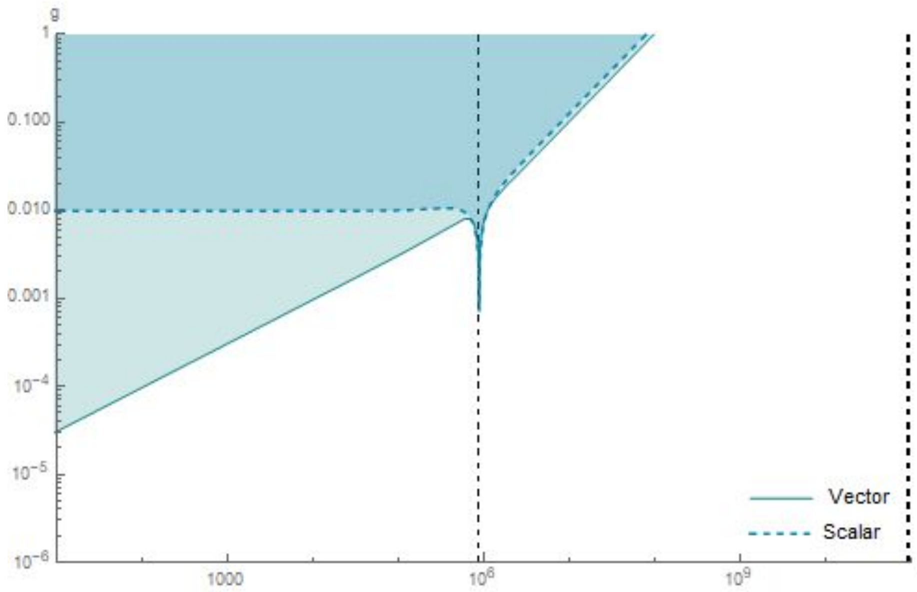
Vertical thick dashed line *- Z boson mass,*

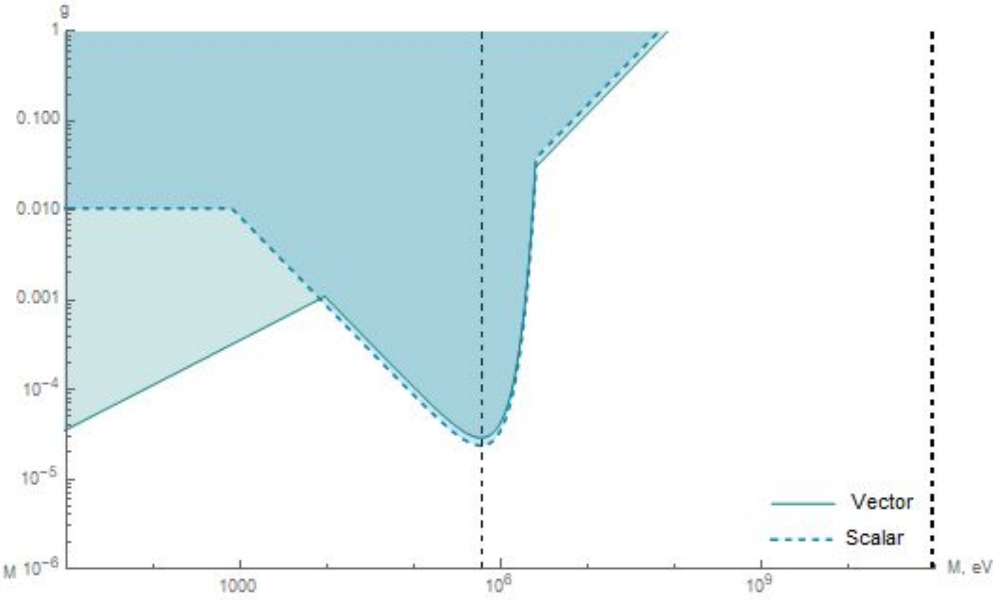
Vertical dashed lines - s-channel resonances

Assumptions:

- ➢ *CVB distribution is reduced to Maxwell-Boltzmann*
- ➢ $m \longrightarrow 0$

Scalar vs vector mediator, TXS 0506+056



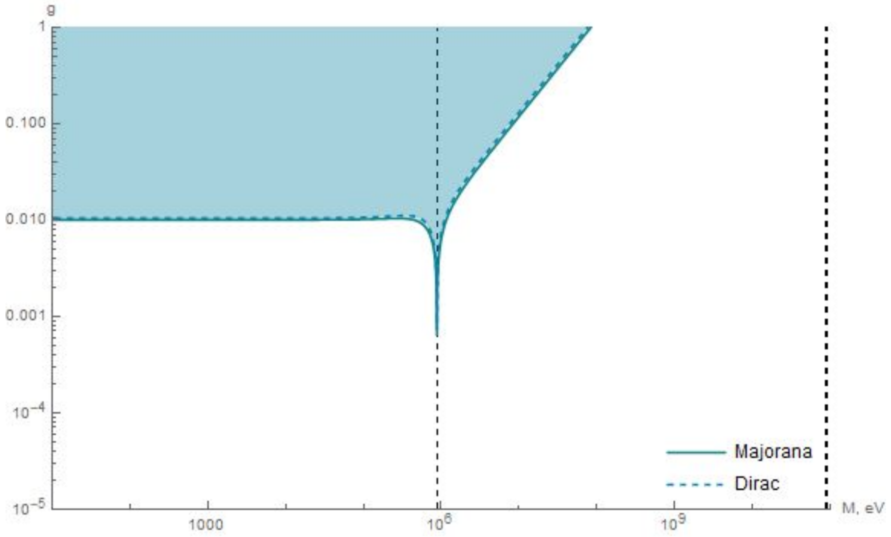


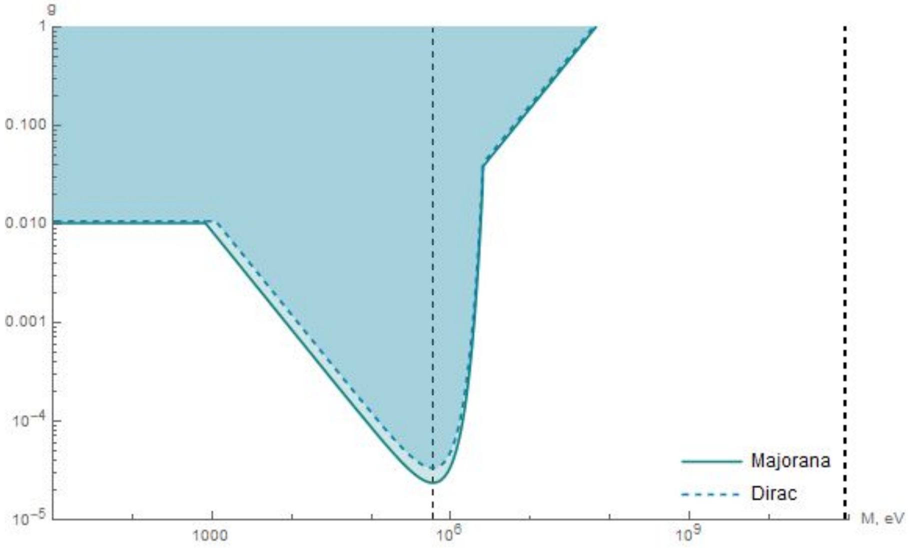
The left panel displays NR CνB regime, neutrino mass $m = 10^{-3}$ eV.

The right panel shows the behavior of the coupling constant with the accounted shape of UR CνB.

The regions above the curves are the regions of exclusion.

Scalar: Majorana vs Dirac





The left panel displays NR CνB regime, neutrino mass $m = 10^{-3}$ eV.

The right panel shows the behavior of the coupling constant with the accounted shape of UR CνB.

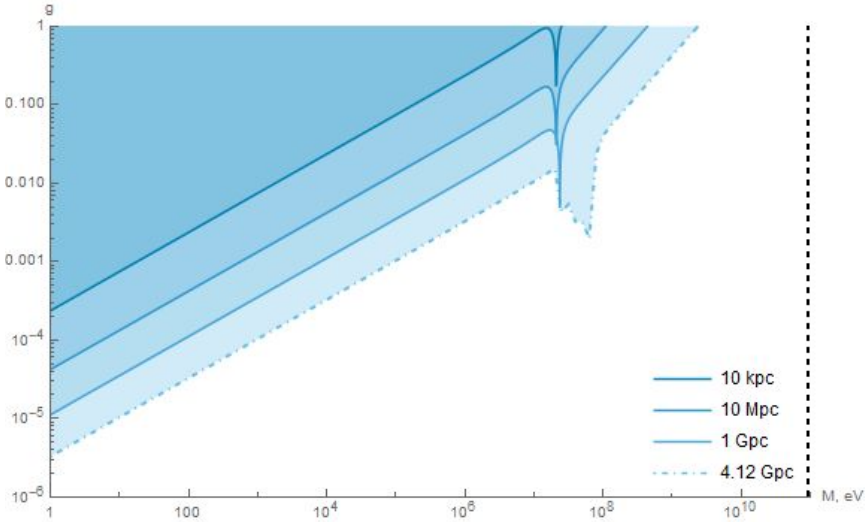
The regions above the curves are the regions of exclusion.

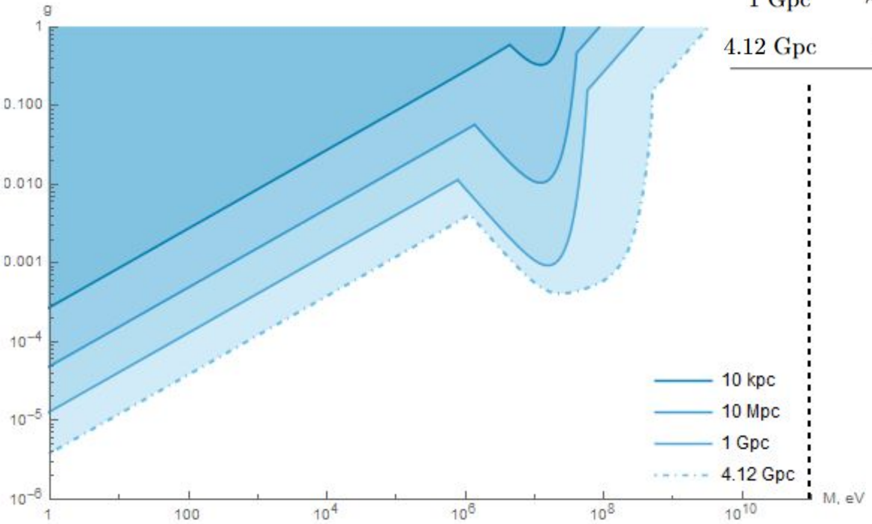
UHEν event with undefined source: KM3-230213A

Exclusion plots for the coupling constant from the mediator mass M for a possible source of KM3-230213A 220 PeV event with uncertain distance: vector mediator, Dirac neutrinos

Representative distances

Distance	Redshift (z)
10 kpc	$\sim 2 \times 10^{-6}$
10 Mpc	$\sim 2.5 \times 10^{-3}$
1 Gpc	~ 0.25
4.12 Gpc	~ 12





NR CνB regime, m = 10^{-3} eV

UR CνB regime

Dashed lines correspond numerical estimate for a very distant source (z = 12) with cosmological expansion taken into account

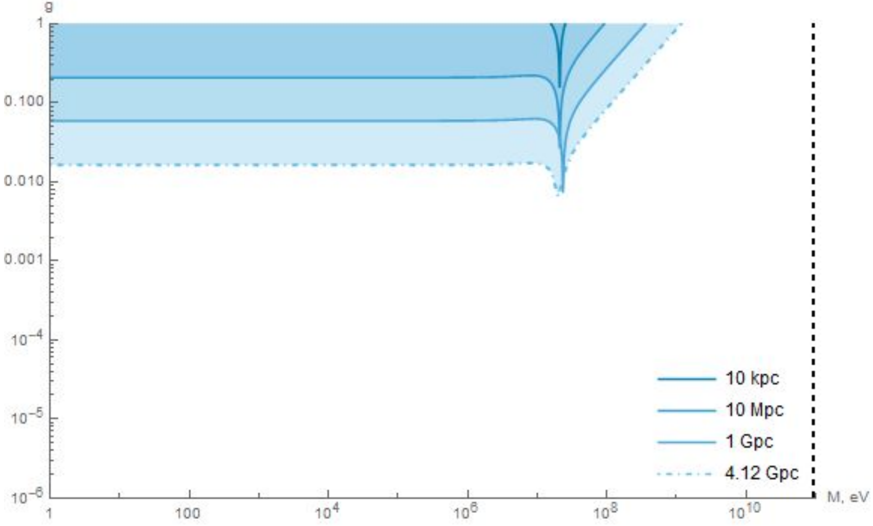
$$\tau(E) = \int_0^z \frac{dz'}{H(z')(1+z')} \Gamma(E(1+z'), z').$$

The KM3-230213A event can provide stronger constraints than previous events if it originates from beyond the local universe (1 Mpc).

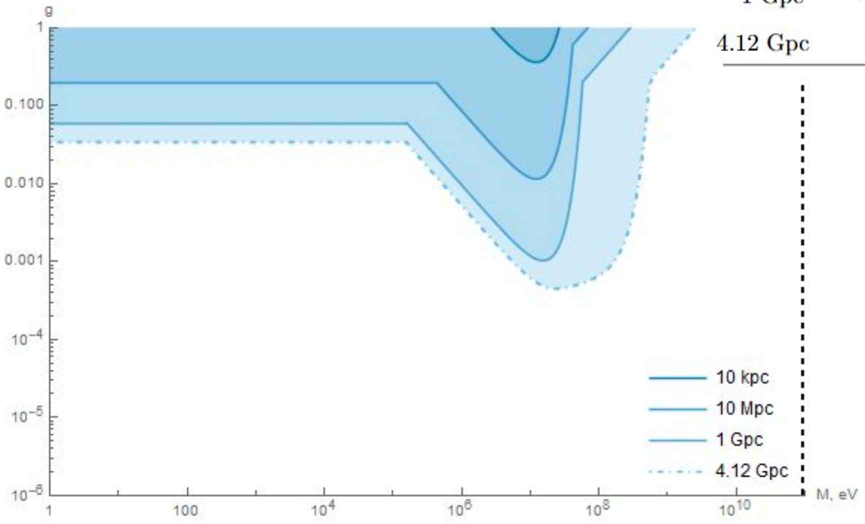
Exclusion plots for the coupling constant from the mediator mass M for a possible source of KM3-230213A 220 PeV event with uncertain distance: Scalar mediator, dirac neutrinos

Representative distances

Distance	Redshift (z)
10 kpc	$\sim 2 \times 10^{-6}$
10 Mpc	$\sim 2.5 \times 10^{-3}$
1 Gpc	~ 0.25
4.12 Gpc	~ 12



NR CνB regime

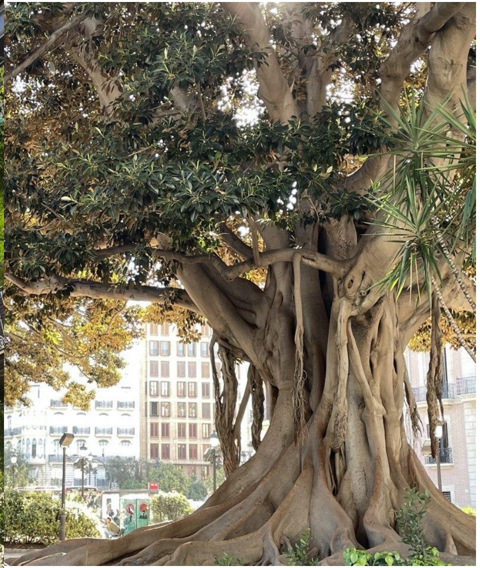


UR CνB regime

The colored regions illustrate excluded values of νSI coupling constant for UR CνB and HEν from corresponding distance.
Dashed lines correspond numerical estimate for a very distant source (z = 12) with cosmological expansion taken into account
NR regime is taken with neutrino mass $m = 10^{-3}$.

The KM3-230213A event can provide stronger constraints than previous events if it originates from beyond the local universe (1 Mpc).

Thank you for attention!



Literature

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[2] Neutrino Self-Interactions: A White Paper - Jeffrey M. Berryman et all

[3] Gauged $L_\mu - L_\tau$ Symmetry at the Electroweak Scale - Julian Heeck, Werner Rodejohann

[4] Sterile Neutrinos -Basudeb Dasgupta, Joachim Kopp

[5] Testing exotic neutrino-neutrino interactions with AGN neutrinos - Petteri Keranen

[6] Shedding light on neutrino self-interactions with solar antineutrino searches - Quan-feng Wu and Xun-Jie Xu

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[10] Constraining the Self-Interacting Neutrino Interpretation of the Hubble Tension - Nikita Blinov, Kevin J. Kelly, Gordan Krnjaic, and Samuel D. McDermott 2019

[11] Toward Powerful Probes of Neutrino Self-Interactions in Supernovae Po-Wen Chang, Ivan Esteban, John F. Beacom, Todd A. Thompson, and Christopher M. Hirata 2022

[12] A multi-messenger study of the blazar PKS 0735+178: a new major neutrino source candidate N. Sahakyan, P. Giommi, P. Padovani, M. Petropoulou, D. Bégué, B. Boccardi, S. Gasparyan

[13] Looking for cosmic neutrino background - C. Yanagisawa

[14] Massive Fermi Gas in the Expanding Universe - A. Trautner

[15] Multimessenger Astronomy and New Neutrino Physics - K. J. Kelly, P. A. N. Machado,

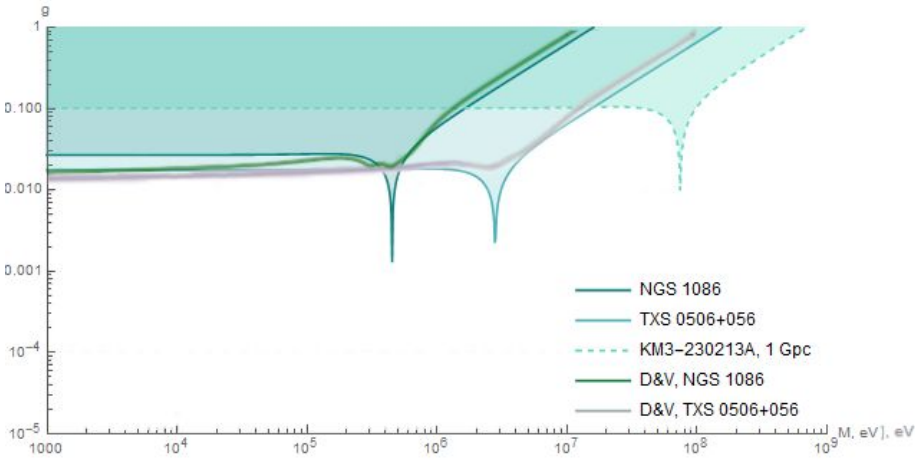
[16] The origin of high-energy astrophysical neutrinos: new results and prospects - Sergey Troitsky

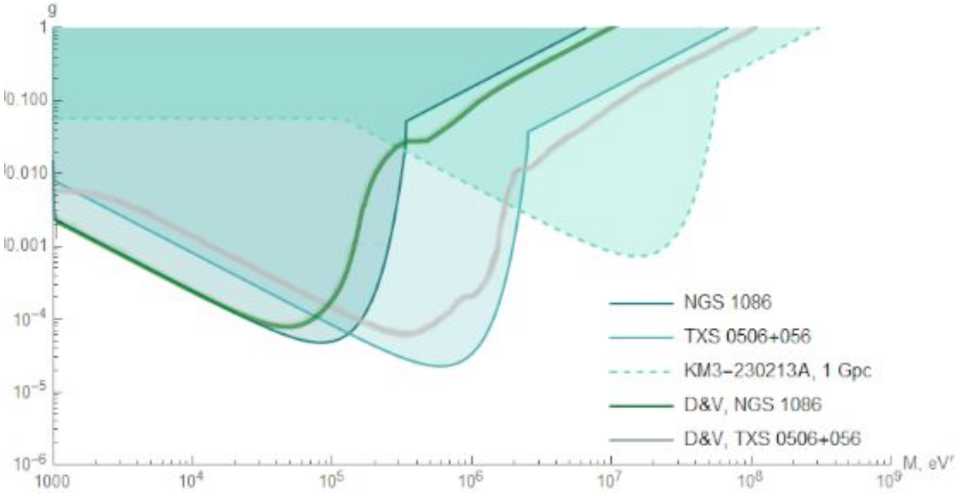
[17] Neutrino Echoes from Multimessenger Transient Sources - K. Murase Ian M. Shoemaker

[18] Testing secret interaction with astrophysical neutrino point sources - Christian Döring and Stefan Vogl

COMPARISON WITH PREVIOUS WORKS

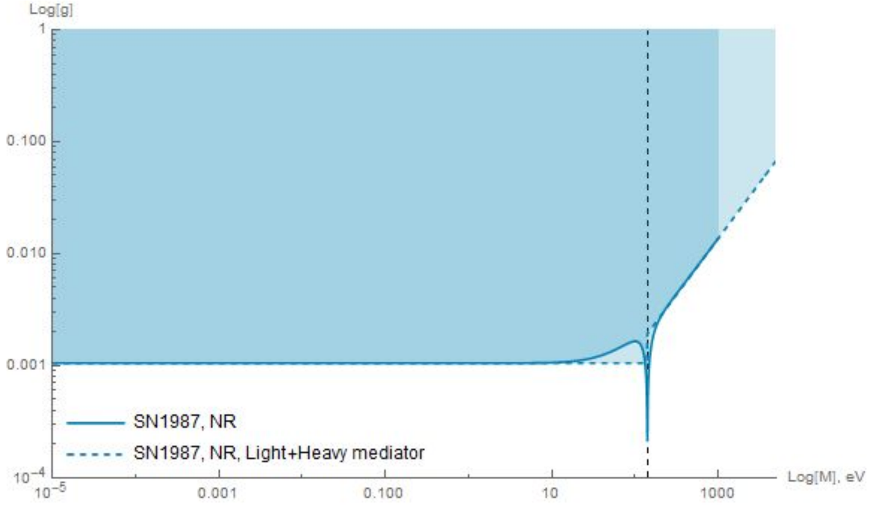
COMPARISON WITH THE MOST RECENT WORK

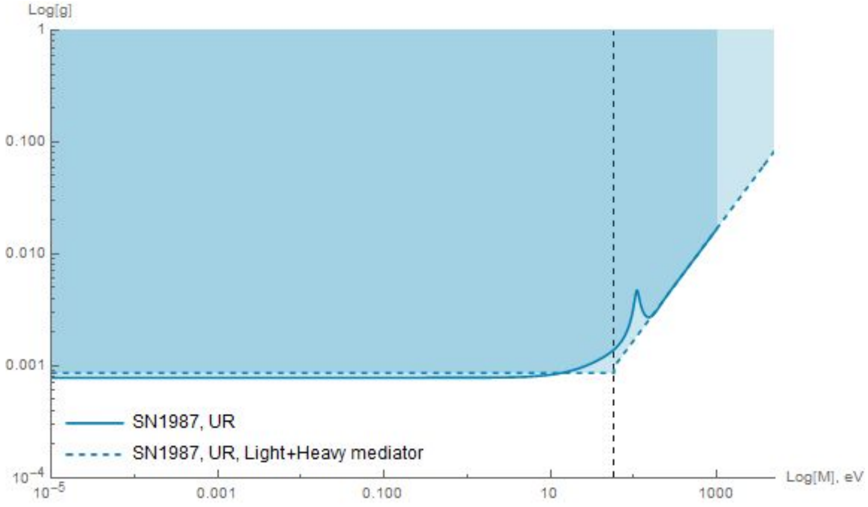




"Testing secret interaction with astrophysical neutrino point sources"
Christian Doring, and Stefan Vogl

COMPARISON WITH THE PIONEERING WORK

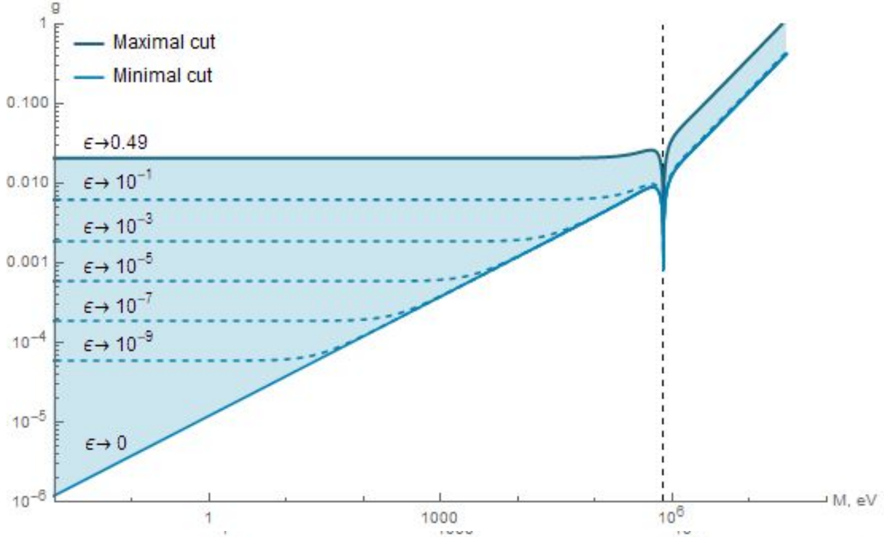




"Supernova 1987A and the secret interactions of neutrinos"
Edward W. Kolb and Michael S. Turner

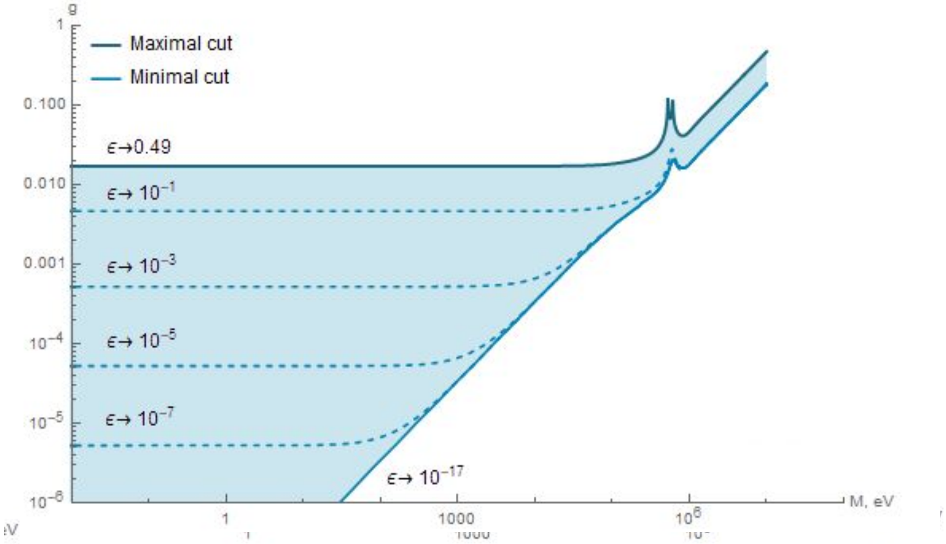
Cutoff parameter = 0.1

CUT-OFF PARAMETER, NR



CUT-OFF PARAMETER, UR

$$g \leq \left(\frac{\lambda|_{g\to 1}}{D} \right)^{1/4}$$



For massless mediator:

The total cross-section is determined by integrating the differential cross-section over the range $-s \leq t \leq 0$.

$-s(1-\epsilon) \leq t \leq s\epsilon$.

Debye screening $\epsilon \sim g^2\,(10^{-11} - 10^{-14})$ for SN1987A.

Angular cutoff $\epsilon \sim 10^{-37}$

Measurements

Laboratory bounds up to O(100 GeV) come from searches for

- Invisible Z boson and Higgs decays (LHC)
  - $Z \to \nu\nu\bar{\nu}\bar{\nu}$
  - $Z \to \nu_\alpha \nu_\beta \phi$
  - $H \to \nu_\alpha \nu_\beta \phi$
- T decays such as $\tau^- \to \overline{\nu}_\tau \nu_\alpha l_\alpha^-$
- Rare meson decays $m^- \to \ell_\alpha^- \nu_\beta \phi; \quad m^- \to \ell_\alpha^- \overline{\nu}_\alpha \nu \overline{\nu}.$
- Neutrinoless double beta decay
  - NEMO-3, KamLAND-Zen, Majorana, CUPID-0, SNO+, CUORE, GERDA, EXO-200
- Pion decays
- Collider searches for new neutrino scattering missing energy channels
  - LHC , Belle-II, NA62, DUNE in future

$$pp \to \ell_\alpha^\pm \ell_\beta^\pm jj + E_T^{\text{miss}} \quad (\alpha,\, \beta = e,\, \mu, \tau)$$

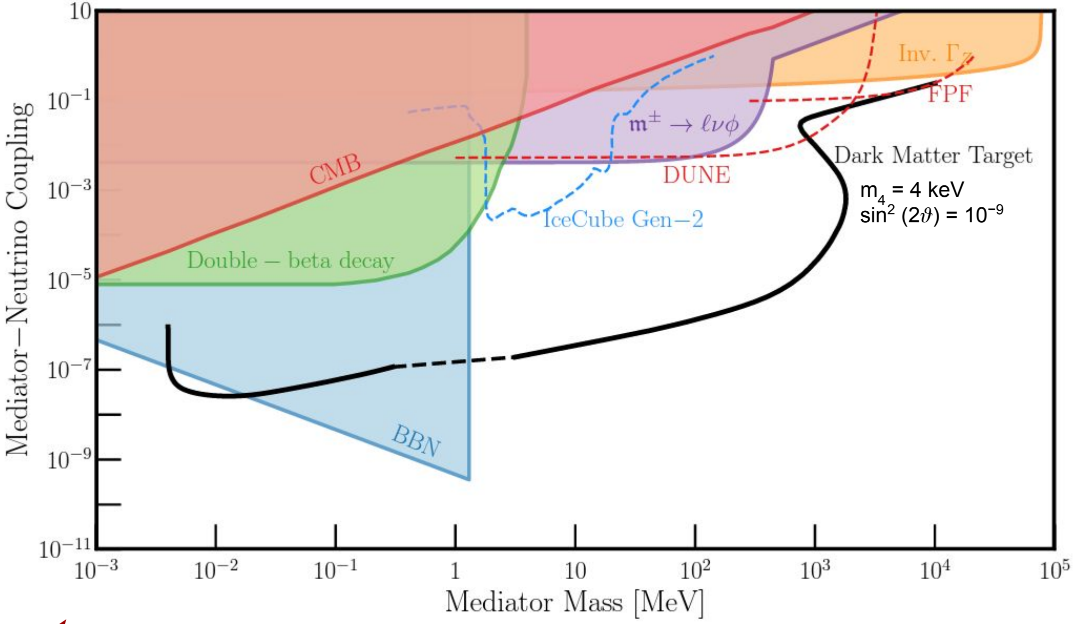
Measurements

νSI can delay neutrino decoupling, change free-streaming of neutrinos

Cosmology scales of eV to MeV

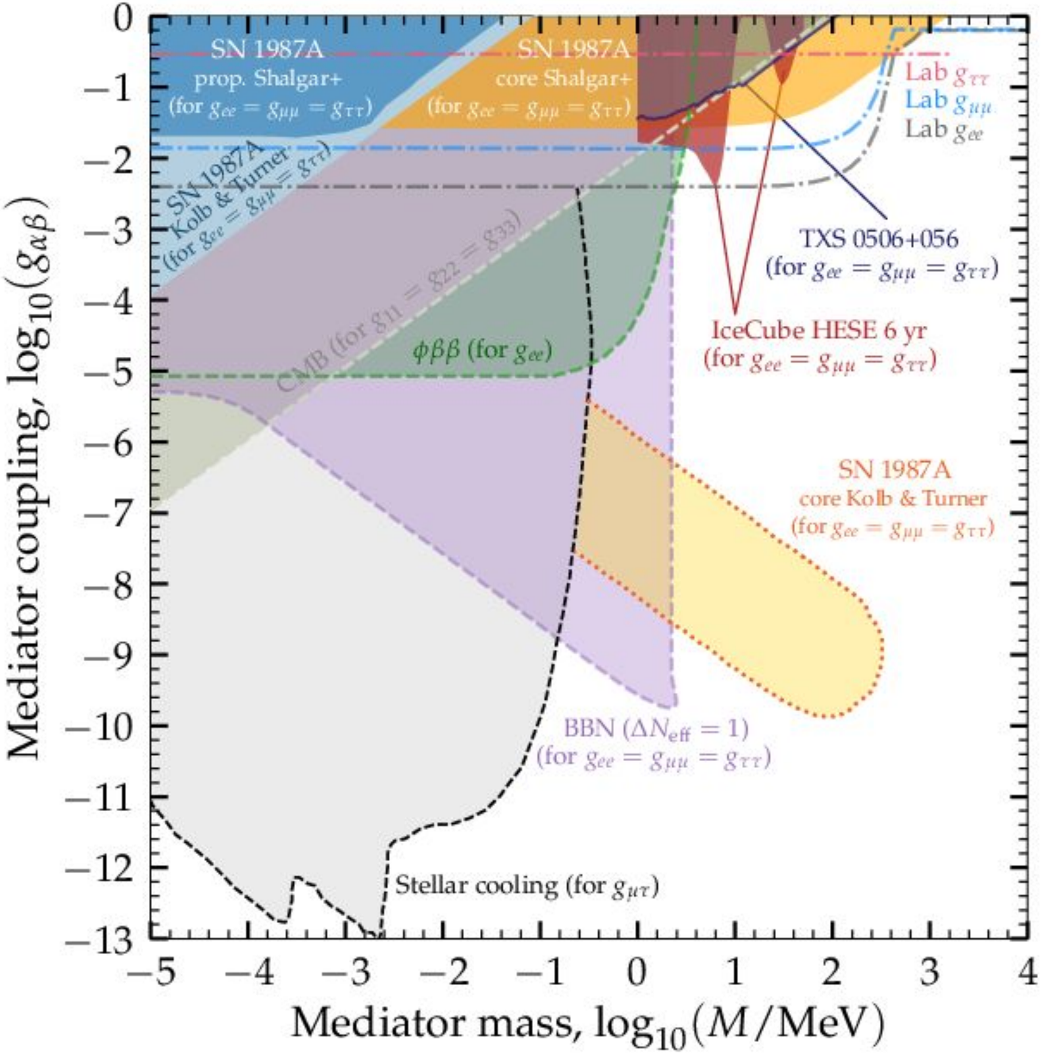
- ★ Big Bang Nucleosynthesis (BBN) -
  neutrinos significantly influenced the era of BBN
  - affects the predicted abundances of light elements
- ★ Cosmic Microwave Background (CMB)
  - shift the CMB power spectra peaks
- ★ Large Scale Structures (LSS)
  - matter clustering on small scales can be altered, altering the matter power spectrum, matter distribution in the Universe
- ★ Hubble expansion rate
  - increases the total radiation energy density at recombination

Measurements



Neutrino Self-Interactions: A White Paper - J. M. Berryman *et all, 2022*

- **Cosmological constraints**
  - Big Bang Nucleosynthesis (BBN)
  - Cosmic Microwave Background (CMB)
  - Dark matter production via a freeze-in mechanism
- **Laboratory bounds**
  - Neutrinoless double beta decay
  - Rare meson, T decays
  - Invisible Z decays
    - e+e− ⟶ γν ̄ν
    - Z ⟶ ννν ̄ν ̄
  - Collider searches for new neutrino scattering (DUNE, FPF)
    - pp ⟶ $l^+_a l^+_b \phi$ + jets



Measurements

Astrophysics scales up to O(100 MeV) (with this analysis up to GeV)

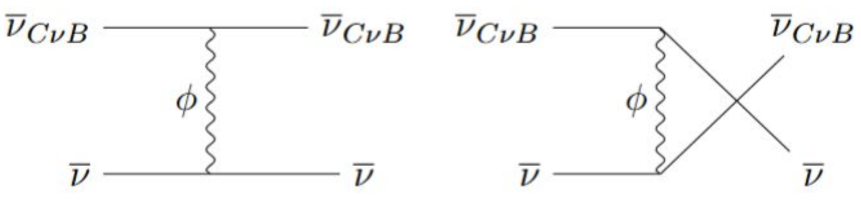
- ★ Supernovae, Blazars, AGN, etc
- ★ Modified observed signal (signal duration, composition, spectrum) of high-energy neutrinos at Ice-Cube, KM3NeT, Baikal GVD
  - ○ due to processes inside the source
  - ○ while traveling to Earth
- ★ Mixing, masses and possible new interaction forms of neutrinos

"Bounds on secret neutrino interactions from high-energy astrophysical neutrinos" - M. Bustamante, C. Rosenstrm, S. Shalgar, and I. Tamborra

"Neutrino Self-Interactions: A White Paper" - Berryman, Jeffrey M. et al

Processes contributing to the HEν scattering on CνB

t+u channel:



t+s channel:

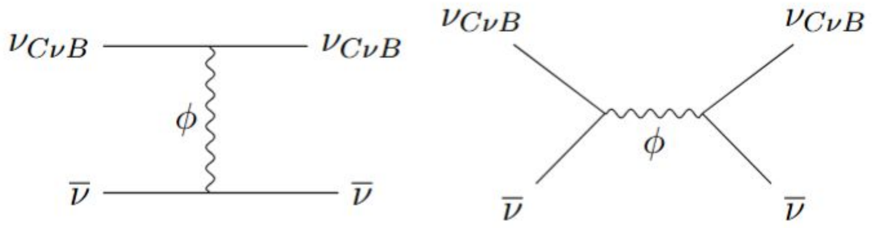
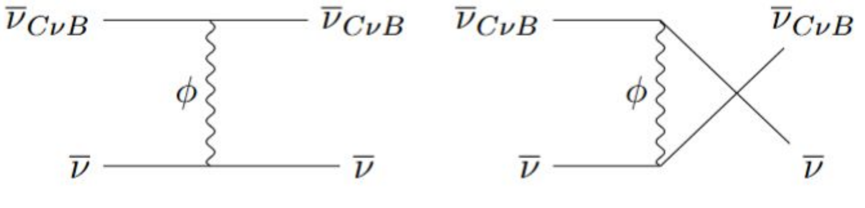


Table III: Differential cross-section, massless neutrino

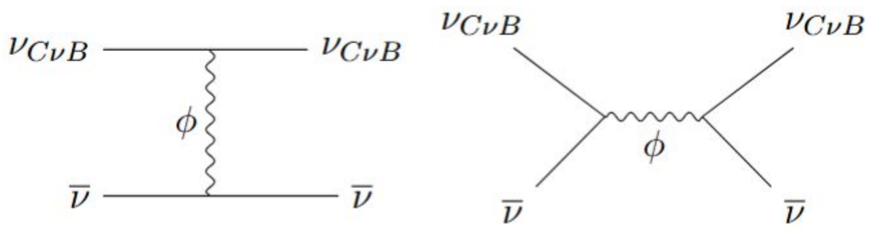
Process	Channel	$(d\sigma/dt)(8\pi s^2/g^4)$
$\overline{\nu}_i\overline{\nu}_i \to \overline{\nu}_i\overline{\nu}_i$	u+t	$\frac{s^2+t^2}{(u-M^2)^2} - \frac{2(-s^2)}{(t-M^2)(u-M^2)} + \frac{u^2+s^2}{(t-M^2)^2}$
$\overline{\nu}_i\nu_i \to \overline{\nu}_i\nu_i$	s+t	$\frac{u^2+s^2}{(t-M^2)^2} - \frac{2(-u^2)}{(s-M^2)(t-M^2)} + \frac{u^2+t^2}{(s-M^2)^2}$
$\overline{\nu}_i\nu_i \to \overline{\nu}_j\nu_j$	s	$\frac{u^2+t^2}{(s-M^2)^2}$
$\overline{\nu}_i\nu_j \to \overline{\nu}_i\nu_j$	t	$\frac{u^2+s^2}{(t-M^2)^2}$

Resonance region: NWA for UR regime

t+u channel:



t+s channel:



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Narrow Width Approximation

$$\frac{1}{(s-M^2)^2+M^2\Gamma^2} \xrightarrow{\Gamma/M\to 0} \frac{\pi}{M\Gamma}\delta(s-M^2).$$

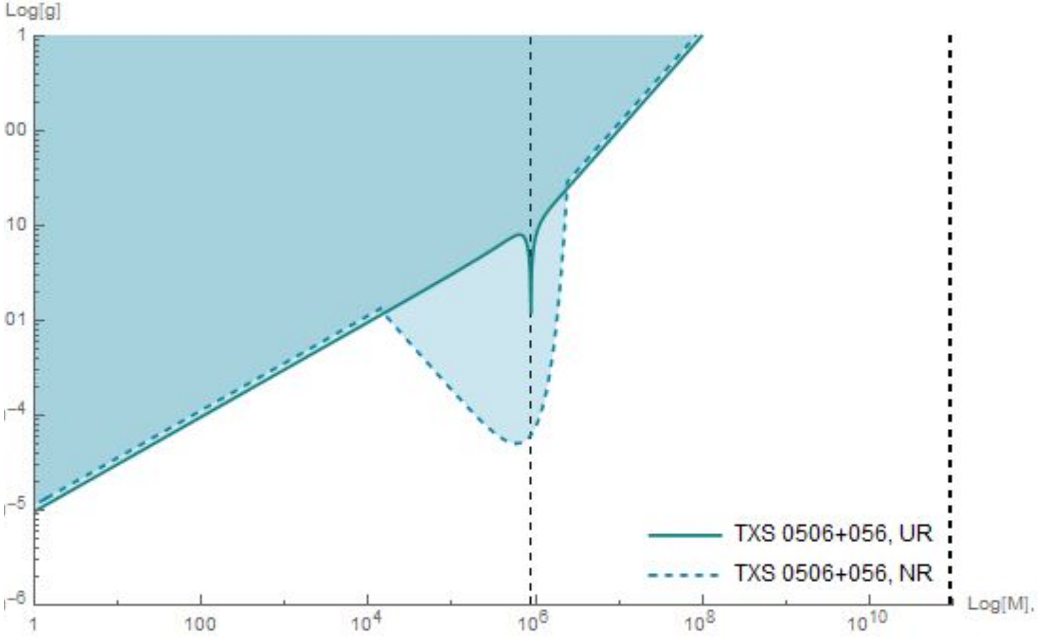
$$\lambda^{-1} = \frac{g^2}{8\pi}\frac{T_{C\nu B}M^2}{(2E)^2}e^{-M^2/4ET_{C\nu B}}$$

$$\Gamma_D \sim \frac{1}{24\pi}g^2 M \text{ per flavor}$$

Details

Comparison of UR and NR CνB regimes

$$g \leq \left(\frac{\lambda|_{g\to 1}}{D} \right)^{1/4}$$



The regions above the curves are the regions of exclusion.

Vertical dashed lines - s-channel resonance - - - $M = \sqrt{s} = \sqrt{2E_\nu m_\nu + m_\nu^2}$

Non-relativistic constraints on coupling constant from SN & B: Neutrino mass dependence

$$g = \left(\frac{\lambda_{MFP}}{\lambda_{SN}}\right)^{1/4}$$

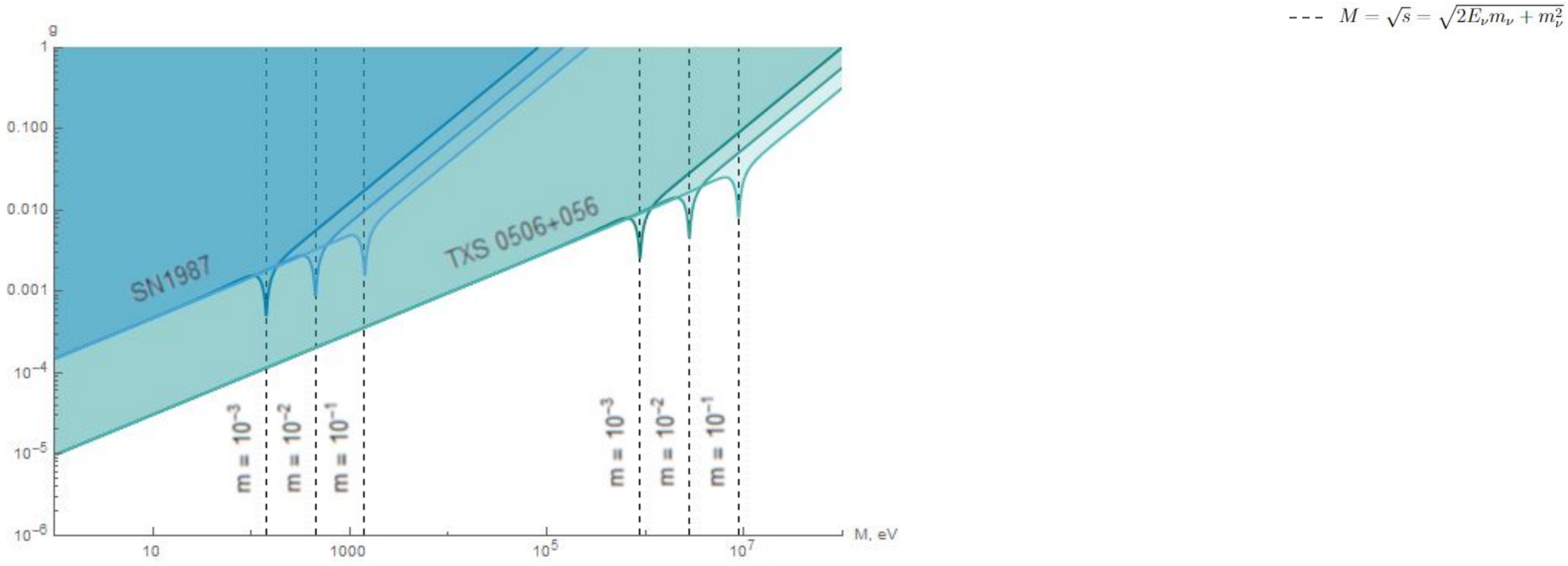
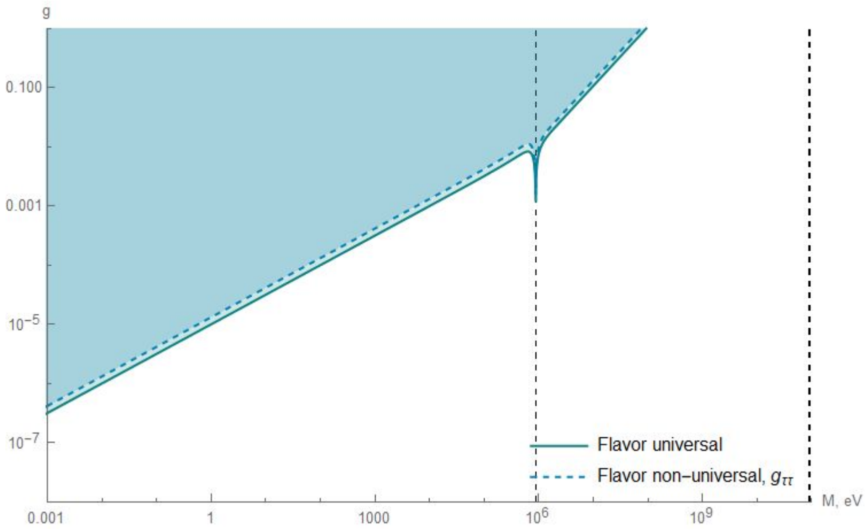


Illustration of sensitivity of a coupling constant exclusion to the neutrino mass variation in NR regime. The plot demonstrates how the position of the s-resonance dip shifts with changes in the neutrino mass. In this scenario, the neutrino mass is assumed to be common for all flavors.

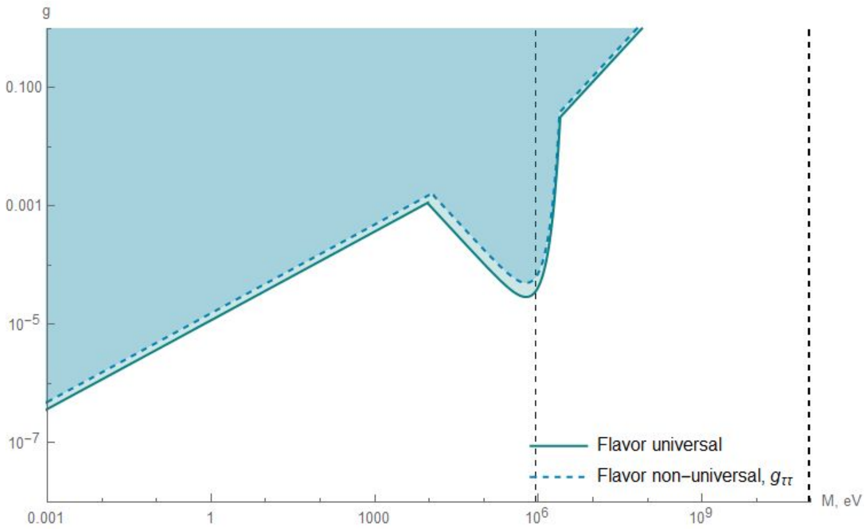
FLAVOR NON-UNIVERSAL COUPLING CONSTANT

flavor Non universal interactions: coupling constant for Supernova

NR



UR



Dependence of the coupling constant from the mediator mass for flavor universal and non-universal (τ) regimes compared. The regions above the curves are the regions of exclusion. NR regime is taken with neutrino mass $m = 10^{-3}$ eV.

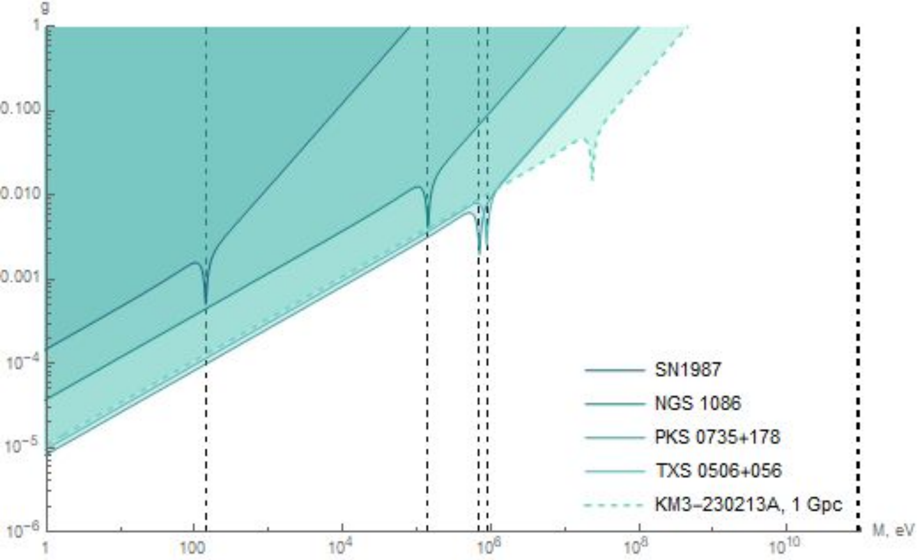
Backup Slides

Exclusion plots for the coupling constant g from the mediator mass M

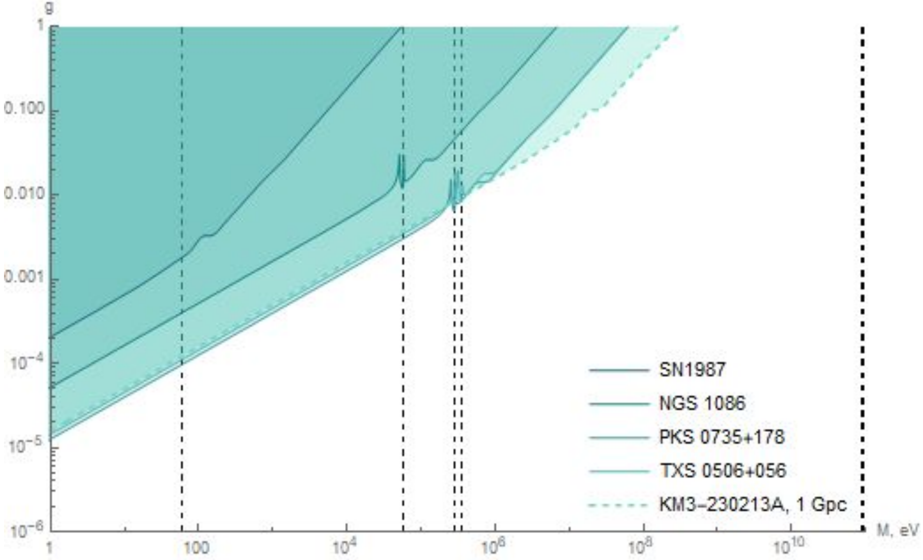
$$g \leq \left(\frac{\lambda|_{g \to 1}}{D} \right)^{1/4}$$

The regions above the curves are the regions of exclusion.

NR CνB regime



UR CνB regime



Toy neutrino mass: $m = 10^{-3}$ eV

Vertical thick dashed line *- Z boson mass,*
Vertical dashed lines - s-channel resonances

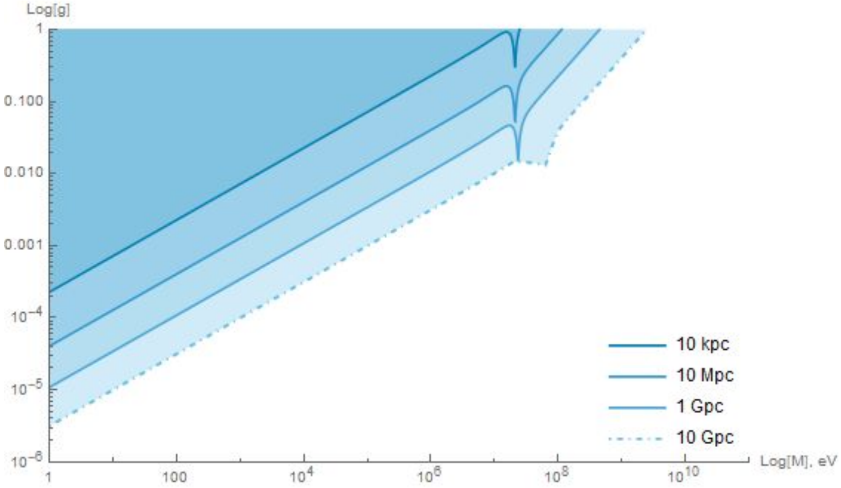
Assumptions:

- *Averaged angle between incident neutrino and background neutrino*
- *CνB distribution is reduced to Maxwell-Boltzmann*
- $m \longrightarrow 0$

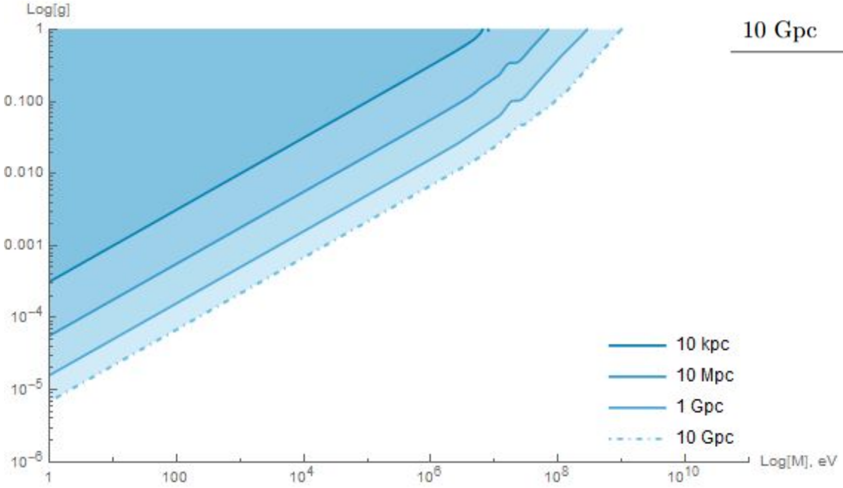
Exclusion plots for the coupling constant from the mediator mass M for a possible source of KM3-230213A 220 PeV event with uncertain distance

Representative distances

Distance	Redshift (z)
10 kpc	$\sim 2 \times 10^{-6}$
10 Mpc	$\sim 2.5 \times 10^{-3}$
1 Gpc	~ 0.25
10 Gpc	~ 12



NR CνB regime



UR CνB regime

The colored regions illustrate excluded values of νSI coupling constant for UR CνB and HEν from corresponding distance.
Dashed lines correspond numerical estimate for a very distant source (z = 12) with cosmological expansion taken into account
NR regime is taken with neutrino mass m = 10^{-3}.

The KM3-230213A event can provide stronger constraints than previous events if it originates from beyond the local universe (1 Mpc).

Results

In this, work we investigated a particular model of NSI with **Dirac neutrinos** and massive vector boson as NSI mediator

We obtained

- ★ Analytical formula for UR (and NR) CnB with full mass dependence
- ★ Constraints on NSI coupling constant by HE neutrinos propagating through the CnB, from SN, Blazars and AGN neutrinos scattering on NR and UR CnB
- ★ Constraints on flavour-non universal NSI
- ★ Applied recent KM3-230213A to constrain NSI

The results are in consistency with the literature, and offer

- ❖ **more precise analysis on the angle cut-off parameter** for the given model,

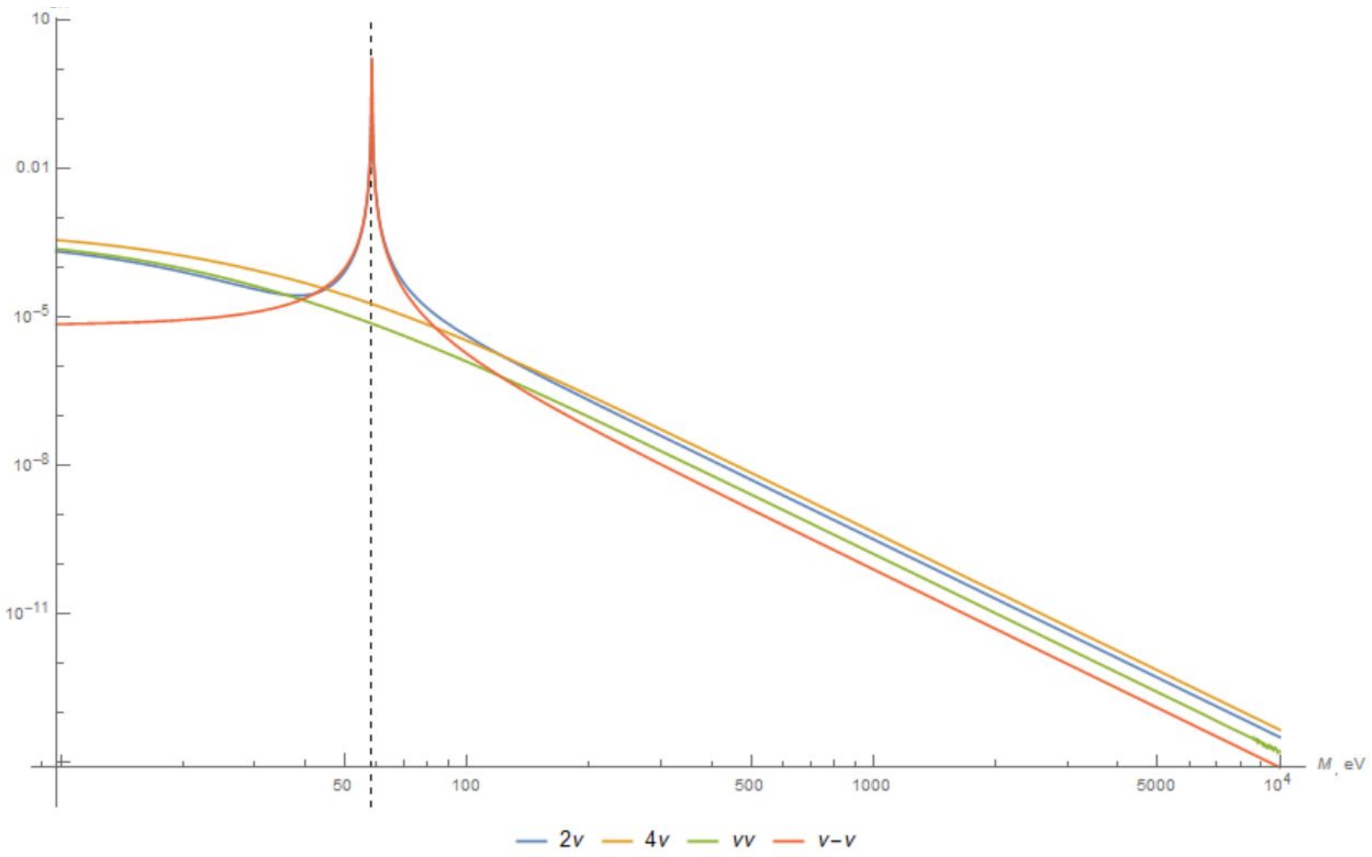
and include

- ❖ **intermediate mass region of the NSI mediator** to the constraints on coupling constant.
- ❖ extended constraints up to mediator mass of GeV scale

Assumptions

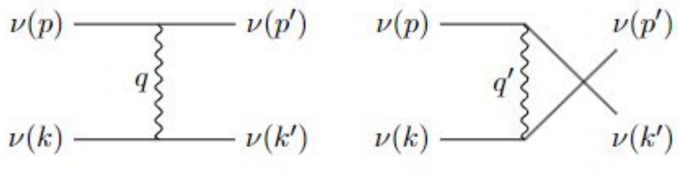
- **Mediator mass:** Full dependence over the relevant interval without splitting into the limits
- **Background regime:** both NR and UR
- We average over angle between incident neutrino and background neutrino
- We adopt CνB-spectrum with temperature 10^{-4} eV, however for calculations, we reduce it to the Maxwell-Boltzmann distribution.
- **Angle cut-off** : s (1–e) < t < - es
- We assume e – μ – τ universality in the non-standard ν – ν interaction

Cross-section vs. mediator mass, Log



Processes contributing to the HEν scattering on CνB

t+u channel:



t+s channel:

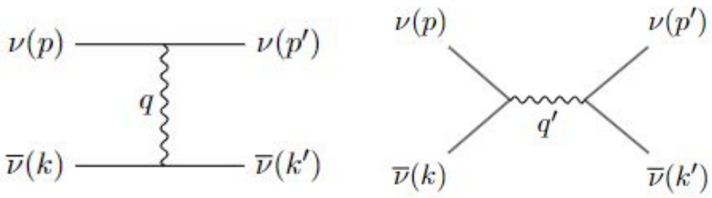


Table II: Differential cross-section

Process	Channel	$(d\sigma/dt)(8\pi s(s-4m^2)/g^4)$
$\overline{\nu}_i\overline{\nu}_i \to \overline{\nu}_i\overline{\nu}_i$	u+t	$\frac{24m^4-8m^2(s+t)+s^2+t^2}{(u-M^2)^2} - \frac{2\left(2m^4-(s-4m^2)^2\right)}{(t-M^2)(u-M^2)} + \frac{(s+t)^2+(s-4m^2)^2-8m^4}{(t-M^2)^2}$
$\overline{\nu}_i\nu_i \to \overline{\nu}_i\nu_i$	s+t	$\frac{(s+t)^2+(s-4m^2)^2-8m^4}{(t-M^2)^2} - \frac{2(4m^4-(s+t)^2)}{(s-M^2)(t-M^2)} + \frac{(s+t)^2+(t-4m^2)^2-8m^4}{(s-M^2)^2}$
$\overline{\nu}_i\nu_i \to \overline{\nu}_j\nu_j$	s	$\frac{(s+t)^2+(t-4m^2)^2-8m^4}{(s-M^2)^2}$
$\overline{\nu}_i\nu_j \to \overline{\nu}_i\nu_j$	t	$\frac{(s+t)^2+(s-4m^2)^2-8m^4}{(t-M^2)^2}$

Asymptotic limits

Heavy massive mediator limit

Process	Channel	$(d\sigma/dt)(8\pi M^4 s^2/g^4)$
$\overline{\nu}_i\overline{\nu}_i \to \overline{\nu}_i\overline{\nu}_i$	u+t	$\frac{1}{2}(4s^2 + t^2 + (s+t)^2)$
$\overline{\nu}_i\nu_i \to \overline{\nu}_i\nu_i$	s+t	$4(s+t)^2 + s^2 + t^2$
$\overline{\nu}_i\nu_i \to \overline{\nu}_j\nu_j$	s	$t^2 + (s+t)^2$
$\overline{\nu}_i\nu_j \to \overline{\nu}_i\nu_j$	t	$s^2 + (s+t)^2$

$$\sigma(s) = g^4 \frac{as}{M^4}$$

Massless mediator limit

Process	Channel	$(d\sigma/dt)(8\pi s^2/g^4)$
$\overline{\nu}_i\overline{\nu}_i \to \overline{\nu}_i\overline{\nu}_i$	u+t	$1 + \frac{s^2}{(s+t)^2} + \frac{s^2}{t^2}$
$\overline{\nu}_i\nu_i \to \overline{\nu}_i\nu_i$	s+t	$2(1 + \frac{(s+t)^2}{s^2} + \frac{(s+t)^2}{t^2})$
$\overline{\nu}_i\nu_i \to \overline{\nu}_j\nu_j$	s	$\frac{(s+t)^2+t^2}{s^2}$
$\overline{\nu}_i\nu_j \to \overline{\nu}_i\nu_j$	t	$\frac{(s+t)^2+s^2}{t^2}$

$$\sigma(s) = g^4 \frac{a}{s}$$

$$d\sigma = d\sigma_{4\nu} + d\sigma_{2\nu} + 2d\sigma_{\overline{\nu}\nu} + 4d\sigma_{\overline{\nu}_i\nu_j}$$

Table III. Differential cross-section for scalar coupling

Process	Channel	$(d\sigma/dt)(8\pi s^2/g^4)$
$\overline{\nu}_i\overline{\nu}_i \to \overline{\nu}_i\overline{\nu}_i$	u+t	$\frac{u^2}{(u-M^2)^2} - \frac{-tu}{(t-M^2)(u-M^2)} + \frac{t^2}{(t-M^2)^2}$
$\overline{\nu}_i\nu_i \to \overline{\nu}_i\nu_i$	s+t	$\frac{t^2}{(t-M^2)^2} - \frac{-ts}{(s-M^2)(t-M^2)} + \frac{s^2}{(s-M^2)^2}$
$\overline{\nu}_i\nu_i \to \overline{\nu}_j\nu_j$	s	$\frac{s^2}{(s-M^2)^2}$
$\overline{\nu}_i\nu_j \to \overline{\nu}_i\nu_j$	t	$\frac{t^2}{(t-M^2)^2}$

Table IV. Cross-section for scalar coupling

Process	Channel	$(d\sigma/dt)(8\pi s^2/g^4)$
$\overline{\nu}_i\overline{\nu}_i \to \overline{\nu}_i\overline{\nu}_i$	u+t	$\frac{u^2}{(u-M^2)^2} - \frac{-tu}{(t-M^2)(u-M^2)} + \frac{t^2}{(t-M^2)^2}$
$\overline{\nu}_i\nu_i \to \overline{\nu}_i\nu_i$	s+t	$\frac{t^2}{(t-M^2)^2} - \frac{-ts}{(s-M^2)(t-M^2)} + \frac{s^2}{(s-M^2)^2}$
$\overline{\nu}_i\nu_i \to \overline{\nu}_j\nu_j$	s	$\frac{s}{8\pi}\frac{1}{(-M^2+s)^2+M^2\Gamma}$
$\overline{\nu}_i\nu_j \to \overline{\nu}_i\nu_j$	t	$\frac{1}{8\pi s^2}\left(M^2 - \frac{M^4}{M^2+s} + s + 2M^2 log\left[\frac{M^2}{M^2+s}\right]\right)$