Dark Models and Bright Tools to Address the Stochastic Gravitational Waves Background Observation

Michele Lucente

TeV Particle Astrophysics 2025, Valencia

November 6, 2025

Based on:
F. Costa, J. Hoefken Zink, M.L., S. Pascoli and S. Rosauro-Alcaraz,
Phys. Lett. B 868 (2025), 139634 — arXiv:2510.00289 [hep-ph]

---

https://github.com/michelelucente/ELENA

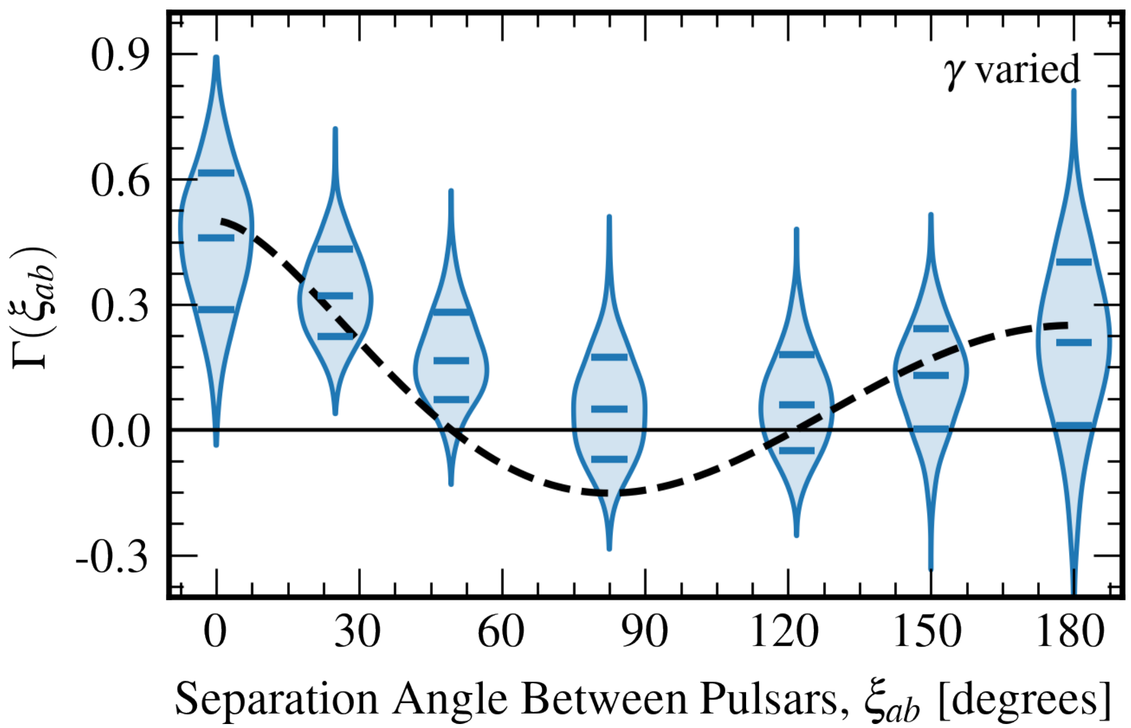
ALMA MATER STUDIORUM
UNIVERSITÀ DI BOLOGNA

**Funded by
the European Union**

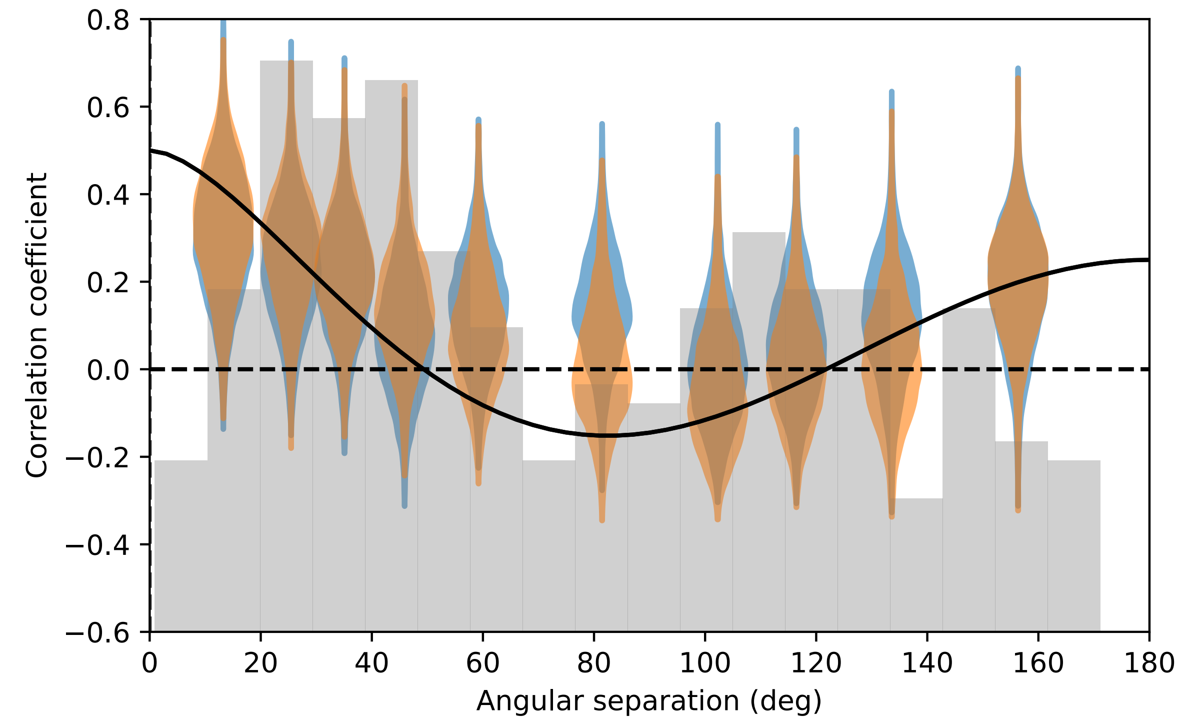
Evidence of a SGWB

PTA collaborations report evidence for a SGWB at 3.5 - 4 σ level

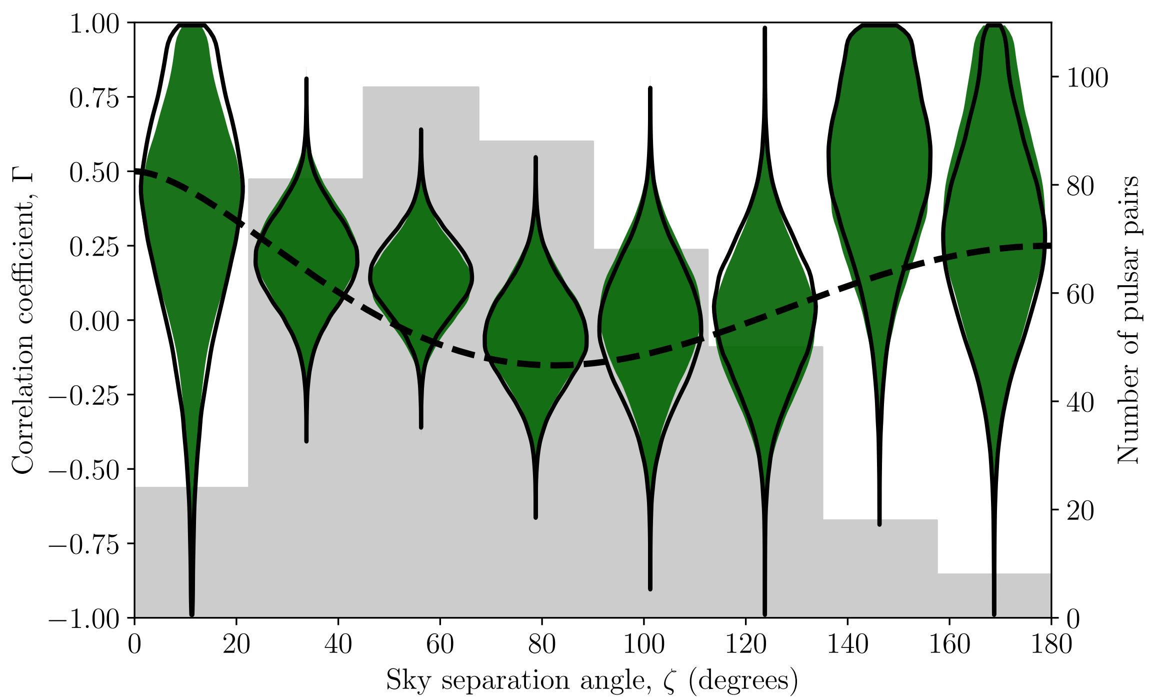
NANOGrav, arXiv:2306.16213 [astro-ph.HE]



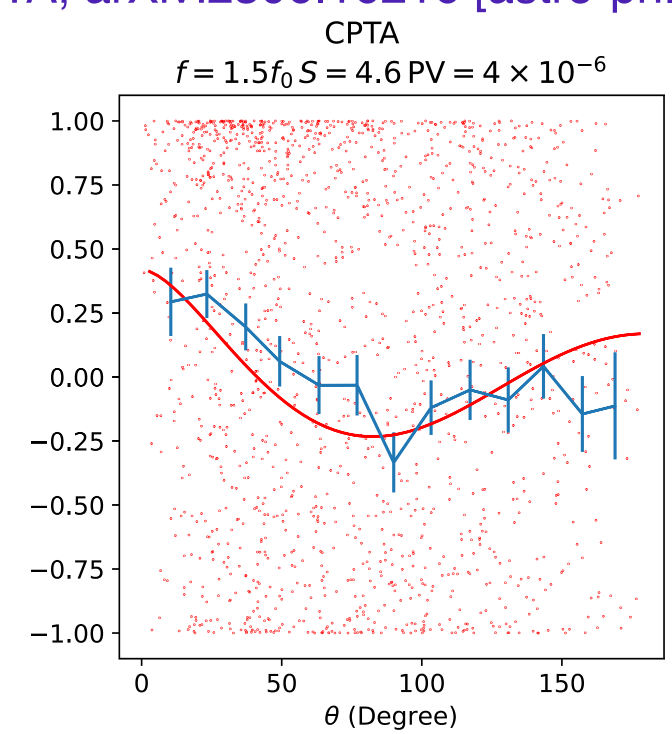
EPTA and InPTA, arXiv:2306.16214 [astro-ph.HE]



PPTA, arXiv:2306.16215 [astro-ph.HE]



CPTA, arXiv:2306.16216 [astro-ph.HE]

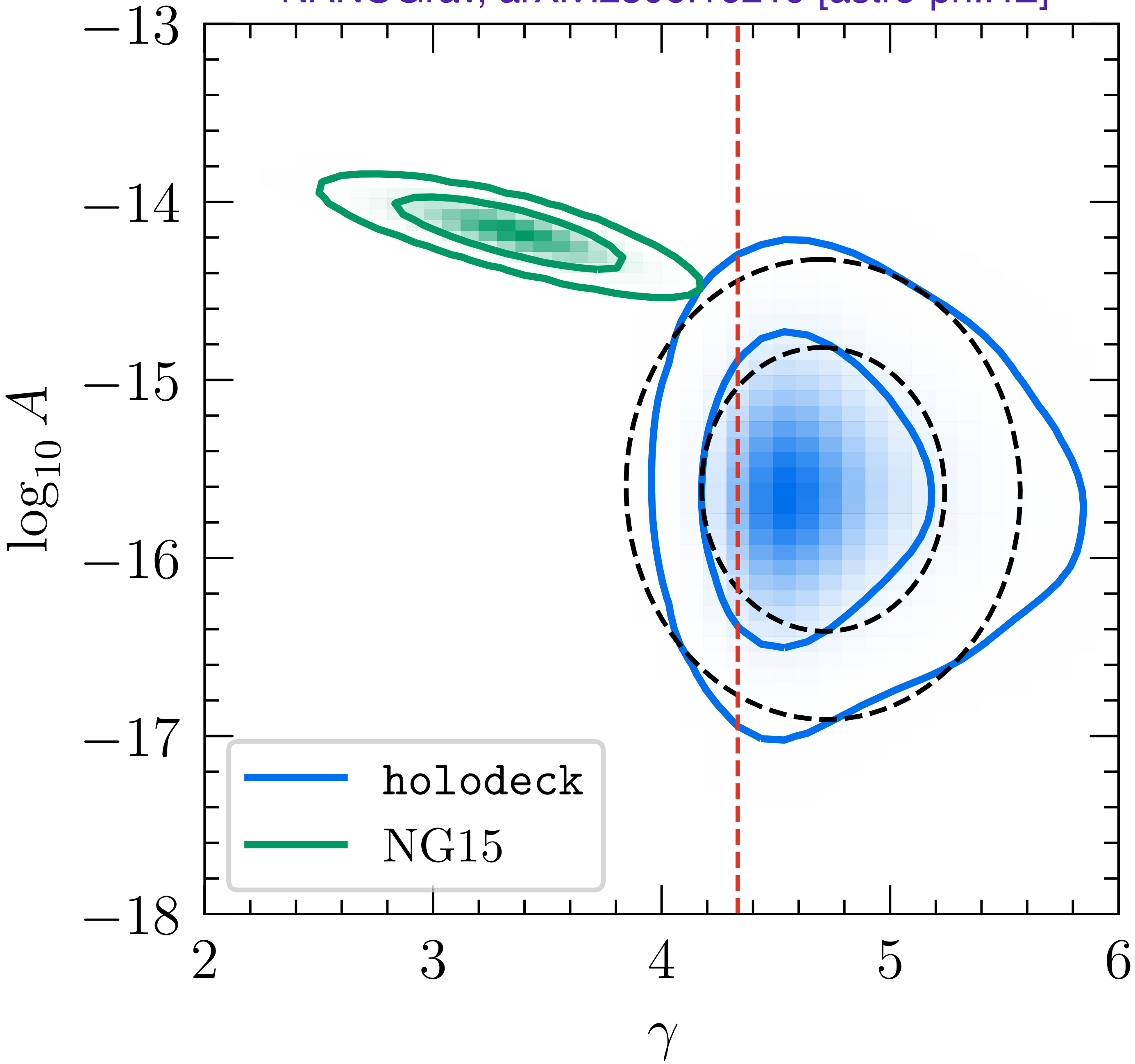


What is the origin of the observed signal?

There is one expected astrophysical source of SGWB

Supermassive Black Holes Binaries

NANOGrav, arXiv:2306.16219 [astro-ph.HE]



The observed signal is in tension with the power-law predicted from astrophysical models

$$\Phi_{\rm BHB}(f) = \frac{A^2_{\rm BHB}}{12\pi^2}\frac{1}{T_{\rm obs}}\left(\frac{f}{\rm yr^{-1}}\right)^{-\gamma_{\rm BHB}} \rm yr^3$$

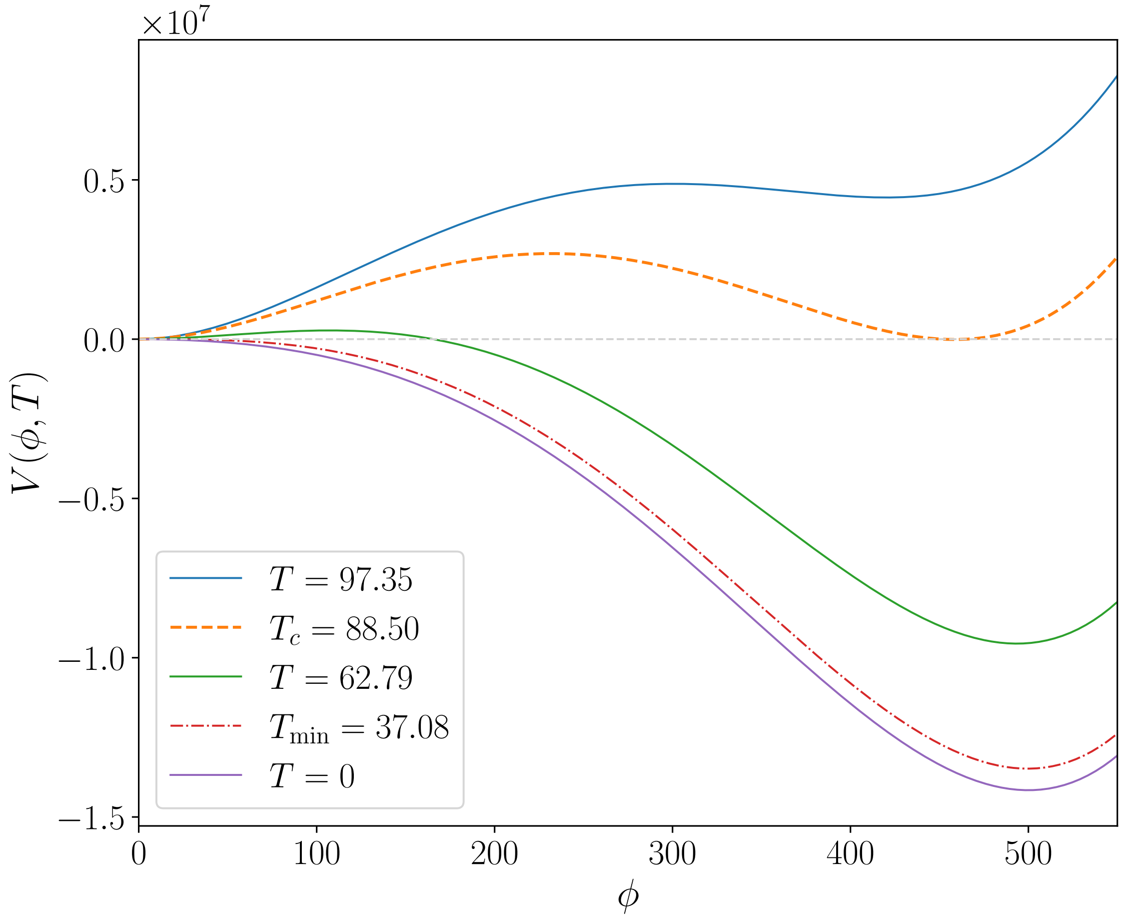
This motivates the exploration of alternative explanations

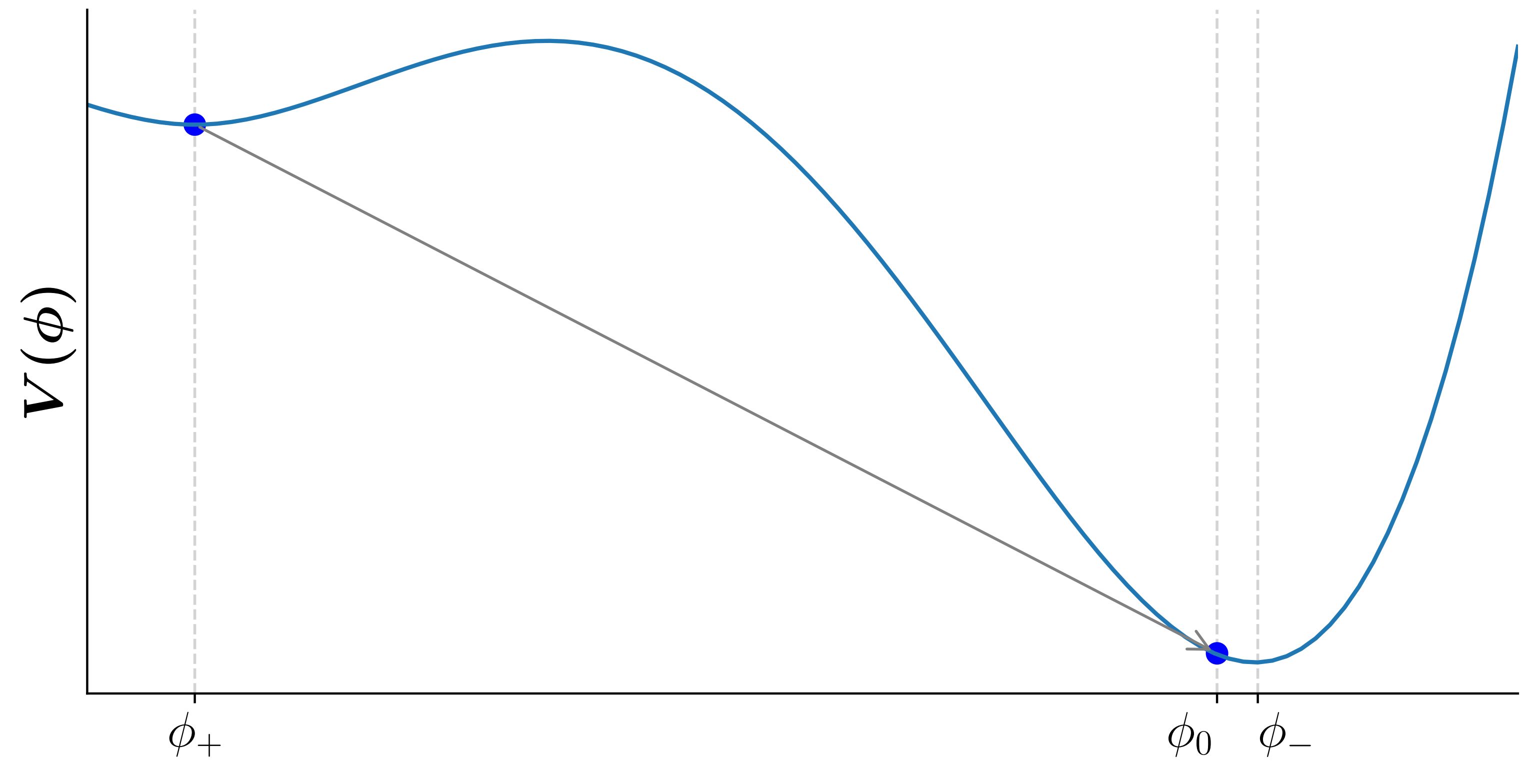
First order phase transitions

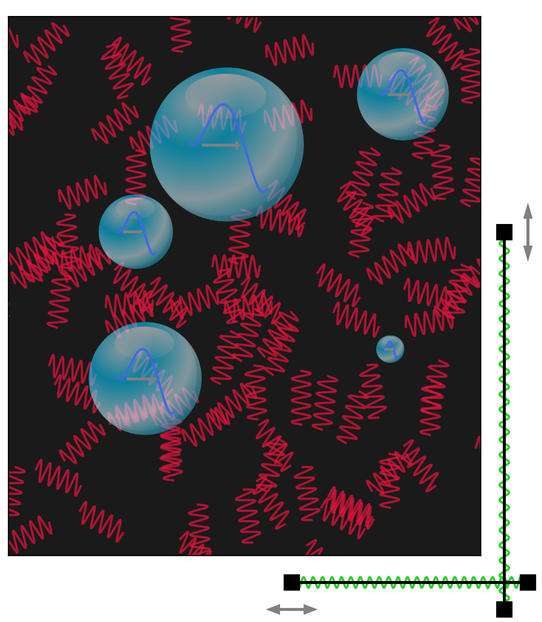
A scalar field in a local minimum can tunnel to the true minimum via bubble nucleation

$$\Gamma(T) \simeq T^4 \left(\frac{S_3}{2\pi T}\right)^{\frac{3}{2}} \exp\left[-\frac{S_3}{T}\right]$$

S. R. Coleman, Phys. Rev. D 15 (1977), 2929-2936







The bubble nucleation can generate GW via bubble collision, sound waves and plasma turbulence

P. Athron, C. Balázs, A. Fowlie, L. Morris and L. Wu, arXiv:2305.02357 [hep-ph]

How to compute the action?

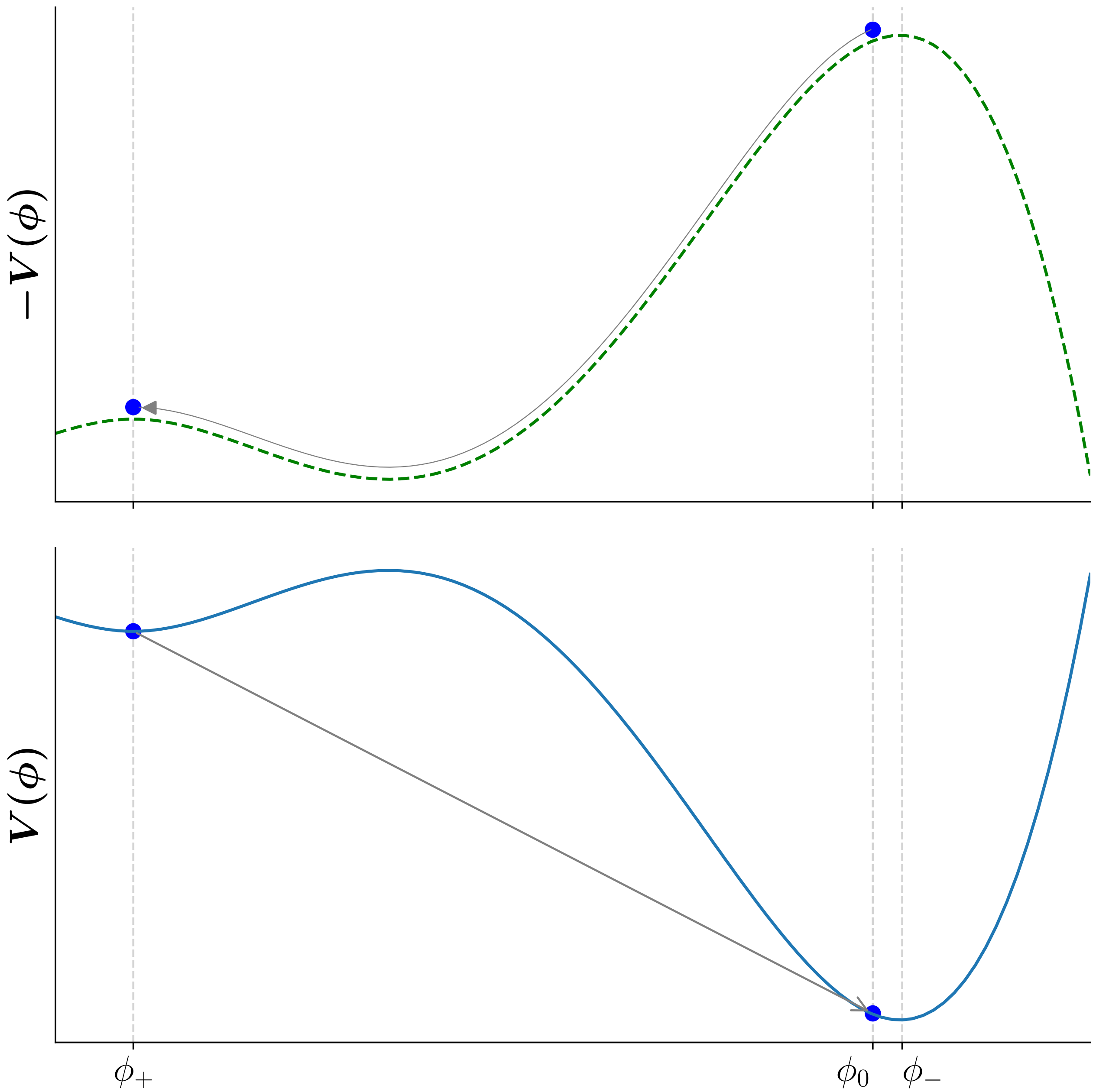
Need to solve the "bounce equation"...

S. R. Coleman, Phys. Rev. D 15 (1977), 2929-2936

$$\boxed{\ddot{\phi}(\rho) + \frac{d-1}{\rho}\dot{\phi}(\rho) = V'(\phi)}$$

$$\dot{\phi}_b(0) = 0$$

$$\lim_{\rho\to\infty} \phi_b(\rho) = \phi_+$$



...and then insert the solution in the action integral

$$S_{E,d} = \frac{2\pi^{d/2}}{\Gamma(d/2)} \int_0^\infty \left[\frac{1}{2}\dot{\phi}_b^2 + V\left(\phi_b\right) - V(\phi_+)\right] \rho^{d-1} \mathrm{d}\rho$$

Overshoot-undershoot method

C. L. Wainwright, arXiv:1109.4189 [hep-ph]

Popular numerical recipe:

1. Solve the equations of motion for a trial value of ϕ_0;
2. Adjust the initial position ϕ_0 depending on the asymptotic behaviour;
3. Iterate until the desired numerical precision is reached.

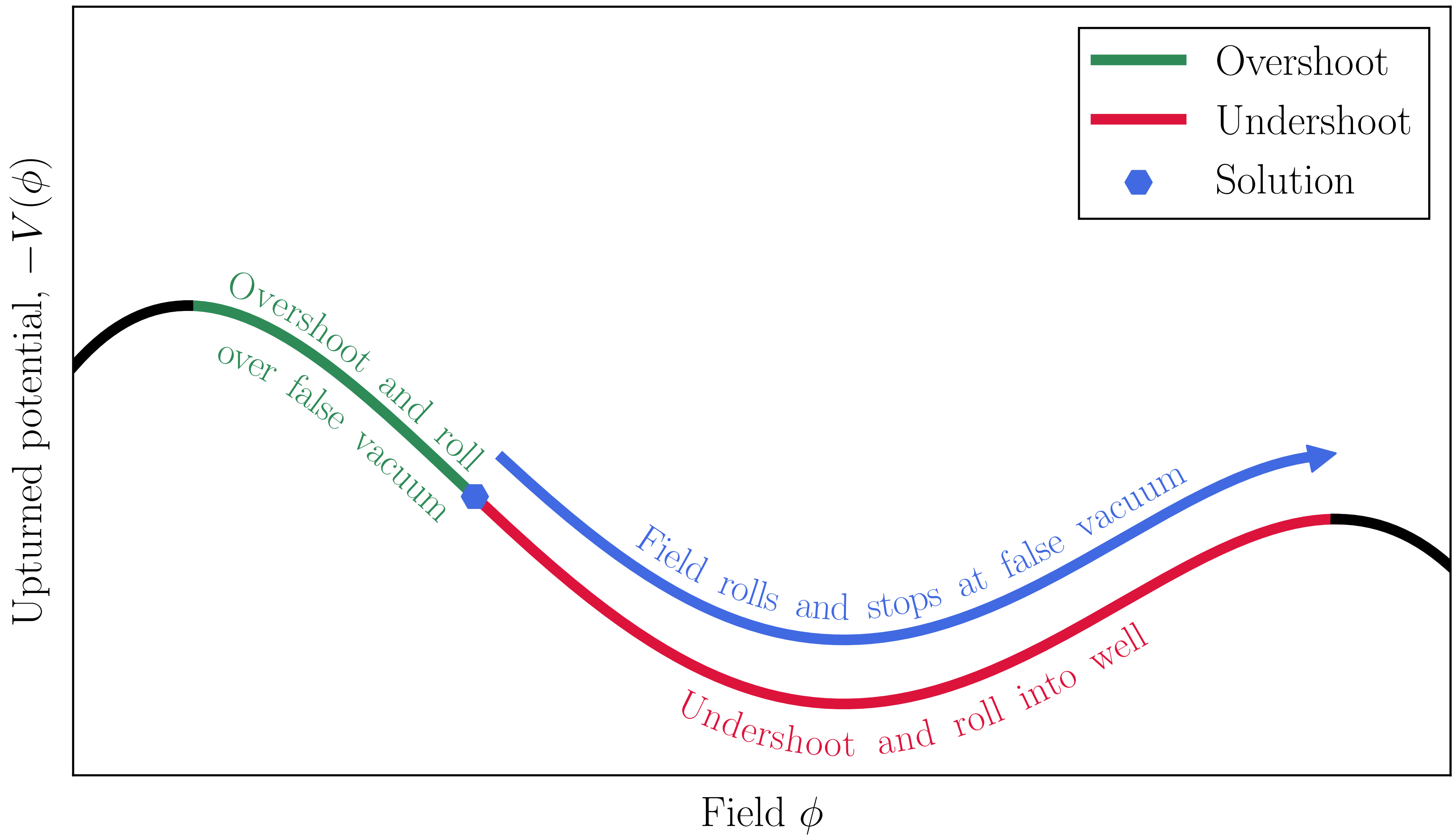


Figure from: P. Athron, C. Balázs, A. Fowlie, L. Morris and L. Wu, arXiv:2305.02357 [hep-ph]

The solution is an unstable point: small deviations from the correct ϕ_0 value can result in large differences of the asymptotic field value

Used in many public codes, but can be numerically demanding

Tunnelling potential formalism

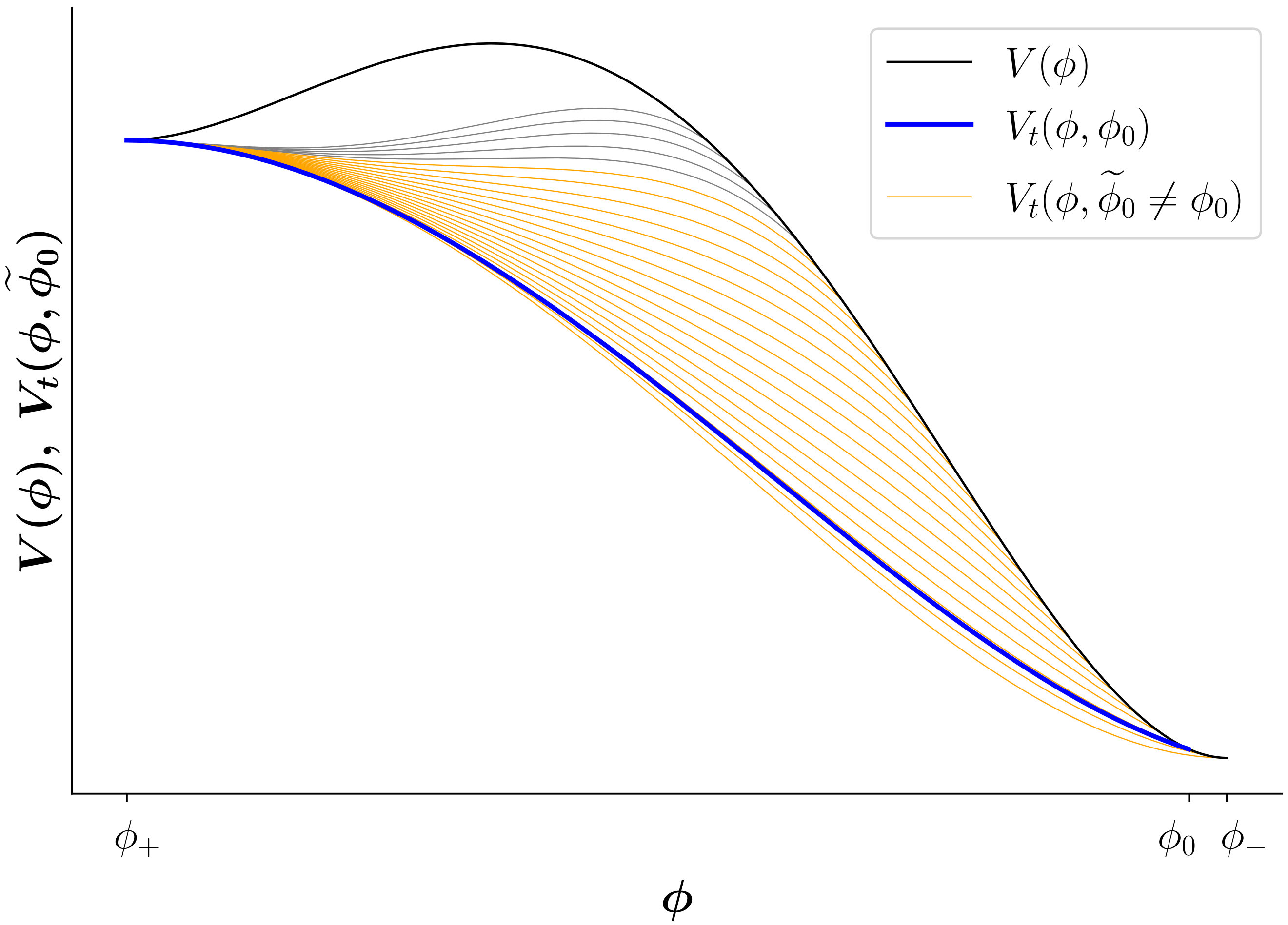
J. R. Espinosa, arXiv:1805.03680 [hep-th]

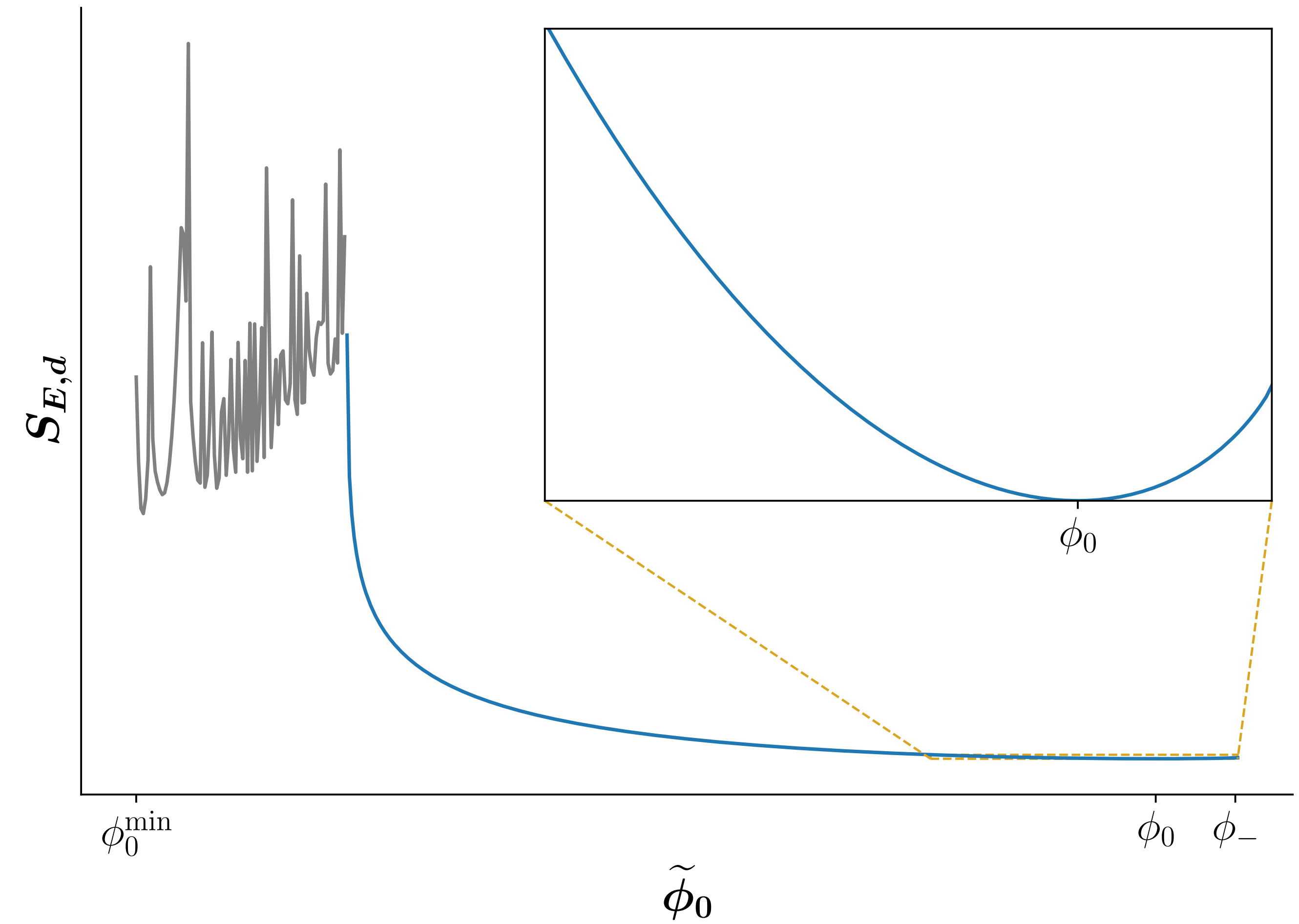
It is possible to redefine the equations of motion

$$\left(V_t'\right)^2 = \frac{d-1}{d}\left[V'V_t' - 2\left(V_t - V\right)V_t''\right]$$

$$V_t(\phi) \equiv V(\phi) - \frac{1}{2}\dot{\phi}_b^2$$

$$V_t\left(\phi_+\right) = V\left(\phi_+\right) \qquad V_t\left(\phi_0\right) = V\left(\phi_0\right)$$



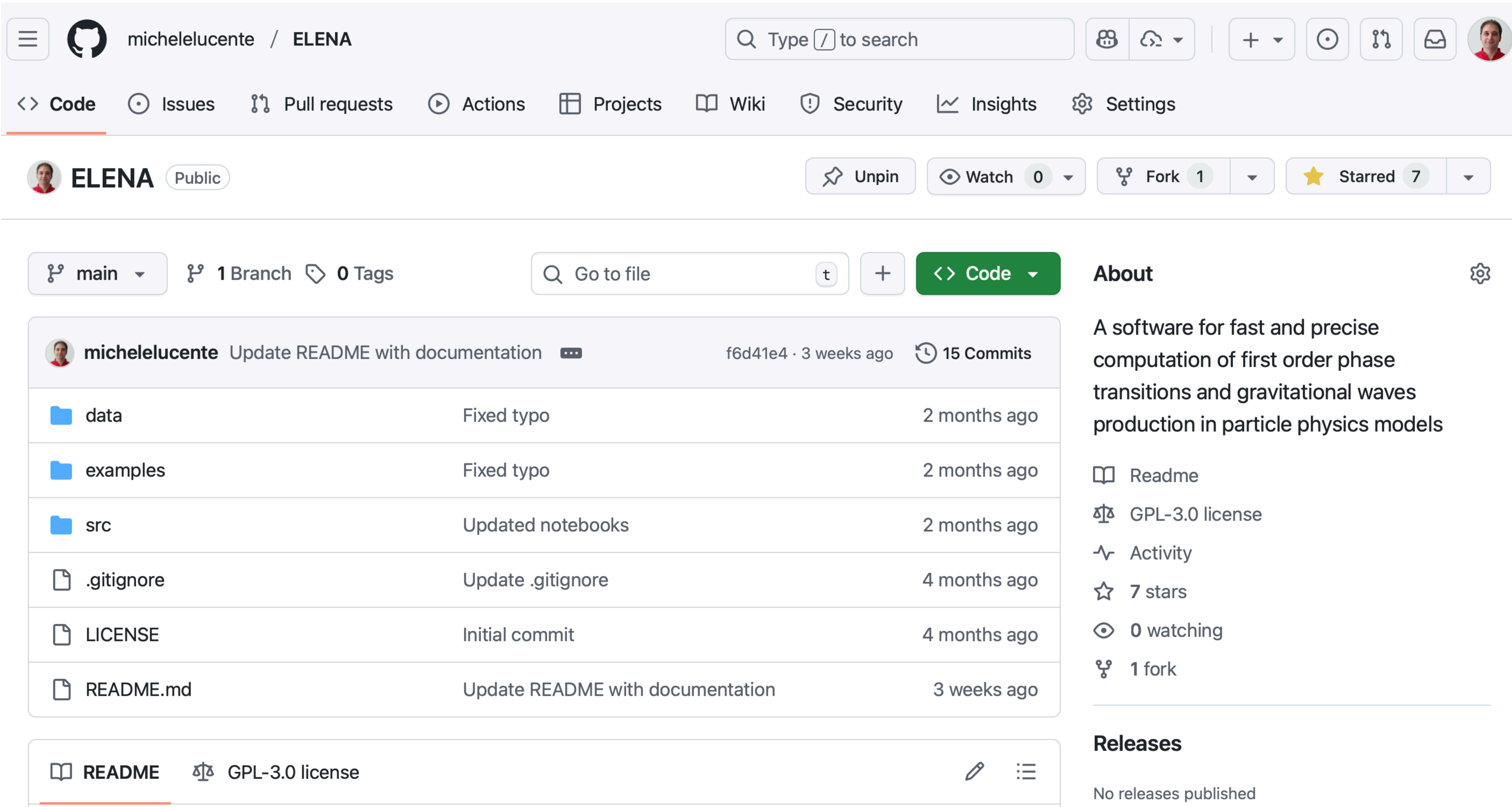


The correct value of ϕ₀ becomes a minimum of the so-called tunnelling action

$$S_{E,d} = \frac{(d-1)^{(d-1)}\,(2\pi)^{\frac{d}{2}}}{\Gamma\left(1+\frac{d}{2}\right)} \int_{\phi_+}^{\phi_0} \frac{\left(V - V_t\right)^{\frac{d}{2}}}{\left|V_t'\right|^{(d-1)}} \mathrm{d}\phi$$

ELENA: EvaLuator of tunnElliNg Actions

We implemented the tunnelling potential formalism in a new public Python code

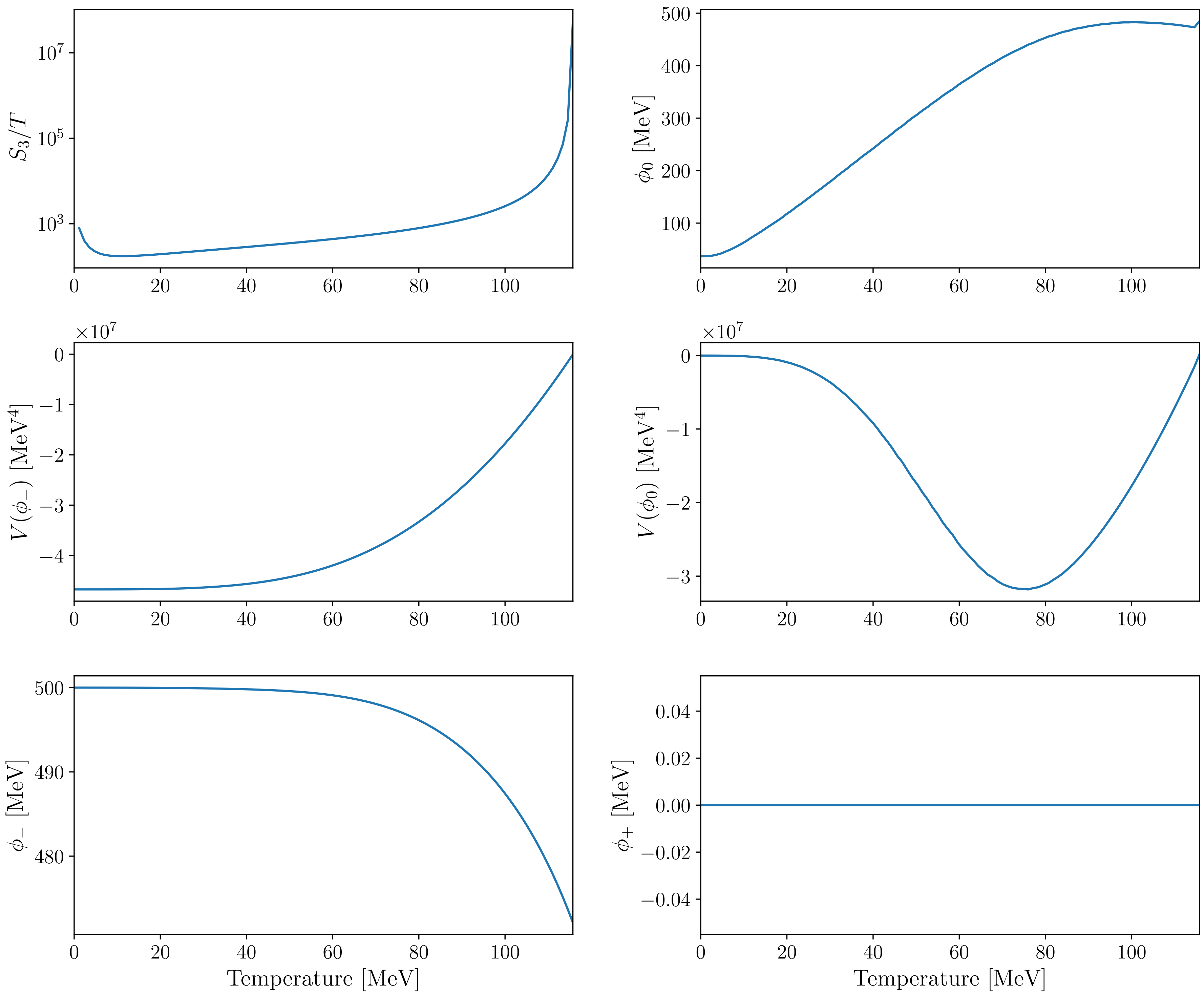


ELENA also provides all the tools to compute the SGWB from FOPT
(from the Lagrangian parameter inputs to the final gravitational waves spectrum)
in a fast and self-contained implementation

https://github.com/michelelucente/ELENA

Going beyond approximations:

The fast computation of the tunnelling enables the use of integral expressions that track the complete evolution of the transition



ELENA employs ~ 20 milliseconds for tunnelling computation on Apple M2 processor

I. Milestone temperatures computation

ELENA only assumes adiabatic expansion

$$\frac{dT}{dt} = -3H(T)\frac{\partial_T V^{\mathrm{T}}(\phi_+(T),T)}{\partial_{TT} V^{\mathrm{T}}(\phi_+(T),T)}$$

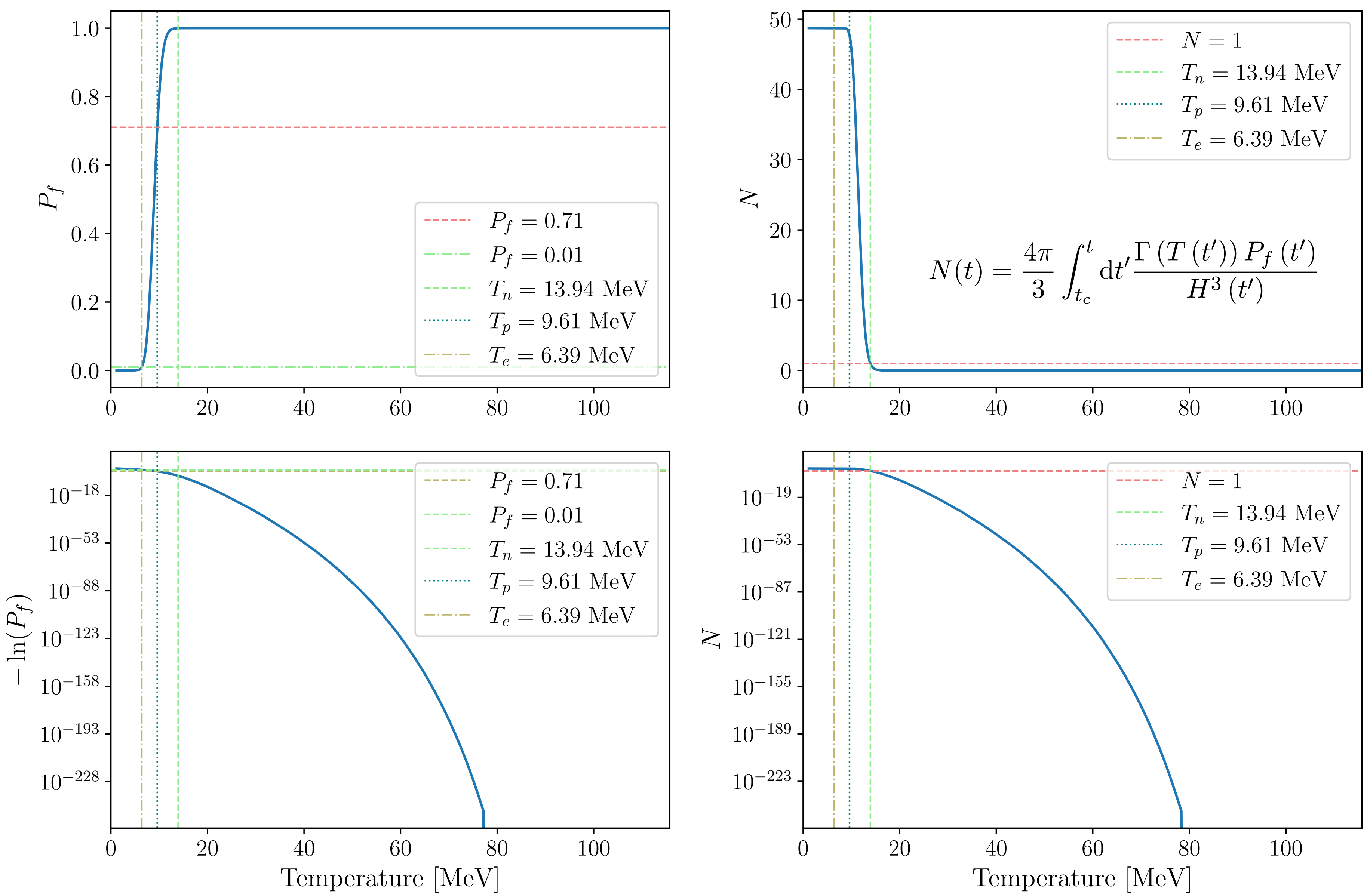
~~Bag equation of state~~

~~Radiation dominated Universe~~

P. Athron, C. Balázs, A. Fowlie, L. Morris and L. Wu, arXiv:2305.02357 [hep-ph]

$$P_f(T) = \exp\left(-\mathcal{V}_t^{ext}(T)\right)$$

$$\mathcal{V}_t^{ext}(t) = \frac{4\pi}{3} v_w^3 \int_{t_0}^{t} dt' \Gamma(t') \left(\frac{a(t')}{a(t)}\right)^3 \left[\int_{t'}^{t} dt'' \frac{a(t)}{a(t'')}\right]^3$$



$$\frac{a(t_1)}{a(t_2)} = \exp\left(\int_{t_2}^{t_1} dt' H(t')\right)$$

Fast and precise computation of

Nucleation (T_n)

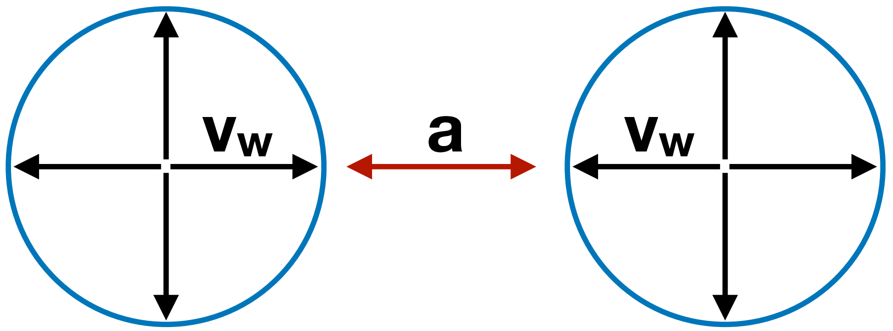
Percolation (T_p)

Completion (T_e)

temperatures

II. Physical volume evolution

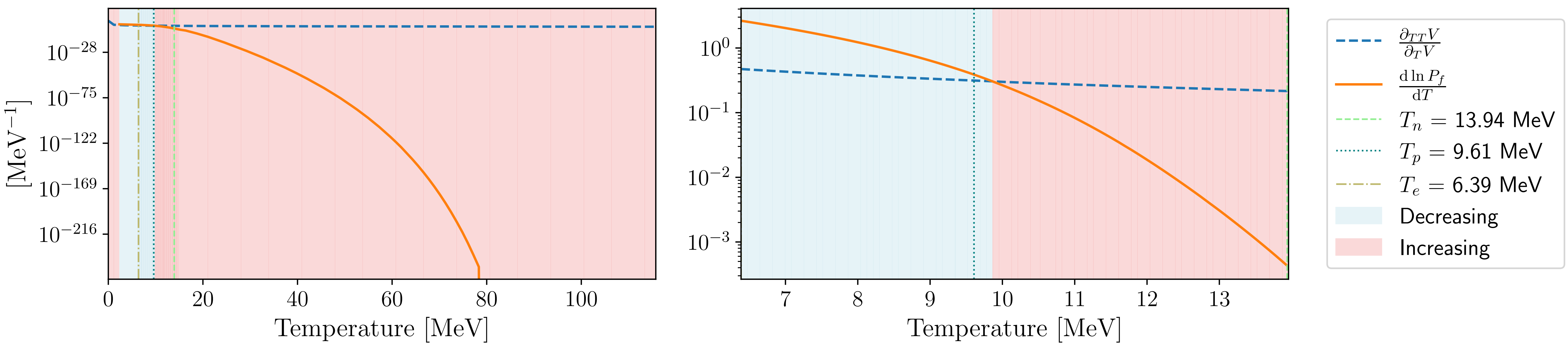
In an expanding Universe, the expansion of bubbles competes with the expansion of space itself



It is essential to check that the physical volume in false vacuum is decreasing at percolation

$$\frac{\mathrm{d}\mathcal{V}_{\mathrm{phys}}}{\mathrm{d}t} = \mathcal{V}_{\mathrm{phys}}(t)\left[\frac{\mathrm{d}}{\mathrm{d}t}\ln\left(P_f(t)\right) + 3H(t)\right] \leq 0 \qquad \mathcal{V}_{\mathrm{phys}}(t) = a^3(t)\,P_f(t)$$

P. Athron, C. Balázs, A. Fowlie, L. Morris and L. Wu, arXiv:2305.02357 [hep-ph],
P. Athron, C. Balázs and L. Morris, arXiv:2212.07559 [hep-ph]

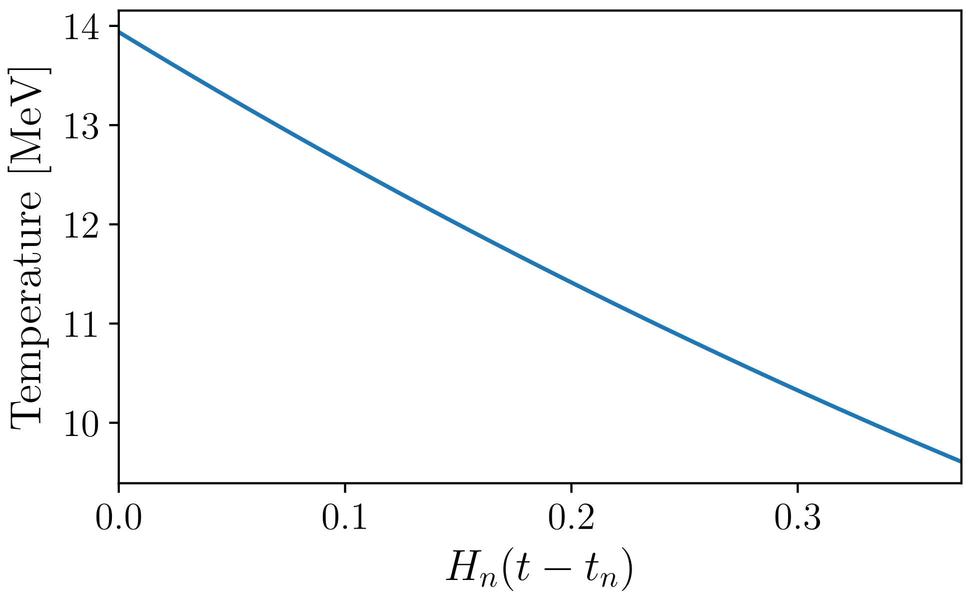


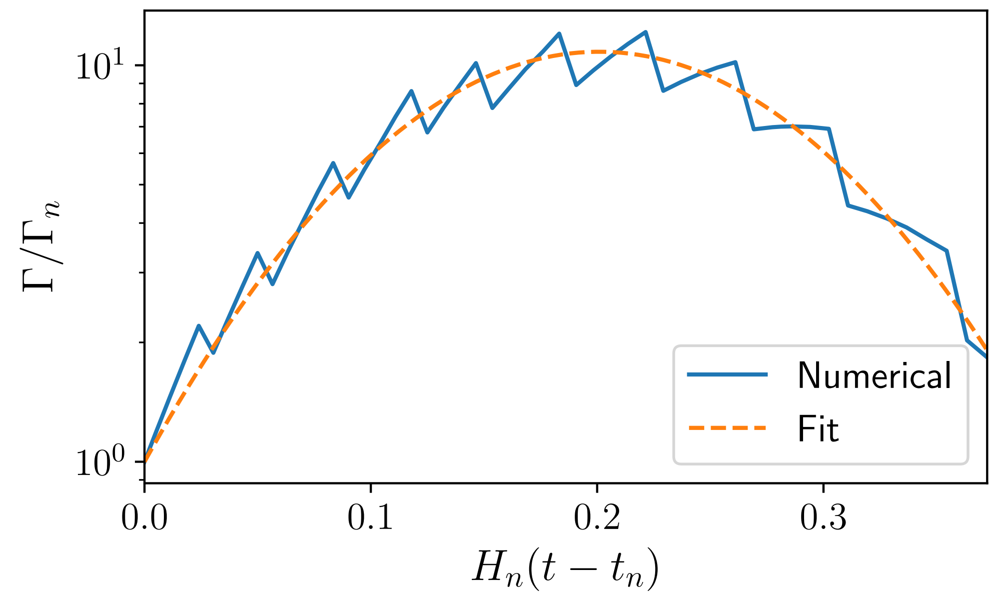
ELENA can readily compute the evolution of physical volume in false vacuum at any temperature

III. Mean bubble separation computation

The inverse duration of the transition β is often used as SGWB input parameter

$$\Gamma(t) = \Gamma_n \exp\left[\beta(t - t_n) - \frac{1}{2}\gamma^2(t - t_n)^2\right]$$



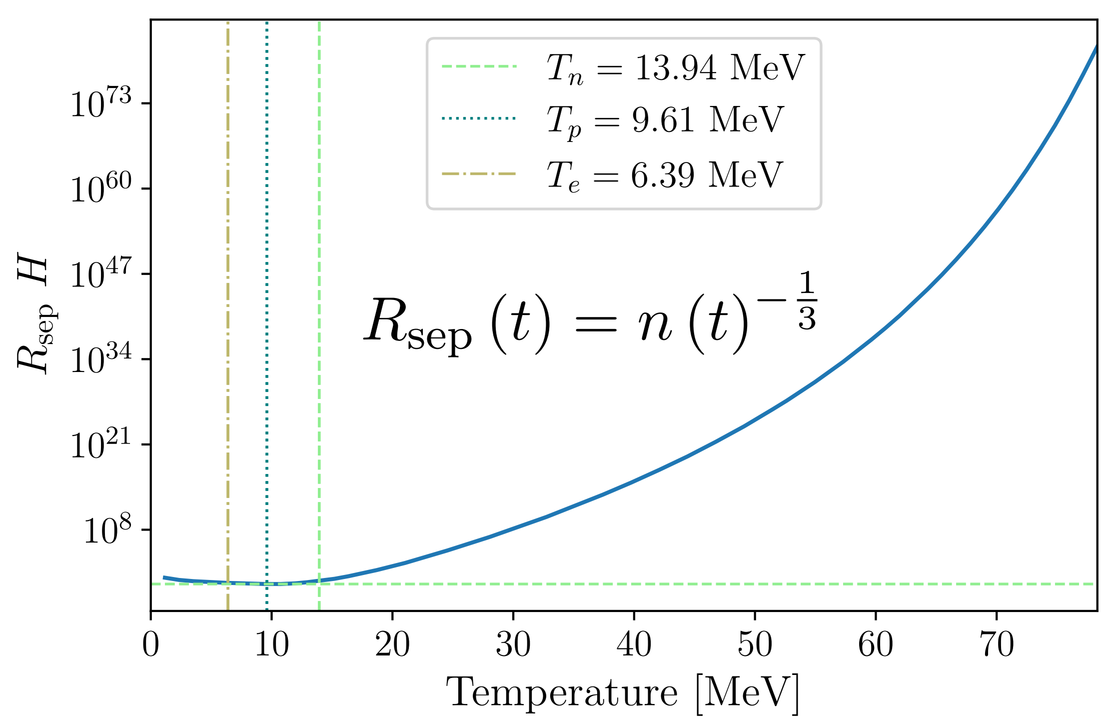


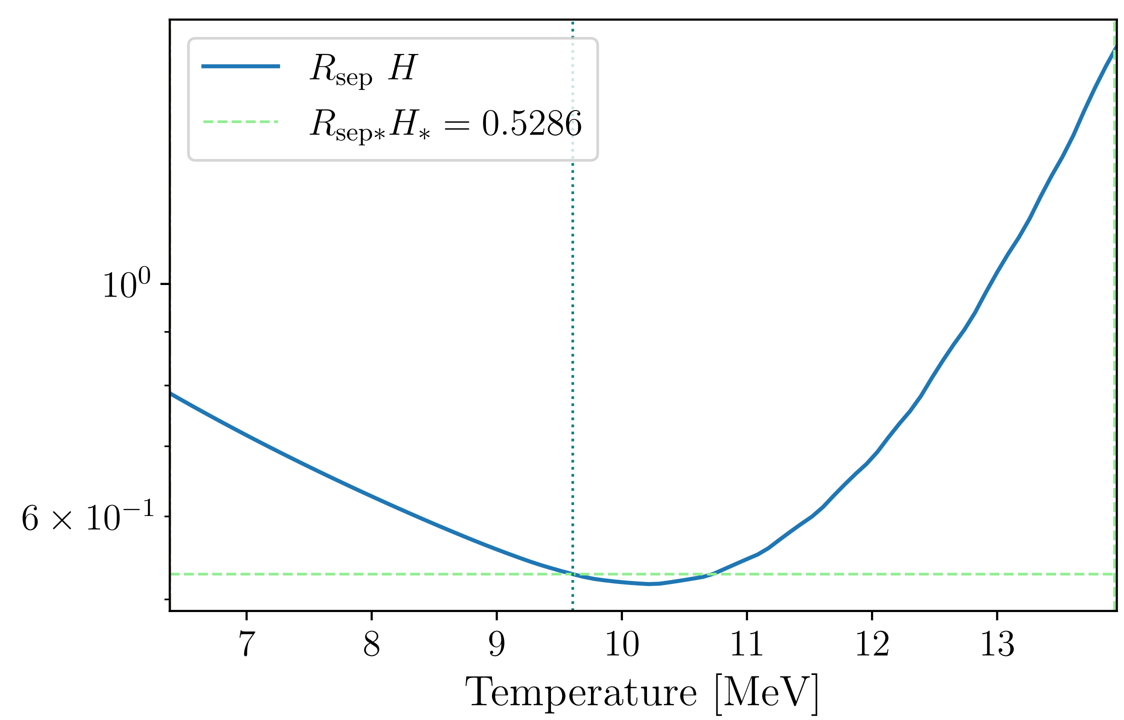
$$\beta = 3.85 \times 10^{-17}\ \mathrm{MeV}$$
$$\gamma = 1.76 \times 10^{-17}\ \mathrm{MeV}$$
$$H_n = 1.62 \times 10^{-18}\ \mathrm{MeV}$$

$$\beta/H_n = 23.71$$
$$\gamma/H_n = 10.86$$
$$\gamma/\beta = 0.46$$

ELENA allows to compute the more general mean bubbles separation parameter

$$n(t) = \int_{t_c}^{t} \mathrm{d}t' \Gamma(t') P_f(t') \frac{a^3(t')}{a^3(t)}$$





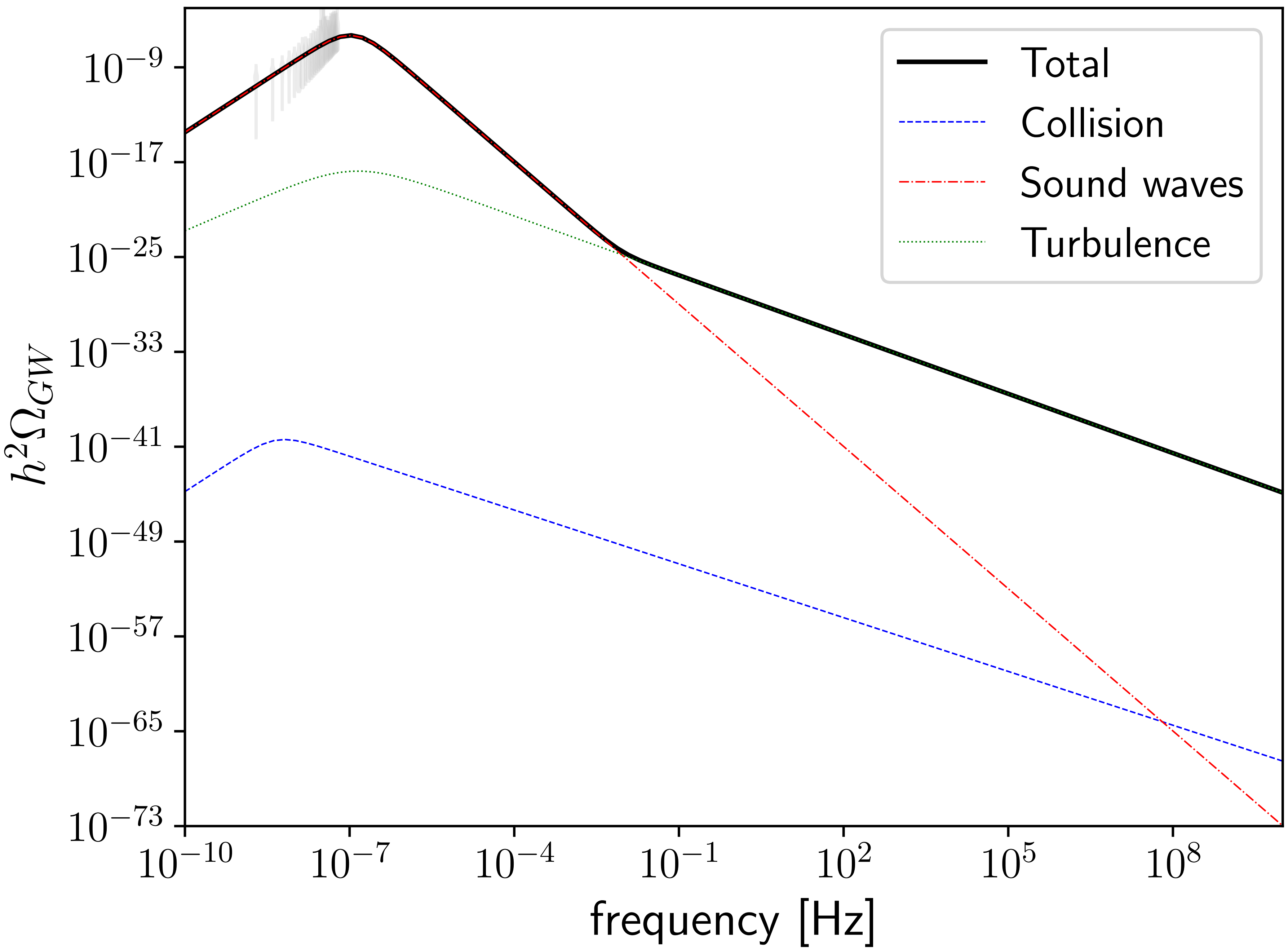
SGWB spectrum computation

ELENA computes the full SGWB spectrum for all the individual sources

Currently implemented the expressions from

J. Ellis, M. Lewicki, J. M. No and V. Vaskonen, arXiv:1903.09642 [hep-ph]

The user can readily use ELENA thermal parameters as input for further fittings



Minimal Dark Sector model for SGWB

A particle physics model for a FOPT must include:

Scalar field to drive the phase transition	Gauge field make the transition 1st order

We demonstrated that a minimal dark sector composed of dark photon Z' and complex scalar ϕ can generate the observed SGWB

F. Costa, J. Hoefken Zink, M. Lucente, S. Pascoli, S. Rosauro-Alcaraz, arXiv:2501.15649

$$\mathcal{L} = \left(D_\mu \varphi\right)^* (D^\mu \varphi) - \frac{1}{4} Z'_{\mu\nu} Z'^{\mu\nu} - V(\varphi^* \varphi) \qquad V = -\mu_\varphi^2 \varphi^* \varphi + \lambda_\varphi (\varphi^* \varphi)^2$$

Assuming $\mu_\varphi^2 > 0$ the U(1) gauge symmetry is spontaneously broken

$$v_\varphi = \mu_\varphi / \sqrt{\lambda_\varphi} \qquad m_{Z'}^2 = g_D^2 v_\varphi^2 \qquad m_\varphi^2 = 2\lambda_\varphi v_\varphi^2$$

For other possible models see e.g.

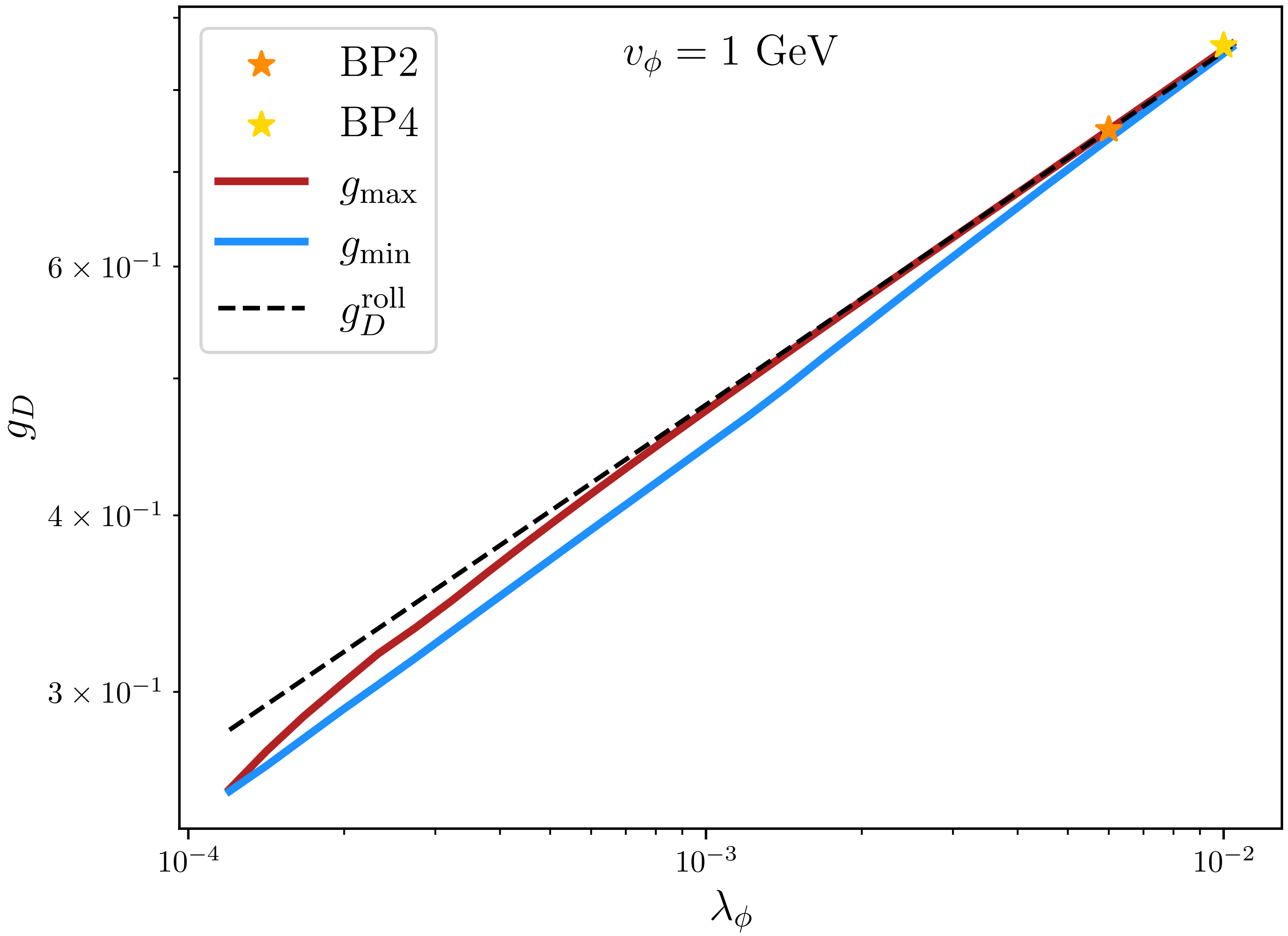
D. Borah, A. Dasgupta and S. K. Kang, arXiv:2105.01007 [hep-ph]; Z. C. Chen, S. L. Li, P. Wu and H. Yu, arXiv:2312.01824 [astro-ph.CO]; A. Conaci, L. Delle Rose, P. S. B. Dev and A. Ghoshal, arXiv:2401.09411 [astro-ph.CO]; J. Gonçalves, D. Marfatia, A. P. Morais and R. Pasechnik, arXiv:2501.11619 [hep-ph]; S. Balan, T. Bringmann, F. Kahlhoefer, J. Matuszak and C. Tasillo,arXiv:2502.19478 [hep-ph]

Dark particles conformal mass ratio

A purely conformal Coleman-Weinberg model (scalar QED) features a specific mass ratio

$$\frac{\overline{M}_S^2}{\overline{M}_V^2} = \frac{3g^2}{8\pi^2}$$

We do not assume a conformal model (μ ≠ 0 in the starting Lagrangian), but we derive a relation for the couplings that give a conformal-like potential



$$g_D = \left\{ \frac{16\pi^2 \lambda_\varphi}{3} \left[1 - \frac{\lambda_\varphi}{8\pi^2} (5 + 2\log 2) \right] \right\}^{1/4}$$

$$\frac{m_\phi^2}{m_{Z'}^2} \simeq \frac{\sqrt{3\lambda_\phi}}{2\pi}$$

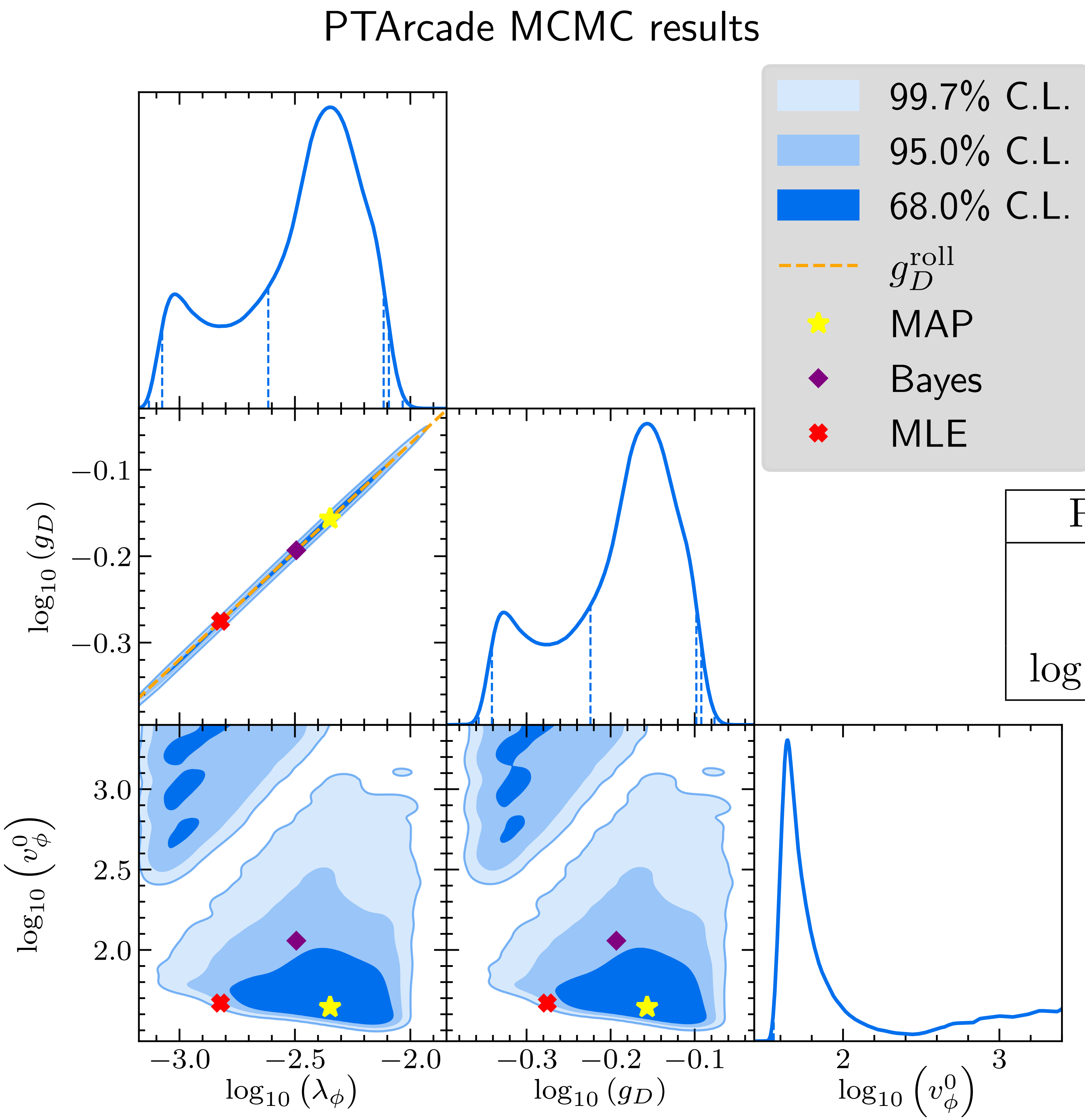
Models accounting for PTA data generally predict a strongly correlated mass spectrum

See B. Sojka and B. Swiezewska, arXiv:2407.07437 [hep-ph] for detailed discussion on radiative vs explicit mass terms

MCMC study of the model with ELENA

We interfaced ELENA with PTArcade to perform a MCMC fit to NANOGrav 15-years data

A. Mitridate, D. Wright, R. von Eckardstein, T. Schröder, J. Nay, K. Olum, K. Schmitz and T. Trickle, arXiv:2306.16377 [hep-ph]

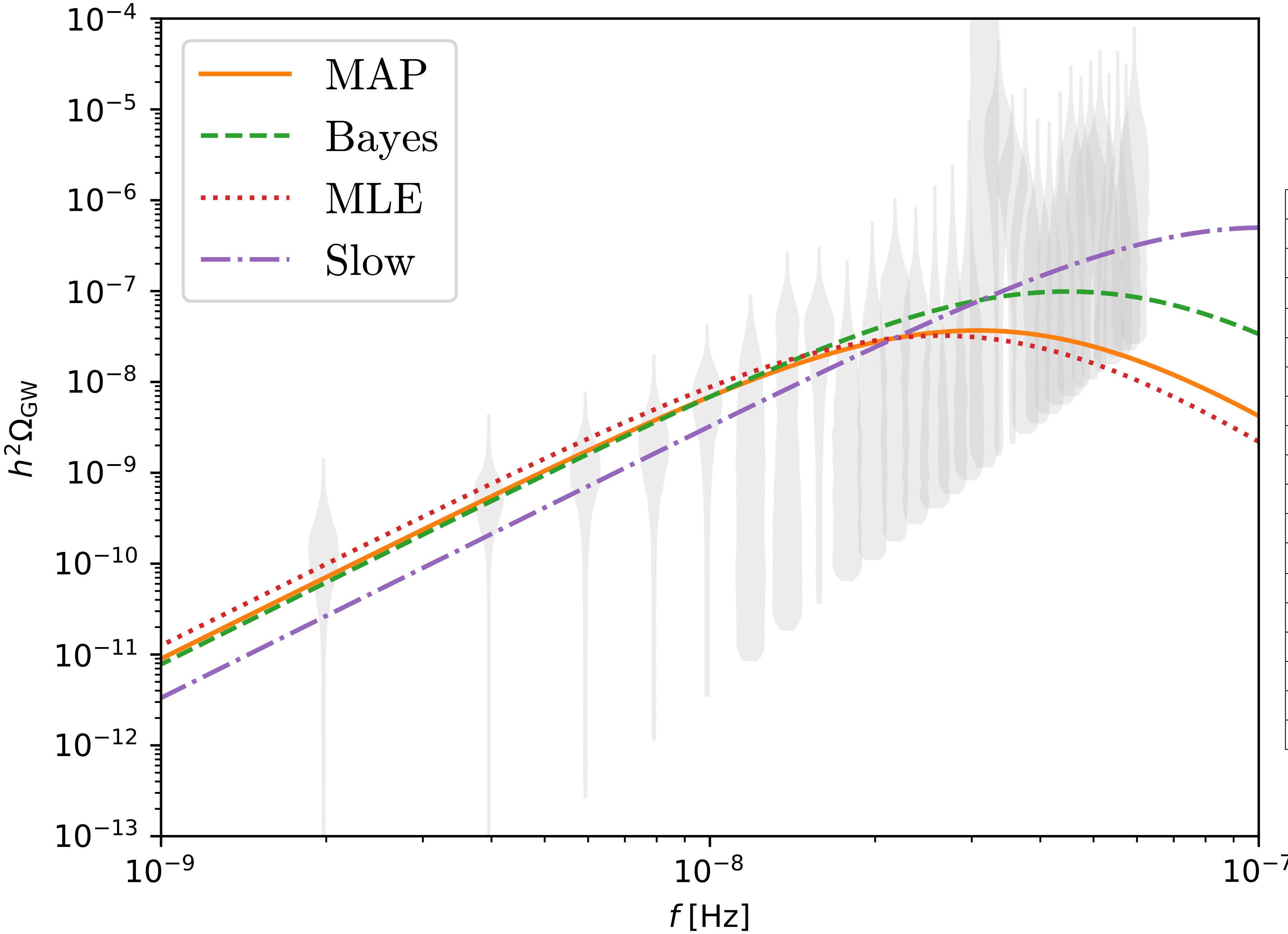


We collected more than 9 millions samples (9,360,965)

Parameter	MAP	Bayes	MLE
$\log_{10}\lambda_\phi$	-2.35	-2.49 ± 0.28	-2.82
$\log_{10} g_D$	-0.16	-0.19 ± 0.07	-0.28
$\log_{10} v_\phi^0$ / MeV	1.64	2.06 ± 0.57	1.67

SGWB predictions

Predicted SGWB spectra in ELENA from the MCMC estimators parameters



Quantity	MAP	Bayes	MLE
λ_ϕ	4.49×10^{-3}	3.21×10^{-3}	1.51×10^{-3}
g_D	0.70	0.64	0.53
v_ϕ^0 (MeV)	44.10	1.14×10^{2}	46.72
m_ϕ (MeV)	4.18	9.14	2.56
$m_{Z'}$ (MeV)	30.75	73.16	24.79
T_{critical} (MeV)	9.50	22.65	7.69
$T_{\mathrm{nucleation}}$ (MeV)	0.91	0.90	0.12
$T_{\mathrm{percolation}}$ (MeV)	0.74	0.68	0.11
$T_{\mathrm{completion}}$ (MeV)	0.68	0.64	0.10
T_{minimal} (MeV)	0.00	0.00	4.13×10^{-2}
$T_{\mathrm{reheating}}$ (MeV)	4.46	10.58	4.03
P_f^{min}	0.00	2.05×10^{-300}	0.00
α	1.31×10^{3}	5.71×10^{4}	1.98×10^{6}
α_∞	39.79	2.62×10^{2}	1.64×10^{3}
α_{eq}	11.07	23.94	47.81
γ_*	4.26×10^{18}	1.19×10^{18}	6.68×10^{17}
γ_{eq}	1.15×10^{2}	2.38×10^{3}	4.14×10^{4}
$R_{\mathrm{sep}*}H_*$	0.14	0.23	0.13

Conclusion

The computation of the nucleation rate in FOPT can be numerically demanding, due to the nature of the bounce equation solutions

We released ELENA, a Python package based on the more efficient tunnelling formalism

ELENA goes beyond common assumptions usually employed in computing SWGB from FOPT

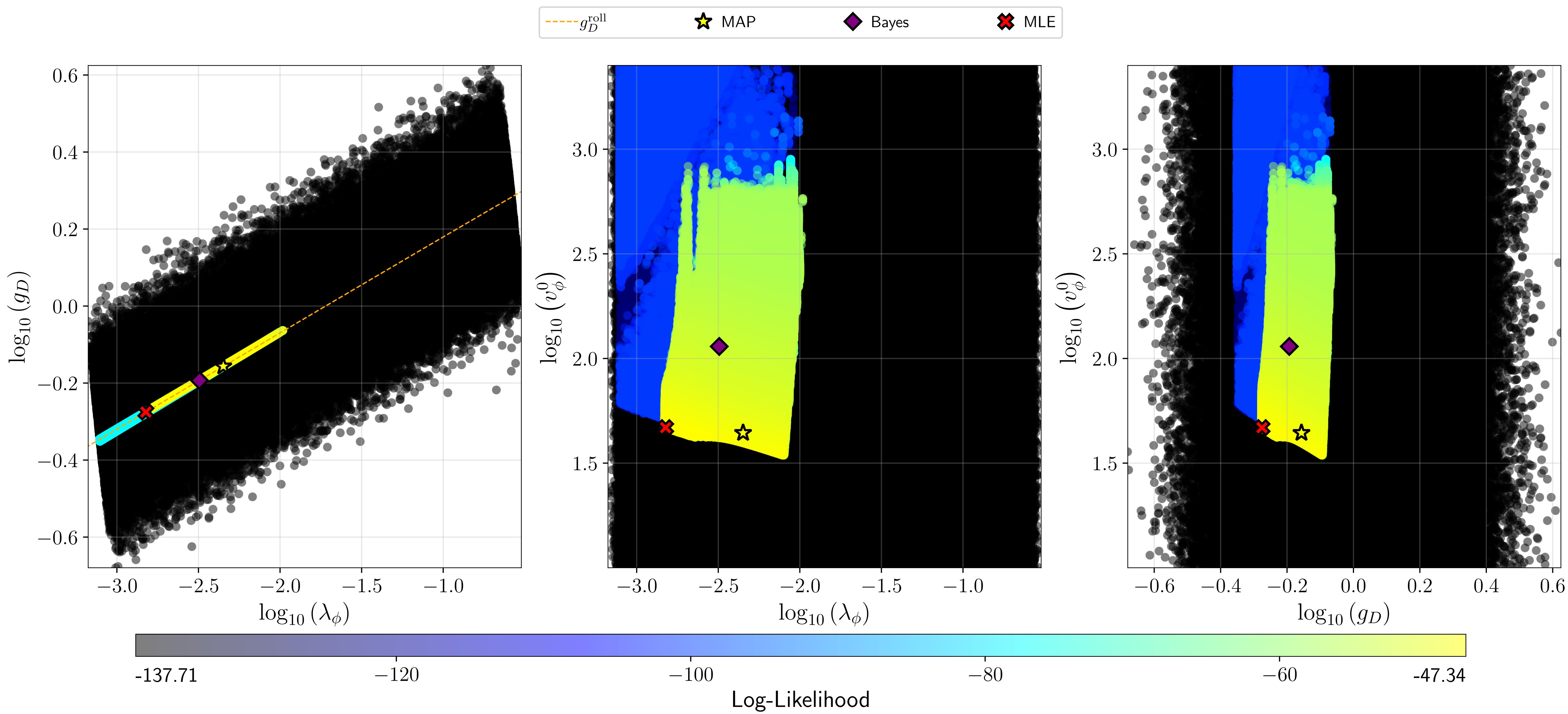
ELENA provides a full pipeline of computation, from Lagrangian parameters to SGWB spectra

We introduced a minimal dark sector model, and interfaced ELENA with PTArcade to perform a MCMC study on NANOGrav data

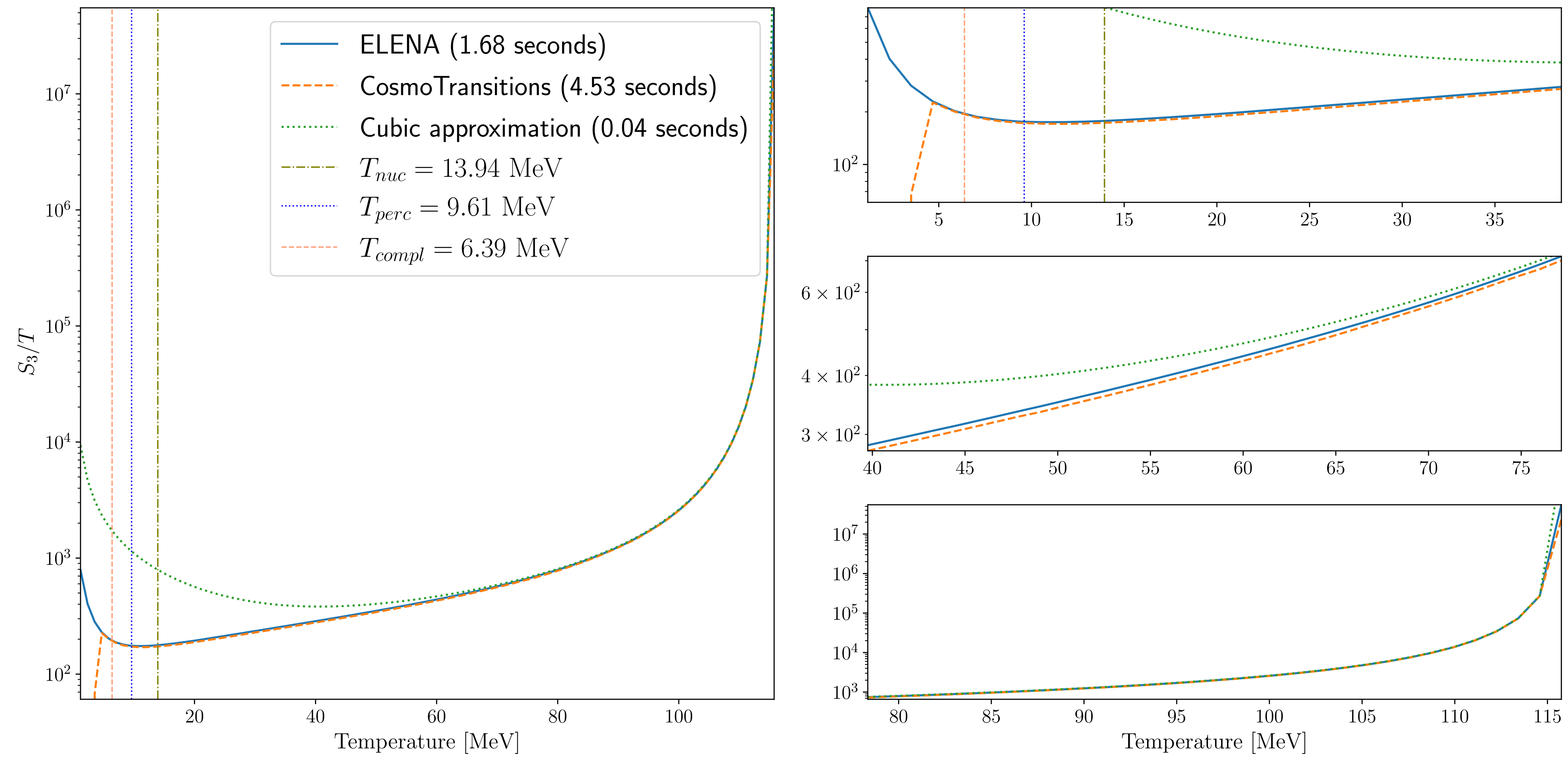
We carefully checked that the model can explain the signal, while complying with several consistency criteria (completion of FOPT, physical true volume evolution, etc.)

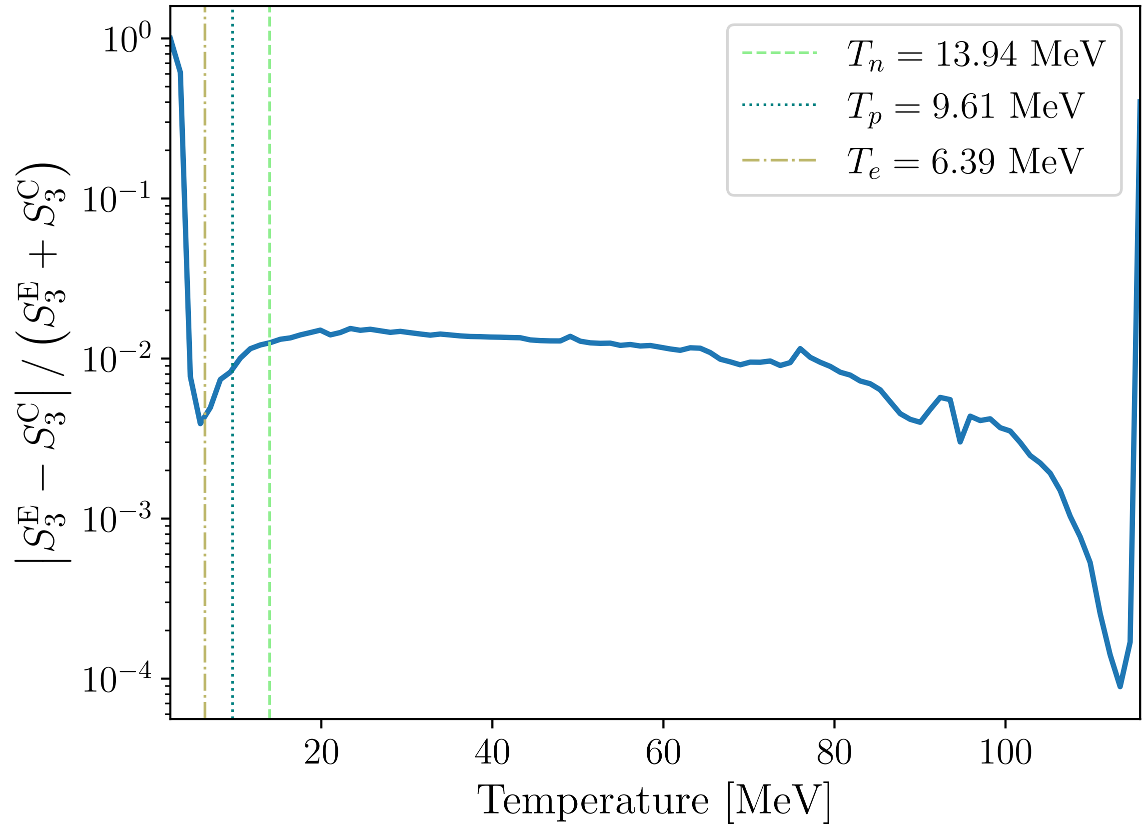
Backup

Log-Likelihood scatter plot

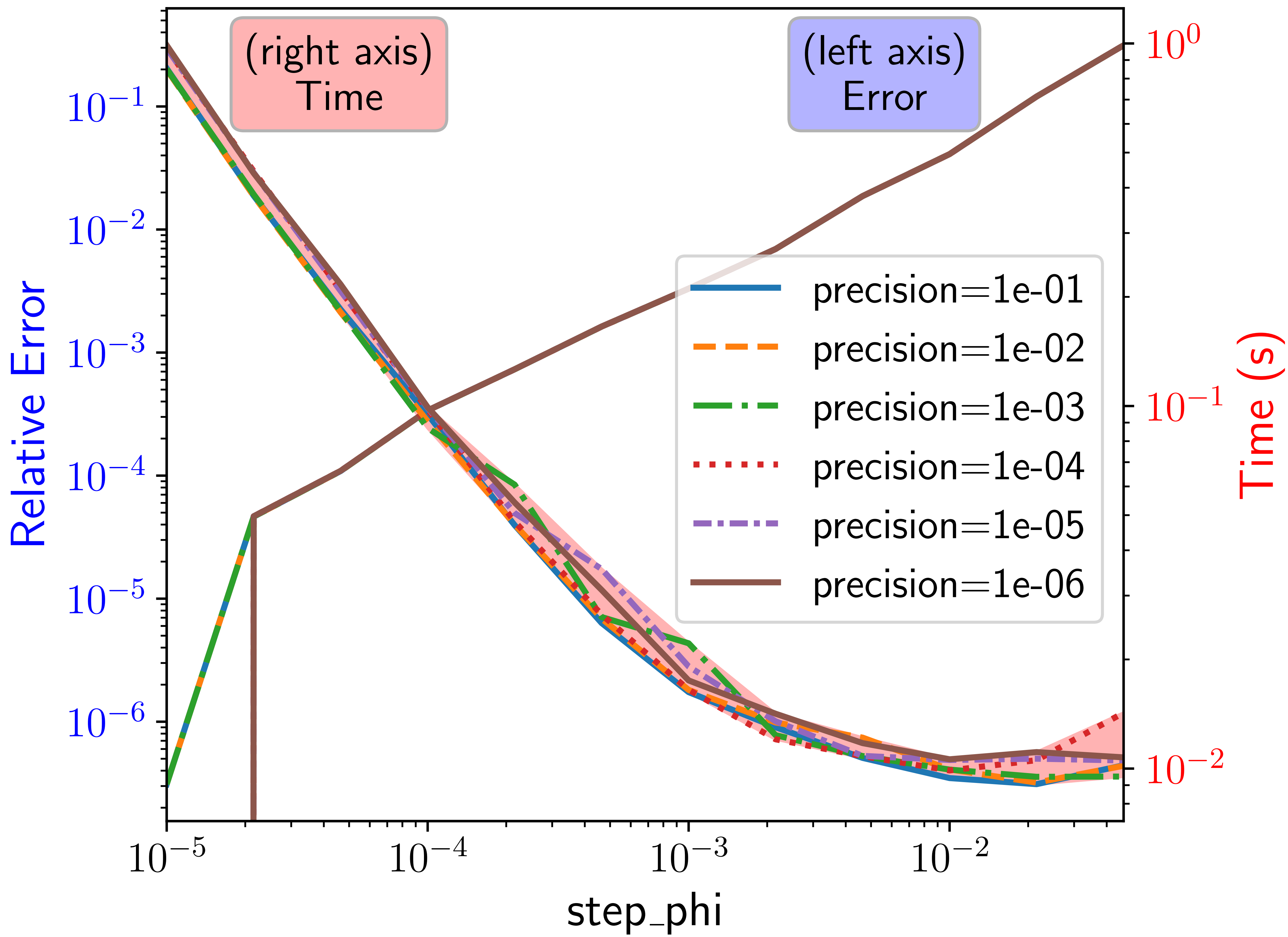


Comparison with CosmoTransitions





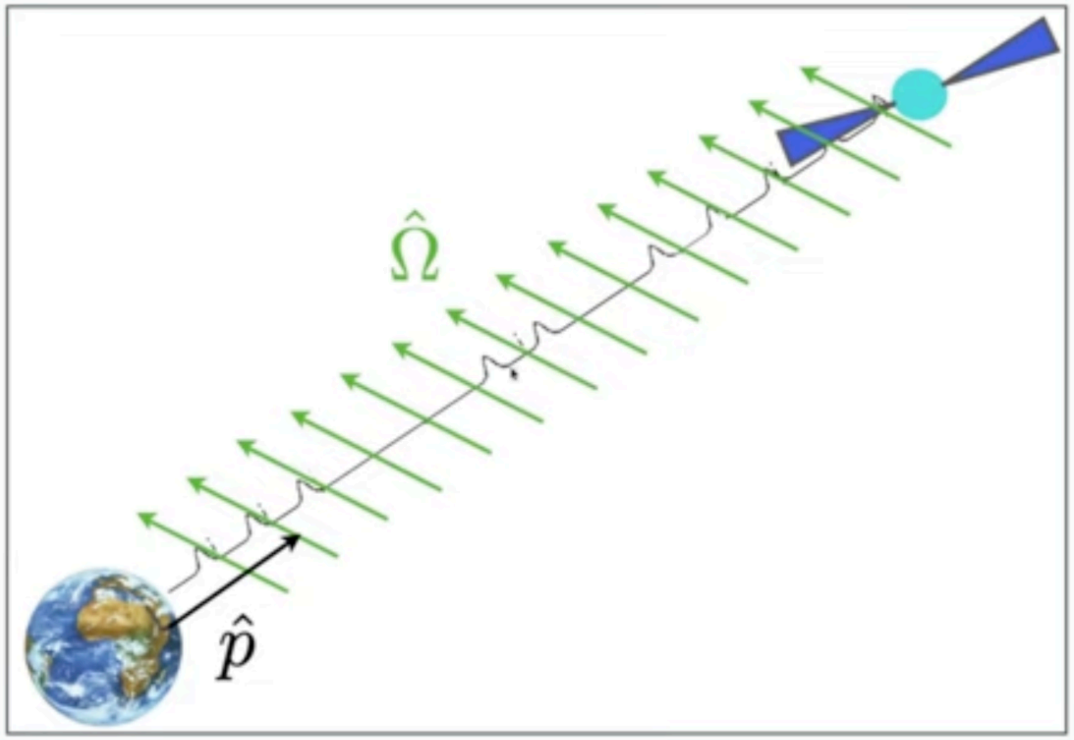
Precision-velocity trade-off in ELENA



Pulsar Timing Arrays

A set of galactic millisecond pulsars, monitored to search for correlations in the pulse time-of-arrival at Earth

Tonia Klein / NANOGrav



Kai Schmitz, Sydney CPPC Seminar 29/08/2024

A monochromatic gravitational wave modifies the pulse period with red/blue shift

$$Z = \frac{1}{2}\frac{\hat{p}^i \hat{p}^j}{1+\hat{\Omega}\cdot\hat{p}}\left[h_{ij}\left(t_{\rm obs}, \mathbf{x}_{\rm earth}\right) - h_{ij}\left(t_{\rm em}, \mathbf{x}_{\rm pulsar}\right)\right]$$

The observable is the timing residual for each pulsar

$$R_a(t) = \int_0^t dt' \; Z(t')$$

International Pulsar Timing Array project

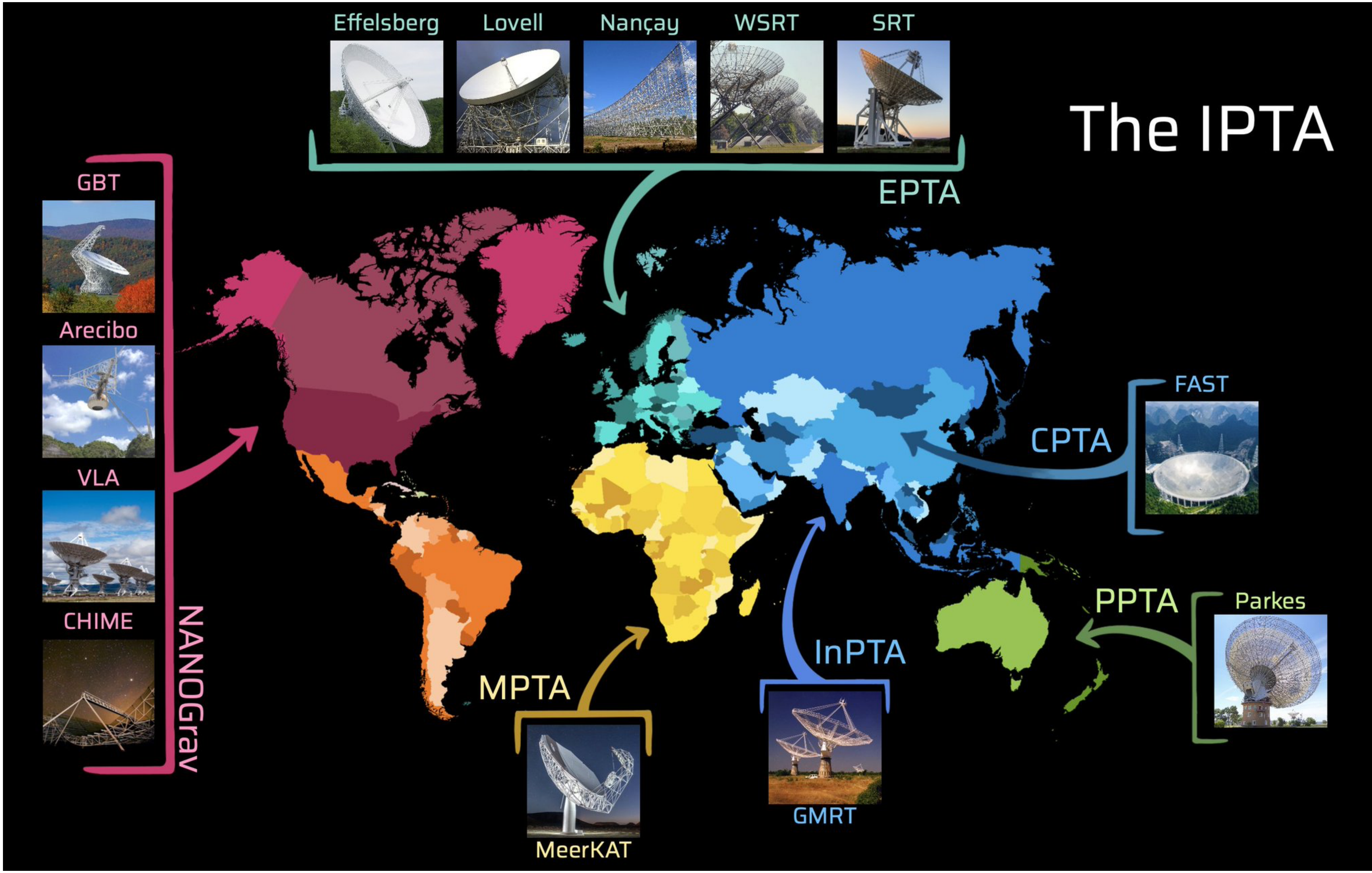
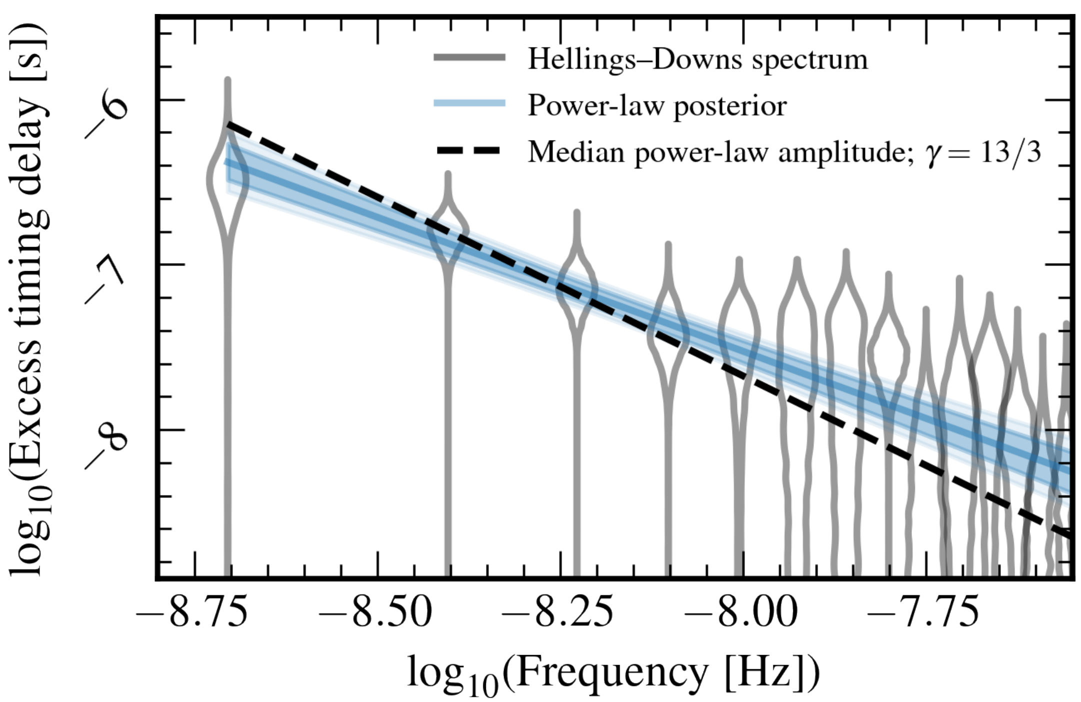


Figure by Thankful Cromartie

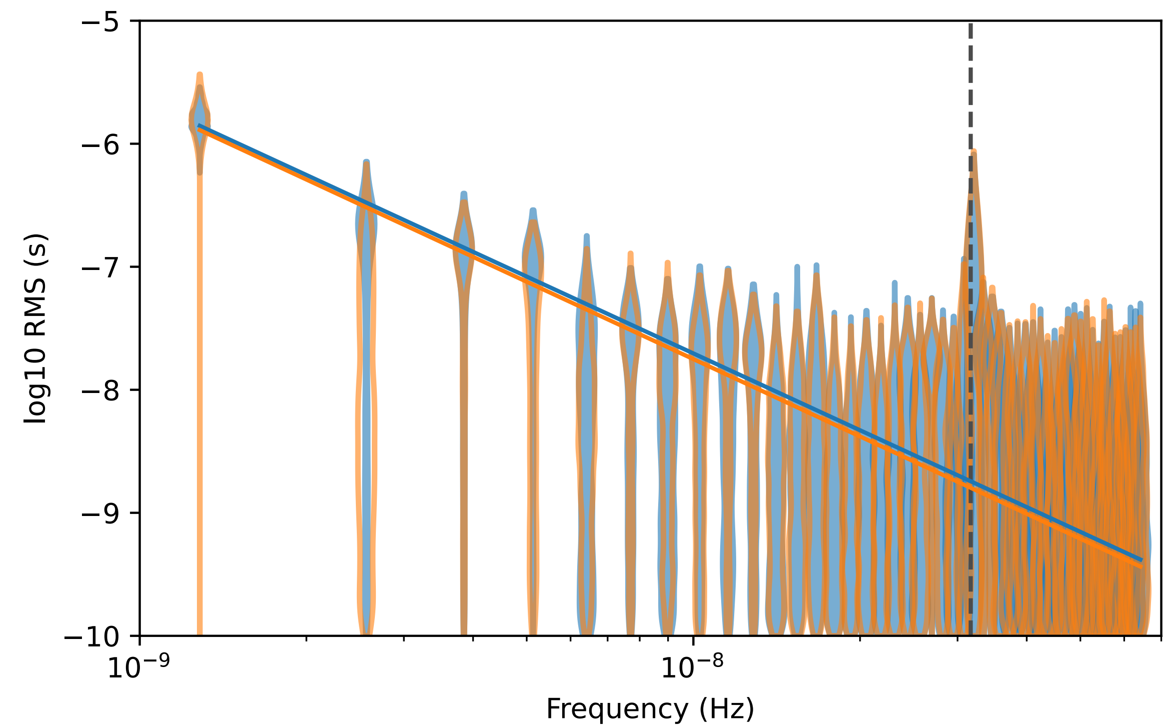
Evidence of a time delay

PTA collaborations observe excess time delay in pulsar timing

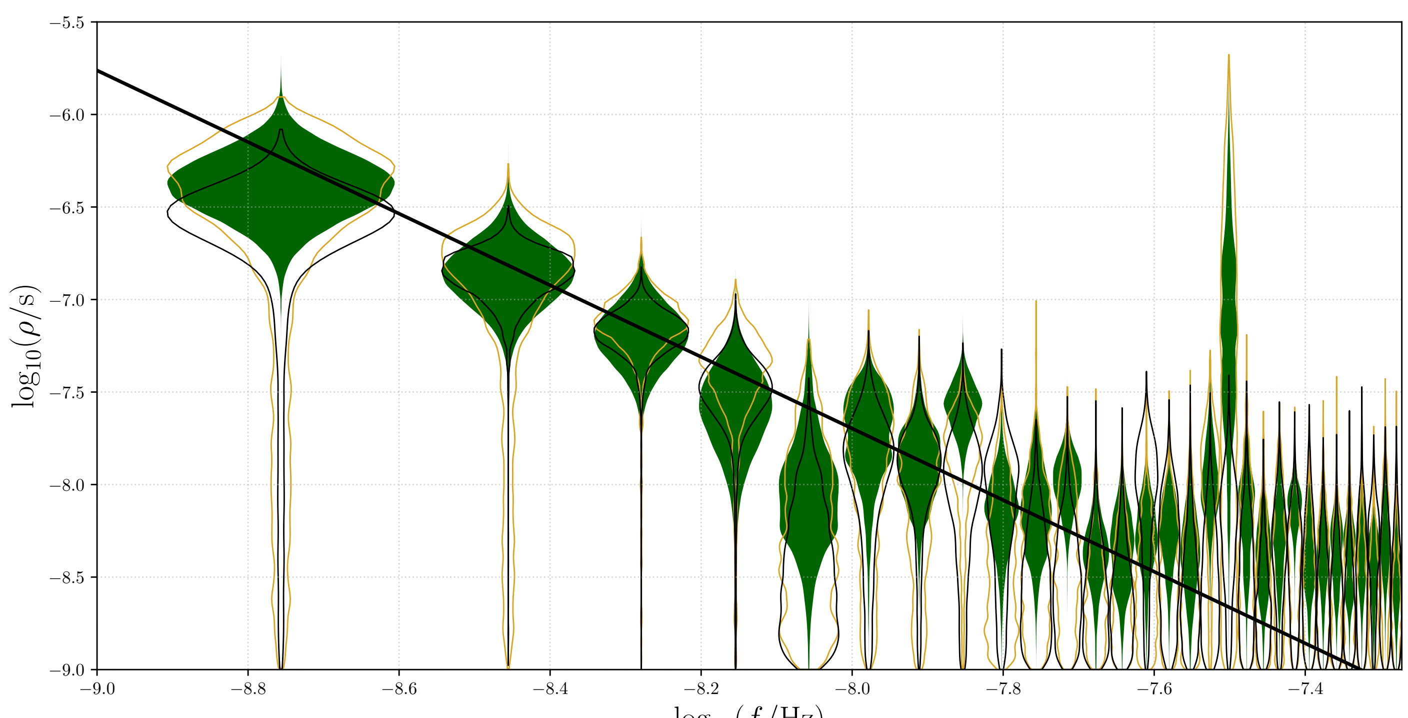
NANOGrav, arXiv:2306.16213 [astro-ph.HE]



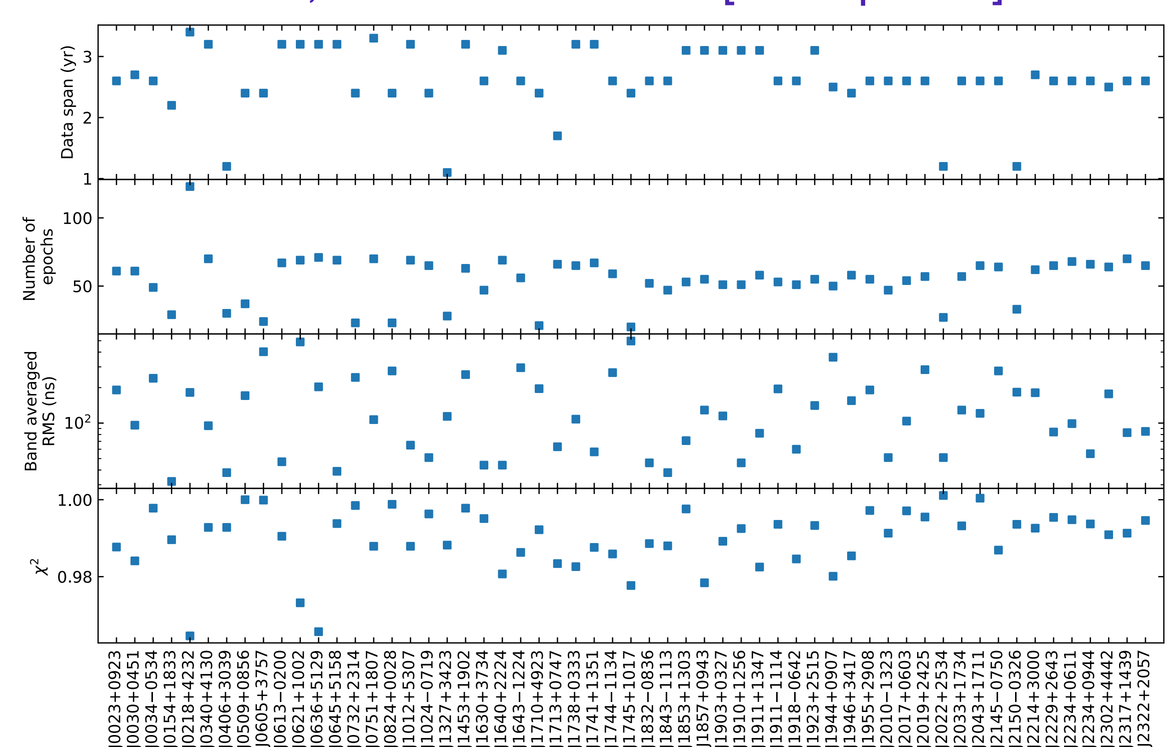
EPTA and InPTA, arXiv:2306.16214 [astro-ph.HE]



PPTA, arXiv:2306.16215 [astro-ph.HE]



CPTA, arXiv:2306.16216 [astro-ph.HE]

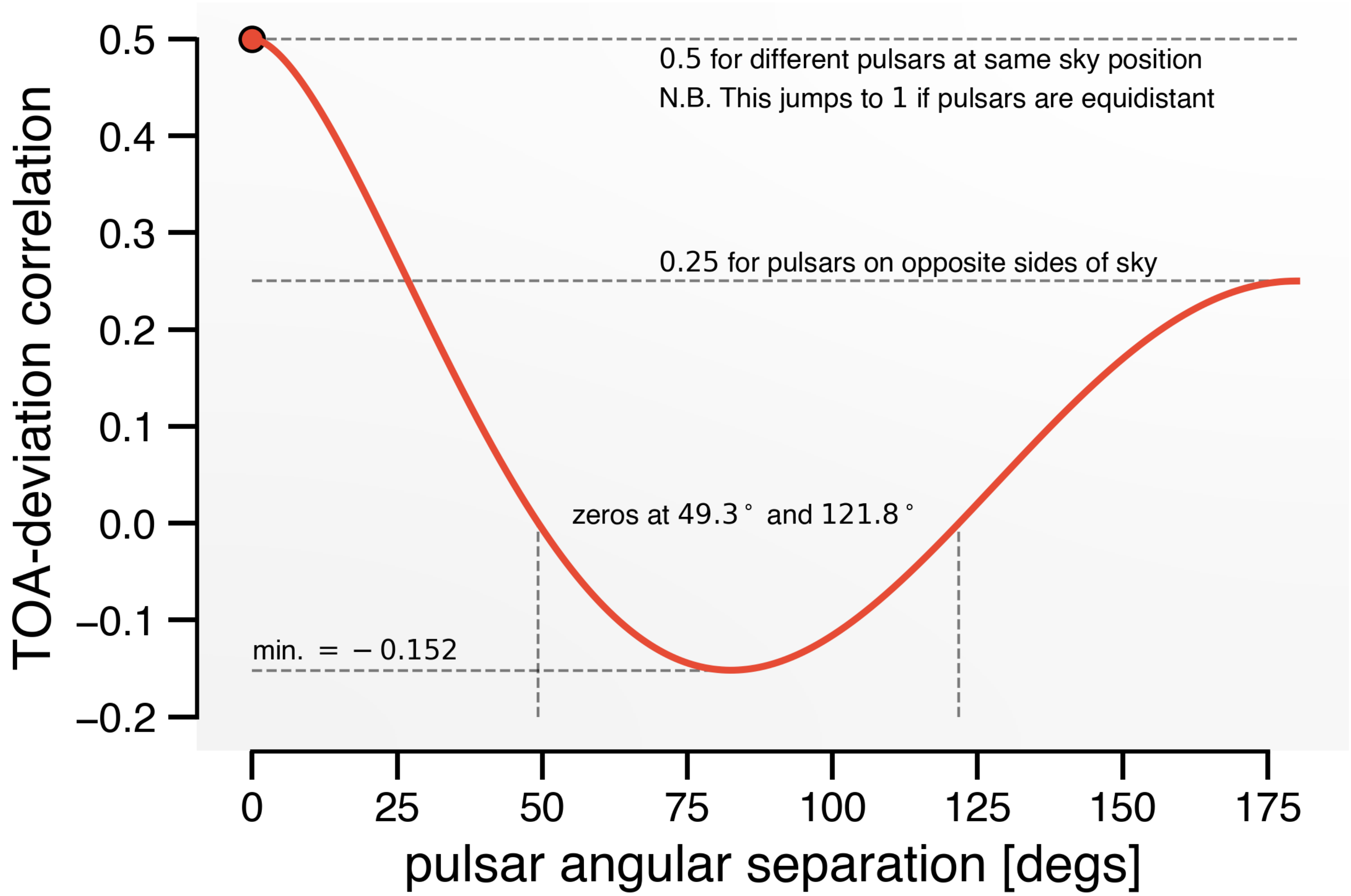


Hellings-Downs curve

The residual timing of a single pulsar is not informative of its origin

If caused by a stochastic GW background, the residuals have a specific correlation among pulsars

R. W. Hellings and G. S. Downs, Astrophys. J. Lett. 265 (1983), L39-L42

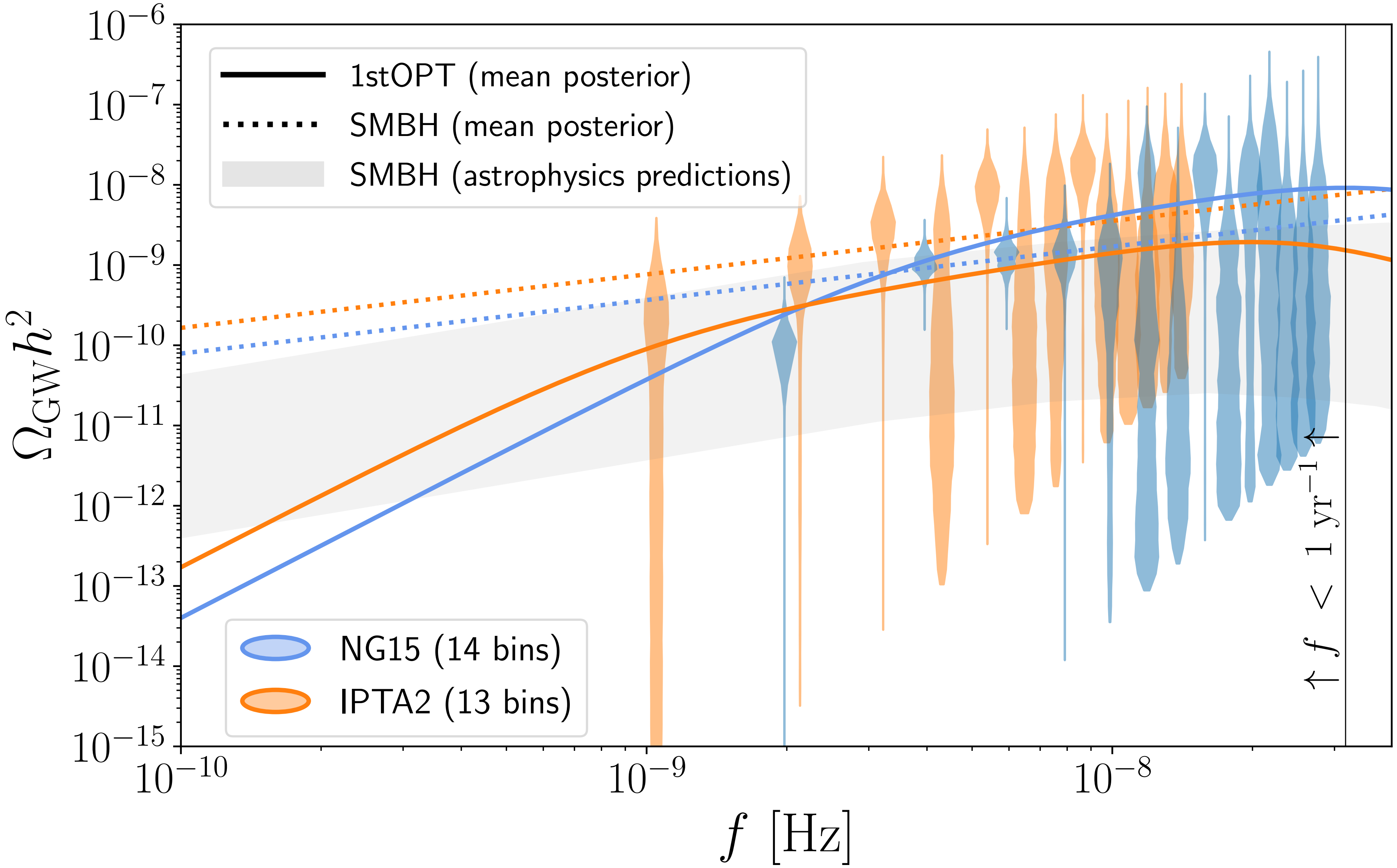


S. R. Taylor, arXiv:2105.13270 [astro-ph.HE]

$$\Gamma_{ij} = \frac{3}{2} x_{ij} \ln(x_{ij}) - \frac{1}{4} x_{ij} + \frac{1}{2} + \frac{1}{2}\delta_{ij} \qquad x_{ij} = \frac{1}{2}\left(1 - \cos\theta_{ij}\right)$$

GW from FOPT

The spectral shape of the SGWB is characteristic of the production mechanism



Y. Gouttenoire, arXiv:2307.04239 [hep-ph]

$$\Omega_b(f) = \mathcal{D}\,\tilde{\Omega}_b\left(\frac{\alpha_*}{1+\alpha_*}\right)^2 (H_* R_*)^2 \mathcal{S}(f/f_b) \qquad \Omega_s(f) = \mathcal{D}\,\tilde{\Omega}_s \Upsilon(\tau_{\mathrm{sw}})\left(\frac{\kappa_s\,\alpha_*}{1+\alpha_*}\right)^2 (H_* R_*) \mathcal{S}(f/f_s)$$

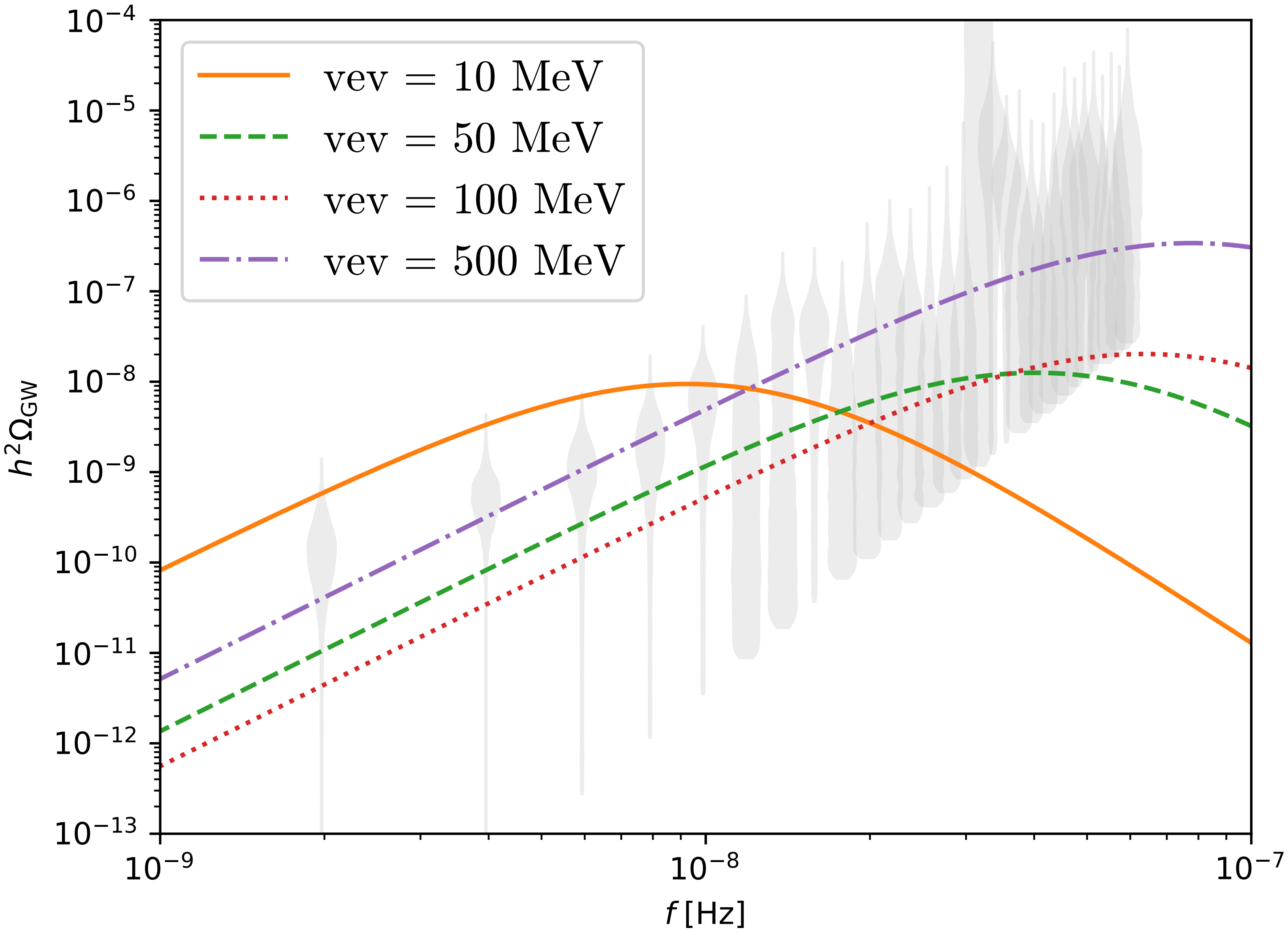
$$\mathcal{D} = \frac{\pi^2}{90}\frac{T_0^4}{M_{\mathrm{Pl}}^2 H_0^2}\, g_* \left(\frac{g_{*,s}^{\mathrm{eq}}}{g_{*,s}}\right)^{4/3} \simeq 1.67\times 10^{-5} \qquad \mathcal{S}(x) = \frac{1}{\mathcal{N}}\frac{(a+b)^c}{(bx^{-a/c}+ax^{b/c})^c}$$

NANOGrav, arXiv:2306.16219 [astro-ph.HE]

The new physics scale

The spectral shape of the SGWB signal observed by PTAs is peaked around 10 nHz

For a FOPT we expect $f_{\text{peak}}\,[\text{Hz}] \approx 10^{-8}\left(\frac{T_*}{100\ \text{MeV}}\right)$



$\lambda = 6 \times 10^{-3}$

$g = 0.74986$

New physics scale lives at the sub-Gev scale

The potential shape

The amplitude of the signal requires a slow transition, typically realised in conformal-like potentials

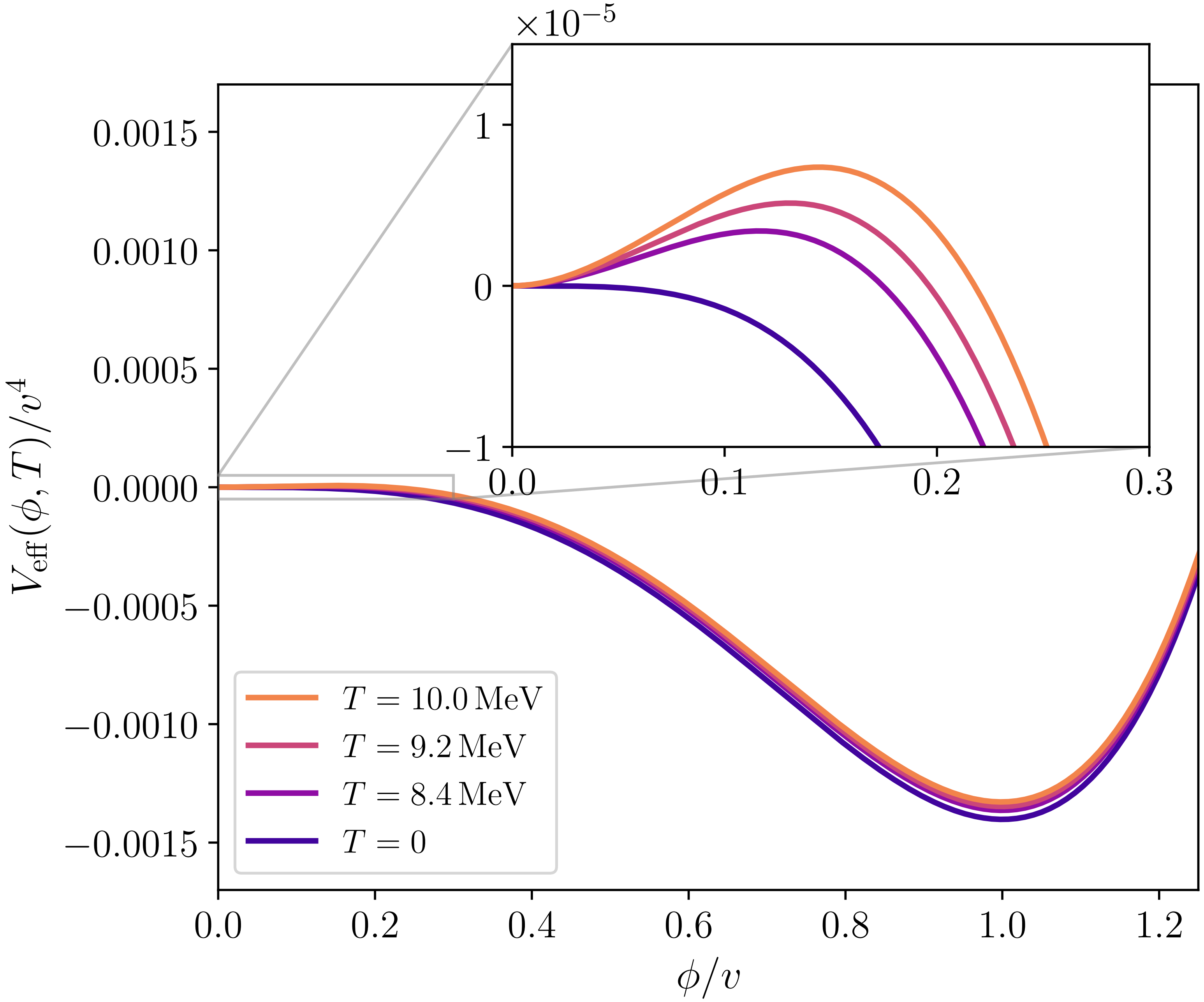
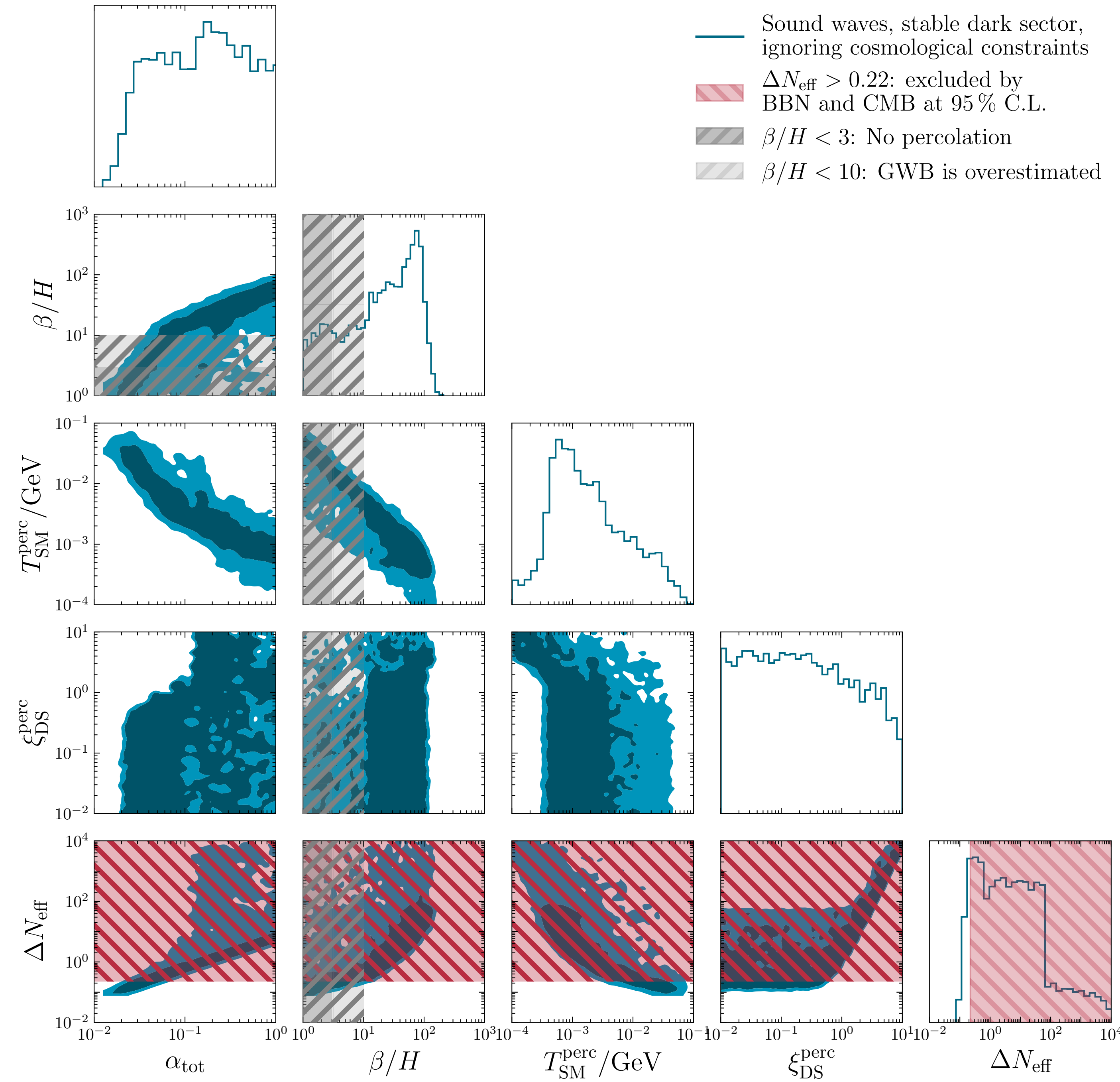


Figure from: S. Balan, T. Bringmann, F. Kahlhoefer, J. Matuszak and C. Tasillo,arXiv:2502.19478 [hep-ph]

A barrier is present until low temperatures to delay the bubble nucleation from falso to true vacuum

Portals with the Standard Model

A fully secluded and stable dark sector to account for PTA data is in tension with cosmology



T. Bringmann, P. F. Depta, T. Konstandin, K. Schmidt-Hoberg and C. Tasillo, arXiv:2306.09411 [astro-ph.CO]

The dark sector must have portals to decay into SM states

Example: Higgs portal ($m_\phi > 2\ m_\mu$)

Consider the most general gauge-invariant scalar potential

$$V_{UV} = -\mu_H^2 H^\dagger H + \lambda_H \left(H^\dagger H\right)^2 + (-\mu_\phi^2 \phi^* \phi) + \lambda_\phi \left(\phi^* \phi\right)^2 + \lambda_{H\phi} \left(H^\dagger H\right) \left(\phi^* \phi\right)$$

We can assume the DS mass term comes from the Higgs sector portal

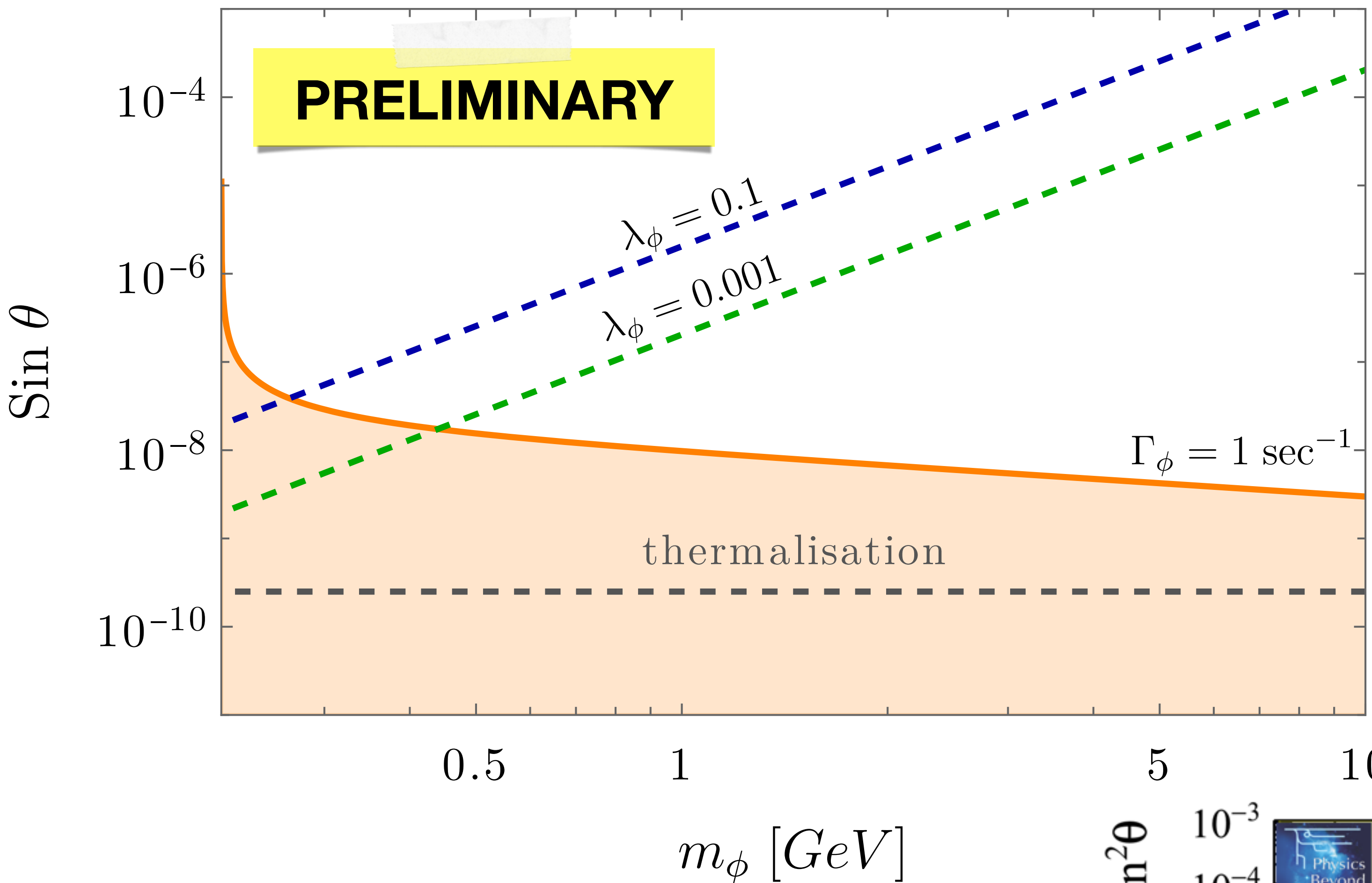
$$\tilde{\mu}_\phi^2 \equiv -\frac{1}{2} \lambda_{H\phi} v_H^2$$

$$\tan 2\theta = \frac{\lambda_{\phi H} v_h v_\varphi}{-\lambda_\varphi v_\varphi^2 + \lambda_H v_h^2} \qquad \sin\theta \simeq \frac{\lambda_{H\phi}}{\lambda_H} \frac{w}{v}$$

$$h_{phys} = \cos\theta h + \sin\theta\phi$$
$$\phi_{phys} = -\sin\theta h + \cos\theta\phi$$

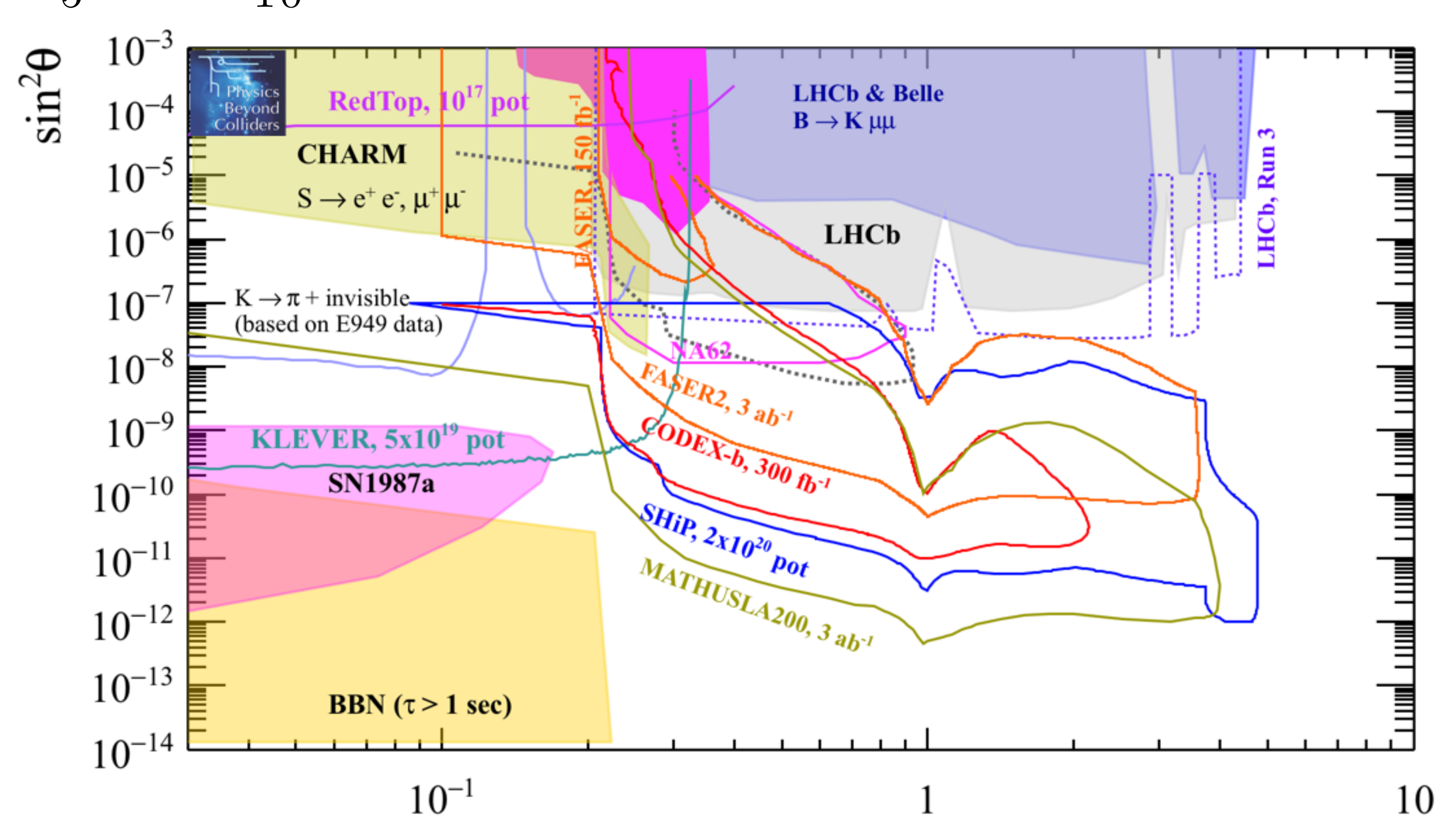
$$m_{h/\varphi}^2 = \lambda_\phi v_\varphi^2 + \lambda_H v_h^2 \pm \sqrt{\left(-\lambda_\phi v_\varphi^2 + \lambda_H v_h^2\right)^2 + \lambda_{\phi H}^2 v_h^2 v_\varphi^2}$$

Higgs portal ($m_\phi > 2\ m_\mu$)



$$\Gamma = \frac{y_\mu^2 \sin^2\theta\ m_\varphi^2}{16\pi}\left(1 - 4\frac{m_\mu^2}{m_\varphi^2}\right)^{3/2}$$

A single dark scalar-Higgs coupling allows for DS thermalisation and decay



Physics Beyond Colliders at CERN: Beyond the Standard Model Working Group Report, arXiv:1901.09966 [hep-ex]

Example: neutrino portal ($m_\phi < 2\, m_\mu$)

The electron Yukawa is too small to allow fast decay via Higgs portal

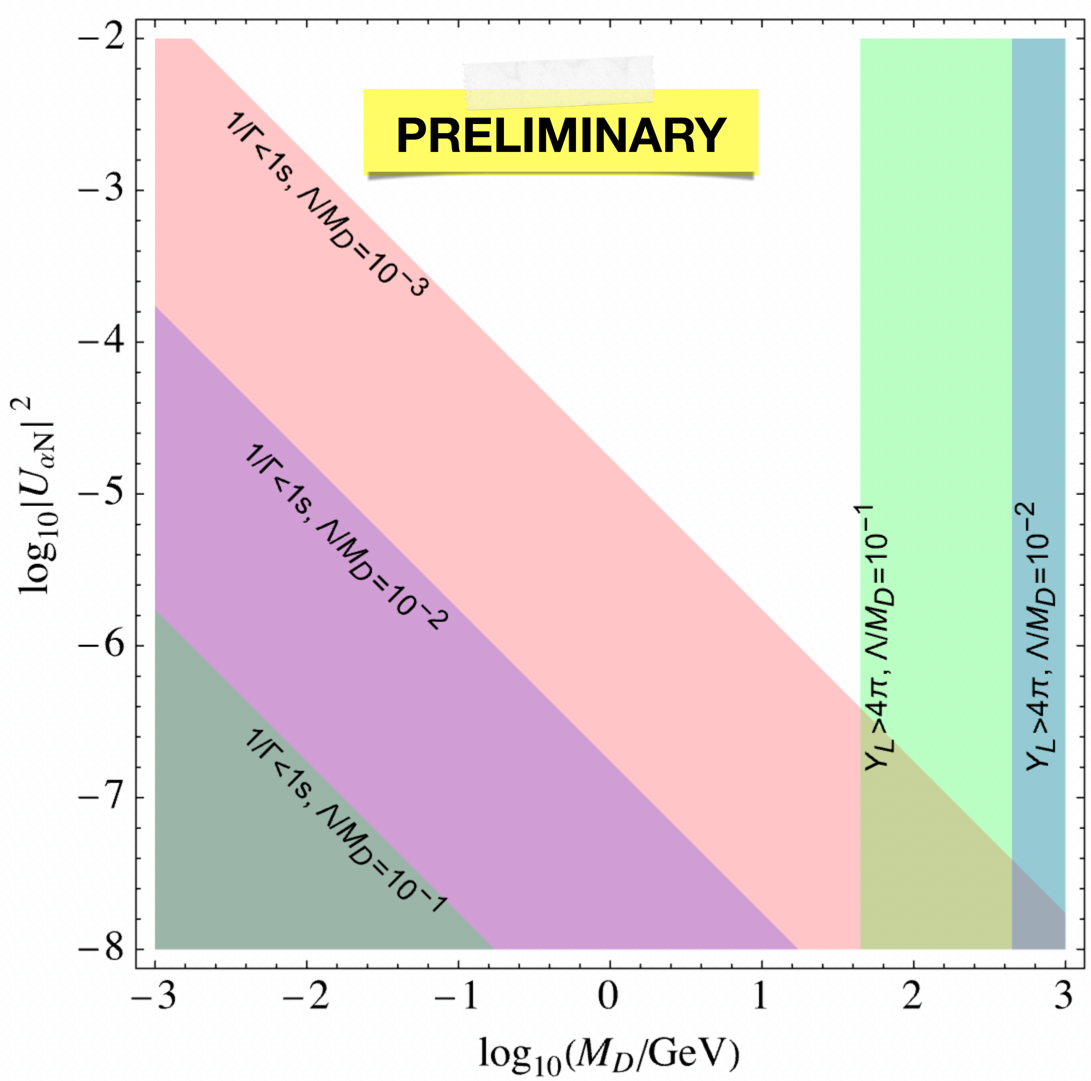
Consider a neutrino portal: add right-handed neutrinos N_R and dark fermions ν_D

$$\mathcal{L} \supset -\bar{L}_L \tilde{H} Y_\nu N_R - \frac{1}{2}\bar{N}_R^c \mu N_R - \bar{\nu}_D Y_{D_L} N_R \phi - \bar{\nu}_D^c Y_{D_R} N_R \phi^* - \bar{\nu}_D M_D \nu_D + h.c.$$

After symmetry breaking the mass matrix is generated

in the basis $n = \begin{pmatrix} \nu_L^c & N_R & \nu_{D_L}^c & \nu_{D_R} \end{pmatrix}$

$$\mathcal{M} = \begin{pmatrix} 0 & m_D & 0 & 0 \\ m_D^T & \mu & \Lambda_{D_L}^T & \Lambda_{D_R}^T \\ 0 & \Lambda_{D_L} & 0 & M_D \\ 0 & \Lambda_{D_R} & M_D^T & 0 \end{pmatrix}$$



The channel $\phi \to \nu\, \nu$ can allow rapid decay of the dark sector

$$\Gamma \simeq \frac{m_\varphi}{128\pi}\frac{v_\varphi}{M_D}\sum_\alpha |U_\alpha|^4 \sum_{X=L,R} Y_{D_X}^4$$