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NEUTRINO OBSERVATORY



WIPAC

Energy-dependent Orbital Compton-Getting Dipole with IceCube Cosmic-Ray Muons

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Solar Dipole: a known anisotropy

due to Earth's revolution around the Sun

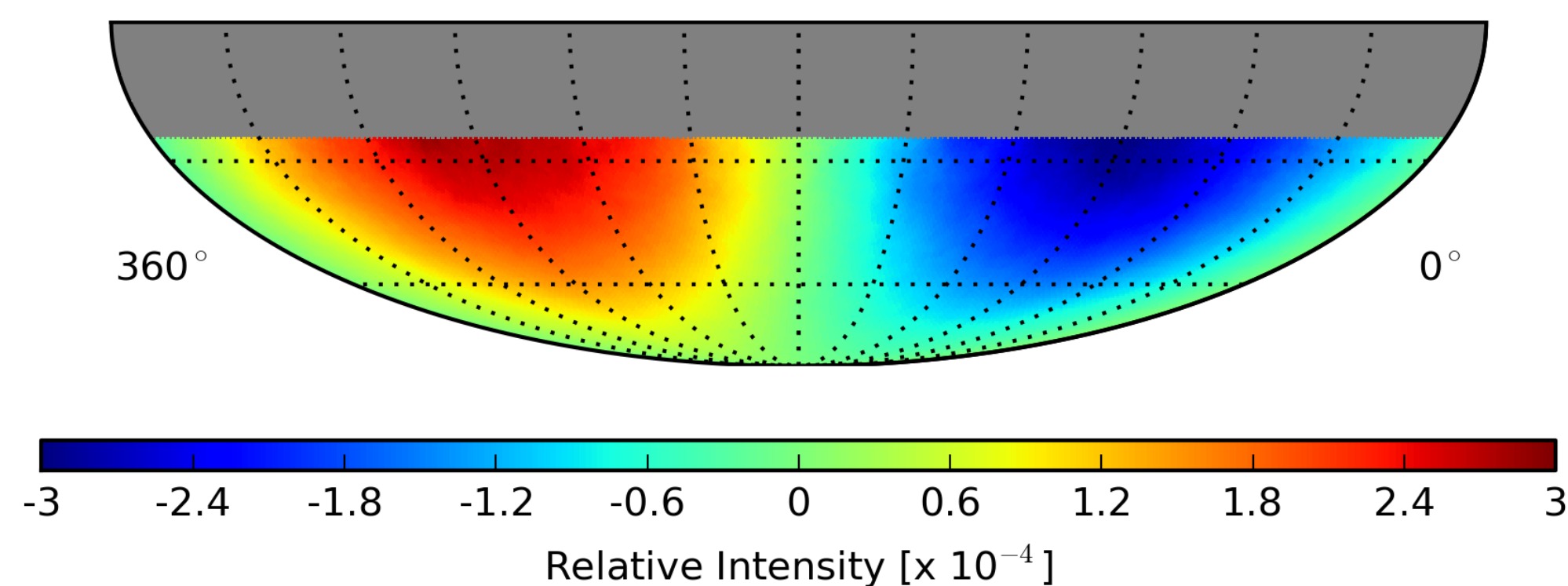
Arthur H. Compton and Ivan A. Getting
Phys. Rev. 47, 817 – Published 1 June 1935
[doi:10.1103/PhysRev.47.817](https://doi.org/10.1103/PhysRev.47.817).



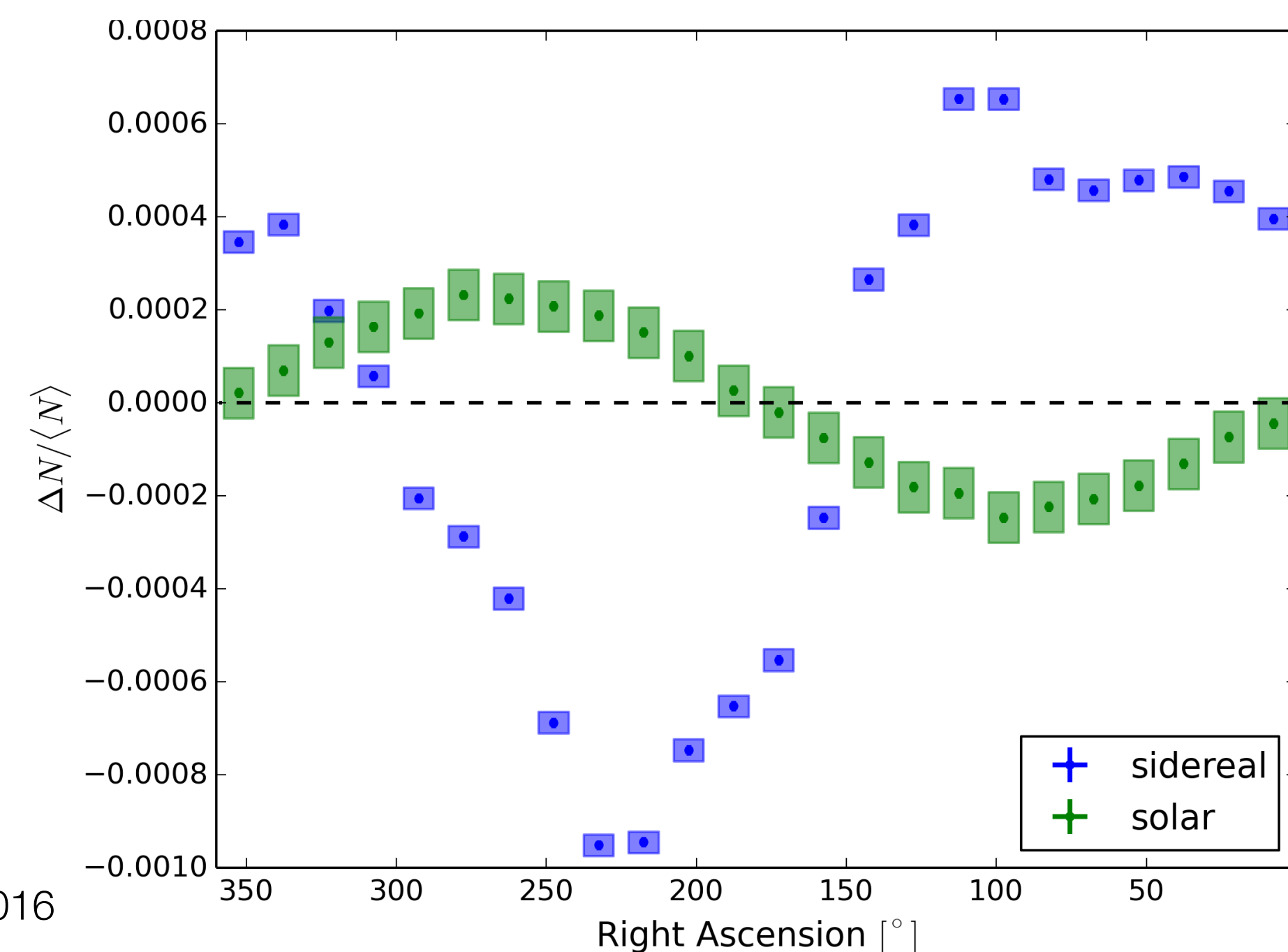
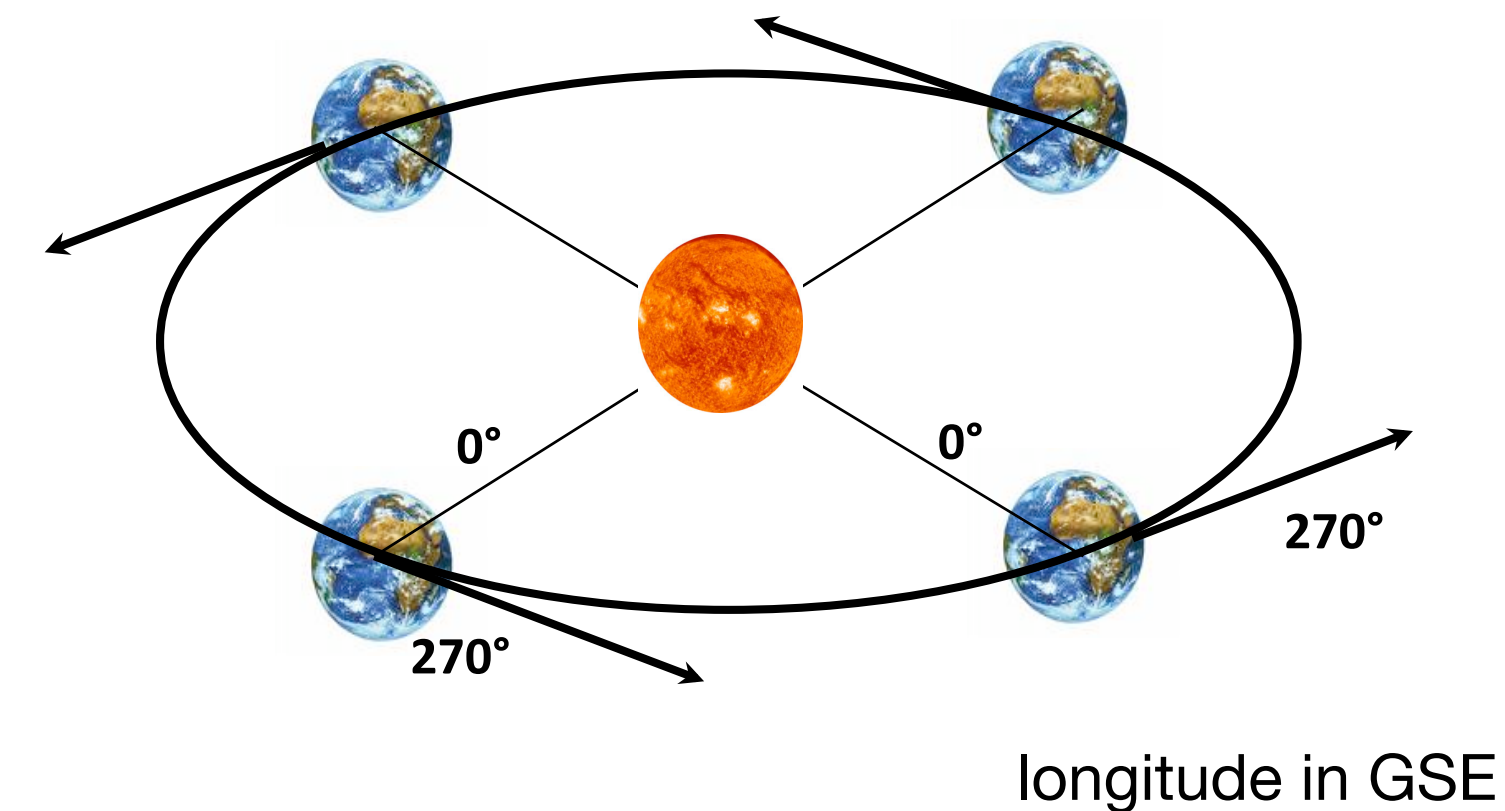
apparent energy-independent $\sim 10^{-4}$ dipole anisotropy due to relative motion of solar system through ISM

motion of solar system around galactic center ~ 29 km/s

$$\frac{\delta I}{I} = \frac{v}{c} (\gamma + 2) \cos(\xi)$$



$$\langle v \rangle = 29.8 \pm 0.5 \text{ km/s}$$



Solar Dipole: a known anisotropy

due to Earth's revolution around the Sun

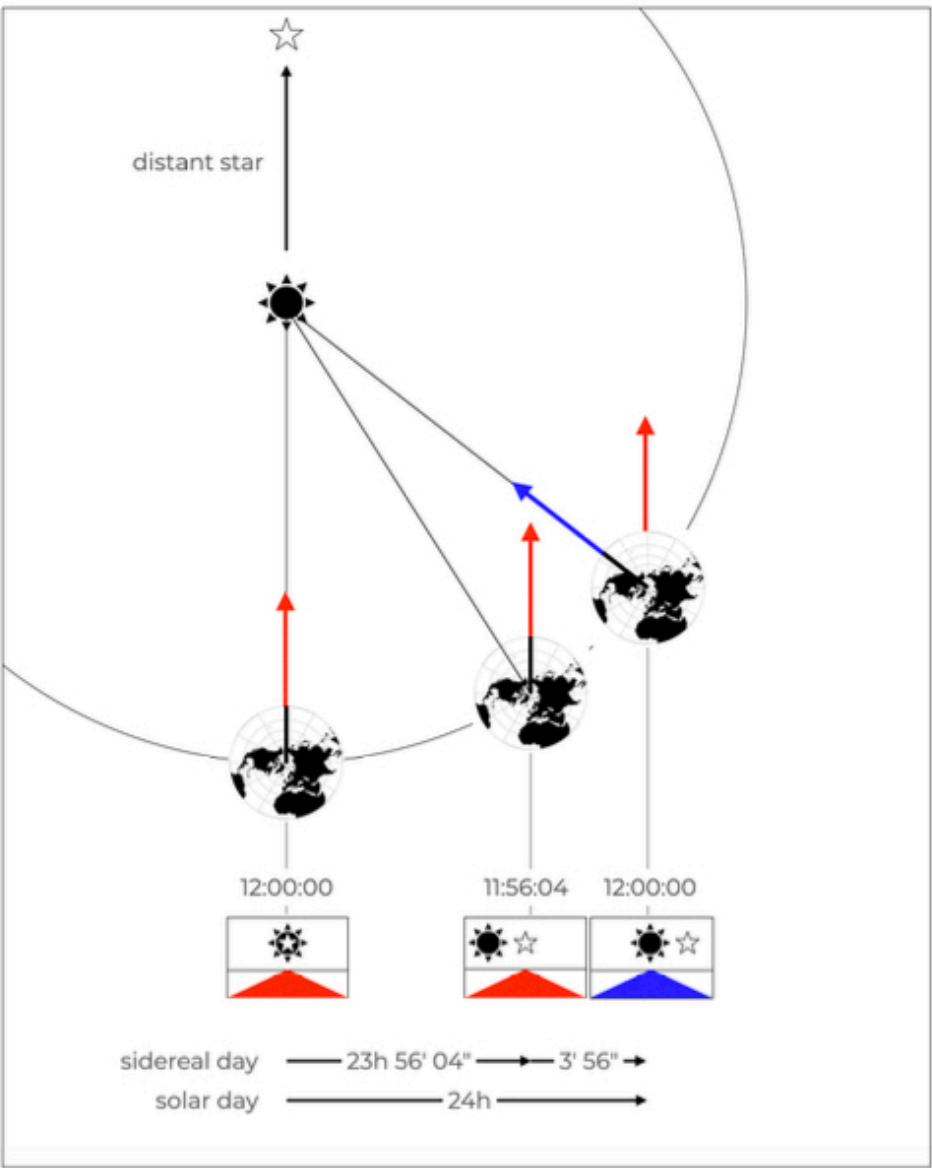
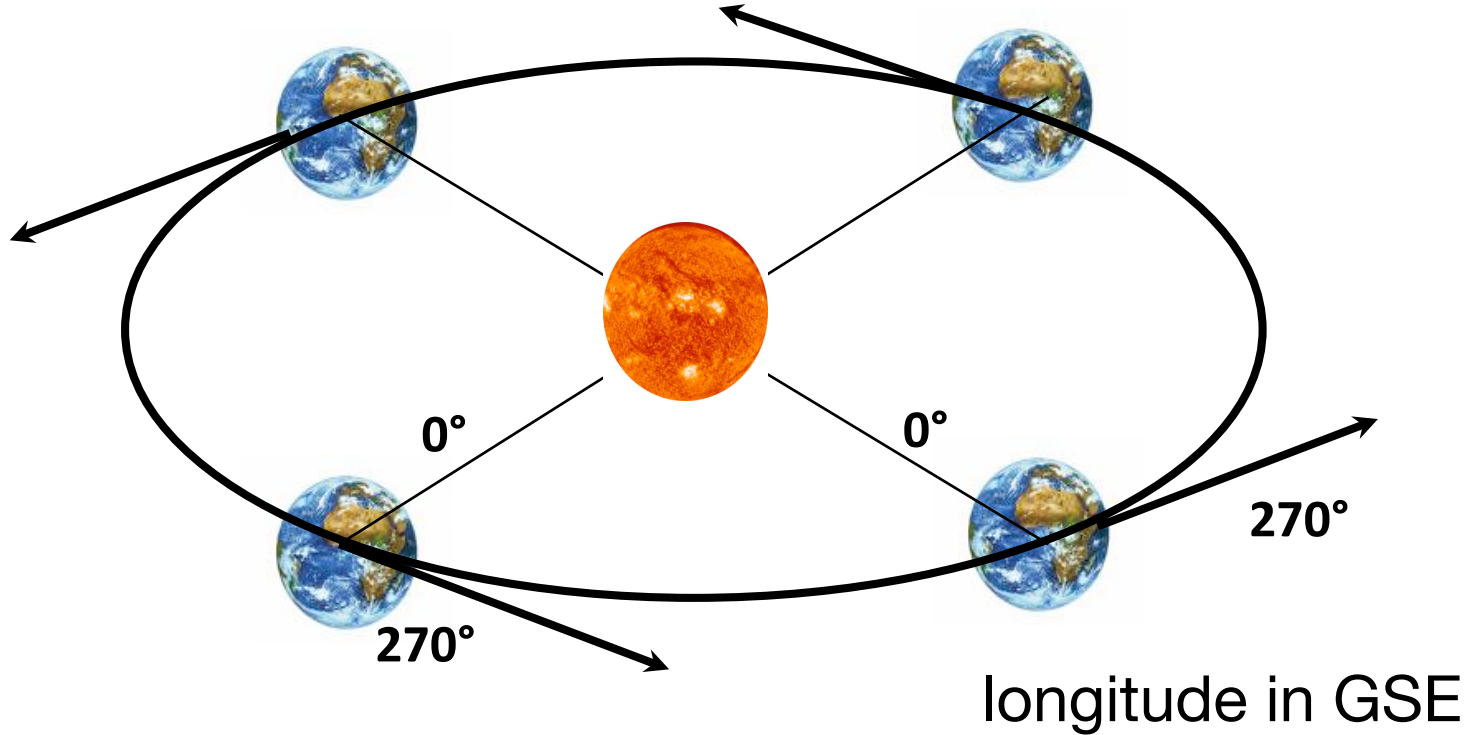
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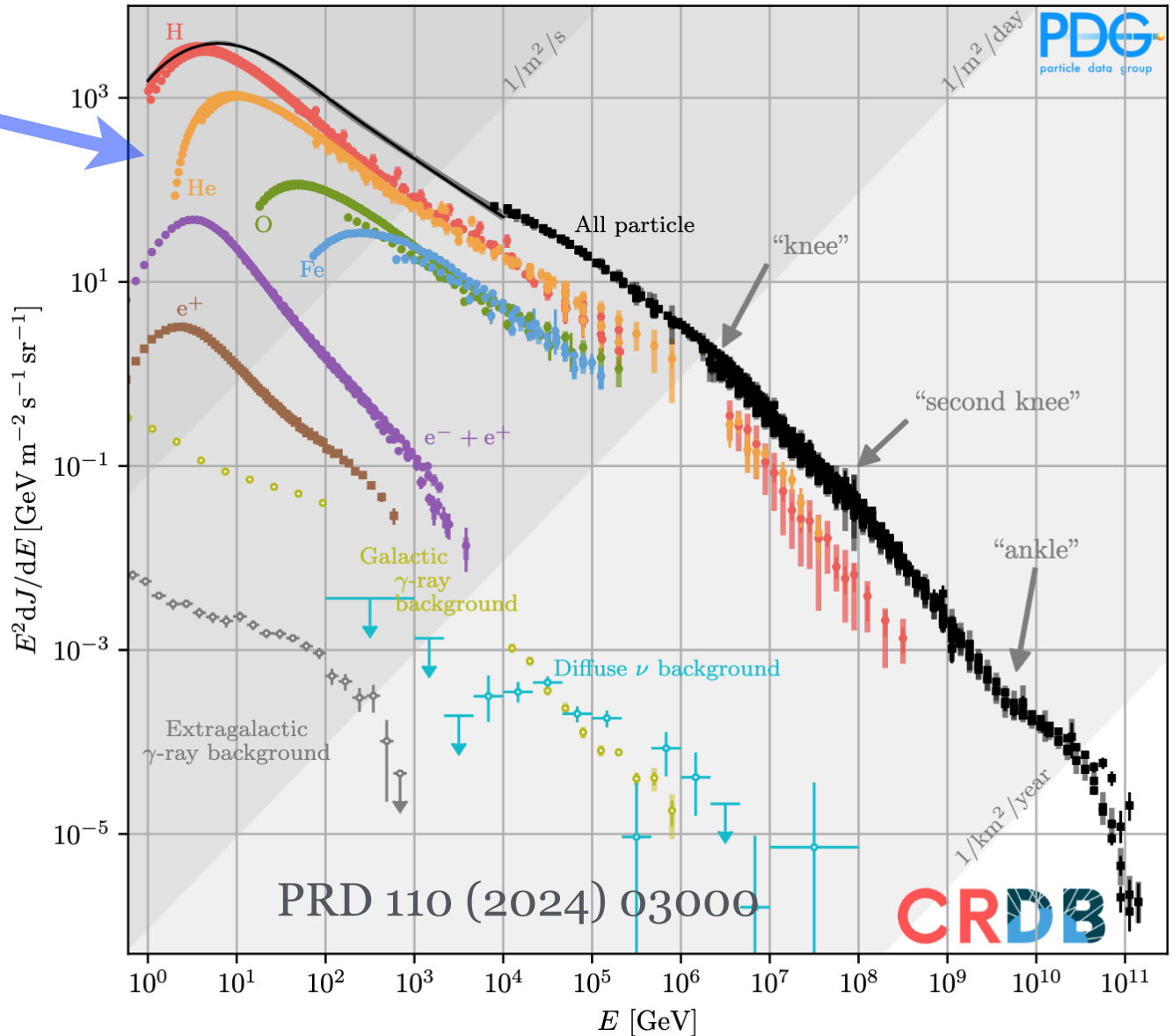
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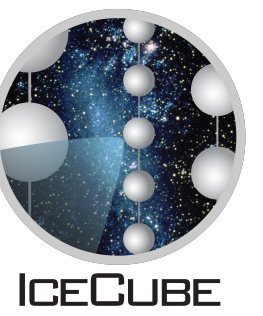
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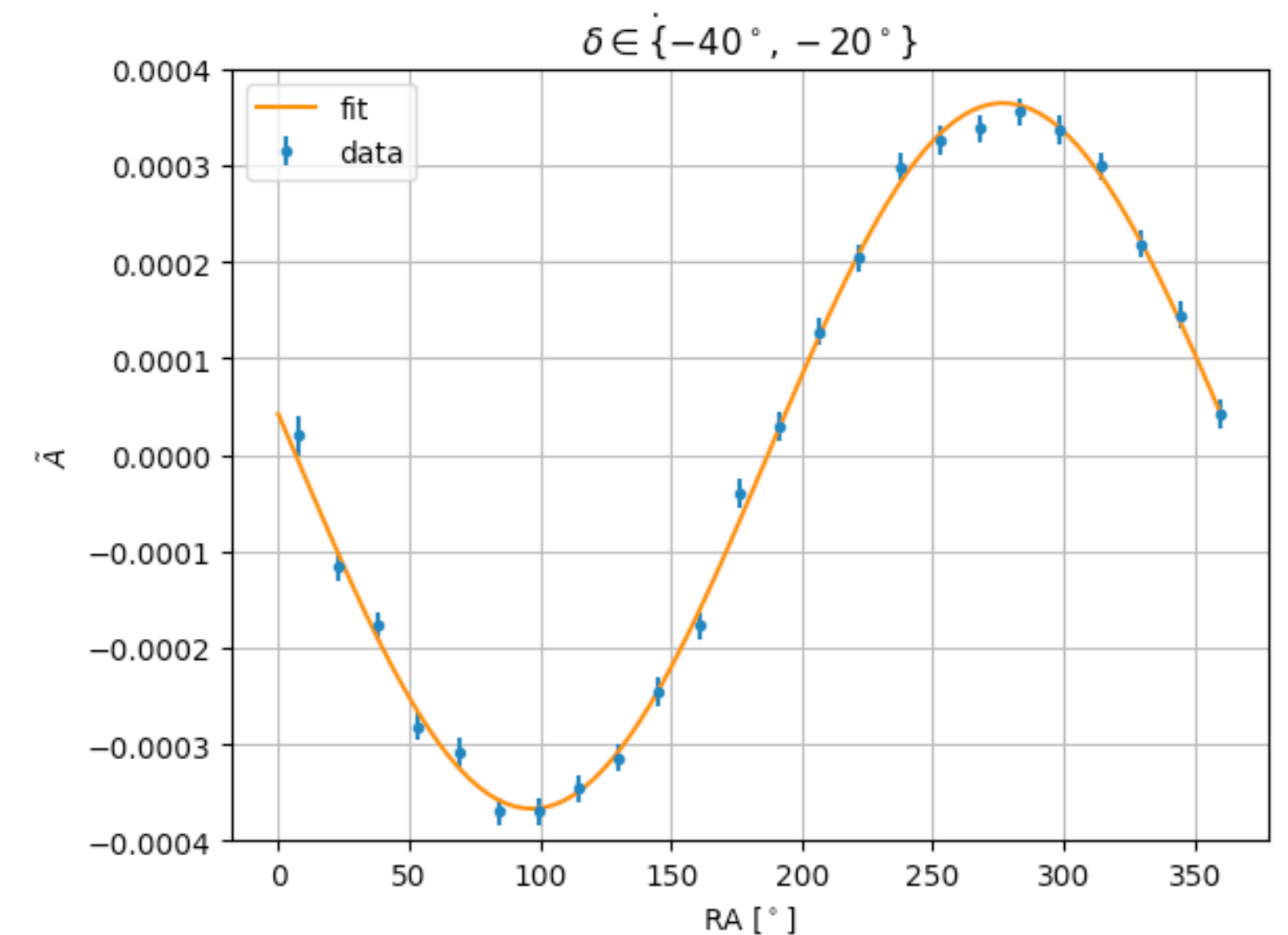
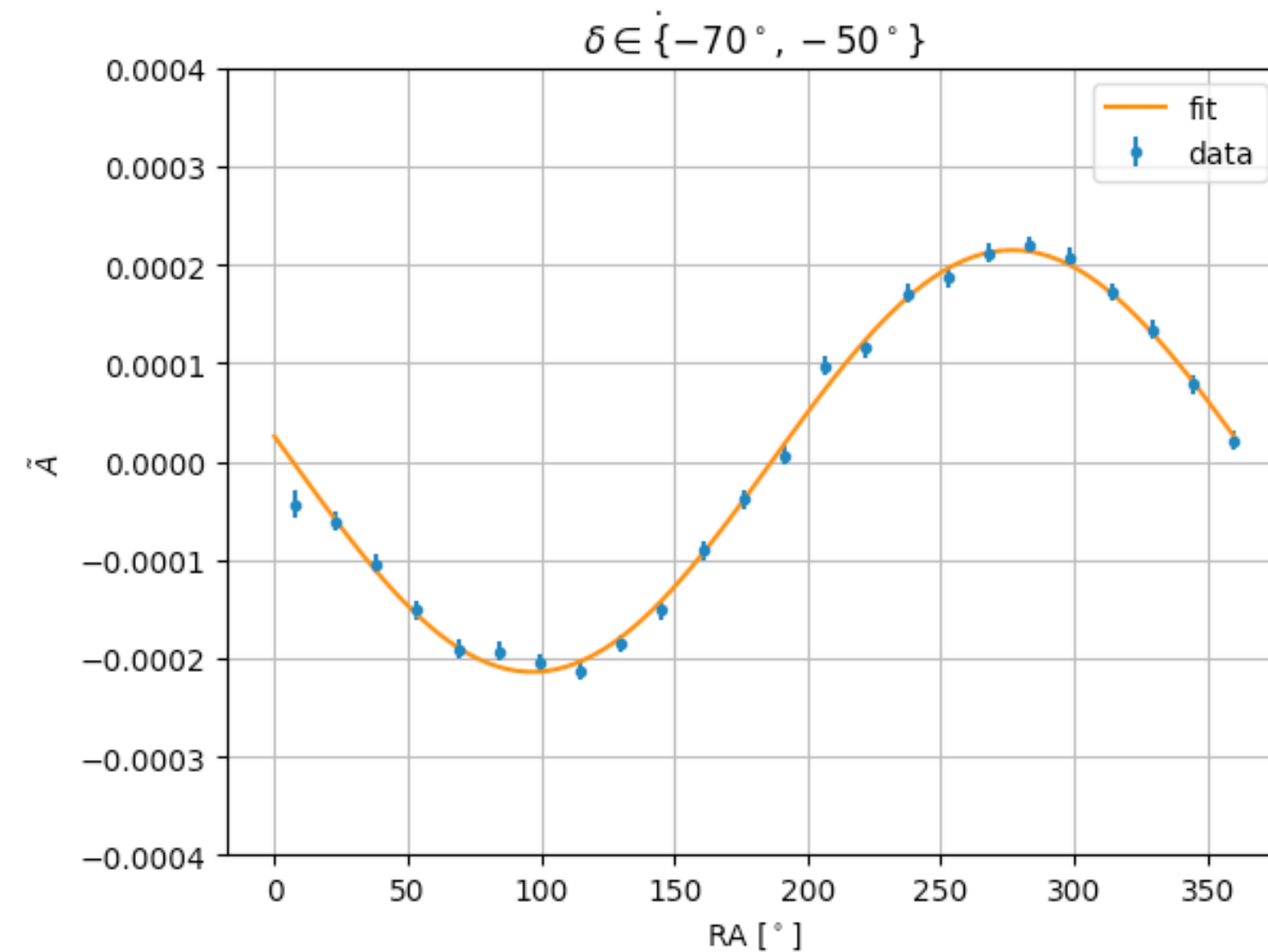
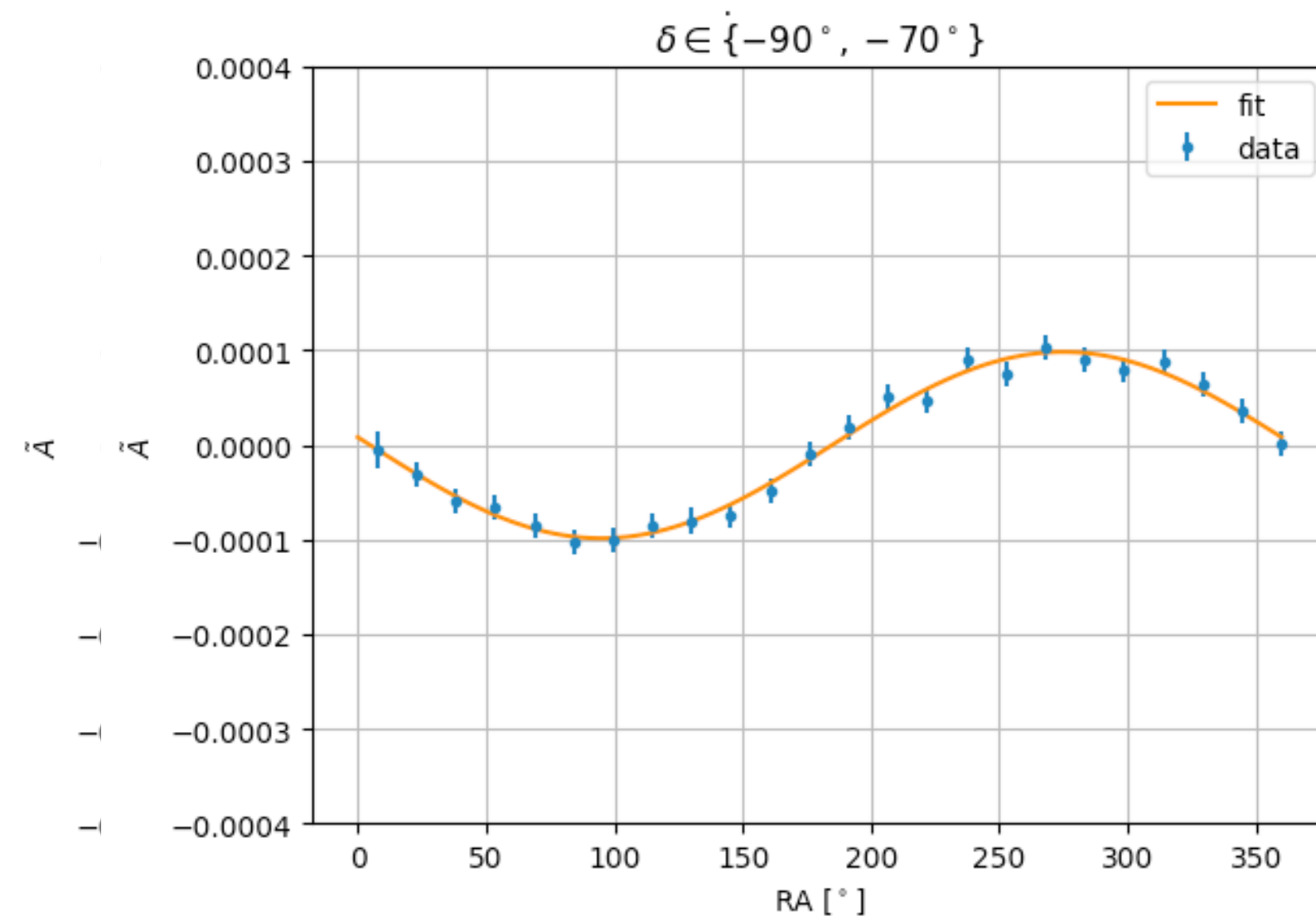
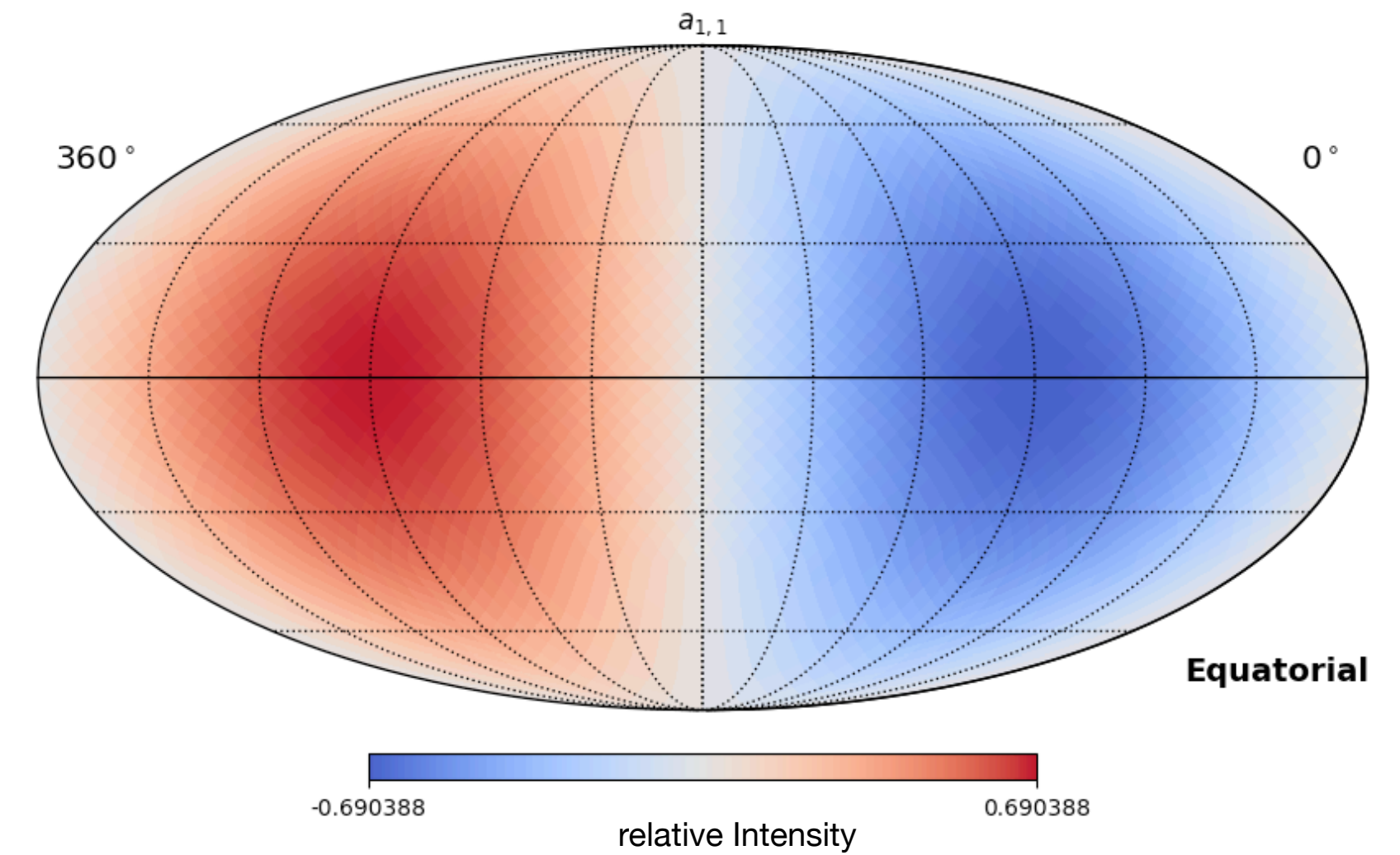
Left: the sidereal time with respect to solar time. As the Earth revolves around the Sun, the sidereal time zero stays fixed in the celestial sky while solar time's reference point moves away.



Declination dependence: 1D Projection (Toy MC)



- Amplitude is maximum towards equator and zero at poles
- 1d measurement corresponds to the average amplitude over declination bands

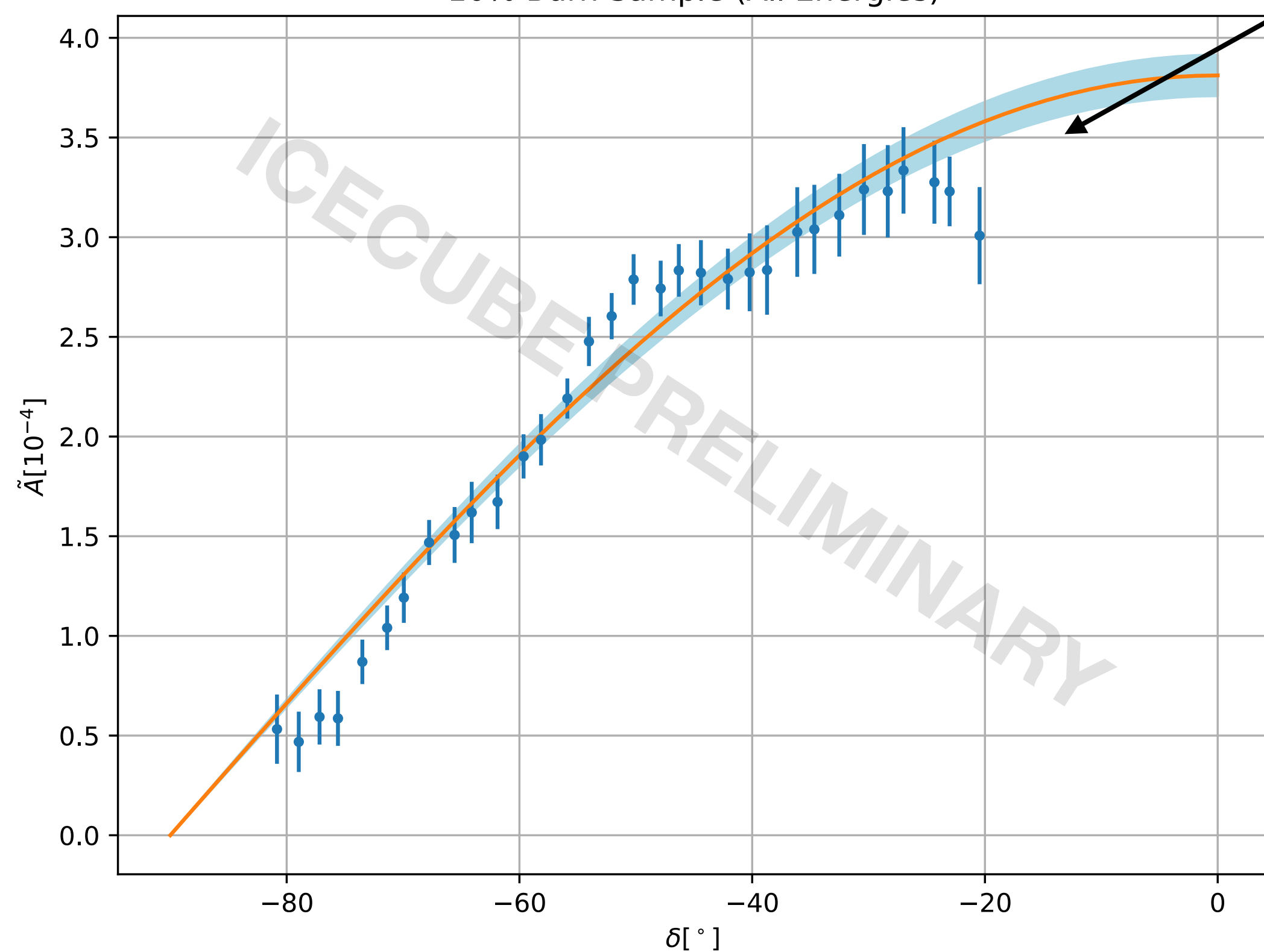


Extrapolation of 1D Amplitude

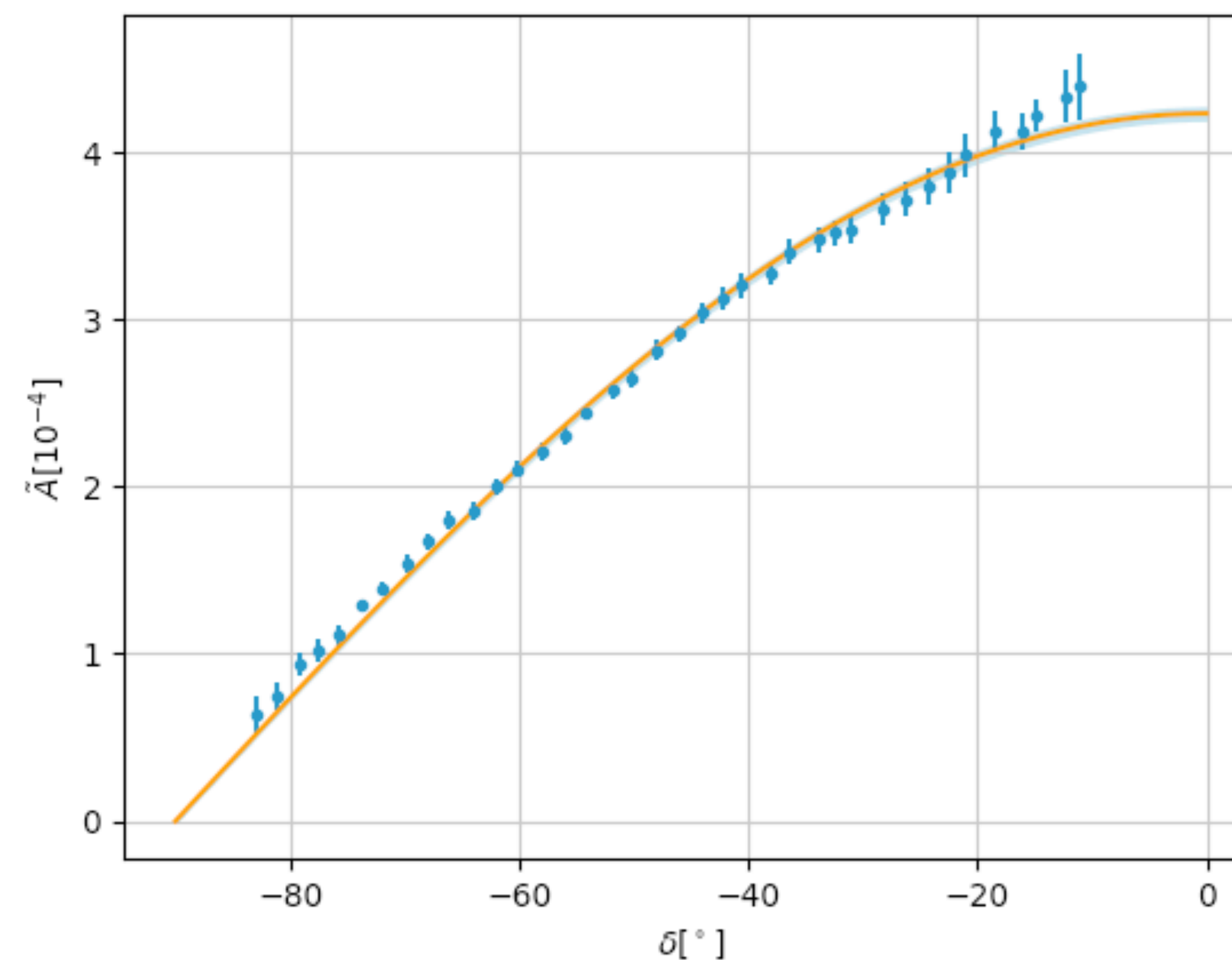
1. Plot amplitude of 1d fit in RA. as a function of declination
2. Fit cosine of declination and extrapolate to horizon

- Low Statistics
- Poor reconstruction

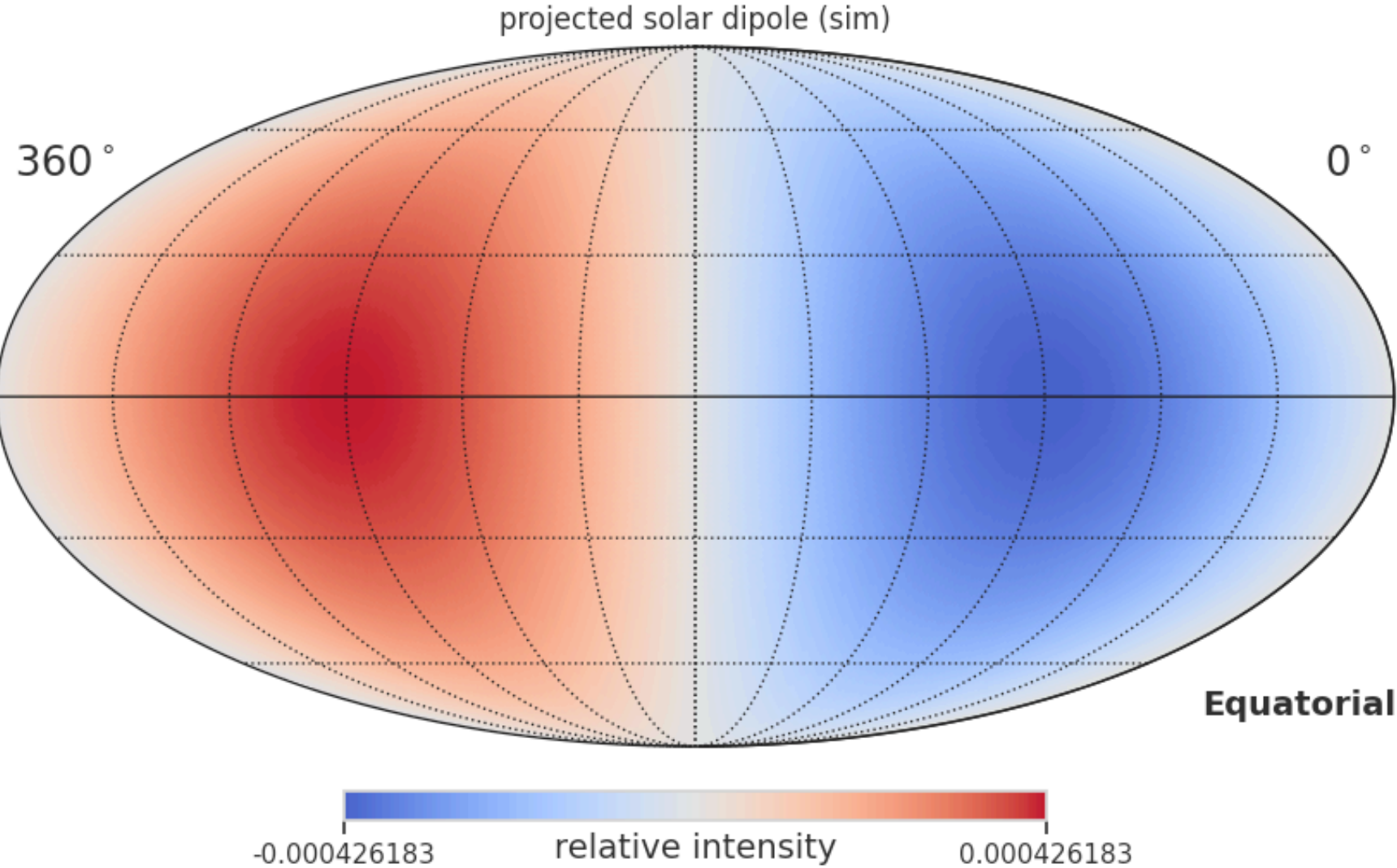
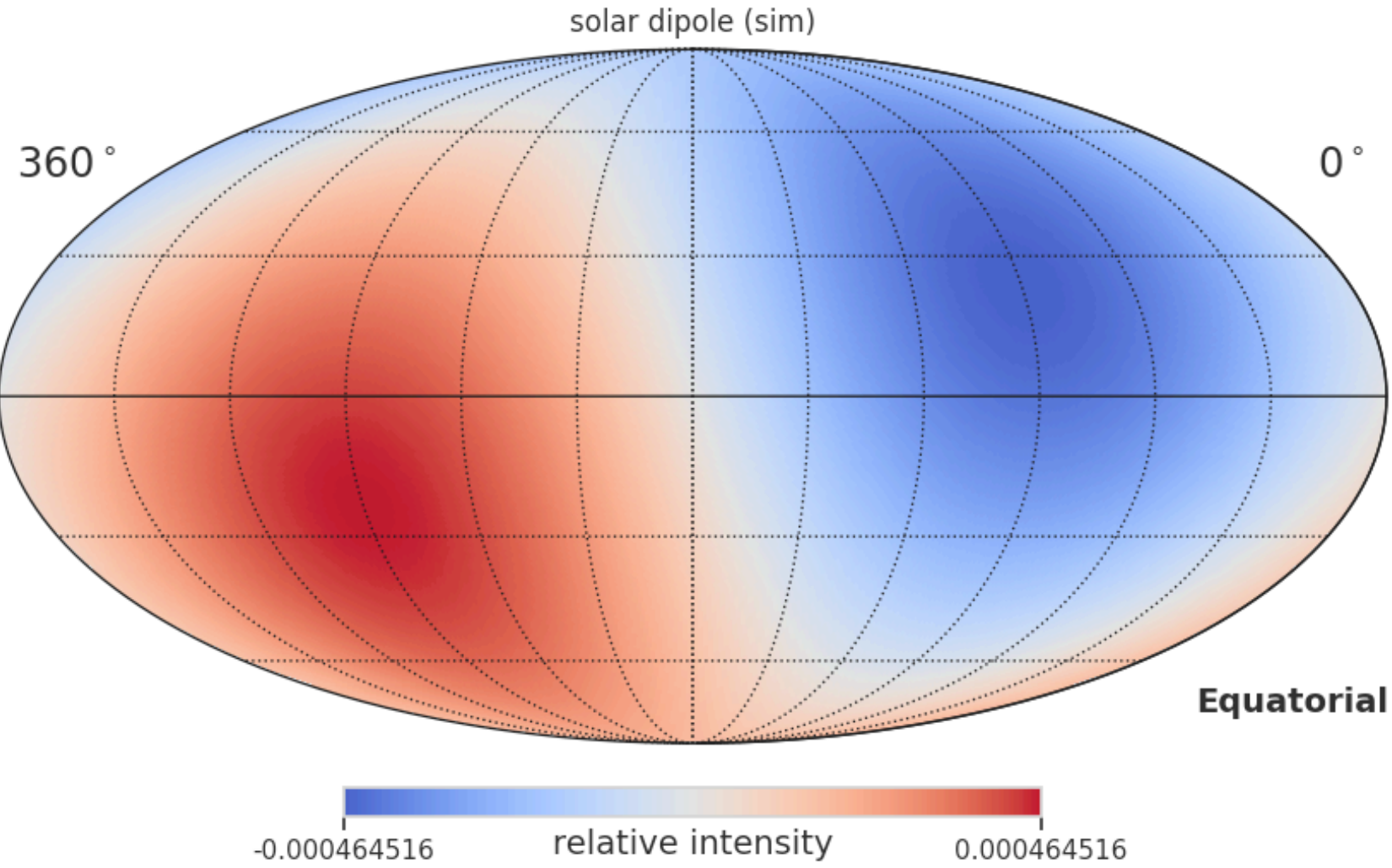
10% Burn Sample (All Energies)



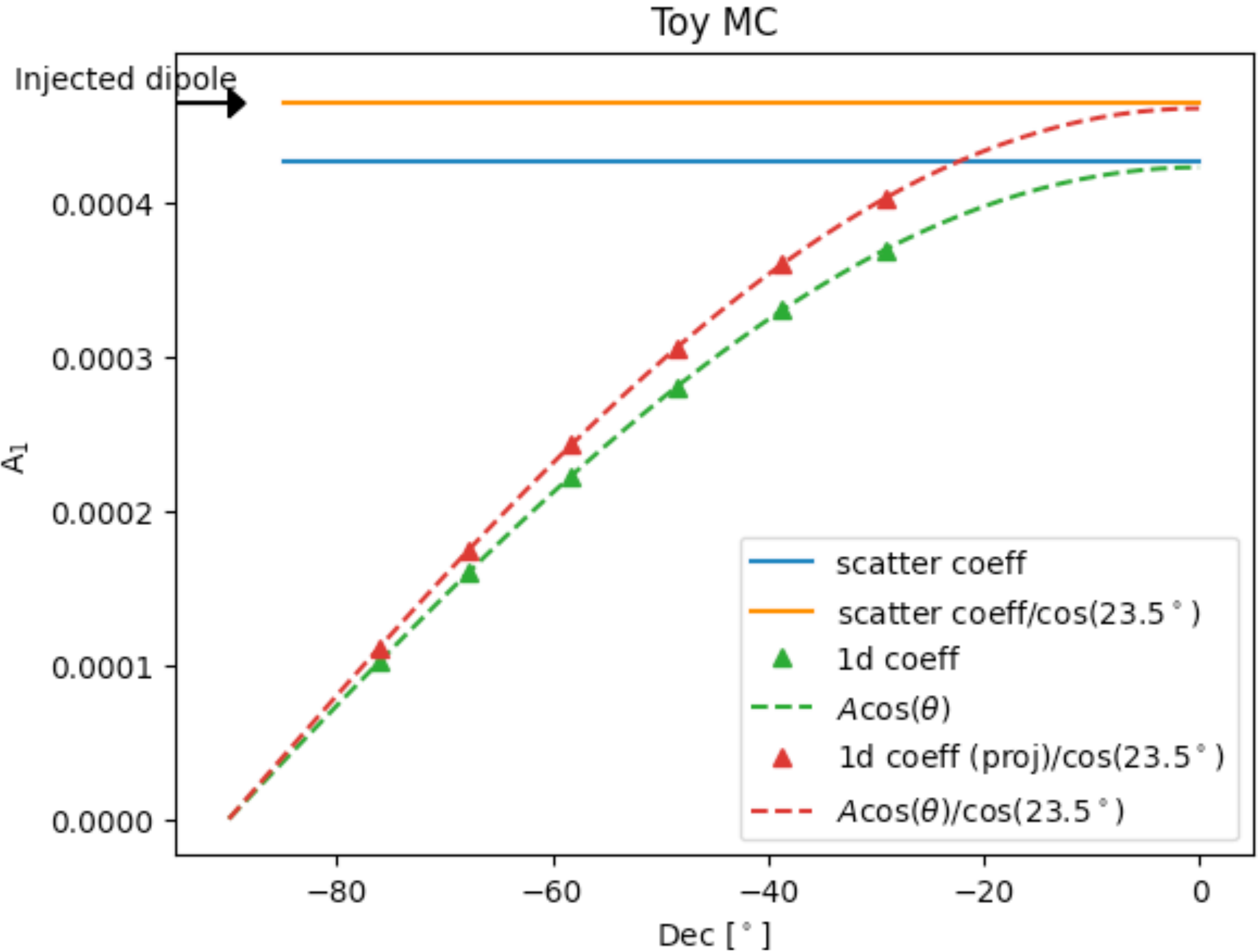
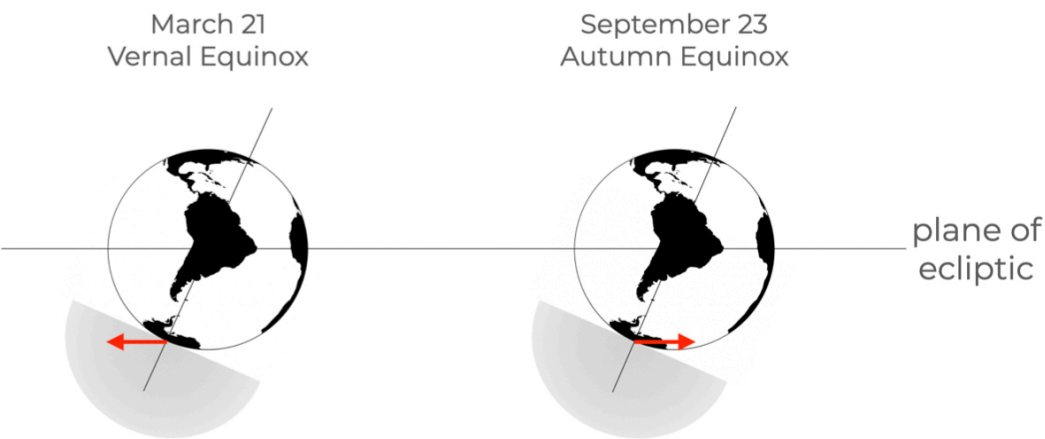
Toy MC



Equatorial Projection Bias



Correct for projection bias
(dipole tilt is known)
 23.5047°



Dipole Fit

Solar Dipole is given by

$$\frac{\delta I_i}{I} = \frac{v}{c}(\gamma + 2) \cos(\xi_i)$$

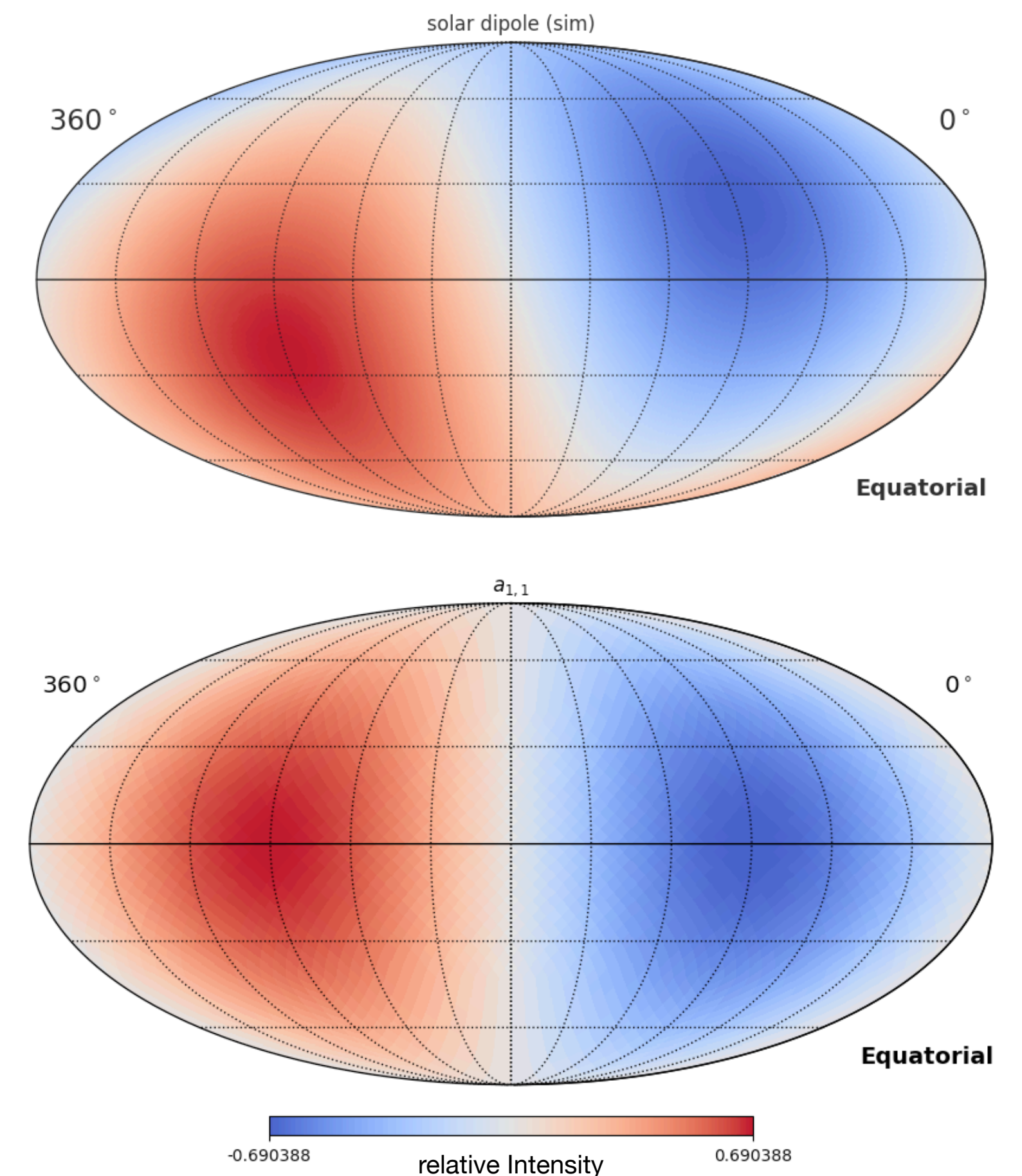
Where ξ_i is the opening angle between velocity direction and CR arrival direction (for pixel i).

Since the $m=0$ component is missing, this reduces to

$$\frac{\delta I_i}{I} = \frac{v}{c}(\gamma + 2) \sin \theta_i \cos(\phi_0 - \phi_i)$$

or, in equatorial coordinates (α, δ) coordinates

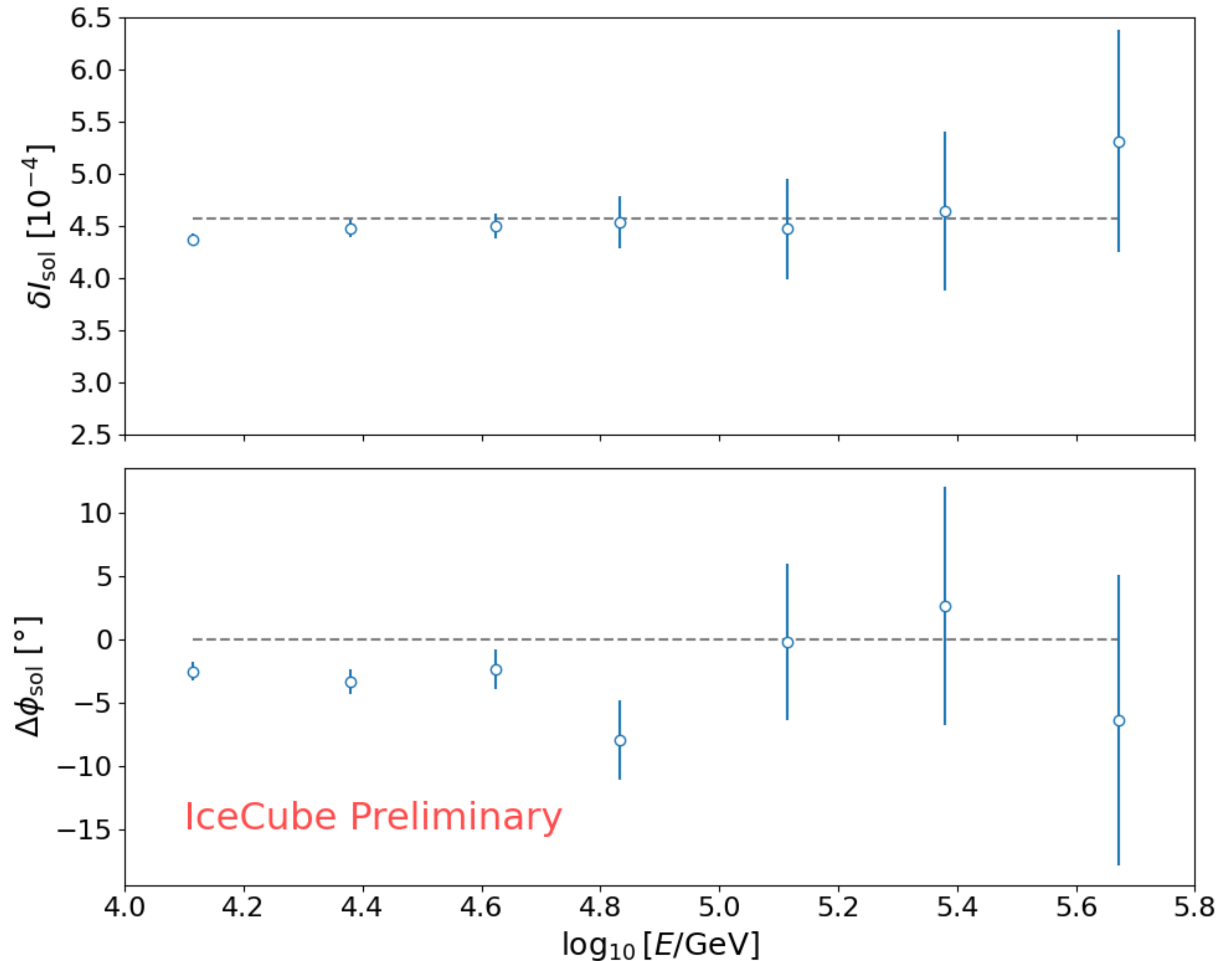
$$\frac{\delta I_i}{I} = \frac{v}{c}(\gamma + 2) \cos \delta_i \cos(\alpha_0 - \alpha_i)$$



Amplitude and Phase vs. Energy

Top: solar dipole amplitudes calculated from 2D fit in the solar reference frame compared to a single power-law hypothesis of $\gamma = -2.6$.

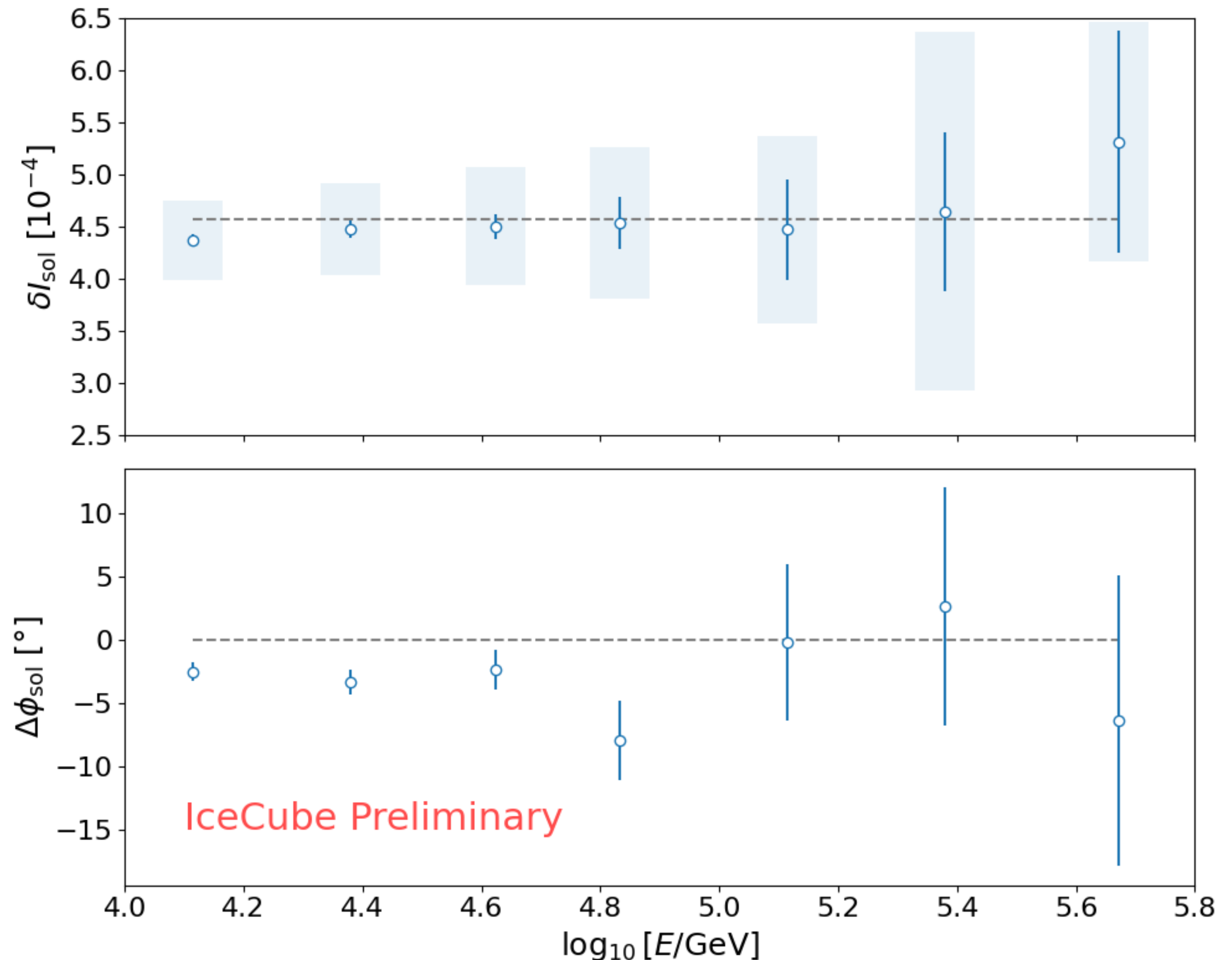
Bottom: phase offset from the direction of the velocity vector. Leakage from the sidereal reference can produce a dipole that interferes constructively or destructively with the solar dipole, changing both the amplitude and direction.



Amplitude and Phase vs. Energy

Top: solar dipole amplitudes calculated from 2D fit in the solar reference frame compared to a single power-law hypothesis of $\gamma = -2.6$ including systematic uncertainties.

Bottom: phase offset from the direction of the velocity vector. Leakage from the sidereal reference can produce a dipole that interferes constructively or destructively with the solar dipole, changing both the amplitude and direction.



Bias from Yearly modulation of solar anisotropy

Because the C-G dipole amplitude is ~3x smaller than the sidereal dipole, the anti-sidereal and spurious sidereal sidebands are comparably smaller.

[PoS\(ICRC2021\)085](#)

anti-sidereal distribution is produced by a **yearly modulation** of the solar-dipole amplitude and it **deforms** the sidereal distribution

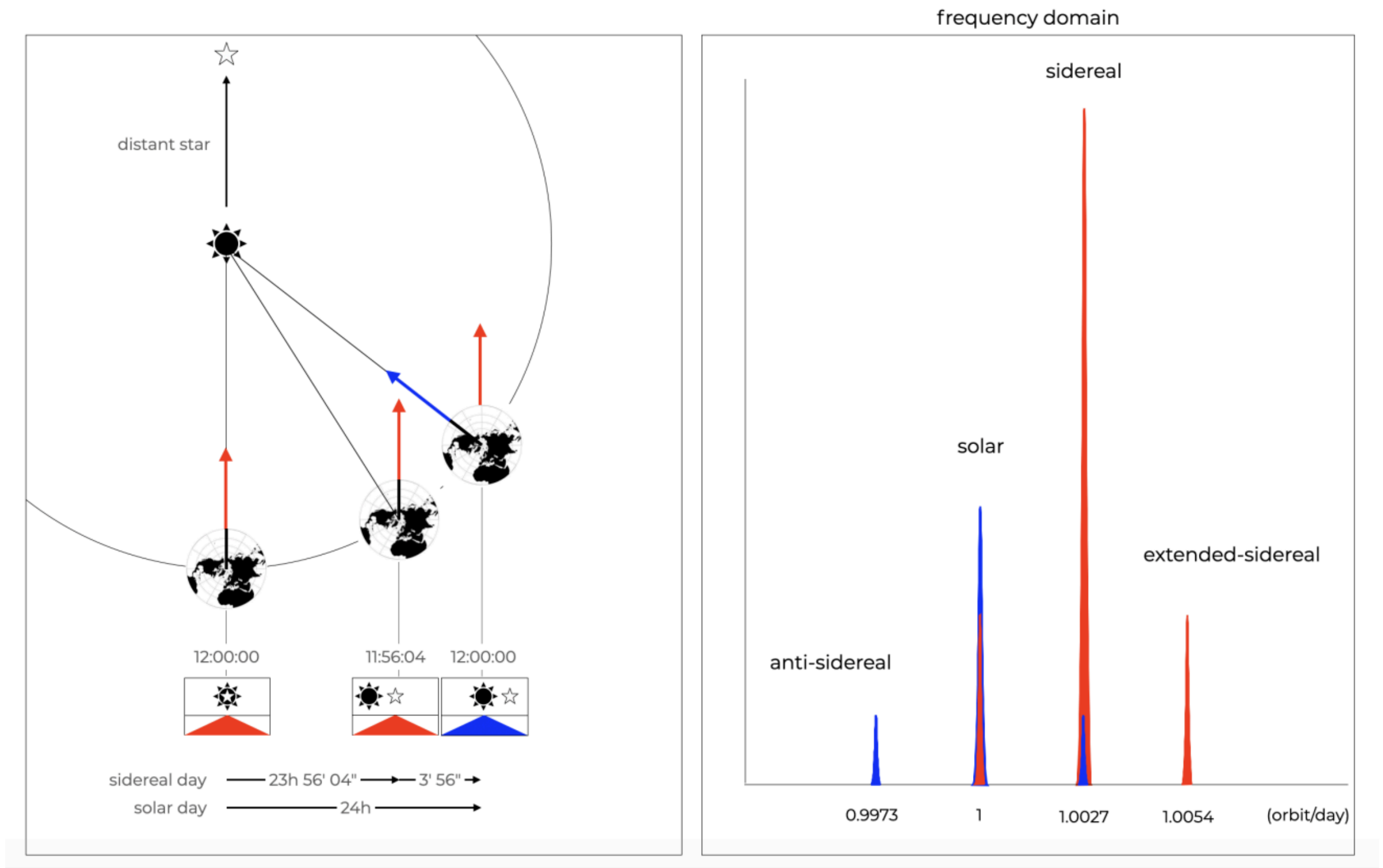


Figure 2: *Left:* the sidereal time with respect to solar time. Time starts at 12:00:00 on the local meridian with the Sun at the same location as a distant star. Then, as the Earth revolves around the Sun, the sidereal time zero point stays fixed in the celestial sky while the solar time's reference point moves away.

Solar Dipole Modulation (Time Dependence)

■

Orbital speed of Earth vs Time is nearly sinusoidal but there is an asymmetry due to the eccentricity of the orbit. The 28-day frequencies observed are caused by variations from the Earth-Moon system.

$$\delta I \approx (\gamma + 2) \frac{\langle v \rangle}{c} + (\gamma + 2) \frac{(v_{\max} - v_{\min})}{c} \cos(\omega t - \phi)$$



Solar Dipole Modulation (Time Dependence)

In addition to the yearly modulation due to the orbital speed, the Earth tilt relative to the velocity vector peaks during the equinoxes and oscillates between $\eta = \pm 23.5^\circ$ by

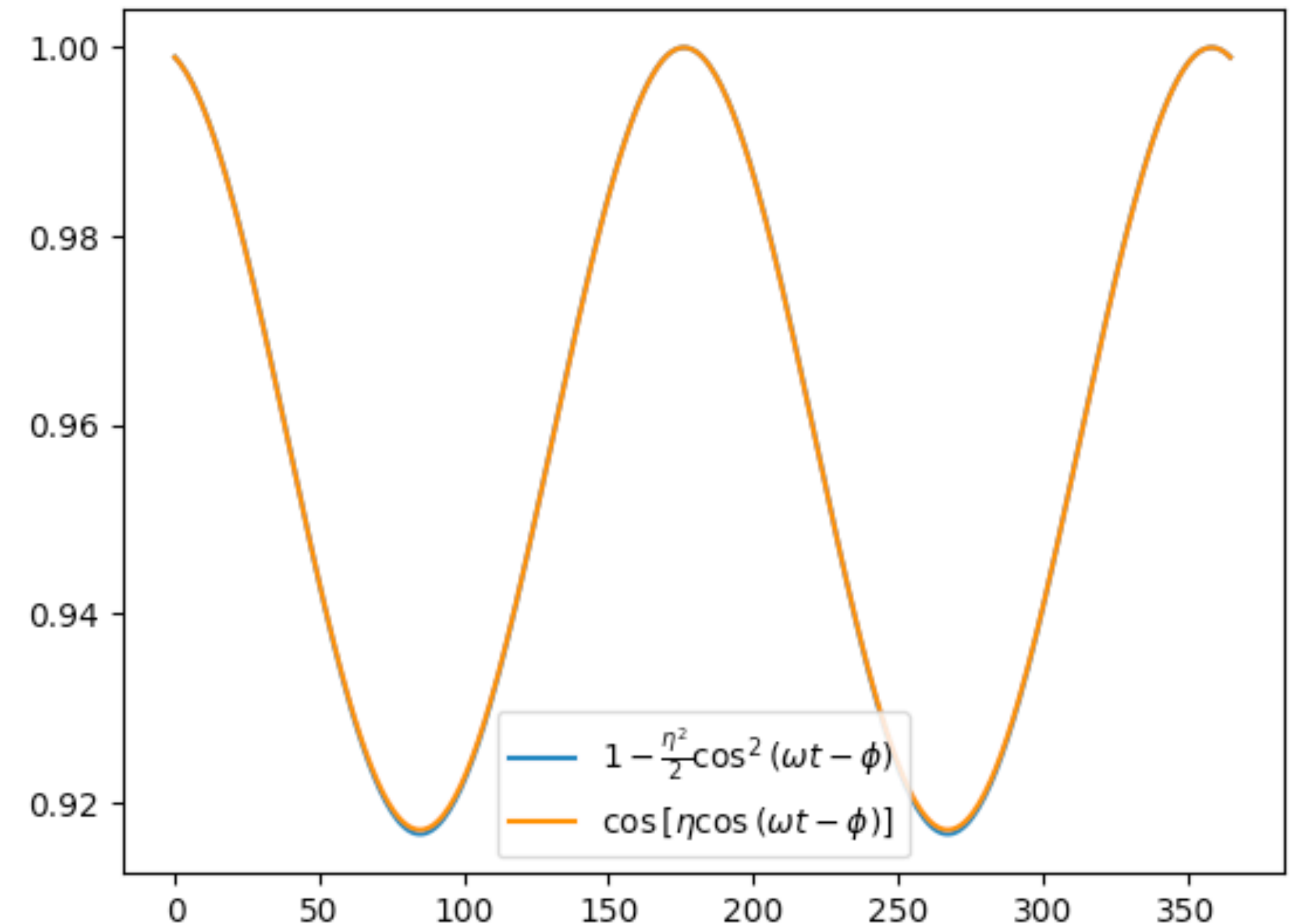
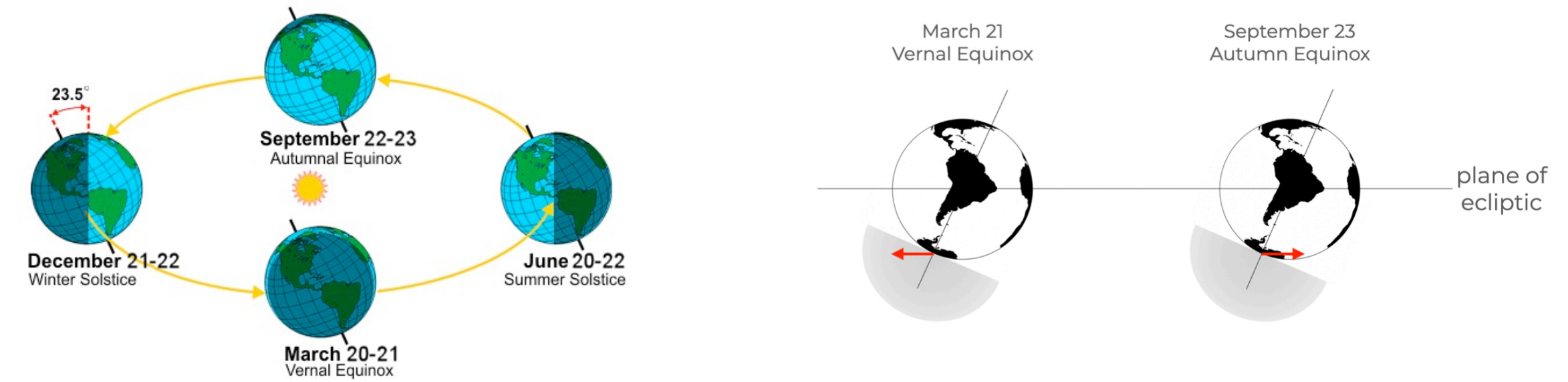
$$\cos [\eta \cos (\omega t - \phi)]$$

$$\approx 1 - \frac{\eta^2}{2} \cos^2 (\omega t - \phi)$$

So the total expected yearly variation is

$$\approx A \cos (\omega t - \phi_1) - \frac{\eta^2}{2} \cos^2 (\omega t - \phi_2) + B$$

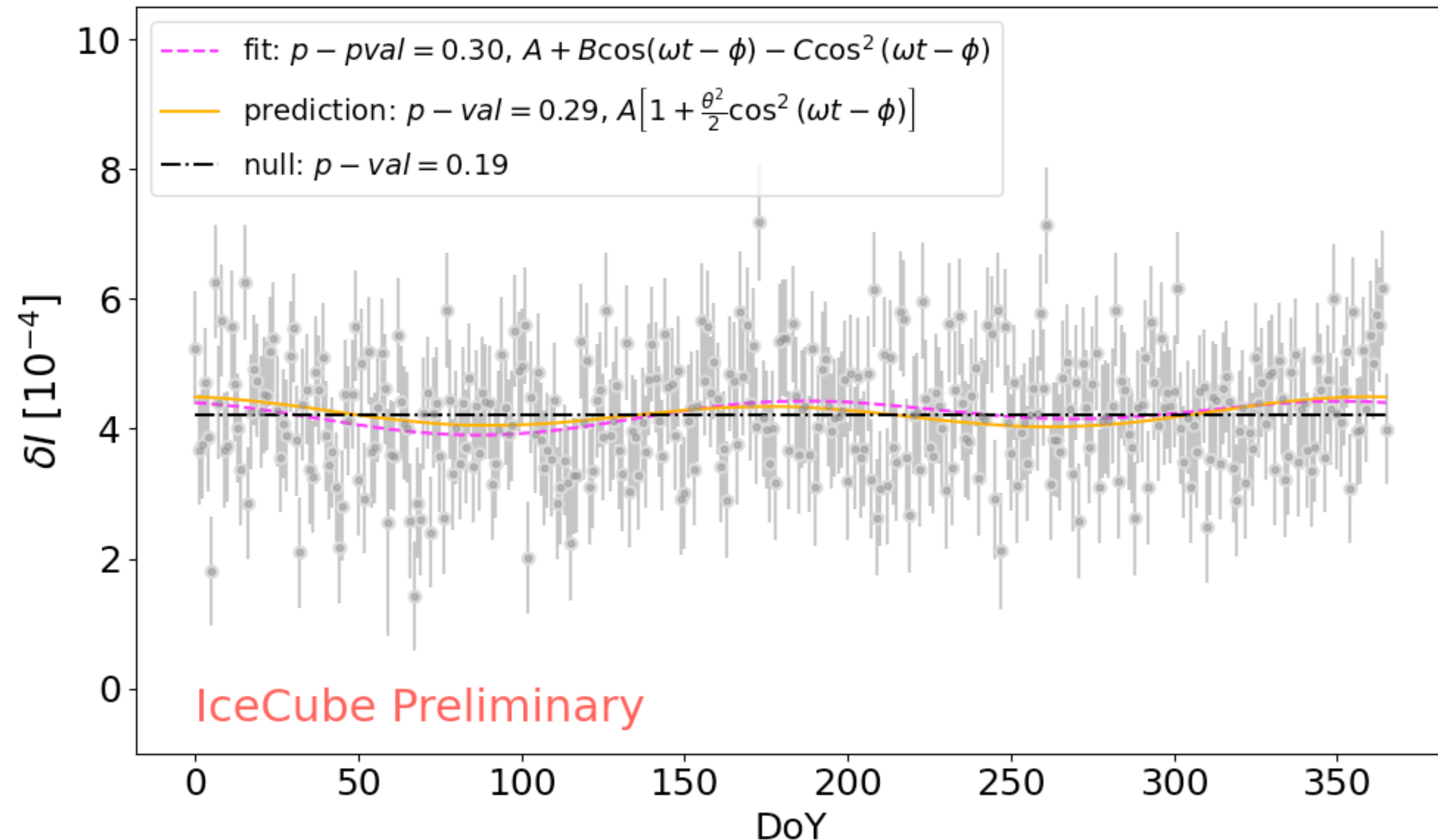
Where B is the mean amplitude and A is the variation due to the orbital speed (~10%)



Solar Dipole Modulation (Time Dependence)

Daily solar amplitude vs. time. We get the best fit for the first and second harmonic representing the variation in orbital speed and tilt shadow, respectively.

The function is a marginally better fit than the null hypothesis but the first order term has the opposite phase as the prediction.



Time series fit

Instead of producing maps in the Solar reference system, we fit daily sidereal maps by pixel as a function of (time, Ra, Dec) and subtract the mean distribution.

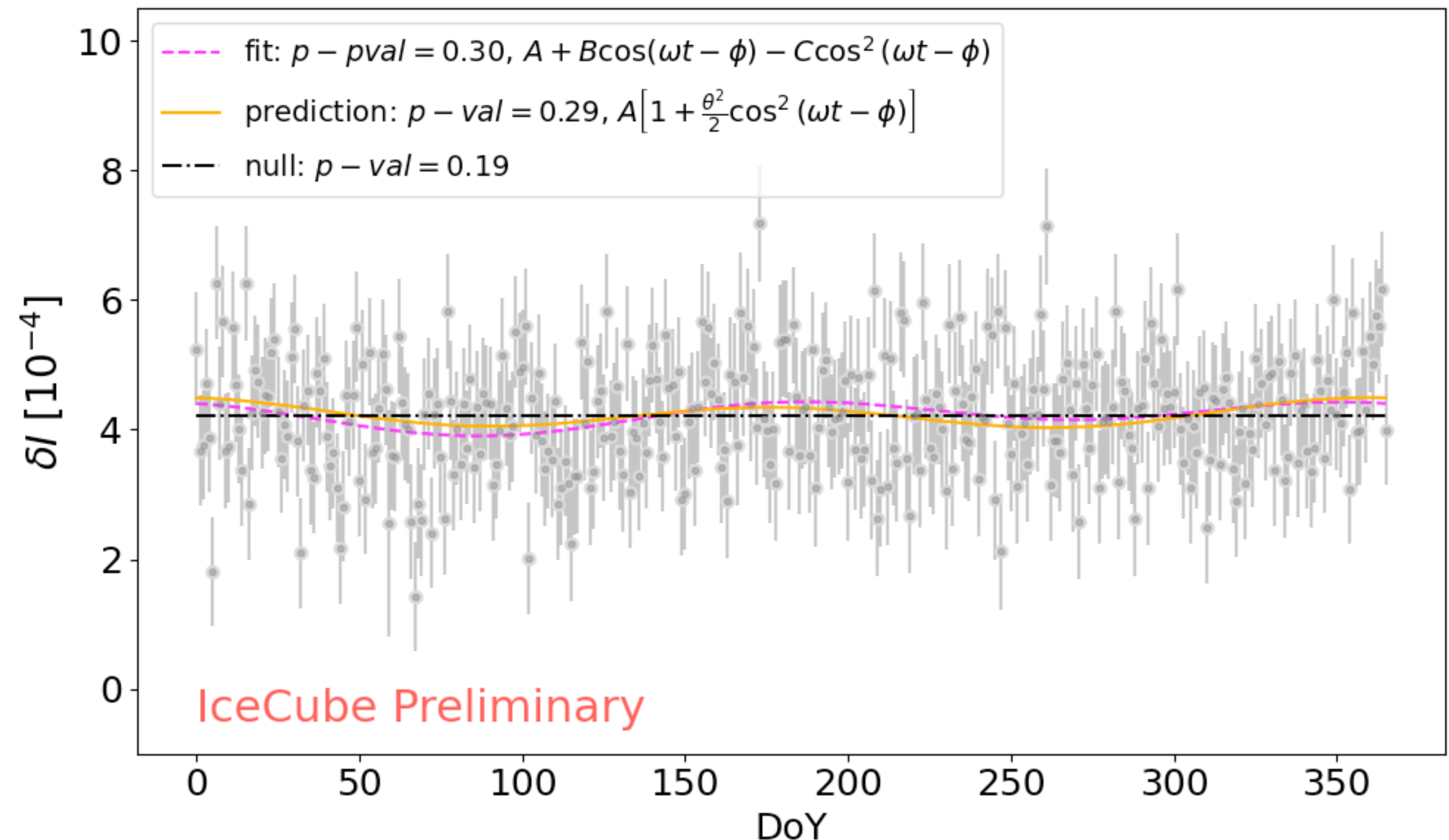
Advantage: daily maps insensitive to gaps in data taking.

The reference map still sensitive to gaps but bias due to leakage is smaller.

We fit

$$f(A, \delta, \alpha, t, \phi) = A \cos(\delta) \cos(\alpha - 2\pi t/365 + \phi)$$

over the time series of pixels for all daily maps with A and ϕ as free parameters.



Time series fit

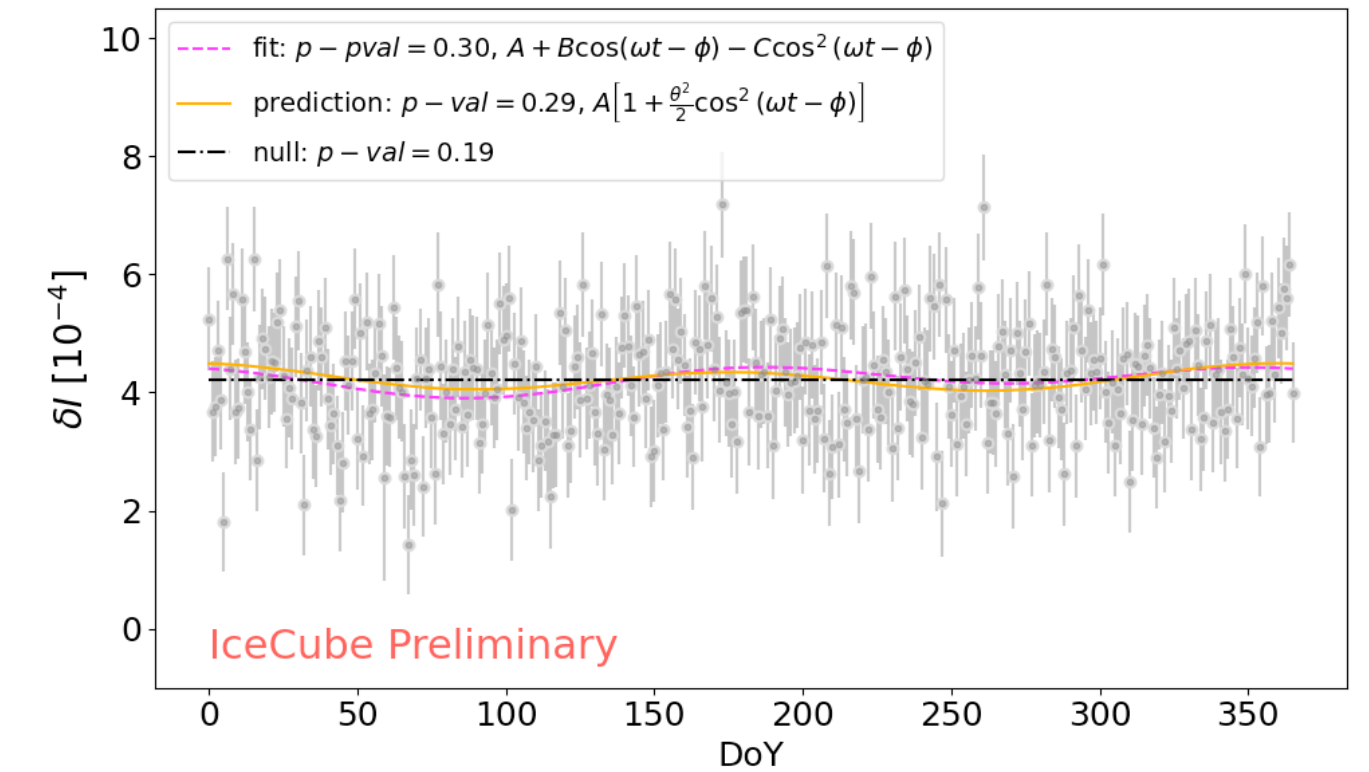
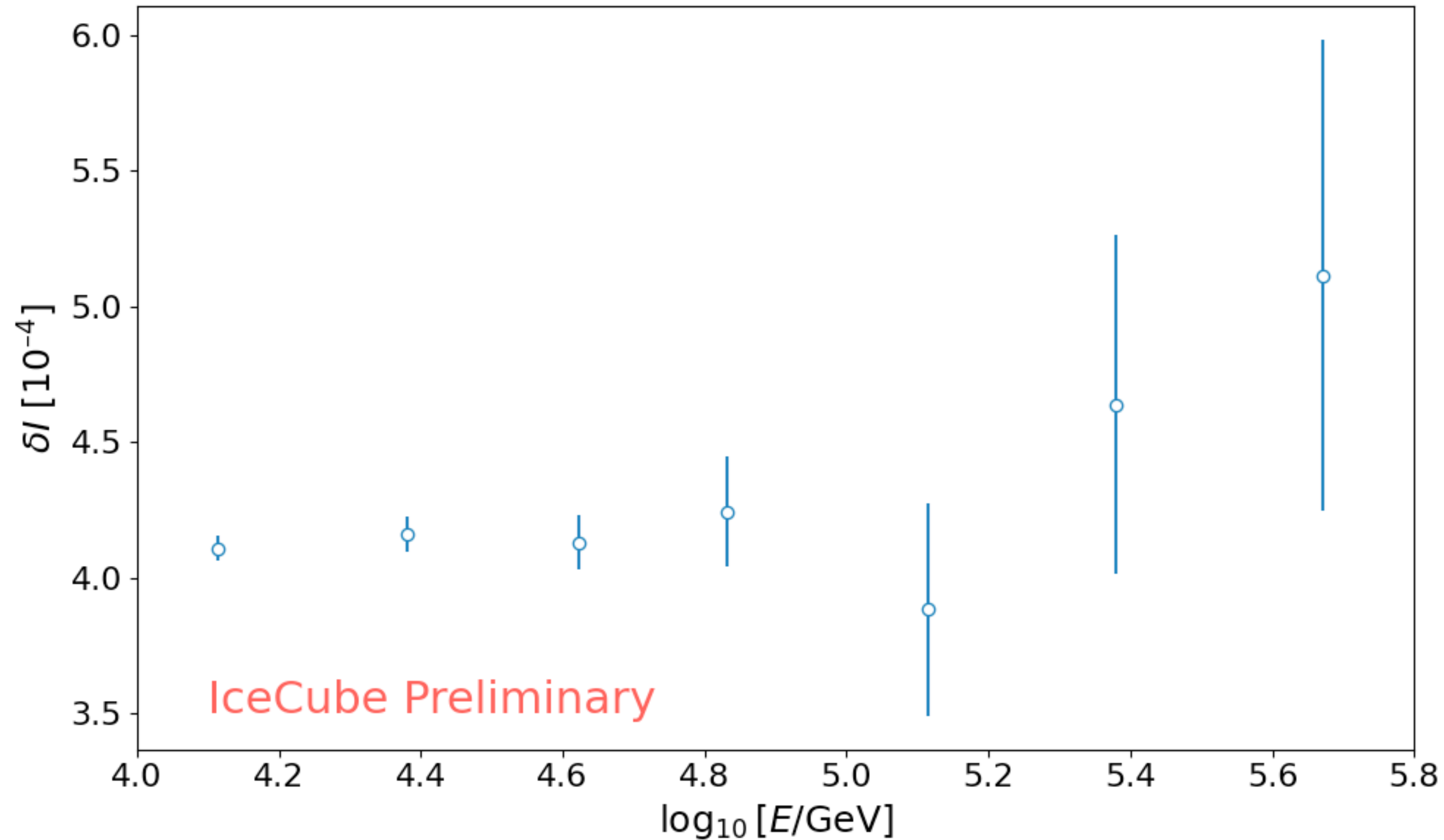
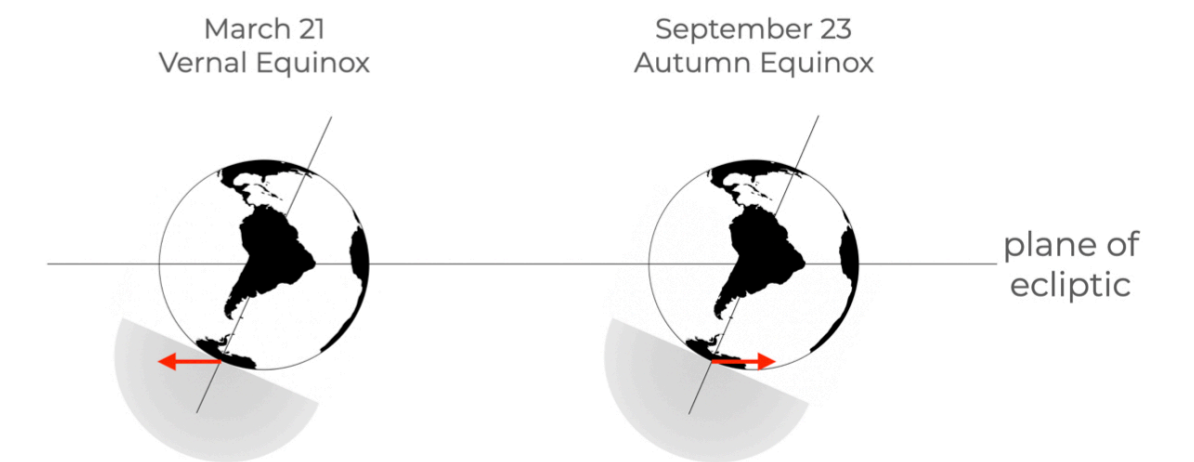
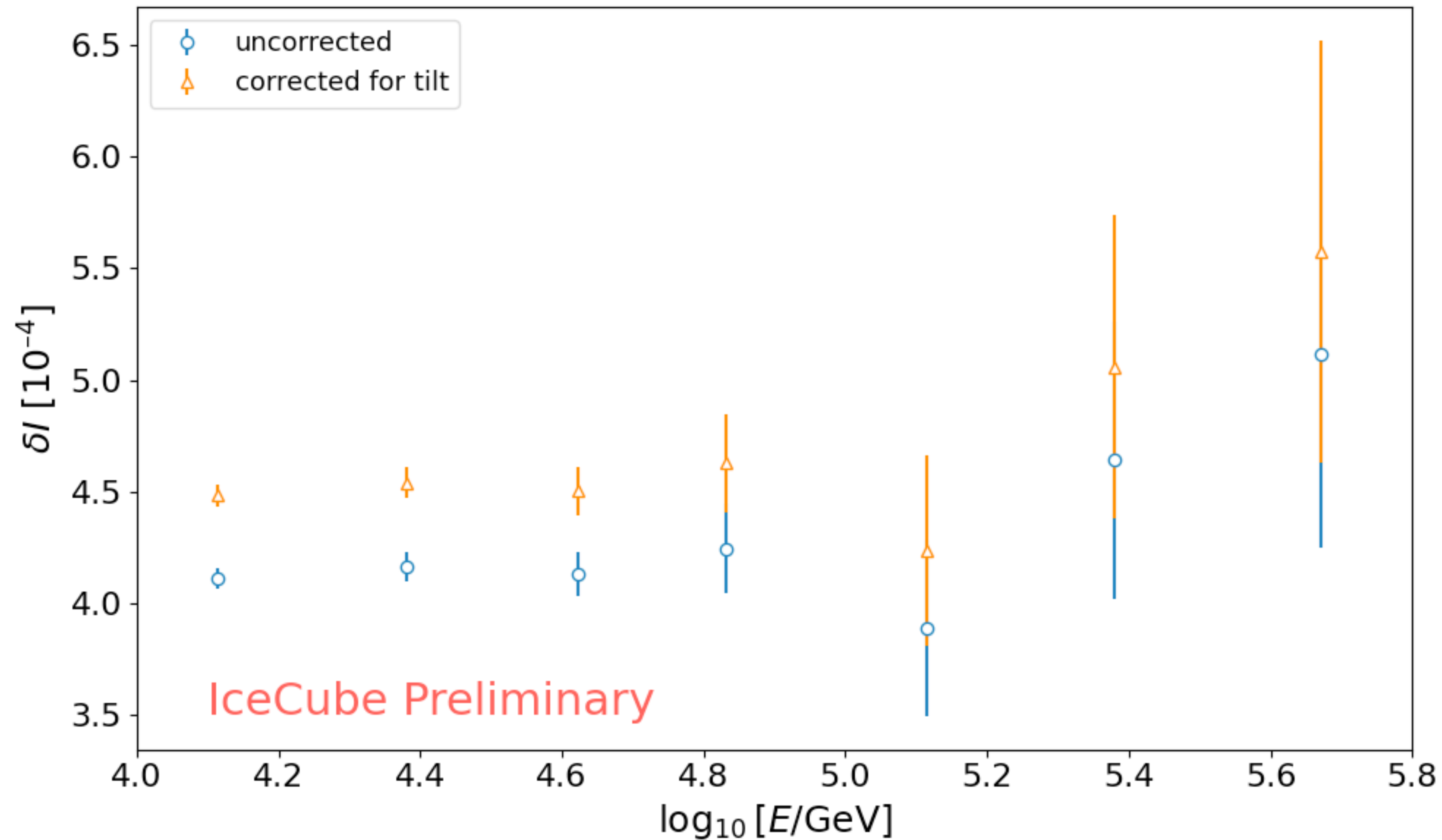


Figure: Projection of Compton-Getting amplitudes vs Energy onto the equatorial plane. Missing North-South dipole component corresponding to $\ell=1, m=0$ term in spherical harmonics.

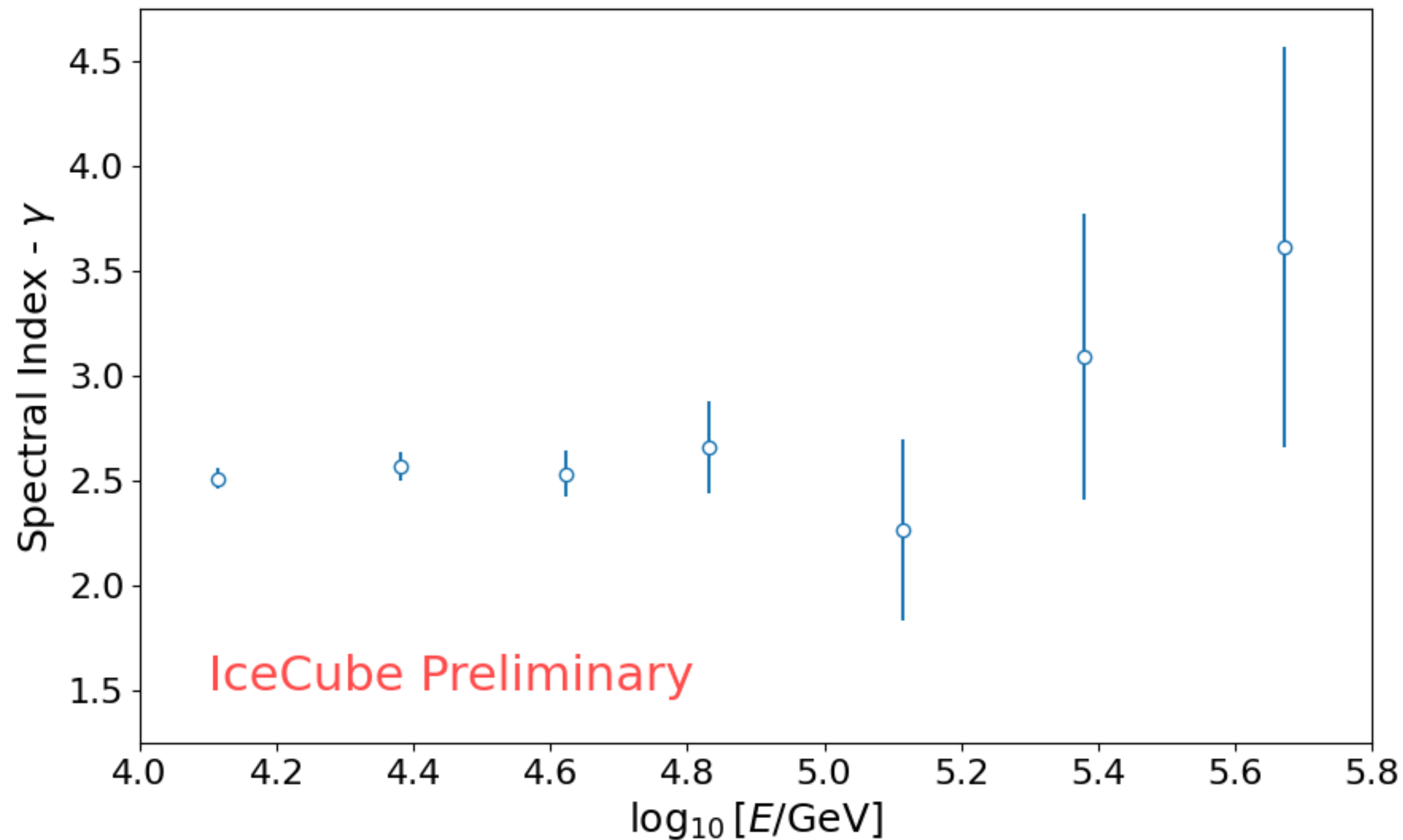
Time series fit



**Correct for projection bias
(dipole tilt is known)
23.5047°**

Figure: Compton-Getting amplitudes vs Energy Corrected for the tilt of Earth axis with respect to the ecliptic $1/\cos(23.5^\circ)$.

Time series fit



Left: Spectral index γ vs Energy for 7 anisotropy bins, calculated from the dipole amplitude according to

$$\gamma = \delta I \frac{c}{v} - 2$$

Errors are statistical corresponding the fit error scaled by the ratio c/v .

Systematic check:

1st six years compared to 2nd six years.

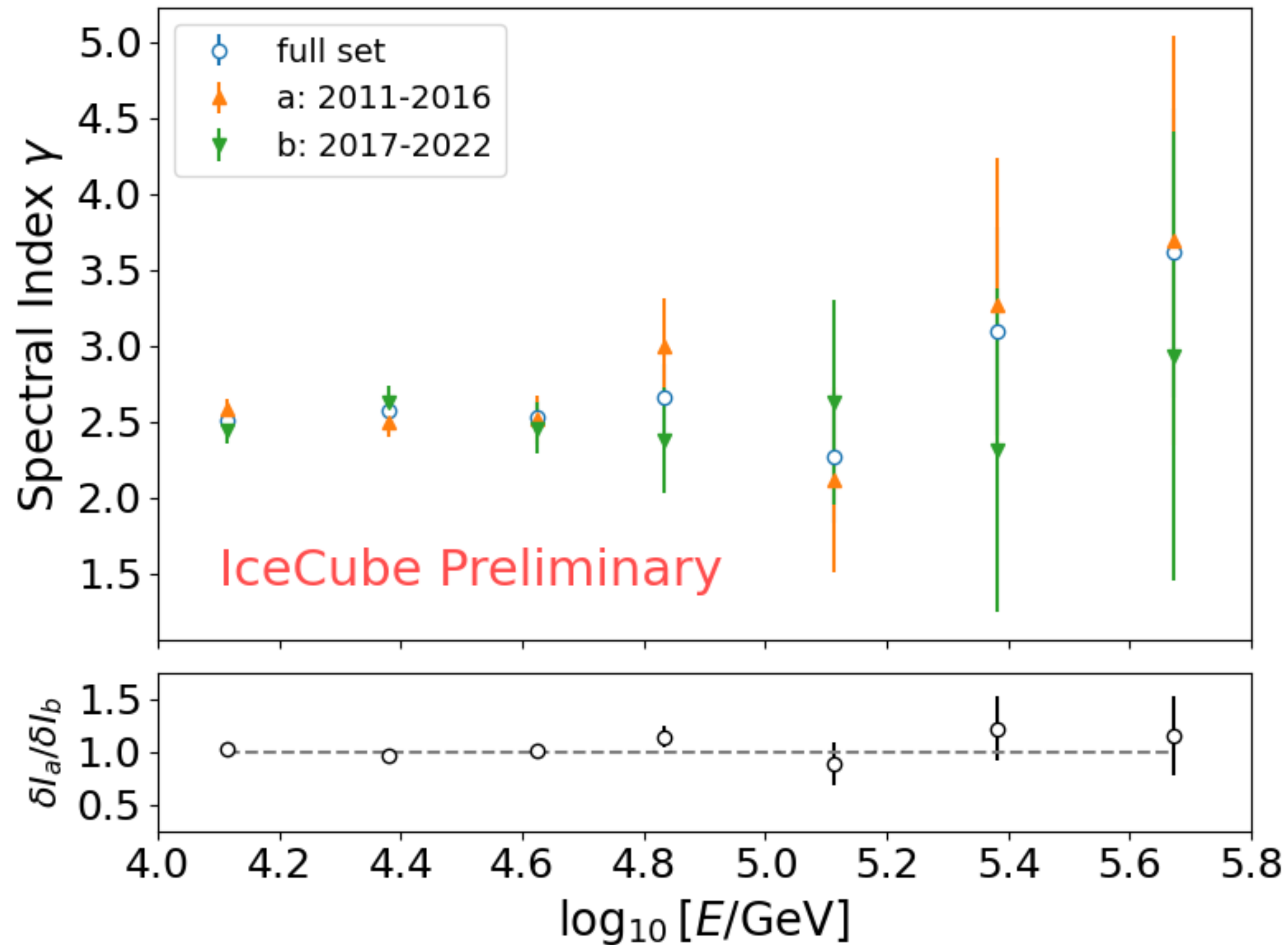
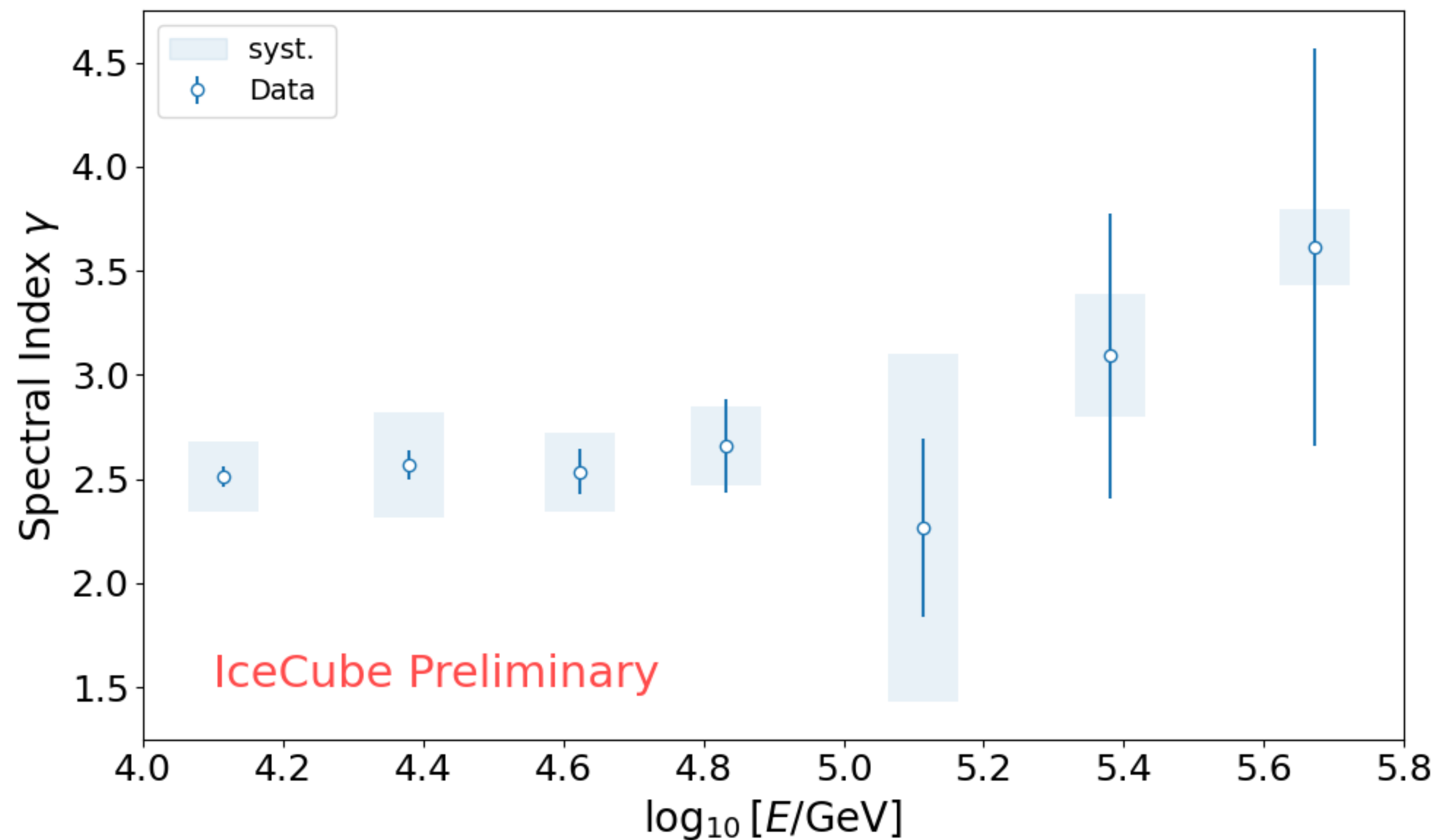


Figure: Dipole measurements differ between first 6 years (2011-2016) vs. second 6 years (2017-2022).

Measurements are consistent within statistical uncertainties.

Time series fit



Systematic uncertainties:
calculated from anti-sidereal dipole
amplitude.

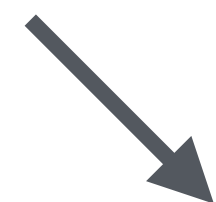
Systematic effects at 100 TeV are
challenging due to rapidly changing
transition (phase flip) and small
amplitude of sidereal dipole relative
to C-G dipole amplitude.

Anti-sidereal Correction Phase



Farley & Storey, Proc. Phys. Soc., 67, 996 1954
Nagashima, et al., Nuovo 907 Cimento C, 6, 550, 1983

$$R(t) = 1 + \underbrace{[A + 2B \cos 2\pi(t - \phi_2)]}_{\text{SEASONAL MODULATION}} \underbrace{\cos 2\pi(Nt - \phi_1)}_{\text{SOLAR VARIATION}} + \underbrace{C \cos 2\pi \{(N + 1)t - \phi_3\}}_{\text{TRUE SIDEREAL VARIATION}},$$



$$R(t) = 1 + \underbrace{A \cos 2\pi(Nt - \phi_1)}_{\text{SOLAR}} + \underbrace{B' \cos 2\pi \{(N + 1)t - (\phi_1 + \phi_2)\}}_{\text{SPURIOUS SIDEREAL}} + \underbrace{C \cos 2\pi \{(N + 1)t - \phi_3\}}_{\text{TRUE SIDEREAL}} + \underbrace{B \cos 2\pi \{(N - 1)t - (\phi_1 - \phi_2)\}}_{\text{PSEUDO-SIDEREAL}}$$

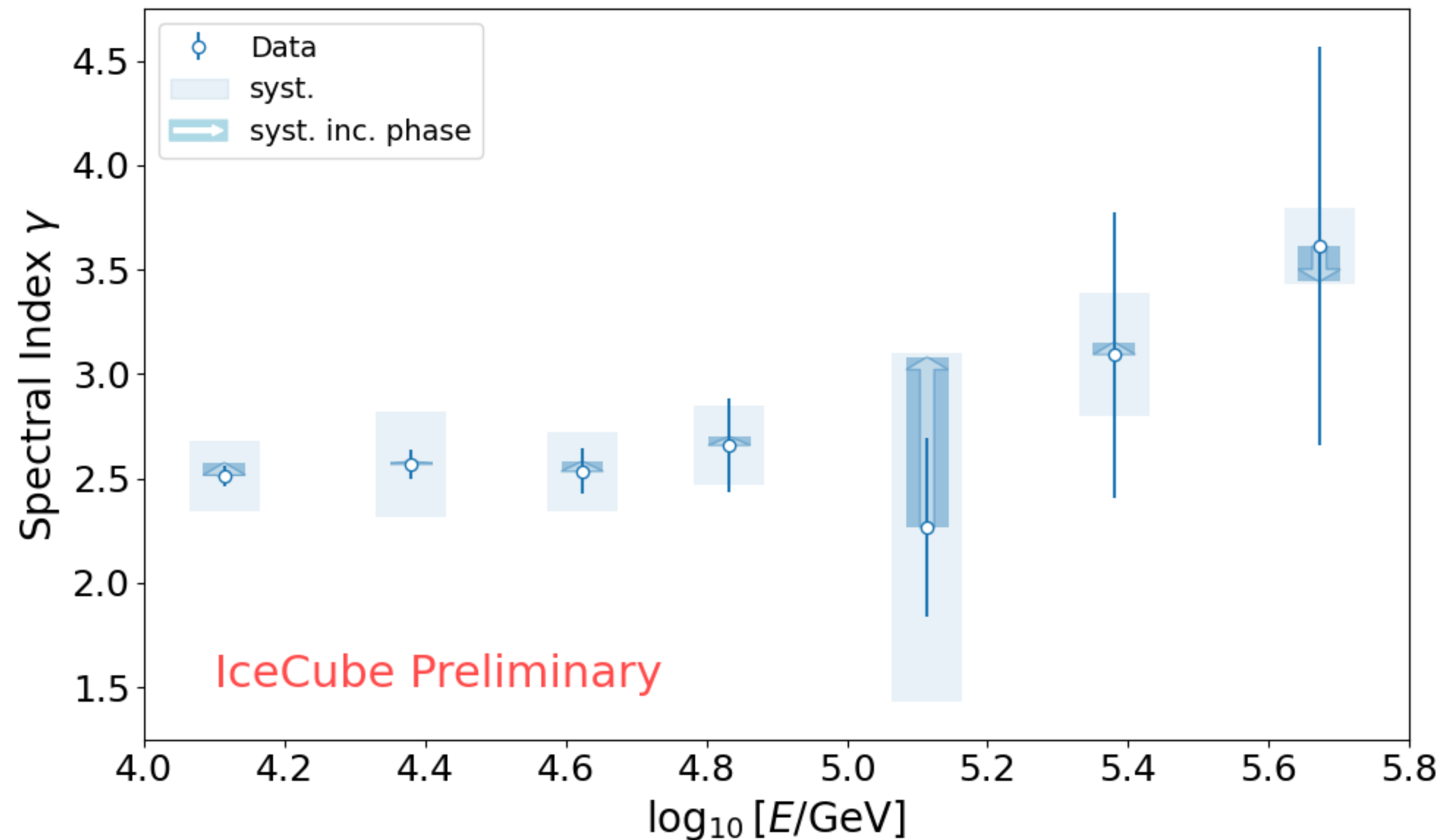
- ϕ_1 phase of solar dipole
- $\phi_1 + \phi_2$ phase of anti-sidereal dipole
- $\alpha = \phi_1 - \phi_2$ phase of sidereal correction term

Systematic uncertainty (time series):

$$\delta I_{\text{as}} \rightarrow \delta I_{\text{as}} \cos \alpha$$

Time series fit

(Systematic Uncertainties)



Systematic uncertainties:

Amplitude is reduced by factor $\cos(\alpha)$.

Direction of bias is determined from angle

$$\alpha = \phi_1 - \phi_2$$

where

$$\phi_1$$

is the phase of solar dipole, and

$$\phi_1 + \phi_2$$

is the phase of anti-sidereal dipole.

Hypotheses

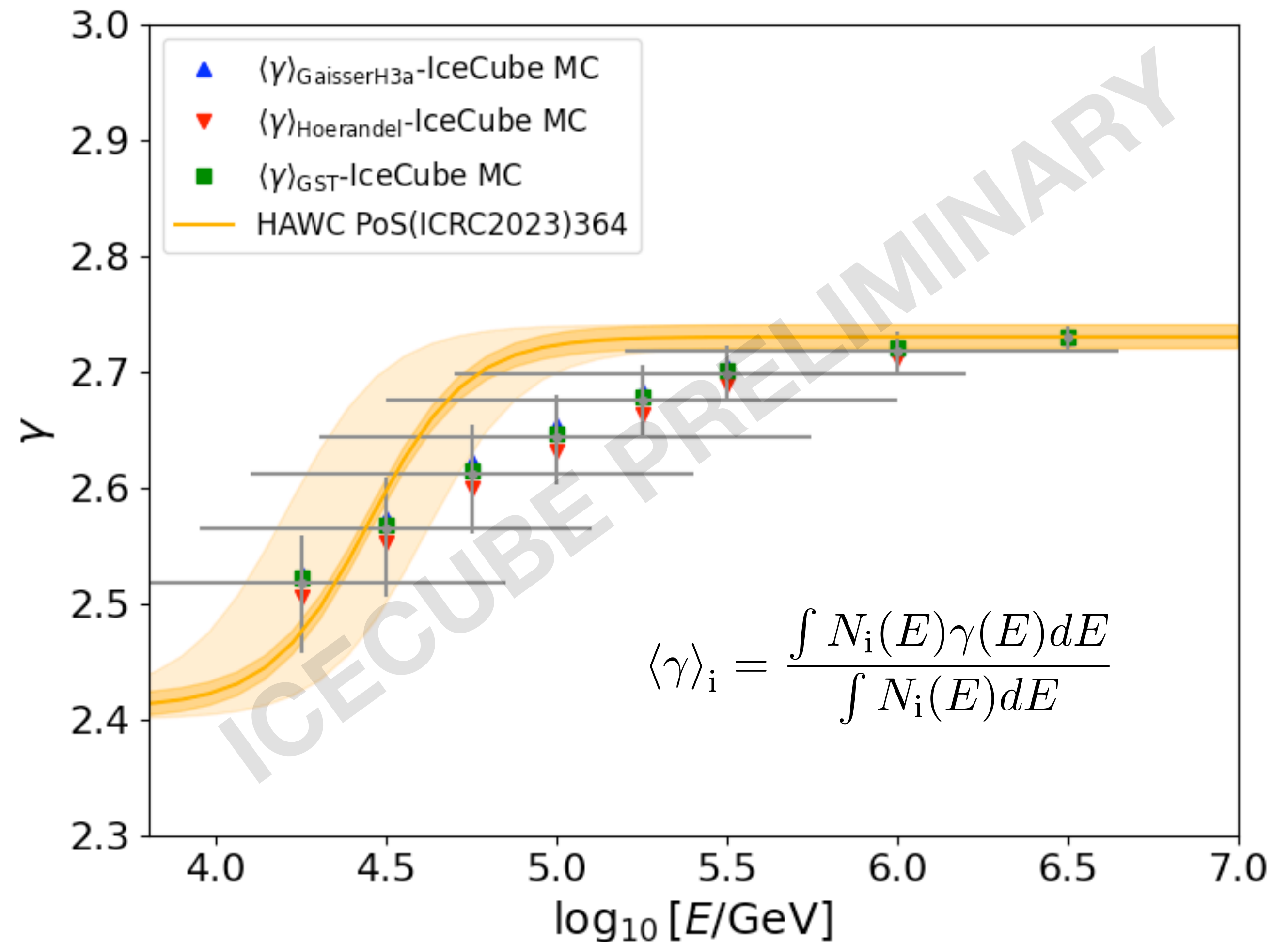
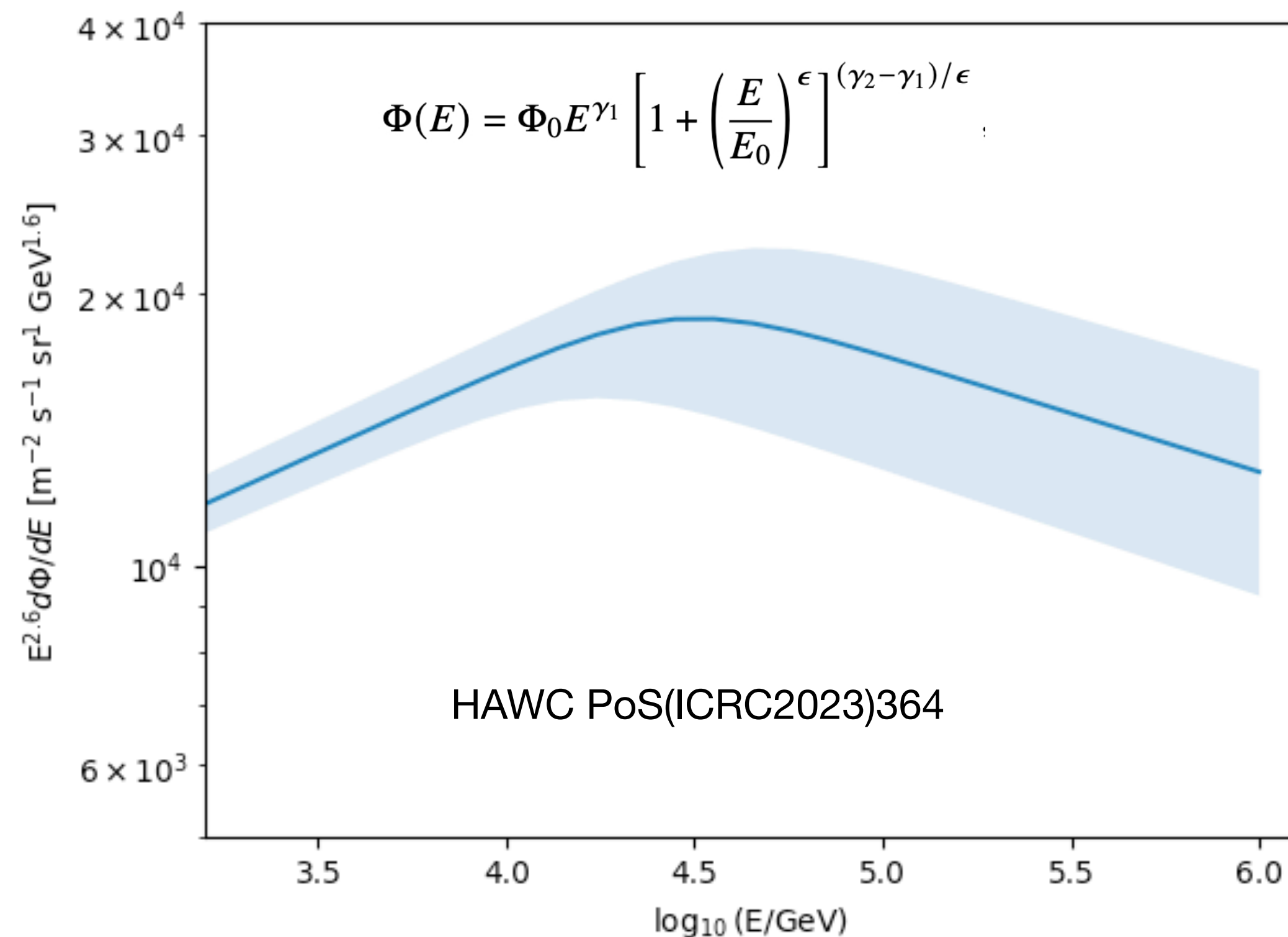
HAWC measurements on the total energy spectrum of cosmic rays

J. A. Morales-Soto^{a,*} and J. C. Arteaga-Velázquez^a for the HAWC collaboration

^aInstituto de Física y Matemáticas,

Universidad Michoacana de San Nicolás de Hidalgo, Morelia, Mexico

E-mail: jorge.morales@umich.mx, juan.arteaga@umich.mx



Time series fit

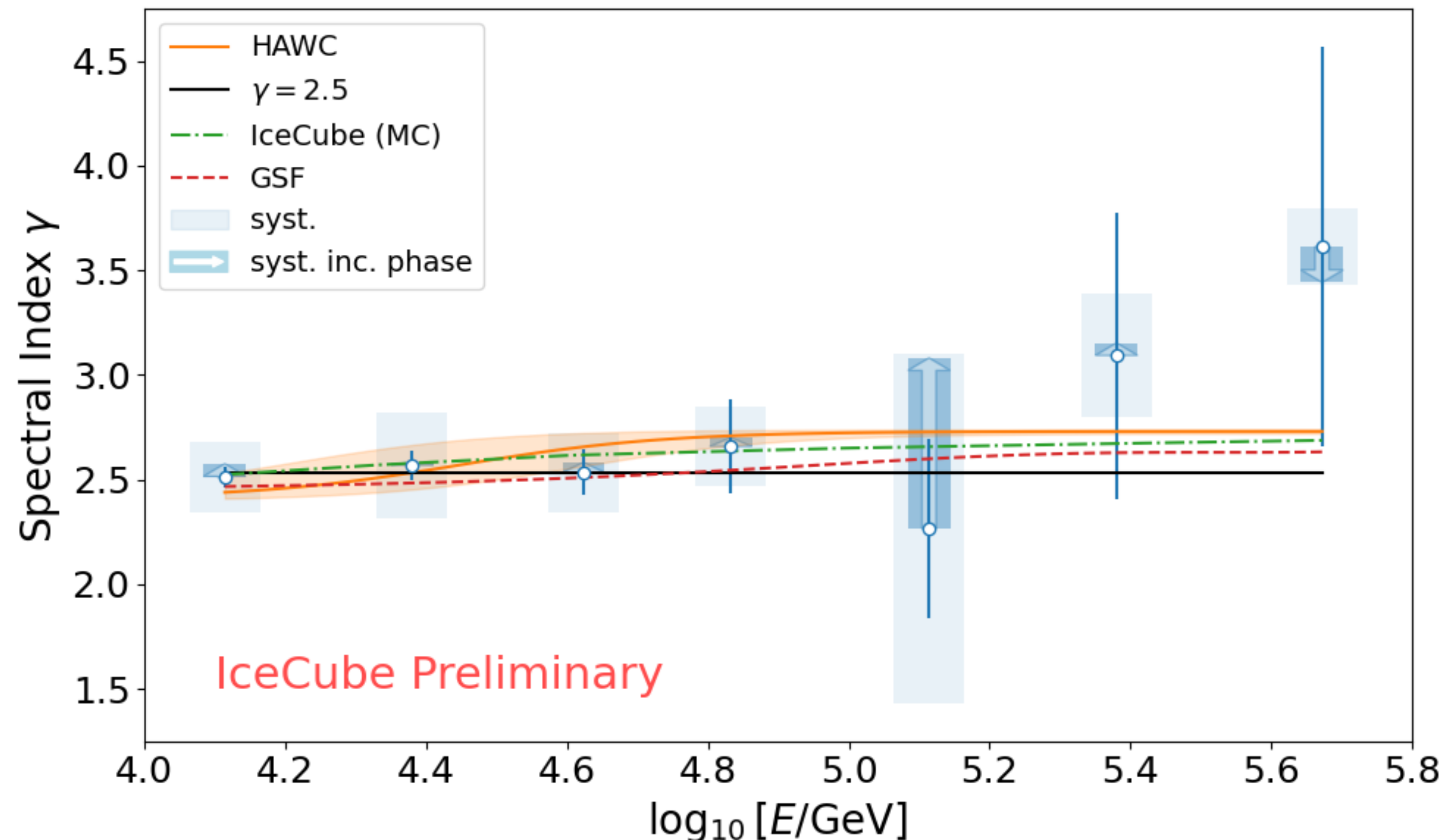
(Hypothesis Test)



Systematic uncertainties: calculated from anti-sidereal dipole amplitude and phase.

Spectral indices are compared to:

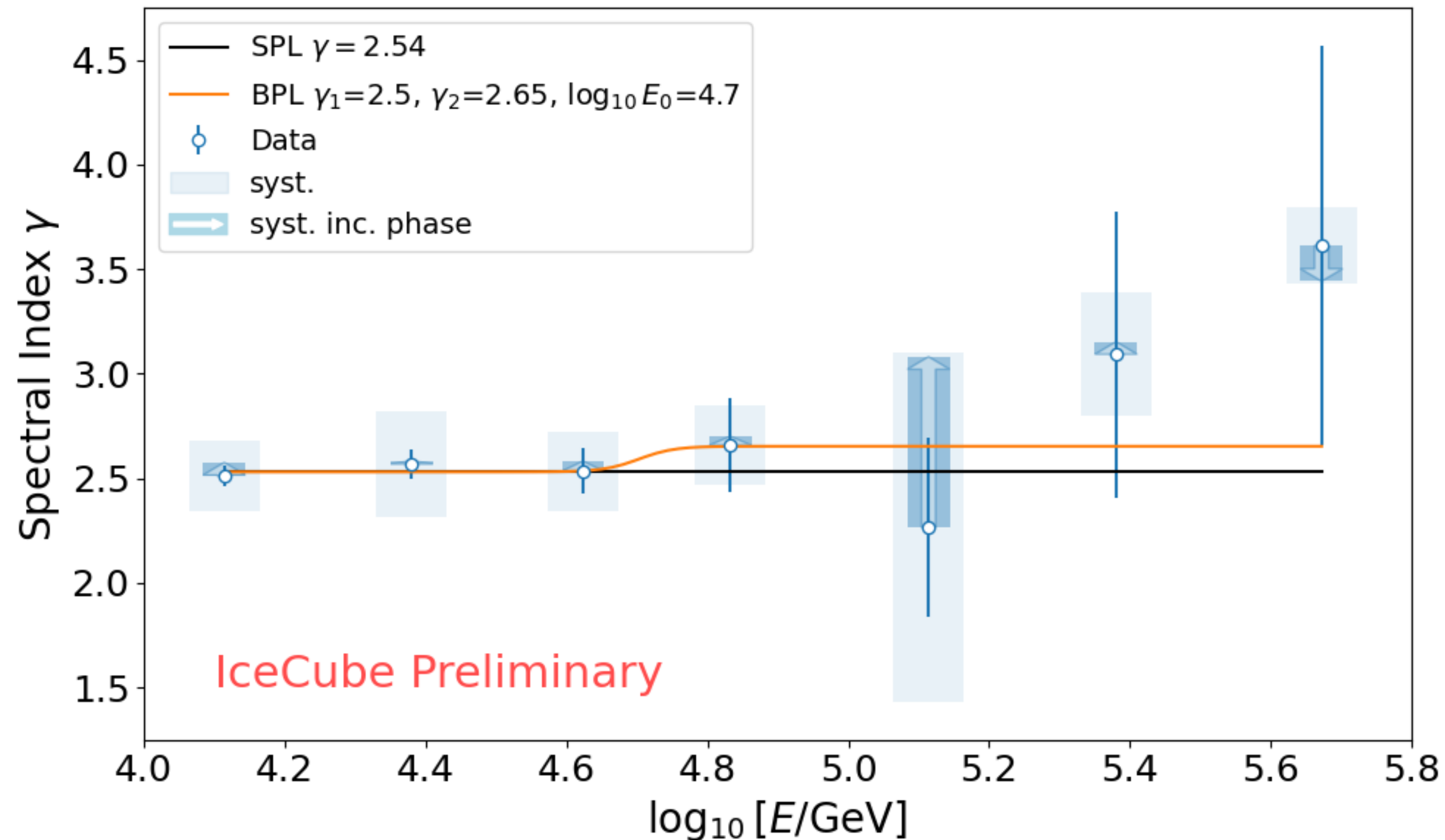
1. Broken power-law fit from HAWC
PoS(ICRC2023)364.
2. HAWC fit convolved with IceCube response
3. GlobalSplineFit2024 convolved with IceCube response.
4. Single power-law fit.
5. Broken power-law fit (described by sigmoid)



$$\gamma_1 + \frac{(\gamma_2 - \gamma_1)}{1 + e^{\epsilon(x_0 - x)}}, x = \log_{10} E$$

Time series fit

(Hypothesis Test)



Systematic uncertainties: calculated from anti-sidereal dipole amplitude and phase.

- Comparison to four different hypotheses.
- Data are consistent with single and broken power law scenarios due to large uncertainties

Model	Fit	χ^2	NDoF	p-val
HAWC	N/A	5.91	7	0.55
I3MC	N/A	2.75	7	0.91
GSF	N/A	4.39	7	0.73
SPL	$\gamma = 2.54 \pm 0.03$	3.09	6	0.8
BPL	$\gamma_1 = 2.53 \pm 0.03, \gamma_2 = 2.65 \pm 0.15, \log_{10} (E_0/\text{GeV}) = 4.7 \pm 0.9$	2.68	4	0.61

Conclusions



Measured the solar dipole amplitude for different energy bins with 12 years of cosmic-ray muon data.

Derived the corresponding spectral index from Compton-Getting formula.

Data are consistent with both single and broken power law scenarios due to large uncertainties.

Best fit for SPL is $\gamma=2.54\pm0.03$, and BPL $\gamma_1=2.53\pm0.03$, $\gamma_2=2.65\pm0.15$, with a break at $\log_{10}(E_0/\text{GeV})=4.70\pm0.87$.

Improved statistics to previous measurement from Tibet of $\gamma=2.99\pm0.54$ at 10 TeV (M. Amenomori, et al., 2008. <https://doi.org/10.1016/j.nuclphysbps.2007.11.044>).

Uncertainties on amplitude and phase of anti-sidereal distribution are smaller than statistical uncertainties for C–G.

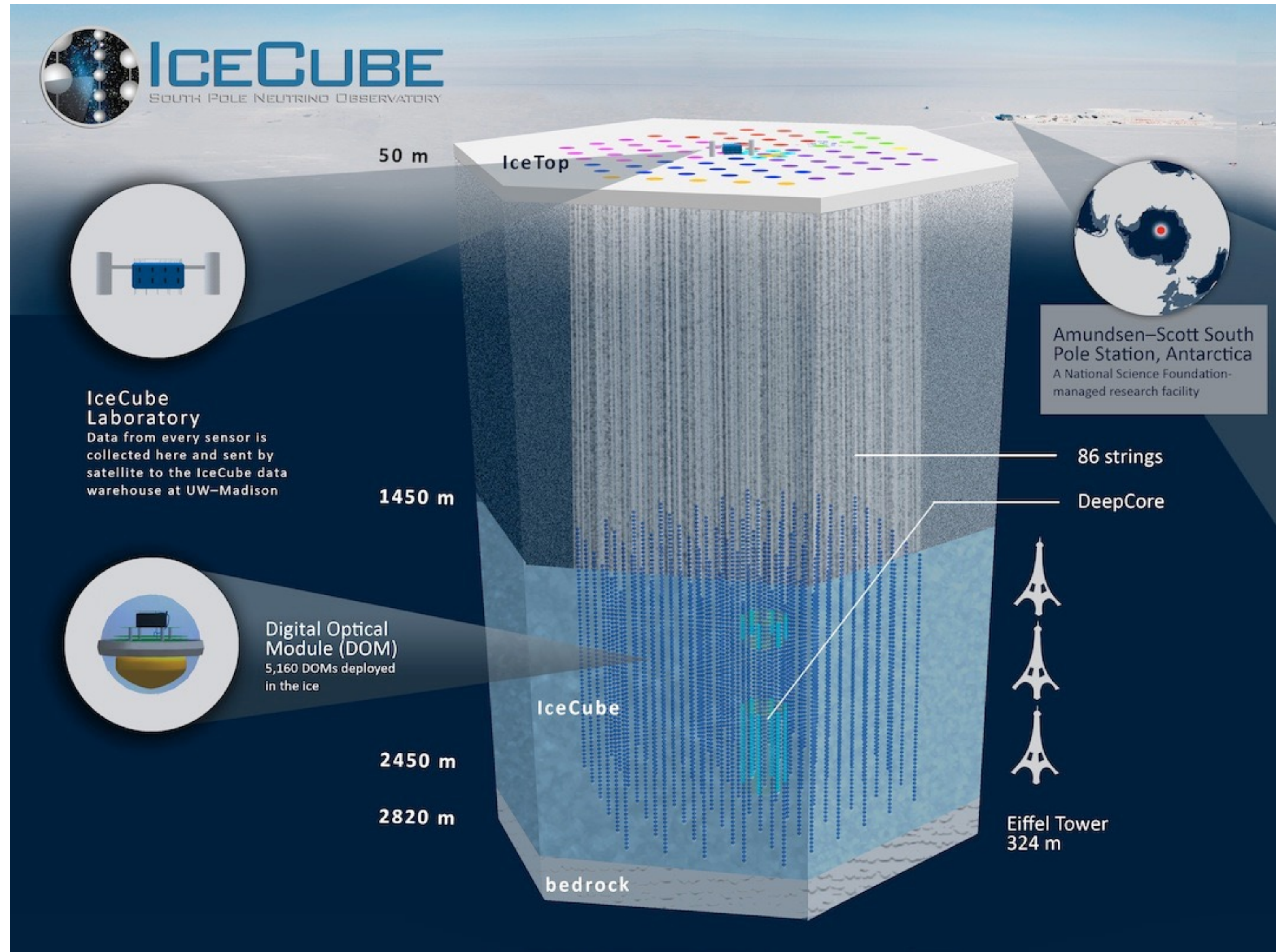
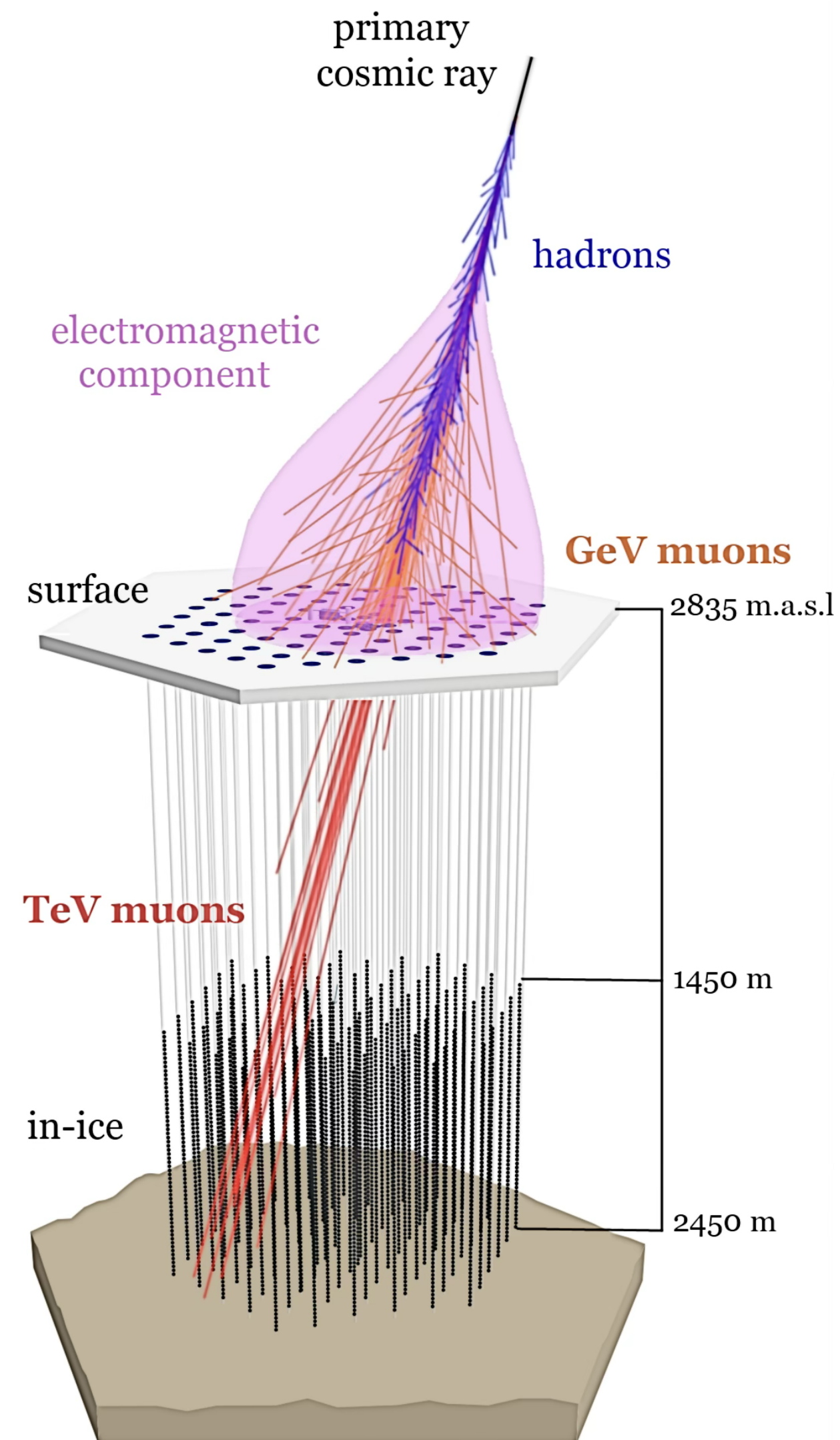
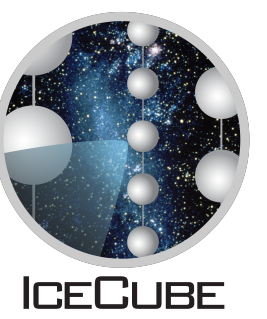
Not sensitive to variation of Earth's orbital speed due to eccentricity.



The presenter acknowledges support from NSF Grant #2209483

Backup

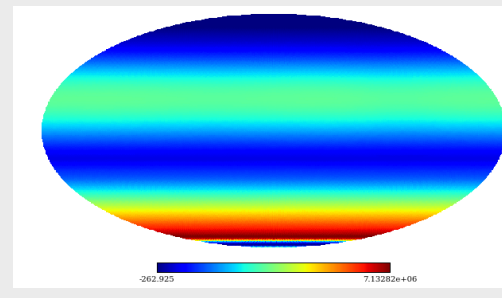
The IceCube Observatory



Method for measuring CR anisotropy

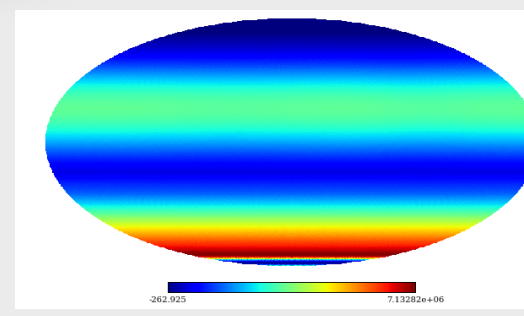
1

Build a binned data map using the equatorial coordinates of the events



2

Construct a “reference” map by integrating acceptance over 24 hours.

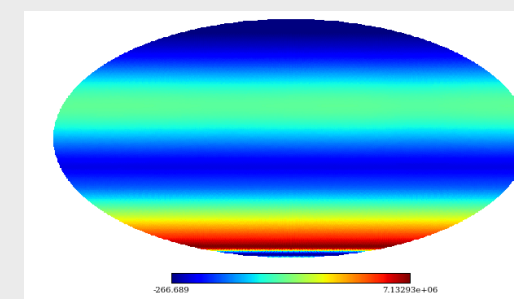


Maximum LLH method (M. Ahlers, et al. 2016):

$$\mathcal{L}(n|I, \mathcal{N}, \mathcal{A}) = \prod_{\tau i} \frac{(\mu_{\tau i})^{n_{\tau i}} e^{-\mu_{\tau i}}}{n_{\tau i}!}$$

3

Correlate pixels to increase sensitivity to different angular scales

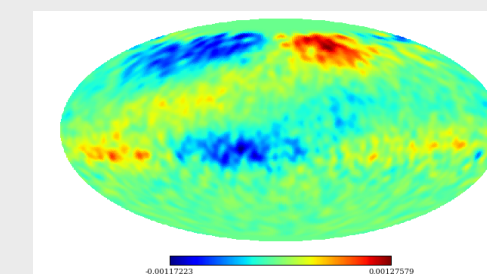


Relative Intensity

$$\delta I(\alpha, \delta)_i = \frac{N(\alpha, \delta)_i - \langle N \rangle(\alpha, \delta)_i}{\langle N \rangle(\alpha, \delta)_i}$$

4

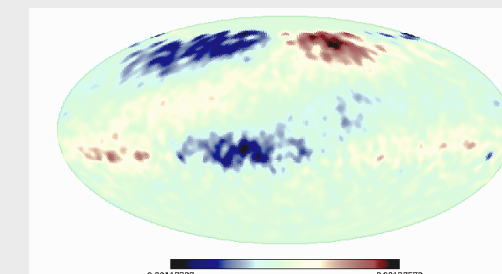
Calculate relative differences between data and reference with significance.



$$s_i = \sqrt{2} \left\{ N_i \log \left[\frac{1 + \alpha}{\alpha} \left(\frac{N_i}{N_i + N_o} \right) \right] + N_o \log \left[(1 + \alpha) \left(\frac{N_o}{N_i + N_o} \right) \right] \right\}^{1/2}$$

5

Calculate statistical significance for each pixel



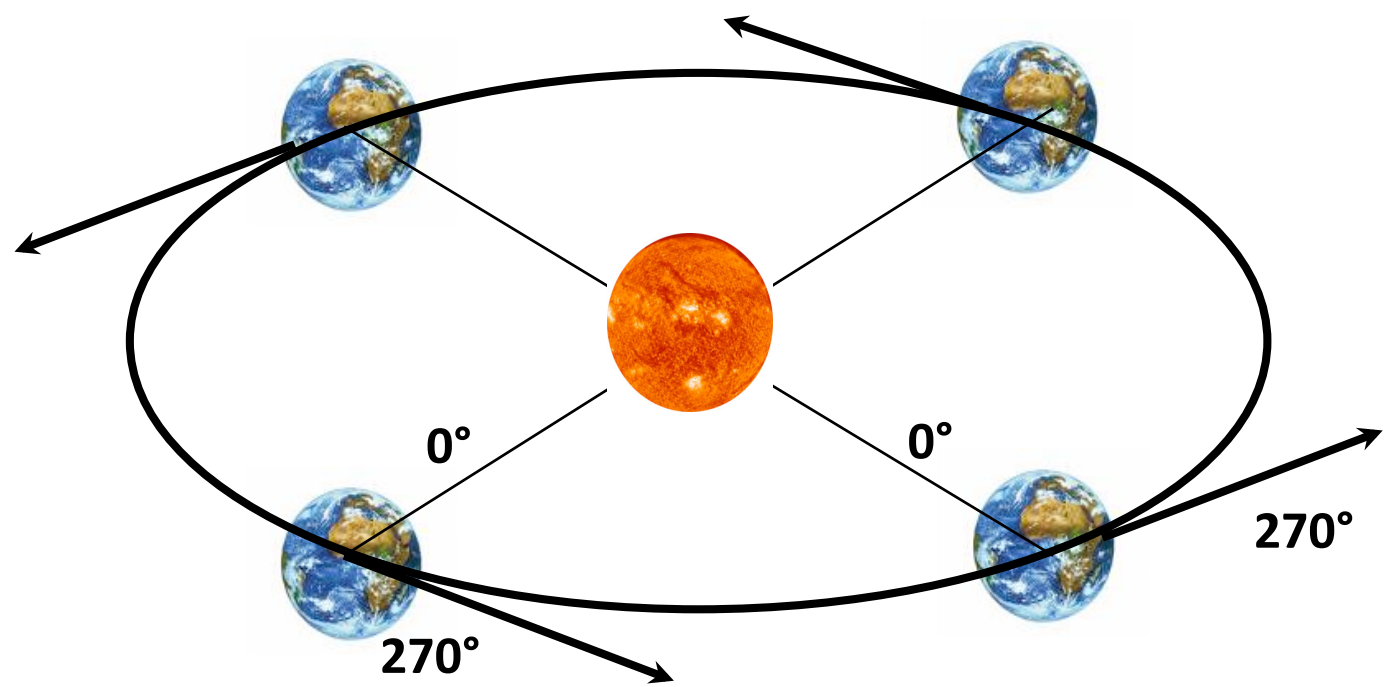
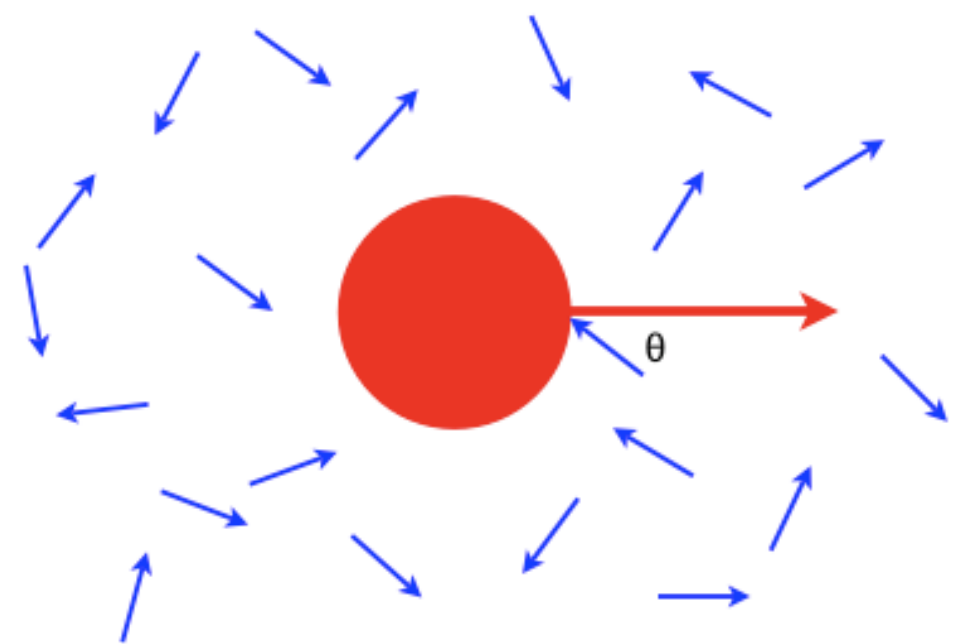
Compton-Getting Effect

Arthur H. Compton and Ivan A. Getting
 Phys. Rev. 47, 817 – June 1935 [doi:10.1103/PhysRev.47.817](https://doi.org/10.1103/PhysRev.47.817).



MIRIAM A. FORMAN, Planet. Space Sci. 1970, Vol. 18, pp. 25 to 31.

Any particle distribution function in phase space is invariant under Lorentz transformation.



$$f'(\mathbf{p}') = f(\mathbf{p})$$

The observer's motion speed is $v \ll c$ (e.g., Earth's orbital speed is about 30 km/sec, much smaller than the speed of light) so

$$\mathbf{p}' = \mathbf{p} + \frac{\gamma - 1}{v} (\mathbf{v} \cdot \mathbf{p}) \mathbf{v} - \gamma \frac{E}{c^2} \mathbf{v} \quad \longrightarrow \quad \mathbf{p}' \approx \mathbf{p} - \frac{E}{c^2} \mathbf{v}$$

And since cosmic-ray particles are ultra-relativistic, $E \approx pc$

$$\mathbf{p}' \approx \mathbf{p} - \frac{p}{c} \mathbf{v}$$

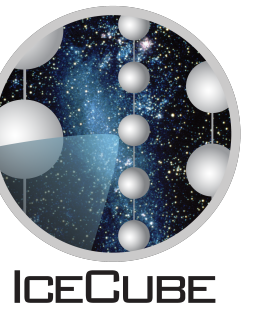


Compton-Getting Effect

By Taylor expansion the Lorentz invariance $f'(\mathbf{p}') = f(\mathbf{p})$ becomes

$$f'(\mathbf{p}') \simeq f_0(\mathbf{p}) - \underbrace{\frac{v}{c}(\mathbf{p} \cdot \mathbf{v})p \frac{\partial f_0}{\partial p}}_{\text{first order term}} + \underbrace{\frac{1}{2} \left(\frac{v}{c}\right)^2 \left[(\mathbf{p} \cdot \mathbf{v})^2 p^2 \frac{\partial^2 f_0}{\partial p^2} + (1 - (\mathbf{p} \cdot \mathbf{v})^2) p \frac{\partial f_0}{\partial p} \right]}_{\text{second order term}}$$

where $f'(\mathbf{p}) \equiv f_0(\mathbf{p})$ is the isotropic cosmic-ray particle distribution function in the *un-primed* reference system.



First Order C-G

The first-order term

$$f'(\mathbf{p}') \simeq f_0(\mathbf{p}) - \frac{v}{c}(\mathbf{p} \cdot \mathbf{v})p \frac{\partial f_0}{\partial p}$$

can be rewritten as

$$f'(\mathbf{p}') \simeq f_0(\mathbf{p}) \left[1 - \frac{v}{c} \cos \theta \frac{p}{f_0} \frac{\partial f_0}{\partial p} \right] = f_0(\mathbf{p}) \left[1 - \frac{v}{c} \cos \theta \frac{\partial \ln f_0}{\partial \ln p} \right]$$



First Order C-G

..or in terms of particle intensity

$$I' \simeq I_0 \left[1 - \frac{v}{c} \cos \theta \frac{\partial \ln I_0}{\partial \ln p} \right]$$

typically expressed as

$$I(E) \equiv \frac{dN}{dE} \propto E^{-\gamma}$$

or in momentum

$$I(p) \equiv \frac{dN}{p^2 dp} \propto p^{-(2+\gamma)}$$

Relative intensity:

$$\frac{dI}{I} \simeq \frac{v}{c} (2 + \gamma) \cos(\theta)$$

Horizontal Multi-pole Fit

The dipole component is given by the $Y_{\ell m}$ terms

$$Y_1^{-1}(\alpha, \delta) = \frac{1}{2} \sqrt{\frac{3}{2\pi}} \cos \delta e^{-i\alpha},$$

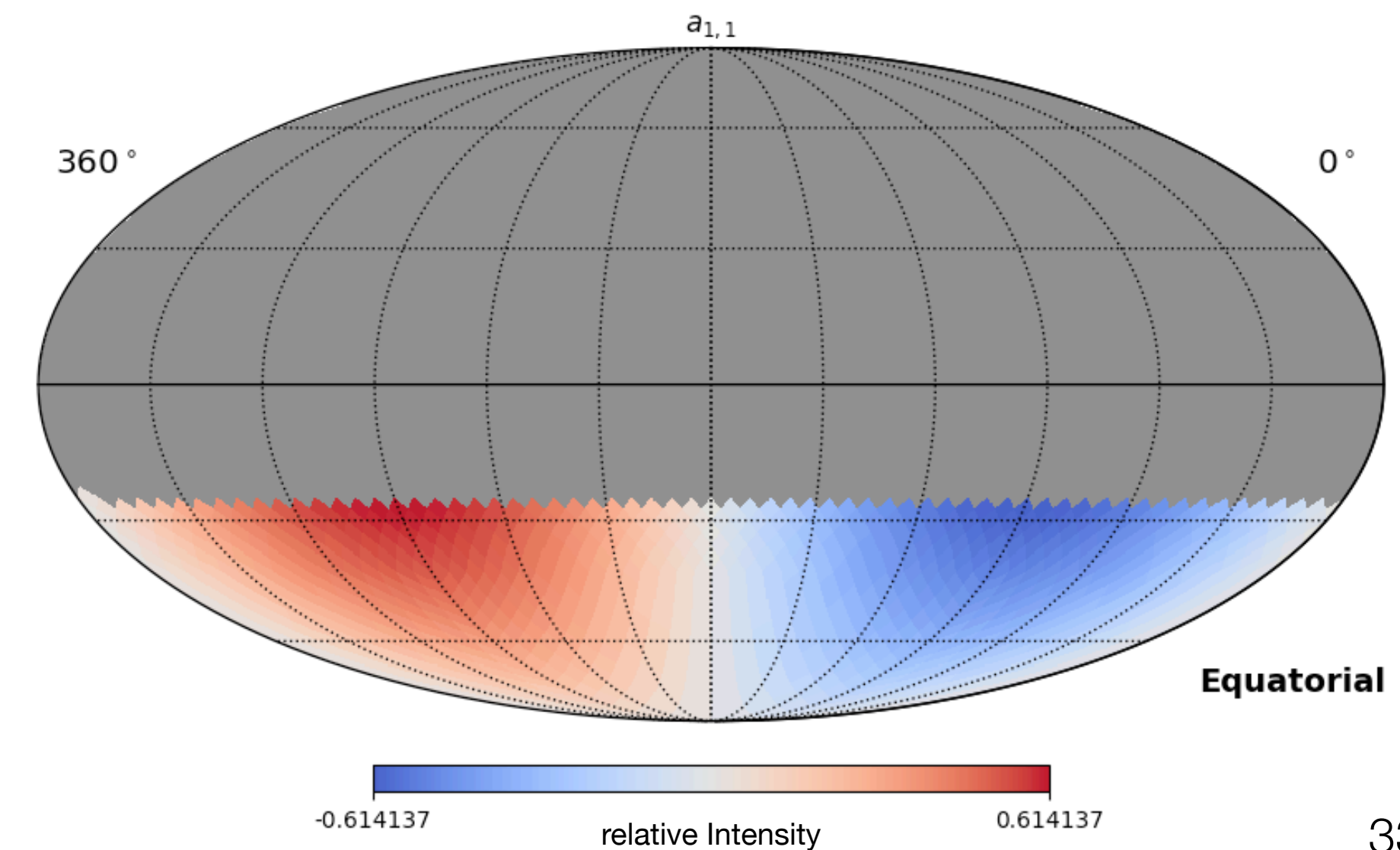
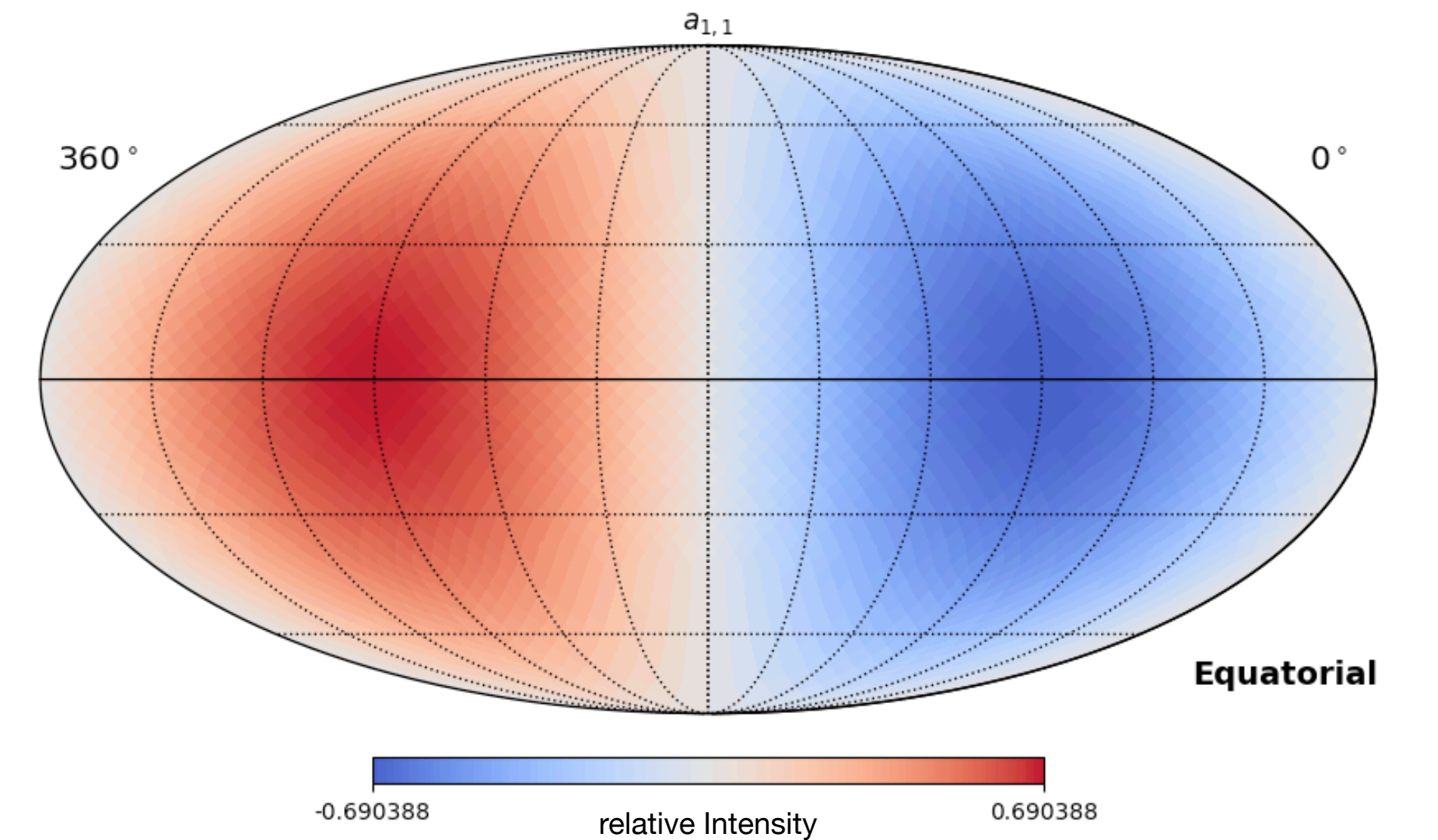
$$Y_1^0(\alpha, \delta) = \frac{1}{2} \sqrt{\frac{3}{\pi}} \sin \delta,$$

$$Y_1^1(\alpha, \delta) = -\frac{1}{2} \sqrt{\frac{3}{2\pi}} \cos \delta e^{i\alpha}$$

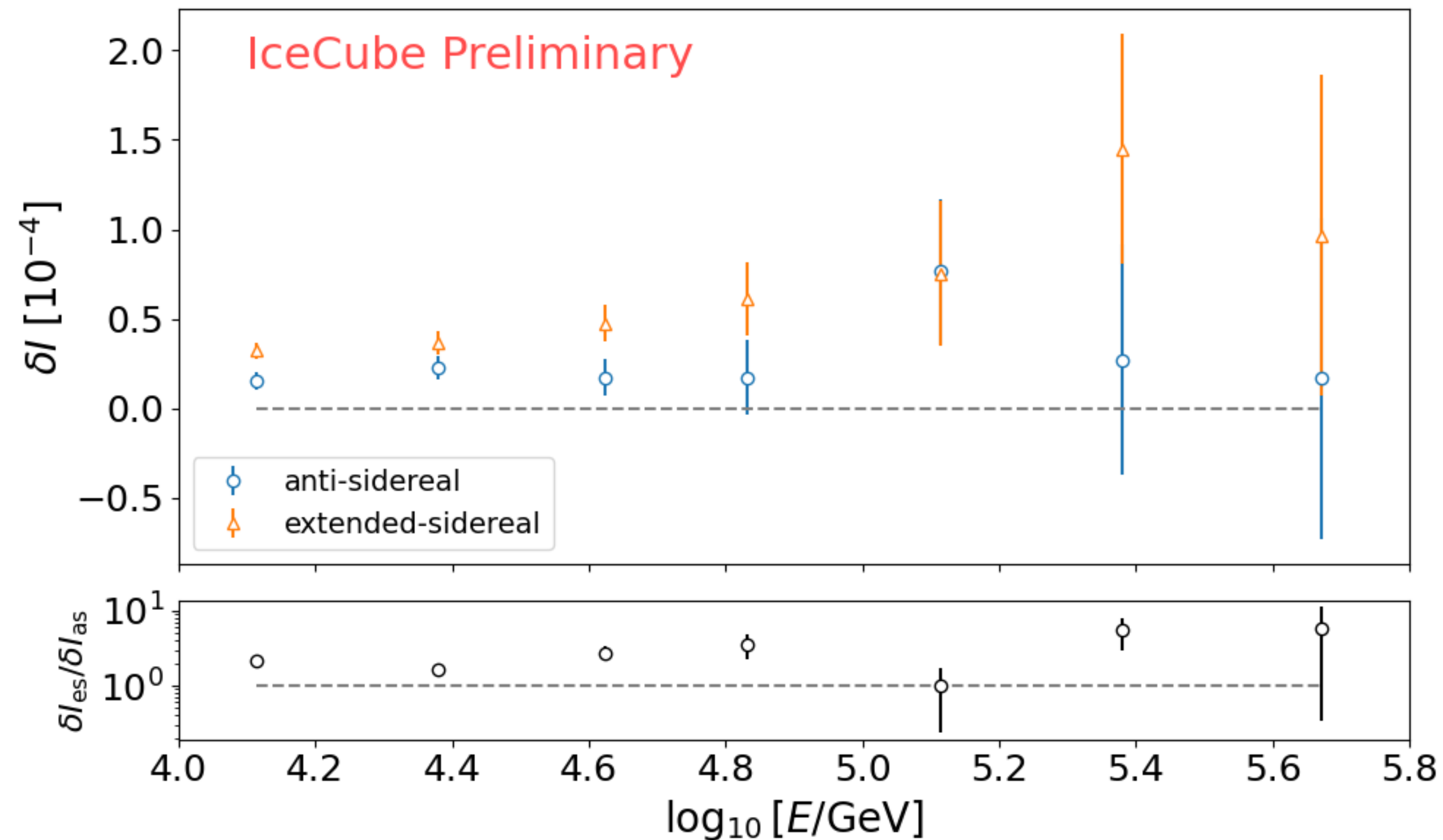
So, for a dipole oriented along the x-y plane,

$$F(\alpha_i, \delta_i) = a_{1,-1} Y_1^{-1} + a_{1,1} Y_1^1 = A_1 \cos(\delta_i) \cos(\alpha_i - \phi_1)$$

We fit parameters: A_1 , ϕ_1 over the pixels in the FoV

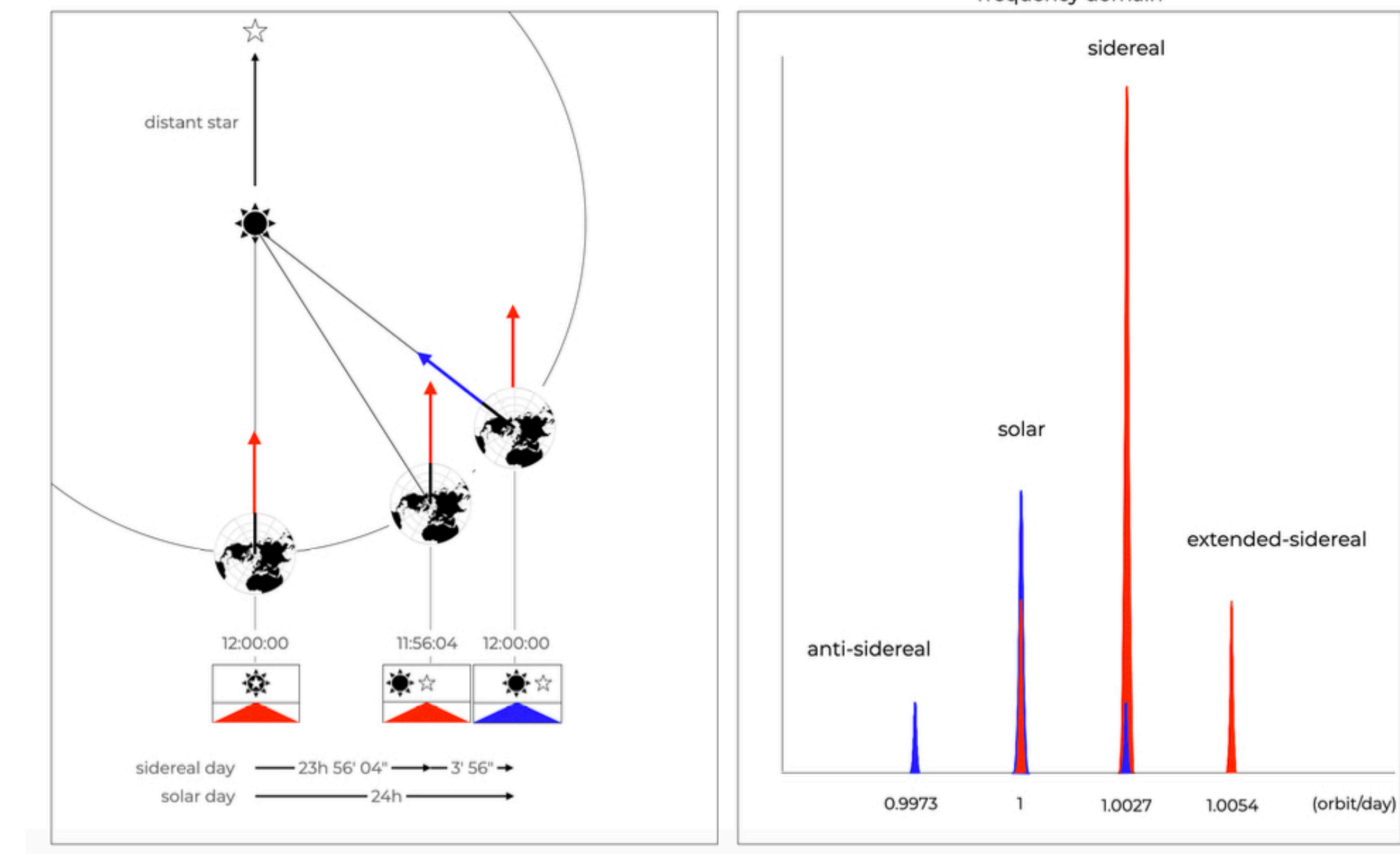


Systematic Biases



extended-sidereal distribution is produced by a **yearly modulation** of the sidereal-dipole amplitude and it **deforms** the solar distribution. The extended-sidereal amplitude is $\sim 3\times$ larger than the anti-sidereal amplitude.

J.C. Díaz-Vélez et al. DOI: <https://doi.org/10.22323/1.395.0085>



Mutual interference between modulations in solar and sidereal frames produces frequency side-bands around both peaks. The other side-bands overlap with sidereal and solar frequencies, producing a deformation of the CRA.

What is up with the bin at 100 TeV?

This is where the dipole flip happens and it is likely very sensitive small variations in energy that may produce an unstable signal.

