Assessing the Diffusion-Drift Transition as the Origin of the Cosmic Ray Knee

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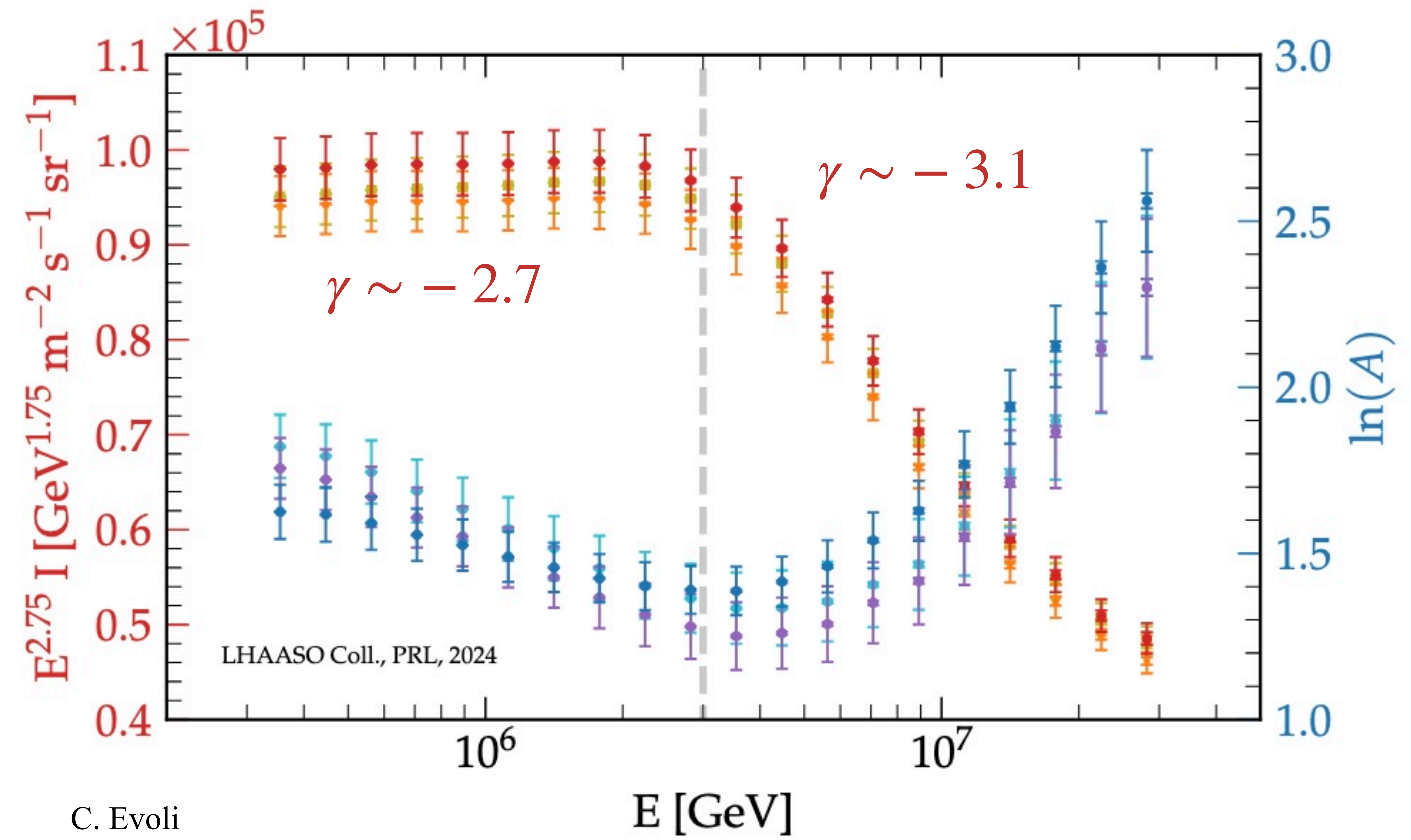
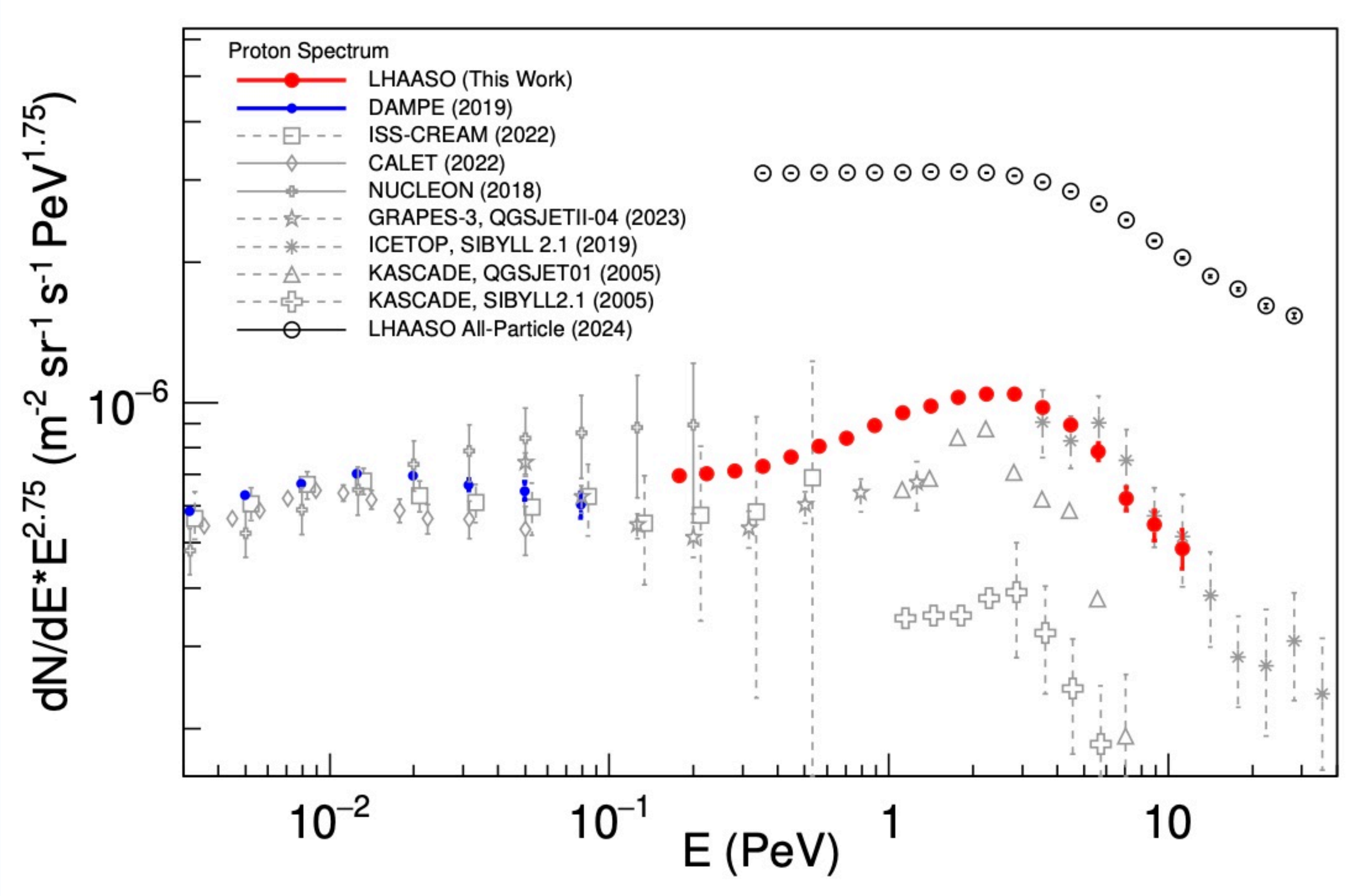
The Cosmic Ray Knee

Recently the cosmic ray **all-particle spectrum** and **composition** has been measured with high precision by LHAASO across the knee **[Cao+2024]**.

They find a knee position in the spectrum at 3.67 ± 0.15 PeV, consistent with the knee in $\langle \ln A \rangle$,and spectral index change $\Delta\gamma \sim 0.4$



Recently a measurement of the proton flux across the knee has also been reported by LHAASO **[Cao+2025]**.



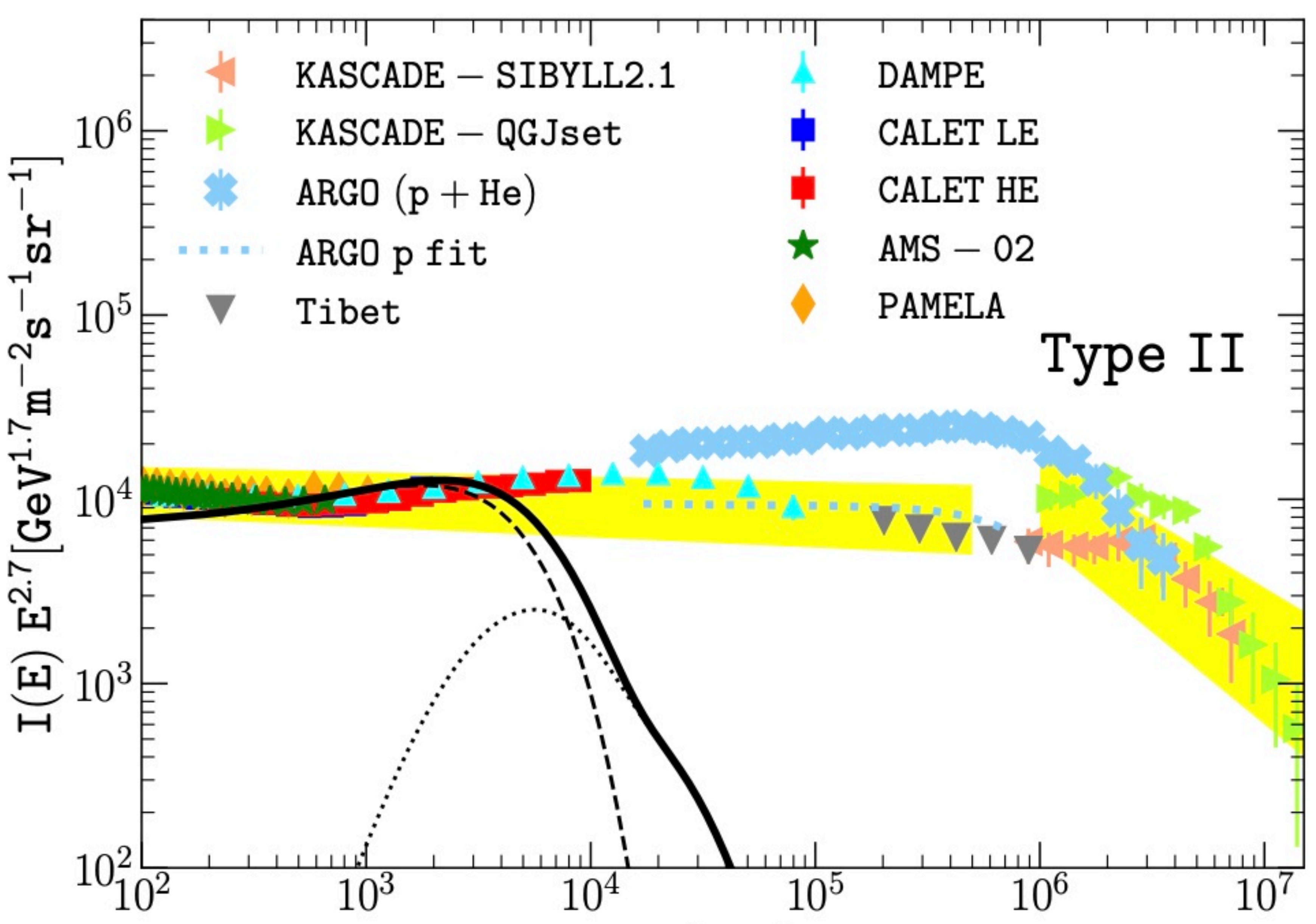
The Cosmic Ray Knee

Source Origin

Knee explained by **maximum energy** achieved by Galactic accelerators.

Sharp spectral break at PeV energies could be attributed to evolution of SNR **[Cristofari+2020]**.

Standard predictions of spectral change imply $\Delta\gamma \gtrsim 2$.

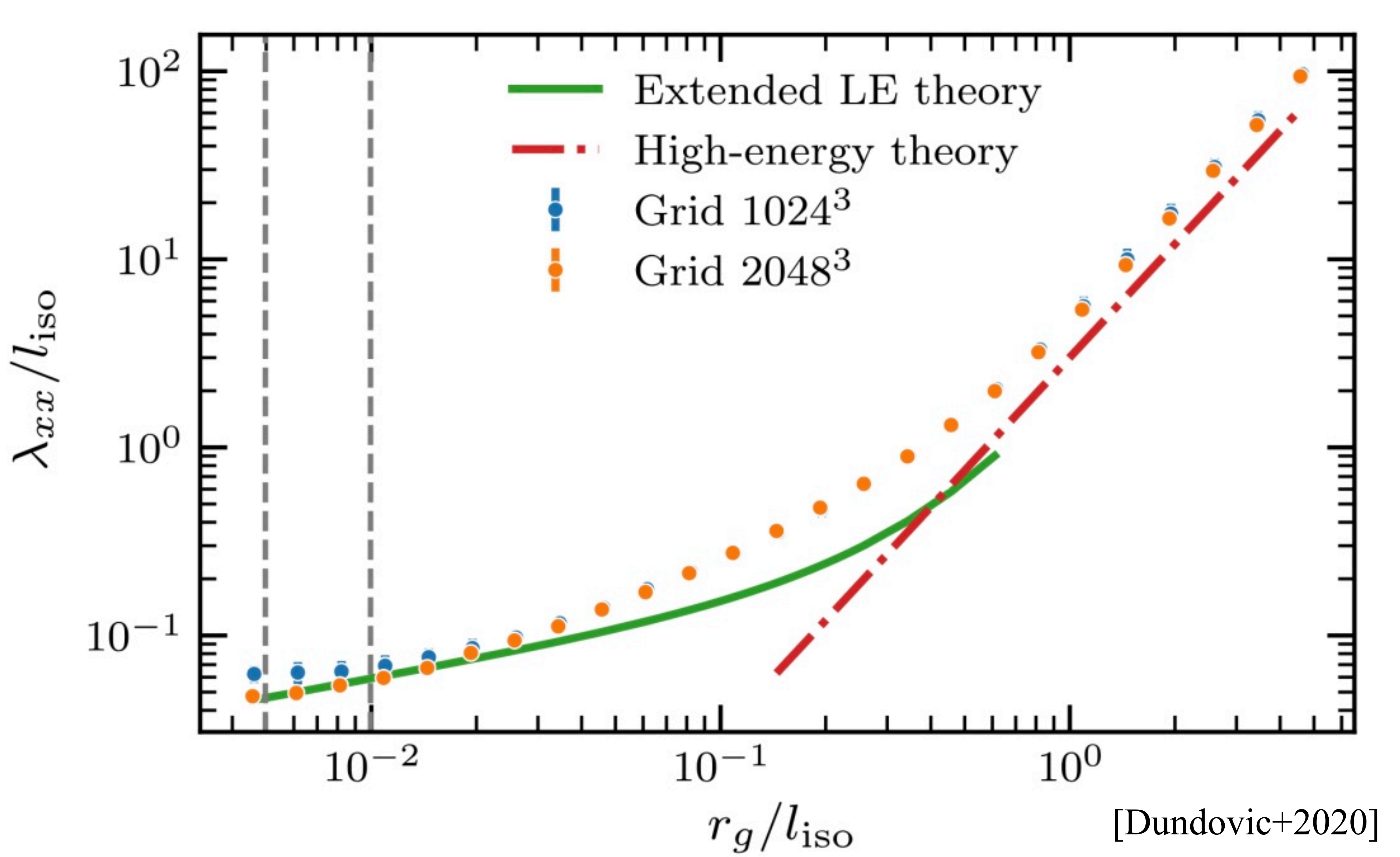


[Cristofari+2020]

Diffusion Origin

Particles above knee exit **diffusion resonant regime** $\left(\tau \propto E^{-1/3}\right)$ and enter **small-pitch-angle scattering** $\left(\tau \propto E^{-2}\right)$

This produces a spectral change $\Delta\gamma \sim 1.7$ **[Dundovic+2020]**



[Dundovic+2020]

Diffusion-Drift Transition as the Origin of the Knee

Diffusion tensor: $D_{ij} = (D_\parallel - D_\perp) b_i b_j + D_\perp \delta_{ij} - D_A \epsilon_{ijk} b_k$

Cosmic ray transport: $-\nabla_i D_{ij}(\mathbf{r}) \nabla_j N(\mathbf{r}) = Q(\mathbf{r})$

Under cylindrical coordinates and azimuthal symmetry: $\left(-\frac{1}{r}\frac{\partial}{\partial r}\left[r D_\perp \frac{\partial}{\partial r} \right] - \frac{\partial}{\partial z}\left[D_\perp \frac{\partial}{\partial z} \right] + u_r \frac{\partial}{\partial r} + u_z \frac{\partial}{\partial z} \right) N(\mathbf{r}) = Q(\mathbf{r})$

$\Longrightarrow$ Drift velocities $u_r = -\frac{\partial(D_A b_\phi)}{\partial z}$, $u_z = \frac{1}{r}\frac{\partial(r D_A b_\phi)}{\partial r}$ associated to Hall diffusion.

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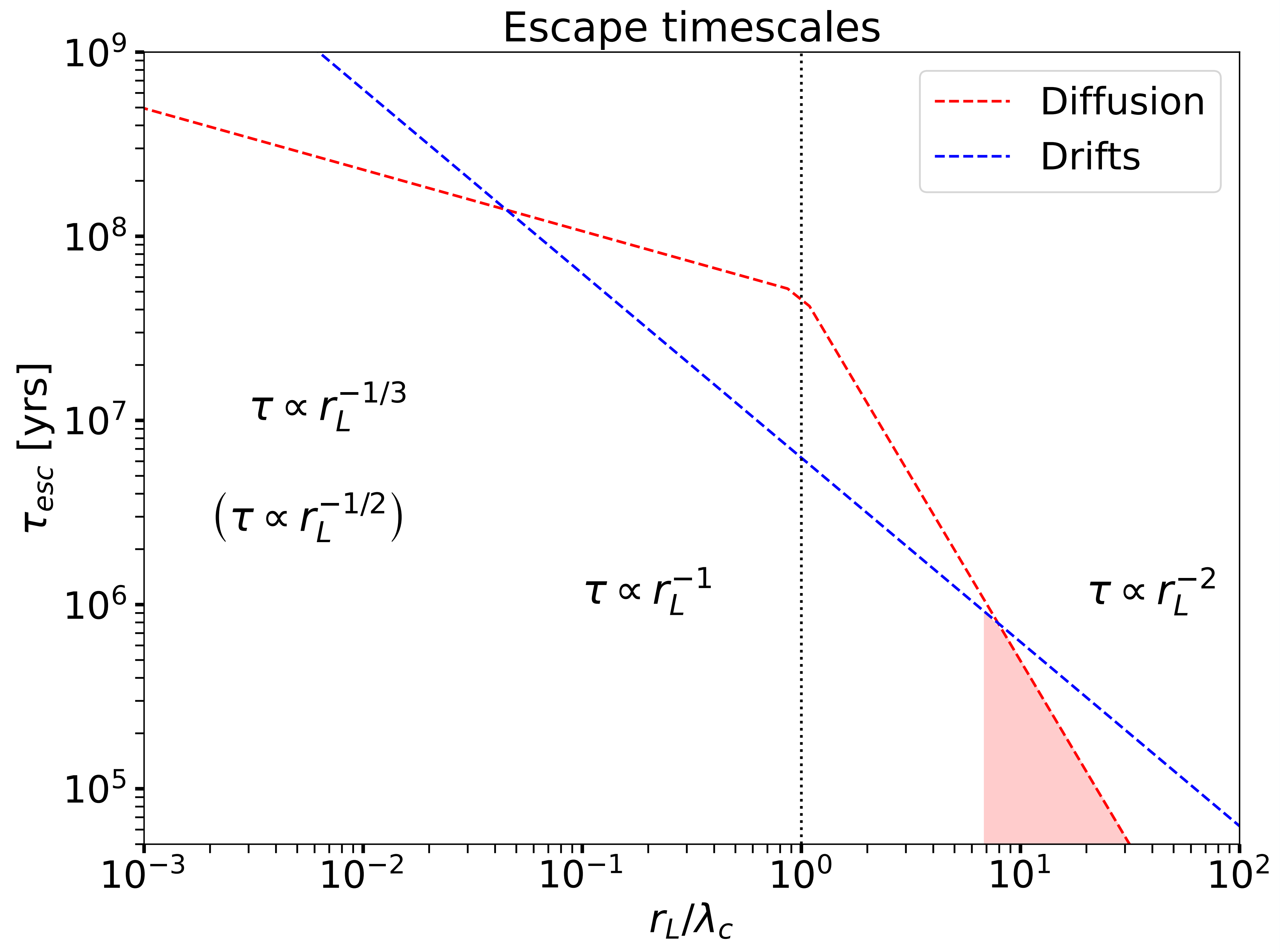
- Original idea of transition from perpendicular diffusion to drifts proposed in **[Ptuskin+1993]**, later re-explored in **[Candia+2003]** in a numerical-analytical hybrid approach.

✦ For how many energy decades can drifts extend?

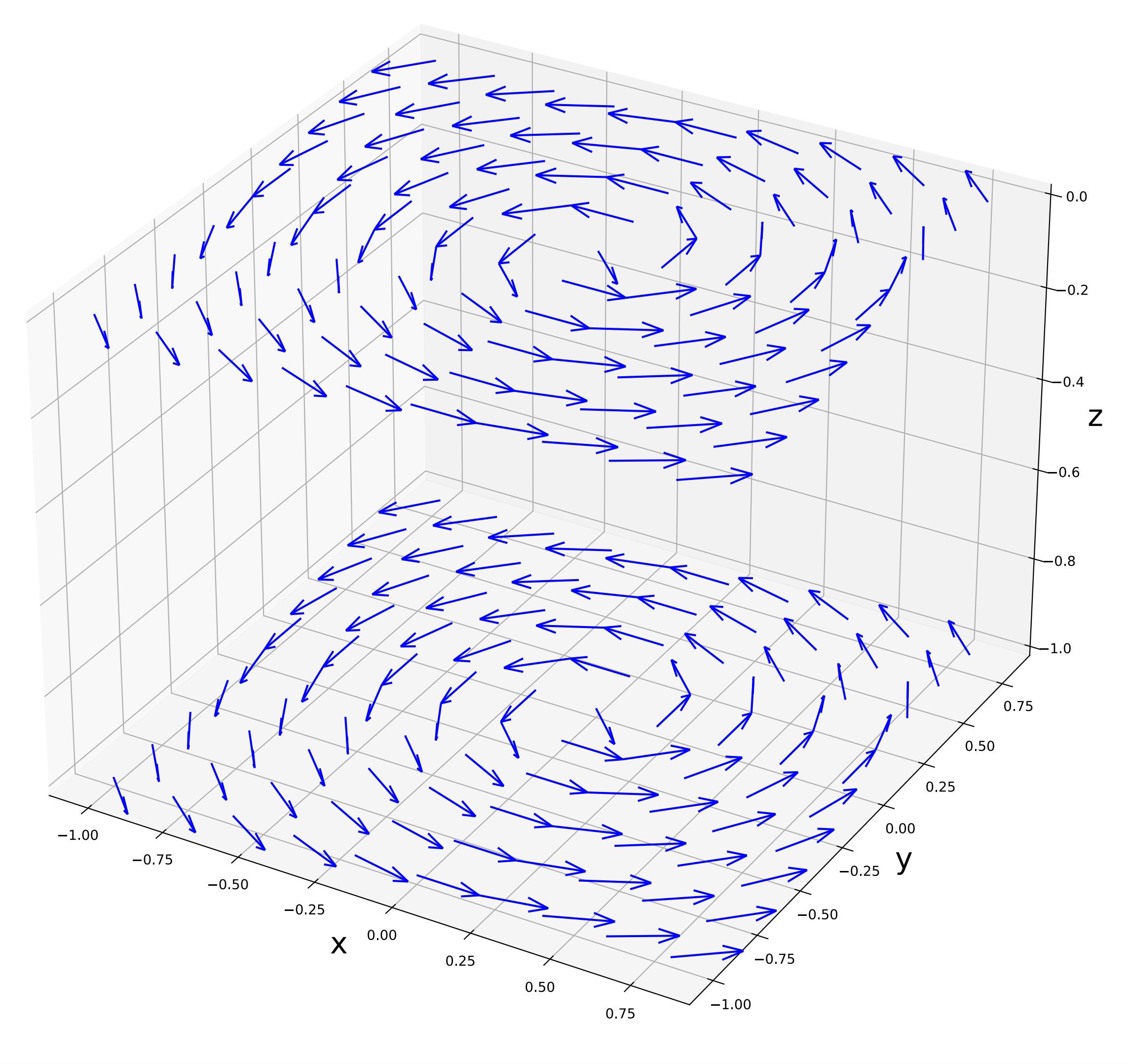
✦ What is the dependency on the structure of the magnetic field?

✦ Can the transition explain spectral and gramme observations simultaneously?



Here: test-particle simulations with synthetic turbulent magnetic fields monitoring diffusive motion, timescale of cosmic ray escape and grammage all together for the first time.

$$\eta = \frac{\delta B_{rms}}{B_0}$$



Galaxy-like magnetic field + turbulence

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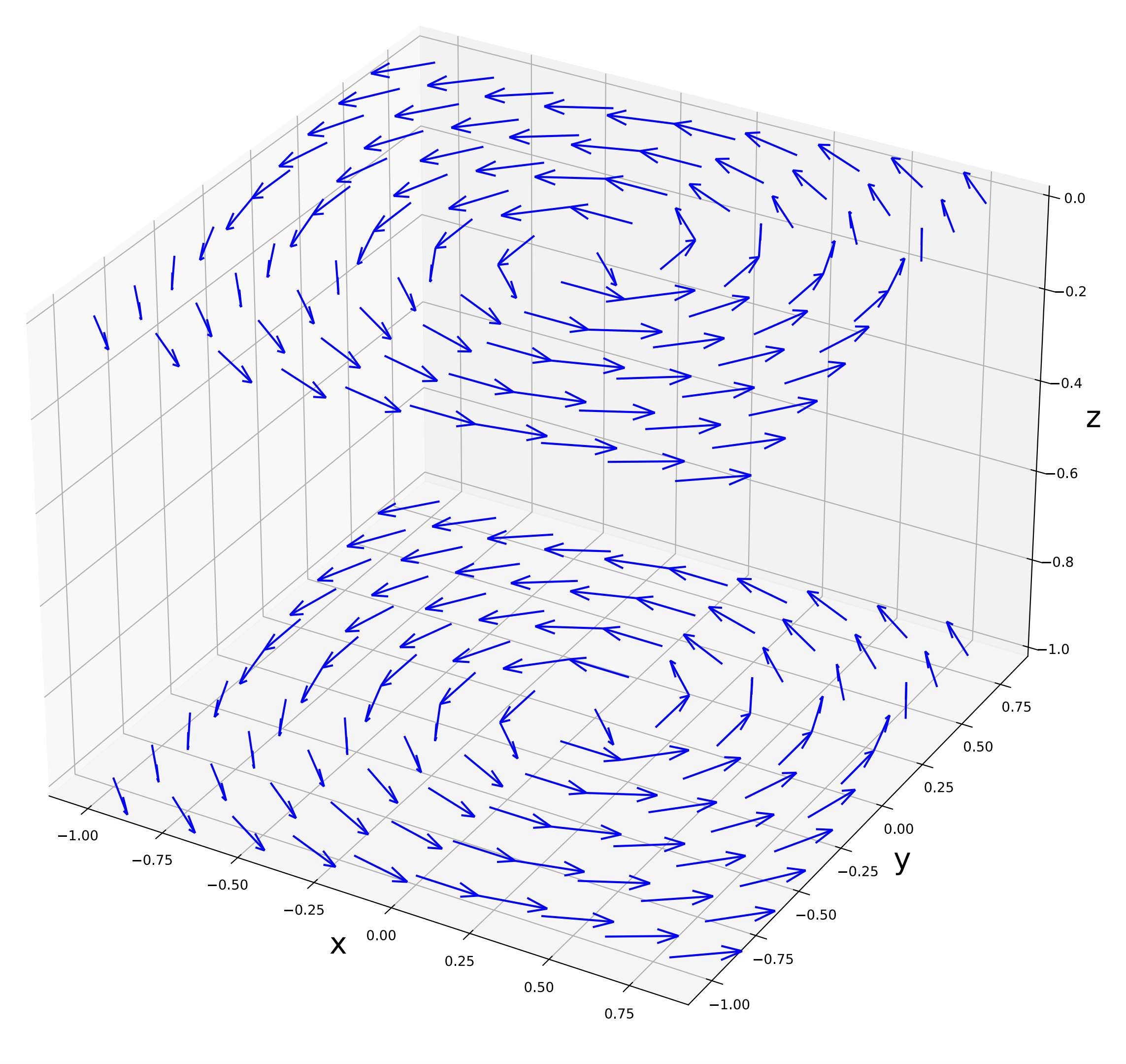
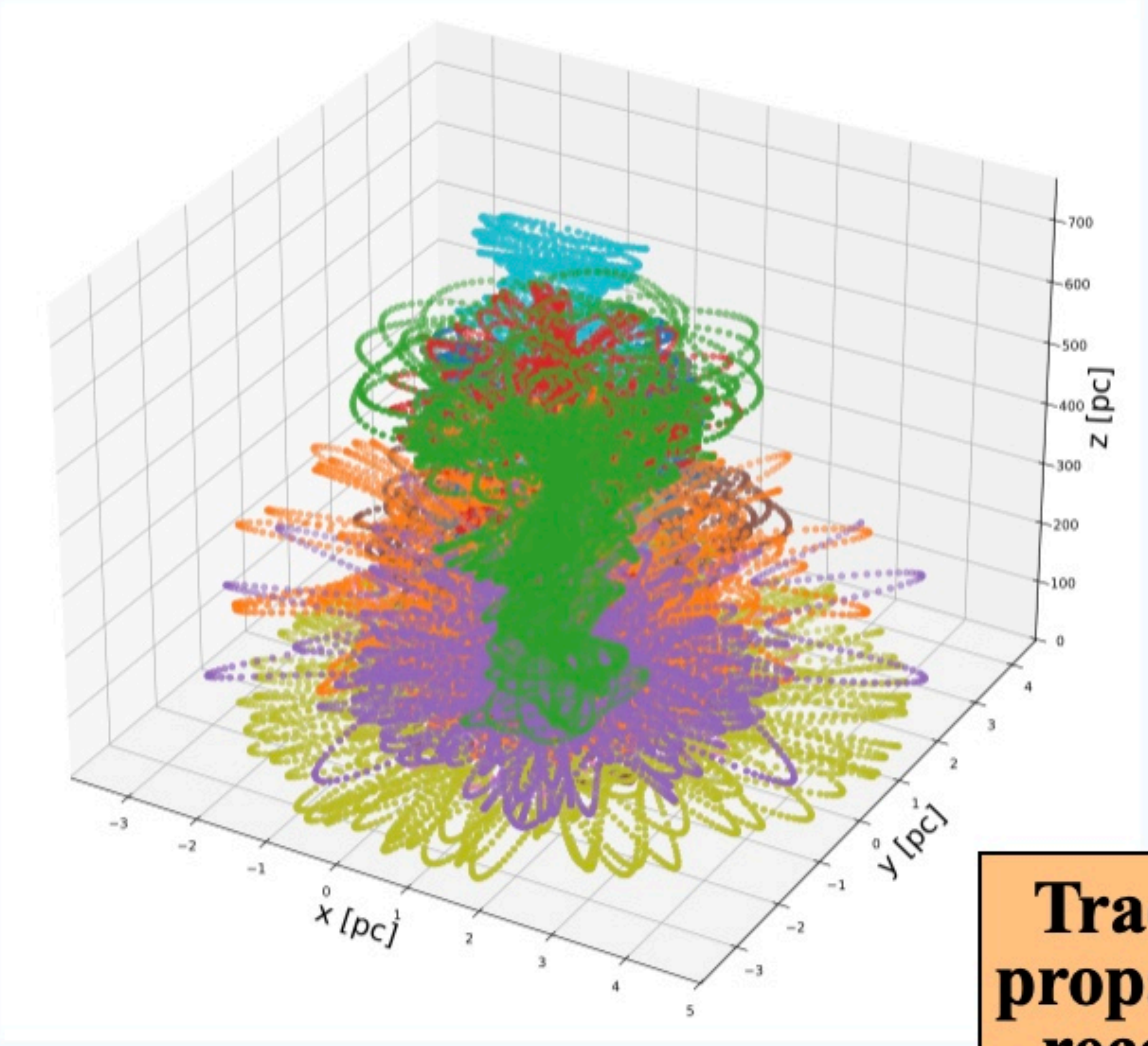
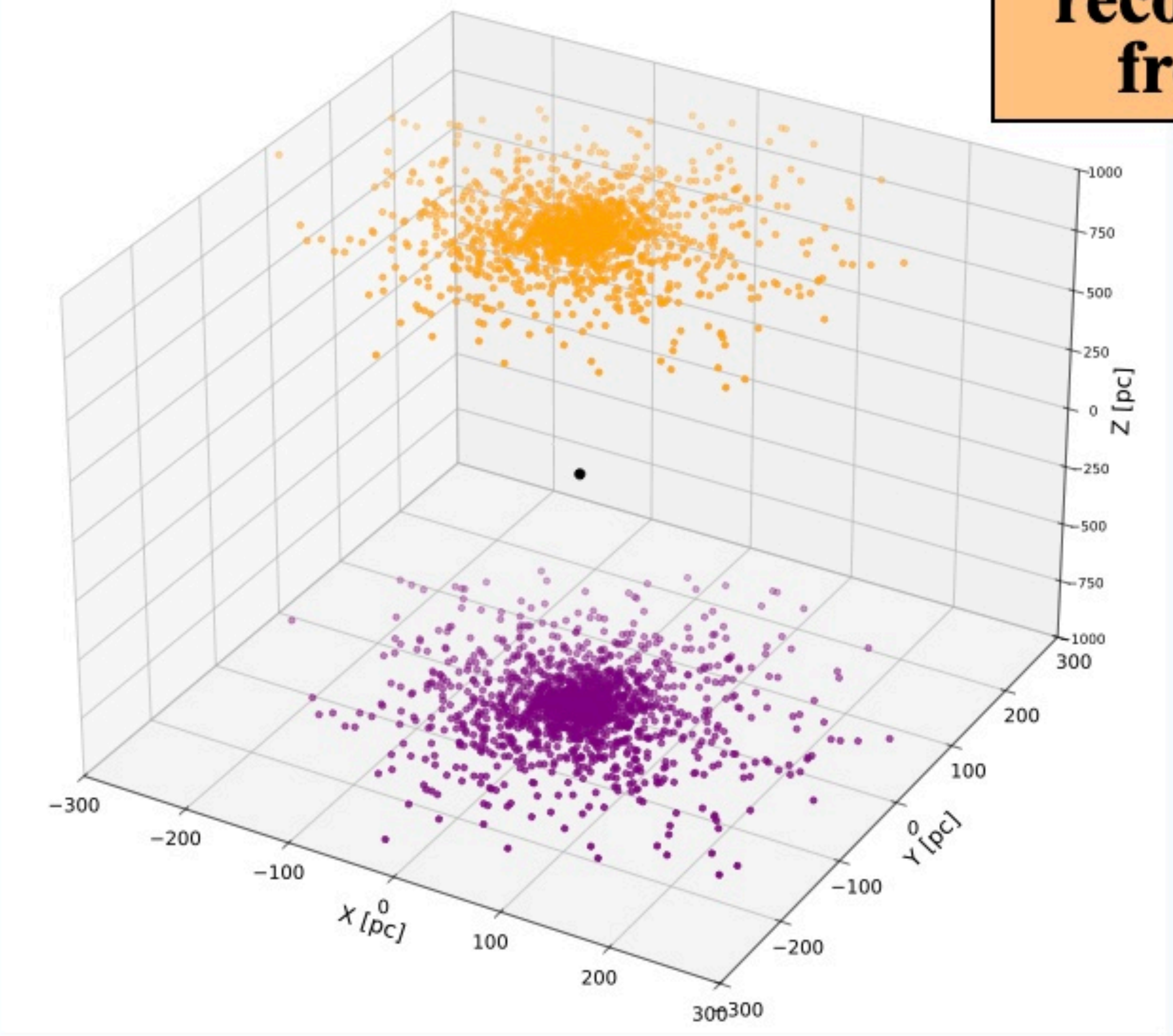


Galaxy-like magnetic field + turbulence

Particle injection

CRPropa [Alves Batista et al., 2022]





Track particle propagation and record escape from halo

[Alves Batista et al., 2022]

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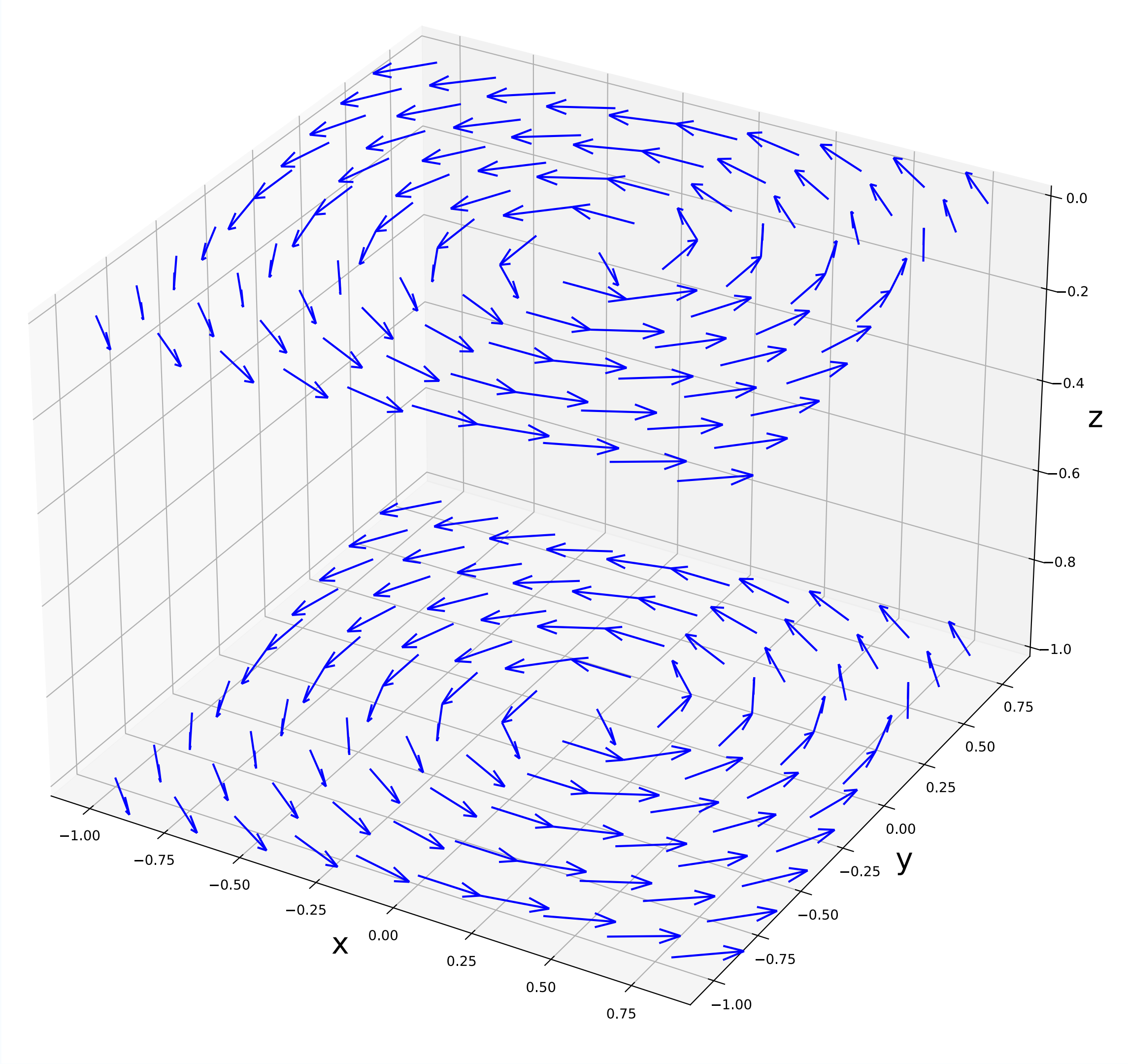
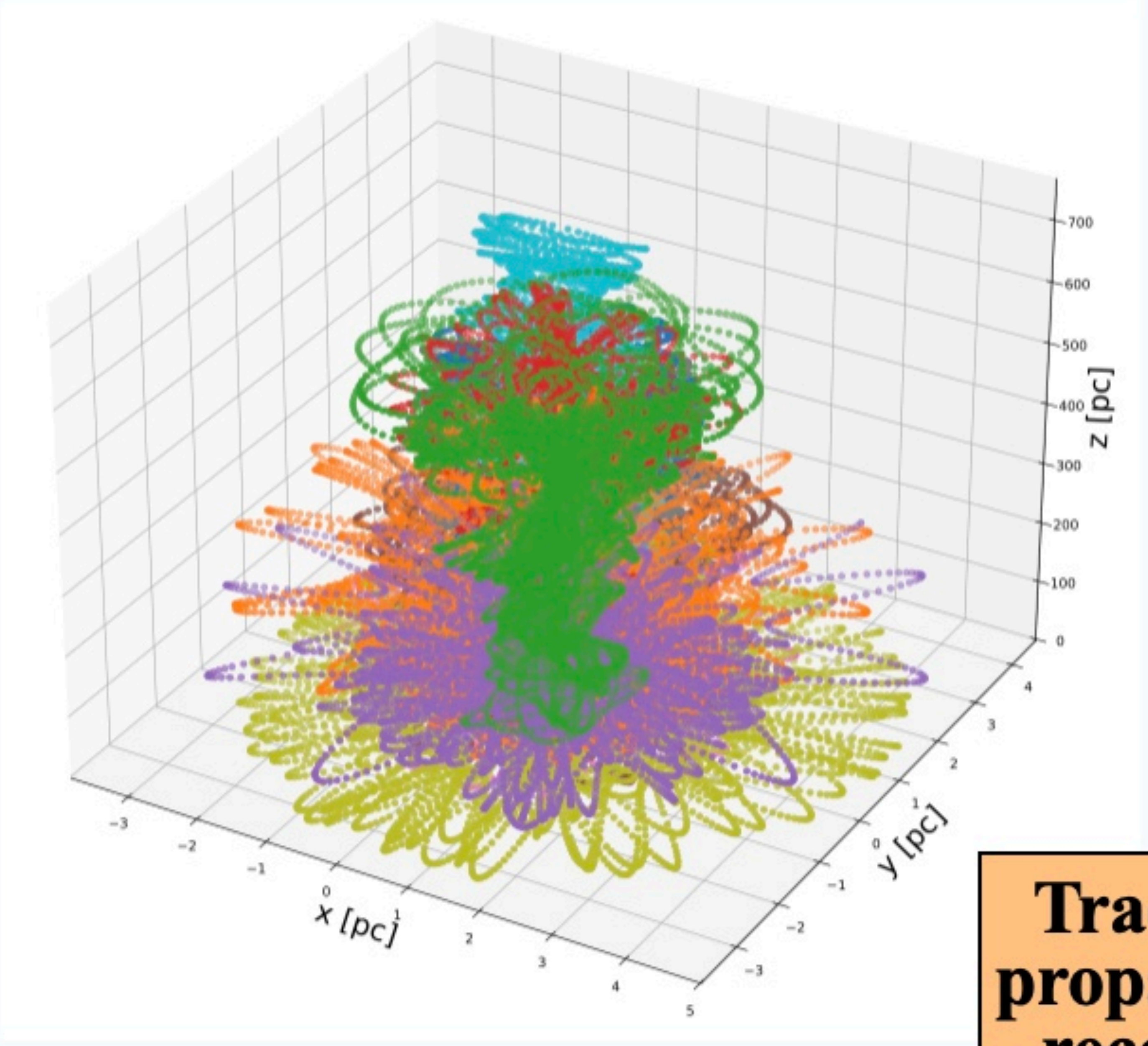


Particle injection



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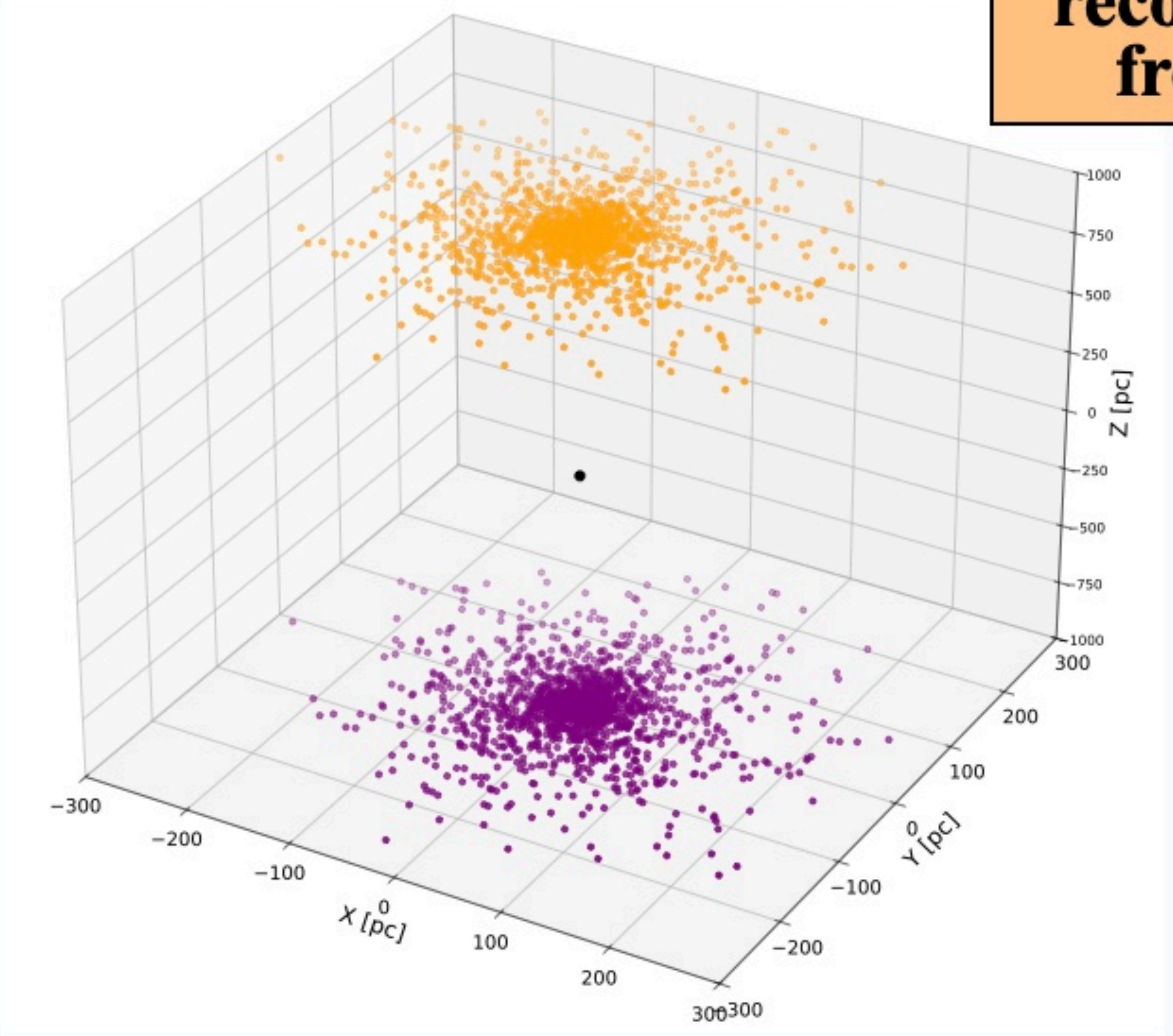
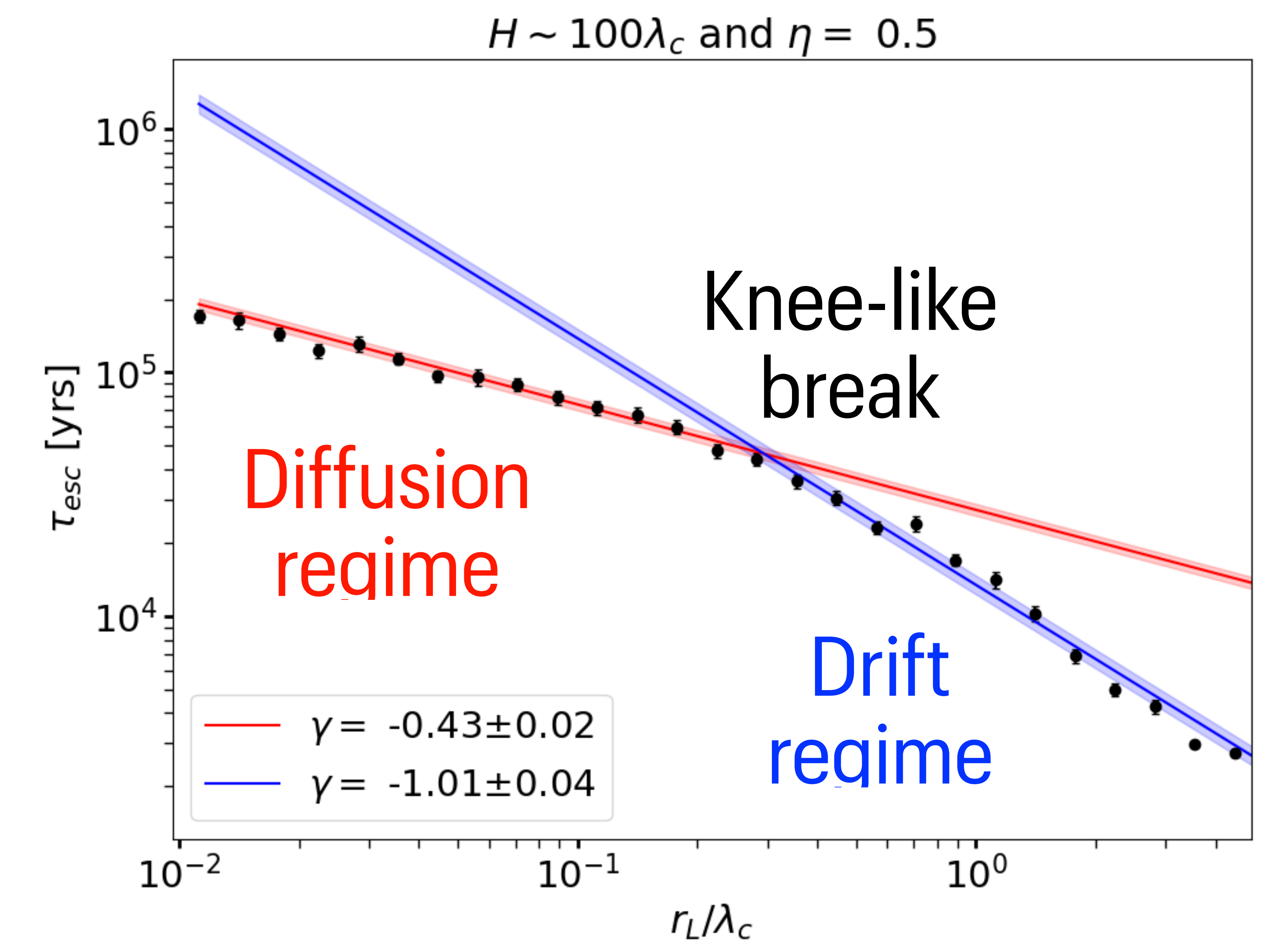
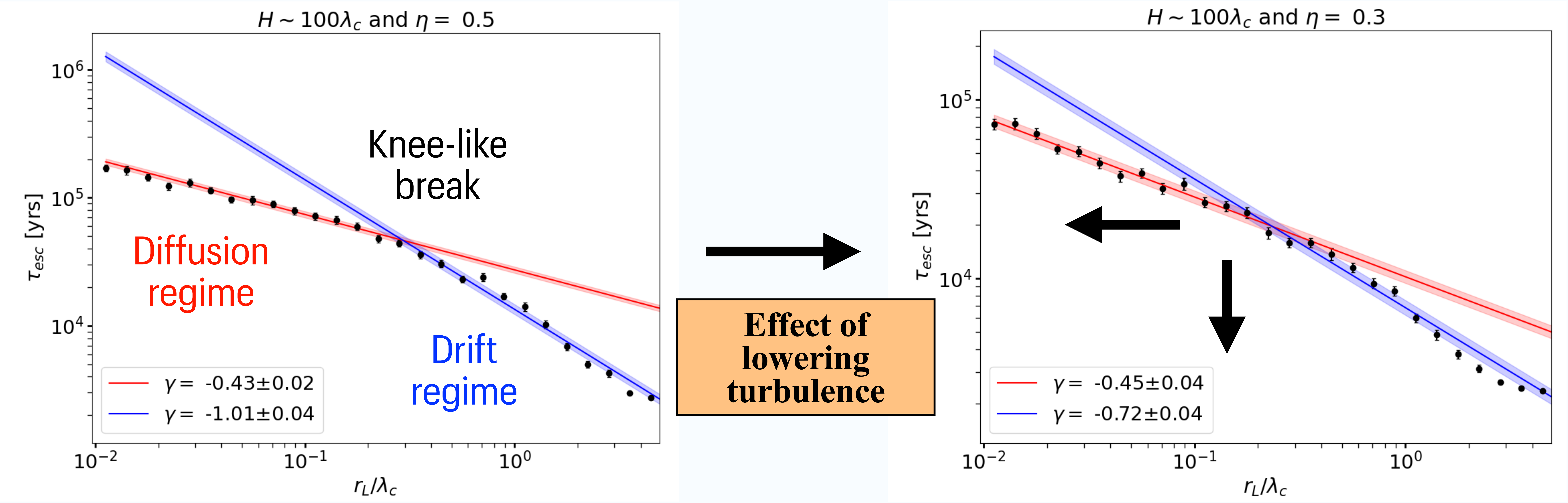


Compute escape times and grammage



Azimuthal field



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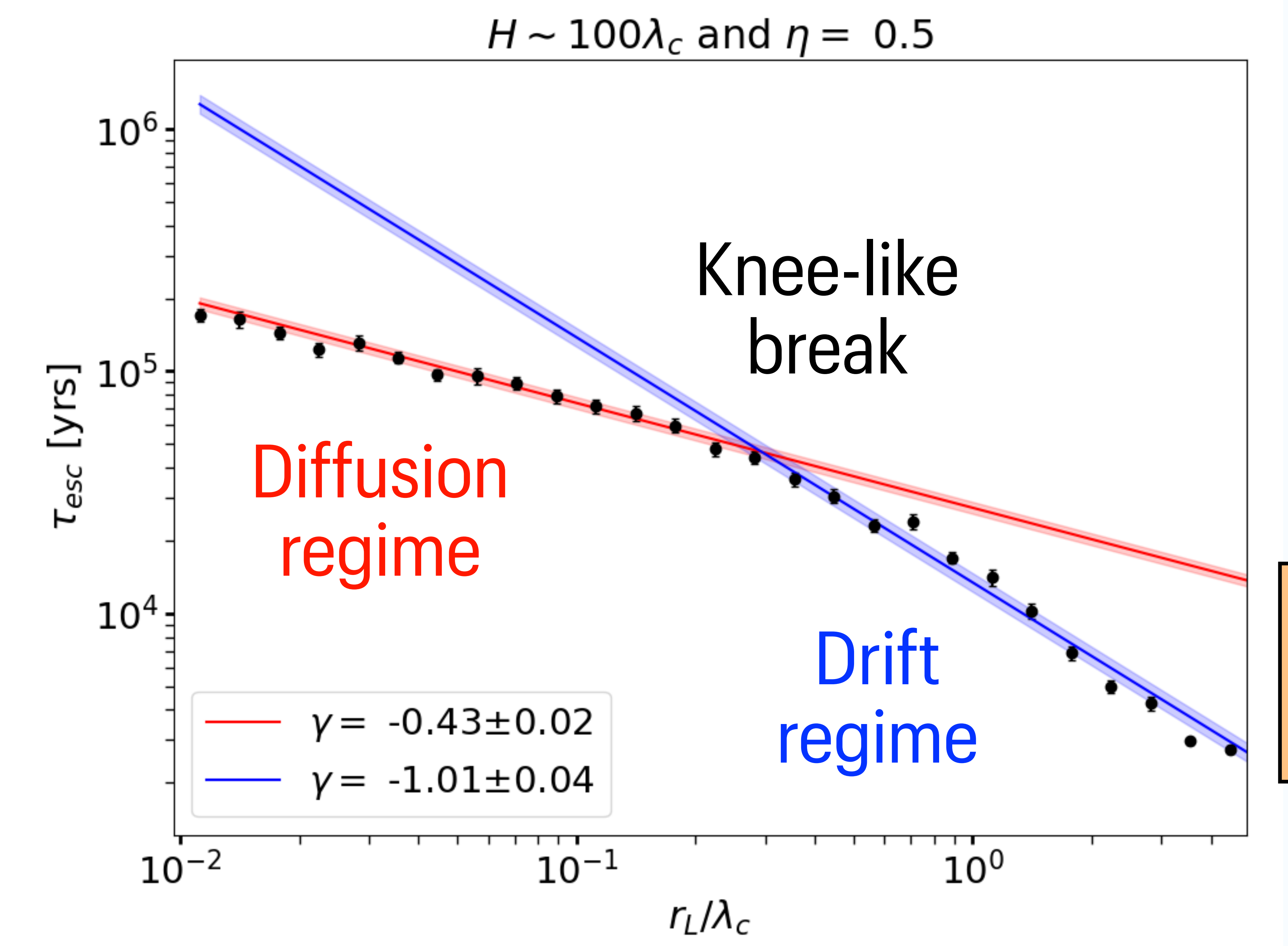
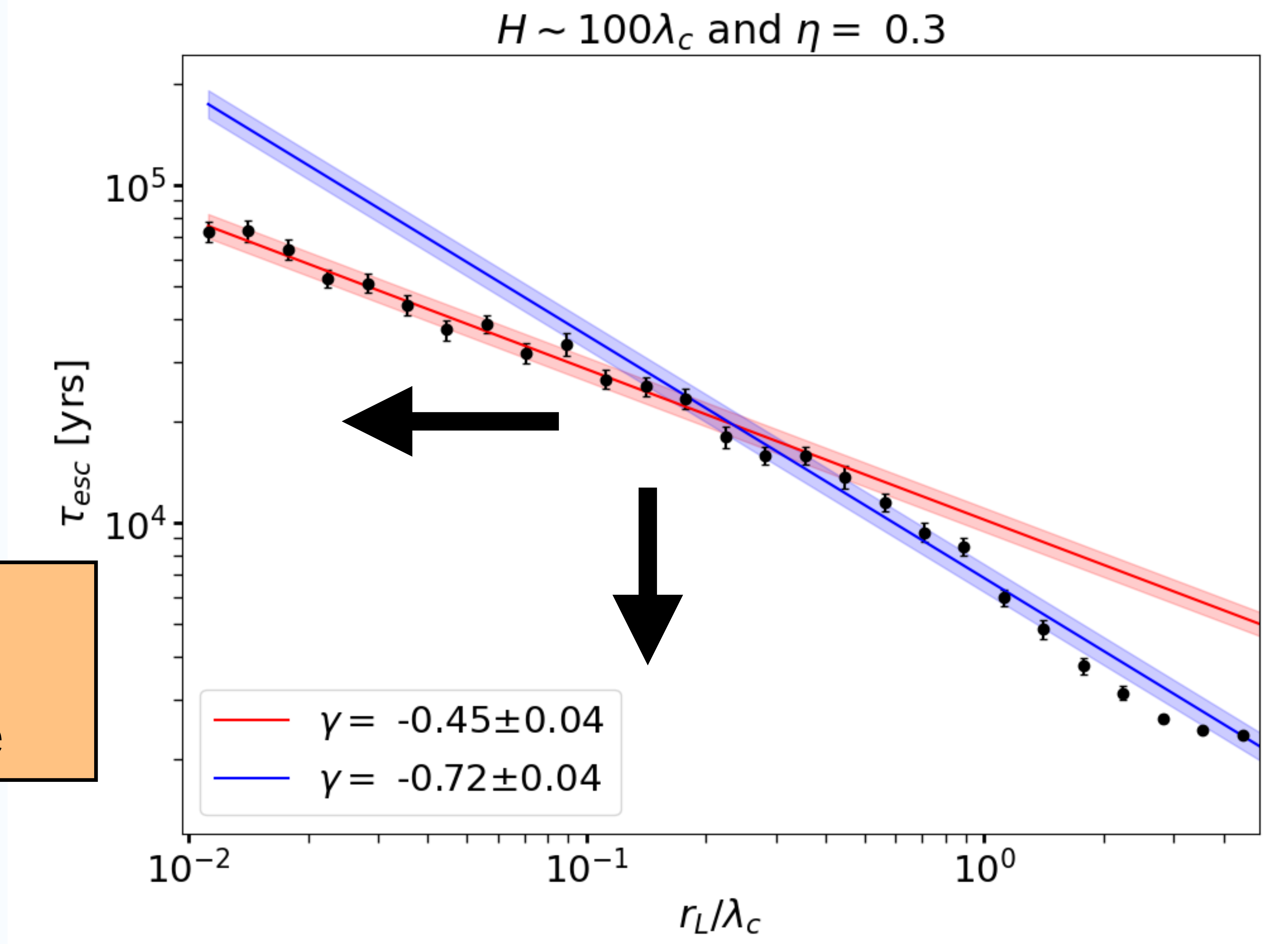
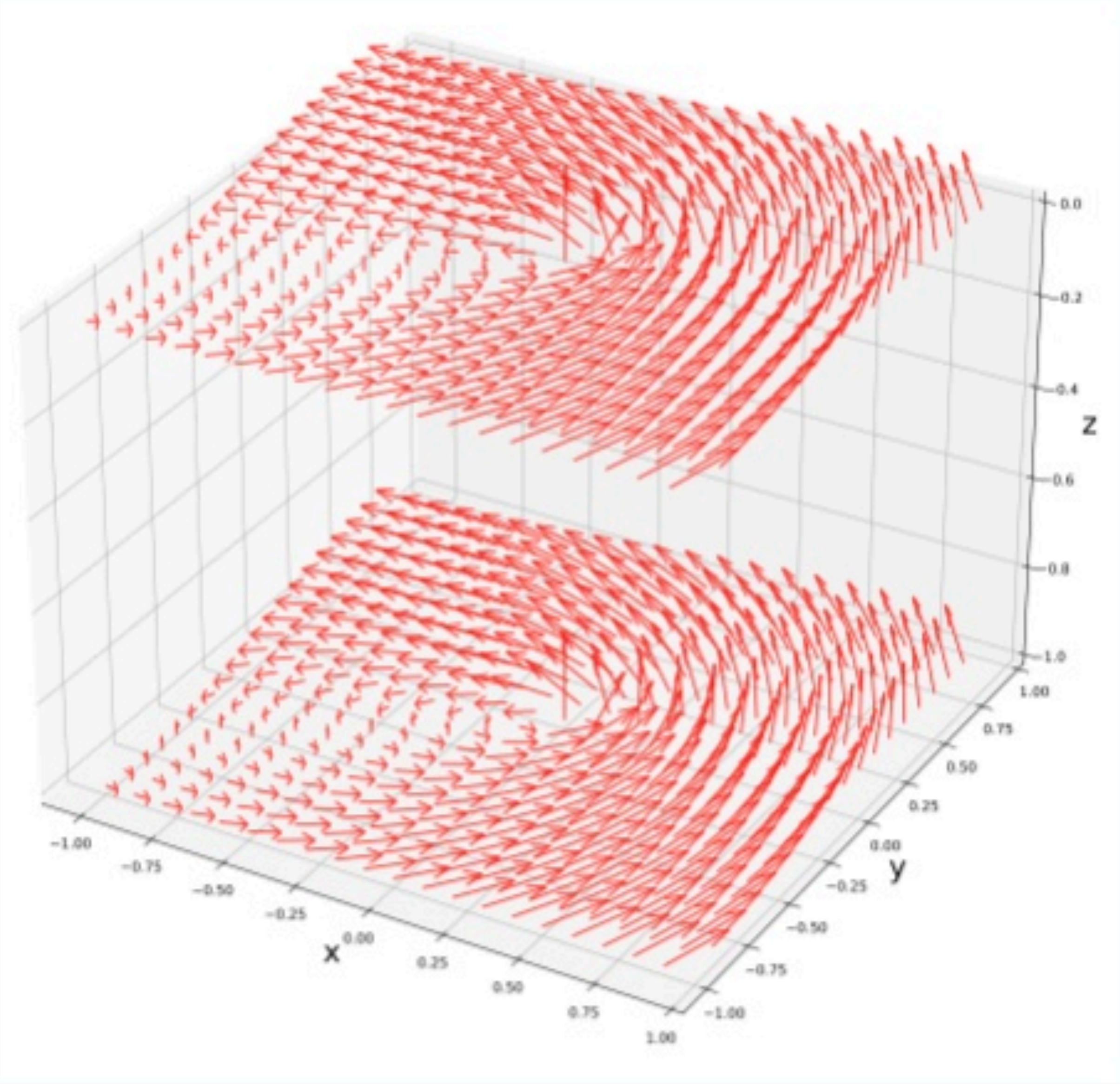


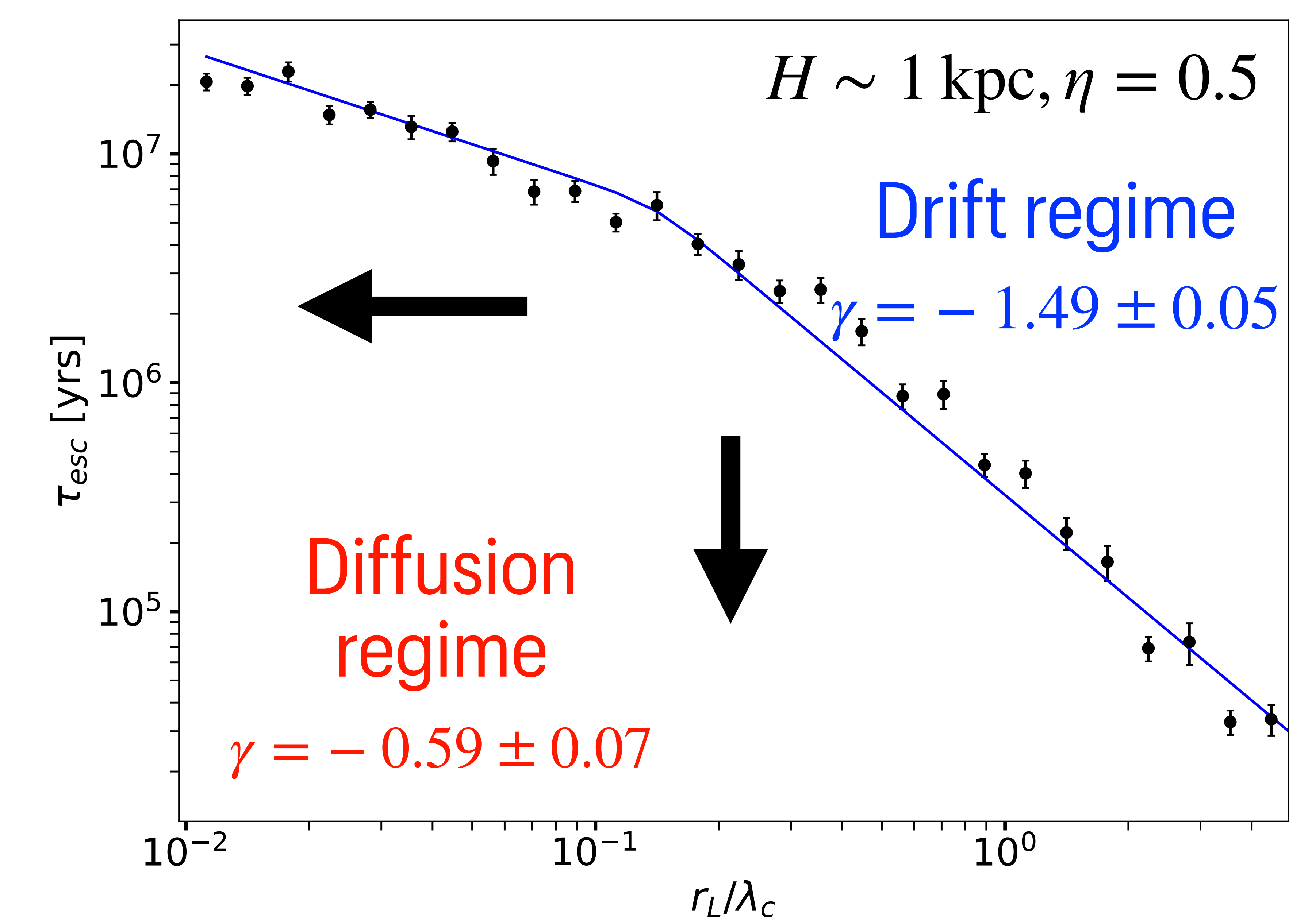
Effect of lowering turbulence



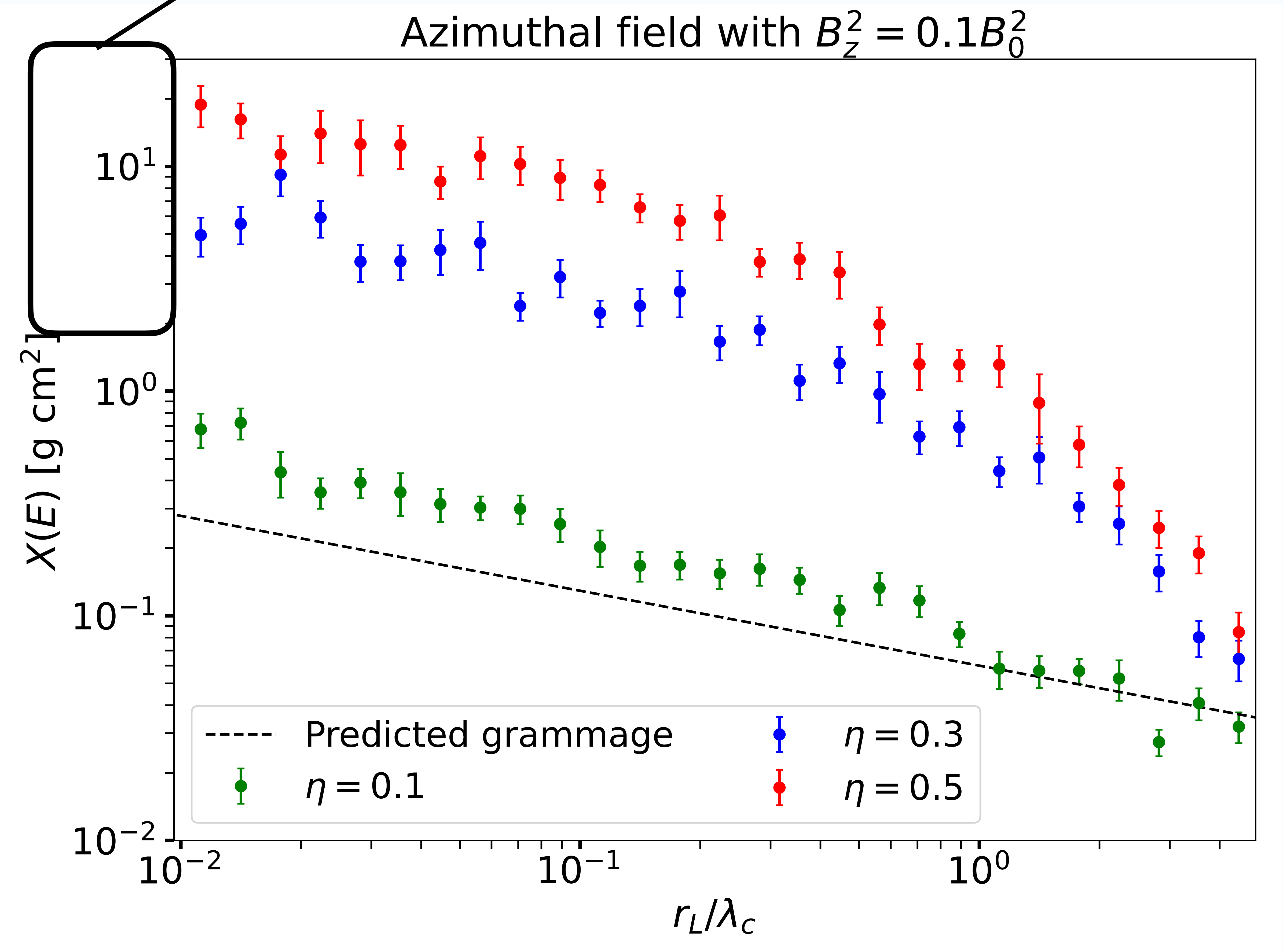
Field with vertical component

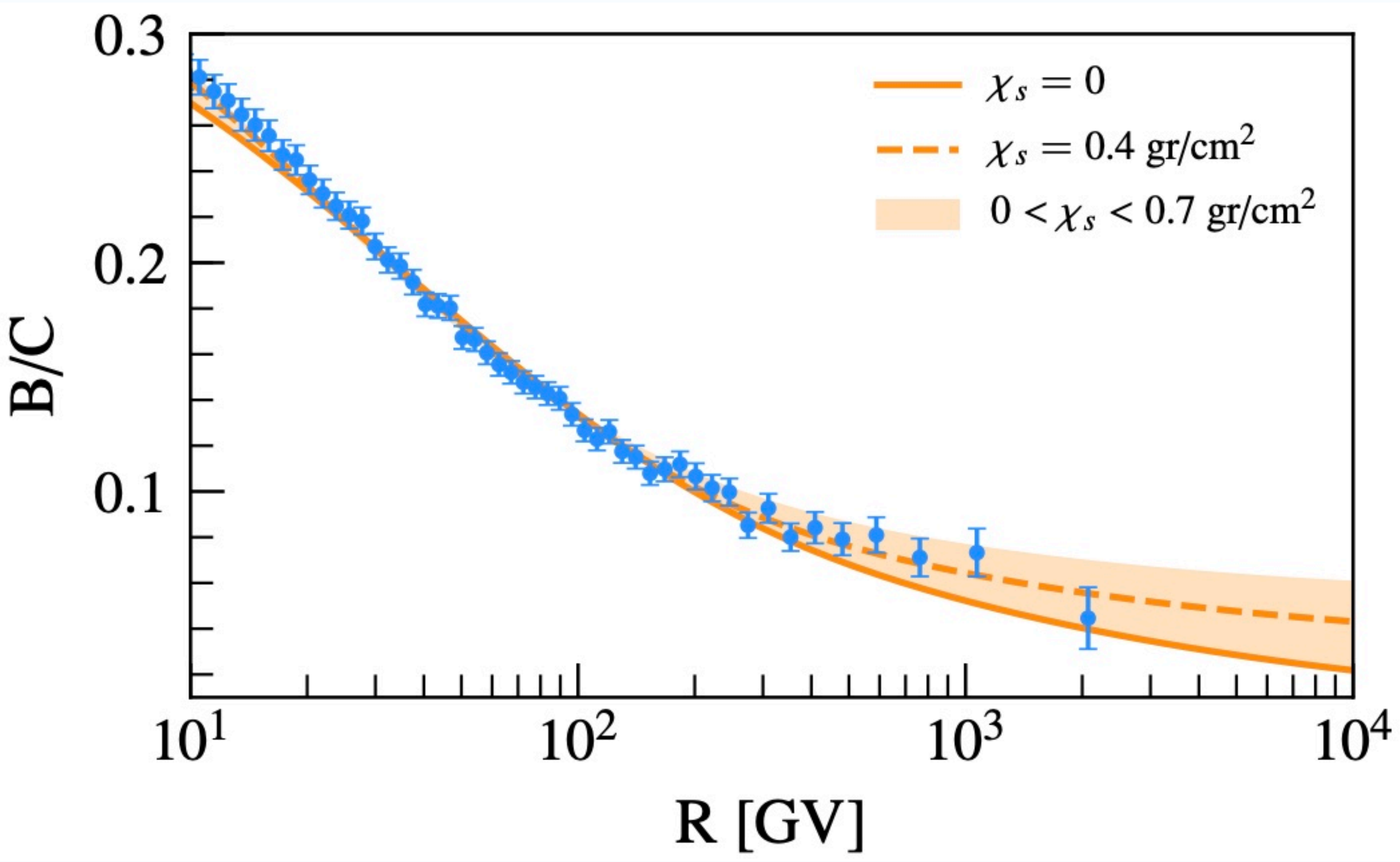


Field with vertical component



Grammage higher than estimates from B/C measurements





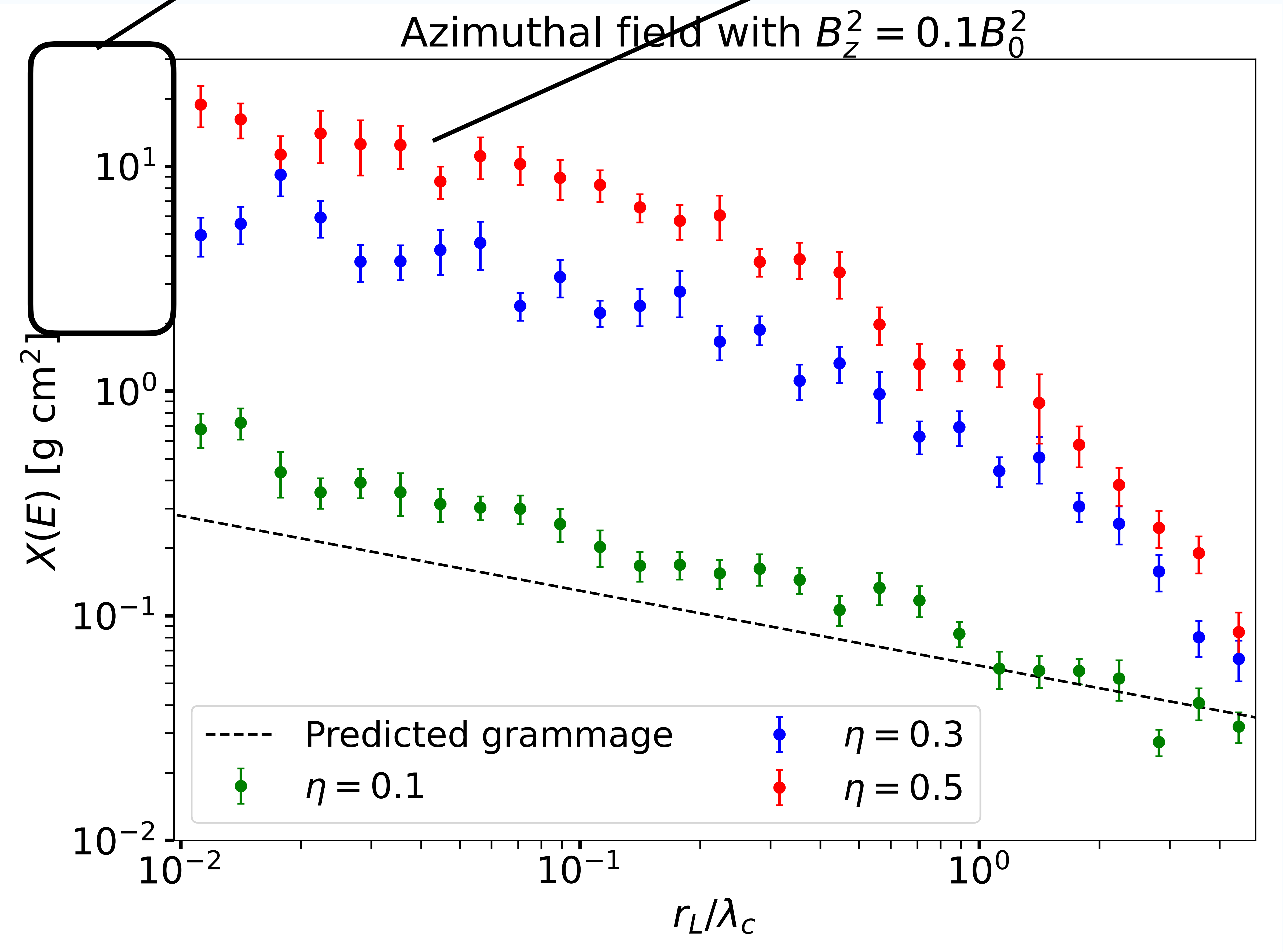
Grammage at TeV inferred from direct observations $\sim 0.6\,\mathrm{g\,cm^{-2}}$

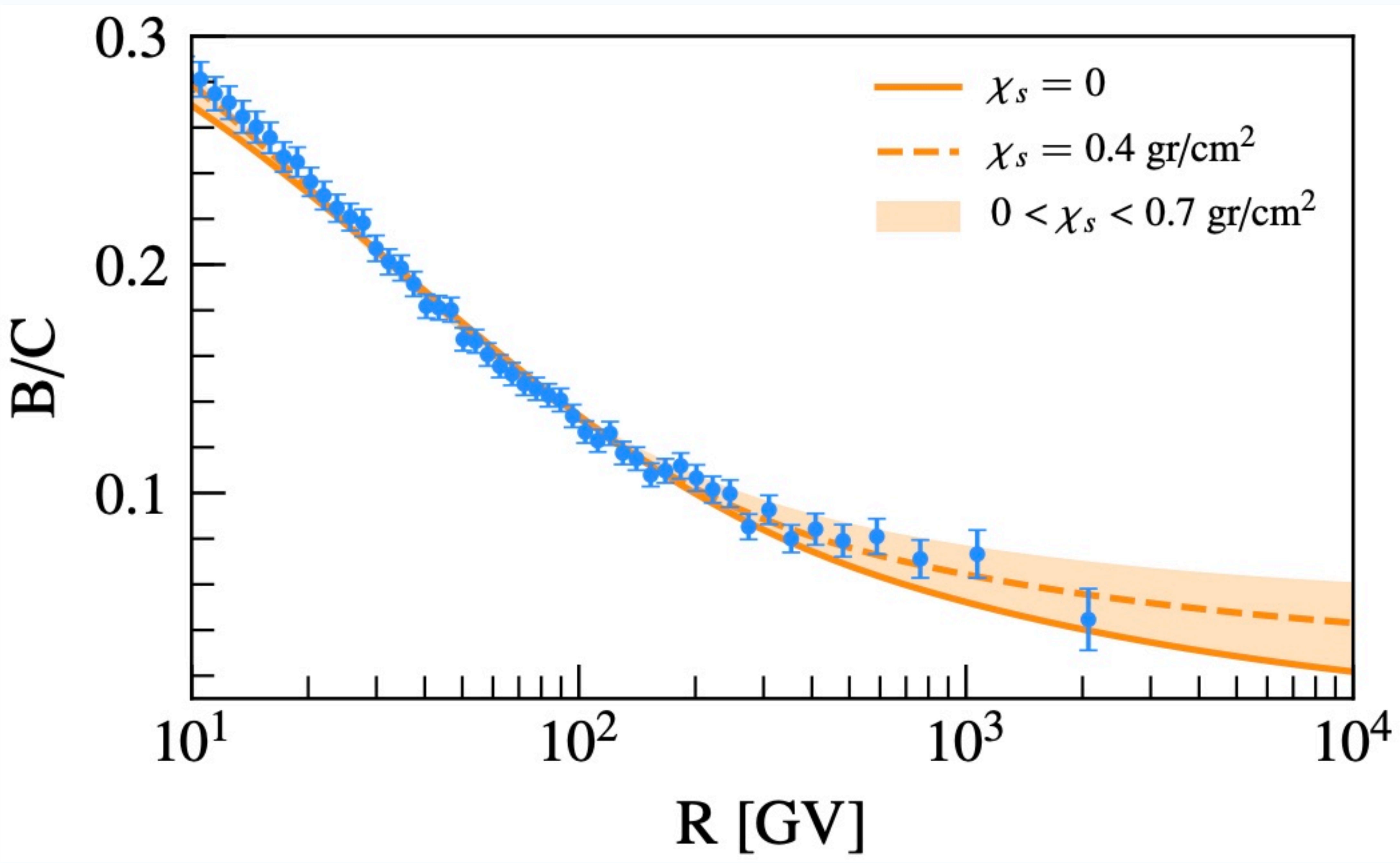
$$X(E = 1\,\mathrm{TeV}) = 5.62\,\mathrm{g\,cm^{-2}} \left(\frac{n}{1\,\mathrm{cm}}\right) \left(\frac{h}{200\,\mathrm{pc}}\right) \left(\frac{H}{1\,\mathrm{kpc}}\right) \left(\frac{\eta}{0.1}\right)^2 \left(\frac{0.1}{\mu}\right)^{1/2} \left(\frac{3\,\mathrm{kpc}}{\lambda_c}\right)^{2/3} \quad \text{for } B_z^2 = \mu B_0^2$$

Gas density; Disc size; Halo size; Turbulence; Vertical field component; Correlation length

Grammage higher than estimates from B/C measurements

Parallel diffusion dominates and makes escape faster





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Gas density (n), Disc size (h), Halo size (H), Turbulence (η), Vertical field component (μ), Correlation length (λ_c)

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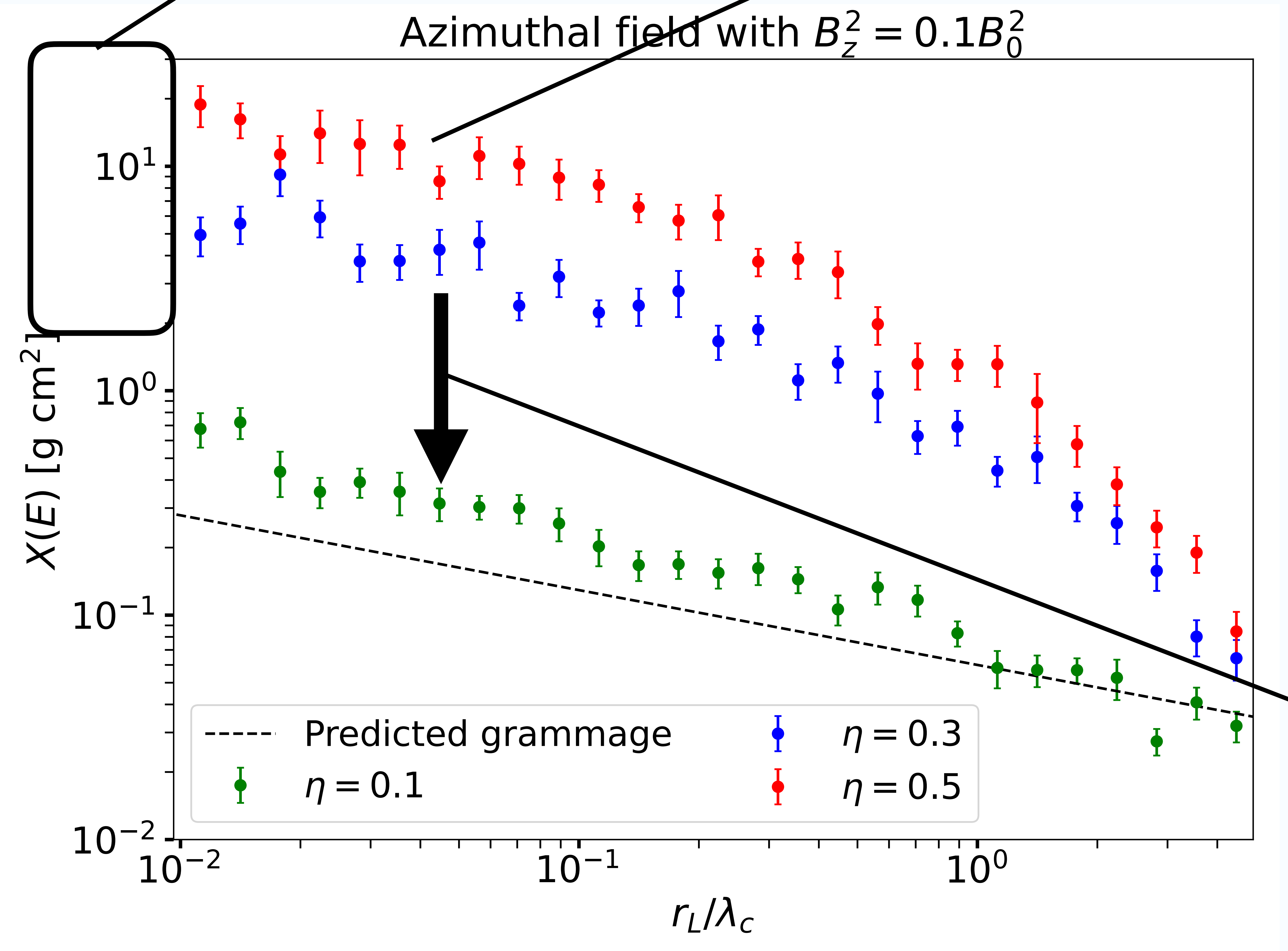

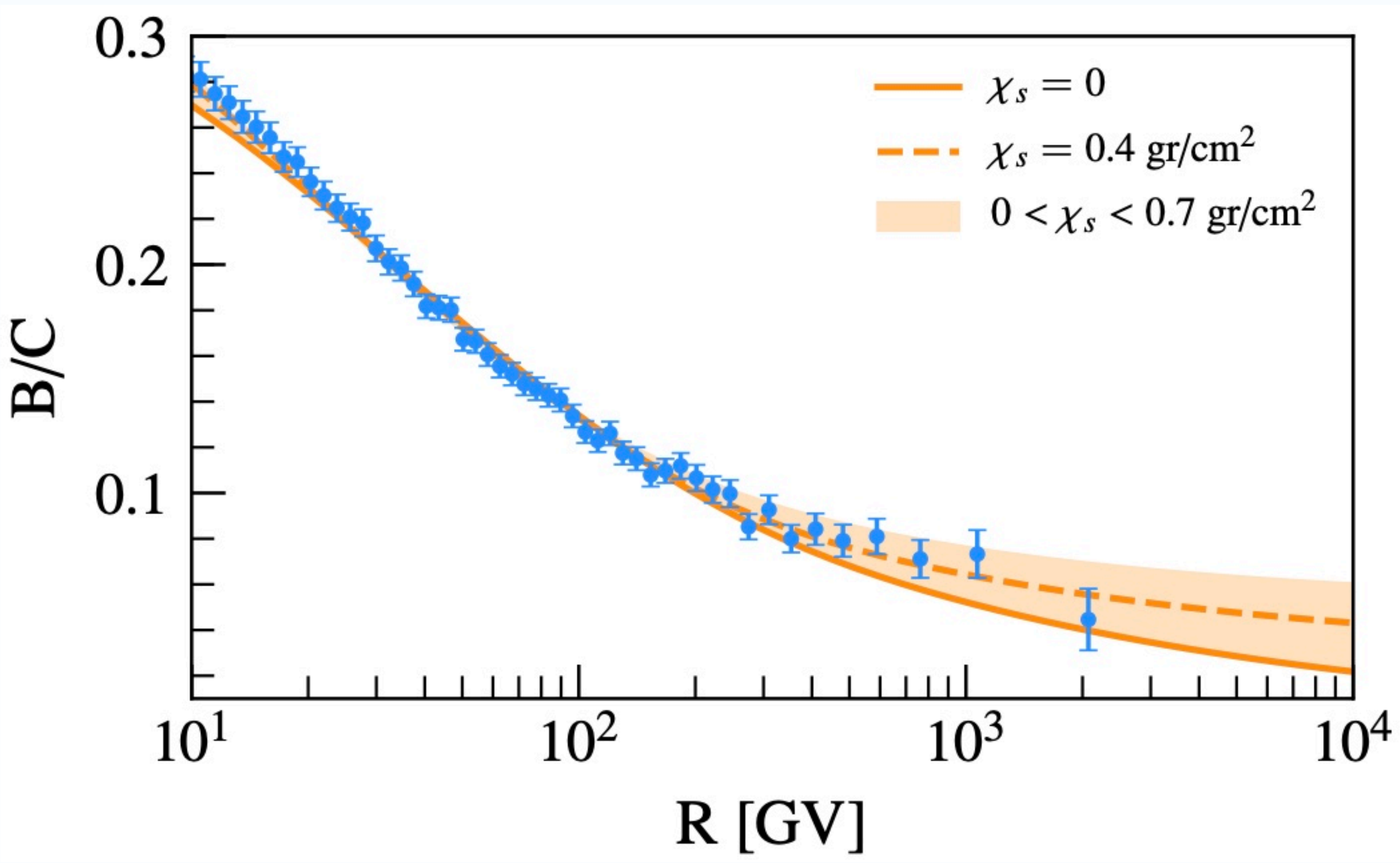

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Lowering turbulence could further decrease grammage

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Gas density; Disc size; Halo size; Turbulence; Vertical field component; Correlation length

Summary and Conclusions

- Transition from perpendicular diffusion to Hall diffusion predicts a **spectral break in agreement with the knee**.
- For the first time, both propagation mechanisms have been characterised as function of particle rigidity, turbulence and magnetic field geometry in test-particle simulations. Results corroborate a **diffusion-drift transition at the knee energy range**.
- In closed-field-lines geometries, perpendicular diffusion is not an efficient escape mechanism as it **predicts higher confinement times in Galactic magnetic fields**.
- Coherent magnetic field structure enhancing **particle escape via parallel diffusion** is necessary for reproduction of grammage, jeopardizing spectral shape of the break.
- **Propagation explanation of cosmic ray knee may be inadequate or requires a more complex coherent magnetic field structure.**

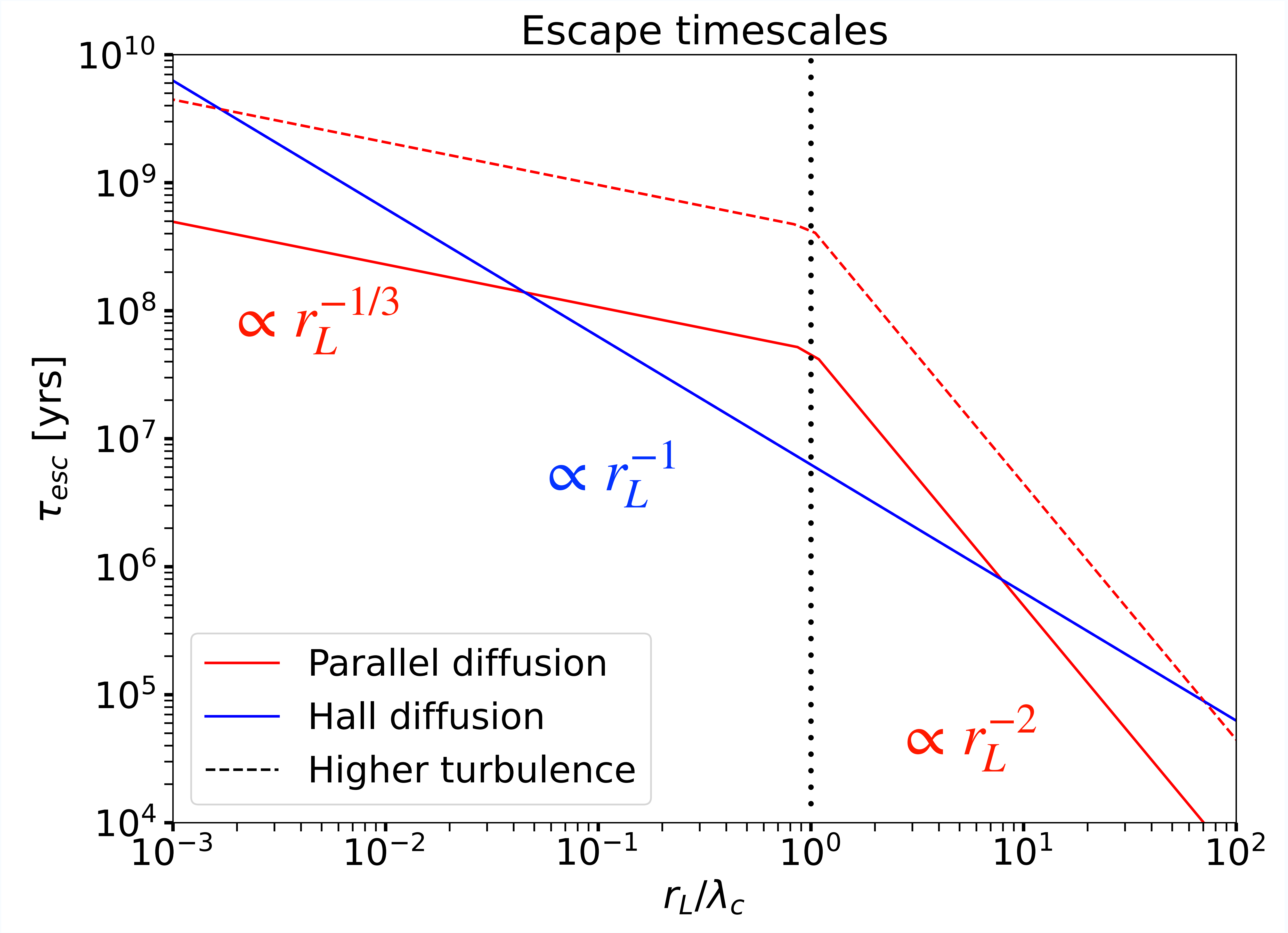
Outlook:

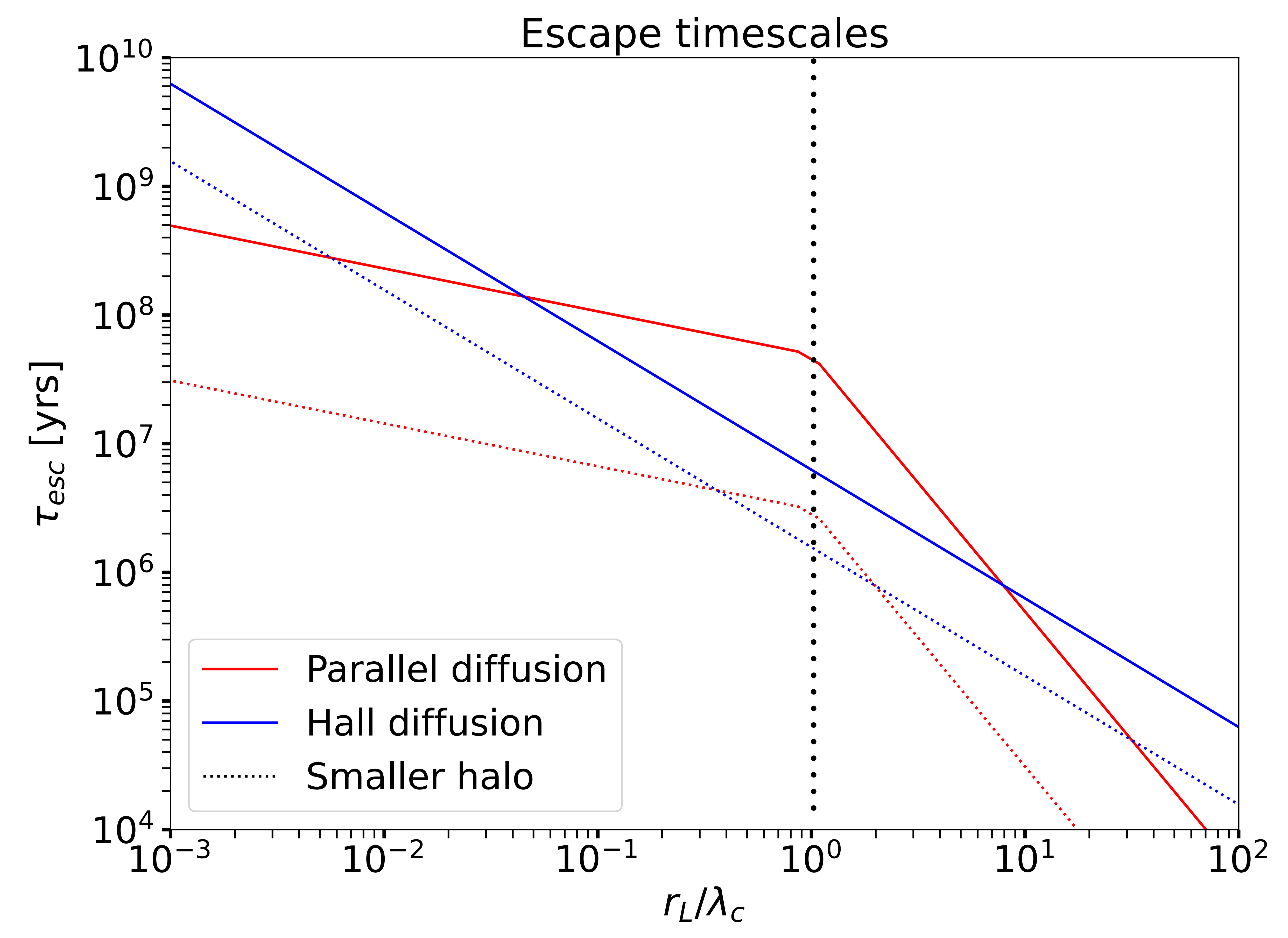
- What is the maximum fraction of vertical magnetic field component $B_z/B_{0,\phi}$ allowed?
- Is the cosmic ray knee explained by source maximum acceleration energy? Is a second population of Galactic sources peaking at PeV energies required to reproduce observations?

Backup Slides

Diffusion-Drift Transition as the Origin of the Knee

- ✦ For how many energy decades can drifts extend?
- ✦ What is the dependency on the structure of the magnetic field?
- ✦ Can the transition explain spectral and gramme observations simultaneously?





Test-Particle Simulations of the Diffusion-Drift Transition

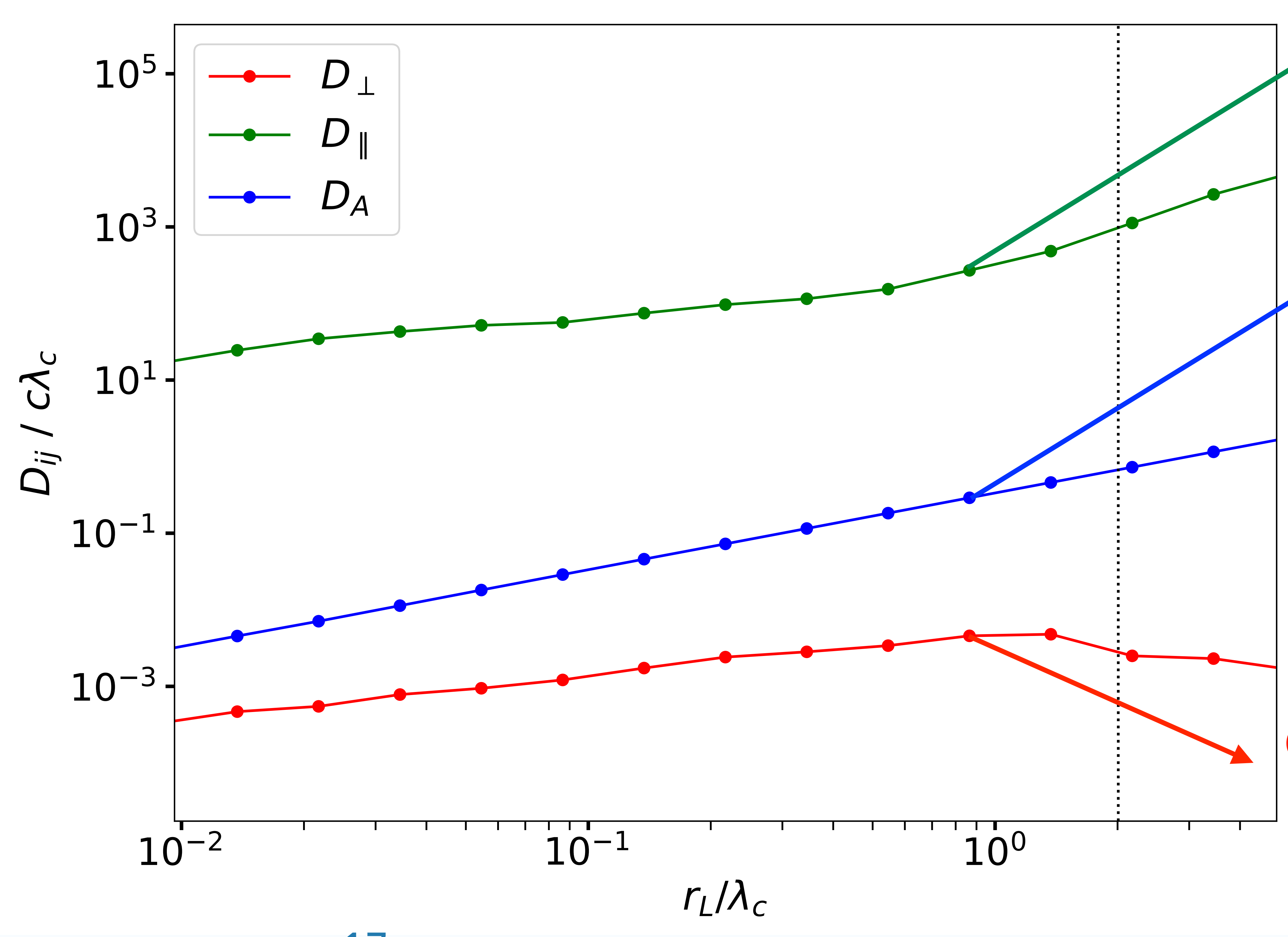
- **Synthetic magnetic fields:** turbulent spectrum assumed to be homogenous, isotropic and follow a power-law. Random components Gaussian-distributed.
- **Test-particle simulations** of TeV-PeV cosmic rays. Particle diffusion measured as function of rigidity and turbulence level.

CRPropa

Turbulent spectrum

$$W(k) \propto \frac{\delta B^2 l_b}{\pi k^2} \frac{(kl_b)^q}{[1+(kl_b)^2]^{(s+q)/2}}$$

- Turbulence level $\eta = \frac{\delta B_{rms}}{B_0}$
- Kolmogorov turbulent spectrum $s = 5/3$, $q = 4$
- Bend-over scale $l_b \sim$ few pc
- Correlation length $\lambda_c = \frac{4\pi}{\delta B^2} \int_0^\infty dr \int_0^\infty dk \frac{\sin(kr)}{kr} k^2 W(k)$



QLT coefficient
$D_\parallel \propto r_L^{1/3}$

Bohm-like coefficient
$D_A \propto r_L$

$v_D \sim \frac{r_L c}{3 r_0}$

Corroboration of anomalous perpendicular behaviour
$D_\perp \propto r_L^{1/2}$

Simulation setup

Magnetic field:

- Spacing of grid $\Delta x \sim 10^{-2}$ pc $\sim 10 \cdot r_L \left(E = 10^{15}\,\text{eV}\right)$

$\Longrightarrow$ Scattering resonance around PeV energy

- Number of grid points $N_{grid} = 2048$
- Size of grid $L = \Delta \cdot N_{grid}$
- Minimum turbulent scale $L_{min} = 2 \cdot \Delta$
- Maximum turbulent scale $L_{max} = L/2$

Test-particle:

- Number of particles $N_p = 10^3$
- Number of position/velocity measurements $N_t = 10^5$
  - Step for position/velocity measurements $\Delta t = 0.1\, r_L/c$
- Minimum and maximum integrations steps $l_{min} = 0.1$ pc and $l_{max} = 100$ Mpc

Mean displacements method

$$D_{ij} = \frac{\left\langle \Delta x_i \Delta x_j \right\rangle}{2\tau}$$

TGK method

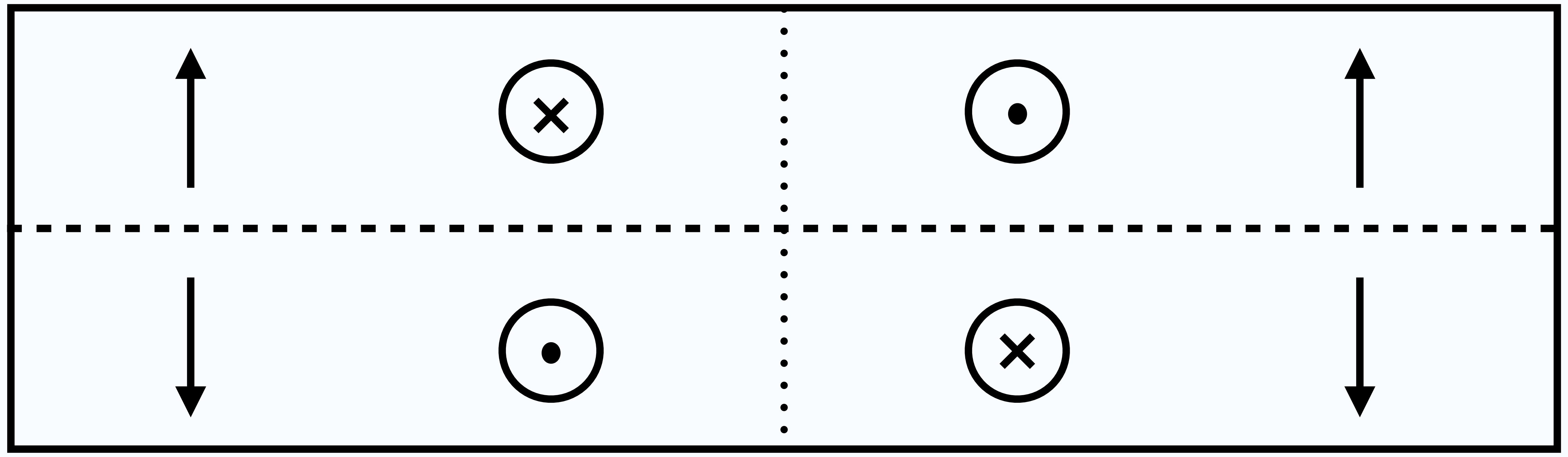
$$D_{ij} = \int_0^\infty dt \left\langle v_i(0) v_j(t) \right\rangle = \int_0^\infty dt\, R_{ij}(t)$$

Ansatz (Bieber & Matthaeus, 1997):

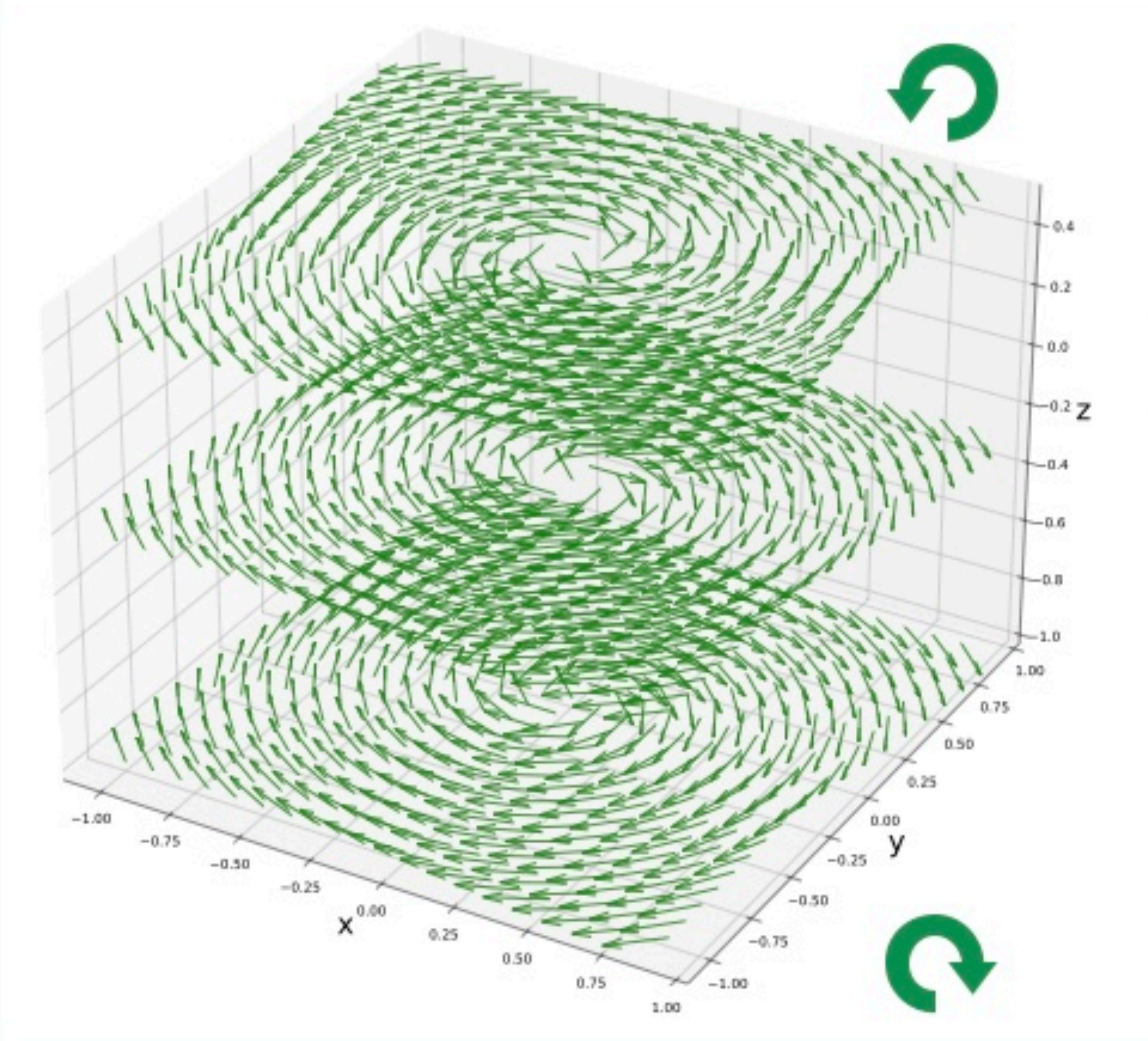
$$R_\parallel(t) = \frac{c^2}{3} e^{-t/\tau_\parallel}$$

$$R_\perp(t) = \frac{c^2}{3} \cos \omega t\, e^{-t/\tau_\perp}$$

$$R_A(t) = -\frac{c^2}{3} \sin \omega t\, e^{-t/\tau_A}$$



Azimuthal antisymmetric field



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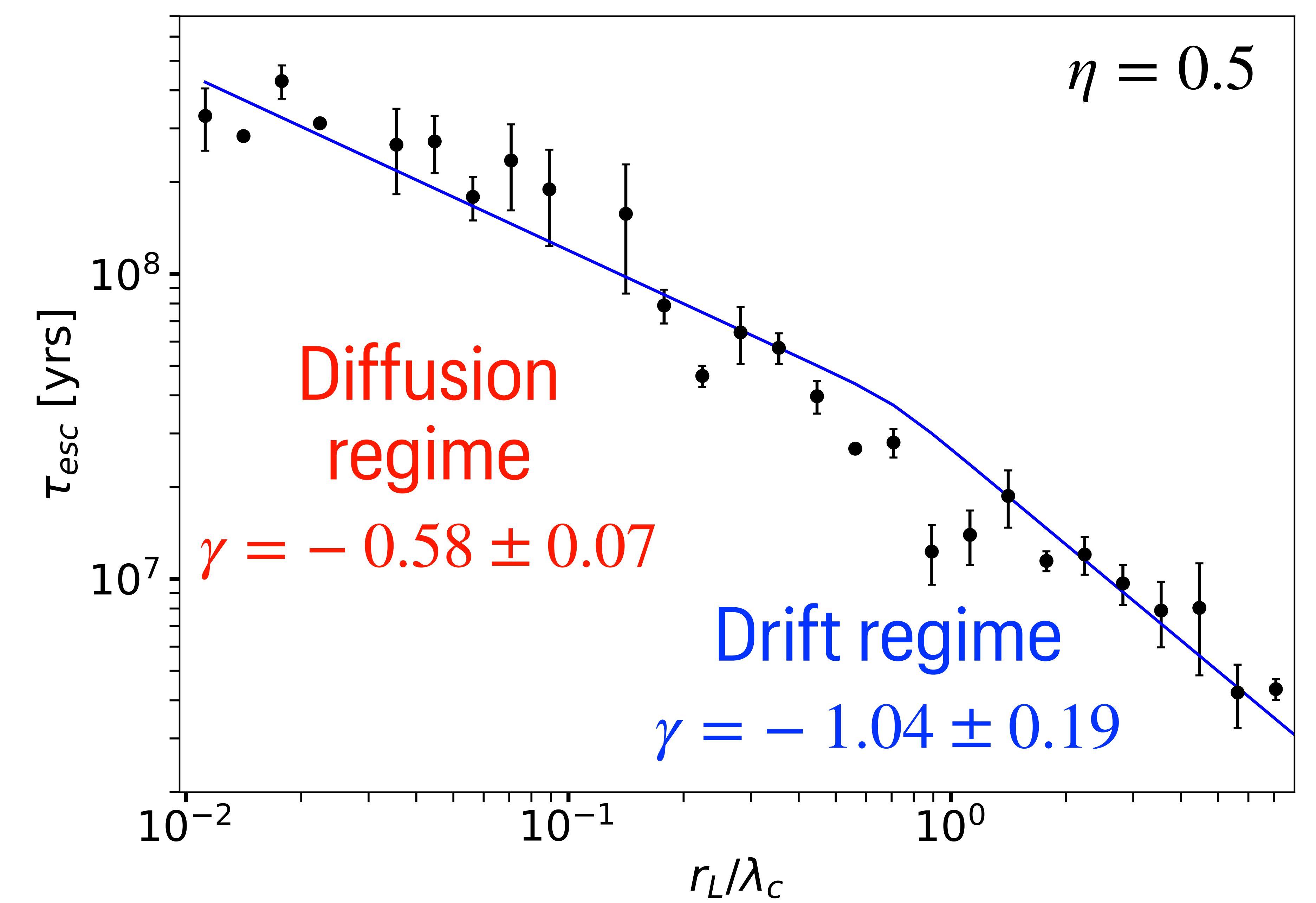
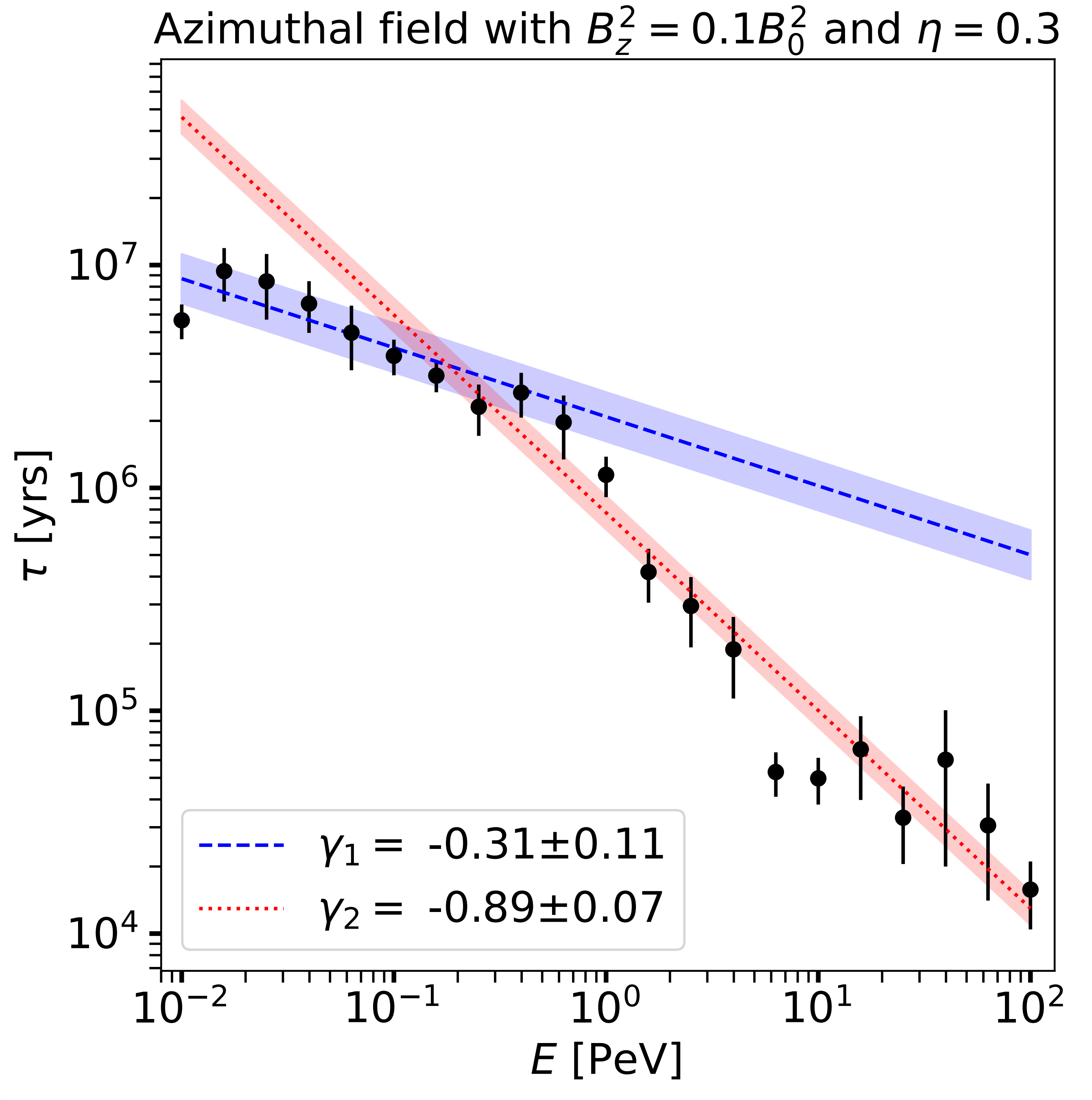
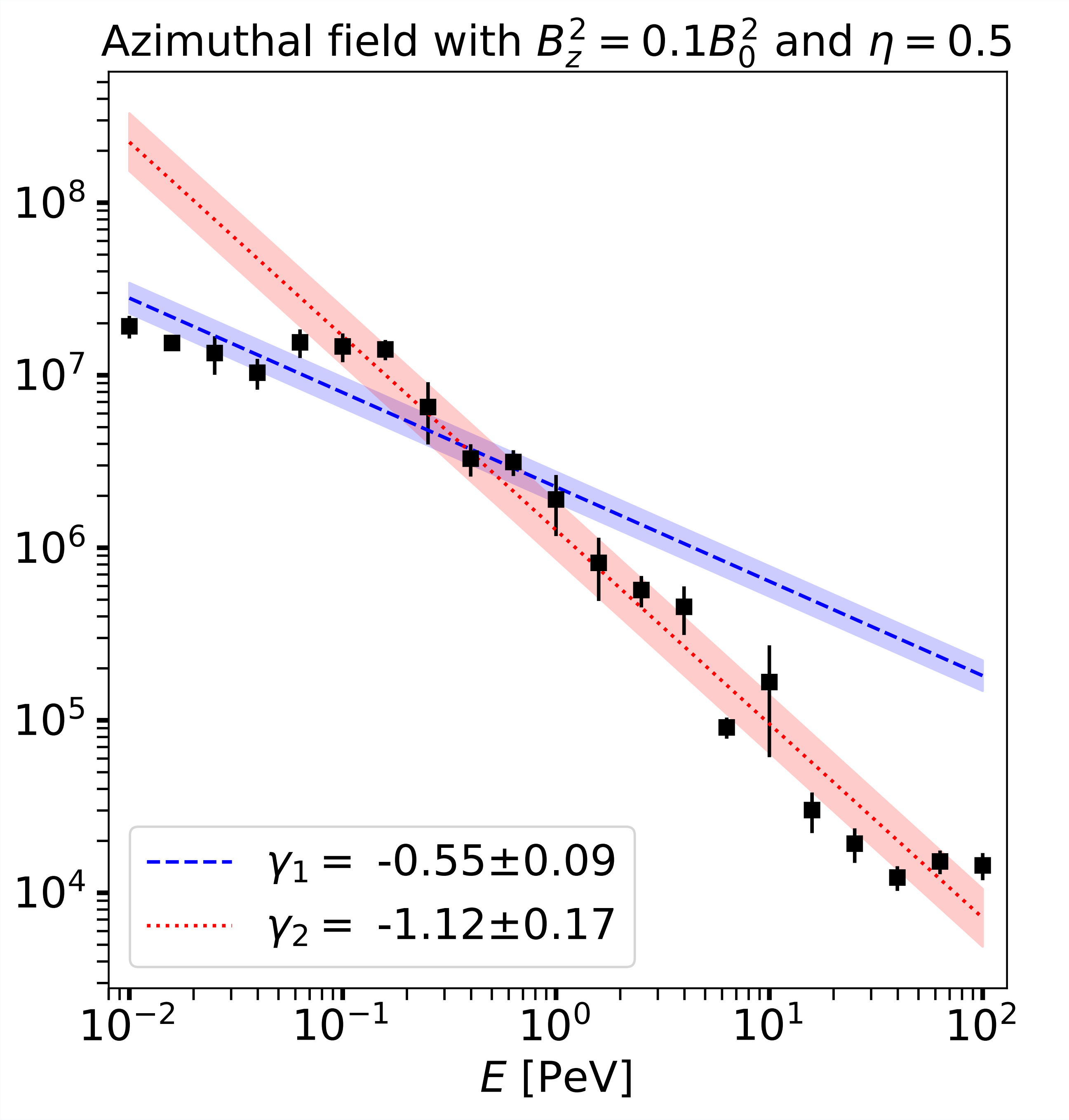
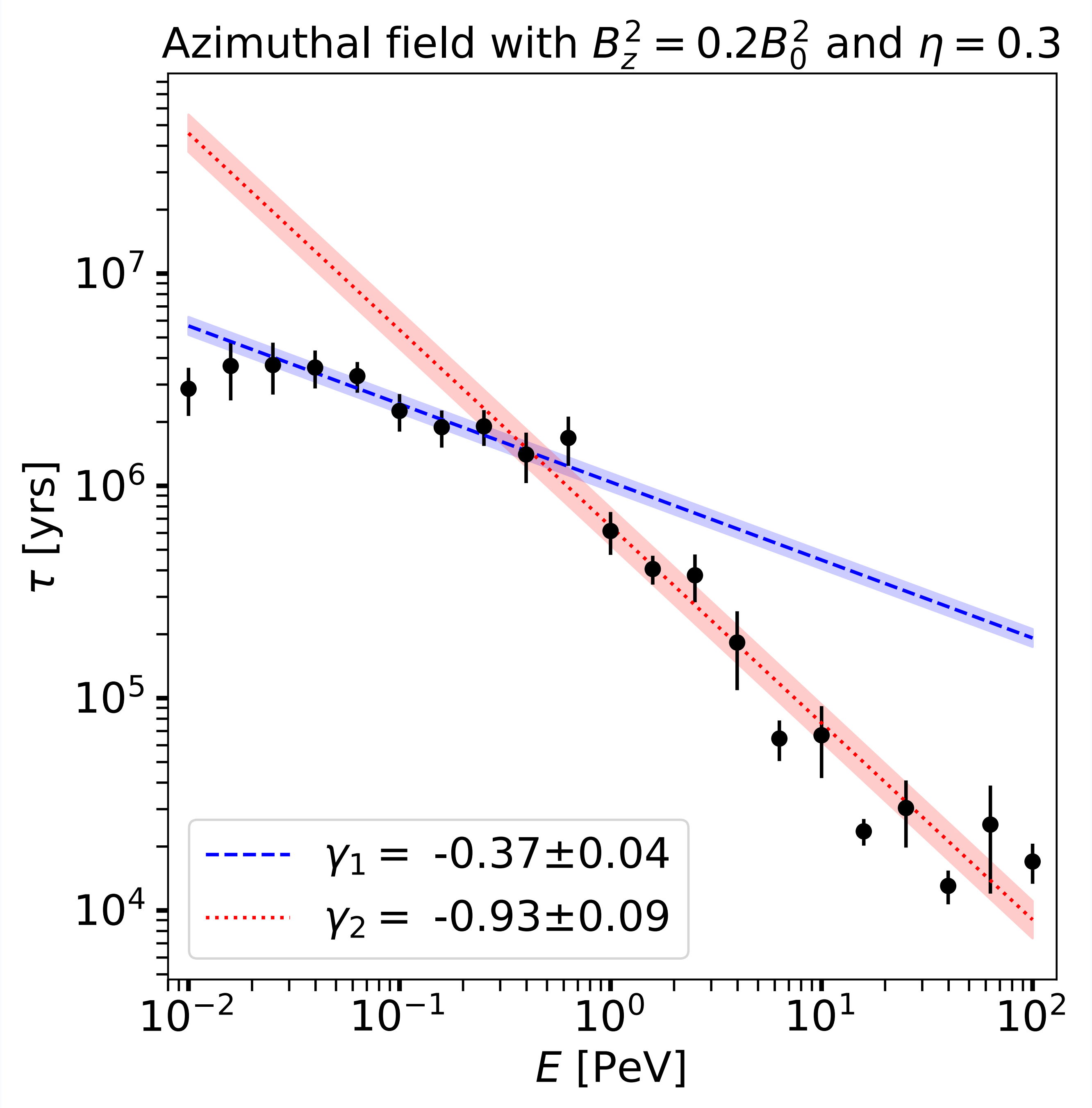


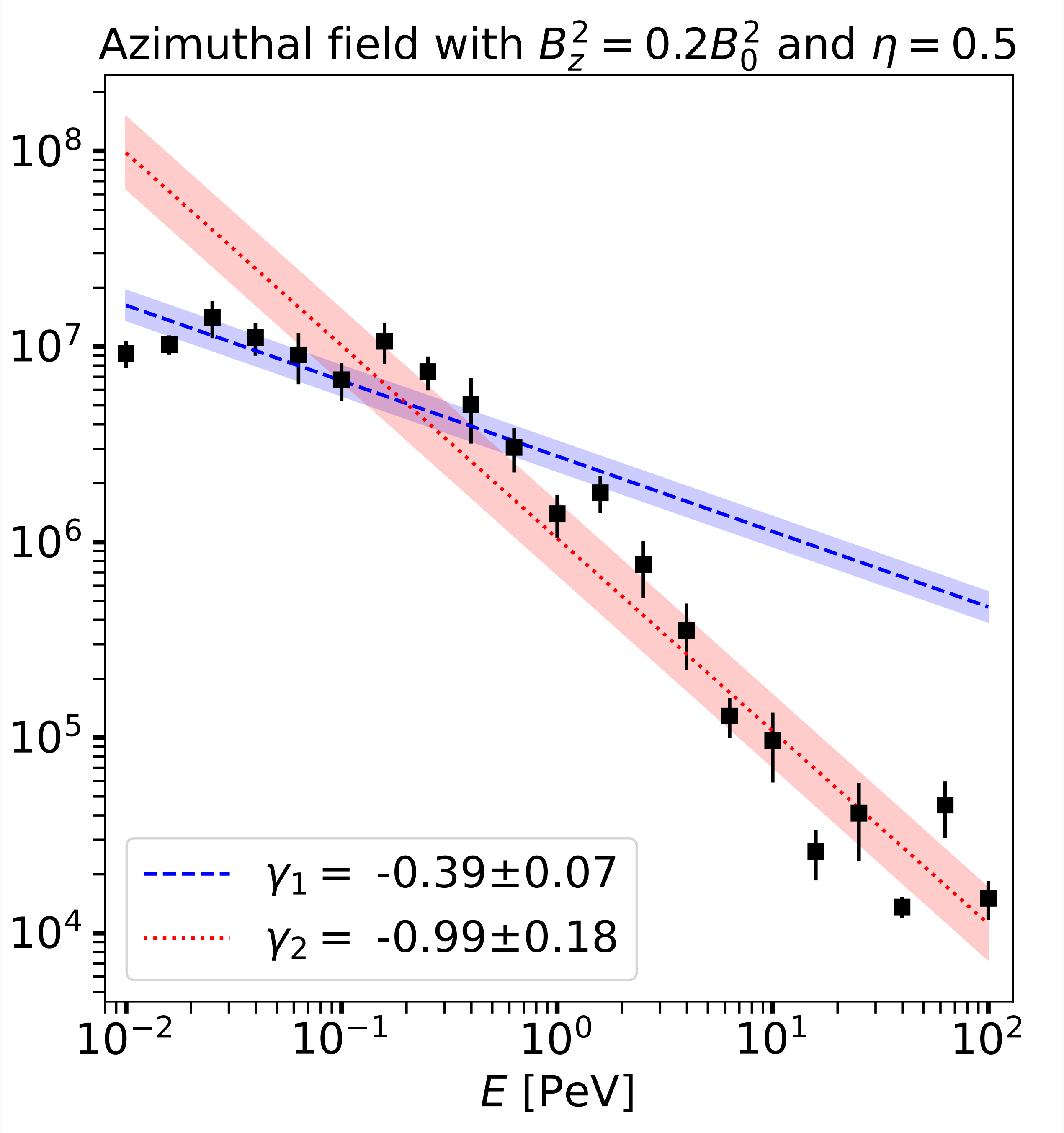


Reducing the turbulence leads to dominant of parallel diffusion over perpendicular diffusion

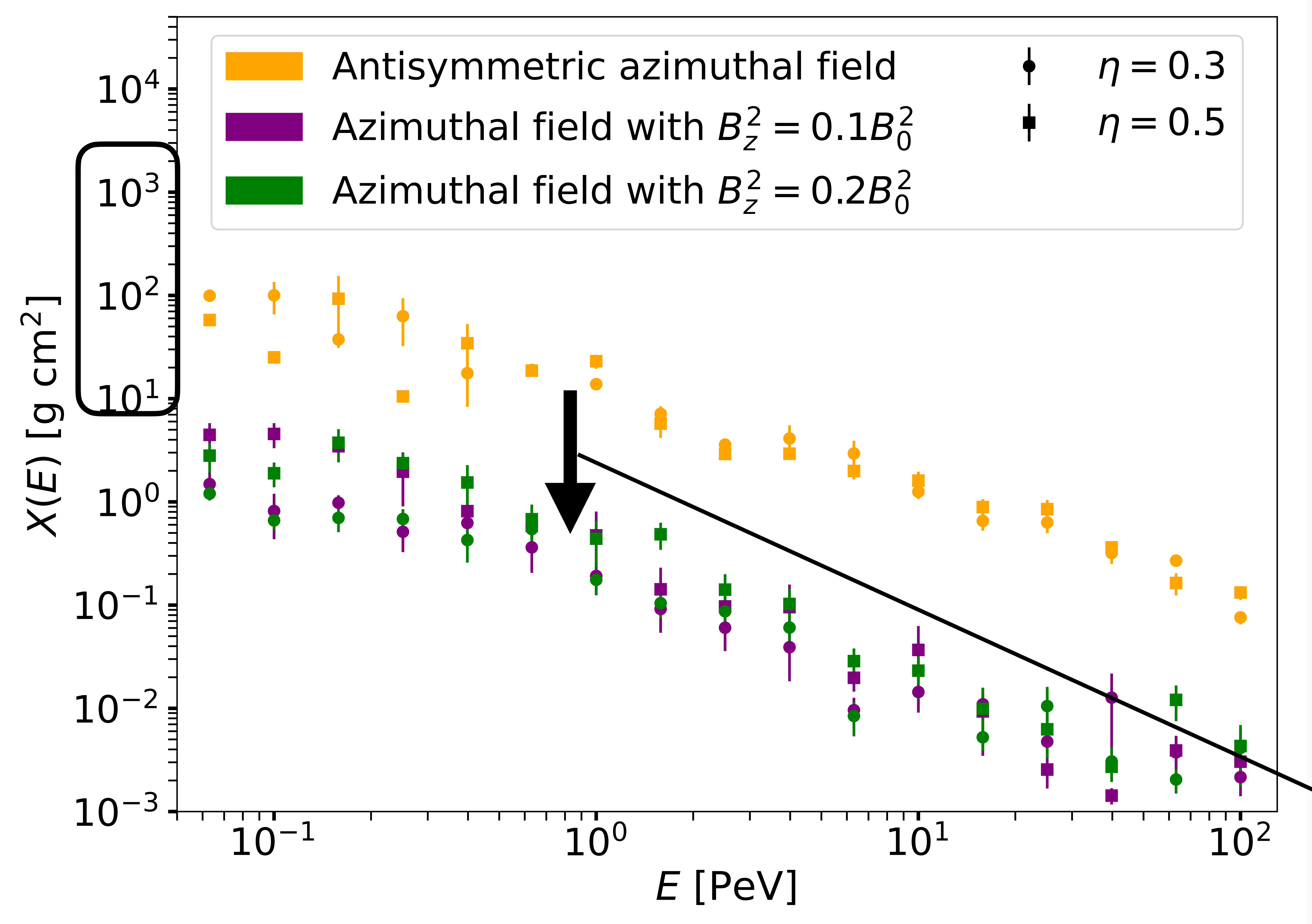


Increasing slightly the vertical component of the magnetic field doesn't reduce the timescales enough

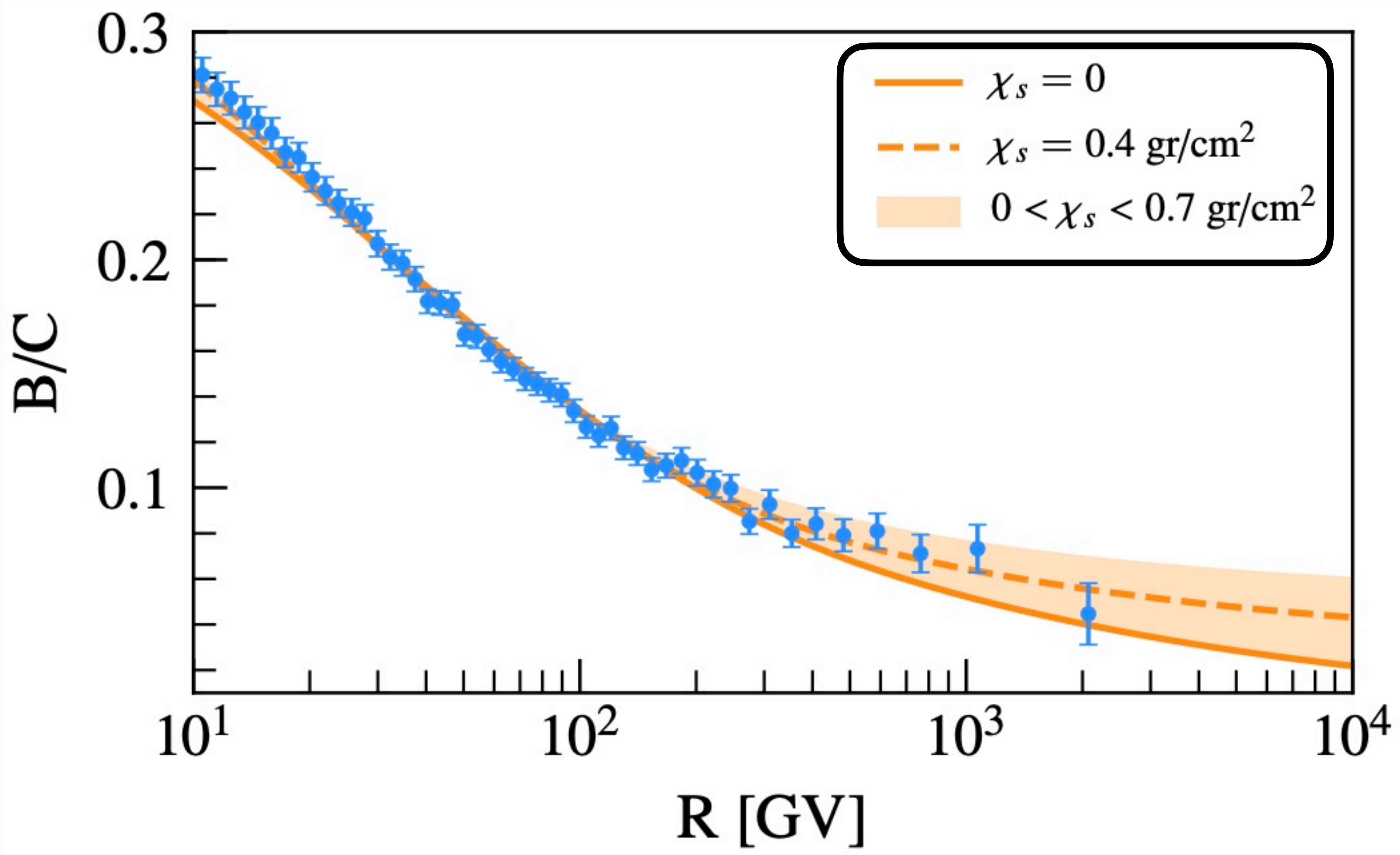




Cosmic Ray Grammage



[Evoli+2019]



Grammage at TeV inferred from direct observations $\sim 0.6\,\mathrm{g\,cm^{-2}}$

Opening of magnetic field lines reduces grammage significantly