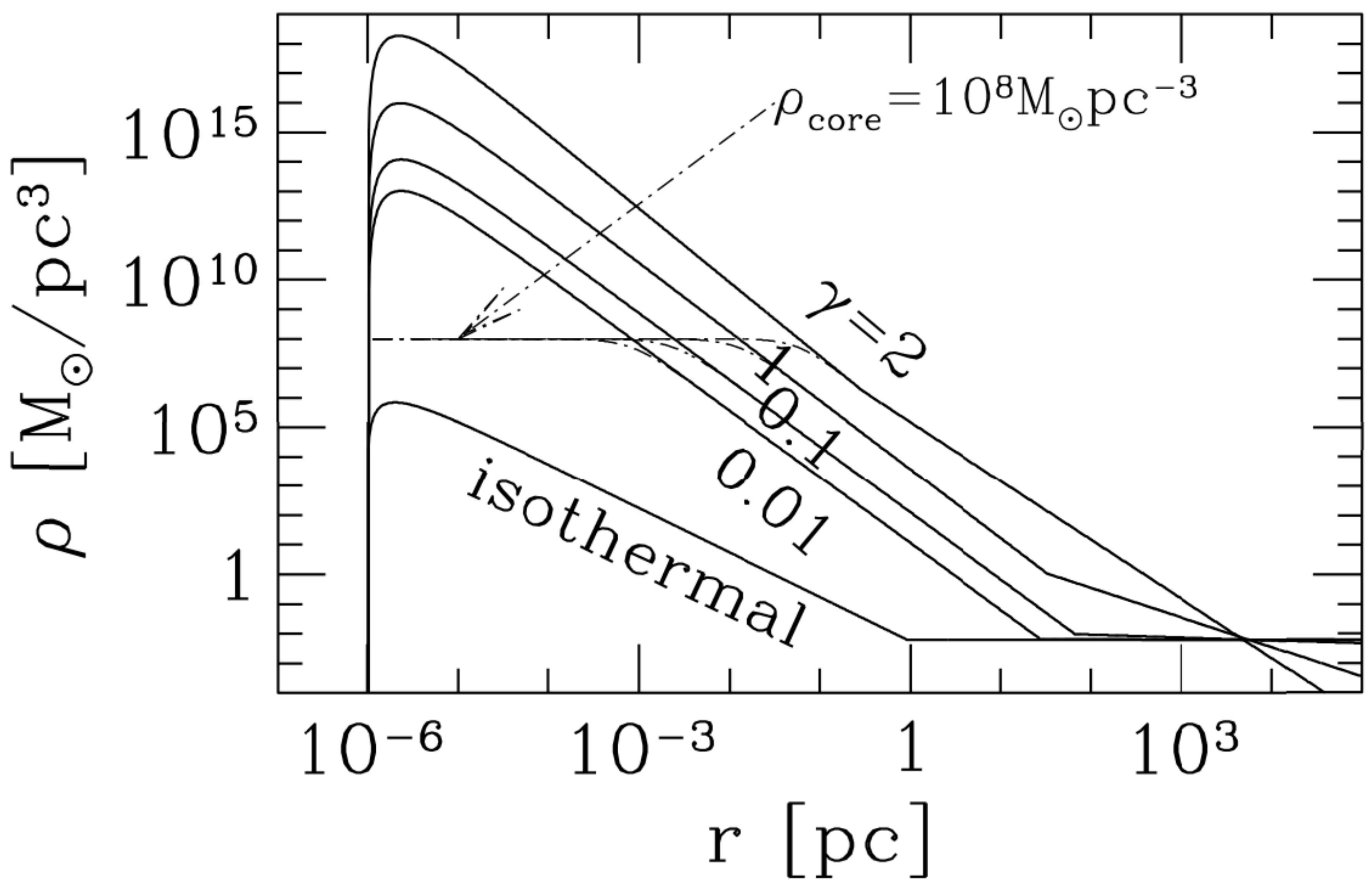
Dark matter spikes from Super-Massive Star collapse in General Relativity

Roberto Caiozzo, November 2025.

Supervisors: Piero Ullio, Gianfranco Bertone, Annalisa Celotti
Collaborators: Rodrigo Vicente, Daniele Gaggero, Bradley Kavanagh

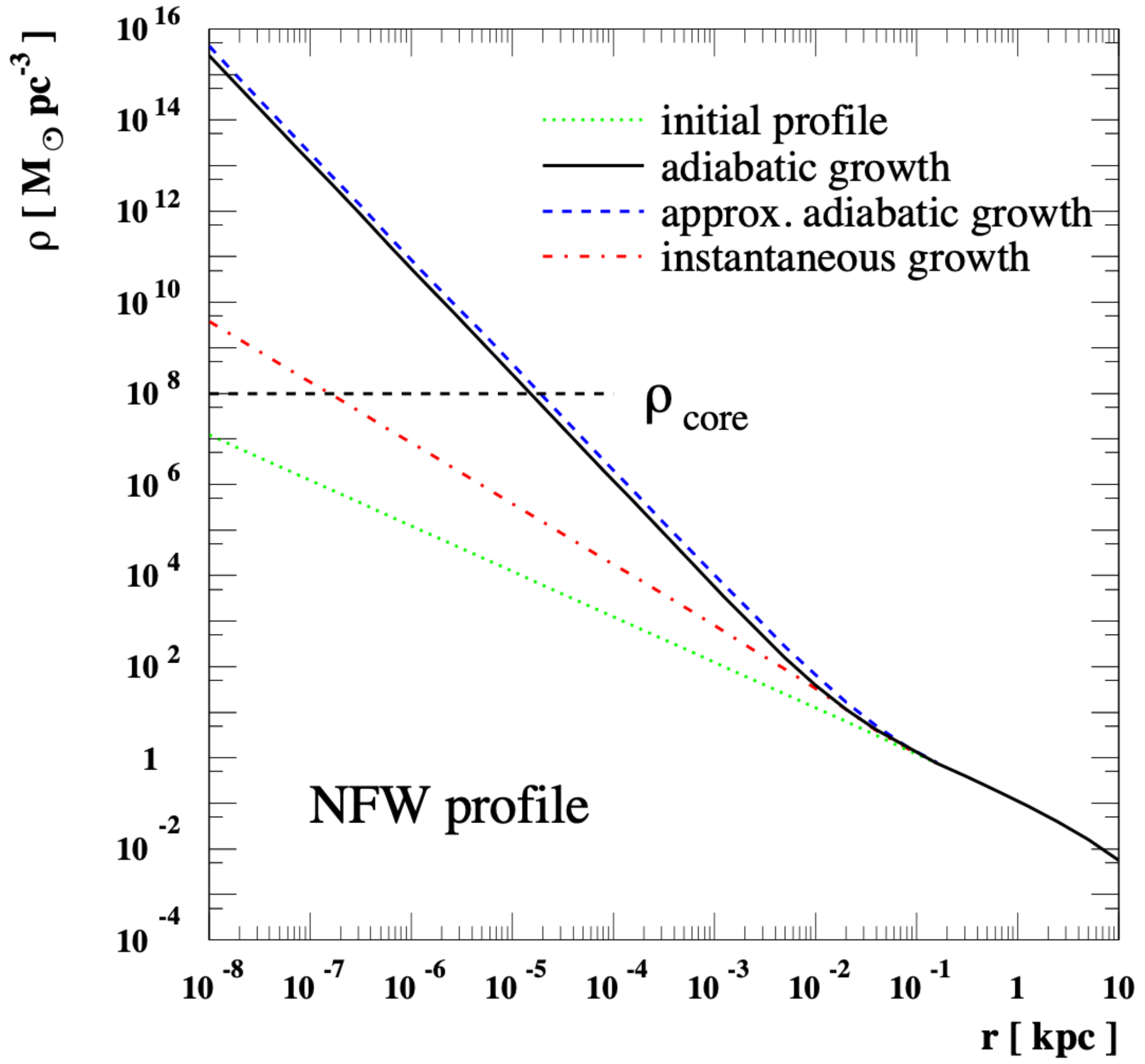
Dark matter spikes

Adiabatic case



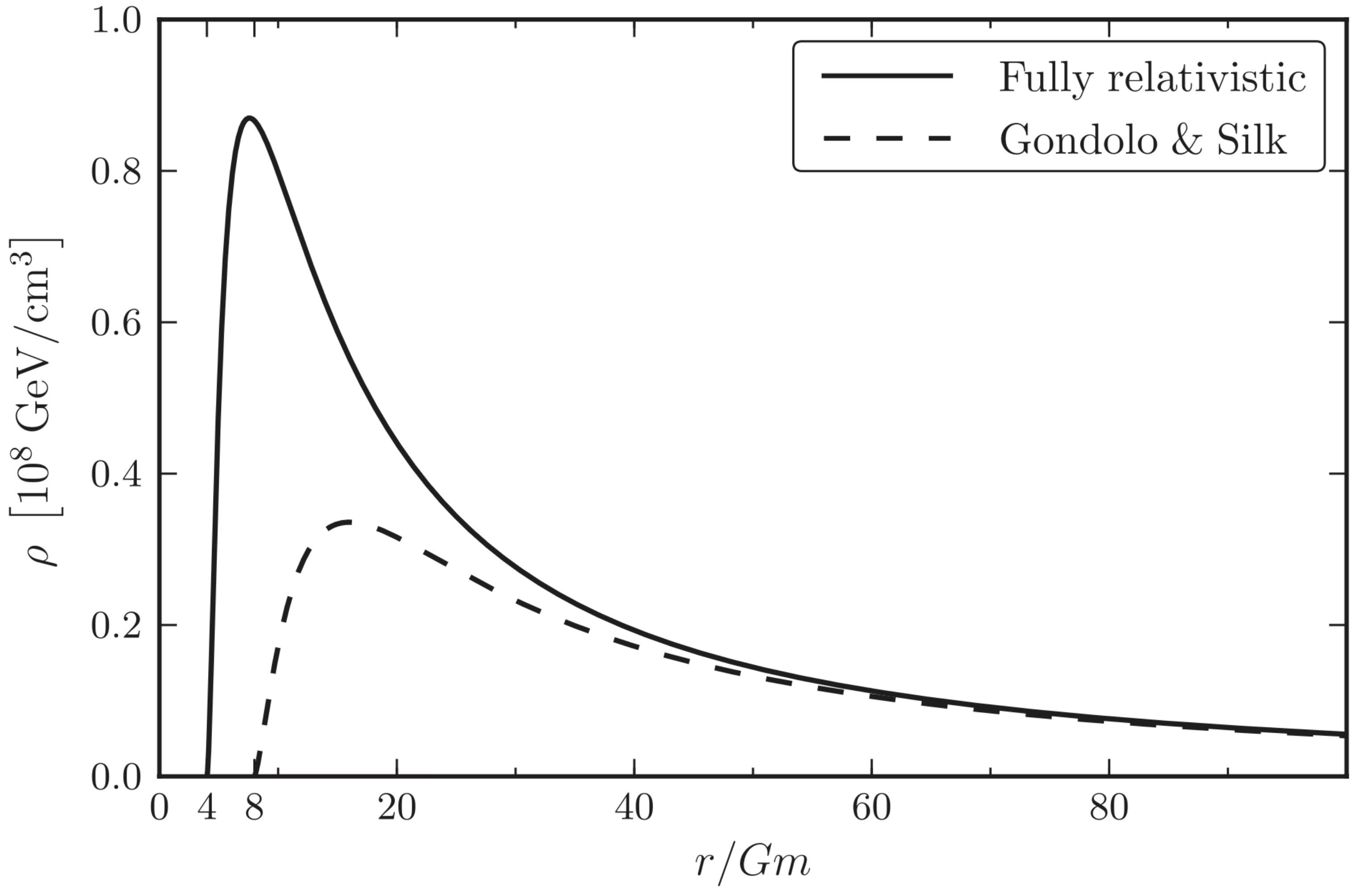
[5] Gondolo and Silk (1999)

Instantaneous case



[6] Ullio, Zhao and Kamionkowski (2001)

Relativistic-Adiabatic case

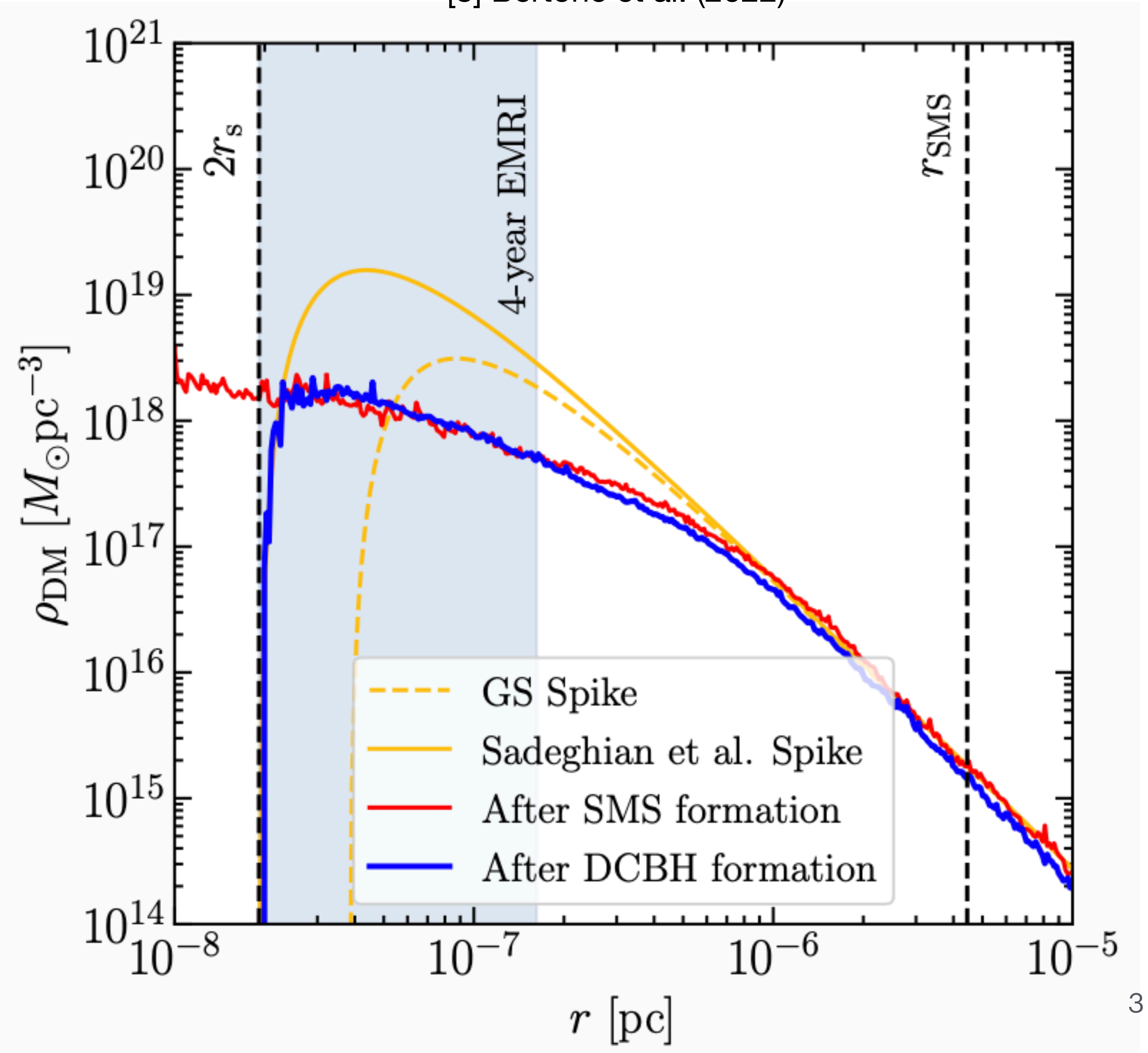


[7] Sadeghian, Ferrer and Will (2013)

SM POP III Stars as SMBH seeds

- JWST is observing many quasars at very high redshift and with masses already at the same range as local SMBHs.
- This suggests POP III stars with masses 100 Solar Masses are likely not enough to seed the SMBH we observe.
- Bertone et al. considered the effect of a much larger seed star (10^5 Solar Masses)
- Our new paper aims to extend these calculations to the full GR case

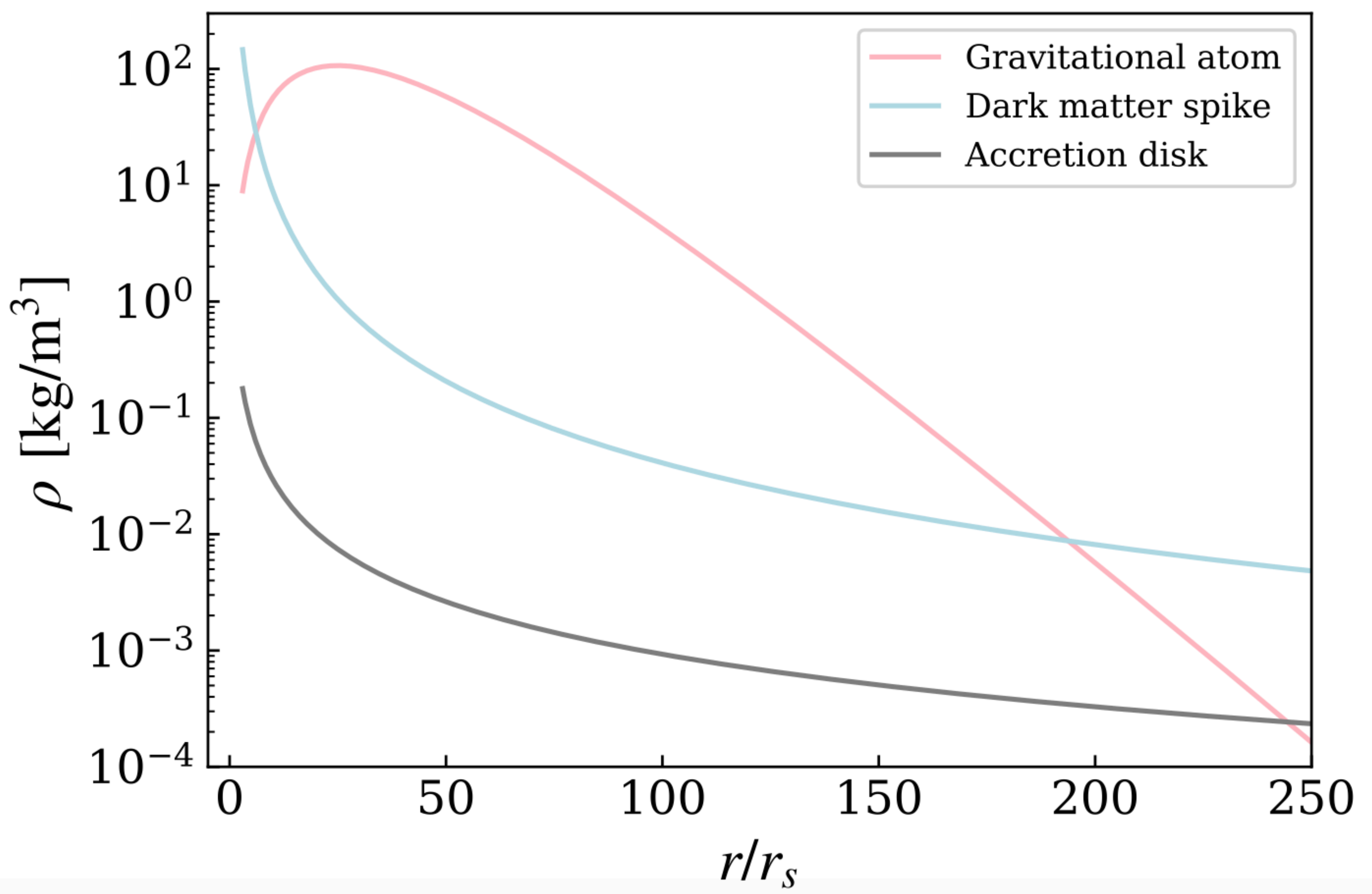
[8] Bertone et al. (2022)



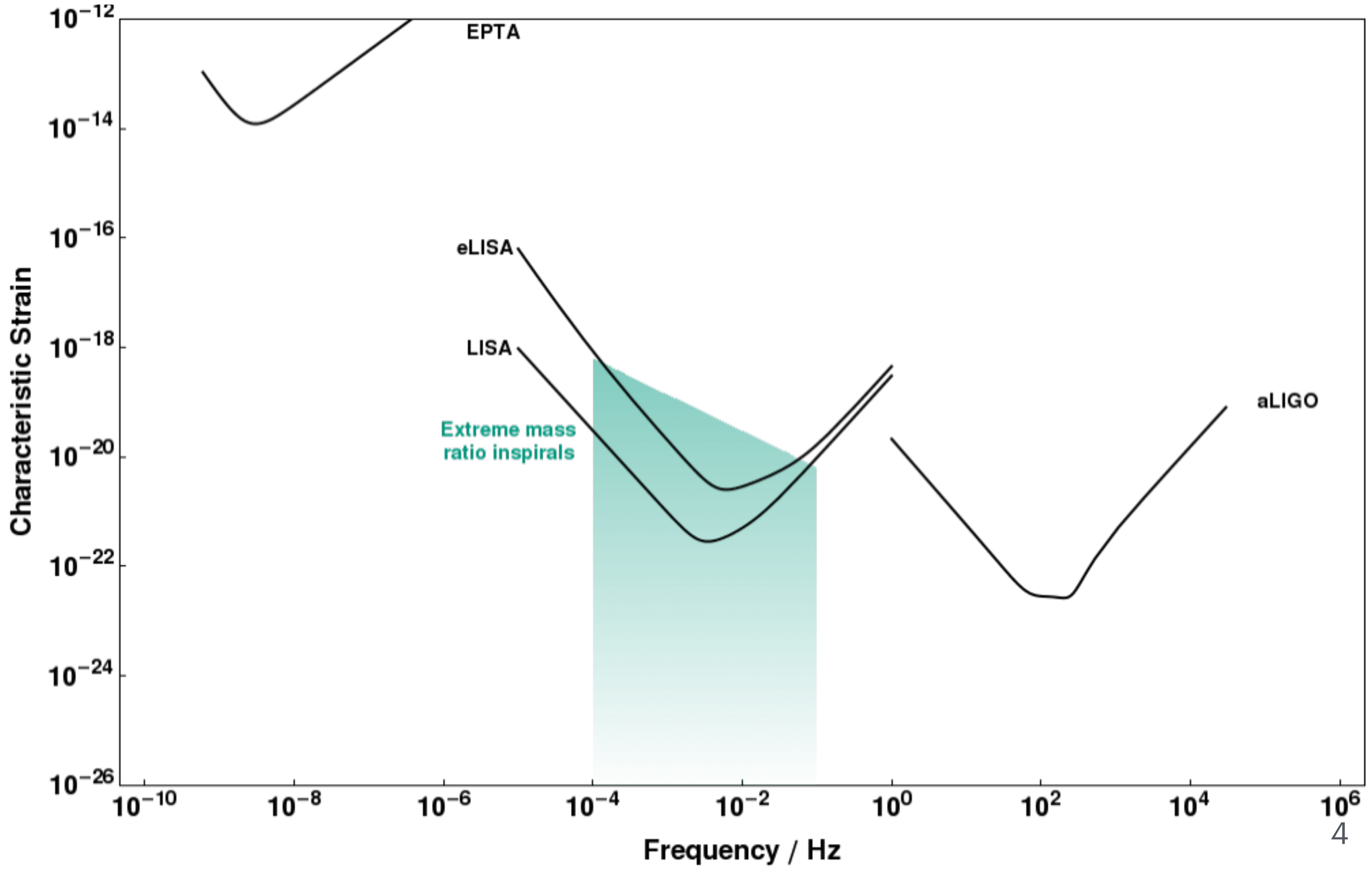
Dark matter Spikes: Why do we care

- Spikes as annihilation boosters
- Gravitational Waves: Accretion and Dynamical Friction (EMRIs)

[9] Cole et al. (2022)

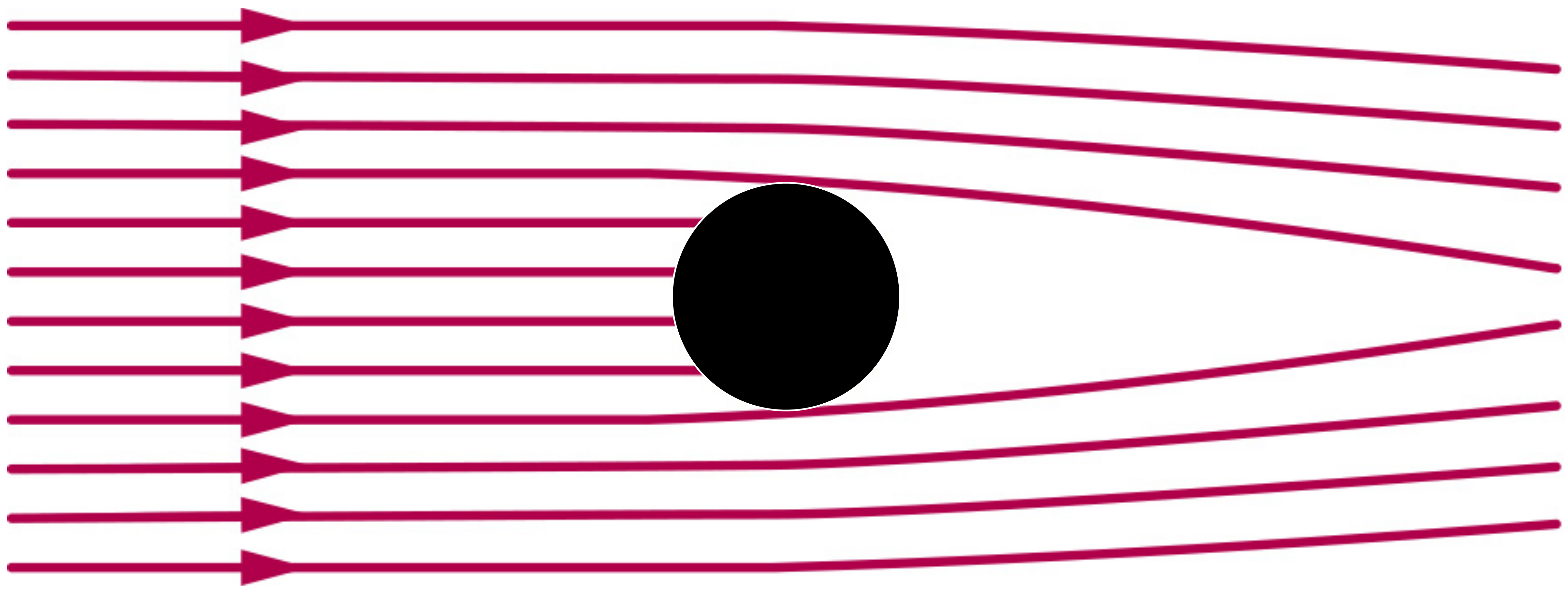


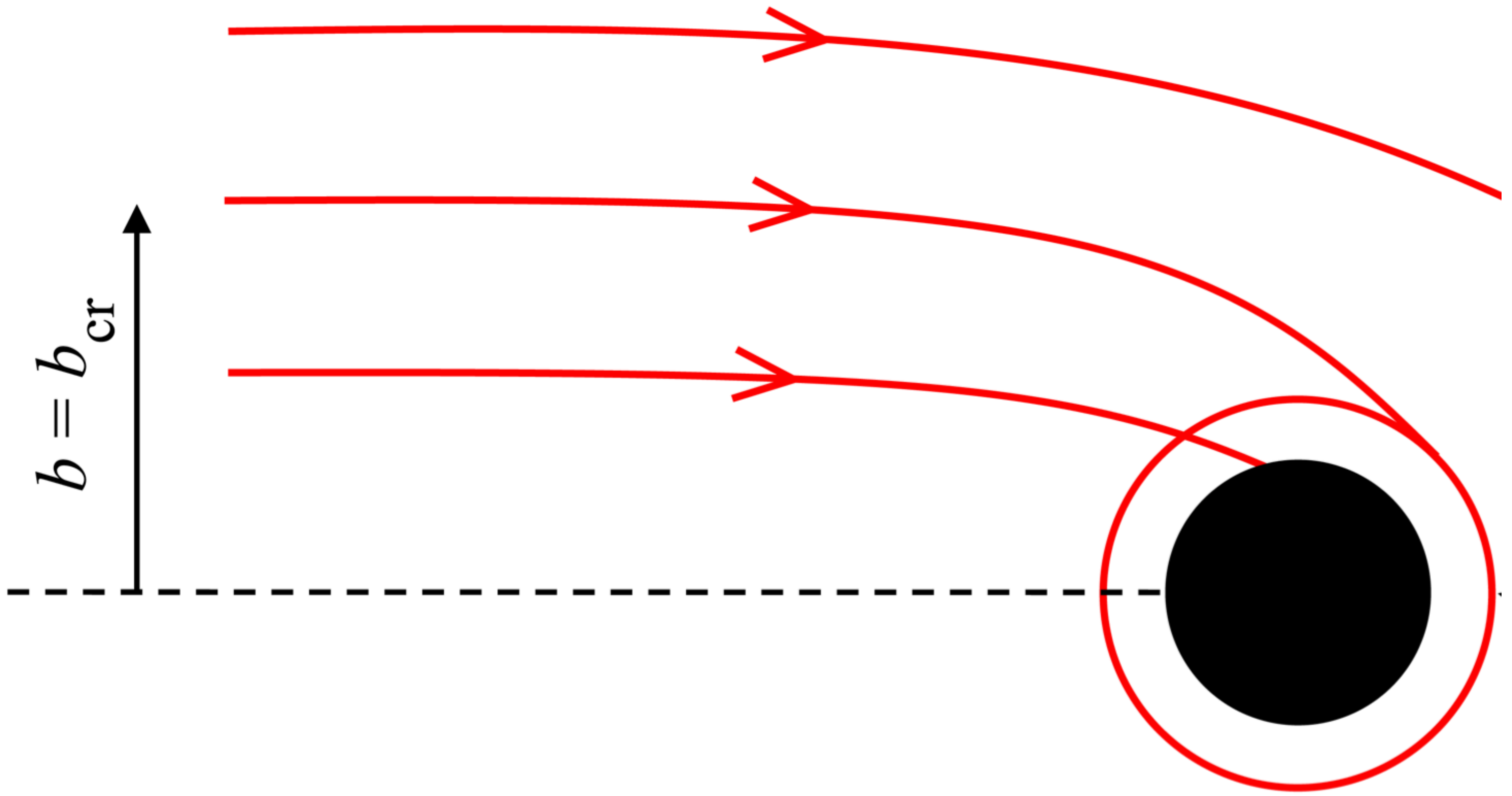
[10] Moore, Cole, Berry (2013)



Relativistic Dark matter Spikes: Why do we care

Relativistic accretion and dynamical friction

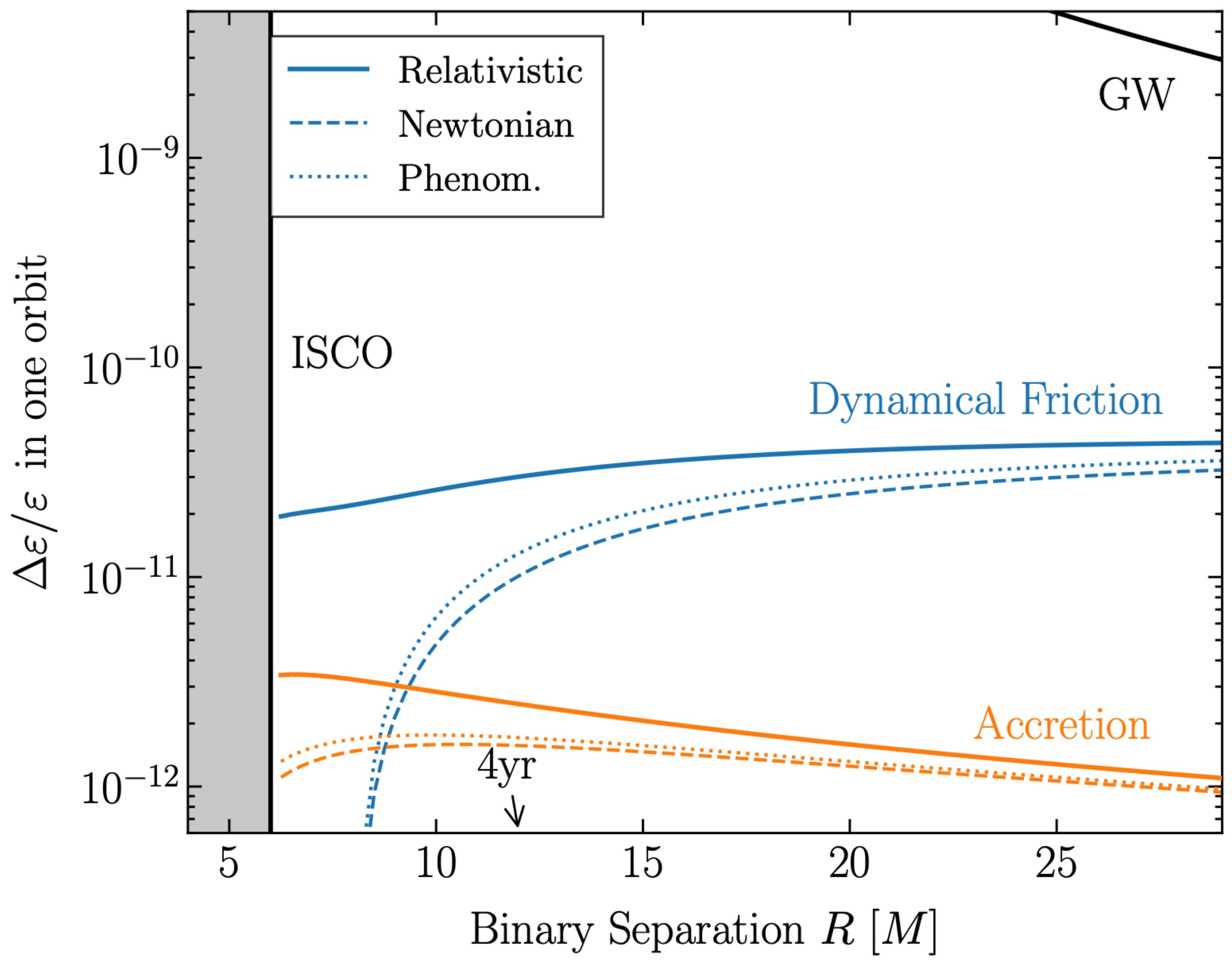


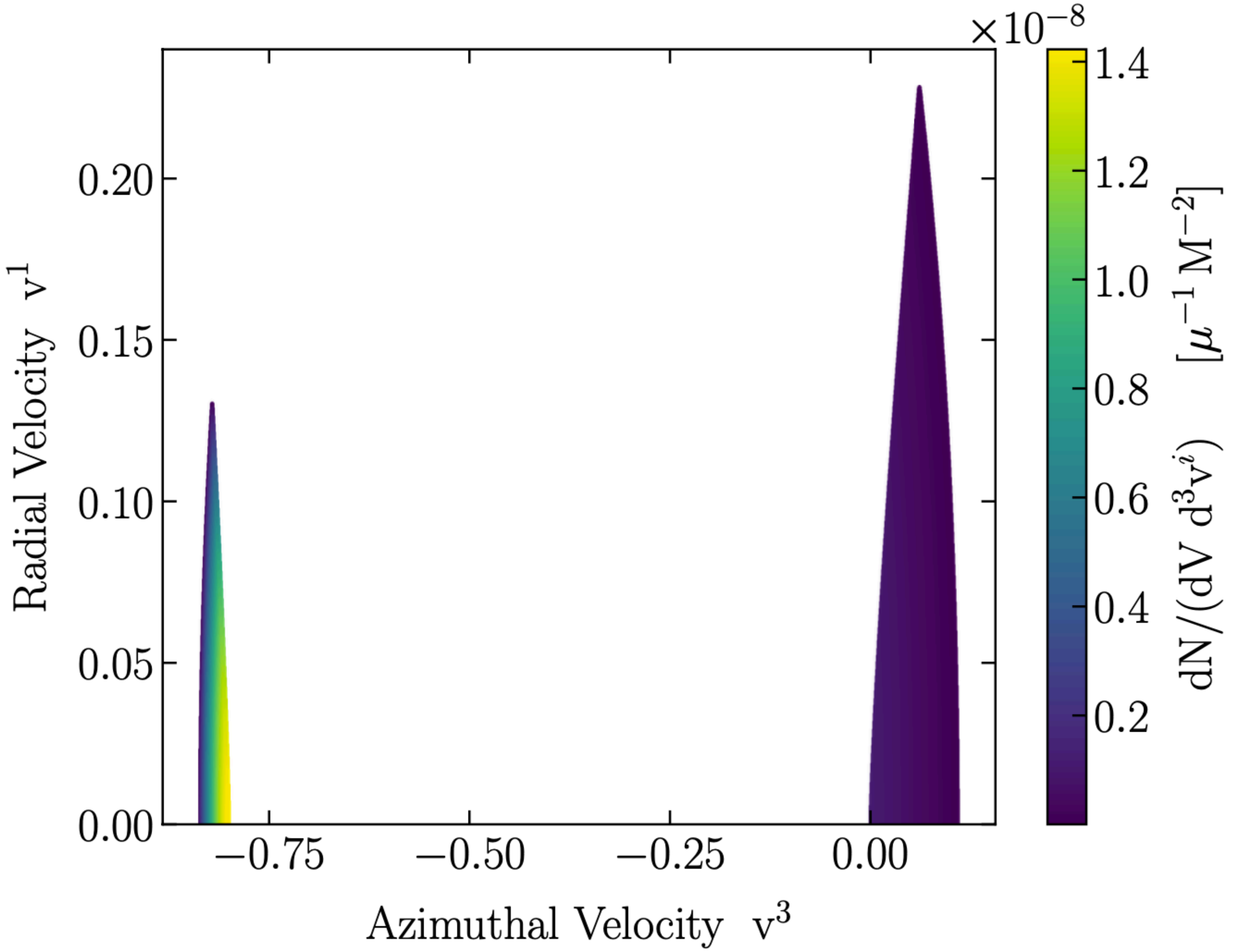


[11] Kogan and Tsupko (2017)

Relativistic Dark matter Spikes: Why do we care

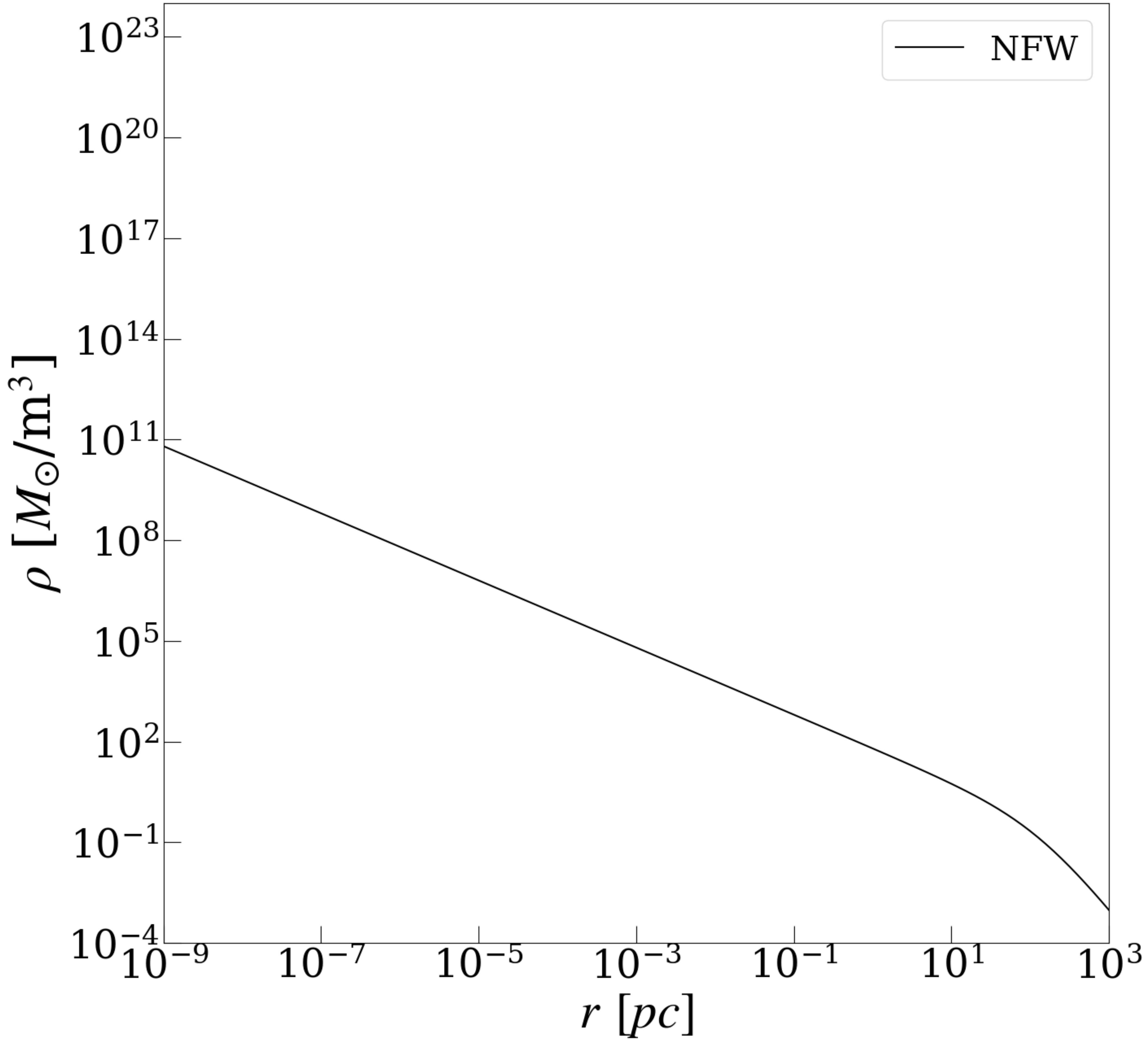
Relativistic accretion and dynamical friction



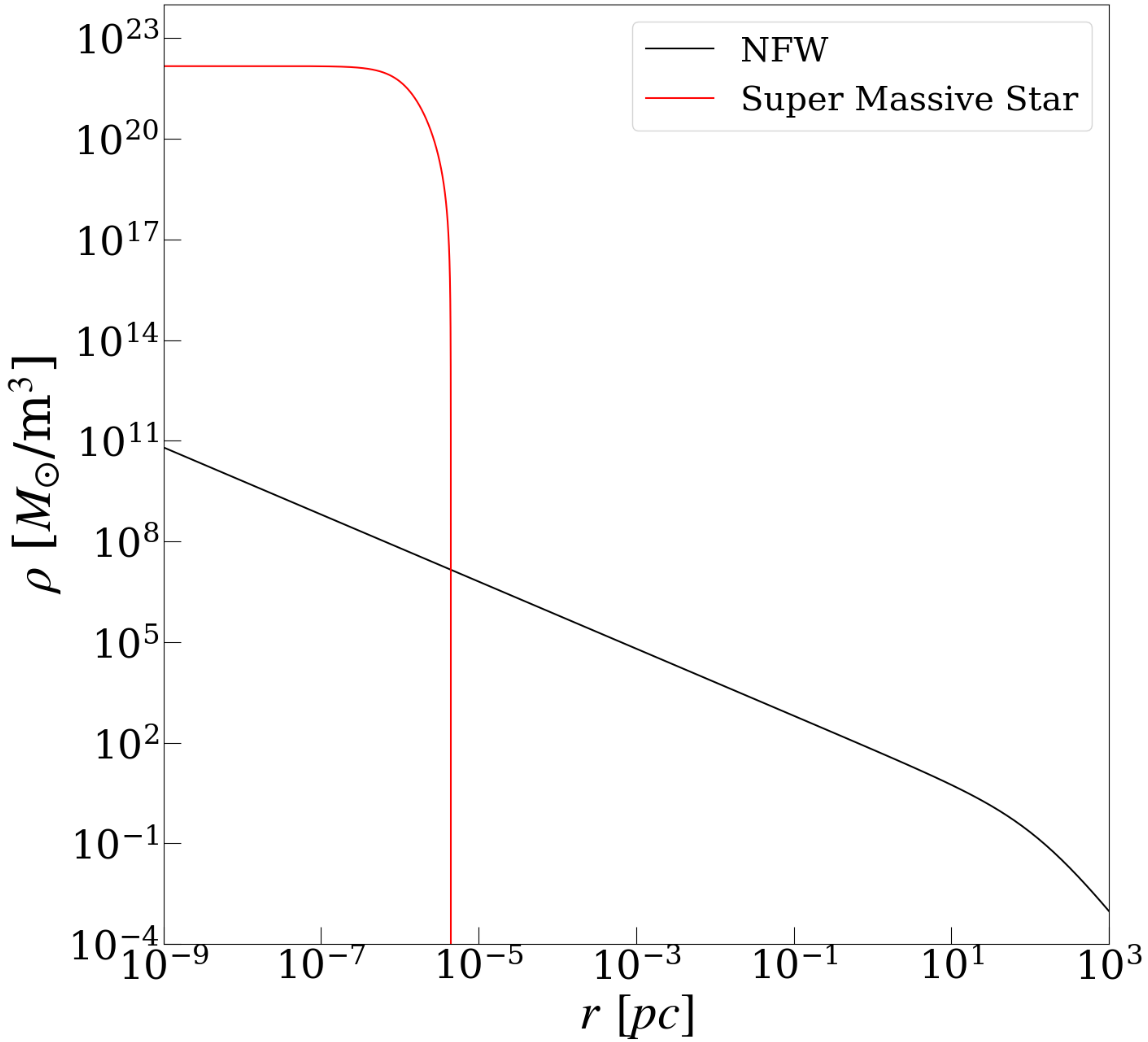


[11] Vicente, Karydas and Bertone (2025)

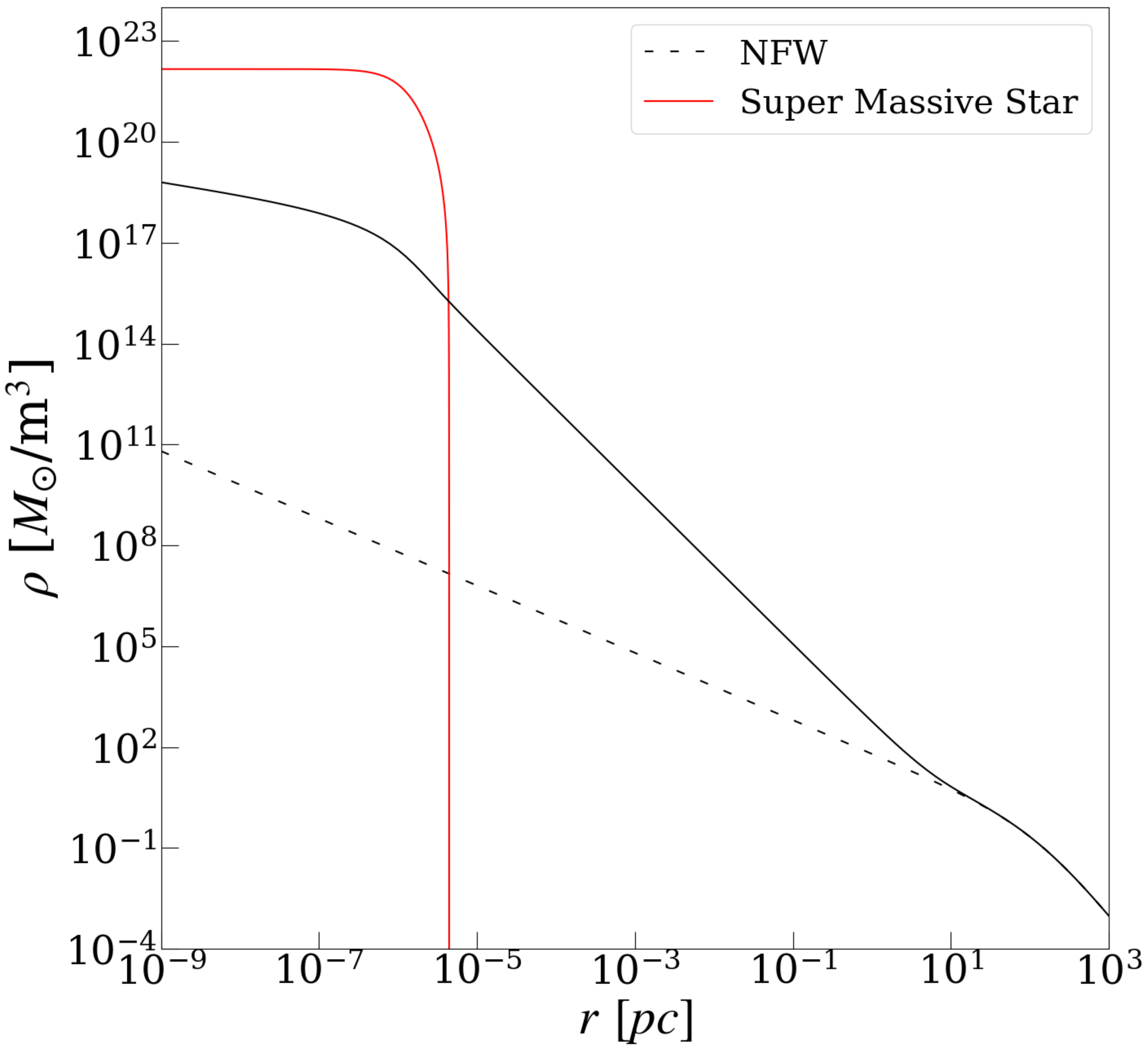
Adiabatic Growth of Pop III star



Adiabatic Growth of Pop III star



Adiabatic Growth of Pop III star



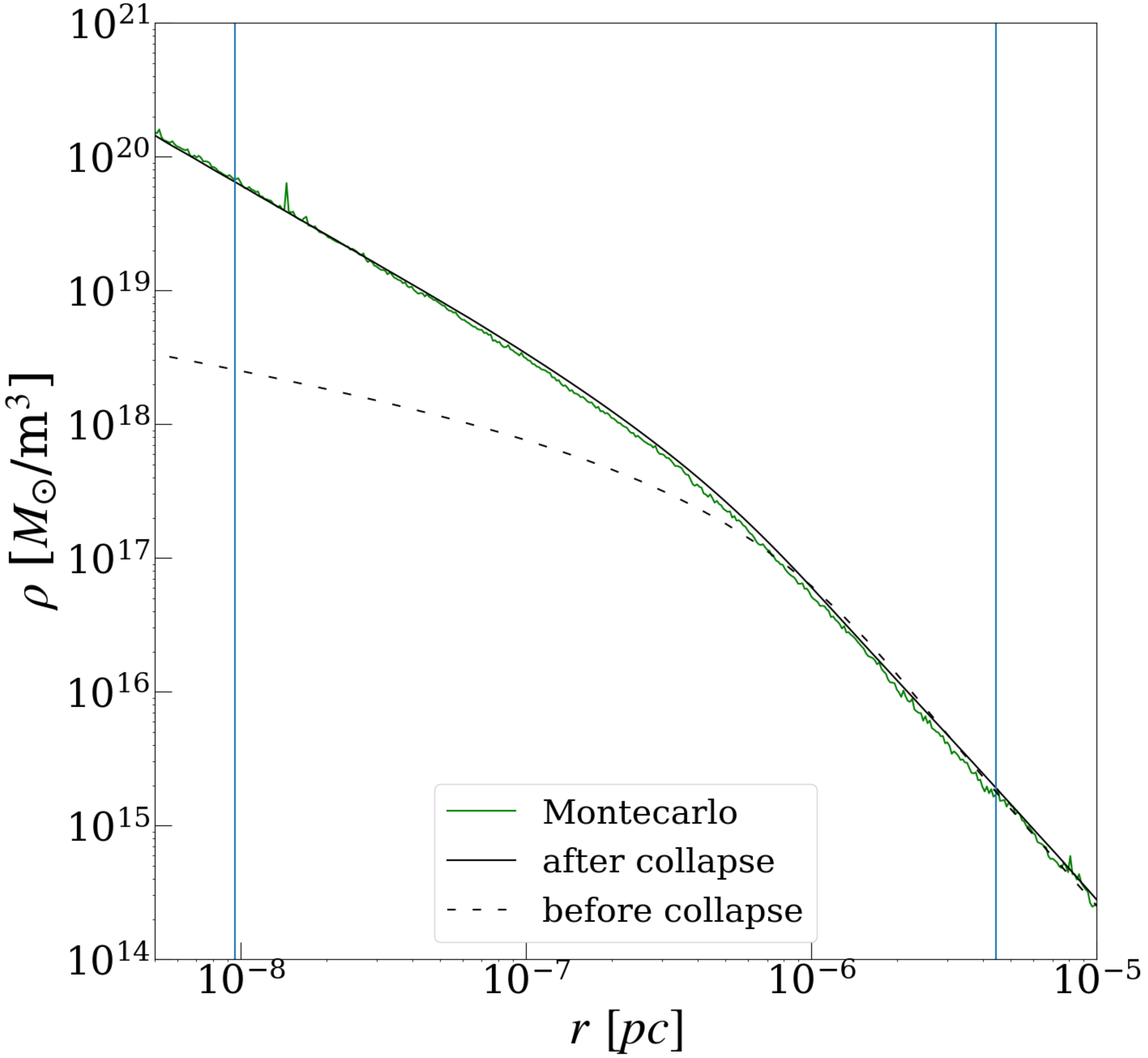
Instant Collapse, Evolving the Distribution function

- Assuming that the system can be well approximated as having reached a steady state equilibrium, we know per the Jeans Theorem that the distribution function should depend only on the Integrals of motion of the particle. In a static spherically symmetric potential this gives $g(E,L) = \dfrac{\mathrm{d}N}{\mathrm{d}E\mathrm{d}L} = \displaystyle\int_{r_p}^{r_a} f(\vec{x},\vec{v}) \dfrac{16\pi^2 L}{v_r m_0}\,\mathrm{d}r = f(E,L)\dfrac{T\,16\pi^2 L}{m_0}$

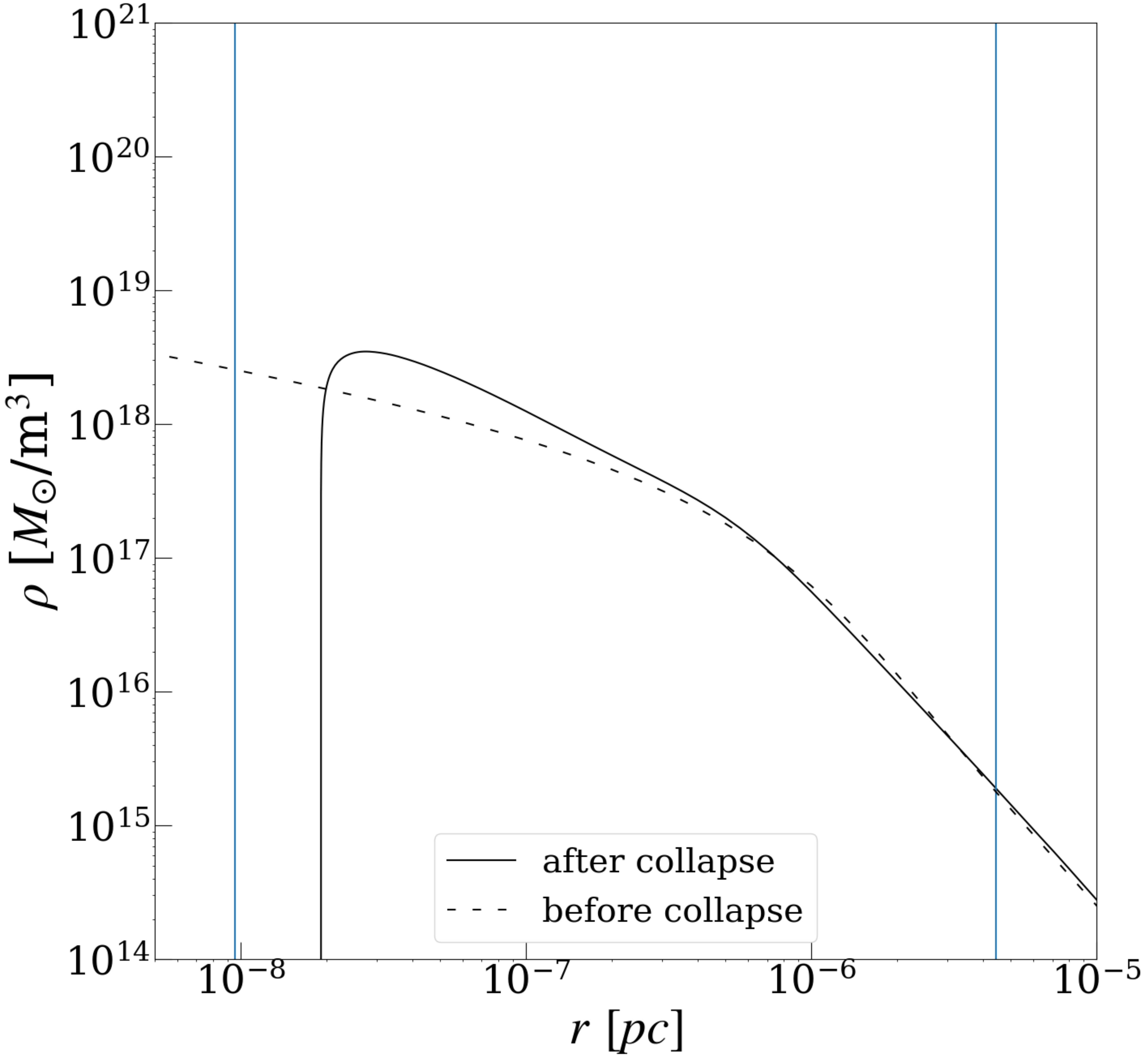
- Liouville's theorem tells us that the distribution function is constant along the trajectories of the system, we also know that we can relate the energy before and after collapse by the equation $E_s\left(E_c, r\right) = E_c - \left(\dfrac{GM}{r} - \Phi_s(r)\right)$

- As the DM evolves in the BH potential the particles will undergo ergodic relaxation until it reaches a new equilibrium configuration such that. $f_c(E,L) = \displaystyle\int_{r_p}^{r_a} \dfrac{f_s\left(E_s, L\right)}{v_r T_c} dr$

Instant Collapse Newtonian



Instant Collapse Newtonian



Instant Collapse GR

- The concept of an instantaneous collapse it's ill defined in GR, for the simple fact that the concept of an "instantaneous" collapse depends on the reference frame.

- If we take a stationary reference frame with respect to the fluid, we can approximate the collapse as instantaneous by taking an hypersurface defined by t=constant and "glue" the star's metric to the Schwarzschild one.

- However it's impossible to evolve the geodesic equations for u^t and u^r due to divergences of the time derivatives of the metric

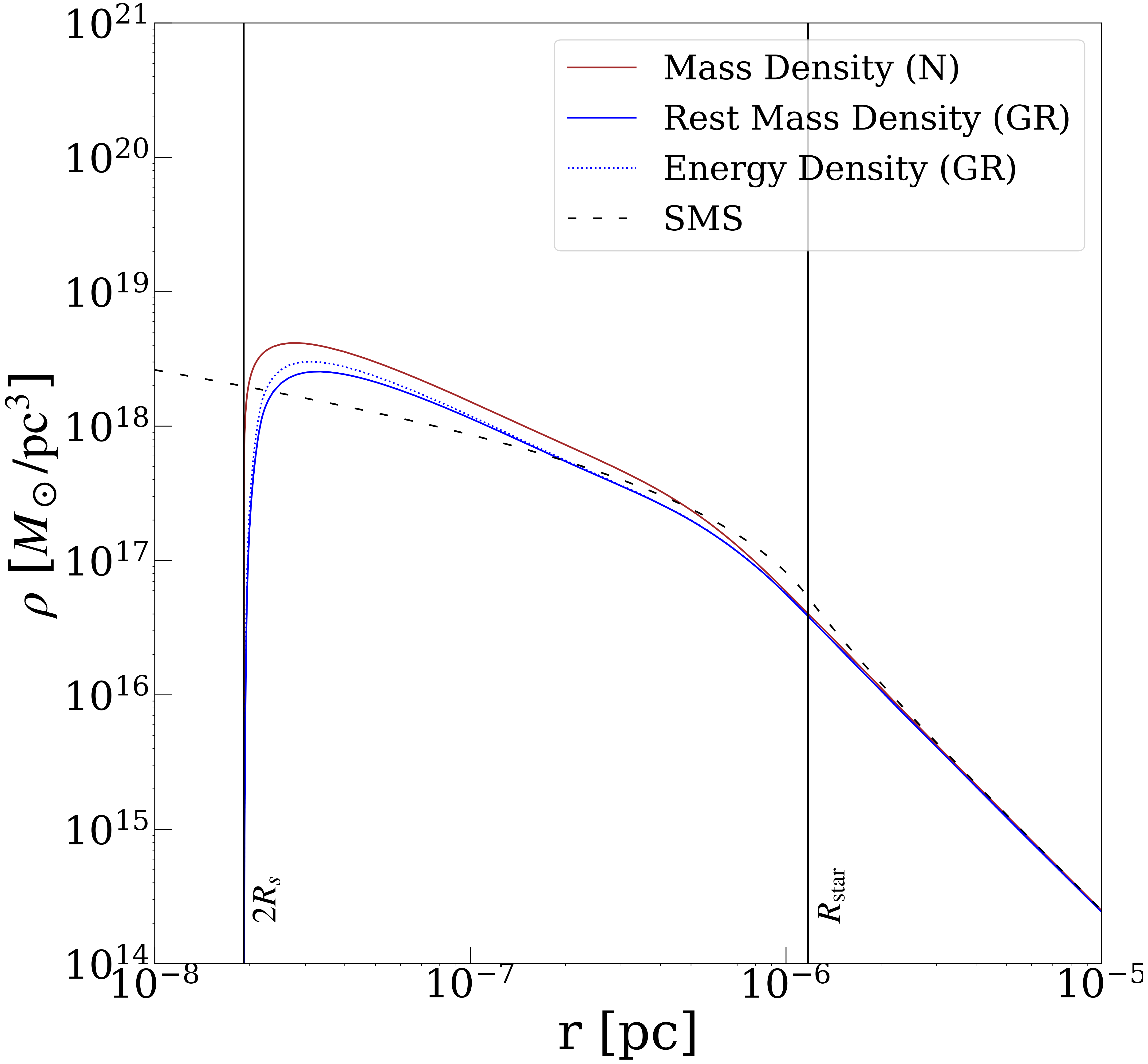
Instant Collapse GR

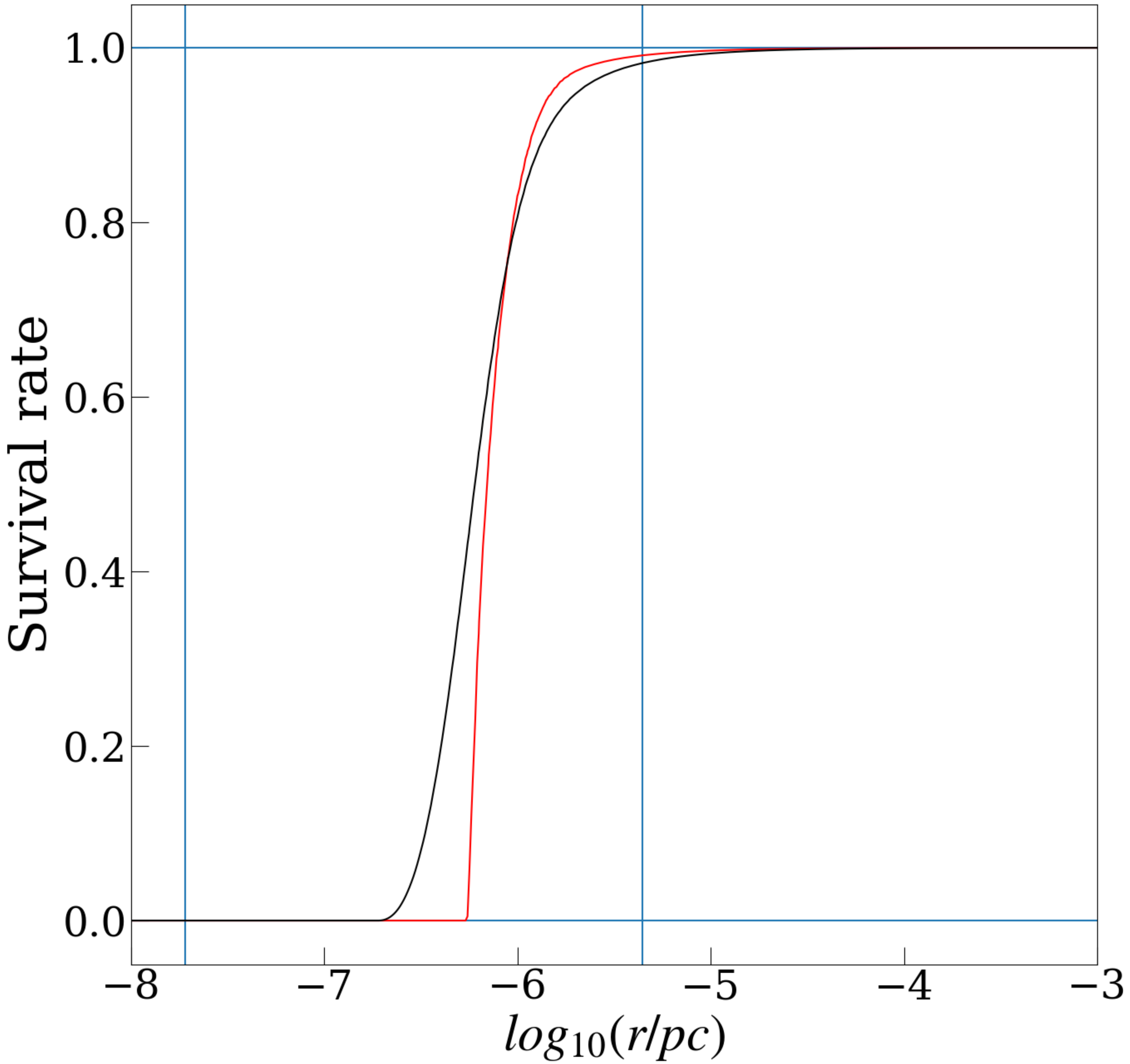
- Other people have already studied similar scenarios when the derivatives of the metric diverge! In these cases what we can use are the so called Israel junction conditions.

- These are usually used to study sudden changes due to thin shells of matter, but they have also been used to study space-like changes in metric for some exotic theories.

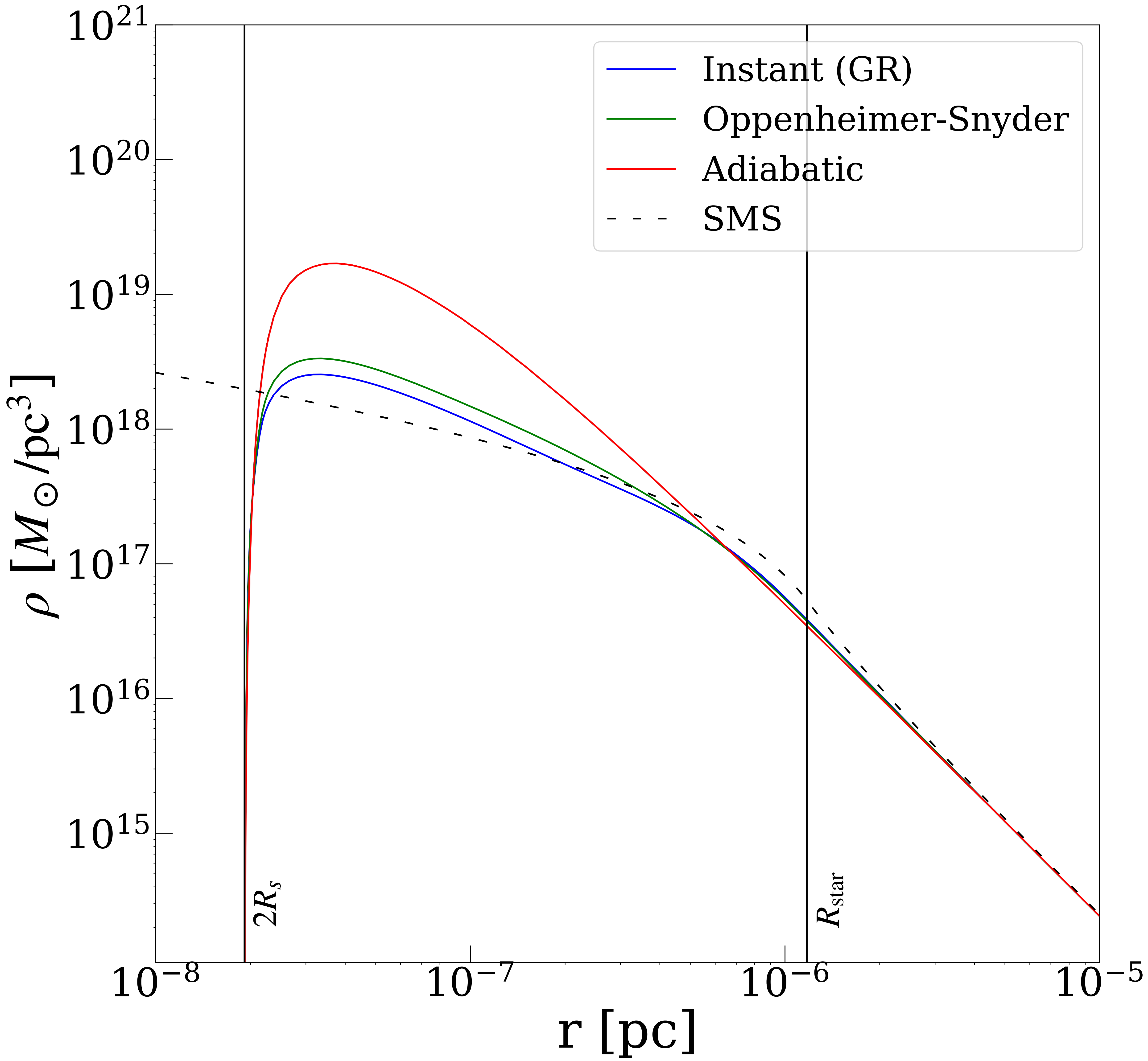
- Imposing the first junction condition, the conservation of the induced metric, gives us the relation $\mathcal{E}_i\left(\mathcal{E}_f, r_i\right) = \mathcal{E}_f\sqrt{\frac{g_{00i}}{g_{00f}}}$

Sudden Collapse GR





Oppenheimer Snyder Collapse



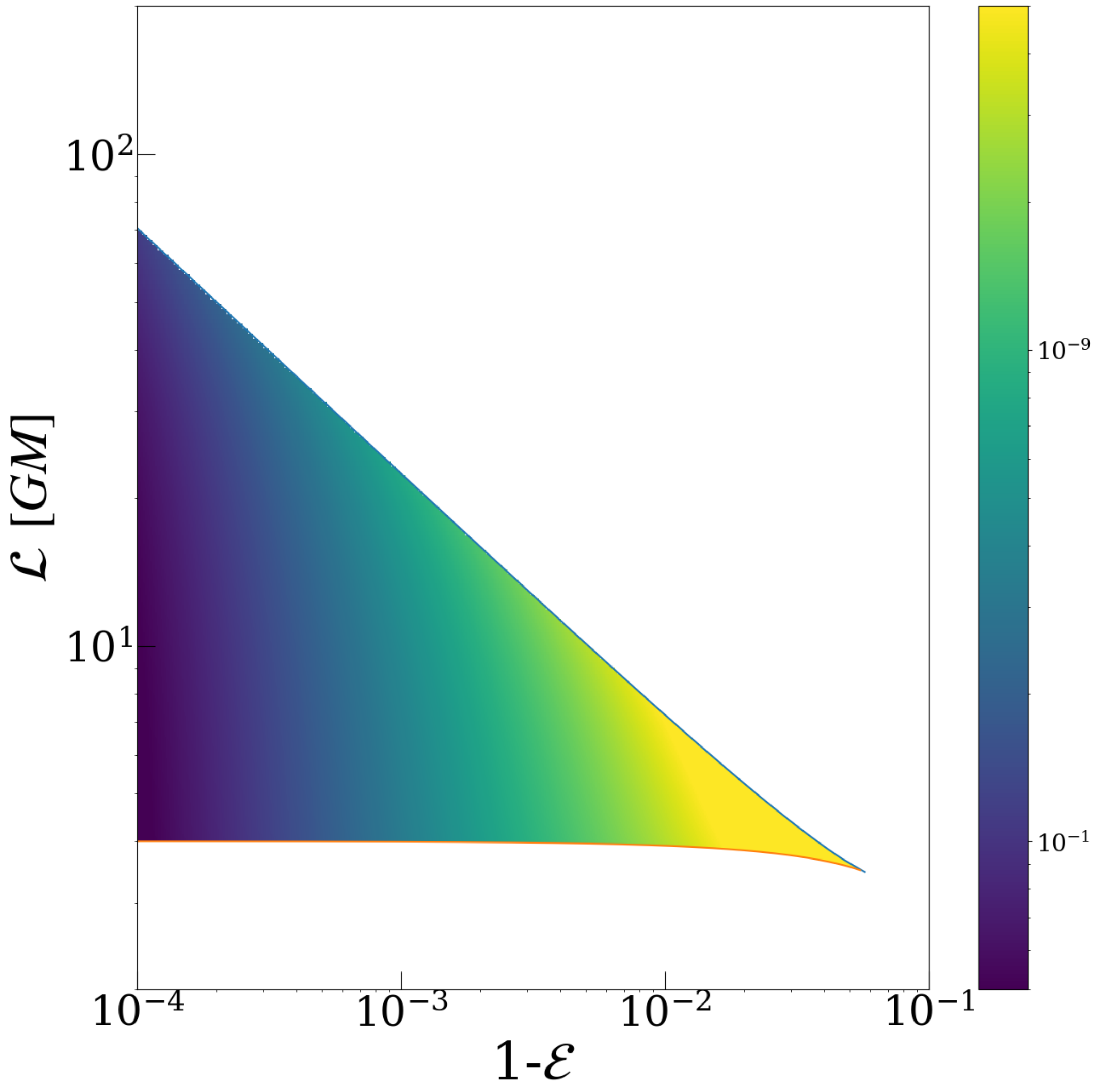
- We also compute the distribution function after an Oppenheimer-Snyder collapse as a toy model to evolve the geodesics instead of the idealised instant collapse.
- For an Oppenheimer Snyder collapse we also consider an space like hypersurface at τ=0. Where the metric becomes

$$d\tau^2 = d\tau_d^2 - a^2(\tau_d)\left(\frac{dR^2}{1 - R_s\frac{R^2}{R^3_{\text{star}}}} + R^2 d\Omega^2\right)$$

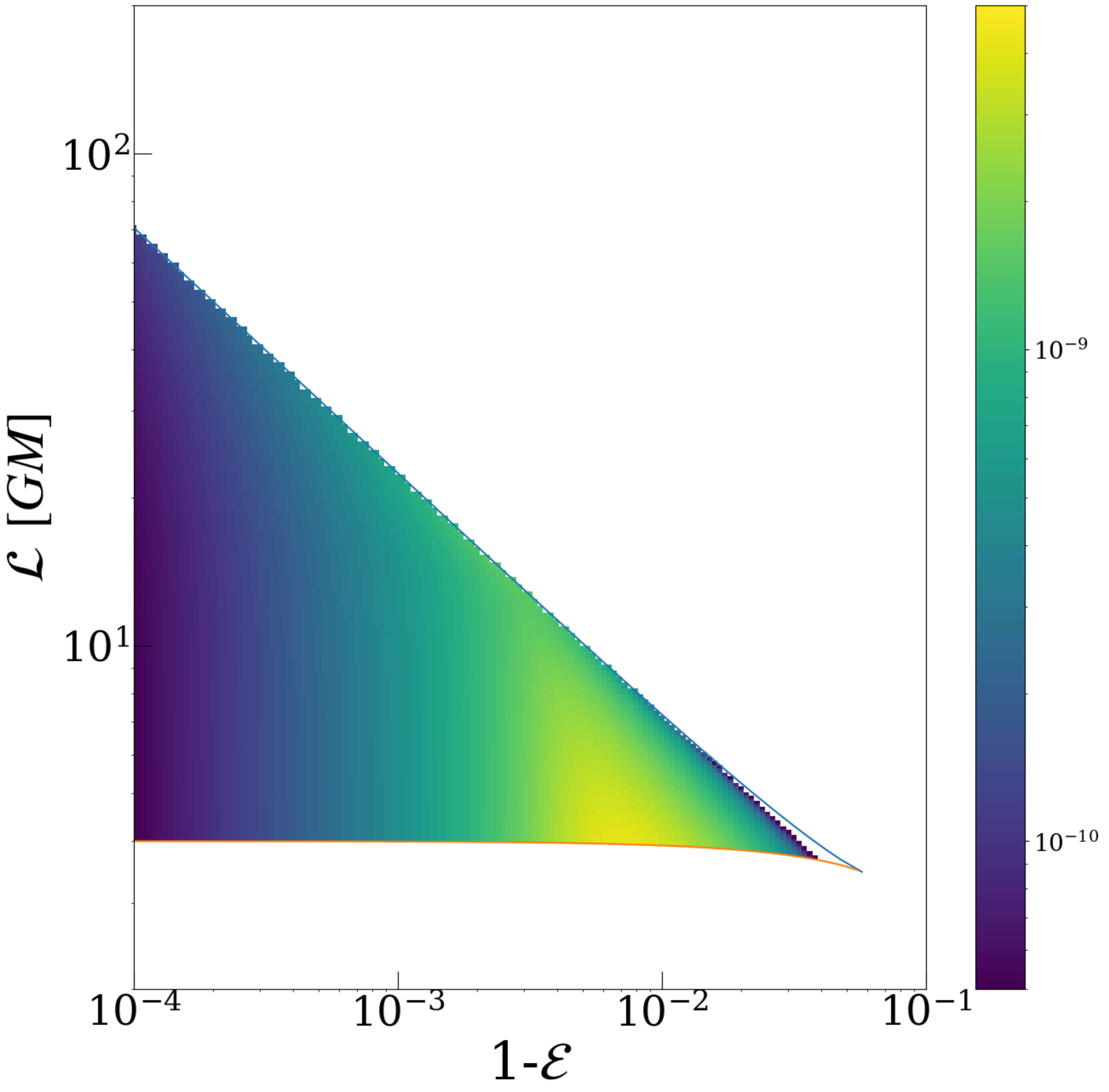
- As expected the result is somewhere in between the instant ad adiabatic collapse.

Comparing Distribution functions

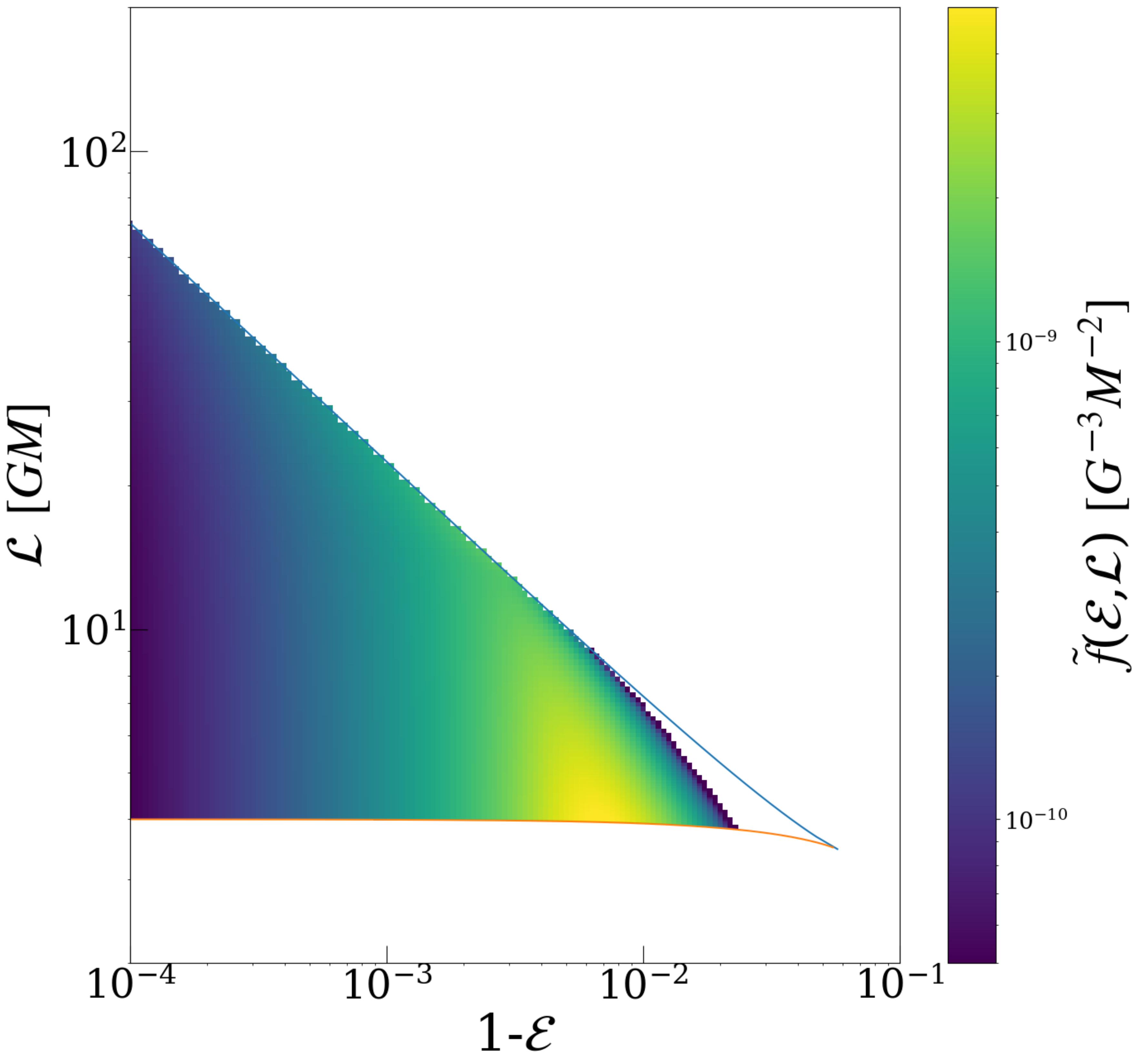
Adiabatic



Oppenheimer Snyder



Instant



Comparing Distribution functions

From the perspective of a static observer at 3 R_s

Adiabatic

Oppenheimer Snyder

Instant

What is next

- Regrowth of the SMBH post collapse will likely erase some of these properties
- We are now starting to look into how these effects impact the GW signals and detection.
- If the results are interesting this could be extended to the Kerr case.
- Baryonic effects and Dynamical friction from infalling compact objects may impact the central regions of the spike.
- Adding effects from self-interactions and checking if these causes any degeneracies.

Thank you for listening!

Any Questions?