A novel method for Forecasting and Recasting Dark Matter Annihilation Limits from Gamma-Ray Observations

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TeV Particle Astrophysics
TeVPA
Valencia 2025

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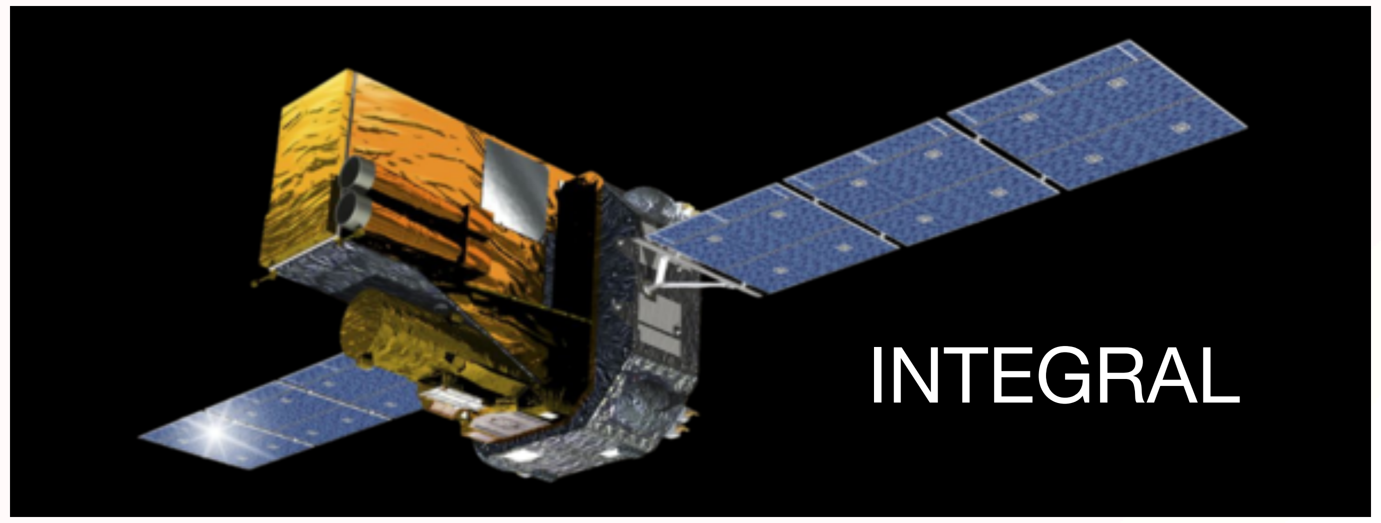
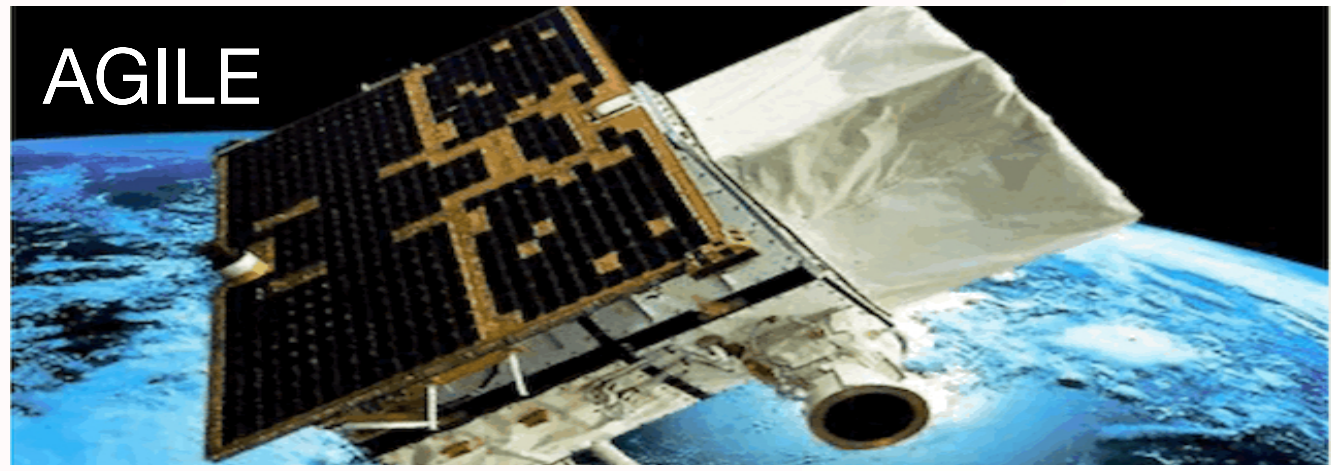
beatriu de pinós

Motivation

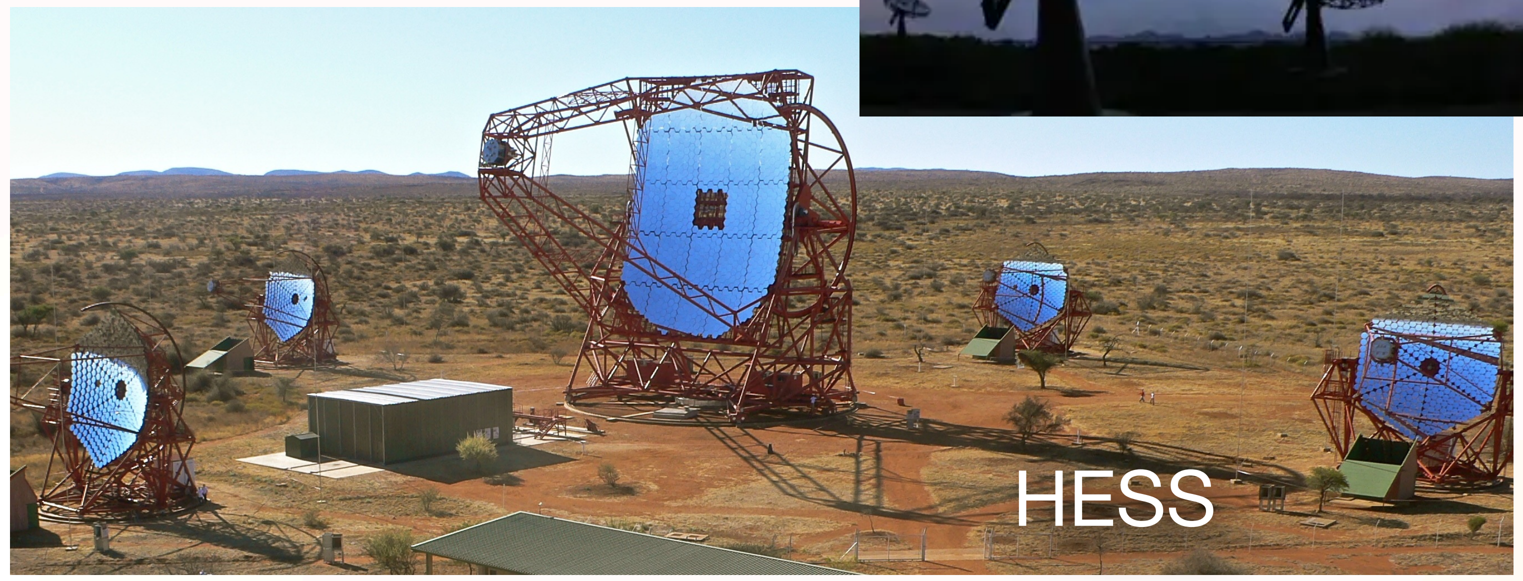
Different γ-ray telescopes have been trying to detect DM in the past decade

Space Telescopes

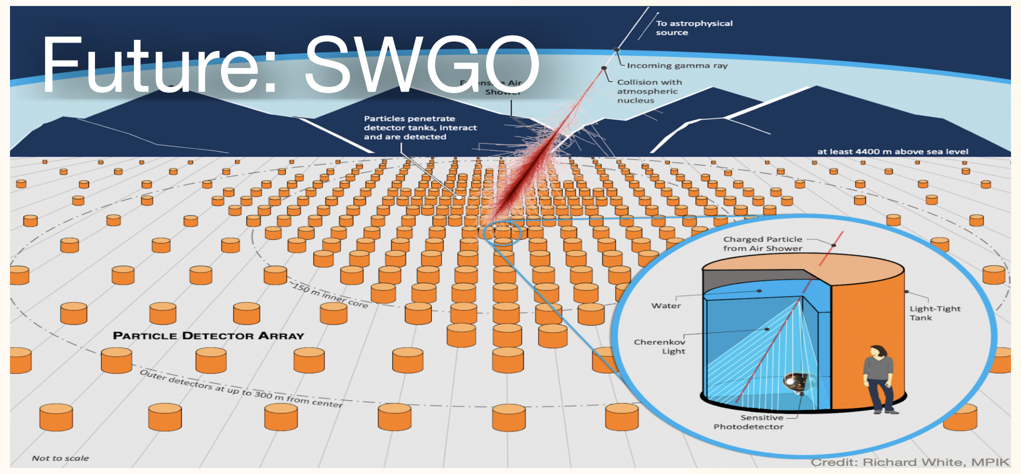




Imaging Atmospheric Cherenkov Telescopes



Water Cherenkov detector

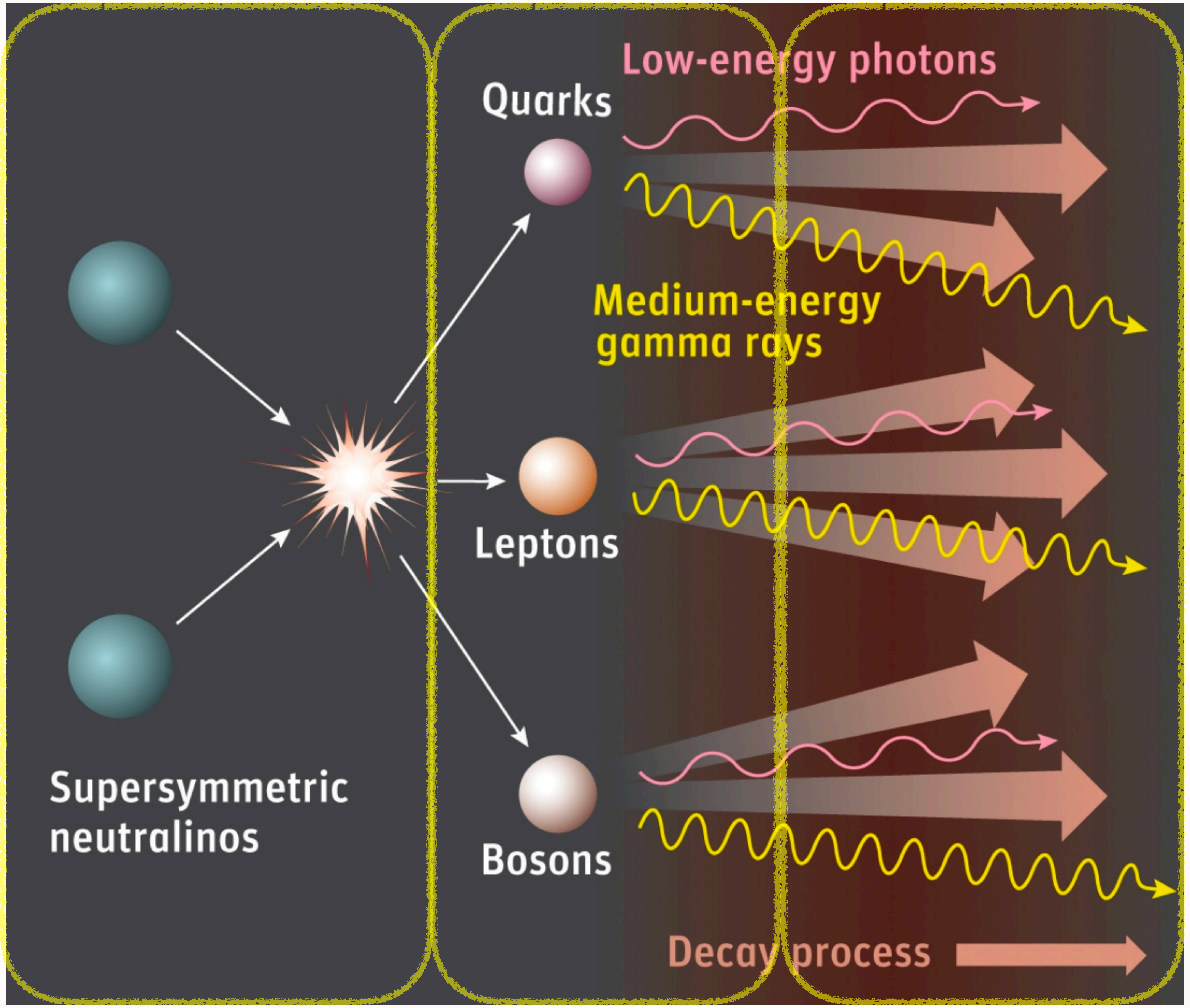


Motivation

Supersymmetry or other models

Standard Model

Astrophysics

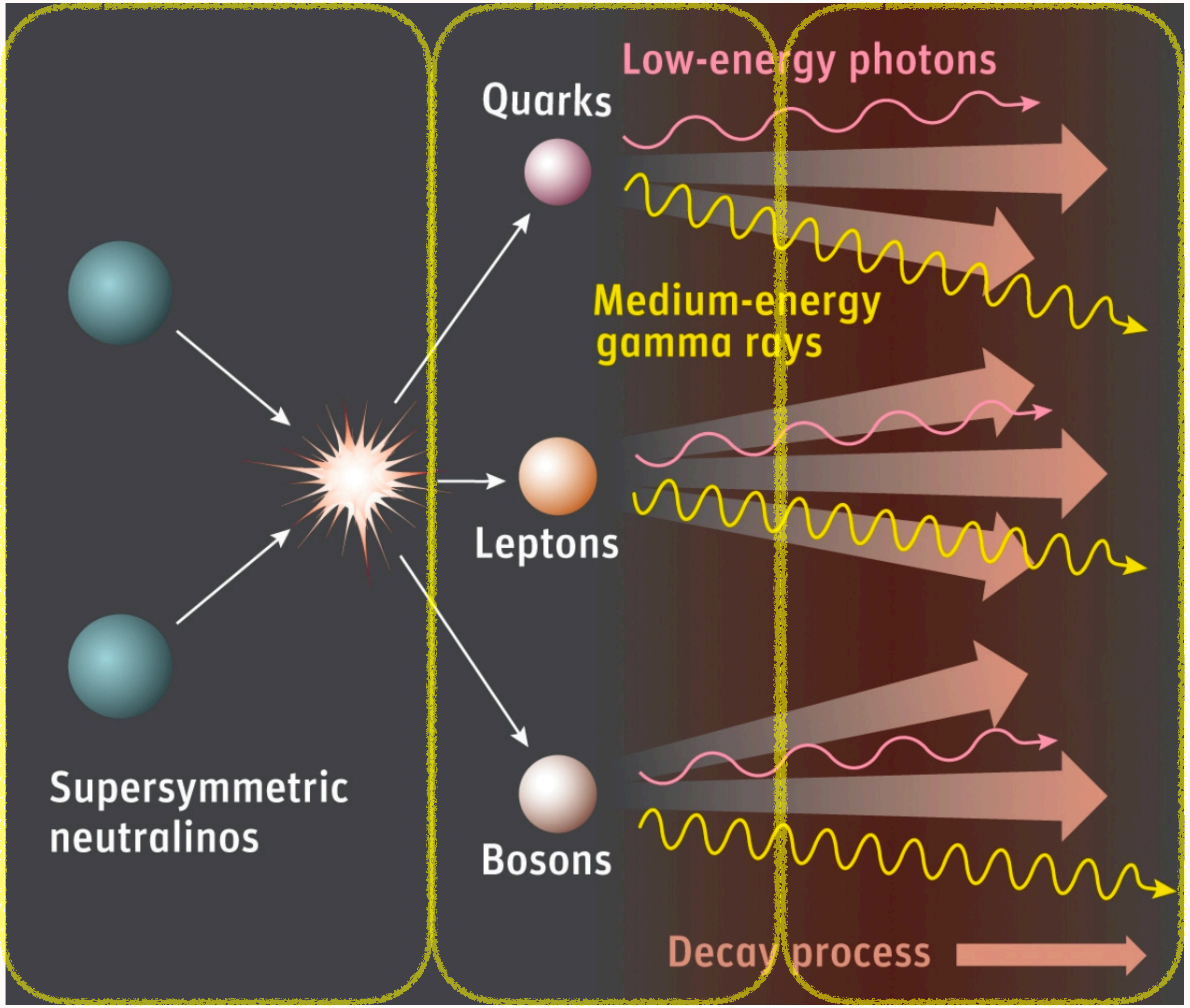


Motivation

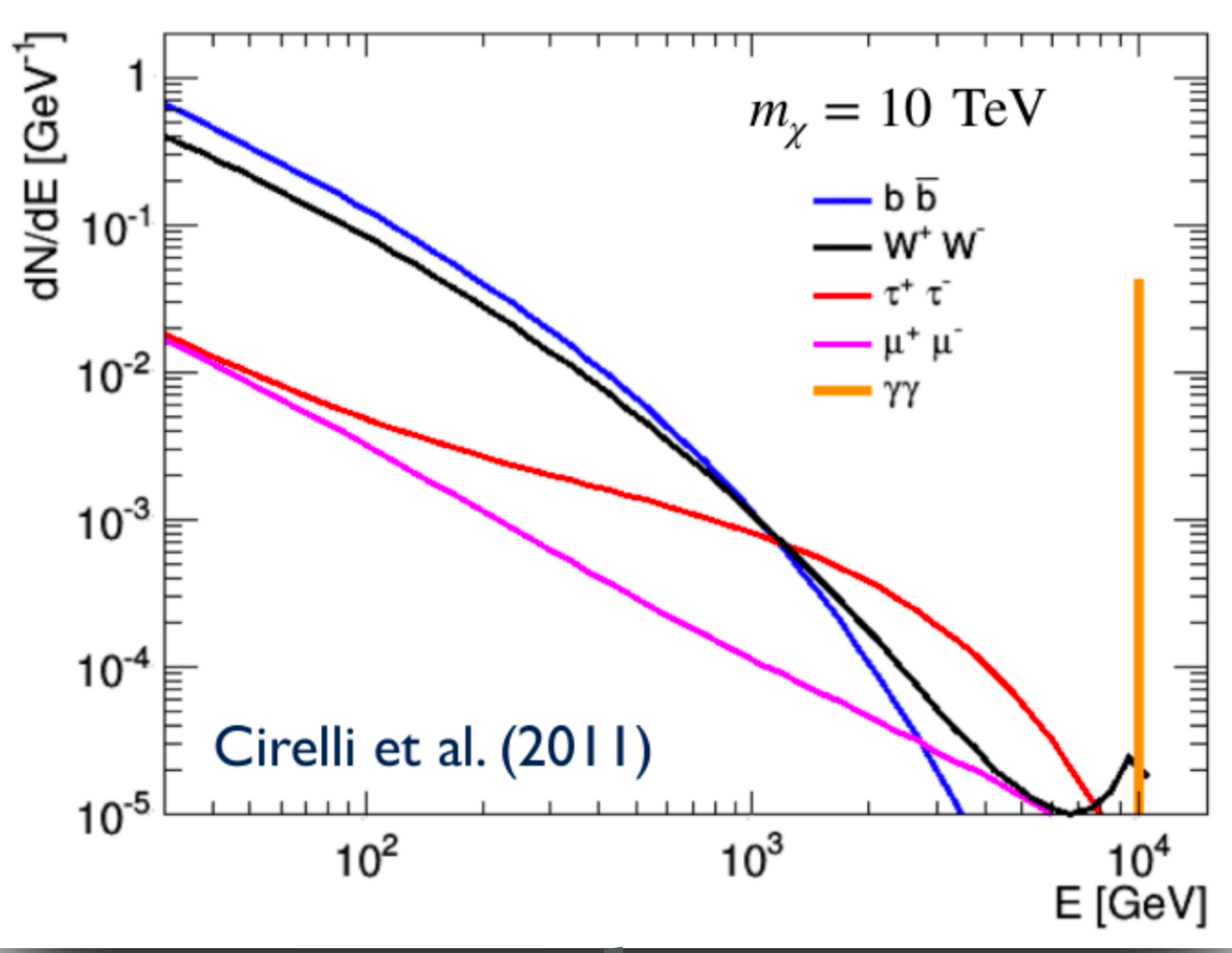
Supersymmetry or other models

Standard Model

Astrophysics



Energy distribution of gamma-rays from DM annihilation



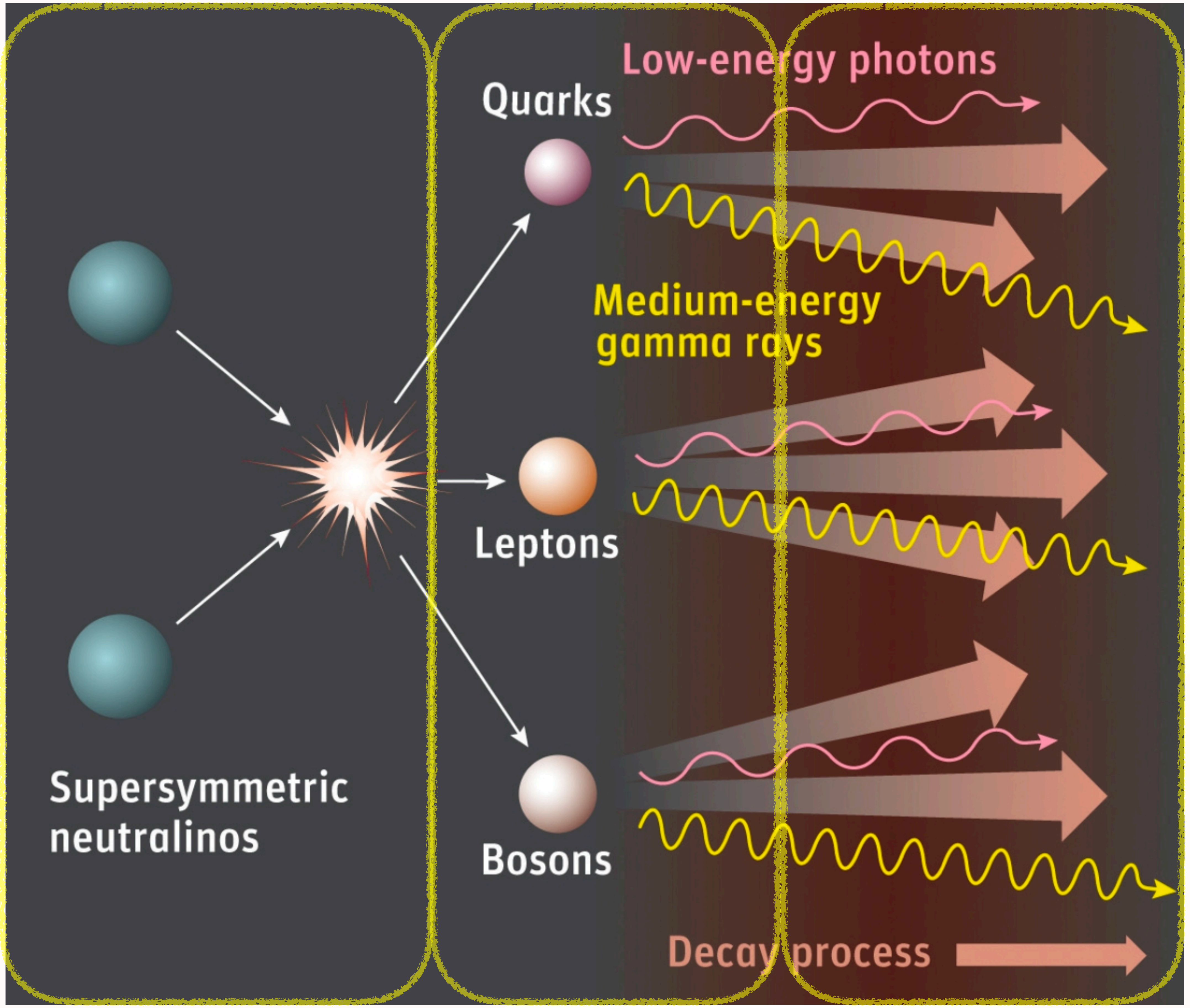
DM cross-section

$$\mathrm{IRF} \otimes \frac{dN_\gamma(E)}{dE} \cdot \frac{T_{obs} J}{8\pi m_\chi^2} \cdot \langle \sigma v \rangle$$

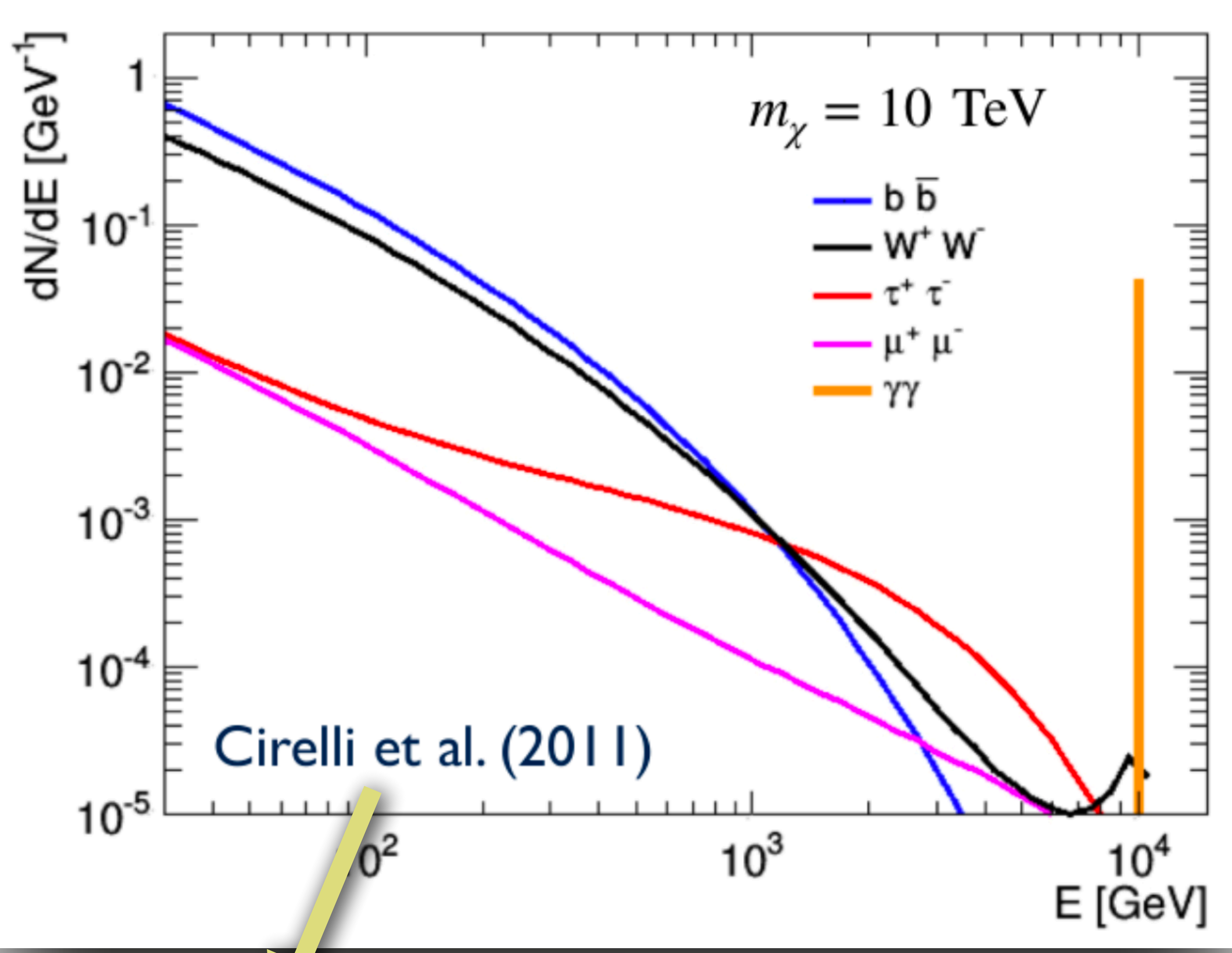
Instrument Response Function

Motivation

Supersymmetry or other models | Standard Model | Astrophysics



Energy distribution of gamma-rays from DM annihilation



PPPC 4 DM ID: a poor particle physicist cookbookfor dark matter indirect detection

M Cirelli, G Corcella, A Hektor, G Hütsi, M Kadastik, P Panci, M Raidal, F Sala, A Strumia

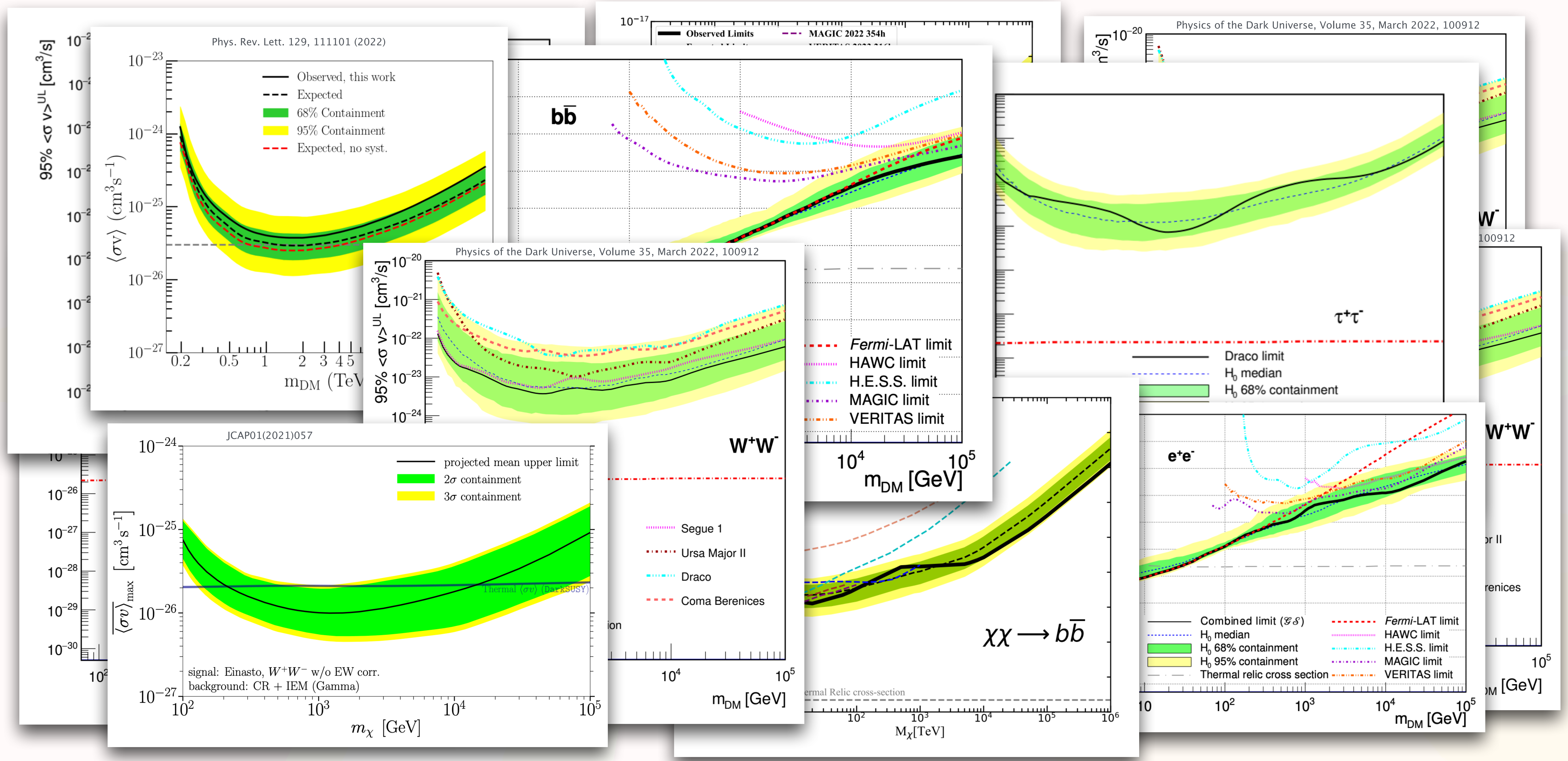
Journal of Cosmology and Astroparticle Physics, 2011 · iopscience.iop.org

Abstract

We provide ingredients and recipes for computing signals of TeV-scale Dark Matter annihilations and decays in the Galaxy and beyond. For each DM channel, we present the energy spectra of at production, computed by high-statistics simulations. We estimate the Monte Carlo uncertainty by comparing the results yielded by the Pythia and Herwig event generators. We then provide the propagation functions for charged particles in the Galaxy, for several DM distribution profiles and sets of propagation parameters. Propagation of

SHOW MORE

Motivation

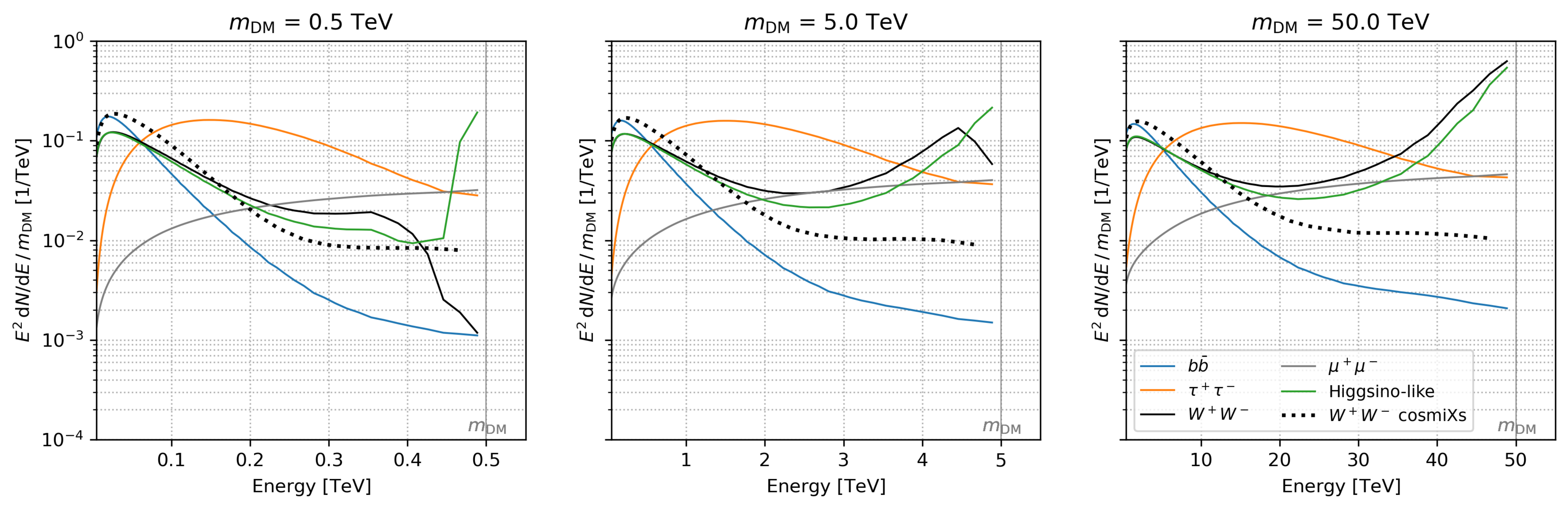


Phys. Rev. Lett. 129, 111101 (2022)
Observed, this work
Expected
68% Containment
95% Containment
Expected, no syst.
$\langle\sigma v\rangle$ (cm^{3}s^{-1})
m_{DM} (TeV
Observed Limits
MAGIC 2022 354h
$b\bar{b}$
Fermi-LAT limit
HAWC limit
H.E.S.S. limit
MAGIC limit
VERITAS limit
m_{DM} [GeV]
Physics of the Dark Universe, Volume 35, March 2022, 100912
95% <σ v>^{UL} [cm^3/s]
W^+W^-
Segue 1
Ursa Major II
Draco
Coma Berenices
$\tau^+\tau^-$
Draco limit
H_0 median
H_0 68% containment
JCAP01(2021)057
projected mean upper limit
2σ containment
3σ containment
Thermal $\langle\sigma v\rangle$ (DarkSUSY)
signal: Einasto, W^+W^- w/o EW corr.
background: CR + IEM (Gamma)
$\overline{\langle\sigma v\rangle}_{max}$ [cm^{3}s^{-1}]
m_χ [GeV]
$\chi\chi \longrightarrow b\bar{b}$
M_χ[TeV]
e^+e^-
Combined limit
H_0 median
H_0 68% containment
H_0 95% containment
Thermal relic cross section
Fermi-LAT limit
HAWC limit
H.E.S.S. limit
MAGIC limit
VERITAS limit

Motivation

Would it be possible to reinterper (*recast*) the published DM ULs from “benchmark” annihilation modes into alternative models?

Example:



Upper/Lower Limits

The standard "rule" for getting upper/lower limits

$$-2 \ln \mathscr{L}(\sigma) = \lambda$$

The likelihood

By definition the probability of the data given your model

parameter of interest σ

A threshold value

Whose value can be 2.71 for a 95% CL if the likelihood is properly *profiled* (Wilks' theorem)

Or obtained through MC simulations

Upper/Lower Limits

The standard “rule” for getting upper/lower limits

$$-2\ln\mathscr{L}(\sigma) = \lambda$$

$$-2\ln\mathscr{L}(\sigma) \equiv 2\sum_i f_i(s_i)$$

Analysis usually performed in bins

Upper/Lower Limits

The standard "rule" for getting upper/lower limits

$$-2\ln\mathscr{L}(\sigma) = \lambda$$

$$-2\ln\mathscr{L}(\sigma) \equiv 2\sum_i f_i(s_i)$$

$$\ln\mathscr{L}_i(s_i \mid D_i) \equiv f_i(s_i)$$

Expected signal count per bin i

$$s_i = K_i \cdot \sigma$$

with K_i a bin-dependent proportional factor

$$K_i \equiv \int_{\Delta E_i'} dE' \int dE\, A_{\gamma,\mathrm{eff}}(E) \cdot \mathscr{G}(E,E') \cdot \frac{dN_\gamma}{dE} \cdot \frac{T_{\mathrm{obs}}\, J}{8\pi m_\chi^2}$$

Upper/Lower Limits

The standard "rule" for getting upper/lower limits

$$-2\ln\mathscr{L}(\sigma) = \lambda$$

$$-2\ln\mathscr{L}(\sigma) \equiv 2\sum_i f_i(s_i) \simeq \sum_i K_i^2\, f_i''(K_i\hat{\sigma})\,(\sigma-\hat{\sigma})^2$$

Taylor expansion around the value $\hat{\sigma}$ that maximizes the likelihood [1]

[1] Cowan, Glen, et al. "Asymptotic formulae for likelihood-based tests of new physics." *The European Physical Journal C* 71.2 (2011): 1554.

Upper/Lower Limits

The standard “rule” for getting upper/lower limits

$$-2 \ln \mathscr{L}(\sigma) = \lambda$$

$$\sum_i K_i^2 f_i''(K_i\hat{\sigma})\,(\sigma - \hat{\sigma})^2 \simeq \lambda$$

$$\sigma^{UL} \simeq \hat{\sigma} + \sqrt{\frac{\lambda}{\sum_i K_i^2 f_i''(K_i\hat{\sigma})}}$$

Upper Limits

$$\sigma^{UL} \simeq \hat{\sigma} + \sqrt{\frac{\lambda}{\sum_i K_i^2 \, f_i''(K_i \hat{\sigma})}}$$

Expected bkg (nuisance)

Off counts

Model-indipendent constant

On counts

$$f_i(s_i) = s_i - n_i \ln\left(s_i + b_i\right) + (1+\alpha)\, b_i - m_i \ln\left(\alpha b_i\right) + C$$

Cash Statistic

Wstat (On/Off) Statistic

Upper Limits

$$\sigma^{UL} \simeq \hat{\sigma} + \sqrt{\frac{\lambda}{\sum_i K_i^2 \, f_i''(K_i \hat{\sigma})}}$$

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Cash Statistic

Wstat (On/Off) Statistic

$$f''(s) = n\frac{(1+\frac{db}{ds})^2 - (s+b)\frac{d^2b}{ds^2}}{(s+b)^2} + m\frac{\left(\frac{db}{ds}\right)^2 - \frac{d^2b}{ds^2}b}{b^2} + (1+\alpha)\frac{d^2b}{ds^2}$$

Upper Limits

$$\sigma^{UL} \simeq \hat{\sigma} + \sqrt{\frac{\lambda}{\sum_i K_i^2\, f_i''(K_i\hat{\sigma})}}$$

On counts

Expected bkg (nuisance)

Off counts

Model-indipendent constant

$$f_i(s_i) = s_i - n_i \ln\left(s_i + b_i\right) + (1+\alpha)\, b_i - m_i \ln\left(\alpha b_i\right) + C$$

Cash Statistic ($s_i - n_i \ln(s_i + b_i)$)

Wstat (On/Off) Statistic ($s_i - n_i \ln(s_i + b_i) + (1+\alpha)\, b_i - m_i \ln(\alpha b_i)$)

$$f''(s) = n\frac{(1+\frac{db}{ds})^2 - (s+b)\frac{d^2b}{ds^2}}{(s+b)^2} + m\frac{\left(\frac{db}{ds}\right)^2 - \frac{d^2b}{ds^2}b}{b^2} + (1+\alpha)\frac{d^2b}{ds^2}$$

No signal hypothesis $\hat{\sigma} = 0$ → $s = 0 \quad n = b \quad m = \alpha b$

$$f'' = \frac{1}{b(1+\alpha^{-1})}$$

Upper Limits - Approximate expression

Cash statistic

$$\sigma^{UL} \simeq \sqrt{\frac{\lambda}{\sum_i K_i^2/b_i}}$$

Wstat (On/Off) statistic

$$\sigma^{UL} \simeq \sqrt{\frac{\lambda}{\sum_i K_i^2/((1+\alpha^{-1})b_i)}}$$

Forecasting Upper Limits

Cash statistic

$$\sigma^{UL} \simeq \sqrt{\frac{\lambda}{\sum_i K_i^2/b_i}}$$

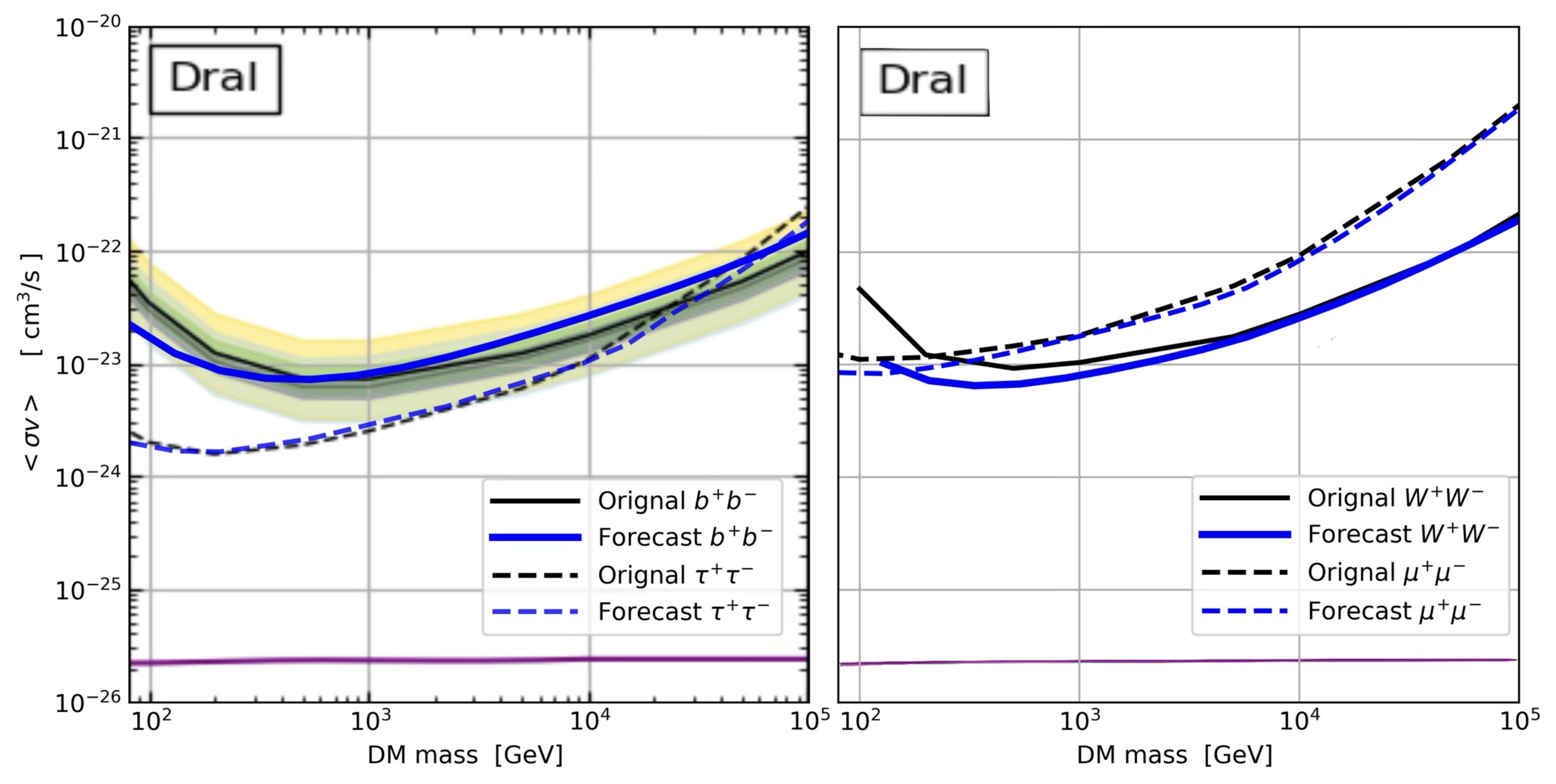
Wstat (On/Off) statistic

$$\sigma^{UL} \simeq \sqrt{\frac{\lambda}{\sum_i K_i^2/((1+\alpha^{-1})b_i)}}$$

1 Forecasting Upper Limits

We adopt the same observational assumptions as in [1]:

- a J-factor of $10^{18.7}$ GeV^2/cm^5 integrated over a cone of radius $0.5°$
- total observation time of 100 hours
- Publicly available IRFs of CTAO



[1] Abe, K., et al. "Prospects for dark matter observations in dwarf spheroidal galaxies with the Cherenkov Telescope Array Observatory." *Monthly Notices of the Royal Astronomical Society* (2025): staf1798.

Recasting Across Models

Cash statistic

$$\sigma^{UL} \simeq \sqrt{\frac{\lambda}{\sum_i K_i^2/b_i}}$$

Wstat (On/Off) statistic

$$\sigma^{UL} \simeq \sqrt{\frac{\lambda}{\sum_i K_i^2/((1+\alpha^{-1})b_i)}}$$

2 Recasting Across Models

Dark Model I

Dark Model 0

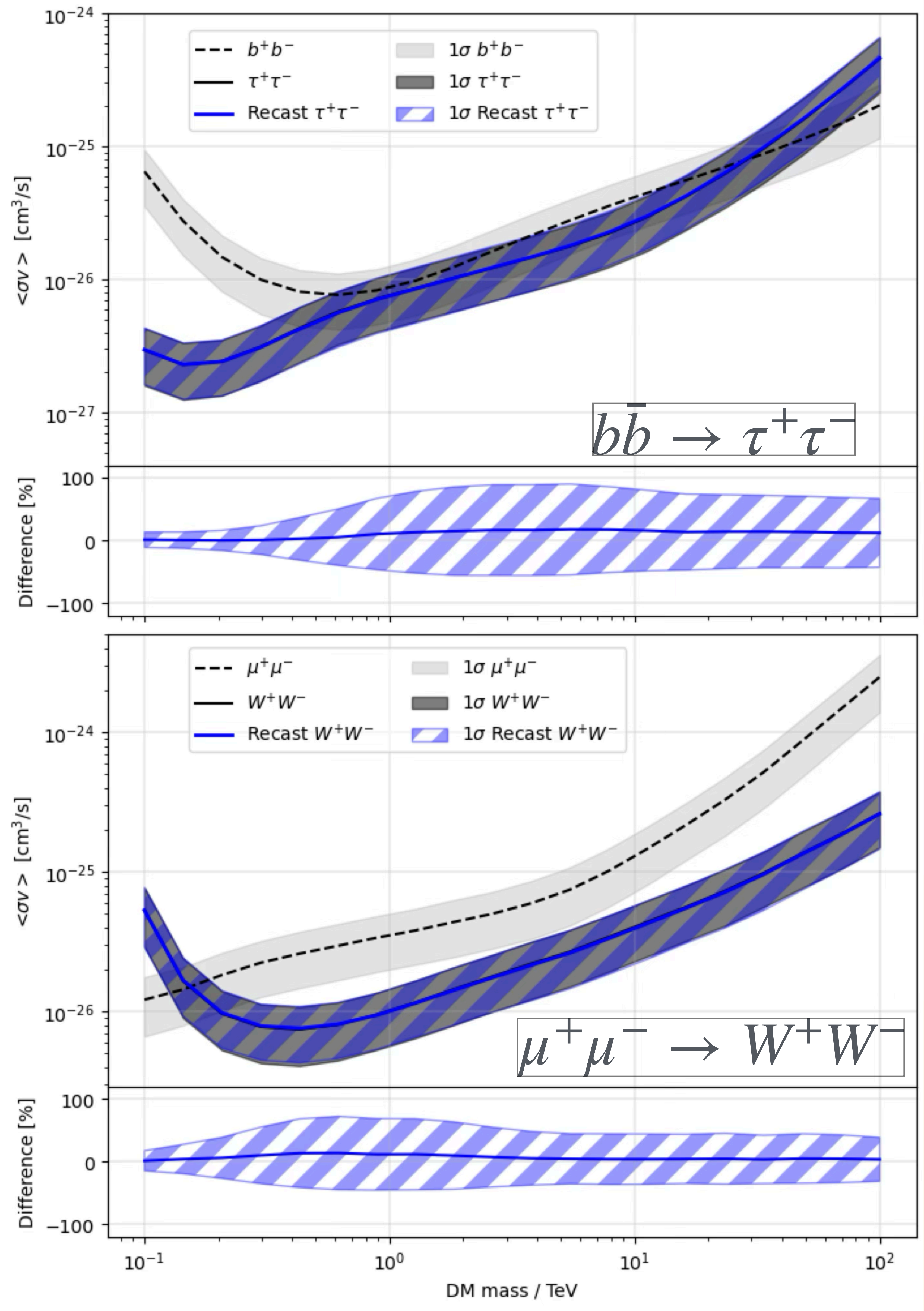
$$\frac{\sigma_I^{UL}}{\sigma_0^{UL}} = \sqrt{\frac{\sum_i K_{0,i}^2/b_i}{\sum_i K_{I,i}^2/b_i}} \longrightarrow \sigma_I^{UL} = \sqrt{\frac{\sum_i K_{0,i}^2/b_i}{\sum_i K_{I,i}^2/b_i}} \cdot \sigma_0^{UL}$$

Recasting Across Models

Validation on MC simulations

We generated 10^5 toy MC realizations under the null hypothesis of no DM signal:

1. We draw Poisson distributed counts n_i (ON region) and m_i(OFF region) in every energy bin
2. Publicly available IRFs of CTAO were adopted
3. Using the binned likelihood, we derived σ^{UL} for each DM mass m_χ and for four annihilation channels: $\tau^+\tau^-$, $b\bar{b}$, $\mu^+\mu^-$, and W^+W^-
4. The ULs for $\tau^+\tau^-$ and W^+W^-were recast from those of $b\bar{b}$ and $\mu^+\mu^-$, respectively



Recasting Across Models - dealing with missing IRF

⚠ Recasting expression required knowledge of the instrument IRF (background b_i and telescope response for computing K_i)

$$\sigma_I^{UL} = \sqrt{\frac{\sum_i K_{0,i}^2 / b_i}{\sum_i K_{I,i}^2 / b_i}} \cdot \sigma_0^{UL}$$

?

Recasting Across Models - dealing with missing IRF

⚠ Recasting expression required knowledge of the instrument IRF (background b_i and telescope response for computing K_i)

$$\sigma_I^{UL} = \sqrt{\frac{\sum_i K_{0,i}^2 / b_i}{\sum_i K_{I,i}^2 / b_i}} \cdot \sigma_0^{UL} \equiv \sqrt{\frac{\sum_i \left(V_i \cdot \Delta N_{\gamma,i}^0\right)^2}{\sum_i \left(V_i \cdot \Delta N_{\gamma,i}^I\right)^2}} \cdot \sigma_0^{UL}$$

?

Recasting Across Models - dealing with missing IRF

⚠ Recasting expression required knowledge of the instrument IRF (background b_i and telescope response for computing K_i)

$$\sigma_I^{UL} = \sqrt{\frac{\sum_i K_{0,i}^2 / b_i}{\sum_i K_{I,i}^2 / b_i}} \cdot \sigma_0^{UL} \equiv \sqrt{\frac{\sum_i \left(V_i \cdot \Delta N_{\gamma,i}^0 \right)^2}{\sum_i \left(V_i \cdot \Delta N_{\gamma,i}^I \right)^2}} \cdot \sigma_0^{UL}$$

?

intrinsic number of photons predicted by the DM model

✓ We can compute it!

$$\Delta N_{\gamma,i} \equiv \int_{\Delta E_i'} dE \, \frac{dN_\gamma}{dE}$$

Recasting Across Models - dealing with missing IRF

Recasting expression required knowledge of the instrument IRF (background b_i and telescope response for computing K_i)

$$\sigma_I^{UL} = \sqrt{\frac{\sum_i K_{0,i}^2 / b_i}{\sum_i K_{I,i}^2 / b_i}} \cdot \sigma_0^{UL} \equiv \sqrt{\frac{\sum_i \left(V_i \cdot \Delta N_{\gamma,i}^{0}\right)^2}{\sum_i \left(V_i \cdot \Delta N_{\gamma,i}^{I}\right)^2}} \cdot \sigma_0^{UL}$$

?

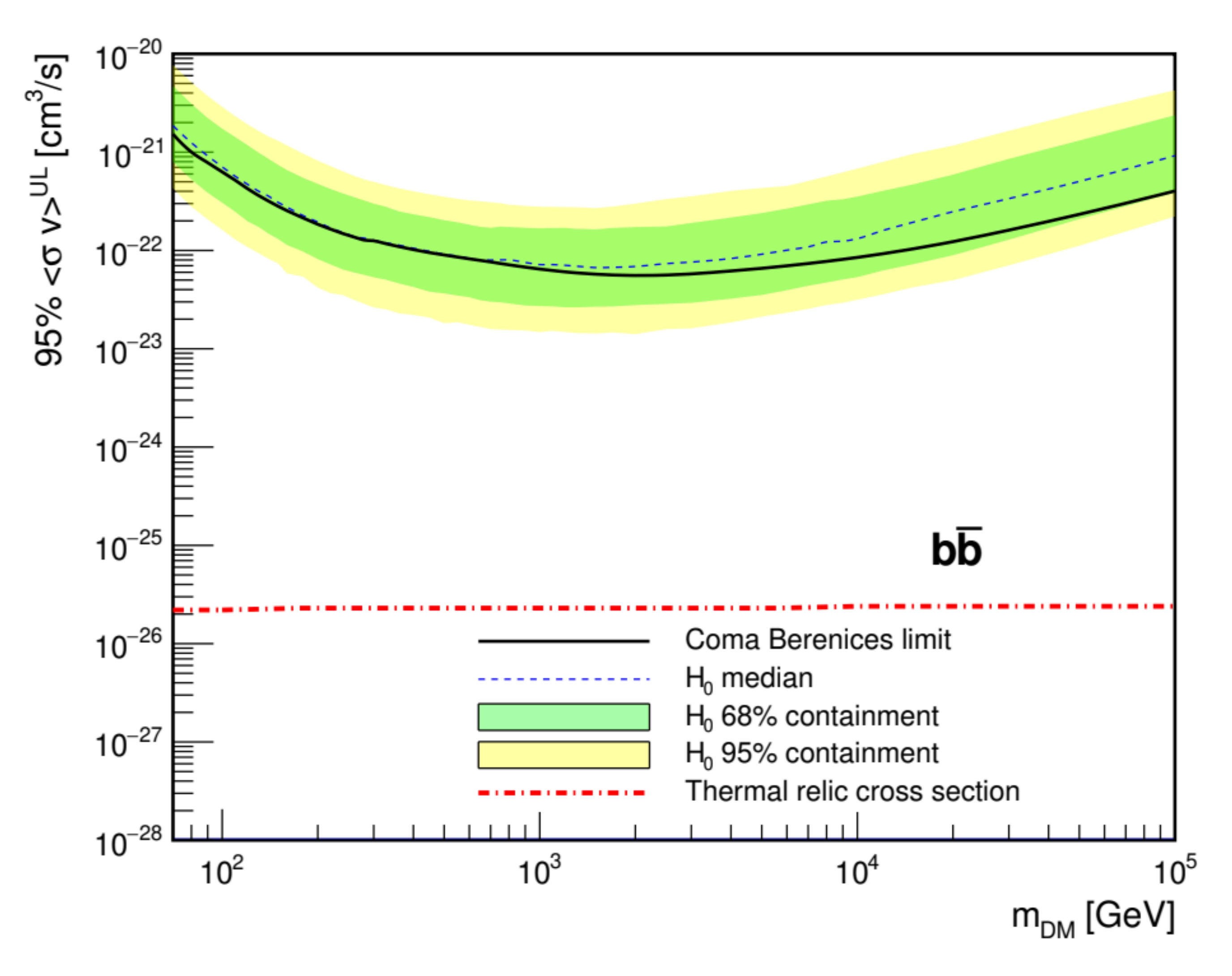
$$V_i \equiv \frac{K_i}{\sqrt{b_i} \cdot \Delta N_{\gamma,i}} \approx \frac{1}{\sqrt{b_i}} A_i$$

V_i are approximately model independent, therefore can be obtained by comparing ULs from 2 channels

Under the assumption of good energy resolution and DM spectrum varying slowly compared to the bin width

Effective area averaged over the bin i

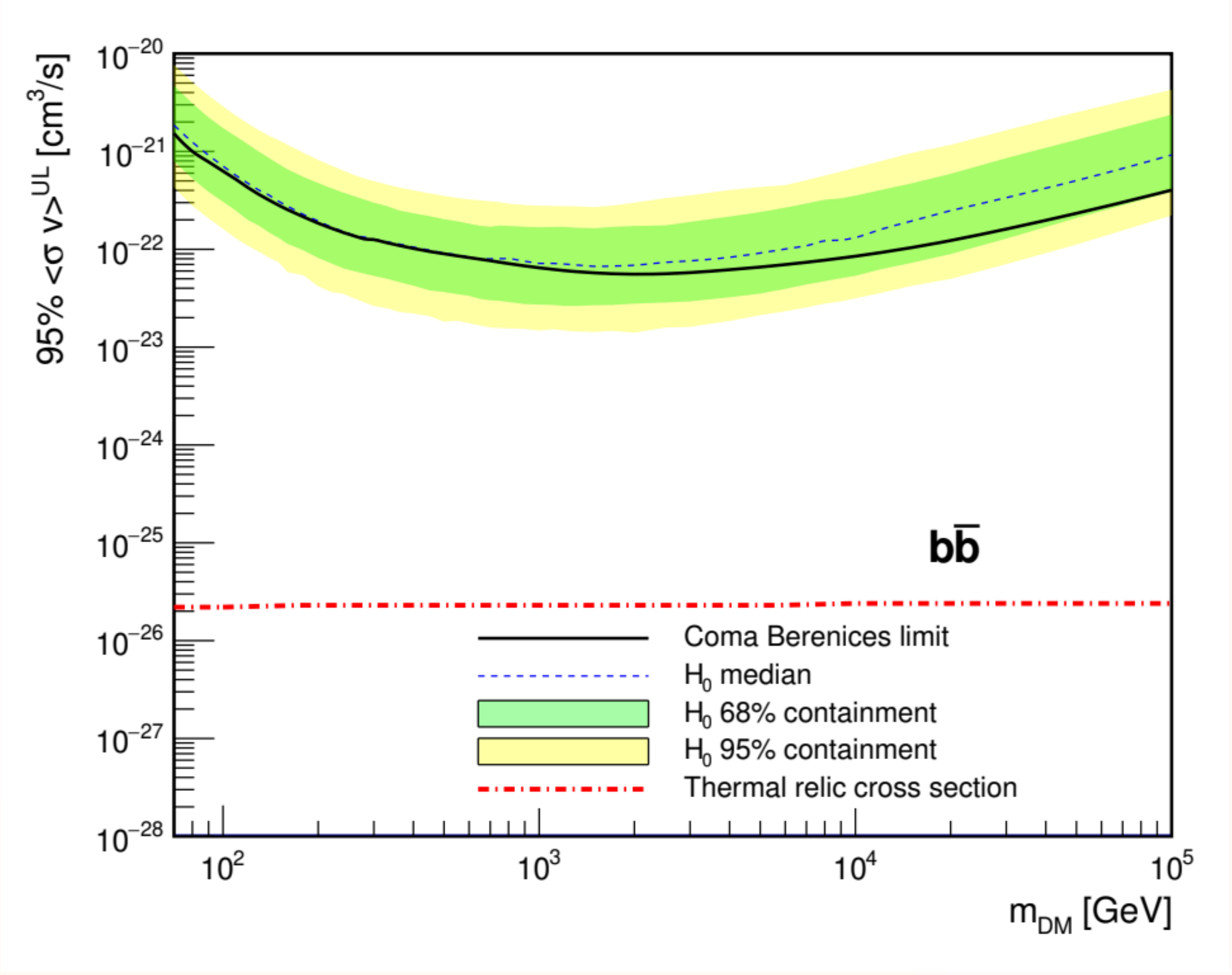
Recasting Across Models - dealing with missing IRF



?

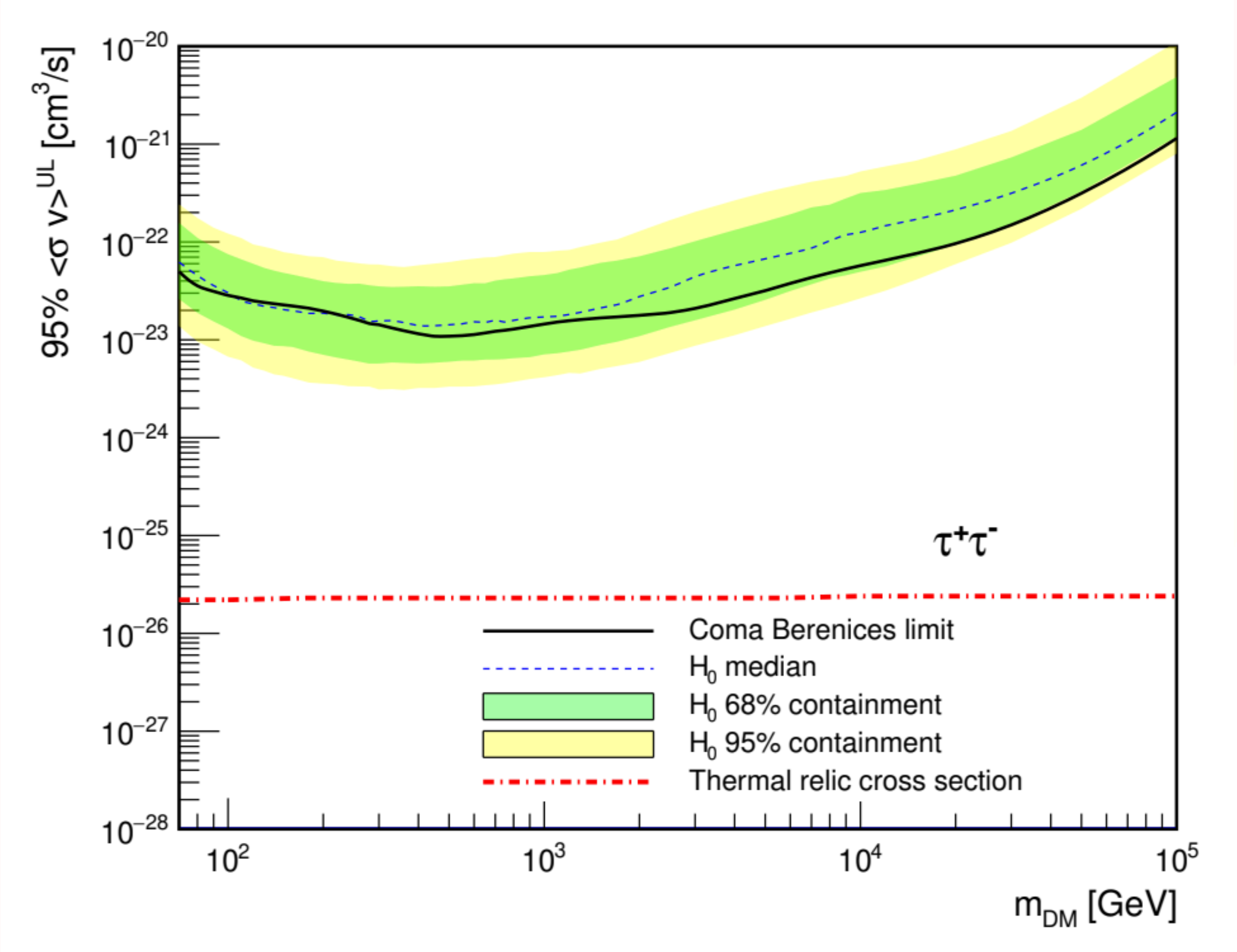
$$W^+W^-$$

Recasting Across Models - dealing with missing IRF



?

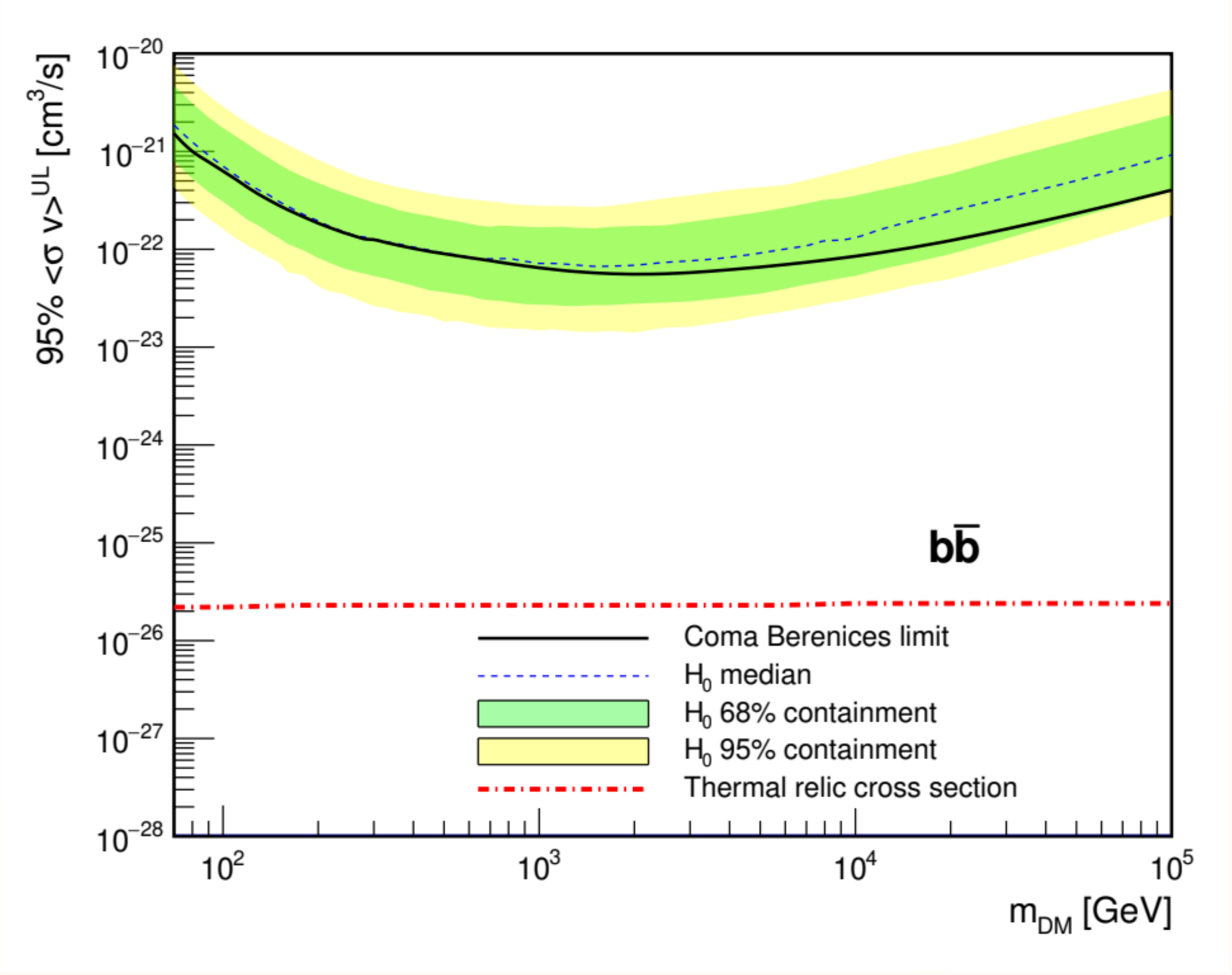
W^+W^-

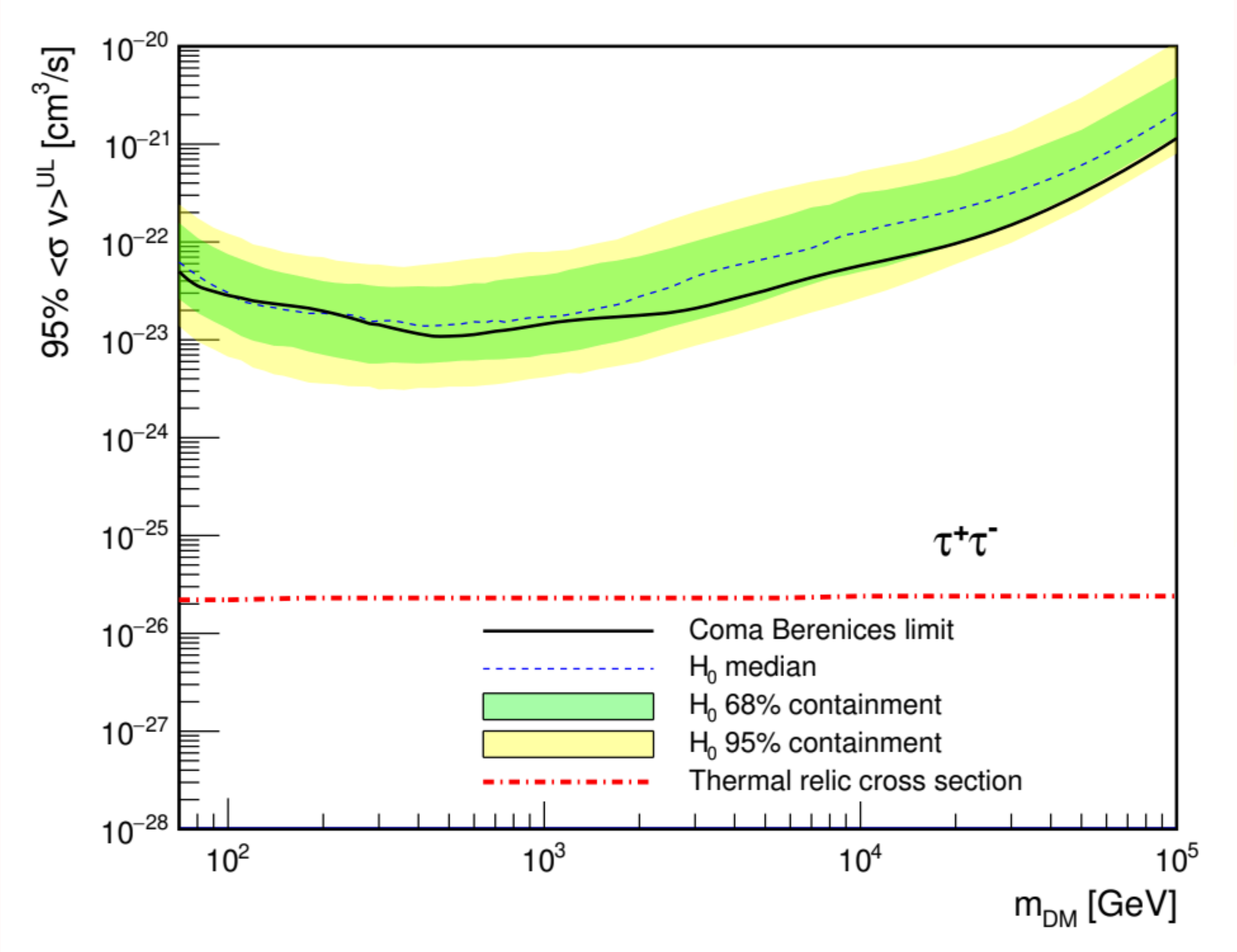


$$\sigma_I^{UL} = \sqrt{\frac{\sum_i \left(V_i \cdot \Delta N^0_{\gamma,i}\right)^2}{\sum_i \left(V_i \cdot \Delta N^I_{\gamma,i}\right)^2}} \cdot \sigma_0^{UL}$$

V_i

Recasting Across Models - dealing with missing IRF

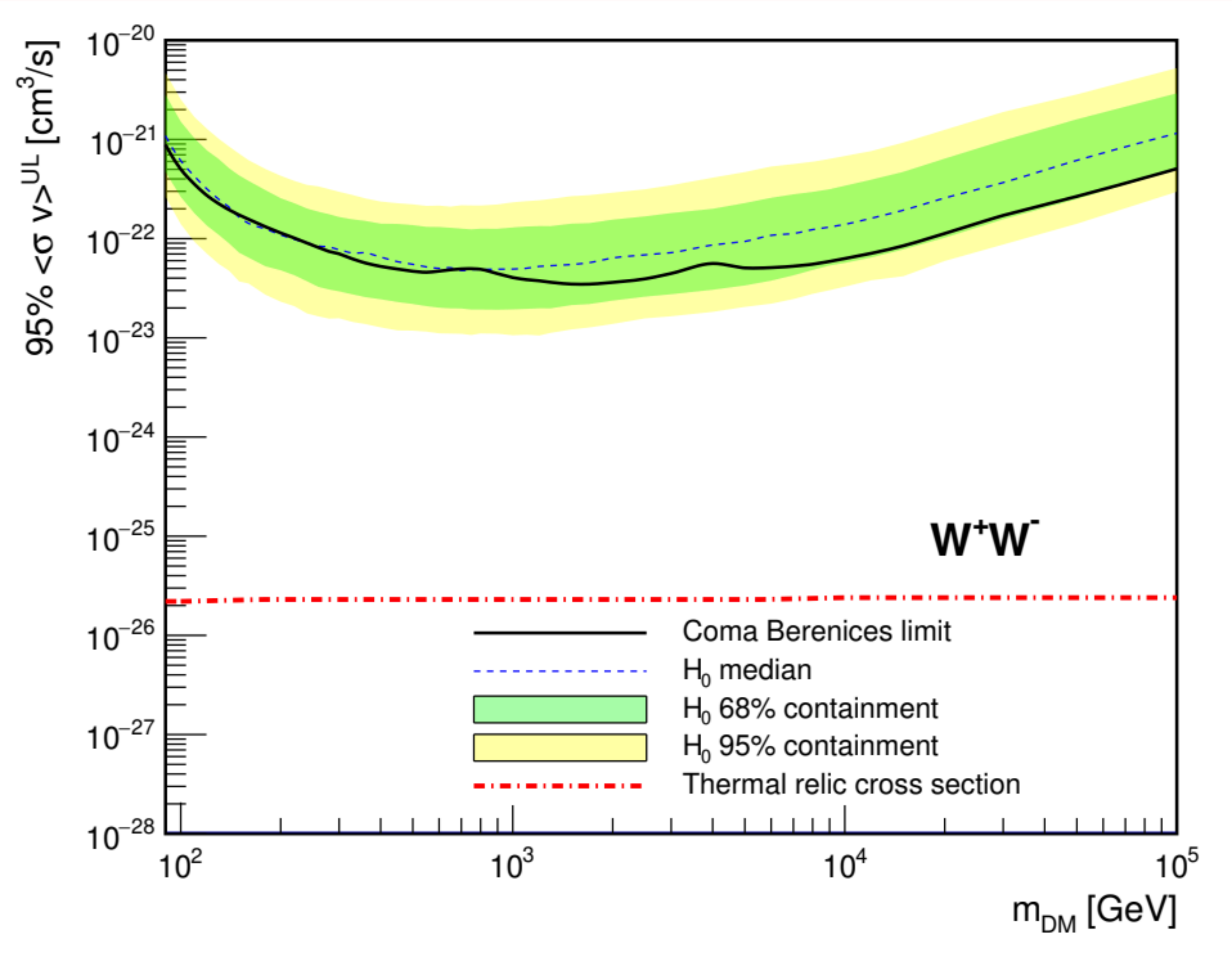




$$\sigma_I^{UL} = \sqrt{\frac{\sum_i \left(V_i \cdot \Delta N^0_{\gamma,i}\right)^2}{\sum_i \left(V_i \cdot \Delta N^I_{\gamma,i}\right)^2}} \cdot \sigma_0^{UL}$$

$$V_i$$

$$\sigma_I^{UL} = \sqrt{\frac{\sum_i \left(V_i \cdot \Delta N^0_{\gamma,i}\right)^2}{\sum_i \left(V_i \cdot \Delta N^I_{\gamma,i}\right)^2}} \cdot \sigma_0^{UL}$$



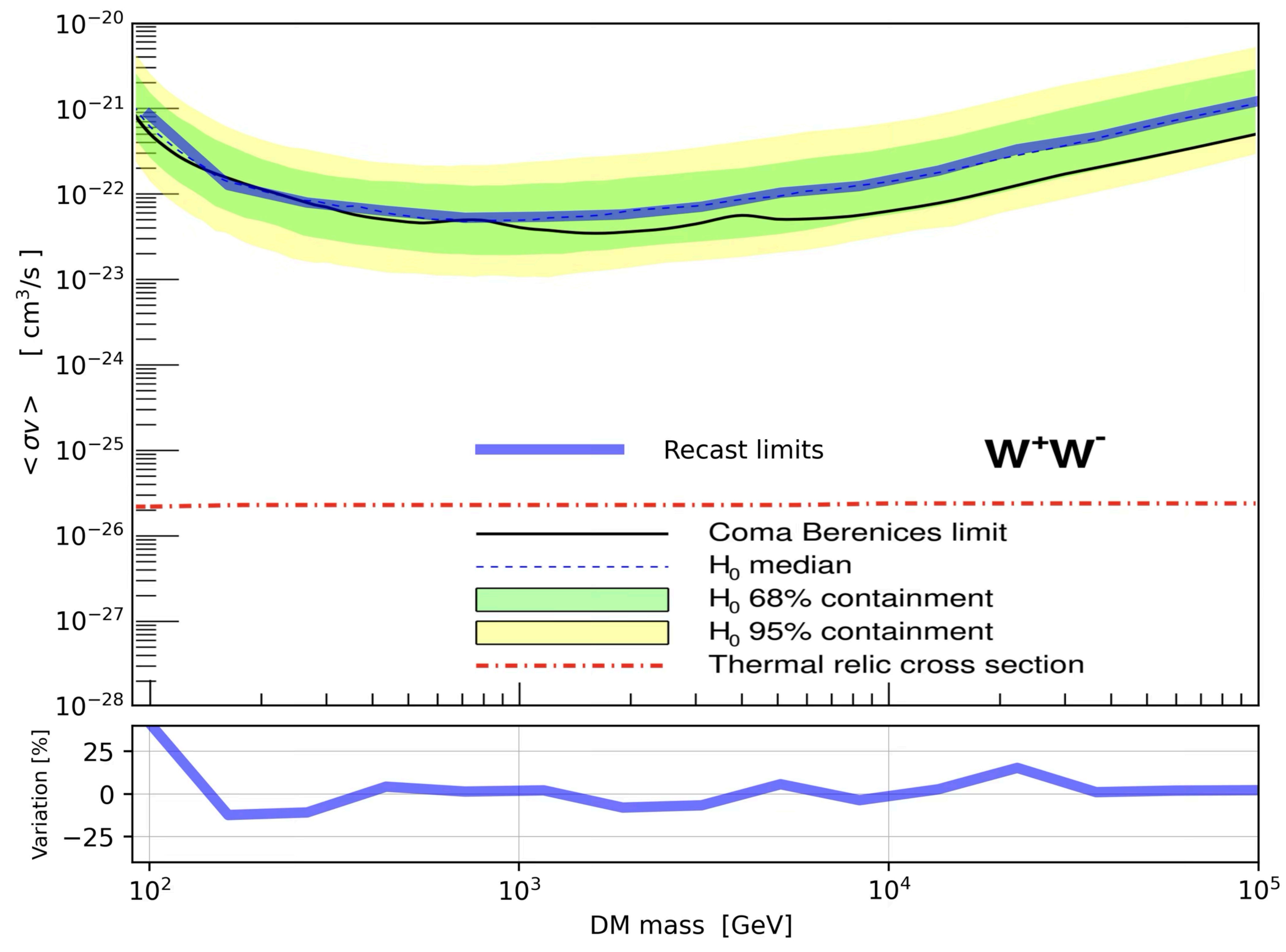
Recasting Across Models

Validation on published ULs from the MAGIC collaboration [1]

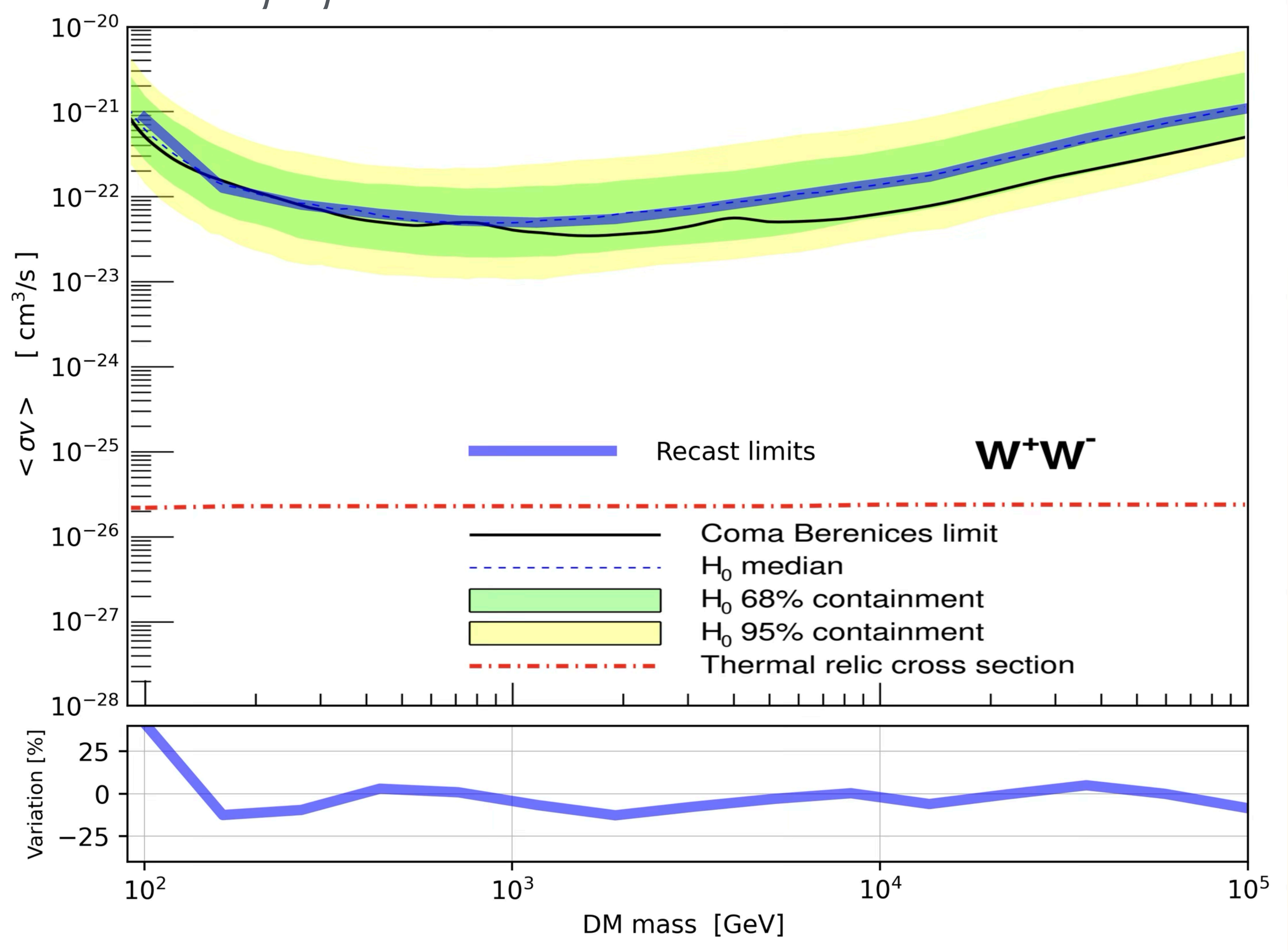
[1] Acciari, V.A., et al. (MAGIC), 2022. Combined searches for dark matter in dwarf spheroidal galaxies observed with the MAGIC telescopes, including new data from Coma Berenices and Draco. Phys. Dark Univ. 35, 100912.

$$b\bar{b} \longrightarrow W^+W^-$$

$\tau^+\tau^-$ as a second benchmark channel



$\mu^+\mu^-$ as a second benchmark channel



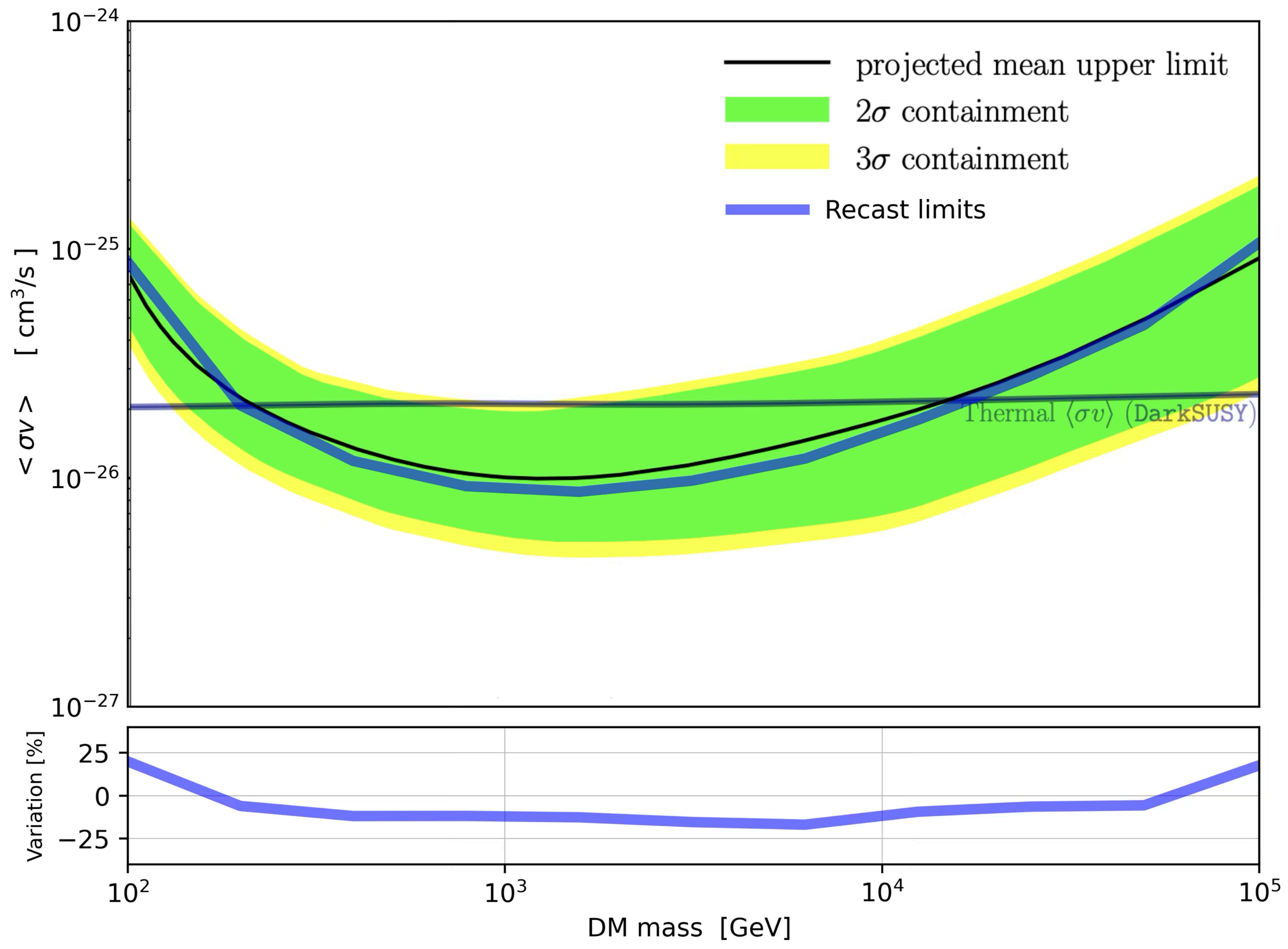
Recasting Across Models

Validation on Published ULs - other insturments

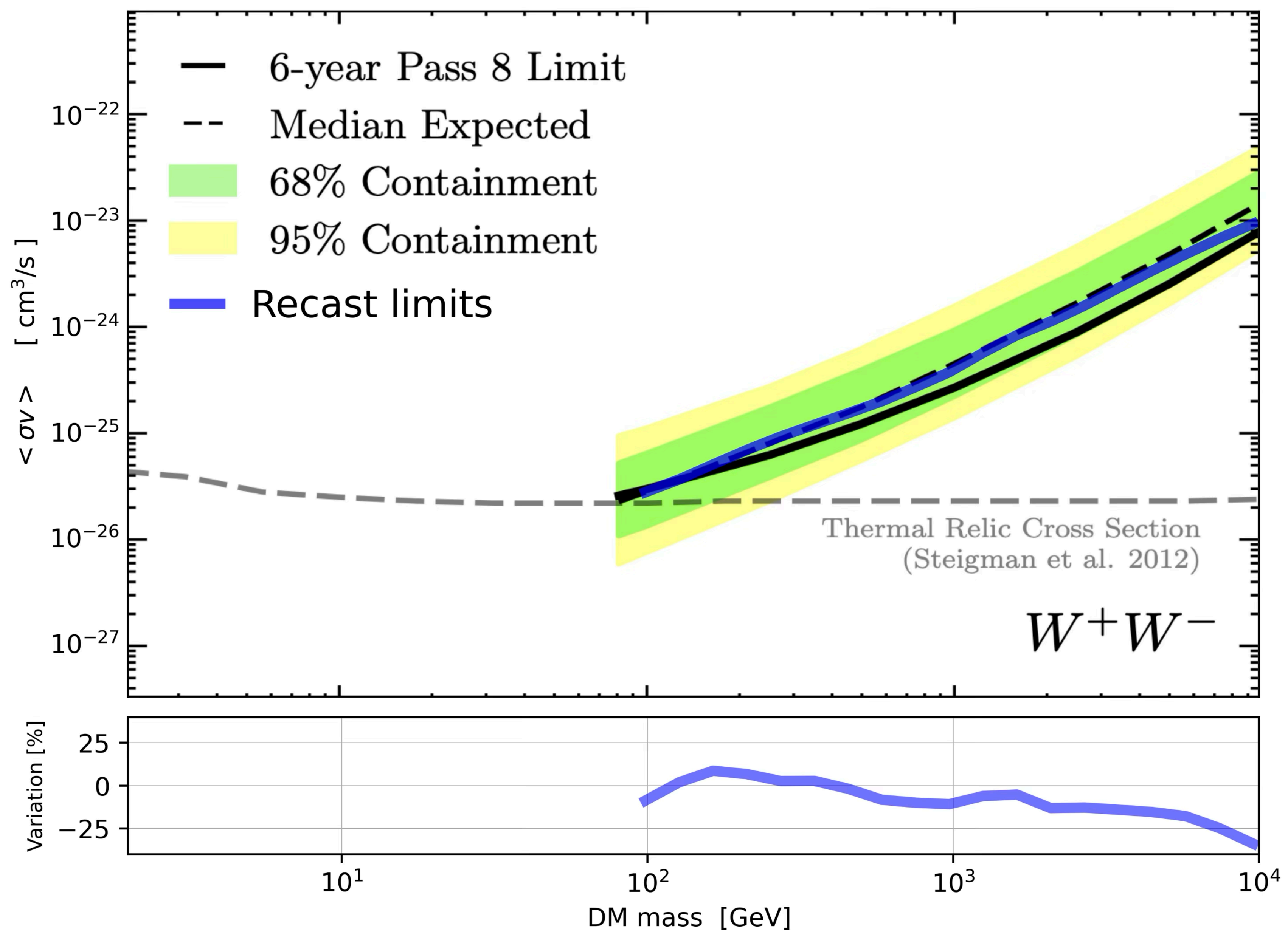
$$b\bar{b} \longrightarrow W^+W^-$$

$\tau^+\tau^-$ as a second benchmark channel

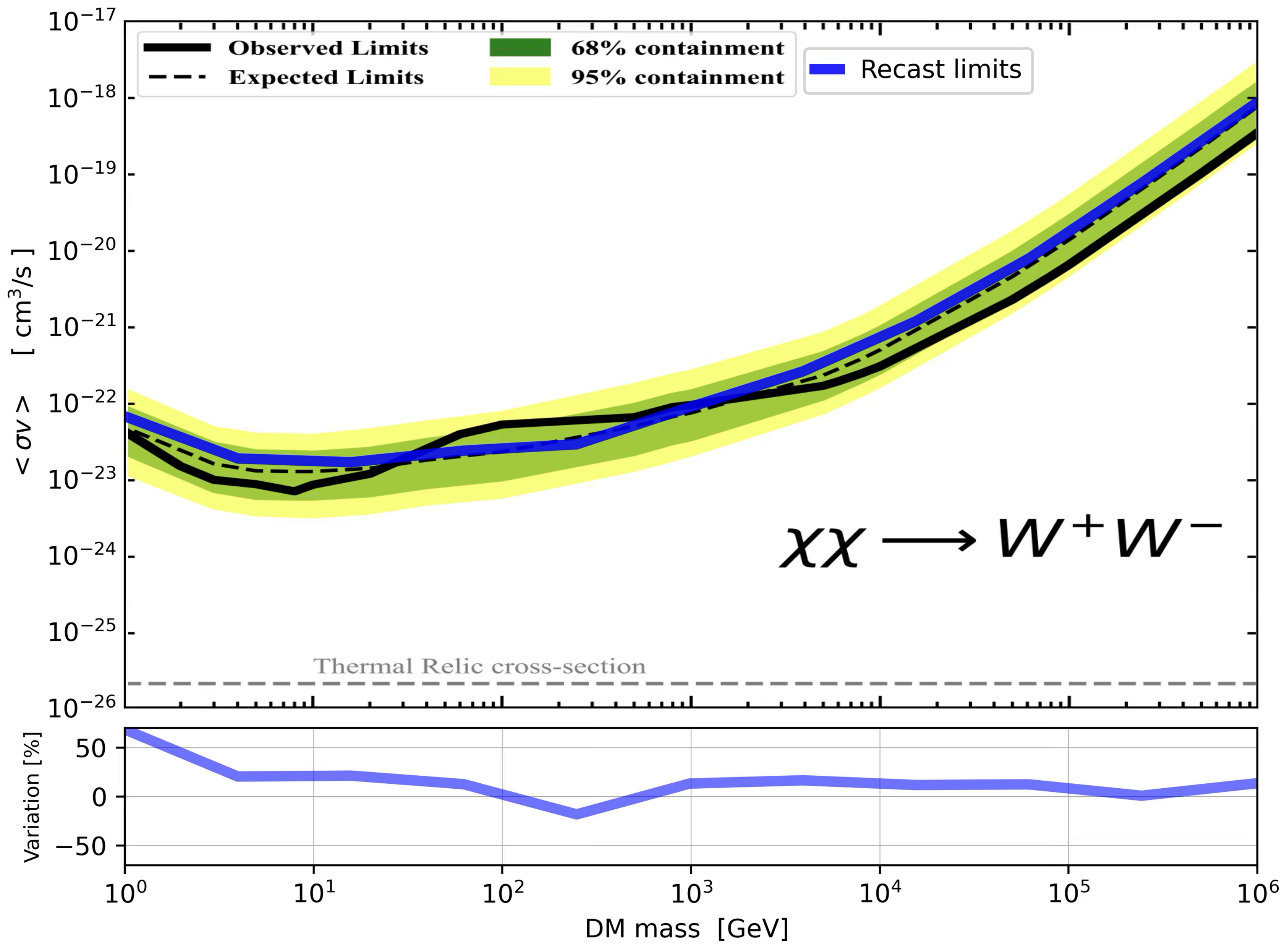
CTAO / GC Projection



***Fermi* - LAT / dSph**



LHAASO / dSph



Acharyya, A., et al. (CTA), 2021. Sensitivity of the Cherenkov Telescope Array to a dark matter signal from the Galactic centre. JCAP 01, 057

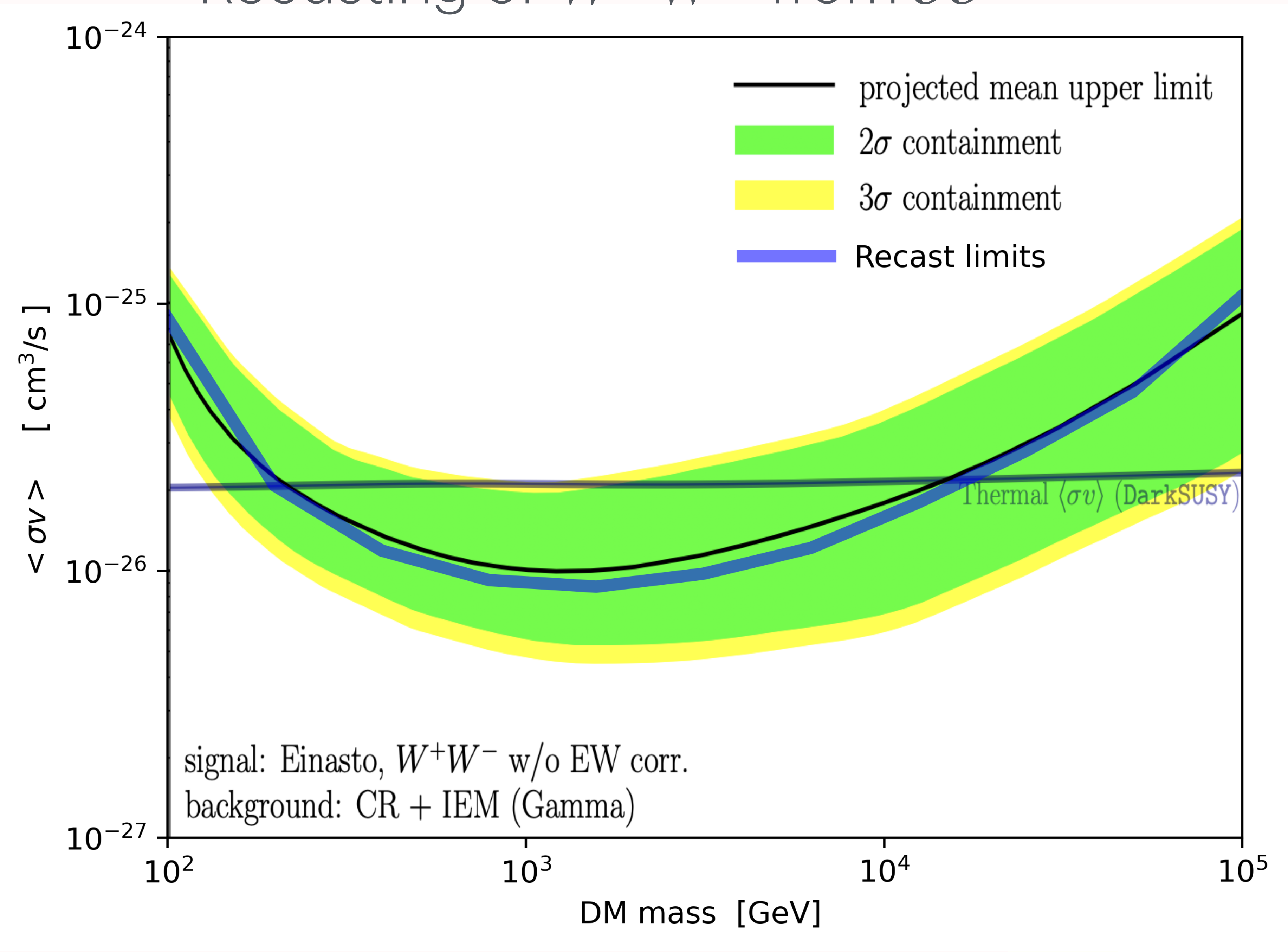
Ackermann, M., et al. (Fermi-LAT), 2015. Searching for Dark Matter Annihilation from Milky Way Dwarf Spheroidal Galaxies with Six Years of Fermi Large Area Telescope Data. Phys. Rev. Lett. 115, 231301.

Cao, Z., et al. (LHAASO), 2024. Constraints on Ultraheavy Dark Matter Properties from Dwarf Spheroidal Galaxies with LHAASO Observations. Phys. Rev. Lett. 133, 061001.

Recasting into new models

Higgsino-like scenario - CTAO project from GC [1]

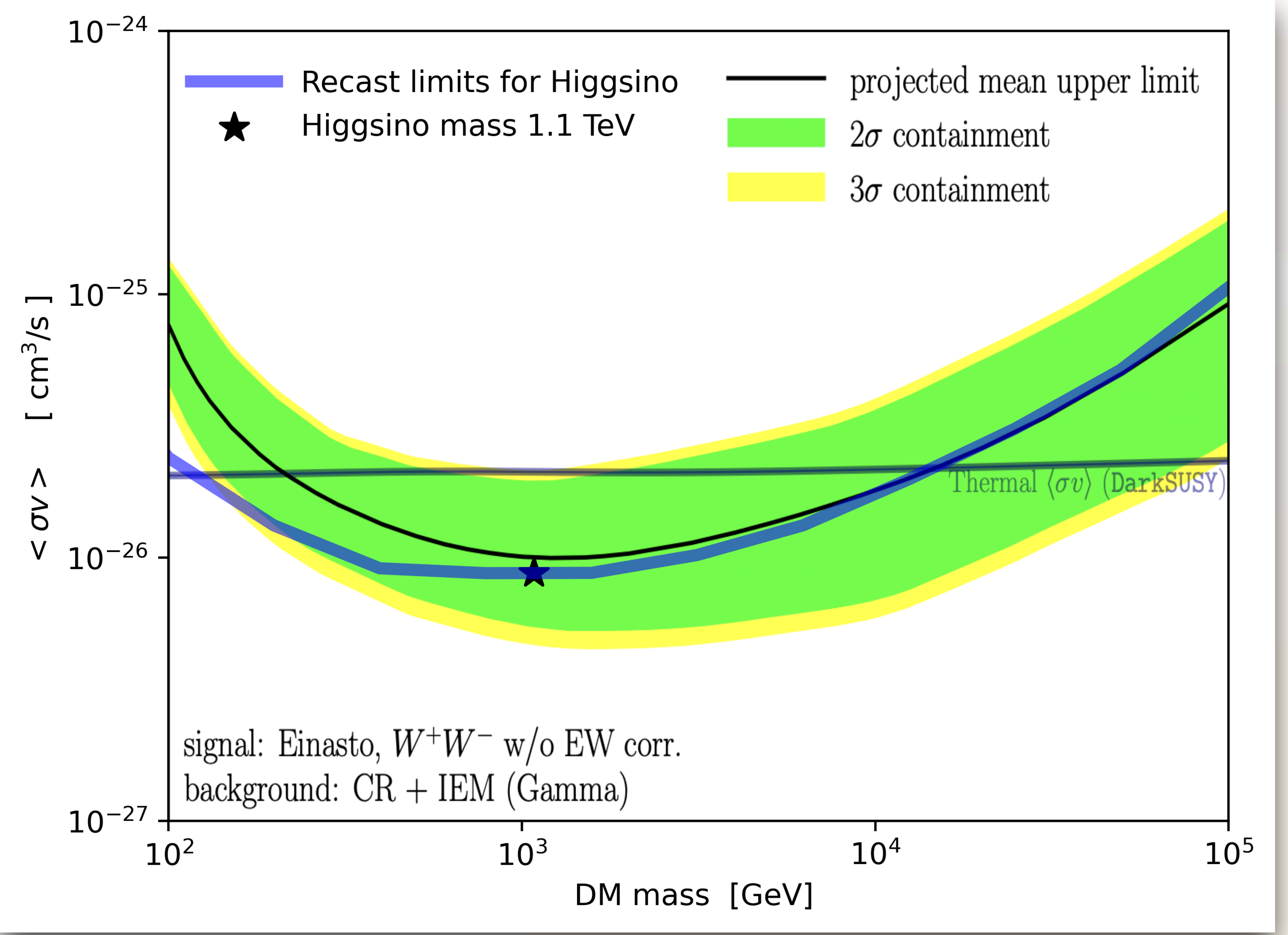
Recasting of W^+W^- from $b\bar{b}$



[1] Acharyya, A., et al. (CTA), 2021. Sensitivity of the Cherenkov Telescope Array to a dark matter signal from the Galactic centre. JCAP 01, 057

Same recasting but...

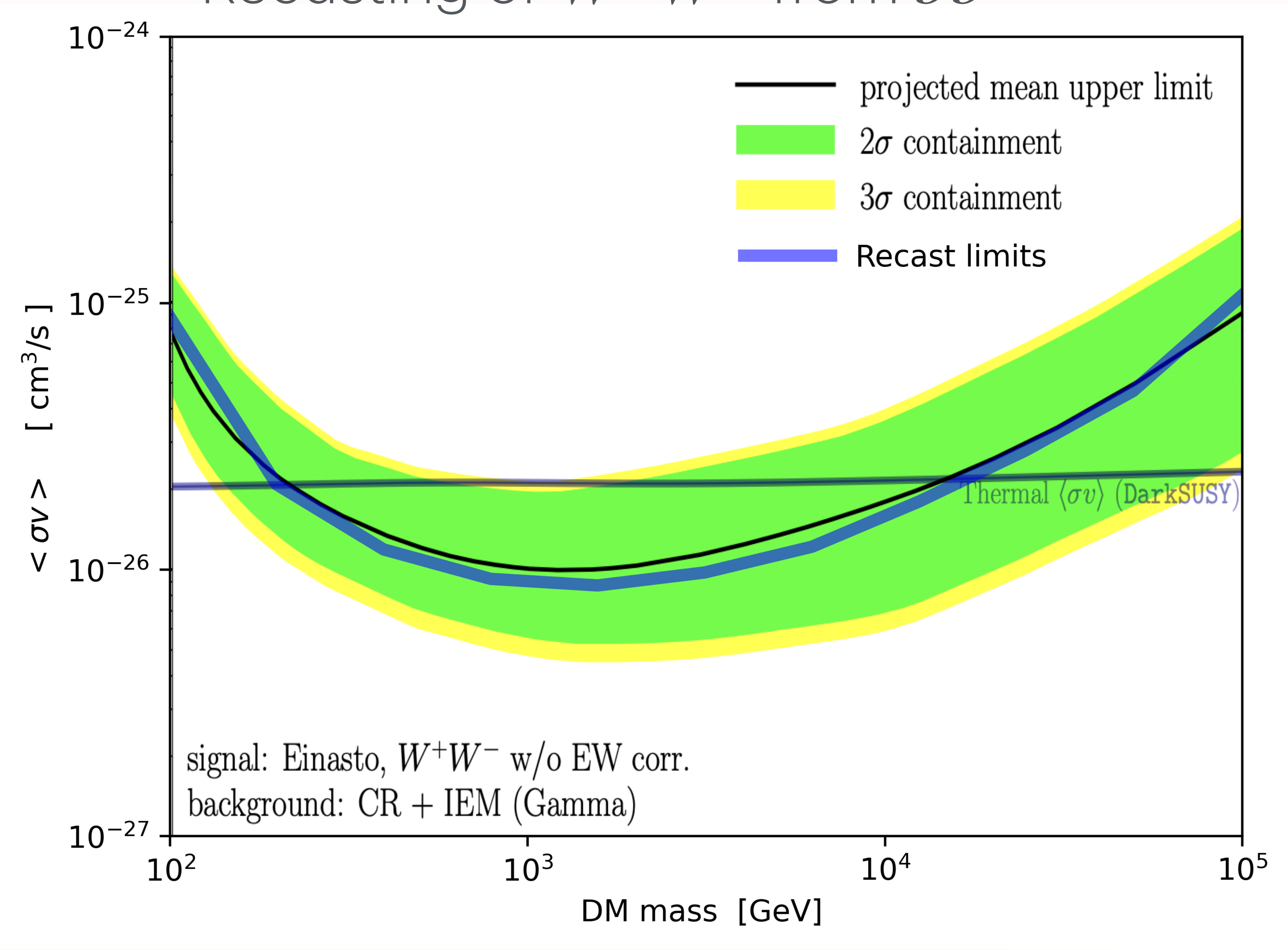
of the *Higgsino-like* spectrum with annihilation into $W^+W^-, ZZ, \gamma\gamma/\gamma Z$ with branching ratios $BR_i = 0.611,0.382,0.008$ respectively



Recasting into new models

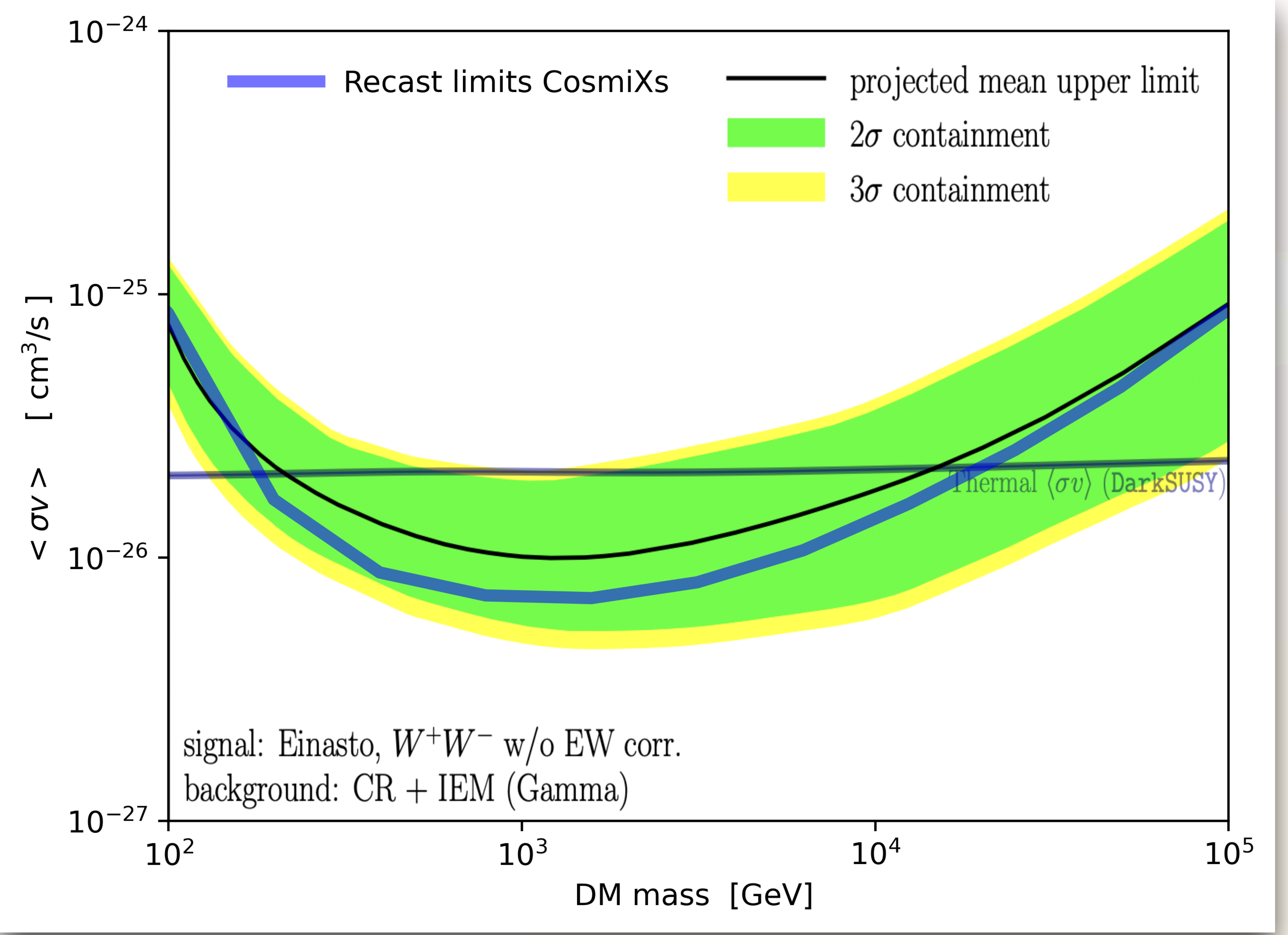
CosmiXs-based DM photon - CTAO project from GC [1]

Recasting of W^+W^- from $b\bar{b}$



Same recasting but…

of the W^+W^- channel with *cosmiXs*-based spectra [2] instead of the *PPPC ones*



[1] Acharyya, A., et al. (CTA), 2021. Sensitivity of the Cherenkov Telescope Array to a dark matter signal from the Galactic centre. JCAP 01, 057

[2] Arina, C., Di Mauro, M., Fornengo, N., Heisig, J., Jueid, A., de Austri, R.R., 2024. Cosmixs: cosmic messenger spectra for indirect dark matter searches. Journal of Cosmology and Astroparticle Physics 2024, 035

Conclusion

- **This novel method enables recasting of dark-matter limits without raw data**, using only published ULs

- **Validated across multiple gamma-ray telescopes**, reproducing official collaboration results within uncertainties.

- **Forecasting of ULs:** provides sensitivity projections for next-generation experiments such as CTAO.

- **General and portable:** adaptable to any instrument or dark-matter channel (if the DM spectrum varies slowly compared to the instrument energy resolution).

- **Code available:** https://github.com/giacomodamico24/DM_recast_limits

Backup - Recast from annihalation to decay

Annihilation

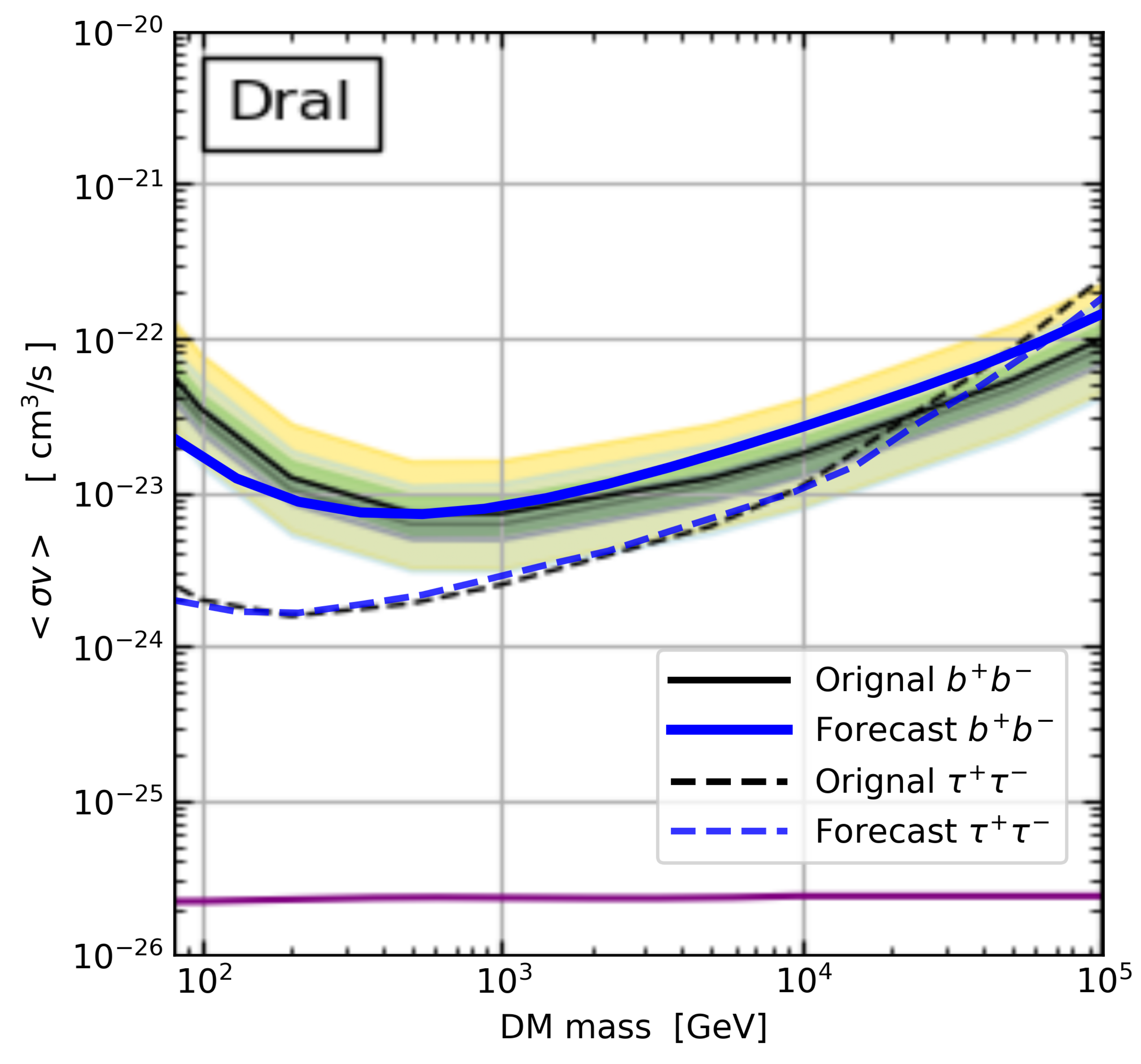
$$\frac{d\Phi}{dE}(E) = J_{\rm ann} \cdot \left(\frac{\langle \sigma v \rangle}{8\pi\, k\, m_\chi^2} \frac{dN_\gamma}{dE} \right)$$

$$J_{\rm ann} \equiv \int_{\Delta\Omega} d\Omega \int_{l.o.s.} dl\, \rho_\chi^2(l,\theta)$$

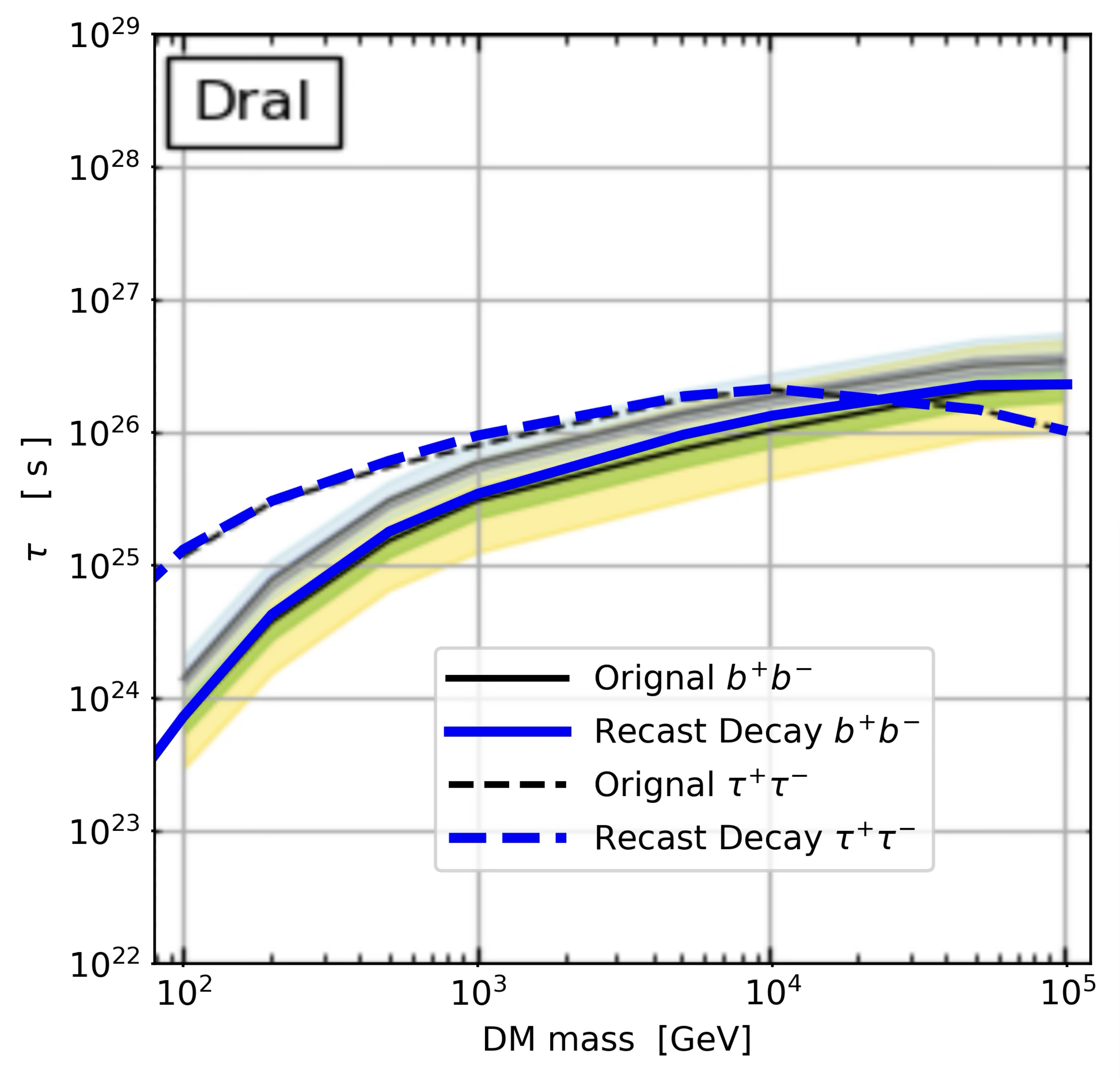
Decay

$$\frac{d\Phi}{dE}(E) = J_{\rm dec} \cdot \left(\frac{1}{4\pi\, m_\chi\, \tau} \frac{dN_\gamma}{dE} \right)$$

$$J_{\rm dec} \equiv \int_{\Delta\Omega} d\Omega \int_{l.o.s.} dl\, \rho_\chi(l,\theta)$$



$$\tau^{\rm UL} = \frac{J_{\rm dec}}{J_{\rm ann}} \cdot \frac{m_\chi}{\sigma^{\rm UL}}$$



Backup - First and second derivative

Cahs case

$$f(s) = s - n\ln(s+b) + C$$

$$f'(s) = 1 - \frac{n}{s+b} \sim 0 \qquad f''(s) = \frac{n}{(s+b)^2} \sim \frac{1}{b}$$

Wstat case

$$f(s) = s - n\ln(s+b) + (1+\alpha)b - m\ln(\alpha b) + C$$

$$b(s) = \frac{n_1(s)+n_2(s)}{2(1+\alpha)} \qquad n_1(s) = n + m - (1+\alpha)s \qquad n_2(s) = \sqrt{n_1^2(s) + 4(1+\alpha)sm}$$

$$\frac{db}{ds} = \frac{2m - n_1 - n_2}{2n_2} \sim -\frac{1}{1+\alpha} \qquad \frac{d^2b}{ds^2} = \frac{(1+\alpha^{-1})(n_1+n_2-2m)(n_2+2m-n_1)}{2\alpha^{-1}n_2^3} \sim \frac{2\,\alpha}{(1+\alpha)^2 b}$$

$$f'(s) = -\frac{n}{s+b}\left(1+\frac{db}{ds}\right) - \frac{m}{b}\frac{db}{ds} + 1 + (1+\alpha)\frac{db}{ds} \sim 0$$

$$f''(s) = n\frac{(1+\frac{db}{ds})^2 - (s+b)\frac{d^2b}{ds^2}}{(s+b)^2} + m\frac{\left(\frac{db}{ds}\right)^2 - \frac{d^2b}{ds^2}b}{b^2} + (1+\alpha)\frac{d^2b}{ds^2} \sim \frac{1}{b(1+\alpha^{-1})}$$

Backup - Impact of Background Knowledge

Cash statistic

$$\sigma^{UL} \simeq \sqrt{\frac{\lambda}{\sum_i K_i^2/b_i}}$$

Wstat (On/Off) statistic

$$\sigma^{UL} \simeq \sqrt{\frac{\lambda}{\sum_i K_i^2/((1+\alpha^{-1})b_i)}}$$

- Impact of Background Knowledge

$$1+\alpha^{-1} > 0 \longrightarrow \sum_i \frac{K_i^2}{b_i} > \sum_i \frac{K_i^2}{b_i(1+\alpha^{-1})} \longrightarrow$$

Cash ULs are more **stringent** than Wstat ULs

Backup - Impact of Background Knowledge

Cash statistic

$$\sigma^{UL} \simeq \sqrt{\frac{\lambda}{\sum_i K_i^2/b_i}}$$

Wstat (On/Off) statistic

$$\sigma^{UL} \simeq \sqrt{\frac{\lambda}{\sum_i K_i^2/((1+\alpha^{-1})b_i)}}$$

- Impact of Background Knowledge

$$1+\alpha^{-1} > 0 \longrightarrow \sum_i \frac{K_i^2}{b_i} > \sum_i \frac{K_i^2}{b_i(1+\alpha^{-1})} \longrightarrow$$

Cash ULs are more **stringent** than Wstat ULs

If OFF exposure infinetely larger than ON one $\longrightarrow$ $\alpha^{-1} \to 0$ $\longrightarrow$ Cash and Wstat ULs converge

Backup - Advantage of Multi-bin Analyses

Cash statistic

$$\sigma^{UL} \simeq \sqrt{\frac{\lambda}{\sum_i K_i^2/b_i}}$$

Wstat (On/Off) statistic

$$\sigma^{UL} \simeq \sqrt{\frac{\lambda}{\sum_i K_i^2/((1+\alpha^{-1})b_i)}}$$

Advantage of Multi-bin Analyses

Cauchy–Schwarz inequality

$$\left(\sum_i X_i Y_i\right)^2 \leq \left(\sum_i X_i^2\right)\left(\sum_i Y_i^2\right)$$

$$X_i = \frac{K_i}{\sqrt{b_i}}, \quad Y_i = \sqrt{b_i}$$

$$\longrightarrow \sqrt{\sum_i \frac{K_i^2}{b_i}} \geq \frac{\sum_i K_i}{\sqrt{\sum_i b_i}} \longrightarrow$$

Multi-bins analysis gives **stringent** ULs than single bin analysis

Validation on MC simulations

We generated 10^5 toy MC realizations under the null hypothesis of no DM signal:

1. We draw Poisson distributed counts n_i (ON region) and m_i(OFF region) in every energy bin
2. Publicly available IRFs of CTAO were adopted
3. Using the binned likelihood, we derived σ^{UL} for each DM mass m_χ and for four annihilation channels: $\tau^+\tau^-$, $b\bar{b}$, $\mu^+\mu^-$, and W^+W^-
4. The factors V_i were inferred using another benchmark channel: W^+W^- (upper case) and $b\bar{b}$ (lower case)
5. The ULs for $\tau^+\tau^-$ and W^+W^-were recast from those of $b\bar{b}$ and $\mu^+\mu^-$, respectively