

Chiral cat code

Enhanced error correction induced by higher-order nonlinearities



Adrià Labay Mora
Roberta Zambrini
Gian Luca Giorgi



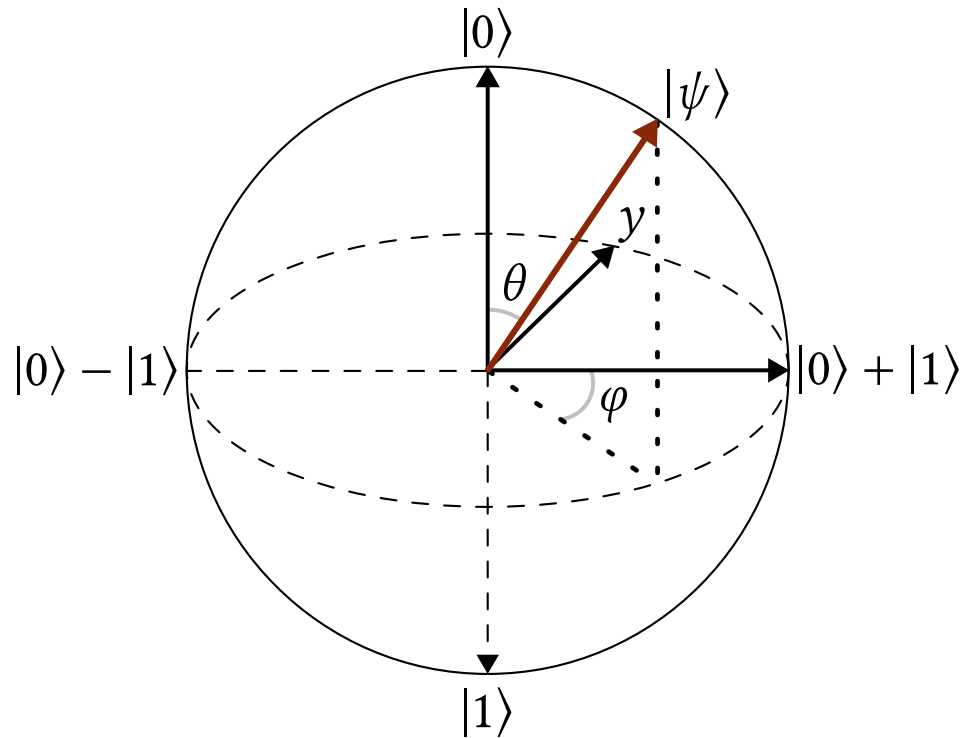
Alberto Mercurio
Vincenzo Savona



ALICE & BOB

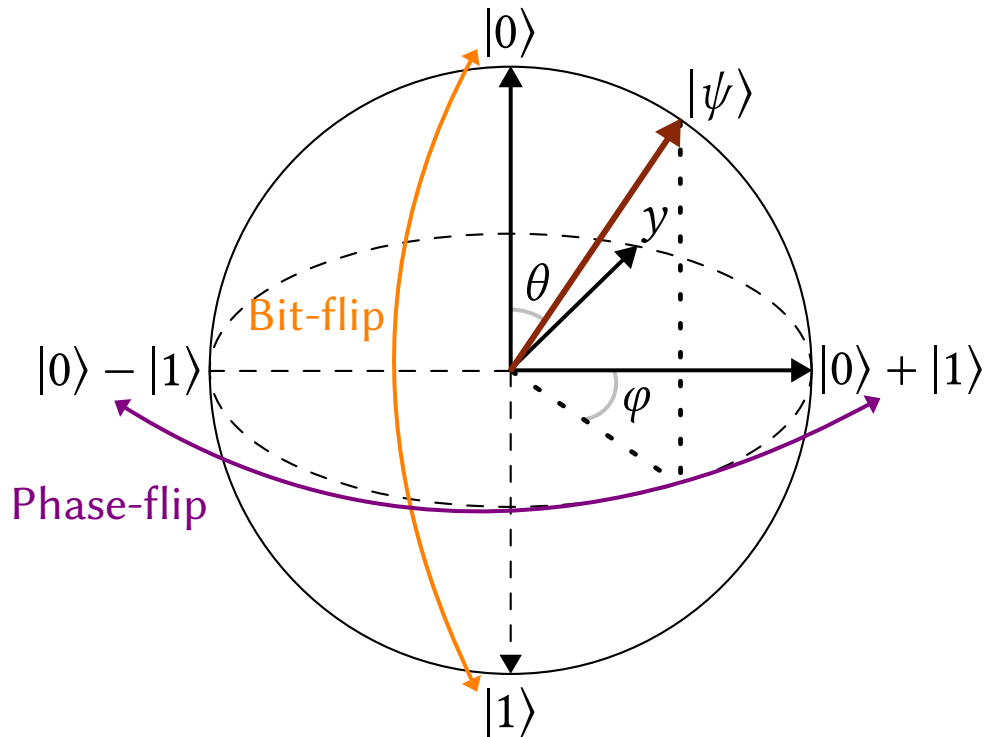
Fabrizio Minganti

Storage of quantum information



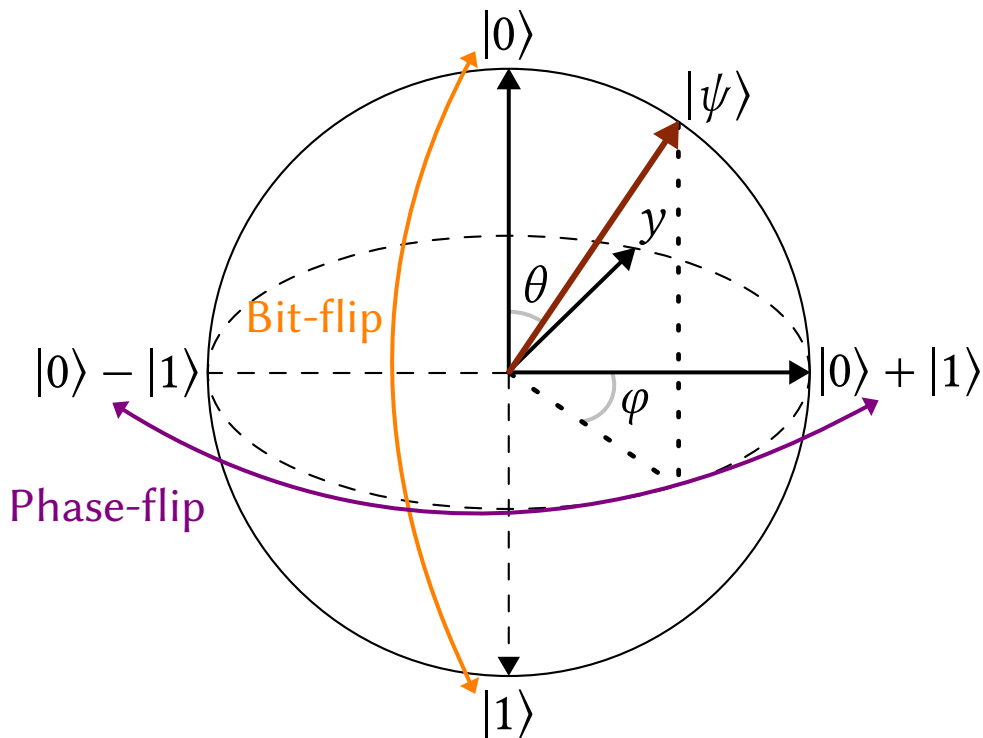
Storage of quantum information

Noise degrades the quantum information



Quantum Error Correction

Redundantly encode information in a larger Hilbert space



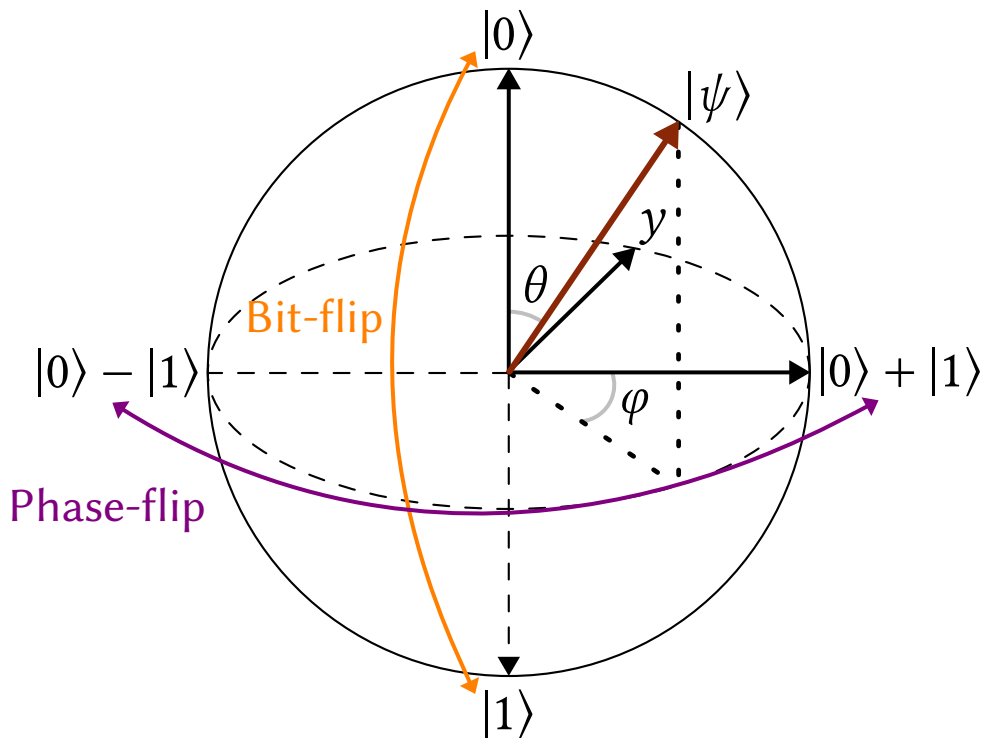
Ex: Three qubit code

$$\begin{array}{lcl} |0\rangle_L = |000\rangle & \xrightarrow{\text{orange}} & |010\rangle \xrightarrow{\text{green}} |000\rangle \\ |1\rangle_L = |111\rangle & & |101\rangle \xrightarrow{\text{green}} |111\rangle \end{array}$$

Protects against single bit-flip errors

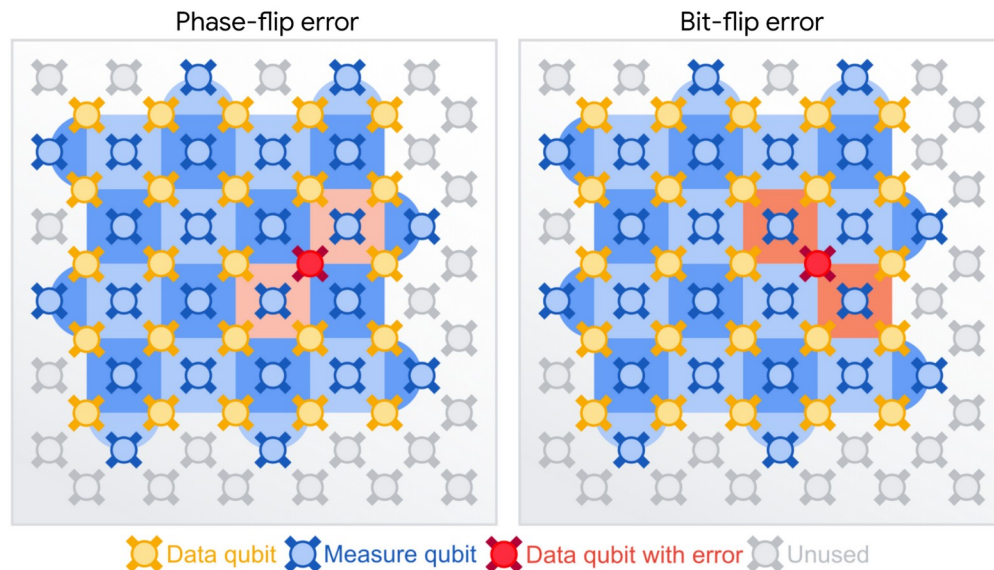
Quantum Error Correction

Redundantly encode information in a larger Hilbert space



Surface code

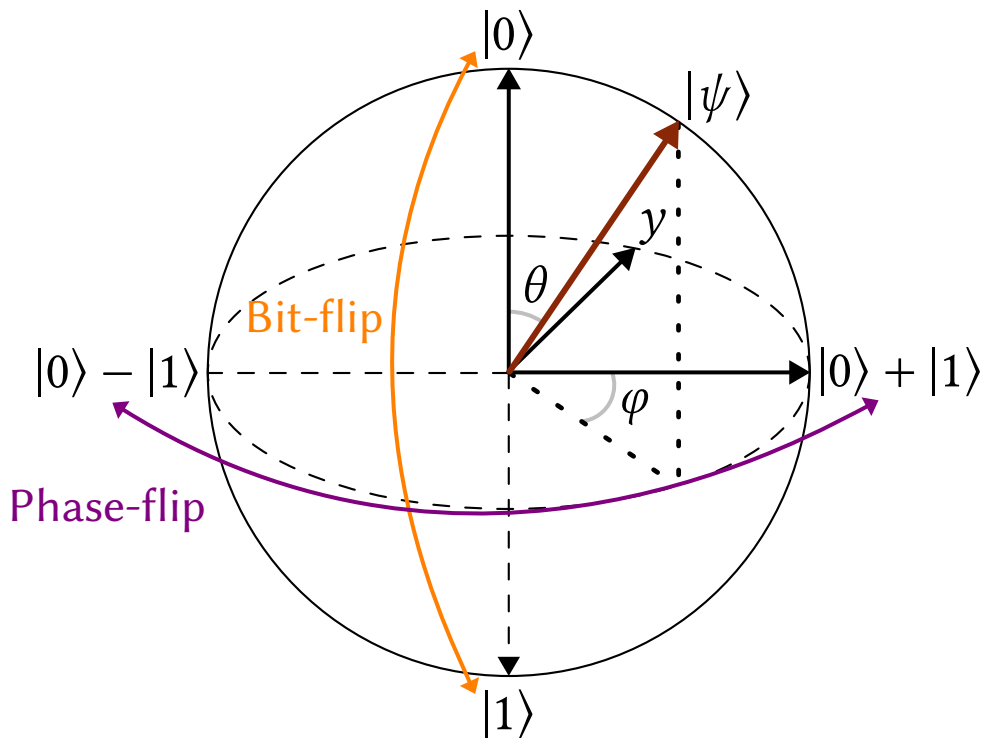
Correct for both bit-flip and phase-flips



Suppressing quantum errors by scaling a surface code logical qubit.
Nature, 2023, 614.7949: 676-681.

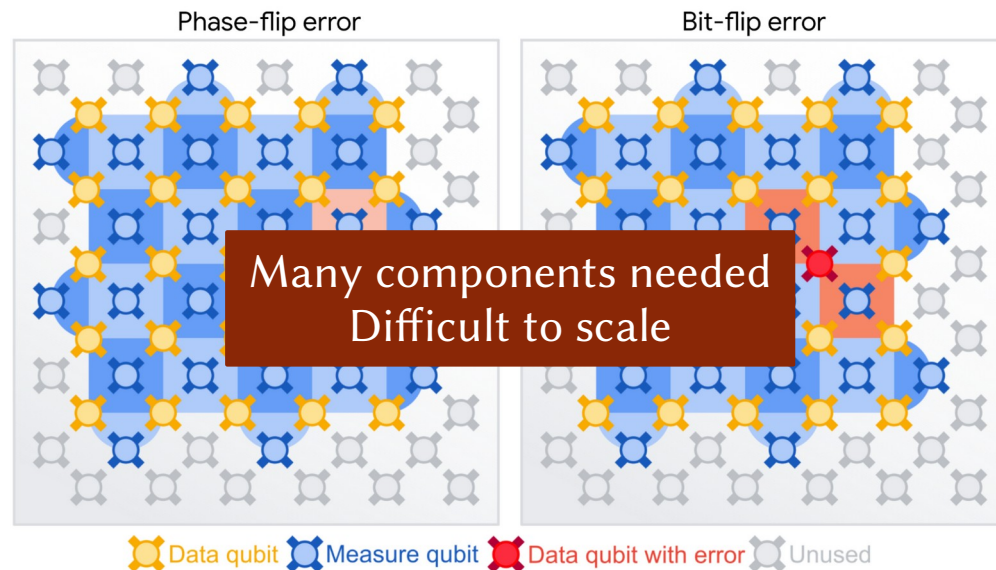
Quantum Error Correction

Redundantly encode information in a larger Hilbert space



Surface code

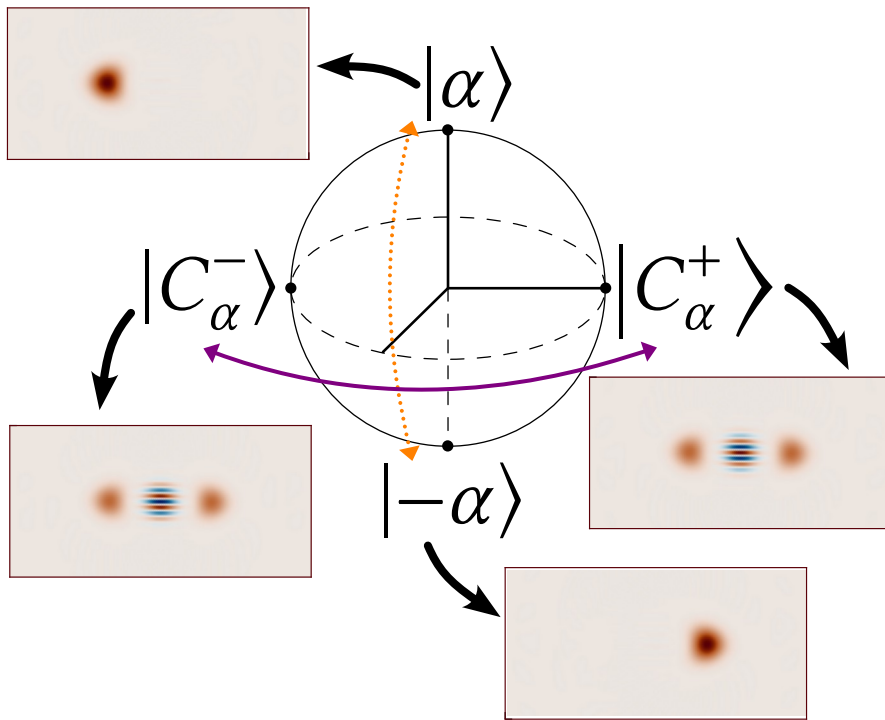
Correct for both bit-flip and phase-flips



Suppressing quantum errors by scaling a surface code logical qubit.
Nature, 2023, 614.7949: 676-681.

Bosonic Quantum Code: Cat Qubits

Intrinsically protect against bit-flip errors



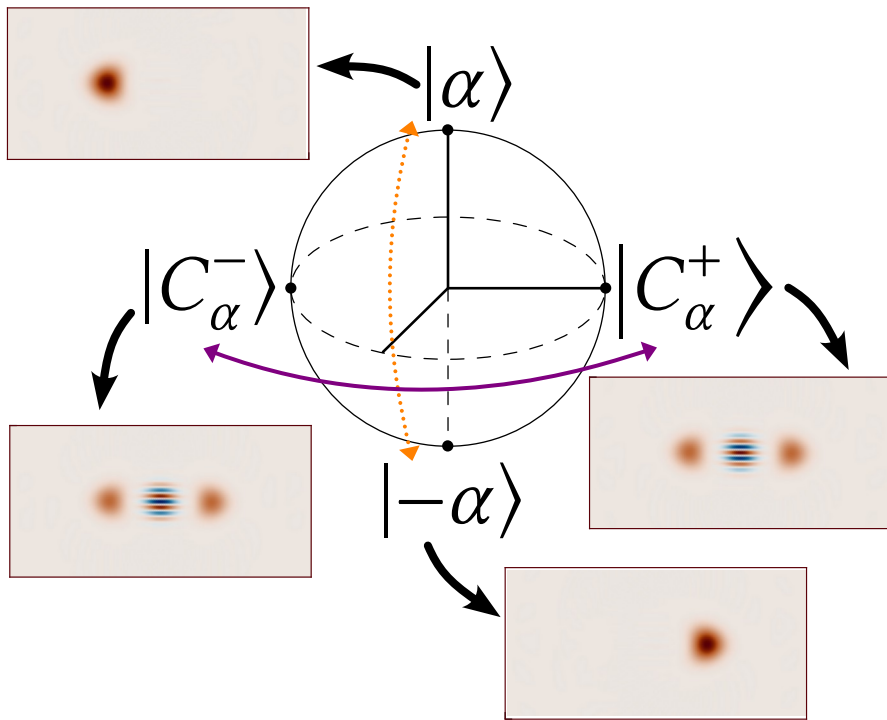
Coherent states used as logical states

$$z: |\alpha\rangle = e^{-|\alpha|^2/2} \sum_{k \geq 0} \frac{\alpha^k}{\sqrt{k!}}$$

$$x: |C_{\alpha}^{\pm}\rangle \propto |\alpha\rangle \pm |-\alpha\rangle$$

Bosonic Quantum Code: Cat Qubits

Intrinsically protect against bit-flip errors



Coherent states used as logical states

$$z: |\alpha\rangle = e^{-|\alpha|^2/2} \sum_{k \geq 0} \frac{\alpha^k}{\sqrt{k!}}$$

$$x: |C_{\alpha}^{\pm}\rangle \propto |\alpha\rangle \pm |-\alpha\rangle$$

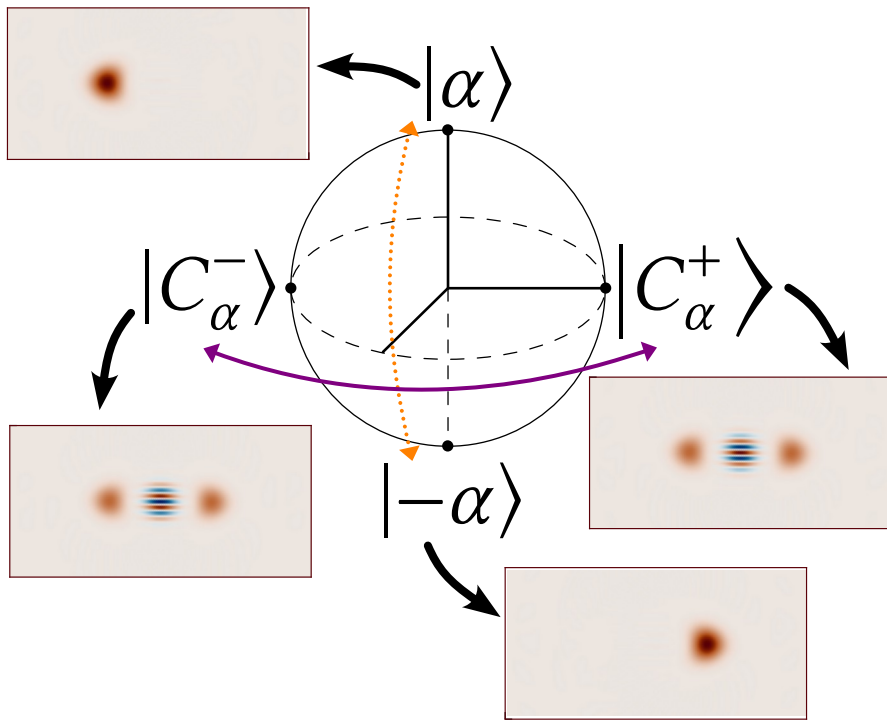
Action of photon-loss events

$$\hat{a} |\alpha\rangle = \alpha |\alpha\rangle \longrightarrow \text{NO ERROR}$$

$$\hat{a} |C_{\alpha}^{\pm}\rangle \propto |C_{\alpha}^{\mp}\rangle \longrightarrow \text{PHASE-FLIP}$$

Bosonic Quantum Code: Cat Qubits

Intrinsically protect against bit-flip errors



Noise-biased qubits

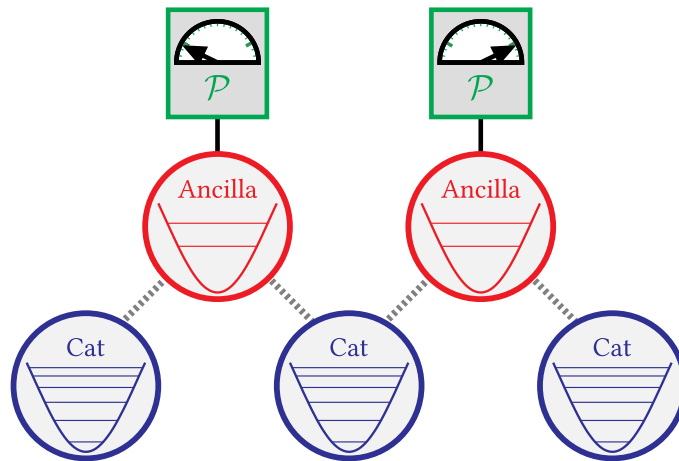
- Bit-flips exponentially suppressed

$$\Gamma_{\text{bf}} \propto e^{-\kappa_{\text{bf}}|\alpha|^2}$$

- Phase-flips only increase linearly

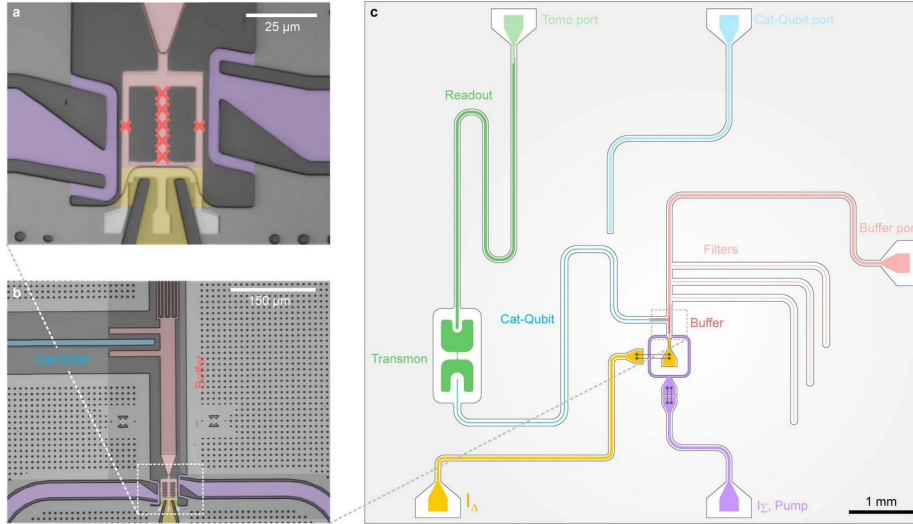
$$\Gamma_{\text{pf}} \propto \kappa_{\text{pf}}|\alpha|^2$$

Repetition code



Superconducting circuits

The two-photon driven transmon



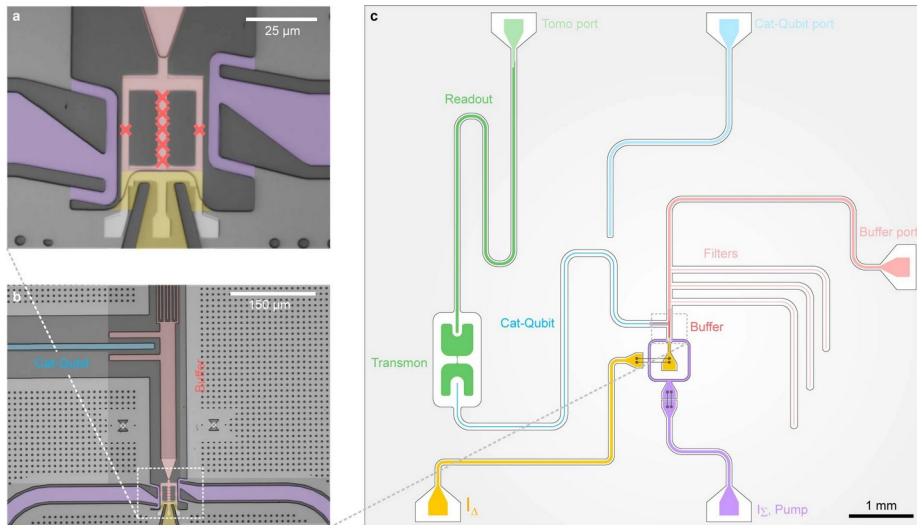
LESCANNE, Raphaël, et al. Nature Physics, 2020, 16.5: 509-513.

KOCH, Jens, et al. Physical Review A—Atomic, Molecular, and Optical Physics, 2007, 76.4: 042319.

$$\hat{H}_t \propto \cos(\hat{\phi}) \sim K_1 \hat{a}^\dagger \hat{a} - K_2 (\hat{a}^\dagger)^2 \hat{a}^2 + K_3 (\hat{a}^\dagger)^3 \hat{a}^3 + \dots$$
$$K_3 \ll K_2$$

Superconducting circuits

The two-photon driven transmon



Lescanne, Raphaël, et al. Nature Physics, 2020, 16.5: 509-513.

Koch, Jens, et al. Physical Review A—Atomic, Molecular, and Optical Physics, 2007, 76.4: 042319.

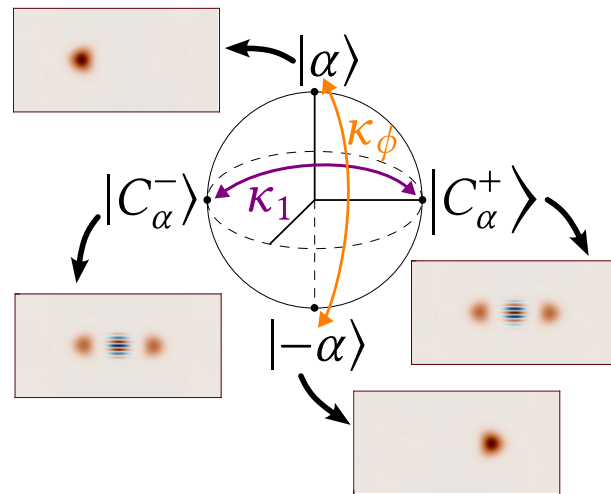
$$\hat{H}_t \propto \cos(\hat{\phi}) \sim K_1 \hat{a}^\dagger \hat{a} - K_2 (\hat{a}^\dagger)^2 \hat{a}^2 + K_3 (\hat{a}^\dagger)^3 \hat{a}^3 + \dots$$

$$\mathcal{L} = -i[\hat{H}_t + \hat{H}_d, \hat{\rho}] + \kappa_1 \mathcal{D}[\hat{a}] + \kappa_\phi \mathcal{D}[\hat{a}^\dagger \hat{a}]$$

$$\hat{H}_d = G(\hat{a}^2 e^{i\omega_d t} + \hat{a}^{\dagger 2} e^{-i\omega_d t})$$

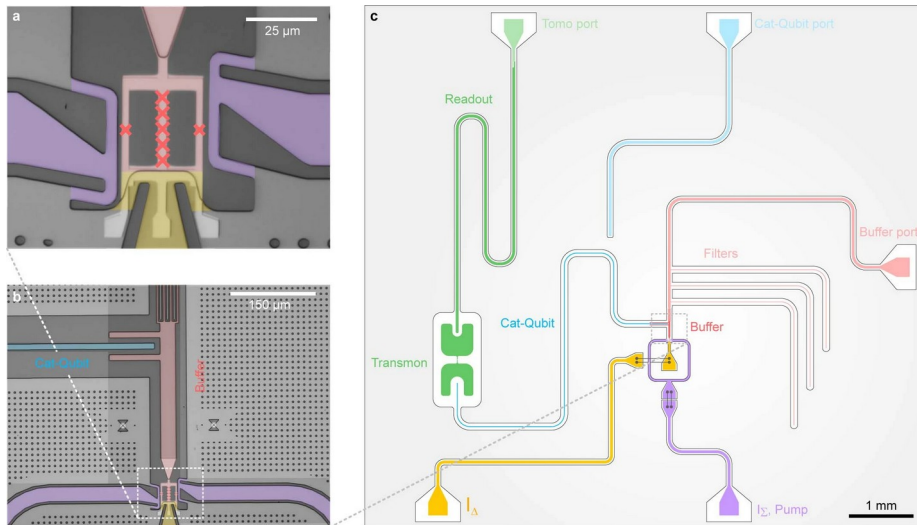
Two main sources of errors:

- Photon-loss κ_1
(energy loss to the environment)
- Dephasing noise κ_ϕ
(coupling to other SC components)



Superconducting circuits

The two-photon driven transmon



Lescanne, Raphaël, et al. Nature Physics, 2020, 16.5: 509-513.

Koch, Jens, et al. Physical Review A—Atomic, Molecular, and Optical Physics, 2007, 76.4: 042319.

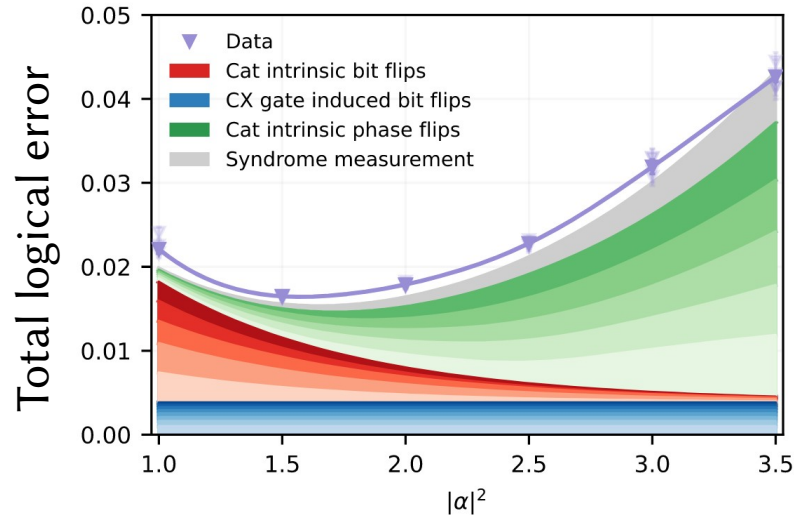
$$\hat{H}_t \propto \cos(\hat{\phi}) \sim K_1 \hat{a}^\dagger \hat{a} - K_2 (\hat{a}^\dagger)^2 \hat{a}^2 + K_3 (\hat{a}^\dagger)^3 \hat{a}^3 + \dots$$

$$\mathcal{L} = -i[\hat{H}_t + \hat{H}_d, \hat{\rho}] + \kappa_1 \mathcal{D}[\hat{a}] + \kappa_\phi \mathcal{D}[\hat{a}^\dagger \hat{a}]$$

$$\hat{H}_d = G(\hat{a}^2 + \hat{a}^{\dagger 2})$$

Two main sources of errors:

- Photon-loss $\mathcal{K}_1$
(energy loss to the environment)
- Dephasing noise $\mathcal{K}_\phi$
(coupling to other SC components)

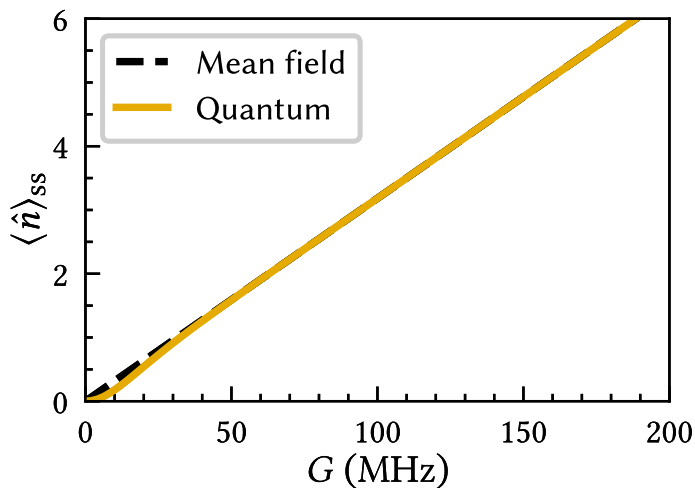


Putterman, Harald, et al. Nature 638.8052 (2025): 927-934.

Noise-biased qubits

Dissipative cats

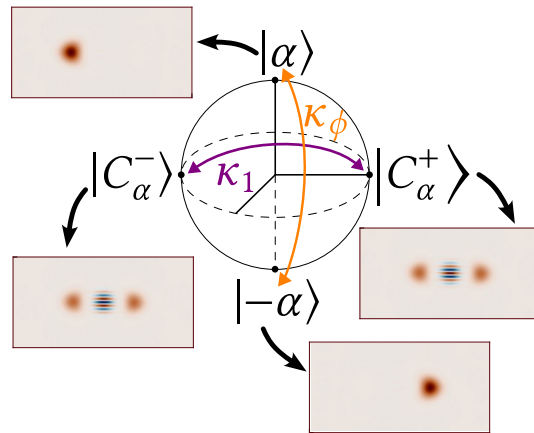
$$\mathcal{L}(\hat{\rho}) = -iG[\hat{a}^{\dagger 2} + \hat{a}^2, \hat{\rho}] + \kappa_2 \mathcal{D}[\hat{a}^2]\hat{\rho}$$



Mirrahimi, Mazhar, et al. New Journal of Physics 16.4 (2014): 045014.
 Leghtas, Zaki, et al. Science 347.6224 (2015): 853-857.
 Berdou, Camille, et al. PRX Quantum 4.2 (2023): 020350.

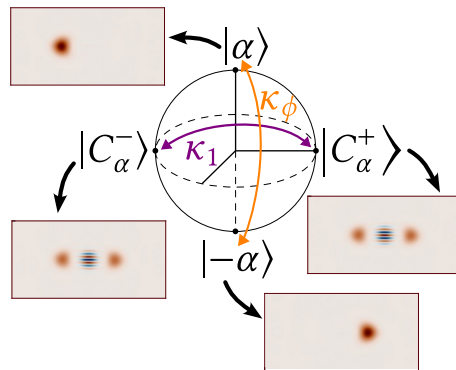
Kerr cats

$$\hat{H} = K_2(\hat{a}^{\dagger})^2\hat{a}^2 + G(\hat{a}^{\dagger 2} + \hat{a}^2)$$



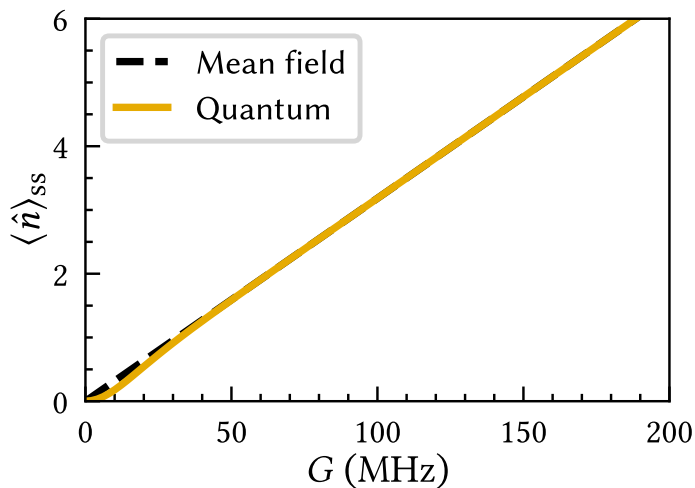
Grimm, Alexander. et. al. (2020). Nature, 584(7820), 205-209.
 Puri, Shruti, Samuel Boutin, and Alexandre Blais. npj Quantum Information 3.1 (2017): 18.
 Ofek, Nissim, et al. Nature 536.7617 (2016): 441-445.

Noise-biased qubits



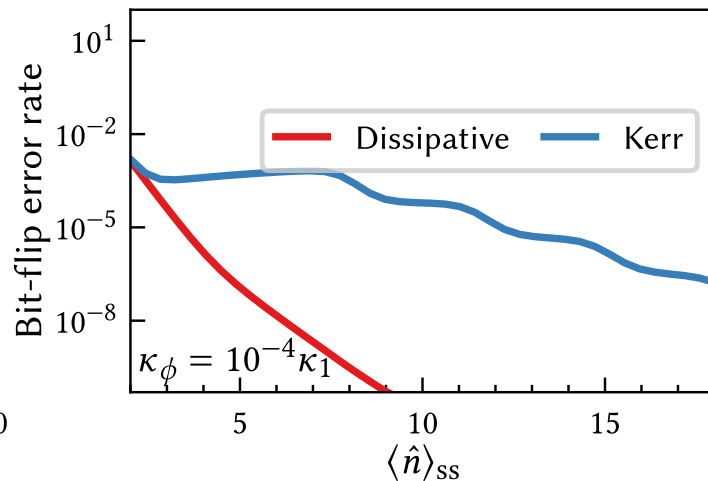
Dissipative cats

$$\mathcal{L}(\hat{\rho}) = -iG[\hat{a}^{\dagger 2} + \hat{a}^2, \hat{\rho}] + \kappa_2 \mathcal{D}[\hat{a}^2] \hat{\rho}$$

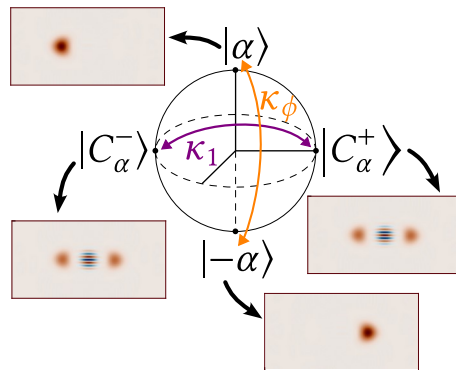


Kerr cats

$$\hat{H} = K_2(\hat{a}^{\dagger})^2 \hat{a}^2 + G(\hat{a}^{\dagger 2} + \hat{a}^2)$$

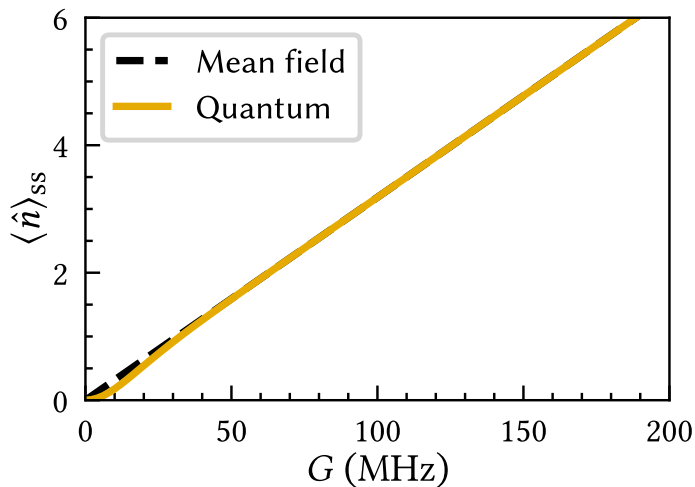


Noise-biased qubits



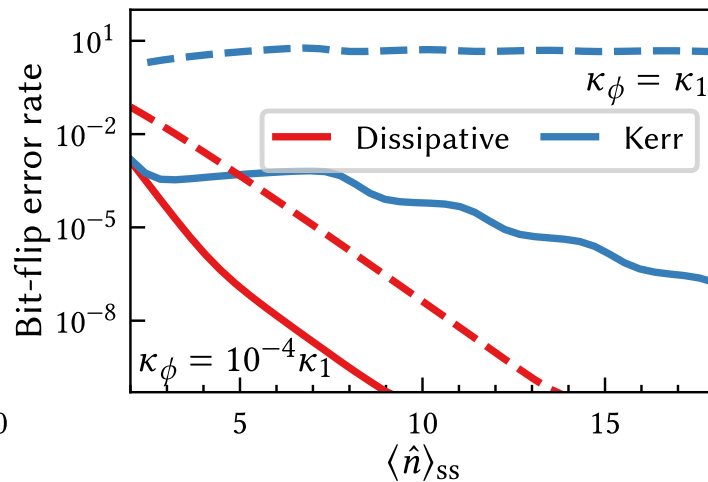
Dissipative cats

$$\mathcal{L}(\hat{\rho}) = -iG[\hat{a}^{\dagger 2} + \hat{a}^2, \hat{\rho}] + \kappa_2 \mathcal{D}[\hat{a}^2] \hat{\rho}$$

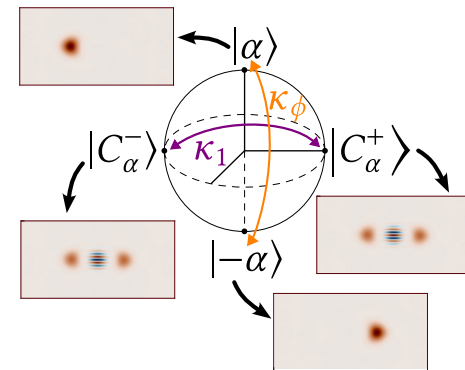


Kerr cats

$$\hat{H} = K_2(\hat{a}^{\dagger})^2 \hat{a}^2 + G(\hat{a}^{\dagger 2} + \hat{a}^2)$$



Noise-biased qubits

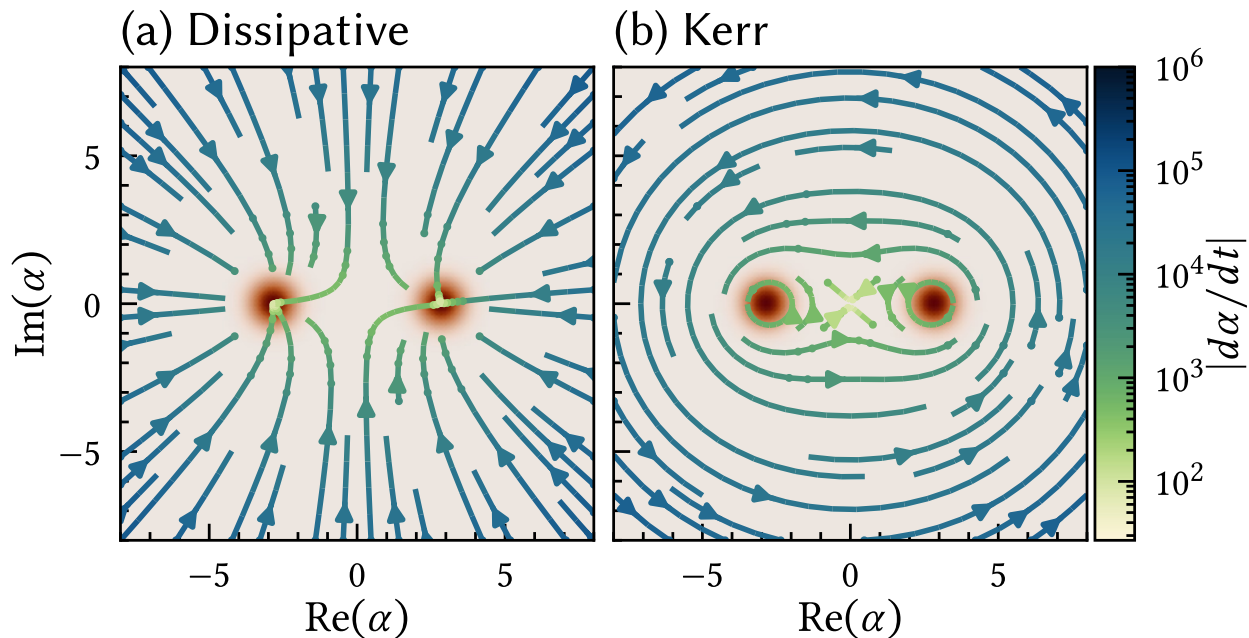


Dissipative cats

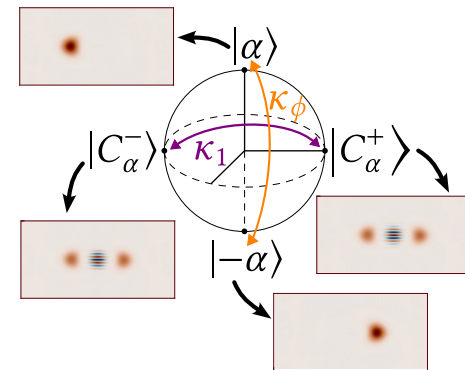
$$\mathcal{L}(\hat{\rho}) = -iG[\hat{a}^{\dagger 2} + \hat{a}^2, \hat{\rho}] + \kappa_2 \mathcal{D}[\hat{a}^2]\hat{\rho}$$

Kerr cats

$$\hat{H} = K_2(\hat{a}^{\dagger})^2 \hat{a}^2 + G(\hat{a}^{\dagger 2} + \hat{a}^2)$$



Noise-biased qubits

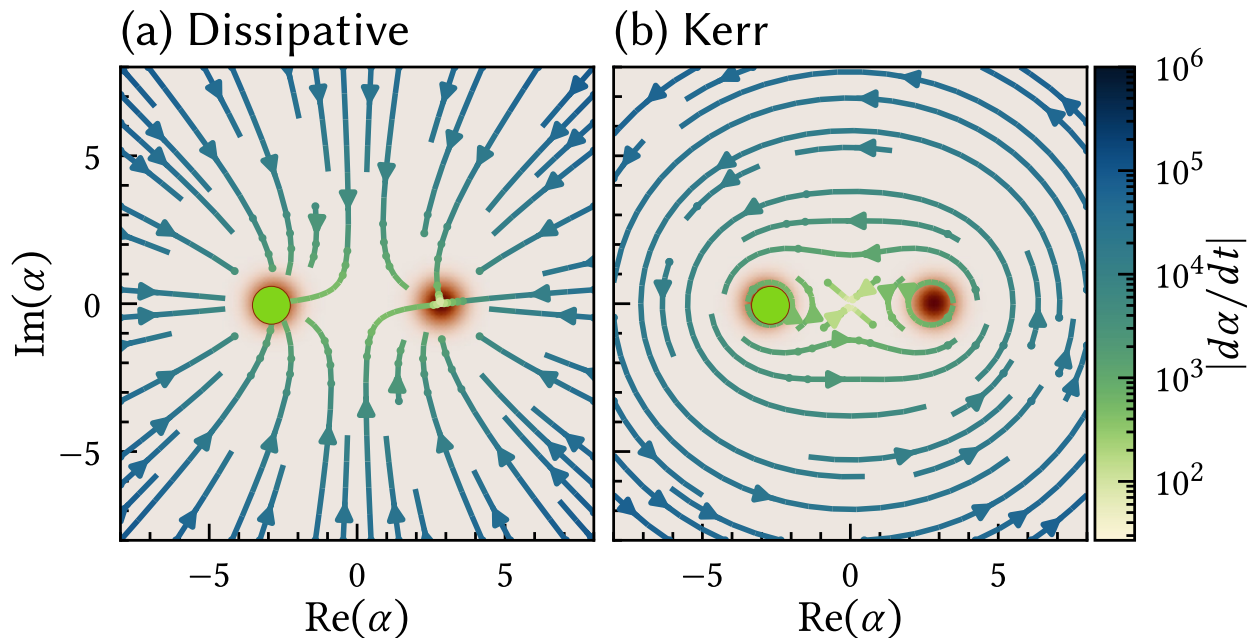


Dissipative cats

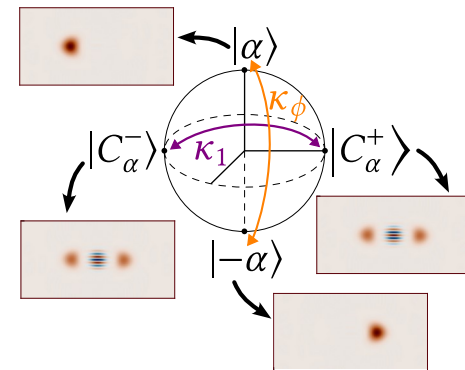
$$\mathcal{L}(\hat{\rho}) = -iG[\hat{a}^{\dagger 2} + \hat{a}^2, \hat{\rho}] + \kappa_2 \mathcal{D}[\hat{a}^2]\hat{\rho}$$

Kerr cats

$$\hat{H} = K_2(\hat{a}^{\dagger})^2 \hat{a}^2 + G(\hat{a}^{\dagger 2} + \hat{a}^2)$$



Noise-biased qubits

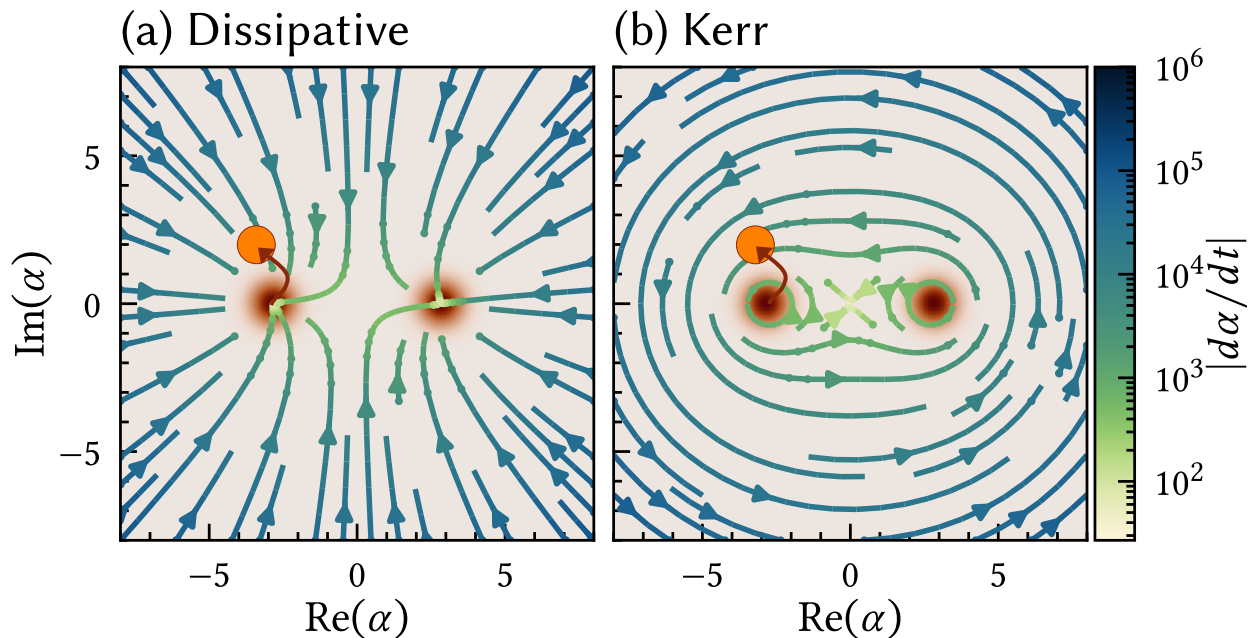


Dissipative cats

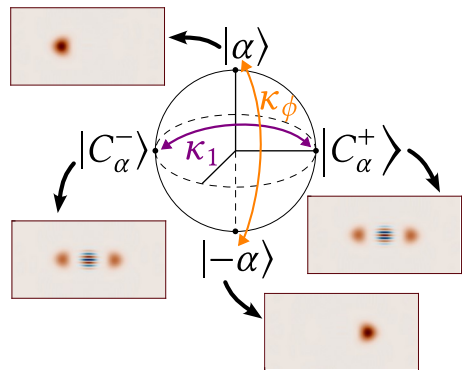
$$\mathcal{L}(\hat{\rho}) = -iG[\hat{a}^{\dagger 2} + \hat{a}^2, \hat{\rho}] + \kappa_2 \mathcal{D}[\hat{a}^2]\hat{\rho}$$

Kerr cats

$$\hat{H} = K_2(\hat{a}^{\dagger})^2 \hat{a}^2 + G(\hat{a}^{\dagger 2} + \hat{a}^2)$$



Noise-biased qubits

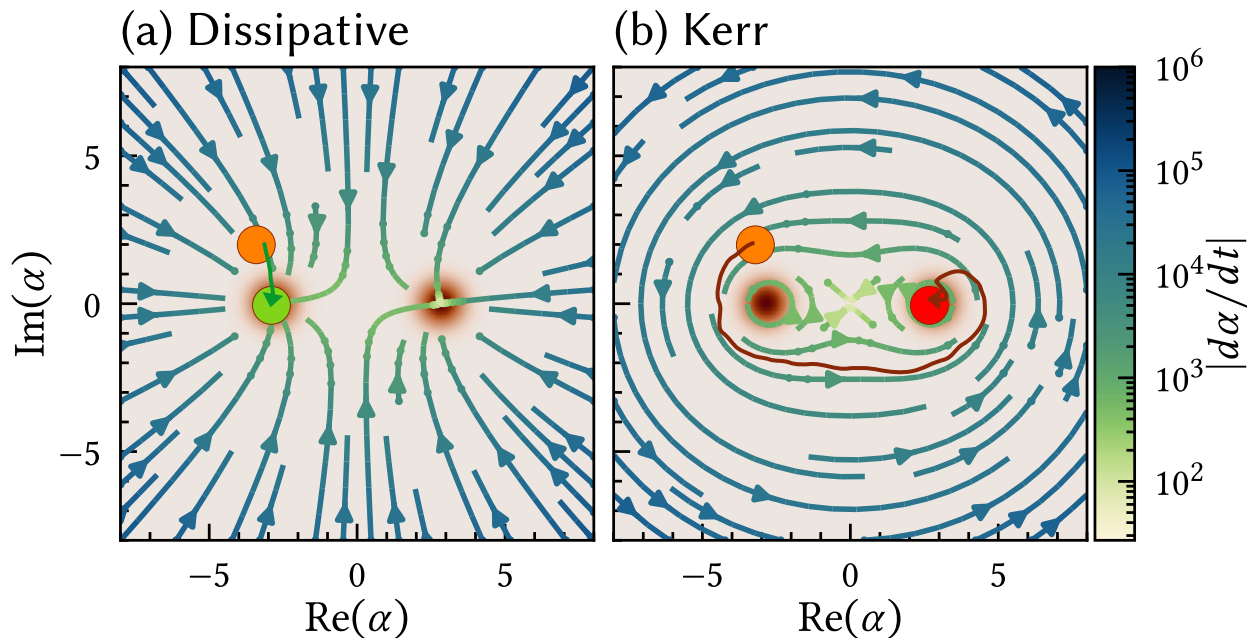


Dissipative cats

$$\mathcal{L}(\hat{\rho}) = -iG[\hat{a}^{\dagger 2} + \hat{a}^2, \hat{\rho}] + \kappa_2 \mathcal{D}[\hat{a}^2]\hat{\rho}$$

Kerr cats

$$\hat{H} = K_2(\hat{a}^{\dagger})^2 \hat{a}^2 + G(\hat{a}^{\dagger 2} + \hat{a}^2)$$



Noise-biased qubits

Dissipative cats

$$\mathcal{L}(\hat{\rho}) = -iG[\hat{a}^{\dagger 2} + \hat{a}^2, \hat{\rho}] + \kappa_2 \mathcal{D}[\hat{a}^2] \hat{\rho}$$

Preserve bit-flip protection even with large dephasing

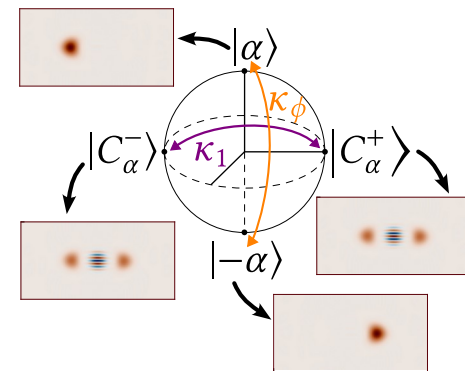
Confinement leads to slow gates

Kerr cats

$$\hat{H} = K_2(\hat{a}^{\dagger})^2 \hat{a}^2 + G(\hat{a}^{\dagger 2} + \hat{a}^2)$$

Poor performance with sizable dephasing noise

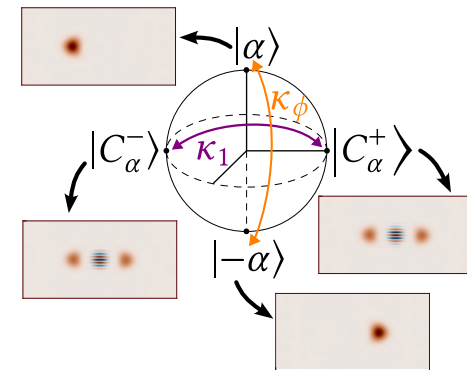
Fast operations



Mirrahimi, Mazyar, et al. New Journal of Physics 16.4 (2014): 045014.
Leghtas, Zaki, et al. Science 347.6224 (2015): 853-857.
Berdou, Camille, et al. PRX Quantum 4.2 (2023): 020350.

Grimm, Alexander. et. al. (2020). Nature, 584(7820), 205-209.
Puri, Shruti, Samuel Boutin, and Alexandre Blais. npj Quantum Information 3.1 (2017): 18.
Ofek, Nissim, et al. Nature 536.7617 (2016): 441-445.

Noise-biased qubits



Dissipative cats

$$\mathcal{L}(\hat{\rho}) = -iG[\hat{a}^{\dagger 2} + \hat{a}^2, \hat{\rho}] + \kappa_2 \mathcal{D}[\hat{a}^2] \hat{\rho}$$

Preserve bit-flip protection even with large dephasing

Confinement leads to slow gates

Kerr cats

$$\hat{H} = K_2(\hat{a}^{\dagger})^2 \hat{a}^2 + G(\hat{a}^{\dagger 2} + \hat{a}^2)$$

Poor performance with sizable dephasing noise

Fast operations

Hybrid cats

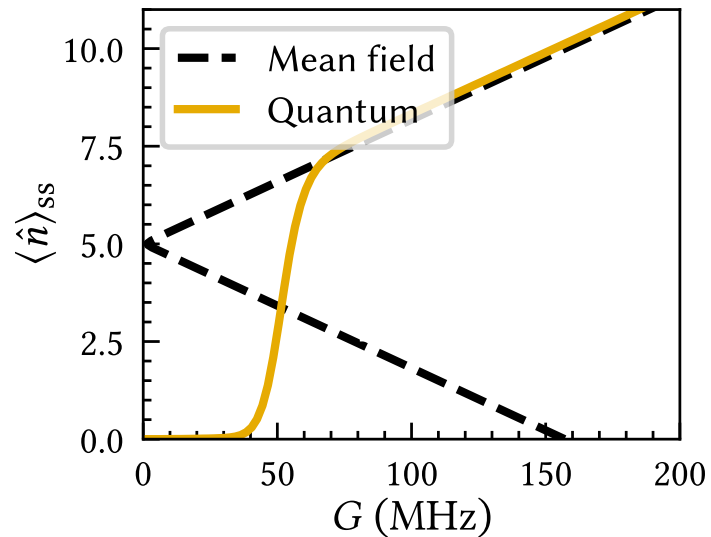
Add small 2-photon dissipation to Kerr cats

Gautier, Ronan, Alain Sarlette, and Mazyar Mirrahimi. PRX Quantum 3.2 (2022): 020339.

The critical cat

A detuned hybrid cat

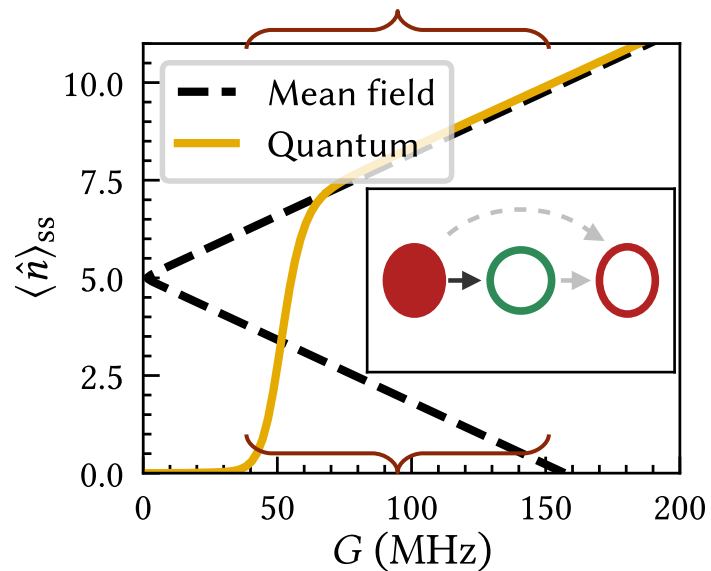
$$\mathcal{L}(\hat{\rho}) = -i[\Delta \hat{a}^\dagger \hat{a} + K_2 \hat{a}^{\dagger 2} \hat{a}^2 + G(\hat{a}^{\dagger 2} + \hat{a}^2), \hat{\rho}] + \kappa_2 \mathcal{D}[\hat{a}^2] \hat{\rho}$$



The critical cat

A detuned hybrid cat

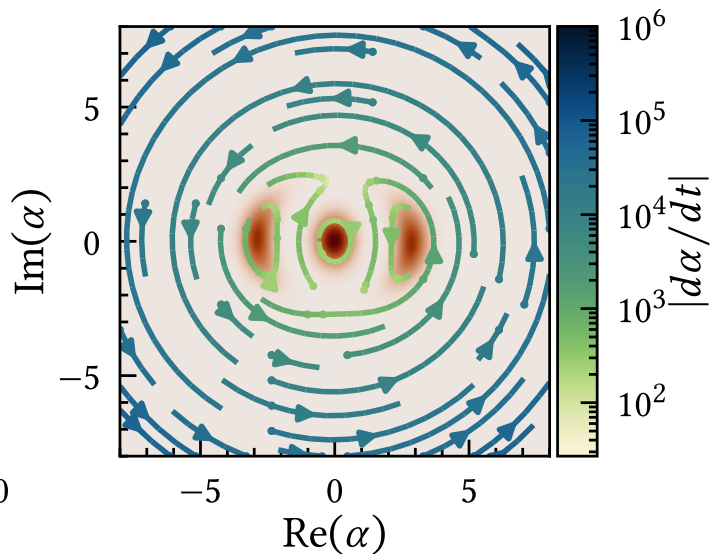
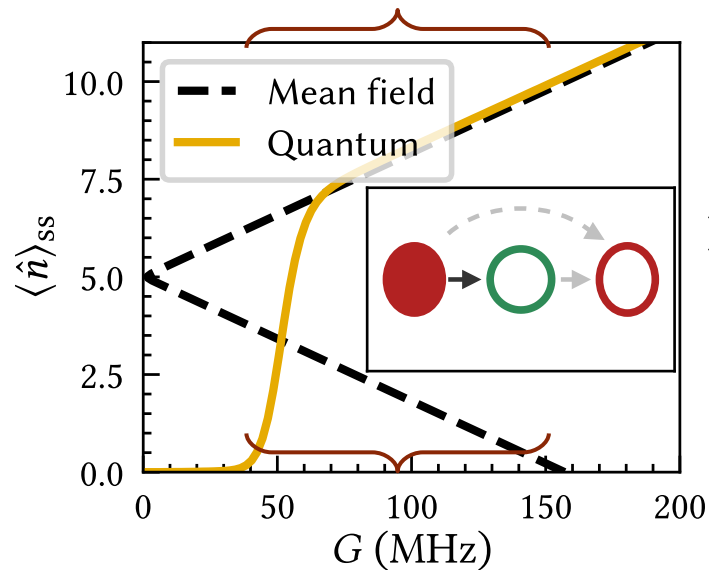
$$\mathcal{L}(\hat{\rho}) = -i[\Delta \hat{a}^\dagger \hat{a} + K_2 \hat{a}^{\dagger 2} \hat{a}^2 + G(\hat{a}^{\dagger 2} + \hat{a}^2), \hat{\rho}] + \kappa_2 \mathcal{D}[\hat{a}^2] \hat{\rho}$$



The critical cat

A detuned hybrid cat

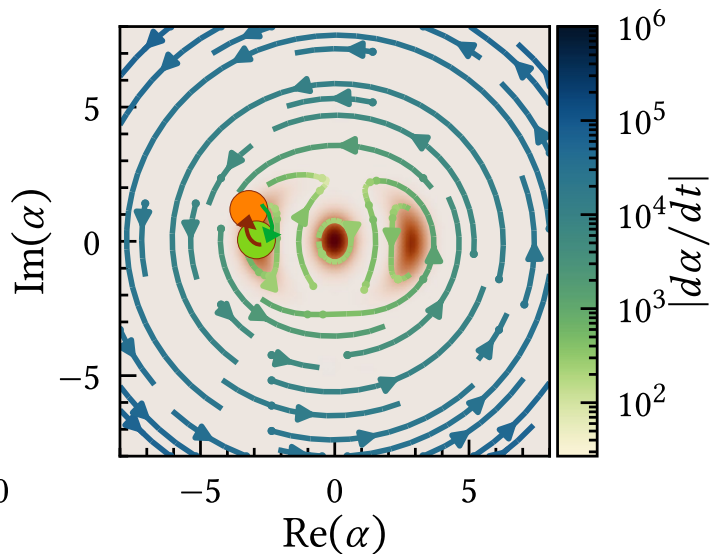
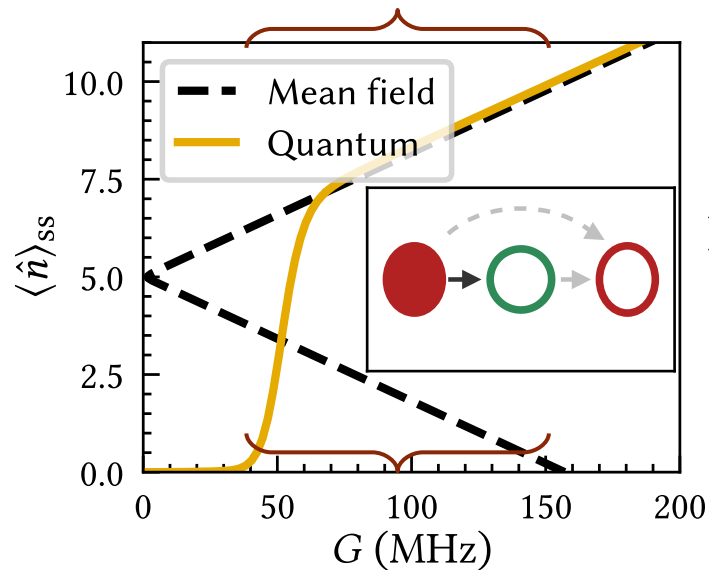
$$\mathcal{L}(\hat{\rho}) = -i[\Delta \hat{a}^\dagger \hat{a} + K_2 \hat{a}^{\dagger 2} \hat{a}^2 + G(\hat{a}^{\dagger 2} + \hat{a}^2), \hat{\rho}] + \kappa_2 \mathcal{D}[\hat{a}^2] \hat{\rho}$$



The critical cat

A detuned hybrid cat

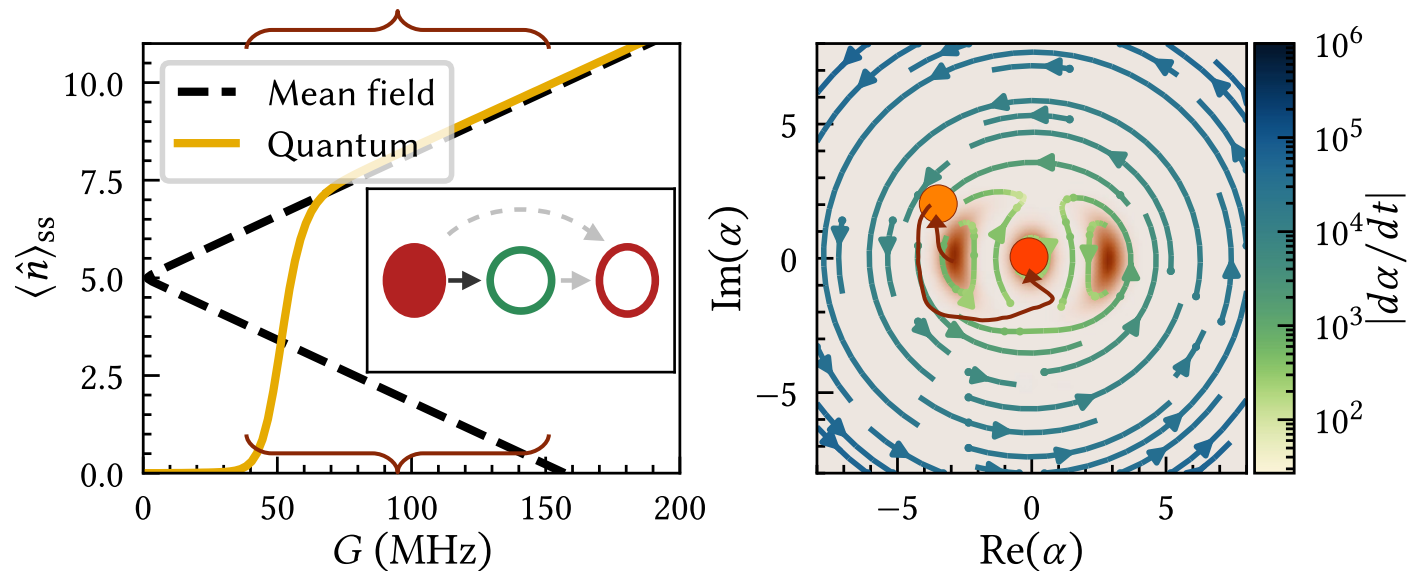
$$\mathcal{L}(\hat{\rho}) = -i[\Delta \hat{a}^\dagger \hat{a} + K_2 \hat{a}^{\dagger 2} \hat{a}^2 + G(\hat{a}^{\dagger 2} + \hat{a}^2), \hat{\rho}] + \kappa_2 \mathcal{D}[\hat{a}^2] \hat{\rho}$$



The critical cat

A detuned hybrid cat

$$\mathcal{L}(\hat{\rho}) = -i[\Delta \hat{a}^\dagger \hat{a} + K_2 \hat{a}^{\dagger 2} \hat{a}^2 + G(\hat{a}^{\dagger 2} + \hat{a}^2), \hat{\rho}] + \kappa_2 \mathcal{D}[\hat{a}^2] \hat{\rho}$$

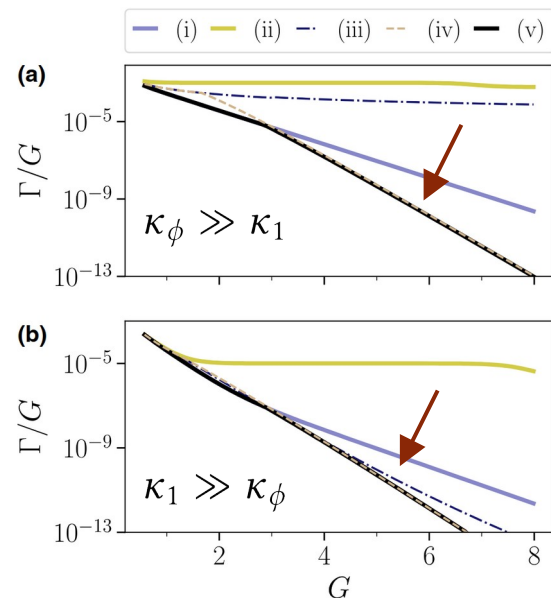
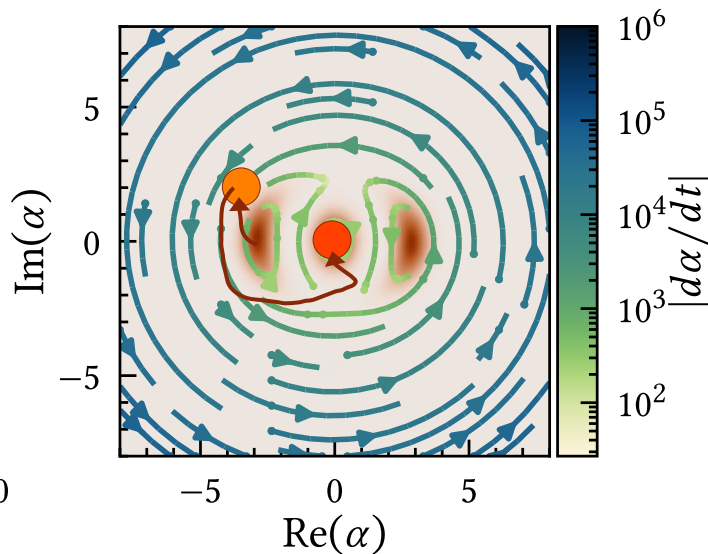
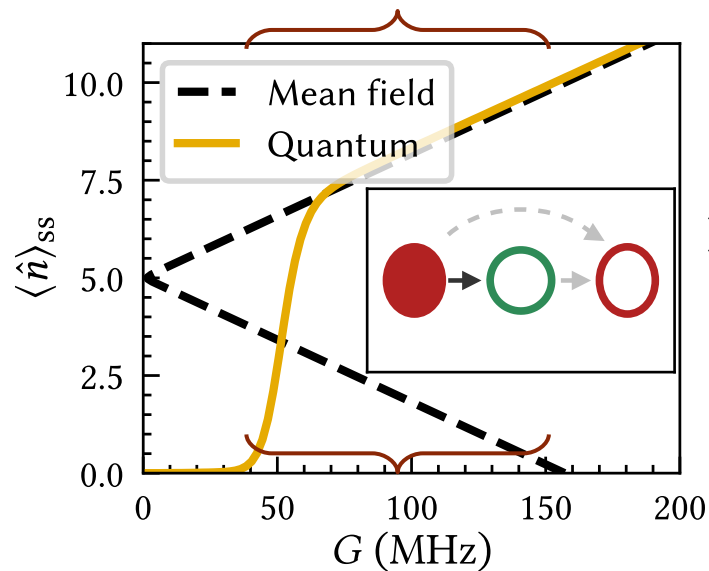


Possibility to detect an error but not correct it

The critical cat

A detuned hybrid cat

$$\mathcal{L}(\hat{\rho}) = -i[\Delta \hat{a}^\dagger \hat{a} + K_2 \hat{a}^{\dagger 2} \hat{a}^2 + G(\hat{a}^{\dagger 2} + \hat{a}^2), \hat{\rho}] + \kappa_2 \mathcal{D}[\hat{a}^2] \hat{\rho}$$



Recovers protection
against dephasing

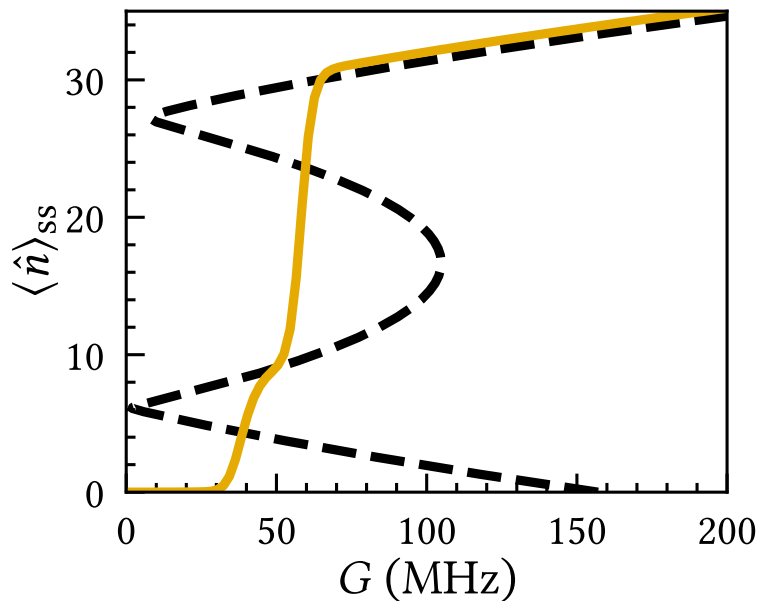
Possibility to detect an error but not correct it

The chiral cat

A critical cat with higher-order nonlinearities

Recall the transmon Hamiltonian $\rightarrow \hat{H}_t \propto \cos(\hat{\phi}) \sim K_1 \hat{a}^\dagger \hat{a} - K_2 (\hat{a}^\dagger)^2 \hat{a}^2 + K_3 (\hat{a}^\dagger)^3 \hat{a}^3 + \dots$

$$\mathcal{L}(\hat{\rho}) = -i[\Delta \hat{a}^\dagger \hat{a} - K_2 \hat{a}^{\dagger 2} \hat{a}^2 + K_3 \hat{a}^{\dagger 3} \hat{a}^3 + G(\hat{a}^{\dagger 2} + \hat{a}^2), \hat{\rho}] + \kappa_2 \mathcal{D}[\hat{a}^2] \hat{\rho}$$

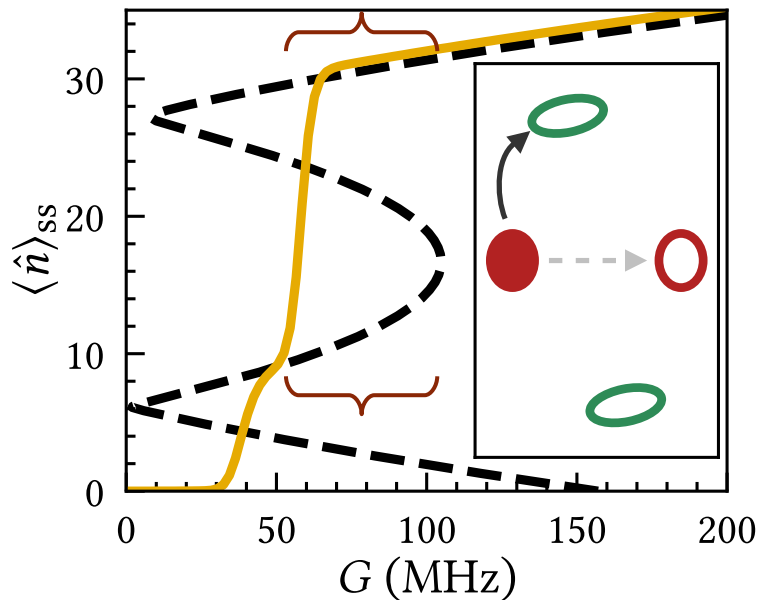


The chiral cat

A critical cat with higher-order nonlinearities

Recall the transmon Hamiltonian $\rightarrow \hat{H} \propto \cos(\hat{\varphi}) \sim K_1 \hat{a}^\dagger \hat{a} + K_2 (\hat{a}^\dagger)^2 \hat{a}^2 + K_3 (\hat{a}^\dagger)^3 \hat{a}^3 + \dots$

$$\mathcal{L}(\hat{\rho}) = -i[\Delta \hat{a}^\dagger \hat{a} - K_2 \hat{a}^{\dagger 2} \hat{a}^2 + K_3 \hat{a}^{\dagger 3} \hat{a}^3 + G(\hat{a}^{\dagger 2} + \hat{a}^2), \hat{\rho}] + \kappa_2 \mathcal{D}[\hat{a}^2] \hat{\rho}$$

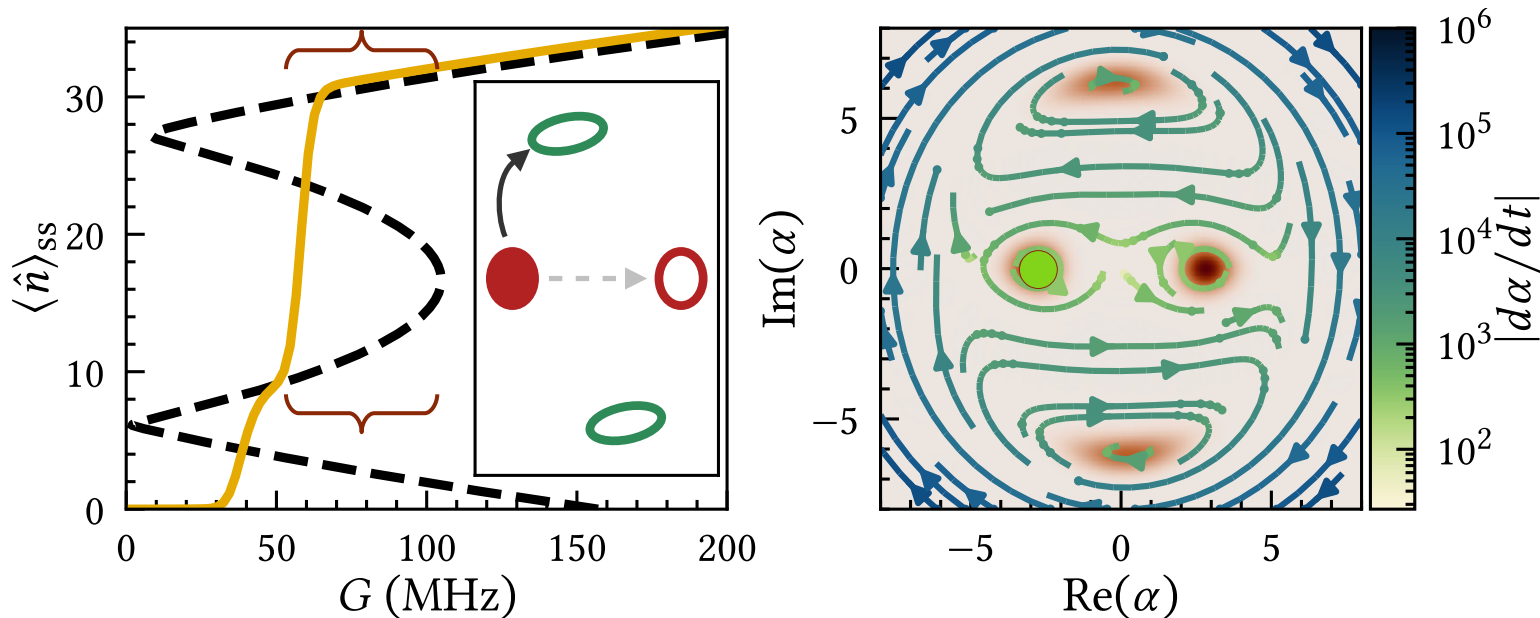


The chiral cat

A critical cat with higher-order nonlinearities

Recall the transmon Hamiltonian $\rightarrow \hat{H} \propto \cos(\hat{\varphi}) \sim K_1 \hat{a}^\dagger \hat{a} + K_2 (\hat{a}^\dagger)^2 \hat{a}^2 + K_3 (\hat{a}^\dagger)^3 \hat{a}^3 + \dots$

$$\mathcal{L}(\hat{\rho}) = -i[\Delta \hat{a}^\dagger \hat{a} - K_2 \hat{a}^{\dagger 2} \hat{a}^2 + K_3 \hat{a}^{\dagger 3} \hat{a}^3 + G(\hat{a}^{\dagger 2} + \hat{a}^2), \hat{\rho}] + \kappa_2 \mathcal{D}[\hat{a}^2] \hat{\rho}$$

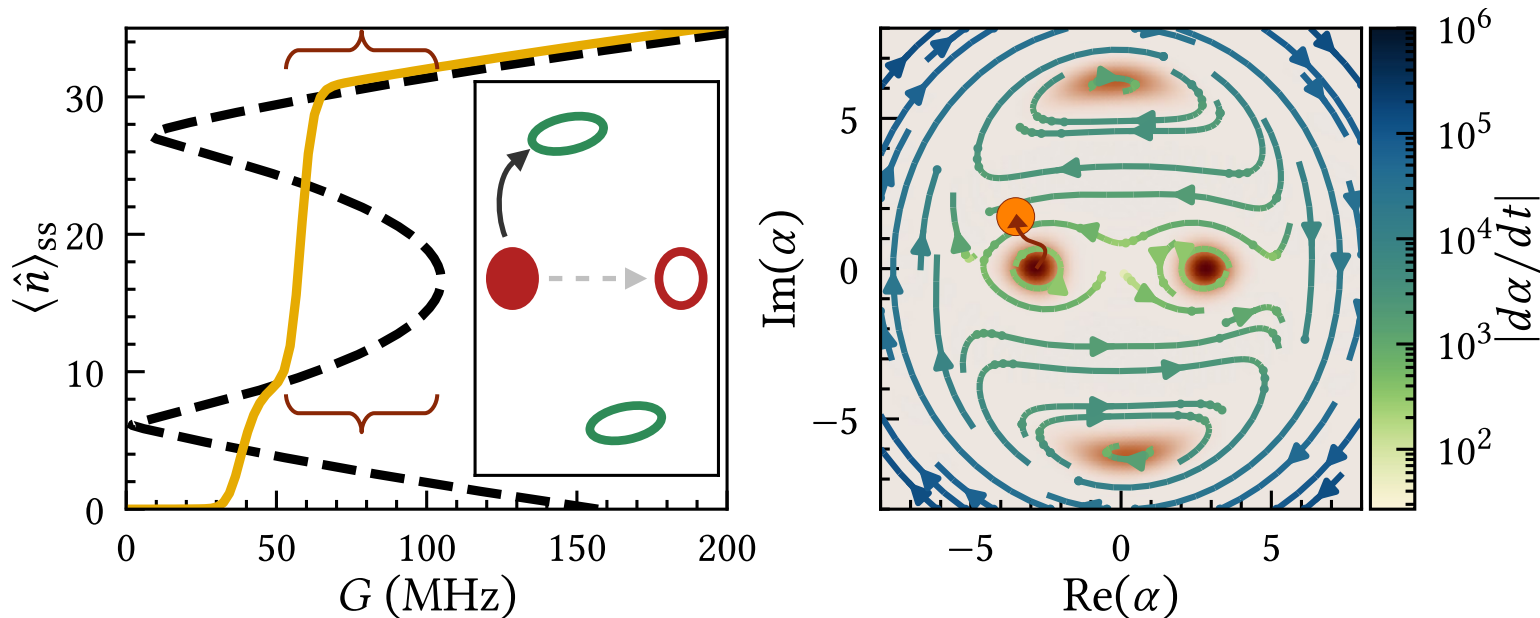


The chiral cat

A critical cat with higher-order nonlinearities

Recall the transmon Hamiltonian $\rightarrow \hat{H} \propto \cos(\hat{\varphi}) \sim K_1 \hat{a}^\dagger \hat{a} + K_2 (\hat{a}^\dagger)^2 \hat{a}^2 + K_3 (\hat{a}^\dagger)^3 \hat{a}^3 + \dots$

$$\mathcal{L}(\hat{\rho}) = -i[\Delta \hat{a}^\dagger \hat{a} - K_2 \hat{a}^{\dagger 2} \hat{a}^2 + K_3 \hat{a}^{\dagger 3} \hat{a}^3 + G(\hat{a}^{\dagger 2} + \hat{a}^2), \hat{\rho}] + \kappa_2 \mathcal{D}[\hat{a}^2] \hat{\rho}$$

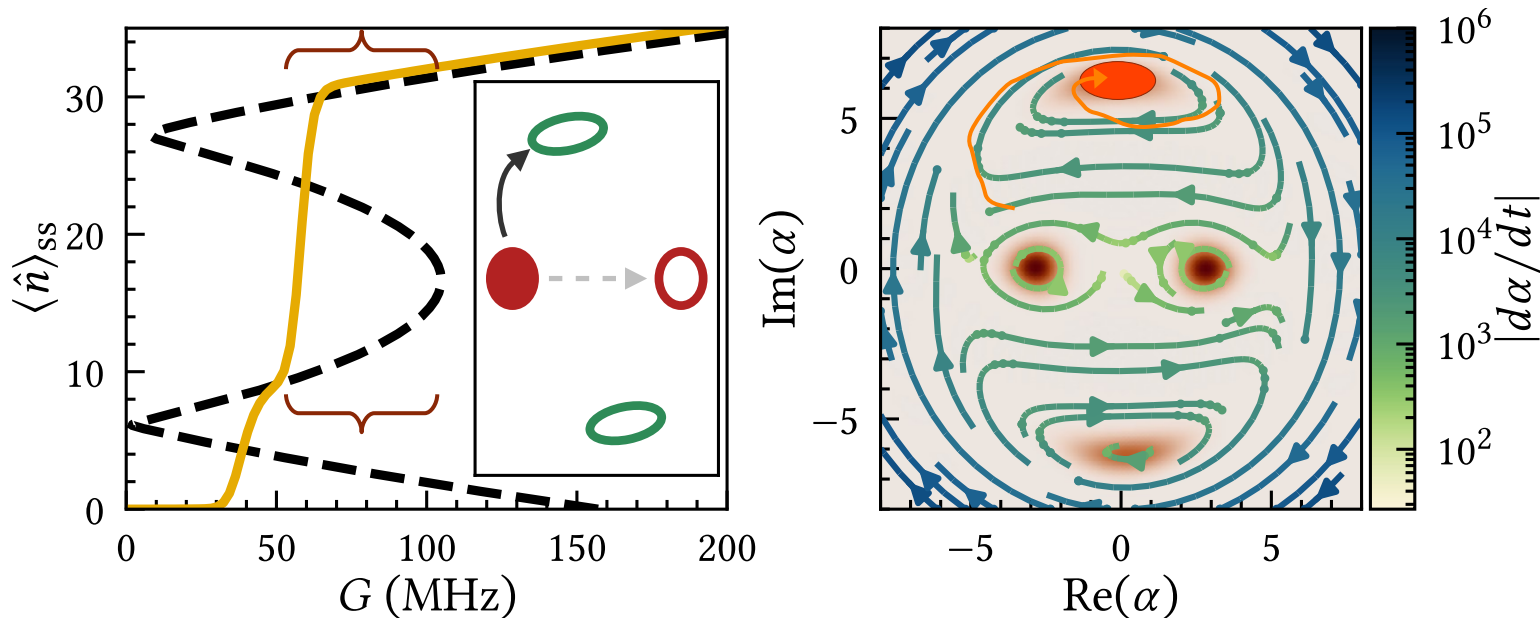


The chiral cat

A critical cat with higher-order nonlinearities

Recall the transmon Hamiltonian $\rightarrow \hat{H} \propto \cos(\hat{\varphi}) \sim K_1 \hat{a}^\dagger \hat{a} + K_2 (\hat{a}^\dagger)^2 \hat{a}^2 + K_3 (\hat{a}^\dagger)^3 \hat{a}^3 + \dots$

$$\mathcal{L}(\hat{\rho}) = -i[\Delta \hat{a}^\dagger \hat{a} - K_2 \hat{a}^{\dagger 2} \hat{a}^2 + K_3 \hat{a}^{\dagger 3} \hat{a}^3 + G(\hat{a}^{\dagger 2} + \hat{a}^2), \hat{\rho}] + \kappa_2 \mathcal{D}[\hat{a}^2] \hat{\rho}$$



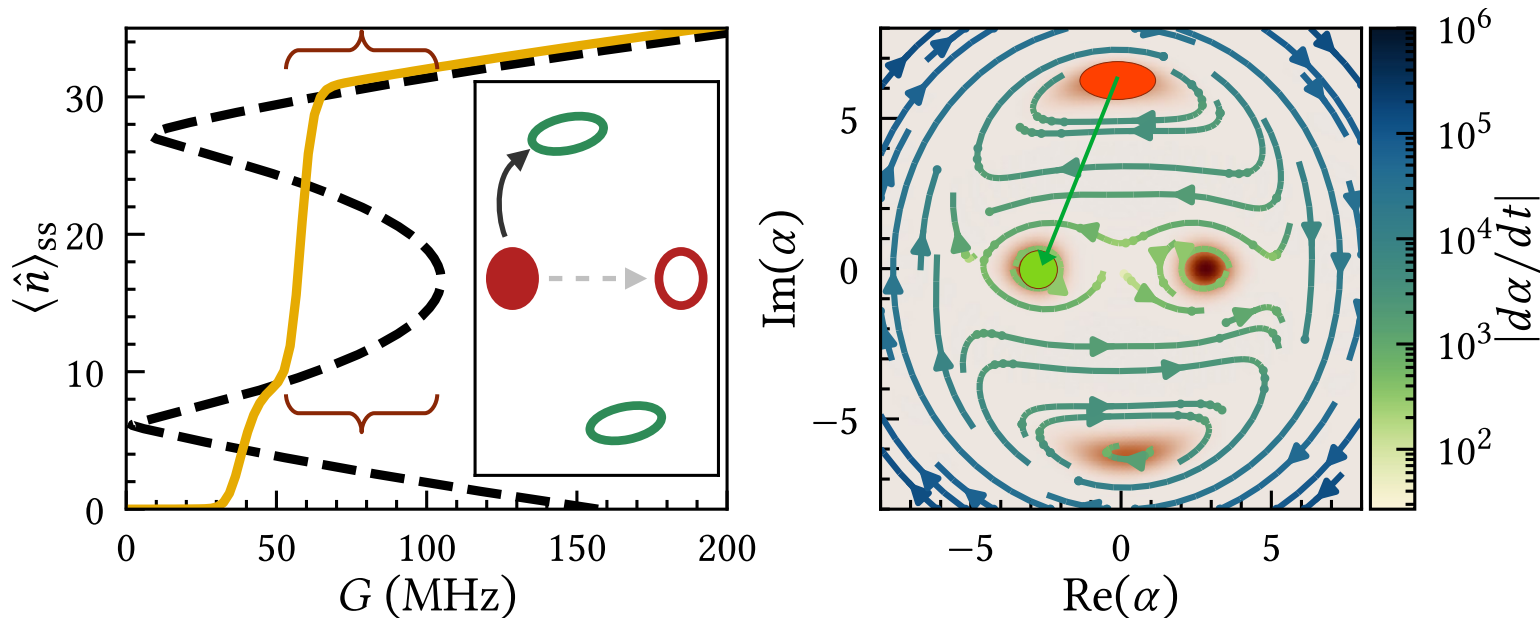
Possibility to detect an error

The chiral cat

A critical cat with higher-order nonlinearities

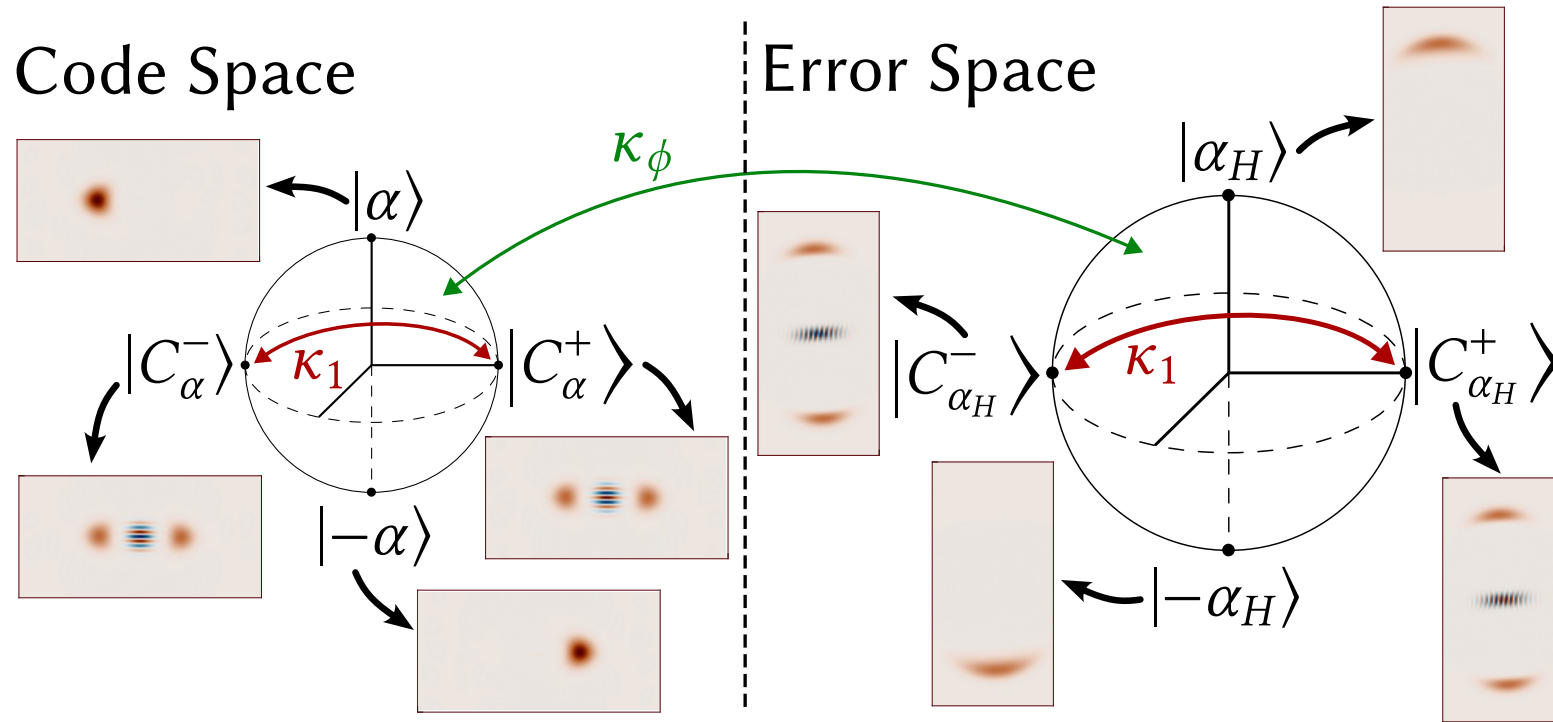
Recall the transmon Hamiltonian $\rightarrow \hat{H} \propto \cos(\hat{\phi}) \sim K_1 \hat{a}^\dagger \hat{a} + K_2 (\hat{a}^\dagger)^2 \hat{a}^2 + K_3 (\hat{a}^\dagger)^3 \hat{a}^3 + \dots$

$$\mathcal{L}(\hat{\rho}) = -i[\Delta \hat{a}^\dagger \hat{a} - K_2 \hat{a}^{\dagger 2} \hat{a}^2 + K_3 \hat{a}^{\dagger 3} \hat{a}^3 + G(\hat{a}^{\dagger 2} + \hat{a}^2), \hat{\rho}] + \kappa_2 \mathcal{D}[\hat{a}^2] \hat{\rho}$$



Possibility to detect an error and correct it

The chiral cat



Two quasi-orthogonal Bloch spheres with different mean photon number

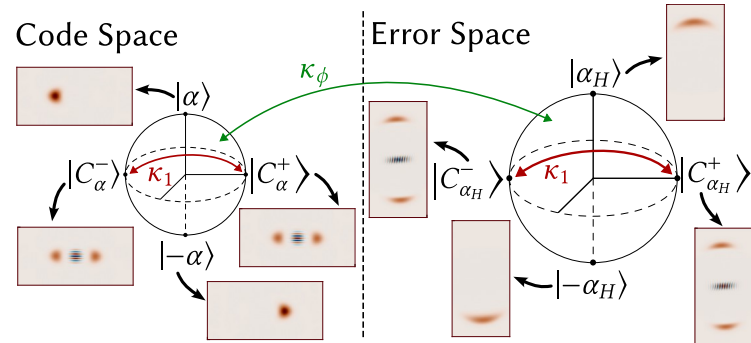
The chiral cat

How to perform the recovery?

$$\begin{cases} \hat{\Pi}_H = |\alpha_H\rangle\langle\alpha_H| + |-\alpha_H\rangle\langle-\alpha_H| \\ \hat{R} = |\alpha\rangle\langle\alpha_H| + |-\alpha\rangle\langle-\alpha_H| \end{cases}$$

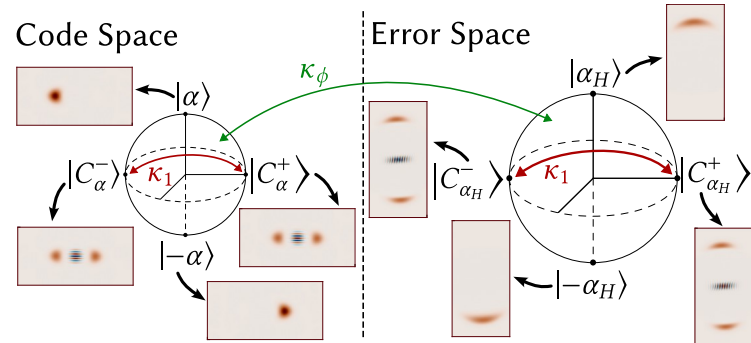
Projector onto the high manifold

Unitary operator from high to low manifold



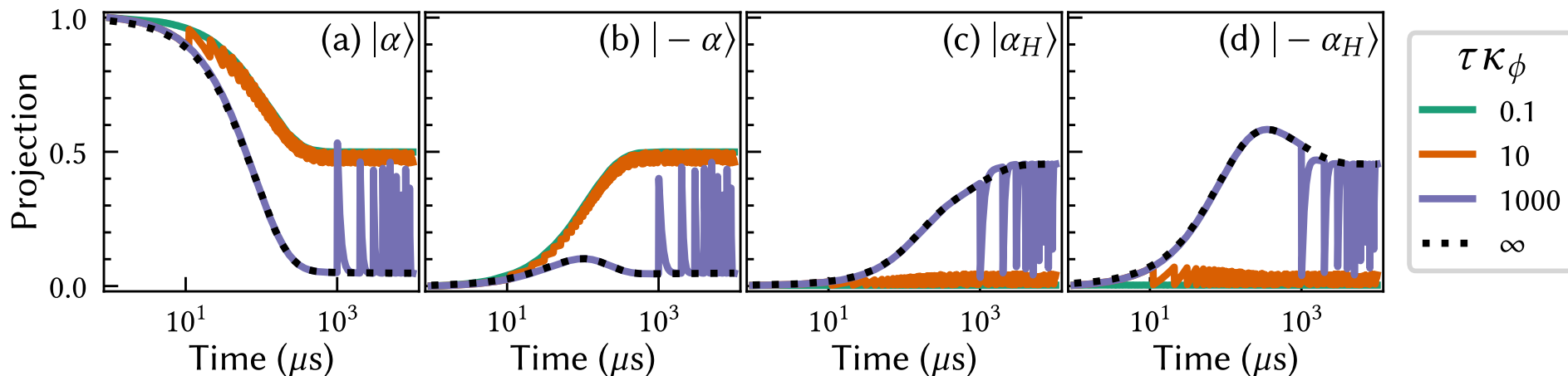
The chiral cat

How to perform the recovery?



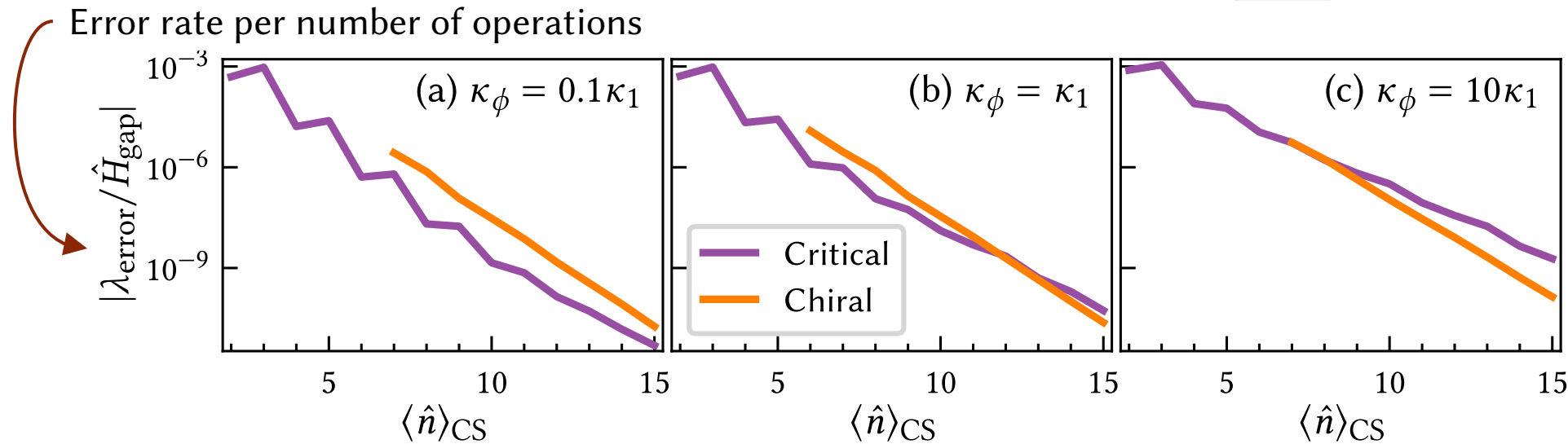
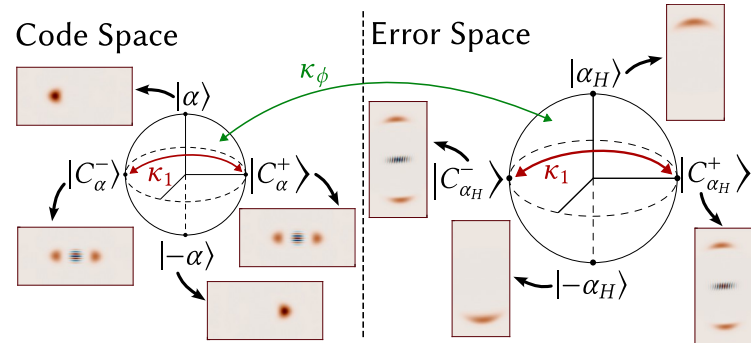
$$\begin{cases} \hat{\Pi}_H = |\alpha_H\rangle\langle\alpha_H| + |-\alpha_H\rangle\langle-\alpha_H| & \text{Projector onto the high manifold} \\ \hat{R} = |\alpha\rangle\langle\alpha_H| + |-\alpha\rangle\langle-\alpha_H| & \text{Unitary operator from high to low manifold} \end{cases}$$

$$\hat{\rho}_R = \mathcal{R}(\hat{\rho}) = \hat{R}\hat{\Pi}_H\hat{\rho}\hat{\Pi}_H\hat{R}^\dagger + (\hat{\mathbb{I}} - \hat{\Pi}_H)\hat{\rho}(\hat{\mathbb{I}} - \hat{\Pi}_H) \longrightarrow \hat{\rho}(t + \tau) = \mathcal{R}[e^{\mathcal{L}\tau} \hat{\rho}(t)]$$



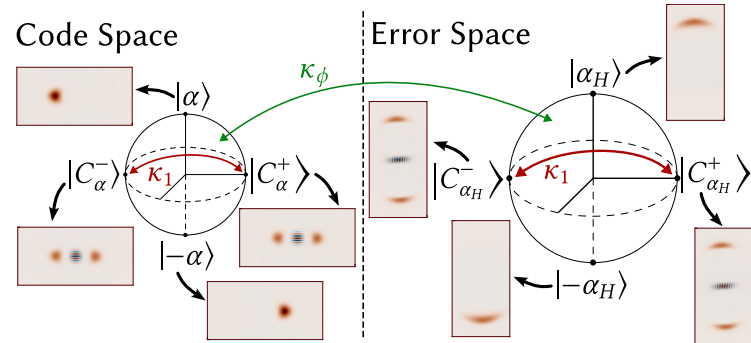
The chiral cat

Protection against dephasing noise

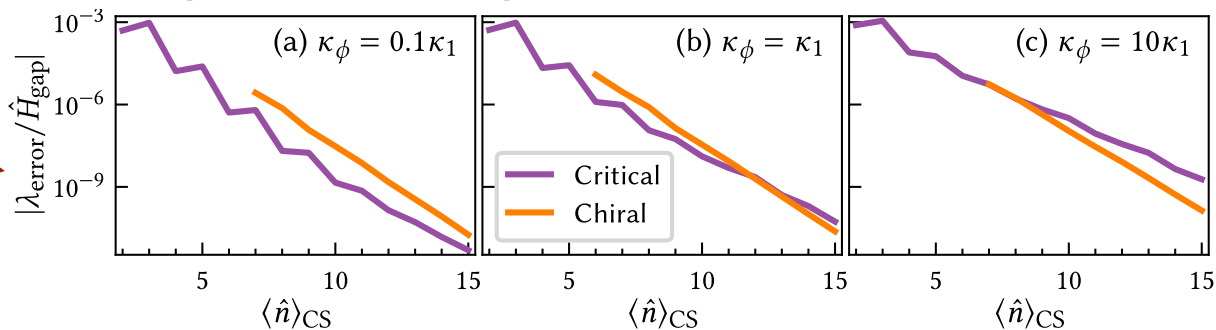


The chiral cat

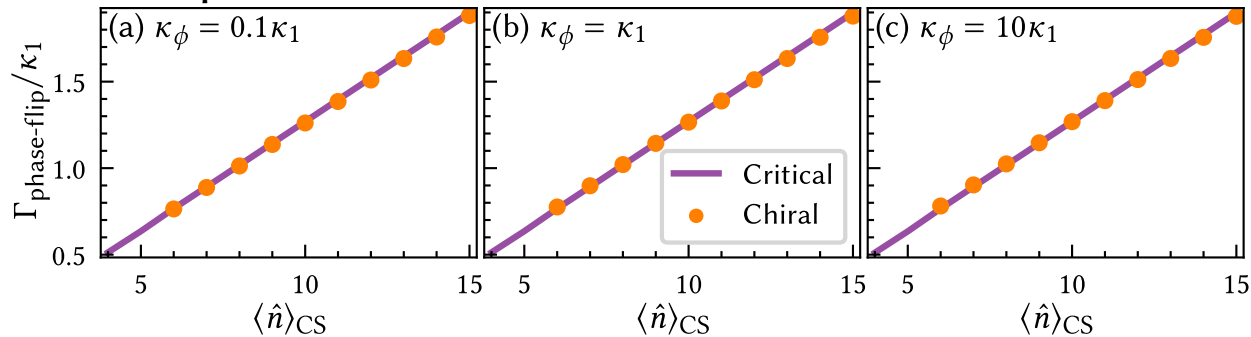
Protection against dephasing noise



Error rate per number of operations



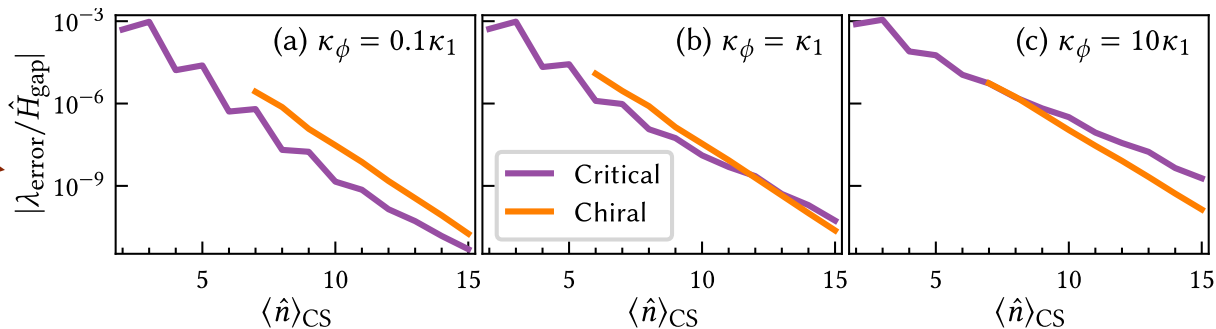
Phase-flip error rate



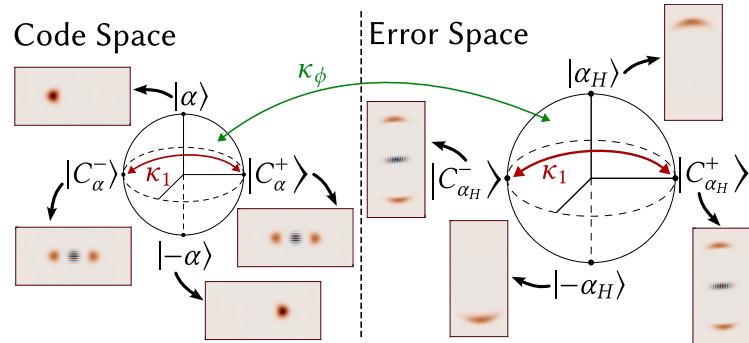
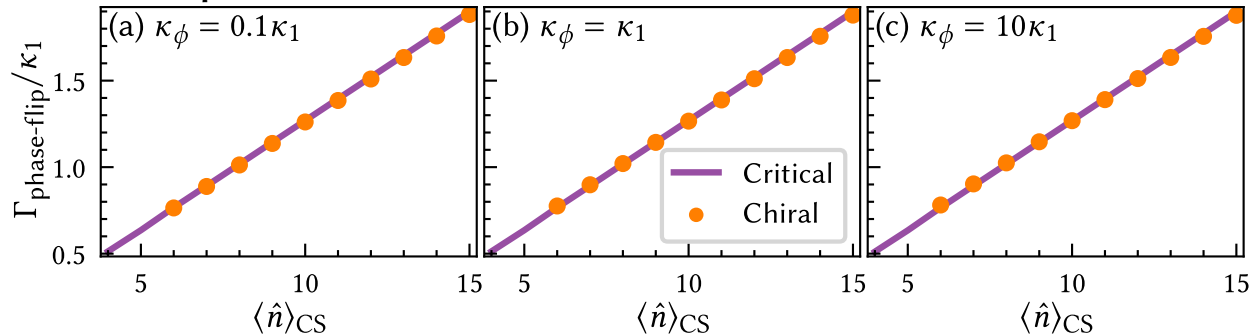
The chiral cat

Protection against dephasing noise

Error rate per number of operations



Phase-flip error rate

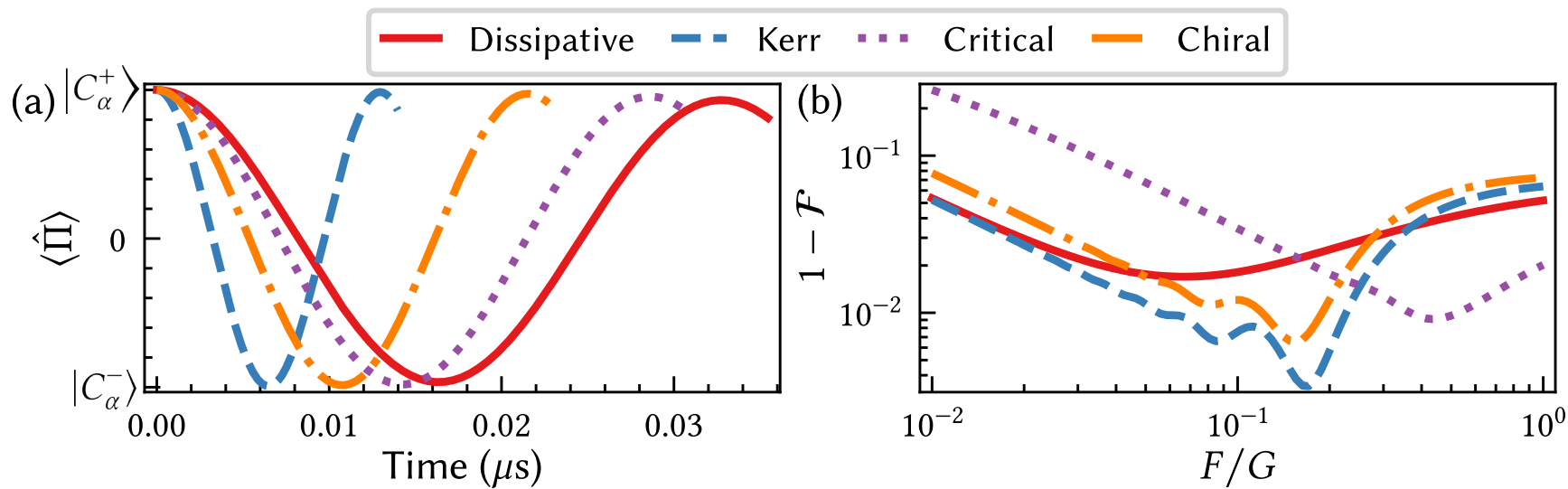
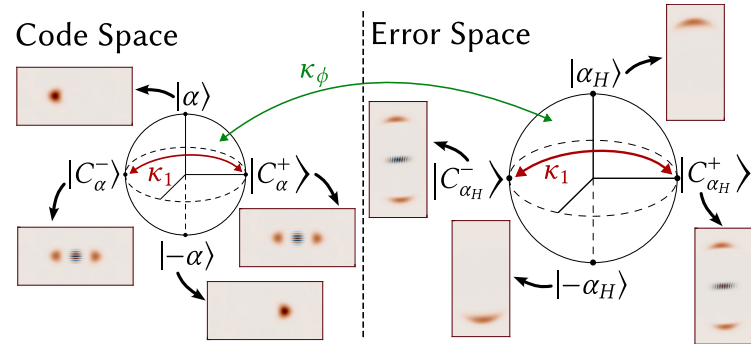


Bit-flip rate of the high manifold

Phase-flip rate of the low manifold

The chiral cat

Z-gate $\rightarrow$ rotation along the z axis $\hat{H}_z = F(\hat{a} + \hat{a}^\dagger)$



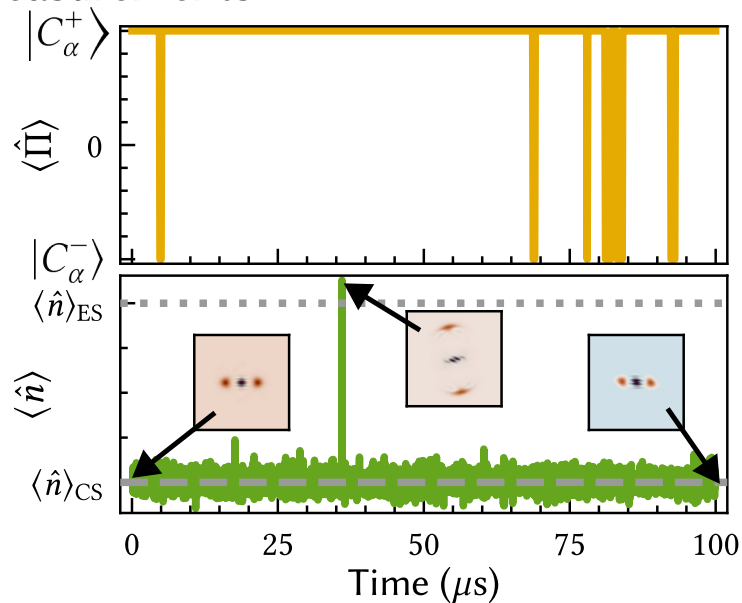
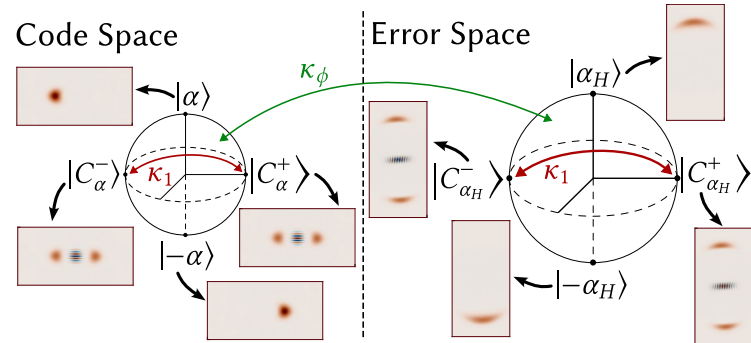
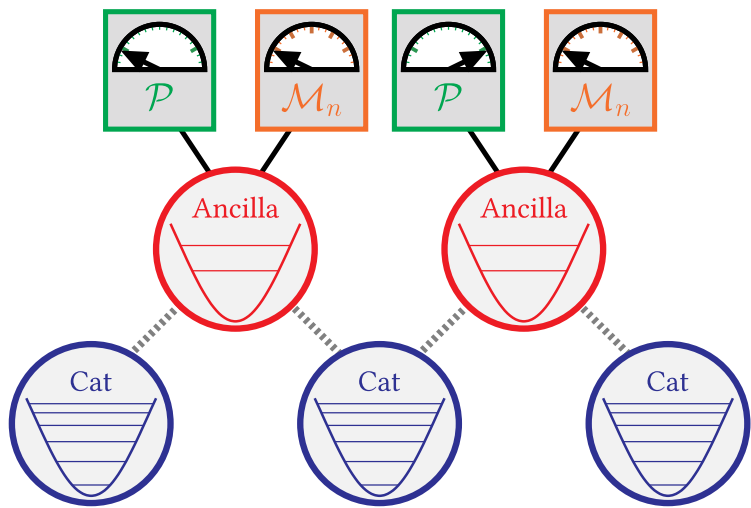
Speed comparable to Kerr cats with minimum error

The chiral cat

Repetition code

Phase-flip errors detected using parity measurements

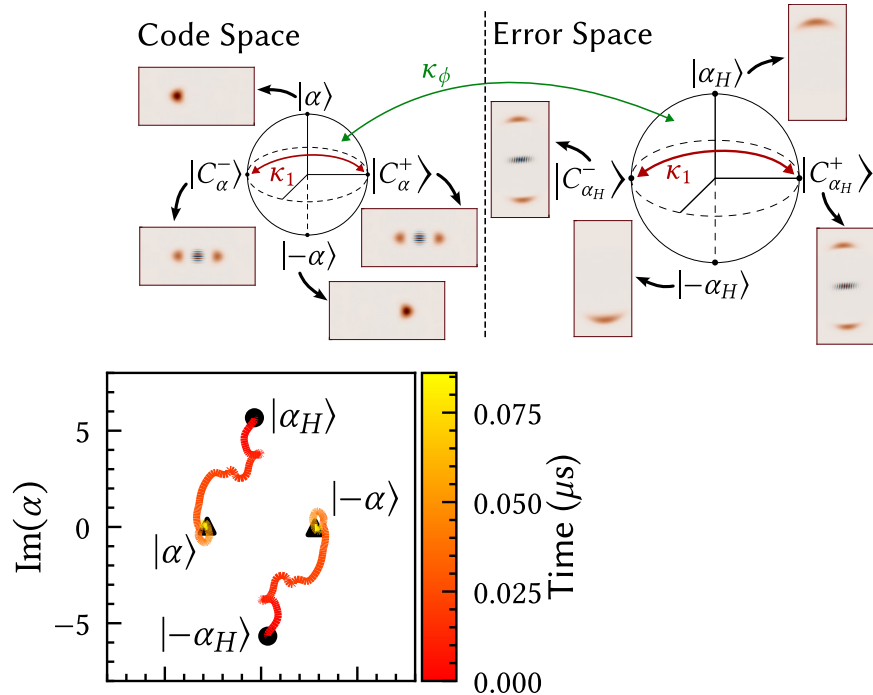
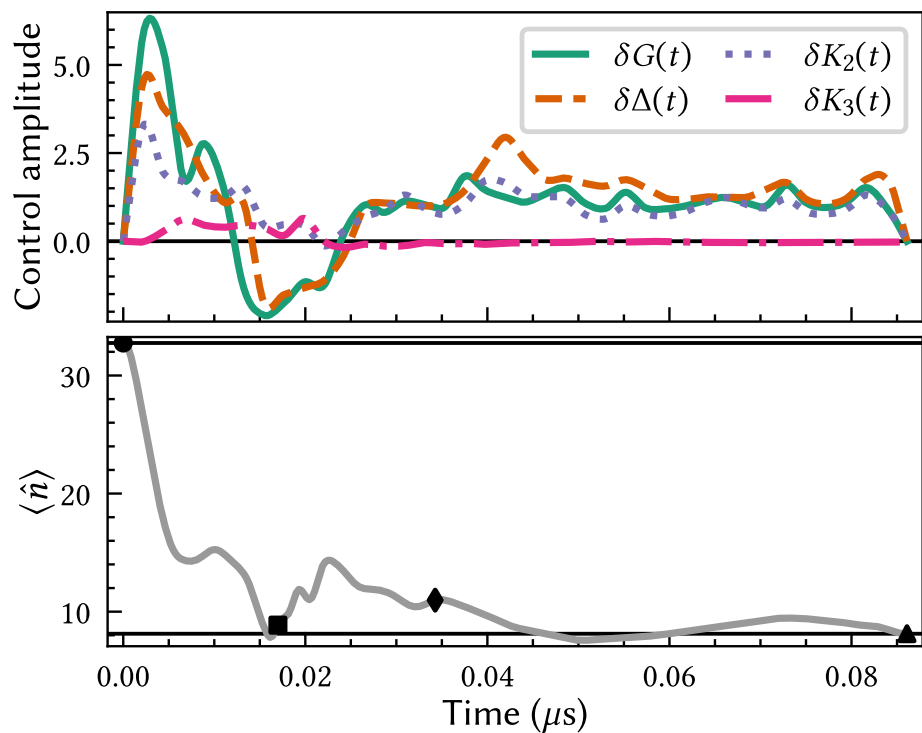
Bit-flip errors detected using photon-number measurements



No extra overhead in terms of components
(same ancilla can be used for both bit- and phase-flip measurements)

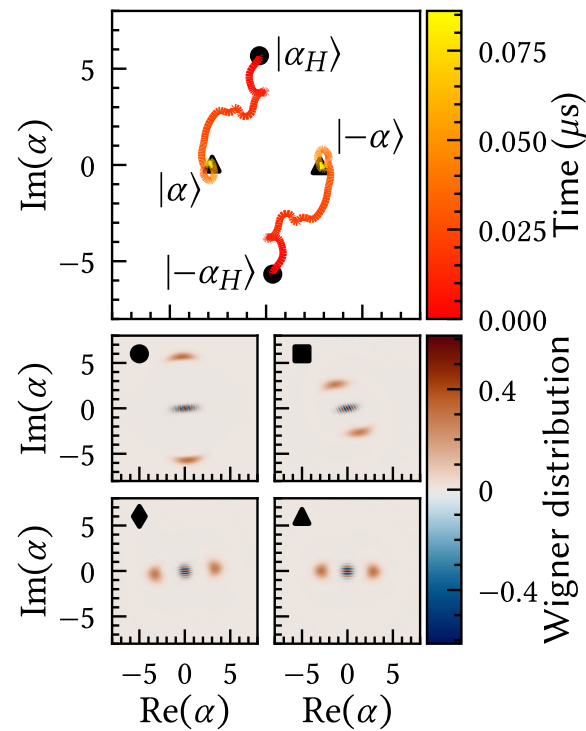
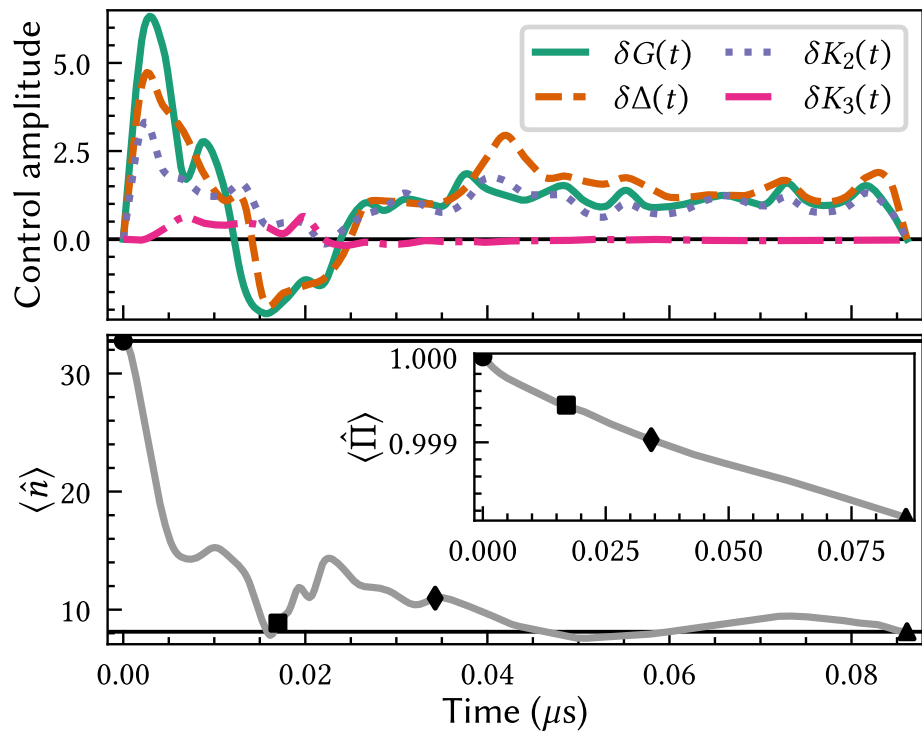
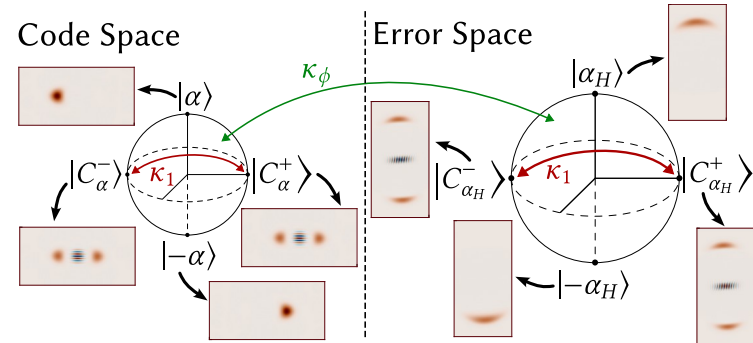
The chiral cat

Realistic recovery using optimal control



The chiral cat

Realistic recovery using optimal control

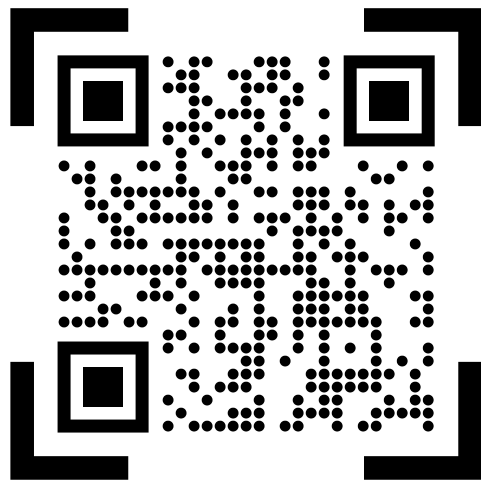


Summary

- Higher-order nonlinearity introduces a high-amplitude manifold where quantum information can be stored
- High-manifold acts as a “bit-flip trap”
- Exponential suppression of bit-flip errors even with large dephasing
 - Phase-flip rate remains the same
- Repetition code can be implemented with no extra overhead
- Fast gates with minimal error comparable to Kerr cats
- Optimal control protocol for recovery operation

Gràcies

Read the full paper



Universitat
de les Illes Balears



CSIC



ALICE & BOB



Quantum
SPAIN



AGENCIA
ESTATAL DE
INVESTIGACIÓN



EXCELENCIA
MARÍA
DE MAEZTU
2023 - 2027



Swiss National
Science Foundation